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AiryAiPrime

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Notations

Traditional name

Derivative of the Airy function Ai

Traditional notation

$\text{Ai}'(z)$

Mathematica StandardForm notation

`AiryAiPrime[z]`

Primary definition

03.07.02.0001.01

$$\text{Ai}'(z) = \frac{z^2}{2\, 3^{2/3}\, \Gamma\left(\frac{2}{3}\right)}\, {}_0F_1\left(;\frac{5}{3};\frac{z^3}{9}\right) - \frac{1}{\sqrt[3]{3}\, \Gamma\left(\frac{1}{3}\right)}\, {}_0F_1\left(;\frac{1}{3};\frac{z^3}{9}\right)$$

03.07.02.0002.01

$$\text{Ai}'(z) = \frac{z^2}{2\, 3^{2/3}\, \Gamma\left(\frac{2}{3}\right)}\, {}_0F_1\left(;\frac{5}{3};\frac{z^3}{9}\right) - \frac{1}{\sqrt[3]{3}\, \Gamma\left(\frac{1}{3}\right)}\, {}_0F_1\left(;\frac{1}{3};\frac{z^3}{9}\right)$$

Specific values

Values at fixed points

03.07.03.0001.01

$$\text{Ai}'(0) = -\frac{1}{\sqrt[3]{3}\, \Gamma\left(\frac{1}{3}\right)}$$

Values at infinities

03.07.03.0002.01

$$\lim_{x \to \infty} \text{Ai}'(x) = 0$$

General characteristics

Domain and analyticity

$\text{Ai}'(z)$ is an entire, and so analytical, function of z, which is defined in the whole complex z plane.

03.07.04.0001.01

$$z \longrightarrow \text{Ai}'(z) :: \mathbb{C} \longrightarrow \mathbb{C}$$

Symmetries and periodicities

Mirror symmetry

03.07.04.0002.01

$$\text{Ai}'(\bar{z}) = \overline{\text{Ai}'(z)}$$

Periodicity

No periodicity

Poles and essential singularities

The function $\text{Ai}'(z)$ has only one singular point at $z = \tilde{\infty}$. It is an essential singular point.

03.07.04.0003.01

$$\mathcal{Sing}_z(\text{Ai}'(z)) = \{\{\tilde{\infty}, \infty\}\}$$

Branch points

The function $\text{Ai}'(z)$ does not have branch points.

03.07.04.0004.01

$$\mathcal{BP}_z(\text{Ai}'(z)) = \{\}$$

Branch cuts

The function $\text{Ai}'(z)$ does not have branch cuts.

03.07.04.0005.01

$$\mathcal{BC}_z(\text{Ai}'(z)) = \{\}$$

Series representations

Generalized power series

Expansions at generic point $z = z_0$

For the function itself

03.07.06.0028.01

$$\text{Ai}'(z) \propto \text{Ai}'(z_0) + \text{Ai}(z_0)\, z_0\, (z - z_0) + \frac{1}{2}\left(\text{Ai}(z_0) + \text{Ai}'(z_0)\, z_0\right)(z - z_0)^2 + \ldots /; (z \to z_0)$$

03.07.06.0029.01

$$\text{Ai}'(z) \propto \text{Ai}'(z_0) + \text{Ai}(z_0)\, z_0\, (z - z_0) + \frac{1}{2}\left(\text{Ai}(z_0) + \text{Ai}'(z_0)\, z_0\right)(z - z_0)^2 + O\left((z - z_0)^3\right)$$

03.07.06.0030.01

$$\begin{aligned}&\mathrm{Ai}'(z) = \\ &\frac{1}{2}\mathrm{Ai}'(z_0) + \sum_{k=0}^{\infty}\frac{1}{k!}\left(\frac{z_0^{-k}}{4}\left(2\sum_{s=0}^{k}\sum_{j=0}^{s}\sum_{i=0}^{s}\frac{(-1)^{j+s}(s-i)!\,(-3j+3s+1)(-3j+3s+2)(-3j-k+3s+3)_{k-2}\left(\frac{2}{3}\right)_s}{i!\,j!\,(s-j)!\,(s-2i)!\left(\frac{2}{3}\right)_i\left(\frac{1}{3}-s\right)_i}\left(-\frac{z_0^3}{9}\right)^i + \right.\right.\\ &\left.\sum_{s=0}^{k}\sum_{j=0}^{s}\sum_{i=0}^{s-1}\frac{(-1)^{j+s}(-i+s-1)!\,(3i-3s+2)(-3j-k+3s+1)_k\left(-\frac{2}{3}\right)_s}{(i-1)!\,j!\,(s-j)!\,(s-2i)!\left(\frac{1}{3}\right)_i\left(\frac{5}{3}-s\right)_i}\left(-\frac{z_0^3}{9}\right)^i\right)\mathrm{Ai}'(z_0) + \\ &\frac{z_0^{2-k}}{4}\left(\sum_{s=0}^{k}\sum_{j=0}^{s}\sum_{i=0}^{s-1}\left((-1)^{j+s-1}(-i+s-1)!\,(-3j+3s+1)(-3j+3s+2)(-3j-k+3s+3)_{k-2}\left(\frac{2}{3}\right)_s\right)\right/\\ &\left.\left(i!\,j!\,(s-j)!\,(-2i+s-1)!\left(\frac{5}{3}\right)_i\left(\frac{1}{3}-s\right)_i\right)\left(-\frac{z_0^3}{9}\right)^i - \\ &\left.\left.\sum_{s=0}^{k}\sum_{j=0}^{s}\sum_{i=0}^{s-1}\frac{(-1)^{j+s-1}(-i+s-1)!\,(-3j-k+3s+1)_k\left(-\frac{2}{3}\right)_s}{i!\,j!\,(s-j)!\,(-2i+s-1)!\left(\frac{1}{3}\right)_i\left(\frac{5}{3}-s\right)_i}\left(-\frac{z_0^3}{9}\right)^i\right)\mathrm{Ai}(z_0)\right)(z-z_0)^k\end{aligned}$$

03.07.06.0031.01

$$\mathrm{Ai}'(z) = \sum_{k=0}^{\infty}\frac{3^{k-\frac{8}{3}}z_0^{-k}}{k!}\left(\Gamma\left(\frac{1}{3}\right){}_2\tilde{F}_3\left(1, \frac{4}{3}; 1-\frac{k}{3}, \frac{4-k}{3}, \frac{5-k}{3}; \frac{z_0^3}{9}\right)z_0^2 + 3\sqrt[3]{3}\,\Gamma\left(-\frac{1}{3}\right){}_2\tilde{F}_3\left(\frac{2}{3}, 1; \frac{1-k}{3}, \frac{2-k}{3}, 1-\frac{k}{3}; \frac{z_0^3}{9}\right)\right)(z-z_0)^k$$

03.07.06.0032.01

$$\mathrm{Ai}'(z) \propto \mathrm{Ai}'(z_0)\,(1+O(z-z_0))$$

Expansions at $z == 0$

For the function itself

03.07.06.0001.02

$$\mathrm{Ai}'(z) \propto -\frac{1}{\sqrt[3]{3}\,\Gamma\left(\frac{1}{3}\right)}\left(1+\frac{z^3}{3}+\frac{z^6}{72}+\ldots\right) + \frac{z^2}{2\,3^{2/3}\,\Gamma\left(\frac{2}{3}\right)}\left(1+\frac{z^3}{15}+\frac{z^6}{720}+\ldots\right) /; (z\to 0)$$

03.07.06.0033.01

$$\mathrm{Ai}'(z) \propto -\frac{1}{\sqrt[3]{3}\,\Gamma\left(\frac{1}{3}\right)}\left(1+\frac{z^3}{3}+\frac{z^6}{72}+O(z^9)\right) + \frac{z^2}{2\,3^{2/3}\,\Gamma\left(\frac{2}{3}\right)}\left(1+\frac{z^3}{15}+\frac{z^6}{720}+O(z^9)\right)$$

03.07.06.0002.01

$$\mathrm{Ai}'(z) = \frac{z^2}{2\,3^{2/3}\,\Gamma\left(\frac{2}{3}\right)}\sum_{k=0}^{\infty}\frac{1}{\left(\frac{5}{3}\right)_k k!}\left(\frac{z^3}{9}\right)^k - \frac{1}{\sqrt[3]{3}\,\Gamma\left(\frac{1}{3}\right)}\sum_{k=0}^{\infty}\frac{1}{\left(\frac{1}{3}\right)_k k!}\left(\frac{z^3}{9}\right)^k$$

03.07.06.0003.01

$$\mathrm{Ai}'(z) = \frac{z^2}{2\,3^{2/3}\,\Gamma\left(\frac{2}{3}\right)}\,{}_0F_1\left(;\frac{5}{3};\frac{z^3}{9}\right) - \frac{1}{\sqrt[3]{3}\,\Gamma\left(\frac{1}{3}\right)}\,{}_0F_1\left(;\frac{1}{3};\frac{z^3}{9}\right)$$

03.07.06.0034.01

$$\mathrm{Ai}'(z) = \frac{1}{\sqrt[3]{3}\,\pi}\sum_{k=0}^{\infty}\frac{\Gamma\left(\frac{k+2}{3}\right)\sin\left(\frac{2\pi(k+2)}{3}\right)}{k!}\left(\sqrt[3]{3}\,z\right)^k$$

03.07.06.0004.02

$$\mathrm{Ai}'(z) \propto -\frac{1}{\sqrt[3]{3}\,\Gamma\left(\frac{1}{3}\right)} + \frac{z^2}{2\,3^{2/3}\,\Gamma\left(\frac{2}{3}\right)} + O\left(z^3\right)$$

03.07.06.0035.01

$$\mathrm{Ai}'(z) = F_\infty(z) \;/;\; \left(\left(F_n(z) = \frac{z^2}{2\,3^{2/3}\,\Gamma\left(\frac{2}{3}\right)}\sum_{k=0}^{n}\frac{\left(\frac{z^3}{9}\right)^k}{\left(\frac{5}{3}\right)_k k!} - \frac{1}{\sqrt[3]{3}\,\Gamma\left(\frac{1}{3}\right)}\sum_{k=0}^{n}\frac{\left(\frac{z^3}{9}\right)^k}{\left(\frac{1}{3}\right)_k k!} = \mathrm{Ai}'(z) - \frac{z^2}{2\,3^{2/3}\,\Gamma\left(\frac{2}{3}\right)(n+1)!\left(\frac{5}{3}\right)_{n+1}}\left(\frac{z^3}{9}\right)^{n+1} {}_1F_2\left(1;\, n+2,\, n+\frac{8}{3};\, \frac{z^3}{9}\right) + \frac{1}{\sqrt[3]{3}\,\Gamma\left(\frac{1}{3}\right)(n+1)!\left(\frac{1}{3}\right)_{n+1}}\left(\frac{z^3}{9}\right)^{n+1} {}_1F_2\left(1;\, n+2,\, n+\frac{4}{3};\, \frac{z^3}{9}\right)\right) \bigwedge n \in \mathbb{N}\right)$$

Summed form of the truncated series expansion.

Asymptotic series expansions

Expansions inside Stokes sectors

In exponential form ||| In exponential form

03.07.06.0014.01

$$\mathrm{Ai}'(z) \propto -\frac{1}{2\sqrt{\pi}}\, e^{-\frac{2}{3}z^{3/2}}\sqrt[4]{z}\left(1 + \frac{7}{48\,z^{3/2}} - \frac{455}{4608\,z^3} + O\left(\frac{1}{z^{9/2}}\right)\right) \;/;\; |\arg(z)| < \pi \bigwedge (|z| \to \infty)$$

03.07.06.0015.01

$$\mathrm{Ai}'(z) \propto -\frac{1}{2\sqrt{\pi}}\, e^{-\frac{2}{3}z^{3/2}}\sqrt[4]{z}\left(\sum_{k=0}^{n}\frac{\left(-\frac{1}{6}\right)_k\left(\frac{7}{6}\right)_k}{k!}\left(-\frac{3}{4\,z^{3/2}}\right)^k + O\left(\frac{1}{z^{\frac{3(n+1)}{2}}}\right)\right) \;/;\; |\arg(z)| < \pi \bigwedge (|z| \to \infty) \bigwedge n \in \mathbb{N}$$

03.07.06.0016.01

$$\mathrm{Ai}'(z) \propto -\frac{1}{2\sqrt{\pi}}\, e^{-\frac{2}{3}z^{3/2}}\sqrt[4]{z}\left(\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{6}\right)_k\left(\frac{7}{6}\right)_k}{k!}\left(-\frac{3}{4\,z^{3/2}}\right)^k\right) \;/;\; |\arg(z)| < \pi \bigwedge (|z| \to \infty)$$

03.07.06.0036.01

$$\mathrm{Ai}'(z) \propto -\frac{e^{-\frac{1}{3}\left(2z^{3/2}\right)}\sqrt[4]{z}}{2\sqrt{\pi}}\left(\sum_{k=0}^{n}\frac{\left(-\frac{1}{12}\right)_k\left(\frac{5}{12}\right)_k\left(\frac{7}{12}\right)_k\left(\frac{13}{12}\right)_k\left(\frac{9}{4z^3}\right)^k}{\left(\frac{1}{2}\right)_k k!} + O\left(\frac{1}{z^{3(n+1)}}\right)\right) - \frac{7\,e^{-\frac{1}{3}\left(2z^{3/2}\right)}}{96\sqrt{\pi}\,z^{5/4}}\left(\sum_{k=0}^{n}\frac{\left(\frac{5}{12}\right)_k\left(\frac{11}{12}\right)_k\left(\frac{13}{12}\right)_k\left(\frac{19}{12}\right)_k\left(\frac{9}{4z^3}\right)^k}{\left(\frac{3}{2}\right)_k k!} + O\left(\frac{1}{z^{3(n+1)}}\right)\right) \;/;\; |\arg(z)| < \pi \bigwedge (|z| \to \infty) \bigwedge n \in \mathbb{N}$$

03.07.06.0005.01

$$\mathrm{Ai}'(z) \propto -\frac{1}{2\sqrt{\pi}} e^{-\frac{2}{3}z^{3/2}} \sqrt[4]{z}\, {}_2F_0\left(-\frac{1}{6}, \frac{7}{6};; -\frac{3}{4z^{3/2}}\right) /; |\arg(z)| < \pi \bigwedge (|z| \to \infty)$$

03.07.06.0006.01

$$\mathrm{Ai}'(z) \propto -\frac{1}{2\sqrt{\pi}} e^{-\frac{2}{3}z^{3/2}} \sqrt[4]{z} \left(1 + O\left(\frac{1}{z^{3/2}}\right)\right) /; |\arg(z)| < \pi \bigwedge (|z| \to \infty)$$

In trigonometric form ||| In trigonometric form

03.07.06.0017.01

$$\mathrm{Ai}'(-z) \propto -\frac{\sqrt[4]{z}}{\sqrt{\pi}} \left(\cos\left(\frac{2z^{3/2}}{3} + \frac{\pi}{4}\right)\left(1 + \frac{455}{4608z^3} - \frac{40415375}{127401984z^6} + O\left(\frac{1}{z^9}\right)\right) - \frac{7}{48z^{3/2}} \sin\left(\frac{2z^{3/2}}{3} + \frac{\pi}{4}\right)\left(1 - \frac{13585}{13824z^3} + \frac{823318925}{127401984z^6} + O\left(\frac{1}{z^9}\right)\right)\right) /; |\arg(z)| < \frac{2\pi}{3} \bigwedge (|z| \to \infty)$$

03.07.06.0018.01

$$\mathrm{Ai}'(-z) \propto -\frac{\sqrt[4]{z}}{\sqrt{\pi}} \left(\cos\left(\frac{2z^{3/2}}{3} + \frac{\pi}{4}\right)\left(\sum_{k=0}^{n} \frac{\left(-\frac{1}{12}\right)_k \left(\frac{5}{12}\right)_k \left(\frac{7}{12}\right)_k \left(\frac{13}{12}\right)_k}{\left(\frac{1}{2}\right)_k k!} \left(-\frac{9}{4z^3}\right)^k + O\left(\frac{1}{z^{3n+3}}\right)\right) - \frac{7}{48z^{3/2}} \sin\left(\frac{2z^{3/2}}{3} + \frac{\pi}{4}\right)\left(\sum_{k=0}^{n} \frac{\left(\frac{5}{12}\right)_k \left(\frac{11}{12}\right)_k \left(\frac{13}{12}\right)_k \left(\frac{19}{12}\right)_k}{\left(\frac{3}{2}\right)_k k!} \left(-\frac{9}{4z^3}\right)^k + O\left(\frac{1}{z^{3n+3}}\right)\right)\right) /; |\arg(z)| < \frac{2\pi}{3} \bigwedge (|z| \to \infty) \bigwedge n \in \mathbb{N}$$

03.07.06.0019.01

$$\mathrm{Ai}'(-z) \propto -\frac{z^{1/4}}{\sqrt{\pi}} \left(\cos\left(\frac{2z^{3/2}}{3} + \frac{\pi}{4}\right) \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{12}\right)_k \left(\frac{5}{12}\right)_k \left(\frac{7}{12}\right)_k \left(\frac{13}{12}\right)_k}{\left(\frac{1}{2}\right)_k k!} \left(-\frac{9}{4z^3}\right)^k - \frac{7}{48z^{3/2}} \sin\left(\frac{2z^{3/2}}{3} + \frac{\pi}{4}\right) \sum_{k=0}^{\infty} \frac{\left(\frac{5}{12}\right)_k \left(\frac{11}{12}\right)_k \left(\frac{13}{12}\right)_k \left(\frac{19}{12}\right)_k}{\left(\frac{3}{2}\right)_k k!} \left(-\frac{9}{4z^3}\right)^k\right) /; |\arg(z)| < \frac{2\pi}{3} \bigwedge (|z| \to \infty)$$

03.07.06.0007.01

$$\mathrm{Ai}'(-z) \propto -\frac{1}{\sqrt{\pi}} \sqrt[4]{z} \left(\cos\left(\frac{2z^{3/2}}{3} + \frac{\pi}{4}\right) {}_4F_1\left(-\frac{1}{12}, \frac{5}{12}, \frac{7}{12}, \frac{13}{12}; \frac{1}{2}; -\frac{9}{4z^3}\right) - \frac{7}{48z^{3/2}} \sin\left(\frac{2z^{3/2}}{3} + \frac{\pi}{4}\right) {}_4F_1\left(\frac{5}{12}, \frac{11}{12}, \frac{13}{12}, \frac{19}{12}; \frac{3}{2}; -\frac{9}{4z^3}\right)\right) /; |\arg(z)| < \frac{2\pi}{3} \bigwedge (|z| \to \infty)$$

03.07.06.0009.01

$$\mathrm{Ai}'(-z) \propto -\frac{1}{\sqrt{\pi}} \sqrt[4]{z} \left(\cos\left(\frac{2z^{3/2}}{3} + \frac{\pi}{4}\right)\left(1 + O\left(\frac{1}{z^3}\right)\right) - \frac{7}{48z^{3/2}} \sin\left(\frac{2z^{3/2}}{3} + \frac{\pi}{4}\right)\left(1 + O\left(\frac{1}{z^3}\right)\right)\right) /; |\arg(z)| < \frac{2\pi}{3} \bigwedge (|z| \to \infty)$$

Expansions for any z in exponential form

Using exponential function with branch cut-containing arguments

03.07.06.0020.01

$$\operatorname{Ai}'(z) \propto \frac{1}{2\sqrt{3\pi}\left(-z^3\right)^{7/12}}\left(-\sqrt[12]{-1}\left((-1)^{1/3} z^2+\left(-z^3\right)^{2/3}\right) e^{\frac{2}{3} i \sqrt{-z^3}}\left(1+\frac{7 i}{48\sqrt{-z^3}}-\frac{455}{4608 z^3}+O\left(\frac{1}{z^{9/2}}\right)\right)+(-1)^{11/12}\left(-(-1)^{2/3} z^2+\left(-z^3\right)^{2/3}\right) e^{-\frac{2}{3} i \sqrt{-z^3}}\left(1-\frac{7 i}{48\sqrt{-z^3}}-\frac{455}{4608 z^3}+O\left(\frac{1}{z^{9/2}}\right)\right)\right) /; (|z| \to \infty)$$

03.07.06.0021.01

$$\operatorname{Ai}'(z) \propto \frac{1}{2\sqrt{3\pi}\left(-z^3\right)^{7/12}}\left((-1)^{11/12} e^{-\frac{2}{3} i \sqrt{-z^3}}\left(\left(-z^3\right)^{2/3}-(-1)^{2/3} z^2\right)\left(\sum_{k=0}^{n} \frac{\left(-\frac{1}{6}\right)_k\left(\frac{7}{6}\right)_k}{k!}\left(\frac{3 i}{4\sqrt{-z^3}}\right)^k+O\left(\frac{1}{z^{\frac{3(n+1)}{2}}}\right)\right)-\sqrt[12]{-1}\, e^{\frac{2}{3} i \sqrt{-z^3}}\left(\left(-z^3\right)^{2/3}+(-1)^{1/3} z^2\right)\left(\sum_{k=0}^{n} \frac{\left(-\frac{1}{6}\right)_k\left(\frac{7}{6}\right)_k}{k!}\left(-\frac{3 i}{4\sqrt{-z^3}}\right)^k+O\left(\frac{1}{z^{\frac{3(n+1)}{2}}}\right)\right)\right) /; (|z| \to \infty) \land n \in \mathbb{N}$$

03.07.06.0022.01

$$\operatorname{Ai}'(z) \propto \frac{1}{2\sqrt{3\pi}\left(-z^3\right)^{7/12}}\left((-1)^{11/12} e^{-\frac{2}{3} i \sqrt{-z^3}}\left(\left(-z^3\right)^{2/3}-(-1)^{2/3} z^2\right) \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{6}\right)_k\left(\frac{7}{6}\right)_k}{k!}\left(\frac{3 i}{4\sqrt{-z^3}}\right)^k-(-1)^{1/12} e^{\frac{2}{3} i \sqrt{-z^3}}\left(\left(-z^3\right)^{2/3}+(-1)^{1/3} z^2\right) \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{6}\right)_k\left(\frac{7}{6}\right)_k}{k!}\left(-\frac{3 i}{4\sqrt{-z^3}}\right)^k\right) /; (|z| \to \infty)$$

03.07.06.0010.01

$$\operatorname{Ai}'(z) \propto \frac{1}{2\sqrt{3\pi}\left(-z^3\right)^{7/12}}\left((-1)^{11/12} e^{-\frac{2}{3} i \sqrt{-z^3}}\left(\left(-z^3\right)^{2/3}-(-1)^{2/3} z^2\right)\,{}_2F_0\left(\frac{7}{6},-\frac{1}{6};;\frac{3 i}{4\sqrt{-z^3}}\right)-\sqrt[12]{-1}\, e^{\frac{2}{3} i \sqrt{-z^3}}\left(\left(-z^3\right)^{2/3}+(-1)^{1/3} z^2\right)\,{}_2F_0\left(\frac{7}{6},-\frac{1}{6};;-\frac{3 i}{4\sqrt{-z^3}}\right)\right) /; (|z| \to \infty)$$

03.07.06.0037.01

$$\operatorname{Ai}'(z) \propto \frac{(-1)^{3/4}}{4\sqrt{3\pi}\left(-z^3\right)^{7/12}}\left(\left(e^{\frac{2 i}{3}\sqrt{-z^3}}\, i\left(\left(-i+\sqrt{3}\right)\left(-z^3\right)^{2/3}+\left(i+\sqrt{3}\right) z^2\right)+e^{-\frac{2 i}{3}\sqrt{-z^3}}\left(\left(i+\sqrt{3}\right)\left(-z^3\right)^{2/3}+\left(-i+\sqrt{3}\right) z^2\right)\right)\left(\sum_{k=0}^{n} \frac{\left(-\frac{1}{12}\right)_k\left(\frac{5}{12}\right)_k\left(\frac{7}{12}\right)_k\left(\frac{13}{12}\right)_k}{\left(\frac{1}{2}\right)_k k!}\left(\frac{9}{4 z^3}\right)^k+O\left(\frac{1}{z^{\frac{3(n+1)}{2}}}\right)\right)-\frac{7}{48\sqrt{-z^3}}\left(e^{\frac{2 i}{3}\sqrt{-z^3}}\left(\left(-i+\sqrt{3}\right)\left(-z^3\right)^{2/3}+\left(i+\sqrt{3}\right) z^2\right)+e^{-\frac{2 i}{3}\sqrt{-z^3}}\, i\left(\left(i+\sqrt{3}\right)\left(-z^3\right)^{2/3}+\left(-i+\sqrt{3}\right) z^2\right)\right)\left(\sum_{k=0}^{n} \frac{\left(\frac{5}{12}\right)_k\left(\frac{11}{12}\right)_k\left(\frac{13}{12}\right)_k\left(\frac{19}{12}\right)_k}{k!\left(\frac{3}{2}\right)_k}\left(\frac{9}{4 z^3}\right)^k+O\left(\frac{1}{z^{\frac{3(n+1)}{2}}}\right)\right)\right) /; (|z| \to \infty) \land n \in \mathbb{N}$$

03.07.06.0038.01

$$\mathrm{Ai}'(z) \propto \frac{(-1)^{3/4}}{4\sqrt{3\pi}\,(-z^3)^{7/12}} \Bigg(\Bigg(e^{\frac{2i}{3}\sqrt{-z^3}}\, i\left(\left(-i+\sqrt{3}\right)(-z^3)^{2/3}+\left(i+\sqrt{3}\right)z^2\right)+e^{-\frac{2i}{3}\sqrt{-z^3}}\left(\left(i+\sqrt{3}\right)(-z^3)^{2/3}+\left(-i+\sqrt{3}\right)z^2\right)\Bigg) \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{12}\right)_k\left(\frac{5}{12}\right)_k\left(\frac{7}{12}\right)_k\left(\frac{13}{12}\right)_k}{\left(\frac{1}{2}\right)_k\, k!}\left(\frac{9}{4z^3}\right)^k-\frac{7}{48\sqrt{-z^3}}\Bigg(e^{\frac{2i}{3}\sqrt{-z^3}}\left(\left(-i+\sqrt{3}\right)(-z^3)^{2/3}+\left(i+\sqrt{3}\right)z^2\right)+ e^{-\frac{2i}{3}\sqrt{-z^3}}\, i\left(\left(i+\sqrt{3}\right)(-z^3)^{2/3}+\left(-i+\sqrt{3}\right)z^2\right)\Bigg)\sum_{k=0}^{\infty}\frac{\left(\frac{5}{12}\right)_k\left(\frac{11}{12}\right)_k\left(\frac{13}{12}\right)_k\left(\frac{19}{12}\right)_k}{k!\left(\frac{3}{2}\right)_k}\left(\frac{9}{4z^3}\right)^k\Bigg)\,/;\,(|z|\to\infty)$$

03.07.06.0039.01

$$\mathrm{Ai}'(z) \propto \frac{(-1)^{3/4}}{4\sqrt{3\pi}\,(-z^3)^{7/12}} \Bigg(\Bigg(e^{\frac{2i}{3}\sqrt{-z^3}}\, i\left(\left(-i+\sqrt{3}\right)(-z^3)^{2/3}+\left(i+\sqrt{3}\right)z^2\right)+e^{-\frac{2i}{3}\sqrt{-z^3}}\left(\left(i+\sqrt{3}\right)(-z^3)^{2/3}+\left(-i+\sqrt{3}\right)z^2\right)\Bigg) {}_4F_1\left(-\frac{1}{12},\frac{5}{12},\frac{7}{12},\frac{13}{12};\frac{1}{2};\frac{9}{4z^3}\right)-\frac{7}{48\sqrt{-z^3}}\Bigg(e^{\frac{2i}{3}\sqrt{-z^3}}\left(\left(-i+\sqrt{3}\right)(-z^3)^{2/3}+\left(i+\sqrt{3}\right)z^2\right)+ e^{-\frac{2i}{3}\sqrt{-z^3}}\, i\left(\left(i+\sqrt{3}\right)(-z^3)^{2/3}+\left(-i+\sqrt{3}\right)z^2\right)\Bigg){}_4F_1\left(\frac{5}{12},\frac{11}{12},\frac{13}{12},\frac{19}{12};\frac{3}{2};\frac{9}{4z^3}\right)\Bigg)\,/;\,(|z|\to\infty)$$

03.07.06.0011.01

$$\mathrm{Ai}'(z) \propto \frac{1}{2\sqrt{3\pi}\,(-z^3)^{7/12}}\left((-1)^{11/12}\, e^{-\frac{2}{3}i\sqrt{-z^3}}\left((-z^3)^{2/3}-(-1)^{2/3}z^2\right)\left(1+O\left(\frac{1}{z^{3/2}}\right)\right)- \sqrt[12]{-1}\; e^{\frac{2i}{3}\sqrt{-z^3}}\left((-z^3)^{2/3}+(-1)^{1/3}z^2\right)\left(1+O\left(\frac{1}{z^{3/2}}\right)\right)\right)\,/;\,(|z|\to\infty)$$

Using exponential function with branch cut-free arguments

03.07.06.0040.01

$$\mathrm{Ai}'(z) \propto -\frac{1}{4\sqrt{6\pi}\,(-z^3)^{7/12}} \Bigg(\Bigg(e^{-\frac{1}{3}(2 z^{3/2})}\Big(-(-1+\sqrt{3})\sqrt[6]{-z^3}\; z^{3/2} + (-1+\sqrt{3}) z^2 + (1+\sqrt{3})\sqrt{-z^3}\;\sqrt{z} + (1+\sqrt{3})(-z^3)^{2/3}\Big) + e^{\frac{2 z^{3/2}}{3}}\Big((-1+\sqrt{3})\sqrt[6]{-z^3}\; z^{3/2} + (-1+\sqrt{3}) z^2 - (1+\sqrt{3})\sqrt{-z^3}\;\sqrt{z} + (1+\sqrt{3})(-z^3)^{2/3}\Big)\Bigg) \left(1 - \frac{455}{4608 z^3} - \frac{40\,415\,375}{127\,401\,984 z^6} - \frac{6\,183\,948\,445\,675}{1\,761\,205\,026\,816 z^9} + O\left(\frac{1}{z^{12}}\right)\right) - \frac{7}{48\sqrt{-z^3}} \Bigg(e^{-\frac{1}{3}(2 z^{3/2})}\Big((1+\sqrt{3})\sqrt[6]{-z^3}\; z^{3/2} + (1+\sqrt{3}) z^2 + (1-\sqrt{3})\sqrt{-z^3}\;\sqrt{z} - (1-\sqrt{3})(-z^3)^{2/3}\Big) + e^{\frac{2 z^{3/2}}{3}}\Big(-(1+\sqrt{3})\sqrt[6]{-z^3}\; z^{3/2} + (1+\sqrt{3}) z^2 - (1-\sqrt{3})\sqrt{-z^3}\;\sqrt{z} - (1-\sqrt{3})(-z^3)^{2/3}\Big)\Bigg) \left(1 + \frac{13\,585}{13\,824 z^3} + \frac{823\,318\,925}{127\,401\,984 z^6} + \frac{189\,935\,559\,402\,875}{1\,761\,205\,026\,816 z^9} + O\left(\frac{1}{z^{12}}\right)\right)\Bigg) /; (|z| \to \infty)$$

03.07.06.0041.01

$$\mathrm{Ai}'(z) \propto -\frac{1}{4\sqrt{6\pi}\,(-z^3)^{7/12}} \Bigg(\Bigg(e^{-\frac{1}{3}(2 z^{3/2})}\Big(-(-1+\sqrt{3})\sqrt[6]{-z^3}\; z^{3/2} + (-1+\sqrt{3}) z^2 + (1+\sqrt{3})\sqrt{-z^3}\;\sqrt{z} + (1+\sqrt{3})(-z^3)^{2/3}\Big) + e^{\frac{2 z^{3/2}}{3}}\Big((-1+\sqrt{3})\sqrt[6]{-z^3}\; z^{3/2} + (-1+\sqrt{3}) z^2 - (1+\sqrt{3})\sqrt{-z^3}\;\sqrt{z} + (1+\sqrt{3})(-z^3)^{2/3}\Big)\Bigg) \left(\sum_{k=0}^{n} \frac{\left(-\frac{1}{12}\right)_k \left(\frac{5}{12}\right)_k \left(\frac{7}{12}\right)_k \left(\frac{13}{12}\right)_k}{\left(\frac{1}{2}\right)_k k!} \left(\frac{9}{4 z^3}\right)^k + O\left(\frac{1}{z^{3n+3}}\right)\right) - \frac{7}{48\sqrt{-z^3}} \Bigg(e^{-\frac{1}{3}(2 z^{3/2})}\Big((1+\sqrt{3})\sqrt[6]{-z^3}\; z^{3/2} + (1+\sqrt{3}) z^2 + (1-\sqrt{3})\sqrt{-z^3}\;\sqrt{z} - (1-\sqrt{3})(-z^3)^{2/3}\Big) + e^{\frac{2 z^{3/2}}{3}}\Big(-(1+\sqrt{3})\sqrt[6]{-z^3}\; z^{3/2} + (1+\sqrt{3}) z^2 - (1-\sqrt{3})\sqrt{-z^3}\;\sqrt{z} - (1-\sqrt{3})(-z^3)^{2/3}\Big)\Bigg) \left(\sum_{k=0}^{n} \frac{\left(\frac{5}{12}\right)_k \left(\frac{11}{12}\right)_k \left(\frac{13}{12}\right)_k \left(\frac{19}{12}\right)_k}{k! \left(\frac{3}{2}\right)_k} \left(\frac{9}{4 z^3}\right)^k + O\left(\frac{1}{z^{3n+3}}\right)\right)\Bigg) /; (|z| \to \infty) \land n \in \mathbb{N}$$

03.07.06.0042.01

$$\mathrm{Ai}'(z) \propto -\frac{1}{4\sqrt{6\pi}\,(-z^3)^{7/12}}\Bigg(\Bigg(e^{-\frac{1}{3}(2z^{3/2})}\Big(-\big(-1+\sqrt{3}\big)\sqrt[6]{-z^3}\,z^{3/2}+\big(-1+\sqrt{3}\big)z^2+\big(1+\sqrt{3}\big)\sqrt{-z^3}\,\sqrt{z}+\big(1+\sqrt{3}\big)\big(-z^3\big)^{2/3}\Big)+ e^{\frac{2z^{3/2}}{3}}\Big(\big(-1+\sqrt{3}\big)\sqrt[6]{-z^3}\,z^{3/2}+\big(-1+\sqrt{3}\big)z^2-\big(1+\sqrt{3}\big)\sqrt{-z^3}\,\sqrt{z}+\big(1+\sqrt{3}\big)\big(-z^3\big)^{2/3}\Big)\Bigg) \sum_{k=0}^{\infty}\frac{\left(-\frac{1}{12}\right)_k\left(\frac{5}{12}\right)_k\left(\frac{7}{12}\right)_k\left(\frac{13}{12}\right)_k}{\left(\frac{1}{2}\right)_k k!}\left(\frac{9}{4z^3}\right)^k-\frac{7}{48\sqrt{-z^3}} \Bigg(e^{-\frac{1}{3}(2z^{3/2})}\Big(\big(1+\sqrt{3}\big)\sqrt[6]{-z^3}\,z^{3/2}+\big(1+\sqrt{3}\big)z^2+\big(1-\sqrt{3}\big)\sqrt{-z^3}\,\sqrt{z}-\big(1-\sqrt{3}\big)\big(-z^3\big)^{2/3}\Big)+ e^{\frac{2z^{3/2}}{3}}\Big(-\big(1+\sqrt{3}\big)\sqrt[6]{-z^3}\,z^{3/2}+\big(1+\sqrt{3}\big)z^2-\big(1-\sqrt{3}\big)\sqrt{-z^3}\,\sqrt{z}-\big(1-\sqrt{3}\big)\big(-z^3\big)^{2/3}\Big)\Bigg) \sum_{k=0}^{\infty}\frac{\left(\frac{5}{12}\right)_k\left(\frac{11}{12}\right)_k\left(\frac{13}{12}\right)_k\left(\frac{19}{12}\right)_k}{k!\left(\frac{3}{2}\right)_k}\left(\frac{9}{4z^3}\right)^k\Bigg) /; (|z|\to\infty)$$

03.07.06.0043.01

$$\mathrm{Ai}'(z) \propto -\frac{1}{4\sqrt{6\pi}\,(-z^3)^{7/12}}\Bigg(\Bigg(e^{-\frac{1}{3}(2z^{3/2})}\Big(-\big(-1+\sqrt{3}\big)\sqrt[6]{-z^3}\,z^{3/2}+\big(-1+\sqrt{3}\big)z^2+\big(1+\sqrt{3}\big)\sqrt{-z^3}\,\sqrt{z}+\big(1+\sqrt{3}\big)\big(-z^3\big)^{2/3}\Big)+ e^{\frac{2z^{3/2}}{3}}\Big(\big(-1+\sqrt{3}\big)\sqrt[6]{-z^3}\,z^{3/2}+\big(-1+\sqrt{3}\big)z^2-\big(1+\sqrt{3}\big)\sqrt{-z^3}\,\sqrt{z}+\big(1+\sqrt{3}\big)\big(-z^3\big)^{2/3}\Big)\Bigg) {}_4F_1\!\left(-\frac{1}{12},\frac{5}{12},\frac{7}{12},\frac{13}{12};\frac{1}{2};\frac{9}{4z^3}\right)-\frac{7}{48\sqrt{-z^3}} \Bigg(e^{-\frac{1}{3}(2z^{3/2})}\Big(\big(1+\sqrt{3}\big)\sqrt[6]{-z^3}\,z^{3/2}+\big(1+\sqrt{3}\big)z^2+\big(1-\sqrt{3}\big)\sqrt{-z^3}\,\sqrt{z}-\big(1-\sqrt{3}\big)\big(-z^3\big)^{2/3}\Big)+ e^{\frac{2z^{3/2}}{3}}\Big(-\big(1+\sqrt{3}\big)\sqrt[6]{-z^3}\,z^{3/2}+\big(1+\sqrt{3}\big)z^2-\big(1-\sqrt{3}\big)\sqrt{-z^3}\,\sqrt{z}-\big(1-\sqrt{3}\big)\big(-z^3\big)^{2/3}\Big)\Bigg) {}_4F_1\!\left(\frac{5}{12},\frac{11}{12},\frac{13}{12},\frac{19}{12};\frac{3}{2};\frac{9}{4z^3}\right)\Bigg) /; (|z|\to\infty)$$

03.07.06.0044.01

$$\text{Ai}'(z) \propto -\frac{1}{4\sqrt{6\pi}\,(-z^3)^{7/12}} \Bigg(\Bigg(e^{-\frac{1}{3}(2 z^{3/2})} \left(-(-1+\sqrt{3})\sqrt[6]{-z^3}\, z^{3/2} + (-1+\sqrt{3}) z^2 + (1+\sqrt{3})\sqrt{-z^3}\,\sqrt{z} + (1+\sqrt{3})(-z^3)^{2/3}\right) + e^{\frac{2 z^{3/2}}{3}} \left((-1+\sqrt{3})\sqrt[6]{-z^3}\, z^{3/2} + (-1+\sqrt{3}) z^2 - (1+\sqrt{3})\sqrt{-z^3}\,\sqrt{z} + (1+\sqrt{3})(-z^3)^{2/3}\right)\Bigg)\left(1 + O\left(\frac{1}{z^3}\right)\right) - \frac{7}{48\sqrt{-z^3}} \left(e^{-\frac{1}{3}(2 z^{3/2})} \left((1+\sqrt{3})\sqrt[6]{-z^3}\, z^{3/2} + (1+\sqrt{3}) z^2 + (1-\sqrt{3})\sqrt{-z^3}\,\sqrt{z} - (1-\sqrt{3})(-z^3)^{2/3}\right) + e^{\frac{2 z^{3/2}}{3}} \left(-(1+\sqrt{3})\sqrt[6]{-z^3}\, z^{3/2} + (1+\sqrt{3}) z^2 - (1-\sqrt{3})\sqrt{-z^3}\,\sqrt{z} - (1-\sqrt{3})(-z^3)^{2/3}\right)\right)\left(1 + O\left(\frac{1}{z^3}\right)\right)\Bigg) \;/;\; (|z| \to \infty)$$

03.07.06.0045.01

$$\text{Ai}'(z) \propto \begin{cases} -\frac{i\, e^{\frac{2 z^{3/2}}{3}} \sqrt[4]{z}}{2\sqrt{\pi}} - \frac{e^{-\frac{2 z^{3/2}}{3}} \sqrt[4]{z}}{2\sqrt{\pi}} & \arg(z) \le -\frac{2\pi}{3} \\ -\frac{e^{-\frac{2 z^{3/2}}{3}} \sqrt[4]{z}}{2\sqrt{\pi}} & -\frac{2\pi}{3} < \arg(z) \le \frac{2\pi}{3} \\ \frac{i\, e^{\frac{2 z^{3/2}}{3}} \sqrt[4]{z}}{2\sqrt{\pi}} - \frac{e^{-\frac{2 z^{3/2}}{3}} \sqrt[4]{z}}{2\sqrt{\pi}} & \text{True} \end{cases} \;/;\; (|z| \to \infty)$$

Expansions for any z in trigonometric form

Using trigonometric functions with branch cut-containing arguments

03.07.06.0023.01

$$\text{Ai}'(z) \propto \frac{1}{2\sqrt{3\pi}\,(-z^3)^{7/12}} \Bigg(\left(\left(z^2 - (-z^3)^{2/3}\right)\cos\left(\frac{\pi}{4} - \frac{2\sqrt{-z^3}}{3}\right) - \sqrt{3}\left((-z^3)^{2/3} + z^2\right)\cos\left(\frac{2\sqrt{-z^3}}{3} + \frac{\pi}{4}\right)\right)\left(1 - \frac{455}{4608 z^3} - \frac{40\,415\,375}{127\,401\,984 z^6} + O\left(\frac{1}{z^9}\right)\right) - \frac{7}{48\sqrt{-z^3}} \left(\left((-z^3)^{2/3} - z^2\right)\cos\left(\frac{2\sqrt{-z^3}}{3} + \frac{\pi}{4}\right) - \sqrt{3}\left((-z^3)^{2/3} + z^2\right)\cos\left(\frac{\pi}{4} - \frac{2\sqrt{-z^3}}{3}\right)\right) \left(1 + \frac{13\,585}{13\,824 z^3} + \frac{823\,318\,925}{127\,401\,984 z^6} + O\left(\frac{1}{z^9}\right)\right)\Bigg) \;/;\; (|z| \to \infty)$$

03.07.06.0024.01

$$\mathrm{Ai}'(z) \propto \frac{1}{2\sqrt{3\pi}\,(-z^3)^{7/12}}\left(\left(\left(z^2-(-z^3)^{2/3}\right)\cos\left(\frac{2\sqrt{-z^3}}{3}-\frac{\pi}{4}\right)-\sqrt{3}\left((-z^3)^{2/3}+z^2\right)\cos\left(\frac{2\sqrt{-z^3}}{3}+\frac{\pi}{4}\right)\right)\right.$$
$$\left(\sum_{k=0}^{n}\frac{\left(-\frac{1}{12}\right)_k\left(\frac{5}{12}\right)_k\left(\frac{7}{12}\right)_k\left(\frac{13}{12}\right)_k}{\left(\frac{1}{2}\right)_k k!}\left(\frac{9}{4z^3}\right)^k+O\left(\frac{1}{z^{\frac{3(n+1)}{2}}}\right)\right)-$$
$$\frac{7}{48\sqrt{-z^3}}\left(\left((-z^3)^{2/3}-z^2\right)\cos\left(\frac{2\sqrt{-z^3}}{3}+\frac{\pi}{4}\right)-\sqrt{3}\left((-z^3)^{2/3}+z^2\right)\cos\left(\frac{2\sqrt{-z^3}}{3}-\frac{\pi}{4}\right)\right)$$
$$\left.\left(\sum_{k=0}^{n}\frac{\left(\frac{5}{12}\right)_k\left(\frac{11}{12}\right)_k\left(\frac{13}{12}\right)_k\left(\frac{19}{12}\right)_k}{k!\left(\frac{3}{2}\right)_k}\left(\frac{9}{4z^3}\right)^k+O\left(\frac{1}{z^{\frac{3(n+1)}{2}}}\right)\right)\right)\;/;\;(|z|\to\infty)\land n\in\mathbb{N}$$

03.07.06.0025.01

$$\mathrm{Ai}'(z) \propto \frac{1}{2\sqrt{3\pi}\,(-z^3)^{7/12}}$$
$$\left(\left(\left(z^2-(-z^3)^{2/3}\right)\cos\left(\frac{2\sqrt{-z^3}}{3}-\frac{\pi}{4}\right)-\sqrt{3}\left((-z^3)^{2/3}+z^2\right)\cos\left(\frac{2\sqrt{-z^3}}{3}+\frac{\pi}{4}\right)\right)\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{12}\right)_k\left(\frac{5}{12}\right)_k\left(\frac{7}{12}\right)_k\left(\frac{13}{12}\right)_k}{\left(\frac{1}{2}\right)_k k!}\left(\frac{9}{4z^3}\right)^k-\right.$$
$$\frac{7}{48\sqrt{-z^3}}\left(\left((-z^3)^{2/3}-z^2\right)\cos\left(\frac{2\sqrt{-z^3}}{3}+\frac{\pi}{4}\right)-\sqrt{3}\left((-z^3)^{2/3}+z^2\right)\cos\left(\frac{2\sqrt{-z^3}}{3}-\frac{\pi}{4}\right)\right)$$
$$\left.\sum_{k=0}^{\infty}\frac{\left(\frac{5}{12}\right)_k\left(\frac{11}{12}\right)_k\left(\frac{13}{12}\right)_k\left(\frac{19}{12}\right)_k}{k!\left(\frac{3}{2}\right)_k}\left(\frac{9}{4z^3}\right)^k\right)\;/;\;(|z|\to\infty)$$

03.07.06.0026.01

$$\mathrm{Ai}'(z) \propto \frac{1}{2\sqrt{3\pi}\,(-z^3)^{7/12}}$$
$$\left(\left(\left(z^2-(-z^3)^{2/3}\right)\cos\left(\frac{2\sqrt{-z^3}}{3}-\frac{\pi}{4}\right)-\sqrt{3}\left((-z^3)^{2/3}+z^2\right)\cos\left(\frac{2\sqrt{-z^3}}{3}+\frac{\pi}{4}\right)\right){}_4F_1\left(-\frac{1}{12},\frac{5}{12},\frac{7}{12},\frac{13}{12};\frac{1}{2};\frac{9}{4z^3}\right)-\right.$$
$$\frac{7}{48\sqrt{-z^3}}\left(\left((-z^3)^{2/3}-z^2\right)\cos\left(\frac{2\sqrt{-z^3}}{3}+\frac{\pi}{4}\right)-\sqrt{3}\left((-z^3)^{2/3}+z^2\right)\cos\left(\frac{2\sqrt{-z^3}}{3}-\frac{\pi}{4}\right)\right)$$
$$\left.{}_4F_1\left(\frac{5}{12},\frac{11}{12},\frac{13}{12},\frac{19}{12};\frac{3}{2};\frac{9}{4z^3}\right)\right)\;/;\;(|z|\to\infty)$$

03.07.06.0027.01

$$\mathrm{Ai}'(z) \propto \frac{1}{2\sqrt{3\pi}\,\left(-z^3\right)^{7/12}}\left(\left(\left(z^2-\left(-z^3\right)^{2/3}\right)\cos\left(\frac{\pi}{4}-\frac{2\sqrt{-z^3}}{3}\right)-\sqrt{3}\,\left(\left(-z^3\right)^{2/3}+z^2\right)\cos\left(\frac{2\sqrt{-z^3}}{3}+\frac{\pi}{4}\right)\right)\left(1+O\left(\frac{1}{z^3}\right)\right)-\frac{7}{48\sqrt{-z^3}}\left(\left(\left(-z^3\right)^{2/3}-z^2\right)\cos\left(\frac{2\sqrt{-z^3}}{3}+\frac{\pi}{4}\right)-\sqrt{3}\,\left(\left(-z^3\right)^{2/3}+z^2\right)\cos\left(\frac{\pi}{4}-\frac{2\sqrt{-z^3}}{3}\right)\right)\left(1+O\left(\frac{1}{z^3}\right)\right)\right)\,/;\,(|z|\to\infty)$$

Using trigonometric functions with branch cut-free arguments

03.07.06.0046.01

$$\mathrm{Ai}'(z) \propto -\frac{1}{2\sqrt{6\pi}\,\left(-z^3\right)^{7/12}}\left(\left(\left(\left(1+\sqrt{3}\right)\left(-z^3\right)^{2/3}+\left(-1+\sqrt{3}\right)z^2\right)\cosh\left(\frac{2z^{3/2}}{3}\right)-\sqrt{z}\,\sqrt[6]{-z^3}\,\left(\left(1+\sqrt{3}\right)\sqrt[3]{-z^3}+\left(1-\sqrt{3}\right)z\right)\sinh\left(\frac{2z^{3/2}}{3}\right)\right)\left(1-\frac{455}{4608\,z^3}-\frac{40\,415\,375}{127\,401\,984\,z^6}-\frac{6\,183\,948\,445\,675}{1\,761\,205\,026\,816\,z^9}+O\left(\frac{1}{z^{12}}\right)\right)-\frac{7}{48\sqrt{-z^3}}\left(\sqrt{z}\,\left(\left(-1+\sqrt{3}\right)\sqrt[3]{-z^3}-\left(1+\sqrt{3}\right)z\right)\sinh\left(\frac{2z^{3/2}}{3}\right)\sqrt[6]{-z^3}+\left(\left(-1+\sqrt{3}\right)\left(-z^3\right)^{2/3}+\left(1+\sqrt{3}\right)z^2\right)\cosh\left(\frac{2z^{3/2}}{3}\right)\right)\left(1+\frac{13\,585}{13\,824\,z^3}+\frac{823\,318\,925}{127\,401\,984\,z^6}+\frac{189\,935\,559\,402\,875}{1\,761\,205\,026\,816\,z^9}+O\left(\frac{1}{z^{12}}\right)\right)\right)\,/;\,(|z|\to\infty)$$

03.07.06.0047.01

$$\mathrm{Ai}'(z) \propto -\frac{1}{2\sqrt{6\pi}\,\left(-z^3\right)^{7/12}}\left(\left(\left(\left(1+\sqrt{3}\right)\left(-z^3\right)^{2/3}+\left(-1+\sqrt{3}\right)z^2\right)\cosh\left(\frac{2z^{3/2}}{3}\right)-\sqrt{z}\,\sqrt[6]{-z^3}\,\left(\left(1+\sqrt{3}\right)\sqrt[3]{-z^3}+\left(1-\sqrt{3}\right)z\right)\sinh\left(\frac{2z^{3/2}}{3}\right)\right)\left(\sum_{k=0}^{n}\frac{\left(-\frac{1}{12}\right)_k\left(\frac{5}{12}\right)_k\left(\frac{7}{12}\right)_k\left(\frac{13}{12}\right)_k}{\left(\frac{1}{2}\right)_k\,k!}\left(\frac{9}{4z^3}\right)^k+O\left(\frac{1}{z^{3n+3}}\right)\right)-\frac{7}{48\sqrt{-z^3}}\left(\sqrt{z}\,\sqrt[6]{-z^3}\,\left(\left(-1+\sqrt{3}\right)\sqrt[3]{-z^3}-\left(1+\sqrt{3}\right)z\right)\sinh\left(\frac{2z^{3/2}}{3}\right)+\left(\left(-1+\sqrt{3}\right)\left(-z^3\right)^{2/3}+\left(1+\sqrt{3}\right)z^2\right)\cosh\left(\frac{2z^{3/2}}{3}\right)\right)\left(\sum_{k=0}^{n}\frac{\left(\frac{5}{12}\right)_k\left(\frac{11}{12}\right)_k\left(\frac{13}{12}\right)_k\left(\frac{19}{12}\right)_k}{k!\left(\frac{3}{2}\right)_k}\left(\frac{9}{4z^3}\right)^k+O\left(\frac{1}{z^{3n+3}}\right)\right)\right)\,/;\,(|z|\to\infty)\land n\in\mathbb{N}$$

03.07.06.0048.01

$$\mathrm{Ai}'(z) \propto -\frac{1}{2\sqrt{6\pi}\,(-z^3)^{7/12}} \left(\left(\left((1+\sqrt{3})(-z^3)^{2/3} + (-1+\sqrt{3})z^2 \right) \cosh\left(\frac{2z^{3/2}}{3}\right) - \sqrt{z}\,\sqrt[6]{-z^3}\left((1+\sqrt{3})\sqrt[3]{-z^3} + (1-\sqrt{3})z \right) \sinh\left(\frac{2z^{3/2}}{3}\right) \right) \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{12}\right)_k \left(\frac{5}{12}\right)_k \left(\frac{7}{12}\right)_k \left(\frac{13}{12}\right)_k}{\left(\frac{1}{2}\right)_k k!} \left(\frac{9}{4z^3}\right)^k - \frac{7}{48\sqrt{-z^3}} \left(\sqrt{z}\,\sqrt[6]{-z^3}\left((-1+\sqrt{3})\sqrt[3]{-z^3} - (1+\sqrt{3})z \right) \sinh\left(\frac{2z^{3/2}}{3}\right) + \left((-1+\sqrt{3})(-z^3)^{2/3} + (1+\sqrt{3})z^2 \right) \cosh\left(\frac{2z^{3/2}}{3}\right) \right) \sum_{k=0}^{\infty} \frac{\left(\frac{5}{12}\right)_k \left(\frac{11}{12}\right)_k \left(\frac{13}{12}\right)_k \left(\frac{19}{12}\right)_k}{k!\left(\frac{3}{2}\right)_k} \left(\frac{9}{4z^3}\right)^k \right) /; (|z| \to \infty)$$

03.07.06.0049.01

$$\mathrm{Ai}'(z) \propto -\frac{1}{2\sqrt{6\pi}\,(-z^3)^{7/12}} \left(\left(\left((1+\sqrt{3})(-z^3)^{2/3} + (-1+\sqrt{3})z^2 \right) \cosh\left(\frac{2z^{3/2}}{3}\right) - \sqrt{z}\,\sqrt[6]{-z^3}\left((1+\sqrt{3})\sqrt[3]{-z^3} + (1-\sqrt{3})z \right) \sinh\left(\frac{2z^{3/2}}{3}\right) \right) {}_4F_1\left(-\frac{1}{12}, \frac{5}{12}, \frac{7}{12}, \frac{13}{12}; \frac{1}{2}; \frac{9}{4z^3}\right) - \frac{7}{48\sqrt{-z^3}} \left(\sqrt{z}\left((-1+\sqrt{3})\sqrt[3]{-z^3} - (1+\sqrt{3})z \right) \sinh\left(\frac{2z^{3/2}}{3}\right) \sqrt[6]{-z^3} + \left((-1+\sqrt{3})(-z^3)^{2/3} + (1+\sqrt{3})z^2 \right) \cosh\left(\frac{2z^{3/2}}{3}\right) \right) {}_4F_1\left(\frac{5}{12}, \frac{11}{12}, \frac{13}{12}, \frac{19}{12}; \frac{3}{2}; \frac{9}{4z^3}\right) \right) /; (|z| \to \infty)$$

03.07.06.0050.01

$$\mathrm{Ai}'(z) \propto -\frac{1}{2\sqrt{6\pi}\,(-z^3)^{7/12}} \left(\left(\left((1+\sqrt{3})(-z^3)^{2/3} + (-1+\sqrt{3})z^2 \right) \cosh\left(\frac{2z^{3/2}}{3}\right) - \sqrt{z}\,\sqrt[6]{-z^3}\left((1+\sqrt{3})\sqrt[3]{-z^3} + (1-\sqrt{3})z \right) \sinh\left(\frac{2z^{3/2}}{3}\right) \right) \left(1 + O\left(\frac{1}{z^3}\right)\right) - \frac{7}{48\sqrt{-z^3}} \left(\sqrt{z}\left((-1+\sqrt{3})\sqrt[3]{-z^3} - (1+\sqrt{3})z \right) \sinh\left(\frac{2z^{3/2}}{3}\right) \sqrt[6]{-z^3} + \left((-1+\sqrt{3})(-z^3)^{2/3} + (1+\sqrt{3})z^2 \right) \cosh\left(\frac{2z^{3/2}}{3}\right) \right) \left(1 + O\left(\frac{1}{z^3}\right)\right) \right) /; (|z| \to \infty)$$

03.07.06.0051.01

$$\mathrm{Ai}'(z) \propto \begin{cases} -\frac{\sqrt[4]{-1}\,\sqrt[4]{z}}{\sqrt{2\pi}}\left(\cosh\left(\frac{2z^{3/2}}{3}\right)+i\sinh\left(\frac{2z^{3/2}}{3}\right)\right) & \arg(z)\le -\frac{2\pi}{3} \\ \frac{\sqrt[4]{z}}{2\sqrt{\pi}}\left(\sinh\left(\frac{2z^{3/2}}{3}\right)-\cosh\left(\frac{2z^{3/2}}{3}\right)\right) & -\frac{2\pi}{3}<\arg(z)\le\frac{2\pi}{3} \\ \frac{(-1)^{3/4}\sqrt[4]{z}}{\sqrt{2\pi}}\left(\cosh\left(\frac{2z^{3/2}}{3}\right)-i\sinh\left(\frac{2z^{3/2}}{3}\right)\right) & \text{True} \end{cases} /;\ (|z|\to\infty)$$

Residue representations

03.07.06.0012.01

$$\mathrm{Ai}'(z) = -\frac{\sqrt[6]{3}}{2\pi}\left(\sum_{j=0}^{\infty}\mathrm{res}_s\left(\left(\Gamma\left(s+\frac{2}{3}\right)\left(3^{-2/3}z\right)^{-3s}\right)\Gamma(s)\right)(-j)+\sum_{j=0}^{\infty}\mathrm{res}_s\left(\left(\Gamma(s)\left(3^{-2/3}z\right)^{-3s}\right)\Gamma\left(s+\frac{2}{3}\right)\right)\left(-j-\frac{2}{3}\right)\right)$$

03.07.06.0013.01

$$\mathrm{Ai}'(z) = \frac{\pi}{9}\left(3^{1/3}z^2\sum_{j=0}^{\infty}\mathrm{res}_s\left(\frac{\left(\frac{z^3}{9}\right)^{-s}}{\Gamma\left(s+\frac{1}{2}\right)\Gamma\left(\frac{5}{3}-s\right)\Gamma\left(\frac{1}{2}-s\right)}\Gamma(s)\right)(-j)-3\,3^{2/3}\sum_{j=0}^{\infty}\mathrm{res}_s\left(\frac{\left(\frac{z^3}{9}\right)^{-s}}{\Gamma\left(s+\frac{1}{2}\right)\Gamma\left(\frac{1}{3}-s\right)\Gamma\left(\frac{1}{2}-s\right)}\Gamma(s)\right)(-j)\right)$$

Integral representations

On the real axis

Of the direct function

03.07.07.0001.01

$$\mathrm{Ai}'(z) = -\frac{1}{\pi}\int_0^{\infty} t\sin\left(\frac{t^3}{3}+z\,t\right)dt\ /;\ \mathrm{Im}(z) = 0$$

03.07.07.0006.01

$$\mathrm{Ai}'(z) = \frac{i}{2\pi}\int_{-\infty}^{\infty} t\,e^{i\left(\frac{t^3}{3}+z\,t\right)}dt\ /;\ \mathrm{Im}(z) = 0$$

Contour integral representations

03.07.07.0002.01

$$\mathrm{Ai}'(z) = -\frac{1}{2\pi i}\int_{\infty\exp(-\pi i/3)}^{\infty\exp(\pi i/3)} t\,e^{\frac{t^3}{3}-z\,t}\,dt$$

03.07.07.0003.01

$$\mathrm{Ai}'(z) = -\frac{\sqrt[6]{3}}{2\pi}\frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty}\Gamma(s)\,\Gamma\left(s+\frac{2}{3}\right)\left(3^{-2/3}z\right)^{-3s}ds\ /;\ 0<\gamma$$

03.07.07.0004.01

$$\mathrm{Ai}'(z) = -\frac{\sqrt[6]{3}}{2\pi}\frac{1}{2\pi i}\int_{\mathcal{L}}\Gamma(s)\,\Gamma\left(s+\frac{2}{3}\right)\left(3^{-2/3}z\right)^{-3s}ds$$

03.07.07.0005.01

$$\mathrm{Ai}'(z) = \frac{\pi}{9}\left(\frac{3^{1/3} z^2}{2\pi i}\int_{\mathcal{L}} \frac{\Gamma(s)}{\Gamma\left(s+\frac{1}{2}\right)\Gamma\left(\frac{5}{3}-s\right)\Gamma\left(\frac{1}{2}-s\right)}\left(\frac{z^3}{9}\right)^{-s} ds - \frac{3\, 3^{2/3}}{2\pi i}\int_{\mathcal{L}} \frac{\Gamma(s)}{\Gamma\left(s+\frac{1}{2}\right)\Gamma\left(\frac{1}{3}-s\right)\Gamma\left(\frac{1}{2}-s\right)}\left(\frac{z^3}{9}\right)^{-s} ds\right)$$

Differential equations

Ordinary linear differential equations and wronskians

For the direct function itself

03.07.13.0001.01

$$z\, w''(z) - w'(z) - z^2 w(z) = 0 \,/;\, w(z) = \mathrm{Ai}'(z) \bigwedge w(0) = -\frac{1}{\sqrt[3]{3}\, \Gamma\left(\frac{1}{3}\right)} \bigwedge w'(0) = 0$$

03.07.13.0002.01

$$z\, w''(z) - w'(z) - z^2 w(z) = 0 \,/;\, w(z) = \mathrm{Ai}'(z)\, c_1 + c_2\, \mathrm{Bi}'(z)$$

03.07.13.0003.01

$$W_z\left(\mathrm{Ai}'(z), \mathrm{Bi}'(z)\right) = -\frac{z}{\pi}$$

03.07.13.0004.01

$$W_z\left(\mathrm{Ai}'(z), \mathrm{Ai}'\left(z\, e^{\frac{2\pi i}{3}}\right)\right) = \frac{z}{2\pi} e^{\frac{\pi i}{6}}$$

03.07.13.0005.01

$$W_z\left(\mathrm{Ai}'(z), \mathrm{Ai}'\left(z\, e^{-\frac{2\pi i}{3}}\right)\right) = \frac{z}{2\pi} e^{-\frac{\pi i}{6}}$$

03.07.13.0006.01

$$W_z\left(\mathrm{Ai}'\left(z\, e^{-\frac{2\pi i}{3}}\right), \mathrm{Ai}'\left(z\, e^{\frac{2\pi i}{3}}\right)\right) = -\frac{z}{2\pi i}$$

03.07.13.0011.01

$$g(z)\, g'(z)\, w''(z) - \left(g'(z)^2 + g(z)\, g''(z)\right) w'(z) - g(z)^2\, g'(z)^3\, w(z) = 0 \,/;\, w(z) = c_1\, \mathrm{Ai}'(g(z)) + c_2\, \mathrm{Bi}'(g(z))$$

03.07.13.0012.01

$$W_z\left(\mathrm{Ai}'(g(z)), \mathrm{Bi}'(g(z))\right) = -\frac{g(z)\, g'(z)}{\pi}$$

03.07.13.0013.01

$$g(z)\, g'(z)\, h(z)^2\, w''(z) - \left(2\, g(z)\, g'(z)\, h'(z) + h(z)\left(g'(z)^2 + g(z)\, g''(z)\right)\right) h(z)\, w'(z) + \left(-g(z)^2\, h(z)^2\, g'(z)^3 + h(z)\, h'(z)\, g'(z)^2 + g(z)\left(h(z)\, h'(z)\, g''(z) + g'(z)\left(2\, h'(z)^2 - h(z)\, h''(z)\right)\right)\right) w(z) = 0 \,/;\, w(z) = c_1\, h(z)\, \mathrm{Ai}'(g(z)) + c_2\, h(z)\, \mathrm{Bi}'(g(z))$$

03.07.13.0014.01

$$W_z\left(h(z)\, \mathrm{Ai}'(g(z)), h(z)\, \mathrm{Bi}'(g(z))\right) = -\frac{g(z)\, h(z)^2\, g'(z)}{\pi}$$

03.07.13.0015.01

$$z^2\, w''(z) + z(-2r - 2s + 1)\, w'(z) + \left(s(2r+s) - a^3 r^2 z^{3r}\right) w(z) = 0 \,/;\, w(z) = c_1\, z^s\, \mathrm{Ai}'(a\, z^r) + c_2\, z^s\, \mathrm{Bi}'(a\, z^r)$$

03.07.13.0016.01

$$W_z\left(z^s \operatorname{Ai}'(a z^r), z^s \operatorname{Bi}'(a z^r)\right) == -\frac{a^2 r z^{2r+2s-1}}{\pi}$$

03.07.13.0017.01

$$w''(z) - 2(\log(r) + \log(s))\, w'(z) + \left(\log(s)(2\log(r) + \log(s)) - a^3 r^{3z} \log^2(r)\right) w(z) = 0 /;\ w(z) = c_1 s^z \operatorname{Ai}'(a r^z) + c_2 s^z \operatorname{Bi}'(a r^z)$$

03.07.13.0018.01

$$W_z\left(s^z \operatorname{Ai}'(a r^z), s^z \operatorname{Bi}'(a r^z)\right) == -\frac{a^2 r^{2z} s^{2z} \log(r)}{\pi}$$

Involving related functions

03.07.13.0007.01

$$w^{(3)}(z) - 4 z\, w'(z) - 2 w(z) == 0 /;\ w(z) == c_1 \operatorname{Ai}(z)^2 + c_2 \operatorname{Bi}(z) \operatorname{Ai}(z) + c_3 \operatorname{Bi}(z)^2$$

03.07.13.0008.01

$$w^{(3)}(z) - 4 z\, w'(z) - 2 w(z) == 0 /;\ w(z) == w_1(z)\, w_2(z) \land w_1''(z) - z\, w_1(z) == 0 \land w_2''(z) - z\, w_2(z) == 0$$

03.07.13.0009.01

$$W_z\left(\operatorname{Ai}'(z)^2, \operatorname{Ai}'(z) \operatorname{Bi}'(z), \operatorname{Bi}'(z)^2\right) == -\frac{2}{\pi^3} z^3$$

Ordinary nonlinear differential equations

03.07.13.0010.01

$$w(z)^2 - z + w'(z) == 0 /;\ w(z) == \frac{\operatorname{Bi}'(z) + c_1 \operatorname{Ai}'(z)}{\operatorname{Bi}(z) + c_1 \operatorname{Ai}(z)}$$

Riccati form of differential equation

Transformations

Transformations and argument simplifications

Argument involving basic arithmetic operations

03.07.16.0001.01

$$\operatorname{Ai}'(c (d z^n)^m) == \frac{1}{2}\left(\frac{(d z^3)^{2m}}{d^{2m} z^{6m}} + 1\right) \operatorname{Ai}'\left(c d^m z^{3m}\right) - \frac{1}{2\sqrt{3}}\left(1 - \frac{(d z^3)^{2m}}{d^{2m} z^{6m}}\right) \operatorname{Bi}'\left(c d^m z^{3m}\right) /;\ \text{IntegerQ}[3 m]$$

03.07.16.0002.01

$$\operatorname{Ai}'\left(\sqrt[3]{z^3}\right) == \frac{1}{2}\left(\frac{(z^3)^{2/3}}{z^2} + 1\right) \operatorname{Ai}'(z) - \frac{1}{2\sqrt{3}}\left(1 - \frac{(z^3)^{2/3}}{z^2}\right) \operatorname{Bi}'(z)$$

03.07.16.0003.01

$$\operatorname{Ai}'\left((-1)^{2/3} z\right) == \frac{1 - i\sqrt{3}}{4}\left(\operatorname{Ai}'(z) - i \operatorname{Bi}'(z)\right)$$

03.07.16.0004.01

$$\operatorname{Ai}'\left(-\left(\sqrt[3]{-1}\, z\right)\right) == \frac{1 + i\sqrt{3}}{4}\left(\operatorname{Ai}'(z) + i \operatorname{Bi}'(z)\right)$$

Identities

Functional identities

03.07.17.0001.01

$$\mathrm{Ai}'(z) + e^{-\frac{2 i \pi}{3}} \mathrm{Ai}'\left(e^{\frac{2 i \pi}{3}} z\right) + e^{\frac{2 i \pi}{3}} \mathrm{Ai}'\left(e^{-\frac{2 i \pi}{3}} z\right) = 0$$

Complex characteristics

Real part

03.07.19.0001.01

$$\mathrm{Re}(\mathrm{Ai}'(x + i y)) = \frac{1}{2}\left(\mathrm{Ai}'\left(x - x \sqrt{-\frac{y^2}{x^2}}\right) + \mathrm{Ai}'\left(x + x \sqrt{-\frac{y^2}{x^2}}\right)\right)$$

Imaginary part

03.07.19.0002.01

$$\mathrm{Im}(\mathrm{Ai}'(x + i y)) = \frac{x}{2 y} \sqrt{-\frac{y^2}{x^2}} \left(\mathrm{Ai}'\left(x - x \sqrt{-\frac{y^2}{x^2}}\right) - \mathrm{Ai}'\left(x + x \sqrt{-\frac{y^2}{x^2}}\right)\right)$$

Absolute value

03.07.19.0003.01

$$\left|\mathrm{Ai}'(x + i y)\right| = \sqrt{\mathrm{Ai}'\left(x - x \sqrt{-\frac{y^2}{x^2}}\right) \mathrm{Ai}'\left(x + x \sqrt{-\frac{y^2}{x^2}}\right)}$$

Argument

03.07.19.0004.01

$$\arg(\mathrm{Ai}'(x + i y)) = \tan^{-1}\left(\frac{1}{2}\left(\mathrm{Ai}'\left(x - x \sqrt{-\frac{y^2}{x^2}}\right) + \mathrm{Ai}'\left(x + x \sqrt{-\frac{y^2}{x^2}}\right)\right), \frac{x}{2 y} \sqrt{-\frac{y^2}{x^2}} \left(\mathrm{Ai}'\left(x - x \sqrt{-\frac{y^2}{x^2}}\right) - \mathrm{Ai}'\left(x + x \sqrt{-\frac{y^2}{x^2}}\right)\right)\right)$$

Conjugate value

03.07.19.0005.01

$$\overline{\mathrm{Ai}'(x + i y)} = \frac{1}{2}\left(\mathrm{Ai}'\left(x - x \sqrt{-\frac{y^2}{x^2}}\right) + \mathrm{Ai}'\left(x + x \sqrt{-\frac{y^2}{x^2}}\right)\right) - \frac{i x}{2 y} \sqrt{-\frac{y^2}{x^2}} \left(\mathrm{Ai}'\left(x - x \sqrt{-\frac{y^2}{x^2}}\right) - \mathrm{Ai}'\left(x + x \sqrt{-\frac{y^2}{x^2}}\right)\right)$$

Signum value

03.07.19.0006.01

$$\operatorname{sgn}(\operatorname{Ai}'(x+i\,y)) = \frac{\frac{i}{y}\sqrt{-\frac{y^2}{x^2}}\;x\left(\operatorname{Ai}'\left(x-x\sqrt{-\frac{y^2}{x^2}}\right)-\operatorname{Ai}'\left(\sqrt{-\frac{y^2}{x^2}}\;x+x\right)\right)+\operatorname{Ai}'\left(\sqrt{-\frac{y^2}{x^2}}\;x+x\right)+\operatorname{Ai}'\left(x-x\sqrt{-\frac{y^2}{x^2}}\right)}{2\sqrt{\operatorname{Ai}'\left(x-x\sqrt{-\frac{y^2}{x^2}}\right)\operatorname{Ai}'\left(\sqrt{-\frac{y^2}{x^2}}\;x+x\right)}}$$

Differentiation

Low-order differentiation

03.07.20.0001.01

$$\frac{\partial \operatorname{Ai}'(z)}{\partial z} = z\operatorname{Ai}(z)$$

03.07.20.0002.01

$$\frac{\partial^2 \operatorname{Ai}'(z)}{\partial z^2} = \operatorname{Ai}(z) + z\operatorname{Ai}'(z)$$

Symbolic differentiation

03.07.20.0005.01

$$\frac{\partial^n \operatorname{Ai}'(z)}{\partial z^n} = \frac{1}{2}\operatorname{Ai}'(z)\,\delta_n + \frac{1}{4}z^{-n}\left(2\sum_{k=0}^{n}\sum_{j=0}^{k}\sum_{i=0}^{k}\frac{(-1)^{j+k}(k-i)!\,(-3j+3k+1)(-3j+3k+2)(-3j+3k-n+3)_{n-2}\left(\frac{2}{3}\right)_k}{i!\,j!\,(k-j)!\,(k-2i)!\left(\frac{2}{3}\right)_i\left(\frac{1}{3}-k\right)_i}\left(-\frac{z^3}{9}\right)^i + \right.$$
$$\left.\sum_{k=0}^{n}\sum_{j=0}^{k}\sum_{i=0}^{k-1}\frac{(-1)^{j+k}(-i+k-1)!\,(3i-3k+2)(-3j+3k-n+1)_n\left(-\frac{2}{3}\right)_k}{(i-1)!\,j!\,(k-j)!\,(k-2i)!\left(\frac{1}{3}\right)_i\left(\frac{5}{3}-k\right)_i}\left(-\frac{z^3}{9}\right)^i\right)\operatorname{Ai}'(z) +$$
$$\frac{1}{4}z^{2-n}\left(\sum_{k=0}^{n}\sum_{j=0}^{k}\sum_{i=0}^{k-1}\frac{(-1)^{j+k-1}(-i+k-1)!\,(-3j+3k+1)(-3j+3k+2)(-3j+3k-n+3)_{n-2}\left(\frac{2}{3}\right)_k}{i!\,j!\,(k-j)!\,(-2i+k-1)!\left(\frac{5}{3}\right)_i\left(\frac{1}{3}-k\right)_i}\left(-\frac{z^3}{9}\right)^i - \right.$$
$$\left.\sum_{k=0}^{n}\sum_{j=0}^{k}\sum_{i=0}^{k-1}\frac{(-1)^{j+k-1}(-i+k-1)!\,(-3j+3k-n+1)_n\left(-\frac{2}{3}\right)_k}{i!\,j!\,(k-j)!\,(-2i+k-1)!\left(\frac{1}{3}\right)_i\left(\frac{5}{3}-k\right)_i}\left(-\frac{z^3}{9}\right)^i\right)\operatorname{Ai}(z)\;/;\;n\in\mathbb{N}$$

03.07.20.0003.02

$$\frac{\partial^n \operatorname{Ai}'(z)}{\partial z^n} = 3^{n-\frac{8}{3}}z^{-n}\left(\Gamma\left(\frac{1}{3}\right)z^2\,{}_2\tilde{F}_3\left(1,\frac{4}{3};1-\frac{n}{3},\frac{4-n}{3},\frac{5-n}{3};\frac{z^3}{9}\right)+3\sqrt[3]{3}\,\Gamma\left(-\frac{1}{3}\right){}_2\tilde{F}_3\left(\frac{2}{3},1;\frac{1-n}{3},\frac{2-n}{3},1-\frac{n}{3};\frac{z^3}{9}\right)\right)/;\;n\in\mathbb{N}$$

Fractional integro-differentiation

03.07.20.0004.01

$$\frac{\partial^\alpha \operatorname{Ai}'(z)}{\partial z^\alpha} = 3^{\alpha-\frac{8}{3}}z^{-\alpha}\left(\Gamma\left(\frac{1}{3}\right)z^2\,{}_2\tilde{F}_3\left(1,\frac{4}{3};1-\frac{\alpha}{3},\frac{4-\alpha}{3},\frac{5-\alpha}{3};\frac{z^3}{9}\right)+3\sqrt[3]{3}\,\Gamma\left(-\frac{1}{3}\right){}_2\tilde{F}_3\left(\frac{2}{3},1;\frac{1-\alpha}{3},\frac{2-\alpha}{3},1-\frac{\alpha}{3};\frac{z^3}{9}\right)\right)$$

Integration

Indefinite integration

Involving only one direct function

03.07.21.0001.01

$$\int \mathrm{Ai}'(a\,z)\,dz = \frac{\mathrm{Ai}(a\,z)}{a}$$

03.07.21.0002.01

$$\int \mathrm{Ai}'(z)\,dz = \mathrm{Ai}(z)$$

Involving one direct function and elementary functions

Involving power function

Involving power

Linear argument

03.07.21.0003.01

$$\int z^{\alpha-1}\,\mathrm{Ai}'(a\,z)\,dz = \frac{a^2\,z^{\alpha+2}\,\Gamma\left(\frac{\alpha}{3}+\frac{2}{3}\right)}{9\,3^{2/3}}\,{}_1\tilde{F}_2\left(\frac{\alpha}{3}+\frac{2}{3};\,\frac{5}{3},\,\frac{\alpha}{3}+\frac{5}{3};\,\frac{a^3\,z^3}{9}\right) - \frac{z^\alpha\,\Gamma\left(\frac{\alpha}{3}\right)}{3\,\sqrt[3]{3}}\,{}_1\tilde{F}_2\left(\frac{\alpha}{3};\,\frac{1}{3},\,\frac{\alpha}{3}+1;\,\frac{a^3\,z^3}{9}\right)$$

03.07.21.0004.01

$$\int z^{\alpha-1}\,\mathrm{Ai}'(z)\,dz = \frac{z^{\alpha+2}}{9\,3^{2/3}}\,\Gamma\left(\frac{\alpha}{3}+\frac{2}{3}\right)\,{}_1\tilde{F}_2\left(\frac{\alpha}{3}+\frac{2}{3};\,\frac{5}{3},\,\frac{\alpha}{3}+\frac{5}{3};\,\frac{z^3}{9}\right) - \frac{z^\alpha}{3\,\sqrt[3]{3}}\,\Gamma\left(\frac{\alpha}{3}\right)\,{}_1\tilde{F}_2\left(\frac{\alpha}{3};\,\frac{1}{3},\,\frac{\alpha}{3}+1;\,\frac{z^3}{9}\right)$$

03.07.21.0005.01

$$\int z^{n+2}\,\mathrm{Ai}'(z)\,dz = -(n+2)\left(z\,\mathrm{Ai}'(z) - n\,\mathrm{Ai}(z)\right)z^{n-1} + \mathrm{Ai}(z)\,z^{n+2} - (n-1)\,n\,(n+2)\int z^{n-2}\,\mathrm{Ai}(z)\,dz\,/;\,n\in\mathbb{N}$$

03.07.21.0006.01

$$\int z\,\mathrm{Ai}'(z)\,dz = \frac{\Gamma\left(\frac{4}{3}\right)}{9\,3^{2/3}\,\Gamma\left(\frac{5}{3}\right)\Gamma\left(\frac{7}{3}\right)}\,z^4\,{}_1F_2\left(\frac{4}{3};\,\frac{5}{3},\,\frac{7}{3};\,\frac{z^3}{9}\right) - \frac{\Gamma\left(\frac{2}{3}\right)}{3\,\sqrt[3]{3}\,\Gamma\left(\frac{1}{3}\right)\Gamma\left(\frac{5}{3}\right)}\,z^2\,{}_1F_2\left(\frac{2}{3};\,\frac{1}{3},\,\frac{5}{3};\,\frac{z^3}{9}\right)$$

03.07.21.0007.01

$$\int z^2\,\mathrm{Ai}'(z)\,dz = z^2\,\mathrm{Ai}(z) - 2\,\mathrm{Ai}'(z)$$

Power arguments

03.07.21.0008.01

$$\int z^{\alpha-1}\,\mathrm{Ai}'(a\,z^r)\,dz =$$

$$\frac{z^\alpha}{9\,3^{2/3}\,r}\left(a^2\,z^{2r}\,\Gamma\left(\frac{1}{3}\left(\frac{\alpha}{r}+2\right)\right)\,{}_1\tilde{F}_2\left(\frac{1}{3}\left(\frac{\alpha}{r}+2\right);\,\frac{5}{3},\,\frac{1}{3}\left(\frac{\alpha}{r}+5\right);\,\frac{1}{9}\,a^3\,z^{3r}\right) - 3\,\sqrt[3]{3}\,\Gamma\left(\frac{\alpha}{3\,r}\right)\,{}_1\tilde{F}_2\left(\frac{\alpha}{3\,r};\,\frac{1}{3},\,\frac{\alpha}{3\,r}+1;\,\frac{1}{9}\,a^3\,z^{3r}\right)\right)$$

Involving exponential function

Involving exp

Linear argument

03.07.21.0009.01

$$\int e^{-\frac{2}{3}(a z)^{3/2}} \operatorname{Ai}'(a z)\, dz = \frac{1}{15\, 3^{2/3}} \left(\frac{1}{a\, \Gamma\left(\frac{5}{3}\right)} \left(6\, {}_1F_1\left(-\frac{5}{6}; \frac{1}{3}; \frac{1}{3} (-4) (a z)^{3/2}\right) - 20 (a z)^{3/2}\, {}_1F_1\left(\frac{1}{6}; \frac{4}{3}; \frac{1}{3} (-4) (a z)^{3/2}\right)\right) + \frac{3 \sqrt[3]{3}\, z}{\Gamma\left(\frac{1}{3}\right)} \left(2 (a z)^{3/2}\, {}_1F_1\left(\frac{5}{6}; \frac{8}{3}; \frac{1}{3} (-4) (a z)^{3/2}\right) - 5\, {}_1F_1\left(-\frac{1}{6}; \frac{5}{3}; \frac{1}{3} (-4) (a z)^{3/2}\right)\right)\right)$$

03.07.21.0010.01

$$\int e^{\frac{2}{3}(a z)^{3/2}} \operatorname{Ai}'(a z)\, dz = \frac{1}{15\, 3^{2/3}} \left(\frac{1}{a\, \Gamma\left(\frac{5}{3}\right)} \left(20\, {}_1F_1\left(\frac{1}{6}; \frac{4}{3}; \frac{4}{3} (a z)^{3/2}\right) (a z)^{3/2} + 6\, {}_1F_1\left(-\frac{5}{6}; \frac{1}{3}; \frac{4}{3} (a z)^{3/2}\right)\right) - \frac{3 \sqrt[3]{3}\, z}{\Gamma\left(\frac{1}{3}\right)} \left(2\, {}_1F_1\left(\frac{5}{6}; \frac{8}{3}; \frac{4}{3} (a z)^{3/2}\right) (a z)^{3/2} + 5\, {}_1F_1\left(-\frac{1}{6}; \frac{5}{3}; \frac{4}{3} (a z)^{3/2}\right)\right)\right)$$

Power arguments

03.07.21.0011.01

$$\int e^{\frac{1}{3}(-2)(a z^r)^{3/2}} \operatorname{Ai}'(a z^r)\, dz = -\frac{1}{3\, 3^{2/3} (2 r + 1)\, \Gamma\left(\frac{1}{3}\right) \Gamma\left(\frac{5}{3}\right)} \left(z \left(3 \sqrt[3]{3}\, (2 r + 1)\, \Gamma\left(\frac{5}{3}\right) {}_2F_2\left(-\frac{1}{6}, \frac{2}{3 r}; -\frac{1}{3}, 1 + \frac{2}{3 r}; \frac{1}{3} (-4) (a z^r)^{3/2}\right) - a^2 z^{2 r}\, \Gamma\left(\frac{1}{3}\right) {}_2F_2\left(\frac{7}{6}, \frac{4}{3} + \frac{2}{3 r}; \frac{7}{3}, \frac{7}{3} + \frac{2}{3 r}; \frac{1}{3} (-4) (a z^r)^{3/2}\right)\right)\right)$$

03.07.21.0012.01

$$\int e^{\frac{2}{3}(a z^r)^{3/2}} \operatorname{Ai}'(a z^r)\, dz = -\frac{1}{3\, 3^{2/3} (2 r + 1)\, \Gamma\left(\frac{1}{3}\right) \Gamma\left(\frac{5}{3}\right)} \left(z \left(3 \sqrt[3]{3}\, (2 r + 1)\, \Gamma\left(\frac{5}{3}\right) {}_2F_2\left(-\frac{1}{6}, \frac{2}{3 r}; -\frac{1}{3}, 1 + \frac{2}{3 r}; \frac{4}{3} (a z^r)^{3/2}\right) - a^2 z^{2 r}\, \Gamma\left(\frac{1}{3}\right) {}_2F_2\left(\frac{7}{6}, \frac{4}{3} + \frac{2}{3 r}; \frac{7}{3}, \frac{7}{3} + \frac{2}{3 r}; \frac{4}{3} (a z^r)^{3/2}\right)\right)\right)$$

Involving exponential function and a power function

Involving exp and power

Linear arguments

03.07.21.0013.01

$$\int z^{\alpha-1} e^{\frac{1}{3}(-2)(a z)^{3/2}} \operatorname{Ai}'(a z)\, dz = \frac{1}{3\, 3^{2/3}\, \alpha (\alpha+2)\, \Gamma\left(\frac{1}{3}\right) \Gamma\left(\frac{5}{3}\right)} \left(z^{\alpha} \left(a^2 z^2 \alpha\, \Gamma\left(\frac{1}{3}\right) {}_2F_2\left(\frac{7}{6}, \frac{2\alpha}{3}+\frac{4}{3}; \frac{7}{3}, \frac{2\alpha}{3}+\frac{7}{3}; \frac{1}{3}(-4)(a z)^{3/2}\right) - 3\sqrt[3]{3}\, (\alpha+2)\, \Gamma\left(\frac{5}{3}\right) {}_2F_2\left(-\frac{1}{6}, \frac{2\alpha}{3}; -\frac{1}{3}, \frac{2\alpha}{3}+1; \frac{1}{3}(-4)(a z)^{3/2}\right)\right)\right)$$

03.07.21.0014.01

$$\int \sqrt{z}\, e^{-\frac{2}{3}(a z)^{3/2}} \operatorname{Ai}'(a z)\, dz = \frac{1}{21 a^2 \sqrt{z}\, \Gamma\left(\frac{1}{3}\right)} \left(e^{\frac{1}{3}(-2)(a z)^{3/2}} \left(6 a^2 \operatorname{Ai}'(a z)\, \Gamma\left(\frac{1}{3}\right) z^2 + 2\sqrt{a z} \left(a^2 z^2 I_{\frac{5}{3}}\left(\frac{2}{3} a^{3/2} z^{3/2}\right) \Gamma\left(\frac{1}{3}\right) \sqrt[3]{a^{3/2} z^{3/2}} - 2\, 3^{2/3} e^{\frac{2}{3}(a z)^{3/2}} - \frac{a^3 z^3\, \Gamma\left(\frac{1}{3}\right)}{\sqrt[3]{a^{3/2} z^{3/2}}} I_{-\frac{5}{3}}\left(\frac{2}{3} a^{3/2} z^{3/2}\right)\right)\right)\right)$$

03.07.21.0015.01

$$\int z^{\alpha-1} e^{\frac{2}{3}(a z)^{3/2}} \operatorname{Ai}'(a z)\, dz = \frac{1}{3\, 3^{2/3}\, \alpha (\alpha+2)\, \Gamma\left(\frac{1}{3}\right) \Gamma\left(\frac{5}{3}\right)} \left(z^{\alpha} \left(a^2 z^2 \alpha\, \Gamma\left(\frac{1}{3}\right) {}_2F_2\left(\frac{7}{6}, \frac{2\alpha}{3}+\frac{4}{3}; \frac{7}{3}, \frac{2\alpha}{3}+\frac{7}{3}; \frac{4}{3}(a z)^{3/2}\right) - 3\sqrt[3]{3}\, (\alpha+2)\, \Gamma\left(\frac{5}{3}\right) {}_2F_2\left(-\frac{1}{6}, \frac{2\alpha}{3}; -\frac{1}{3}, \frac{2\alpha}{3}+1; \frac{4}{3}(a z)^{3/2}\right)\right)\right)$$

03.07.21.0016.01

$$\int \sqrt{z}\, e^{\frac{2}{3}(a z)^{3/2}} \operatorname{Ai}'(a z)\, dz = \frac{1}{21 a^2 \sqrt{z}\, \Gamma\left(\frac{1}{3}\right)} \left(6 a^2 e^{\frac{2}{3}(a z)^{3/2}} \operatorname{Ai}'(a z)\, \Gamma\left(\frac{1}{3}\right) z^2 + 2\sqrt{a z} \left(-a^2 e^{\frac{2}{3}(a z)^{3/2}} z^2 I_{\frac{5}{3}}\left(\frac{2}{3} a^{3/2} z^{3/2}\right) \Gamma\left(\frac{1}{3}\right) \sqrt[3]{a^{3/2} z^{3/2}} + 2\, 3^{2/3} + \frac{a^3\, \Gamma\left(\frac{1}{3}\right)}{\sqrt[3]{a^{3/2} z^{3/2}}} e^{\frac{2}{3}(a z)^{3/2}} z^3 I_{-\frac{5}{3}}\left(\frac{2}{3} a^{3/2} z^{3/2}\right)\right)\right)$$

Power arguments

03.07.21.0017.01

$$\int z^{\alpha-1} e^{\frac{1}{3}(-2)(a z^r)^{3/2}} \operatorname{Ai}'(a z^r)\, dz = \frac{1}{3\, 3^{2/3}\, \alpha (2r+\alpha)\, \Gamma\left(\frac{1}{3}\right) \Gamma\left(\frac{5}{3}\right)} \left(z^{\alpha} \left(a^2 z^{2r} \alpha\, \Gamma\left(\frac{1}{3}\right) {}_2F_2\left(\frac{7}{6}, \frac{2\alpha}{3r}+\frac{4}{3}; \frac{7}{3}, \frac{2\alpha}{3r}+\frac{7}{3}; \frac{1}{3}(-4)(a z^r)^{3/2}\right) - 3\sqrt[3]{3}\, (2r+\alpha)\, \Gamma\left(\frac{5}{3}\right) {}_2F_2\left(-\frac{1}{6}, \frac{2\alpha}{3r}; -\frac{1}{3}, \frac{2\alpha}{3r}+1; \frac{1}{3}(-4)(a z^r)^{3/2}\right)\right)\right)$$

03.07.21.0018.01

$$\int z^{\alpha-1} e^{\frac{2}{3}(a z^r)^{3/2}} \operatorname{Ai}'(a z^r)\, dz = \frac{1}{3\, 3^{2/3} \alpha (2r+\alpha) \Gamma\left(\frac{1}{3}\right) \Gamma\left(\frac{5}{3}\right)} \left(z^\alpha \left(a^2 z^{2r} \alpha\, \Gamma\left(\frac{1}{3}\right) {}_2F_2\left(\frac{7}{6}, \frac{2\alpha}{3r}+\frac{4}{3}; \frac{7}{3}, \frac{2\alpha}{3r}+\frac{7}{3}; \frac{4}{3}(a z^r)^{3/2}\right) - 3\sqrt[3]{3}\, (2r+\alpha)\, \Gamma\left(\frac{5}{3}\right) {}_2F_2\left(-\frac{1}{6}, \frac{2\alpha}{3r}; -\frac{1}{3}, \frac{2\alpha}{3r}+1; \frac{4}{3}(a z^r)^{3/2}\right)\right)\right)$$

Involving hyperbolic functions

Involving sinh

Linear argument

03.07.21.0019.01

$$\int \sinh\left(\frac{2}{3}(a z)^{3/2}\right) \operatorname{Ai}'(a z)\, dz = \frac{1}{10\left(a^{3/2} z^{3/2}\right)^{2/3}} \left(e^{\frac{1}{3}(-2)(a z)^{3/2}} z \left(\left(-1+e^{\frac{4}{3}(a z)^{3/2}}\right)\left(5 \operatorname{Ai}'(a z)\left(a^{3/2} z^{3/2}\right)^{2/3} + a^2 z^2 \left(\frac{a z}{\left(a^{3/2} z^{3/2}\right)^{2/3}} I_{\frac{4}{3}}\left(\frac{2}{3} a^{3/2} z^{3/2}\right) - I_{-\frac{4}{3}}\left(\frac{2}{3} a^{3/2} z^{3/2}\right)\right)\right) - 2\left(1+e^{\frac{4}{3}(a z)^{3/2}}\right)\sqrt{a z}\left(a^{3/2} z^{3/2}\right)^{2/3} \operatorname{Ai}(a z)\right)\right)$$

03.07.21.0020.01

$$\int \sinh\left(\frac{2}{3}(a z)^{3/2}+b\right) \operatorname{Ai}'(a z)\, dz = \frac{1}{30\, a \left(a^{3/2} z^{3/2}\right)^{2/3} \Gamma\left(\frac{5}{3}\right)} \left(e^{-\frac{1}{3} 2 (a z)^{3/2} - b} \left(-6\left(1+e^{\frac{4}{3}(a z)^{3/2}+2b}\right)\left(a^{3/2} z^{3/2}\right)^{2/3} \operatorname{Ai}(a z)\, \Gamma\left(\frac{5}{3}\right)(a z)^{3/2} + 2\sqrt[3]{3}\, e^{\frac{2}{3}(a z)^{3/2}} \left(a^{3/2} z^{3/2}\right)^{2/3} - 2\sqrt[3]{3}\, e^{\frac{2}{3}\left((a z)^{3/2}+3b\right)} \left(a^{3/2} z^{3/2}\right)^{2/3} + 15\, a\left(-1+e^{\frac{4}{3}(a z)^{3/2}+2b}\right) z \left(a^{3/2} z^{3/2}\right)^{2/3} \operatorname{Ai}'(a z)\, \Gamma\left(\frac{5}{3}\right) + \frac{3 a^4 z^4\, \Gamma\left(\frac{5}{3}\right)}{\left(a^{3/2} z^{3/2}\right)^{2/3}} e^{\frac{4}{3}(a z)^{3/2}+2b} I_{\frac{4}{3}}\left(\frac{2}{3} a^{3/2} z^{3/2}\right) - 3 a^3 e^{\frac{4}{3}(a z)^{3/2}+2b} z^3 I_{-\frac{4}{3}}\left(\frac{2}{3} a^{3/2} z^{3/2}\right)\Gamma\left(\frac{5}{3}\right) + 3 a^3 z^3 I_{-\frac{4}{3}}\left(\frac{2}{3} a^{3/2} z^{3/2}\right)\Gamma\left(\frac{5}{3}\right) - \frac{3 a^4 z^4\, \Gamma\left(\frac{5}{3}\right)}{\left(a^{3/2} z^{3/2}\right)^{2/3}} I_{\frac{4}{3}}\left(\frac{2}{3} a^{3/2} z^{3/2}\right)\right)\right)$$

Power arguments

03.07.21.0021.01

$$\int \sinh\left(\frac{2}{3}(a z^r)^{3/2}\right) \mathrm{Ai}'(a z^r)\, dz = \frac{1}{6\, 3^{2/3}\,(2r+1)\,\Gamma\left(\frac{1}{3}\right)\Gamma\left(\frac{5}{3}\right)} \left(z\left(-a^2\,\Gamma\left(\frac{1}{3}\right)\left({}_2F_2\left(\frac{7}{6}, \frac{4}{3}+\frac{2}{3r}; \frac{7}{3}, \frac{7}{3}+\frac{2}{3r}; \frac{1}{3}(-4)(a z^r)^{3/2}\right) - {}_2F_2\left(\frac{7}{6}, \frac{4}{3}+\frac{2}{3r}; \frac{7}{3}, \frac{7}{3}+\frac{2}{3r}; \frac{4}{3}(a z^r)^{3/2}\right)\right) z^{2r} - 3\sqrt[3]{3}\,(2r+1)\,\Gamma\left(\frac{5}{3}\right) {}_2F_2\left(-\frac{1}{6}, \frac{2}{3r}; -\frac{1}{3}, 1+\frac{2}{3r}; \frac{4}{3}(a z^r)^{3/2}\right) + 3\sqrt[3]{3}\,(2r+1)\,\Gamma\left(\frac{5}{3}\right) {}_2F_2\left(-\frac{1}{6}, \frac{2}{3r}; -\frac{1}{3}, 1+\frac{2}{3r}; \frac{1}{3}(-4)(a z^r)^{3/2}\right)\right)\right)$$

03.07.21.0022.01

$$\int \sinh\left(\frac{2}{3}(a z^r)^{3/2} + b\right) \mathrm{Ai}'(a z^r)\, dz = \frac{1}{6\, 3^{2/3}\,(2r+1)\,\Gamma\left(\frac{1}{3}\right)\Gamma\left(\frac{5}{3}\right)} \left(e^{-b} z\left(-a^2\,\Gamma\left(\frac{1}{3}\right)\left({}_2F_2\left(\frac{7}{6}, \frac{4}{3}+\frac{2}{3r}; \frac{7}{3}, \frac{7}{3}+\frac{2}{3r}; \frac{1}{3}(-4)(a z^r)^{3/2}\right) - e^{2b}\, {}_2F_2\left(\frac{7}{6}, \frac{4}{3}+\frac{2}{3r}; \frac{7}{3}, \frac{7}{3}+\frac{2}{3r}; \frac{4}{3}(a z^r)^{3/2}\right)\right) z^{2r} - 3\sqrt[3]{3}\, e^{2b}\,(2r+1)\,\Gamma\left(\frac{5}{3}\right) {}_2F_2\left(-\frac{1}{6}, \frac{2}{3r}; -\frac{1}{3}, 1+\frac{2}{3r}; \frac{4}{3}(a z^r)^{3/2}\right) + 3\sqrt[3]{3}\,(2r+1)\,\Gamma\left(\frac{5}{3}\right) {}_2F_2\left(-\frac{1}{6}, \frac{2}{3r}; -\frac{1}{3}, 1+\frac{2}{3r}; \frac{1}{3}(-4)(a z^r)^{3/2}\right)\right)\right)$$

Involving cosh

Linear argument

03.07.21.0023.01

$$\int \cosh\left(\frac{2}{3}(a z)^{3/2}\right) \mathrm{Ai}'(a z)\, dz = \frac{1}{30\, a\left(a^{3/2} z^{3/2}\right)^{2/3}\Gamma\left(\frac{5}{3}\right)} \left(e^{\frac{1}{3}(-2)(a z)^{3/2}} \left(-6\left(-1+e^{\frac{4}{3}(a z)^{3/2}}\right)\left(a^{3/2} z^{3/2}\right)^{2/3} \mathrm{Ai}(a z)\,\Gamma\left(\frac{5}{3}\right)(a z)^{3/2} - 4\sqrt[3]{3}\, e^{\frac{2}{3}(a z)^{3/2}}\left(a^{3/2} z^{3/2}\right)^{2/3} + 15\, a\left(1+e^{\frac{4}{3}(a z)^{3/2}}\right) z\left(a^{3/2} z^{3/2}\right)^{2/3} \mathrm{Ai}'(a z)\,\Gamma\left(\frac{5}{3}\right) + \frac{3 a^4 z^4\,\Gamma\left(\frac{5}{3}\right)}{\left(a^{3/2} z^{3/2}\right)^{2/3}} e^{\frac{4}{3}(a z)^{3/2}} I_{\frac{4}{3}}\left(\frac{2}{3} a^{3/2} z^{3/2}\right) + \frac{3 a^4 z^4\,\Gamma\left(\frac{5}{3}\right)}{\left(a^{3/2} z^{3/2}\right)^{2/3}} I_{\frac{4}{3}}\left(\frac{2}{3} a^{3/2} z^{3/2}\right) - 3 a^3 e^{\frac{4}{3}(a z)^{3/2}} z^3 I_{-\frac{4}{3}}\left(\frac{2}{3} a^{3/2} z^{3/2}\right)\Gamma\left(\frac{5}{3}\right) - 3 a^3 z^3 I_{-\frac{4}{3}}\left(\frac{2}{3} a^{3/2} z^{3/2}\right)\Gamma\left(\frac{5}{3}\right)\right)\right)$$

03.07.21.0024.01

$$\int \cosh\left(\frac{2}{3}(a z)^{3/2}+b\right)\operatorname{Ai}'(a z)\,dz = \frac{1}{30\,a\left(a^{3/2} z^{3/2}\right)^{2/3}\Gamma\left(\frac{5}{3}\right)} \left(e^{-\frac{1}{3}2(a z)^{3/2}-b}\left(-6\left(-1+e^{\frac{4}{3}(a z)^{3/2}+2b}\right)\left(a^{3/2} z^{3/2}\right)^{2/3}\operatorname{Ai}(a z)\,\Gamma\left(\frac{5}{3}\right)(a z)^{3/2} - 2\sqrt[3]{3}\,e^{\frac{2}{3}(a z)^{3/2}}\left(a^{3/2} z^{3/2}\right)^{2/3} - 2\sqrt[3]{3}\,e^{\frac{2}{3}\left((a z)^{3/2}+3b\right)} \left(a^{3/2} z^{3/2}\right)^{2/3} + 15\,a\left(1+e^{\frac{4}{3}(a z)^{3/2}+2b}\right) z\left(a^{3/2} z^{3/2}\right)^{2/3}\operatorname{Ai}'(a z)\,\Gamma\left(\frac{5}{3}\right) + \frac{3\,a^4\,\Gamma\left(\frac{5}{3}\right)}{\left(a^{3/2} z^{3/2}\right)^{2/3}} e^{\frac{4}{3}(a z)^{3/2}+2b} z^4\, I_{\frac{4}{3}}\left(\frac{2}{3} a^{3/2} z^{3/2}\right) + \frac{3\,a^4 z^4\,\Gamma\left(\frac{5}{3}\right)}{\left(a^{3/2} z^{3/2}\right)^{2/3}} I_{\frac{4}{3}}\left(\frac{2}{3} a^{3/2} z^{3/2}\right) - 3\,a^3\,e^{\frac{4}{3}(a z)^{3/2}+2b} z^3\, I_{-\frac{4}{3}}\left(\frac{2}{3} a^{3/2} z^{3/2}\right)\Gamma\left(\frac{5}{3}\right) - 3\,a^3 z^3\, I_{-\frac{4}{3}}\left(\frac{2}{3} a^{3/2} z^{3/2}\right)\Gamma\left(\frac{5}{3}\right)\right)\right)$$

Power arguments

03.07.21.0025.01

$$\int \cosh\left(\frac{2}{3}(a z^r)^{3/2}\right)\operatorname{Ai}'(a z^r)\,dz = \frac{1}{6\,3^{2/3}(2r+1)\,\Gamma\left(\frac{1}{3}\right)\Gamma\left(\frac{5}{3}\right)} \left(z\left(a^2\,\Gamma\left(\frac{1}{3}\right)\left({}_2F_2\left(\frac{7}{6}, \frac{4}{3}+\frac{2}{3r}; \frac{7}{3}, \frac{7}{3}+\frac{2}{3r}; \frac{4}{3}(a z^r)^{3/2}\right) + {}_2F_2\left(\frac{7}{6}, \frac{4}{3}+\frac{2}{3r}; \frac{7}{3}, \frac{7}{3}+\frac{2}{3r}; \frac{1}{3}(-4)(a z^r)^{3/2}\right)\right) z^{2r} - 3\sqrt[3]{3}\,(2r+1)\,\Gamma\left(\frac{5}{3}\right){}_2F_2\left(-\frac{1}{6}, \frac{2}{3r}; -\frac{1}{3}, 1+\frac{2}{3r}; \frac{4}{3}(a z^r)^{3/2}\right) - 3\sqrt[3]{3}\,(2r+1)\,\Gamma\left(\frac{5}{3}\right){}_2F_2\left(-\frac{1}{6}, \frac{2}{3r}; -\frac{1}{3}, 1+\frac{2}{3r}; \frac{1}{3}(-4)(a z^r)^{3/2}\right)\right)\right)$$

03.07.21.0026.01

$$\int \cosh\left(\frac{2}{3}(a z^r)^{3/2}+b\right)\operatorname{Ai}'(a z^r)\,dz = \frac{1}{6\,3^{2/3}(2r+1)\,\Gamma\left(\frac{1}{3}\right)\Gamma\left(\frac{5}{3}\right)} \left(e^{-b} z\left(a^2\,\Gamma\left(\frac{1}{3}\right)\left(e^{2b}\,{}_2F_2\left(\frac{7}{6}, \frac{4}{3}+\frac{2}{3r}; \frac{7}{3}, \frac{7}{3}+\frac{2}{3r}; \frac{4}{3}(a z^r)^{3/2}\right) + {}_2F_2\left(\frac{7}{6}, \frac{4}{3}+\frac{2}{3r}; \frac{7}{3}, \frac{7}{3}+\frac{2}{3r}; \frac{1}{3}(-4)(a z^r)^{3/2}\right)\right) z^{2r} - 3\sqrt[3]{3}\,e^{2b}(2r+1)\,\Gamma\left(\frac{5}{3}\right){}_2F_2\left(-\frac{1}{6}, \frac{2}{3r}; -\frac{1}{3}, 1+\frac{2}{3r}; \frac{4}{3}(a z^r)^{3/2}\right) - 3\sqrt[3]{3}\,(2r+1)\,\Gamma\left(\frac{5}{3}\right){}_2F_2\left(-\frac{1}{6}, \frac{2}{3r}; -\frac{1}{3}, 1+\frac{2}{3r}; \frac{1}{3}(-4)(a z^r)^{3/2}\right)\right)\right)$$

Involving hyperbolic functions and a power function

Involving sinh and power

Linear argument

03.07.21.0027.01

$$\int z^{\alpha-1}\sinh\left(\frac{2}{3}(a\,z)^{3/2}\right)\mathrm{Ai}'(a\,z)\,dz = \frac{1}{6\,3^{2/3}\,\alpha\,(\alpha+2)\,\Gamma\left(\frac{1}{3}\right)\Gamma\left(\frac{5}{3}\right)}\left(z^{\alpha}\left(a^2\,\alpha\,\Gamma\left(\frac{1}{3}\right)\left({}_2F_2\left(\frac{7}{6},\frac{2\alpha}{3}+\frac{4}{3};\frac{7}{3},\frac{2\alpha}{3}+\frac{7}{3};\frac{4}{3}(a\,z)^{3/2}\right)-{}_2F_2\left(\frac{7}{6},\frac{2\alpha}{3}+\frac{4}{3};\frac{7}{3},\frac{2\alpha}{3}+\frac{7}{3};\frac{1}{3}(-4)(a\,z)^{3/2}\right)\right)z^2-3\sqrt[3]{3}\,(\alpha+2)\,\Gamma\left(\frac{5}{3}\right){}_2F_2\left(-\frac{1}{6},\frac{2\alpha}{3};-\frac{1}{3},\frac{2\alpha}{3}+1;\frac{4}{3}(a\,z)^{3/2}\right)+3\sqrt[3]{3}\,(\alpha+2)\,\Gamma\left(\frac{5}{3}\right){}_2F_2\left(-\frac{1}{6},\frac{2\alpha}{3};-\frac{1}{3},\frac{2\alpha}{3}+1;\frac{1}{3}(-4)(a\,z)^{3/2}\right)\right)\right)$$

03.07.21.0028.01

$$\int z^{\alpha-1}\sinh\left(\frac{2}{3}(a\,z)^{3/2}+b\right)\mathrm{Ai}'(a\,z)\,dz = \frac{1}{6\,3^{2/3}\,\alpha\,(\alpha+2)\,\Gamma\left(\frac{1}{3}\right)\Gamma\left(\frac{5}{3}\right)}\left(e^{-b}\,z^{\alpha}\left(-a^2\,\alpha\,\Gamma\left(\frac{1}{3}\right)\left({}_2F_2\left(\frac{7}{6},\frac{2\alpha}{3}+\frac{4}{3};\frac{7}{3},\frac{2\alpha}{3}+\frac{7}{3};\frac{1}{3}(-4)(a\,z)^{3/2}\right)-e^{2b}\,{}_2F_2\left(\frac{7}{6},\frac{2\alpha}{3}+\frac{4}{3};\frac{7}{3},\frac{2\alpha}{3}+\frac{7}{3};\frac{4}{3}(a\,z)^{3/2}\right)\right)z^2-3\sqrt[3]{3}\,e^{2b}\,(\alpha+2)\,\Gamma\left(\frac{5}{3}\right){}_2F_2\left(-\frac{1}{6},\frac{2\alpha}{3};-\frac{1}{3},\frac{2\alpha}{3}+1;\frac{4}{3}(a\,z)^{3/2}\right)+3\sqrt[3]{3}\,(\alpha+2)\,\Gamma\left(\frac{5}{3}\right){}_2F_2\left(-\frac{1}{6},\frac{2\alpha}{3};-\frac{1}{3},\frac{2\alpha}{3}+1;\frac{1}{3}(-4)(a\,z)^{3/2}\right)\right)\right)$$

Power arguments

03.07.21.0029.01

$$\int z^{\alpha-1}\sinh\left(\frac{2}{3}(a\,z^r)^{3/2}\right)\mathrm{Ai}'(a\,z^r)\,dz = \frac{1}{6\,3^{2/3}\,\alpha\,(2\,r+\alpha)\,\Gamma\left(\frac{1}{3}\right)\Gamma\left(\frac{5}{3}\right)}\left(z^{\alpha}\left(-a^2\,\alpha\,\Gamma\left(\frac{1}{3}\right)\left({}_2F_2\left(\frac{7}{6},\frac{2\alpha}{3r}+\frac{4}{3};\frac{7}{3},\frac{2\alpha}{3r}+\frac{7}{3};\frac{1}{3}(-4)(a\,z^r)^{3/2}\right)-{}_2F_2\left(\frac{7}{6},\frac{2\alpha}{3r}+\frac{4}{3};\frac{7}{3},\frac{2\alpha}{3r}+\frac{7}{3};\frac{4}{3}(a\,z^r)^{3/2}\right)\right)z^{2r}-3\sqrt[3]{3}\,(2\,r+\alpha)\,\Gamma\left(\frac{5}{3}\right){}_2F_2\left(-\frac{1}{6},\frac{2\alpha}{3r};-\frac{1}{3},\frac{2\alpha}{3r}+1;\frac{4}{3}(a\,z^r)^{3/2}\right)+3\sqrt[3]{3}\,(2\,r+\alpha)\,\Gamma\left(\frac{5}{3}\right){}_2F_2\left(-\frac{1}{6},\frac{2\alpha}{3r};-\frac{1}{3},\frac{2\alpha}{3r}+1;\frac{1}{3}(-4)(a\,z^r)^{3/2}\right)\right)\right)$$

03.07.21.0030.01

$$\int z^{\alpha-1}\sinh\left(\frac{2}{3}(a\,z^r)^{3/2}+b\right)\mathrm{Ai}'(a\,z^r)\,dz = \frac{1}{6\,3^{2/3}\,\alpha\,(2\,r+\alpha)\,\Gamma\left(\frac{1}{3}\right)\Gamma\left(\frac{5}{3}\right)}\left(e^{-b}\,z^{\alpha}\left(-a^2\,\alpha\,\Gamma\left(\frac{1}{3}\right)\left({}_2F_2\left(\frac{7}{6},\frac{2\alpha}{3r}+\frac{4}{3};\frac{7}{3},\frac{2\alpha}{3r}+\frac{7}{3};\frac{1}{3}(-4)(a\,z^r)^{3/2}\right)-e^{2b}\,{}_2F_2\left(\frac{7}{6},\frac{2\alpha}{3r}+\frac{4}{3};\frac{7}{3},\frac{2\alpha}{3r}+\frac{7}{3};\frac{4}{3}(a\,z^r)^{3/2}\right)\right)z^{2r}-3\sqrt[3]{3}\,e^{2b}\,(2\,r+\alpha)\,\Gamma\left(\frac{5}{3}\right){}_2F_2\left(-\frac{1}{6},\frac{2\alpha}{3r};-\frac{1}{3},\frac{2\alpha}{3r}+1;\frac{4}{3}(a\,z^r)^{3/2}\right)+3\sqrt[3]{3}\,(2\,r+\alpha)\,\Gamma\left(\frac{5}{3}\right){}_2F_2\left(-\frac{1}{6},\frac{2\alpha}{3r};-\frac{1}{3},\frac{2\alpha}{3r}+1;\frac{1}{3}(-4)(a\,z^r)^{3/2}\right)\right)\right)$$

Involving cosh and power

Linear argument

03.07.21.0031.01

$$\int z^{\alpha-1}\cosh\left(\frac{2}{3}(a z)^{3/2}\right)\mathrm{Ai}'(a z)\,dz = \frac{1}{6\,3^{2/3}\,\alpha\,(\alpha+2)\,\Gamma\left(\frac{1}{3}\right)\Gamma\left(\frac{5}{3}\right)}\left(z^{\alpha}\left(a^2\,\alpha\,\Gamma\left(\frac{1}{3}\right)\left({}_2F_2\left(\frac{7}{6},\frac{2\alpha}{3}+\frac{4}{3};\frac{7}{3},\frac{2\alpha}{3}+\frac{7}{3};\frac{4}{3}(a z)^{3/2}\right)+{}_2F_2\left(\frac{7}{6},\frac{2\alpha}{3}+\frac{4}{3};\frac{7}{3},\frac{2\alpha}{3}+\frac{7}{3};\frac{1}{3}(-4)(a z)^{3/2}\right)\right)z^2 - 3\sqrt[3]{3}\,(\alpha+2)\,\Gamma\left(\frac{5}{3}\right){}_2F_2\left(-\frac{1}{6},\frac{2\alpha}{3};-\frac{1}{3},\frac{2\alpha}{3}+1;\frac{4}{3}(a z)^{3/2}\right) - 3\sqrt[3]{3}\,(\alpha+2)\,\Gamma\left(\frac{5}{3}\right){}_2F_2\left(-\frac{1}{6},\frac{2\alpha}{3};-\frac{1}{3},\frac{2\alpha}{3}+1;\frac{1}{3}(-4)(a z)^{3/2}\right)\right)\right)$$

03.07.21.0032.01

$$\int z^{\alpha-1}\cosh\left(\frac{2}{3}(a z)^{3/2}+b\right)\mathrm{Ai}'(a z)\,dz = \frac{1}{6\,3^{2/3}\,\alpha\,(\alpha+2)\,\Gamma\left(\frac{1}{3}\right)\Gamma\left(\frac{5}{3}\right)}\left(e^{-b}z^{\alpha}\left(a^2\,\alpha\,\Gamma\left(\frac{1}{3}\right)\left(e^{2b}\,{}_2F_2\left(\frac{7}{6},\frac{2\alpha}{3}+\frac{4}{3};\frac{7}{3},\frac{2\alpha}{3}+\frac{7}{3};\frac{4}{3}(a z)^{3/2}\right)+{}_2F_2\left(\frac{7}{6},\frac{2\alpha}{3}+\frac{4}{3};\frac{7}{3},\frac{2\alpha}{3}+\frac{7}{3};\frac{1}{3}(-4)(a z)^{3/2}\right)\right)z^2 - 3\sqrt[3]{3}\,e^{2b}(\alpha+2)\,\Gamma\left(\frac{5}{3}\right){}_2F_2\left(-\frac{1}{6},\frac{2\alpha}{3};-\frac{1}{3},\frac{2\alpha}{3}+1;\frac{4}{3}(a z)^{3/2}\right) - 3\sqrt[3]{3}\,(\alpha+2)\,\Gamma\left(\frac{5}{3}\right){}_2F_2\left(-\frac{1}{6},\frac{2\alpha}{3};-\frac{1}{3},\frac{2\alpha}{3}+1;\frac{1}{3}(-4)(a z)^{3/2}\right)\right)\right)$$

Power arguments

03.07.21.0033.01

$$\int z^{\alpha-1}\cosh\left(\frac{2}{3}(a z^r)^{3/2}\right)\mathrm{Ai}'(a z^r)\,dz = \frac{1}{6\,3^{2/3}\,\alpha\,(2r+\alpha)\,\Gamma\left(\frac{1}{3}\right)\Gamma\left(\frac{5}{3}\right)}\left(z^{\alpha}\left(a^2\,\alpha\,\Gamma\left(\frac{1}{3}\right)\left({}_2F_2\left(\frac{7}{6},\frac{2\alpha}{3r}+\frac{4}{3};\frac{7}{3},\frac{2\alpha}{3r}+\frac{7}{3};\frac{4}{3}(a z^r)^{3/2}\right)+{}_2F_2\left(\frac{7}{6},\frac{2\alpha}{3r}+\frac{4}{3};\frac{7}{3},\frac{2\alpha}{3r}+\frac{7}{3};\frac{1}{3}(-4)(a z^r)^{3/2}\right)\right)z^{2r} - 3\sqrt[3]{3}\,(2r+\alpha)\,\Gamma\left(\frac{5}{3}\right){}_2F_2\left(-\frac{1}{6},\frac{2\alpha}{3r};-\frac{1}{3},\frac{2\alpha}{3r}+1;\frac{4}{3}(a z^r)^{3/2}\right) - 3\sqrt[3]{3}\,(2r+\alpha)\,\Gamma\left(\frac{5}{3}\right){}_2F_2\left(-\frac{1}{6},\frac{2\alpha}{3r};-\frac{1}{3},\frac{2\alpha}{3r}+1;\frac{1}{3}(-4)(a z^r)^{3/2}\right)\right)\right)$$

03.07.21.0034.01

$$\int z^{\alpha-1}\cosh\left(\frac{2}{3}(a z^r)^{3/2}+b\right)\mathrm{Ai}'(a z^r)\,dz = \frac{1}{6\,3^{2/3}\,\alpha\,(2r+\alpha)\,\Gamma\left(\frac{1}{3}\right)\Gamma\left(\frac{5}{3}\right)}\left(e^{-b}z^{\alpha}\left(a^2\,\alpha\,\Gamma\left(\frac{1}{3}\right)\left(e^{2b}\,{}_2F_2\left(\frac{7}{6},\frac{2\alpha}{3r}+\frac{4}{3};\frac{7}{3},\frac{2\alpha}{3r}+\frac{7}{3};\frac{4}{3}(a z^r)^{3/2}\right)+{}_2F_2\left(\frac{7}{6},\frac{2\alpha}{3r}+\frac{4}{3};\frac{7}{3},\frac{2\alpha}{3r}+\frac{7}{3};\frac{1}{3}(-4)(a z^r)^{3/2}\right)\right)z^{2r} - 3\sqrt[3]{3}\,e^{2b}(2r+\alpha)\,\Gamma\left(\frac{5}{3}\right){}_2F_2\left(-\frac{1}{6},\frac{2\alpha}{3r};-\frac{1}{3},\frac{2\alpha}{3r}+1;\frac{4}{3}(a z^r)^{3/2}\right) - 3\sqrt[3]{3}\,(2r+\alpha)\,\Gamma\left(\frac{5}{3}\right){}_2F_2\left(-\frac{1}{6},\frac{2\alpha}{3r};-\frac{1}{3},\frac{2\alpha}{3r}+1;\frac{1}{3}(-4)(a z^r)^{3/2}\right)\right)\right)$$

Involving functions of the direct function

Involving elementary functions of the direct function

Involving powers of the direct function

Linear arguments

03.07.21.0035.01

$$\int \mathrm{Ai}'(a\,z)^2\,dz = \frac{1}{3\,a}\left(-a^2\,z^2\,\mathrm{Ai}(a\,z)^2 + 2\,\mathrm{Ai}'(a\,z)\,\mathrm{Ai}(a\,z) + a\,z\,\mathrm{Ai}'(a\,z)^2\right)$$

Power arguments

03.07.21.0036.01

$$\int \mathrm{Ai}'(a\,z^r)^2\,dz = \frac{z}{4\,\sqrt[3]{2}\,3^{2/3}\,\pi^{3/2}\,r}\,G_{2,4}^{3,1}\!\left(\left(\frac{2}{3}\right)^{2/3} a\,z^r,\ \frac{1}{3}\,\middle|\,\begin{array}{c} 1-\frac{1}{3\,r},\ \frac{7}{6} \\ 0,\ \frac{2}{3},\ \frac{4}{3},\ -\frac{1}{3\,r}\end{array}\right)$$

Involving products of the direct function

Linear arguments

03.07.21.0037.01

$$\int \mathrm{Ai}'(-a\,z)\,\mathrm{Ai}'(a\,z)\,dz = \frac{z}{4\,\sqrt[3]{2}\,3^{2/3}\,\pi^{3/2}}\,G_{0,4}^{3,0}\!\left(-\frac{a\,z}{\sqrt[3]{2}\,3^{2/3}},\ \frac{1}{6}\,\middle|\,0,\ \frac{1}{3},\ \frac{2}{3},\ -\frac{1}{6}\right)$$

Power arguments

03.07.21.0038.01

$$\int \mathrm{Ai}'(-a\,z^r)\,\mathrm{Ai}'(a\,z^r)\,dz = -\frac{z}{4\,\sqrt[3]{2}\,3^{2/3}\,\pi^{3/2}\,r}\,G_{2,6}^{4,1}\!\left(-\frac{a\,z^r}{\sqrt[3]{2}\,3^{2/3}},\ \frac{1}{6}\,\middle|\,\begin{array}{c} 1-\frac{1}{6\,r},\ -\frac{1}{6} \\ 0,\ \frac{1}{3},\ \frac{2}{3},\ \frac{5}{6},\ -\frac{1}{6},\ -\frac{1}{6\,r}\end{array}\right)$$

Involving functions of the direct function and elementary functions

Involving elementary functions of the direct function and elementary functions

Involving powers of the direct function and a power function

Linear arguments

03.07.21.0039.01

$$\int z^{\alpha-1}\,\mathrm{Ai}'(a\,z)^2\,dz = \frac{z^\alpha}{4\,\sqrt[3]{2}\,3^{2/3}\,\pi^{3/2}}\,G_{2,4}^{3,1}\!\left(\left(\frac{2}{3}\right)^{2/3} a\,z,\ \frac{1}{3}\,\middle|\,\begin{array}{c} 1-\frac{\alpha}{3},\ \frac{7}{6} \\ 0,\ \frac{2}{3},\ \frac{4}{3},\ -\frac{\alpha}{3}\end{array}\right)$$

03.07.21.0040.01

$$\int z\,\mathrm{Ai}'(a z)^2\,dz = \frac{1}{10 a^2}\left(-\left(2 a^3 z^3+3\right)\mathrm{Ai}(a z)^2+6 a z\,\mathrm{Ai}'(a z)\,\mathrm{Ai}(a z)+2 a^2 z^2\,\mathrm{Ai}'(a z)^2\right)$$

03.07.21.0041.01

$$\int z^2\,\mathrm{Ai}'(a z)^2\,dz = \frac{1}{7 a^3}\left(-a^4\,\mathrm{Ai}(a z)^2 z^4+4 a^2\,\mathrm{Ai}(a z)\,\mathrm{Ai}'(a z)\,z^2+\left(a^3 z^3-4\right)\mathrm{Ai}'(a z)^2\right)$$

03.07.21.0042.01

$$\int z^3\,\mathrm{Ai}'(a z)^2\,dz = \frac{1}{18 a^4}\left(-a^2 z^2\left(2 a^3 z^3+5\right)\mathrm{Ai}(a z)^2+10\left(a^3 z^3+1\right)\mathrm{Ai}'(a z)\,\mathrm{Ai}(a z)+2 a z\left(a^3 z^3-5\right)\mathrm{Ai}'(a z)^2\right)$$

Power arguments

03.07.21.0043.01

$$\int z^{\alpha-1}\,\mathrm{Ai}'(a z^r)^2\,dz = \frac{z^\alpha}{4\sqrt[3]{2}\,3^{2/3}\,\pi^{3/2}\,r}\,G_{2,4}^{3,1}\!\left(\left(\frac{2}{3}\right)^{2/3} a z^r,\ \frac{1}{3}\ \middle|\ \begin{matrix}1-\frac{\alpha}{3 r},\ \frac{7}{6}\\ 0,\ \frac{2}{3},\ \frac{4}{3},\ -\frac{\alpha}{3 r}\end{matrix}\right)$$

Involving products of the direct function and a power function

Linear arguments

03.07.21.0044.01

$$\int z^{\alpha-1}\,\mathrm{Ai}'(-a z)\,\mathrm{Ai}'(a z)\,dz = \frac{z^\alpha}{4\sqrt[3]{2}\,3^{2/3}\,\pi^{3/2}}\,G_{1,5}^{3,1}\!\left(-\frac{a z}{\sqrt[3]{2}\,3^{2/3}},\ \frac{1}{6}\ \middle|\ \begin{matrix}1-\frac{\alpha}{6}\\ 0,\ \frac{1}{3},\ \frac{2}{3},\ \frac{5}{6},\ -\frac{\alpha}{6}\end{matrix}\right)$$

Power arguments

03.07.21.0045.01

$$\int z^{\alpha-1}\,\mathrm{Ai}'(-a z^r)\,\mathrm{Ai}'(a z^r)\,dz = -\frac{z^\alpha}{4\sqrt[3]{2}\,3^{2/3}\,\pi^{3/2}\,r}\,G_{2,6}^{4,1}\!\left(-\frac{a z^r}{\sqrt[3]{2}\,3^{2/3}},\ \frac{1}{6}\ \middle|\ \begin{matrix}1-\frac{\alpha}{6 r},\ -\frac{1}{6}\\ 0,\ \frac{1}{3},\ \frac{2}{3},\ \frac{5}{6},\ -\frac{1}{6},\ -\frac{\alpha}{6 r}\end{matrix}\right)$$

Involving direct function and Bessel-type functions

Involving Bessel functions

Involving Bessel I

Linear argument

03.07.21.0046.01

$$\int I_\nu\!\left(\frac{2}{3}(a z)^{3/2}\right)\mathrm{Ai}'(a z)\,dz = -\frac{2^{\nu-\frac{7}{3}}\,3^{-\nu-\frac{1}{6}}\left((a z)^{3/2}\right)^\nu}{a\,\pi^{3/2}}\,G_{3,5}^{2,3}\!\left(\left(\frac{2}{3}\right)^{2/3} a z,\ \frac{1}{3}\ \middle|\ \begin{matrix}\frac{1}{6}(4-3\nu),\ \frac{1}{6}(7-3\nu),\ 1-\frac{\nu}{2}\\ \frac{1}{3},\ 1,\ \frac{1}{3}-\nu,\ 1-\nu,\ -\frac{\nu}{2}\end{matrix}\right)$$

Power arguments

03.07.21.0047.01

$$\int I_\nu\left(\frac{2}{3}(a z^r)^{3/2}\right) \mathrm{Ai}'(a z^r)\, dz = -\frac{2^{\nu-\frac{5}{3}} 3^{-\nu-\frac{5}{6}} z \left((a z^r)^{3/2}\right)^\nu}{\pi^{3/2} r} G_{3,5}^{2,3}\left(\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3} \middle| \begin{matrix} \frac{1}{6}(2-3\nu), \frac{1}{6}(5-3\nu), -\frac{\nu}{2}-\frac{1}{3r}+1 \\ 0, \frac{2}{3}, \frac{2}{3}-\nu, -\nu, -\frac{3r\nu+2}{6r} \end{matrix}\right)$$

Involving Bessel I and power

Linear argument

03.07.21.0048.01

$$\int z^{\alpha-1} I_\nu\left(\frac{2}{3}(a z)^{3/2}\right) \mathrm{Ai}'(a z)\, dz = -\frac{2^{\nu-\frac{5}{3}} 3^{-\nu-\frac{5}{6}} z^\alpha \left((a z)^{3/2}\right)^\nu}{\pi^{3/2}} G_{3,5}^{2,3}\left(\left(\frac{2}{3}\right)^{2/3} a z, \frac{1}{3} \middle| \begin{matrix} \frac{1}{6}(2-3\nu), \frac{1}{6}(5-3\nu), \frac{1}{6}(-2\alpha-3\nu+6) \\ 0, \frac{2}{3}, \frac{1}{6}(-2\alpha-3\nu), \frac{2}{3}-\nu, -\nu \end{matrix}\right)$$

03.07.21.0049.01

$$\int z^{3/2} I_\nu\left(\frac{2}{3}(a z)^{3/2}\right) \mathrm{Ai}'(a z)\, dz = -\frac{2^{\nu-\frac{5}{3}} 3^{-\nu-\frac{5}{6}} z^{5/2} \left((a z)^{3/2}\right)^\nu}{\pi^{3/2}} G_{3,5}^{2,3}\left(\left(\frac{2}{3}\right)^{2/3} a z, \frac{1}{3} \middle| \begin{matrix} \frac{1}{6}(1-3\nu), \frac{1}{6}(2-3\nu), \frac{1}{6}(5-3\nu) \\ 0, \frac{2}{3}, \frac{1}{6}(-3\nu-5), \frac{2}{3}-\nu, -\nu \end{matrix}\right)$$

03.07.21.0050.01

$$\int z^{-3/2} I_\nu\left(\frac{2}{3}(a z)^{3/2}\right) \mathrm{Ai}'(a z)\, dz = -\frac{2^{\nu-\frac{5}{3}} 3^{-\nu-\frac{5}{6}} \left((a z)^{3/2}\right)^\nu}{\pi^{3/2} \sqrt{z}} G_{3,5}^{2,3}\left(\left(\frac{2}{3}\right)^{2/3} a z, \frac{1}{3} \middle| \begin{matrix} \frac{1}{6}(2-3\nu), \frac{1}{6}(5-3\nu), \frac{1}{6}(7-3\nu) \\ 0, \frac{2}{3}, \frac{1}{6}(1-3\nu), \frac{2}{3}-\nu, -\nu \end{matrix}\right)$$

Power arguments

03.07.21.0051.01

$$\int z^{\alpha-1} I_\nu\left(\frac{2}{3}(a z^r)^{3/2}\right) \mathrm{Ai}'(a z^r)\, dz = -\frac{2^{\nu-\frac{5}{3}} 3^{-\nu-\frac{5}{6}} z^\alpha \left((a z^r)^{3/2}\right)^\nu}{\pi^{3/2} r} G_{3,5}^{2,3}\left(\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3} \middle| \begin{matrix} \frac{1}{6}(2-3\nu), \frac{1}{6}(5-3\nu), -\frac{\alpha}{3r}-\frac{\nu}{2}+1 \\ 0, \frac{2}{3}, \frac{2}{3}-\nu, -\nu, -\frac{2\alpha+3r\nu}{6r} \end{matrix}\right)$$

Involving Bessel K

Linear argument

03.07.21.0052.01

$$\int K_\nu\left(\frac{2}{3}(a z)^{3/2}\right) \mathrm{Ai}'(a z)\, dz =$$

$$\frac{1}{a\sqrt{\pi}}\left(2^{-\nu-\frac{10}{3}} 3^{-\nu-\frac{1}{6}} \left((a z)^{3/2}\right)^{-\nu} \csc(\pi\nu) \left(4^\nu \left((a z)^{3/2}\right)^{2\nu} G_{3,5}^{2,3}\left(\left(\frac{2}{3}\right)^{2/3} a z, \frac{1}{3} \middle| \begin{matrix} \frac{1}{6}(4-3\nu), \frac{1}{6}(7-3\nu), 1-\frac{\nu}{2} \\ \frac{1}{3}, 1, \frac{1}{3}-\nu, 1-\nu, -\frac{\nu}{2} \end{matrix}\right) - \right.\right.$$

$$\left.\left. 9^\nu G_{3,5}^{2,3}\left(\left(\frac{2}{3}\right)^{2/3} a z, \frac{1}{3} \middle| \begin{matrix} \frac{\nu+2}{2}, \frac{1}{6}(3\nu+4), \frac{1}{6}(3\nu+7) \\ \frac{1}{3}, 1, \frac{\nu}{2}, \nu+\frac{1}{3}, \nu+1 \end{matrix}\right)\right)\right)$$

03.07.21.0053.01

$$\int K_0\left(\frac{2}{3}(a z)^{3/2}\right)\mathrm{Ai}'(a z)\,dz = \frac{1}{8\sqrt[3]{2}\,\sqrt[6]{3}\,a\,\pi^{3/2}}\left(\left(2\log\left((a z)^{3/2}\right)-3\log(a z)\right)G_{2,4}^{2,2}\left(\left(\frac{2}{3}\right)^{2/3} a z,\frac{1}{3}\,\middle|\,\begin{matrix}\frac{2}{3},\frac{7}{6}\\ \frac{1}{3},1,0,\frac{1}{3}\end{matrix}\right)-2\pi\,G_{3,5}^{4,1}\left(\left(\frac{2}{3}\right)^{2/3} a z,\frac{1}{3}\,\middle|\,\begin{matrix}1,\frac{2}{3},\frac{7}{6}\\ \frac{1}{3},\frac{1}{3},1,1,0\end{matrix}\right)\right)$$

03.07.21.0054.01

$$\int K_1\left(\frac{2}{3}(a z)^{3/2}\right)\mathrm{Ai}'(a z)\,dz = \frac{1}{8\sqrt[3]{2}\,\sqrt[6]{3}\,a\,\pi^{3/2}}\left(\left(3\log(a z)-2\log\left((a z)^{3/2}\right)\right)G_{3,5}^{2,3}\left(\left(\frac{2}{3}\right)^{2/3} a z,\frac{1}{3}\,\middle|\,\begin{matrix}\frac{2}{3},1,\frac{7}{6}\\ \frac{5}{6},\frac{3}{2},-\frac{1}{6},0,\frac{1}{2}\end{matrix}\right)-2\pi\,G_{3,5}^{4,1}\left(\left(\frac{2}{3}\right)^{2/3} a z,\frac{1}{3}\,\middle|\,\begin{matrix}1,\frac{2}{3},\frac{7}{6}\\ -\frac{1}{6},\frac{1}{2},\frac{5}{6},\frac{3}{2},0\end{matrix}\right)\right)$$

03.07.21.0055.01

$$\int K_2\left(\frac{2}{3}(a z)^{3/2}\right)\mathrm{Ai}'(a z)\,dz = \frac{1}{8\sqrt[3]{2}\,\sqrt[6]{3}\,a\,\pi^{3/2}}\left(2\pi\,G_{3,5}^{5,0}\left(\left(\frac{2}{3}\right)^{2/3} a z,\frac{1}{3}\,\middle|\,\begin{matrix}\frac{2}{3},1,\frac{7}{6}\\ -\frac{2}{3},0,0,\frac{4}{3},2\end{matrix}\right)+\left(2\log\left((a z)^{3/2}\right)-3\log(a z)\right)G_{3,5}^{2,3}\left(\left(\frac{2}{3}\right)^{2/3} a z,\frac{1}{3}\,\middle|\,\begin{matrix}\frac{2}{3},1,\frac{7}{6}\\ \frac{4}{3},2,-\frac{2}{3},0,0\end{matrix}\right)\right)$$

Power arguments

03.07.21.0056.01

$$\int K_\nu\left(\frac{2}{3}(a z^r)^{3/2}\right)\mathrm{Ai}'(a z^r)\,dz = \frac{1}{\sqrt{\pi}\,r}\left(2^{-\nu-\frac{8}{3}}\,3^{-\nu-\frac{5}{6}}\,z\left((a z^r)^{3/2}\right)^{-\nu}\csc(\pi\nu)\left(4^\nu\left((a z^r)^{3/2}\right)^{2\nu}G_{3,5}^{2,3}\left(\left(\frac{2}{3}\right)^{2/3} a z^r,\frac{1}{3}\,\middle|\,\begin{matrix}\frac{1}{6}(2-3\nu),\frac{1}{6}(5-3\nu),-\frac{\nu}{2}-\frac{1}{3r}+1\\ 0,\frac{2}{3},\frac{2}{3}-\nu,-\nu,-\frac{3r\nu+2}{6r}\end{matrix}\right)-9^\nu\,G_{3,5}^{2,3}\left(\left(\frac{2}{3}\right)^{2/3} a z^r,\frac{1}{3}\,\middle|\,\begin{matrix}\frac{\nu}{2}-\frac{1}{3r}+1,\frac{1}{6}(3\nu+2),\frac{1}{6}(3\nu+5)\\ 0,\frac{2}{3},\nu,\nu+\frac{2}{3},\frac{3r\nu-2}{6r}\end{matrix}\right)\right)\right)$$

03.07.21.0057.01

$$\int K_0\left(\frac{2}{3}(a z^r)^{3/2}\right)\mathrm{Ai}'(a z^r)\,dz = \frac{1}{4\,2^{2/3}\,3^{5/6}\,\pi^{3/2}\,r}\left(z\left(\left(2\log\left((a z^r)^{3/2}\right)-3\log(a z^r)\right)G_{3,5}^{2,3}\left(\left(\frac{2}{3}\right)^{2/3} a z^r,\frac{1}{3}\,\middle|\,\begin{matrix}\frac{1}{3},\frac{5}{6},1-\frac{1}{3r}\\ 0,\frac{2}{3},0,\frac{2}{3},-\frac{1}{3r}\end{matrix}\right)-2\pi\,G_{3,5}^{4,1}\left(\left(\frac{2}{3}\right)^{2/3} a z^r,\frac{1}{3}\,\middle|\,\begin{matrix}1-\frac{1}{3r},\frac{1}{3},\frac{5}{6}\\ 0,0,\frac{2}{3},\frac{2}{3},-\frac{1}{3r}\end{matrix}\right)\right)\right)$$

03.07.21.0058.01

$$\int K_1\left(\frac{2}{3}(a z^r)^{3/2}\right)\mathrm{Ai}'(a z^r)\,dz = \frac{1}{4\,2^{2/3}\,3^{5/6}\,\pi^{3/2}\,r}\left(z\left(\left(3\log(a z^r)-2\log\left((a z^r)^{3/2}\right)\right)G_{3,5}^{2,3}\left(\left(\frac{2}{3}\right)^{2/3} a z^r,\frac{1}{3}\,\middle|\,\begin{matrix}\frac{1}{3},\frac{5}{6},1-\frac{1}{3r}\\ \frac{1}{2},\frac{7}{6},-\frac{1}{2},\frac{1}{6},-\frac{1}{3r}\end{matrix}\right)-2\pi\,G_{3,5}^{4,1}\left(\left(\frac{2}{3}\right)^{2/3} a z^r,\frac{1}{3}\,\middle|\,\begin{matrix}1-\frac{1}{3r},\frac{1}{3},\frac{5}{6}\\ -\frac{1}{2},\frac{1}{6},\frac{1}{2},\frac{7}{6},-\frac{1}{3r}\end{matrix}\right)\right)\right)$$

03.07.21.0059.01

$$\int K_2\left(\frac{2}{3}(a z^r)^{3/2}\right) \operatorname{Ai}'(a z^r)\, dz = \frac{1}{4\, 2^{2/3}\, 3^{5/6}\, \pi^{3/2}\, r} \left(z\left(\left(2\log\left((a z^r)^{3/2}\right) - 3\log(a z^r)\right) G_{3,5}^{2,3}\left(\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3} \middle| \begin{array}{c} \frac{1}{3}, \frac{5}{6}, 1-\frac{1}{3r} \\ 1, \frac{5}{3}, -1, -\frac{1}{3}, -\frac{1}{3r} \end{array}\right) - 2\pi\, G_{3,5}^{4,1}\left(\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3} \middle| \begin{array}{c} 1-\frac{1}{3r}, \frac{1}{3}, \frac{5}{6} \\ -1, -\frac{1}{3}, 1, \frac{5}{3}, -\frac{1}{3r} \end{array}\right)\right)\right)$$

Involving Bessel *K* and power

Linear argument

03.07.21.0060.01

$$\int z^{\alpha-1} K_\nu\left(\frac{2}{3}(a z)^{3/2}\right) \operatorname{Ai}'(a z)\, dz = \frac{1}{\sqrt{\pi}} \left(2^{-\nu-\frac{8}{3}}\, 3^{-\nu-\frac{5}{6}}\, z^\alpha \left((a z)^{3/2}\right)^{-\nu} \csc(\pi\nu) \left(4^\nu \left((a z)^{3/2}\right)^{2\nu} G_{3,5}^{2,3}\left(\left(\frac{2}{3}\right)^{2/3} a z, \frac{1}{3} \middle| \begin{array}{c} \frac{1}{6}(2-3\nu), \frac{1}{6}(5-3\nu), \frac{1}{6}(-2\alpha-3\nu+6) \\ 0, \frac{2}{3}, \frac{1}{6}(-2\alpha-3\nu), \frac{2}{3}-\nu, -\nu \end{array}\right) - 9^\nu\, G_{3,5}^{2,3}\left(\left(\frac{2}{3}\right)^{2/3} a z, \frac{1}{3} \middle| \begin{array}{c} \frac{1}{6}(3\nu+2), \frac{1}{6}(3\nu+5), \frac{1}{6}(-2\alpha+3\nu+6) \\ 0, \frac{2}{3}, \nu, \nu+\frac{2}{3}, \frac{1}{6}(3\nu-2\alpha) \end{array}\right)\right)\right)$$

03.07.21.0061.01

$$\int z^{\alpha-1} K_0\left(\frac{2}{3}(a z)^{3/2}\right) \operatorname{Ai}'(a z)\, dz = -\frac{z^\alpha}{4\, 2^{2/3}\, 3^{5/6}\, \pi^{3/2}} \left(2\pi\, G_{3,5}^{4,1}\left(\left(\frac{2}{3}\right)^{2/3} a z, \frac{1}{3} \middle| \begin{array}{c} 1-\frac{\alpha}{3}, \frac{1}{3}, \frac{5}{6} \\ 0, 0, \frac{2}{3}, \frac{2}{3}, -\frac{\alpha}{3} \end{array}\right) + \left(3\log(a z) - 2\log\left((a z)^{3/2}\right)\right) G_{3,5}^{2,3}\left(\left(\frac{2}{3}\right)^{2/3} a z, \frac{1}{3} \middle| \begin{array}{c} \frac{1}{3}, \frac{5}{6}, 1-\frac{\alpha}{3} \\ 0, \frac{2}{3}, 0, \frac{2}{3}, -\frac{\alpha}{3} \end{array}\right)\right)$$

03.07.21.0062.01

$$\int z^{\alpha-1} K_1\left(\frac{2}{3}(a z)^{3/2}\right) \operatorname{Ai}'(a z)\, dz = -\frac{1}{4\, 2^{2/3}\, 3^{5/6}\, \pi^{3/2}} \left(z^\alpha \left(2\pi\, G_{3,5}^{4,1}\left(\left(\frac{2}{3}\right)^{2/3} a z, \frac{1}{3} \middle| \begin{array}{c} 1-\frac{\alpha}{3}, \frac{1}{3}, \frac{5}{6} \\ -\frac{1}{2}, \frac{1}{6}, \frac{1}{2}, \frac{7}{6}, -\frac{\alpha}{3} \end{array}\right) + \left(2\log\left((a z)^{3/2}\right) - 3\log(a z)\right) G_{3,5}^{2,3}\left(\left(\frac{2}{3}\right)^{2/3} a z, \frac{1}{3} \middle| \begin{array}{c} \frac{1}{3}, \frac{5}{6}, 1-\frac{\alpha}{3} \\ \frac{1}{2}, \frac{7}{6}, -\frac{1}{2}, \frac{1}{6}, -\frac{\alpha}{3} \end{array}\right)\right)\right)$$

03.07.21.0063.01

$$\int z^{\alpha-1} K_2\left(\frac{2}{3}(a z)^{3/2}\right) \operatorname{Ai}'(a z)\, dz = -\frac{1}{4\, 2^{2/3}\, 3^{5/6}\, \pi^{3/2}} \left(z^\alpha \left(2\pi\, G_{3,5}^{4,1}\left(\left(\frac{2}{3}\right)^{2/3} a z, \frac{1}{3} \middle| \begin{array}{c} 1-\frac{\alpha}{3}, \frac{1}{3}, \frac{5}{6} \\ -1, -\frac{1}{3}, 1, \frac{5}{3}, -\frac{\alpha}{3} \end{array}\right) + \left(3\log(a z) - 2\log\left((a z)^{3/2}\right)\right) G_{3,5}^{2,3}\left(\left(\frac{2}{3}\right)^{2/3} a z, \frac{1}{3} \middle| \begin{array}{c} \frac{1}{3}, \frac{5}{6}, 1-\frac{\alpha}{3} \\ 1, \frac{5}{3}, -1, -\frac{1}{3}, -\frac{\alpha}{3} \end{array}\right)\right)\right)$$

03.07.21.0064.01

$$\int z^{3/2} K_2\left(\frac{2}{3}(a z)^{3/2}\right) \operatorname{Ai}'(a z)\, dz = \frac{1}{4\, 2^{2/3}\, 3^{5/6}\, \pi^{3/2}} \left(z^{5/2} \left(\left(2\log\left((a z)^{3/2}\right) - 3\log(a z)\right) G_{3,5}^{2,3}\left(\left(\frac{2}{3}\right)^{2/3} a z, \frac{1}{3} \middle| \begin{array}{c} \frac{1}{6}, \frac{1}{3}, \frac{5}{6} \\ 1, \frac{5}{3}, -1, -\frac{5}{6}, -\frac{1}{3} \end{array}\right) - 2\pi\, G_{3,5}^{4,1}\left(\left(\frac{2}{3}\right)^{2/3} a z, \frac{1}{3} \middle| \begin{array}{c} \frac{1}{6}, \frac{1}{3}, \frac{5}{6} \\ -1, -\frac{1}{3}, 1, \frac{5}{3}, -\frac{5}{6} \end{array}\right)\right)\right)$$

03.07.21.0065.01

$$\int z^{-3/2} K_2\left(\frac{2}{3}(a z)^{3/2}\right) \text{Ai}'(a z)\, dz = \frac{1}{4\, 2^{2/3}\, 3^{5/6}\, \pi^{3/2} \sqrt{z}} \left(\left(2\log\left((a z)^{3/2}\right) - 3\log(a z)\right) G_{3,5}^{2,3}\left(\left(\frac{2}{3}\right)^{2/3} a z, \frac{1}{3} \,\middle|\, \begin{matrix} \frac{1}{3}, \frac{5}{6}, \frac{7}{6} \\ 1, \frac{5}{3}, -1, -\frac{1}{3}, \frac{1}{6} \end{matrix}\right) - 2\pi\, G_{3,5}^{4,1}\left(\left(\frac{2}{3}\right)^{2/3} a z, \frac{1}{3} \,\middle|\, \begin{matrix} \frac{7}{6}, \frac{1}{3}, \frac{5}{6} \\ -1, -\frac{1}{3}, 1, \frac{5}{3}, \frac{1}{6} \end{matrix}\right)\right)$$

Power arguments

03.07.21.0066.01

$$\int z^{\alpha-1} K_\nu\left(\frac{2}{3}(a z^r)^{3/2}\right) \text{Ai}'(a z^r)\, dz = \frac{1}{2}\pi \csc(\pi \nu) \left(\frac{2^{\nu-\frac{5}{3}}\, 3^{-\nu-\frac{5}{6}}\, z^\alpha \left((a z^r)^{3/2}\right)^\nu}{\pi^{3/2}\, r} G_{3,5}^{2,3}\left(\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3} \,\middle|\, \begin{matrix} \frac{1}{6}(2-3\nu), \frac{1}{6}(5-3\nu), -\frac{\alpha}{3r} - \frac{\nu}{2} + 1 \\ 0, \frac{2}{3}, \frac{2}{3} - \nu, -\nu, -\frac{2\alpha+3r\nu}{6r} \end{matrix}\right) - \frac{2^{-\nu-\frac{5}{3}}\, 3^{\nu-\frac{5}{6}}\, z^\alpha \left((a z^r)^{3/2}\right)^{-\nu}}{\pi^{3/2}\, r} G_{3,5}^{2,3}\left(\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3} \,\middle|\, \begin{matrix} -\frac{\alpha}{3r} + \frac{\nu}{2} + 1, \frac{1}{6}(3\nu+2), \frac{1}{6}(3\nu+5) \\ 0, \frac{2}{3}, \frac{\nu}{2} - \frac{\alpha}{3r}, \nu, \nu + \frac{2}{3} \end{matrix}\right)\right)$$

03.07.21.0067.01

$$\int z^{\alpha-1} K_0\left(\frac{2}{3}(a z^r)^{3/2}\right) \text{Ai}'(a z^r)\, dz = -\frac{\sqrt[6]{3}}{2\, 2^{2/3} \sqrt{\pi}} \left(\frac{z^\alpha}{3r} G_{3,5}^{4,1}\left(\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3} \,\middle|\, \begin{matrix} 1 - \frac{\alpha}{3r}, \frac{1}{3}, \frac{5}{6} \\ 0, 0, \frac{2}{3}, \frac{2}{3}, -\frac{\alpha}{3r} \end{matrix}\right) + \frac{z^\alpha \left(\frac{3}{2}\log(a z^r) - \log\left((a z^r)^{3/2}\right)\right)}{3\pi r} G_{3,5}^{2,3}\left(\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3} \,\middle|\, \begin{matrix} \frac{1}{3}, \frac{5}{6}, 1 - \frac{\alpha}{3r} \\ 0, \frac{2}{3}, 0, \frac{2}{3}, -\frac{\alpha}{3r} \end{matrix}\right)\right)$$

03.07.21.0068.01

$$\int z^{\alpha-1} K_1\left(\frac{2}{3}(a z^r)^{3/2}\right) \text{Ai}'(a z^r)\, dz = -\frac{\sqrt[6]{3}}{2\, 2^{2/3} \sqrt{\pi}} \left(\frac{z^\alpha}{3r} G_{3,5}^{4,1}\left(\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3} \,\middle|\, \begin{matrix} 1 - \frac{\alpha}{3r}, \frac{1}{3}, \frac{5}{6} \\ -\frac{1}{2}, \frac{1}{6}, \frac{1}{2}, \frac{7}{6}, -\frac{\alpha}{3r} \end{matrix}\right) - \frac{z^\alpha \left(\frac{3}{2}\log(a z^r) - \log\left((a z^r)^{3/2}\right)\right)}{3\pi r} G_{3,5}^{2,3}\left(\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3} \,\middle|\, \begin{matrix} \frac{1}{3}, \frac{5}{6}, 1 - \frac{\alpha}{3r} \\ \frac{1}{2}, \frac{7}{6}, -\frac{1}{2}, \frac{1}{6}, -\frac{\alpha}{3r} \end{matrix}\right)\right)$$

03.07.21.0069.01

$$\int z^{\alpha-1} K_2\left(\frac{2}{3}(a z^r)^{3/2}\right) \text{Ai}'(a z^r)\, dz = -\frac{\sqrt[6]{3}}{2\, 2^{2/3} \sqrt{\pi}} \left(\frac{z^\alpha}{3r} G_{3,5}^{4,1}\left(\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3} \,\middle|\, \begin{matrix} 1 - \frac{\alpha}{3r}, \frac{1}{3}, \frac{5}{6} \\ -1, -\frac{1}{3}, 1, \frac{5}{3}, -\frac{\alpha}{3r} \end{matrix}\right) + \frac{z^\alpha \left(\frac{3}{2}\log(a z^r) - \log\left((a z^r)^{3/2}\right)\right)}{3\pi r} G_{3,5}^{2,3}\left(\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3} \,\middle|\, \begin{matrix} \frac{1}{3}, \frac{5}{6}, 1 - \frac{\alpha}{3r} \\ 1, \frac{5}{3}, -1, -\frac{1}{3}, -\frac{\alpha}{3r} \end{matrix}\right)\right)$$

Involving other Airy functions

Involving Ai

Linear arguments

03.07.21.0070.01

$$\int \text{Ai}(a\,z)\,\text{Ai}'(a\,z)\,dz = \frac{\text{Ai}(a\,z)^2}{2\,a}$$

Power arguments

03.07.21.0071.01

$$\int \text{Ai}(a\,z^r)\,\text{Ai}'(a\,z^r)\,dz = -\frac{z}{12\,\pi^{3/2}\,r}\,G_{2,4}^{3,1}\!\left(\left(\frac{2}{3}\right)^{2/3} a\,z^r,\,\frac{1}{3}\,\middle|\,\begin{array}{c} 1-\frac{1}{3r},\,\frac{1}{2} \\ 0,\,\frac{1}{3},\,\frac{2}{3},\,-\frac{1}{3r} \end{array}\right)$$

Involving Ai and power

Linear arguments

03.07.21.0072.01

$$\int z^{\alpha-1}\,\text{Ai}(a\,z)\,\text{Ai}'(a\,z)\,dz = -\frac{z^\alpha}{12\,\pi^{3/2}}\,G_{2,4}^{3,1}\!\left(\left(\frac{2}{3}\right)^{2/3} a\,z,\,\frac{1}{3}\,\middle|\,\begin{array}{c} 1-\frac{\alpha}{3},\,\frac{1}{2} \\ 0,\,\frac{1}{3},\,\frac{2}{3},\,-\frac{\alpha}{3} \end{array}\right)$$

03.07.21.0073.01

$$\int z\,\text{Ai}(a\,z)\,\text{Ai}'(a\,z)\,dz = \frac{\text{Ai}'(a\,z)^2}{2\,a^2}$$

03.07.21.0074.01

$$\int z^2\,\text{Ai}(a\,z)\,\text{Ai}'(a\,z)\,dz = \frac{a^2\,z^2\,\text{Ai}(a\,z)^2 - 2\,\text{Ai}'(a\,z)\,\text{Ai}(a\,z) + 2\,a\,z\,\text{Ai}'(a\,z)^2}{6\,a^3}$$

03.07.21.0075.01

$$\int z^3\,\text{Ai}(a\,z)\,\text{Ai}'(a\,z)\,dz = \frac{1}{10\,a^4}\left(\left(2\,a^3\,z^3+3\right)\text{Ai}(a\,z)^2 - 6\,a\,z\,\text{Ai}'(a\,z)\,\text{Ai}(a\,z) + 3\,a^2\,z^2\,\text{Ai}'(a\,z)^2\right)$$

Power arguments

03.07.21.0076.01

$$\int z^{\alpha-1}\,\text{Ai}(a\,z^r)\,\text{Ai}'(a\,z^r)\,dz = -\frac{z^\alpha}{12\,\pi^{3/2}\,r}\,G_{2,4}^{3,1}\!\left(\left(\frac{2}{3}\right)^{2/3} a\,z^r,\,\frac{1}{3}\,\middle|\,\begin{array}{c} 1-\frac{\alpha}{3r},\,\frac{1}{2} \\ 0,\,\frac{1}{3},\,\frac{2}{3},\,-\frac{\alpha}{3r} \end{array}\right)$$

Involving Ai and exp

Power arguments

03.07.21.0077.01

$$\int e^{-\frac{2}{3}z^3}\left(z\,\mathrm{Ai}(z^2)-\mathrm{Ai}'(z^2)\right)dz = \frac{1}{60\,3^{2/3}}\left(-\frac{4z^5}{\Gamma\left(\frac{5}{3}\right)}\,{}_2F_2\left(\frac{7}{6},\frac{5}{3};\frac{7}{3},\frac{8}{3};-\frac{4}{3}z^3\right)-\frac{5\sqrt[3]{3}\,z^4}{\Gamma\left(\frac{4}{3}\right)}\,{}_2F_2\left(\frac{5}{6},\frac{4}{3};\frac{5}{3},\frac{7}{3};-\frac{4}{3}z^3\right)+\frac{30z^2}{\Gamma\left(\frac{2}{3}\right)}\,{}_2F_2\left(\frac{1}{6},\frac{2}{3};\frac{1}{3},\frac{5}{3};-\frac{4}{3}z^3\right)+\frac{60\sqrt[3]{3}\,z}{\Gamma\left(\frac{1}{3}\right)}\,{}_2F_2\left(-\frac{1}{6},\frac{1}{3};-\frac{1}{3},\frac{4}{3};-\frac{4}{3}z^3\right)\right)$$

Involving Bi

Linear arguments

03.07.21.0078.01

$$\int \mathrm{Bi}(a z)\,\mathrm{Ai}'(a z)\,dz = \frac{a z\,\mathrm{Ai}'(a z)\,\mathrm{Bi}(a z)+\mathrm{Ai}(a z)\left(\mathrm{Bi}(a z)-a z\,\mathrm{Bi}'(a z)\right)}{2a}$$

Power arguments

03.07.21.0079.01

$$\int \mathrm{Bi}(a z^r)\,\mathrm{Ai}'(a z^r)\,dz = -\frac{z}{12\pi^{3/2}r}\left(6\sqrt{\pi}\,r+G_{2,4}^{2,2}\left(\left(\frac{2}{3}\right)^{2/3}a z^r,\frac{1}{3}\,\middle|\,\begin{matrix}\frac{1}{2},1-\frac{1}{3r}\\ \frac{1}{3},\frac{2}{3},0,-\frac{1}{3r}\end{matrix}\right)\right)$$

Involving Bi and power

Linear arguments

03.07.21.0080.01

$$\int z^{\alpha-1}\,\mathrm{Bi}(a z)\,\mathrm{Ai}'(a z)\,dz = -\frac{z^\alpha}{12\pi^{3/2}\alpha}\left(\alpha\,G_{2,4}^{2,2}\left(\left(\frac{2}{3}\right)^{2/3}a z,\frac{1}{3}\,\middle|\,\begin{matrix}\frac{1}{2},1-\frac{\alpha}{3}\\ \frac{1}{3},\frac{2}{3},0,-\frac{\alpha}{3}\end{matrix}\right)+6\sqrt{\pi}\right)$$

03.07.21.0081.01

$$\int z\,\mathrm{Bi}(a z)\,\mathrm{Ai}'(a z)\,dz = \frac{1}{4}\left(\mathrm{Ai}'(a z)\left(\mathrm{Bi}(a z)\,z^2+\frac{2\,\mathrm{Bi}'(a z)}{a^2}\right)-z^2\,\mathrm{Ai}(a z)\,\mathrm{Bi}'(a z)\right)$$

03.07.21.0082.01

$$\int z^2\,\mathrm{Bi}(a z)\,\mathrm{Ai}'(a z)\,dz = \frac{1}{6a^3}\left(\mathrm{Ai}'(a z)\left(\left(a^3z^3-1\right)\mathrm{Bi}(a z)+2 a z\,\mathrm{Bi}'(a z)\right)-\mathrm{Ai}(a z)\left(\left(a^3z^3+1\right)\mathrm{Bi}'(a z)-a^2z^2\,\mathrm{Bi}(a z)\right)\right)$$

03.07.21.0083.01

$$\int z^3\,\mathrm{Bi}(a z)\,\mathrm{Ai}'(a z)\,dz = \frac{1}{40a^4}\left(a z\,\mathrm{Ai}'(a z)\left(\left(5a^3z^3-12\right)\mathrm{Bi}(a z)+12 a z\,\mathrm{Bi}'(a z)\right)+\mathrm{Ai}(a z)\left(4\left(2a^3z^3+3\right)\mathrm{Bi}(a z)-a z\left(5a^3z^3+12\right)\mathrm{Bi}'(a z)\right)\right)$$

Power arguments

03.07.21.0084.01

$$\int z^{\alpha-1}\,\mathrm{Bi}(a\,z^r)\,\mathrm{Ai}'(a\,z^r)\,dz = -\frac{z^\alpha}{12\,\pi^{3/2}\,r}\,G_{2,4}^{2,2}\!\left(\left(\frac{2}{3}\right)^{2/3} a\,z^r,\ \frac{1}{3}\ \middle|\ \begin{matrix}\frac{1}{2},\ 1-\frac{\alpha}{3r}\\ \frac{1}{3},\ \frac{2}{3},\ 0,\ -\frac{\alpha}{3r}\end{matrix}\right) - \frac{z^\alpha}{2\,\pi\,\alpha}$$

Integral transforms

Fourier exp transforms

03.07.22.0001.01

$$\mathcal{F}_t[\mathrm{Ai}'(t)](z) = -\frac{z}{4\sqrt{2}\,\pi^{3/2}}\left(4\,\pi\,i\,e^{-\frac{i z^3}{3}} + \sqrt[6]{3}\,z\left(2\,\Gamma\!\left(\frac{2}{3}\right) - 3\,\Gamma\!\left(\frac{5}{3}\right)\right){}_1F_2\!\left(1;\ \frac{2}{3},\ \frac{7}{6};\ -\frac{z^6}{36}\right)\right)$$

Inverse Fourier exp transforms

03.07.22.0002.01

$$\mathcal{F}_t^{-1}[\mathrm{Ai}'(t)](z) = \frac{z}{4\sqrt{2}\,\pi^{3/2}}\left(4\,\pi\,i\,e^{\frac{i z^3}{3}} + \sqrt[6]{3}\,z\left(3\,\Gamma\!\left(\frac{5}{3}\right) - 2\,\Gamma\!\left(\frac{2}{3}\right)\right){}_1F_2\!\left(1;\ \frac{2}{3},\ \frac{7}{6};\ -\frac{z^6}{36}\right)\right)$$

Fourier cos transforms

03.07.22.0003.01

$$\mathcal{F}c_t[\mathrm{Ai}'(t)](z) = -\frac{1}{3\sqrt{2}\,\pi^{3/2}}\left(2\,\pi\sin\!\left(\frac{z^3}{3}\right)z - 3\sqrt[6]{3}\,\Gamma\!\left(\frac{2}{3}\right){}_1F_2\!\left(1;\ \frac{2}{3},\ \frac{7}{6};\ -\frac{z^6}{36}\right)z^2 + 3\,3^{5/6}\,\Gamma\!\left(\frac{4}{3}\right){}_1F_2\!\left(1;\ \frac{1}{3},\ \frac{5}{6};\ -\frac{z^6}{36}\right)\right)$$

Fourier sin transforms

03.07.22.0004.01

$$\mathcal{F}s_t[\mathrm{Ai}'(t)](z) = -\frac{1}{12\sqrt{2}\,\pi^{3/2}}\left(3\sqrt[6]{3}\,\Gamma\!\left(\frac{2}{3}\right){}_1F_2\!\left(1;\ \frac{7}{6},\ \frac{5}{3};\ -\frac{z^6}{36}\right)z^5 - 2\,3^{5/6}\,\Gamma\!\left(\frac{1}{3}\right){}_1F_2\!\left(1;\ \frac{5}{6},\ \frac{4}{3};\ -\frac{z^6}{36}\right)z^3 + 8\,\pi\cos\!\left(\frac{z^3}{3}\right)z\right)$$

Laplace transforms

03.07.22.0005.01

$$\mathcal{L}_t[\mathrm{Ai}'(t)](z) = \frac{z}{72\,\pi}\,e^{-\frac{z^3}{3}}\left(9\left(-3\,i+\sqrt{3}\right)\Gamma\!\left(\frac{5}{3}\right)\Gamma\!\left(\frac{1}{3},\ -\frac{z^3}{3}\right) - 4\sqrt[3]{-1}\,\sqrt{3}\,\Gamma\!\left(\frac{1}{3}\right)\Gamma\!\left(-\frac{1}{3},\ -\frac{z^3}{3}\right)\right)$$

Mellin transforms

03.07.22.0006.01

$$\mathcal{M}_t[\mathrm{Ai}'(t)](z) = -\frac{1}{2\,\pi}\,3^{\frac{4z-5}{6}}\,\Gamma\!\left(\frac{z}{3}\right)\Gamma\!\left(\frac{z+2}{3}\right) /;\ \mathrm{Re}(z) > 0$$

Hankel transforms

03.07.22.0007.01

$$\mathcal{H}_{t;\nu}[\mathrm{Ai}'(t)](z) = 2^{-\nu-6}\,3^{-\frac{\nu}{3}}\,z^{\nu+\frac{1}{2}}\Bigg(z^2\,\Gamma\left(\frac{\nu}{3}+\frac{11}{6}\right)\Bigg(\frac{16}{\Gamma\left(\frac{\nu}{3}+1\right)\Gamma\left(\frac{\nu+2}{3}\right)\Gamma\left(\frac{\nu+4}{3}\right)}\,\Gamma\left(\frac{\nu}{3}+\frac{7}{6}\right){}_4F_5\left(\frac{\nu}{6}+\frac{7}{12},\frac{\nu}{6}+\frac{11}{12},\frac{\nu}{6}+\frac{13}{12},\frac{\nu}{6}+\frac{17}{12};\frac{2}{3},\frac{4}{3},\frac{\nu}{3}+\frac{2}{3},\frac{\nu}{3}+1,\frac{\nu}{3}+\frac{4}{3};-\frac{z^6}{36}\right)-\frac{3^{\nu+\frac{17}{6}}\,z^2}{\pi\,\Gamma(\nu+3)}\,\Gamma\left(\frac{\nu}{3}+\frac{5}{2}\right){}_4F_5\left(\frac{\nu}{6}+\frac{11}{12},\frac{\nu}{6}+\frac{5}{4},\frac{\nu}{6}+\frac{17}{12},\frac{\nu}{6}+\frac{7}{4};\frac{4}{3},\frac{5}{3},\frac{\nu}{3}+1,\frac{\nu}{3}+\frac{4}{3},\frac{\nu}{3}+\frac{5}{3};-\frac{z^6}{36}\right)\Bigg)-\frac{32\,3^{\nu+\frac{1}{6}}}{\pi\,\Gamma(\nu+1)}\,\Gamma\left(\frac{\nu}{3}+\frac{1}{2}\right)\Gamma\left(\frac{\nu}{3}+\frac{7}{6}\right){}_4F_5\left(\frac{\nu}{6}+\frac{1}{4},\frac{\nu}{6}+\frac{7}{12},\frac{\nu}{6}+\frac{3}{4},\frac{\nu}{6}+\frac{13}{12};\frac{1}{3},\frac{2}{3},\frac{\nu}{3}+\frac{1}{3},\frac{\nu}{3}+\frac{2}{3},\frac{\nu}{3}+1;-\frac{z^6}{36}\right)\Bigg)\;/;\;\mathrm{Re}(\nu)>-\frac{3}{2}$$

Representations through more general functions

Through hypergeometric functions

Involving ${}_0F_1$

03.07.26.0001.01

$$\mathrm{Ai}'(z) = \frac{z^2}{2\,3^{2/3}\,\Gamma\left(\frac{2}{3}\right)}\,{}_0F_1\left(;\frac{5}{3};\frac{z^3}{9}\right)-\frac{1}{\sqrt[3]{3}\,\Gamma\left(\frac{1}{3}\right)}\,{}_0F_1\left(;\frac{1}{3};\frac{z^3}{9}\right)$$

Through Meijer *G*

Classical cases for the direct function itself

03.07.26.0002.01

$$\mathrm{Ai}'(z) = \frac{1}{9}\left(\sqrt[3]{3}\,\pi\,z^2\,G_{1,3}^{1,0}\left(\frac{z^3}{9}\,\middle|\,\begin{matrix}\frac{1}{2}\\ 0,-\frac{2}{3},\frac{1}{2}\end{matrix}\right)-3\,3^{2/3}\,\pi\,G_{1,3}^{1,0}\left(\frac{z^3}{9}\,\middle|\,\begin{matrix}\frac{1}{2}\\ 0,\frac{2}{3},\frac{1}{2}\end{matrix}\right)\right)$$

03.07.26.0027.01

$$\mathrm{Ai}'(z) = -\frac{\sqrt[6]{3}}{2\,\pi}\,G_{0,2}^{2,0}\left(\frac{z^3}{9}\,\middle|\,0,\frac{2}{3}\right)\;/;\;-\frac{\pi}{3}<\arg(z)\leq\frac{\pi}{3}$$

Classical cases involving exp

03.07.26.0028.01

$$e^{-\frac{1}{3}\left(2\,z^{3/2}\right)}\,\mathrm{Ai}'(z) = -\frac{\sqrt[6]{3}}{2\,\sqrt[3]{2}\,\sqrt{\pi}}\,G_{1,2}^{2,0}\left(\frac{4\,z^{3/2}}{3}\,\middle|\,\begin{matrix}\frac{7}{6}\\ 0,\frac{4}{3}\end{matrix}\right)\;/;\;-\frac{2\,\pi}{3}<\arg(z)\leq\frac{2\,\pi}{3}$$

03.07.26.0029.01

$$e^{\frac{2\,z^{3/2}}{3}}\,\mathrm{Ai}'(z) = \frac{\sqrt[6]{3}}{4\,\sqrt[3]{2}\,\pi^{3/2}}\,G_{1,2}^{2,1}\left(\frac{4\,z^{3/2}}{3}\,\middle|\,\begin{matrix}\frac{7}{6}\\ 0,\frac{4}{3}\end{matrix}\right)\;/;\;-\frac{2\,\pi}{3}<\arg(z)\leq\frac{2\,\pi}{3}$$

03.07.26.0030.01

$$e^{-z}\,\mathrm{Ai}'\left(\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) = -\frac{\sqrt[6]{3}}{2\sqrt[3]{2}\,\sqrt{\pi}}\,G_{1,2}^{2,0}\left(2\,z \,\middle|\, \begin{matrix} \frac{7}{6} \\ 0,\ \frac{4}{3} \end{matrix}\right)$$

03.07.26.0031.01

$$e^{z}\,\mathrm{Ai}'\left(\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) = \frac{\sqrt[6]{3}}{4\sqrt[3]{2}\,\pi^{3/2}}\,G_{1,2}^{2,1}\left(2\,z \,\middle|\, \begin{matrix} \frac{7}{6} \\ 0,\ \frac{4}{3} \end{matrix}\right)$$

Classical cases involving $_0F_1$

03.07.26.0003.01

$$\mathrm{Ai}'(z)\,{}_0F_1\left(;\,b;\,\frac{z^3}{9}\right) = -\frac{2^{b-\frac{8}{3}}\,\sqrt[6]{3}\,\Gamma(b)}{\pi^{3/2}}\,G_{2,4}^{2,2}\left(\frac{4\,z^3}{9} \,\middle|\, \begin{matrix} \frac{1}{6}(5-3\,b),\ \frac{1}{6}(8-3\,b) \\ 0,\ \frac{2}{3},\ 1-b,\ \frac{5}{3}-b \end{matrix}\right) /;\ -\frac{\pi}{3} < \arg(z) \leq \frac{\pi}{3}$$

03.07.26.0023.01

$$\mathrm{Ai}'\left(3^{2/3}\sqrt[3]{z}\right)\,{}_0F_1(;\,b;\,z) = -\frac{2^{b-\frac{8}{3}}\,\sqrt[6]{3}\,\Gamma(b)}{\pi^{3/2}}\,G_{2,4}^{2,2}\left(4\,z \,\middle|\, \begin{matrix} \frac{1}{6}(5-3\,b),\ \frac{1}{6}(8-3\,b) \\ 0,\ \frac{2}{3},\ 1-b,\ \frac{5}{3}-b \end{matrix}\right)$$

Classical cases involving $_0\tilde{F}_1$

03.07.26.0004.01

$$\mathrm{Ai}'(z)\,{}_0\tilde{F}_1\left(;\,b;\,\frac{z^3}{9}\right) = -\frac{2^{b-\frac{8}{3}}\,\sqrt[6]{3}}{\pi^{3/2}}\,G_{2,4}^{2,2}\left(\frac{4\,z^3}{9} \,\middle|\, \begin{matrix} \frac{1}{6}(5-3\,b),\ \frac{1}{6}(8-3\,b) \\ 0,\ \frac{2}{3},\ 1-b,\ \frac{5}{3}-b \end{matrix}\right) /;\ -\frac{\pi}{3} < \arg(z) \leq \frac{\pi}{3}$$

03.07.26.0024.01

$$\mathrm{Ai}'\left(3^{2/3}\sqrt[3]{z}\right)\,{}_0\tilde{F}_1(;\,b;\,z) = -\frac{2^{b-\frac{8}{3}}\,\sqrt[6]{3}}{\pi^{3/2}}\,G_{2,4}^{2,2}\left(4\,z \,\middle|\, \begin{matrix} \frac{1}{6}(5-3\,b),\ \frac{1}{6}(8-3\,b) \\ 0,\ \frac{2}{3},\ 1-b,\ \frac{5}{3}-b \end{matrix}\right)$$

Generalized cases for the direct function itself

03.07.26.0005.01

$$\mathrm{Ai}'(z) = -\frac{\sqrt[6]{3}}{2\,\pi}\,G_{0,2}^{2,0}\left(3^{-2/3}\,z,\ \frac{1}{3} \,\middle|\, 0,\ \frac{2}{3}\right)$$

Generalized cases involving exp

03.07.26.0006.01

$$\exp\left(-\frac{2\,z^{3/2}}{3}\right)\mathrm{Ai}'(z) = -\frac{\sqrt[6]{3}}{2\sqrt[3]{2}\,\sqrt{\pi}}\,G_{1,2}^{2,0}\left(\frac{2\sqrt[3]{2}\,z}{3^{2/3}},\ \frac{2}{3} \,\middle|\, \begin{matrix} \frac{7}{6} \\ 0,\ \frac{4}{3} \end{matrix}\right)$$

03.07.26.0007.01

$$\exp\left(\frac{2\,z^{3/2}}{3}\right)\mathrm{Ai}'(z) = \frac{\sqrt[6]{3}}{4\sqrt[3]{2}\,\pi^{3/2}}\,G_{1,2}^{2,1}\left(\frac{2\sqrt[3]{2}\,z}{3^{2/3}},\ \frac{2}{3} \,\middle|\, \begin{matrix} \frac{7}{6} \\ 0,\ \frac{4}{3} \end{matrix}\right)$$

Generalized cases involving cosh

03.07.26.0008.01

$$\cosh\left(\frac{2 z^{3/2}}{3}\right) \operatorname{Ai}'(z) = -\frac{1}{4\pi} \sqrt[6]{\frac{3}{2}}\, G_{2,4}^{2,2}\left(\left(\frac{2}{3}\right)^{2/3} z, \frac{1}{3} \,\middle|\, \begin{matrix} \frac{7}{12}, \frac{13}{12} \\ 0, \frac{2}{3}, \frac{1}{2}, \frac{7}{6} \end{matrix}\right)$$

03.07.26.0032.01

$$\cosh(z) \operatorname{Ai}'\left(\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) = -\frac{\sqrt[6]{\frac{3}{2}}}{4\pi} G_{2,4}^{2,2}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} \frac{7}{12}, \frac{13}{12} \\ 0, \frac{2}{3}, \frac{1}{2}, \frac{7}{6} \end{matrix}\right)$$

Generalized cases involving sinh

03.07.26.0009.01

$$\sinh\left(\frac{2 z^{3/2}}{3}\right) \operatorname{Ai}'(z) = -\frac{1}{4\pi} \sqrt[6]{\frac{3}{2}}\, G_{2,4}^{2,2}\left(\left(\frac{2}{3}\right)^{2/3} z, \frac{1}{3} \,\middle|\, \begin{matrix} \frac{7}{12}, \frac{13}{12} \\ \frac{1}{2}, \frac{7}{6}, 0, \frac{2}{3} \end{matrix}\right)$$

03.07.26.0033.01

$$\sinh(z) \operatorname{Ai}'\left(\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) = -\frac{\sqrt[6]{\frac{3}{2}}}{4\pi} G_{2,4}^{2,2}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} \frac{7}{12}, \frac{13}{12} \\ \frac{1}{2}, \frac{7}{6}, 0, \frac{2}{3} \end{matrix}\right)$$

Generalized cases for powers of Ai′

03.07.26.0010.01

$$\operatorname{Ai}'(z)^2 = \frac{1}{4\pi^{3/2}} \sqrt[3]{\frac{3}{2}}\, G_{1,3}^{3,0}\left(\left(\frac{2}{3}\right)^{2/3} z, \frac{1}{3} \,\middle|\, \begin{matrix} \frac{7}{6} \\ 0, \frac{2}{3}, \frac{4}{3} \end{matrix}\right)$$

Generalized cases involving Ai

03.07.26.0011.01

$$\operatorname{Ai}(z) \operatorname{Ai}'(z) = -\frac{1}{4\pi^{3/2}} G_{1,3}^{3,0}\left(\left(\frac{2}{3}\right)^{2/3} z, \frac{1}{3} \,\middle|\, \begin{matrix} \frac{1}{2} \\ 0, \frac{1}{3}, \frac{2}{3} \end{matrix}\right)$$

Generalized cases involving Bi

03.07.26.0012.01

$$\operatorname{Ai}'(z) \operatorname{Bi}(z) = -\frac{1}{4\pi^{3/2}} G_{1,3}^{2,1}\left(\left(\frac{2}{3}\right)^{2/3} z, \frac{1}{3} \,\middle|\, \begin{matrix} \frac{1}{2} \\ \frac{1}{3}, \frac{2}{3}, 0 \end{matrix}\right) - \frac{1}{2\pi}$$

03.07.26.0013.01

$$\operatorname{Ai}'(z) \operatorname{Bi}(z) = \frac{\sqrt{3}}{4\pi^{3/2}} G_{2,4}^{3,1}\left(\left(\frac{2}{3}\right)^{2/3} z, \frac{1}{3} \,\middle|\, \begin{matrix} -\frac{2}{3}, \frac{1}{2} \\ -\frac{2}{3}, 0, \frac{2}{3}, \frac{1}{3} \end{matrix}\right) - 2\, G_{2,4}^{2,2}\left(\left(\frac{2}{3}\right)^{2/3} z, \frac{1}{3} \,\middle|\, \begin{matrix} \frac{1}{2}, 1 \\ \frac{1}{3}, 1, 0, \frac{2}{3} \end{matrix}\right)$$

03.07.26.0034.01

$$\operatorname{Ai}'(z) \operatorname{Bi}(z) = \frac{1}{4\pi^{3/2}} G_{2,4}^{3,1}\left(\left(\frac{2}{3}\right)^{2/3} z, \frac{1}{3} \,\middle|\, \begin{matrix} \frac{1}{2}, 1 \\ \frac{1}{3}, \frac{2}{3}, 0, 1 \end{matrix}\right) - \frac{1}{\sqrt{3}\, \Gamma\left(\frac{1}{3}\right) \Gamma\left(\frac{2}{3}\right)}$$

Generalized cases involving Bi′

03.07.26.0014.01

$$\mathrm{Ai}'(z)\,\mathrm{Bi}'(z) = \frac{1}{4\pi^{3/2}}\sqrt[3]{\frac{3}{2}}\; G_{1,3}^{2,1}\!\left(\left(\frac{2}{3}\right)^{2/3} z, \frac{1}{3} \,\middle|\, \begin{matrix} \frac{7}{6} \\ 0, \frac{4}{3}, \frac{2}{3} \end{matrix}\right)$$

Generalized cases involving ${}_0F_1$

03.07.26.0015.01

$$\mathrm{Ai}'(z)\,{}_0F_1\!\left(;b;\frac{z^3}{9}\right) = -\frac{2^{b-\frac{8}{3}}\sqrt[6]{3}\,\Gamma(b)}{\pi^{3/2}}\, G_{2,4}^{2,2}\!\left(\left(\frac{2}{3}\right)^{2/3} z, \frac{1}{3} \,\middle|\, \begin{matrix} \frac{1}{6}(5-3b), \frac{1}{6}(8-3b) \\ 0, \frac{2}{3}, 1-b, \frac{5}{3}-b \end{matrix}\right)$$

03.07.26.0035.01

$$\mathrm{Ai}'\!\left(3^{2/3}\sqrt[3]{z}\right){}_0F_1(;b;z) = -\frac{2^{b-\frac{8}{3}}\sqrt[6]{3}\,\Gamma(b)}{\pi^{3/2}}\, G_{2,4}^{2,2}\!\left(2^{2/3}\sqrt[3]{z}, \frac{1}{3} \,\middle|\, \begin{matrix} \frac{1}{6}(5-3b), \frac{1}{6}(8-3b) \\ 0, \frac{2}{3}, 1-b, \frac{5}{3}-b \end{matrix}\right)$$

Generalized cases involving ${}_0\tilde{F}_1$

03.07.26.0016.01

$$\mathrm{Ai}'(z)\,{}_0\tilde{F}_1\!\left(;b;\frac{z^3}{9}\right) = -\frac{2^{b-\frac{8}{3}}\sqrt[6]{3}}{\pi^{3/2}}\, G_{2,4}^{2,2}\!\left(\left(\frac{2}{3}\right)^{2/3} z, \frac{1}{3} \,\middle|\, \begin{matrix} \frac{1}{6}(5-3b), \frac{1}{6}(8-3b) \\ 0, \frac{2}{3}, 1-b, \frac{5}{3}-b \end{matrix}\right)$$

03.07.26.0036.01

$$\mathrm{Ai}'\!\left(3^{2/3}\sqrt[3]{z}\right){}_0\tilde{F}_1(;b;z) = -\frac{2^{b-\frac{8}{3}}\sqrt[6]{3}}{\pi^{3/2}}\, G_{2,4}^{2,2}\!\left(2^{2/3}\sqrt[3]{z}, \frac{1}{3} \,\middle|\, \begin{matrix} \frac{1}{6}(5-3b), \frac{1}{6}(8-3b) \\ 0, \frac{2}{3}, 1-b, \frac{5}{3}-b \end{matrix}\right)$$

Generalized cases involving Bessel I

03.07.26.0017.01

$$\mathrm{Ai}'(z)\,I_\nu\!\left(\frac{2z^{3/2}}{3}\right) = -\frac{\sqrt[6]{3}\,z^{-\frac{3\nu}{2}}\left(z^{3/2}\right)^\nu}{2\,2^{2/3}\,\pi^{3/2}}\, G_{2,4}^{2,2}\!\left(\left(\frac{2}{3}\right)^{2/3} z, \frac{1}{3} \,\middle|\, \begin{matrix} \frac{1}{3}, \frac{5}{6} \\ \frac{\nu}{2}, \frac{1}{6}(3\nu+4), -\frac{\nu}{2}, \frac{1}{6}(4-3\nu) \end{matrix}\right)$$

03.07.26.0025.01

$$\mathrm{Ai}'\!\left(\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) I_\nu(z) = -\frac{\sqrt[6]{3}}{2\,2^{2/3}\,\pi^{3/2}}\, G_{2,4}^{2,2}\!\left(z^{2/3}, \frac{1}{3} \,\middle|\, \begin{matrix} \frac{1}{3}, \frac{5}{6} \\ \frac{\nu}{2}, \frac{\nu}{2}+\frac{2}{3}, -\frac{\nu}{2}, \frac{2}{3}-\frac{\nu}{2} \end{matrix}\right)$$

Generalized cases involving Bessel K

03.07.26.0037.01

$$\mathrm{Ai}'(z)\,K_\nu\!\left(\frac{2z^{3/2}}{3}\right) = -\frac{\sqrt[6]{3}}{2\,2^{2/3}\sqrt{\pi}}\, G_{2,4}^{4,0}\!\left(\left(\frac{2}{3}\right)^{2/3} z, \frac{1}{3} \,\middle|\, \begin{matrix} \frac{1}{3}, \frac{5}{6} \\ \frac{\nu}{2}, -\frac{\nu}{2}, \frac{1}{6}(3\nu+4), \frac{1}{6}(4-3\nu) \end{matrix}\right) /;\ -\frac{1}{3}(2\pi) < \arg(z) \leq \frac{2\pi}{3}$$

03.07.26.0018.01

$$\mathrm{Ai}'(z)\,K_\nu\!\left(\frac{2z^{3/2}}{3}\right) = -\frac{\sqrt[6]{3}\,\csc(\pi\nu)}{4\,2^{2/3}\sqrt{\pi}}\left(G_{2,4}^{2,2}\!\left(\left(\frac{2}{3}\right)^{2/3} z, \frac{1}{3} \,\middle|\, \begin{matrix} \frac{1}{3}, \frac{5}{6} \\ -\frac{\nu}{2}, \frac{2}{3}-\frac{\nu}{2}, \frac{\nu}{2}, \frac{\nu}{2}+\frac{2}{3} \end{matrix}\right) - G_{2,4}^{2,2}\!\left(\left(\frac{2}{3}\right)^{2/3} z, \frac{1}{3} \,\middle|\, \begin{matrix} \frac{1}{3}, \frac{5}{6} \\ \frac{\nu}{2}, \frac{\nu}{2}+\frac{2}{3}, -\frac{\nu}{2}, \frac{2}{3}-\frac{\nu}{2} \end{matrix}\right)\right) /;$$

$$-\frac{2\pi}{3} < \arg(z) \leq \frac{2\pi}{3}$$

03.07.26.0026.01

$$\text{Ai}'\left(\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) K_\nu(z) = -\frac{\sqrt[6]{3}}{2\, 2^{2/3} \sqrt{\pi}} G_{2,4}^{4,0}\left(z^{2/3}, \frac{1}{3} \,\middle|\, \begin{matrix} \frac{1}{3}, \frac{5}{6} \\ \frac{\nu}{2}, -\frac{\nu}{2}, \frac{1}{6}(3\nu+4), \frac{1}{6}(4-3\nu) \end{matrix}\right)$$

Through other functions

Involving Bessel functions

03.07.26.0019.01

$$\text{Ai}'(z) = \frac{1}{3} z \left(J_{-\frac{2}{3}}\left(\frac{2}{3}(-z)^{3/2}\right) - J_{\frac{2}{3}}\left(\frac{2}{3}(-z)^{3/2}\right)\right) /; \text{Re}(z) \le 0$$

03.07.26.0020.01

$$\text{Ai}'(z) = \frac{1}{3} z \left(I_{\frac{2}{3}}\left(\frac{2 z^{3/2}}{3}\right) - I_{-\frac{2}{3}}\left(\frac{2 z^{3/2}}{3}\right)\right) /; \text{Re}(z) \ge 0$$

03.07.26.0021.01

$$\text{Ai}'(z) = \frac{1}{3}\left(z^2 \left(z^{3/2}\right)^{-\frac{2}{3}} I_{\frac{2}{3}}\left(\frac{2 z^{3/2}}{3}\right) - \left(z^{3/2}\right)^{2/3} I_{-\frac{2}{3}}\left(\frac{2 z^{3/2}}{3}\right)\right)$$

03.07.26.0022.01

$$\text{Ai}'(z) = -\frac{z}{\sqrt{3}\,\pi} K_{\frac{2}{3}}\left(\frac{2 z^{3/2}}{3}\right) /; \text{Re}(z) \ge 0$$

Representations through equivalent functions

With related functions

03.07.27.0001.01

$$\text{Ai}'\left(e^{-\frac{2 i \pi}{3}} z\right) + e^{-\frac{i \pi}{3}} \text{Ai}'\left(e^{\frac{2 i \pi}{3}} z\right) = e^{\frac{5 i \pi}{6}} \text{Bi}'(z)$$

03.07.27.0002.01

$$\text{Ai}'\left(e^{\frac{2 i \pi}{3}} z\right) = \frac{1}{2} e^{-\frac{i \pi}{3}} \left(\text{Ai}'(z) - i\, \text{Bi}'(z)\right)$$

03.07.27.0003.01

$$\text{Ai}'\left(e^{-\frac{1}{3}(2 \pi i)} z\right) = \frac{1}{2} e^{\frac{i \pi}{3}} \left(\text{Ai}'(z) + i\, \text{Bi}'(z)\right)$$

Zeros

03.07.30.0001.01

$$\text{Ai}'(z) = 0 /; z = z_k \wedge k \in \mathbb{N}$$

03.07.30.0002.01

$$\text{Im}(z_k) = 0 \wedge \text{Re}(z_k) < 0 /; \text{Ai}'(z_k) = 0$$

On real axis, $\text{Ai}'(z)$ has an infinite number of zeros, all of which are negative. In complex plane, $\text{Ai}'(z)$ has no other zeros.

History

- G. B. Airy (1838), H. Jeffreys (1928, 1942)
- J. C. P. Miller (1946) suggested the notations Ai, Bi

Applications of Ai′ include quantum mechanics of linear potential, electrodynamics, combinatorics, analysis of the complexity of algorithms, optical theory of the rainbow, solid state physics, and semiconductors in electric fields.

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