AiryBiPrime

Notations

Traditional name

Derivative of the Airy function Bi

Traditional notation

$\mathrm{Bi}'(z)$

Mathematica StandardForm notation

`AiryBiPrime[z]`

Primary definition

03.08.02.0001.01

$$\mathrm{Bi}'(z) = \frac{z^2}{2\sqrt[6]{3}\,\Gamma\left(\frac{2}{3}\right)}\,{}_0F_1\left(;\frac{5}{3};\frac{z^3}{9}\right) + \frac{\sqrt[6]{3}}{\Gamma\left(\frac{1}{3}\right)}\,{}_0F_1\left(;\frac{1}{3};\frac{z^3}{9}\right)$$

Specific values

Values at fixed points

03.08.03.0001.01

$$\mathrm{Bi}'(0) = \frac{1}{\Gamma\left(\frac{1}{3}\right)}\sqrt[6]{3}$$

Values at infinities

03.08.03.0002.01

$$\lim_{x\to\infty} \mathrm{Bi}'(x) = \infty$$

03.08.03.0003.01

$$\lim_{x\to-\infty} \mathrm{Bi}'(x) = 0$$

General characteristics

Domain and analyticity

$\text{Bi}'(z)$ is an entire analytical function of z, which is defined in the whole complex z-plane.

03.08.04.0001.01

$$z \longrightarrow \text{Bi}'(z) :: \mathbb{C} \longrightarrow \mathbb{C}$$

Symmetries and periodicities

Mirror symmetry

03.08.04.0002.01

$$\text{Bi}'(\bar{z}) = \overline{\text{Bi}'(z)}$$

Periodicity

No periodicity

Poles and essential singularities

The function $\text{Bi}'(z)$ has only one singular point at $z = \tilde{\infty}$. It is an essential singular point.

03.08.04.0003.01

$$\mathcal{Sing}_z(\text{Bi}'(z)) = \{\{\tilde{\infty}, \infty\}\}$$

Branch points

The function $\text{Bi}'(z)$ does not have branch points.

03.08.04.0004.01

$$\mathcal{BP}_z(\text{Bi}'(z)) = \{\}$$

Branch cuts

The function $\text{Bi}'(z)$ does not have branch cuts.

03.08.04.0005.01

$$\mathcal{BC}_z(\text{Bi}'(z)) = \{\}$$

Series representations

Generalized power series

Expansions at generic point $z = z_0$

For the function itself

03.08.06.0031.01

$$\text{Bi}'(z) \propto \text{Bi}'(z_0) + z_0 \text{Bi}(z_0)(z - z_0) + \frac{1}{2}\left(\text{Bi}(z_0) + z_0 \text{Bi}'(z_0)\right)(z - z_0)^2 + \ldots /; (z \to z_0)$$

03.08.06.0032.01

$$\text{Bi}'(z) \propto \text{Bi}'(z_0) + z_0 \text{Bi}(z_0)(z - z_0) + \frac{1}{2}\left(\text{Bi}(z_0) + z_0 \text{Bi}'(z_0)\right)(z - z_0)^2 + O\left((z - z_0)^3\right)$$

03.08.06.0033.01

$$\mathrm{Bi}'(z) = \frac{1}{2}\mathrm{Bi}'(z_0) + \sum_{k=0}^{\infty}\frac{1}{k!}\left(\frac{z_0^{-k}}{4}\left(2\sum_{s=0}^{k}\sum_{j=0}^{s}\sum_{i=0}^{s}\frac{(-1)^{j+s}(s-i)!\,(-3j+3s+1)(-3j+3s+2)(-3j-k+3s+3)_{k-2}\left(\frac{2}{3}\right)_s}{i!\,j!\,(s-j)!\,(s-2i)!\left(\frac{2}{3}\right)_i\left(\frac{1}{3}-s\right)_i}\left(-\frac{z_0^3}{9}\right)^i + \sum_{s=0}^{k}\sum_{j=0}^{s}\sum_{i=0}^{s-1}\frac{(-1)^{j+s}(-i+s-1)!\,(3i-3s+2)(-3j-k+3s+1)_k\left(-\frac{2}{3}\right)_s}{(i-1)!\,j!\,(s-j)!\,(s-2i)!\left(\frac{1}{3}\right)_i\left(\frac{5}{3}-s\right)_i}\left(-\frac{z_0^3}{9}\right)^i\right)\mathrm{Bi}'(z_0) + \frac{z_0^{2-k}}{4}\left(\sum_{s=0}^{k}\sum_{j=0}^{s}\sum_{i=0}^{s-1}\left((-1)^{j+s-1}(-i+s-1)!\,(-3j+3s+1)(-3j+3s+2)(-3j-k+3s+3)_{k-2}\left(\frac{2}{3}\right)_s\right)\Big/\left(i!\,j!\,(s-j)!\,(-2i+s-1)!\left(\frac{5}{3}\right)_i\left(\frac{1}{3}-s\right)_i\right)\left(-\frac{z_0^3}{9}\right)^i - \sum_{s=0}^{k}\sum_{j=0}^{s}\sum_{i=0}^{s-1}\frac{(-1)^{j+s-1}(-i+s-1)!\,(-3j-k+3s+1)_k\left(-\frac{2}{3}\right)_s}{i!\,j!\,(s-j)!\,(-2i+s-1)!\left(\frac{1}{3}\right)_i\left(\frac{5}{3}-s\right)_i}\left(-\frac{z_0^3}{9}\right)^i\right)\mathrm{Bi}(z_0)\right)(z-z_0)^k$$

03.08.06.0034.01

$$\mathrm{Bi}'(z) = \sum_{k=0}^{\infty}\frac{3^{k-\frac{13}{6}}z_0^{-k}}{k!}\left(z_0^2\,\Gamma\left(\frac{1}{3}\right){}_2\tilde{F}_3\left(1,\frac{4}{3};1-\frac{k}{3},\frac{4-k}{3},\frac{5-k}{3};\frac{z_0^3}{9}\right) + 9\sqrt[3]{3}\,\Gamma\left(\frac{2}{3}\right){}_2\tilde{F}_3\left(\frac{2}{3},1;\frac{1-k}{3},\frac{2-k}{3},1-\frac{k}{3};\frac{z_0^3}{9}\right)\right)(z-z_0)^k$$

03.08.06.0035.01

$$\mathrm{Bi}'(z) \propto \mathrm{Bi}'(z_0)\,(1+O(z-z_0))$$

Expansions at $z == 0$

For the function itself

03.08.06.0001.02

$$\mathrm{Bi}'(z) \propto \frac{\sqrt[6]{3}}{\Gamma\left(\frac{1}{3}\right)}\left(1+\frac{z^3}{3}+\frac{z^6}{72}+\ldots\right) + \frac{z^2}{2\sqrt[6]{3}\,\Gamma\left(\frac{2}{3}\right)}\left(1+\frac{z^3}{15}+\frac{z^6}{720}+\ldots\right)\ /;\ (z\to 0)$$

03.08.06.0036.01

$$\mathrm{Bi}'(z) \propto \frac{\sqrt[6]{3}}{\Gamma\left(\frac{1}{3}\right)}\left(1+\frac{z^3}{3}+\frac{z^6}{72}+O\left(z^9\right)\right) + \frac{z^2}{2\sqrt[6]{3}\,\Gamma\left(\frac{2}{3}\right)}\left(1+\frac{z^3}{15}+\frac{z^6}{720}+O\left(z^9\right)\right)$$

03.08.06.0002.01

$$\mathrm{Bi}'(z) = \frac{\sqrt[6]{3}}{\Gamma\left(\frac{1}{3}\right)}\sum_{k=0}^{\infty}\frac{1}{\left(\frac{1}{3}\right)_k k!}\left(\frac{z^3}{9}\right)^k + \frac{z^2}{2\sqrt[6]{3}\,\Gamma\left(\frac{2}{3}\right)}\sum_{k=0}^{\infty}\frac{1}{\left(\frac{5}{3}\right)_k k!}\left(\frac{z^3}{9}\right)^k$$

03.08.06.0003.01

$$\mathrm{Bi}'(z) = \frac{\sqrt[6]{3}}{\Gamma\left(\frac{1}{3}\right)}\,{}_0F_1\left(;\frac{1}{3};\frac{z^3}{9}\right) + \frac{z^2}{2\sqrt[6]{3}\,\Gamma\left(\frac{2}{3}\right)}\,{}_0F_1\left(;\frac{5}{3};\frac{z^3}{9}\right)$$

03.08.06.0037.01

$$\mathrm{Bi}'(z) = \frac{3^{1/6}}{\pi} \sum_{k=0}^{\infty} \frac{\Gamma\left(\frac{k+2}{3}\right) \left|\sin\left(\frac{2\pi(k+2)}{3}\right)\right|}{k!} \left(\sqrt[3]{3}\, z\right)^k$$

03.08.06.0004.02

$$\mathrm{Bi}'(z) \propto \frac{\sqrt[6]{3}}{\Gamma\left(\frac{1}{3}\right)} + \frac{z^2}{2\sqrt[6]{3}\,\Gamma\left(\frac{2}{3}\right)} + O\left(z^3\right)$$

03.08.06.0038.01

$$\mathrm{Bi}'(z) = F_\infty(z) \;/;$$
$$\left(\left(F_n(z) = \frac{\sqrt[6]{3}}{\Gamma\left(\frac{1}{3}\right)} \sum_{k=0}^{n} \frac{\left(\frac{z^3}{9}\right)^k}{\left(\frac{1}{3}\right)_k k!} + \frac{z^2}{2\sqrt[6]{3}\,\Gamma\left(\frac{2}{3}\right)} \sum_{k=0}^{n} \frac{\left(\frac{z^3}{9}\right)^k}{\left(\frac{5}{3}\right)_k k!} = \mathrm{Bi}'(z) - \frac{\sqrt[6]{3}}{\Gamma\left(\frac{1}{3}\right)(n+1)!\left(\frac{1}{3}\right)_{n+1}} \left(\frac{z^3}{9}\right)^{n+1} {}_1F_2\left(1; n+2, n+\frac{4}{3}; \frac{z^3}{9}\right) - \frac{z^2}{2\sqrt[6]{3}\,\Gamma\left(\frac{2}{3}\right)(n+1)!\left(\frac{5}{3}\right)_{n+1}} \left(\frac{z^3}{9}\right)^{n+1} {}_1F_2\left(1; n+2, n+\frac{8}{3}; \frac{z^3}{9}\right)\right) \bigwedge n \in \mathbb{N}\right)$$

Summed form of the truncated series expansion.

Asymptotic series expansions

Expansions inside Stokes sectors

In exponential form ||| In exponential form

03.08.06.0017.01

$$\mathrm{Bi}'(z) \propto \frac{e^{\frac{2z^{3/2}}{3}} \sqrt[4]{z}}{\sqrt{\pi}} \left(1 - \frac{7}{48 z^{3/2}} - \frac{455}{4608 z^3} + O\left(\frac{1}{z^{9/2}}\right)\right) /; |\arg(z)| < \frac{\pi}{3} \bigwedge (|z| \to \infty)$$

03.08.06.0018.01

$$\mathrm{Bi}'(z) \propto \frac{e^{\frac{2z^{3/2}}{3}} \sqrt[4]{z}}{\sqrt{\pi}} \left(\sum_{k=0}^{n} \frac{\left(-\frac{1}{6}\right)_k \left(\frac{7}{6}\right)_k}{k!} \left(\frac{3}{4 z^{3/2}}\right)^k + O\left(\frac{1}{z^{\frac{3(n+1)}{2}}}\right)\right) /; |\arg(z)| < \frac{\pi}{3} \bigwedge (|z| \to \infty) \bigwedge n \in \mathbb{N}$$

03.08.06.0019.01

$$\mathrm{Bi}'(z) \propto \frac{e^{\frac{2z^{3/2}}{3}} \sqrt[4]{z}}{\sqrt{\pi}} \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{6}\right)_k \left(\frac{7}{6}\right)_k}{k!} \left(\frac{3}{4 z^{3/2}}\right)^k /; |\arg(z)| < \frac{\pi}{3} \bigwedge (|z| \to \infty)$$

03.08.06.0039.01

$$\mathrm{Bi}'(z) \propto \frac{e^{\frac{2 z^{3/2}}{3}} \sqrt[4]{z}}{\sqrt{\pi}} \left(\sum_{k=0}^{n} \frac{\left(-\frac{1}{12}\right)_k \left(\frac{5}{12}\right)_k \left(\frac{7}{12}\right)_k \left(\frac{13}{12}\right)_k \left(\frac{9}{4 z^3}\right)^k}{\left(\frac{1}{2}\right)_k k!} + O\left(\frac{1}{z^{3(n+1)}}\right) \right) - \frac{7 e^{\frac{2 z^{3/2}}{3}}}{48 \sqrt{\pi} z^{5/4}} \left(\sum_{k=0}^{n} \frac{\left(\frac{5}{12}\right)_k \left(\frac{11}{12}\right)_k \left(\frac{13}{12}\right)_k \left(\frac{19}{12}\right)_k \left(\frac{9}{4 z^3}\right)^k}{\left(\frac{3}{2}\right)_k k!} + O\left(\frac{1}{z^{3(n+1)}}\right) \right) /; |\arg(z)| < \pi \bigwedge (|z| \to \infty) \bigwedge n \in \mathbb{N}$$

03.08.06.0005.01

$$\mathrm{Bi}'(z) \propto \frac{1}{\sqrt{\pi}} e^{\frac{2}{3} z^{3/2}} \sqrt[4]{z} \; {}_2F_0\left(-\frac{1}{6}, \frac{7}{6};; \frac{3}{4 z^{3/2}}\right) /; |\arg(z)| < \frac{\pi}{3} \bigwedge (|z| \to \infty)$$

03.08.06.0006.01

$$\mathrm{Bi}'(z) \propto \frac{1}{\sqrt{\pi}} e^{\frac{2}{3} z^{3/2}} \sqrt[4]{z} \left(1 + O\left(\frac{1}{z^{3/2}}\right)\right) /; |\arg(z)| < \frac{\pi}{3} \bigwedge (|z| \to \infty)$$

In trigonometric form ||| In trigonometric form

03.08.06.0020.01

$$\mathrm{Bi}'(-z) \propto \frac{\sqrt[4]{z}}{\sqrt{\pi}} \left(\sin\left(\frac{2 z^{3/2}}{3} + \frac{\pi}{4}\right) \left(1 + \frac{455}{4608 z^3} - \frac{40\,415\,375}{127\,401\,984 z^6} + O\left(\frac{1}{z^9}\right)\right) + \frac{7}{48 z^{3/2}} \cos\left(\frac{2 z^{3/2}}{3} + \frac{\pi}{4}\right) \left(1 - \frac{13\,585}{13\,824 z^3} + \frac{823\,318\,925}{127\,401\,984 z^6} + O\left(\frac{1}{z^9}\right)\right)\right) /; |\arg(z)| < \frac{2\pi}{3} \bigwedge (|z| \to \infty)$$

03.08.06.0021.01

$$\mathrm{Bi}'(-z) \propto \frac{\sqrt[4]{z}}{\sqrt{\pi}} \left(\sin\left(\frac{2 z^{3/2}}{3} + \frac{\pi}{4}\right) \left(\sum_{k=0}^{n} \frac{\left(-\frac{1}{12}\right)_k \left(\frac{5}{12}\right)_k \left(\frac{7}{12}\right)_k \left(\frac{13}{12}\right)_k}{\left(\frac{1}{2}\right)_k k!} \left(-\frac{9}{4 z^3}\right)^k + O\left(\frac{1}{z^{3n+3}}\right) \right) + \frac{7}{48 z^{3/2}} \cos\left(\frac{2 z^{3/2}}{3} + \frac{\pi}{4}\right) \left(\sum_{k=0}^{n} \frac{\left(\frac{5}{12}\right)_k \left(\frac{11}{12}\right)_k \left(\frac{13}{12}\right)_k \left(\frac{19}{12}\right)_k}{\left(\frac{3}{2}\right)_k k!} \left(-\frac{9}{4 z^3}\right)^k + O\left(\frac{1}{z^{3n+3}}\right) \right) \right) /; |\arg(z)| < \frac{2\pi}{3} \bigwedge (|z| \to \infty) \bigwedge n \in \mathbb{N}$$

03.08.06.0022.01

$$\mathrm{Bi}'(-z) \propto \frac{\sqrt[4]{z}}{\sqrt{\pi}} \left(\sin\left(\frac{2 z^{3/2}}{3} + \frac{\pi}{4}\right) \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{12}\right)_k \left(\frac{5}{12}\right)_k \left(\frac{7}{12}\right)_k \left(\frac{13}{12}\right)_k}{\left(\frac{1}{2}\right)_k k!} \left(-\frac{9}{4 z^3}\right)^k + \frac{7}{48 z^{3/2}} \cos\left(\frac{2 z^{3/2}}{3} + \frac{\pi}{4}\right) \sum_{k=0}^{\infty} \frac{\left(\frac{5}{12}\right)_k \left(\frac{11}{12}\right)_k \left(\frac{13}{12}\right)_k \left(\frac{19}{12}\right)_k}{\left(\frac{3}{2}\right)_k k!} \left(-\frac{9}{4 z^3}\right)^k \right) /; |\arg(z)| < \frac{2\pi}{3} \bigwedge (|z| \to \infty)$$

03.08.06.0007.01

$$\mathrm{Bi}'(-z) \propto \frac{\sqrt[4]{z}}{\sqrt{\pi}} \left(\frac{7}{48 z^{3/2}} \cos\left(\frac{2 z^{3/2}}{3} + \frac{\pi}{4}\right) {}_4F_1\left(\frac{5}{12}, \frac{11}{12}, \frac{13}{12}, \frac{19}{12}; \frac{3}{2}; -\frac{9}{4 z^3}\right) + {}_4F_1\left(-\frac{1}{12}, \frac{5}{12}, \frac{7}{12}, \frac{13}{12}; \frac{1}{2}; -\frac{9}{4 z^3}\right) \sin\left(\frac{2 z^{3/2}}{3} + \frac{\pi}{4}\right)\right) /; |\arg(z)| < \frac{2\pi}{3} \bigwedge (|z| \to \infty)$$

03.08.06.0008.01

$$\mathrm{Bi}'(-z) \propto \frac{\sqrt[4]{z}}{\sqrt{\pi}} \sin\left(\frac{2 z^{3/2}}{3} + \frac{\pi}{4}\right)\left(1 + O\left(\frac{1}{z^3}\right)\right) /; |\arg(z)| < \frac{2\pi}{3} \bigwedge (|z| \to \infty)$$

03.08.06.0009.01

$$\mathrm{Bi}'\left(e^{\frac{\pi i}{3}} z\right) \propto e^{-\frac{i\pi}{6}} \sqrt{\frac{2}{\pi}} \sqrt[4]{z} \left(\cos\left(\frac{2 z^{3/2}}{3} - \frac{1}{2} i \log(2) + \frac{\pi}{4}\right) {}_4F_1\left(-\frac{1}{12}, \frac{5}{12}, \frac{7}{12}, \frac{13}{12}; \frac{1}{2}; -\frac{9}{4 z^3}\right) - \frac{7}{48 z^{3/2}} \sin\left(\frac{2 z^{3/2}}{3} - \frac{1}{2} i \log(2) + \frac{\pi}{4}\right) {}_4F_1\left(\frac{5}{12}, \frac{11}{12}, \frac{13}{12}, \frac{19}{12}; \frac{3}{2}; -\frac{9}{4 z^3}\right)\right) /; |\arg(z)| < \frac{2\pi}{3} \bigwedge (|z| \to \infty)$$

03.08.06.0010.01

$$\mathrm{Bi}'\left(e^{\frac{\pi i}{3}} z\right) \propto e^{-\frac{i\pi}{6}} \sqrt{\frac{2}{\pi}} \sqrt[4]{z} \left(\cos\left(\frac{2 z^{3/2}}{3} - \frac{1}{2} i \log(2) + \frac{\pi}{4}\right)\left(1 + O\left(\frac{1}{z^3}\right)\right) - \frac{7}{48 z^{3/2}} \sin\left(\frac{2 z^{3/2}}{3} - \frac{1}{2} i \log(2) + \frac{\pi}{4}\right)\left(1 + O\left(\frac{1}{z^3}\right)\right)\right) /; |\arg(z)| < \frac{2\pi}{3} \bigwedge (|z| \to \infty)$$

03.08.06.0011.01

$$\mathrm{Bi}'\left(e^{-\frac{\pi i}{3}} z\right) \propto e^{\frac{i\pi}{6}} \sqrt{\frac{2}{\pi}} \sqrt[4]{z} \left(\cos\left(\frac{2 z^{3/2}}{3} + \frac{1}{2} i \log(2) + \frac{\pi}{4}\right) {}_4F_1\left(-\frac{1}{12}, \frac{5}{12}, \frac{7}{12}, \frac{13}{12}; \frac{1}{2}; -\frac{9}{4 z^3}\right) - \frac{7}{48 z^{3/2}} \sin\left(\frac{2 z^{3/2}}{3} + \frac{1}{2} i \log(2) + \frac{\pi}{4}\right) {}_4F_1\left(\frac{5}{12}, \frac{11}{12}, \frac{13}{12}, \frac{19}{12}; \frac{3}{2}; -\frac{9}{4 z^3}\right)\right) /; |\arg(z)| < \frac{2\pi}{3} \bigwedge (|z| \to \infty)$$

03.08.06.0012.01

$$\mathrm{Bi}'\left(e^{-\frac{\pi i}{3}} z\right) \propto e^{\frac{i\pi}{6}} \sqrt{\frac{2}{\pi}} \sqrt[4]{z} \left(\cos\left(\frac{2 z^{3/2}}{3} + \frac{1}{2} i \log(2) + \frac{\pi}{4}\right)\left(1 + O\left(\frac{1}{z^3}\right)\right) - \frac{7}{48 z^{3/2}} \sin\left(\frac{2 z^{3/2}}{3} + \frac{1}{2} i \log(2) + \frac{\pi}{4}\right)\left(1 + O\left(\frac{1}{z^3}\right)\right)\right) /; |\arg(z)| < \frac{2\pi}{3} \bigwedge (|z| \to \infty)$$

Expansions for any z in exponential form

Using exponential function with branch cut-containing arguments

03.08.06.0023.01

$$\mathrm{Bi}'(z) \propto \frac{1}{2\sqrt{\pi}\,(-z^3)^{7/12}} \left(\sqrt[12]{-1}\, e^{\frac{2}{3} i \sqrt{-z^3}} \left((-z^3)^{2/3} + (-1)^{-2/3} z^2\right)\left(1 + \frac{7 i}{48\sqrt{-z^3}} - \frac{455}{4608 z^3} + O\left(\frac{1}{z^{9/2}}\right)\right) - (-1)^{11/12} e^{\frac{1}{3}(-2) i \sqrt{-z^3}} \left((-z^3)^{2/3} + (-1)^{2/3} z^2\right)\left(1 - \frac{7 i}{48\sqrt{-z^3}} - \frac{455}{4608 z^3} + O\left(\frac{1}{z^{9/2}}\right)\right)\right) /; (|z| \to \infty)$$

03.08.06.0024.01

$$\mathrm{Bi}'(z) \propto \frac{1}{2\sqrt{\pi}\,(-z^3)^{7/12}} \left(-(-1)^{11/12} e^{\frac{1}{3}(-2)\,i\sqrt{-z^3}} \left((-z^3)^{2/3} + (-1)^{2/3} z^2\right) \left(\sum_{k=0}^{n} \frac{\left(-\frac{1}{6}\right)_k \left(\frac{7}{6}\right)_k}{k!} \left(\frac{3\,i}{4\sqrt{-z^3}}\right)^k + O\left(\frac{1}{z^{\frac{3(n+1)}{2}}}\right)\right) + \sqrt[12]{-1}\, e^{\frac{2}{3} i \sqrt{-z^3}} \left((-z^3)^{2/3} + (-1)^{-2/3} z^2\right) \left(\sum_{k=0}^{n} \frac{\left(-\frac{1}{6}\right)_k \left(\frac{7}{6}\right)_k}{k!} \left(-\frac{3\,i}{4\sqrt{-z^3}}\right)^k + O\left(\frac{1}{z^{\frac{3(n+1)}{2}}}\right)\right)\right) /;\ (|z| \to \infty) \land n \in \mathbb{N}$$

03.08.06.0025.01

$$\mathrm{Bi}'(z) \propto \frac{1}{2\sqrt{\pi}\,(-z^3)^{7/12}} \left(-(-1)^{11/12} e^{\frac{1}{3}(-2)\,i\sqrt{-z^3}} \left((-z^3)^{2/3} + (-1)^{2/3} z^2\right) \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{6}\right)_k \left(\frac{7}{6}\right)_k}{k!} \left(\frac{3\,i}{4\sqrt{-z^3}}\right)^k + \sqrt[12]{-1}\, e^{\frac{2}{3} i \sqrt{-z^3}} \left((-z^3)^{2/3} + (-1)^{-2/3} z^2\right) \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{6}\right)_k \left(\frac{7}{6}\right)_k}{k!} \left(-\frac{3\,i}{4\sqrt{-z^3}}\right)^k \right) /;\ (|z| \to \infty)$$

03.08.06.0013.01

$$\mathrm{Bi}'(z) \propto \frac{1}{2\sqrt{\pi}\,(-z^3)^{7/12}} \left(\sqrt[12]{-1}\, e^{\frac{2}{3} i \sqrt{-z^3}} \left((-z^3)^{2/3} + (-1)^{-2/3} z^2\right) {}_2F_0\left(\frac{7}{6}, -\frac{1}{6};; -\frac{3\,i}{4\sqrt{-z^3}}\right) - (-1)^{11/12} e^{-\frac{2}{3} i \sqrt{-z^3}} \left((-z^3)^{2/3} + (-1)^{2/3} z^2\right) {}_2F_0\left(\frac{7}{6}, -\frac{1}{6};; \frac{3\,i}{4\sqrt{-z^3}}\right)\right) /;\ (|z| \to \infty)$$

03.08.06.0040.01

$$\mathrm{Bi}'(z) \propto \frac{\sqrt[4]{-1}}{4\sqrt{\pi}\,(-z^3)^{7/12}} \left(\left(e^{\frac{2i}{3}\sqrt{-z^3}} \left(\left(-i + \sqrt{3}\right)(-z^3)^{2/3} - \left(i + \sqrt{3}\right) z^2\right) + e^{-\frac{2i}{3}\sqrt{-z^3}}\, i \left(\left(-i + \sqrt{3}\right) z^2 - \left(i + \sqrt{3}\right)(-z^3)^{2/3}\right)\right) \left(\sum_{k=0}^{n} \frac{\left(-\frac{1}{12}\right)_k \left(\frac{5}{12}\right)_k \left(\frac{7}{12}\right)_k \left(\frac{13}{12}\right)_k}{k!\left(\frac{1}{2}\right)_k} \left(\frac{9}{4z^3}\right)^k + O\left(\frac{1}{z^{3n+3}}\right)\right) + \frac{7}{48\sqrt{-z^3}} \left(i\, e^{\frac{2i}{3}\sqrt{-z^3}} \left(\left(-i + \sqrt{3}\right)(-z^3)^{2/3} - \left(i + \sqrt{3}\right) z^2\right) + e^{-\frac{2i}{3}\sqrt{-z^3}} \left(\left(-i + \sqrt{3}\right) z^2 - \left(i + \sqrt{3}\right)(-z^3)^{2/3}\right)\right) \left(\sum_{k=0}^{n} \frac{\left(\frac{5}{12}\right)_k \left(\frac{11}{12}\right)_k \left(\frac{13}{12}\right)_k \left(\frac{19}{12}\right)_k}{k!\left(\frac{3}{2}\right)_k} \left(\frac{9}{4z^3}\right)^k + O\left(\frac{1}{z^{3n+3}}\right)\right)\right) /;\ (|z| \to \infty) \land n \in \mathbb{N}$$

03.08.06.0041.01

$$\text{Bi}'(z) \propto \frac{\sqrt[4]{-1}}{4\sqrt{\pi}\,(-z^3)^{7/12}} \Bigg(\Big(e^{\frac{2i}{3}\sqrt{-z^3}} \left(\left(-i+\sqrt{3}\right)(-z^3)^{2/3} - \left(i+\sqrt{3}\right) z^2 \right) + e^{-\frac{2i}{3}\sqrt{-z^3}}\, i \left(\left(-i+\sqrt{3}\right) z^2 - \left(i+\sqrt{3}\right)(-z^3)^{2/3} \right) \Big) \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{12}\right)_k \left(\frac{5}{12}\right)_k \left(\frac{7}{12}\right)_k \left(\frac{13}{12}\right)_k}{k!\left(\frac{1}{2}\right)_k} \left(\frac{9}{4z^3}\right)^k + \frac{7}{48\sqrt{-z^3}} \Big(i\, e^{\frac{2i}{3}\sqrt{-z^3}} \left(\left(-i+\sqrt{3}\right)(-z^3)^{2/3} - \left(i+\sqrt{3}\right) z^2 \right) + e^{-\frac{2i}{3}\sqrt{-z^3}} \left(\left(-i+\sqrt{3}\right) z^2 - \left(i+\sqrt{3}\right)(-z^3)^{2/3} \right) \Big) \sum_{k=0}^{\infty} \frac{\left(\frac{5}{12}\right)_k \left(\frac{11}{12}\right)_k \left(\frac{13}{12}\right)_k \left(\frac{19}{12}\right)_k}{k!\left(\frac{3}{2}\right)_k} \left(\frac{9}{4z^3}\right)^k \Bigg) \;/;\; (|z| \to \infty)$$

03.08.06.0042.01

$$\text{Bi}'(z) \propto \frac{\sqrt[4]{-1}}{4\sqrt{\pi}\,(-z^3)^{7/12}} \Bigg(\Big(e^{\frac{2i}{3}\sqrt{-z^3}} \left(\left(-i+\sqrt{3}\right)(-z^3)^{2/3} - \left(i+\sqrt{3}\right) z^2 \right) + e^{-\frac{2i}{3}\sqrt{-z^3}}\, i \left(\left(-i+\sqrt{3}\right) z^2 - \left(i+\sqrt{3}\right)(-z^3)^{2/3} \right) \Big)\, {}_4F_1\!\left(-\frac{1}{12}, \frac{5}{12}, \frac{7}{12}, \frac{13}{12}; \frac{1}{2}; \frac{9}{4z^3}\right) + \frac{7}{48\sqrt{-z^3}} \Big(i\, e^{\frac{2i}{3}\sqrt{-z^3}} \left(\left(-i+\sqrt{3}\right)(-z^3)^{2/3} - \left(i+\sqrt{3}\right) z^2 \right) + e^{-\frac{2i}{3}\sqrt{-z^3}} \left(\left(-i+\sqrt{3}\right) z^2 - \left(i+\sqrt{3}\right)(-z^3)^{2/3} \right) \Big)\, {}_4F_1\!\left(\frac{5}{12}, \frac{11}{12}, \frac{13}{12}, \frac{19}{12}; \frac{3}{2}; \frac{9}{4z^3}\right) \Bigg) \;/;\; (|z| \to \infty)$$

03.08.06.0014.01

$$\text{Bi}'(z) \propto \frac{1}{2\sqrt{\pi}\,(-z^3)^{7/12}} \left(\sqrt[12]{-1}\, e^{\frac{2i}{3}\sqrt{-z^3}} \left((-z^3)^{2/3} + (-1)^{-2/3} z^2 \right) \left(1 + O\!\left(\frac{1}{z^{3/2}}\right)\right) - (-1)^{11/12} e^{-\frac{2i}{3}\sqrt{-z^3}} \left((-z^3)^{2/3} + (-1)^{2/3} z^2 \right) \left(1 + O\!\left(\frac{1}{z^{3/2}}\right)\right) \right) \;/;\; (|z| \to \infty)$$

Using exponential function with branch cut-free arguments

03.08.06.0043.01

$$\text{Bi}'(z) \propto \frac{1}{4\sqrt{2\pi}\, z\,(-z^3)^{5/12}} \Bigg(\Big(e^{\frac{2z^{3/2}}{3}} \left(\left(1+\sqrt{3}\right)\sqrt[3]{-z^3}\, z^{3/2} + \left(-1+\sqrt{3}\right) z^{5/2} + \left(1+\sqrt{3}\right)\sqrt{-z^3}\, z + \left(-1+\sqrt{3}\right)(-z^3)^{5/6} \right) + e^{-\frac{1}{3}(2z^{3/2})} \left(-\left(1+\sqrt{3}\right)\sqrt[3]{-z^3}\, z^{3/2} - \left(-1+\sqrt{3}\right) z^{5/2} + \left(1+\sqrt{3}\right)\sqrt{-z^3}\, z + \left(-1+\sqrt{3}\right)(-z^3)^{5/6} \right) \Big) \left(1 - \frac{455}{4608 z^3} - \frac{40\,415\,375}{127\,401\,984 z^6} - \frac{6\,183\,948\,445\,675}{1\,761\,205\,026\,816 z^9} + O\!\left(\frac{1}{z^{12}}\right)\right) - \frac{7}{96 z^{3/2}} \Big(e^{\frac{2z^{3/2}}{3}} \left(\left(1+\sqrt{3}\right)\sqrt[3]{-z^3}\, z^{3/2} + \left(-1+\sqrt{3}\right) z^{5/2} + \left(1+\sqrt{3}\right)\sqrt{-z^3}\, z + \left(-1+\sqrt{3}\right)(-z^3)^{5/6} \right) + e^{-\frac{1}{3}(2z^{3/2})} \left(\left(1+\sqrt{3}\right)\sqrt[3]{-z^3}\, z^{3/2} + \left(-1+\sqrt{3}\right) z^{5/2} - \left(1+\sqrt{3}\right)\sqrt{-z^3}\, z - \left(-1+\sqrt{3}\right)(-z^3)^{5/6} \right) \Big) \left(1 + \frac{13\,585}{13\,824 z^3} + \frac{823\,318\,925}{127\,401\,984 z^6} + \frac{189\,935\,559\,402\,875}{1\,761\,205\,026\,816 z^9} + O\!\left(\frac{1}{z^{12}}\right)\right) \Bigg) \;/;\; (|z| \to \infty)$$

03.08.06.0044.01

$$\operatorname{Bi}'(z) \propto \frac{1}{4\sqrt{2\pi}\, z\,(-z^3)^{5/12}} \Bigg(\bigg(e^{\frac{2 z^{3/2}}{3}} \Big((1+\sqrt{3})\sqrt[3]{-z^3}\, z^{3/2} + (-1+\sqrt{3}) z^{5/2} + (1+\sqrt{3})\sqrt{-z^3}\, z + (-1+\sqrt{3})(-z^3)^{5/6}\Big) + e^{-\frac{1}{3}(2 z^{3/2})} \Big(-(1+\sqrt{3})\sqrt[3]{-z^3}\, z^{3/2} - (-1+\sqrt{3}) z^{5/2} + (1+\sqrt{3})\sqrt{-z^3}\, z + (-1+\sqrt{3})(-z^3)^{5/6}\Big)\bigg) \left(\sum_{k=0}^{n} \frac{\left(-\frac{1}{12}\right)_k \left(\frac{5}{12}\right)_k \left(\frac{7}{12}\right)_k \left(\frac{13}{12}\right)_k}{k!\left(\frac{1}{2}\right)_k} \left(\frac{9}{4 z^3}\right)^k + O\!\left(\frac{1}{z^{3n+3}}\right)\right) - \frac{7}{96 z^{3/2}} \bigg(e^{\frac{2 z^{3/2}}{3}} \Big((1+\sqrt{3})\sqrt[3]{-z^3}\, z^{3/2} + (-1+\sqrt{3}) z^{5/2} + (1+\sqrt{3})\sqrt{-z^3}\, z + (-1+\sqrt{3})(-z^3)^{5/6}\Big) + e^{-\frac{1}{3}(2 z^{3/2})} \Big((1+\sqrt{3})\sqrt[3]{-z^3}\, z^{3/2} + (-1+\sqrt{3}) z^{5/2} - (1+\sqrt{3})\sqrt{-z^3}\, z - (-1+\sqrt{3})(-z^3)^{5/6}\Big)\bigg) \left(\sum_{k=0}^{n} \frac{\left(\frac{5}{12}\right)_k \left(\frac{11}{12}\right)_k \left(\frac{13}{12}\right)_k \left(\frac{19}{12}\right)_k}{k!\left(\frac{3}{2}\right)_k} \left(\frac{9}{4 z^3}\right)^k + O\!\left(\frac{1}{z^{3n+3}}\right)\right)\Bigg) \;/;\; (|z| \to \infty) \land n \in \mathbb{N}$$

03.08.06.0045.01

$$\operatorname{Bi}'(z) \propto \frac{1}{4\sqrt{2\pi}\, z\,(-z^3)^{5/12}} \Bigg(\bigg(e^{\frac{2 z^{3/2}}{3}} \Big((1+\sqrt{3})\sqrt[3]{-z^3}\, z^{3/2} + (-1+\sqrt{3}) z^{5/2} + (1+\sqrt{3})\sqrt{-z^3}\, z + (-1+\sqrt{3})(-z^3)^{5/6}\Big) + e^{-\frac{1}{3}(2 z^{3/2})} \Big(-(1+\sqrt{3})\sqrt[3]{-z^3}\, z^{3/2} - (-1+\sqrt{3}) z^{5/2} + (1+\sqrt{3})\sqrt{-z^3}\, z + (-1+\sqrt{3})(-z^3)^{5/6}\Big)\bigg) \sum_{k=0}^{\infty} \frac{\left(-\frac{1}{12}\right)_k \left(\frac{5}{12}\right)_k \left(\frac{7}{12}\right)_k \left(\frac{13}{12}\right)_k}{k!\left(\frac{1}{2}\right)_k} \left(\frac{9}{4 z^3}\right)^k - \frac{7}{96 z^{3/2}} \bigg(e^{\frac{2 z^{3/2}}{3}} \Big((1+\sqrt{3})\sqrt[3]{-z^3}\, z^{3/2} + (-1+\sqrt{3}) z^{5/2} + (1+\sqrt{3})\sqrt{-z^3}\, z + (-1+\sqrt{3})(-z^3)^{5/6}\Big) + e^{-\frac{1}{3}(2 z^{3/2})} \Big((1+\sqrt{3})\sqrt[3]{-z^3}\, z^{3/2} + (-1+\sqrt{3}) z^{5/2} - (1+\sqrt{3})\sqrt{-z^3}\, z - (-1+\sqrt{3})(-z^3)^{5/6}\Big)\bigg) \sum_{k=0}^{\infty} \frac{\left(\frac{5}{12}\right)_k \left(\frac{11}{12}\right)_k \left(\frac{13}{12}\right)_k \left(\frac{19}{12}\right)_k}{k!\left(\frac{3}{2}\right)_k} \left(\frac{9}{4 z^3}\right)^k\Bigg) \;/;\; (|z| \to \infty)$$

03.08.06.0046.01

$$\mathrm{Bi}'(z) \propto \frac{1}{4\sqrt{2\pi}\, z\left(-z^3\right)^{5/12}} \Bigg(\Bigg(e^{\frac{2z^{3/2}}{3}} \left(\left(1+\sqrt{3}\right)\sqrt[3]{-z^3}\, z^{3/2} + \left(-1+\sqrt{3}\right) z^{5/2} + \left(1+\sqrt{3}\right)\sqrt{-z^3}\, z + \left(-1+\sqrt{3}\right)\left(-z^3\right)^{5/6}\right) + e^{-\frac{1}{3}\left(2z^{3/2}\right)} \left(-\left(1+\sqrt{3}\right)\sqrt[3]{-z^3}\, z^{3/2} - \left(-1+\sqrt{3}\right) z^{5/2} + \left(1+\sqrt{3}\right)\sqrt{-z^3}\, z + \left(-1+\sqrt{3}\right)\left(-z^3\right)^{5/6}\right)\Bigg) {}_4F_1\left(-\frac{1}{12}, \frac{5}{12}, \frac{7}{12}, \frac{13}{12}; \frac{1}{2}; \frac{9}{4z^3}\right) - \frac{7}{96 z^{3/2}} \Bigg(e^{\frac{2z^{3/2}}{3}} \left(\left(1+\sqrt{3}\right)\sqrt[3]{-z^3}\, z^{3/2} + \left(-1+\sqrt{3}\right) z^{5/2} + \left(1+\sqrt{3}\right)\sqrt{-z^3}\, z + \left(-1+\sqrt{3}\right)\left(-z^3\right)^{5/6}\right) + e^{-\frac{1}{3}\left(2z^{3/2}\right)} \left(\left(1+\sqrt{3}\right)\sqrt[3]{-z^3}\, z^{3/2} + \left(-1+\sqrt{3}\right) z^{5/2} - \left(1+\sqrt{3}\right)\sqrt{-z^3}\, z - \left(-1+\sqrt{3}\right)\left(-z^3\right)^{5/6}\right)\Bigg) {}_4F_1\left(\frac{5}{12}, \frac{11}{12}, \frac{13}{12}, \frac{19}{12}; \frac{3}{2}; \frac{9}{4z^3}\right)\Bigg) /; (|z| \to \infty)$$

03.08.06.0047.01

$$\mathrm{Bi}'(z) \propto \frac{1}{4\sqrt{2\pi}\, z\left(-z^3\right)^{5/12}} \Bigg(\Bigg(e^{\frac{2z^{3/2}}{3}} \left(\left(1+\sqrt{3}\right)\sqrt[3]{-z^3}\, z^{3/2} + \left(-1+\sqrt{3}\right) z^{5/2} + \left(1+\sqrt{3}\right)\sqrt{-z^3}\, z + \left(-1+\sqrt{3}\right)\left(-z^3\right)^{5/6}\right) + e^{-\frac{1}{3}\left(2z^{3/2}\right)} \left(-\left(1+\sqrt{3}\right)\sqrt[3]{-z^3}\, z^{3/2} - \left(-1+\sqrt{3}\right) z^{5/2} + \left(1+\sqrt{3}\right)\sqrt{-z^3}\, z + \left(-1+\sqrt{3}\right)\left(-z^3\right)^{5/6}\right)\Bigg)\left(1 + O\left(\frac{1}{z^3}\right)\right) - \frac{7}{96 z^{3/2}} \Bigg(e^{\frac{2z^{3/2}}{3}} \left(\left(1+\sqrt{3}\right)\sqrt[3]{-z^3}\, z^{3/2} + \left(-1+\sqrt{3}\right) z^{5/2} + \left(1+\sqrt{3}\right)\sqrt{-z^3}\, z + \left(-1+\sqrt{3}\right)\left(-z^3\right)^{5/6}\right) + e^{-\frac{1}{3}\left(2z^{3/2}\right)} \left(\left(1+\sqrt{3}\right)\sqrt[3]{-z^3}\, z^{3/2} + \left(-1+\sqrt{3}\right) z^{5/2} - \left(1+\sqrt{3}\right)\sqrt{-z^3}\, z - \left(-1+\sqrt{3}\right)\left(-z^3\right)^{5/6}\right)\Bigg)\left(1 + O\left(\frac{1}{z^3}\right)\right)\Bigg) /; (|z| \to \infty)$$

03.08.06.0048.01

$$\mathrm{Bi}'(z) \propto \begin{cases} \frac{e^{\frac{2z^{3/2}}{3}}\sqrt[4]{z}}{2\sqrt{\pi}} + \frac{i\, e^{-\frac{2z^{3/2}}{3}}\sqrt[4]{z}}{2\sqrt{\pi}} & \arg(z) \le -\frac{2\pi}{3} \\ \frac{e^{\frac{2z^{3/2}}{3}}\sqrt[4]{z}}{\sqrt{\pi}} + \frac{i\, e^{-\frac{2z^{3/2}}{3}}\sqrt[4]{z}}{2\sqrt{\pi}} & -\frac{2\pi}{3} < \arg(z) \le 0 \\ \frac{e^{\frac{2z^{3/2}}{3}}\sqrt[4]{z}}{\sqrt{\pi}} - \frac{i\, e^{-\frac{2z^{3/2}}{3}}\sqrt[4]{z}}{2\sqrt{\pi}} & 0 < \arg(z) \le \frac{2\pi}{3} \\ \frac{e^{\frac{2z^{3/2}}{3}}\sqrt[4]{z}}{2\sqrt{\pi}} - \frac{i\, e^{-\frac{2z^{3/2}}{3}}\sqrt[4]{z}}{2\sqrt{\pi}} & \text{True} \end{cases} /; (|z| \to \infty)$$

Expansions for any z in trigonometric form

Using trigonometric functions with branch cut-containing arguments

03.08.06.0026.01

$$\mathrm{Bi}'(z) \propto \frac{1}{2\sqrt{\pi}\left(-z^3\right)^{7/12}}\left(\frac{7}{48\sqrt{-z^3}}\left(\left(\left(-z^3\right)^{2/3}+z^2\right)\cos\left(\frac{2\sqrt{-z^3}}{3}+\frac{\pi}{4}\right)-\sqrt{3}\left(\left(-z^3\right)^{2/3}-z^2\right)\cos\left(\frac{\pi}{4}-\frac{2\sqrt{-z^3}}{3}\right)\right)\left(1+\frac{13585}{13824\,z^3}+\frac{823318925}{127401984\,z^6}+O\left(\frac{1}{z^9}\right)\right)+\left(\sqrt{3}\left(\left(-z^3\right)^{2/3}-z^2\right)\cos\left(\frac{2\sqrt{-z^3}}{3}+\frac{\pi}{4}\right)+\left(\left(-z^3\right)^{2/3}+z^2\right)\cos\left(\frac{\pi}{4}-\frac{2\sqrt{-z^3}}{3}\right)\right)\left(1-\frac{455}{4608\,z^3}-\frac{40415375}{127401984\,z^6}+O\left(\frac{1}{z^9}\right)\right)\right)\ /;\ (|z|\to\infty)$$

03.08.06.0027.01

$$\mathrm{Bi}'(z) \propto \frac{1}{2\sqrt{\pi}\left(-z^3\right)^{7/12}}\left(\left(\sqrt{3}\left(\left(-z^3\right)^{2/3}-z^2\right)\cos\left(\frac{2\sqrt{-z^3}}{3}+\frac{\pi}{4}\right)+\left(\left(-z^3\right)^{2/3}+z^2\right)\cos\left(\frac{\pi}{4}-\frac{2\sqrt{-z^3}}{3}\right)\right)\left(\sum_{k=0}^{n}\frac{\left(-\frac{1}{12}\right)_k\left(\frac{5}{12}\right)_k\left(\frac{7}{12}\right)_k\left(\frac{13}{12}\right)_k}{k!\left(\frac{1}{2}\right)_k}\left(\frac{9}{4z^3}\right)^k+O\left(\frac{1}{z^{3n+3}}\right)\right)+\frac{7}{48\sqrt{-z^3}}\left(\left(\left(-z^3\right)^{2/3}+z^2\right)\cos\left(\frac{2\sqrt{-z^3}}{3}+\frac{\pi}{4}\right)-\sqrt{3}\left(\left(-z^3\right)^{2/3}-z^2\right)\cos\left(\frac{\pi}{4}-\frac{2\sqrt{-z^3}}{3}\right)\right)\left(\sum_{k=0}^{n}\frac{\left(\frac{5}{12}\right)_k\left(\frac{11}{12}\right)_k\left(\frac{13}{12}\right)_k\left(\frac{19}{12}\right)_k}{k!\left(\frac{3}{2}\right)_k}\left(\frac{9}{4z^3}\right)^k+O\left(\frac{1}{z^{3n+3}}\right)\right)\right)\ /;\ (|z|\to\infty)\land n\in\mathbb{N}$$

03.08.06.0028.01

$$\mathrm{Bi}'(z) \propto \frac{1}{2\sqrt{\pi}\left(-z^3\right)^{7/12}}\left(\left(\sqrt{3}\left(\left(-z^3\right)^{2/3}-z^2\right)\cos\left(\frac{2\sqrt{-z^3}}{3}+\frac{\pi}{4}\right)+\left(\left(-z^3\right)^{2/3}+z^2\right)\cos\left(\frac{\pi}{4}-\frac{2\sqrt{-z^3}}{3}\right)\right)\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{12}\right)_k\left(\frac{5}{12}\right)_k\left(\frac{7}{12}\right)_k\left(\frac{13}{12}\right)_k}{k!\left(\frac{1}{2}\right)_k}\left(\frac{9}{4z^3}\right)^k+\frac{t}{48\sqrt{-z^3}}\left(\left(\left(-z^3\right)^{2/3}+z^2\right)\cos\left(\frac{2\sqrt{-z^3}}{3}+\frac{\pi}{4}\right)-\sqrt{3}\left(\left(-z^3\right)^{2/3}-z^2\right)\cos\left(\frac{\pi}{4}-\frac{2\sqrt{-z^3}}{3}\right)\right)\sum_{k=0}^{\infty}\frac{\left(\frac{5}{12}\right)_k\left(\frac{11}{12}\right)_k\left(\frac{13}{12}\right)_k\left(\frac{19}{12}\right)_k}{k!\left(\frac{3}{2}\right)_k}\left(\frac{9}{4z^3}\right)^k\right)\ /;\ (|z|\to\infty)$$

03.08.06.0029.01

$$\mathrm{Bi}'(z) \propto \frac{1}{2\sqrt{\pi}\,(-z^3)^{7/12}} \left(\frac{7}{48\sqrt{-z^3}} \left(\left((-z^3)^{2/3} + z^2\right) \cos\left(\frac{2\sqrt{-z^3}}{3} + \frac{\pi}{4}\right) - \sqrt{3}\left((-z^3)^{2/3} - z^2\right) \cos\left(\frac{\pi}{4} - \frac{2\sqrt{-z^3}}{3}\right)\right) {}_4F_1\left(\frac{5}{12}, \frac{11}{12}, \frac{13}{12}, \frac{19}{12}; \frac{3}{2}; \frac{9}{4z^3}\right) + \left(\sqrt{3}\left((-z^3)^{2/3} - z^2\right) \cos\left(\frac{2\sqrt{-z^3}}{3} + \frac{\pi}{4}\right) + \left((-z^3)^{2/3} + z^2\right) \cos\left(\frac{\pi}{4} - \frac{2\sqrt{-z^3}}{3}\right)\right) {}_4F_1\left(-\frac{1}{12}, \frac{5}{12}, \frac{7}{12}, \frac{13}{12}; \frac{1}{2}; \frac{9}{4z^3}\right)\right) /; (|z| \to \infty)$$

03.08.06.0030.01

$$\mathrm{Bi}'(z) \propto \frac{1}{2\sqrt{\pi}\,(-z^3)^{7/12}} \left(\frac{7}{48\sqrt{-z^3}} \left(\left((-z^3)^{2/3} + z^2\right) \cos\left(\frac{2\sqrt{-z^3}}{3} + \frac{\pi}{4}\right) - \sqrt{3}\left((-z^3)^{2/3} - z^2\right) \cos\left(\frac{\pi}{4} - \frac{2\sqrt{-z^3}}{3}\right)\right) \left(1 + O\left(\frac{1}{z^9}\right)\right) + \left(\sqrt{3}\left((-z^3)^{2/3} - z^2\right) \cos\left(\frac{2\sqrt{-z^3}}{3} + \frac{\pi}{4}\right) + \left((-z^3)^{2/3} + z^2\right) \cos\left(\frac{\pi}{4} - \frac{2\sqrt{-z^3}}{3}\right)\right) \left(1 + O\left(\frac{1}{z^9}\right)\right)\right) /; (|z| \to \infty)$$

Using trigonometric functions with branch cut-free arguments

03.08.06.0049.01

$$\mathrm{Bi}'(z) \propto \frac{1}{2\sqrt{2\pi}\, z\,(-z^3)^{5/12}} \left(\left(z^{3/2}\left((1+\sqrt{3})\sqrt[3]{-z^3} + (-1+\sqrt{3})z\right) \sinh\left(\frac{2z^{3/2}}{3}\right) + \sqrt{-z^3}\left((-1+\sqrt{3})\sqrt[3]{-z^3} + (1+\sqrt{3})z\right) \cosh\left(\frac{2z^{3/2}}{3}\right)\right) \left(1 - \frac{455}{4608 z^3} - \frac{40\,415\,375}{127\,401\,984 z^6} - \frac{6\,183\,948\,445\,675}{1\,761\,205\,026\,816 z^9} + O\left(\frac{1}{z^{12}}\right)\right) - \frac{7}{48 z^{3/2}} \left(z^{3/2}\left((1+\sqrt{3})\sqrt[3]{-z^3} + (-1+\sqrt{3})z\right) \cosh\left(\frac{2z^{3/2}}{3}\right) + \sqrt{-z^3}\left((-1+\sqrt{3})\sqrt[3]{-z^3} + (1+\sqrt{3})z\right) \sinh\left(\frac{2z^{3/2}}{3}\right)\right) \left(1 + \frac{13\,585}{13\,824 z^3} + \frac{823\,318\,925}{127\,401\,984 z^6} + \frac{189\,935\,559\,402\,875}{1\,761\,205\,026\,816 z^9} + O\left(\frac{1}{z^{12}}\right)\right)\right) /; (|z| \to \infty)$$

03.08.06.0050.01

$$\mathrm{Bi}'(z) \propto \frac{1}{2\sqrt{2\pi}\, z\left(-z^3\right)^{5/12}} \left(\left(z^{3/2}\left(\left(1+\sqrt{3}\right)\sqrt[3]{-z^3}+\left(-1+\sqrt{3}\right)z\right)\sinh\left(\frac{2z^{3/2}}{3}\right)+\sqrt{-z^3}\left(\left(-1+\sqrt{3}\right)\sqrt[3]{-z^3}+\left(1+\sqrt{3}\right)z\right)\cosh\left(\frac{2z^{3/2}}{3}\right)\right)\left(\sum_{k=0}^{n}\frac{\left(-\frac{1}{12}\right)_k\left(\frac{5}{12}\right)_k\left(\frac{7}{12}\right)_k\left(\frac{13}{12}\right)_k}{k!\left(\frac{1}{2}\right)_k}\left(\frac{9}{4z^3}\right)^k+O\left(\frac{1}{z^{3n+3}}\right)\right)-\frac{7}{48z^{3/2}}\left(z^{3/2}\left(\left(1+\sqrt{3}\right)\sqrt[3]{-z^3}+\left(-1+\sqrt{3}\right)z\right)\cosh\left(\frac{2z^{3/2}}{3}\right)+\sqrt{-z^3}\left(\left(-1+\sqrt{3}\right)\sqrt[3]{-z^3}+\left(1+\sqrt{3}\right)z\right)\sinh\left(\frac{2z^{3/2}}{3}\right)\right)\left(\sum_{k=0}^{n}\frac{\left(\frac{5}{12}\right)_k\left(\frac{11}{12}\right)_k\left(\frac{13}{12}\right)_k\left(\frac{19}{12}\right)_k}{k!\left(\frac{3}{2}\right)_k}\left(\frac{9}{4z^3}\right)^k+O\left(\frac{1}{z^{3n+3}}\right)\right)\right)\;/;\;(|z|\to\infty)\land n\in\mathbb{N}$$

03.08.06.0051.01

$$\mathrm{Bi}'(z) \propto \frac{1}{2\sqrt{2\pi}\, z\left(-z^3\right)^{5/12}} \left(\left(z^{3/2}\left(\left(1+\sqrt{3}\right)\sqrt[3]{-z^3}+\left(-1+\sqrt{3}\right)z\right)\sinh\left(\frac{2z^{3/2}}{3}\right)+\sqrt{-z^3}\left(\left(-1+\sqrt{3}\right)\sqrt[3]{-z^3}+\left(1+\sqrt{3}\right)z\right)\cosh\left(\frac{2z^{3/2}}{3}\right)\right)\sum_{k=0}^{\infty}\frac{\left(-\frac{1}{12}\right)_k\left(\frac{5}{12}\right)_k\left(\frac{7}{12}\right)_k\left(\frac{13}{12}\right)_k}{k!\left(\frac{1}{2}\right)_k}\left(\frac{9}{4z^3}\right)^k-\frac{7}{48z^{3/2}}\left(z^{3/2}\left(\left(1+\sqrt{3}\right)\sqrt[3]{-z^3}+\left(-1+\sqrt{3}\right)z\right)\cosh\left(\frac{2z^{3/2}}{3}\right)+\sqrt{-z^3}\left(\left(-1+\sqrt{3}\right)\sqrt[3]{-z^3}+\left(1+\sqrt{3}\right)z\right)\sinh\left(\frac{2z^{3/2}}{3}\right)\right)\sum_{k=0}^{\infty}\frac{\left(\frac{5}{12}\right)_k\left(\frac{11}{12}\right)_k\left(\frac{13}{12}\right)_k\left(\frac{19}{12}\right)_k}{k!\left(\frac{3}{2}\right)_k}\left(\frac{9}{4z^3}\right)^k\right)\;/;\;(|z|\to\infty)$$

03.08.06.0052.01

$$\mathrm{Bi}'(z) \propto \frac{1}{2\sqrt{2\pi}\, z\left(-z^3\right)^{5/12}} \left(\left(z^{3/2}\left(\left(1+\sqrt{3}\right)\sqrt[3]{-z^3}+\left(-1+\sqrt{3}\right)z\right)\sinh\left(\frac{2z^{3/2}}{3}\right)+\sqrt{-z^3}\left(\left(-1+\sqrt{3}\right)\sqrt[3]{-z^3}+\left(1+\sqrt{3}\right)z\right)\cosh\left(\frac{2z^{3/2}}{3}\right)\right){}_4F_1\left(-\frac{1}{12},\frac{5}{12},\frac{7}{12},\frac{13}{12};\frac{1}{2};\frac{9}{4z^3}\right)-\frac{7}{48z^{3/2}}\left(z^{3/2}\left(\left(1+\sqrt{3}\right)\sqrt[3]{-z^3}+\left(-1+\sqrt{3}\right)z\right)\cosh\left(\frac{2z^{3/2}}{3}\right)+\sqrt{-z^3}\left(\left(-1+\sqrt{3}\right)\sqrt[3]{-z^3}+\left(1+\sqrt{3}\right)z\right)\sinh\left(\frac{2z^{3/2}}{3}\right)\right){}_4F_1\left(\frac{5}{12},\frac{11}{12},\frac{13}{12},\frac{19}{12};\frac{3}{2};\frac{9}{4z^3}\right)\right)\;/;\;(|z|\to\infty)$$

03.08.06.0053.01

$$\mathrm{Bi}'(z) \propto \frac{1}{2\sqrt{2\pi}\, z\,(-z^3)^{5/12}} \left(\left(z^{3/2}\left(\left(1+\sqrt{3}\right)\sqrt[3]{-z^3} + \left(-1+\sqrt{3}\right)z\right)\sinh\left(\frac{2z^{3/2}}{3}\right) + \sqrt{-z^3}\left(\left(-1+\sqrt{3}\right)\sqrt[3]{-z^3} + \left(1+\sqrt{3}\right)z\right)\cosh\left(\frac{2z^{3/2}}{3}\right)\right)\left(1+O\left(\frac{1}{z^3}\right)\right) - \frac{7}{48 z^{3/2}}\left(z^{3/2}\left(\left(1+\sqrt{3}\right)\sqrt[3]{-z^3} + \left(-1+\sqrt{3}\right)z\right)\cosh\left(\frac{2z^{3/2}}{3}\right) + \sqrt{-z^3}\left(\left(-1+\sqrt{3}\right)\sqrt[3]{-z^3} + \left(1+\sqrt{3}\right)z\right)\sinh\left(\frac{2z^{3/2}}{3}\right)\right)\left(1+O\left(\frac{1}{z^3}\right)\right)\right) /;\ (|z| \to \infty)$$

03.08.06.0054.01

$$\mathrm{Bi}'(z) \propto \begin{cases} \frac{\sqrt[4]{-1}\,\sqrt[4]{z}}{\sqrt{2\pi}}\left(\cosh\left(\frac{2z^{3/2}}{3}\right) - i\sinh\left(\frac{2z^{3/2}}{3}\right)\right) & \arg(z) \le -\frac{2\pi}{3} \\ \frac{i\sqrt[4]{z}}{2\sqrt{\pi}}\left((1-2i)\cosh\left(\frac{2z^{3/2}}{3}\right) - (1+2i)\sinh\left(\frac{2z^{3/2}}{3}\right)\right) & -\frac{2\pi}{3} < \arg(z) \le 0 \\ -\frac{i\sqrt[4]{z}}{2\sqrt{\pi}}\left((1+2i)\cosh\left(\frac{2z^{3/2}}{3}\right) - (1-2i)\sinh\left(\frac{2z^{3/2}}{3}\right)\right) & 0 < \arg(z) \le \frac{2\pi}{3} \\ -\frac{(-1)^{3/4}\sqrt[4]{z}}{\sqrt{2\pi}}\left(\cosh\left(\frac{2z^{3/2}}{3}\right) + i\sinh\left(\frac{2z^{3/2}}{3}\right)\right) & \text{True} \end{cases} /;\ (|z| \to \infty)$$

Residue representations

03.08.06.0015.01

$$\mathrm{Bi}'(z) = -2\pi\sqrt[6]{3}\left(\sum_{j=0}^{\infty}\mathrm{res}_s\left(\frac{\Gamma\left(s+\frac{2}{3}\right)\left(3^{-2/3}z\right)^{-3s}}{\Gamma\left(s-\frac{1}{6}\right)\Gamma\left(s+\frac{1}{3}\right)\Gamma\left(\frac{7}{6}-s\right)\Gamma\left(\frac{2}{3}-s\right)}\Gamma(s)\right)(-j) + \sum_{j=0}^{\infty}\mathrm{res}_s\left(\frac{\Gamma(s)\left(3^{-2/3}z\right)^{-3s}}{\Gamma\left(s-\frac{1}{6}\right)\Gamma\left(s+\frac{1}{3}\right)\Gamma\left(\frac{7}{6}-s\right)\Gamma\left(\frac{2}{3}-s\right)}\Gamma\left(s+\frac{2}{3}\right)\right)\left(-j-\frac{2}{3}\right)\right)$$

03.08.06.0016.01

$$\mathrm{Bi}'(z) = \frac{\pi z^2}{3\sqrt[6]{3}}\sum_{j=0}^{\infty}\mathrm{res}_s\left(\frac{\left(\frac{z^3}{9}\right)^{-s}}{\Gamma\left(s+\frac{1}{2}\right)\Gamma\left(\frac{5}{3}-s\right)\Gamma\left(\frac{1}{2}-s\right)}\Gamma(s)\right)(-j) + 3^{1/6}\pi\sum_{j=0}^{\infty}\mathrm{res}_s\left(\frac{\left(\frac{z^3}{9}\right)^{-s}}{\Gamma\left(s+\frac{1}{2}\right)\Gamma\left(\frac{1}{3}-s\right)\Gamma\left(\frac{1}{2}-s\right)}\Gamma(s)\right)(-j)$$

Integral representations

On the real axis

Of the direct function

03.08.07.0001.01

$$\mathrm{Bi}'(z) = \frac{1}{\pi}\int_0^{\infty} t\left(\cos\left(\frac{t^3}{3}+z\,t\right) + e^{z\,t-\frac{t^3}{3}}\right)dt /;\ z < 0$$

Contour integral representations

03.08.07.0002.01

$$\operatorname{Bi}'(z) = -\frac{1}{2\pi}\int_{-\infty}^{\infty e^{\frac{\pi i}{3}}} t\, e^{\frac{t^3}{3}-z t}\, dt - \frac{1}{2\pi}\int_{-\infty}^{\infty e^{-\frac{\pi i}{3}}} t\, e^{\frac{t^3}{3}-z t}\, dt$$

03.08.07.0003.01

$$\operatorname{Bi}'(z) = -\frac{2\pi\sqrt[6]{3}}{2\pi i}\int_{\mathcal{L}} \frac{\Gamma(s)\,\Gamma\left(s+\frac{2}{3}\right)\left(3^{-2/3} z\right)^{-3s}}{\Gamma\left(s-\frac{1}{6}\right)\Gamma\left(s+\frac{1}{3}\right)\Gamma\left(\frac{7}{6}-s\right)\Gamma\left(\frac{2}{3}-s\right)}\, ds$$

03.08.07.0004.01

$$\operatorname{Bi}'(z) = \frac{\pi z^2}{3\sqrt[6]{3}}\,\frac{z}{2\pi i}\int_{\mathcal{L}} \frac{\Gamma(s)}{\Gamma\left(s+\frac{1}{2}\right)\Gamma\left(\frac{5}{3}-s\right)\Gamma\left(\frac{1}{2}-s\right)}\left(\frac{z^3}{9}\right)^{-s} ds + \frac{3^{1/6}\pi}{2\pi i}\int_{\mathcal{L}} \frac{\Gamma(s)}{\Gamma\left(s+\frac{1}{2}\right)\Gamma\left(\frac{1}{3}-s\right)\Gamma\left(\frac{1}{2}-s\right)}\left(\frac{z^3}{9}\right)^{-s} ds$$

Differential equations

Ordinary linear differential equations and wronskians

For the direct function itself

03.08.13.0001.01

$$w''(z)\, z - w'(z) - z^2 w(z) = 0 \;/;\; w(z) = \operatorname{Bi}'(z) \bigwedge w(0) = \frac{\sqrt[6]{3}}{\Gamma\left(\frac{1}{3}\right)} \bigwedge w'(0) = 0$$

03.08.13.0002.01

$$w''(z)\, z - w'(z) - z^2 w(z) = 0 \;/;\; w(z) = \operatorname{Ai}'(z)\, c_1 + c_2 \operatorname{Bi}'(z)$$

03.08.13.0003.01

$$W_z\left(\operatorname{Ai}'(z), \operatorname{Bi}'(z)\right) = -\frac{z}{\pi}$$

03.08.13.0008.01

$$g(z)\, g'(z)\, w''(z) - \left(g'(z)^2 + g(z)\, g''(z)\right) w'(z) - g(z)^2\, g'(z)^3\, w(z) = 0 \;/;\; w(z) = c_1 \operatorname{Ai}'(g(z)) + c_2 \operatorname{Bi}'(g(z))$$

03.08.13.0009.01

$$W_z\left(\operatorname{Ai}'(g(z)), \operatorname{Bi}'(g(z))\right) = -\frac{g(z)\, g'(z)}{\pi}$$

03.08.13.0010.01

$$g(z)\, g'(z)\, h(z)^2\, w''(z) - \left(2 g(z)\, g'(z)\, h'(z) + h(z)\left(g'(z)^2 + g(z)\, g''(z)\right)\right) h(z)\, w'(z) + \left(-g(z)^2 h(z)^2 g'(z)^3 + h(z)\, h'(z)\, g'(z)^2 + g(z)\left(h(z)\, h'(z)\, g''(z) + g'(z)\left(2 h'(z)^2 - h(z)\, h''(z)\right)\right)\right) w(z) = 0 \;/;\; w(z) = c_1 h(z) \operatorname{Ai}'(g(z)) + c_2 h(z) \operatorname{Bi}'(g(z))$$

03.08.13.0011.01

$$W_z\left(h(z) \operatorname{Ai}'(g(z)), h(z) \operatorname{Bi}'(g(z))\right) = -\frac{g(z)\, h(z)^2\, g'(z)}{\pi}$$

03.08.13.0012.01

$$z^2 w''(z) + z(-2r - 2s + 1)\, w'(z) + \left(s(2r+s) - a^3 r^2 z^{3r}\right) w(z) = 0 \;/;\; w(z) = c_1 z^s \operatorname{Ai}'(a z^r) + c_2 z^s \operatorname{Bi}'(a z^r)$$

03.08.13.0013.01

$$W_z\left(z^s \operatorname{Ai}'(a z^r), z^s \operatorname{Bi}'(a z^r)\right) = -\frac{a^2 r z^{2r+2s-1}}{\pi}$$

03.08.13.0014.01

$$w''(z) - 2(\log(r) + \log(s))\, w'(z) + \left(\log(s)(2\log(r) + \log(s)) - a^3 r^{3z} \log^2(r)\right) w(z) = 0 \;/;\; w(z) = c_1 s^z \operatorname{Ai}'(a r^z) + c_2 s^z \operatorname{Bi}'(a r^z)$$

03.08.13.0015.01

$$W_z\left(s^z \operatorname{Ai}'(a r^z), s^z \operatorname{Bi}'(a r^z)\right) = -\frac{a^2 r^{2z} s^{2z} \log(r)}{\pi}$$

Involving related functions

03.08.13.0004.01

$$w^{(3)}(z) - 4 z\, w'(z) - 2 w(z) = 0 \;/;\; w(z) = c_1 \operatorname{Ai}(z)^2 + c_2 \operatorname{Bi}(z) \operatorname{Ai}(z) + c_3 \operatorname{Bi}(z)^2$$

03.08.13.0005.01

$$w^{(3)}(z) - 4 z\, w'(z) - 2 w(z) = 0 \;/;\; w(z) = w_1(z)\, w_2(z) \land w_1''(z) - z\, w_1(z) = 0 \land w_2''(z) - z\, w_2(z) = 0$$

03.08.13.0006.01

$$W_z\left(\operatorname{Ai}'(z)^2, \operatorname{Ai}'(z) \operatorname{Bi}'(z), \operatorname{Bi}'(z)^2\right) = -\frac{2 z^3}{\pi^3}$$

Ordinary nonlinear differential equations

03.08.13.0007.01

$$w(z)^2 - z + w'(z) = 0 \;/;\; w(z) = \frac{\operatorname{Bi}'(z) + c_1 \operatorname{Ai}'(z)}{\operatorname{Bi}(z) + c_1 \operatorname{Ai}(z)}$$

Riccati form of differential equation

Transformations

Transformations and argument simplifications

Argument involving basic arithmetic operations

03.08.16.0001.01

$$\operatorname{Bi}'(c (d z^n)^m) = \frac{1}{2}\left(\left(\frac{(d z^3)^{2m}}{d^{2m} z^{6m}} + 1\right) \operatorname{Bi}'\left(c\, d^m z^{3m}\right) - \sqrt{3}\left(1 - \frac{(d z^3)^{2m}}{d^{2m} z^{6m}}\right) \operatorname{Ai}'\left(c\, d^m z^{3m}\right)\right) /;\; 3m \in \mathbb{Z}$$

03.08.16.0002.01

$$\operatorname{Bi}'\left(\sqrt[3]{z^3}\right) = \frac{1}{2}\left(\left(\frac{(z^3)^{2/3}}{z^2} + 1\right) \operatorname{Bi}'(z) - \sqrt{3}\left(1 - \frac{(z^3)^{2/3}}{z^2}\right) \operatorname{Ai}'(z)\right)$$

03.08.16.0003.01

$$\operatorname{Bi}'\left((-1)^{2/3} z\right) = \frac{1}{4}\left(-i - \sqrt{3}\right)\left(3 \operatorname{Ai}'(z) + i \operatorname{Bi}'(z)\right)$$

03.08.16.0004.01

$$\operatorname{Bi}'\left(-\left(\sqrt[3]{-1}\, z\right)\right) = \frac{1}{4}\left(i - \sqrt{3}\right)\left(3 \operatorname{Ai}'(z) - i \operatorname{Bi}'(z)\right)$$

Identities

Functional identities

03.08.17.0001.01

$$\mathrm{Bi}'(z) + e^{-\frac{2 i \pi}{3}} \mathrm{Bi}'\left(e^{\frac{2 i \pi}{3}} z\right) + e^{\frac{2 i \pi}{3}} \mathrm{Bi}'\left(e^{-\frac{2 i \pi}{3}} z\right) == 0$$

Complex characteristics

Real part

03.08.19.0001.01

$$\mathrm{Re}\left(\mathrm{Bi}'(x + i y)\right) == \frac{1}{2} \left(\mathrm{Bi}'\left(x - x \sqrt{-\frac{y^2}{x^2}}\right) + \mathrm{Bi}'\left(x + x \sqrt{-\frac{y^2}{x^2}}\right)\right)$$

Imaginary part

03.08.19.0002.01

$$\mathrm{Im}\left(\mathrm{Bi}'(x + i y)\right) == \frac{x}{2 y} \sqrt{-\frac{y^2}{x^2}} \left(\mathrm{Bi}'\left(x - x \sqrt{-\frac{y^2}{x^2}}\right) - \mathrm{Bi}'\left(x + x \sqrt{-\frac{y^2}{x^2}}\right)\right)$$

Absolute value

03.08.19.0003.01

$$\left|\mathrm{Bi}'(x + i y)\right| == \sqrt{\mathrm{Bi}'\left(x - x \sqrt{-\frac{y^2}{x^2}}\right) \mathrm{Bi}'\left(x + x \sqrt{-\frac{y^2}{x^2}}\right)}$$

Argument

03.08.19.0004.01

$$\arg\left(\mathrm{Bi}'(x + i y)\right) == \tan^{-1}\left(\frac{1}{2} \left(\mathrm{Bi}'\left(x - x \sqrt{-\frac{y^2}{x^2}}\right) + \mathrm{Bi}'\left(x + x \sqrt{-\frac{y^2}{x^2}}\right)\right), \frac{x}{2 y} \sqrt{-\frac{y^2}{x^2}} \left(\mathrm{Bi}'\left(x - x \sqrt{-\frac{y^2}{x^2}}\right) - \mathrm{Bi}'\left(x + x \sqrt{-\frac{y^2}{x^2}}\right)\right)\right)$$

Conjugate value

03.08.19.0005.01

$$\overline{\mathrm{Bi}'(x + i y)} == \frac{1}{2} \left(\mathrm{Bi}'\left(x - x \sqrt{-\frac{y^2}{x^2}}\right) + \mathrm{Bi}'\left(x + x \sqrt{-\frac{y^2}{x^2}}\right)\right) - \frac{i x}{2 y} \sqrt{-\frac{y^2}{x^2}} \left(\mathrm{Bi}'\left(x - x \sqrt{-\frac{y^2}{x^2}}\right) - \mathrm{Bi}'\left(x + x \sqrt{-\frac{y^2}{x^2}}\right)\right)$$

Signum value

03.08.19.0006.01

$$\operatorname{sgn}\left(\operatorname{Bi}'(x+i\,y)\right) = \frac{\frac{i\,x}{y}\sqrt{-\frac{y^2}{x^2}}\left(\operatorname{Bi}'\left(x-x\sqrt{-\frac{y^2}{x^2}}\right)-\operatorname{Bi}'\left(\sqrt{-\frac{y^2}{x^2}}\;x+x\right)\right)+\operatorname{Bi}'\left(\sqrt{-\frac{y^2}{x^2}}\;x+x\right)+\operatorname{Bi}'\left(x-x\sqrt{-\frac{y^2}{x^2}}\right)}{2\sqrt{\operatorname{Bi}'\left(x-x\sqrt{-\frac{y^2}{x^2}}\right)\operatorname{Bi}'\left(\sqrt{-\frac{y^2}{x^2}}\;x+x\right)}}$$

Differentiation

Low-order differentiation

03.08.20.0001.01

$$\frac{\partial \operatorname{Bi}'(z)}{\partial z} = z\operatorname{Bi}(z)$$

03.08.20.0002.01

$$\frac{\partial^2 \operatorname{Bi}'(z)}{\partial z^2} = \operatorname{Bi}(z)+z\operatorname{Bi}'(z)$$

Symbolic differentiation

03.08.20.0005.01

$$\frac{\partial^n \operatorname{Bi}'(z)}{\partial z^n} = \frac{1}{2}\operatorname{Bi}'(z)\,\delta_n+\frac{1}{4}z^{-n}\left(2\sum_{k=0}^{n}\sum_{j=0}^{k}\sum_{i=0}^{k}\frac{(-1)^{j+k}\,(k-i)!\,(-3\,j+3\,k+1)\,(-3\,j+3\,k+2)\,(-3\,j+3\,k-n+3)_{n-2}\left(\frac{2}{3}\right)_k}{i!\,j!\,(k-j)!\,(k-2\,i)!\left(\frac{2}{3}\right)_i\left(\frac{1}{3}-k\right)_i}\left(-\frac{z^3}{9}\right)^i+\right.$$
$$\left.\sum_{k=0}^{n}\sum_{j=0}^{k}\sum_{i=0}^{k-1}\frac{(-1)^{j+k}\,(-i+k-1)!\,(3\,i-3\,k+2)\,(-3\,j+3\,k-n+1)_n\left(-\frac{2}{3}\right)_k}{(i-1)!\,j!\,(k-j)!\,(k-2\,i)!\left(\frac{1}{3}\right)_i\left(\frac{5}{3}-k\right)_i}\left(-\frac{z^3}{9}\right)^i\right)\operatorname{Bi}'(z)+$$
$$\frac{1}{4}z^{2-n}\left(\sum_{k=0}^{n}\sum_{j=0}^{k}\sum_{i=0}^{k-1}\frac{(-1)^{j+k-1}\,(-i+k-1)!\,(-3\,j+3\,k+1)\,(-3\,j+3\,k+2)\,(-3\,j+3\,k-n+3)_{n-2}\left(\frac{2}{3}\right)_k}{i!\,j!\,(k-j)!\,(-2\,i+k-1)!\left(\frac{5}{3}\right)_i\left(\frac{1}{3}-k\right)_i}\left(-\frac{z^3}{9}\right)^i-\right.$$
$$\left.\sum_{k=0}^{n}\sum_{j=0}^{k}\sum_{i=0}^{k-1}\frac{(-1)^{j+k-1}\,(-i+k-1)!\,(-3\,j+3\,k-n+1)_n\left(-\frac{2}{3}\right)_k}{i!\,j!\,(k-j)!\,(-2\,i+k-1)!\left(\frac{1}{3}\right)_i\left(\frac{5}{3}-k\right)_i}\left(-\frac{z^3}{9}\right)^i\right)\operatorname{Bi}(z)\;/;\;n\in\mathbb{N}$$

03.08.20.0003.02

$$\frac{\partial^n \operatorname{Bi}'(z)}{\partial z^n} = 3^{n-\frac{13}{6}}\,z^{-n}\left(\Gamma\left(\frac{1}{3}\right){}_2\tilde{F}_3\left(1,\frac{4}{3};1-\frac{n}{3},\frac{4-n}{3},\frac{5-n}{3};\frac{z^3}{9}\right)z^2+9\sqrt[3]{3}\;\Gamma\left(\frac{2}{3}\right){}_2\tilde{F}_3\left(\frac{2}{3},1;\frac{1-n}{3},\frac{2-n}{3},1-\frac{n}{3};\frac{z^3}{9}\right)\right)/;\;n\in\mathbb{N}$$

Fractional integro-differentiation

03.08.20.0004.01

$$\frac{\partial^\alpha \operatorname{Bi}'(z)}{\partial z^\alpha} = 3^{\alpha-\frac{13}{6}}\,z^{-\alpha}\left(\Gamma\left(\frac{1}{3}\right)z^2\,{}_2\tilde{F}_3\left(1,\frac{4}{3};1-\frac{\alpha}{3},\frac{4-\alpha}{3},\frac{5-\alpha}{3};\frac{z^3}{9}\right)+9\sqrt[3]{3}\;\Gamma\left(\frac{2}{3}\right){}_2\tilde{F}_3\left(\frac{2}{3},1;\frac{1-\alpha}{3},\frac{2-\alpha}{3},1-\frac{\alpha}{3};\frac{z^3}{9}\right)\right)$$

Integration

Indefinite integration

Involving only one direct function

03.08.21.0001.01

$$\int \mathrm{Bi}'(a\,z)\,dz = \frac{\mathrm{Bi}(a\,z)}{a}$$

03.08.21.0002.01

$$\int \mathrm{Bi}'(z)\,dz = \mathrm{Bi}(z)$$

Involving one direct function and elementary functions

Involving power function

Involving power

Linear argument

03.08.21.0003.01

$$\int z^{\alpha-1}\,\mathrm{Bi}'(a\,z)\,dz = \frac{z^{\alpha}}{9\,3^{5/6}}\left(3^{2/3}\,a^2\,\Gamma\!\left(\frac{\alpha+2}{3}\right)\,{}_1\tilde{F}_2\!\left(\frac{\alpha+2}{3};\,\frac{5}{3},\,\frac{\alpha+5}{3};\,\frac{a^3\,z^3}{9}\right)z^2 + 9\,\Gamma\!\left(\frac{\alpha}{3}\right)\,{}_1\tilde{F}_2\!\left(\frac{\alpha}{3};\,\frac{1}{3},\,\frac{\alpha}{3}+1;\,\frac{a^3\,z^3}{9}\right)\right)$$

03.08.21.0004.01

$$\int z^{\alpha-1}\,\mathrm{Bi}'(z)\,dz = \frac{z^{\alpha}}{3^{5/6}}\,\Gamma\!\left(\frac{\alpha}{3}\right)\,{}_1\tilde{F}_2\!\left(\frac{\alpha}{3};\,\frac{1}{3},\,\frac{\alpha}{3}+1;\,\frac{z^3}{9}\right) + \frac{z^{\alpha+2}}{9\,\sqrt[6]{3}}\,\Gamma\!\left(\frac{\alpha}{3}+\frac{2}{3}\right)\,{}_1\tilde{F}_2\!\left(\frac{\alpha}{3}+\frac{2}{3};\,\frac{5}{3},\,\frac{\alpha}{3}+\frac{5}{3};\,\frac{z^3}{9}\right)$$

03.08.21.0005.01

$$\int z^{n+2}\,\mathrm{Bi}'(z)\,dz = -(n+2)\left(z\,\mathrm{Bi}'(z) - n\,\mathrm{Bi}(z)\right)z^{n-1} + \mathrm{Bi}(z)\,z^{n+2} - (n-1)\,n\,(n+2)\int z^{n-2}\,\mathrm{Bi}(z)\,dz \;/;\; n \in \mathbb{N}$$

03.08.21.0006.01

$$\int z\,\mathrm{Bi}'(z)\,dz = \frac{z^4}{12\,\sqrt[6]{3}\,\Gamma\!\left(\frac{5}{3}\right)}\,{}_1F_2\!\left(\frac{4}{3};\,\frac{5}{3},\,\frac{7}{3};\,\frac{z^3}{9}\right) + \frac{z^2}{2\,\Gamma\!\left(\frac{1}{3}\right)}\,\sqrt[6]{3}\,{}_1F_2\!\left(\frac{2}{3};\,\frac{1}{3},\,\frac{5}{3};\,\frac{z^3}{9}\right)$$

03.08.21.0007.01

$$\int z^2\,\mathrm{Bi}'(z)\,dz = z^2\,\mathrm{Bi}(z) - 2\,\mathrm{Bi}'(z)$$

Power arguments

03.08.21.0008.01

$$\int z^{\alpha-1}\,\mathrm{Bi}'(a\,z^r)\,dz =$$

$$\frac{z^{\alpha}}{9\,3^{5/6}\,r}\left(3^{2/3}\,a^2\,\Gamma\!\left(\frac{1}{3}\left(\frac{\alpha}{r}+2\right)\right)\,{}_1\tilde{F}_2\!\left(\frac{1}{3}\left(\frac{\alpha}{r}+2\right);\,\frac{5}{3},\,\frac{1}{3}\left(\frac{\alpha}{r}+5\right);\,\frac{1}{9}\,a^3\,z^{3\,r}\right)z^{2\,r} + 9\,\Gamma\!\left(\frac{\alpha}{3\,r}\right)\,{}_1\tilde{F}_2\!\left(\frac{\alpha}{3\,r};\,\frac{1}{3},\,\frac{\alpha}{3\,r}+1;\,\frac{1}{9}\,a^3\,z^{3\,r}\right)\right)$$

Involving exponential function

Involving exp

Linear argument

03.08.21.0009.01

$$\int e^{-\frac{2}{3}(a z)^{3/2}} \operatorname{Bi}'(a z)\, dz = \frac{\sqrt[6]{3}\, z}{5\,\Gamma\left(\frac{1}{3}\right)} \left(5\, {}_1F_1\left(-\frac{1}{6}; \frac{5}{3}; \frac{1}{3}(-4)(a z)^{3/2}\right) - 2\,(a z)^{3/2}\, {}_1F_1\left(\frac{5}{6}; \frac{8}{3}; \frac{1}{3}(-4)(a z)^{3/2}\right)\right) - \frac{2}{15\,\sqrt[6]{3}\, a\,\Gamma\left(\frac{5}{3}\right)} \left(10\, {}_1F_1\left(\frac{1}{6}; \frac{4}{3}; \frac{1}{3}(-4)(a z)^{3/2}\right)(a z)^{3/2} - 3\, {}_1F_1\left(-\frac{5}{6}; \frac{1}{3}; \frac{1}{3}(-4)(a z)^{3/2}\right)\right)$$

03.08.21.0010.01

$$\int e^{\frac{2}{3}(a z)^{3/2}} \operatorname{Bi}'(a z)\, dz = \frac{2}{15\,\sqrt[6]{3}\, a\,\Gamma\left(\frac{5}{3}\right)} \left(10\, {}_1F_1\left(\frac{1}{6}; \frac{4}{3}; \frac{4}{3}(a z)^{3/2}\right)(a z)^{3/2} + 3\, {}_1F_1\left(-\frac{5}{6}; \frac{1}{3}; \frac{4}{3}(a z)^{3/2}\right)\right) + \frac{\sqrt[6]{3}\, z}{5\,\Gamma\left(\frac{1}{3}\right)} \left(2\, {}_1F_1\left(\frac{5}{6}; \frac{8}{3}; \frac{4}{3}(a z)^{3/2}\right)(a z)^{3/2} + 5\, {}_1F_1\left(-\frac{1}{6}; \frac{5}{3}; \frac{4}{3}(a z)^{3/2}\right)\right)$$

Power arguments

03.08.21.0011.01

$$\int e^{-\frac{2}{3}(a z^r)^{3/2}} \operatorname{Bi}'(a z^r)\, dz = \frac{1}{3\, 3^{5/6}(2 r+1)\,\Gamma\left(\frac{1}{3}\right)\Gamma\left(\frac{5}{3}\right)} \left(z\left(3^{2/3} a^2\, \Gamma\left(\frac{1}{3}\right) {}_2F_2\left(\frac{7}{6}, \frac{4}{3}+\frac{2}{3 r}; \frac{7}{3}, \frac{7}{3}+\frac{2}{3 r}; \frac{1}{3}(-4)(a z^r)^{3/2}\right) z^{2 r} + 9\,(2 r+1)\, \Gamma\left(\frac{5}{3}\right) {}_2F_2\left(-\frac{1}{6}, \frac{2}{3 r}; -\frac{1}{3}, 1+\frac{2}{3 r}; \frac{1}{3}(-4)(a z^r)^{3/2}\right)\right)\right)$$

03.08.21.0012.01

$$\int e^{\frac{2}{3}(a z^r)^{3/2}} \operatorname{Bi}'(a z^r)\, dz = \frac{1}{3\, 3^{5/6}(2 r+1)\,\Gamma\left(\frac{1}{3}\right)\Gamma\left(\frac{5}{3}\right)} \left(z\left(3^{2/3} a^2\, \Gamma\left(\frac{1}{3}\right) {}_2F_2\left(\frac{7}{6}, \frac{4}{3}+\frac{2}{3 r}; \frac{7}{3}, \frac{7}{3}+\frac{2}{3 r}; \frac{4}{3}(a z^r)^{3/2}\right) z^{2 r} + 9\,(2 r+1)\, \Gamma\left(\frac{5}{3}\right) {}_2F_2\left(-\frac{1}{6}, \frac{2}{3 r}; -\frac{1}{3}, 1+\frac{2}{3 r}; \frac{4}{3}(a z^r)^{3/2}\right)\right)\right)$$

Involving exponential function and a power function

Involving exp and power

Linear arguments

03.08.21.0013.01

$$\int z^{\alpha-1} e^{-\frac{2}{3}(a z)^{3/2}} \operatorname{Bi}'(a z) \, dz = \frac{\sqrt[6]{3}\, z^{\alpha}}{\alpha\, \Gamma\left(\frac{1}{3}\right)} \, {}_2F_2\left(-\frac{1}{6}, \frac{2\alpha}{3}; -\frac{1}{3}, \frac{2\alpha}{3}+1; \frac{1}{3}(-4)(a z)^{3/2}\right) + \frac{a^2 z^{\alpha+2}}{3\sqrt[6]{3}\, (\alpha+2)\, \Gamma\left(\frac{5}{3}\right)} \, {}_2F_2\left(\frac{7}{6}, \frac{2\alpha}{3}+\frac{4}{3}; \frac{7}{3}, \frac{2\alpha}{3}+\frac{7}{3}; \frac{1}{3}(-4)(a z)^{3/2}\right)$$

03.08.21.0014.01

$$\int \sqrt{z}\, e^{-\frac{2}{3}(a z)^{3/2}} \operatorname{Bi}'(a z) \, dz = \frac{1}{21 a^2 \sqrt{z}\, \Gamma\left(\frac{1}{3}\right)} \left(2 e^{\frac{1}{3}(-2)(a z)^{3/2}} \left(3 a^2 \operatorname{Bi}'(a z)\, \Gamma\left(\frac{1}{3}\right) z^2 + \sqrt[6]{3}\, \sqrt{a z} \left(\sqrt[3]{3}\, a^2 z^2 I_{\frac{5}{3}}\left(\frac{2}{3} a^{3/2} z^{3/2}\right) \Gamma\left(\frac{1}{3}\right) \sqrt[3]{a^{3/2} z^{3/2}} + 6 e^{\frac{2}{3}(a z)^{3/2}} + \frac{\sqrt[3]{3}\, a^3 z^3\, \Gamma\left(\frac{1}{3}\right)}{\sqrt[3]{a^{3/2} z^{3/2}}} I_{-\frac{5}{3}}\left(\frac{2}{3} a^{3/2} z^{3/2}\right)\right)\right)\right)$$

03.08.21.0015.01

$$\int z^{\alpha-1} e^{\frac{2}{3}(a z)^{3/2}} \operatorname{Bi}'(a z) \, dz = \frac{\sqrt[6]{3}\, z^{\alpha}}{\alpha\, \Gamma\left(\frac{1}{3}\right)} \, {}_2F_2\left(-\frac{1}{6}, \frac{2\alpha}{3}; -\frac{1}{3}, \frac{2\alpha}{3}+1; \frac{4}{3}(a z)^{3/2}\right) + \frac{a^2 z^{\alpha+2}}{3\sqrt[6]{3}\, (\alpha+2)\, \Gamma\left(\frac{5}{3}\right)} \, {}_2F_2\left(\frac{7}{6}, \frac{2\alpha}{3}+\frac{4}{3}; \frac{7}{3}, \frac{2\alpha}{3}+\frac{7}{3}; \frac{4}{3}(a z)^{3/2}\right)$$

03.08.21.0016.01

$$\int \sqrt{z}\, e^{\frac{2}{3}(a z)^{3/2}} \operatorname{Bi}'(a z) \, dz = -\frac{1}{21 a^2 \sqrt{z}\, \Gamma\left(\frac{1}{3}\right)} \left(2\left(\sqrt[6]{3}\, \sqrt{a z} \left(\sqrt[3]{3}\, a^2 e^{\frac{2}{3}(a z)^{3/2}} z^2 I_{\frac{5}{3}}\left(\frac{2}{3} a^{3/2} z^{3/2}\right) \Gamma\left(\frac{1}{3}\right) \sqrt[3]{a^{3/2} z^{3/2}} + \frac{\sqrt[3]{3}\, a^3\, \Gamma\left(\frac{1}{3}\right)}{\sqrt[3]{a^{3/2} z^{3/2}}} e^{\frac{2}{3}(a z)^{3/2}} z^3 I_{-\frac{5}{3}}\left(\frac{2}{3} a^{3/2} z^{3/2}\right) + 6\right) - 3 a^2 e^{\frac{2}{3}(a z)^{3/2}} z^2 \operatorname{Bi}'(a z)\, \Gamma\left(\frac{1}{3}\right)\right)\right)$$

Power arguments

03.08.21.0017.01

$$\int z^{\alpha-1} e^{\frac{1}{3}(-2)(a z^r)^{3/2}} \operatorname{Bi}'(a z^r) \, dz = \frac{1}{3\, 3^{5/6} \alpha (2r+\alpha)\, \Gamma\left(\frac{1}{3}\right) \Gamma\left(\frac{5}{3}\right)} \left(z^{\alpha}\left(3^{2/3} a^2 \alpha\, \Gamma\left(\frac{1}{3}\right) {}_2F_2\left(\frac{7}{6}, \frac{2\alpha}{3r}+\frac{4}{3}; \frac{7}{3}, \frac{2\alpha}{3r}+\frac{7}{3}; \frac{1}{3}(-4)(a z^r)^{3/2}\right) z^{2r} + 9(2r+\alpha)\, \Gamma\left(\frac{5}{3}\right) {}_2F_2\left(-\frac{1}{6}, \frac{2\alpha}{3r}; -\frac{1}{3}, \frac{2\alpha}{3r}+1; \frac{1}{3}(-4)(a z^r)^{3/2}\right)\right)\right)$$

03.08.21.0018.01

$$\int z^{\alpha-1}\, e^{\frac{2}{3}(a z^r)^{3/2}}\, \text{Bi}'(a z^r)\, dz = \frac{1}{3\, 3^{5/6}\, \alpha\, (2 r+\alpha)\, \Gamma\left(\frac{1}{3}\right) \Gamma\left(\frac{5}{3}\right)} \left(z^{\alpha} \left(3^{2/3} a^2\, \alpha\, \Gamma\left(\frac{1}{3}\right) {}_2F_2\left(\frac{7}{6}, \frac{2\alpha}{3 r}+\frac{4}{3}; \frac{7}{3}, \frac{2\alpha}{3 r}+\frac{7}{3}; \frac{4}{3} (a z^r)^{3/2}\right) z^{2 r}+9\, (2 r+\alpha)\, \Gamma\left(\frac{5}{3}\right) {}_2F_2\left(-\frac{1}{6}, \frac{2\alpha}{3 r}; -\frac{1}{3}, \frac{2\alpha}{3 r}+1; \frac{4}{3} (a z^r)^{3/2}\right)\right)\right)$$

Involving hyperbolic functions

Involving sinh

Linear argument

03.08.21.0019.01

$$\int \sinh\left(\frac{2}{3} (a z)^{3/2}\right) \text{Bi}'(a z)\, dz = -\frac{1}{10 \left(a^{3/2} z^{3/2}\right)^{2/3}} \left(e^{\frac{1}{3} (-2) (a z)^{3/2}}\, z \left(2 \left(1+e^{\frac{4}{3} (a z)^{3/2}}\right) \sqrt{a z}\, \left(a^{3/2} z^{3/2}\right)^{2/3} \text{Bi}(a z) - \left(-1+e^{\frac{4}{3} (a z)^{3/2}}\right) \left(5\, \text{Bi}'(a z) \left(a^{3/2} z^{3/2}\right)^{2/3} - \sqrt{3}\, a^2 z^2\, I_{-\frac{4}{3}}\left(\frac{2}{3} a^{3/2} z^{3/2}\right) - \frac{\sqrt{3}\, a^3 z^3}{\left(a^{3/2} z^{3/2}\right)^{2/3}} I_{\frac{4}{3}}\left(\frac{2}{3} a^{3/2} z^{3/2}\right)\right)\right)\right)$$

03.08.21.0020.01

$$\int \sinh\left(\frac{2}{3} (a z)^{3/2}+b\right) \text{Bi}'(a z)\, dz = \frac{1}{30\, a \left(a^{3/2} z^{3/2}\right)^{2/3} \Gamma\left(\frac{5}{3}\right)} \left(e^{-\frac{1}{3} 2 (a z)^{3/2}-b} \left(-6 \left(1+e^{\frac{4}{3} (a z)^{3/2}+2 b}\right) \left(a^{3/2} z^{3/2}\right)^{2/3} \text{Bi}(a z)\, \Gamma\left(\frac{5}{3}\right) (a z)^{3/2} + 15\, a \left(-1+e^{\frac{4}{3} (a z)^{3/2}+2 b}\right) z \left(a^{3/2} z^{3/2}\right)^{2/3} \text{Bi}'(a z)\, \Gamma\left(\frac{5}{3}\right) - \sqrt{3} \left(\left(3\, a^{5/2} \left(-1+e^{\frac{4}{3} (a z)^{3/2}+2 b}\right) I_{\frac{4}{3}}\left(\frac{2}{3} a^{3/2} z^{3/2}\right) \Gamma\left(\frac{5}{3}\right) z^{5/2} + 2 \sqrt[3]{3}\, e^{\frac{2}{3} (a z)^{3/2}} \left(-1+e^{2 b}\right) \sqrt[3]{a^{3/2} z^{3/2}}\right) \sqrt[3]{a^{3/2} z^{3/2}} + 3\, a^3 \left(-1+e^{\frac{4}{3} (a z)^{3/2}+2 b}\right) z^3\, I_{-\frac{4}{3}}\left(\frac{2}{3} a^{3/2} z^{3/2}\right) \Gamma\left(\frac{5}{3}\right)\right)\right)\right)$$

Power arguments

03.08.21.0021.01

$$\int \sinh\left(\frac{2}{3} (a z^r)^{3/2}\right) \text{Bi}'(a z^r)\, dz = -\frac{1}{6\, 3^{5/6}\, (2 r+1)\, \Gamma\left(\frac{1}{3}\right) \Gamma\left(\frac{5}{3}\right)} \left(z \left(3^{2/3} a^2\, \Gamma\left(\frac{1}{3}\right) \left({}_2F_2\left(\frac{7}{6}, \frac{4}{3}+\frac{2}{3 r}; \frac{7}{3}, \frac{7}{3}+\frac{2}{3 r}; \frac{1}{3} (-4) (a z^r)^{3/2}\right) - {}_2F_2\left(\frac{7}{6}, \frac{4}{3}+\frac{2}{3 r}; \frac{7}{3}, \frac{7}{3}+\frac{2}{3 r}; \frac{4}{3} (a z^r)^{3/2}\right)\right) z^{2 r} - 9\, (2 r+1)\, \Gamma\left(\frac{5}{3}\right) {}_2F_2\left(-\frac{1}{6}, \frac{2}{3 r}; -\frac{1}{3}, 1+\frac{2}{3 r}; \frac{4}{3} (a z^r)^{3/2}\right) + 9\, (2 r+1)\, \Gamma\left(\frac{5}{3}\right) {}_2F_2\left(-\frac{1}{6}, \frac{2}{3 r}; -\frac{1}{3}, 1+\frac{2}{3 r}; \frac{1}{3} (-4) (a z^r)^{3/2}\right)\right)\right)$$

03.08.21.0022.01

$$\int \sinh\left(\frac{2}{3}(a z^r)^{3/2}+b\right)\mathrm{Bi}'(a z^r)\,dz = \frac{1}{6\,3^{5/6}(2r+1)\,\Gamma\left(\frac{1}{3}\right)\Gamma\left(\frac{5}{3}\right)}\left(e^{-b} z\left(-3^{2/3} a^2\,\Gamma\left(\frac{1}{3}\right)\left({}_2F_2\left(\frac{7}{6},\frac{4}{3}+\frac{2}{3r};\frac{7}{3},\frac{7}{3}+\frac{2}{3r};\frac{1}{3}(-4)(a z^r)^{3/2}\right)-e^{2b}\,{}_2F_2\left(\frac{7}{6},\frac{4}{3}+\frac{2}{3r};\frac{7}{3},\frac{7}{3}+\frac{2}{3r};\frac{4}{3}(a z^r)^{3/2}\right)\right)z^{2r}+9e^{2b}(2r+1)\,\Gamma\left(\frac{5}{3}\right){}_2F_2\left(-\frac{1}{6},\frac{2}{3r};-\frac{1}{3},1+\frac{2}{3r};\frac{4}{3}(a z^r)^{3/2}\right)-9(2r+1)\,\Gamma\left(\frac{5}{3}\right){}_2F_2\left(-\frac{1}{6},\frac{2}{3r};-\frac{1}{3},1+\frac{2}{3r};\frac{1}{3}(-4)(a z^r)^{3/2}\right)\right)\right)$$

Involving cosh

Linear argument

03.08.21.0023.01

$$\int \cosh\left(\frac{2}{3}(a z)^{3/2}\right)\mathrm{Bi}'(a z)\,dz = \frac{1}{30\,a\left(a^{3/2} z^{3/2}\right)^{2/3}\Gamma\left(\frac{5}{3}\right)}\left(e^{\frac{1}{3}(-2)(a z)^{3/2}}\left(-6\left(-1+e^{\frac{4}{3}(a z)^{3/2}}\right)\left(a^{3/2} z^{3/2}\right)^{2/3}\mathrm{Bi}(a z)\,\Gamma\left(\frac{5}{3}\right)(a z)^{3/2}+15\,a\left(1+e^{\frac{4}{3}(a z)^{3/2}}\right)z\left(a^{3/2} z^{3/2}\right)^{2/3}\mathrm{Bi}'(a z)\,\Gamma\left(\frac{5}{3}\right)-\sqrt{3}\left(\left(3a^{5/2}\left(1+e^{\frac{4}{3}(a z)^{3/2}}\right)I_{\frac{4}{3}}\left(\frac{2}{3}a^{3/2} z^{3/2}\right)\Gamma\left(\frac{5}{3}\right)z^{5/2}+4\sqrt[3]{3}\;e^{\frac{2}{3}(a z)^{3/2}}\sqrt[3]{a^{3/2} z^{3/2}}\right)\sqrt[3]{a^{3/2} z^{3/2}}+3a^3\left(1+e^{\frac{4}{3}(a z)^{3/2}}\right)z^3\,I_{-\frac{4}{3}}\left(\frac{2}{3}a^{3/2} z^{3/2}\right)\Gamma\left(\frac{5}{3}\right)\right)\right)\right)$$

03.08.21.0024.01

$$\int \cosh\left(\frac{2}{3}(a z)^{3/2}+b\right)\mathrm{Bi}'(a z)\,dz = \frac{1}{30\,a\left(a^{3/2} z^{3/2}\right)^{2/3}\Gamma\left(\frac{5}{3}\right)}\left(e^{-\frac{1}{3}2(a z)^{3/2}-b}\left(-6\left(-1+e^{\frac{4}{3}(a z)^{3/2}+2b}\right)\left(a^{3/2} z^{3/2}\right)^{2/3}\mathrm{Bi}(a z)\,\Gamma\left(\frac{5}{3}\right)(a z)^{3/2}+15\,a\left(1+e^{\frac{4}{3}(a z)^{3/2}+2b}\right)z\left(a^{3/2} z^{3/2}\right)^{2/3}\mathrm{Bi}'(a z)\,\Gamma\left(\frac{5}{3}\right)-\sqrt{3}\left(\left(3a^{5/2}\left(1+e^{\frac{4}{3}(a z)^{3/2}+2b}\right)I_{\frac{4}{3}}\left(\frac{2}{3}a^{3/2} z^{3/2}\right)\Gamma\left(\frac{5}{3}\right)z^{5/2}+2\sqrt[3]{3}\;e^{\frac{2}{3}(a z)^{3/2}}\left(1+e^{2b}\right)\sqrt[3]{a^{3/2} z^{3/2}}\right)\sqrt[3]{a^{3/2} z^{3/2}}+3a^3\left(1+e^{\frac{4}{3}(a z)^{3/2}+2b}\right)z^3\,I_{-\frac{4}{3}}\left(\frac{2}{3}a^{3/2} z^{3/2}\right)\Gamma\left(\frac{5}{3}\right)\right)\right)\right)$$

Power arguments

03.08.21.0025.01

$$\int \cosh\left(\frac{2}{3}(a z^r)^{3/2}\right) \mathrm{Bi}'(a z^r)\, dz = \frac{1}{6\, 3^{5/6} (2r+1)\, \Gamma\left(\frac{1}{3}\right) \Gamma\left(\frac{5}{3}\right)} \left(z \left(3^{2/3} a^2\, \Gamma\left(\frac{1}{3}\right) \left({}_2F_2\left(\frac{7}{6}, \frac{4}{3} + \frac{2}{3r}; \frac{7}{3}, \frac{7}{3} + \frac{2}{3r}; \frac{4}{3}(a z^r)^{3/2}\right) + {}_2F_2\left(\frac{7}{6}, \frac{4}{3} + \frac{2}{3r}; \frac{7}{3}, \frac{7}{3} + \frac{2}{3r}; \frac{1}{3}(-4)(a z^r)^{3/2}\right)\right) z^{2r} + 9(2r+1)\, \Gamma\left(\frac{5}{3}\right) {}_2F_2\left(-\frac{1}{6}, \frac{2}{3r}; -\frac{1}{3}, 1 + \frac{2}{3r}; \frac{4}{3}(a z^r)^{3/2}\right) + 9(2r+1)\, \Gamma\left(\frac{5}{3}\right) {}_2F_2\left(-\frac{1}{6}, \frac{2}{3r}; -\frac{1}{3}, 1 + \frac{2}{3r}; \frac{1}{3}(-4)(a z^r)^{3/2}\right)\right)\right)$$

03.08.21.0026.01

$$\int \cosh\left(\frac{2}{3}(a z^r)^{3/2} + b\right) \mathrm{Bi}'(a z^r)\, dz = \frac{1}{6\, 3^{5/6} (2r+1)\, \Gamma\left(\frac{1}{3}\right) \Gamma\left(\frac{5}{3}\right)} \left(e^{-b} z \left(3^{2/3} a^2\, \Gamma\left(\frac{1}{3}\right) \left(e^{2b}\, {}_2F_2\left(\frac{7}{6}, \frac{4}{3} + \frac{2}{3r}; \frac{7}{3}, \frac{7}{3} + \frac{2}{3r}; \frac{4}{3}(a z^r)^{3/2}\right) + {}_2F_2\left(\frac{7}{6}, \frac{4}{3} + \frac{2}{3r}; \frac{7}{3}, \frac{7}{3} + \frac{2}{3r}; \frac{1}{3}(-4)(a z^r)^{3/2}\right)\right) z^{2r} + 9 e^{2b} (2r+1)\, \Gamma\left(\frac{5}{3}\right) {}_2F_2\left(-\frac{1}{6}, \frac{2}{3r}; -\frac{1}{3}, 1 + \frac{2}{3r}; \frac{4}{3}(a z^r)^{3/2}\right) + 9(2r+1)\, \Gamma\left(\frac{5}{3}\right) {}_2F_2\left(-\frac{1}{6}, \frac{2}{3r}; -\frac{1}{3}, 1 + \frac{2}{3r}; \frac{1}{3}(-4)(a z^r)^{3/2}\right)\right)\right)$$

Involving hyperbolic functions and a power function

Involving sinh and power

Linear argument

03.08.21.0027.01

$$\int z^{\alpha-1} \sinh\left(\frac{2}{3}(a z)^{3/2}\right) \mathrm{Bi}'(a z)\, dz = -\frac{1}{6\, 3^{5/6} \alpha (\alpha+2)\, \Gamma\left(\frac{1}{3}\right) \Gamma\left(\frac{5}{3}\right)} \left(z^\alpha \left(3^{2/3} a^2 \alpha\, \Gamma\left(\frac{1}{3}\right) \left({}_2F_2\left(\frac{7}{6}, \frac{2\alpha}{3} + \frac{4}{3}; \frac{7}{3}, \frac{2\alpha}{3} + \frac{7}{3}; \frac{1}{3}(-4)(a z)^{3/2}\right) - {}_2F_2\left(\frac{7}{6}, \frac{2\alpha}{3} + \frac{4}{3}; \frac{7}{3}, \frac{2\alpha}{3} + \frac{7}{3}; \frac{4}{3}(a z)^{3/2}\right)\right) z^2 - 9(\alpha+2)\, \Gamma\left(\frac{5}{3}\right) {}_2F_2\left(-\frac{1}{6}, \frac{2\alpha}{3}; -\frac{1}{3}, \frac{2\alpha}{3} + 1; \frac{4}{3}(a z)^{3/2}\right) + 9(\alpha+2)\, \Gamma\left(\frac{5}{3}\right) {}_2F_2\left(-\frac{1}{6}, \frac{2\alpha}{3}; -\frac{1}{3}, \frac{2\alpha}{3} + 1; \frac{1}{3}(-4)(a z)^{3/2}\right)\right)\right)$$

03.08.21.0028.01

$$\int z^{\alpha-1} \sinh\left(\frac{2}{3}(a z)^{3/2} + b\right) \mathrm{Bi}'(a z)\, dz = \frac{1}{6\, 3^{5/6} \alpha (\alpha+2)\, \Gamma\left(\frac{1}{3}\right) \Gamma\left(\frac{5}{3}\right)} \left(e^{-b} z^\alpha \left(-3^{2/3} a^2 \alpha\, \Gamma\left(\frac{1}{3}\right) \left({}_2F_2\left(\frac{7}{6}, \frac{2\alpha}{3} + \frac{4}{3}; \frac{7}{3}, \frac{2\alpha}{3} + \frac{7}{3}; \frac{1}{3}(-4)(a z)^{3/2}\right) - e^{2b}\, {}_2F_2\left(\frac{7}{6}, \frac{2\alpha}{3} + \frac{4}{3}; \frac{7}{3}, \frac{2\alpha}{3} + \frac{7}{3}; \frac{4}{3}(a z)^{3/2}\right)\right) z^2 + 9 e^{2b} (\alpha+2)\, \Gamma\left(\frac{5}{3}\right) {}_2F_2\left(-\frac{1}{6}, \frac{2\alpha}{3}; -\frac{1}{3}, \frac{2\alpha}{3} + 1; \frac{4}{3}(a z)^{3/2}\right) - 9(\alpha+2)\, \Gamma\left(\frac{5}{3}\right) {}_2F_2\left(-\frac{1}{6}, \frac{2\alpha}{3}; -\frac{1}{3}, \frac{2\alpha}{3} + 1; \frac{1}{3}(-4)(a z)^{3/2}\right)\right)\right)$$

Power arguments

03.08.21.0029.01

$$\int z^{\alpha-1} \sinh\left(\frac{2}{3} (a z^r)^{3/2}\right) \operatorname{Bi}'(a z^r) \, dz = -\frac{1}{6\, 3^{5/6} \alpha (2 r+\alpha) \Gamma\left(\frac{1}{3}\right) \Gamma\left(\frac{5}{3}\right)} \left(z^\alpha \left(3^{2/3} a^2 \alpha \Gamma\left(\frac{1}{3}\right) \left({}_2F_2\left(\frac{7}{6}, \frac{2\alpha}{3r}+\frac{4}{3}; \frac{7}{3}, \frac{2\alpha}{3r}+\frac{7}{3}; \frac{1}{3}(-4)(a z^r)^{3/2}\right) - {}_2F_2\left(\frac{7}{6}, \frac{2\alpha}{3r}+\frac{4}{3}; \frac{7}{3}, \frac{2\alpha}{3r}+\frac{7}{3}; \frac{4}{3}(a z^r)^{3/2}\right)\right) z^{2r} - 9 (2r+\alpha) \Gamma\left(\frac{5}{3}\right) {}_2F_2\left(-\frac{1}{6}, \frac{2\alpha}{3r}; -\frac{1}{3}, \frac{2\alpha}{3r}+1; \frac{4}{3}(a z^r)^{3/2}\right) + 9 (2r+\alpha) \Gamma\left(\frac{5}{3}\right) {}_2F_2\left(-\frac{1}{6}, \frac{2\alpha}{3r}; -\frac{1}{3}, \frac{2\alpha}{3r}+1; \frac{1}{3}(-4)(a z^r)^{3/2}\right)\right)\right)$$

03.08.21.0030.01

$$\int z^{\alpha-1} \sinh\left(\frac{2}{3} (a z^r)^{3/2} + b\right) \operatorname{Bi}'(a z^r) \, dz = \frac{1}{6\, 3^{5/6} \alpha (2 r+\alpha) \Gamma\left(\frac{1}{3}\right) \Gamma\left(\frac{5}{3}\right)} \left(e^{-b} z^\alpha \left(-3^{2/3} a^2 \alpha \Gamma\left(\frac{1}{3}\right) \left({}_2F_2\left(\frac{7}{6}, \frac{2\alpha}{3r}+\frac{4}{3}; \frac{7}{3}, \frac{2\alpha}{3r}+\frac{7}{3}; \frac{1}{3}(-4)(a z^r)^{3/2}\right) - e^{2b} {}_2F_2\left(\frac{7}{6}, \frac{2\alpha}{3r}+\frac{4}{3}; \frac{7}{3}, \frac{2\alpha}{3r}+\frac{7}{3}; \frac{4}{3}(a z^r)^{3/2}\right)\right) z^{2r} + 9 e^{2b} (2r+\alpha) \Gamma\left(\frac{5}{3}\right) {}_2F_2\left(-\frac{1}{6}, \frac{2\alpha}{3r}; -\frac{1}{3}, \frac{2\alpha}{3r}+1; \frac{4}{3}(a z^r)^{3/2}\right) - 9 (2r+\alpha) \Gamma\left(\frac{5}{3}\right) {}_2F_2\left(-\frac{1}{6}, \frac{2\alpha}{3r}; -\frac{1}{3}, \frac{2\alpha}{3r}+1; \frac{1}{3}(-4)(a z^r)^{3/2}\right)\right)\right)$$

Involving cosh and power

Linear argument

03.08.21.0031.01

$$\int z^{\alpha-1} \cosh\left(\frac{2}{3} (a z)^{3/2}\right) \operatorname{Bi}'(a z) \, dz = \frac{1}{6\, 3^{5/6} \alpha (\alpha+2) \Gamma\left(\frac{1}{3}\right) \Gamma\left(\frac{5}{3}\right)} \left(z^\alpha \left(3^{2/3} a^2 \alpha \Gamma\left(\frac{1}{3}\right) \left({}_2F_2\left(\frac{7}{6}, \frac{2\alpha}{3}+\frac{4}{3}; \frac{7}{3}, \frac{2\alpha}{3}+\frac{7}{3}; \frac{4}{3}(a z)^{3/2}\right) + {}_2F_2\left(\frac{7}{6}, \frac{2\alpha}{3}+\frac{4}{3}; \frac{7}{3}, \frac{2\alpha}{3}+\frac{7}{3}; \frac{1}{3}(-4)(a z)^{3/2}\right)\right) z^2 + 9 (\alpha+2) \Gamma\left(\frac{5}{3}\right) {}_2F_2\left(-\frac{1}{6}, \frac{2\alpha}{3}; -\frac{1}{3}, \frac{2\alpha}{3}+1; \frac{4}{3}(a z)^{3/2}\right) + 9 (\alpha+2) \Gamma\left(\frac{5}{3}\right) {}_2F_2\left(-\frac{1}{6}, \frac{2\alpha}{3}; -\frac{1}{3}, \frac{2\alpha}{3}+1; \frac{1}{3}(-4)(a z)^{3/2}\right)\right)\right)$$

03.08.21.0032.01

$$\int z^{\alpha-1}\cosh\left(\frac{2}{3}(a\,z)^{3/2}+b\right)\mathrm{Bi}'(a\,z)\,dz = \frac{1}{6\,3^{5/6}\,\alpha\,(\alpha+2)\,\Gamma\left(\frac{1}{3}\right)\Gamma\left(\frac{5}{3}\right)}\left(e^{-b}\,z^{\alpha}\left(3^{2/3}\,a^2\,\alpha\,\Gamma\left(\frac{1}{3}\right)\left(e^{2b}\,{}_2F_2\left(\frac{7}{6},\frac{2\alpha}{3}+\frac{4}{3};\frac{7}{3},\frac{2\alpha}{3}+\frac{7}{3};\frac{4}{3}(a\,z)^{3/2}\right)+{}_2F_2\left(\frac{7}{6},\frac{2\alpha}{3}+\frac{4}{3};\frac{7}{3},\frac{2\alpha}{3}+\frac{7}{3};\frac{1}{3}(-4)(a\,z)^{3/2}\right)\right)z^2+9\,e^{2b}\,(\alpha+2)\,\Gamma\left(\frac{5}{3}\right){}_2F_2\left(-\frac{1}{6},\frac{2\alpha}{3};-\frac{1}{3},\frac{2\alpha}{3}+1;\frac{4}{3}(a\,z)^{3/2}\right)+9\,(\alpha+2)\,\Gamma\left(\frac{5}{3}\right){}_2F_2\left(-\frac{1}{6},\frac{2\alpha}{3};-\frac{1}{3},\frac{2\alpha}{3}+1;\frac{1}{3}(-4)(a\,z)^{3/2}\right)\right)\right)$$

Power arguments

03.08.21.0033.01

$$\int z^{\alpha-1}\cosh\left(\frac{2}{3}(a\,z^r)^{3/2}\right)\mathrm{Bi}'(a\,z^r)\,dz = \frac{1}{6\,3^{5/6}\,\alpha\,(2\,r+\alpha)\,\Gamma\left(\frac{1}{3}\right)\Gamma\left(\frac{5}{3}\right)}\left(z^{\alpha}\left(3^{2/3}\,a^2\,\alpha\,\Gamma\left(\frac{1}{3}\right)\left({}_2F_2\left(\frac{7}{6},\frac{2\alpha}{3r}+\frac{4}{3};\frac{7}{3},\frac{2\alpha}{3r}+\frac{7}{3};\frac{4}{3}(a\,z^r)^{3/2}\right)+{}_2F_2\left(\frac{7}{6},\frac{2\alpha}{3r}+\frac{4}{3};\frac{7}{3},\frac{2\alpha}{3r}+\frac{7}{3};\frac{1}{3}(-4)(a\,z^r)^{3/2}\right)\right)z^{2r}+9\,(2\,r+\alpha)\,\Gamma\left(\frac{5}{3}\right){}_2F_2\left(-\frac{1}{6},\frac{2\alpha}{3r};-\frac{1}{3},\frac{2\alpha}{3r}+1;\frac{4}{3}(a\,z^r)^{3/2}\right)+9\,(2\,r+\alpha)\,\Gamma\left(\frac{5}{3}\right){}_2F_2\left(-\frac{1}{6},\frac{2\alpha}{3r};-\frac{1}{3},\frac{2\alpha}{3r}+1;\frac{1}{3}(-4)(a\,z^r)^{3/2}\right)\right)\right)$$

03.08.21.0034.01

$$\int z^{\alpha-1}\cosh\left(\frac{2}{3}(a\,z^r)^{3/2}+b\right)\mathrm{Bi}'(a\,z^r)\,dz = \frac{1}{6\,3^{5/6}\,\alpha\,(2\,r+\alpha)\,\Gamma\left(\frac{1}{3}\right)\Gamma\left(\frac{5}{3}\right)}\left(e^{-b}\,z^{\alpha}\left(3^{2/3}\,a^2\,\alpha\,\Gamma\left(\frac{1}{3}\right)\left(e^{2b}\,{}_2F_2\left(\frac{7}{6},\frac{2\alpha}{3r}+\frac{4}{3};\frac{7}{3},\frac{2\alpha}{3r}+\frac{7}{3};\frac{4}{3}(a\,z^r)^{3/2}\right)+{}_2F_2\left(\frac{7}{6},\frac{2\alpha}{3r}+\frac{4}{3};\frac{7}{3},\frac{2\alpha}{3r}+\frac{7}{3};\frac{1}{3}(-4)(a\,z^r)^{3/2}\right)\right)z^{2r}+9\,e^{2b}\,(2\,r+\alpha)\,\Gamma\left(\frac{5}{3}\right){}_2F_2\left(-\frac{1}{6},\frac{2\alpha}{3r};-\frac{1}{3},\frac{2\alpha}{3r}+1;\frac{4}{3}(a\,z^r)^{3/2}\right)+9\,(2\,r+\alpha)\,\Gamma\left(\frac{5}{3}\right){}_2F_2\left(-\frac{1}{6},\frac{2\alpha}{3r};-\frac{1}{3},\frac{2\alpha}{3r}+1;\frac{1}{3}(-4)(a\,z^r)^{3/2}\right)\right)\right)$$

Involving functions of the direct function

Involving elementary functions of the direct function

Involving powers of the direct function

Linear arguments

03.08.21.0035.01

$$\int \mathrm{Bi}'(a\,z)^2\,dz = \frac{-a^2\,z^2\,\mathrm{Bi}(a\,z)^2+2\,\mathrm{Bi}'(a\,z)\,\mathrm{Bi}(a\,z)+a\,z\,\mathrm{Bi}'(a\,z)^2}{3\,a}$$

Power arguments

03.08.21.0036.01

$$\int \mathrm{Bi}'(a z^r)^2 \, dz = \frac{z}{4\sqrt[3]{2}\, 3^{2/3}\, \pi^{3/2}\, r} \left(8 \pi^2 G_{2,4}^{1,1}\left(\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3} \,\middle|\, \begin{matrix} 1-\frac{1}{3r}, \frac{7}{6} \\ \frac{2}{3}, 0, \frac{4}{3}, -\frac{1}{3r} \end{matrix} \right) + 3\, G_{2,4}^{3,1}\left(\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3} \,\middle|\, \begin{matrix} 1-\frac{1}{3r}, \frac{7}{6} \\ 0, \frac{2}{3}, \frac{4}{3}, -\frac{1}{3r} \end{matrix} \right) \right)$$

Involving products of the direct function

Linear arguments

03.08.21.0037.01

$$\int \mathrm{Bi}'(-a z)\, \mathrm{Bi}'(a z) \, dz = \frac{2 a^2 z^3}{9\, \Gamma\left(\frac{2}{3}\right) \Gamma\left(\frac{4}{3}\right)} \, {}_0F_3\left(; \frac{2}{3}, \frac{4}{3}, \frac{3}{2}; -\frac{1}{324} a^6 z^6\right) + \frac{3}{4 a \pi^{3/2}} G_{0,4}^{3,0}\left(\frac{a z}{\sqrt[3]{2}\, 3^{2/3}}, \frac{1}{6} \,\middle|\, \frac{1}{6}, \frac{1}{2}, \frac{5}{6}, 0 \right)$$

Power arguments

03.08.21.0038.01

$$\int \mathrm{Bi}'(-a z^r)\, \mathrm{Bi}'(a z^r) \, dz = \\ \frac{z}{4\sqrt[3]{2}\, 3^{2/3}\, \pi^{3/2}\, r} \left(8 \pi^2 G_{1,5}^{1,1}\left(\frac{a z^r}{\sqrt[3]{2}\, 3^{2/3}}, \frac{1}{6} \,\middle|\, \begin{matrix} 1-\frac{1}{6r} \\ \frac{1}{3}, 0, \frac{2}{3}, \frac{5}{6}, -\frac{1}{6r} \end{matrix} \right) - 3\, G_{2,6}^{4,1}\left(\frac{a z^r}{\sqrt[3]{2}\, 3^{2/3}}, \frac{1}{6} \,\middle|\, \begin{matrix} 1-\frac{1}{6r}, -\frac{1}{6} \\ 0, \frac{1}{3}, \frac{2}{3}, \frac{5}{6}, -\frac{1}{6}, -\frac{1}{6r} \end{matrix} \right) \right)$$

Involving functions of the direct function and elementary functions

Involving elementary functions of the direct function and elementary functions

Involving powers of the direct function and a power function

Linear arguments

03.08.21.0039.01

$$\int z^{\alpha-1} \mathrm{Bi}'(a z)^2 \, dz = \frac{z^\alpha \sqrt[3]{\frac{3}{2}}}{4 \pi^{3/2}} G_{2,4}^{3,1}\left(\left(\frac{2}{3}\right)^{2/3} a z, \frac{1}{3} \,\middle|\, \begin{matrix} 1-\frac{\alpha}{3}, \frac{7}{6} \\ 0, \frac{2}{3}, \frac{4}{3}, -\frac{\alpha}{3} \end{matrix} \right) + \frac{4}{9} a^2 z^{\alpha+2}\, \Gamma\left(\frac{\alpha+2}{3}\right) {}_2\tilde{F}_3\left(\frac{1}{2}, \frac{\alpha+2}{3}; \frac{1}{3}, \frac{5}{3}, \frac{\alpha+5}{3}; \frac{4 a^3 z^3}{9} \right)$$

03.08.21.0040.01

$$\int z\, \mathrm{Bi}'(a z)^2 \, dz = \frac{1}{10 a^2} \left(-\left(2 a^3 z^3 + 3\right) \mathrm{Bi}(a z)^2 + 6 a z\, \mathrm{Bi}'(a z)\, \mathrm{Bi}(a z) + 2 a^2 z^2\, \mathrm{Bi}'(a z)^2 \right)$$

03.08.21.0041.01

$$\int z^2\, \mathrm{Bi}'(a z)^2 \, dz = \frac{1}{7 a^3} \left(-a^4\, \mathrm{Bi}(a z)^2 z^4 + 4 a^2\, \mathrm{Bi}(a z)\, \mathrm{Bi}'(a z)\, z^2 + \left(a^3 z^3 - 4\right) \mathrm{Bi}'(a z)^2 \right)$$

03.08.21.0042.01

$$\int z^3\,\mathrm{Bi}'(a\,z)^2\,dz = \frac{1}{18\,a^4}\left(-a^2\,z^2\left(2\,a^3\,z^3+5\right)\mathrm{Bi}(a\,z)^2+10\left(a^3\,z^3+1\right)\mathrm{Bi}'(a\,z)\,\mathrm{Bi}(a\,z)+2\,a\,z\left(a^3\,z^3-5\right)\mathrm{Bi}'(a\,z)^2\right)$$

Power arguments

03.08.21.0043.01

$$\int z^{\alpha-1}\,\mathrm{Bi}'(a\,z^r)^2\,dz = \frac{z^\alpha}{4\sqrt[3]{2}\;3^{2/3}\,\pi^{3/2}\,r}\left(8\,\pi^2\,G_{2,4}^{1,1}\!\left(\left(\frac{2}{3}\right)^{2/3}a\,z^r,\frac{1}{3}\,\middle|\,\begin{matrix}1-\frac{\alpha}{3r},\frac{7}{6}\\ \frac{2}{3},0,\frac{4}{3},-\frac{\alpha}{3r}\end{matrix}\right)+3\,G_{2,4}^{3,1}\!\left(\left(\frac{2}{3}\right)^{2/3}a\,z^r,\frac{1}{3}\,\middle|\,\begin{matrix}1-\frac{\alpha}{3r},\frac{7}{6}\\ 0,\frac{2}{3},\frac{4}{3},-\frac{\alpha}{3r}\end{matrix}\right)\right)$$

Involving products of the direct function and a power function

Linear arguments

03.08.21.0044.01

$$\int z^{\alpha-1}\,\mathrm{Bi}'(-a\,z)\,\mathrm{Bi}'(a\,z)\,dz =$$

$$\frac{z^\alpha}{72\,\pi^{3/2}}\left(8\,a^2\,\pi^2\,\Gamma\!\left(\frac{\alpha+2}{6}\right){}_1\tilde{F}_4\!\left(\frac{\alpha+2}{6};\frac{1}{2},\frac{2}{3},\frac{4}{3},\frac{\alpha+8}{6};-\frac{1}{324}\,a^6\,z^6\right)z^2+9\,2^{2/3}\sqrt[3]{3}\;G_{1,5}^{3,1}\!\left(\frac{a\,z}{\sqrt[3]{2}\;3^{2/3}},\frac{1}{6}\,\middle|\,\begin{matrix}1-\frac{\alpha}{6}\\ 0,\frac{1}{3},\frac{2}{3},\frac{5}{6},-\frac{\alpha}{6}\end{matrix}\right)\right)$$

Power arguments

03.08.21.0045.01

$$\int z^{\alpha-1}\,\mathrm{Bi}'(-a\,z^r)\,\mathrm{Bi}'(a\,z^r)\,dz =$$

$$\frac{z^\alpha}{4\sqrt[3]{2}\;3^{2/3}\,\pi^{3/2}\,r}\left(8\,\pi^2\,G_{1,5}^{1,1}\!\left(\frac{a\,z^r}{\sqrt[3]{2}\;3^{2/3}},\frac{1}{6}\,\middle|\,\begin{matrix}1-\frac{\alpha}{6r}\\ \frac{1}{3},0,\frac{2}{3},\frac{5}{6},-\frac{\alpha}{6r}\end{matrix}\right)-3\,G_{2,6}^{4,1}\!\left(\frac{a\,z^r}{\sqrt[3]{2}\;3^{2/3}},\frac{1}{6}\,\middle|\,\begin{matrix}1-\frac{\alpha}{6r},-\frac{1}{6}\\ 0,\frac{1}{3},\frac{2}{3},\frac{5}{6},-\frac{1}{6},-\frac{\alpha}{6r}\end{matrix}\right)\right)$$

Involving direct function and Bessel-type functions

Involving Bessel functions

Involving Bessel I

Linear argument

03.08.21.0046.01

$$\int I_\nu\!\left(\frac{2}{3}\,(a\,z)^{3/2}\right)\mathrm{Bi}'(a\,z)\,dz = \frac{2^{\nu-\frac{2}{3}}\,3^{-\nu-\frac{5}{6}}\,z\left((a\,z)^{3/2}\right)^\nu}{\sqrt{\pi}}\,G_{4,6}^{2,3}\!\left(-\left(\frac{2}{3}\right)^{2/3}a\,z,\frac{1}{3}\,\middle|\,\begin{matrix}\frac{1}{6}(2-3\,\nu),\frac{1}{6}(4-3\,\nu),\frac{1}{6}(5-3\,\nu),\frac{1}{3}\\ 0,\frac{2}{3},\frac{1}{3},\frac{1}{6}(-3\,\nu-2),\frac{2}{3}-\nu,-\nu\end{matrix}\right)$$

Power arguments

03.08.21.0047.01

$$\int I_\nu\left(\frac{2}{3}(a z^r)^{3/2}\right)\mathrm{Bi}'(a z^r)\,dz = \frac{2^{\nu-\frac{2}{3}}\,3^{-\nu-\frac{5}{6}}\,z\left((a z^r)^{3/2}\right)^\nu}{\sqrt{\pi}\,r}\,G_{4,6}^{2,3}\left(-\left(\frac{2}{3}\right)^{2/3} a z^r,\,\frac{1}{3}\,\middle|\,\begin{matrix}\frac{1}{6}(2-3\nu),\,\frac{1}{6}(5-3\nu),\,-\frac{\nu}{2}-\frac{1}{3r}+1,\,\frac{1}{3}\\ 0,\,\frac{2}{3},\,\frac{1}{3},\,\frac{2}{3}-\nu,\,-\nu,\,-\frac{3r\nu+2}{6r}\end{matrix}\right)$$

Involving Bessel I and power

Linear argument

03.08.21.0048.01

$$\int z^{\alpha-1} I_\nu\left(\frac{2}{3}(a z)^{3/2}\right)\mathrm{Bi}'(a z)\,dz = \frac{2^{\nu-\frac{2}{3}}\,3^{-\nu-\frac{5}{6}}\,z^\alpha\left((a z)^{3/2}\right)^\nu}{\sqrt{\pi}}\,G_{4,6}^{2,3}\left(-\left(\frac{2}{3}\right)^{2/3} a z,\,\frac{1}{3}\,\middle|\,\begin{matrix}\frac{1}{6}(2-3\nu),\,\frac{1}{6}(5-3\nu),\,\frac{1}{6}(-2\alpha-3\nu+6),\,\frac{1}{3}\\ 0,\,\frac{2}{3},\,\frac{1}{3},\,\frac{1}{6}(-2\alpha-3\nu),\,\frac{2}{3}-\nu,\,-\nu\end{matrix}\right)$$

03.08.21.0049.01

$$\int z^{3/2} I_\nu\left(\frac{2}{3}(a z)^{3/2}\right)\mathrm{Bi}'(a z)\,dz = \frac{2^{\nu-\frac{2}{3}}\,3^{-\nu-\frac{5}{6}}\,z^{5/2}\left((a z)^{3/2}\right)^\nu}{\sqrt{\pi}}\,G_{4,6}^{2,3}\left(-\left(\frac{2}{3}\right)^{2/3} a z,\,\frac{1}{3}\,\middle|\,\begin{matrix}\frac{1}{6}(1-3\nu),\,\frac{1}{6}(2-3\nu),\,\frac{1}{6}(5-3\nu),\,\frac{1}{3}\\ 0,\,\frac{2}{3},\,\frac{1}{3},\,\frac{1}{6}(-3\nu-5),\,\frac{2}{3}-\nu,\,-\nu\end{matrix}\right)$$

03.08.21.0050.01

$$\int z^{-3/2} I_\nu\left(\frac{2}{3}(a z)^{3/2}\right)\mathrm{Bi}'(a z)\,dz = \frac{2^{\nu-\frac{2}{3}}\,3^{-\nu-\frac{5}{6}}\left((a z)^{3/2}\right)^\nu}{\sqrt{\pi}\,\sqrt{z}}\,G_{4,6}^{2,3}\left(-\left(\frac{2}{3}\right)^{2/3} a z,\,\frac{1}{3}\,\middle|\,\begin{matrix}\frac{1}{6}(2-3\nu),\,\frac{1}{6}(5-3\nu),\,\frac{1}{6}(7-3\nu),\,\frac{1}{3}\\ 0,\,\frac{2}{3},\,\frac{1}{3},\,\frac{1}{6}(1-3\nu),\,\frac{2}{3}-\nu,\,-\nu\end{matrix}\right)$$

Power arguments

03.08.21.0051.01

$$\int z^{\alpha-1} I_\nu\left(\frac{2}{3}(a z^r)^{3/2}\right)\mathrm{Bi}'(a z^r)\,dz = \frac{2^{\nu-\frac{2}{3}}\,3^{-\nu-\frac{5}{6}}\,z^\alpha\left((a z^r)^{3/2}\right)^\nu}{\sqrt{\pi}\,r}\,G_{4,6}^{2,3}\left(-\left(\frac{2}{3}\right)^{2/3} a z^r,\,\frac{1}{3}\,\middle|\,\begin{matrix}\frac{1}{6}(2-3\nu),\,\frac{1}{6}(5-3\nu),\,-\frac{\alpha}{3r}-\frac{\nu}{2}+1,\,\frac{1}{3}\\ 0,\,\frac{2}{3},\,\frac{1}{3},\,\frac{2}{3}-\nu,\,-\nu,\,-\frac{2\alpha+3r\nu}{6r}\end{matrix}\right)$$

Involving Bessel K

Linear argument

03.08.21.0052.01

$$\int K_\nu\left(\frac{2}{3}(a z)^{3/2}\right)\mathrm{Bi}'(a z)\,dz =$$

$$-2^{-\nu-\frac{5}{3}}\,3^{-\nu-\frac{5}{6}}\sqrt{\pi}\,z\left((a z)^{3/2}\right)^{-\nu}\csc(\pi\nu)\left(4^\nu\left((a z)^{3/2}\right)^{2\nu} G_{4,6}^{2,3}\left(-\left(\frac{2}{3}\right)^{2/3} a z,\,\frac{1}{3}\,\middle|\,\begin{matrix}\frac{1}{6}(2-3\nu),\,\frac{1}{6}(4-3\nu),\,\frac{1}{6}(5-3\nu),\,\frac{1}{3}\\ 0,\,\frac{2}{3},\,\frac{1}{3},\,\frac{1}{6}(-3\nu-2),\,\frac{2}{3}-\nu,\,-\nu\end{matrix}\right)-\right.$$

$$\left.9^\nu\,G_{4,6}^{2,3}\left(-\left(\frac{2}{3}\right)^{2/3} a z,\,\frac{1}{3}\,\middle|\,\begin{matrix}\frac{1}{6}(3\nu+2),\,\frac{1}{6}(3\nu+4),\,\frac{1}{6}(3\nu+5),\,\frac{1}{3}\\ 0,\,\frac{2}{3},\,\frac{1}{3},\,\nu,\,\nu+\frac{2}{3},\,\frac{1}{6}(3\nu-2)\end{matrix}\right)\right)$$

03.08.21.0053.01

$$\int K_0\left(\frac{2}{3}(a\,z)^{3/2}\right)\mathrm{Bi}'(a\,z)\,dz = -\frac{1}{2\,2^{2/3}\,3^{5/6}\,\pi^{3/2}}\left(z\left(\left(2\,\pi\log\left((a\,z)^{3/2}\right)-3\,\pi\log(-a\,z)\right)G_{2,4}^{2,1}\left(-\left(\frac{2}{3}\right)^{2/3}a\,z,\,\frac{1}{3}\,\middle|\,\begin{array}{c}\frac{5}{6},\frac{1}{3}\\ 0,\frac{2}{3},-\frac{1}{3},0\end{array}\right)+G_{4,6}^{4,3}\left(-\left(\frac{2}{3}\right)^{2/3}a\,z,\,\frac{1}{3}\,\middle|\,\begin{array}{c}\frac{1}{3},\frac{2}{3},\frac{5}{6},-\frac{1}{6}\\ 0,0,\frac{2}{3},\frac{2}{3},-\frac{1}{3},-\frac{1}{6}\end{array}\right)+\pi^2\,G_{6,8}^{4,3}\left(-\left(\frac{2}{3}\right)^{2/3}a\,z,\,\frac{1}{3}\,\middle|\,\begin{array}{c}\frac{1}{3},\frac{2}{3},\frac{5}{6},-\frac{1}{6},\frac{1}{6},\frac{1}{2}\\ 0,0,\frac{2}{3},\frac{2}{3},-\frac{1}{3},-\frac{1}{6},\frac{1}{6},\frac{1}{2}\end{array}\right)\right)\right)$$

03.08.21.0054.01

$$\int K_1\left(\frac{2}{3}(a\,z)^{3/2}\right)\mathrm{Bi}'(a\,z)\,dz = \frac{1}{24\,2^{2/3}\,3^{5/6}\,\pi^{3/2}}\left(z\,(a\,z)^{3/2}\left(\left(8\,\pi\log\left((a\,z)^{3/2}\right)-12\,\pi\log(-a\,z)\right)G_{2,4}^{2,1}\left(-\left(\frac{2}{3}\right)^{2/3}a\,z,\,\frac{1}{3}\,\middle|\,\begin{array}{c}\frac{5}{6},\frac{1}{3}\\ 0,\frac{2}{3},-\frac{1}{3},0\end{array}\right)+G_{4,6}^{5,2}\left(-\left(\frac{2}{3}\right)^{2/3}a\,z,\,\frac{1}{3}\,\middle|\,\begin{array}{c}\frac{1}{3},\frac{5}{6},\frac{2}{3},\frac{11}{6}\\ -1,-\frac{1}{3},-\frac{1}{3},1,\frac{5}{3},\frac{11}{6}\end{array}\right)+2\,G_{4,6}^{5,2}\left(-\left(\frac{2}{3}\right)^{2/3}a\,z,\,\frac{1}{3}\,\middle|\,\begin{array}{c}\frac{1}{3},\frac{5}{6},-\frac{1}{6},\frac{2}{3}\\ -1,-\frac{1}{3},-\frac{1}{3},1,\frac{5}{3},-\frac{1}{6}\end{array}\right)+G_{4,6}^{5,2}\left(-\left(\frac{2}{3}\right)^{2/3}a\,z,\,\frac{1}{3}\,\middle|\,\begin{array}{c}\frac{1}{3},\frac{5}{6},-\frac{13}{6},\frac{2}{3}\\ -1,-\frac{1}{3},-\frac{1}{3},1,\frac{5}{3},-\frac{13}{6}\end{array}\right)+4\,\pi^2\,G_{6,8}^{5,2}\left(-\left(\frac{2}{3}\right)^{2/3}a\,z,\,\frac{1}{3}\,\middle|\,\begin{array}{c}\frac{1}{3},\frac{5}{6},-\frac{1}{6},\frac{1}{6},\frac{1}{2},\frac{2}{3}\\ -1,-\frac{1}{3},-\frac{1}{3},1,\frac{5}{3},-\frac{1}{6},\frac{1}{6},\frac{1}{2}\end{array}\right)+12\,\pi\log(-a\,z)\,G_{3,5}^{2,2}\left(-\left(\frac{2}{3}\right)^{2/3}a\,z,\,\frac{1}{3}\,\middle|\,\begin{array}{c}\frac{2}{3},\frac{5}{6},\frac{1}{3}\\ 1,\frac{5}{3},-1,-\frac{1}{3},-\frac{1}{3}\end{array}\right)-8\,\pi\log\left((a\,z)^{3/2}\right)G_{3,5}^{2,2}\left(-\left(\frac{2}{3}\right)^{2/3}a\,z,\,\frac{1}{3}\,\middle|\,\begin{array}{c}\frac{2}{3},\frac{5}{6},\frac{1}{3}\\ 1,\frac{5}{3},-1,-\frac{1}{3},-\frac{1}{3}\end{array}\right)+4\,G_{4,6}^{4,3}\left(-\left(\frac{2}{3}\right)^{2/3}a\,z,\,\frac{1}{3}\,\middle|\,\begin{array}{c}\frac{1}{3},\frac{2}{3},\frac{5}{6},-\frac{1}{6}\\ 0,0,\frac{2}{3},\frac{2}{3},-\frac{1}{3},-\frac{1}{6}\end{array}\right)+4\,\pi^2\,G_{6,8}^{4,3}\left(-\left(\frac{2}{3}\right)^{2/3}a\,z,\,\frac{1}{3}\,\middle|\,\begin{array}{c}\frac{1}{3},\frac{2}{3},\frac{5}{6},-\frac{1}{6},\frac{1}{6},\frac{1}{2}\\ 0,0,\frac{2}{3},\frac{2}{3},-\frac{1}{3},-\frac{1}{6},\frac{1}{6},\frac{1}{2}\end{array}\right)\right)\right)$$

03.08.21.0055.01

$$\int K_2\left(\frac{2}{3}(a\,z)^{3/2}\right)\mathrm{Bi}'(a\,z)\,dz = -\frac{1}{8\,2^{2/3}\,3^{5/6}\,a\,\pi^{3/2}}\left(4\,a\,\pi\,z\left(2\log\left((a\,z)^{3/2}\right)-3\log(-a\,z)\right)G_{3,5}^{2,2}\left(-\left(\frac{2}{3}\right)^{2/3}a\,z,\,\frac{1}{3}\,\middle|\,\begin{array}{c}\frac{2}{3},\frac{5}{6},\frac{1}{3}\\ 1,\frac{5}{3},-1,-\frac{1}{3},-\frac{1}{3}\end{array}\right)-a\,z\left(G_{4,6}^{5,2}\left(-\left(\frac{2}{3}\right)^{2/3}a\,z,\,\frac{1}{3}\,\middle|\,\begin{array}{c}\frac{1}{3},\frac{5}{6},\frac{2}{3},\frac{11}{6}\\ -1,-\frac{1}{3},-\frac{1}{3},1,\frac{5}{3},\frac{11}{6}\end{array}\right)+2\,G_{4,6}^{5,2}\left(-\left(\frac{2}{3}\right)^{2/3}a\,z,\,\frac{1}{3}\,\middle|\,\begin{array}{c}\frac{1}{3},\frac{5}{6},-\frac{1}{6},\frac{2}{3}\\ -1,-\frac{1}{3},-\frac{1}{3},1,\frac{5}{3},-\frac{1}{6}\end{array}\right)+G_{4,6}^{5,2}\left(-\left(\frac{2}{3}\right)^{2/3}a\,z,\,\frac{1}{3}\,\middle|\,\begin{array}{c}\frac{1}{3},\frac{5}{6},-\frac{13}{6},\frac{2}{3}\\ -1,-\frac{1}{3},-\frac{1}{3},1,\frac{5}{3},-\frac{13}{6}\end{array}\right)+4\,\pi^2\,G_{6,8}^{5,2}\left(-\left(\frac{2}{3}\right)^{2/3}a\,z,\,\frac{1}{3}\,\middle|\,\begin{array}{c}\frac{1}{3},\frac{5}{6},-\frac{1}{6},\frac{1}{6},\frac{1}{2},\frac{2}{3}\\ -1,-\frac{1}{3},-\frac{1}{3},1,\frac{5}{3},-\frac{1}{6},\frac{1}{6},\frac{1}{2}\end{array}\right)\right)\right)$$

Power arguments

03.08.21.0056.01

$$\int K_\nu\left(\frac{2}{3}(a z^r)^{3/2}\right)\mathrm{Bi}'(a z^r)\,dz = -\frac{1}{r}\left(2^{-\nu-\frac{5}{3}} 3^{-\nu-\frac{5}{6}}\sqrt{\pi}\, z\left((a z^r)^{3/2}\right)^{-\nu}\csc(\pi\nu)\left(4^\nu\left((a z^r)^{3/2}\right)^{2\nu} G_{4,6}^{2,3}\left(-\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3} \middle| \begin{matrix} \frac{1}{6}(2-3\nu), \frac{1}{6}(5-3\nu), -\frac{\nu}{2}-\frac{1}{3r}+1, \frac{1}{3} \\ 0, \frac{2}{3}, \frac{1}{3}, \frac{2}{3}-\nu, -\nu, -\frac{3r\nu+2}{6r} \end{matrix}\right) - 9^\nu G_{4,6}^{2,3}\left(-\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3} \middle| \begin{matrix} \frac{\nu}{2}-\frac{1}{3r}+1, \frac{1}{6}(3\nu+2), \frac{1}{6}(3\nu+5), \frac{1}{3} \\ 0, \frac{2}{3}, \frac{1}{3}, \nu, \nu+\frac{2}{3}, \frac{3r\nu-2}{6r} \end{matrix}\right)\right)\right)$$

03.08.21.0057.01

$$\int K_0\left(\frac{2}{3}(a z^r)^{3/2}\right)\mathrm{Bi}'(a z^r)\,dz = -\frac{1}{2\, 2^{2/3} 3^{5/6}\pi^{3/2} r}\left(z\left(G_{4,6}^{4,3}\left(-\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3} \middle| \begin{matrix} \frac{1}{3}, \frac{5}{6}, 1-\frac{1}{3r}, -\frac{1}{6} \\ 0, 0, \frac{2}{3}, \frac{2}{3}, -\frac{1}{6}, -\frac{1}{3r} \end{matrix}\right) + \pi\left(\left(2\log\left((a z^r)^{3/2}\right) - 3\log(-a z^r)\right) G_{4,6}^{2,3}\left(-\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3} \middle| \begin{matrix} \frac{1}{3}, \frac{5}{6}, 1-\frac{1}{3r}, \frac{1}{3} \\ 0, \frac{2}{3}, 0, \frac{1}{3}, \frac{2}{3}, -\frac{1}{3r} \end{matrix}\right) + \pi\, G_{6,8}^{4,3}\left(-\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3} \middle| \begin{matrix} \frac{1}{3}, \frac{5}{6}, 1-\frac{1}{3r}, -\frac{1}{6}, \frac{1}{6}, \frac{1}{2} \\ 0, 0, \frac{2}{3}, \frac{2}{3}, -\frac{1}{6}, \frac{1}{6}, \frac{1}{2}, -\frac{1}{3r} \end{matrix}\right)\right)\right)\right)$$

03.08.21.0058.01

$$\int K_1\left(\frac{2}{3}(a z^r)^{3/2}\right)\mathrm{Bi}'(a z^r)\,dz = \frac{1}{24\, 2^{2/3} 3^{5/6}\pi^{3/2} r}\left(z (a z^r)^{3/2}\left(4\pi\left(2\log\left((a z^r)^{3/2}\right) - 3\log(-a z^r)\right) G_{4,6}^{2,3}\left(-\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3} \middle| \begin{matrix} \frac{1}{3}, \frac{5}{6}, 1-\frac{1}{3r}, \frac{1}{3} \\ 0, \frac{2}{3}, 0, \frac{1}{3}, \frac{2}{3}, -\frac{1}{3r} \end{matrix}\right) + 4\pi\left(2\log\left((a z^r)^{3/2}\right) - 3\log(-a z^r)\right) G_{4,6}^{2,3}\left(-\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3} \middle| \begin{matrix} \frac{1}{3}, \frac{5}{6}, 1-\frac{1}{3r}, \frac{4}{3} \\ 1, \frac{5}{3}, -1, -\frac{1}{3}, \frac{4}{3}, -\frac{1}{3r} \end{matrix}\right) - G_{4,6}^{4,3}\left(-\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3} \middle| \begin{matrix} \frac{1}{3}, \frac{5}{6}, 1-\frac{1}{3r}, \frac{11}{6} \\ -1, -\frac{1}{3}, 1, \frac{5}{3}, \frac{11}{6}, -\frac{1}{3r} \end{matrix}\right) - 2\, G_{4,6}^{4,3}\left(-\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3} \middle| \begin{matrix} \frac{1}{3}, \frac{5}{6}, 1-\frac{1}{3r}, -\frac{1}{6} \\ -1, -\frac{1}{3}, 1, \frac{5}{3}, -\frac{1}{6}, -\frac{1}{3r} \end{matrix}\right) - G_{4,6}^{4,3}\left(-\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3} \middle| \begin{matrix} \frac{1}{3}, \frac{5}{6}, 1-\frac{1}{3r}, -\frac{13}{6} \\ -1, -\frac{1}{3}, 1, \frac{5}{3}, -\frac{13}{6}, -\frac{1}{3r} \end{matrix}\right) - 4\pi^2 G_{6,8}^{4,3}\left(-\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3} \middle| \begin{matrix} \frac{1}{3}, \frac{5}{6}, 1-\frac{1}{3r}, -\frac{1}{6}, \frac{1}{6}, \frac{1}{2} \\ -1, -\frac{1}{3}, 1, \frac{5}{3}, -\frac{1}{6}, \frac{1}{6}, \frac{1}{2}, -\frac{1}{3r} \end{matrix}\right) + 4\left(G_{4,6}^{4,3}\left(-\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3} \middle| \begin{matrix} \frac{1}{3}, \frac{5}{6}, 1-\frac{1}{3r}, -\frac{1}{6} \\ 0, 0, \frac{2}{3}, \frac{2}{3}, -\frac{1}{6}, -\frac{1}{3r} \end{matrix}\right) + \pi^2 G_{6,8}^{4,3}\left(-\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3} \middle| \begin{matrix} \frac{1}{3}, \frac{5}{6}, 1-\frac{1}{3r}, -\frac{1}{6}, \frac{1}{6}, \frac{1}{2} \\ 0, 0, \frac{2}{3}, \frac{2}{3}, -\frac{1}{6}, \frac{1}{6}, \frac{1}{2}, -\frac{1}{3r} \end{matrix}\right)\right)\right)\right)$$

03.08.21.0059.01

$$\int K_2\left(\frac{2}{3}(a z^r)^{3/2}\right) \mathrm{Bi}'(a z^r)\, dz = -\frac{1}{8\, 2^{2/3}\, 3^{5/6}\, \pi^{3/2}\, r}\left(z\left(4\pi\left(3\log(-a z^r) - 2\log\left((a z^r)^{3/2}\right)\right) G_{4,6}^{2,3}\left(-\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3} \,\middle|\, \begin{matrix} \frac{1}{3}, \frac{5}{6}, 1-\frac{1}{3r}, \frac{4}{3} \\ 1, \frac{5}{3}, -1, -\frac{1}{3}, \frac{4}{3}, -\frac{1}{3r} \end{matrix}\right) + G_{4,6}^{4,3}\left(-\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3} \,\middle|\, \begin{matrix} \frac{1}{3}, \frac{5}{6}, 1-\frac{1}{3r}, \frac{11}{6} \\ -1, -\frac{1}{3}, 1, \frac{5}{3}, \frac{11}{6}, -\frac{1}{3r} \end{matrix}\right) + 2\, G_{4,6}^{4,3}\left(-\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3} \,\middle|\, \begin{matrix} \frac{1}{3}, \frac{5}{6}, 1-\frac{1}{3r}, -\frac{1}{6} \\ -1, -\frac{1}{3}, 1, \frac{5}{3}, -\frac{1}{6}, -\frac{1}{3r} \end{matrix}\right) + G_{4,6}^{4,3}\left(-\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3} \,\middle|\, \begin{matrix} \frac{1}{3}, \frac{5}{6}, 1-\frac{1}{3r}, -\frac{13}{6} \\ -1, -\frac{1}{3}, 1, \frac{5}{3}, -\frac{13}{6}, -\frac{1}{3r} \end{matrix}\right) + 4\pi^2\, G_{6,8}^{4,3}\left(-\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3} \,\middle|\, \begin{matrix} \frac{1}{3}, \frac{5}{6}, 1-\frac{1}{3r}, -\frac{1}{6}, \frac{1}{6}, \frac{1}{2} \\ -1, -\frac{1}{3}, 1, \frac{5}{3}, -\frac{1}{6}, \frac{1}{6}, \frac{1}{2}, -\frac{1}{3r} \end{matrix}\right)\right)\right)$$

Involving Bessel K and power

Linear argument

03.08.21.0060.01

$$\int z^{\alpha-1} K_\nu\left(\frac{2}{3}(a z)^{3/2}\right) \mathrm{Bi}'(a z)\, dz = -2^{-\nu-\frac{5}{3}}\, 3^{-\nu-\frac{5}{6}} \sqrt{\pi}\, z^\alpha \left((a z)^{3/2}\right)^{-\nu} \csc(\pi\nu)\left(4^\nu \left((a z)^{3/2}\right)^{2\nu} G_{4,6}^{2,3}\left(-\left(\frac{2}{3}\right)^{2/3} a z, \frac{1}{3} \,\middle|\, \begin{matrix} \frac{1}{6}(2-3\nu), \frac{1}{6}(5-3\nu), \frac{1}{6}(-2\alpha-3\nu+6), \frac{1}{3} \\ 0, \frac{2}{3}, \frac{1}{3}, \frac{1}{6}(-2\alpha-3\nu), \frac{2}{3}-\nu, -\nu \end{matrix}\right) - 9^\nu\, G_{4,6}^{2,3}\left(-\left(\frac{2}{3}\right)^{2/3} a z, \frac{1}{3} \,\middle|\, \begin{matrix} \frac{1}{6}(3\nu+2), \frac{1}{6}(3\nu+5), \frac{1}{6}(-2\alpha+3\nu+6), \frac{1}{3} \\ 0, \frac{2}{3}, \frac{1}{3}, \nu, \nu+\frac{2}{3}, \frac{1}{6}(3\nu-2\alpha) \end{matrix}\right)\right)$$

03.08.21.0061.01

$$\int z^{\alpha-1} K_0\left(\frac{2}{3}(a z)^{3/2}\right) \mathrm{Bi}'(a z)\, dz = -\frac{1}{2\, 2^{2/3}\, 3^{5/6}\, \pi^{3/2}}\left(z^\alpha\left(\left(2\pi\log\left((a z)^{3/2}\right) - 3\pi\log(-a z)\right) G_{3,5}^{2,2}\left(-\left(\frac{2}{3}\right)^{2/3} a z, \frac{1}{3} \,\middle|\, \begin{matrix} \frac{5}{6}, 1-\frac{\alpha}{3}, \frac{1}{3} \\ 0, \frac{2}{3}, 0, \frac{2}{3}, -\frac{\alpha}{3} \end{matrix}\right) + G_{4,6}^{4,3}\left(-\left(\frac{2}{3}\right)^{2/3} a z, \frac{1}{3} \,\middle|\, \begin{matrix} \frac{1}{3}, \frac{5}{6}, 1-\frac{\alpha}{3}, -\frac{1}{6} \\ 0, 0, \frac{2}{3}, \frac{2}{3}, -\frac{1}{6}, -\frac{\alpha}{3} \end{matrix}\right) + \pi^2\, G_{6,8}^{4,3}\left(-\left(\frac{2}{3}\right)^{2/3} a z, \frac{1}{3} \,\middle|\, \begin{matrix} \frac{1}{3}, \frac{5}{6}, 1-\frac{\alpha}{3}, -\frac{1}{6}, \frac{1}{6}, \frac{1}{2} \\ 0, 0, \frac{2}{3}, \frac{2}{3}, -\frac{1}{6}, \frac{1}{6}, \frac{1}{2}, -\frac{\alpha}{3} \end{matrix}\right)\right)\right)$$

03.08.21.0062.01

$$\begin{aligned}
&\int z^{\alpha-1} K_1\!\left(\frac{2}{3}(a\,z)^{3/2}\right) \mathrm{Bi}'(a\,z)\,dz = \\
&\frac{1}{24\,2^{2/3}\,3^{5/6}\,\pi^{3/2}} \left(z^{\alpha} (a\,z)^{3/2} \left(G_{3,5}^{4,2}\!\left(-\left(\frac{2}{3}\right)^{2/3} a\,z, \frac{1}{3} \,\middle|\, \begin{array}{c} \frac{1}{3}, 1-\frac{\alpha}{3}, \frac{5}{6} \\ -1, -\frac{1}{3}, 1, \frac{5}{3}, -\frac{\alpha}{3} \end{array}\right) + \left(8\,\pi \log\!\left((a\,z)^{3/2}\right) - 12\,\pi \log(-a\,z)\right)\right.\right. \\
&\quad G_{3,5}^{2,2}\!\left(-\left(\frac{2}{3}\right)^{2/3} a\,z, \frac{1}{3} \,\middle|\, \begin{array}{c} \frac{5}{6}, 1-\frac{\alpha}{3}, \frac{1}{3} \\ 0, \frac{2}{3}, 0, \frac{2}{3}, -\frac{\alpha}{3} \end{array}\right) + 12\,\pi \log(-a\,z)\, G_{3,5}^{2,2}\!\left(-\left(\frac{2}{3}\right)^{2/3} a\,z, \frac{1}{3} \,\middle|\, \begin{array}{c} \frac{5}{6}, 1-\frac{\alpha}{3}, \frac{1}{3} \\ 1, \frac{5}{3}, -1, -\frac{1}{3}, -\frac{\alpha}{3} \end{array}\right) - \\
&\quad 8\,\pi \log\!\left((a\,z)^{3/2}\right) G_{3,5}^{2,2}\!\left(-\left(\frac{2}{3}\right)^{2/3} a\,z, \frac{1}{3} \,\middle|\, \begin{array}{c} \frac{5}{6}, 1-\frac{\alpha}{3}, \frac{1}{3} \\ 1, \frac{5}{3}, -1, -\frac{1}{3}, -\frac{\alpha}{3} \end{array}\right) - 2\,G_{4,6}^{4,3}\!\left(-\left(\frac{2}{3}\right)^{2/3} a\,z, \frac{1}{3} \,\middle|\, \begin{array}{c} \frac{1}{3}, \frac{5}{6}, 1-\frac{\alpha}{3}, -\frac{1}{6} \\ -1, -\frac{1}{3}, 1, \frac{5}{3}, -\frac{1}{6}, -\frac{\alpha}{3} \end{array}\right) + \\
&\quad 4\,G_{4,6}^{4,3}\!\left(-\left(\frac{2}{3}\right)^{2/3} a\,z, \frac{1}{3} \,\middle|\, \begin{array}{c} \frac{1}{3}, \frac{5}{6}, 1-\frac{\alpha}{3}, -\frac{1}{6} \\ 0, 0, \frac{2}{3}, \frac{2}{3}, -\frac{1}{6}, -\frac{\alpha}{3} \end{array}\right) - G_{4,6}^{4,3}\!\left(-\left(\frac{2}{3}\right)^{2/3} a\,z, \frac{1}{3} \,\middle|\, \begin{array}{c} \frac{1}{3}, \frac{5}{6}, 1-\frac{\alpha}{3}, -\frac{13}{6} \\ -1, -\frac{1}{3}, 1, \frac{5}{3}, -\frac{13}{6}, -\frac{\alpha}{3} \end{array}\right) - \\
&\quad \left.\left. 4\,\pi^2\, G_{6,8}^{4,3}\!\left(-\left(\frac{2}{3}\right)^{2/3} a\,z, \frac{1}{3} \,\middle|\, \begin{array}{c} \frac{1}{3}, \frac{5}{6}, 1-\frac{\alpha}{3}, -\frac{1}{6}, \frac{1}{6}, \frac{1}{2} \\ -1, -\frac{1}{3}, 1, \frac{5}{3}, -\frac{1}{6}, \frac{1}{6}, \frac{1}{2}, -\frac{\alpha}{3} \end{array}\right) + 4\,\pi^2\, G_{6,8}^{4,3}\!\left(-\left(\frac{2}{3}\right)^{2/3} a\,z, \frac{1}{3} \,\middle|\, \begin{array}{c} \frac{1}{3}, \frac{5}{6}, 1-\frac{\alpha}{3}, -\frac{1}{6}, \frac{1}{6}, \frac{1}{2} \\ 0, 0, \frac{2}{3}, \frac{2}{3}, -\frac{1}{6}, \frac{1}{6}, \frac{1}{2}, -\frac{\alpha}{3} \end{array}\right)\right)\right)
\end{aligned}$$

03.08.21.0063.01

$$\begin{aligned}
&\int z^{\alpha-1} K_2\!\left(\frac{2}{3}(a\,z)^{3/2}\right) \mathrm{Bi}'(a\,z)\,dz = \\
&\frac{1}{8\,2^{2/3}\,3^{5/6}\,\pi^{3/2}} \left(z^{\alpha} \left(G_{3,5}^{4,2}\!\left(-\left(\frac{2}{3}\right)^{2/3} a\,z, \frac{1}{3} \,\middle|\, \begin{array}{c} \frac{1}{3}, 1-\frac{\alpha}{3}, \frac{5}{6} \\ -1, -\frac{1}{3}, 1, \frac{5}{3}, -\frac{\alpha}{3} \end{array}\right) + 4\,\pi \left(3 \log(-a\,z) - 2 \log\!\left((a\,z)^{3/2}\right)\right)\right.\right. \\
&\quad G_{3,5}^{2,2}\!\left(-\left(\frac{2}{3}\right)^{2/3} a\,z, \frac{1}{3} \,\middle|\, \begin{array}{c} \frac{5}{6}, 1-\frac{\alpha}{3}, \frac{1}{3} \\ 1, \frac{5}{3}, -1, -\frac{1}{3}, -\frac{\alpha}{3} \end{array}\right) - 2\,G_{4,6}^{4,3}\!\left(-\left(\frac{2}{3}\right)^{2/3} a\,z, \frac{1}{3} \,\middle|\, \begin{array}{c} \frac{1}{3}, \frac{5}{6}, 1-\frac{\alpha}{3}, -\frac{1}{6} \\ -1, -\frac{1}{3}, 1, \frac{5}{3}, -\frac{1}{6}, -\frac{\alpha}{3} \end{array}\right) - \\
&\quad \left.\left. G_{4,6}^{4,3}\!\left(-\left(\frac{2}{3}\right)^{2/3} a\,z, \frac{1}{3} \,\middle|\, \begin{array}{c} \frac{1}{3}, \frac{5}{6}, 1-\frac{\alpha}{3}, -\frac{13}{6} \\ -1, -\frac{1}{3}, 1, \frac{5}{3}, -\frac{13}{6}, -\frac{\alpha}{3} \end{array}\right) - 4\,\pi^2\, G_{6,8}^{4,3}\!\left(-\left(\frac{2}{3}\right)^{2/3} a\,z, \frac{1}{3} \,\middle|\, \begin{array}{c} \frac{1}{3}, \frac{5}{6}, 1-\frac{\alpha}{3}, -\frac{1}{6}, \frac{1}{6}, \frac{1}{2} \\ -1, -\frac{1}{3}, 1, \frac{5}{3}, -\frac{1}{6}, \frac{1}{6}, \frac{1}{2}, -\frac{\alpha}{3} \end{array}\right)\right)\right)
\end{aligned}$$

03.08.21.0064.01

$$\begin{aligned}
&\int z^{3/2} K_2\!\left(\frac{2}{3}(a\,z)^{3/2}\right) \mathrm{Bi}'(a\,z)\,dz = \\
&\frac{1}{8\,2^{2/3}\,3^{5/6}\,\pi^{3/2}} \left(z^{5/2} \left(-4\,\pi^2\, G_{5,7}^{4,2}\!\left(-\left(\frac{2}{3}\right)^{2/3} a\,z, \frac{1}{3} \,\middle|\, \begin{array}{c} \frac{1}{3}, \frac{5}{6}, -\frac{1}{6}, \frac{1}{6}, \frac{1}{2} \\ -1, -\frac{1}{3}, 1, \frac{5}{3}, -\frac{5}{6}, -\frac{1}{6}, \frac{1}{2} \end{array}\right) + G_{3,5}^{4,2}\!\left(-\left(\frac{2}{3}\right)^{2/3} a\,z, \frac{1}{3} \,\middle|\, \begin{array}{c} \frac{1}{6}, \frac{1}{3}, \frac{5}{6} \\ -1, -\frac{1}{3}, 1, \frac{5}{3}, -\frac{5}{6} \end{array}\right) +\right.\right. \\
&\quad 4\,\pi \left(3 \log(-a\,z) - 2 \log\!\left((a\,z)^{3/2}\right)\right) G_{3,5}^{2,2}\!\left(-\left(\frac{2}{3}\right)^{2/3} a\,z, \frac{1}{3} \,\middle|\, \begin{array}{c} \frac{1}{6}, \frac{5}{6}, \frac{1}{3} \\ 1, \frac{5}{3}, -1, -\frac{5}{6}, -\frac{1}{3} \end{array}\right) - \\
&\quad \left.\left. 2\,G_{4,6}^{4,3}\!\left(-\left(\frac{2}{3}\right)^{2/3} a\,z, \frac{1}{3} \,\middle|\, \begin{array}{c} \frac{1}{6}, \frac{1}{3}, \frac{5}{6}, -\frac{1}{6} \\ -1, -\frac{1}{3}, 1, \frac{5}{3}, -\frac{5}{6}, -\frac{1}{6} \end{array}\right) - G_{4,6}^{4,3}\!\left(-\left(\frac{2}{3}\right)^{2/3} a\,z, \frac{1}{3} \,\middle|\, \begin{array}{c} \frac{1}{6}, \frac{1}{3}, \frac{5}{6}, -\frac{13}{6} \\ -1, -\frac{1}{3}, 1, \frac{5}{3}, -\frac{13}{6}, -\frac{5}{6} \end{array}\right)\right)\right)
\end{aligned}$$

03.08.21.0065.01

$$\int z^{-3/2} K_2\left(\frac{2}{3}(a z)^{3/2}\right) \mathrm{Bi}'(a z)\, dz = \frac{1}{8\, 2^{2/3}\, 3^{5/6}\, \pi^{3/2} \sqrt{z}} \left(G_{3,5}^{4,2}\left(-\left(\frac{2}{3}\right)^{2/3} a z, \frac{1}{3} \,\middle|\, \begin{matrix} \frac{1}{3}, \frac{7}{6}, \frac{5}{6} \\ -1, -\frac{1}{3}, 1, \frac{5}{3}, \frac{1}{6} \end{matrix}\right) + 4\pi \left(3\log(-a z) - 2\log\left((a z)^{3/2}\right)\right) G_{3,5}^{2,2}\left(-\left(\frac{2}{3}\right)^{2/3} a z, \frac{1}{3} \,\middle|\, \begin{matrix} \frac{5}{6}, \frac{7}{6}, \frac{1}{3} \\ 1, \frac{5}{3}, -1, -\frac{1}{3}, \frac{1}{6} \end{matrix}\right) - 2\, G_{4,6}^{4,3}\left(-\left(\frac{2}{3}\right)^{2/3} a z, \frac{1}{3} \,\middle|\, \begin{matrix} \frac{1}{3}, \frac{5}{6}, \frac{7}{6}, -\frac{1}{6} \\ -1, -\frac{1}{3}, 1, \frac{5}{3}, -\frac{1}{6}, \frac{1}{6} \end{matrix}\right) - G_{4,6}^{4,3}\left(-\left(\frac{2}{3}\right)^{2/3} a z, \frac{1}{3} \,\middle|\, \begin{matrix} \frac{1}{3}, \frac{5}{6}, \frac{7}{6}, -\frac{13}{6} \\ -1, -\frac{1}{3}, 1, \frac{5}{3}, -\frac{13}{6}, \frac{1}{6} \end{matrix}\right) - 4\pi^2\, G_{6,8}^{4,3}\left(-\left(\frac{2}{3}\right)^{2/3} a z, \frac{1}{3} \,\middle|\, \begin{matrix} \frac{1}{3}, \frac{5}{6}, \frac{7}{6}, -\frac{1}{6}, \frac{1}{6}, \frac{1}{2} \\ -1, -\frac{1}{3}, 1, \frac{5}{3}, -\frac{1}{6}, \frac{1}{6}, \frac{1}{6}, \frac{1}{2} \end{matrix}\right)\right)$$

Power arguments

03.08.21.0066.01

$$\int z^{\alpha-1} K_\nu\left(\frac{2}{3}(a z^r)^{3/2}\right) \mathrm{Bi}'(a z^r)\, dz = -\frac{1}{r}\left(2^{-\nu-\frac{5}{3}}\, 3^{-\nu-\frac{5}{6}} \sqrt{\pi}\, z^\alpha \left((a z^r)^{3/2}\right)^{-\nu} \csc(\pi\nu) \left(4^\nu \left((a z^r)^{3/2}\right)^{2\nu} G_{4,6}^{2,3}\left(-\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3} \,\middle|\, \begin{matrix} \frac{1}{6}(2-3\nu), \frac{1}{6}(5-3\nu), -\frac{\alpha}{3r} - \frac{\nu}{2} + 1, \frac{1}{3} \\ 0, \frac{2}{3}, \frac{1}{3}, \frac{2}{3} - \nu, -\nu, -\frac{2\alpha+3r\nu}{6r} \end{matrix}\right) - 9^\nu\, G_{4,6}^{2,3}\left(-\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3} \,\middle|\, \begin{matrix} -\frac{\alpha}{3r} + \frac{\nu}{2} + 1, \frac{1}{6}(3\nu+2), \frac{1}{6}(3\nu+5), \frac{1}{3} \\ 0, \frac{2}{3}, \frac{1}{3}, \frac{\nu}{2} - \frac{\alpha}{3r}, \nu, \nu + \frac{2}{3} \end{matrix}\right)\right)\right)$$

03.08.21.0067.01

$$\int z^{\alpha-1} K_0\left(\frac{2}{3}(a z^r)^{3/2}\right) \mathrm{Bi}'(a z^r)\, dz = -\frac{1}{2\, 2^{2/3}\, 3^{5/6}\, \pi^{3/2}\, r}\left(z^\alpha \left(G_{4,6}^{4,3}\left(-\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3} \,\middle|\, \begin{matrix} \frac{1}{3}, \frac{5}{6}, 1 - \frac{\alpha}{3r}, -\frac{1}{6} \\ 0, 0, \frac{2}{3}, \frac{2}{3}, -\frac{1}{6}, -\frac{\alpha}{3r} \end{matrix}\right) + \pi\left(\left(2\log\left((a z^r)^{3/2}\right) - 3\log(-a z^r)\right) G_{4,6}^{2,3}\left(-\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3} \,\middle|\, \begin{matrix} \frac{1}{3}, \frac{5}{6}, 1 - \frac{\alpha}{3r}, \frac{1}{3} \\ 0, \frac{2}{3}, 0, \frac{1}{3}, \frac{2}{3}, -\frac{\alpha}{3r} \end{matrix}\right) + \pi\, G_{6,8}^{4,3}\left(-\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3} \,\middle|\, \begin{matrix} \frac{1}{3}, \frac{5}{6}, 1 - \frac{\alpha}{3r}, -\frac{1}{6}, \frac{1}{6}, \frac{1}{2} \\ 0, 0, \frac{2}{3}, \frac{2}{3}, -\frac{1}{6}, \frac{1}{6}, \frac{1}{2}, -\frac{\alpha}{3r} \end{matrix}\right)\right)\right)\right)$$

03.08.21.0068.01

$$\int z^{\alpha-1} K_1\!\left(\frac{2}{3}(a z^r)^{3/2}\right) \operatorname{Bi}'(a z^r)\, dz = \frac{1}{24\, 2^{2/3}\, 3^{5/6}\, \pi^{3/2}\, r} \Bigg(z^\alpha (a z^r)^{3/2} \Bigg(4\pi \left(2\log\left((a z^r)^{3/2}\right) - 3\log(-a z^r)\right) G_{4,6}^{2,3}\!\left(-\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3} \,\middle|\, \begin{matrix} \frac{1}{3}, \frac{5}{6}, 1-\frac{\alpha}{3r}, \frac{1}{3} \\ 0, \frac{2}{3}, 0, \frac{1}{3}, \frac{2}{3}, -\frac{\alpha}{3r} \end{matrix} \right) +$$

$$4\pi \left(2\log\left((a z^r)^{3/2}\right) - 3\log(-a z^r)\right) G_{4,6}^{2,3}\!\left(-\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3} \,\middle|\, \begin{matrix} \frac{1}{3}, \frac{5}{6}, 1-\frac{\alpha}{3r}, \frac{4}{3} \\ 1, \frac{5}{3}, -1, -\frac{1}{3}, \frac{4}{3}, -\frac{\alpha}{3r} \end{matrix} \right) -$$

$$G_{4,6}^{4,3}\!\left(-\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3} \,\middle|\, \begin{matrix} \frac{1}{3}, \frac{5}{6}, 1-\frac{\alpha}{3r}, \frac{11}{6} \\ -1, -\frac{1}{3}, 1, \frac{5}{3}, \frac{11}{6}, -\frac{\alpha}{3r} \end{matrix} \right) - 2\, G_{4,6}^{4,3}\!\left(-\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3} \,\middle|\, \begin{matrix} \frac{1}{3}, \frac{5}{6}, 1-\frac{\alpha}{3r}, -\frac{1}{6} \\ -1, -\frac{1}{3}, 1, \frac{5}{3}, -\frac{1}{6}, -\frac{\alpha}{3r} \end{matrix} \right) -$$

$$G_{4,6}^{4,3}\!\left(-\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3} \,\middle|\, \begin{matrix} \frac{1}{3}, \frac{5}{6}, 1-\frac{\alpha}{3r}, -\frac{13}{6} \\ -1, -\frac{1}{3}, 1, \frac{5}{3}, -\frac{13}{6}, -\frac{\alpha}{3r} \end{matrix} \right) - 4\pi^2\, G_{6,8}^{4,3}\!\left(-\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3} \,\middle|\, \begin{matrix} \frac{1}{3}, \frac{5}{6}, 1-\frac{\alpha}{3r}, -\frac{1}{6}, \frac{1}{6}, \frac{1}{2} \\ -1, -\frac{1}{3}, 1, \frac{5}{3}, -\frac{1}{6}, \frac{1}{6}, \frac{1}{2}, -\frac{\alpha}{3r} \end{matrix} \right) +$$

$$4 \left(G_{4,6}^{4,3}\!\left(-\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3} \,\middle|\, \begin{matrix} \frac{1}{3}, \frac{5}{6}, 1-\frac{\alpha}{3r}, -\frac{1}{6} \\ 0, 0, \frac{2}{3}, \frac{2}{3}, -\frac{1}{6}, -\frac{\alpha}{3r} \end{matrix} \right) + \pi^2\, G_{6,8}^{4,3}\!\left(-\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3} \,\middle|\, \begin{matrix} \frac{1}{3}, \frac{5}{6}, 1-\frac{\alpha}{3r}, -\frac{1}{6}, \frac{1}{6}, \frac{1}{2} \\ 0, 0, \frac{2}{3}, \frac{2}{3}, -\frac{1}{6}, \frac{1}{6}, \frac{1}{2}, -\frac{\alpha}{3r} \end{matrix} \right)\right)\Bigg)\Bigg)$$

03.08.21.0069.01

$$\int z^{\alpha-1} K_2\!\left(\frac{2}{3}(a z^r)^{3/2}\right) \operatorname{Bi}'(a z^r)\, dz = -\frac{1}{8\, 2^{2/3}\, 3^{5/6}\, \pi^{3/2}\, r} \Bigg(z^\alpha \Bigg(4\pi \left(3\log(-a z^r) - 2\log\left((a z^r)^{3/2}\right)\right) G_{4,6}^{2,3}\!\left(-\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3} \,\middle|\, \begin{matrix} \frac{1}{3}, \frac{5}{6}, 1-\frac{\alpha}{3r}, \frac{4}{3} \\ 1, \frac{5}{3}, -1, -\frac{1}{3}, \frac{4}{3}, -\frac{\alpha}{3r} \end{matrix} \right) +$$

$$G_{4,6}^{4,3}\!\left(-\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3} \,\middle|\, \begin{matrix} \frac{1}{3}, \frac{5}{6}, 1-\frac{\alpha}{3r}, \frac{11}{6} \\ -1, -\frac{1}{3}, 1, \frac{5}{3}, \frac{11}{6}, -\frac{\alpha}{3r} \end{matrix} \right) + 2\, G_{4,6}^{4,3}\!\left(-\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3} \,\middle|\, \begin{matrix} \frac{1}{3}, \frac{5}{6}, 1-\frac{\alpha}{3r}, -\frac{1}{6} \\ -1, -\frac{1}{3}, 1, \frac{5}{3}, -\frac{1}{6}, -\frac{\alpha}{3r} \end{matrix} \right) +$$

$$G_{4,6}^{4,3}\!\left(-\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3} \,\middle|\, \begin{matrix} \frac{1}{3}, \frac{5}{6}, 1-\frac{\alpha}{3r}, -\frac{13}{6} \\ -1, -\frac{1}{3}, 1, \frac{5}{3}, -\frac{13}{6}, -\frac{\alpha}{3r} \end{matrix} \right) + 4\pi^2\, G_{6,8}^{4,3}\!\left(-\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3} \,\middle|\, \begin{matrix} \frac{1}{3}, \frac{5}{6}, 1-\frac{\alpha}{3r}, -\frac{1}{6}, \frac{1}{6}, \frac{1}{2} \\ -1, -\frac{1}{3}, 1, \frac{5}{3}, -\frac{1}{6}, \frac{1}{6}, \frac{1}{2}, -\frac{\alpha}{3r} \end{matrix} \right)\Bigg)\Bigg)$$

Involving other Airy functions

Involving Ai

Linear arguments

03.08.21.0070.01

$$\int \operatorname{Ai}(a z) \operatorname{Bi}'(a z)\, dz = \frac{\operatorname{Ai}(a z)\left(\operatorname{Bi}(a z) + a z \operatorname{Bi}'(a z)\right) - a z \operatorname{Ai}'(a z) \operatorname{Bi}(a z)}{2a}$$

Power arguments

03.08.21.0071.01

$$\int \mathrm{Ai}(a\,z^r)\,\mathrm{Bi}'(a\,z^r)\,dz = \frac{z}{2\,\pi} - \frac{z}{12\,\pi^{3/2}\,r}\,G_{2,4}^{2,2}\!\left(\left(\frac{2}{3}\right)^{2/3} a\,z^r,\ \frac{1}{3}\ \middle|\ \begin{matrix} \frac{1}{2},\ 1-\frac{1}{3\,r} \\ \frac{1}{3},\ \frac{2}{3},\ 0,\ -\frac{1}{3\,r} \end{matrix}\right)$$

Involving Ai and power

Linear arguments

03.08.21.0072.01

$$\int z^{\alpha-1}\,\mathrm{Ai}(a\,z)\,\mathrm{Bi}'(a\,z)\,dz = \frac{z^{\alpha}}{12\,\pi^{3/2}\,\alpha}\left(6\,\sqrt{\pi} - \alpha\,G_{2,4}^{2,2}\!\left(\left(\frac{2}{3}\right)^{2/3} a\,z,\ \frac{1}{3}\ \middle|\ \begin{matrix} \frac{1}{2},\ 1-\frac{\alpha}{3} \\ \frac{1}{3},\ \frac{2}{3},\ 0,\ -\frac{\alpha}{3} \end{matrix}\right)\right)$$

03.08.21.0073.01

$$\int z\,\mathrm{Ai}(a\,z)\,\mathrm{Bi}'(a\,z)\,dz = \frac{1}{4}\left(\mathrm{Ai}(a\,z)\,\mathrm{Bi}'(a\,z)\,z^2 + \left(\frac{2\,\mathrm{Bi}'(a\,z)}{a^2} - z^2\,\mathrm{Bi}(a\,z)\right)\mathrm{Ai}'(a\,z)\right)$$

03.08.21.0074.01

$$\int z^2\,\mathrm{Ai}(a\,z)\,\mathrm{Bi}'(a\,z)\,dz = \frac{1}{6\,a^3}\left(\mathrm{Ai}(a\,z)\left(a^2\,\mathrm{Bi}(a\,z)\,z^2 + \left(a^3\,z^3-1\right)\mathrm{Bi}'(a\,z)\right) - \mathrm{Ai}'(a\,z)\left(\left(a^3\,z^3+1\right)\mathrm{Bi}(a\,z) - 2\,a\,z\,\mathrm{Bi}'(a\,z)\right)\right)$$

03.08.21.0075.01

$$\int z^3\,\mathrm{Ai}(a\,z)\,\mathrm{Bi}'(a\,z)\,dz =$$

$$\frac{1}{40\,a^4}\left(a\,z\,\mathrm{Ai}'(a\,z)\left(12\,a\,z\,\mathrm{Bi}'(a\,z) - \left(5\,a^3\,z^3+12\right)\mathrm{Bi}(a\,z)\right) + \mathrm{Ai}(a\,z)\left(4\left(2\,a^3\,z^3+3\right)\mathrm{Bi}(a\,z) + a\,z\left(5\,a^3\,z^3-12\right)\mathrm{Bi}'(a\,z)\right)\right)$$

Power arguments

03.08.21.0076.01

$$\int z^{\alpha-1}\,\mathrm{Ai}(a\,z^r)\,\mathrm{Bi}'(a\,z^r)\,dz = \frac{z^{\alpha}}{2\,\pi\,\alpha} - \frac{z^{\alpha}}{12\,\pi^{3/2}\,r}\,G_{2,4}^{2,2}\!\left(\left(\frac{2}{3}\right)^{2/3} a\,z^r,\ \frac{1}{3}\ \middle|\ \begin{matrix} \frac{1}{2},\ 1-\frac{\alpha}{3\,r} \\ \frac{1}{3},\ \frac{2}{3},\ 0,\ -\frac{\alpha}{3\,r} \end{matrix}\right)$$

Involving Bi

Linear arguments

03.08.21.0077.01

$$\int \mathrm{Bi}(a\,z)\,\mathrm{Bi}'(a\,z)\,dz = \frac{\mathrm{Bi}(a\,z)^2}{2\,a}$$

Power arguments

03.08.21.0078.01

$$\int \operatorname{Bi}(a z^r)\operatorname{Bi}'(a z^r)\,dz = \frac{z}{12\pi^{3/2} r}\left(6\sqrt{3\pi}\, r + \sqrt{3}\, G_{2,4}^{2,2}\!\left(\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3}\,\middle|\, \begin{matrix} \frac{1}{2}, 1-\frac{1}{3r} \\ \frac{1}{3}, \frac{2}{3}, 0, -\frac{1}{3r}\end{matrix}\right) + 8\pi^2 G_{5,7}^{2,3}\!\left(\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3}\,\middle|\, \begin{matrix} \frac{1}{2}, 1, 1-\frac{1}{3r}, \frac{5}{6}, \frac{4}{3} \\ \frac{2}{3}, 1, 0, \frac{1}{3}, \frac{5}{6}, \frac{4}{3}, -\frac{1}{3r}\end{matrix}\right)\right)$$

Involving Bi and power

Linear arguments

03.08.21.0079.01

$$\int z^{\alpha-1}\operatorname{Bi}(a z)\operatorname{Bi}'(a z)\,dz = \frac{z^\alpha}{12\pi^{3/2}\alpha}\left(\sqrt{3}\,\alpha\, G_{2,4}^{2,2}\!\left(\left(\frac{2}{3}\right)^{2/3} a z, \frac{1}{3}\,\middle|\, \begin{matrix} \frac{1}{2}, 1-\frac{\alpha}{3} \\ \frac{1}{3}, \frac{2}{3}, 0, -\frac{\alpha}{3}\end{matrix}\right) + 8\pi^2\alpha\, G_{5,7}^{2,3}\!\left(\left(\frac{2}{3}\right)^{2/3} a z, \frac{1}{3}\,\middle|\, \begin{matrix} \frac{1}{2}, 1, 1-\frac{\alpha}{3}, \frac{5}{6}, \frac{4}{3} \\ \frac{2}{3}, 1, 0, \frac{1}{3}, \frac{5}{6}, \frac{4}{3}, -\frac{\alpha}{3}\end{matrix}\right) + 6\sqrt{3\pi}\right)$$

03.08.21.0080.01

$$\int z\operatorname{Bi}(a z)\operatorname{Bi}'(a z)\,dz = \frac{\operatorname{Bi}'(a z)^2}{2a^2}$$

03.08.21.0081.01

$$\int z^2\operatorname{Bi}(a z)\operatorname{Bi}'(a z)\,dz = \frac{a^2 z^2\operatorname{Bi}(a z)^2 - 2\operatorname{Bi}'(a z)\operatorname{Bi}(a z) + 2 a z \operatorname{Bi}'(a z)^2}{6a^3}$$

03.08.21.0082.01

$$\int z^3\operatorname{Bi}(a z)\operatorname{Bi}'(a z)\,dz = \frac{1}{10a^4}\left(\left(2a^3 z^3+3\right)\operatorname{Bi}(a z)^2 - 6 a z\operatorname{Bi}'(a z)\operatorname{Bi}(a z) + 3a^2 z^2\operatorname{Bi}'(a z)^2\right)$$

Power arguments

03.08.21.0083.01

$$\int z^{\alpha-1}\operatorname{Bi}(a z^r)\operatorname{Bi}'(a z^r)\,dz = \frac{z^\alpha}{12\pi^{3/2} r\alpha}\left(6\sqrt{3\pi}\, r + \sqrt{3}\,\alpha\, G_{2,4}^{2,2}\!\left(\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3}\,\middle|\, \begin{matrix} \frac{1}{2}, 1-\frac{\alpha}{3r} \\ \frac{1}{3}, \frac{2}{3}, 0, -\frac{\alpha}{3r}\end{matrix}\right) + 8\pi^2\alpha\, G_{5,7}^{2,3}\!\left(\left(\frac{2}{3}\right)^{2/3} a z^r, \frac{1}{3}\,\middle|\, \begin{matrix} \frac{1}{2}, 1, 1-\frac{\alpha}{3r}, \frac{5}{6}, \frac{4}{3} \\ \frac{2}{3}, 1, 0, \frac{1}{3}, \frac{5}{6}, \frac{4}{3}, -\frac{\alpha}{3r}\end{matrix}\right)\right)$$

Involving Ai′

Linear arguments

03.08.21.0084.01

$$\int \operatorname{Ai}'(a z)\operatorname{Bi}'(-a z)\,dz = -\frac{1}{4a\pi^{3/2}}\, G_{1,5}^{3,1}\!\left(\frac{a z}{\sqrt[3]{2}\, 3^{2/3}}, \frac{1}{6}\,\middle|\, \begin{matrix} 1 \\ \frac{1}{6}, \frac{5}{6}, 1, 0, \frac{1}{2}\end{matrix}\right)$$

03.08.21.0085.01

$$\int \mathrm{Ai}'(a\,z)\,\mathrm{Bi}'(a\,z)\,dz = \frac{1}{3\,a}\left(\mathrm{Ai}(a\,z)\left(\mathrm{Bi}'(a\,z) - a^2\,z^2\,\mathrm{Bi}(a\,z)\right) + \mathrm{Ai}'(a\,z)\left(\mathrm{Bi}(a\,z) + a\,z\,\mathrm{Bi}'(a\,z)\right)\right)$$

Power arguments

03.08.21.0086.01

$$\int \mathrm{Ai}'(a\,z^r)\,\mathrm{Bi}'(-a\,z^r)\,dz = -\frac{z}{4\sqrt[3]{2}\;3^{2/3}\,\pi^{3/2}\,r}\,G_{2,6}^{4,1}\!\left(\frac{a\,z^r}{\sqrt[3]{2}\;3^{2/3}},\,\frac{1}{6}\,\middle|\,\begin{array}{c}1-\frac{1}{6r},\,\frac{1}{3}\\ 0,\,\frac{1}{3},\,\frac{2}{3},\,\frac{5}{6},\,\frac{1}{3},\,-\frac{1}{6r}\end{array}\right)$$

03.08.21.0087.01

$$\int \mathrm{Ai}'(a\,z^r)\,\mathrm{Bi}'(a\,z^r)\,dz = -\frac{\sqrt{\pi}\;z}{\sqrt[3]{2}\;3^{2/3}\,r}\,G_{3,5}^{2,1}\!\left(\left(\frac{2}{3}\right)^{2/3} a\,z^r,\,\frac{1}{3}\,\middle|\,\begin{array}{c}1-\frac{1}{3r},\,\frac{1}{6},\,\frac{7}{6}\\ 0,\,\frac{4}{3},\,\frac{1}{6},\,\frac{2}{3},\,-\frac{1}{3r}\end{array}\right)$$

Involving Ai′ and power

Linear arguments

03.08.21.0088.01

$$\int z^{\alpha-1}\,\mathrm{Ai}'(a\,z)\,\mathrm{Bi}'(-a\,z)\,dz = -\frac{z^\alpha}{4\sqrt[3]{2}\;3^{2/3}\,\pi^{3/2}}\,G_{1,5}^{3,1}\!\left(\frac{a\,z}{\sqrt[3]{2}\;3^{2/3}},\,\frac{1}{6}\,\middle|\,\begin{array}{c}1-\frac{\alpha}{6}\\ 0,\,\frac{2}{3},\,\frac{5}{6},\,\frac{1}{3},\,-\frac{\alpha}{6}\end{array}\right)$$

03.08.21.0089.01

$$\int z^{\alpha-1}\,\mathrm{Ai}'(a\,z)\,\mathrm{Bi}'(a\,z)\,dz = -\frac{\sqrt{\pi}\;z^\alpha}{\sqrt[3]{2}\;3^{2/3}}\,G_{3,5}^{2,1}\!\left(\left(\frac{2}{3}\right)^{2/3} a\,z,\,\frac{1}{3}\,\middle|\,\begin{array}{c}1-\frac{\alpha}{3},\,\frac{1}{6},\,\frac{7}{6}\\ 0,\,\frac{4}{3},\,\frac{1}{6},\,\frac{2}{3},\,-\frac{\alpha}{3}\end{array}\right)$$

03.08.21.0090.01

$$\int z\,\mathrm{Ai}'(a\,z)\,\mathrm{Bi}'(a\,z)\,dz = \frac{1}{10\,a^2}\left(a\,z\,\mathrm{Ai}'(a\,z)\left(3\,\mathrm{Bi}(a\,z) + 2\,a\,z\,\mathrm{Bi}'(a\,z)\right) + \mathrm{Ai}(a\,z)\left(3\,a\,z\,\mathrm{Bi}'(a\,z) - \left(2\,a^3\,z^3 + 3\right)\mathrm{Bi}(a\,z)\right)\right)$$

03.08.21.0091.01

$$\int z^2\,\mathrm{Ai}'(a\,z)\,\mathrm{Bi}'(a\,z)\,dz = \frac{1}{7\,a^3}\left(\mathrm{Ai}(a\,z)\left(2\,a^2\,z^2\,\mathrm{Bi}'(a\,z) - a^4\,z^4\,\mathrm{Bi}(a\,z)\right) + \mathrm{Ai}'(a\,z)\left(2\,a^2\,\mathrm{Bi}(a\,z)\,z^2 + \left(a^3\,z^3 - 4\right)\mathrm{Bi}'(a\,z)\right)\right)$$

03.08.21.0092.01

$$\int z^3\,\mathrm{Ai}'(a\,z)\,\mathrm{Bi}'(a\,z)\,dz = \frac{1}{18\,a^4}\left(\mathrm{Ai}'(a\,z)\left(5\left(a^3\,z^3+1\right)\mathrm{Bi}(a\,z) + 2\,a\,z\left(a^3\,z^3-5\right)\mathrm{Bi}'(a\,z)\right) + \mathrm{Ai}(a\,z)\left(5\left(a^3\,z^3+1\right)\mathrm{Bi}'(a\,z) - a^2\,z^2\left(2\,a^3\,z^3+5\right)\mathrm{Bi}(a\,z)\right)\right)$$

Power arguments

03.08.21.0093.01

$$\int z^{\alpha-1}\,\mathrm{Ai}'(a\,z^r)\,\mathrm{Bi}'(-a\,z^r)\,dz = -\frac{z^\alpha}{4\sqrt[3]{2}\;3^{2/3}\,\pi^{3/2}\,r}\,G_{2,6}^{4,1}\!\left(\frac{a\,z^r}{\sqrt[3]{2}\;3^{2/3}},\,\frac{1}{6}\,\middle|\,\begin{array}{c}1-\frac{\alpha}{6r},\,\frac{1}{3}\\ 0,\,\frac{1}{3},\,\frac{2}{3},\,\frac{5}{6},\,\frac{1}{3},\,-\frac{\alpha}{6r}\end{array}\right)$$

03.08.21.0094.01

$$\int z^{\alpha-1}\,\mathrm{Ai}'(a\,z^r)\,\mathrm{Bi}'(a\,z^r)\,dz = -\frac{\sqrt{\pi}\;z^{\alpha}}{\sqrt[3]{2}\;3^{2/3}\,r}\,G_{3,5}^{2,1}\!\left(\left(\frac{2}{3}\right)^{2/3} a\,z^r,\ \frac{1}{3}\,\middle|\,\begin{matrix}1-\frac{\alpha}{3r},\ \frac{1}{6},\ \frac{7}{6}\\ 0,\ \frac{4}{3},\ \frac{1}{6},\ \frac{2}{3},\ -\frac{\alpha}{3r}\end{matrix}\right)$$

Integral transforms

Laplace transforms

03.08.22.0001.01

$$\mathcal{L}_t[\mathrm{Bi}'(t)](z) = -\frac{1}{6\,\pi\,z}\,e^{-\frac{z^3}{3}}\left(z\left(\Gamma\!\left(\frac{1}{3}\right)\Gamma\!\left(-\frac{1}{3},\,-\frac{z^3}{3}\right)+2\sqrt{3}\,\pi\right)\sqrt[3]{-z^3}+\left(\Gamma\!\left(-\frac{1}{3}\right)\Gamma\!\left(\frac{1}{3},\,-\frac{z^3}{3}\right)+2\sqrt{3}\,\pi\right)\left(-z^3\right)^{2/3}\right)$$

Representations through more general functions

Through hypergeometric functions

Involving $_0F_1$

03.08.26.0001.01

$$\mathrm{Bi}'(z) = \frac{\sqrt[6]{3}}{\Gamma\left(\frac{1}{3}\right)}\,{}_0F_1\!\left(;\frac{1}{3};\frac{z^3}{9}\right)+\frac{z^2}{2\sqrt[6]{3}\;\Gamma\left(\frac{2}{3}\right)}\,{}_0F_1\!\left(;\frac{5}{3};\frac{z^3}{9}\right)$$

Through Meijer *G*

Classical cases for the direct function itself

03.08.26.0002.01

$$\mathrm{Bi}'(z) = \sqrt[6]{3}\,\pi\,G_{1,3}^{1,0}\!\left(\frac{z^3}{9}\,\middle|\,\begin{matrix}\frac{1}{2}\\ 0,\ \frac{2}{3},\ \frac{1}{2}\end{matrix}\right)+\frac{\pi\,z^2}{3\sqrt[6]{3}}\,G_{1,3}^{1,0}\!\left(\frac{z^3}{9}\,\middle|\,\begin{matrix}\frac{1}{2}\\ 0,\ -\frac{2}{3},\ \frac{1}{2}\end{matrix}\right)$$

03.08.26.0026.01

$$\mathrm{Bi}'(z) = -2\,\pi\sqrt[6]{3}\;G_{2,4}^{2,0}\!\left(\frac{z^3}{9}\,\middle|\,\begin{matrix}-\frac{1}{6},\ \frac{1}{3}\\ 0,\ \frac{2}{3},\ -\frac{1}{6},\ \frac{1}{3}\end{matrix}\right)/;\ -\frac{\pi}{3}<\arg(z)\leq\frac{\pi}{3}$$

Classical cases involving exp

03.08.26.0027.01

$$e^{-\frac{1}{3}\left(2\,z^{3/2}\right)}\,\mathrm{Bi}'(z) = \frac{\sqrt[6]{3}}{2\sqrt[3]{2}\,\sqrt{\pi}}\,G_{2,3}^{2,1}\!\left(\frac{4\,z^{3/2}}{3}\,\middle|\,\begin{matrix}\frac{7}{6},\ -\frac{1}{3}\\ 0,\ \frac{4}{3},\ -\frac{1}{3}\end{matrix}\right)/;\ -\frac{2\,\pi}{3}<\arg(z)\leq\frac{2\,\pi}{3}$$

03.08.26.0028.01

$$e^{\frac{2\,z^{3/2}}{3}}\,\mathrm{Bi}'(z) = -\frac{\sqrt[6]{3}\,\sqrt{\pi}}{\sqrt[3]{2}}\,G_{2,3}^{2,0}\!\left(\frac{4\,z^{3/2}}{3}\,\middle|\,\begin{matrix}-\frac{1}{3},\ \frac{7}{6}\\ 0,\ \frac{4}{3},\ -\frac{1}{3}\end{matrix}\right)/;\ -\frac{2\,\pi}{3}<\arg(z)\leq\frac{2\,\pi}{3}$$

03.08.26.0029.01

$$e^{-z}\,\mathrm{Bi}'\left(\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) = \frac{\sqrt[6]{3}}{2\sqrt[3]{2}\,\sqrt{\pi}}\, G_{2,3}^{2,1}\!\left(2z \,\middle|\, \begin{matrix} \frac{7}{6}, -\frac{1}{3} \\ 0, \frac{4}{3}, -\frac{1}{3} \end{matrix}\right)$$

03.08.26.0030.01

$$e^{z}\,\mathrm{Bi}'\left(\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) = -\frac{\sqrt[6]{3}\,\sqrt{\pi}}{\sqrt[3]{2}}\, G_{2,3}^{2,0}\!\left(2z \,\middle|\, \begin{matrix} -\frac{1}{3}, \frac{7}{6} \\ 0, \frac{4}{3}, -\frac{1}{3} \end{matrix}\right)$$

Classical cases involving ${}_0F_1$

03.08.26.0003.01

$$\mathrm{Bi}'(z)\,{}_0F_1\!\left(;b;\frac{z^3}{9}\right) = 2^{b-\frac{2}{3}}\sqrt[6]{3}\,\sqrt{\pi}\,\Gamma(b)\, G_{4,6}^{2,2}\!\left(\frac{4z^3}{9} \,\middle|\, \begin{matrix} \frac{1}{6}(5-3b), \frac{1}{6}(8-3b), \frac{1}{3}, \frac{5}{6} \\ 0, \frac{2}{3}, \frac{1}{3}, \frac{5}{6}, 1-b, \frac{5}{3}-b \end{matrix}\right) /;\ -\frac{\pi}{3} < \arg(z) \leq \frac{\pi}{3}$$

03.08.26.0021.01

$$\mathrm{Bi}'\left(3^{2/3}\sqrt[3]{z}\right){}_0F_1(;b;z) = 2^{b-\frac{2}{3}}\sqrt[6]{3}\,\sqrt{\pi}\,\Gamma(b)\, G_{4,6}^{2,2}\!\left(4z \,\middle|\, \begin{matrix} \frac{1}{6}(5-3b), \frac{1}{6}(8-3b), \frac{1}{3}, \frac{5}{6} \\ 0, \frac{2}{3}, \frac{1}{3}, \frac{5}{6}, 1-b, \frac{5}{3}-b \end{matrix}\right)$$

Classical cases involving ${}_0\tilde{F}_1$

03.08.26.0004.01

$$\mathrm{Bi}'(z)\,{}_0\tilde{F}_1\!\left(;b;\frac{z^3}{9}\right) = 2^{b-\frac{2}{3}}\sqrt[6]{3}\,\sqrt{\pi}\, G_{4,6}^{2,2}\!\left(\frac{4z^3}{9} \,\middle|\, \begin{matrix} \frac{1}{6}(5-3b), \frac{1}{6}(8-3b), \frac{1}{3}, \frac{5}{6} \\ 0, \frac{2}{3}, \frac{1}{3}, \frac{5}{6}, 1-b, \frac{5}{3}-b \end{matrix}\right) /;\ -\frac{\pi}{3} < \arg(z) \leq \frac{\pi}{3}$$

03.08.26.0022.01

$$\mathrm{Bi}'\left(3^{2/3}\sqrt[3]{z}\right){}_0\tilde{F}_1(;b;z) = 2^{b-\frac{2}{3}}\sqrt[6]{3}\,\sqrt{\pi}\, G_{4,6}^{2,2}\!\left(4z \,\middle|\, \begin{matrix} \frac{1}{6}(5-3b), \frac{1}{6}(8-3b), \frac{1}{3}, \frac{5}{6} \\ 0, \frac{2}{3}, \frac{1}{3}, \frac{5}{6}, 1-b, \frac{5}{3}-b \end{matrix}\right)$$

Generalized cases for the direct function itself

03.08.26.0005.01

$$\mathrm{Bi}'(z) = -2\pi\sqrt[6]{3}\, G_{2,4}^{2,0}\!\left(3^{-2/3}z, \frac{1}{3} \,\middle|\, \begin{matrix} -\frac{1}{6}, \frac{1}{3} \\ 0, \frac{2}{3}, -\frac{1}{6}, \frac{1}{3} \end{matrix}\right)$$

Generalized cases involving exp

03.08.26.0006.01

$$\exp\left(-\frac{2z^{3/2}}{3}\right)\mathrm{Bi}'(z) = \frac{\sqrt[6]{3}}{2\sqrt[3]{2}\,\sqrt{\pi}}\, G_{2,3}^{2,1}\!\left(\frac{2\sqrt[3]{2}\,z}{3^{2/3}}, \frac{2}{3} \,\middle|\, \begin{matrix} \frac{7}{6}, -\frac{1}{3} \\ 0, \frac{4}{3}, -\frac{1}{3} \end{matrix}\right)$$

03.08.26.0007.01

$$\exp\left(\frac{2z^{3/2}}{3}\right)\mathrm{Bi}'(z) = -\frac{\sqrt[6]{3}\,\sqrt{\pi}}{\sqrt[3]{2}}\, G_{2,3}^{2,0}\!\left(\frac{2\sqrt[3]{2}\,z}{3^{2/3}}, \frac{2}{3} \,\middle|\, \begin{matrix} -\frac{1}{3}, \frac{7}{6} \\ 0, \frac{4}{3}, -\frac{1}{3} \end{matrix}\right)$$

Generalized cases involving cosh

03.08.26.0008.01

$$\cosh\left(\frac{2 z^{3/2}}{3}\right) \mathrm{Bi}'(z) = \sqrt[6]{\frac{3}{2}}\, \pi\, G_{4,6}^{2,2}\left(\left(\frac{2}{3}\right)^{2/3} z, \frac{1}{3} \,\middle|\, \begin{array}{c} \frac{7}{12}, \frac{13}{12}, \frac{1}{3}, \frac{5}{6} \\ 0, \frac{2}{3}, \frac{1}{3}, \frac{1}{2}, \frac{5}{6}, \frac{7}{6} \end{array}\right)$$

03.08.26.0031.01

$$\cosh(z)\, \mathrm{Bi}'\left(\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) = \sqrt[6]{\frac{3}{2}}\, \pi\, G_{4,6}^{2,2}\left(z, \frac{1}{2} \,\middle|\, \begin{array}{c} \frac{7}{12}, \frac{13}{12}, \frac{1}{3}, \frac{5}{6} \\ 0, \frac{2}{3}, \frac{1}{3}, \frac{1}{2}, \frac{5}{6}, \frac{7}{6} \end{array}\right)$$

Generalized cases involving sinh

03.08.26.0009.01

$$\sinh\left(\frac{2 z^{3/2}}{3}\right) \mathrm{Bi}'(z) = \sqrt[6]{\frac{3}{2}}\, \pi\, G_{4,6}^{2,2}\left(\left(\frac{2}{3}\right)^{2/3} z, \frac{1}{3} \,\middle|\, \begin{array}{c} \frac{7}{12}, \frac{13}{12}, \frac{5}{6}, \frac{4}{3} \\ \frac{1}{2}, \frac{7}{6}, 0, \frac{2}{3}, \frac{5}{6}, \frac{4}{3} \end{array}\right)$$

03.08.26.0032.01

$$\sinh(z)\, \mathrm{Bi}'\left(\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) = \sqrt[6]{\frac{3}{2}}\, \pi\, G_{4,6}^{2,2}\left(z, \frac{1}{2} \,\middle|\, \begin{array}{c} \frac{7}{12}, \frac{13}{12}, \frac{5}{6}, \frac{4}{3} \\ \frac{1}{2}, \frac{7}{6}, 0, \frac{2}{3}, \frac{5}{6}, \frac{4}{3} \end{array}\right)$$

Generalized cases for powers of Bi′

03.08.26.0010.01

$$\mathrm{Bi}'(z)^2 = \sqrt[3]{\frac{2}{3}}\, \sqrt{\pi}\, z \left(G_{3,5}^{2,1}\left(\left(\frac{2}{3}\right)^{2/3} z, \frac{1}{3} \,\middle|\, \begin{array}{c} \frac{5}{6}, 0, \frac{1}{2} \\ -\frac{1}{3}, \frac{1}{3}, 0, \frac{1}{2}, 1 \end{array}\right) + G_{3,5}^{2,1}\left(\left(\frac{2}{3}\right)^{2/3} z, \frac{1}{3} \,\middle|\, \begin{array}{c} \frac{5}{6}, \frac{2}{3}, \frac{7}{6} \\ \frac{1}{3}, 1, -\frac{1}{3}, \frac{2}{3}, \frac{7}{6} \end{array}\right)\right)$$

03.08.26.0023.01

$$\mathrm{Bi}'(z)^2 = \sqrt[3]{\frac{3}{2}}\, \sqrt{\pi} \left(G_{3,5}^{2,1}\left(\left(\frac{2}{3}\right)^{2/3} z, \frac{1}{3} \,\middle|\, \begin{array}{c} \frac{7}{6}, 1, \frac{3}{2} \\ \frac{2}{3}, \frac{4}{3}, 0, 1, \frac{3}{2} \end{array}\right) + G_{3,5}^{2,1}\left(\left(\frac{2}{3}\right)^{2/3} z, \frac{1}{3} \,\middle|\, \begin{array}{c} \frac{7}{6}, \frac{1}{3}, \frac{5}{6} \\ 0, \frac{2}{3}, \frac{1}{3}, \frac{5}{6}, \frac{4}{3} \end{array}\right)\right)$$

Generalized cases involving Ai

03.08.26.0011.01

$$\mathrm{Ai}(z)\, \mathrm{Bi}'(z) = \frac{1}{2\pi} - \frac{1}{4\pi^{3/2}}\, G_{1,3}^{2,1}\left(\left(\frac{2}{3}\right)^{2/3} z, \frac{1}{3} \,\middle|\, \begin{array}{c} \frac{1}{2} \\ \frac{1}{3}, \frac{2}{3}, 0 \end{array}\right)$$

Generalized cases involving Ai′

03.08.26.0012.01

$$\mathrm{Ai}'(z)\, \mathrm{Bi}'(z) = \frac{1}{4\pi^{3/2}} \sqrt[3]{\frac{3}{2}}\, G_{1,3}^{2,1}\left(\left(\frac{2}{3}\right)^{2/3} z, \frac{1}{3} \,\middle|\, \begin{array}{c} \frac{7}{6} \\ 0, \frac{4}{3}, \frac{2}{3} \end{array}\right)$$

Generalized cases involving Bi

03.08.26.0013.01

$$\mathrm{Bi}(z)\, \mathrm{Bi}'(z) = \frac{3}{4\pi^{3/2}}\, G_{1,3}^{3,0}\left(\left(\frac{2}{3}\right)^{2/3} z, \frac{1}{3} \,\middle|\, \begin{array}{c} \frac{1}{2} \\ 0, \frac{1}{3}, \frac{2}{3} \end{array}\right) + 2\sqrt{\pi}\, G_{2,4}^{2,0}\left(\left(\frac{2}{3}\right)^{2/3} z, \frac{1}{3} \,\middle|\, \begin{array}{c} 1, \frac{1}{2} \\ \frac{2}{3}, \frac{1}{3}, 0, 1 \end{array}\right)$$

Generalized cases involving ${}_0F_1$

03.08.26.0014.01

$$\mathrm{Bi}'(z)\,{}_0F_1\left(;b;\frac{z^3}{9}\right)=2^{b-\frac{2}{3}}\sqrt[6]{3}\,\sqrt{\pi}\,\Gamma(b)\,G_{4,6}^{2,2}\left(\left(\frac{2}{3}\right)^{2/3}z,\frac{1}{3}\,\middle|\,\begin{array}{c}\frac{1}{6}(5-3b),\frac{1}{6}(8-3b),\frac{1}{3},\frac{5}{6}\\0,\frac{2}{3},\frac{1}{3},\frac{5}{6},1-b,\frac{5}{3}-b\end{array}\right)$$

03.08.26.0033.01

$$\mathrm{Bi}'\left(3^{2/3}\sqrt[3]{z}\right){}_0F_1(;b;z)=2^{b-\frac{2}{3}}\sqrt[6]{3}\,\sqrt{\pi}\,\Gamma(b)\,G_{4,6}^{2,2}\left(2^{2/3}\sqrt[3]{z},\frac{1}{3}\,\middle|\,\begin{array}{c}\frac{1}{6}(5-3b),\frac{1}{6}(8-3b),\frac{1}{3},\frac{5}{6}\\0,\frac{2}{3},\frac{1}{3},\frac{5}{6},1-b,\frac{5}{3}-b\end{array}\right)$$

Generalized cases involving ${}_0\tilde{F}_1$

03.08.26.0015.01

$$\mathrm{Bi}'(z)\,{}_0\tilde{F}_1\left(;b;\frac{z^3}{9}\right)=2^{b-\frac{2}{3}}\sqrt[6]{3}\,\sqrt{\pi}\,G_{4,6}^{2,2}\left(\left(\frac{2}{3}\right)^{2/3}z,\frac{1}{3}\,\middle|\,\begin{array}{c}\frac{1}{6}(5-3b),\frac{1}{6}(8-3b),\frac{1}{3},\frac{5}{6}\\0,\frac{2}{3},\frac{1}{3},\frac{5}{6},1-b,\frac{5}{3}-b\end{array}\right)$$

03.08.26.0034.01

$$\mathrm{Bi}'\left(3^{2/3}\sqrt[3]{z}\right){}_0\tilde{F}_1(;b;z)=2^{b-\frac{2}{3}}\sqrt[6]{3}\,\sqrt{\pi}\,G_{4,6}^{2,2}\left(2^{2/3}\sqrt[3]{z},\frac{1}{3}\,\middle|\,\begin{array}{c}\frac{1}{6}(5-3b),\frac{1}{6}(8-3b),\frac{1}{3},\frac{5}{6}\\0,\frac{2}{3},\frac{1}{3},\frac{5}{6},1-b,\frac{5}{3}-b\end{array}\right)$$

Generalized cases involving Bessel I

03.08.26.0016.01

$$\mathrm{Bi}'(z)\,I_\nu\left(\frac{2z^{3/2}}{3}\right)=\sqrt[3]{2}\,\sqrt[6]{3}\,\sqrt{\pi}\,z^{-\frac{3\nu}{2}}\left(z^{3/2}\right)^\nu G_{4,6}^{2,2}\left(\left(\frac{2}{3}\right)^{2/3}z,\frac{1}{3}\,\middle|\,\begin{array}{c}\frac{1}{3},\frac{5}{6},\frac{1}{6}(3\nu+2),\frac{\nu}{2}+\frac{5}{6}\\\frac{\nu}{2},\frac{1}{6}(3\nu+4),-\frac{\nu}{2},\frac{1}{6}(4-3\nu),\frac{1}{6}(3\nu+2),\frac{1}{6}(3\nu+5)\end{array}\right)$$

03.08.26.0024.01

$$\mathrm{Bi}'\left(\left(\frac{3}{2}\right)^{2/3}z^{2/3}\right)I_\nu(z)=\sqrt[3]{2}\,\sqrt[6]{3}\,\sqrt{\pi}\,G_{4,6}^{2,2}\left(z^{2/3},\frac{1}{3}\,\middle|\,\begin{array}{c}\frac{1}{3},\frac{5}{6},\frac{1}{6}(3\nu+2),\frac{\nu}{2}+\frac{5}{6}\\\frac{\nu}{2},\frac{1}{6}(3\nu+4),-\frac{\nu}{2},\frac{1}{6}(4-3\nu),\frac{1}{6}(3\nu+2),\frac{1}{6}(3\nu+5)\end{array}\right)$$

Generalized cases involving Bessel K

03.08.26.0017.01

$$\mathrm{Bi}'(z)\,K_\nu\left(\frac{2z^{3/2}}{3}\right)=\frac{\sqrt[6]{3}\,\pi^{3/2}\csc(\pi\nu)}{2^{2/3}}\left(G_{4,6}^{2,2}\left(\left(\frac{2}{3}\right)^{2/3}z,\frac{1}{3}\,\middle|\,\begin{array}{c}\frac{1}{3},\frac{5}{6},\frac{1}{3}-\frac{\nu}{2},\frac{5}{6}-\frac{\nu}{2}\\-\frac{\nu}{2},\frac{2}{3}-\frac{\nu}{2},\frac{1}{3}-\frac{\nu}{2},\frac{5}{6}-\frac{\nu}{2},\frac{\nu}{2},\frac{\nu}{2}+\frac{2}{3}\end{array}\right)-\right.$$
$$\left.G_{4,6}^{2,2}\left(\left(\frac{2}{3}\right)^{2/3}z,\frac{1}{3}\,\middle|\,\begin{array}{c}\frac{1}{3},\frac{5}{6},\frac{\nu}{2}+\frac{1}{3},\frac{\nu}{2}+\frac{5}{6}\\\frac{\nu}{2},\frac{\nu}{2}+\frac{2}{3},-\frac{\nu}{2},\frac{2}{3}-\frac{\nu}{2},\frac{\nu}{2}+\frac{1}{3},\frac{\nu}{2}+\frac{5}{6}\end{array}\right)\right)/;-\frac{2\pi}{3}<\arg(z)\le\frac{2\pi}{3}$$

03.08.26.0025.01

$$\mathrm{Bi}'\left(\left(\frac{3}{2}\right)^{2/3}z^{2/3}\right)K_\nu(z)=$$
$$\frac{\sqrt[6]{3}\,\pi^{3/2}\csc(\pi\nu)}{2^{2/3}}\left(G_{4,6}^{2,2}\left(z^{2/3},\frac{1}{3}\,\middle|\,\begin{array}{c}\frac{1}{3},\frac{5}{6},\frac{1}{3}-\frac{\nu}{2},\frac{5}{6}-\frac{\nu}{2}\\-\frac{\nu}{2},\frac{2}{3}-\frac{\nu}{2},\frac{1}{3}-\frac{\nu}{2},\frac{5}{6}-\frac{\nu}{2},\frac{\nu}{2},\frac{\nu}{2}+\frac{2}{3}\end{array}\right)-G_{4,6}^{2,2}\left(z^{2/3},\frac{1}{3}\,\middle|\,\begin{array}{c}\frac{1}{3},\frac{5}{6},\frac{\nu}{2}+\frac{1}{3},\frac{\nu}{2}+\frac{5}{6}\\\frac{\nu}{2},\frac{\nu}{2}+\frac{2}{3},-\frac{\nu}{2},\frac{2}{3}-\frac{\nu}{2},\frac{\nu}{2}+\frac{1}{3},\frac{\nu}{2}+\frac{5}{6}\end{array}\right)\right)$$

Through other functions

Involving Bessel functions

03.08.26.0018.01

$$\mathrm{Bi}'(z) = -\frac{z}{\sqrt{3}}\left(J_{\frac{2}{3}}\left(\frac{2}{3}(-z)^{3/2}\right) + J_{-\frac{2}{3}}\left(\frac{2}{3}(-z)^{3/2}\right)\right) /; \mathrm{Re}(z) \leq 0$$

03.08.26.0019.01

$$\mathrm{Bi}'(z) = \frac{z}{\sqrt{3}}\left(I_{\frac{2}{3}}\left(\frac{2 z^{3/2}}{3}\right) + I_{-\frac{2}{3}}\left(\frac{2 z^{3/2}}{3}\right)\right) /; \mathrm{Re}(z) \geq 0$$

03.08.26.0020.01

$$\mathrm{Bi}'(z) = \frac{1}{\sqrt{3}}\left(\left(z^{3/2}\right)^{2/3} I_{-\frac{2}{3}}\left(\frac{2 z^{3/2}}{3}\right) + z^2 \left(z^{3/2}\right)^{-\frac{2}{3}} I_{\frac{2}{3}}\left(\frac{2 z^{3/2}}{3}\right)\right)$$

Representations through equivalent functions

With related functions

03.08.27.0001.01

$$\mathrm{Bi}'(z) = e^{\frac{5\pi i}{6}} \mathrm{Ai}'\left(e^{\frac{2\pi i}{3}} z\right) + e^{-\frac{5\pi i}{6}} \mathrm{Ai}'\left(e^{-\frac{2\pi i}{3}} z\right)$$

03.08.27.0002.01

$$\mathrm{Bi}'(z) = 2\,(-1)^{5/6}\, \mathrm{Ai}'\left((-1)^{2/3} z\right) - i\, \mathrm{Ai}'(z)$$

03.08.27.0003.01

$$\mathrm{Bi}'(z) = i\, \mathrm{Ai}'(z) - 2\sqrt[6]{-1}\, \mathrm{Ai}'\left(-\sqrt[3]{-1}\, z\right)$$

Zeros

03.08.30.0001.01

$$\mathrm{Bi}'(z) = 0 /; z = z_k \wedge k \in \mathbb{N}$$

03.08.30.0002.01

$$\mathrm{Im}(z_k) = 0 \wedge \mathrm{Re}(z_k) < 0 /; \mathrm{Bi}'(z_k) = 0$$

On the real axis $\mathrm{Bi}'(z)$ has an infinite number of zeros, all of which are negative.

03.08.30.0003.01

$$\frac{\pi}{3} < |\arg(z_k)| < \frac{\pi}{2} /; \mathrm{Bi}'(z_k) = 0$$

Equation $\mathrm{Bi}'(x) = 0$ has only negative real solutions and solutions in the sector $\frac{\pi}{3} < |\mathrm{Arg}(x)| < \frac{\pi}{2}$.

History

– G. B. Airy (1838), H. Jeffreys (1928, 1942)
– J. C. P. Miller (1946) suggested the notations Ai, Bi.

Applications of Bi' include quantum mechanics of linear potential, electrodynamics, combinatorics, analysis of the complexity of algorithms, optical theory of the rainbow, solid state physics, and semiconductors in electric fields.

Copyright