ArcCos

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Notations

Traditional name

Inverse cosine

Traditional notation

$\cos^{-1}(z)$

Mathematica StandardForm notation

`ArcCos[z]`

Primary definition

01.13.02.0001.01

$$\cos^{-1}(z) = \frac{\pi}{2} + i \log\left(i z + \sqrt{1 - z^2}\right)$$

The function $\cos^{-1}(z)$ can also be defined by the formula $\cos^{-1}(z) = \frac{\pi}{2} - \sin^{-1}(z)$.

The function $\cos^{-1}(z)$ can be defined also as the inverse function for $\cos(w)$: $w = \cos^{-1}(z)$ if and only if $\cos(w) = z$.

Specific values

Values at fixed points

01.13.03.0001.01

$$\cos^{-1}(0) = \frac{\pi}{2}$$

01.13.03.0002.01

$$\cos^{-1}\left(\frac{\sqrt{3} - 1}{2\sqrt{2}}\right) = \frac{5\pi}{12}$$

01.13.03.0003.01

$$\cos^{-1}\left(-\frac{\sqrt{3} - 1}{2\sqrt{2}}\right) = \frac{7\pi}{12}$$

01.13.03.0004.01

$$\cos^{-1}\left(\frac{\sqrt{5}-1}{4}\right) == \frac{2\pi}{5}$$

01.13.03.0005.01

$$\cos^{-1}\left(-\frac{\sqrt{5}-1}{4}\right) == \frac{3\pi}{5}$$

01.13.03.0006.01

$$\cos^{-1}\left(\frac{\sqrt{2-\sqrt{2}}}{2}\right) == \frac{3\pi}{8}$$

01.13.03.0007.01

$$\cos^{-1}\left(-\frac{\sqrt{2-\sqrt{2}}}{2}\right) == \frac{5\pi}{8}$$

01.13.03.0008.01

$$\cos^{-1}\left(\frac{1}{2}\right) == \frac{\pi}{3}$$

01.13.03.0009.01

$$\cos^{-1}\left(-\frac{1}{2}\right) == \frac{2\pi}{3}$$

01.13.03.0010.01

$$\cos^{-1}\left(\frac{1}{2}\sqrt{\frac{5-\sqrt{5}}{2}}\right) == \frac{3\pi}{10}$$

01.13.03.0011.01

$$\cos^{-1}\left(-\frac{1}{2}\sqrt{\frac{5-\sqrt{5}}{2}}\right) == \frac{7\pi}{10}$$

01.13.03.0012.01

$$\cos^{-1}\left(\frac{\sqrt{2}}{2}\right) == \frac{\pi}{4}$$

01.13.03.0013.01

$$\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) == \frac{3\pi}{4}$$

01.13.03.0014.01

$$\cos^{-1}\left(\frac{\sqrt{5}+1}{4}\right) == \frac{\pi}{5}$$

01.13.03.0015.01

$$\cos^{-1}\left(-\frac{\sqrt{5}+1}{4}\right) == \frac{4\pi}{5}$$

01.13.03.0016.01

$$\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) == \frac{\pi}{6}$$

01.13.03.0017.01

$$\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) == \frac{5\pi}{6}$$

01.13.03.0018.01

$$\cos^{-1}\left(\frac{\sqrt{2+\sqrt{2}}}{2}\right) == \frac{\pi}{8}$$

01.13.03.0019.01

$$\cos^{-1}\left(-\frac{\sqrt{2+\sqrt{2}}}{2}\right) == \frac{7\pi}{8}$$

01.13.03.0020.01

$$\cos^{-1}\left(\frac{1}{2}\sqrt{\frac{5+\sqrt{5}}{2}}\right) == \frac{\pi}{10}$$

01.13.03.0021.01

$$\cos^{-1}\left(-\frac{1}{2}\sqrt{\frac{5+\sqrt{5}}{2}}\right) == \frac{9\pi}{10}$$

01.13.03.0022.01

$$\cos^{-1}\left(\frac{1+\sqrt{3}}{2\sqrt{2}}\right) == \frac{\pi}{12}$$

01.13.03.0023.01

$$\cos^{-1}\left(-\frac{1+\sqrt{3}}{2\sqrt{2}}\right) == \frac{11\pi}{12}$$

01.13.03.0024.01

$$\cos^{-1}(1) == 0$$

01.13.03.0025.01

$$\cos^{-1}(-1) == \pi$$

01.13.03.0026.01

$$\cos^{-1}(i) == \frac{\pi}{2} + i\log\left(\sqrt{2} - 1\right)$$

01.13.03.0027.01

$$\cos^{-1}(-i) == \frac{\pi}{2} + i\log\left(1 + \sqrt{2}\right)$$

Values at infinities

01.13.03.0028.01

$\cos^{-1}(\infty) == i\,\infty$

01.13.03.0029.01

$\cos^{-1}(-\infty) == -i\,\infty$

01.13.03.0030.01

$\cos^{-1}(i\,\infty) == -i\,\infty$

01.13.03.0031.01

$\cos^{-1}(-i\,\infty) == i\,\infty$

01.13.03.0032.01

$\cos^{-1}(\tilde{\infty}) == \tilde{\infty}$

General characteristics

Domain and analyticity

$\cos^{-1}(z)$ is an analytical function of z, which is defined over the whole complex z-plane.

01.13.04.0001.01

$z \longrightarrow \cos^{-1}(z) :: \mathbb{C} \longrightarrow \mathbb{C}$

Symmetries and periodicities

Mirror symmetry

01.13.04.0002.01

$\cos^{-1}(\bar{z}) == \overline{\cos^{-1}(z)} \;/;\; z \notin (-\infty, -1) \bigwedge z \notin (1, \infty)$

Periodicity

No periodicity

Poles and essential singularities

The function $\cos^{-1}(z)$ does not have poles and essential singularities.

01.13.04.0003.01

$\mathcal{Sing}_z\left(\cos^{-1}(z)\right) == \{\}$

Branch points

The function $\cos^{-1}(z)$ has three branch points: $z = \pm 1$, $z = \tilde{\infty}$.

01.13.04.0004.01

$\mathcal{BP}_z\left(\cos^{-1}(z)\right) == \{1, -1, \tilde{\infty}\}$

01.13.04.0005.01

$\mathcal{R}_z\left(\cos^{-1}(z), 1\right) == 2$

01.13.04.0006.01

$\mathcal{R}_z\left(\cos^{-1}(z), -1\right) == 2$

01.13.04.0007.01

$$\mathcal{R}_z\left(\cos^{-1}(z),\ \tilde{\infty}\right) = \log$$

Branch cuts

The function $\cos^{-1}(z)$ is a single-valued function on the z-plane cut along the intervals $(-\infty,\ -1]$ and $[1,\ \infty)$.
The function $\cos^{-1}(z)$ is continuous from above on the interval $(-\infty, -1]$ and from below on the interval $[1,\ \infty)$.

01.13.04.0008.01

$$\mathcal{BC}_z\left(\cos^{-1}(z)\right) = \{\{(-\infty,\ -1],\ -i\},\ \{[1,\ \infty),\ i\}\}$$

01.13.04.0009.01

$$\lim_{\epsilon \to +0} \cos^{-1}(x + i\,\epsilon) = \cos^{-1}(x) \ /;\ x < -1$$

01.13.04.0010.01

$$\lim_{\epsilon \to +0} \cos^{-1}(x - i\,\epsilon) = \cos^{-1}(-x) + \pi \ /;\ x < -1$$

01.13.04.0011.01

$$\lim_{\epsilon \to +0} \cos^{-1}(x - i\,\epsilon) = \cos^{-1}(x) \ /;\ x > 1$$

01.13.04.0012.01

$$\lim_{\epsilon \to +0} \cos^{-1}(x + i\,\epsilon) = -\cos^{-1}(x) \ /;\ x > 1$$

Analytic continuations

The analytic continuation of $\cos^{-1}$ has infinitely many sheets; the values of $\tilde{\cos}^{-1}$ are $\tilde{\cos}^{-1}(z) = \cos^{-1}(z) + 2\,k\,\pi\ /;\ k \in \mathbb{Z}$.

Series representations

Generalized power series

Expansions at generic point $z = z_0$

For the function itself

01.13.06.0019.01

$$\cos^{-1}(z) \propto \left(\frac{1}{1-z_0}\right)^{\frac{1}{2}\left\lfloor \frac{\arg(z_0-z)}{2\pi} \right\rfloor} (1-z_0)^{\frac{1}{2}\left\lfloor \frac{\arg(z_0-z)}{2\pi} \right\rfloor} \left(-2\,\pi\, i\, i^{\left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor \left\lfloor \frac{\arg(z_0+1)+\pi}{2\pi} \right\rfloor + \right.$$
$$\left. \left(\frac{1}{z_0+1}\right)^{\frac{1}{2}\left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} (z_0+1)^{\frac{1}{2}\left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} \left(\cos^{-1}(z_0) - \frac{z-z_0}{\sqrt{1-z_0^2}} - \frac{z_0\,(z-z_0)^2}{2\left(1-z_0^2\right)^{3/2}} + \ldots\right)\right) /;\ (z \to z_0)$$

01.13.06.0020.01

$$\cos^{-1}(z) \propto \left(\frac{1}{1-z_0}\right)^{\frac{1}{2}\left\lfloor \frac{\arg(z_0-z)}{2\pi} \right\rfloor} (1-z_0)^{\frac{1}{2}\left\lfloor \frac{\arg(z_0-z)}{2\pi} \right\rfloor} \left(-2\pi i\, i^{\left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor \left\lfloor \frac{\arg(z_0+1)+\pi}{2\pi} \right\rfloor + \left(\frac{1}{z_0+1}\right)^{\frac{1}{2}\left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} (z_0+1)^{\frac{1}{2}\left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} \left(\cos^{-1}(z_0) - \frac{z-z_0}{\sqrt{1-z_0^2}} - \frac{z_0 (z-z_0)^2}{2\left(1-z_0^2\right)^{3/2}} + O\left((z-z_0)^3\right)\right)\right)$$

01.13.06.0021.01

$$\cos^{-1}(z) = \left(\frac{1}{1-z_0}\right)^{\frac{1}{2}\left\lfloor \frac{\arg(z_0-z)}{2\pi} \right\rfloor} (1-z_0)^{\frac{1}{2}\left\lfloor \frac{\arg(z_0-z)}{2\pi} \right\rfloor} \left(-2\pi i\, i^{\left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor \left\lfloor \frac{\arg(z_0+1)+\pi}{2\pi} \right\rfloor + \left(\frac{1}{z_0+1}\right)^{\frac{1}{2}\left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} (z_0+1)^{\frac{1}{2}\left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} \left(\pi - \frac{\sqrt{\pi}}{2} \sum_{k=0}^{\infty} \sum_{j=0}^{k} \frac{\left(-\frac{1}{2}\right)_{k-j} (1-z_0)^{j-k+\frac{1}{2}} (z_0+1)^{\frac{1}{2}-j}}{(k-j)!\, j!} \, {}_2\tilde{F}_1\left(1, 1; \frac{3}{2}-j; \frac{1}{2}(z_0+1)\right) (z-z_0)^k\right)\right)$$

01.13.06.0022.01

$$\cos^{-1}(z) = \left(\frac{1}{1-z_0}\right)^{\frac{1}{2}\left\lfloor \frac{\arg(z_0-z)}{2\pi} \right\rfloor} (1-z_0)^{\frac{1}{2}\left\lfloor \frac{\arg(z_0-z)}{2\pi} \right\rfloor} \left(-2\pi i\, i^{\left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor \left\lfloor \frac{\arg(z_0+1)+\pi}{2\pi} \right\rfloor + \left(\frac{1}{z_0+1}\right)^{\frac{1}{2}\left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} (z_0+1)^{\frac{1}{2}\left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} \left(\frac{\pi}{2} - \frac{\sqrt{\pi}}{2} \sum_{k=0}^{\infty} \frac{2^k z_0^{1-k}}{k!} \, {}_3\tilde{F}_2\left(\frac{1}{2}, \frac{1}{2}, 1; 1-\frac{k}{2}, \frac{3-k}{2}; z_0^2\right) (z-z_0)^k\right)\right)$$

01.13.06.0023.01

$$\cos^{-1}(z) \propto \left(\frac{1}{1-z_0}\right)^{\frac{1}{2}\left\lfloor \frac{\arg(z_0-z)}{2\pi} \right\rfloor} (1-z_0)^{\frac{1}{2}\left\lfloor \frac{\arg(z_0-z)}{2\pi} \right\rfloor} \left(-2\pi i\, i^{\left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor \left\lfloor \frac{\arg(z_0+1)+\pi}{2\pi} \right\rfloor + \left(\frac{1}{z_0+1}\right)^{\frac{1}{2}\left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} (z_0+1)^{\frac{1}{2}\left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} \cos^{-1}(z_0)\right) (1+O(z-z_0))$$

Expansions on branch cuts

For the function itself

In the left half-plane

01.13.06.0024.01

$$\cos^{-1}(z) \propto -2\pi i\, i^{\left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor} \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor + e^{\pi i \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor} \left(\cos^{-1}(x) - \frac{z-x}{\sqrt{1-x^2}} - \frac{x (z-x)^2}{2\left(1-x^2\right)^{3/2}} + \ldots\right) /; (z \to x) \land x \in \mathbb{R} \land x < -1$$

01.13.06.0025.01

$$\cos^{-1}(z) \propto -2\pi i \, i^{\left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor} \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor + e^{\pi i \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor} \left(\cos^{-1}(x) - \frac{z-x}{\sqrt{1-x^2}} - \frac{x(z-x)^2}{2(1-x^2)^{3/2}} + O\left((z-x)^3\right) \right) /; x \in \mathbb{R} \land x < -1$$

01.13.06.0026.01

$$\cos^{-1}(z) = -2\pi i \, i^{\left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor} \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor + e^{\pi i \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor} \left(\pi - \frac{\sqrt{\pi}}{2} \sum_{k=0}^{\infty} \sum_{j=0}^{k} \frac{\left(-\frac{1}{2}\right)_{k-j} (1-x)^{j-k+\frac{1}{2}} (x+1)^{\frac{1}{2}-j}}{(k-j)!\, j!} \, {}_2\tilde{F}_1\left(1, 1; \frac{3}{2} - j; \frac{x+1}{2}\right) (z-x)^k \right) /; x \in \mathbb{R} \land x < -1$$

01.13.06.0027.01

$$\cos^{-1}(z) = -2\pi i \, i^{\left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor} \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor + e^{\pi i \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor} \left(\frac{\pi}{2} - \frac{1}{2} \sqrt{\pi} \sum_{k=0}^{\infty} \frac{2^k x^{1-k}}{k!} \, {}_3\tilde{F}_2\left(\frac{1}{2}, \frac{1}{2}, 1; 1 - \frac{k}{2}, \frac{3-k}{2}; x^2\right) (z-x)^k \right) /; x \in \mathbb{R} \land x < -1$$

01.13.06.0028.01

$$\cos^{-1}(z) \propto -2\pi i \, i^{\left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor} \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor + e^{\pi i \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor} \cos^{-1}(x) (1 + O(z-x)) /; x \in \mathbb{R} \land x < -1$$

In the right half-plane

01.13.06.0029.01

$$\cos^{-1}(z) \propto e^{\pi i \left\lfloor \frac{\arg(x-z)}{2\pi} \right\rfloor} \left(\cos^{-1}(x) - \frac{z-x}{\sqrt{1-x^2}} - \frac{x(z-x)^2}{2(1-x^2)^{3/2}} + \ldots \right) /; (z \to x) \land x \in \mathbb{R} \land x > 1$$

01.13.06.0030.01

$$\cos^{-1}(z) \propto e^{\pi i \left\lfloor \frac{\arg(x-z)}{2\pi} \right\rfloor} \left(\cos^{-1}(x) - \frac{z-x}{\sqrt{1-x^2}} - \frac{x(z-x)^2}{2(1-x^2)^{3/2}} + O\left((z-x)^3\right) \right) /; x \in \mathbb{R} \land x > 1$$

01.13.06.0031.01

$$\cos^{-1}(z) = e^{\pi i \left\lfloor \frac{\arg(x-z)}{2\pi} \right\rfloor} \left(\pi - \frac{1}{2} \sqrt{\pi} \sum_{k=0}^{\infty} \sum_{j=0}^{k} \frac{\left(-\frac{1}{2}\right)_{k-j} (1-x)^{j-k+\frac{1}{2}} (x+1)^{\frac{1}{2}-j}}{(k-j)!\, j!} \, {}_2\tilde{F}_1\left(1, 1; \frac{3}{2} - j; \frac{x+1}{2}\right) (z-x)^k \right) /; x \in \mathbb{R} \land x > 1$$

01.13.06.0032.01

$$\cos^{-1}(z) = e^{\pi i \left\lfloor \frac{\arg(x-z)}{2\pi} \right\rfloor} \left(\frac{\pi}{2} - \frac{1}{2} \sqrt{\pi} \sum_{k=0}^{\infty} \frac{2^k x^{1-k}}{k!} \, {}_3\tilde{F}_2\left(\frac{1}{2}, \frac{1}{2}, 1; 1 - \frac{k}{2}, \frac{3-k}{2}; x^2\right) (z-x)^k \right) /; x \in \mathbb{R} \land x > 1$$

01.13.06.0033.01

$$\cos^{-1}(z) \propto e^{\pi i \left\lfloor \frac{\arg(x-z)}{2\pi} \right\rfloor} \cos^{-1}(x) (1 + O(z-x)) /; x \in \mathbb{R} \land x > 1$$

Expansions at $z = 0$

For the function itself

01.13.06.0001.02

$$\cos^{-1}(z) \propto \frac{\pi}{2} - z - \frac{z^3}{6} - \frac{3 z^5}{40} - \ldots \;/;\; (z \to 0)$$

01.13.06.0034.01

$$\cos^{-1}(z) \propto \frac{\pi}{2} - z - \frac{z^3}{6} - \frac{3 z^5}{40} + O\left(z^7\right)$$

01.13.06.0002.01

$$\cos^{-1}(z) = \frac{\pi}{2} - \sum_{k=0}^{\infty} \frac{\left(\frac{1}{2}\right)_k z^{2k+1}}{(2k+1)\, k!} \;/;\; |z| < 1$$

01.13.06.0003.02

$$\cos^{-1}(z) = \frac{\pi}{2} - z\, {}_2F_1\left(\frac{1}{2}, \frac{1}{2}; \frac{3}{2}; z^2\right)$$

01.13.06.0004.02

$$\cos^{-1}(z) \propto \frac{\pi}{2} + O(z)$$

01.13.06.0035.01

$$\cos^{-1}(z) = F_\infty(z) \;/;$$
$$\left(\left(F_n(z) = \frac{\pi}{2} - \sum_{k=0}^{n} \frac{\left(\frac{1}{2}\right)_k z^{2k+1}}{(2k+1)\, k!} = \cos^{-1}(z) + \frac{z^{2n+3}}{2\sqrt{\pi}}\, \Gamma\left(n + \frac{3}{2}\right)^2 {}_3\tilde{F}_2\left(1, n + \frac{3}{2}, n + \frac{3}{2}; n + 2, n + \frac{5}{2}; z^2\right)\right) \bigwedge n \in \mathbb{N}\right)$$

Summed form of the truncated series expansion.

For small integer powers of the function

For the second power

01.13.06.0036.01

$$\cos^{-1}(z)^2 \propto \frac{\pi^2}{4} - \pi z \left(1 + \frac{z^2}{6} + \frac{3 z^4}{40} + \ldots\right) + z^2 \left(1 + \frac{z^2}{3} + \frac{8 z^4}{45} + \ldots\right) \;/;\; (z \to 0)$$

01.13.06.0037.01

$$\cos^{-1}(z)^2 \propto \frac{\pi^2}{4} - \pi z \left(1 + \frac{z^2}{6} + \frac{3 z^4}{40} + O\left(z^6\right)\right) + z^2 \left(1 + \frac{z^2}{3} + \frac{8 z^4}{45} + O\left(z^6\right)\right)$$

01.13.06.0038.01

$$\cos^{-1}(z)^2 = \frac{\pi^2}{4} - \pi z \sum_{k=0}^{\infty} \frac{\left(\frac{1}{2}\right)_k z^{2k}}{(2k+1)\, k!} + z^2 \sum_{k=0}^{\infty} \frac{2^{2k}\, k!^2\, z^{2k}}{(2k+1)!\, (k+1)} \;/;\; |z| < 1$$

01.13.06.0039.01

$$\cos^{-1}(z)^2 = \frac{\pi^2}{4} - \pi z\, {}_2F_1\left(\frac{1}{2}, \frac{1}{2}; \frac{3}{2}; z^2\right) + z^2\, {}_3F_2\left(1, 1, 1; \frac{3}{2}, 2; z^2\right)$$

01.13.06.0040.01

$$\cos^{-1}(z)^2 \propto \frac{\pi^2}{4} - \pi z\, (1 + O(z))$$

01.13.06.0041.01

$$\cos^{-1}(z)^2 = F_\infty(z) /; \left(\left(F_n(z) = \frac{\pi^2}{4} + z^2 \sum_{k=0}^{n} \frac{2^{2k}\, k!^2\, z^{2k}}{(2k+1)!\,(k+1)} - \pi z \sum_{k=0}^{n} \frac{\left(\frac{1}{2}\right)_k z^{2k}}{(2k+1)\,k!} = \cos^{-1}(z)^2 + \frac{1}{2}\sqrt{\pi}\,\Gamma\left(n+\frac{3}{2}\right)^2 z^{2n+3}\, {}_3\tilde{F}_2\left(1, n+\frac{3}{2}, n+\frac{3}{2}; n+2, n+\frac{5}{2}; z^2\right) - \frac{1}{2}\sqrt{\pi}\,\Gamma(n+2)^2 z^{2n+4}\, {}_3\tilde{F}_2\left(1, n+2, n+2; n+\frac{5}{2}, n+3; z^2\right)\right) \bigwedge n \in \mathbb{N}\right)$$

Summed form of the truncated series expansion.

Expansions at $z = 1$

For the function itself

01.13.06.0005.02

$$\cos^{-1}(z) \propto \sqrt{2}\,\sqrt{1-z}\left(1 + \frac{1-z}{12} + \frac{3}{160}(1-z)^2 + \ldots\right) /; (z \to 1)$$

01.13.06.0042.01

$$\cos^{-1}(z) \propto \sqrt{2}\,\sqrt{1-z}\left(1 + \frac{1-z}{12} + \frac{3}{160}(1-z)^2 + O\left((z-1)^3\right)\right)$$

01.13.06.0043.01

$$\cos^{-1}(z) = 2\sin^{-1}\left(\sqrt{\frac{1-z}{2}}\right)$$

The last formula allows to express the function at the singular points $z = 1$ through the inverse sine function value at the regular point $\tilde{z} = 0$.

01.13.06.0006.02

$$\cos^{-1}(z) = \sqrt{2}\,\sqrt{1-z} \sum_{k=0}^{\infty} \frac{\left(\frac{1}{2}\right)_k (1-z)^k}{2^k\,(2k+1)\,k!} /; |z-1| < 2$$

01.13.06.0007.01

$$\cos^{-1}(z) = \sqrt{2}\,\sqrt{1-z}\; {}_2F_1\left(\frac{1}{2}, \frac{1}{2}; \frac{3}{2}; \frac{1-z}{2}\right)$$

01.13.06.0008.02

$$\cos^{-1}(z) \propto \sqrt{2}\,\sqrt{1-z}\,(1 + O(z-1))$$

01.13.06.0044.01

$$\cos^{-1}(z) = F_\infty(z) /; \left(\left(F_n(z) = \sqrt{2}\,\sqrt{1-z} \sum_{k=0}^{n} \frac{\left(\frac{1}{2}\right)_k (1-z)^k}{2^k\,(2k+1)\,k!} = \cos^{-1}(z) - \frac{\Gamma\left(n+\frac{3}{2}\right)^2 (2-2z)^{n+\frac{3}{2}}}{\pi\,(2n+3)!}\; {}_3F_2\left(1, n+\frac{3}{2}, n+\frac{3}{2}; n+2, n+\frac{5}{2}; \frac{1-z}{2}\right)\right) \bigwedge n \in \mathbb{N}\right)$$

Summed form of the truncated series expansion.

For small integer powers of the function

For the second power

01.13.06.0045.01

$$\cos^{-1}(z)^2 \propto -2\,(z-1)\left(1-\frac{z-1}{6}+\frac{2}{45}\,(z-1)^2+\ldots\right) /;\ (z\to 1)$$

01.13.06.0046.01

$$\cos^{-1}(z)^2 \propto -2\,(z-1)\left(1-\frac{z-1}{6}+\frac{2}{45}\,(z-1)^2+O\!\left((z-1)^3\right)\right)$$

01.13.06.0047.01

$$\cos^{-1}(z)^2 = -2\,(z-1)\left(\sum_{k=0}^{\infty}\frac{\left(\frac{1}{2}\right)_k (1-z)^k}{2^k\,(2k+1)\,k!}\right)^2 /;\ |z-1|<2$$

01.13.06.0048.01

$$\cos^{-1}(z)^2 = 4\sin^{-1}\!\left(\sqrt{\frac{1-z}{2}}\right)^2$$

The last formula allows to express the function at the singular points $z = 1$ through the inverse sine function value at the regular point $\tilde{z} = 0$.

01.13.06.0049.01

$$\cos^{-1}(z)^2 = -2\,(z-1)\,{}_2F_1\!\left(\frac{1}{2},\frac{1}{2};\frac{3}{2};\frac{1-z}{2}\right)^2$$

01.13.06.0050.01

$$\cos^{-1}(z)^2 \propto -2\,(z-1)\,(1+O(z-1))$$

01.13.06.0051.01

$$\cos^{-1}(z)^2 = F_\infty(z) /;\ \left(\left(F_n(z) = -2\,(z-1)\left(\sum_{k=0}^{n}\frac{\left(\frac{1}{2}\right)_k (1-z)^k}{2^k\,(2k+1)\,k!}\right)^2 = \cos^{-1}(z)^2-\frac{2^{\frac{1}{2}-n}\,\Gamma\!\left(n+\frac{3}{2}\right)\cos^{-1}(z)\,(1-z)^{n+\frac{3}{2}}}{(2n+3)\sqrt{\pi}\,(n+1)!}\,{}_3F_2\!\left(1,n+\frac{3}{2},n+\frac{3}{2};n+2,n+\frac{5}{2};\frac{1-z}{2}\right)+\frac{2^{-2n-1}\,\Gamma\!\left(n+\frac{3}{2}\right)^2(1-z)^{2n+3}}{(2n+3)^2\,\pi\,(n+1)!^2}\,{}_3F_2\!\left(1,n+\frac{3}{2},n+\frac{3}{2};n+2,n+\frac{5}{2};\frac{1-z}{2}\right)^2\right)\bigwedge n\in\mathbb{N}\right)$$

Summed form of the truncated series expansion.

Expansions at $z = -1$

For the function itself

01.13.06.0009.02

$$\cos^{-1}(z) \propto \pi - \sqrt{2}\,\sqrt{1+z}\left(1 + \frac{1+z}{12} + \frac{3}{160}(1+z)^2 + \ldots\right) /; (z \to -1)$$

01.13.06.0052.01

$$\cos^{-1}(z) \propto \pi - \sqrt{2}\,\sqrt{1+z}\left(1 + \frac{1+z}{12} + \frac{3}{160}(1+z)^2 + O\left((z+1)^3\right)\right)$$

01.13.06.0053.01

$$\cos^{-1}(z) = 2\cos^{-1}\left(\sqrt{\frac{z+1}{2}}\right)$$

The last formula allows to express the function at the singular points $z = -1$ through a function value at the regular point $\tilde{z} = 0$.

01.13.06.0010.02

$$\cos^{-1}(z) = \pi - \sqrt{2}\,\sqrt{z+1}\sum_{k=0}^{\infty} \frac{\left(\frac{1}{2}\right)_k (z+1)^k}{2^k (2k+1)\, k!} /; |z+1| < 2$$

01.13.06.0011.01

$$\cos^{-1}(z) = \pi - \sqrt{2}\,\sqrt{z+1}\; {}_2F_1\left(\frac{1}{2}, \frac{1}{2}; \frac{3}{2}; \frac{z+1}{2}\right)$$

01.13.06.0012.02

$$\cos^{-1}(z) \propto \pi - \sqrt{2}\,\sqrt{z+1}\,(1 + O(z+1))$$

01.13.06.0054.01

$$\cos^{-1}(z) = F_\infty(z) /; \left(\left(F_n(z) = \pi - \sqrt{2}\,\sqrt{z+1}\sum_{k=0}^{n} \frac{\left(\frac{1}{2}\right)_k (z+1)^k}{2^k (2k+1)\, k!} = \cos^{-1}(z) + \frac{\Gamma\left(n+\frac{3}{2}\right)^2 (2+2z)^{n+\frac{3}{2}}}{\pi (2n+3)!}\; {}_3F_2\left(1, n+\frac{3}{2}, n+\frac{3}{2}; n+2, n+\frac{5}{2}; \frac{z+1}{2}\right)\right) \land n \in \mathbb{N}\right)$$

Summed form of the truncated series expansion.

For small integer powers of the function

For the second power

01.13.06.0055.01

$$\cos^{-1}(z)^2 \propto \pi^2 - \pi\, 2\sqrt{2}\,\sqrt{z+1}\left(1 + \frac{z+1}{12} + \frac{3}{160}(z+1)^2 + \ldots\right) + 2(z+1)\left(1 + \frac{z+1}{6} + \frac{2}{45}(z+1)^2 + \ldots\right) /; (z \to -1)$$

01.13.06.0056.01

$$\cos^{-1}(z)^2 \propto \pi^2 - \pi\, 2\sqrt{2}\,\sqrt{z+1}\left(1 + \frac{z+1}{12} + \frac{3}{160}(z+1)^2 + O\left((z+1)^3\right)\right) + 2(z+1)\left(1 + \frac{z+1}{6} + \frac{2}{45}(z+1)^2 + O\left((z+1)^3\right)\right)$$

01.13.06.0057.01

$$\cos^{-1}(z)^2 = \pi^2 - \pi\, 2\sqrt{2}\,\sqrt{z+1}\,\sum_{k=0}^{\infty} \frac{\left(\frac{1}{2}\right)_k (z+1)^k}{2^k (2k+1)\, k!} + 2(z+1)\left(\sum_{k=0}^{\infty} \frac{\left(\frac{1}{2}\right)_k (z+1)^k}{2^k (2k+1)\, k!}\right)^2 \;/;\; |z+1| < 2$$

01.13.06.0058.01

$$\cos^{-1}(z)^2 = 4\cos^{-1}\left(\sqrt{\frac{z+1}{2}}\right)^2$$

The last formula allows to express the function at the singular points $z = -1$ through a function value at the regular point $\tilde{z} = 0$.

01.13.06.0059.01

$$\cos^{-1}(z)^2 = \pi^2 - \pi\, 2\sqrt{2}\,\sqrt{z+1}\; {}_2F_1\left(\frac{1}{2}, \frac{1}{2}; \frac{3}{2}; \frac{z+1}{2}\right) + 2(z+1)\; {}_2F_1\left(\frac{1}{2}, \frac{1}{2}; \frac{3}{2}; \frac{z+1}{2}\right)^2$$

01.13.06.0060.01

$$\cos^{-1}(z)^2 \propto \pi^2 - \pi\, 2\sqrt{2}\,\sqrt{z+1}\,(1 + O(z+1)) + 2(z+1)(1 + O(z+1))$$

01.13.06.0061.01

$$\cos^{-1}(z)^2 = F_\infty(z) \;/;\; \left(\left(F_n(z) = \pi^2 - \pi\, 2\sqrt{2}\,\sqrt{z+1}\,\sum_{k=0}^{n} \frac{\left(\frac{1}{2}\right)_k (z+1)^k}{2^k (2k+1)\, k!} + 2(z+1)\left(\sum_{k=0}^{n} \frac{\left(\frac{1}{2}\right)_k (z+1)^k}{2^k (2k+1)\, k!}\right)^2 = \right.\right.$$

$$\cos^{-1}(z)^2 + \frac{(z+1)^{2n+3}\,\Gamma\left(n+\frac{3}{2}\right)^4}{\pi^{3/2}\,(2n+3)!}\; {}_3F_2\left(1, n+\frac{3}{2}, n+\frac{3}{2}; n+2, n+\frac{5}{2}; \frac{z+1}{2}\right)^2 +$$

$$\left.\left.\frac{2^{\frac{1}{2}-n}\,(z+1)^{n+\frac{3}{2}}\cos^{-1}(z)\,\Gamma\left(n+\frac{3}{2}\right)}{(2n+3)\sqrt{\pi}\,(n+1)!}\; {}_3F_2\left(1, n+\frac{3}{2}, n+\frac{3}{2}; n+2, n+\frac{5}{2}; \frac{z+1}{2}\right)\right) \bigwedge n \in \mathbb{N}\right)$$

Summed form of the truncated series expansion.

Expansions at $z = \infty$

For the function itself

01.13.06.0013.02

$$\cos^{-1}(z) \propto \frac{\pi}{2} - \frac{z}{2\sqrt{-z^2}}\left(\log\left(-4z^2\right) - \frac{1}{2z^2} - \frac{3}{16z^4} - \ldots\right) \;/;\; (|z| \to \infty)$$

01.13.06.0062.01

$$\cos^{-1}(z) \propto \frac{\pi}{2} - \frac{z}{2\sqrt{-z^2}}\left(\log\left(-4z^2\right) - \frac{1}{2z^2} - \frac{3}{16z^4} + O\left(\frac{1}{z^6}\right)\right)$$

01.13.06.0014.01

$$\cos^{-1}(z) = \frac{\pi}{2} - \frac{z}{2\sqrt{-z^2}}\left(\log\left(-4z^2\right) - \sum_{k=1}^{\infty} \frac{\left(\frac{1}{2}\right)_k z^{-2k}}{k\, k!}\right) \;/;\; |z| > 1$$

01.13.06.0015.01

$$\cos^{-1}(z) = \frac{\pi}{2} - \frac{z}{2\sqrt{-z^2}}\left(\log(-4z^2) - \frac{1}{2z^2}\, {}_3F_2\!\left(\frac{3}{2}, 1, 1; 2, 2; \frac{1}{z^2}\right)\right) /;\, z \notin (-1, 1)$$

01.13.06.0016.02

$$\cos^{-1}(z) \propto \frac{\pi}{2} - \frac{z\log(-4z^2)}{2\sqrt{-z^2}} + \frac{1}{4z\sqrt{-z^2}}\left(1 + O\!\left(\frac{1}{z^2}\right)\right)$$

01.13.06.0063.01

$$\cos^{-1}(z) \propto \begin{cases} i\log(z) & \arg(z) \le 0 \\ -i\log(z) & \text{True} \end{cases} /;\, (|z| \to \infty)$$

01.13.06.0064.01

$$\cos^{-1}(z) = F_\infty(z) /;\, \left(\left(F_n(z) = \frac{\pi}{2} - \frac{z}{2\sqrt{-z^2}}\left(\log(-4z^2) - \frac{1}{2z^2}\sum_{k=0}^{n} \frac{\left(\frac{3}{2}\right)_k z^{-2k}}{(k+1)^2\, k!}\right) = \cos^{-1}(z) - \frac{3z^{-2(n+2)}\left(\frac{5}{2}\right)_n}{4(n+2)^2\,(n+1)!}\, {}_3F_2\!\left(1, n+\frac{5}{2}, n+2; n+3, n+3; \frac{1}{z^2}\right)\right) \bigwedge n \in \mathbb{N}\right)$$

Summed form of the truncated series expansion.

For small integer powers of the function

For the second power

01.13.06.0065.01

$$\cos^{-1}(z)^2 \propto -\frac{1}{4}\log^2(-z^2) - \frac{z\pi}{2\sqrt{-z^2}}\left(\log(-4z^2) - \frac{1}{2z^2} - \frac{3}{16z^4} - \ldots\right) + \log(-z^2)\left(-\log(2) + \frac{1}{4z^2} + \frac{3}{32z^4} + \ldots\right) + \left(\frac{\pi^2}{4} - \log^2(2) + \frac{\log(2)}{2z^2} + \frac{3\log(2) - 1}{16z^4} + \ldots\right) /;\, (|z| \to \infty)$$

01.13.06.0066.01

$$\cos^{-1}(z)^2 \propto -\frac{1}{4}\log^2(-z^2) - \frac{z\pi}{2\sqrt{-z^2}}\left(\log(-4z^2) - \frac{1}{2z^2} - \frac{3}{16z^4} + O\!\left(\frac{1}{z^6}\right)\right) + \log(-z^2)\left(-\log(2) + \frac{1}{4z^2} + \frac{3}{32z^4} + O\!\left(\frac{1}{z^6}\right)\right) + \left(\frac{\pi^2}{4} - \log^2(2) + \frac{\log(2)}{2z^2} + \frac{3\log(2) - 1}{16z^4} + O\!\left(\frac{1}{z^6}\right)\right)$$

01.13.06.0067.01

$$\cos^{-1}(z)^2 = -\frac{1}{4}\log^2(-4z^2) - \frac{\pi z\log(-4z^2)}{2\sqrt{-z^2}} + \frac{\pi^2}{4} + \frac{1}{4z^2}\left(\frac{\pi z}{\sqrt{-z^2}} + \log(-z^2)\right)\sum_{k=0}^{\infty} \frac{\left(\frac{3}{2}\right)_k z^{-2k}}{(k+1)^2\, k!} - \frac{1}{4z^2}\sum_{k=0}^{\infty} \frac{\left(\frac{3}{2}\right)_k \left(\psi\!\left(-k-\frac{1}{2}\right) - \psi(k+1)\right) z^{-2k}}{(k+1)^2\, k!} + \frac{1}{2z^2}\sum_{k=0}^{\infty} \frac{\left(\frac{3}{2}\right)_k z^{-2k}}{(k+1)^3\, k!} /;\, |z| > 1$$

01.13.06.0068.01

$$\cos^{-1}(z)^2 = -\frac{1}{4}\log^2(-4z^2) - \frac{\pi z \log(-4z^2)}{2\sqrt{-z^2}} + \frac{\pi^2}{4} + \frac{1}{4z^2}\left(\frac{\pi z}{\sqrt{-z^2}} + \log(-4z^2)\right)\sum_{k=0}^{\infty}\frac{\left(\frac{3}{2}\right)_k z^{-2k}}{(k+1)^2\,k!} - \frac{1}{16z^4}\left(\sum_{k=0}^{\infty}\frac{\left(\frac{3}{2}\right)_k z^{-2k}}{(k+1)^2\,k!}\right)^2 /; |z| > 1$$

01.13.06.0069.01

$$\cos^{-1}(z)^2 = -\frac{1}{4}\log^2(-4z^2) + \frac{\pi\sqrt{-z^2}\log(-4z^2)}{2z} + \frac{\pi^2}{4} - \log^2\left(\frac{1}{2}\left(\sqrt{1-\frac{1}{z^2}}+1\right)\right) + \frac{\pi\sqrt{-z^2} - z\log(-z^2)}{z}\log\left(\frac{1}{2}\left(\sqrt{1-\frac{1}{z^2}}+1\right)\right) + 2\,\mathrm{Li}_2\left(\frac{1}{2}\left(1-\sqrt{1-\frac{1}{z^2}}\right)\right) - \frac{1}{4z^2}\sum_{k=0}^{\infty}\frac{\left(\frac{3}{2}\right)_k\left(\psi\left(-k-\frac{1}{2}\right)-\psi(k+1)\right)z^{-2k}}{(k+1)^2\,k!} /; |z| > 1$$

01.13.06.0070.01

$$\cos^{-1}(z)^2 = -\frac{1}{4}\log^2(-4z^2) - \frac{\pi z\log(-4z^2)}{2\sqrt{-z^2}} + \frac{\pi^2}{4} + \frac{1}{2z^2}\,{}_4F_3\left(\frac{3}{2},1,1,1;2,2,2;\frac{1}{z^2}\right) + \frac{1}{4z^2}\left(\frac{\pi z}{\sqrt{-z^2}} + \log(-z^2)\right){}_3F_2\left(\frac{3}{2},1,1;2,2;\frac{1}{z^2}\right) - \frac{1}{4z^2}\sum_{k=0}^{\infty}\left(\frac{3}{2}\right)_k\frac{\left(\psi\left(-k-\frac{1}{2}\right)-\psi(k+1)\right)z^{-2k}}{(k+1)^2\,k!} /; |z| > 1$$

01.13.06.0071.01

$$\cos^{-1}(z)^2 = -\frac{1}{4}\log^2(-4z^2) - \frac{\pi z\log(-4z^2)}{2\sqrt{-z^2}} + \frac{1}{4z^2}\left(\frac{\pi z}{\sqrt{-z^2}} + \log(-4z^2)\right){}_3F_2\left(\frac{3}{2},1,1;2,2;\frac{1}{z^2}\right) + \frac{\pi^2}{4} - \frac{1}{16z^4}\,{}_3F_2\left(\frac{3}{2},1,1;2,2;\frac{1}{z^2}\right)^2 /; z \notin (-1, 0)$$

01.13.06.0072.01

$$\cos^{-1}(z)^2 \propto -\frac{1}{4}\log^2(-z^2) - \frac{z\pi\log(-4z^2)}{2\sqrt{-z^2}} + \frac{\pi}{4z\sqrt{-z^2}}\left(1+O\left(\frac{1}{z^2}\right)\right) - \log(-z^2)\log(2)\left(1+O\left(\frac{1}{z^2}\right)\right) + \frac{\pi^2}{4} - \log^2(2)\left(1+O\left(\frac{1}{z^2}\right)\right)$$

01.13.06.0073.01

$$\cos^{-1}(z)^2 \propto -\log^2(z) /; (|z| \to \infty)$$

01.13.06.0074.01

$$\cos^{-1}(z)^2 = F_\infty(z) /; \left(\left(F_n(z) = \frac{\pi^2}{4} - \frac{1}{4}\log^2(-4z^2) - \frac{\pi z \log(-4z^2)}{2\sqrt{-z^2}} + \frac{1}{4z^2}\left(\frac{\pi z}{\sqrt{-z^2}} + \log(-4z^2)\right)\sum_{k=0}^{n}\frac{\left(\frac{3}{2}\right)_k z^{-2k}}{(k+1)^2\, k!} - \frac{1}{16z^4}\left(\sum_{k=0}^{n}\frac{\left(\frac{3}{2}\right)_k z^{-2k}}{(k+1)^2\, k!}\right)^2 = \frac{z^{-4(n+2)}}{16(n+2)^4 (n+1)!^2}\left(4(n+2)^2 z^{2(n+2)} \cos^{-1}(z)(n+1)! - 3\left(\frac{5}{2}\right)_n {}_3F_2\left(1, n+2, n+\frac{5}{2}; n+3, n+3; \frac{1}{z^2}\right)\right)^2\right) \bigwedge n \in \mathbb{N}\right)$$

Summed form of the truncated series expansion.

Residue representations

01.13.06.0017.01

$$\cos^{-1}(z) = \frac{1}{2z\sqrt{\pi}} \sum_{j=1}^{\infty} \operatorname{res}_s\left(\frac{\Gamma\left(-s-\frac{1}{2}\right)^2 (-z^2)^{-s}}{\Gamma\left(\frac{1}{2}-s\right)}\Gamma(s+1)\right)(-j) + \frac{\pi}{2} /; |z| < 1$$

01.13.06.0018.01

$$\cos^{-1}(z) = \frac{z}{2\sqrt{\pi}} \sum_{j=0}^{\infty} \operatorname{res}_s\left(\frac{\Gamma(s)(-z^2)^{-s}}{\Gamma\left(\frac{3}{2}-s\right)}\Gamma\left(\frac{1}{2}-s\right)^2\right)\left(\frac{1}{2}+j\right) + \frac{\pi}{2} /; |z| > 1$$

Integral representations

On the real axis

Of the direct function

01.13.07.0001.01

$$\cos^{-1}(z) = \int_z^1 \frac{1}{\sqrt{1-t^2}}\, dt$$

Contour integral representations

01.13.07.0002.01

$$\cos^{-1}(z) = \frac{\pi}{2} - \frac{z}{\left(2\sqrt{\pi}\right)2\pi i}\int_{\mathcal{L}} \frac{\Gamma(s)\Gamma\left(\frac{1}{2}-s\right)^2}{\Gamma\left(\frac{3}{2}-s\right)}(-z^2)^{-s}\, ds /; \left|\arg(-z^2)\right| < \pi$$

01.13.07.0003.01

$$\cos^{-1}(z) = \frac{\pi}{2} - \frac{z}{\left(2\sqrt{\pi}\right)2\pi i}\int_{\mathcal{L}} \Gamma(s)\Gamma\left(s+\frac{1}{2}\right)\Gamma\left(\frac{1}{2}-s\right)^2 (1-z^2)^{-s}\, ds /; \left|\arg(1-z^2)\right| < \pi$$

01.13.07.0004.01

$$\cos^{-1}(z) = \frac{\pi}{2} - \frac{z}{\left(2\sqrt{\pi}\right) 2 \pi i} \int_{\gamma - i\infty}^{\gamma + i\infty} \frac{\Gamma(s)\, \Gamma\left(\frac{1}{2} - s\right)^2}{\Gamma\left(\frac{3}{2} - s\right)} \left(-z^2\right)^{-s} ds \;/;\; 0 < \gamma < \frac{1}{2} \bigwedge \left|\arg\left(-z^2\right)\right| < \pi$$

01.13.07.0005.01

$$\cos^{-1}(z) = \frac{\pi}{2} - \frac{z}{\left(2\sqrt{\pi}\right) 2 \pi i} \int_{\gamma - i\infty}^{\gamma + i\infty} \Gamma(s)\, \Gamma\left(s + \frac{1}{2}\right) \Gamma\left(\frac{1}{2} - s\right)^2 \left(1 - z^2\right)^{-s} ds \;/;\; 0 < \gamma < \frac{1}{2} \bigwedge \left|\arg\left(1 - z^2\right)\right| < \pi$$

Continued fraction representations

01.13.10.0001.01

$$\cos^{-1}(z) = \frac{\pi}{2} - \cfrac{z\sqrt{1 - z^2}}{1 - \cfrac{1 \times 2 z^2}{3 - \cfrac{1 \times 2 z^2}{5 - \cfrac{3 \times 4 z^2}{7 - \cfrac{3 \times 4 z^2}{9 - \cfrac{5 \times 6 z^2}{11 - \ldots}}}}}} \;/;\; z \notin (-\infty, -1) \bigwedge z \notin (1, \infty)$$

01.13.10.0002.01

$$\cos^{-1}(z) = \frac{\pi}{2} - \frac{z\sqrt{1 - z^2}}{1 + K_k\left(-2\left(2\left\lfloor \frac{k+1}{2} \right\rfloor - 1\right)\left\lfloor \frac{k+1}{2} \right\rfloor z^2,\, 2k + 1\right)_1^{\infty}} \;/;\; z \notin (-\infty, -1) \bigwedge z \notin (1, \infty)$$

Differential equations

Ordinary linear differential equations and wronskians

For the direct function itself

01.13.13.0001.01

$$\left(1 - z^2\right) w''(z) - z\, w'(z) = 0 \;/;\; w(z) = \cos^{-1}(z) \bigwedge w(0) = \frac{\pi}{2} \bigwedge w'(0) = -1$$

01.13.13.0002.01

$$\left(1 - z^2\right) w''(z) - z\, w'(z) = 0 \;/;\; w(z) = c_1 + c_2 \cos^{-1}(z)$$

01.13.13.0003.01

$$W_z\left(1, \cos^{-1}(z)\right) = -\frac{1}{\sqrt{1 - z^2}}$$

01.13.13.0004.01

$$\sqrt{1 - z^2}\; w'(z) = -1 \;/;\; w(z) = \cos^{-1}(z) \bigwedge w(0) = \frac{\pi}{2}$$

Transformations

Transformations and argument simplifications

Argument involving basic arithmetic operations

Involving $\cos^{-1}(-z)$

Involving $\cos^{-1}(-z)$ and $\cos^{-1}(z)$

01.13.16.0001.01

$$\cos^{-1}(-z) = \pi - \cos^{-1}(z)$$

Involving $\cos^{-1}(c\,z)$

Involving $\cos^{-1}(i\,z)$ and $\cos^{-1}(1+2\,z^2)$

01.13.16.0018.01

$$\cos^{-1}(i\,z) = \frac{1}{2}\cos^{-1}(2\,z^2+1) + \frac{\pi}{2} \;/;\; 0 < \arg(z) \leq \pi$$

01.13.16.0019.01

$$\cos^{-1}(i\,z) = \frac{\pi}{2} - \frac{1}{2}\cos^{-1}(2\,z^2+1) \;/;\; -\pi < \arg(z) \leq 0$$

01.13.16.0020.01

$$\cos^{-1}(i\,z) = \frac{i\sqrt{-z^2}}{2\,z}\cos^{-1}(2\,z^2+1) + \frac{\pi}{2}$$

Involving $\cos^{-1}(-i\,z)$ and $\cos^{-1}(1+2\,z^2)$

01.13.16.0021.01

$$\cos^{-1}(-i\,z) = -\frac{1}{2}\cos^{-1}(2\,z^2+1) + \frac{\pi}{2} \;/;\; 0 < \arg(z) \leq \pi$$

01.13.16.0022.01

$$\cos^{-1}(-i\,z) = \frac{\pi}{2} + \frac{1}{2}\cos^{-1}(2\,z^2+1) \;/;\; -\pi < \arg(z) \leq 0$$

01.13.16.0023.01

$$\cos^{-1}(-i\,z) = \frac{\pi}{2} - \frac{i\sqrt{-z^2}}{2\,z}\cos^{-1}(2\,z^2+1)$$

Involving $\cos^{-1}\left(\sqrt{z^2}\right)$

Involving $\cos^{-1}\left(\sqrt{z^2}\right)$ and $\cos^{-1}(z)$

01.13.16.0024.01

$$\cos^{-1}\left(\sqrt{z^2}\right) = \cos^{-1}(z) \;/;\; -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

01.13.16.0025.01

$$\cos^{-1}\left(\sqrt{z^2}\right) = \pi - \cos^{-1}(z) \;/;\; \frac{\pi}{2} < \arg(z) \leq \pi \bigvee -\pi < \arg(z) \leq -\frac{\pi}{2}$$

01.13.16.0006.01

$$\cos^{-1}\left(\sqrt{z^2}\right) = \frac{\pi}{2} - \frac{\sqrt{z^2}}{z}\left(\frac{\pi}{2} - \cos^{-1}(z)\right)$$

Involving $\cos^{-1}\left(a\,(b\,z^c)^m\right)$

Involving $\cos^{-1}(a\,(b\,z^c)^m)$ and $\cos^{-1}(a\,b^m\,z^{m\,c})$

01.13.16.0002.01

$$\cos^{-1}(a\,(b\,z^c)^m) = \frac{\pi}{2} - \frac{(b\,z^c)^m}{b^m\,z^{m\,c}}\left(\frac{\pi}{2} - \cos^{-1}(a\,b^m\,z^{m\,c})\right) \;/;\; 2\,m \in \mathbb{Z}$$

Involving $\cos^{-1}\left(1 - 2\,z^2\right)$

Involving $\cos^{-1}\left(1 - 2\,z^2\right)$ and $\cos^{-1}(z)$

01.13.16.0026.01

$$\cos^{-1}\left(1 - 2\,z^2\right) = \pi - 2\cos^{-1}(z) \;/;\; -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

01.13.16.0027.01

$$\cos^{-1}\left(1 - 2\,z^2\right) = 2\cos^{-1}(z) - \pi \;/;\; \frac{\pi}{2} < \arg(z) \leq \pi \bigvee -\pi < \arg(z) \leq -\frac{\pi}{2}$$

01.13.16.0003.01

$$\cos^{-1}\left(1 - 2\,z^2\right) = \frac{2\sqrt{z^2}}{z}\left(\frac{\pi}{2} - \cos^{-1}(z)\right)$$

Involving $\cos^{-1}\left(2\,z^2 - 1\right)$

Involving $\cos^{-1}\left(2\,z^2 - 1\right)$ and $\cos^{-1}(z)$

01.13.16.0028.01

$$\cos^{-1}\left(2\,z^2 - 1\right) = 2\cos^{-1}(z) \;/;\; -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

01.13.16.0029.01

$$\cos^{-1}\left(2\,z^2 - 1\right) = 2\,\pi - 2\cos^{-1}(z) \;/;\; \frac{\pi}{2} < \arg(z) \leq \pi \bigvee -\pi < \arg(z) \leq -\frac{\pi}{2}$$

01.13.16.0004.02

$$\cos^{-1}(2 z^2 - 1) = \pi \left(1 - \frac{\sqrt{z^2}}{z}\right) + \frac{2 \sqrt{z^2}}{z} \cos^{-1}(z)$$

Involving $\cos^{-1}\left(\frac{z^2-2}{z^2}\right)$

Involving $\cos^{-1}\left(\frac{z^2-2}{z^2}\right)$ and $\cos^{-1}\left(\frac{1}{z}\right)$

01.13.16.0030.01

$$\cos^{-1}\left(\frac{z^2 - 2}{z^2}\right) = \pi - 2 \cos^{-1}\left(\frac{1}{z}\right) /; -\frac{\pi}{2} \leq \arg(z) < \frac{\pi}{2}$$

01.13.16.0031.01

$$\cos^{-1}\left(\frac{z^2 - 2}{z^2}\right) = 2 \cos^{-1}\left(\frac{1}{z}\right) - \pi /; \frac{\pi}{2} \leq \arg(z) \leq \pi \bigvee -\pi < \arg(z) < -\frac{\pi}{2}$$

01.13.16.0032.01

$$\cos^{-1}\left(\frac{z^2 - 2}{z^2}\right) = \sqrt{\frac{1}{z^2}} z \left(\pi - 2 \cos^{-1}\left(\frac{1}{z}\right)\right)$$

Involving $\cos^{-1}\left(\frac{2-z^2}{z^2}\right)$

Involving $\cos^{-1}\left(\frac{z^2-2}{z^2}\right)$ and $\cos^{-1}\left(\frac{1}{z}\right)$

01.13.16.0033.01

$$\cos^{-1}\left(\frac{2 - z^2}{z^2}\right) = 2 \cos^{-1}\left(\frac{1}{z}\right) /; -\frac{\pi}{2} \leq \arg(z) < \frac{\pi}{2}$$

01.13.16.0034.01

$$\cos^{-1}\left(\frac{2 - z^2}{z^2}\right) = 2 \pi - 2 \cos^{-1}\left(\frac{1}{z}\right) /; \frac{\pi}{2} \leq \arg(z) \leq \pi \bigvee -\pi < \arg(z) < -\frac{\pi}{2}$$

01.13.16.0035.01

$$\cos^{-1}\left(\frac{2 - z^2}{z^2}\right) = \pi \left(1 - \sqrt{\frac{1}{z^2}} z\right) + 2 z \sqrt{\frac{1}{z^2}} \cos^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\sqrt{1-z}\right)$

Involving $\cos^{-1}\left(\sqrt{1-z}\right)$ and $\cos^{-1}\left(\sqrt{z}\right)$

01.13.16.0036.01

$$\cos^{-1}\left(\sqrt{1-z}\right) = \frac{\pi}{2} - \cos^{-1}\left(\sqrt{z}\right)$$

Involving $\cos^{-1}\left(\sqrt{\frac{1+c\,z}{2}}\right)$

Involving $\cos^{-1}\left(\sqrt{\frac{1+z}{2}}\right)$ and $\cos^{-1}(z)$

01.13.16.0005.01

$$\cos^{-1}\left(\sqrt{\frac{z+1}{2}}\right) = \frac{1}{2}\cos^{-1}(z)$$

Involving $\cos^{-1}\left(\sqrt{\frac{1-z}{2}}\right)$ and $\cos^{-1}(z)$

01.13.16.0037.01

$$\cos^{-1}\left(\sqrt{\frac{1-z}{2}}\right) = \frac{\pi}{2} - \frac{1}{2}\cos^{-1}(z)$$

Involving $\cos^{-1}\left(\frac{\sqrt{z-1}}{\sqrt{z}}\right)$

Involving $\cos^{-1}\left(\frac{\sqrt{z-1}}{\sqrt{z}}\right)$ and $\cos^{-1}\left(\frac{1}{\sqrt{z}}\right)$

01.13.16.0038.01

$$\cos^{-1}\left(\frac{\sqrt{z-1}}{\sqrt{z}}\right) = \frac{\pi}{2} - \cos^{-1}\left(\frac{1}{\sqrt{z}}\right) /;\ |\arg(z)| < \pi$$

01.13.16.0039.01

$$\cos^{-1}\left(\frac{\sqrt{z-1}}{\sqrt{z}}\right) = \cos^{-1}\left(\frac{1}{\sqrt{z}}\right) - \frac{\pi}{2} /;\ (z \in \mathbb{R} \land z < 0)$$

01.13.16.0040.01

$$\cos^{-1}\left(\frac{\sqrt{z-1}}{\sqrt{z}}\right) = \sqrt{\frac{1}{z}}\,\sqrt{z}\left(\frac{\pi}{2} - \cos^{-1}\left(\frac{1}{\sqrt{z}}\right)\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{z-1}}{\sqrt{z}}\right)$ and $\cos^{-1}\left(\sqrt{\frac{1}{z}}\right)$

01.13.16.0041.01

$$\cos^{-1}\left(\frac{\sqrt{z-1}}{\sqrt{z}}\right) = \frac{\pi}{2} - \cos^{-1}\left(\sqrt{\frac{1}{z}}\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{-z}}\right)$

Involving $\cos^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{-z}}\right)$ and $\cos^{-1}\left(\frac{1}{\sqrt{z}}\right)$

01.13.16.0042.01

$$\cos^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{-z}}\right) = \frac{\pi}{2} - \cos^{-1}\left(\frac{1}{\sqrt{z}}\right) /; z \notin (-\infty, 1)$$

01.13.16.0043.01

$$\cos^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{-z}}\right) = \cos^{-1}\left(\frac{1}{\sqrt{z}}\right) + \frac{\pi}{2} /; (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.16.0044.01

$$\cos^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{-z}}\right) = \cos^{-1}\left(\frac{1}{\sqrt{z}}\right) - \frac{\pi}{2} /; (z \in \mathbb{R} \land z < 0)$$

01.13.16.0045.01

$$\cos^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{-z}}\right) = \frac{\pi}{2}\left(-\sqrt{\frac{z}{z-1}}\sqrt{\frac{z-1}{z}} + \sqrt{\frac{z}{z-1}}\sqrt{z}\sqrt{\frac{z-1}{z^2}} + 1\right) - \sqrt{\frac{z-1}{z^2}}\sqrt{z}\sqrt{\frac{z}{z-1}}\cos^{-1}\left(\frac{1}{\sqrt{z}}\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{-z}}\right)$ and $\cos^{-1}\left(\sqrt{\frac{1}{z}}\right)$

01.13.16.0046.01

$$\cos^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{-z}}\right) = \frac{\pi}{2} - \cos^{-1}\left(\sqrt{\frac{1}{z}}\right) /; z \notin (0, 1)$$

01.13.16.0047.01

$$\cos^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{-z}}\right) = \cos^{-1}\left(\sqrt{\frac{1}{z}}\right) + \frac{\pi}{2} /; (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.16.0048.01

$$\cos^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{-z}}\right) = \frac{\pi}{2} - \sqrt{\frac{z-1}{z}}\sqrt{\frac{z}{z-1}}\cos^{-1}\left(\sqrt{\frac{1}{z}}\right)$$

Involving $\cos^{-1}\left(\sqrt{\frac{z-1}{z}}\right)$

Involving $\cos^{-1}\left(\sqrt{\frac{z-1}{z}}\right)$ and $\cos^{-1}\left(\frac{1}{\sqrt{z}}\right)$

01.13.16.0049.01

$$\cos^{-1}\left(\sqrt{\frac{z-1}{z}}\right) = \frac{\pi}{2} - \cos^{-1}\left(\frac{1}{\sqrt{z}}\right) /; |\arg(z)| < \pi$$

01.13.16.0050.01

$$\cos^{-1}\left(\sqrt{\frac{z-1}{z}}\right) = \cos^{-1}\left(\frac{1}{\sqrt{z}}\right) - \frac{\pi}{2} /; (z \in \mathbb{R} \land z < 0)$$

01.13.16.0051.01

$$\cos^{-1}\left(\sqrt{\frac{z-1}{z}}\right) = \sqrt{\frac{1}{z}} \sqrt{z} \left(\frac{\pi}{2} - \cos^{-1}\left(\frac{1}{\sqrt{z}}\right)\right)$$

Involving $\cos^{-1}\left(\sqrt{\frac{z-1}{z}}\right)$ and $\cos^{-1}\left(\sqrt{\frac{1}{z}}\right)$

01.13.16.0052.01

$$\cos^{-1}\left(\sqrt{\frac{z-1}{z}}\right) = \frac{\pi}{2} - \cos^{-1}\left(\sqrt{\frac{1}{z}}\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{z+a}}{\sqrt{2z}}\right)$

Involving $\cos^{-1}\left(\frac{\sqrt{z-1}}{\sqrt{2z}}\right)$ and $\cos^{-1}\left(\frac{1}{z}\right)$

01.13.16.0053.01

$$\cos^{-1}\left(\frac{\sqrt{z-1}}{\sqrt{2z}}\right) = \frac{\pi}{2} - \frac{1}{2}\cos^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{z+1}}{\sqrt{2z}}\right)$ and $\cos^{-1}\left(\frac{1}{z}\right)$

01.13.16.0054.01

$$\cos^{-1}\left(\frac{\sqrt{z+1}}{\sqrt{2z}}\right) = \frac{1}{2}\cos^{-1}\left(\frac{1}{z}\right) /; z \notin (-1, 0)$$

01.13.16.0055.01

$$\cos^{-1}\left(\frac{\sqrt{z+1}}{\sqrt{2z}}\right) = \pi - \frac{1}{2}\cos^{-1}\left(\frac{1}{z}\right) /; (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.16.0056.01

$$\cos^{-1}\left(\frac{\sqrt{z+1}}{\sqrt{2z}}\right) = \frac{\pi}{2}\left(1 - \sqrt{\frac{z+1}{z}} \sqrt{\frac{z}{z+1}}\right) + \frac{1}{2}\sqrt{\frac{z}{z+1}} \sqrt{\frac{z+1}{z}} \cos^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{a-z}}{\sqrt{-2z}}\right)$

Involving $\cos^{-1}\left(\frac{\sqrt{-z-1}}{\sqrt{-2z}}\right)$ and $\cos^{-1}\left(\frac{1}{z}\right)$

01.13.16.0057.01

$$\cos^{-1}\left(\frac{\sqrt{-z-1}}{\sqrt{-2z}}\right) == \frac{1}{2}\cos^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{-2z}}\right)$ and $\cos^{-1}\left(\frac{1}{z}\right)$

01.13.16.0058.01

$$\cos^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{-2z}}\right) == \frac{\pi}{2} - \frac{1}{2}\cos^{-1}\left(\frac{1}{z}\right) /; z \notin (0,\, 1)$$

01.13.16.0059.01

$$\cos^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{-2z}}\right) == \frac{1}{2}\cos^{-1}\left(\frac{1}{z}\right) + \frac{\pi}{2} /; (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.16.0060.01

$$\cos^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{-2z}}\right) == \frac{\pi}{2} - \frac{1}{2}\sqrt{\frac{z-1}{z}}\sqrt{\frac{z}{z-1}}\cos^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\sqrt{\frac{z+a}{2z}}\right)$

Involving $\cos^{-1}\left(\sqrt{\frac{z-1}{2z}}\right)$ and $\cos^{-1}\left(\frac{1}{z}\right)$

01.13.16.0061.01

$$\cos^{-1}\left(\sqrt{\frac{z-1}{2z}}\right) == \frac{\pi}{2} - \frac{1}{2}\cos^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\sqrt{\frac{z+1}{2z}}\right)$ and $\cos^{-1}\left(\frac{1}{z}\right)$

01.13.16.0062.01

$$\cos^{-1}\left(\sqrt{\frac{z+1}{2z}}\right) == \frac{1}{2}\cos^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\sqrt{1-z^2}\right)$

Involving $\cos^{-1}\left(\sqrt{1-z^2}\right)$ and $\cos^{-1}(z)$

01.13.16.0063.01

$$\cos^{-1}\left(\sqrt{1-z^2}\right) = \frac{\pi}{2} - \cos^{-1}(z) \;/;\; -\frac{\pi}{2} < \arg(z) \le \frac{\pi}{2}$$

01.13.16.0064.01

$$\cos^{-1}\left(\sqrt{1-z^2}\right) = \cos^{-1}(z) - \frac{\pi}{2} \;/;\; \frac{\pi}{2} < \arg(z) \le \pi \bigvee -\pi < \arg(z) \le -\frac{\pi}{2}$$

01.13.16.0007.01

$$\cos^{-1}\left(\sqrt{1-z^2}\right) = \frac{\sqrt{z^2}}{z}\left(\frac{\pi}{2} - \cos^{-1}(z)\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{z^2-1}}{z}\right)$

Involving $\cos^{-1}\left(\frac{\sqrt{z^2-1}}{z}\right)$ and $\cos^{-1}\left(\frac{1}{z}\right)$

01.13.16.0065.01

$$\cos^{-1}\left(\frac{\sqrt{z^2-1}}{z}\right) = \frac{\pi}{2} - \cos^{-1}\left(\frac{1}{z}\right) \;/;\; \text{Re}(z) > 0$$

01.13.16.0066.01

$$\cos^{-1}\left(\frac{\sqrt{z^2-1}}{z}\right) = \frac{3\pi}{2} - \cos^{-1}\left(\frac{1}{z}\right) \;/;\; \text{Re}(z) < 0$$

01.13.16.0067.01

$$\cos^{-1}\left(\frac{\sqrt{z^2-1}}{z}\right) = -\frac{\pi}{2} + \cos^{-1}\left(\frac{1}{z}\right) \;/;\; (i\,z \in \mathbb{R} \land i\,z < 0)$$

01.13.16.0068.01

$$\cos^{-1}\left(\frac{\sqrt{z^2-1}}{z}\right) = \cos^{-1}\left(\frac{1}{z}\right) + \frac{\pi}{2} \;/;\; (i\,z \in \mathbb{R} \land i\,z > 0)$$

01.13.16.0069.01

$$\cos^{-1}\left(\frac{\sqrt{z^2-1}}{z}\right) = \frac{\pi}{2}\left(2 - \frac{\sqrt{z^2}}{z}\right) - \cos^{-1}\left(\frac{1}{z}\right) \;/;\; \text{Re}(z) \neq 0$$

01.13.16.0070.01

$$\cos^{-1}\left(\frac{\sqrt{z^2-1}}{z}\right) == \frac{\pi}{2}\left(\sqrt{\frac{1}{z^2}}\sqrt{z^2} - \frac{\sqrt{z^2}}{z} + 1\right) - \sqrt{z^2}\sqrt{\frac{1}{z^2}}\cos^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{z^2-1}}{\sqrt{z^2}}\right)$

Involving $\cos^{-1}\left(\frac{\sqrt{z^2-1}}{\sqrt{z^2}}\right)$ and $\cos^{-1}\left(\frac{1}{z}\right)$

01.13.16.0071.01

$$\cos^{-1}\left(\frac{\sqrt{z^2-1}}{\sqrt{z^2}}\right) == \frac{\pi}{2} - \cos^{-1}\left(\frac{1}{z}\right) /; -\frac{\pi}{2} \leq \arg(z) < \frac{\pi}{2}$$

01.13.16.0072.01

$$\cos^{-1}\left(\frac{\sqrt{z^2-1}}{\sqrt{z^2}}\right) == -\frac{\pi}{2} + \cos^{-1}\left(\frac{1}{z}\right) /; \frac{\pi}{2} \leq \arg(z) \leq \pi \bigvee -\pi < \arg(z) < -\frac{\pi}{2}$$

01.13.16.0073.01

$$\cos^{-1}\left(\frac{\sqrt{z^2-1}}{\sqrt{z^2}}\right) == z\sqrt{\frac{1}{z^2}}\left(\frac{\pi}{2} - \cos^{-1}\left(\frac{1}{z}\right)\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{1-z^2}}{\sqrt{-z^2}}\right)$

Involving $\cos^{-1}\left(\frac{\sqrt{1-z^2}}{\sqrt{-z^2}}\right)$ and $\cos^{-1}\left(\frac{1}{z}\right)$

01.13.16.0074.01

$$\cos^{-1}\left(\frac{\sqrt{1-z^2}}{\sqrt{-z^2}}\right) == \frac{\pi}{2} - \cos^{-1}\left(\frac{1}{z}\right) /; -\frac{\pi}{2} \leq \arg(z) < 0 \bigvee 0 < \arg(z) < \frac{\pi}{2} \bigvee (z \in \mathbb{R} \land z > 1)$$

01.13.16.0075.01

$$\cos^{-1}\left(\frac{\sqrt{1-z^2}}{\sqrt{-z^2}}\right) == \cos^{-1}\left(\frac{1}{z}\right) - \frac{\pi}{2} /; \frac{\pi}{2} \leq \arg(z) < \pi \bigvee -\pi < \arg(z) < -\frac{\pi}{2} \bigvee (z \in \mathbb{R} \land z < -1)$$

01.13.16.0076.01

$$\cos^{-1}\left(\frac{\sqrt{1-z^2}}{\sqrt{-z^2}}\right) == \cos^{-1}\left(\frac{1}{z}\right) + \frac{\pi}{2} /; (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.16.0077.01

$$\cos^{-1}\left(\frac{\sqrt{1-z^2}}{\sqrt{-z^2}}\right) = \frac{3\pi}{2} - \cos^{-1}\left(\frac{1}{z}\right) /; (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.16.0078.01

$$\cos^{-1}\left(\frac{\sqrt{1-z^2}}{\sqrt{-z^2}}\right) = \frac{\pi}{2}\left(\sqrt{\frac{z^2}{z^2-1}}\left(z\sqrt{\frac{z^2-1}{z^4}} - \sqrt{\frac{z^2-1}{z^2}}\right) + 1\right) - z\sqrt{\frac{z^2}{z^2-1}}\sqrt{\frac{z^2-1}{z^4}}\cos^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\sqrt{\frac{z^2-1}{z^2}}\right)$

Involving $\cos^{-1}\left(\sqrt{\frac{z^2-1}{z^2}}\right)$ and $\cos^{-1}\left(\frac{1}{z}\right)$

01.13.16.0079.01

$$\cos^{-1}\left(\sqrt{\frac{z^2-1}{z^2}}\right) = \frac{\pi}{2} - \cos^{-1}\left(\frac{1}{z}\right) /; -\frac{\pi}{2} \leq \arg(z) < \frac{\pi}{2}$$

01.13.16.0080.01

$$\cos^{-1}\left(\sqrt{\frac{z^2-1}{z^2}}\right) = -\frac{\pi}{2} + \cos^{-1}\left(\frac{1}{z}\right) /; \frac{\pi}{2} \leq \arg(z) \leq \pi \bigvee -\pi < \arg(z) < -\frac{\pi}{2}$$

01.13.16.0081.01

$$\cos^{-1}\left(\sqrt{\frac{z^2-1}{z^2}}\right) = z\sqrt{\frac{1}{z^2}}\left(\frac{\pi}{2} - \cos^{-1}\left(\frac{1}{z}\right)\right)$$

Involving $\cos^{-1}\left(2z\sqrt{1-z^2}\right)$

Involving $\cos^{-1}\left(2z\sqrt{1-z^2}\right)$ and $\cos^{-1}(z)$

01.13.16.0009.02

$$\cos^{-1}\left(2z\sqrt{1-z^2}\right) = 2\cos^{-1}(z) - \frac{\pi}{2} /; \frac{\pi}{4} \leq |\arg(z)| \leq \frac{3\pi}{4}$$

01.13.16.0082.01

$$\cos^{-1}\left(2z\sqrt{1-z^2}\right) = \frac{\pi\sqrt{1-2z^2}\sqrt{z^4-z^2}}{2\sqrt{-z^2}\sqrt{z^2-1}\sqrt{2z^2-1}} \left(-\frac{\sqrt{z^2}}{z} + \sqrt{\frac{1}{z}}\sqrt{z}\sqrt{\frac{1}{\sqrt{2}\,z+1}}\sqrt{\sqrt{2}\,z+1} - \sqrt{-\frac{1}{z}}\sqrt{-z}\sqrt{\frac{1}{1-\sqrt{2}\,z}}\sqrt{1-\sqrt{2}\,z} - \frac{\sqrt{z^4-z^2}}{z\sqrt{z^2-1}} + 2\right) - \frac{2\sqrt{1-2z^2}\sqrt{z^4-z^2}}{\sqrt{-z^2}\sqrt{z^2-1}\sqrt{2z^2-1}}\cos^{-1}(z) + \frac{\pi}{2}$$

Involving $\cos^{-1}\left(\frac{2\sqrt{z^2-1}}{z^2}\right)$

Involving $\cos^{-1}\left(\frac{2\sqrt{z^2-1}}{z^2}\right)$ and $\cos^{-1}\left(\frac{1}{z}\right)$

01.13.16.0083.01

$$\cos^{-1}\left(\frac{2\sqrt{z^2-1}}{z^2}\right) = 2\cos^{-1}\left(\frac{1}{z}\right) - \frac{\pi}{2} \;/;\; \frac{\pi}{4} \le |\arg(z)| < \frac{\pi}{2} \bigvee \mathrm{Re}(z) > 0 \bigwedge |z| \ge \sqrt{2}$$

01.13.16.0084.01

$$\cos^{-1}\left(\frac{2\sqrt{z^2-1}}{z^2}\right) = \frac{\pi}{2}\left(1 - \frac{2\sqrt{z^2}}{z}\right) + \frac{2\sqrt{z^2}}{z}\cos^{-1}\left(\frac{1}{z}\right) \;/;\; \frac{\pi}{4} \le |\arg(z)| \le \frac{3\pi}{4} \bigvee |z| \ge \sqrt{2}$$

01.13.16.0085.01

$$\cos^{-1}\left(\frac{2\sqrt{z^2-1}}{z^2}\right) = \frac{\pi\sqrt{z^2}}{2z}\left(\frac{\sqrt{z^2}}{z} - \sqrt{\frac{z}{z+\sqrt{2}}}\sqrt{\frac{z+\sqrt{2}}{z}} + \sqrt{\frac{z-\sqrt{2}}{z}}\sqrt{\frac{z}{z-\sqrt{2}}}\right) - \frac{2\sqrt{z^2-2}}{z}\sqrt{\frac{z^2}{z^2-2}}\left(\frac{\pi}{2} - \cos^{-1}\left(\frac{1}{z}\right)\right)$$

01.13.16.0086.01

$$\cos^{-1}\left(\frac{2\sqrt{z^2-1}}{z^2}\right) = \left(\left(z^3\sqrt{\frac{1-z^2}{z^4}}\sqrt{z^2-2}\sqrt{z^2-1}\sqrt{\frac{1}{z}}\sqrt{-\frac{z+1}{z}}\right)\right.$$

$$\left(\pi\left(\frac{\sqrt{\frac{1-z^2}{z^2}}\sqrt{\frac{1-z^2}{z^4}}z^3}{1-z^2}+\sqrt{\frac{1}{z^2}}z-\sqrt{\frac{1}{z}}\sqrt{\frac{z}{z+\sqrt{2}}}\sqrt{\frac{z+\sqrt{2}}{z}}\sqrt{z}+\right.\right.$$

$$\left.\left.\left.\sqrt{1-\frac{\sqrt{2}}{z}}\sqrt{-\frac{1}{z}}\sqrt{-z}\sqrt{\frac{z}{z-\sqrt{2}}}-2\right)+4\cos^{-1}\left(\frac{1}{z}\right)\right)\right)\Big/\left(2\sqrt{1-z}\,(z+1)\sqrt{-z^4+3z^2-2}\right)+\frac{\pi}{2}$$

Involving $\cos^{-1}\left(\sqrt{\left(1-\sqrt{1+c\,z^2}\right)/2}\right)$

Involving $\cos^{-1}\left(\sqrt{\left(1-\sqrt{1+z^2}\right)/2}\right)$ and $\cos^{-1}(i\,z)$

01.13.16.0087.01

$$\cos^{-1}\left(\sqrt{\frac{1-\sqrt{1+z^2}}{2}}\right) = \frac{\pi}{4}+\frac{1}{2}\cos^{-1}(i\,z)\;/;\;-\pi<\arg(z)\leq 0$$

01.13.16.0088.01

$$\cos^{-1}\left(\sqrt{\frac{1-\sqrt{1+z^2}}{2}}\right) = \frac{3\pi}{4}-\frac{1}{2}\cos^{-1}(i\,z)\;/;\;0<\arg(z)\leq \pi$$

Involving $\cos^{-1}\left(\sqrt{\left(1-\sqrt{1-z^2}\right)/2}\right)$ and $\cos^{-1}(z)$

01.13.16.0089.01

$$\cos^{-1}\left(\sqrt{\frac{1-\sqrt{1-z^2}}{2}}\right) = \frac{\pi}{4}+\frac{1}{2}\cos^{-1}(z)\;/;\;-\frac{\pi}{2}<\arg(z)\leq \frac{\pi}{2}$$

01.13.16.0090.01

$$\cos^{-1}\left(\sqrt{\frac{1-\sqrt{1-z^2}}{2}}\right) = \frac{3\pi}{4} - \frac{1}{2}\cos^{-1}(z) \;/;\; \frac{\pi}{2} < \arg(z) \leq \pi \bigvee -\pi < \arg(z) \leq -\frac{\pi}{2}$$

01.13.16.0008.01

$$\cos^{-1}\left(\sqrt{\frac{1-\sqrt{1-z^2}}{2}}\right) = \frac{\pi}{2}\left(1 - \frac{\sqrt{z^2}}{2z}\right) + \frac{\sqrt{z^2}}{2z}\cos^{-1}(z)$$

Involving $\cos^{-1}\left(z\sqrt{1-\sqrt{1-z^2}}\Big/\sqrt{2z^2}\right)$

Involving $\cos^{-1}\left(z\sqrt{1-\sqrt{1-z^2}}\Big/\sqrt{2z^2}\right)$ and $\cos^{-1}(z)$

01.13.16.0091.01

$$\cos^{-1}\left(\frac{z\sqrt{1-\sqrt{1-z^2}}}{\sqrt{2}\sqrt{z^2}}\right) = \frac{\pi}{4} + \frac{1}{2}\cos^{-1}(z)$$

Involving $\cos^{-1}\left(z\sqrt{\left(1-\sqrt{1-z^2}\right)\Big/\left(2z^2\right)}\right)$

Involving $\cos^{-1}\left(z\sqrt{\left(1-\sqrt{1-z^2}\right)\Big/\left(2z^2\right)}\right)$ and $\cos^{-1}(z)$

01.13.16.0092.01

$$\cos^{-1}\left(z\sqrt{\frac{1-\sqrt{1-z^2}}{2z^2}}\right) = \frac{\pi}{4} + \frac{1}{2}\cos^{-1}(z)$$

Involving $\cos^{-1}\left(\sqrt{z-\sqrt{z^2-1}}\Big/\sqrt{2z}\right)$

Involving $\cos^{-1}\left(\sqrt{z-\sqrt{z^2-1}}\Big/\sqrt{2z}\right)$ and $\cos^{-1}\left(\frac{1}{z}\right)$

01.13.16.0093.01

$$\cos^{-1}\left(\frac{\sqrt{z-\sqrt{z^2-1}}}{\sqrt{2z}}\right) = \frac{1}{2}\cos^{-1}\left(\frac{1}{z}\right)+\frac{\pi}{4} /; -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

01.13.16.0094.01

$$\cos^{-1}\left(\frac{\sqrt{z-\sqrt{z^2-1}}}{\sqrt{2z}}\right) = \frac{1}{2}\cos^{-1}\left(\frac{1}{z}\right)-\frac{\pi}{4} /; \frac{\pi}{2} < \arg(z) < \pi \bigvee -\pi < \arg(z) < -\frac{\pi}{2} \bigvee (z \in \mathbb{R} \land z < -1)$$

01.13.16.0095.01

$$\cos^{-1}\left(\frac{\sqrt{z-\sqrt{z^2-1}}}{\sqrt{2z}}\right) = \frac{5\pi}{4}-\frac{1}{2}\cos^{-1}\left(\frac{1}{z}\right) /; (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.16.0096.01

$$\cos^{-1}\left(\frac{\sqrt{z-\sqrt{z^2-1}}}{\sqrt{2z}}\right) = \frac{\pi}{4}-\frac{1}{2}\cos^{-1}\left(\frac{1}{z}\right) /; (i z \in \mathbb{R} \land i z > 0)$$

01.13.16.0097.01

$$\cos^{-1}\left(\frac{\sqrt{z-\sqrt{z^2-1}}}{\sqrt{2z}}\right) = \frac{1}{4}\pi\left(\frac{\sqrt{z^2}-z}{\sqrt{\frac{z}{z+1}}}\sqrt{\frac{1}{z}}\sqrt{\frac{1}{z+1}} - \sqrt{\frac{i}{z}}\sqrt{\frac{1}{z}}\sqrt{-i z}\sqrt{z}\sqrt{\frac{1}{z+1}}\sqrt{z+1}+2\right)+$$
$$\frac{1}{2}\sqrt{\frac{1}{z}}\sqrt{\frac{i}{z}}\sqrt{z}\sqrt{-i z}\sqrt{\frac{1}{z+1}}\sqrt{z+1}\cos^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\sqrt{\left(z-\sqrt{z^2-1}\right)\Big/(2z)}\right)$

Involving $\cos^{-1}\left(\sqrt{\left(z-\sqrt{z^2-1}\right)\Big/(2z)}\right)$ and $\cos^{-1}\left(\frac{1}{z}\right)$

01.13.16.0098.01

$$\cos^{-1}\left(\sqrt{\frac{z-\sqrt{z^2-1}}{2z}}\right) = \frac{1}{2}\cos^{-1}\left(\frac{1}{z}\right) + \frac{\pi}{4} \;/;\; \mathrm{Re}(z) > 0$$

01.13.16.0099.01

$$\cos^{-1}\left(\sqrt{\frac{z-\sqrt{z^2-1}}{2z}}\right) = \frac{1}{2}\cos^{-1}\left(\frac{1}{z}\right) - \frac{\pi}{4} \;/;\; \mathrm{Re}(z) < 0$$

01.13.16.0100.01

$$\cos^{-1}\left(\sqrt{\frac{z-\sqrt{z^2-1}}{2z}}\right) = \frac{3\pi}{4} - \frac{1}{2}\cos^{-1}\left(\frac{1}{z}\right) \;/;\; (i\,z \in \mathbb{R} \land i\,z < 0)$$

01.13.16.0101.01

$$\cos^{-1}\left(\sqrt{\frac{z-\sqrt{z^2-1}}{2z}}\right) = \frac{\pi}{4} - \frac{1}{2}\cos^{-1}\left(\frac{1}{z}\right) \;/;\; (i\,z \in \mathbb{R} \land i\,z > 0)$$

01.13.16.0102.01

$$\cos^{-1}\left(\sqrt{\frac{z-\sqrt{z^2-1}}{2z}}\right) = \frac{\pi z}{4\sqrt{z^2}} + \frac{1}{2}\cos^{-1}\left(\frac{1}{z}\right) \;/;\; \mathrm{Re}(z) \neq 0$$

01.13.16.0103.01

$$\cos^{-1}\left(\sqrt{\frac{z-\sqrt{z^2-1}}{2z}}\right) = \frac{\pi}{4}\left(1 - \sqrt{\frac{1}{z^2}}\sqrt{z^2} + \frac{\sqrt{z^2}}{z}\right) + \frac{1}{2}\sqrt{z^2}\sqrt{\frac{1}{z^2}}\cos^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(c\,z^{-r}\sqrt{-1+\frac{z^{2r}}{c^2}}\right)$

Involving $\cos^{-1}\left(c\,z^{-r}\sqrt{-1+\frac{z^{2r}}{c^2}}\right)$ and $\cos^{-1}\left(\frac{c}{z^r}\right)$

01.13.16.0104.01

$$\cos^{-1}\left(c\,z^{-r}\sqrt{\frac{z^{2r}}{c^2}-1}\right) = \frac{\pi}{2} - \sqrt{c^2 z^{-2r}}\sqrt{\frac{z^{2r}}{c^2}}\left(\frac{\pi\sqrt{c^2 z^{-2r}}\,z^r}{2c} + \cos^{-1}\left(\frac{c}{z^r}\right) - \frac{\pi}{2}\right)$$

Products, sums, and powers of the direct function

Sums of the direct function

01.13.16.0010.01

$$\cos^{-1}(x)+\cos^{-1}(y)=\pi\,(1-\operatorname{sgn}(x+y))+\cos^{-1}\left(x\,y-\sqrt{1-x^2}\;\sqrt{1-y^2}\right)\operatorname{sgn}(x+y)\;/;\;-1<x<1\bigvee -1<y<1\bigvee x\,y>0$$

01.13.16.0105.01

$$\cos^{-1}(x)+\cos^{-1}(y)=\pi-\operatorname{sgn}(x+y)\cos^{-1}\left(\sqrt{1-x^2}\;\sqrt{1-y^2}-x\,y\right)\;/;\;x>-1\bigwedge y>-1\bigvee x<1\bigwedge y<1$$

01.13.16.0011.01

$$\cos^{-1}(x)+\cos^{-1}(y)=\pi\,(\operatorname{sgn}(x+y)+1)-\operatorname{sgn}(x+y)\cos^{-1}\left(x\,y-\sqrt{1-x^2}\;\sqrt{1-y^2}\right)\;/;\;x<-1\bigwedge y>1\bigvee x>1\bigwedge y<-1$$

01.13.16.0106.01

$$\cos^{-1}(x)+\cos^{-1}(y)=\pi-\sin^{-1}\left(y\sqrt{1-x^2}+x\sqrt{1-y^2}\right)\;/;\;x\,y\leq 0\bigvee x^2+y^2\leq 1$$

01.13.16.0107.01

$$\cos^{-1}(x)+\cos^{-1}(y)=(1-\operatorname{sgn}(x))\,\pi+\sin^{-1}\left(y\sqrt{1-x^2}+x\sqrt{1-y^2}\right)\;/;\;x\,y>0\bigwedge x^2+y^2>1$$

01.13.16.0108.01

$$\cos^{-1}(x)+\cos^{-1}(y)=\pi-\tan^{-1}\left(\frac{y\sqrt{1-x^2}+x\sqrt{1-y^2}}{\sqrt{1-y^2}\;\sqrt{1-x^2}-x\,y}\right)\;/;\;x\,y<0\bigvee x^2+y^2\leq 1$$

01.13.16.0109.01

$$\cos^{-1}(x)+\cos^{-1}(y)=-\tan^{-1}\left(\frac{y\sqrt{1-x^2}+x\sqrt{1-y^2}}{\sqrt{1-y^2}\;\sqrt{1-x^2}-x\,y}\right)+\pi\,(1-\operatorname{sgn}(x))\;/;\;x\,y>0\bigwedge x^2+y^2>1$$

01.13.16.0110.01

$$\cos^{-1}(x)+\cos^{-1}(y)=$$

$$-\frac{\sqrt{1-x^2}\;\sqrt{1-y^2}-x\,y}{\sqrt{\left(\sqrt{1-x^2}\;\sqrt{1-y^2}-x\,y\right)^2}}\sin^{-1}\left(\sqrt{1-y^2}\;x+\sqrt{1-x^2}\;y\right)+\frac{1}{2}\,\pi\left(\frac{\sqrt{1-x^2}\;\sqrt{1-y^2}-x\,y}{\sqrt{\left(\sqrt{1-x^2}\;\sqrt{1-y^2}-x\,y\right)^2}}+1\right)+$$

$$\pi\left\lfloor\frac{-\arg\left(i\,x+\sqrt{1-x^2}\right)-\arg\left(i\,y+\sqrt{1-y^2}\right)+\pi}{2\,\pi}\right\rfloor\left(\frac{\sqrt{1-x^2}\;\sqrt{1-y^2}-x\,y}{\sqrt{\left(\sqrt{1-x^2}\;\sqrt{1-y^2}-x\,y\right)^2}}+1\right)-$$

$$\pi\left(1-\frac{\sqrt{1-x^2}\;\sqrt{1-y^2}-x\,y}{\sqrt{\left(\sqrt{1-x^2}\;\sqrt{1-y^2}-x\,y\right)^2}}\right)\left\lfloor\frac{\arg\left(i\,x+\sqrt{1-x^2}\right)+\arg\left(i\,y+\sqrt{1-y^2}\right)}{2\,\pi}\right\rfloor$$

01.13.16.0111.01

$$\cos^{-1}(x)+\cos^{-1}(y)=\cos^{-1}\left((-1)^{\left\lfloor \frac{1}{2}-\frac{\arg\left(\sqrt{1-x^2}\,\sqrt{1-y^2}-x\,y\right)}{\pi}\right\rfloor}\left(\sqrt{1-y^2}\,x+\sqrt{1-x^2}\,y\right)\right)+$$
$$\frac{1}{2}\pi\left(2\left(-1+(-1)^{\left\lfloor \frac{1}{2}-\frac{\arg\left(\sqrt{1-x^2}\,\sqrt{1-y^2}-x\,y\right)}{\pi}\right\rfloor}\right)\left\lfloor\frac{\arg\left(i\,x+\sqrt{1-x^2}\right)+\arg\left(i\,y+\sqrt{1-y^2}\right)}{2\,\pi}\right\rfloor+(-1)^{\left\lfloor \frac{1}{2}-\frac{\arg\left(\sqrt{1-x^2}\,\sqrt{1-y^2}-x\,y\right)}{\pi}\right\rfloor}+\right.$$
$$\left.2\left(1+(-1)^{\left\lfloor \frac{1}{2}-\frac{\arg\left(\sqrt{1-x^2}\,\sqrt{1-y^2}-x\,y\right)}{\pi}\right\rfloor}\right)\left\lfloor\frac{1}{2}-\frac{\arg\left(i\,x+\sqrt{1-x^2}\right)+\arg\left(i\,y+\sqrt{1-y^2}\right)}{2\,\pi}\right\rfloor\right)$$

Differences of the direct function

01.13.16.0012.01

$$\cos^{-1}(x)-\cos^{-1}(y)=-\operatorname{sgn}(x-y)\cos^{-1}\left(x\,y+\sqrt{1-x^2}\,\sqrt{1-y^2}\right)\text{ /; }-1<x<1\bigvee -1<y<1\bigvee x\,y<0$$

01.13.16.0112.01

$$\cos^{-1}(x)-\cos^{-1}(y)=-\operatorname{sgn}(x-y)\cos^{-1}\left(\sqrt{1-x^2}\,\sqrt{1-y^2}+x\,y\right)\text{ /; }x>-1\bigwedge y<1\bigvee x<1\bigwedge y>-1$$

01.13.16.0013.01

$$\cos^{-1}(x)-\cos^{-1}(y)=\operatorname{sgn}(x-y)\cos^{-1}\left(x\,y+\sqrt{1-x^2}\,\sqrt{1-y^2}\right)\text{ /; }x>1\bigwedge y>1\bigvee x<-1\bigwedge y<-1$$

01.13.16.0113.01

$$\cos^{-1}(x)-\cos^{-1}(y)=-\sin^{-1}\left(x\sqrt{1-y^2}-y\sqrt{1-x^2}\right)\text{ /; }x\,y\geq 0\bigvee x^2+y^2\leq 1$$

01.13.16.0114.01

$$\cos^{-1}(x)-\cos^{-1}(y)=-\operatorname{sgn}(x)\,\pi+\sin^{-1}\left(x\sqrt{1-y^2}-y\sqrt{1-x^2}\right)\text{ /; }x\,y<0\bigwedge x^2+y^2>1$$

01.13.16.0115.01

$$\cos^{-1}(x)-\cos^{-1}(y)=-\tan^{-1}\left(\frac{x\sqrt{1-y^2}-y\sqrt{1-x^2}}{\sqrt{1-y^2}\,\sqrt{1-x^2}+x\,y}\right)\text{ /; }x\,y>0\bigvee x^2+y^2\leq 1$$

01.13.16.0116.01

$$\cos^{-1}(x)-\cos^{-1}(y)=-\tan^{-1}\left(\frac{x\sqrt{1-y^2}-y\sqrt{1-x^2}}{\sqrt{1-y^2}\,\sqrt{1-x^2}+x\,y}\right)-\pi\operatorname{sgn}(x)\text{ /; }x\,y<0\bigwedge x^2+y^2>1$$

01.13.16.0117.01

$$\cos^{-1}(x) - \cos^{-1}(y) = -\frac{\sqrt{1-x^2}\,\sqrt{1-y^2} + x\,y}{\sqrt{\left(\sqrt{1-x^2}\,\sqrt{1-y^2} + x\,y\right)^2}} \sin^{-1}\!\left(x\sqrt{1-y^2} - y\sqrt{1-x^2}\right) - \frac{\pi}{2}\left(1 - \frac{\sqrt{1-x^2}\,\sqrt{1-y^2} + x\,y}{\sqrt{\left(\sqrt{1-x^2}\,\sqrt{1-y^2} + x\,y\right)^2}}\right) - \pi\left(1 - \frac{\sqrt{1-x^2}\,\sqrt{1-y^2} + x\,y}{\sqrt{\left(\sqrt{1-x^2}\,\sqrt{1-y^2} + x\,y\right)^2}}\right)\left\lfloor \frac{\arg\left(i\,x + \sqrt{1-x^2}\right) + \arg\left(-i\,y + \sqrt{1-y^2}\right)}{2\,\pi} \right\rfloor + \pi\left(1 + \frac{\sqrt{1-x^2}\,\sqrt{1-y^2} + x\,y}{\sqrt{\left(\sqrt{1-x^2}\,\sqrt{1-y^2} + x\,y\right)^2}}\right)\left\lfloor \frac{\pi - \arg\left(i\,x + \sqrt{1-x^2}\right) - \arg\left(-i\,y + \sqrt{1-y^2}\right)}{2\,\pi} \right\rfloor$$

01.13.16.0118.01

$$\cos^{-1}(x) - \cos^{-1}(y) = \cos^{-1}\!\left((-1)^{\left\lfloor \frac{1}{2} - \frac{\arg\left(x\,y + \sqrt{1-x^2}\,\sqrt{1-y^2}\right)}{\pi} \right\rfloor}\left(x\sqrt{1-y^2} - \sqrt{1-x^2}\,y\right)\right) + \frac{1}{2}\pi\left(2\left(-1 + (-1)^{\left\lfloor \frac{1}{2} - \frac{\arg\left(x\,y + \sqrt{1-x^2}\,\sqrt{1-y^2}\right)}{\pi} \right\rfloor}\right)\left\lfloor \frac{\arg\left(i\,x + \sqrt{1-x^2}\right) + \arg\left(\sqrt{1-y^2} - i\,y\right)}{2\,\pi} \right\rfloor + (-1)^{\left\lfloor \frac{1}{2} - \frac{\arg\left(x\,y + \sqrt{1-x^2}\,\sqrt{1-y^2}\right)}{\pi} \right\rfloor} + 2\left(1 + (-1)^{\left\lfloor \frac{1}{2} - \frac{\arg\left(x\,y + \sqrt{1-x^2}\,\sqrt{1-y^2}\right)}{\pi} \right\rfloor}\right)\left\lfloor \frac{1}{2} - \frac{\arg\left(i\,x + \sqrt{1-x^2}\right) + \arg\left(\sqrt{1-y^2} - i\,y\right)}{2\,\pi} \right\rfloor - 2\right)$$

Linear combinations of the direct function

01.13.16.0119.01

$$a\cos^{-1}(x)+b\cos^{-1}(y)=$$

$$\frac{1}{2}\pi(a+b)-2\,i\,\pi\left(\left\lfloor\frac{-\arg\left(\left(i\,x+\sqrt{1-x^2}\right)^{i\,a}\right)-\arg\left(\left(i\,y+\sqrt{1-y^2}\right)^{i\,b}\right)+\pi}{2\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Re}\left(a\log\left(i\,x+\sqrt{1-x^2}\right)\right)}{2\pi}\right\rfloor+\right.$$

$$\left.\left\lfloor\frac{\pi-\operatorname{Re}\left(b\log\left(i\,y+\sqrt{1-y^2}\right)\right)}{2\pi}\right\rfloor\right)+\log\left(\left(i\,x+\sqrt{1-x^2}\right)^{i\,a}\left(i\,y+\sqrt{1-y^2}\right)^{i\,b}\right)$$

01.13.16.0120.01

$$a\cos^{-1}(x)+b\cos^{-1}(y)=$$

$$\frac{1}{2}\pi(a+b)-i\,(-1)^{\left\lfloor\frac{\arg\left(\left(i\,x+\sqrt{1-x^2}\right)^{i\,a}\left(i\,y+\sqrt{1-y^2}\right)^{i\,b}-1\right)}{2\pi}+\frac{1}{2}-\frac{\arg\left(\left(i\,y+\sqrt{1-y^2}\right)^{i\,b}\left(i\,x+\sqrt{1-x^2}\right)^{i\,a}+1\right)}{2\pi}\right\rfloor+\left\lfloor-\frac{\arg\left(\left(i\,x+\sqrt{1-x^2}\right)^{i\,a}\left(i\,y+\sqrt{1-y^2}\right)^{i\,b}\right)}{\pi}\right\rfloor}$$

$$\cos^{-1}\left(\frac{1}{2}\left(i\,x+\sqrt{1-x^2}\right)^{-i\,a}\left(i\,y+\sqrt{1-y^2}\right)^{-i\,b}\left(\left(i\,y+\sqrt{1-y^2}\right)^{2\,i\,b}\left(i\,x+\sqrt{1-x^2}\right)^{2\,i\,a}+1\right)\right)-$$

$$2\,i\,\pi\left(\left\lfloor\frac{-\arg\left(\left(i\,x+\sqrt{1-x^2}\right)^{i\,a}\right)-\arg\left(\left(i\,y+\sqrt{1-y^2}\right)^{i\,b}\right)+\pi}{2\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Re}\left(a\log\left(i\,x+\sqrt{1-x^2}\right)\right)}{2\pi}\right\rfloor+\right.$$

$$\left.\left\lfloor\frac{\pi-\operatorname{Re}\left(b\log\left(i\,y+\sqrt{1-y^2}\right)\right)}{2\pi}\right\rfloor\right)+\left(1-(-1)^{\left\lfloor-\frac{\arg\left(\left(i\,y+\sqrt{1-y^2}\right)^{i\,b}\left(i\,x+\sqrt{1-x^2}\right)^{i\,a}+1\right)}{2\pi}\right\rfloor-\left\lfloor-\frac{\arg\left(\left(i\,x+\sqrt{1-x^2}\right)^{i\,a}\left(i\,y+\sqrt{1-y^2}\right)^{i\,b}\right)}{2\pi}\right\rfloor}\right)i\,\pi$$

Related transformations

Sums involving the direct function

Involving log(*z*)

01.13.16.0121.01

$$\cos^{-1}(x)+\log(y)=-2\,i\,\pi\left(\left\lfloor\frac{-\arg\left(\left(i\,x+\sqrt{1-x^2}\right)^{i}\right)-\arg(y)+\pi}{2\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}(\log(y))}{2\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Re}\left(\log\left(i\,x+\sqrt{1-x^2}\right)\right)}{2\pi}\right\rfloor\right)+$$

$$\log\left(\left(i\,x+\sqrt{1-x^2}\right)^{i}y\right)+\frac{\pi}{2}$$

01.13.16.0122.01

$$\cos^{-1}(x) + \log(y) = -2\, i\, \pi \left(\left\lfloor \frac{-\arg\left(\left(i\, x + \sqrt{1 - x^2}\right)^{i}\right) - \arg(y) + \pi}{2\pi} \right\rfloor + \left\lfloor \frac{\pi - \operatorname{Im}(\log(y))}{2\pi} \right\rfloor + \left\lfloor \frac{\pi - \operatorname{Re}\left(\log\left(i\, x + \sqrt{1 - x^2}\right)\right)}{2\pi} \right\rfloor \right) +$$
$$i \left(1 - (-1)^{\left\lfloor -\frac{\arg\left(y \left(i\, x + \sqrt{1 - x^2}\right)^{i} + 1\right)}{2\pi} \right\rfloor - \left\lfloor -\frac{\arg\left(\left(i\, x + \sqrt{1 - x^2}\right)^{i} y\right)}{2\pi} \right\rfloor} \right) \pi -$$
$$i\, (-1)^{\left\lfloor \frac{\arg\left(\left(i\, x + \sqrt{1 - x^2}\right)^{i} y - 1\right)}{2\pi} + \frac{1}{2} - \frac{\arg\left(y \left(i\, x + \sqrt{1 - x^2}\right)^{i} + 1\right)}{2\pi} \right\rfloor + \left\lfloor -\frac{\arg\left(\left(i\, x + \sqrt{1 - x^2}\right)^{i} y\right)}{\pi} \right\rfloor} \cos^{-1}\left(\frac{\left(i\, x + \sqrt{1 - x^2}\right)^{-i} \left(y^2 \left(i\, x + \sqrt{1 - x^2}\right)^{2 i} + 1\right)}{2\, y} \right) + \frac{\pi}{2}$$

Involving $\sin^{-1}(z)$

01.13.16.0014.01

$$\cos^{-1}(x) + \sin^{-1}(y) = \frac{\pi}{2} - \operatorname{sgn}(x - y) \cos^{-1}\left(x\, y + \sqrt{1 - x^2}\, \sqrt{1 - y^2}\right) /;\, -1 < x < 1 \lor -1 < y < 1 \lor x\, y < 0$$

01.13.16.0123.01

$$\cos^{-1}(x) + \sin^{-1}(y) = \frac{\pi}{2} - \operatorname{sgn}(x - y) \cos^{-1}\left(x\, y + \sqrt{1 - x^2}\, \sqrt{1 - y^2}\right) /;\, x < 1 \land y > -1 \lor x > -1 \land y < 1$$

01.13.16.0124.01

$$\cos^{-1}(x) + \sin^{-1}(y) = \operatorname{sgn}(x - y) \cos^{-1}\left(\sqrt{1 - x^2}\, \sqrt{1 - y^2} + x\, y\right) + \frac{\pi}{2} /;\, x > 1 \land y > 1 \lor x < -1 \land y < -1$$

01.13.16.0125.01

$$\cos^{-1}(x) + \sin^{-1}(y) = \sin^{-1}\left(-x \sqrt{1 - y^2} + y \sqrt{1 - x^2}\right) + \frac{\pi}{2} /;\, x\, y \geq 0 \bigvee x^2 + y^2 \leq 1$$

01.13.16.0126.01

$$\cos^{-1}(x) + \sin^{-1}(y) = \frac{\pi}{2} + \operatorname{sgn}(y)\, \pi + \sin^{-1}\left(x \sqrt{1 - y^2} - y \sqrt{1 - x^2}\right) /;\, x\, y < 0 \bigwedge x^2 + y^2 > 1$$

01.13.16.0127.01

$$\cos^{-1}(x) + \sin^{-1}(y) = -\tan^{-1}\left(\frac{x \sqrt{1 - y^2} - y \sqrt{1 - x^2}}{\sqrt{1 - y^2}\, \sqrt{1 - x^2} + x\, y} \right) + \frac{\pi}{2} /;\, x\, y > 0 \bigvee x^2 + y^2 \leq 1$$

01.13.16.0128.01

$$\cos^{-1}(x) + \sin^{-1}(y) = -\tan^{-1}\left(\frac{x \sqrt{1 - y^2} - y \sqrt{1 - x^2}}{\sqrt{1 - y^2}\, \sqrt{1 - x^2} + x\, y} \right) + \pi \operatorname{sgn}(y) + \frac{\pi}{2} /;\, x\, y < 0 \bigwedge x^2 + y^2 > 1$$

01.13.16.0129.01

$$\cos^{-1}(x)+\sin^{-1}(y) == \frac{x\,y+\sqrt{1-x^2}\,\sqrt{1-y^2}}{\sqrt{\left(x\,y+\sqrt{1-x^2}\,\sqrt{1-y^2}\right)^2}}\sin^{-1}\left(\sqrt{1-x^2}\,y-x\sqrt{1-y^2}\right)+\frac{\pi\left(x\,y+\sqrt{1-x^2}\,\sqrt{1-y^2}\right)}{2\sqrt{\left(x\,y+\sqrt{1-x^2}\,\sqrt{1-y^2}\right)^2}}+\pi\left(\frac{x\,y+\sqrt{1-x^2}\,\sqrt{1-y^2}}{\sqrt{\left(x\,y+\sqrt{1-x^2}\,\sqrt{1-y^2}\right)^2}}+1\right)\left\lfloor\frac{-\arg\left(i\,x+\sqrt{1-x^2}\right)-\arg\left(\sqrt{1-y^2}-i\,y\right)+\pi}{2\pi}\right\rfloor-\pi\left(1-\frac{x\,y+\sqrt{1-x^2}\,\sqrt{1-y^2}}{\sqrt{\left(x\,y+\sqrt{1-x^2}\,\sqrt{1-y^2}\right)^2}}\right)\left\lfloor\frac{\arg\left(i\,x+\sqrt{1-x^2}\right)+\arg\left(\sqrt{1-y^2}-i\,y\right)}{2\pi}\right\rfloor$$

01.13.16.0130.01

$$\cos^{-1}(x)+\sin^{-1}(y) = -\cos^{-1}\left((-1)^{\left\lfloor\frac{1}{2}-\frac{\arg\left(x\,y+\sqrt{1-x^2}\,\sqrt{1-y^2}\right)}{\pi}\right\rfloor}\left(\sqrt{1-x^2}\,y-x\sqrt{1-y^2}\right)\right)-\frac{1}{2}\pi\left(2\left(-1+(-1)^{\left\lfloor\frac{1}{2}-\frac{\arg\left(x\,y+\sqrt{1-x^2}\,\sqrt{1-y^2}\right)}{\pi}\right\rfloor}\right)\left\lfloor\frac{\arg\left(\sqrt{1-x^2}-i\,x\right)+\arg\left(i\,y+\sqrt{1-y^2}\right)}{2\pi}\right\rfloor+(-1)^{\left\lfloor\frac{1}{2}-\frac{\arg\left(x\,y+\sqrt{1-x^2}\,\sqrt{1-y^2}\right)}{\pi}\right\rfloor}+2\left(1+(-1)^{\left\lfloor\frac{1}{2}-\frac{\arg\left(x\,y+\sqrt{1-x^2}\,\sqrt{1-y^2}\right)}{\pi}\right\rfloor}\right)\left\lfloor\frac{1}{2}-\frac{\arg\left(\sqrt{1-x^2}-i\,x\right)+\arg\left(i\,y+\sqrt{1-y^2}\right)}{2\pi}\right\rfloor-3\right)$$

Involving $\tan^{-1}(z)$

01.13.16.0131.01

$$\cos^{-1}(x)+\tan^{-1}(y)=\frac{\pi}{2}-\frac{1}{2\sqrt{\frac{\left(x-\sqrt{1-x^2}\,y\right)^2}{y^2+1}}\,\sqrt{y^2+1}}\left(\left(x-\sqrt{1-x^2}\,y\right)\left(\pi-2\sin^{-1}\left(\frac{x\,y+\sqrt{1-x^2}}{\sqrt{y^2+1}}\right)\right)+\right.$$
$$2\pi\left(x-\sqrt{1-x^2}\,y+\sqrt{\frac{-y^2x^2+x^2-2\sqrt{1-x^2}\,y\,x+y^2}{y^2+1}}\,\sqrt{y^2+1}\right)\left\lfloor\frac{\arg\left(i\,x+\sqrt{1-x^2}\right)+\arg\left(\frac{i+y}{\sqrt{y^2+1}}\right)}{2\pi}\right\rfloor-$$
$$\left.2\pi\left(-x+\sqrt{1-x^2}\,y+\sqrt{\frac{-y^2x^2+x^2-2\sqrt{1-x^2}\,y\,x+y^2}{y^2+1}}\,\sqrt{y^2+1}\right)\left\lfloor-\frac{\arg\left(i\,x+\sqrt{1-x^2}\right)+\arg\left(\frac{i+y}{\sqrt{y^2+1}}\right)-\pi}{2\pi}\right\rfloor\right)$$

01.13.16.0132.01

$$\cos^{-1}(x)+\tan^{-1}(y)=$$
$$\cos^{-1}\left(\frac{(-1)^{\left\lfloor\frac{1}{2}-\frac{\arg\left(\frac{x-\sqrt{1-x^2}\,y}{\sqrt{y^2+1}}\right)}{\pi}\right\rfloor}\left(-x\,y-\sqrt{1-x^2}\right)}{\sqrt{y^2+1}}\right)-\frac{1}{2}\pi\left(2\left(1+(-1)^{\left\lfloor\frac{1}{2}-\frac{\arg\left(\frac{x-\sqrt{1-x^2}\,y}{\sqrt{y^2+1}}\right)}{\pi}\right\rfloor}\right)\left\lfloor\frac{\arg\left(i\,x+\sqrt{1-x^2}\right)+\arg\left(\frac{i+y}{\sqrt{y^2+1}}\right)}{2\pi}\right\rfloor+\right.$$
$$\left.(-1)^{\left\lfloor\frac{1}{2}-\frac{\arg\left(\frac{x-\sqrt{1-x^2}\,y}{\sqrt{y^2+1}}\right)}{\pi}\right\rfloor}-2\left(-1+(-1)^{\left\lfloor\frac{1}{2}-\frac{\arg\left(\frac{x-\sqrt{1-x^2}\,y}{\sqrt{y^2+1}}\right)}{\pi}\right\rfloor}\right)\left\lfloor\frac{1}{2}-\frac{\arg\left(i\,x+\sqrt{1-x^2}\right)+\arg\left(\frac{i+y}{\sqrt{y^2+1}}\right)}{2\pi}\right\rfloor\right)$$

Involving $\cot^{-1}(z)$

01.13.16.0133.01

$$\cos^{-1}(x)+\cot^{-1}(y)=\frac{\pi}{2}-\left(-\left(\sqrt{1-x^2}-x\,y\right)\left(\pi-2\sin^{-1}\left(\frac{\frac{x}{y}+\sqrt{1-x^2}}{\sqrt{1+\frac{1}{y^2}}}\right)\right)+2\pi\left(x\,y+\sqrt{\frac{(y^2-1)\,x^2-2\sqrt{1-x^2}\,y\,x+1}{y^2+1}}\sqrt{1+\frac{1}{y^2}}\,y-\sqrt{1-x^2}\right)\left\lfloor\frac{\arg\left(i\,x+\sqrt{1-x^2}\right)+\arg\left(\frac{i+\frac{1}{y}}{\sqrt{1+\frac{1}{y^2}}}\right)}{2\pi}\right\rfloor-2\pi\left(-x\,y+\sqrt{\frac{(y^2-1)\,x^2-2\sqrt{1-x^2}\,y\,x+1}{y^2+1}}\sqrt{1+\frac{1}{y^2}}\,y+\sqrt{1-x^2}\right)\left\lfloor-\frac{\arg\left(i\,x+\sqrt{1-x^2}\right)+\arg\left(\frac{i+\frac{1}{y}}{\sqrt{1+\frac{1}{y^2}}}\right)-\pi}{2\pi}\right\rfloor\right)\Bigg/\left(2\sqrt{1+\frac{1}{y^2}}\,y\sqrt{\frac{(y^2-1)\,x^2-2\sqrt{1-x^2}\,y\,x+1}{y^2+1}}\right)$$

01.13.16.0134.01

$$\cos^{-1}(x)+\cot^{-1}(y)=$$

$$\cos^{-1}\left(\frac{(-1)^{\left\lfloor \frac{1}{2}-\frac{\arg\left(\frac{x-\frac{\sqrt{1-x^2}}{y}}{\sqrt{1+\frac{1}{y^2}}}\right)}{\pi}\right\rfloor}\left(-\frac{x}{y}-\sqrt{1-x^2}\right)}{\sqrt{1+\frac{1}{y^2}}}\right)-\frac{1}{2}\pi\left(2\left(1+(-1)^{\left\lfloor \frac{1}{2}-\frac{\arg\left(\frac{x-\frac{\sqrt{1-x^2}}{y}}{\sqrt{1+\frac{1}{y^2}}}\right)}{\pi}\right\rfloor}\right)\left\lfloor\frac{\arg\left(i\,x+\sqrt{1-x^2}\right)+\arg\left(\frac{i+\frac{1}{y}}{\sqrt{1+\frac{1}{y^2}}}\right)}{2\pi}\right\rfloor+\right.$$

$$\left.(-1)^{\left\lfloor \frac{1}{2}-\frac{\arg\left(\frac{x-\frac{\sqrt{1-x^2}}{y}}{\sqrt{1+\frac{1}{y^2}}}\right)}{\pi}\right\rfloor}-2\left(-1+(-1)^{\left\lfloor \frac{1}{2}-\frac{\arg\left(\frac{x-\frac{\sqrt{1-x^2}}{y}}{\sqrt{1+\frac{1}{y^2}}}\right)}{\pi}\right\rfloor}\right)\left\lfloor\frac{1}{2}-\frac{\arg\left(i\,x+\sqrt{1-x^2}\right)+\arg\left(\frac{i+\frac{1}{y}}{\sqrt{1+\frac{1}{y^2}}}\right)}{2\pi}\right\rfloor\right)$$

Involving $\csc^{-1}(z)$

01.13.16.0135.01

$$\cos^{-1}(x)+\csc^{-1}(y)=-\frac{\frac{x}{y}+\sqrt{1-x^2}\,\sqrt{1-\frac{1}{y^2}}}{\sqrt{\left(\frac{x}{y}+\sqrt{1-x^2}\,\sqrt{1-\frac{1}{y^2}}\right)^2}}\sin^{-1}\left(x\sqrt{1-\frac{1}{y^2}}-\frac{\sqrt{1-x^2}}{y}\right)-$$

$$\pi\left\lfloor\frac{\arg\left(i\,x+\sqrt{1-x^2}\right)+\arg\left(\sqrt{1-\frac{1}{y^2}}-\frac{i}{y}\right)}{2\,\pi}\right\rfloor\left(1-\frac{x+\sqrt{1-x^2}\,y\sqrt{1-\frac{1}{y^2}}}{\sqrt{\left(\frac{x}{y}+\sqrt{1-x^2}\,\sqrt{1-\frac{1}{y^2}}\right)^2}\,y}\right)+$$

$$\frac{\pi}{2}\,\frac{x+\sqrt{1-x^2}\,y\sqrt{1-\frac{1}{y^2}}}{\sqrt{\left(\frac{x}{y}+\sqrt{1-x^2}\,\sqrt{1-\frac{1}{y^2}}\right)^2}\,y}+$$

$$\pi\left(\frac{\frac{x}{y}+\sqrt{1-x^2}\,\sqrt{1-\frac{1}{y^2}}}{\sqrt{\left(\frac{x}{y}+\sqrt{1-x^2}\,\sqrt{1-\frac{1}{y^2}}\right)^2}}+1\right)\left\lfloor-\frac{\arg\left(i\,x+\sqrt{1-x^2}\right)+\arg\left(\sqrt{1-\frac{1}{y^2}}-\frac{i}{y}\right)-\pi}{2\,\pi}\right\rfloor$$

01.13.16.0136.01

$$\cos^{-1}(x)+\csc^{-1}(y)=-\cos^{-1}\left((-1)^{\left\lfloor \frac{1}{2}-\frac{\arg\left(\frac{x}{y}+\sqrt{1-x^2}\sqrt{1-\frac{1}{y^2}}\right)}{\pi}\right\rfloor}\left(\frac{\sqrt{1-x^2}}{y}-x\sqrt{1-\frac{1}{y^2}}\right)\right)-$$
$$\frac{1}{2}\pi\left(2\left(-1+(-1)^{\left\lfloor \frac{1}{2}-\frac{\arg\left(\frac{x}{y}+\sqrt{1-x^2}\sqrt{1-\frac{1}{y^2}}\right)}{\pi}\right\rfloor}\right)\left\lfloor\frac{\arg\left(\sqrt{1-x^2}-i\,x\right)+\arg\left(\sqrt{1-\frac{1}{y^2}}+\frac{i}{y}\right)}{2\pi}\right\rfloor+(-1)^{\left\lfloor \frac{1}{2}-\frac{\arg\left(\frac{x}{y}+\sqrt{1-x^2}\sqrt{1-\frac{1}{y^2}}\right)}{\pi}\right\rfloor}+\right.$$
$$\left.2\left(1+(-1)^{\left\lfloor \frac{1}{2}-\frac{\arg\left(\frac{x}{y}+\sqrt{1-x^2}\sqrt{1-\frac{1}{y^2}}\right)}{\pi}\right\rfloor}\right)\left\lfloor\frac{1}{2}-\frac{\arg\left(\sqrt{1-x^2}-i\,x\right)+\arg\left(\sqrt{1-\frac{1}{y^2}}+\frac{i}{y}\right)}{2\pi}\right\rfloor-3\right)$$

Involving $\sec^{-1}(z)$

01.13.16.0137.01

$$\cos^{-1}(x)+\sec^{-1}(y)=-\frac{\sqrt{1-x^2}\,\sqrt{1-\frac{1}{y^2}}-\frac{x}{y}}{\sqrt{\left(\sqrt{1-x^2}\,\sqrt{1-\frac{1}{y^2}}-\frac{x}{y}\right)^2}}\sin^{-1}\left(\sqrt{1-\frac{1}{y^2}}\,x+\frac{\sqrt{1-x^2}}{y}\right)-$$

$$\pi\left\lfloor\frac{\arg\left(i\,x+\sqrt{1-x^2}\right)+\arg\left(\sqrt{1-\frac{1}{y^2}}+\frac{i}{y}\right)}{2\pi}\right\rfloor\left(\frac{x-\sqrt{1-x^2}\,\sqrt{1-\frac{1}{y^2}}\,y}{\sqrt{\left(\sqrt{1-x^2}\,\sqrt{1-\frac{1}{y^2}}-\frac{x}{y}\right)^2}\,y}+1\right)+$$

$$\frac{\pi}{2}\left(1-\frac{x-\sqrt{1-x^2}\,\sqrt{1-\frac{1}{y^2}}\,y}{\sqrt{\left(\sqrt{1-x^2}\,\sqrt{1-\frac{1}{y^2}}-\frac{x}{y}\right)^2}\,y}\right)+$$

$$\pi\left(\frac{\sqrt{1-x^2}\,\sqrt{1-\frac{1}{y^2}}-\frac{x}{y}}{\sqrt{\left(\sqrt{1-x^2}\,\sqrt{1-\frac{1}{y^2}}-\frac{x}{y}\right)^2}}+1\right)\left\lfloor-\frac{\arg\left(i\,x+\sqrt{1-x^2}\right)+\arg\left(\sqrt{1-\frac{1}{y^2}}+\frac{i}{y}\right)-\pi}{2\pi}\right\rfloor$$

01.13.16.0138.01

$$\cos^{-1}(x)+\sec^{-1}(y)=\cos^{-1}\left((-1)^{\left\lfloor \frac{1}{2}-\frac{\arg\left(\sqrt{1-x^2}\sqrt{1-\frac{1}{y^2}}-\frac{x}{y}\right)}{\pi}\right\rfloor}\left(\sqrt{1-\frac{1}{y^2}}\,x+\frac{\sqrt{1-x^2}}{y}\right)\right)+$$
$$\frac{1}{2}\pi\left(2\left(-1+(-1)^{\left\lfloor \frac{1}{2}-\frac{\arg\left(\sqrt{1-x^2}\sqrt{1-\frac{1}{y^2}}-\frac{x}{y}\right)}{\pi}\right\rfloor}\right)\left\lfloor\frac{\arg\left(i\,x+\sqrt{1-x^2}\right)+\arg\left(\sqrt{1-\frac{1}{y^2}}+\frac{i}{y}\right)}{2\,\pi}\right\rfloor+(-1)^{\left\lfloor \frac{1}{2}-\frac{\arg\left(\sqrt{1-x^2}\sqrt{1-\frac{1}{y^2}}-\frac{x}{y}\right)}{\pi}\right\rfloor}+\right.$$
$$\left.2\left(1+(-1)^{\left\lfloor \frac{1}{2}-\frac{\arg\left(\sqrt{1-x^2}\sqrt{1-\frac{1}{y^2}}-\frac{x}{y}\right)}{\pi}\right\rfloor}\right)\left\lfloor\frac{1}{2}-\frac{\arg\left(i\,x+\sqrt{1-x^2}\right)+\arg\left(\sqrt{1-\frac{1}{y^2}}+\frac{i}{y}\right)}{2\,\pi}\right\rfloor\right)$$

Involving $\sinh^{-1}(z)$

01.13.16.0139.01

$$\cos^{-1}(x)+\sinh^{-1}(y)=-2\,i\,\pi$$
$$\left(\left\lfloor\frac{-\arg\left(y+\sqrt{y^2+1}\right)-\arg\left(\left(i\,x+\sqrt{1-x^2}\right)^{i}\right)+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}\left(\log\left(y+\sqrt{y^2+1}\right)\right)}{2\,\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Re}\left(\log\left(i\,x+\sqrt{1-x^2}\right)\right)}{2\,\pi}\right\rfloor\right)+$$
$$\log\left(\left(i\,x+\sqrt{1-x^2}\right)^{i}\left(y+\sqrt{y^2+1}\right)\right)+\frac{\pi}{2}$$

01.13.16.0140.01

$$\cos^{-1}(x)+\sinh^{-1}(y)=-2\,i\,\pi\left(\left\lfloor\frac{-\arg\left(y+\sqrt{y^2+1}\right)-\arg\left(\left(i\,x+\sqrt{1-x^2}\right)^{i}\right)+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}\left(\log\left(y+\sqrt{y^2+1}\right)\right)}{2\,\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Re}\left(\log\left(i\,x+\sqrt{1-x^2}\right)\right)}{2\,\pi}\right\rfloor\right)+$$

$$i\left(1-(-1)^{\left\lfloor-\frac{\arg\left(\left(y+\sqrt{y^2+1}\right)\left(i\,x+\sqrt{1-x^2}\right)^{i}+1\right)}{2\,\pi}\right\rfloor-\left\lfloor-\frac{\arg\left(\left(i\,x+\sqrt{1-x^2}\right)^{i}\left(y+\sqrt{y^2+1}\right)\right)}{2\,\pi}\right\rfloor}\right)\pi-$$

$$i\,(-1)^{\left\lfloor\frac{\arg\left(\left(i\,x+\sqrt{1-x^2}\right)^{i}\left(y+\sqrt{y^2+1}\right)-1\right)}{2\,\pi}+\frac{1}{2}-\frac{\arg\left(\left(y+\sqrt{y^2+1}\right)\left(i\,x+\sqrt{1-x^2}\right)^{i}+1\right)}{2\,\pi}\right\rfloor+\left\lfloor-\frac{\arg\left(\left(i\,x+\sqrt{1-x^2}\right)^{i}\left(y+\sqrt{y^2+1}\right)\right)}{\pi}\right\rfloor}$$

$$\cos^{-1}\left(\frac{\left(i\,x+\sqrt{1-x^2}\right)^{-i}\left(\left(y+\sqrt{y^2+1}\right)^2\left(i\,x+\sqrt{1-x^2}\right)^{2\,i}+1\right)}{2\left(y+\sqrt{y^2+1}\right)}\right)+\frac{\pi}{2}$$

01.13.16.0141.01

$$\cos^{-1}(x)+i\sinh^{-1}(y)=\frac{i\left(i\,x\,y+\sqrt{1-x^2}\,\sqrt{y^2+1}\right)}{\sqrt{\left(i\,x\,y+\sqrt{1-x^2}\,\sqrt{y^2+1}\right)^2}}\sinh^{-1}\left(i\sqrt{y^2+1}\,x+\sqrt{1-x^2}\,y\right)-$$

$$\pi\left\lfloor\frac{\arg\left(i\,x+\sqrt{1-x^2}\right)+\arg\left(y+\sqrt{y^2+1}\right)}{2\,\pi}\right\rfloor\left(1-\frac{i\,x\,y+\sqrt{1-x^2}\,\sqrt{y^2+1}}{\sqrt{\left(i\,x\,y+\sqrt{1-x^2}\,\sqrt{y^2+1}\right)^2}}\right)+$$

$$\frac{1}{2}\pi\frac{i\,x\,y+\sqrt{1-x^2}\,\sqrt{y^2+1}}{\sqrt{\left(i\,x\,y+\sqrt{1-x^2}\,\sqrt{y^2+1}\right)^2}}+$$

$$\pi\left(\frac{i\,x\,y+\sqrt{1-x^2}\,\sqrt{y^2+1}}{\sqrt{\left(i\,x\,y+\sqrt{1-x^2}\,\sqrt{y^2+1}\right)^2}}+1\right)\left\lfloor\frac{-\arg\left(i\,x+\sqrt{1-x^2}\right)-\arg\left(y+\sqrt{y^2+1}\right)+\pi}{2\,\pi}\right\rfloor$$

01.13.16.0142.01

$$\cos^{-1}(x)+i\sinh^{-1}(y)=\cos^{-1}\left((-1)^{\left\lfloor\frac{1}{2}-\frac{\arg\left(i\,x\,y+\sqrt{1-x^2}\,\sqrt{y^2+1}\right)}{\pi}\right\rfloor}\left(x\sqrt{y^2+1}-i\sqrt{1-x^2}\,y\right)\right)+$$
$$\frac{1}{2}\pi\left(2\left(-1+(-1)^{\left\lfloor\frac{1}{2}-\frac{\arg\left(i\,x\,y+\sqrt{1-x^2}\,\sqrt{y^2+1}\right)}{\pi}\right\rfloor}\right)\left\lfloor\frac{\arg\left(i\,x+\sqrt{1-x^2}\right)+\arg\left(y+\sqrt{y^2+1}\right)}{2\pi}\right\rfloor+(-1)^{\left\lfloor\frac{1}{2}-\frac{\arg\left(i\,x\,y+\sqrt{1-x^2}\,\sqrt{y^2+1}\right)}{\pi}\right\rfloor}+\right.$$
$$\left.2\left(1+(-1)^{\left\lfloor\frac{1}{2}-\frac{\arg\left(i\,x\,y+\sqrt{1-x^2}\,\sqrt{y^2+1}\right)}{\pi}\right\rfloor}\right)\left\lfloor\frac{1}{2}-\frac{\arg\left(i\,x+\sqrt{1-x^2}\right)+\arg\left(y+\sqrt{y^2+1}\right)}{2\pi}\right\rfloor-1\right)$$

Involving $\cosh^{-1}(z)$

01.13.16.0143.01

$$\cos^{-1}(x)+\cosh^{-1}(y)=$$
$$-2\,i\,\pi\left(\left\lfloor\frac{-\arg\left(y+\sqrt{y-1}\,\sqrt{y+1}\right)-\arg\left(\left(i\,x+\sqrt{1-x^2}\right)^{i}\right)+\pi}{2\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}\left(\log\left(y+\sqrt{y-1}\,\sqrt{y+1}\right)\right)}{2\pi}\right\rfloor+\right.$$
$$\left.\left\lfloor\frac{\pi-\operatorname{Re}\left(\log\left(i\,x+\sqrt{1-x^2}\right)\right)}{2\pi}\right\rfloor\right)+\log\left(\left(i\,x+\sqrt{1-x^2}\right)^{i}\left(y+\sqrt{y-1}\,\sqrt{y+1}\right)\right)+\frac{\pi}{2}$$

01.13.16.0144.01

$$\cos^{-1}(x)+\cosh^{-1}(y)=$$

$$-2\,i\,\pi\left(\left\lfloor\frac{-\arg\left(y+\sqrt{y-1}\,\sqrt{y+1}\right)-\arg\left(\left(i\,x+\sqrt{1-x^2}\right)^{i}\right)+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}\left(\log\left(y+\sqrt{y-1}\,\sqrt{y+1}\right)\right)}{2\,\pi}\right\rfloor+\right.$$

$$\left.\left\lfloor\frac{\pi-\operatorname{Re}\left(\log\left(i\,x+\sqrt{1-x^2}\right)\right)}{2\,\pi}\right\rfloor\right)+i\left(1-(-1)^{\left\lfloor-\frac{\arg\left(\left(y+\sqrt{y-1}\,\sqrt{y+1}\right)\left(i\,x+\sqrt{1-x^2}\right)^{i}+1\right)}{2\,\pi}\right\rfloor-\left\lfloor-\frac{\arg\left(\left(i\,x+\sqrt{1-x^2}\right)^{i}\left(y+\sqrt{y-1}\,\sqrt{y+1}\right)\right)}{2\,\pi}\right\rfloor}\right)\pi-$$

$$i\,(-1)^{\left\lfloor\frac{\arg\left(\left(i\,x+\sqrt{1-x^2}\right)^{i}\left(y+\sqrt{y-1}\,\sqrt{y+1}\right)-1\right)}{2\,\pi}+\frac{1}{2}-\frac{\arg\left(\left(y+\sqrt{y-1}\,\sqrt{y+1}\right)\left(i\,x+\sqrt{1-x^2}\right)^{i}+1\right)}{2\,\pi}\right\rfloor+\left\lfloor-\frac{\arg\left(\left(i\,x+\sqrt{1-x^2}\right)^{i}\left(y+\sqrt{y-1}\,\sqrt{y+1}\right)\right)}{\pi}\right\rfloor}$$

$$\cos^{-1}\left(\frac{\left(i\,x+\sqrt{1-x^2}\right)^{-i}\left(\left(y+\sqrt{y-1}\,\sqrt{y+1}\right)^{2}\left(i\,x+\sqrt{1-x^2}\right)^{2\,i}+1\right)}{2\left(y+\sqrt{y-1}\,\sqrt{y+1}\right)}\right)+\frac{\pi}{2}$$

01.13.16.0145.01

$$
\begin{aligned}
&\cos^{-1}(x) + i\cosh^{-1}(y) = \\
&\frac{1}{2}\pi\left(-\frac{1}{2}\left(1+(-1)^{-\left\lfloor -\frac{\arg(1-y)}{2\pi}\right\rfloor}\right)\left(2\left(-1+(-1)^{\left\lfloor \frac{1}{2}-\frac{\arg\left(x\,y+\sqrt{1-x^2}\,\sqrt{1-y^2}\right)}{\pi}\right\rfloor}\right)\left\lfloor \frac{\arg\left(\sqrt{1-x^2}-i\,x\right)+\arg\left(i\,y+\sqrt{1-y^2}\right)}{2\pi}\right\rfloor +\right.\right. \\
&\quad (-1)^{\left\lfloor \frac{1}{2}-\frac{\arg\left(x\,y+\sqrt{1-x^2}\,\sqrt{1-y^2}\right)}{\pi}\right\rfloor} + 2\left(1+(-1)^{\left\lfloor \frac{1}{2}-\frac{\arg\left(x\,y+\sqrt{1-x^2}\,\sqrt{1-y^2}\right)}{\pi}\right\rfloor}\right)\left.\left\lfloor \frac{1}{2}-\frac{\arg\left(\sqrt{1-x^2}-i\,x\right)+\arg\left(i\,y+\sqrt{1-y^2}\right)}{2\pi}\right\rfloor\right) + \\
&\quad \frac{1}{2}\left(1-(-1)^{-\left\lfloor -\frac{\arg(1-y)}{2\pi}\right\rfloor}\right)\left(2\left(-1+(-1)^{\left\lfloor \frac{1}{2}-\frac{\arg\left(\sqrt{1-x^2}\,\sqrt{1-y^2}-x\,y\right)}{\pi}\right\rfloor}\right)\left\lfloor \frac{\arg\left(i\,x+\sqrt{1-x^2}\right)+\arg\left(i\,y+\sqrt{1-y^2}\right)}{2\pi}\right\rfloor +\right. \\
&\quad (-1)^{\left\lfloor \frac{1}{2}-\frac{\arg\left(\sqrt{1-x^2}\,\sqrt{1-y^2}-x\,y\right)}{\pi}\right\rfloor} + 2\left(1+(-1)^{\left\lfloor \frac{1}{2}-\frac{\arg\left(\sqrt{1-x^2}\,\sqrt{1-y^2}-x\,y\right)}{\pi}\right\rfloor}\right)\left.\left\lfloor \frac{1}{2}-\frac{\arg\left(i\,x+\sqrt{1-x^2}\right)+\arg\left(i\,y+\sqrt{1-y^2}\right)}{2\pi}\right\rfloor\right) + \\
&\quad \left. 2\right) - \cos^{-1}\left((-1)^{\left\lfloor \frac{1}{2}-\frac{\arg\left(-x\,y+i\,\sqrt{1-x^2}\,\sqrt{y-1}\,\sqrt{y+1}\right)}{\pi}\right\rfloor}\left(-i\,\sqrt{y-1}\,\sqrt{y+1}\,x-\sqrt{1-x^2}\,y\right)\right)
\end{aligned}
$$

Involving $\tanh^{-1}(z)$

01.13.16.0146.01

$$\cos^{-1}(x) + \tanh^{-1}(y) =$$

$$-2\,i\,\pi \left(\left\lfloor \frac{-\frac{1}{2}\arg(y+1) - \arg\left(\frac{\left(i\,x+\sqrt{1-x^2}\right)^i}{\sqrt{1-y}}\right) + \pi}{2\,\pi} \right\rfloor + \left\lfloor \frac{\pi - \frac{1}{2}\operatorname{Im}(\log(y+1))}{2\,\pi} \right\rfloor + \left\lfloor \frac{\pi - \operatorname{Im}\left(\log\left(\frac{\left(i\,x+\sqrt{1-x^2}\right)^i}{\sqrt{1-y}}\right)\right)}{2\,\pi} \right\rfloor \right) -$$

$$2\,i\,\pi \left(\left\lfloor \frac{-\arg\left(\left(i\,x+\sqrt{1-x^2}\right)^i\right) + \frac{1}{2}\arg(1-y) + \pi}{2\,\pi} \right\rfloor + \left\lfloor \frac{\frac{1}{2}\operatorname{Im}(\log(1-y)) + \pi}{2\,\pi} \right\rfloor + \left\lfloor \frac{\pi - \operatorname{Re}\left(\log\left(i\,x+\sqrt{1-x^2}\right)\right)}{2\,\pi} \right\rfloor \right) +$$

$$\log\left(\frac{\left(i\,x+\sqrt{1-x^2}\right)^i \sqrt{y+1}}{\sqrt{1-y}}\right) + \frac{\pi}{2}$$

01.13.16.0147.01

$$\cos^{-1}(x)+\tanh^{-1}(y)=$$

$$-2\,i\,\pi\left(\left\lfloor\frac{-\frac{1}{2}\arg(y+1)-\arg\left(\frac{\left(i\,x+\sqrt{1-x^2}\right)^i}{\sqrt{1-y}}\right)+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\pi-\frac{1}{2}\,\mathrm{Im}(\log(y+1))}{2\,\pi}\right\rfloor+\left\lfloor\frac{\pi-\mathrm{Im}\left(\log\left(\frac{\left(i\,x+\sqrt{1-x^2}\right)^i}{\sqrt{1-y}}\right)\right)}{2\,\pi}\right\rfloor\right)-$$

$$2\,i\,\pi\left(\left\lfloor\frac{-\arg\left(\left(i\,x+\sqrt{1-x^2}\right)^i\right)+\frac{1}{2}\arg(1-y)+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\frac{1}{2}\,\mathrm{Im}(\log(1-y))+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\pi-\mathrm{Re}\left(\log\left(i\,x+\sqrt{1-x^2}\right)\right)}{2\,\pi}\right\rfloor\right)+$$

$$i\left(1-(-1)^{\left\lfloor-\frac{\arg\left(\frac{\sqrt{y+1}\,\left(i\,x+\sqrt{1-x^2}\right)^i}{\sqrt{1-y}}+1\right)}{2\,\pi}\right\rfloor-\left\lfloor-\frac{\arg\left(\frac{\left(i\,x+\sqrt{1-x^2}\right)^i\,\sqrt{y+1}}{\sqrt{1-y}}\right)}{2\,\pi}\right\rfloor}\right)\pi-$$

$$i\,(-1)^{\left\lfloor\frac{\arg\left(\frac{\left(i\,x+\sqrt{1-x^2}\right)^i\,\sqrt{y+1}}{\sqrt{1-y}}-1\right)}{2\,\pi}+\frac{1}{2}-\frac{\arg\left(\frac{\sqrt{y+1}\,\left(i\,x+\sqrt{1-x^2}\right)^i}{\sqrt{1-y}}+1\right)}{2\,\pi}\right\rfloor+\left\lfloor-\frac{\arg\left(\frac{\left(i\,x+\sqrt{1-x^2}\right)^i\,\sqrt{y+1}}{\sqrt{1-y}}\right)}{\pi}\right\rfloor}$$

$$\cos^{-1}\left(\frac{\left(i\,x+\sqrt{1-x^2}\right)^{-i}\,\sqrt{1-y}\,\left(\frac{(y+1)\left(i\,x+\sqrt{1-x^2}\right)^{2\,i}}{1-y}+1\right)}{2\,\sqrt{y+1}}\right)+\frac{\pi}{2}$$

01.13.16.0148.01

$$\cos^{-1}(x)+i\tanh^{-1}(y)=\frac{\pi}{2}-\frac{1}{2\sqrt{\frac{\left(x-i\sqrt{1-x^2}\;y\right)^2}{1-y^2}}\;\sqrt{1-y^2}}\left(\left(x-i\sqrt{1-x^2}\;y\right)\left(\pi-2\sin^{-1}\left(\frac{i\,x\,y+\sqrt{1-x^2}}{\sqrt{1-y^2}}\right)\right)+\right.$$

$$2\pi\left(x-i\sqrt{1-x^2}\;y+\sqrt{\frac{y^2x^2+x^2-2\,i\sqrt{1-x^2}\;y\,x-y^2}{1-y^2}}\;\sqrt{1-y^2}\right)\left\lfloor\frac{\arg\left(i\,x+\sqrt{1-x^2}\right)+\arg\left(\frac{i+i\,y}{\sqrt{1-y^2}}\right)}{2\pi}\right\rfloor-$$

$$\left.2\pi\left(-x+i\sqrt{1-x^2}\;y+\sqrt{\frac{y^2x^2+x^2-2\,i\sqrt{1-x^2}\;y\,x-y^2}{1-y^2}}\;\sqrt{1-y^2}\right)\left\lfloor-\frac{\arg\left(i\,x+\sqrt{1-x^2}\right)+\arg\left(\frac{i+i\,y}{\sqrt{1-y^2}}\right)-\pi}{2\pi}\right\rfloor\right)$$

01.13.16.0149.01

$$\cos^{-1}(x)+i\tanh^{-1}(y)=$$

$$\cos^{-1}\left(\frac{(-1)^{\left\lfloor\frac{1}{2}-\frac{\arg\left(\frac{x-i\sqrt{1-x^2}\;y}{\sqrt{1-y^2}}\right)}{\pi}\right\rfloor}\left(-i\,x\,y-\sqrt{1-x^2}\right)}{\sqrt{1-y^2}}\right)-\frac{1}{2}\pi\left(2\left(1+(-1)^{\left\lfloor\frac{1}{2}-\frac{\arg\left(\frac{x-i\sqrt{1-x^2}\;y}{\sqrt{1-y^2}}\right)}{\pi}\right\rfloor}\right)\left\lfloor\frac{\arg\left(i\,x+\sqrt{1-x^2}\right)+\arg\left(\frac{i+i\,y}{\sqrt{1-y^2}}\right)}{2\pi}\right\rfloor+\right.$$

$$\left.(-1)^{\left\lfloor\frac{1}{2}-\frac{\arg\left(\frac{x-i\sqrt{1-x^2}\;y}{\sqrt{1-y^2}}\right)}{\pi}\right\rfloor}-2\left(-1+(-1)^{\left\lfloor\frac{1}{2}-\frac{\arg\left(\frac{x-i\sqrt{1-x^2}\;y}{\sqrt{1-y^2}}\right)}{\pi}\right\rfloor}\right)\left\lfloor\frac{1}{2}-\frac{\arg\left(i\,x+\sqrt{1-x^2}\right)+\arg\left(\frac{i+i\,y}{\sqrt{1-y^2}}\right)}{2\pi}\right\rfloor\right)$$

Involving $\coth^{-1}(z)$

01.13.16.0150.01

$$\cos^{-1}(x)+\coth^{-1}(y)=$$

$$-2\,i\,\pi\left(\left\lfloor\frac{-\frac{1}{2}\arg\left(1+\frac{1}{y}\right)-\arg\left(\frac{\left(i\,x+\sqrt{1-x^2}\right)^i}{\sqrt{1-\frac{1}{y}}}\right)+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\pi-\frac{1}{2}\operatorname{Im}\left(\log\left(1+\frac{1}{y}\right)\right)}{2\,\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}\left(\log\left(\frac{\left(i\,x+\sqrt{1-x^2}\right)^i}{\sqrt{1-\frac{1}{y}}}\right)\right)}{2\,\pi}\right\rfloor\right)-$$

$$2\,i\,\pi\left(\left\lfloor\frac{-\arg\left(\left(i\,x+\sqrt{1-x^2}\right)^i\right)+\frac{1}{2}\arg\left(1-\frac{1}{y}\right)+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\frac{1}{2}\operatorname{Im}\left(\log\left(1-\frac{1}{y}\right)\right)+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Re}\left(\log\left(i\,x+\sqrt{1-x^2}\right)\right)}{2\,\pi}\right\rfloor\right)+$$

$$\log\left(\frac{\left(i\,x+\sqrt{1-x^2}\right)^i\sqrt{1+\frac{1}{y}}}{\sqrt{1-\frac{1}{y}}}\right)+\frac{\pi}{2}$$

01.13.16.0151.01

$$\cos^{-1}(x)+\coth^{-1}(y)=$$

$$-2\,i\,\pi\left(\left\lfloor\frac{-\frac{1}{2}\arg\left(1+\frac{1}{y}\right)-\arg\left(\frac{\left(i\,x+\sqrt{1-x^2}\right)^{i}}{\sqrt{1-\frac{1}{y}}}\right)+\pi}{2\pi}\right\rfloor+\left\lfloor\frac{\pi-\frac{1}{2}\operatorname{Im}\left(\log\left(1+\frac{1}{y}\right)\right)}{2\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}\left(\log\left(\frac{\left(i\,x+\sqrt{1-x^2}\right)^{i}}{\sqrt{1-\frac{1}{y}}}\right)\right)}{2\pi}\right\rfloor\right)-$$

$$2\,i\,\pi\left(\left\lfloor\frac{-\arg\left(\left(i\,x+\sqrt{1-x^2}\right)^{i}\right)+\frac{1}{2}\arg\left(1-\frac{1}{y}\right)+\pi}{2\pi}\right\rfloor+\left\lfloor\frac{\frac{1}{2}\operatorname{Im}\left(\log\left(1-\frac{1}{y}\right)\right)+\pi}{2\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Re}\left(\log\left(i\,x+\sqrt{1-x^2}\right)\right)}{2\pi}\right\rfloor\right)+$$

$$i\left(1-(-1)^{\left\lfloor-\frac{\arg\left(\frac{\sqrt{1+\frac{1}{y}}\left(i\,x+\sqrt{1-x^2}\right)^{i}}{\sqrt{1-\frac{1}{y}}}+1\right)}{2\pi}\right\rfloor-\left\lfloor-\frac{\arg\left(\frac{\left(i\,x+\sqrt{1-x^2}\right)^{i}\sqrt{1+\frac{1}{y}}}{\sqrt{1-\frac{1}{y}}}\right)}{2\pi}\right\rfloor}\right)\pi-$$

$$i\,(-1)^{\left\lfloor\frac{\arg\left(\frac{\left(i\,x+\sqrt{1-x^2}\right)^{i}\sqrt{1+\frac{1}{y}}}{\sqrt{1-\frac{1}{y}}}-1\right)}{2\pi}+\frac{1}{2}-\frac{\arg\left(\frac{\sqrt{1+\frac{1}{y}}\left(i\,x+\sqrt{1-x^2}\right)^{i}}{\sqrt{1-\frac{1}{y}}}+1\right)}{2\pi}\right\rfloor+\left\lfloor-\frac{\arg\left(\frac{\left(i\,x+\sqrt{1-x^2}\right)^{i}\sqrt{1+\frac{1}{y}}}{\sqrt{1-\frac{1}{y}}}\right)}{\pi}\right\rfloor}$$

$$\cos^{-1}\left(\frac{\left(i\,x+\sqrt{1-x^2}\right)^{-i}\left(\frac{\left(1+\frac{1}{y}\right)\left(i\,x+\sqrt{1-x^2}\right)^{2i}}{1-\frac{1}{y}}+1\right)\sqrt{1-\frac{1}{y}}}{2\sqrt{1+\frac{1}{y}}}\right)+\frac{\pi}{2}$$

01.13.16.0152.01

$$\cos^{-1}(x)+i\coth^{-1}(y)=\frac{\pi}{2}+\frac{i}{2\sqrt{1-\frac{1}{y^2}}\;y\sqrt{\frac{\left(i\,x\,y+\sqrt{1-x^2}\right)^2}{1-y^2}}}\left(\left(i\,x\,y+\sqrt{1-x^2}\right)\left(\pi-2\sin^{-1}\left(\frac{\frac{i\,x}{y}+\sqrt{1-x^2}}{\sqrt{1-\frac{1}{y^2}}}\right)\right)+\right.$$

$$2\pi\left(i\,x\,y+i\sqrt{\frac{\left(-y^2-1\right)x^2+2\,i\sqrt{1-x^2}\;y\,x+1}{1-y^2}}\sqrt{1-\frac{1}{y^2}}\;y+\sqrt{1-x^2}\right)\left\lfloor\frac{\arg\left(i\,x+\sqrt{1-x^2}\right)+\arg\left(\frac{i+\frac{i}{y}}{\sqrt{1-\frac{1}{y^2}}}\right)}{2\pi}\right\rfloor+$$

$$2\pi\left(i\,x\,y-i\sqrt{1-\frac{1}{y^2}}\sqrt{\frac{\left(-y^2-1\right)x^2+2\,i\sqrt{1-x^2}\;y\,x+1}{1-y^2}}\;y+\sqrt{1-x^2}\right)$$

$$\left.\left\lfloor-\frac{\arg\left(i\,x+\sqrt{1-x^2}\right)+\arg\left(\frac{i+\frac{i}{y}}{\sqrt{1-\frac{1}{y^2}}}\right)-\pi}{2\pi}\right\rfloor\right)$$

01.13.16.0153.01

$$\cos^{-1}(x)+i\coth^{-1}(y)=$$

$$\cos^{-1}\left(\frac{(-1)^{\left\lfloor \frac{1}{2}-\frac{\arg\left(\frac{x-\frac{i\sqrt{1-x^2}}{y}}{\sqrt{1-\frac{1}{y^2}}}\right)}{\pi}\right\rfloor}\left(-\frac{i x}{y}-\sqrt{1-x^2}\right)}{\sqrt{1-\frac{1}{y^2}}}\right)-\frac{1}{2}\pi\left(2\left(1+(-1)^{\left\lfloor \frac{1}{2}-\frac{\arg\left(\frac{x-\frac{i\sqrt{1-x^2}}{y}}{\sqrt{1-\frac{1}{y^2}}}\right)}{\pi}\right\rfloor}\right)\left\lfloor\frac{\arg\left(i x+\sqrt{1-x^2}\right)+\arg\left(\frac{i+\frac{i}{y}}{\sqrt{1-\frac{1}{y^2}}}\right)}{2\pi}\right\rfloor+\right.$$

$$\left.(-1)^{\left\lfloor \frac{1}{2}-\frac{\arg\left(\frac{x-\frac{i\sqrt{1-x^2}}{y}}{\sqrt{1-\frac{1}{y^2}}}\right)}{\pi}\right\rfloor}-2\left(-1+(-1)^{\left\lfloor \frac{1}{2}-\frac{\arg\left(\frac{x-\frac{i\sqrt{1-x^2}}{y}}{\sqrt{1-\frac{1}{y^2}}}\right)}{\pi}\right\rfloor}\right)\left\lfloor\frac{1}{2}-\frac{\arg\left(i x+\sqrt{1-x^2}\right)+\arg\left(\frac{i+\frac{i}{y}}{\sqrt{1-\frac{1}{y^2}}}\right)}{2\pi}\right\rfloor\right)$$

Involving $\operatorname{csch}^{-1}(z)$

01.13.16.0154.01

$$\cos^{-1}(x)+\operatorname{csch}^{-1}(y)=-2 i\pi\left(\left\lfloor\frac{-\arg\left(\left(i x+\sqrt{1-x^2}\right)^{i}\right)-\arg\left(\sqrt{1+\frac{1}{y^2}}+\frac{1}{y}\right)+\pi}{2\pi}\right\rfloor+\right.$$

$$\left.\left\lfloor\frac{\pi-\operatorname{Im}\left(\log\left(\sqrt{1+\frac{1}{y^2}}+\frac{1}{y}\right)\right)}{2\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Re}\left(\log\left(i x+\sqrt{1-x^2}\right)\right)}{2\pi}\right\rfloor\right)+\log\left(\left(i x+\sqrt{1-x^2}\right)^{i}\left(\sqrt{1+\frac{1}{y^2}}+\frac{1}{y}\right)\right)+\frac{\pi}{2}$$

01.13.16.0155.01

$$\cos^{-1}(x)+\operatorname{csch}^{-1}(y)=-2\,i\,\pi\left(\left\lfloor\frac{-\arg\left(\left(i\,x+\sqrt{1-x^2}\right)^{i}\right)-\arg\left(\sqrt{1+\frac{1}{y^2}}+\frac{1}{y}\right)+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}\left(\log\left(\sqrt{1+\frac{1}{y^2}}+\frac{1}{y}\right)\right)}{2\,\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Re}\left(\log\left(i\,x+\sqrt{1-x^2}\right)\right)}{2\,\pi}\right\rfloor\right)+i\left(1-(-1)^{\left\lfloor-\frac{\arg\left(\left(\sqrt{1+\frac{1}{y^2}}+\frac{1}{y}\right)\left(\left(i\,x+\sqrt{1-x^2}\right)^{i}+1\right)\right)}{2\,\pi}\right\rfloor-\left\lfloor-\frac{\arg\left(\left(i\,x+\sqrt{1-x^2}\right)^{i}\left(\sqrt{1+\frac{1}{y^2}}+\frac{1}{y}\right)\right)}{2\,\pi}\right\rfloor}\right)\pi-i\,(-1)^{\left\lfloor\frac{\arg\left(\left(i\,x+\sqrt{1-x^2}\right)^{i}\left(\sqrt{1+\frac{1}{y^2}}+\frac{1}{y}\right)-1\right)}{2\,\pi}+\frac{1}{2}-\frac{\arg\left(\left(\sqrt{1+\frac{1}{y^2}}+\frac{1}{y}\right)\left(\left(i\,x+\sqrt{1-x^2}\right)^{i}+1\right)\right)}{2\,\pi}\right\rfloor+\left\lfloor-\frac{\arg\left(\left(i\,x+\sqrt{1-x^2}\right)^{i}\left(\sqrt{1+\frac{1}{y^2}}+\frac{1}{y}\right)\right)}{\pi}\right\rfloor}\cos^{-1}\left(\frac{\left(i\,x+\sqrt{1-x^2}\right)^{-i}\left(\left(\sqrt{1+\frac{1}{y^2}}+\frac{1}{y}\right)^{2}\left(i\,x+\sqrt{1-x^2}\right)^{2\,i}+1\right)}{2\left(\sqrt{1+\frac{1}{y^2}}+\frac{1}{y}\right)}\right)+\frac{\pi}{2}$$

01.13.16.0156.01

$$\cos^{-1}(x)+i\,\operatorname{csch}^{-1}(y)=-\frac{\frac{i\,x}{y}+\sqrt{1-x^2}\,\sqrt{1+\frac{1}{y^2}}}{\sqrt{\left(\frac{i\,x}{y}+\sqrt{1-x^2}\,\sqrt{1+\frac{1}{y^2}}\right)^2}}\sin^{-1}\left(x\sqrt{1+\frac{1}{y^2}}-\frac{i\sqrt{1-x^2}}{y}\right)-$$
$$\pi\left\lfloor\frac{\arg\left(i\,x+\sqrt{1-x^2}\right)+\arg\left(\sqrt{1+\frac{1}{y^2}}+\frac{1}{y}\right)}{2\pi}\right\rfloor\left(1-\frac{i\left(x-i\sqrt{1-x^2}\,\sqrt{1+\frac{1}{y^2}}\,y\right)}{\sqrt{\left(\frac{i\,x}{y}+\sqrt{1-x^2}\,\sqrt{1+\frac{1}{y^2}}\right)^2}\,y}\right)+$$
$$\frac{1}{2}\pi\frac{i\left(x-i\sqrt{1-x^2}\,\sqrt{1+\frac{1}{y^2}}\,y\right)}{\sqrt{\left(\frac{i\,x}{y}+\sqrt{1-x^2}\,\sqrt{1+\frac{1}{y^2}}\right)^2}\,y}+$$
$$\pi\left(\frac{\frac{i\,x}{y}+\sqrt{1-x^2}\,\sqrt{1+\frac{1}{y^2}}}{\sqrt{\left(\frac{i\,x}{y}+\sqrt{1-x^2}\,\sqrt{1+\frac{1}{y^2}}\right)^2}}+1\right)\left\lfloor-\frac{\arg\left(i\,x+\sqrt{1-x^2}\right)+\arg\left(\sqrt{1+\frac{1}{y^2}}+\frac{1}{y}\right)-\pi}{2\pi}\right\rfloor$$

01.13.16.0157.01

$$\cos^{-1}(x)+i\operatorname{csch}^{-1}(y)=\cos^{-1}\left((-1)^{\left\lfloor \frac{1}{2}-\frac{\arg\left(\frac{i x}{y}+\sqrt{1-x^2}\sqrt{1+\frac{1}{y^2}}\right)}{\pi}\right\rfloor}\left(x\sqrt{1+\frac{1}{y^2}}-\frac{i\sqrt{1-x^2}}{y}\right)\right)+$$

$$\frac{1}{2}\pi\left(2\left(-1+(-1)^{\left\lfloor \frac{1}{2}-\frac{\arg\left(\frac{i x}{y}+\sqrt{1-x^2}\sqrt{1+\frac{1}{y^2}}\right)}{\pi}\right\rfloor}\right)\left\lfloor\frac{\arg\left(i x+\sqrt{1-x^2}\right)+\arg\left(\sqrt{1+\frac{1}{y^2}}+\frac{1}{y}\right)}{2\pi}\right\rfloor+(-1)^{\left\lfloor \frac{1}{2}-\frac{\arg\left(\frac{i x}{y}+\sqrt{1-x^2}\sqrt{1+\frac{1}{y^2}}\right)}{\pi}\right\rfloor}+\right.$$

$$\left.2\left(1+(-1)^{\left\lfloor \frac{1}{2}-\frac{\arg\left(\frac{i x}{y}+\sqrt{1-x^2}\sqrt{1+\frac{1}{y^2}}\right)}{\pi}\right\rfloor}\right)\left\lfloor\frac{1}{2}-\frac{\arg\left(i x+\sqrt{1-x^2}\right)+\arg\left(\sqrt{1+\frac{1}{y^2}}+\frac{1}{y}\right)}{2\pi}\right\rfloor-1\right)$$

Involving $\operatorname{sech}^{-1}(z)$

01.13.16.0158.01

$$\cos^{-1}(x)+\operatorname{sech}^{-1}(y)=$$

$$-2 i\pi\left(\left\lfloor\frac{-\arg\left(\left(i x+\sqrt{1-x^2}\right)^i\right)-\arg\left(\sqrt{\frac{1}{y}-1}\sqrt{1+\frac{1}{y}}+\frac{1}{y}\right)+\pi}{2\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}\left(\log\left(\sqrt{\frac{1}{y}-1}\sqrt{1+\frac{1}{y}}+\frac{1}{y}\right)\right)}{2\pi}\right\rfloor+\right.$$

$$\left.\left\lfloor\frac{\pi-\operatorname{Re}\left(\log\left(i x+\sqrt{1-x^2}\right)\right)}{2\pi}\right\rfloor\right)+\log\left(\left(i x+\sqrt{1-x^2}\right)^i\left(\sqrt{\frac{1}{y}-1}\sqrt{1+\frac{1}{y}}+\frac{1}{y}\right)\right)+\frac{\pi}{2}$$

01.13.16.0159.01

$$\cos^{-1}(x)+\operatorname{sech}^{-1}(y)=$$

$$-2\,i\,\pi\left(\left\lfloor\frac{-\arg\left(\left(i\,x+\sqrt{1-x^2}\right)^{i}\right)-\arg\left(\sqrt{\frac{1}{y}-1}\,\sqrt{1+\frac{1}{y}}+\frac{1}{y}\right)+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}\left(\log\left(\sqrt{\frac{1}{y}-1}\,\sqrt{1+\frac{1}{y}}+\frac{1}{y}\right)\right)}{2\,\pi}\right\rfloor+\right.$$

$$\left.\left\lfloor\frac{\pi-\operatorname{Re}\left(\log\left(i\,x+\sqrt{1-x^2}\right)\right)}{2\,\pi}\right\rfloor\right)+i\left(1-(-1)^{\left\lfloor-\frac{\arg\left(\left(\sqrt{\frac{1}{y}-1}\,\sqrt{1+\frac{1}{y}}+\frac{1}{y}\right)\left(i\,x+\sqrt{1-x^2}\right)^{i}+1\right)}{2\,\pi}\right\rfloor-\left\lfloor-\frac{\arg\left(\left(i\,x+\sqrt{1-x^2}\right)^{i}\left(\sqrt{\frac{1}{y}-1}\,\sqrt{1+\frac{1}{y}}+\frac{1}{y}\right)\right)}{2\,\pi}\right\rfloor}\right)\pi-$$

$$i\,(-1)^{\left\lfloor\frac{\arg\left(\left(i\,x+\sqrt{1-x^2}\right)^{i}\left(\sqrt{\frac{1}{y}-1}\,\sqrt{1+\frac{1}{y}}+\frac{1}{y}\right)-1\right)}{2\,\pi}+\frac{1}{2}-\frac{\arg\left(\left(\sqrt{\frac{1}{y}-1}\,\sqrt{1+\frac{1}{y}}+\frac{1}{y}\right)\left(i\,x+\sqrt{1-x^2}\right)^{i}+1\right)}{2\,\pi}\right\rfloor+\left\lfloor-\frac{\arg\left(\left(i\,x+\sqrt{1-x^2}\right)^{i}\left(\sqrt{\frac{1}{y}-1}\,\sqrt{1+\frac{1}{y}}+\frac{1}{y}\right)\right)}{\pi}\right\rfloor}$$

$$\cos^{-1}\left(\frac{\left(i\,x+\sqrt{1-x^2}\right)^{-i}\left(\left(\sqrt{\frac{1}{y}-1}\,\sqrt{1+\frac{1}{y}}+\frac{1}{y}\right)^{2}\left(i\,x+\sqrt{1-x^2}\right)^{2\,i}+1\right)}{2\left(\sqrt{\frac{1}{y}-1}\,\sqrt{1+\frac{1}{y}}+\frac{1}{y}\right)}\right)+\frac{\pi}{2}$$

01.13.16.0160.01

$$\cos^{-1}(x) + i\,\text{sech}^{-1}(y) =$$

$$\frac{1}{2}\pi\left(-\frac{1}{2}\left(1+(-1)^{-\left\lfloor -\frac{\arg\left(1-\frac{1}{y}\right)}{2\pi}\right\rfloor}\right)\left(2\left(-1+(-1)^{\left\lfloor \frac{1}{2}-\frac{\arg\left(\frac{x}{y}+\sqrt{1-x^2}\,\sqrt{1-\frac{1}{y^2}}\right)}{\pi}\right\rfloor}\right)\left\lfloor \frac{\arg\left(\sqrt{1-x^2}-i\,x\right)+\arg\left(\sqrt{1-\frac{1}{y^2}}+\frac{i}{y}\right)}{2\pi}\right\rfloor + \right.\right.$$

$$\left.(-1)^{\left\lfloor \frac{1}{2}-\frac{\arg\left(\frac{x}{y}+\sqrt{1-x^2}\,\sqrt{1-\frac{1}{y^2}}\right)}{\pi}\right\rfloor}+2\left(1+(-1)^{\left\lfloor \frac{1}{2}-\frac{\arg\left(\frac{x}{y}+\sqrt{1-x^2}\,\sqrt{1-\frac{1}{y^2}}\right)}{\pi}\right\rfloor}\right)\left\lfloor \frac{1}{2}-\frac{\arg\left(\sqrt{1-x^2}-i\,x\right)+\arg\left(\sqrt{1-\frac{1}{y^2}}+\frac{i}{y}\right)}{2\pi}\right\rfloor\right)+$$

$$\frac{1}{2}\left(1-(-1)^{-\left\lfloor -\frac{\arg\left(1-\frac{1}{y}\right)}{2\pi}\right\rfloor}\right)\left(2\left(-1+(-1)^{\left\lfloor \frac{1}{2}-\frac{\arg\left(\sqrt{1-x^2}\,\sqrt{1-\frac{1}{y^2}}-\frac{x}{y}\right)}{\pi}\right\rfloor}\right)\left\lfloor \frac{\arg\left(i\,x+\sqrt{1-x^2}\right)+\arg\left(\sqrt{1-\frac{1}{y^2}}+\frac{i}{y}\right)}{2\pi}\right\rfloor + \right.$$

$$\left.(-1)^{\left\lfloor \frac{1}{2}-\frac{\arg\left(\sqrt{1-x^2}\,\sqrt{1-\frac{1}{y^2}}-\frac{x}{y}\right)}{\pi}\right\rfloor}+2\left(1+(-1)^{\left\lfloor \frac{1}{2}-\frac{\arg\left(\sqrt{1-x^2}\,\sqrt{1-\frac{1}{y^2}}-\frac{x}{y}\right)}{\pi}\right\rfloor}\right)\left\lfloor \frac{1}{2}-\frac{\arg\left(i\,x+\sqrt{1-x^2}\right)+\arg\left(\sqrt{1-\frac{1}{y^2}}+\frac{i}{y}\right)}{2\pi}\right\rfloor\right)+$$

$$\left.2\right) - \cos^{-1}\left((-1)^{\left\lfloor \frac{1}{2}-\frac{\arg\left(i\,\sqrt{1-x^2}\,\sqrt{\frac{1}{y}-1}\,\sqrt{1+\frac{1}{y}}-\frac{x}{y}\right)}{\pi}\right\rfloor}\left(-i\,\sqrt{\frac{1}{y}-1}\,\sqrt{1+\frac{1}{y}}\,x-\frac{\sqrt{1-x^2}}{y}\right)\right)$$

Differences involving the direct function

Involving log(*z*)

01.13.16.0161.01

$$\cos^{-1}(x) - \log(y) = -2\,i\,\pi \left(\left\lfloor \frac{-\arg\left(\left(i\,x + \sqrt{1-x^2}\right)^{i}\right) - \arg\left(\frac{1}{y}\right) + \pi}{2\,\pi} \right\rfloor + \left\lfloor \frac{\operatorname{Im}(\log(y)) + \pi}{2\,\pi} \right\rfloor + \left\lfloor \frac{\pi - \operatorname{Re}\left(\log\left(i\,x + \sqrt{1-x^2}\right)\right)}{2\,\pi} \right\rfloor \right) + \log\left(\frac{\left(i\,x + \sqrt{1-x^2}\right)^{i}}{y} \right) + \frac{\pi}{2}$$

01.13.16.0162.01

$$\cos^{-1}(x) - \log(y) = -2\,i\,\pi \left(\left\lfloor \frac{-\arg\left(\left(i\,x + \sqrt{1-x^2}\right)^{i}\right) - \arg\left(\frac{1}{y}\right) + \pi}{2\,\pi} \right\rfloor + \left\lfloor \frac{\operatorname{Im}(\log(y)) + \pi}{2\,\pi} \right\rfloor + \left\lfloor \frac{\pi - \operatorname{Re}\left(\log\left(i\,x + \sqrt{1-x^2}\right)\right)}{2\,\pi} \right\rfloor \right) +$$

$$i \left(1 - (-1)^{\left\lfloor -\frac{\arg\left(\frac{\left(i\,x+\sqrt{1-x^2}\right)^{i}}{y} + 1\right)}{2\,\pi} \right\rfloor - \left\lfloor -\frac{\arg\left(\frac{\left(i\,x+\sqrt{1-x^2}\right)^{i}}{y}\right)}{2\,\pi} \right\rfloor} \right) \pi -$$

$$i\,(-1)^{\left\lfloor \frac{\arg\left(\frac{\left(i\,x+\sqrt{1-x^2}\right)^{i}}{y} - 1\right)}{2\,\pi} + \frac{1}{2} - \frac{\arg\left(\frac{\left(i\,x+\sqrt{1-x^2}\right)^{i}}{y} + 1\right)}{2\,\pi} \right\rfloor + \left\lfloor -\frac{\arg\left(\frac{\left(i\,x+\sqrt{1-x^2}\right)^{i}}{y}\right)}{\pi} \right\rfloor} \cos^{-1}\left(\frac{1}{2} \left(i\,x + \sqrt{1-x^2}\right)^{-i} \left(\frac{\left(i\,x + \sqrt{1-x^2}\right)^{2\,i}}{y^2} + 1 \right) y \right) + \frac{\pi}{2}$$

Involving $\sin^{-1}(z)$

01.13.16.0017.01

$$\cos^{-1}(x) - \sin^{-1}(y) = \pi \left(\operatorname{sgn}(x+y) + \frac{1}{2} \right) - \operatorname{sgn}(x+y) \cos^{-1}\left(x\,y - \sqrt{1-x^2}\,\sqrt{1-y^2} \right) /;\; x < -1 \land y > 1 \lor x > 1 \land y < -1$$

01.13.16.0016.01

$$\cos^{-1}(x) - \sin^{-1}(y) = \pi \left(\frac{1}{2} - \operatorname{sgn}(x+y) \right) + \cos^{-1}\left(x\,y - \sqrt{1-x^2}\,\sqrt{1-y^2} \right) \operatorname{sgn}(x+y) /;\; -1 < x < 1 \lor -1 < y < 1 \lor x\,y > 0$$

01.13.16.0163.01

$$\cos^{-1}(x) - \sin^{-1}(y) = \frac{\pi}{2} - \operatorname{sgn}(x+y) \cos^{-1}\left(\sqrt{1-x^2}\,\sqrt{1-y^2} - x\,y \right) /;\; x > -1 \land y > -1 \lor x < 1 \land y < 1$$

01.13.16.0164.01

$$\cos^{-1}(x)-\sin^{-1}(y)=\cos^{-1}\left(\sqrt{1-x^2}\ \sqrt{1-y^2}-x\,y\right)\operatorname{sgn}(x+y)+\frac{\pi}{2}\ /;\ x<-1\land y>1\lor x>1\land y<-1$$

01.13.16.0165.01

$$\cos^{-1}(x)-\sin^{-1}(y)=\frac{\pi}{2}-\sin^{-1}\left(\sqrt{1-y^2}\ x+\sqrt{1-x^2}\ y\right)\ /;\ x\,y\leq 0\bigvee x^2+y^2\leq 1$$

01.13.16.0166.01

$$\cos^{-1}(x)-\sin^{-1}(y)=\sin^{-1}\left(\sqrt{1-y^2}\ x+\sqrt{1-x^2}\ y\right)+\pi\,(-\operatorname{sgn}(x))+\frac{\pi}{2}\ /;\ x\,y>0\bigwedge x^2+y^2>1$$

01.13.16.0167.01

$$\cos^{-1}(x)-\sin^{-1}(y)=\frac{\pi}{2}-\tan^{-1}\left(\frac{\sqrt{1-y^2}\ x+\sqrt{1-x^2}\ y}{\sqrt{1-y^2}\ \sqrt{1-x^2}-x\,y}\right)\ /;\ x\,y<0\bigvee x^2+y^2\leq 1$$

01.13.16.0168.01

$$\cos^{-1}(x)-\sin^{-1}(y)=-\tan^{-1}\left(\frac{\sqrt{1-y^2}\ x+\sqrt{1-x^2}\ y}{\sqrt{1-y^2}\ \sqrt{1-x^2}-x\,y}\right)-\operatorname{sgn}(x)\,\pi+\frac{\pi}{2}\ /;\ x\,y>0\bigwedge x^2+y^2>1$$

01.13.16.0169.01

$$\cos^{-1}(x)-\sin^{-1}(y)=-\frac{\sqrt{1-x^2}\ \sqrt{1-y^2}-x\,y}{\sqrt{\left(\sqrt{1-x^2}\ \sqrt{1-y^2}-x\,y\right)^2}}\sin^{-1}\left(\sqrt{1-y^2}\ x+\sqrt{1-x^2}\ y\right)\frac{\pi\left(\sqrt{1-x^2}\ \sqrt{1-y^2}-x\,y\right)}{2\sqrt{\left(\sqrt{1-x^2}\ \sqrt{1-y^2}-x\,y\right)^2}}+$$
$$\pi\left(\frac{\sqrt{1-x^2}\ \sqrt{1-y^2}-x\,y}{\sqrt{\left(\sqrt{1-x^2}\ \sqrt{1-y^2}-x\,y\right)^2}}+1\right)\left\lfloor\frac{-\arg\left(i\,x+\sqrt{1-x^2}\right)-\arg\left(i\,y+\sqrt{1-y^2}\right)+\pi}{2\,\pi}\right\rfloor-$$
$$\pi\left(1-\frac{\sqrt{1-x^2}\ \sqrt{1-y^2}-x\,y}{\sqrt{\left(\sqrt{1-x^2}\ \sqrt{1-y^2}-x\,y\right)^2}}\right)\left\lfloor\frac{\arg\left(i\,x+\sqrt{1-x^2}\right)+\arg\left(i\,y+\sqrt{1-y^2}\right)}{2\,\pi}\right\rfloor$$

01.13.16.0170.01

$$\cos^{-1}(x) - \sin^{-1}(y) = -\cos^{-1}\left((-1)^{\left\lfloor \frac{1}{2} - \frac{\arg\left(\sqrt{1-x^2}\sqrt{1-y^2} - x y\right)}{\pi} \right\rfloor}\left(-\sqrt{1-y^2}\, x - \sqrt{1-x^2}\, y\right)\right) - \frac{1}{2}\pi\left(2\left(-1 + (-1)^{\left\lfloor \frac{1}{2} - \frac{\arg\left(\sqrt{1-x^2}\sqrt{1-y^2} - x y\right)}{\pi} \right\rfloor}\right)\left\lfloor \frac{\arg\left(\sqrt{1-x^2} - i x\right) + \arg\left(\sqrt{1-y^2} - i y\right)}{2\pi} \right\rfloor + (-1)^{\left\lfloor \frac{1}{2} - \frac{\arg\left(\sqrt{1-x^2}\sqrt{1-y^2} - x y\right)}{\pi} \right\rfloor} + 2\left(1 + (-1)^{\left\lfloor \frac{1}{2} - \frac{\arg\left(\sqrt{1-x^2}\sqrt{1-y^2} - x y\right)}{\pi} \right\rfloor}\right)\left\lfloor \frac{1}{2} - \frac{\arg\left(\sqrt{1-x^2} - i x\right) + \arg\left(\sqrt{1-y^2} - i y\right)}{2\pi} \right\rfloor - 3\right)$$

Involving $\tan^{-1}(z)$

01.13.16.0171.01

$$\cos^{-1}(x) - \tan^{-1}(y) = \frac{\pi}{2} - \frac{1}{2\sqrt{\frac{\left(x+\sqrt{1-x^2}\, y\right)^2}{y^2+1}}\sqrt{y^2+1}}\left(\left(x + \sqrt{1-x^2}\, y\right)\left(\pi - 2\sin^{-1}\left(\frac{\sqrt{1-x^2} - x y}{\sqrt{y^2+1}}\right)\right) + 2\pi\left(x + \sqrt{1-x^2}\, y + \sqrt{\frac{-y^2 x^2 + x^2 + 2\sqrt{1-x^2}\, y x + y^2}{y^2+1}}\sqrt{y^2+1}\right)\left\lfloor \frac{\arg\left(i x + \sqrt{1-x^2}\right) + \arg\left(\frac{i-y}{\sqrt{y^2+1}}\right)}{2\pi} \right\rfloor - 2\pi\left(-x - \sqrt{1-x^2}\, y + \sqrt{\frac{-y^2 x^2 + x^2 + 2\sqrt{1-x^2}\, y x + y^2}{y^2+1}}\sqrt{y^2+1}\right)\left\lfloor -\frac{\arg\left(i x + \sqrt{1-x^2}\right) + \arg\left(\frac{i-y}{\sqrt{y^2+1}}\right) - \pi}{2\pi} \right\rfloor\right)$$

01.13.16.0172.01

$$\cos^{-1}(x)-\tan^{-1}(y)=\cos^{-1}\left(\frac{(-1)^{\left\lfloor \frac{1}{2}-\frac{\arg\left(\frac{x+\sqrt{1-x^2}\,y}{\sqrt{y^2+1}}\right)}{\pi}\right\rfloor}\left(x\,y-\sqrt{1-x^2}\right)}{\sqrt{y^2+1}}\right)-\frac{1}{2}\pi\left(2\left(1+(-1)^{\left\lfloor \frac{1}{2}-\frac{\arg\left(\frac{x+\sqrt{1-x^2}\,y}{\sqrt{y^2+1}}\right)}{\pi}\right\rfloor}\right)\left\lfloor\frac{\arg\left(i\,x+\sqrt{1-x^2}\right)+\arg\left(\frac{i-y}{\sqrt{y^2+1}}\right)}{2\pi}\right\rfloor+\right.$$
$$\left.(-1)^{\left\lfloor \frac{1}{2}-\frac{\arg\left(\frac{x+\sqrt{1-x^2}\,y}{\sqrt{y^2+1}}\right)}{\pi}\right\rfloor}-2\left(-1+(-1)^{\left\lfloor \frac{1}{2}-\frac{\arg\left(\frac{x+\sqrt{1-x^2}\,y}{\sqrt{y^2+1}}\right)}{\pi}\right\rfloor}\right)\left\lfloor\frac{1}{2}-\frac{\arg\left(i\,x+\sqrt{1-x^2}\right)+\arg\left(\frac{i-y}{\sqrt{y^2+1}}\right)}{2\pi}\right\rfloor\right)$$

Involving $\cot^{-1}(z)$

01.13.16.0173.01

$$\cos^{-1}(x)-\cot^{-1}(y)=\frac{\pi}{2}-\frac{1}{2\sqrt{1+\frac{1}{y^2}}\,y\sqrt{\frac{\left(x\,y+\sqrt{1-x^2}\right)^2}{y^2+1}}}\left(\left(x\,y+\sqrt{1-x^2}\right)\left(\pi-2\sin^{-1}\left(\frac{\sqrt{1-x^2}-\frac{x}{y}}{\sqrt{1+\frac{1}{y^2}}}\right)\right)+\right.$$
$$2\pi\left(x\,y+\sqrt{\frac{\left(y^2-1\right)x^2+2\sqrt{1-x^2}\,y\,x+1}{y^2+1}}\sqrt{1+\frac{1}{y^2}}\,y+\sqrt{1-x^2}\right)\left\lfloor\frac{\arg\left(i\,x+\sqrt{1-x^2}\right)+\arg\left(\frac{i-\frac{1}{y}}{\sqrt{1+\frac{1}{y^2}}}\right)}{2\pi}\right\rfloor+$$
$$\left.2\pi\left(x\,y-\sqrt{1+\frac{1}{y^2}}\sqrt{\frac{\left(y^2-1\right)x^2+2\sqrt{1-x^2}\,y\,x+1}{y^2+1}}\,y+\sqrt{1-x^2}\right)\left\lfloor-\frac{\arg\left(i\,x+\sqrt{1-x^2}\right)+\arg\left(\frac{i-\frac{1}{y}}{\sqrt{1+\frac{1}{y^2}}}\right)-\pi}{2\pi}\right\rfloor\right)$$

01.13.16.0174.01

$$\cos^{-1}(x) - \cot^{-1}(y) =$$

$$\cos^{-1}\left(\frac{(-1)^{\left\lfloor \frac{1}{2} - \frac{\arg\left(\frac{x+\frac{\sqrt{1-x^2}}{y}}{\sqrt{1+\frac{1}{y^2}}}\right)}{\pi} \right\rfloor}\left(\frac{x}{y} - \sqrt{1-x^2}\right)}{\sqrt{1+\frac{1}{y^2}}}\right) - \frac{1}{2}\pi\left(2\left(1+(-1)^{\left\lfloor \frac{1}{2} - \frac{\arg\left(\frac{x+\frac{\sqrt{1-x^2}}{y}}{\sqrt{1+\frac{1}{y^2}}}\right)}{\pi} \right\rfloor}\right)\left\lfloor \frac{\arg\left(i\,x+\sqrt{1-x^2}\right)+\arg\left(\frac{i-\frac{1}{y}}{\sqrt{1+\frac{1}{y^2}}}\right)}{2\pi} \right\rfloor + \right.$$

$$\left. (-1)^{\left\lfloor \frac{1}{2} - \frac{\arg\left(\frac{x+\frac{\sqrt{1-x^2}}{y}}{\sqrt{1+\frac{1}{y^2}}}\right)}{\pi} \right\rfloor} - 2\left(-1+(-1)^{\left\lfloor \frac{1}{2} - \frac{\arg\left(\frac{x+\frac{\sqrt{1-x^2}}{y}}{\sqrt{1+\frac{1}{y^2}}}\right)}{\pi} \right\rfloor}\right)\left\lfloor \frac{1}{2} - \frac{\arg\left(i\,x+\sqrt{1-x^2}\right)+\arg\left(\frac{i-\frac{1}{y}}{\sqrt{1+\frac{1}{y^2}}}\right)}{2\pi} \right\rfloor\right)$$

Involving $\csc^{-1}(z)$

01.13.16.0175.01

$$\cos^{-1}(x)-\csc^{-1}(y)=-\frac{\sqrt{1-x^2}\,\sqrt{1-\frac{1}{y^2}}-\frac{x}{y}}{\sqrt{\left(\sqrt{1-x^2}\,\sqrt{1-\frac{1}{y^2}}-\frac{x}{y}\right)^2}}\sin^{-1}\!\left(\sqrt{1-\frac{1}{y^2}}\;x+\frac{\sqrt{1-x^2}}{y}\right)-$$

$$\pi\left\lfloor\frac{\arg\!\left(i\,x+\sqrt{1-x^2}\right)+\arg\!\left(\sqrt{1-\frac{1}{y^2}}+\frac{i}{y}\right)}{2\,\pi}\right\rfloor\left(\frac{x-\sqrt{1-x^2}\,\sqrt{1-\frac{1}{y^2}}\;y}{\sqrt{\left(\sqrt{1-x^2}\,\sqrt{1-\frac{1}{y^2}}-\frac{x}{y}\right)^2}\;y}+1\right)-$$

$$\frac{1}{2}\,\pi\,\frac{x-\sqrt{1-x^2}\,\sqrt{1-\frac{1}{y^2}}\;y}{\sqrt{\left(\sqrt{1-x^2}\,\sqrt{1-\frac{1}{y^2}}-\frac{x}{y}\right)^2}\;y}+$$

$$\pi\left(\frac{\sqrt{1-x^2}\,\sqrt{1-\frac{1}{y^2}}-\frac{x}{y}}{\sqrt{\left(\sqrt{1-x^2}\,\sqrt{1-\frac{1}{y^2}}-\frac{x}{y}\right)^2}}+1\right)\left\lfloor-\frac{\arg\!\left(i\,x+\sqrt{1-x^2}\right)+\arg\!\left(\sqrt{1-\frac{1}{y^2}}+\frac{i}{y}\right)-\pi}{2\,\pi}\right\rfloor$$

01.13.16.0176.01

$$\cos^{-1}(x)-\csc^{-1}(y)=-\cos^{-1}\left((-1)^{\left\lfloor \frac{1}{2}-\frac{\arg\left(\sqrt{1-x^2}\,\sqrt{1-\frac{1}{y^2}}\,-\frac{x}{y}\right)}{\pi}\right\rfloor}\left(-x\sqrt{1-\frac{1}{y^2}}-\frac{\sqrt{1-x^2}}{y}\right)\right)-$$

$$\frac{1}{2}\pi\left(2\left(-1+(-1)^{\left\lfloor \frac{1}{2}-\frac{\arg\left(\sqrt{1-x^2}\,\sqrt{1-\frac{1}{y^2}}\,-\frac{x}{y}\right)}{\pi}\right\rfloor}\right)\left\lfloor\frac{\arg\left(\sqrt{1-x^2}-i\,x\right)+\arg\left(\sqrt{1-\frac{1}{y^2}}-\frac{i}{y}\right)}{2\pi}\right\rfloor+(-1)^{\left\lfloor \frac{1}{2}-\frac{\arg\left(\sqrt{1-x^2}\,\sqrt{1-\frac{1}{y^2}}\,-\frac{x}{y}\right)}{\pi}\right\rfloor}+\right.$$

$$\left.2\left(1+(-1)^{\left\lfloor \frac{1}{2}-\frac{\arg\left(\sqrt{1-x^2}\,\sqrt{1-\frac{1}{y^2}}\,-\frac{x}{y}\right)}{\pi}\right\rfloor}\right)\left\lfloor\frac{1}{2}-\frac{\arg\left(\sqrt{1-x^2}-i\,x\right)+\arg\left(\sqrt{1-\frac{1}{y^2}}-\frac{i}{y}\right)}{2\pi}\right\rfloor-3\right)$$

Involving $\sec^{-1}(z)$

01.13.16.0177.01

$$\cos^{-1}(x)-\sec^{-1}(y)=-\frac{\frac{x}{y}+\sqrt{1-x^2}\,\sqrt{1-\frac{1}{y^2}}}{\sqrt{\left(\frac{x}{y}+\sqrt{1-x^2}\,\sqrt{1-\frac{1}{y^2}}\right)^2}}\sin^{-1}\left(x\sqrt{1-\frac{1}{y^2}}-\frac{\sqrt{1-x^2}}{y}\right)-\pi\left\lfloor\frac{\arg\left(i\,x+\sqrt{1-x^2}\right)+\arg\left(\sqrt{1-\frac{1}{y^2}}-\frac{i}{y}\right)}{2\pi}\right\rfloor\left(1-\frac{x+\sqrt{1-x^2}\,y\sqrt{1-\frac{1}{y^2}}}{\sqrt{\left(\frac{x}{y}+\sqrt{1-x^2}\,\sqrt{1-\frac{1}{y^2}}\right)^2}\,y}\right)-\frac{1}{2}\pi\left(1-\frac{x+\sqrt{1-x^2}\,y\sqrt{1-\frac{1}{y^2}}}{\sqrt{\left(\frac{x}{y}+\sqrt{1-x^2}\,\sqrt{1-\frac{1}{y^2}}\right)^2}\,y}\right)+\pi\left(\frac{\frac{x}{y}+\sqrt{1-x^2}\,\sqrt{1-\frac{1}{y^2}}}{\sqrt{\left(\frac{x}{y}+\sqrt{1-x^2}\,\sqrt{1-\frac{1}{y^2}}\right)^2}}+1\right)\left\lfloor-\frac{\arg\left(i\,x+\sqrt{1-x^2}\right)+\arg\left(\sqrt{1-\frac{1}{y^2}}-\frac{i}{y}\right)-\pi}{2\pi}\right\rfloor$$

01.13.16.0178.01

$$\cos^{-1}(x)-\sec^{-1}(y)=\cos^{-1}\left((-1)^{\left\lfloor \frac{1}{2}-\frac{\arg\left(\frac{x}{y}+\sqrt{1-x^2}\,\sqrt{1-\frac{1}{y^2}}\right)}{\pi}\right\rfloor}\left(x\sqrt{1-\frac{1}{y^2}}-\frac{\sqrt{1-x^2}}{y}\right)\right)+$$

$$\frac{1}{2}\pi\left(2\left(-1+(-1)^{\left\lfloor \frac{1}{2}-\frac{\arg\left(\frac{x}{y}+\sqrt{1-x^2}\,\sqrt{1-\frac{1}{y^2}}\right)}{\pi}\right\rfloor}\right)\left\lfloor\frac{\arg\left(i\,x+\sqrt{1-x^2}\right)+\arg\left(\sqrt{1-\frac{1}{y^2}}-\frac{i}{y}\right)}{2\pi}\right\rfloor+(-1)^{\left\lfloor \frac{1}{2}-\frac{\arg\left(\frac{x}{y}+\sqrt{1-x^2}\,\sqrt{1-\frac{1}{y^2}}\right)}{\pi}\right\rfloor}+\right.$$

$$\left.2\left(1+(-1)^{\left\lfloor \frac{1}{2}-\frac{\arg\left(\frac{x}{y}+\sqrt{1-x^2}\,\sqrt{1-\frac{1}{y^2}}\right)}{\pi}\right\rfloor}\right)\left\lfloor\frac{1}{2}-\frac{\arg\left(i\,x+\sqrt{1-x^2}\right)+\arg\left(\sqrt{1-\frac{1}{y^2}}-\frac{i}{y}\right)}{2\pi}\right\rfloor-2\right)$$

Involving $\sinh^{-1}(z)$

01.13.16.0179.01

$$\cos^{-1}(x)-\sinh^{-1}(y)=$$

$$-2\,i\,\pi\left(\left\lfloor\frac{-\arg\left(\frac{1}{y+\sqrt{y^2+1}}\right)-\arg\left(\left(i\,x+\sqrt{1-x^2}\right)^{i}\right)+\pi}{2\pi}\right\rfloor+\left\lfloor\frac{\operatorname{Im}\left(\log\left(y+\sqrt{y^2+1}\right)\right)+\pi}{2\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Re}\left(\log\left(i\,x+\sqrt{1-x^2}\right)\right)}{2\pi}\right\rfloor\right)+$$

$$\log\left(\frac{\left(i\,x+\sqrt{1-x^2}\right)^{i}}{y+\sqrt{y^2+1}}\right)+\frac{\pi}{2}$$

01.13.16.0180.01

$$\cos^{-1}(x)-\sinh^{-1}(y)=$$

$$-2\,i\,\pi\left(\left\lfloor\frac{-\arg\left(\frac{1}{y+\sqrt{y^2+1}}\right)-\arg\left(\left(i\,x+\sqrt{1-x^2}\right)^{i}\right)+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\operatorname{Im}\left(\log\left(y+\sqrt{y^2+1}\right)\right)+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Re}\left(\log\left(i\,x+\sqrt{1-x^2}\right)\right)}{2\,\pi}\right\rfloor\right)+$$

$$i\left(1-(-1)^{\left\lfloor-\frac{\arg\left(\frac{\left(i\,x+\sqrt{1-x^2}\right)^{i}}{y+\sqrt{y^2+1}}+1\right)}{2\,\pi}\right\rfloor-\left\lfloor-\frac{\arg\left(\frac{\left(i\,x+\sqrt{1-x^2}\right)^{i}}{y+\sqrt{y^2+1}}\right)}{2\,\pi}\right\rfloor}\right)\pi-i\,(-1)^{\left\lfloor\frac{\arg\left(\frac{\left(i\,x+\sqrt{1-x^2}\right)^{i}}{y+\sqrt{y^2+1}}-1\right)}{2\,\pi}+\frac{1}{2}-\frac{\arg\left(\frac{\left(i\,x+\sqrt{1-x^2}\right)^{i}}{y+\sqrt{y^2+1}}+1\right)}{2\,\pi}\right\rfloor+\left\lfloor-\frac{\arg\left(\frac{\left(i\,x+\sqrt{1-x^2}\right)^{i}}{y+\sqrt{y^2+1}}\right)}{\pi}\right\rfloor}$$

$$\cos^{-1}\left(\frac{1}{2}\left(i\,x+\sqrt{1-x^2}\right)^{-i}\left(y+\sqrt{y^2+1}\right)\left(\frac{\left(i\,x+\sqrt{1-x^2}\right)^{2\,i}}{\left(y+\sqrt{y^2+1}\right)^2}+1\right)\right)+\frac{\pi}{2}$$

01.13.16.0181.01

$$\cos^{-1}(x)-i\sinh^{-1}(y)=$$

$$-\frac{i\left(\sqrt{1-x^2}\,\sqrt{y^2+1}-i\,x\,y\right)}{\sqrt{\left(\sqrt{1-x^2}\,\sqrt{y^2+1}-i\,x\,y\right)^2}}\sinh^{-1}\left(\sqrt{1-x^2}\,y-i\,x\sqrt{y^2+1}\right)+\frac{1}{2}\pi\frac{\sqrt{1-x^2}\,\sqrt{y^2+1}-i\,x\,y}{\sqrt{\left(\sqrt{1-x^2}\,\sqrt{y^2+1}-i\,x\,y\right)^2}}-$$

$$\pi\left\lfloor\frac{\arg\left(i\,x+\sqrt{1-x^2}\right)+\arg\left(\sqrt{y^2+1}-y\right)}{2\,\pi}\right\rfloor\left(1-\frac{\sqrt{1-x^2}\,\sqrt{y^2+1}-i\,x\,y}{\sqrt{\left(\sqrt{1-x^2}\,\sqrt{y^2+1}-i\,x\,y\right)^2}}\right)+$$

$$\pi\left(\frac{\sqrt{1-x^2}\,\sqrt{y^2+1}-i\,x\,y}{\sqrt{\left(\sqrt{1-x^2}\,\sqrt{y^2+1}-i\,x\,y\right)^2}}+1\right)\left\lfloor\frac{-\arg\left(i\,x+\sqrt{1-x^2}\right)-\arg\left(\sqrt{y^2+1}-y\right)+\pi}{2\,\pi}\right\rfloor$$

01.13.16.0182.01

$$\cos^{-1}(x) - i\sinh^{-1}(y) = \cos^{-1}\left((-1)^{\left\lfloor \frac{1}{2} - \frac{\arg\left(\sqrt{1-x^2}\sqrt{y^2+1} - i x y\right)}{\pi} \right\rfloor}\left(\sqrt{y^2+1}\, x + i\sqrt{1-x^2}\, y\right)\right) + \frac{1}{2}\pi\left(2\left(-1 + (-1)^{\left\lfloor \frac{1}{2} - \frac{\arg\left(\sqrt{1-x^2}\sqrt{y^2+1} - i x y\right)}{\pi} \right\rfloor}\right)\left\lfloor \frac{\arg\left(i x + \sqrt{1-x^2}\right) + \arg\left(\sqrt{y^2+1} - y\right)}{2\pi} \right\rfloor + (-1)^{\left\lfloor \frac{1}{2} - \frac{\arg\left(\sqrt{1-x^2}\sqrt{y^2+1} - i x y\right)}{\pi} \right\rfloor} + 2\left(1 + (-1)^{\left\lfloor \frac{1}{2} - \frac{\arg\left(\sqrt{1-x^2}\sqrt{y^2+1} - i x y\right)}{\pi} \right\rfloor}\right)\left\lfloor \frac{1}{2} - \frac{\arg\left(i x + \sqrt{1-x^2}\right) + \arg\left(\sqrt{y^2+1} - y\right)}{2\pi} \right\rfloor - 1\right)$$

Involving $\cosh^{-1}(z)$

01.13.16.0183.01

$$\cos^{-1}(x) - \cosh^{-1}(y) = -2 i\pi\left(\left\lfloor \frac{-\arg\left(\frac{1}{y+\sqrt{y-1}\sqrt{y+1}}\right) - \arg\left(\left(i x + \sqrt{1-x^2}\right)^i\right) + \pi}{2\pi} \right\rfloor + \left\lfloor \frac{\operatorname{Im}\left(\log\left(y + \sqrt{y-1}\sqrt{y+1}\right)\right) + \pi}{2\pi} \right\rfloor + \left\lfloor \frac{\pi - \operatorname{Re}\left(\log\left(i x + \sqrt{1-x^2}\right)\right)}{2\pi} \right\rfloor\right) + \log\left(\frac{\left(i x + \sqrt{1-x^2}\right)^i}{y + \sqrt{y-1}\sqrt{y+1}}\right) + \frac{\pi}{2}$$

01.13.16.0184.01

$$\cos^{-1}(x)-\cosh^{-1}(y)=-2\,i\,\pi\left(\left\lfloor\frac{-\arg\left(\frac{1}{y+\sqrt{y-1}\,\sqrt{y+1}}\right)-\arg\left(\left(i\,x+\sqrt{1-x^2}\right)^{i}\right)+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\operatorname{Im}\left(\log\left(y+\sqrt{y-1}\,\sqrt{y+1}\right)\right)+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Re}\left(\log\left(i\,x+\sqrt{1-x^2}\right)\right)}{2\,\pi}\right\rfloor\right)+ i\left(1-(-1)^{\left\lfloor-\frac{\arg\left(\frac{\left(i\,x+\sqrt{1-x^2}\right)^{i}}{y+\sqrt{y-1}\,\sqrt{y+1}}+1\right)}{2\,\pi}\right\rfloor-\left\lfloor-\frac{\arg\left(\frac{\left(i\,x+\sqrt{1-x^2}\right)^{i}}{y+\sqrt{y-1}\,\sqrt{y+1}}\right)}{2\,\pi}\right\rfloor}\right)\pi-i\,(-1)^{\left\lfloor\frac{\arg\left(\frac{\left(i\,x+\sqrt{1-x^2}\right)^{i}}{y+\sqrt{y-1}\,\sqrt{y+1}}-1\right)}{2\,\pi}+\frac{1}{2}-\frac{\arg\left(\frac{\left(i\,x+\sqrt{1-x^2}\right)^{i}}{y+\sqrt{y-1}\,\sqrt{y+1}}+1\right)}{2\,\pi}\right\rfloor+\left\lfloor-\frac{\arg\left(\frac{\left(i\,x+\sqrt{1-x^2}\right)^{i}}{y+\sqrt{y-1}\,\sqrt{y+1}}\right)}{\pi}\right\rfloor} \cos^{-1}\left(\frac{1}{2}\left(i\,x+\sqrt{1-x^2}\right)^{-i}\left(y+\sqrt{y-1}\,\sqrt{y+1}\right)\left(\frac{\left(i\,x+\sqrt{1-x^2}\right)^{2\,i}}{\left(y+\sqrt{y-1}\,\sqrt{y+1}\right)^2}+1\right)\right)+\frac{\pi}{2}$$

01.13.16.0185.01

$$\cos^{-1}(x)-i\cosh^{-1}(y)=\cos^{-1}\left((-1)^{\left\lfloor \frac{1}{2}-\frac{\arg\left(x\,y+i\sqrt{1-x^2}\,\sqrt{y-1}\,\sqrt{y+1}\right)}{\pi}\right\rfloor}\left(i\,x\sqrt{y-1}\,\sqrt{y+1}-\sqrt{1-x^2}\,y\right)\right)-$$

$$\frac{1}{2}\pi\left(\frac{1}{2}\left(1-(-1)^{-\left\lfloor -\frac{\arg(1-y)}{2\pi}\right\rfloor}\right)\left(2\left(-1+(-1)^{\left\lfloor \frac{1}{2}-\frac{\arg\left(x\,y+\sqrt{1-x^2}\,\sqrt{1-y^2}\right)}{\pi}\right\rfloor}\right)\left\lfloor \frac{\arg\left(\sqrt{1-x^2}-i\,x\right)+\arg\left(i\,y+\sqrt{1-y^2}\right)}{2\pi}\right\rfloor+\right.\right.$$

$$\left.(-1)^{\left\lfloor \frac{1}{2}-\frac{\arg\left(x\,y+\sqrt{1-x^2}\,\sqrt{1-y^2}\right)}{\pi}\right\rfloor}+2\left(1+(-1)^{\left\lfloor \frac{1}{2}-\frac{\arg\left(x\,y+\sqrt{1-x^2}\,\sqrt{1-y^2}\right)}{\pi}\right\rfloor}\right)\left\lfloor \frac{1}{2}-\frac{\arg\left(\sqrt{1-x^2}-i\,x\right)+\arg\left(i\,y+\sqrt{1-y^2}\right)}{2\pi}\right\rfloor\right)-$$

$$\frac{1}{2}\left(1+(-1)^{-\left\lfloor -\frac{\arg(1-y)}{2\pi}\right\rfloor}\right)\left(2\left(-1+(-1)^{\left\lfloor \frac{1}{2}-\frac{\arg\left(\sqrt{1-x^2}\,\sqrt{1-y^2}-x\,y\right)}{\pi}\right\rfloor}\right)\left\lfloor \frac{\arg\left(i\,x+\sqrt{1-x^2}\right)+\arg\left(i\,y+\sqrt{1-y^2}\right)}{2\pi}\right\rfloor+\right.$$

$$\left.\left.(-1)^{\left\lfloor \frac{1}{2}-\frac{\arg\left(\sqrt{1-x^2}\,\sqrt{1-y^2}-x\,y\right)}{\pi}\right\rfloor}+2\left(1+(-1)^{\left\lfloor \frac{1}{2}-\frac{\arg\left(\sqrt{1-x^2}\,\sqrt{1-y^2}-x\,y\right)}{\pi}\right\rfloor}\right)\left\lfloor \frac{1}{2}-\frac{\arg\left(i\,x+\sqrt{1-x^2}\right)+\arg\left(i\,y+\sqrt{1-y^2}\right)}{2\pi}\right\rfloor\right)\right)$$

Involving $\tanh^{-1}(z)$

01.13.16.0186.01

$$\cos^{-1}(x) - \tanh^{-1}(y) = -2\, i\, \pi \left(\left\lfloor \frac{\frac{1}{2}\arg(y+1) - \arg\left(\left(i\, x + \sqrt{1-x^2}\right)^i \sqrt{1-y}\right) + \pi}{2\pi} \right\rfloor + \left\lfloor \frac{\frac{1}{2}\operatorname{Im}(\log(y+1)) + \pi}{2\pi} \right\rfloor + \left\lfloor \frac{\pi - \operatorname{Im}\left(\log\left(\left(i\, x + \sqrt{1-x^2}\right)^i \sqrt{1-y}\right)\right)}{2\pi} \right\rfloor \right) - 2\, i\, \pi \left(\left\lfloor \frac{-\arg\left(\left(i\, x + \sqrt{1-x^2}\right)^i\right) - \frac{1}{2}\arg(1-y) + \pi}{2\pi} \right\rfloor + \left\lfloor \frac{\pi - \frac{1}{2}\operatorname{Im}(\log(1-y))}{2\pi} \right\rfloor + \left\lfloor \frac{\pi - \operatorname{Re}\left(\log\left(i\, x + \sqrt{1-x^2}\right)\right)}{2\pi} \right\rfloor \right) + \log\left(\frac{\left(i\, x + \sqrt{1-x^2}\right)^i \sqrt{1-y}}{\sqrt{y+1}} \right) + \frac{\pi}{2}$$

01.13.16.0187.01

$$\cos^{-1}(x)-\tanh^{-1}(y)=-2\,i\,\pi\left(\left\lfloor\frac{\frac{1}{2}\arg(y+1)-\arg\left(\left(i\,x+\sqrt{1-x^2}\right)^{i}\sqrt{1-y}\right)+\pi}{2\pi}\right\rfloor+\left\lfloor\frac{\frac{1}{2}\operatorname{Im}(\log(y+1))+\pi}{2\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}\left(\log\left(\left(i\,x+\sqrt{1-x^2}\right)^{i}\sqrt{1-y}\right)\right)}{2\pi}\right\rfloor\right)-$$

$$2\,i\,\pi\left(\left\lfloor\frac{-\arg\left(\left(i\,x+\sqrt{1-x^2}\right)^{i}\right)-\frac{1}{2}\arg(1-y)+\pi}{2\pi}\right\rfloor+\left\lfloor\frac{\pi-\frac{1}{2}\operatorname{Im}(\log(1-y))}{2\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Re}\left(\log\left(i\,x+\sqrt{1-x^2}\right)\right)}{2\pi}\right\rfloor\right)+$$

$$i\left(1-(-1)^{\left\lfloor-\frac{\arg\left(\frac{\sqrt{1-y}\left(i\,x+\sqrt{1-x^2}\right)^{i}}{\sqrt{y+1}}+1\right)}{2\pi}\right\rfloor-\left\lfloor-\frac{\arg\left(\frac{\left(i\,x+\sqrt{1-x^2}\right)^{i}\sqrt{1-y}}{\sqrt{y+1}}\right)}{2\pi}\right\rfloor}\right)\pi-$$

$$i\,(-1)^{\left\lfloor\frac{\arg\left(\frac{\left(i\,x+\sqrt{1-x^2}\right)^{i}\sqrt{1-y}}{\sqrt{y+1}}-1\right)}{2\pi}+\frac{1}{2}-\frac{\arg\left(\frac{\sqrt{1-y}\left(i\,x+\sqrt{1-x^2}\right)^{i}}{\sqrt{y+1}}+1\right)}{2\pi}\right\rfloor+\left\lfloor-\frac{\arg\left(\frac{\left(i\,x+\sqrt{1-x^2}\right)^{i}\sqrt{1-y}}{\sqrt{y+1}}\right)}{\pi}\right\rfloor}$$

$$\cos^{-1}\left(\frac{\left(i\,x+\sqrt{1-x^2}\right)^{-i}\sqrt{y+1}\left(\frac{(1-y)\left(i\,x+\sqrt{1-x^2}\right)^{2i}}{y+1}+1\right)}{2\sqrt{1-y}}\right)+\frac{\pi}{2}$$

01.13.16.0188.01

$$\cos^{-1}(x) - i\tanh^{-1}(y) = \frac{\pi}{2} - \frac{1}{2\sqrt{\frac{\left(x+i\sqrt{1-x^2}\,y\right)^2}{1-y^2}}\,\sqrt{1-y^2}}\Bigg(\left(x+i\sqrt{1-x^2}\,y\right)\left(\pi - 2\sin^{-1}\left(\frac{\sqrt{1-x^2}-i\,x\,y}{\sqrt{1-y^2}}\right)\right) +$$

$$2\pi\left(x+i\sqrt{1-x^2}\,y+\sqrt{\frac{y^2x^2+x^2+2\,i\sqrt{1-x^2}\,y\,x-y^2}{1-y^2}}\,\sqrt{1-y^2}\right)\left\lfloor\frac{\arg\left(i\,x+\sqrt{1-x^2}\right)+\arg\left(\frac{i-i\,y}{\sqrt{1-y^2}}\right)}{2\pi}\right\rfloor -$$

$$2\pi\left(-x-i\sqrt{1-x^2}\,y+\sqrt{\frac{y^2x^2+x^2+2\,i\sqrt{1-x^2}\,y\,x-y^2}{1-y^2}}\,\sqrt{1-y^2}\right)\left\lfloor-\frac{\arg\left(i\,x+\sqrt{1-x^2}\right)+\arg\left(\frac{i-i\,y}{\sqrt{1-y^2}}\right)-\pi}{2\pi}\right\rfloor\Bigg)$$

01.13.16.0189.01

$$\cos^{-1}(x) - i\tanh^{-1}(y) =$$

$$\cos^{-1}\left(\frac{(-1)^{\left\lfloor\frac{1}{2}-\frac{\arg\left(\frac{x+i\sqrt{1-x^2}\,y}{\sqrt{1-y^2}}\right)}{\pi}\right\rfloor}\left(i\,x\,y-\sqrt{1-x^2}\right)}{\sqrt{1-y^2}}\right) - \frac{1}{2}\pi\Bigg(2\left(1+(-1)^{\left\lfloor\frac{1}{2}-\frac{\arg\left(\frac{x+i\sqrt{1-x^2}\,y}{\sqrt{1-y^2}}\right)}{\pi}\right\rfloor}\right)\left\lfloor\frac{\arg\left(i\,x+\sqrt{1-x^2}\right)+\arg\left(\frac{i-i\,y}{\sqrt{1-y^2}}\right)}{2\pi}\right\rfloor +$$

$$(-1)^{\left\lfloor\frac{1}{2}-\frac{\arg\left(\frac{x+i\sqrt{1-x^2}\,y}{\sqrt{1-y^2}}\right)}{\pi}\right\rfloor} - 2\left(-1+(-1)^{\left\lfloor\frac{1}{2}-\frac{\arg\left(\frac{x+i\sqrt{1-x^2}\,y}{\sqrt{1-y^2}}\right)}{\pi}\right\rfloor}\right)\left\lfloor\frac{1}{2}-\frac{\arg\left(i\,x+\sqrt{1-x^2}\right)+\arg\left(\frac{i-i\,y}{\sqrt{1-y^2}}\right)}{2\pi}\right\rfloor\Bigg)$$

Involving $\coth^{-1}(z)$

01.13.16.0190.01

$$\cos^{-1}(x)-\coth^{-1}(y)=-2\,i\,\pi\left(\left\lfloor\frac{\frac{1}{2}\arg\left(1+\frac{1}{y}\right)-\arg\left(\left(i\,x+\sqrt{1-x^2}\right)^{i}\sqrt{1-\frac{1}{y}}\right)+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\frac{1}{2}\operatorname{Im}\left(\log\left(1+\frac{1}{y}\right)\right)+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}\left(\log\left(\left(i\,x+\sqrt{1-x^2}\right)^{i}\sqrt{1-\frac{1}{y}}\right)\right)}{2\,\pi}\right\rfloor\right)-2\,i\,\pi\left(\left\lfloor\frac{-\arg\left(\left(i\,x+\sqrt{1-x^2}\right)^{i}\right)-\frac{1}{2}\arg\left(1-\frac{1}{y}\right)+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\pi-\frac{1}{2}\operatorname{Im}\left(\log\left(1-\frac{1}{y}\right)\right)}{2\,\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Re}\left(\log\left(i\,x+\sqrt{1-x^2}\right)\right)}{2\,\pi}\right\rfloor\right)+\log\left(\frac{\left(i\,x+\sqrt{1-x^2}\right)^{i}\sqrt{1-\frac{1}{y}}}{\sqrt{1+\frac{1}{y}}}\right)+\frac{\pi}{2}$$

01.13.16.0191.01

$$\cos^{-1}(x) - \coth^{-1}(y) = -2\,i\,\pi \left(\left\lfloor \frac{\frac{1}{2}\arg\left(1+\frac{1}{y}\right) - \arg\left(\left(i\,x+\sqrt{1-x^2}\right)^{i}\sqrt{1-\frac{1}{y}}\right) + \pi}{2\,\pi} \right\rfloor + \left\lfloor \frac{\frac{1}{2}\operatorname{Im}\left(\log\left(1+\frac{1}{y}\right)\right) + \pi}{2\,\pi} \right\rfloor + \left\lfloor \frac{\pi - \operatorname{Im}\left(\log\left(\left(i\,x+\sqrt{1-x^2}\right)^{i}\sqrt{1-\frac{1}{y}}\right)\right)}{2\,\pi} \right\rfloor \right) - 2\,i\,\pi \left(\left\lfloor \frac{-\arg\left(\left(i\,x+\sqrt{1-x^2}\right)^{i}\right) - \frac{1}{2}\arg\left(1-\frac{1}{y}\right) + \pi}{2\,\pi} \right\rfloor + \left\lfloor \frac{\pi - \frac{1}{2}\operatorname{Im}\left(\log\left(1-\frac{1}{y}\right)\right)}{2\,\pi} \right\rfloor + \left\lfloor \frac{\pi - \operatorname{Re}\left(\log\left(i\,x+\sqrt{1-x^2}\right)\right)}{2\,\pi} \right\rfloor \right) + i\left(1 - (-1)^{\left\lfloor -\frac{\arg\left(\frac{\sqrt{1-\frac{1}{y}}\left(i\,x+\sqrt{1-x^2}\right)^{i}}{\sqrt{1+\frac{1}{y}}}+1\right)}{2\,\pi} \right\rfloor - \left\lfloor -\frac{\arg\left(\frac{\left(i\,x+\sqrt{1-x^2}\right)^{i}\sqrt{1-\frac{1}{y}}}{\sqrt{1+\frac{1}{y}}}\right)}{2\,\pi} \right\rfloor}\right)\pi - i\,(-1)^{\left\lfloor \frac{\arg\left(\frac{\left(i\,x+\sqrt{1-x^2}\right)^{i}\sqrt{1-\frac{1}{y}}}{\sqrt{1+\frac{1}{y}}}-1\right)}{2\,\pi} + \frac{1}{2} - \frac{\arg\left(\frac{\sqrt{1-\frac{1}{y}}\left(i\,x+\sqrt{1-x^2}\right)^{i}}{\sqrt{1+\frac{1}{y}}}+1\right)}{2\,\pi} \right\rfloor + \left\lfloor -\frac{\arg\left(\frac{\left(i\,x+\sqrt{1-x^2}\right)^{i}\sqrt{1-\frac{1}{y}}}{\sqrt{1+\frac{1}{y}}}\right)}{\pi} \right\rfloor} \cos^{-1}\left(\frac{\left(i\,x+\sqrt{1-x^2}\right)^{-i}\left(\frac{\left(1-\frac{1}{y}\right)\left(i\,x+\sqrt{1-x^2}\right)^{2\,i}}{1+\frac{1}{y}}+1\right)\sqrt{1+\frac{1}{y}}}{2\sqrt{1-\frac{1}{y}}}\right) + \frac{\pi}{2}$$

01.13.16.0192.01

$$\cos^{-1}(x) - i\coth^{-1}(y) = \frac{\pi}{2} - \frac{1}{2\sqrt{1-\frac{1}{y^2}}\sqrt{\frac{\left(i\sqrt{1-x^2}+x\,y\right)^2}{y^2-1}}}\left(\left(x+\frac{i\sqrt{1-x^2}}{y}\right)\left(\pi - 2\sin^{-1}\left(\frac{\sqrt{1-x^2}-\frac{i\,x}{y}}{\sqrt{1-\frac{1}{y^2}}}\right)\right)+\right.$$
$$2\pi\left(x+\sqrt{1-\frac{1}{y^2}}\sqrt{\frac{(y^2+1)\,x^2+2\,i\sqrt{1-x^2}\;y\,x-1}{y^2-1}}+\frac{i\sqrt{1-x^2}}{y}\right)\left\lfloor\frac{\arg\left(i\,x+\sqrt{1-x^2}\right)+\arg\left(\frac{i-\frac{i}{y}}{\sqrt{1-\frac{1}{y^2}}}\right)}{2\pi}\right\rfloor -$$
$$\left.2\pi\left(-x+\sqrt{1-\frac{1}{y^2}}\sqrt{\frac{(y^2+1)\,x^2+2\,i\sqrt{1-x^2}\;y\,x-1}{y^2-1}}-\frac{i\sqrt{1-x^2}}{y}\right)\left\lfloor-\frac{\arg\left(i\,x+\sqrt{1-x^2}\right)+\arg\left(\frac{i-\frac{i}{y}}{\sqrt{1-\frac{1}{y^2}}}\right)-\pi}{2\pi}\right\rfloor\right)$$

01.13.16.0193.01

$$\cos^{-1}(x) - i \coth^{-1}(y) =$$

$$\cos^{-1}\left(\frac{(-1)^{\left\lfloor \frac{1}{2} - \frac{\arg\left(\frac{x + \frac{i \sqrt{1-x^2}}{y}}{\sqrt{1-\frac{1}{y^2}}}\right)}{\pi} \right\rfloor} \left(\frac{i x}{y} - \sqrt{1-x^2}\right)}{\sqrt{1-\frac{1}{y^2}}}\right) - \frac{1}{2} \pi \left(2 \left(1 + (-1)^{\left\lfloor \frac{1}{2} - \frac{\arg\left(\frac{x + \frac{i \sqrt{1-x^2}}{y}}{\sqrt{1-\frac{1}{y^2}}}\right)}{\pi} \right\rfloor}\right) \left\lfloor \frac{\arg\left(i x + \sqrt{1-x^2}\right) + \arg\left(\frac{i - \frac{i}{y}}{\sqrt{1-\frac{1}{y^2}}}\right)}{2 \pi} \right\rfloor + \right.$$

$$\left. (-1)^{\left\lfloor \frac{1}{2} - \frac{\arg\left(\frac{x + \frac{i \sqrt{1-x^2}}{y}}{\sqrt{1-\frac{1}{y^2}}}\right)}{\pi} \right\rfloor} - 2 \left(-1 + (-1)^{\left\lfloor \frac{1}{2} - \frac{\arg\left(\frac{x + \frac{i \sqrt{1-x^2}}{y}}{\sqrt{1-\frac{1}{y^2}}}\right)}{\pi} \right\rfloor}\right) \left\lfloor \frac{1}{2} - \frac{\arg\left(i x + \sqrt{1-x^2}\right) + \arg\left(\frac{i - \frac{i}{y}}{\sqrt{1-\frac{1}{y^2}}}\right)}{2 \pi} \right\rfloor \right)$$

Involving $\operatorname{csch}^{-1}(z)$

01.13.16.0194.01

$$\cos^{-1}(x)-\operatorname{csch}^{-1}(y)=$$
$$-2\,i\,\pi\left(\left\lfloor\frac{-\arg\left(\left(i\,x+\sqrt{1-x^2}\right)^{i}\right)-\arg\left(\frac{1}{\sqrt{1+\frac{1}{y^2}}+\frac{1}{y}}\right)+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\operatorname{Im}\left(\log\left(\sqrt{1+\frac{1}{y^2}}+\frac{1}{y}\right)\right)+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Re}\left(\log\left(i\,x+\sqrt{1-x^2}\right)\right)}{2\,\pi}\right\rfloor\right)+$$
$$\log\left(\frac{\left(i\,x+\sqrt{1-x^2}\right)^{i}}{\sqrt{1+\frac{1}{y^2}}+\frac{1}{y}}\right)+\frac{\pi}{2}$$

01.13.16.0195.01

$$\cos^{-1}(x)-\operatorname{csch}^{-1}(y)=$$
$$-2\,i\,\pi\left(\left\lfloor\frac{-\arg\left(\left(i\,x+\sqrt{1-x^2}\right)^{i}\right)-\arg\left(\frac{1}{\sqrt{1+\frac{1}{y^2}}+\frac{1}{y}}\right)+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\operatorname{Im}\left(\log\left(\sqrt{1+\frac{1}{y^2}}+\frac{1}{y}\right)\right)+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Re}\left(\log\left(i\,x+\sqrt{1-x^2}\right)\right)}{2\,\pi}\right\rfloor\right)+$$
$$i\left(1-(-1)^{\left\lfloor-\frac{\arg\left(\frac{\left(i\,x+\sqrt{1-x^2}\right)^{i}}{\sqrt{1+\frac{1}{y^2}}+\frac{1}{y}}\right)+1}{2\,\pi}\right\rfloor-\left\lfloor-\frac{\arg\left(\frac{\left(i\,x+\sqrt{1-x^2}\right)^{i}}{\sqrt{1+\frac{1}{y^2}}+\frac{1}{y}}\right)}{2\,\pi}\right\rfloor}\right)\pi-i\,(-1)^{\left\lfloor\frac{\arg\left(\frac{\left(i\,x+\sqrt{1-x^2}\right)^{i}}{\sqrt{1+\frac{1}{y^2}}+\frac{1}{y}}\right)-1}{2\,\pi}+\frac{1}{2}-\frac{\arg\left(\frac{\left(i\,x+\sqrt{1-x^2}\right)^{i}}{\sqrt{1+\frac{1}{y^2}}+\frac{1}{y}}\right)+1}{2\,\pi}\right\rfloor+\left\lfloor-\frac{\arg\left(\frac{\left(i\,x+\sqrt{1-x^2}\right)^{i}}{\sqrt{1+\frac{1}{y^2}}+\frac{1}{y}}\right)}{\pi}\right\rfloor}$$
$$\cos^{-1}\left(\frac{1}{2}\left(i\,x+\sqrt{1-x^2}\right)^{-i}\left(\frac{\left(i\,x+\sqrt{1-x^2}\right)^{2\,i}}{\left(\sqrt{1+\frac{1}{y^2}}+\frac{1}{y}\right)^{2}}+1\right)\left(\sqrt{1+\frac{1}{y^2}}+\frac{1}{y}\right)\right)+\frac{\pi}{2}$$

01.13.16.0196.01

$$\cos^{-1}(x) - i\,\operatorname{csch}^{-1}(y) = -\frac{\sqrt{1-x^2}\,\sqrt{1+\frac{1}{y^2}} - \frac{i\,x}{y}}{\sqrt{\left(\sqrt{1-x^2}\,\sqrt{1+\frac{1}{y^2}} - \frac{i\,x}{y}\right)^2}} \sin^{-1}\!\left(x\,\sqrt{1+\frac{1}{y^2}} + \frac{i\,\sqrt{1-x^2}}{y}\right) -$$

$$\pi\left\lfloor \frac{\arg\left(i\,x+\sqrt{1-x^2}\right) + \arg\left(\sqrt{1+\frac{1}{y^2}} - \frac{1}{y}\right)}{2\,\pi} \right\rfloor \left(\frac{i\left(x + i\,\sqrt{1-x^2}\;y\,\sqrt{1+\frac{1}{y^2}}\right)}{\sqrt{\left(\sqrt{1-x^2}\,\sqrt{1+\frac{1}{y^2}} - \frac{i\,x}{y}\right)^2}\;y} + 1\right) -$$

$$\frac{1}{2}\,\pi\,\frac{i\left(x + i\,\sqrt{1-x^2}\;y\,\sqrt{1+\frac{1}{y^2}}\right)}{\sqrt{\left(\sqrt{1-x^2}\,\sqrt{1+\frac{1}{y^2}} - \frac{i\,x}{y}\right)^2}\;y} +$$

$$\pi\left(\frac{\sqrt{1-x^2}\,\sqrt{1+\frac{1}{y^2}} - \frac{i\,x}{y}}{\sqrt{\left(\sqrt{1-x^2}\,\sqrt{1+\frac{1}{y^2}} - \frac{i\,x}{y}\right)^2}} + 1\right) \left\lfloor -\frac{\arg\left(i\,x+\sqrt{1-x^2}\right) + \arg\left(\sqrt{1+\frac{1}{y^2}} - \frac{1}{y}\right) - \pi}{2\,\pi} \right\rfloor$$

01.13.16.0197.01

$$\cos^{-1}(x) - i\,\text{csch}^{-1}(y) = \cos^{-1}\left((-1)^{\left\lfloor \frac{1}{2} - \frac{\arg\left(\sqrt{1-x^2}\,\sqrt{1+\frac{1}{y^2}} - \frac{i\,x}{y}\right)}{\pi} \right\rfloor} \left(x\sqrt{1+\frac{1}{y^2}} + \frac{i\sqrt{1-x^2}}{y}\right)\right) + \frac{1}{2}\pi\left(2\left(-1+(-1)^{\left\lfloor \frac{1}{2} - \frac{\arg\left(\sqrt{1-x^2}\,\sqrt{1+\frac{1}{y^2}} - \frac{i\,x}{y}\right)}{\pi} \right\rfloor}\right)\left\lfloor \frac{\arg\left(i\,x+\sqrt{1-x^2}\right)+\arg\left(\sqrt{1+\frac{1}{y^2}}-\frac{1}{y}\right)}{2\pi} \right\rfloor + (-1)^{\left\lfloor \frac{1}{2} - \frac{\arg\left(\sqrt{1-x^2}\,\sqrt{1+\frac{1}{y^2}} - \frac{i\,x}{y}\right)}{\pi} \right\rfloor} + 2\left(1+(-1)^{\left\lfloor \frac{1}{2} - \frac{\arg\left(\sqrt{1-x^2}\,\sqrt{1+\frac{1}{y^2}} - \frac{i\,x}{y}\right)}{\pi} \right\rfloor}\right)\left\lfloor \frac{1}{2} - \frac{\arg\left(i\,x+\sqrt{1-x^2}\right)+\arg\left(\sqrt{1+\frac{1}{y^2}}-\frac{1}{y}\right)}{2\pi} \right\rfloor - 1\right)$$

Involving $\text{sech}^{-1}(z)$

01.13.16.0198.01

$$\cos^{-1}(x) - \text{sech}^{-1}(y) = -2\,i\,\pi\left(\left\lfloor \frac{-\arg\left(\left(i\,x+\sqrt{1-x^2}\right)^i\right) - \arg\left(\frac{1}{\sqrt{\frac{1}{y}-1}\,\sqrt{1+\frac{1}{y}}+\frac{1}{y}}\right)+\pi}{2\pi} \right\rfloor + \left\lfloor \frac{\text{Im}\left(\log\left(\sqrt{\frac{1}{y}-1}\,\sqrt{1+\frac{1}{y}}+\frac{1}{y}\right)\right)+\pi}{2\pi} \right\rfloor + \left\lfloor \frac{\pi - \text{Re}\left(\log\left(i\,x+\sqrt{1-x^2}\right)\right)}{2\pi} \right\rfloor\right) + \log\left(\frac{\left(i\,x+\sqrt{1-x^2}\right)^i}{\sqrt{\frac{1}{y}-1}\,\sqrt{1+\frac{1}{y}}+\frac{1}{y}}\right) + \frac{\pi}{2}$$

01.13.16.0199.01

$$\cos^{-1}(x)-\operatorname{sech}^{-1}(y)=-2\,i\,\pi\left(\left\lfloor\frac{-\arg\left(\left(i\,x+\sqrt{1-x^2}\right)^{i}\right)-\arg\left(\frac{1}{\sqrt{\frac{1}{y}-1}\,\sqrt{1+\frac{1}{y}}+\frac{1}{y}}\right)+\pi}{2\,\pi}\right\rfloor+\right.$$

$$\left.\left\lfloor\frac{\operatorname{Im}\left(\log\left(\sqrt{\frac{1}{y}-1}\,\sqrt{1+\frac{1}{y}}+\frac{1}{y}\right)\right)+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Re}\left(\log\left(i\,x+\sqrt{1-x^2}\right)\right)}{2\,\pi}\right\rfloor\right)+$$

$$i\left(1-(-1)^{\left\lfloor-\frac{\arg\left(\frac{\left(i\,x+\sqrt{1-x^2}\right)^{i}}{\sqrt{\frac{1}{y}-1}\,\sqrt{1+\frac{1}{y}}+\frac{1}{y}}+1\right)}{2\,\pi}\right\rfloor-\left\lfloor-\frac{\arg\left(\frac{\left(i\,x+\sqrt{1-x^2}\right)^{i}}{\sqrt{\frac{1}{y}-1}\,\sqrt{1+\frac{1}{y}}+\frac{1}{y}}\right)}{2\,\pi}\right\rfloor}\right)\pi-i\,(-1)^{\left\lfloor\frac{\arg\left(\frac{\left(i\,x+\sqrt{1-x^2}\right)^{i}}{\sqrt{\frac{1}{y}-1}\,\sqrt{1+\frac{1}{y}}+\frac{1}{y}}-1\right)}{2\,\pi}+\frac{1}{2}-\frac{\arg\left(\frac{\left(i\,x+\sqrt{1-x^2}\right)^{i}}{\sqrt{\frac{1}{y}-1}\,\sqrt{1+\frac{1}{y}}+\frac{1}{y}}+1\right)}{2\,\pi}\right\rfloor+\left\lfloor-\frac{\arg\left(\frac{\left(i\,x+\sqrt{1-x^2}\right)^{i}}{\sqrt{\frac{1}{y}-1}\,\sqrt{1+\frac{1}{y}}+\frac{1}{y}}\right)}{\pi}\right\rfloor}$$

$$\cos^{-1}\left(\frac{1}{2}\left(i\,x+\sqrt{1-x^2}\right)^{-i}\left(\frac{\left(i\,x+\sqrt{1-x^2}\right)^{2\,i}}{\left(\sqrt{\frac{1}{y}-1}\,\sqrt{1+\frac{1}{y}}+\frac{1}{y}\right)^{2}}+1\right)\left(\sqrt{\frac{1}{y}-1}\,\sqrt{1+\frac{1}{y}}+\frac{1}{y}\right)\right)+\frac{\pi}{2}$$

01.13.16.0200.01

$$\cos^{-1}(x) - i\,\text{sech}^{-1}(y) = \cos^{-1}\left((-1)^{\left\lfloor \frac{1}{2} - \frac{\arg\left(i \sqrt{1-x^2} \sqrt{\frac{1}{y}-1} \sqrt{1+\frac{1}{y}} + \frac{x}{y}\right)}{\pi} \right\rfloor} \left(i\,x \sqrt{\frac{1}{y}-1} \sqrt{1+\frac{1}{y}} - \frac{\sqrt{1-x^2}}{y} \right) \right) -$$

$$\frac{1}{2}\pi \left(\frac{1}{2}\left(1 - (-1)^{-\left\lfloor -\frac{\arg\left(1-\frac{1}{y}\right)}{2\pi} \right\rfloor} \right) \left(2\left(-1 + (-1)^{\left\lfloor \frac{1}{2} - \frac{\arg\left(\frac{x}{y} + \sqrt{1-x^2}\sqrt{1-\frac{1}{y^2}}\right)}{\pi} \right\rfloor} \right) \left\lfloor \frac{\arg\left(\sqrt{1-x^2} - i\,x\right) + \arg\left(\sqrt{1-\frac{1}{y^2}} + \frac{i}{y}\right)}{2\pi} \right\rfloor + \right.\right.$$

$$\left. (-1)^{\left\lfloor \frac{1}{2} - \frac{\arg\left(\frac{x}{y} + \sqrt{1-x^2}\sqrt{1-\frac{1}{y^2}}\right)}{\pi} \right\rfloor} + 2\left(1 + (-1)^{\left\lfloor \frac{1}{2} - \frac{\arg\left(\frac{x}{y} + \sqrt{1-x^2}\sqrt{1-\frac{1}{y^2}}\right)}{\pi} \right\rfloor} \right) \left\lfloor \frac{1}{2} - \frac{\arg\left(\sqrt{1-x^2} - i\,x\right) + \arg\left(\sqrt{1-\frac{1}{y^2}} + \frac{i}{y}\right)}{2\pi} \right\rfloor \right) -$$

$$\frac{1}{2}\left(1 + (-1)^{-\left\lfloor -\frac{\arg\left(1-\frac{1}{y}\right)}{2\pi} \right\rfloor} \right) \left(2\left(-1 + (-1)^{\left\lfloor \frac{1}{2} - \frac{\arg\left(\sqrt{1-x^2}\sqrt{1-\frac{1}{y^2}} - \frac{x}{y}\right)}{\pi} \right\rfloor} \right) \left\lfloor \frac{\arg\left(i\,x + \sqrt{1-x^2}\right) + \arg\left(\sqrt{1-\frac{1}{y^2}} + \frac{i}{y}\right)}{2\pi} \right\rfloor + \right.$$

$$\left.\left. (-1)^{\left\lfloor \frac{1}{2} - \frac{\arg\left(\sqrt{1-x^2}\sqrt{1-\frac{1}{y^2}} - \frac{x}{y}\right)}{\pi} \right\rfloor} + 2\left(1 + (-1)^{\left\lfloor \frac{1}{2} - \frac{\arg\left(\sqrt{1-x^2}\sqrt{1-\frac{1}{y^2}} - \frac{x}{y}\right)}{\pi} \right\rfloor} \right) \left\lfloor \frac{1}{2} - \frac{\arg\left(i\,x + \sqrt{1-x^2}\right) + \arg\left(\sqrt{1-\frac{1}{y^2}} + \frac{i}{y}\right)}{2\pi} \right\rfloor \right)\right)$$

Linear combinations involving the direct function

Involving log(*z*)

01.13.16.0201.01

$$a\cos^{-1}(x)+b\log(y)=\frac{\pi a}{2}-2i\pi\left(\left\lfloor\frac{-\arg\left(\left(ix+\sqrt{1-x^2}\right)^{ia}\right)-\arg\left(y^b\right)+\pi}{2\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}(b\log(y))}{2\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Re}\left(a\log\left(ix+\sqrt{1-x^2}\right)\right)}{2\pi}\right\rfloor\right)+\log\left(\left(ix+\sqrt{1-x^2}\right)^{ia}y^b\right)$$

01.13.16.0202.01

$$a\cos^{-1}(x)+b\log(y)=\frac{\pi a}{2}-i(-1)^{\left\lfloor\frac{\arg\left(\left(ix+\sqrt{1-x^2}\right)^{ia}y^b-1\right)}{2\pi}+\frac{1}{2}-\frac{\arg\left(y^b\left(ix+\sqrt{1-x^2}\right)^{ia}+1\right)}{2\pi}\right\rfloor+\left\lfloor-\frac{\arg\left(\left(ix+\sqrt{1-x^2}\right)^{ia}y^b\right)}{\pi}\right\rfloor}\cos^{-1}\left(\frac{1}{2}\left(ix+\sqrt{1-x^2}\right)^{-ia}y^{-b}\left(y^{2b}\left(ix+\sqrt{1-x^2}\right)^{2ia}+1\right)\right)-2i\pi\left(\left\lfloor\frac{-\arg\left(\left(ix+\sqrt{1-x^2}\right)^{ia}\right)-\arg\left(y^b\right)+\pi}{2\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}(b\log(y))}{2\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Re}\left(a\log\left(ix+\sqrt{1-x^2}\right)\right)}{2\pi}\right\rfloor\right)+\left(1-(-1)^{\left\lfloor-\frac{\arg\left(y^b\left(ix+\sqrt{1-x^2}\right)^{ia}+1\right)}{2\pi}\right\rfloor-\left\lfloor-\frac{\arg\left(\left(ix+\sqrt{1-x^2}\right)^{ia}y^b\right)}{2\pi}\right\rfloor}\right)i\pi$$

Involving $\sin^{-1}(z)$

01.13.16.0203.01

$$a\cos^{-1}(x)+b\sin^{-1}(y)=\frac{\pi a}{2}-2i\pi\left(\left\lfloor\frac{-\arg\left(\left(ix+\sqrt{1-x^2}\right)^{ia}\right)-\arg\left(\left(iy+\sqrt{1-y^2}\right)^{-ib}\right)+\pi}{2\pi}\right\rfloor+\left\lfloor\frac{\operatorname{Re}\left(b\log\left(iy+\sqrt{1-y^2}\right)\right)+\pi}{2\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Re}\left(a\log\left(ix+\sqrt{1-x^2}\right)\right)}{2\pi}\right\rfloor\right)+\log\left(\left(ix+\sqrt{1-x^2}\right)^{ia}\left(iy+\sqrt{1-y^2}\right)^{-ib}\right)$$

01.13.16.0204.01

$$a\cos^{-1}(x)+b\sin^{-1}(y)=\frac{\pi a}{2}-i\,(-1)^{\left\lfloor \frac{\arg\left(\left(i\,x+\sqrt{1-x^2}\right)^{i\,a}\left(i\,y+\sqrt{1-y^2}\right)^{-i\,b}-1\right)}{2\pi}+\frac{1}{2}-\frac{\arg\left(\left(i\,y+\sqrt{1-y^2}\right)^{-i\,b}\left(i\,x+\sqrt{1-x^2}\right)^{i\,a}+1\right)}{2\pi}\right\rfloor+\left\lfloor -\frac{\arg\left(\left(i\,x+\sqrt{1-x^2}\right)^{i\,a}\left(i\,y+\sqrt{1-y^2}\right)^{-i\,b}\right)}{\pi}\right\rfloor}$$
$$\cos^{-1}\left(\frac{1}{2}\left(i\,x+\sqrt{1-x^2}\right)^{-i\,a}\left(i\,y+\sqrt{1-y^2}\right)^{i\,b}\left(\left(i\,y+\sqrt{1-y^2}\right)^{-2\,i\,b}\left(i\,x+\sqrt{1-x^2}\right)^{2\,i\,a}+1\right)\right)-$$
$$2\,i\,\pi\left(\left\lfloor\frac{-\arg\left(\left(i\,x+\sqrt{1-x^2}\right)^{i\,a}\right)-\arg\left(\left(i\,y+\sqrt{1-y^2}\right)^{-i\,b}\right)+\pi}{2\pi}\right\rfloor+\left\lfloor\frac{\mathrm{Re}\left(b\log\left(i\,y+\sqrt{1-y^2}\right)\right)+\pi}{2\pi}\right\rfloor+\right.$$
$$\left.\left\lfloor\frac{\pi-\mathrm{Re}\left(a\log\left(i\,x+\sqrt{1-x^2}\right)\right)}{2\pi}\right\rfloor\right)+\left(1-(-1)^{\left\lfloor-\frac{\arg\left(\left(i\,y+\sqrt{1-y^2}\right)^{-i\,b}\left(i\,x+\sqrt{1-x^2}\right)^{i\,a}+1\right)}{2\pi}\right\rfloor-\left\lfloor-\frac{\arg\left(\left(i\,x+\sqrt{1-x^2}\right)^{i\,a}\left(i\,y+\sqrt{1-y^2}\right)^{-i\,b}\right)}{2\pi}\right\rfloor}\right)i\,\pi$$

Involving $\tan^{-1}(z)$

01.13.16.0205.01

$$a\cos^{-1}(x)+b\tan^{-1}(y)=$$
$$\frac{\pi a}{2}-2\,i\,\pi\left(\left\lfloor\frac{-\arg\left((i\,y+1)^{-\frac{1}{2}(i\,b)}\right)-\arg\left(\left(i\,x+\sqrt{1-x^2}\right)^{i\,a}(1-i\,y)^{\frac{i\,b}{2}}\right)+\pi}{2\pi}\right\rfloor+\left\lfloor\frac{\frac{1}{2}\mathrm{Re}(b\log(i\,y+1))+\pi}{2\pi}\right\rfloor+\right.$$
$$\left.\left\lfloor\frac{\pi-\mathrm{Im}\left(\log\left(\left(i\,x+\sqrt{1-x^2}\right)^{i\,a}(1-i\,y)^{\frac{i\,b}{2}}\right)\right)}{2\pi}\right\rfloor\right)-$$
$$2\,i\,\pi\left(\left\lfloor\frac{-\arg\left(\left(i\,x+\sqrt{1-x^2}\right)^{i\,a}\right)-\arg\left((1-i\,y)^{\frac{i\,b}{2}}\right)+\pi}{2\pi}\right\rfloor+\left\lfloor\frac{\pi-\mathrm{Re}\left(a\log\left(i\,x+\sqrt{1-x^2}\right)\right)}{2\pi}\right\rfloor+\left\lfloor\frac{\pi-\frac{1}{2}\mathrm{Re}(b\log(1-i\,y))}{2\pi}\right\rfloor\right)+$$
$$\log\left(\left(i\,x+\sqrt{1-x^2}\right)^{i\,a}(1-i\,y)^{\frac{i\,b}{2}}(i\,y+1)^{-\frac{1}{2}(i\,b)}\right)$$

01.13.16.0206.01

$$a\cos^{-1}(x)+b\tan^{-1}(y)=$$

$$\frac{\pi a}{2}-i\,(-1)^{\left\lfloor \frac{\arg\left(\left(i x+\sqrt{1-x^2}\right)^{i a}(1-i y)^{\frac{i b}{2}}(i y+1)^{-\frac{1}{2}(i b)}-1\right)}{2\pi}+\frac{1}{2}-\frac{\arg\left((i y+1)^{-\frac{1}{2}(i b)}\left(i x+\sqrt{1-x^2}\right)^{i a}(1-i y)^{\frac{i b}{2}}+1\right)}{2\pi}\right\rfloor+\left\lfloor -\frac{\arg\left(\left(i x+\sqrt{1-x^2}\right)^{i a}(1-i y)^{\frac{i b}{2}}(i y+1)^{-\frac{1}{2}(i b)}\right)}{\pi}\right\rfloor}$$

$$\cos^{-1}\left(\frac{1}{2}\left(i x+\sqrt{1-x^2}\right)^{-i a}\left((i y+1)^{-i b}(1-i y)^{i b}\left(i x+\sqrt{1-x^2}\right)^{2 i a}+1\right)(1-i y)^{-\frac{1}{2}(i b)}(i y+1)^{\frac{i b}{2}}\right)-$$

$$2 i\pi\left(\left\lfloor\frac{-\arg\left((i y+1)^{-\frac{1}{2}(i b)}\right)-\arg\left(\left(i x+\sqrt{1-x^2}\right)^{i a}(1-i y)^{\frac{i b}{2}}\right)+\pi}{2\pi}\right\rfloor+\right.$$

$$\left.\left\lfloor\frac{\frac{1}{2}\operatorname{Re}(b\log(i y+1))+\pi}{2\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}\left(\log\left(\left(i x+\sqrt{1-x^2}\right)^{i a}(1-i y)^{\frac{i b}{2}}\right)\right)}{2\pi}\right\rfloor\right)-$$

$$2 i\pi\left(\left\lfloor\frac{-\arg\left(\left(i x+\sqrt{1-x^2}\right)^{i a}\right)-\arg\left((1-i y)^{\frac{i b}{2}}\right)+\pi}{2\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Re}\left(a\log\left(i x+\sqrt{1-x^2}\right)\right)}{2\pi}\right\rfloor+\left\lfloor\frac{\pi-\frac{1}{2}\operatorname{Re}(b\log(1-i y))}{2\pi}\right\rfloor\right)+$$

$$\left(1-(-1)^{\left\lfloor -\frac{\arg\left((i y+1)^{-\frac{1}{2}(i b)}\left(i x+\sqrt{1-x^2}\right)^{i a}(1-i y)^{\frac{i b}{2}}+1\right)}{2\pi}\right\rfloor-\left\lfloor -\frac{\arg\left(\left(i x+\sqrt{1-x^2}\right)^{i a}(1-i y)^{\frac{i b}{2}}(i y+1)^{-\frac{1}{2}(i b)}\right)}{2\pi}\right\rfloor}\right) i\pi$$

Involving $\cot^{-1}(z)$

01.13.16.0207.01

$$a\cos^{-1}(x)+b\cot^{-1}(y)=\frac{\pi a}{2}-2\,i\,\pi\left(\left\lfloor\frac{-\arg\left(\left(1+\frac{i}{y}\right)^{-\frac{1}{2}(i\,b)}\right)-\arg\left(\left(i\,x+\sqrt{1-x^2}\right)^{i\,a}\left(1-\frac{i}{y}\right)^{\frac{i\,b}{2}}\right)+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\frac{1}{2}\operatorname{Re}\left(b\log\left(1+\frac{i}{y}\right)\right)+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}\left(\log\left(\left(i\,x+\sqrt{1-x^2}\right)^{i\,a}\left(1-\frac{i}{y}\right)^{\frac{i\,b}{2}}\right)\right)}{2\,\pi}\right\rfloor\right)-2\,i\,\pi\left(\left\lfloor\frac{-\arg\left(\left(i\,x+\sqrt{1-x^2}\right)^{i\,a}\right)-\arg\left(\left(1-\frac{i}{y}\right)^{\frac{i\,b}{2}}\right)+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Re}\left(a\log\left(i\,x+\sqrt{1-x^2}\right)\right)}{2\,\pi}\right\rfloor+\left\lfloor\frac{\pi-\frac{1}{2}\operatorname{Re}\left(b\log\left(1-\frac{i}{y}\right)\right)}{2\,\pi}\right\rfloor\right)+\log\left(\left(i\,x+\sqrt{1-x^2}\right)^{i\,a}\left(1-\frac{i}{y}\right)^{\frac{i\,b}{2}}\left(1+\frac{i}{y}\right)^{-\frac{1}{2}(i\,b)}\right)$$

01.13.16.0208.01

$$a\cos^{-1}(x)+b\cot^{-1}(y)=$$

$$\frac{\pi a}{2}-i\,(-1)^{\left\lfloor \frac{\arg\left(\left(i\,x+\sqrt{1-x^2}\right)^{i\,a}\left(1-\frac{i}{y}\right)^{\frac{i\,b}{2}}\left(1+\frac{i}{y}\right)^{-\frac{1}{2}(i\,b)}-1\right)}{2\pi}+\frac{1}{2}-\frac{\arg\left(\left(i\,x+\sqrt{1-x^2}\right)^{i\,a}\left(1+\frac{i}{y}\right)^{-\frac{1}{2}(i\,b)}\left(1-\frac{i}{y}\right)^{\frac{i\,b}{2}}+1\right)}{2\pi}\right\rfloor+\left\lfloor -\frac{\arg\left(\left(i\,x+\sqrt{1-x^2}\right)^{i\,a}\left(1-\frac{i}{y}\right)^{\frac{i\,b}{2}}\left(1+\frac{i}{y}\right)^{-\frac{1}{2}(i\,b)}\right)}{\pi}\right\rfloor}$$

$$\cos^{-1}\left(\frac{1}{2}\left(i\,x+\sqrt{1-x^2}\right)^{-i\,a}\left(\left(1+\frac{i}{y}\right)^{-i\,b}\left(1-\frac{i}{y}\right)^{i\,b}\left(i\,x+\sqrt{1-x^2}\right)^{2\,i\,a}+1\right)\left(1-\frac{i}{y}\right)^{-\frac{1}{2}(i\,b)}\left(1+\frac{i}{y}\right)^{\frac{i\,b}{2}}\right)-$$

$$2\,i\,\pi\left(\left\lfloor\frac{-\arg\left(\left(1+\frac{i}{y}\right)^{-\frac{1}{2}(i\,b)}\right)-\arg\left(\left(i\,x+\sqrt{1-x^2}\right)^{i\,a}\left(1-\frac{i}{y}\right)^{\frac{i\,b}{2}}\right)+\pi}{2\pi}\right\rfloor+\right.$$

$$\left.\left\lfloor\frac{\frac{1}{2}\operatorname{Re}\left(b\log\left(1+\frac{i}{y}\right)\right)+\pi}{2\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}\left(\log\left(\left(i\,x+\sqrt{1-x^2}\right)^{i\,a}\left(1-\frac{i}{y}\right)^{\frac{i\,b}{2}}\right)\right)}{2\pi}\right\rfloor\right)-$$

$$2\,i\,\pi\left(\left\lfloor\frac{-\arg\left(\left(i\,x+\sqrt{1-x^2}\right)^{i\,a}\right)-\arg\left(\left(1-\frac{i}{y}\right)^{\frac{i\,b}{2}}\right)+\pi}{2\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Re}\left(a\log\left(i\,x+\sqrt{1-x^2}\right)\right)}{2\pi}\right\rfloor+\left\lfloor\frac{\pi-\frac{1}{2}\operatorname{Re}\left(b\log\left(1-\frac{i}{y}\right)\right)}{2\pi}\right\rfloor\right)+$$

$$\left(1-(-1)^{\left\lfloor-\frac{\arg\left(\left(i\,x+\sqrt{1-x^2}\right)^{i\,a}\left(1+\frac{i}{y}\right)^{-\frac{1}{2}(i\,b)}\left(1-\frac{i}{y}\right)^{\frac{i\,b}{2}}+1\right)}{2\pi}\right\rfloor-\left\lfloor-\frac{\arg\left(\left(i\,x+\sqrt{1-x^2}\right)^{i\,a}\left(1-\frac{i}{y}\right)^{\frac{i\,b}{2}}\left(1+\frac{i}{y}\right)^{-\frac{1}{2}(i\,b)}\right)}{2\pi}\right\rfloor}\right)i\,\pi$$

Involving $\csc^{-1}(z)$

01.13.16.0209.01

$$a\cos^{-1}(x)+b\csc^{-1}(y)=$$

$$\frac{\pi a}{2}-2 i \pi\left(\left\lfloor\frac{-\arg\left(\left(i x+\sqrt{1-x^2}\right)^{i a}\right)-\arg\left(\left(\sqrt{1-\frac{1}{y^2}}+\frac{i}{y}\right)^{-i b}\right)+\pi}{2 \pi}\right\rfloor+\left\lfloor\frac{\operatorname{Re}\left(b \log\left(\sqrt{1-\frac{1}{y^2}}+\frac{i}{y}\right)\right)+\pi}{2 \pi}\right\rfloor+\right.$$

$$\left.\left\lfloor\frac{\pi-\operatorname{Re}\left(a \log\left(i x+\sqrt{1-x^2}\right)\right)}{2 \pi}\right\rfloor\right)+\log\left(\left(i x+\sqrt{1-x^2}\right)^{i a}\left(\sqrt{1-\frac{1}{y^2}}+\frac{i}{y}\right)^{-i b}\right)$$

01.13.16.0210.01

$$a\cos^{-1}(x)+b\csc^{-1}(y)=\frac{\pi a}{2}-i\,(-1)^{\left\lfloor\frac{\arg\left(\left(i x+\sqrt{1-x^2}\right)^{i a}\left(\sqrt{1-\frac{1}{y^2}}+\frac{i}{y}\right)^{-i b}-1\right)}{2 \pi}+\frac{1}{2}-\frac{\arg\left(\left(\sqrt{1-\frac{1}{y^2}}+\frac{i}{y}\right)^{-i b}\left(i x+\sqrt{1-x^2}\right)^{i a}+1\right)}{2 \pi}\right\rfloor+\left\lfloor-\frac{\arg\left(\left(i x+\sqrt{1-x^2}\right)^{i a}\left(\sqrt{1-\frac{1}{y^2}}+\frac{i}{y}\right)^{-i b}\right)}{\pi}\right\rfloor}$$

$$\cos^{-1}\left(\frac{1}{2}\left(i x+\sqrt{1-x^2}\right)^{-i a}\left(\left(\sqrt{1-\frac{1}{y^2}}+\frac{i}{y}\right)^{-2 i b}\left(i x+\sqrt{1-x^2}\right)^{2 i a}+1\right)\left(\sqrt{1-\frac{1}{y^2}}+\frac{i}{y}\right)^{i b}\right)-$$

$$2 i \pi\left(\left\lfloor\frac{-\arg\left(\left(i x+\sqrt{1-x^2}\right)^{i a}\right)-\arg\left(\left(\sqrt{1-\frac{1}{y^2}}+\frac{i}{y}\right)^{-i b}\right)+\pi}{2 \pi}\right\rfloor+\left\lfloor\frac{\operatorname{Re}\left(b \log\left(\sqrt{1-\frac{1}{y^2}}+\frac{i}{y}\right)\right)+\pi}{2 \pi}\right\rfloor+\right.$$

$$\left.\left\lfloor\frac{\pi-\operatorname{Re}\left(a \log\left(i x+\sqrt{1-x^2}\right)\right)}{2 \pi}\right\rfloor\right)+\left(1-(-1)^{\left\lfloor-\frac{\arg\left(\left(\sqrt{1-\frac{1}{y^2}}+\frac{i}{y}\right)^{-i b}\left(i x+\sqrt{1-x^2}\right)^{i a}+1\right)}{2 \pi}\right\rfloor-\left\lfloor-\frac{\arg\left(\left(i x+\sqrt{1-x^2}\right)^{i a}\left(\sqrt{1-\frac{1}{y^2}}+\frac{i}{y}\right)^{-i b}\right)}{2 \pi}\right\rfloor}\right) i \pi$$

Involving $\sec^{-1}(z)$

01.13.16.0211.01

$$a\cos^{-1}(x)+b\sec^{-1}(y)=\frac{1}{2}\pi(a+b)-2i\pi\left(\left\lfloor\frac{-\arg\left(\left(i x+\sqrt{1-x^2}\right)^{i a}\right)-\arg\left(\left(\sqrt{1-\frac{1}{y^2}}+\frac{i}{y}\right)^{i b}\right)+\pi}{2\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Re}\left(a\log\left(i x+\sqrt{1-x^2}\right)\right)}{2\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Re}\left(b\log\left(\sqrt{1-\frac{1}{y^2}}+\frac{i}{y}\right)\right)}{2\pi}\right\rfloor\right)+\log\left(\left(i x+\sqrt{1-x^2}\right)^{i a}\left(\sqrt{1-\frac{1}{y^2}}+\frac{i}{y}\right)^{i b}\right)$$

01.13.16.0212.01

$$a\cos^{-1}(x)+b\sec^{-1}(y)=\frac{1}{2}\pi(a+b)-i\,(-1)^{\left\lfloor\frac{\arg\left(\left(i x+\sqrt{1-x^2}\right)^{i a}\left(\sqrt{1-\frac{1}{y^2}}+\frac{i}{y}\right)^{i b}-1\right)}{2\pi}+\frac{1}{2}-\frac{\arg\left(\left(\sqrt{1-\frac{1}{y^2}}+\frac{i}{y}\right)^{i b}\left(i x+\sqrt{1-x^2}\right)^{i a}+1\right)}{2\pi}\right\rfloor+\left\lfloor-\frac{\arg\left(\left(i x+\sqrt{1-x^2}\right)^{i a}\left(\sqrt{1-\frac{1}{y^2}}+\frac{i}{y}\right)^{i b}\right)}{\pi}\right\rfloor}$$

$$\cos^{-1}\left(\frac{1}{2}\left(i x+\sqrt{1-x^2}\right)^{-i a}\left(\left(\sqrt{1-\frac{1}{y^2}}+\frac{i}{y}\right)^{2 i b}\left(i x+\sqrt{1-x^2}\right)^{2 i a}+1\right)\left(\sqrt{1-\frac{1}{y^2}}+\frac{i}{y}\right)^{-i b}\right)-$$

$$2i\pi\left(\left\lfloor\frac{-\arg\left(\left(i x+\sqrt{1-x^2}\right)^{i a}\right)-\arg\left(\left(\sqrt{1-\frac{1}{y^2}}+\frac{i}{y}\right)^{i b}\right)+\pi}{2\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Re}\left(a\log\left(i x+\sqrt{1-x^2}\right)\right)}{2\pi}\right\rfloor+\right.$$

$$\left.\left\lfloor\frac{\pi-\operatorname{Re}\left(b\log\left(\sqrt{1-\frac{1}{y^2}}+\frac{i}{y}\right)\right)}{2\pi}\right\rfloor\right)+\left(1-(-1)^{\left\lfloor-\frac{\arg\left(\left(\sqrt{1-\frac{1}{y^2}}+\frac{i}{y}\right)^{i b}\left(i x+\sqrt{1-x^2}\right)^{i a}+1\right)}{2\pi}\right\rfloor-\left\lfloor-\frac{\arg\left(\left(i x+\sqrt{1-x^2}\right)^{i a}\left(\sqrt{1-\frac{1}{y^2}}+\frac{i}{y}\right)^{i b}\right)}{2\pi}\right\rfloor}\right)i\pi$$

Involving $\sinh^{-1}(z)$

01.13.16.0213.01

$$a\cos^{-1}(x)+b\sinh^{-1}(y)=$$
$$\frac{\pi a}{2}-2\,i\,\pi\left(\left\lfloor\frac{-\arg\left(\left(y+\sqrt{y^2+1}\right)^b\right)-\arg\left(\left(i\,x+\sqrt{1-x^2}\right)^{i\,a}\right)+\pi}{2\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}\left(b\log\left(y+\sqrt{y^2+1}\right)\right)}{2\pi}\right\rfloor+\right.$$
$$\left.\left\lfloor\frac{\pi-\operatorname{Re}\left(a\log\left(i\,x+\sqrt{1-x^2}\right)\right)}{2\pi}\right\rfloor\right)+\log\left(\left(i\,x+\sqrt{1-x^2}\right)^{i\,a}\left(y+\sqrt{y^2+1}\right)^b\right)$$

01.13.16.0214.01

$$a\cos^{-1}(x)+b\sinh^{-1}(y)=\frac{\pi a}{2}-i\,(-1)^{\left\lfloor\frac{\arg\left(\left(i\,x+\sqrt{1-x^2}\right)^{i\,a}\left(y+\sqrt{y^2+1}\right)^b-1\right)}{2\pi}+\frac{1}{2}-\frac{\arg\left(\left(y+\sqrt{y^2+1}\right)^b\left(i\,x+\sqrt{1-x^2}\right)^{i\,a}+1\right)}{2\pi}\right\rfloor+\left\lfloor-\frac{\arg\left(\left(i\,x+\sqrt{1-x^2}\right)^{i\,a}\left(y+\sqrt{y^2+1}\right)^b\right)}{\pi}\right\rfloor}$$
$$\cos^{-1}\left(\frac{1}{2}\left(i\,x+\sqrt{1-x^2}\right)^{-i\,a}\left(y+\sqrt{y^2+1}\right)^{-b}\left(\left(y+\sqrt{y^2+1}\right)^{2\,b}\left(i\,x+\sqrt{1-x^2}\right)^{2\,i\,a}+1\right)\right)-$$
$$2\,i\,\pi\left(\left\lfloor\frac{-\arg\left(\left(y+\sqrt{y^2+1}\right)^b\right)-\arg\left(\left(i\,x+\sqrt{1-x^2}\right)^{i\,a}\right)+\pi}{2\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}\left(b\log\left(y+\sqrt{y^2+1}\right)\right)}{2\pi}\right\rfloor+\right.$$
$$\left.\left\lfloor\frac{\pi-\operatorname{Re}\left(a\log\left(i\,x+\sqrt{1-x^2}\right)\right)}{2\pi}\right\rfloor\right)+\left(1-(-1)^{\left\lfloor-\frac{\arg\left(\left(y+\sqrt{y^2+1}\right)^b\left(i\,x+\sqrt{1-x^2}\right)^{i\,a}+1\right)}{2\pi}\right\rfloor-\left\lfloor-\frac{\arg\left(\left(i\,x+\sqrt{1-x^2}\right)^{i\,a}\left(y+\sqrt{y^2+1}\right)^b\right)}{2\pi}\right\rfloor}\right)i\,\pi$$

Involving $\cosh^{-1}(z)$

01.13.16.0215.01

$$a\cos^{-1}(x)+b\cosh^{-1}(y)=$$
$$\frac{\pi a}{2}-2\,i\,\pi\left(\left\lfloor\frac{-\arg\left(\left(y+\sqrt{y-1}\,\sqrt{y+1}\right)^b\right)-\arg\left(\left(i\,x+\sqrt{1-x^2}\right)^{i\,a}\right)+\pi}{2\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}\left(b\log\left(y+\sqrt{y-1}\,\sqrt{y+1}\right)\right)}{2\pi}\right\rfloor+\right.$$
$$\left.\left\lfloor\frac{\pi-\operatorname{Re}\left(a\log\left(i\,x+\sqrt{1-x^2}\right)\right)}{2\pi}\right\rfloor\right)+\log\left(\left(i\,x+\sqrt{1-x^2}\right)^{i\,a}\left(y+\sqrt{y-1}\,\sqrt{y+1}\right)^b\right)$$

01.13.16.0216.01

$$a\cos^{-1}(x)+b\cosh^{-1}(y)=$$

$$\frac{\pi a}{2}-i\,(-1)^{\left\lfloor \frac{\arg\left(\left(i\,x+\sqrt{1-x^2}\right)^{i\,a}\left(y+\sqrt{y-1}\,\sqrt{y+1}\right)^{b}-1\right)}{2\pi}+\frac{1}{2}-\frac{\arg\left(\left(y+\sqrt{y-1}\,\sqrt{y+1}\right)^{b}\left(i\,x+\sqrt{1-x^2}\right)^{i\,a}+1\right)}{2\pi}\right\rfloor+\left\lfloor -\frac{\arg\left(\left(i\,x+\sqrt{1-x^2}\right)^{i\,a}\left(y+\sqrt{y-1}\,\sqrt{y+1}\right)^{b}\right)}{\pi}\right\rfloor}$$

$$\cos^{-1}\left(\frac{1}{2}\left(i\,x+\sqrt{1-x^2}\right)^{-i\,a}\left(y+\sqrt{y-1}\,\sqrt{y+1}\right)^{-b}\left(\left(y+\sqrt{y-1}\,\sqrt{y+1}\right)^{2\,b}\left(i\,x+\sqrt{1-x^2}\right)^{2\,i\,a}+1\right)\right)-$$

$$2\,i\,\pi\left(\left\lfloor\frac{-\arg\left(\left(y+\sqrt{y-1}\,\sqrt{y+1}\right)^{b}\right)-\arg\left(\left(i\,x+\sqrt{1-x^2}\right)^{i\,a}\right)+\pi}{2\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}\left(b\log\left(y+\sqrt{y-1}\,\sqrt{y+1}\right)\right)}{2\pi}\right\rfloor+\right.$$

$$\left.\left\lfloor\frac{\pi-\operatorname{Re}\left(a\log\left(i\,x+\sqrt{1-x^2}\right)\right)}{2\pi}\right\rfloor\right)+\left(1-(-1)^{\left\lfloor -\frac{\arg\left(\left(y+\sqrt{y-1}\,\sqrt{y+1}\right)^{b}\left(i\,x+\sqrt{1-x^2}\right)^{i\,a}+1\right)}{2\pi}\right\rfloor-\left\lfloor -\frac{\arg\left(\left(i\,x+\sqrt{1-x^2}\right)^{i\,a}\left(y+\sqrt{y-1}\,\sqrt{y+1}\right)^{b}\right)}{2\pi}\right\rfloor}\right)i\,\pi$$

Involving $\tanh^{-1}(z)$

01.13.16.0217.01

$$a\cos^{-1}(x)+b\tanh^{-1}(y)=\frac{\pi a}{2}-2\,i\,\pi\left(\left\lfloor\frac{-\arg\left((y+1)^{b/2}\right)-\arg\left(\left(i\,x+\sqrt{1-x^2}\right)^{i\,a}(1-y)^{-\frac{b}{2}}\right)+\pi}{2\pi}\right\rfloor+\right.$$

$$\left.\left\lfloor\frac{\pi-\frac{1}{2}\operatorname{Im}(b\log(y+1))}{2\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}\left(\log\left(\left(i\,x+\sqrt{1-x^2}\right)^{i\,a}(1-y)^{-\frac{b}{2}}\right)\right)}{2\pi}\right\rfloor\right)-$$

$$2\,i\,\pi\left(\left\lfloor\frac{-\arg\left(\left(i\,x+\sqrt{1-x^2}\right)^{i\,a}\right)-\arg\left((1-y)^{-\frac{b}{2}}\right)+\pi}{2\pi}\right\rfloor+\left\lfloor\frac{\frac{1}{2}\operatorname{Im}(b\log(1-y))+\pi}{2\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Re}\left(a\log\left(i\,x+\sqrt{1-x^2}\right)\right)}{2\pi}\right\rfloor\right)+$$

$$\log\left(\left(i\,x+\sqrt{1-x^2}\right)^{i\,a}(1-y)^{-\frac{b}{2}}(y+1)^{b/2}\right)$$

01.13.16.0218.01

$$a\cos^{-1}(x)+b\tanh^{-1}(y)=\frac{\pi a}{2}-i\,(-1)^{\left\lfloor \frac{\arg\left(\left(i x+\sqrt{1-x^2}\right)^{i a}(1-y)^{-\frac{b}{2}}(y+1)^{b/2}-1\right)}{2\pi}+\frac{1}{2}-\frac{\arg\left(\left(i x+\sqrt{1-x^2}\right)^{i a}(1-y)^{-\frac{b}{2}}(y+1)^{b/2}+1\right)}{2\pi}\right\rfloor+\left\lfloor -\frac{\arg\left(\left(i x+\sqrt{1-x^2}\right)^{i a}(1-y)^{-\frac{b}{2}}(y+1)^{b/2}\right)}{\pi}\right\rfloor}$$
$$\cos^{-1}\left(\frac{1}{2}\left(i x+\sqrt{1-x^2}\right)^{-i a}(1-y)^{b/2}(y+1)^{-\frac{b}{2}}\left((y+1)^b(1-y)^{-b}\left(i x+\sqrt{1-x^2}\right)^{2 i a}+1\right)\right)-$$
$$2 i\pi\left(\left\lfloor\frac{-\arg\left((y+1)^{b/2}\right)-\arg\left(\left(i x+\sqrt{1-x^2}\right)^{i a}(1-y)^{-\frac{b}{2}}\right)+\pi}{2\pi}\right\rfloor+\right.$$
$$\left.\left\lfloor\frac{\pi-\frac{1}{2}\operatorname{Im}(b\log(y+1))}{2\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}\left(\log\left(\left(i x+\sqrt{1-x^2}\right)^{i a}(1-y)^{-\frac{b}{2}}\right)\right)}{2\pi}\right\rfloor\right)-$$
$$2 i\pi\left(\left\lfloor\frac{-\arg\left(\left(i x+\sqrt{1-x^2}\right)^{i a}\right)-\arg\left((1-y)^{-\frac{b}{2}}\right)+\pi}{2\pi}\right\rfloor+\left\lfloor\frac{\frac{1}{2}\operatorname{Im}(b\log(1-y))+\pi}{2\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Re}\left(a\log\left(i x+\sqrt{1-x^2}\right)\right)}{2\pi}\right\rfloor\right)+$$
$$\left(1-(-1)^{\left\lfloor-\frac{\arg\left(\left(i x+\sqrt{1-x^2}\right)^{i a}(1-y)^{-\frac{b}{2}}(y+1)^{b/2}+1\right)}{2\pi}\right\rfloor-\left\lfloor-\frac{\arg\left(\left(i x+\sqrt{1-x^2}\right)^{i a}(1-y)^{-\frac{b}{2}}(y+1)^{b/2}\right)}{2\pi}\right\rfloor}\right) i\pi$$

Involving $\coth^{-1}(z)$

01.13.16.0219.01

$$a\cos^{-1}(x)+b\coth^{-1}(y)=\frac{\pi a}{2}-2\,i\,\pi\left(\left\lfloor\frac{-\arg\left(\left(1+\frac{1}{y}\right)^{b/2}\right)-\arg\left(\left(i\,x+\sqrt{1-x^2}\right)^{i\,a}\left(1-\frac{1}{y}\right)^{-\frac{b}{2}}\right)+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\pi-\frac{1}{2}\operatorname{Im}\left(b\log\left(1+\frac{1}{y}\right)\right)}{2\,\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}\left(\log\left(\left(i\,x+\sqrt{1-x^2}\right)^{i\,a}\left(1-\frac{1}{y}\right)^{-\frac{b}{2}}\right)\right)}{2\,\pi}\right\rfloor\right)-2\,i\,\pi\left(\left\lfloor\frac{-\arg\left(\left(i\,x+\sqrt{1-x^2}\right)^{i\,a}\right)-\arg\left(\left(1-\frac{1}{y}\right)^{-\frac{b}{2}}\right)+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\frac{1}{2}\operatorname{Im}\left(b\log\left(1-\frac{1}{y}\right)\right)+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Re}\left(a\log\left(i\,x+\sqrt{1-x^2}\right)\right)}{2\,\pi}\right\rfloor\right)+\log\left(\left(i\,x+\sqrt{1-x^2}\right)^{i\,a}\left(1-\frac{1}{y}\right)^{-\frac{b}{2}}\left(1+\frac{1}{y}\right)^{b/2}\right)$$

01.13.16.0220.01

$$a\cos^{-1}(x)+b\coth^{-1}(y)=\frac{\pi a}{2}-i\,(-1)^{\left\lfloor \frac{\arg\left(\left(i\,x+\sqrt{1-x^2}\right)^{i\,a}\left(1-\frac{1}{y}\right)^{-\frac{b}{2}}\left(1+\frac{1}{y}\right)^{b/2}-1\right)}{2\pi}+\frac{1}{2}-\frac{\arg\left(\left(i\,x+\sqrt{1-x^2}\right)^{i\,a}\left(1-\frac{1}{y}\right)^{-\frac{b}{2}}\left(1+\frac{1}{y}\right)^{b/2}+1\right)}{2\pi}\right\rfloor+\left\lfloor -\frac{\arg\left(\left(i\,x+\sqrt{1-x^2}\right)^{i\,a}\left(1-\frac{1}{y}\right)^{-\frac{b}{2}}\left(1+\frac{1}{y}\right)^{b/2}\right)}{\pi}\right\rfloor}$$

$$\cos^{-1}\left(\frac{1}{2}\left(i\,x+\sqrt{1-x^2}\right)^{-i\,a}\left(\left(1+\frac{1}{y}\right)^{b}\left(1-\frac{1}{y}\right)^{-b}\left(i\,x+\sqrt{1-x^2}\right)^{2\,i\,a}+1\right)\left(1-\frac{1}{y}\right)^{b/2}\left(1+\frac{1}{y}\right)^{-\frac{b}{2}}\right)-$$

$$2\,i\,\pi\left(\left\lfloor\frac{-\arg\left(\left(1+\frac{1}{y}\right)^{b/2}\right)-\arg\left(\left(i\,x+\sqrt{1-x^2}\right)^{i\,a}\left(1-\frac{1}{y}\right)^{-\frac{b}{2}}\right)+\pi}{2\pi}\right\rfloor+\right.$$

$$\left.\left\lfloor\frac{\pi-\frac{1}{2}\operatorname{Im}\left(b\log\left(1+\frac{1}{y}\right)\right)}{2\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}\left(\log\left(\left(i\,x+\sqrt{1-x^2}\right)^{i\,a}\left(1-\frac{1}{y}\right)^{-\frac{b}{2}}\right)\right)}{2\pi}\right\rfloor\right)-$$

$$2\,i\,\pi\left(\left\lfloor\frac{-\arg\left(\left(i\,x+\sqrt{1-x^2}\right)^{i\,a}\right)-\arg\left(\left(1-\frac{1}{y}\right)^{-\frac{b}{2}}\right)+\pi}{2\pi}\right\rfloor+\left\lfloor\frac{\frac{1}{2}\operatorname{Im}\left(b\log\left(1-\frac{1}{y}\right)\right)+\pi}{2\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Re}\left(a\log\left(i\,x+\sqrt{1-x^2}\right)\right)}{2\pi}\right\rfloor\right)+$$

$$\left(1-(-1)^{\left\lfloor -\frac{\arg\left(\left(i\,x+\sqrt{1-x^2}\right)^{i\,a}\left(1-\frac{1}{y}\right)^{-\frac{b}{2}}\left(1+\frac{1}{y}\right)^{b/2}+1\right)}{2\pi}\right\rfloor-\left\lfloor -\frac{\arg\left(\left(i\,x+\sqrt{1-x^2}\right)^{i\,a}\left(1-\frac{1}{y}\right)^{-\frac{b}{2}}\left(1+\frac{1}{y}\right)^{b/2}\right)}{2\pi}\right\rfloor}\right)i\,\pi$$

Involving $\operatorname{csch}^{-1}(z)$

01.13.16.0221.01

$$a\cos^{-1}(x)+b\operatorname{csch}^{-1}(y)=$$
$$\frac{\pi a}{2}-2i\pi\left(\left\lfloor\frac{-\arg\left(\left(ix+\sqrt{1-x^2}\right)^{ia}\right)-\arg\left(\left(\sqrt{1+\frac{1}{y^2}}+\frac{1}{y}\right)^b\right)+\pi}{2\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}\left(b\log\left(\sqrt{1+\frac{1}{y^2}}+\frac{1}{y}\right)\right)}{2\pi}\right\rfloor+\right.$$
$$\left.\left\lfloor\frac{\pi-\operatorname{Re}\left(a\log\left(ix+\sqrt{1-x^2}\right)\right)}{2\pi}\right\rfloor\right)+\log\left(\left(ix+\sqrt{1-x^2}\right)^{ia}\left(\sqrt{1+\frac{1}{y^2}}+\frac{1}{y}\right)^b\right)$$

01.13.16.0222.01

$$a\cos^{-1}(x)+b\operatorname{csch}^{-1}(y)=\frac{\pi a}{2}-i\,(-1)^{\left\lfloor\frac{\arg\left(\left(ix+\sqrt{1-x^2}\right)^{ia}\left(\sqrt{1+\frac{1}{y^2}}+\frac{1}{y}\right)^b-1\right)}{2\pi}+\frac{1}{2}-\frac{\arg\left(\left(\sqrt{1+\frac{1}{y^2}}+\frac{1}{y}\right)^b\left(ix+\sqrt{1-x^2}\right)^{ia}+1\right)}{2\pi}\right\rfloor+\left\lfloor-\frac{\arg\left(\left(ix+\sqrt{1-x^2}\right)^{ia}\left(\sqrt{1+\frac{1}{y^2}}+\frac{1}{y}\right)^b\right)}{\pi}\right\rfloor}$$
$$\cos^{-1}\left(\frac{1}{2}\left(ix+\sqrt{1-x^2}\right)^{-ia}\left(\left(\sqrt{1+\frac{1}{y^2}}+\frac{1}{y}\right)^{2b}\left(ix+\sqrt{1-x^2}\right)^{2ia}+1\right)\left(\sqrt{1+\frac{1}{y^2}}+\frac{1}{y}\right)^{-b}\right)-$$
$$2i\pi\left(\left\lfloor\frac{-\arg\left(\left(ix+\sqrt{1-x^2}\right)^{ia}\right)-\arg\left(\left(\sqrt{1+\frac{1}{y^2}}+\frac{1}{y}\right)^b\right)+\pi}{2\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}\left(b\log\left(\sqrt{1+\frac{1}{y^2}}+\frac{1}{y}\right)\right)}{2\pi}\right\rfloor+\right.$$
$$\left.\left\lfloor\frac{\pi-\operatorname{Re}\left(a\log\left(ix+\sqrt{1-x^2}\right)\right)}{2\pi}\right\rfloor\right)+\left(1-(-1)^{\left\lfloor-\frac{\arg\left(\left(\sqrt{1+\frac{1}{y^2}}+\frac{1}{y}\right)^b\left(ix+\sqrt{1-x^2}\right)^{ia}+1\right)}{2\pi}\right\rfloor-\left\lfloor-\frac{\arg\left(\left(ix+\sqrt{1-x^2}\right)^{ia}\left(\sqrt{1+\frac{1}{y^2}}+\frac{1}{y}\right)^b\right)}{2\pi}\right\rfloor}\right)i\pi$$

Involving $\operatorname{sech}^{-1}(z)$

01.13.16.0223.01

$$a\cos^{-1}(x)+b\,\operatorname{sech}^{-1}(y)=$$
$$\frac{\pi a}{2}-2\,i\,\pi\left(\left\lfloor\frac{-\arg\left(\left(i\,x+\sqrt{1-x^2}\right)^{i\,a}\right)-\arg\left(\left(\sqrt{\frac{1}{y}-1}\,\sqrt{1+\frac{1}{y}}+\frac{1}{y}\right)^{b}\right)+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}\left(b\log\left(\sqrt{\frac{1}{y}-1}\,\sqrt{1+\frac{1}{y}}+\frac{1}{y}\right)\right)}{2\,\pi}\right\rfloor+\right.$$
$$\left.\left\lfloor\frac{\pi-\operatorname{Re}\left(a\log\left(i\,x+\sqrt{1-x^2}\right)\right)}{2\,\pi}\right\rfloor\right)+\log\left(\left(i\,x+\sqrt{1-x^2}\right)^{i\,a}\left(\sqrt{\frac{1}{y}-1}\,\sqrt{1+\frac{1}{y}}+\frac{1}{y}\right)^{b}\right)$$

01.13.16.0224.01

$$a\cos^{-1}(x)+b\,\operatorname{sech}^{-1}(y)=$$
$$\frac{\pi a}{2}-2\,i\,\pi\left(\left\lfloor\frac{-\arg\left(\left(i\,x+\sqrt{1-x^2}\right)^{i\,a}\right)-\arg\left(\left(\sqrt{\frac{1}{y}-1}\,\sqrt{1+\frac{1}{y}}+\frac{1}{y}\right)^{b}\right)+\pi}{2\,\pi}\right\rfloor+\left\lfloor\frac{\pi-\operatorname{Im}\left(b\log\left(\sqrt{\frac{1}{y}-1}\,\sqrt{1+\frac{1}{y}}+\frac{1}{y}\right)\right)}{2\,\pi}\right\rfloor+\right.$$
$$\left.\left\lfloor\frac{\pi-\operatorname{Re}\left(a\log\left(i\,x+\sqrt{1-x^2}\right)\right)}{2\,\pi}\right\rfloor\right)+i\left(1-(-1)^{\left\lfloor-\frac{\arg\left(\left(\sqrt{\frac{1}{y}-1}\,\sqrt{1+\frac{1}{y}}+\frac{1}{y}\right)^{b}\left(i\,x+\sqrt{1-x^2}\right)^{i\,a}+1\right)}{2\,\pi}\right\rfloor-\left\lfloor-\frac{\arg\left(\left(i\,x+\sqrt{1-x^2}\right)^{i\,a}\left(\sqrt{\frac{1}{y}-1}\,\sqrt{1+\frac{1}{y}}+\frac{1}{y}\right)^{b}\right)}{2\,\pi}\right\rfloor}\right)\pi-$$
$$i\,(-1)^{\left\lfloor\frac{\arg\left(\left(i\,x+\sqrt{1-x^2}\right)^{i\,a}\left(\sqrt{\frac{1}{y}-1}\,\sqrt{1+\frac{1}{y}}+\frac{1}{y}\right)^{b}-1\right)}{2\,\pi}+\frac{1}{2}-\frac{\arg\left(\left(\sqrt{\frac{1}{y}-1}\,\sqrt{1+\frac{1}{y}}+\frac{1}{y}\right)^{b}\left(i\,x+\sqrt{1-x^2}\right)^{i\,a}+1\right)}{2\,\pi}\right\rfloor+\left\lfloor-\frac{\arg\left(\left(i\,x+\sqrt{1-x^2}\right)^{i\,a}\left(\sqrt{\frac{1}{y}-1}\,\sqrt{1+\frac{1}{y}}+\frac{1}{y}\right)^{b}\right)}{\pi}\right\rfloor}$$
$$\cos^{-1}\left(\frac{1}{2}\left(i\,x+\sqrt{1-x^2}\right)^{-i\,a}\left(\left(\sqrt{\frac{1}{y}-1}\,\sqrt{1+\frac{1}{y}}+\frac{1}{y}\right)^{2\,b}\left(i\,x+\sqrt{1-x^2}\right)^{2\,i\,a}+1\right)\left(\sqrt{\frac{1}{y}-1}\,\sqrt{1+\frac{1}{y}}+\frac{1}{y}\right)^{-b}\right)$$

Identities

Functional identities

01.13.17.0001.01

$$\cos^2(w(z_1)+w(z_2))-2\,z_1\,z_2\cos(w(z_1)+w(z_2))+z_1^2+z_2^2=1\;/;\;w(z)=\cos^{-1}(z)$$

Complex characteristics

Real part

01.13.19.0001.01

$$\operatorname{Re}\left(\cos^{-1}(x+i\,y)\right)=\cos^{-1}\left(\frac{1}{2}\sqrt{(x+1)^2+y^2}-\frac{1}{2}\sqrt{(x-1)^2+y^2}\right)\;/;\;x+i\,y\notin(-\infty,-1)\bigwedge x+i\,y\notin(1,\infty)$$

01.13.19.0002.01

$$\operatorname{Re}\left(\cos^{-1}(x+i\,y)\right)=\frac{1}{2}\left(\pi-2\tan^{-1}\left(\sqrt[4]{4\,x^2\,y^2+\left(-x^2+y^2+1\right)^2}\cos\left(\frac{1}{2}\tan^{-1}\left(-x^2+y^2+1,\,-2\,x\,y\right)\right)-y,\right.\right.$$
$$\left.\left.\sin\left(\frac{1}{2}\tan^{-1}\left(-x^2+y^2+1,\,-2\,x\,y\right)\right)\sqrt[4]{4\,x^2\,y^2+\left(-x^2+y^2+1\right)^2}+x\right)\right)$$

Imaginary part

01.13.19.0003.01

$$\operatorname{Im}\left(\cos^{-1}(x+i\,y)\right)=-\operatorname{sgn}(y)\log\left(X+\sqrt{X^2-1}\right)\;/;$$
$$X=\frac{1}{2}\sqrt{(x-1)^2+y^2}+\frac{1}{2}\sqrt{(x+1)^2+y^2}\bigwedge x+i\,y\notin(-\infty,-1)\bigwedge x+i\,y\notin(1,\infty)$$

01.13.19.0004.01

$$\operatorname{Im}\left(\cos^{-1}(x+i\,y)\right)=\log\left(\sqrt{\left(\left(y-\sqrt[4]{x^4+2\left(y^2-1\right)x^2+\left(y^2+1\right)^2}\cos\left(\frac{1}{2}\tan^{-1}\left(-x^2+y^2+1,\,-2\,x\,y\right)\right)\right)^2+\right.}\right.$$
$$\left.\left.\left(\sin\left(\frac{1}{2}\tan^{-1}\left(-x^2+y^2+1,\,-2\,x\,y\right)\right)\sqrt[4]{x^4+2\left(y^2-1\right)x^2+\left(y^2+1\right)^2}+x\right)^2\right)\right)$$

Absolute value

01.13.19.0005.01

$$\left|\cos^{-1}(x+i\,y)\right|=$$
$$\sqrt{\left(\frac{1}{4}\left(\pi-2\tan^{-1}\left(\sqrt[4]{4\,x^2\,y^2+\left(-x^2+y^2+1\right)^2}\cos\left(\frac{1}{2}\tan^{-1}\left(-x^2+y^2+1,\,-2\,x\,y\right)\right)-y,\;\sin\left(\frac{1}{2}\tan^{-1}\left(-x^2+y^2+1,\,-2\,x\,y\right)\right)\right.\right.\right.}$$
$$\left.\left.\sqrt[4]{4\,x^2\,y^2+\left(-x^2+y^2+1\right)^2}+x\right)\right)\text{^}2+$$
$$\log^2\left(\sqrt{\left(\left(y-\sqrt[4]{x^4+2\left(y^2-1\right)x^2+\left(y^2+1\right)^2}\cos\left(\frac{1}{2}\tan^{-1}\left(-x^2+y^2+1,\,-2\,x\,y\right)\right)\right)^2+\right.}\right.$$
$$\left.\left.\left.\left(\sin\left(\frac{1}{2}\tan^{-1}\left(-x^2+y^2+1,\,-2\,x\,y\right)\right)\sqrt[4]{x^4+2\left(y^2-1\right)x^2+\left(y^2+1\right)^2}+x\right)^2\right)\right)\right)$$

Argument

01.13.19.0006.01

$$\arg\left(\cos^{-1}(x+i\,y)\right) = \tan^{-1}\left(\pi - 2\tan^{-1}\left(\sqrt[4]{4x^2y^2+\left(-x^2+y^2+1\right)^2}\cos\left(\frac{1}{2}\tan^{-1}\left(-x^2+y^2+1,-2\,x\,y\right)\right)-y,\right.\right.$$
$$\left.\sin\left(\frac{1}{2}\tan^{-1}\left(-x^2+y^2+1,-2\,x\,y\right)\right)\sqrt[4]{4x^2y^2+\left(-x^2+y^2+1\right)^2}+x\right),$$
$$2\log\left(\sqrt{\left(\left(y-\sqrt[4]{x^4+2\left(y^2-1\right)x^2+\left(y^2+1\right)^2}\cos\left(\frac{1}{2}\tan^{-1}\left(-x^2+y^2+1,-2\,x\,y\right)\right)\right)^2+\right.}\right.$$
$$\left.\left.\left.\left(\sin\left(\frac{1}{2}\tan^{-1}\left(-x^2+y^2+1,-2\,x\,y\right)\right)\sqrt[4]{x^4+2\left(y^2-1\right)x^2+\left(y^2+1\right)^2}+x\right)^2\right)\right)\right)$$

Conjugate value

01.13.19.0007.01

$$\overline{\cos^{-1}(x+i\,y)} = \frac{1}{2}\left(-2\tan^{-1}\left(\sqrt[4]{x^4+2\left(y^2-1\right)x^2+\left(y^2+1\right)^2}\cos\left(\frac{1}{2}\tan^{-1}\left(-x^2+y^2+1,-2\,x\,y\right)\right)-y,\right.\right.$$
$$\left.\sin\left(\frac{1}{2}\tan^{-1}\left(-x^2+y^2+1,-2\,x\,y\right)\right)\sqrt[4]{x^4+2\left(y^2-1\right)x^2+\left(y^2+1\right)^2}+x\right)-$$
$$2\,i\log\left(\sqrt{\left(\left(y-\sqrt[4]{x^4+2\left(y^2-1\right)x^2+\left(y^2+1\right)^2}\cos\left(\frac{1}{2}\tan^{-1}\left(-x^2+y^2+1,-2\,x\,y\right)\right)\right)^2+\right.}\right.$$
$$\left.\left.\left.\left(\sin\left(\frac{1}{2}\tan^{-1}\left(-x^2+y^2+1,-2\,x\,y\right)\right)\sqrt[4]{x^4+2\left(y^2-1\right)x^2+\left(y^2+1\right)^2}+x\right)\text{^}2\right)\right)+\pi\right)$$

Signum value

01.13.19.0008.01

$$\operatorname{sgn}(\cos^{-1}(x+i\,y)) = \left(-2\tan^{-1}\left(\sqrt[4]{x^4+2(y^2-1)x^2+(y^2+1)^2}\cos\left(\frac{1}{2}\tan^{-1}(-x^2+y^2+1,-2\,x\,y)\right)-y,\right.\right.$$
$$\left.\sin\left(\frac{1}{2}\tan^{-1}(-x^2+y^2+1,-2\,x\,y)\right)\sqrt[4]{x^4+2(y^2-1)x^2+(y^2+1)^2}+x\right)+$$
$$2\,i\log\left(\sqrt{\left(\left(y-\sqrt[4]{x^4+2(y^2-1)x^2+(y^2+1)^2}\cos\left(\frac{1}{2}\tan^{-1}(-x^2+y^2+1,-2\,x\,y)\right)\right)^2+\right.}\right.$$
$$\left.\left.\overline{\left.\left(\sin\left(\frac{1}{2}\tan^{-1}(-x^2+y^2+1,-2\,x\,y)\right)\sqrt[4]{x^4+2(y^2-1)x^2+(y^2+1)^2}+x\right)^2\right)}\right)+\pi\right)\Big/$$
$$\left(2\sqrt{\left(\frac{1}{4}\left(\pi-2\tan^{-1}\left(\sqrt[4]{4\,x^2\,y^2+(-x^2+y^2+1)^2}\cos\left(\frac{1}{2}\tan^{-1}(-x^2+y^2+1,-2\,x\,y)\right)-y,\right.\right.\right.}\right.$$
$$\left.\left.\sin\left(\frac{1}{2}\tan^{-1}(-x^2+y^2+1,-2\,x\,y)\right)\sqrt[4]{4\,x^2\,y^2+(-x^2+y^2+1)^2}+x\right)\right)^2+$$
$$\log^2\left(\sqrt{\left(\left(y-\sqrt[4]{x^4+2(y^2-1)x^2+(y^2+1)^2}\cos\left(\frac{1}{2}\tan^{-1}(-x^2+y^2+1,-2\,x\,y)\right)\right)^2+\right.}\right.$$
$$\left.\left.\left.\overline{\left.\left(\sin\left(\frac{1}{2}\tan^{-1}(-x^2+y^2+1,-2\,x\,y)\right)\sqrt[4]{x^4+2(y^2-1)x^2+(y^2+1)^2}+x\right)^2\right)}\right)\right)\right)$$

Differentiation

Low-order differentiation

01.13.20.0001.01

$$\frac{\partial\cos^{-1}(z)}{\partial z} = -\frac{1}{\sqrt{1-z^2}}$$

01.13.20.0002.01

$$\frac{\partial^2\cos^{-1}(z)}{\partial z^2} = -\frac{z}{(1-z^2)^{3/2}}$$

Symbolic differentiation

01.13.20.0005.01

$$\frac{\partial^n\cos^{-1}(z)}{\partial z^n} = \delta_n\cos^{-1}(z) - \frac{(-1)^{n-1}}{(1-z^2)^{n-\frac{1}{2}}}\sum_{k=0}^{n-1}\frac{(1-n)_k\left(\frac{1}{2}\right)_k 2^{2k-n+1} z^{2k-n+1}(z^2-1)^{-k+n-1}}{(2\,k-n+1)!}\ /;\ n\in\mathbb{N}$$

01.13.20.0003.02

$$\frac{\partial^n \cos^{-1}(z)}{\partial z^n} = \frac{\delta_n \pi}{2} - 2^{n-1} \sqrt{\pi}\, z^{1-n}\, {}_3\tilde{F}_2\left(\frac{1}{2}, \frac{1}{2}, 1; 1-\frac{n}{2}, \frac{3-n}{2}; z^2\right) /; n \in \mathbb{N}$$

01.13.20.0006.01

$$\frac{\partial^n \cos^{-1}(z)}{\partial z^n} = -\frac{(-i)^{n-1} (n-1)!}{\left(1-z^2\right)^{n/2}} P_{n-1}\left(\frac{i z}{\sqrt{1-z^2}}\right) /; n \in \mathbb{N}^+$$

Brychkov Yu.A. (2006)

01.13.20.0007.01

$$\frac{\partial^n \cos^{-1}(z)}{\partial z^n} = (-1)^n 2^{1-n} \left(1-z^2\right)^{\frac{1}{2}-n} (n-1)!\, C_{n-1}^{1-n}(z) /; n \in \mathbb{Z} \land n \geq 2$$

Brychkov Yu.A. (2006)

Fractional integro-differentiation

01.13.20.0004.01

$$\frac{\partial^\alpha \cos^{-1}(z)}{\partial z^\alpha} = \frac{\pi z^{-\alpha}}{2\,\Gamma(1-\alpha)} - 2^{\alpha-1} \sqrt{\pi}\, z^{1-\alpha}\, {}_3\tilde{F}_2\left(\frac{1}{2}, \frac{1}{2}, 1; 1-\frac{\alpha}{2}, \frac{3}{2}-\frac{\alpha}{2}; z^2\right)$$

Integration

Indefinite integration

For the direct function itself

01.13.21.0001.01

$$\int \cos^{-1}(z)\, dz = z \cos^{-1}(z) - \sqrt{1-z^2}$$

01.13.21.0002.01

$$\int \frac{\cos^{-1}(z)}{z}\, dz = -\frac{i}{2} \cos^{-1}(z)^2 + \log\left(1 + e^{2 i \cos^{-1}(z)}\right) \cos^{-1}(z) - \frac{i}{2} \operatorname{Li}_2\left(-e^{2 i \cos^{-1}(z)}\right)$$

01.13.21.0003.01

$$\int \frac{\cos^{-1}(z)}{\sqrt{z}}\, dz = 2\sqrt{z} \left(\cos^{-1}(z) + \frac{\sqrt{z+1}}{\sqrt{\frac{z}{2z-2}}\, \sqrt{1-z^2}} \left(2 E\left(\sin^{-1}\left(\sqrt{z+1}\right) \middle| \frac{1}{2}\right) - F\left(\sin^{-1}\left(\sqrt{z+1}\right) \middle| \frac{1}{2}\right)\right)\right)$$

01.13.21.0004.01

$$\int z^{\alpha-1} \cos^{-1}(z)\, dz = \frac{\cos^{-1}(z)\, z^\alpha}{\alpha} + \frac{z^{\alpha+1}}{\alpha(\alpha+1)}\, {}_2F_1\left(\frac{\alpha+1}{2}, \frac{1}{2}; \frac{\alpha+3}{2}; z^2\right)$$

01.13.21.0005.01

$$\int \cos^{-1}(b + a z)\, dz = z \cos^{-1}(b + a z) - \frac{b \sin^{-1}(b + a z)}{a} - \frac{\sqrt{1-(b + a z)^2}}{a}$$

01.13.21.0006.01

$$\int z\cos^{-1}(b+a\,z)\,dz = \frac{2\,a^2\cos^{-1}(b+a\,z)\,z^2+\left(2\,b^2+1\right)\sin^{-1}(b+a\,z)+(3\,b-a\,z)\sqrt{1-(b+a\,z)^2}}{4\,a^2}$$

01.13.21.0007.01

$$\int \frac{\cos^{-1}(a\,z+b)}{z}\,dz = -\frac{i}{2}\cos^{-1}(b+a\,z)^2 - 4\,i\sin^{-1}\left(\frac{\sqrt{1-b}}{\sqrt{2}}\right)\tan^{-1}\left(\frac{b+1}{\sqrt{b^2-1}}\tan\left(\frac{1}{2}\cos^{-1}(b+a\,z)\right)\right)+$$
$$\left(\cos^{-1}(b+a\,z)-2\sin^{-1}\left(\frac{\sqrt{1-b}}{\sqrt{2}}\right)\right)\log\left(e^{i\cos^{-1}(b+a\,z)}\left(\sqrt{b^2-1}-b\right)+1\right)+$$
$$\left(\cos^{-1}(b+a\,z)+2\sin^{-1}\left(\frac{\sqrt{1-b}}{\sqrt{2}}\right)\right)\log\left(1-\left(b+\sqrt{b^2-1}\right)e^{i\cos^{-1}(b+a\,z)}\right)-$$
$$i\left(\mathrm{Li}_2\left(\left(b+\sqrt{b^2-1}\right)e^{i\cos^{-1}(b+a\,z)}\right)+\mathrm{Li}_2\left(-\left(\sqrt{b^2-1}-b\right)e^{i\cos^{-1}(b+a\,z)}\right)\right)$$

01.13.21.0008.01

$$\int \frac{1}{\cos^{-1}(z)}\,dz = -\mathrm{Si}\left(\cos^{-1}(z)\right)$$

01.13.21.0009.01

$$\int \cos^{-1}(z)^n\,dz = \frac{1}{2}\,i\left(\left(i\cos^{-1}(z)\right)^{-n-1}\cos^{-1}(z)^{n+1}\,\Gamma\left(n+1,\,i\cos^{-1}(z)\right)-\left(-i\cos^{-1}(z)\right)^{-n-1}\cos^{-1}(z)^{n+1}\,\Gamma\left(n+1,\,-i\cos^{-1}(z)\right)\right)$$

01.13.21.0010.01

$$\int z\cos^{-1}(z)^n\,dz =$$
$$\frac{1}{4}\,i\left(2^{-n-1}\left(i\cos^{-1}(z)\right)^{-n-1}\cos^{-1}(z)^{n+1}\,\Gamma\left(n+1,\,2\,i\cos^{-1}(z)\right)-2^{-n-1}\left(-i\cos^{-1}(z)\right)^{-n-1}\cos^{-1}(z)^{n+1}\,\Gamma\left(n+1,\,-2\,i\cos^{-1}(z)\right)\right)$$

Definite integration

For the direct function itself

01.13.21.0011.01

$$\int_0^1 t\cos^{-1}(t)\,dt = \frac{\pi}{8}$$

01.13.21.0012.01

$$\int_0^1 \frac{\cos^{-1}(t)}{\sqrt{t}}\,dt = \frac{\sqrt{\pi}\,\Gamma\left(\frac{3}{4}\right)}{\Gamma\left(\frac{5}{4}\right)}$$

01.13.21.0013.01

$$\int_0^1 t^a\cos^{-1}(t)\,dt = \frac{\sqrt{\pi}\,\Gamma\left(\frac{a+2}{2}\right)}{(a+1)^2\,\Gamma\left(\frac{a+1}{2}\right)} \;/;\; \mathrm{Re}(a) > -1$$

Involving the direct function

01.13.21.0014.01

$$\int_0^1 \log(t)\cos^{-1}(t)\,dt = \log(2) - 2$$

Representations through more general functions

Through hypergeometric functions

Involving $_2F_1$

01.13.26.0001.01

$$\cos^{-1}(z) = \frac{\pi}{2} - z\,{}_2F_1\left(\frac{1}{2}, \frac{1}{2}; \frac{3}{2}; z^2\right)$$

01.13.26.0002.01

$$\cos^{-1}(z) = \sqrt{2}\,\sqrt{1-z}\,{}_2F_1\left(\frac{1}{2}, \frac{1}{2}; \frac{3}{2}; \frac{1-z}{2}\right)$$

01.13.26.0003.01

$$\cos^{-1}(z) = \pi - \sqrt{2}\,\sqrt{z+1}\,{}_2F_1\left(\frac{1}{2}, \frac{1}{2}; \frac{3}{2}; \frac{z+1}{2}\right)$$

Involving $_pF_q$

01.13.26.0004.01

$$\cos^{-1}(z) = \frac{\pi}{2} - \frac{z}{2\sqrt{-z^2}}\left(\log\left(-4z^2\right) - \frac{1}{2z^2}\,{}_3F_2\left(\frac{3}{2}, 1, 1; 2, 2; \frac{1}{z^2}\right)\right) /; z \notin (-1, 1)$$

Through Meijer G

Classical cases for the direct function itself

01.13.26.0005.01

$$\cos^{-1}(z) = \frac{\pi}{2} - \frac{z}{2\sqrt{\pi}}\,G_{2,2}^{1,2}\left(-z^2 \,\middle|\, \begin{matrix} \frac{1}{2}, \frac{1}{2} \\ 0, -\frac{1}{2} \end{matrix}\right)$$

01.13.26.0006.01

$$\cos^{-1}(z) = \frac{\pi}{2} + \frac{1}{2\sqrt{\pi}\,z}\,G_{2,2}^{1,2}\left(-z^2 \,\middle|\, \begin{matrix} \frac{3}{2}, \frac{3}{2} \\ 1, \frac{1}{2} \end{matrix}\right)$$

01.13.26.0007.01

$$\cos^{-1}(z) = \frac{\pi}{2} + \frac{\sqrt{-z^2}}{2z\sqrt{\pi}}\,G_{2,2}^{1,2}\left(-z^2 \,\middle|\, \begin{matrix} 1, 1 \\ \frac{1}{2}, 0 \end{matrix}\right)$$

01.13.26.0008.01

$$\cos^{-1}(z) = \frac{\pi}{2} - \frac{i}{2\sqrt{\pi}}\,G_{2,2}^{1,2}\left(-z^2 \,\middle|\, \begin{matrix} 1, 1 \\ \frac{1}{2}, 0 \end{matrix}\right) /; 0 < \arg(z) \le \pi$$

01.13.26.0010.01

$$\cos^{-1}\left(\sqrt{z}\right) = \frac{\pi}{2} - \frac{\sqrt{z}}{2\sqrt{\pi}}\,G_{2,2}^{1,2}\left(-z \,\middle|\, \begin{matrix} \frac{1}{2}, \frac{1}{2} \\ 0, -\frac{1}{2} \end{matrix}\right)$$

01.13.26.0035.01

$$\cos^{-1}\left(\sqrt{z}\right)-\frac{\pi}{2}+\sum_{k=0}^{n}\frac{\left(\frac{1}{2}\right)_k z^{k+\frac{1}{2}}}{(2k+1)\,k!}=\frac{(-1)^{n-1}\sqrt{-z}}{2\sqrt{\pi}\sqrt{z}}\,G_{3,3}^{1,3}\left(-z\,\middle|\,\begin{matrix}1,1,n+\frac{3}{2}\\ n+\frac{3}{2},0,\frac{1}{2}\end{matrix}\right)/;\,n\in\mathbb{N}$$

01.13.26.0036.01

$$\cos^{-1}\left(\sqrt{z}\right)-\frac{\pi}{2}+\frac{\sqrt{z}\,\log(-4z)}{2\sqrt{-z}}-\frac{\sqrt{z}}{2\sqrt{-z}}\sum_{k=1}^{n}\frac{\left(\frac{1}{2}\right)_k z^{-k}}{k\,k!}=\frac{(-1)^{n-1}\sqrt{z}}{2\sqrt{\pi}\sqrt{-z}}\,G_{3,3}^{1,3}\left(-\frac{1}{z}\,\middle|\,\begin{matrix}\frac{1}{2},1,n+1\\ n+1,0,0\end{matrix}\right)/;\,n\in\mathbb{N}$$

Classical cases involving algebraic functions in the arguments

01.13.26.0011.01

$$\cos^{-1}\left(\sqrt{z+1}-\sqrt{z}\right)=\frac{1}{2\sqrt{2\pi}}\,G_{3,3}^{2,2}\left(z\,\middle|\,\begin{matrix}\frac{1}{2},1,1\\ \frac{1}{4},\frac{3}{4},0\end{matrix}\right)$$

01.13.26.0012.01

$$\cos^{-1}\left(\frac{\sqrt{z+1}-1}{\sqrt{z}}\right)=\frac{1}{2\sqrt{2\pi}}\,G_{3,3}^{2,2}\left(z\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4},1\\ 0,\frac{1}{2},0\end{matrix}\right)/;\,z\notin(-\infty,0)$$

01.13.26.0013.01

$$\cos^{-1}\left(\frac{1}{\sqrt{z+1}+\sqrt{z}}\right)=\frac{1}{2\sqrt{2\pi}}\,G_{3,3}^{2,2}\left(z\,\middle|\,\begin{matrix}\frac{1}{2},1,1\\ \frac{1}{4},\frac{3}{4},0\end{matrix}\right)/;\,z\notin(-\infty,0)$$

01.13.26.0014.01

$$\cos^{-1}\left(\frac{\sqrt{z}}{\sqrt{z+1}+1}\right)=\frac{1}{2\sqrt{2\pi}}\,G_{3,3}^{2,2}\left(z\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4},1\\ 0,\frac{1}{2},0\end{matrix}\right)/;\,z\notin(-\infty,0)$$

Classical cases involving unit step θ

01.13.26.0015.01

$$\theta(1-|z|)\cos^{-1}\left(\sqrt{z}\right)=\frac{\sqrt{\pi}}{2}\,G_{2,2}^{2,0}\left(z\,\middle|\,\begin{matrix}1,1\\ 0,\frac{1}{2}\end{matrix}\right)/;\,z\notin(-1,0)$$

01.13.26.0016.01

$$\theta(|z|-1)\cos^{-1}\left(\frac{1}{\sqrt{z}}\right)=\frac{\sqrt{\pi}}{2}\,G_{2,2}^{0,2}\left(z\,\middle|\,\begin{matrix}\frac{1}{2},1\\ 0,0\end{matrix}\right)$$

Classical cases for powers of $\cos^{-1}$

01.13.26.0037.01

$$\cos^{-1}\left(\sqrt{z}\right)^2=\frac{\pi^2}{4}-\frac{\sqrt{z}\sqrt{\pi}}{2\sqrt{-z}}\,G_{2,2}^{1,2}\left(-z\,\middle|\,\begin{matrix}1,1\\ \frac{1}{2},0\end{matrix}\right)-\frac{1}{2}\sqrt{\pi}\,G_{3,3}^{1,3}\left(-z\,\middle|\,\begin{matrix}1,1,1\\ 1,0,\frac{1}{2}\end{matrix}\right)$$

Generalized cases for the direct function itself

01.13.26.0017.01

$$\cos^{-1}(z)=\frac{1}{2z\sqrt{\pi}}\,G_{2,2}^{1,2}\left(\sqrt{-z^2},\frac{1}{2}\,\middle|\,\begin{matrix}\frac{3}{2},\frac{3}{2}\\ 1,\frac{1}{2}\end{matrix}\right)+\frac{\pi}{2}$$

01.13.26.0018.01

$$\cos^{-1}(z) = \frac{\pi}{2} + \frac{i}{2\sqrt{\pi}} G_{2,2}^{1,2}\left(i z, \frac{1}{2} \,\middle|\, \begin{matrix} 1, 1 \\ \frac{1}{2}, 0 \end{matrix}\right)$$

01.13.26.0038.01

$$\cos^{-1}(z) - \frac{\pi}{2} + \sum_{k=0}^{n} \frac{\left(\frac{1}{2}\right)_k z^{2k+1}}{(2k+1)k!} = \frac{(-1)^{n-\frac{1}{2}}}{2\sqrt{\pi}} G_{3,3}^{1,3}\left(i z, \frac{1}{2} \,\middle|\, \begin{matrix} 1, 1, n+\frac{3}{2} \\ n+\frac{3}{2}, 0, \frac{1}{2} \end{matrix}\right) /; n \in \mathbb{N}$$

01.13.26.0039.01

$$\cos^{-1}(z) - \frac{\pi}{2} + \frac{z \log(-4z^2)}{2\sqrt{-z^2}} - \frac{z}{2\sqrt{-z^2}} \sum_{k=1}^{n} \frac{\left(\frac{1}{2}\right)_k z^{-2k}}{k\,k!} = \frac{(-1)^{n-1} z}{2\sqrt{\pi}\sqrt{-z^2}} G_{3,3}^{1,3}\left(-\frac{i}{z}, \frac{1}{2} \,\middle|\, \begin{matrix} \frac{1}{2}, 1, n+1 \\ n+1, 0, 0 \end{matrix}\right) /; n \in \mathbb{N}$$

Generalized cases involving algebraic functions in the arguments

01.13.26.0019.01

$$\cos^{-1}\left(\sqrt{z^2+1} - z\right) = \frac{1}{2\sqrt{2\pi}} G_{3,3}^{2,2}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} \frac{1}{2}, 1, 1 \\ \frac{1}{4}, \frac{3}{4}, 0 \end{matrix}\right) /; z \notin (-\infty, 0)$$

01.13.26.0020.01

$$\cos^{-1}\left(\frac{\sqrt{z^2+1} - 1}{z}\right) = \frac{1}{2\sqrt{2\pi}} G_{3,3}^{2,2}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} \frac{1}{4}, \frac{3}{4}, 1 \\ 0, \frac{1}{2}, 0 \end{matrix}\right)$$

01.13.26.0021.01

$$\cos^{-1}\left(\frac{1}{\sqrt{z^2+1} + z}\right) = \frac{1}{2\sqrt{2\pi}} G_{3,3}^{2,2}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} \frac{1}{2}, 1, 1 \\ \frac{1}{4}, \frac{3}{4}, 0 \end{matrix}\right) /; z \notin (-\infty, 0)$$

01.13.26.0022.01

$$\cos^{-1}\left(\frac{z}{\sqrt{z^2+1} + 1}\right) = \frac{1}{2\sqrt{2\pi}} G_{3,3}^{2,2}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} \frac{1}{4}, \frac{3}{4}, 1 \\ 0, \frac{1}{2}, 0 \end{matrix}\right)$$

Generalized cases involving unit step θ

01.13.26.0023.01

$$\theta(1-|z|)\cos^{-1}(z) = \frac{1}{2}\sqrt{\pi}\, G_{2,2}^{2,0}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} 1, 1 \\ 0, \frac{1}{2} \end{matrix}\right)$$

01.13.26.0024.01

$$\theta(|z|-1)\cos^{-1}\left(\frac{1}{z}\right) = \frac{1}{2}\sqrt{\pi}\, G_{2,2}^{0,2}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} \frac{1}{2}, 1 \\ 0, 0 \end{matrix}\right)$$

Generalized cases for powers of $\cos^{-1}$

01.13.26.0040.01

$$\cos^{-1}(z)^2 = \frac{\pi^2}{4} + \frac{i\sqrt{\pi}}{2} G_{2,2}^{1,2}\left(i z, \frac{1}{2} \,\middle|\, \begin{matrix} 1, 1 \\ \frac{1}{2}, 0 \end{matrix}\right) - \frac{\sqrt{\pi}}{2} G_{3,3}^{1,3}\left(i z, \frac{1}{2} \,\middle|\, \begin{matrix} 1, 1, 1 \\ 1, 0, \frac{1}{2} \end{matrix}\right)$$

Through other functions

Involving inverse Jacobi functions

01.13.26.0025.01

$$\cos^{-1}(z) = \text{cd}^{-1}(z \mid 0)$$

01.13.26.0026.01

$$\cos^{-1}(z) = \text{cn}^{-1}(z \mid 0)$$

01.13.26.0027.01

$$\cos^{-1}(z) = \text{dc}^{-1}\left(\frac{1}{z} \,\middle|\, 0\right)$$

01.13.26.0028.01

$$\cos^{-1}(z) = \frac{\pi}{2} - \text{ds}^{-1}\left(\frac{1}{z} \,\middle|\, 0\right)$$

01.13.26.0029.01

$$\cos^{-1}(z) = \text{nc}^{-1}\left(\frac{1}{z} \,\middle|\, 0\right)$$

01.13.26.0030.01

$$\cos^{-1}(z) = \frac{\pi}{2} - \text{ns}^{-1}\left(\frac{1}{z} \,\middle|\, 0\right)$$

01.13.26.0031.01

$$\cos^{-1}(z) = \frac{\pi}{2} - \text{sd}^{-1}(z \mid 0)$$

01.13.26.0032.01

$$\cos^{-1}(z) = \frac{\pi}{2} - \text{sn}^{-1}(z \mid 0)$$

Involving some hypergeometric-type functions

01.13.26.0033.01

$$\cos^{-1}(z) = \frac{\pi}{2} - \frac{\sqrt{z^2}}{2\,z} \, B_{z^2}\left(\frac{1}{2}, \frac{1}{2}\right)$$

01.13.26.0034.01

$$\cos^{-1}(z) = \frac{1}{2\,z}\left(\pi\left(z - \sqrt{z^2}\right) + \sqrt{z^2}\, B_{1-z^2}\left(\frac{1}{2}, \frac{1}{2}\right)\right)$$

Representations through equivalent functions

With inverse function

Involving $\cos^{-1}(\cos(z))$

01.13.27.0001.01

$$\cos^{-1}(\cos(z)) = z \,/;\, 0 < \text{Re}(z) < \pi \lor (\text{Re}(z) = 0 \land \text{Im}(z) \geq 0) \lor (\text{Re}(z) = \pi \land \text{Im}(z) \leq 0)$$

01.13.27.0002.01

$$\cos^{-1}(\cos(z)) = -z \,/;\, -\pi < \text{Re}(z) < 0 \lor (\text{Re}(z) = 0 \land \text{Im}(z) \leq 0) \lor (\text{Re}(z) = -\pi \land \text{Im}(z) \geq 0)$$

01.13.27.0287.01

$$\cos^{-1}(\cos(z)) = \sqrt{z^2} \,/;\, |\text{Re}(z)| < \pi \lor \text{Re}(z) = -\pi \land \text{Im}(z) \geq 0 \lor \text{Re}(z) = \pi \land \text{Im}(z) \leq 0$$

01.13.27.0003.01

$$\cos^{-1}(\cos(z)) = (-1)^k \left(z - \pi k - \frac{\pi}{2}\right) + \frac{\pi}{2} /; (k\pi < \operatorname{Re}(z) < (k+1)\pi \bigvee (\operatorname{Re}(z) = k\pi \bigwedge \operatorname{Im}(z) \geq 0) \bigvee (\operatorname{Re}(z) = (k+1)\pi \bigwedge \operatorname{Im}(z) \leq 0)) \bigwedge k \in \mathbb{Z}$$

01.13.27.0004.01

$$\cos^{-1}(\cos(z)) = \frac{\pi}{2}\left(1 - (-1)^{\left\lfloor -\frac{\operatorname{Re}(z)}{\pi} \right\rfloor}\right) + (-1)^{\left\lfloor -\frac{\operatorname{Re}(z)}{\pi} \right\rfloor}\left(\left(1 + (-1)^{\left\lfloor \frac{\operatorname{Re}(z)}{\pi} \right\rfloor + \left\lfloor -\frac{\operatorname{Re}(z)}{\pi} \right\rfloor}\right)\theta(\operatorname{Im}(z)) - 1\right)\left(z + \pi\left\lfloor -\frac{\operatorname{Re}(z)}{\pi} \right\rfloor\right)$$

01.13.27.2507.01

$$\cos^{-1}(\cos(z)) = \begin{cases} (-1)^{\left\lfloor \frac{\operatorname{Re}(z)}{\pi} \right\rfloor}\left(-z + \pi\left\lfloor \frac{\operatorname{Re}(z)}{\pi} \right\rfloor - \frac{\pi}{2}\right) + \frac{\pi}{2} & \frac{\operatorname{Re}(z)}{\pi} \in \mathbb{Z} \bigwedge \operatorname{Im}(z) \leq 0 \\ (-1)^{\left\lfloor \frac{\operatorname{Re}(z)}{\pi} \right\rfloor}\left(z - \pi\left\lfloor \frac{\operatorname{Re}(z)}{\pi} \right\rfloor - \frac{\pi}{2}\right) + \frac{\pi}{2} & \text{True} \end{cases}$$

01.13.27.2508.01

$$\cos^{-1}(\cos(z)) = \sec^{-1}(\sec(z))$$

Involving $\cos(\cos^{-1}(z))$

01.13.27.0005.01

$$\cos(\cos^{-1}(z)) = z$$

Involving $\cos^{-1}(\sin(z))$

01.13.27.2509.01

$$\cos^{-1}(\sin(z)) = \frac{\pi}{2} - z /; -\frac{\pi}{2} < \operatorname{Re}(z) < \frac{\pi}{2} \bigvee \left(\operatorname{Re}(z) = \frac{\pi}{2} \bigwedge \operatorname{Im}(z) \leq 0\right) \bigvee \left(\operatorname{Re}(z) = -\frac{\pi}{2} \bigwedge \operatorname{Im}(z) \geq 0\right)$$

01.13.27.2510.01

$$\cos^{-1}(\sin(z)) = \frac{\pi}{2} - z /; \frac{\pi}{2} < \operatorname{Re}(z) < \frac{3\pi}{2} \bigvee \left(\operatorname{Re}(z) = \frac{\pi}{2} \bigwedge \operatorname{Im}(z) \geq 0\right) \bigvee \left(\operatorname{Re}(z) = \frac{3\pi}{2} \bigwedge \operatorname{Im}(z) \leq 0\right)$$

01.13.27.2511.01

$$\cos^{-1}(\sin(z)) = \frac{\pi}{2} - z /; \frac{\pi}{2} < \operatorname{Re}(z) < \frac{3\pi}{2} \bigvee \left(\operatorname{Re}(z) = \frac{\pi}{2} \bigwedge \operatorname{Im}(z) \geq 0\right) \bigvee \left(\operatorname{Re}(z) = \frac{3\pi}{2} \bigwedge \operatorname{Im}(z) \leq 0\right)$$

01.13.27.2512.01

$$\cos^{-1}(\sin(z)) = \left(\frac{1}{2} + (-1)^k k\right)\pi - (-1)^k z /; \left(k\pi - \frac{\pi}{2} < \operatorname{Re}(z) < \pi k + \frac{\pi}{2} \bigvee \left(\operatorname{Re}(z) = \pi k + \frac{\pi}{2} \bigwedge \operatorname{Im}(z) \leq 0\right) \bigvee \left(\operatorname{Re}(z) = k\pi - \frac{\pi}{2} \bigwedge \operatorname{Im}(z) \geq 0\right)\right) \bigwedge k \in \mathbb{Z}$$

01.13.27.2513.01

$$\cos^{-1}(\sin(z)) = \frac{1}{2}\left((-1)^{\left\lfloor \frac{\operatorname{Re}(z)}{\pi} - \frac{1}{2} \right\rfloor}\left(-2z + 2\pi\left\lfloor \frac{\operatorname{Re}(z)}{\pi} - \frac{1}{2} \right\rfloor + \pi\right)\left(\left(1 + (-1)^{\left\lfloor \frac{\operatorname{Re}(z)}{\pi} - \frac{1}{2} \right\rfloor + \left\lfloor \frac{1}{2} - \frac{\operatorname{Re}(z)}{\pi} \right\rfloor}\right)\theta(-\operatorname{Im}(z)) - 1\right) - (-1)^{\left\lfloor \frac{\operatorname{Re}(z)}{\pi} - \frac{1}{2} \right\rfloor}\pi + \pi\right)$$

01.13.27.2514.01

$$\cos^{-1}(\sin(z)) = \begin{cases} (-1)^{\left\lfloor \frac{\operatorname{Re}(z)}{\pi} + \frac{1}{2} \right\rfloor}\left(z - \pi\left\lfloor \frac{\operatorname{Re}(z)}{\pi} - \frac{1}{2} \right\rfloor\right) + \frac{\pi}{2} & \frac{\operatorname{Re}(z)}{\pi} + \frac{1}{2} \in \mathbb{Z} \bigwedge \operatorname{Im}(z) \leq 0 \\ (-1)^{\left\lfloor \frac{\operatorname{Re}(z)}{\pi} + \frac{1}{2} \right\rfloor}\left(\pi\left\lfloor \frac{\operatorname{Re}(z)}{\pi} + \frac{1}{2} \right\rfloor - z\right) + \frac{\pi}{2} & \text{True} \end{cases}$$

01.13.27.2515.01

$$\cos^{-1}(\sin(z)) = \sec^{-1}(\csc(z))$$

With related functions

Involving log

01.13.27.0006.01

$$\cos^{-1}(z) = \frac{\pi}{2} + i \log\left(i z + \sqrt{1 - z^2}\right)$$

01.13.27.0007.01

$$\cos^{-1}(z) = -i \log\left(z + \sqrt{z^2 - 1}\right) /; \operatorname{Re}(z) \operatorname{Im}(z) > 0 \lor \operatorname{Re}(z) = 0 \lor -1 < z < 1$$

01.13.27.0008.01

$$\cos^{-1}(z) = \frac{\sqrt{1 - z}}{\sqrt{z - 1}} \log\left(z + \sqrt{z - 1} \sqrt{z + 1}\right)$$

01.13.27.0009.01

$$\cos^{-1}(z) = i \left(\log(2) - 2 \log\left(\sqrt{z + 1} + i \sqrt{1 - z}\right)\right)$$

01.13.27.0010.01

$$\cos^{-1}(z) = -2 i \log\left(\sqrt{\frac{z + 1}{2}} + i \sqrt{\frac{1 - z}{2}}\right)$$

Involving $\sin^{-1}$

Involving $\cos^{-1}(z)$

Involving $\cos^{-1}(z)$ and $\sin^{-1}(z)$

01.13.27.0011.01

$$\cos^{-1}(z) = \frac{\pi}{2} - \sin^{-1}(z)$$

Involving $\cos^{-1}(z)$ and $\sin^{-1}(1 - 2 z^2)$

01.13.27.0288.01

$$\cos^{-1}(z) = \frac{\pi}{4} + \frac{1}{2} \sin^{-1}\left(1 - 2 z^2\right) /; -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

01.13.27.0289.01

$$\cos^{-1}(z) = \frac{3 \pi}{4} - \frac{1}{2} \sin^{-1}\left(1 - 2 z^2\right) /; \frac{\pi}{2} < \arg(z) \leq \pi \bigvee -\pi < \arg(z) \leq -\frac{\pi}{2}$$

01.13.27.0290.01

$$\cos^{-1}(z) = \frac{1}{2} \pi \left(1 - \frac{\sqrt{z^2}}{2 z}\right) + \frac{\sqrt{z^2}}{2 z} \sin^{-1}\left(1 - 2 z^2\right)$$

Involving $\cos^{-1}(z)$ and $\sin^{-1}(2 z^2 - 1)$

01.13.27.0291.01

$$\cos^{-1}(z) = \frac{\pi}{4} - \frac{1}{2} \sin^{-1}\left(2 z^2 - 1\right) /; -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

01.13.27.0292.01

$$\cos^{-1}(z) = \frac{3\pi}{4} + \frac{1}{2}\sin^{-1}(2z^2 - 1) /; \frac{\pi}{2} < \arg(z) \leq \pi \bigvee -\pi < \arg(z) \leq -\frac{\pi}{2}$$

01.13.27.0293.01

$$\cos^{-1}(z) = \frac{\pi}{2}\left(1 - \frac{\sqrt{z^2}}{2z}\right) - \frac{\sqrt{z^2}}{2z}\sin^{-1}(2z^2 - 1)$$

Involving $\cos^{-1}(z)$ and $\sin^{-1}\left(\sqrt{\frac{1+z}{2}}\right)$

01.13.27.0294.01

$$\cos^{-1}(z) = \pi - 2\sin^{-1}\left(\sqrt{\frac{1+z}{2}}\right)$$

Involving $\cos^{-1}(z)$ and $\sin^{-1}\left(\sqrt{\frac{1-z}{2}}\right)$

01.13.27.0295.01

$$\cos^{-1}(z) = 2\sin^{-1}\left(\sqrt{\frac{1-z}{2}}\right)$$

Involving $\cos^{-1}(z)$ and $\sin^{-1}\left(\sqrt{z^2}\right)$

01.13.27.0296.01

$$\cos^{-1}(z) = \frac{\pi}{2} - \sin^{-1}\left(\sqrt{z^2}\right) /; -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

01.13.27.0297.01

$$\cos^{-1}(z) = \sin^{-1}\left(\sqrt{z^2}\right) + \frac{\pi}{2} /; \frac{\pi}{2} < \arg(z) \leq \pi \bigvee -\pi < \arg(z) \leq -\frac{\pi}{2}$$

01.13.27.0298.01

$$\cos^{-1}(z) = \frac{\pi}{2} - \frac{\sqrt{z^2}}{z}\sin^{-1}\left(\sqrt{z^2}\right)$$

Involving $\cos^{-1}(z)$ and $\sin^{-1}\left(\sqrt{1-z^2}\right)$

01.13.27.0299.01

$$\cos^{-1}(z) = \sin^{-1}\left(\sqrt{1-z^2}\right) /; -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

01.13.27.0300.01

$$\cos^{-1}(z) = \pi - \sin^{-1}\left(\sqrt{1-z^2}\right) /; \frac{\pi}{2} < \arg(z) \leq \pi \bigvee -\pi < \arg(z) \leq -\frac{\pi}{2}$$

01.13.27.0301.01

$$\cos^{-1}(z) = \frac{\pi}{2}\left(1 - \frac{\sqrt{z^2}}{z}\right) + \frac{\sqrt{z^2}}{z}\sin^{-1}\left(\sqrt{1-z^2}\right)$$

Involving $\cos^{-1}(z)$ and $\sin^{-1}\left(2 z \sqrt{1-z^2}\right)$

01.13.27.0302.01

$$\cos^{-1}(z) = \frac{\pi}{2} - \frac{1}{2}\sin^{-1}\left(2 z \sqrt{1-z^2}\right) /; \frac{\pi}{4} \leq |\arg(z)| \leq \frac{3\pi}{4}$$

01.13.27.0303.01

$$\cos^{-1}(z) = \frac{\pi}{4}\left(-\frac{\sqrt{z^2}}{z} + \sqrt{\frac{1}{z}}\sqrt{z}\sqrt{\frac{1}{\sqrt{2}\, z+1}}\sqrt{\sqrt{2}\, z+1} - \sqrt{-\frac{1}{z}}\sqrt{-z}\sqrt{\frac{1}{1-\sqrt{2}\, z}}\sqrt{1-\sqrt{2}\, z} - \frac{\sqrt{z^4-z^2}}{z\sqrt{z^2-1}} + 2\right) + \frac{\sqrt{-z^2}\sqrt{z^2-1}\sqrt{2z^2-1}}{2\sqrt{1-2z^2}\sqrt{z^4-z^2}}\sin^{-1}\left(2 z \sqrt{1-z^2}\right)$$

Involving $\cos^{-1}(-z)$

Involving $\cos^{-1}(-z)$ and $\sin^{-1}(z)$

01.13.27.0014.01

$$\cos^{-1}(-z) = \sin^{-1}(z) + \frac{\pi}{2}$$

Involving $\cos^{-1}\left(\sqrt{z}\right)$

Involving $\cos^{-1}\left(\sqrt{z}\right)$ and $\sin^{-1}\left(\sqrt{z}\right)$

01.13.27.0304.01

$$\cos^{-1}\left(\sqrt{z}\right) = \frac{\pi}{2} - \sin^{-1}\left(\sqrt{z}\right)$$

Involving $\cos^{-1}\left(\sqrt{z}\right)$ and $\sin^{-1}\left(1 \Big/ \sqrt{\frac{1}{z}}\right)$

01.13.27.0305.01

$$\cos^{-1}\left(\sqrt{z}\right) = \frac{\pi}{2} - \sin^{-1}\left(1 \Big/ \sqrt{\frac{1}{z}}\right) /; z \notin (-\infty, 0)$$

01.13.27.0306.01

$$\cos^{-1}\left(\sqrt{z}\right) = \frac{\pi}{2} + \sin^{-1}\left(1 \Big/ \sqrt{\frac{1}{z}}\right) /; (z \in \mathbb{R} \land z < 0)$$

01.13.27.0307.01

$$\cos^{-1}\left(\sqrt{z}\right) = \frac{\pi}{2} - \sqrt{z}\,\sqrt{\frac{1}{z}}\,\sin^{-1}\left(1 \Big/ \sqrt{\frac{1}{z}}\right)$$

Involving $\cos^{-1}\left(\frac{1}{\sqrt{z}}\right)$

Involving $\cos^{-1}\left(\frac{1}{\sqrt{z}}\right)$ and $\sin^{-1}\left(\frac{1}{\sqrt{z}}\right)$

01.13.27.0308.01

$$\cos^{-1}\left(\frac{1}{\sqrt{z}}\right) = \frac{\pi}{2} - \sin^{-1}\left(\frac{1}{\sqrt{z}}\right)$$

Involving $\cos^{-1}\left(\frac{1}{\sqrt{z}}\right)$ and $\sin^{-1}\left(\sqrt{\frac{1}{z}}\right)$

01.13.27.0309.01

$$\cos^{-1}\left(\frac{1}{\sqrt{z}}\right) = -\sin^{-1}\left(\sqrt{\frac{1}{z}}\right) + \frac{\pi}{2} /; z \notin (-\infty, 0)$$

01.13.27.0310.01

$$\cos^{-1}\left(\frac{1}{\sqrt{z}}\right) = \sin^{-1}\left(\sqrt{\frac{1}{z}}\right) + \frac{\pi}{2} /; (z \in \mathbb{R} \land z < 0)$$

01.13.27.0311.01

$$\cos^{-1}\left(\frac{1}{\sqrt{z}}\right) = \frac{\pi}{2} - \sqrt{z}\,\sqrt{\frac{1}{z}}\,\sin^{-1}\left(\sqrt{\frac{1}{z}}\right)$$

Involving $\cos^{-1}\left(\sqrt{z^2}\right)$

Involving $\cos^{-1}\left(\sqrt{z^2}\right)$ and $\sin^{-1}(z)$

01.13.27.0312.01

$$\cos^{-1}\left(\sqrt{z^2}\right) = \frac{\pi}{2} - \sin^{-1}(z) /; -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

01.13.27.0313.01

$$\cos^{-1}\left(\sqrt{z^2}\right) = \frac{\pi}{2} + \sin^{-1}(z) /; \frac{\pi}{2} < \arg(z) \leq \pi \bigvee -\pi < \arg(z) \leq -\frac{\pi}{2}$$

01.13.27.0314.01

$$\cos^{-1}\left(\sqrt{z^2}\right) = \frac{\pi}{2} - \frac{\sqrt{z^2}}{z} \sin^{-1}(z)$$

Involving $\cos^{-1}\left(a\,(b\,z^c)^m\right)$

Involving $\cos^{-1}(a\,(b\,z^c)^m)$ and $\sin^{-1}(a\,b^m\,z^{m\,c})$

01.13.27.0315.01

$$\cos^{-1}(a\,(b\,z^c)^m) = \frac{\pi}{2} - \frac{(b\,z^c)^m}{b^m\,z^{m\,c}} \sin^{-1}(a\,b^m\,z^{m\,c})\;/;\;2\,m \in \mathbb{Z}$$

Involving $\cos^{-1}\left(1-2\,z^2\right)$

Involving $\cos^{-1}\left(1-2\,z^2\right)$ and $\sin^{-1}(z)$

01.13.27.0316.01

$$\cos^{-1}\left(1-2\,z^2\right) = 2\sin^{-1}(z)\;/;\;-\frac{\pi}{2} < \arg(z) \le \frac{\pi}{2}$$

01.13.27.0317.01

$$\cos^{-1}\left(1-2\,z^2\right) = -2\sin^{-1}(z)\;/;\;\frac{\pi}{2} < \arg(z) \le \pi \bigvee -\pi < \arg(z) \le -\frac{\pi}{2}$$

01.13.27.0015.01

$$\cos^{-1}\left(1-2\,z^2\right) = \frac{2\sqrt{z^2}}{z} \sin^{-1}(z)$$

Involving $\cos^{-1}\left(2\,z^2-1\right)$

Involving $\cos^{-1}\left(2\,z^2-1\right)$ and $\sin^{-1}(z)$

01.13.27.0318.01

$$\cos^{-1}\left(2\,z^2-1\right) = \pi - 2\sin^{-1}(z)\;/;\;-\frac{\pi}{2} < \arg(z) \le \frac{\pi}{2}$$

01.13.27.0319.01

$$\cos^{-1}\left(2\,z^2-1\right) = \pi + 2\sin^{-1}(z)\;/;\;\frac{\pi}{2} < \arg(z) \le \pi \bigvee -\pi < \arg(z) \le -\frac{\pi}{2}$$

01.13.27.0320.01

$$\cos^{-1}\left(2\,z^2-1\right) = \pi - \frac{2\sqrt{z^2}}{z} \sin^{-1}(z)$$

Involving $\cos^{-1}\left(\frac{z^2-2}{z^2}\right)$

Involving $\cos^{-1}\left(\frac{z^2-2}{z^2}\right)$ and $\sin^{-1}\left(\frac{1}{z}\right)$

01.13.27.0321.01

$$\cos^{-1}\left(\frac{z^2-2}{z^2}\right) = 2\sin^{-1}\left(\frac{1}{z}\right) /; -\frac{\pi}{2} \leq \arg(z) < \frac{\pi}{2}$$

01.13.27.0322.01

$$\cos^{-1}\left(\frac{z^2-2}{z^2}\right) = -2\sin^{-1}\left(\frac{1}{z}\right) /; \frac{\pi}{2} \leq \arg(z) \leq \pi \bigvee -\pi < \arg(z) < -\frac{\pi}{2}$$

01.13.27.0323.01

$$\cos^{-1}\left(\frac{z^2-2}{z^2}\right) = 2\sqrt{\frac{1}{z^2}}\, z \sin^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\frac{2-z^2}{z^2}\right)$

Involving $\cos^{-1}\left(\frac{2-z^2}{z^2}\right)$ and $\sin^{-1}\left(\frac{1}{z}\right)$

01.13.27.0324.01

$$\cos^{-1}\left(\frac{2-z^2}{z^2}\right) = -2\sin^{-1}\left(\frac{1}{z}\right) + \pi /; -\frac{\pi}{2} \leq \arg(z) < \frac{\pi}{2}$$

01.13.27.0325.01

$$\cos^{-1}\left(\frac{2-z^2}{z^2}\right) = 2\sin^{-1}\left(\frac{1}{z}\right) + \pi /; \frac{\pi}{2} \leq \arg(z) \leq \pi \bigvee -\pi < \arg(z) < -\frac{\pi}{2}$$

01.13.27.0326.01

$$\cos^{-1}\left(\frac{2-z^2}{z^2}\right) = \pi - 2\sqrt{\frac{1}{z^2}}\, z \sin^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\sqrt{1-z}\right)$

Involving $\cos^{-1}\left(\sqrt{1-z}\right)$ and $\sin^{-1}\left(\sqrt{z}\right)$

01.13.27.0012.01

$$\cos^{-1}\left(\sqrt{1-z}\right) = \sin^{-1}\left(\sqrt{z}\right)$$

Involving $\cos^{-1}\left(\sqrt{\frac{1+c\,z}{2}}\right)$

Involving $\cos^{-1}\left(\sqrt{\frac{1+z}{2}}\right)$ and $\sin^{-1}(z)$

01.13.27.0327.01

$$\cos^{-1}\left(\sqrt{\frac{z+1}{2}}\right) = \frac{\pi}{4} - \frac{1}{2}\sin^{-1}(z)$$

Involving $\cos^{-1}\left(\sqrt{\frac{1-z}{2}}\right)$ and $\sin^{-1}(z)$

01.13.27.0013.01

$$\cos^{-1}\left(\sqrt{\frac{1-z}{2}}\right) = \frac{1}{2}\sin^{-1}(z) + \frac{\pi}{4}$$

Involving $\cos^{-1}\left(\frac{\sqrt{z-1}}{\sqrt{z}}\right)$

Involving $\cos^{-1}\left(\frac{\sqrt{z-1}}{\sqrt{z}}\right)$ and $\sin^{-1}\left(\frac{1}{\sqrt{z}}\right)$

01.13.27.0328.01

$$\cos^{-1}\left(\frac{\sqrt{z-1}}{\sqrt{z}}\right) = \sin^{-1}\left(\frac{1}{\sqrt{z}}\right) /; |\arg(z)| < \pi$$

01.13.27.0329.01

$$\cos^{-1}\left(\frac{\sqrt{z-1}}{\sqrt{z}}\right) = -\sin^{-1}\left(\frac{1}{\sqrt{z}}\right) /; (z \in \mathbb{R} \land z < 0)$$

01.13.27.0330.01

$$\cos^{-1}\left(\frac{\sqrt{z-1}}{\sqrt{z}}\right) = \sqrt{\frac{1}{z}}\sqrt{z}\,\sin^{-1}\left(\frac{1}{\sqrt{z}}\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{z-1}}{\sqrt{z}}\right)$ and $\sin^{-1}\left(\sqrt{\frac{1}{z}}\right)$

01.13.27.0331.01

$$\cos^{-1}\left(\frac{\sqrt{z-1}}{\sqrt{z}}\right) = \sin^{-1}\left(\sqrt{\frac{1}{z}}\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{-z}}\right)$

Involving $\cos^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{-z}}\right)$ and $\sin^{-1}\left(\frac{1}{\sqrt{z}}\right)$

01.13.27.0332.01

$$\cos^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{-z}}\right) = \sin^{-1}\left(\frac{1}{\sqrt{z}}\right) /; z \notin (-\infty, 1)$$

01.13.27.0333.01

$$\cos^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{-z}}\right) = \pi - \sin^{-1}\left(\frac{1}{\sqrt{z}}\right) /; (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.0334.01

$$\cos^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{-z}}\right) = -\sin^{-1}\left(\frac{1}{\sqrt{z}}\right) /; (z \in \mathbb{R} \land z < 0)$$

01.13.27.0335.01

$$\cos^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{-z}}\right) = \frac{\pi}{2}\left(1 - \sqrt{\frac{z-1}{z}} \sqrt{\frac{z}{z-1}}\right) + \sqrt{\frac{-1+z}{z^2}} \sqrt{z} \sqrt{\frac{z}{-1+z}} \sin^{-1}\left(\frac{1}{\sqrt{z}}\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{-z}}\right)$ and $\sin^{-1}\left(\sqrt{\frac{1}{z}}\right)$

01.13.27.0336.01

$$\cos^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{-z}}\right) = \sin^{-1}\left(\sqrt{\frac{1}{z}}\right) /; z \notin (0, 1)$$

01.13.27.0337.01

$$\cos^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{-z}}\right) = \pi - \sin^{-1}\left(\sqrt{\frac{1}{z}}\right) /; (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.0338.01

$$\cos^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{-z}}\right) = \frac{\pi}{2}\left(1 - \sqrt{\frac{z-1}{z}} \sqrt{\frac{z}{z-1}}\right) + \sqrt{\frac{-1+z}{z}} \sqrt{\frac{z}{-1+z}} \sin^{-1}\left(\sqrt{\frac{1}{z}}\right)$$

Involving $\cos^{-1}\left(\sqrt{\frac{z-1}{z}}\right)$

Involving $\cos^{-1}\left(\sqrt{\frac{z-1}{z}}\right)$ and $\sin^{-1}\left(\frac{1}{\sqrt{z}}\right)$

01.13.27.0339.01

$$\cos^{-1}\left(\sqrt{\frac{z-1}{z}}\right) = \sin^{-1}\left(\frac{1}{\sqrt{z}}\right) /; |\arg(z)| < \pi$$

01.13.27.0340.01

$$\cos^{-1}\left(\sqrt{\frac{z-1}{z}}\right) = -\sin^{-1}\left(\frac{1}{\sqrt{z}}\right) /; (z \in \mathbb{R} \land z < 0)$$

01.13.27.0341.01

$$\cos^{-1}\left(\sqrt{\frac{z-1}{z}}\right) == \sqrt{\frac{1}{z}}\,\sqrt{z}\,\sin^{-1}\left(\frac{1}{\sqrt{z}}\right)$$

Involving $\cos^{-1}\left(\sqrt{\frac{z-1}{z}}\right)$ and $\sin^{-1}\left(\sqrt{\frac{1}{z}}\right)$

01.13.27.0342.01

$$\cos^{-1}\left(\sqrt{\frac{z-1}{z}}\right) == \sin^{-1}\left(\sqrt{\frac{1}{z}}\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{z+a}}{\sqrt{2z}}\right)$

Involving $\cos^{-1}\left(\frac{\sqrt{z+1}}{\sqrt{2z}}\right)$ and $\sin^{-1}\left(\frac{1}{z}\right)$

01.13.27.0343.01

$$\cos^{-1}\left(\frac{\sqrt{z+1}}{\sqrt{2z}}\right) == \frac{\pi}{4} - \frac{1}{2}\sin^{-1}\left(\frac{1}{z}\right) /;\ z \notin (-1,\ 0)$$

01.13.27.0344.01

$$\cos^{-1}\left(\frac{\sqrt{z+1}}{\sqrt{2z}}\right) == \frac{1}{2}\sin^{-1}\left(\frac{1}{z}\right) + \frac{3\pi}{4} /;\ (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.0345.01

$$\cos^{-1}\left(\frac{\sqrt{z+1}}{\sqrt{2z}}\right) == \frac{1}{2}\pi\left(1 - \frac{1}{2}\sqrt{1+\frac{1}{z}}\,\sqrt{\frac{z}{z+1}}\right) - \frac{1}{2}\sqrt{1+\frac{1}{z}}\,\sqrt{\frac{z}{z+1}}\,\sin^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{z-1}}{\sqrt{2z}}\right)$ and $\sin^{-1}\left(\frac{1}{z}\right)$

01.13.27.0346.01

$$\cos^{-1}\left(\frac{\sqrt{z-1}}{\sqrt{2z}}\right) == \frac{\pi}{4} + \frac{1}{2}\sin^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{a-z}}{\sqrt{-2z}}\right)$

Involving $\cos^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{-2z}}\right)$ and $\sin^{-1}\left(\frac{1}{z}\right)$

01.13.27.0347.01

$$\cos^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{-2z}}\right) == \frac{1}{2}\sin^{-1}\left(\frac{1}{z}\right) + \frac{\pi}{4} /;\ z \notin (0,\ 1)$$

01.13.27.0348.01

$$\cos^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{-2z}}\right) = \frac{3\pi}{4} - \frac{1}{2}\sin^{-1}\left(\frac{1}{z}\right) /; (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.0349.01

$$\cos^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{-2z}}\right) = \frac{1}{4}\pi\left(2 - \sqrt{\frac{z-1}{z}}\sqrt{\frac{z}{z-1}}\right) + \frac{1}{2}\sqrt{\frac{z}{z-1}}\sqrt{\frac{z-1}{z}}\sin^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{-z-1}}{\sqrt{-2z}}\right)$ and $\sin^{-1}\left(\frac{1}{z}\right)$

01.13.27.0350.01

$$\cos^{-1}\left(\frac{\sqrt{-z-1}}{\sqrt{-2z}}\right) = -\frac{1}{2}\sin^{-1}\left(\frac{1}{z}\right) + \frac{\pi}{4}$$

Involving $\cos^{-1}\left(\sqrt{\frac{z+a}{2z}}\right)$

Involving $\cos^{-1}\left(\sqrt{\frac{z-1}{2z}}\right)$ and $\sin^{-1}\left(\frac{1}{z}\right)$

01.13.27.0351.01

$$\cos^{-1}\left(\sqrt{\frac{z-1}{2z}}\right) = \frac{\pi}{4} + \frac{1}{2}\sin^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\sqrt{\frac{z+1}{2z}}\right)$ and $\sin^{-1}\left(\frac{1}{z}\right)$

01.13.27.0352.01

$$\cos^{-1}\left(\sqrt{\frac{z+1}{2z}}\right) = -\frac{1}{2}\sin^{-1}\left(\frac{1}{z}\right) + \frac{\pi}{4}$$

Involving $\cos^{-1}\left(\sqrt{1-z^2}\right)$

Involving $\cos^{-1}\left(\sqrt{1-z^2}\right)$ and $\sin^{-1}(z)$

01.13.27.0353.01

$$\cos^{-1}\left(\sqrt{1-z^2}\right) = \sin^{-1}(z) /; -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

01.13.27.0354.01

$$\cos^{-1}\left(\sqrt{1-z^2}\right) = -\sin^{-1}(z) /; \frac{\pi}{2} < \arg(z) \leq \pi \bigvee -\pi < \arg(z) \leq -\frac{\pi}{2}$$

01.13.27.0016.01

$$\cos^{-1}\left(\sqrt{1-z^2}\right) = \frac{\sqrt{z^2}}{z} \sin^{-1}(z)$$

Involving $\cos^{-1}\left(\frac{\sqrt{z^2-1}}{z}\right)$

Involving $\cos^{-1}\left(\frac{\sqrt{z^2-1}}{z}\right)$ and $\sin^{-1}\left(\frac{1}{z}\right)$

01.13.27.0355.01

$$\cos^{-1}\left(\frac{\sqrt{z^2-1}}{z}\right) = \sin^{-1}\left(\frac{1}{z}\right) /; \operatorname{Re}(z) > 0$$

01.13.27.0356.01

$$\cos^{-1}\left(\frac{\sqrt{z^2-1}}{z}\right) = \sin^{-1}\left(\frac{1}{z}\right) + \pi /; \operatorname{Re}(z) < 0$$

01.13.27.0357.01

$$\cos^{-1}\left(\frac{\sqrt{z^2-1}}{z}\right) = -\sin^{-1}\left(\frac{1}{z}\right) /; (i\, z \in \mathbb{R} \land i\, z < 0)$$

01.13.27.0358.01

$$\cos^{-1}\left(\frac{\sqrt{z^2-1}}{z}\right) = \pi - \sin^{-1}\left(\frac{1}{z}\right) /; (i\, z \in \mathbb{R} \land i\, z > 0)$$

01.13.27.0359.01

$$\cos^{-1}\left(\frac{\sqrt{z^2-1}}{z}\right) = \frac{\pi}{2}\left(1 - \frac{\sqrt{z^2}}{z}\right) + \sin^{-1}\left(\frac{1}{z}\right) /; \operatorname{Re}(z) \neq 0$$

01.13.27.0360.01

$$\cos^{-1}\left(\frac{\sqrt{z^2-1}}{z}\right) = \frac{\pi}{2}\left(1 - \frac{\sqrt{z^2}}{z}\right) + \sqrt{z^2}\, \sqrt{\frac{1}{z^2}}\, \sin^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{z^2-1}}{\sqrt{z^2}}\right)$

Involving $\cos^{-1}\left(\frac{\sqrt{z^2-1}}{\sqrt{z^2}}\right)$ and $\sin^{-1}\left(\frac{1}{z}\right)$

01.13.27.0361.01

$$\cos^{-1}\left(\frac{\sqrt{z^2-1}}{\sqrt{z^2}}\right) = \sin^{-1}\left(\frac{1}{z}\right) /; -\frac{\pi}{2} \leq \arg(z) < \frac{\pi}{2}$$

01.13.27.0362.01

$$\cos^{-1}\left(\frac{\sqrt{z^2-1}}{\sqrt{z^2}}\right) = -\sin^{-1}\left(\frac{1}{z}\right) /; \frac{\pi}{2} \leq \arg(z) \leq \pi \bigvee -\pi < \arg(z) < -\frac{\pi}{2}$$

01.13.27.0363.01

$$\cos^{-1}\left(\frac{\sqrt{z^2-1}}{\sqrt{z^2}}\right) = z\sqrt{\frac{1}{z^2}}\sin^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{1-z^2}}{\sqrt{-z^2}}\right)$

Involving $\cos^{-1}\left(\frac{\sqrt{1-z^2}}{\sqrt{-z^2}}\right)$ and $\sin^{-1}\left(\frac{1}{z}\right)$

01.13.27.0364.01

$$\cos^{-1}\left(\frac{\sqrt{1-z^2}}{\sqrt{-z^2}}\right) = \sin^{-1}\left(\frac{1}{z}\right) /; -\frac{\pi}{2} \leq \arg(z) < 0 \bigvee 0 < \arg(z) < \frac{\pi}{2} \bigvee (z \in \mathbb{R} \land z > 1)$$

01.13.27.0365.01

$$\cos^{-1}\left(\frac{\sqrt{1-z^2}}{\sqrt{-z^2}}\right) = -\sin^{-1}\left(\frac{1}{z}\right) /; \frac{\pi}{2} \leq \arg(z) < \pi \bigvee -\pi < \arg(z) < -\frac{\pi}{2} \bigvee (z \in \mathbb{R} \land z < -1)$$

01.13.27.0366.01

$$\cos^{-1}\left(\frac{\sqrt{1-z^2}}{\sqrt{-z^2}}\right) = -\sin^{-1}\left(\frac{1}{z}\right) + \pi /; (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.0367.01

$$\cos^{-1}\left(\frac{\sqrt{1-z^2}}{\sqrt{-z^2}}\right) = \sin^{-1}\left(\frac{1}{z}\right) + \pi /; (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.0368.01

$$\cos^{-1}\left(\frac{\sqrt{1-z^2}}{\sqrt{-z^2}}\right) = \frac{1}{2}\pi\left(1 - \sqrt{1-\frac{1}{z^2}}\sqrt{\frac{z^2}{z^2-1}}\right) + z\sqrt{\frac{z^2-1}{z^4}}\sqrt{\frac{z^2}{z^2-1}}\sin^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\sqrt{\frac{z^2-1}{z^2}}\right)$

Involving $\cos^{-1}\left(\sqrt{\frac{z^2-1}{z^2}}\right)$ and $\sin^{-1}\left(\frac{1}{z}\right)$

01.13.27.0369.01

$$\cos^{-1}\left(\sqrt{\frac{z^2-1}{z^2}}\right) = \sin^{-1}\left(\frac{1}{z}\right) /; -\frac{\pi}{2} \leq \arg(z) < \frac{\pi}{2}$$

01.13.27.0370.01

$$\cos^{-1}\left(\sqrt{\frac{z^2-1}{z^2}}\right) = -\sin^{-1}\left(\frac{1}{z}\right) /; \frac{\pi}{2} \leq \arg(z) \leq \pi \bigvee -\pi < \arg(z) < -\frac{\pi}{2}$$

01.13.27.0371.01

$$\cos^{-1}\left(\sqrt{\frac{z^2-1}{z^2}}\right) = z\sqrt{\frac{1}{z^2}} \sin^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(2z\sqrt{1-z^2}\right)$

Involving $\cos^{-1}\left(2z\sqrt{1-z^2}\right)$ and $\sin^{-1}(z)$

01.13.27.0372.01

$$\cos^{-1}\left(2z\sqrt{1-z^2}\right) = \frac{\pi}{2} - 2\sin^{-1}(z) /; \frac{\pi}{4} \leq |\arg(z)| \leq \frac{3\pi}{4}$$

01.13.27.0373.01

$$\cos^{-1}\left(2z\sqrt{1-z^2}\right) = \frac{\pi\sqrt{1-2z^2}\sqrt{z^4-z^2}}{2\sqrt{-z^2}\sqrt{z^2-1}\sqrt{2z^2-1}} \left(-\frac{\sqrt{z^2}}{z} + \sqrt{\frac{1}{z}}\sqrt{z}\sqrt{\frac{1}{\sqrt{2}\,z+1}}\sqrt{\sqrt{2}\,z+1} - \sqrt{-\frac{1}{z}}\sqrt{-z}\sqrt{\frac{1}{1-\sqrt{2}\,z}}\sqrt{1-\sqrt{2}\,z} - \frac{\sqrt{z^4-z^2}}{z\sqrt{z^2-1}}\right) + \frac{2\sqrt{1-2z^2}\sqrt{z^4-z^2}}{\sqrt{-z^2}\sqrt{z^2-1}\sqrt{2z^2-1}} \sin^{-1}(z) + \frac{\pi}{2}$$

Involving $\cos^{-1}\left(\frac{2\sqrt{z^2-1}}{z^2}\right)$

Involving $\cos^{-1}\left(\frac{2\sqrt{z^2-1}}{z^2}\right)$ and $\sin^{-1}\left(\frac{1}{z}\right)$

01.13.27.0374.01

$$\cos^{-1}\left(\frac{2\sqrt{z^2-1}}{z^2}\right) = \frac{\pi}{2} - 2\sin^{-1}\left(\frac{1}{z}\right) /; \frac{\pi}{4} \leq |\arg(z)| < \frac{\pi}{2} \bigvee |z| \geq \sqrt{2} \bigwedge |\arg(z)| < \frac{\pi}{4}$$

01.13.27.0375.01

$$\cos^{-1}\left(\frac{2\sqrt{z^2-1}}{z^2}\right) = \frac{\pi}{2} - \frac{2\sqrt{z^2}}{z}\sin^{-1}\left(\frac{1}{z}\right) /; \frac{\pi}{4} \leq |\arg(z)| \leq \frac{3\pi}{4} \bigvee |z| \geq \sqrt{2}$$

01.13.27.0376.01

$$\cos^{-1}\left(\frac{2\sqrt{z^2-1}}{z^2}\right) = \frac{\pi}{2} + \frac{z^3\sqrt{z^2-2}\sqrt{z^2-1}}{2\sqrt{1-z}\,(z+1)\sqrt{-z^4+3z^2-2}}\sqrt{\frac{1-z^2}{z^4}}\sqrt{\frac{1}{z}}\sqrt{-\frac{z+1}{z}}\left(\pi\left(\frac{z^3}{1-z^2}\sqrt{\frac{1-z^2}{z^2}}\sqrt{\frac{1-z^2}{z^4}} + \sqrt{\frac{1}{z^2}}\,z - \sqrt{\frac{1}{z}}\sqrt{\frac{z}{z+\sqrt{2}}}\sqrt{\frac{z+\sqrt{2}}{z}}\sqrt{z} + \sqrt{1-\frac{\sqrt{2}}{z}}\sqrt{-\frac{1}{z}}\sqrt{-z}\sqrt{\frac{z}{z-\sqrt{2}}}\right) - 4\sin^{-1}\left(\frac{1}{z}\right)\right)$$

Involving $\cos^{-1}\left(\sqrt{\left(1-\sqrt{1+c\,z^2}\right)/2}\right)$

Involving $\cos^{-1}\left(\sqrt{\left(1-\sqrt{1+z^2}\right)/2}\right)$ and $\sin^{-1}(i\,z)$

01.13.27.0377.01

$$\cos^{-1}\left(\sqrt{\frac{1-\sqrt{1+z^2}}{2}}\right) = \frac{\pi}{2} - \frac{1}{2}\sin^{-1}(i\,z) /; -\pi < \arg(z) \leq 0$$

01.13.27.0378.01

$$\cos^{-1}\left(\sqrt{\frac{1}{2}\left(1-\sqrt{z^2+1}\right)}\right) = \frac{1}{2}\sin^{-1}(i\,z) + \frac{\pi}{2} /; 0 < \arg(z) \leq \pi$$

Involving $\cos^{-1}\left(\sqrt{\left(1-\sqrt{1-z^2}\right)/2}\right)$ and $\sin^{-1}(z)$

01.13.27.0379.01

$$\cos^{-1}\left(\sqrt{\frac{1-\sqrt{1-z^2}}{2}}\right) = \frac{\pi}{2} - \frac{1}{2}\sin^{-1}(z) \;/;\; -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

01.13.27.0380.01

$$\cos^{-1}\left(\sqrt{\frac{1-\sqrt{1-z^2}}{2}}\right) = \frac{\pi}{2} + \frac{1}{2}\sin^{-1}(z) \;/;\; \frac{\pi}{2} < \arg(z) \leq \pi \bigvee -\pi < \arg(z) \leq -\frac{\pi}{2}$$

01.13.27.0017.01

$$\cos^{-1}\left(\sqrt{\frac{1-\sqrt{1-z^2}}{2}}\right) = \frac{\pi}{2} - \frac{\sqrt{z^2}}{2z}\sin^{-1}(z)$$

Involving $\cos^{-1}\left(z\sqrt{1-\sqrt{1-z^2}}\,\big/\sqrt{2z^2}\right)$

Involving $\cos^{-1}\left(z\sqrt{1-\sqrt{1-z^2}}\,\big/\sqrt{2z^2}\right)$ and $\sin^{-1}(z)$

01.13.27.2516.01

$$\cos^{-1}\left(\frac{z\sqrt{1-\sqrt{1-z^2}}}{\sqrt{2}\sqrt{z^2}}\right) = \frac{\pi}{2} - \frac{1}{2}\sin^{-1}(z)$$

Involving $\cos^{-1}\left(z\sqrt{\left(1-\sqrt{1-z^2}\right)\big/\left(2z^2\right)}\right)$

Involving $\cos^{-1}\left(z\sqrt{\left(1-\sqrt{1-z^2}\right)\big/\left(2z^2\right)}\right)$ and $\sin^{-1}(z)$

01.13.27.0381.01

$$\cos^{-1}\left(z\sqrt{\frac{1-\sqrt{1-z^2}}{2z^2}}\right) = \frac{\pi}{2} - \frac{1}{2}\sin^{-1}(z)$$

Involving $\cos^{-1}\left(\sqrt{z-\sqrt{z^2-1}}\Big/\sqrt{2z}\right)$

Involving $\cos^{-1}\left(\sqrt{z-\sqrt{z^2-1}}\Big/\sqrt{2z}\right)$ and $\sin^{-1}\left(\frac{1}{z}\right)$

01.13.27.0382.01

$$\cos^{-1}\left(\frac{\sqrt{z-\sqrt{z^2-1}}}{\sqrt{2z}}\right) = \frac{1}{2}\left(\pi - \sin^{-1}\left(\frac{1}{z}\right)\right) /; -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

01.13.27.0383.01

$$\cos^{-1}\left(\frac{\sqrt{z-\sqrt{z^2-1}}}{\sqrt{2z}}\right) = -\frac{1}{2}\sin^{-1}\left(\frac{1}{z}\right) /; \frac{\pi}{2} < \arg(z) < \pi \bigvee -\pi < \arg(z) < -\frac{\pi}{2} \bigvee (z \in \mathbb{R} \land z < -1)$$

01.13.27.0384.01

$$\cos^{-1}\left(\frac{\sqrt{z-\sqrt{z^2-1}}}{\sqrt{2z}}\right) = \frac{1}{2}\sin^{-1}\left(\frac{1}{z}\right) + \pi /; (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.0385.01

$$\cos^{-1}\left(\frac{\sqrt{z-\sqrt{z^2-1}}}{\sqrt{2z}}\right) = \frac{1}{2}\sin^{-1}\left(\frac{1}{z}\right) /; (i\,z \in \mathbb{R} \land i\,z > 0)$$

01.13.27.0386.01

$$\cos^{-1}\left(\frac{\sqrt{z-\sqrt{z^2-1}}}{\sqrt{2z}}\right) =$$

$$\frac{1}{4}\pi\left(\frac{1}{\sqrt{\frac{z}{z+1}}}\sqrt{\frac{1}{z}}\sqrt{\frac{1}{z+1}}\left(\sqrt{z^2}-z\right)+2\right) - \frac{1}{2}\sqrt{\frac{i}{z}}\sqrt{\frac{1}{z}}\sqrt{-i\,z}\sqrt{z}\sqrt{\frac{1}{z+1}}\sqrt{z+1}\sin^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\sqrt{\left(z-\sqrt{z^2-1}\right)/(2z)}\right)$

Involving $\cos^{-1}\left(\sqrt{\left(z-\sqrt{z^2-1}\right)/(2z)}\right)$ and $\sin^{-1}\left(\frac{1}{z}\right)$

01.13.27.0387.01

$$\cos^{-1}\left(\sqrt{\frac{z-\sqrt{z^2-1}}{2z}}\right) = \frac{1}{2}\left(\pi-\sin^{-1}\left(\frac{1}{z}\right)\right) /; \operatorname{Re}(z)>0$$

01.13.27.0388.01

$$\cos^{-1}\left(\sqrt{\frac{z-\sqrt{z^2-1}}{2z}}\right) = -\frac{1}{2}\sin^{-1}\left(\frac{1}{z}\right) /; \operatorname{Re}(z)<0$$

01.13.27.0389.01

$$\cos^{-1}\left(\sqrt{\frac{z-\sqrt{z^2-1}}{2z}}\right) = \frac{1}{2}\left(\sin^{-1}\left(\frac{1}{z}\right)+\pi\right) /; (i\,z\in\mathbb{R}\land i\,z<0)$$

01.13.27.0390.01

$$\cos^{-1}\left(\sqrt{\frac{z-\sqrt{z^2-1}}{2z}}\right) = \frac{1}{2}\sin^{-1}\left(\frac{1}{z}\right) /; (i\,z\in\mathbb{R}\land i\,z>0)$$

01.13.27.2517.01

$$\cos^{-1}\left(\sqrt{\frac{z-\sqrt{z^2-1}}{2z}}\right) = -\frac{1}{2}\sin^{-1}\left(\frac{1}{z}\right)+\frac{\pi}{4}\left(\frac{\sqrt{z^2}}{z}+1\right) /; \operatorname{Re}(z)\neq 0$$

01.13.27.0391.01

$$\cos^{-1}\left(\sqrt{\frac{z-\sqrt{z^2-1}}{2z}}\right) = -\frac{1}{2}\sqrt{z^2}\sqrt{\frac{1}{z^2}}\sin^{-1}\left(\frac{1}{z}\right)+\frac{1}{4}\pi\left(\frac{\sqrt{z^2}}{z}+1\right)$$

Involving $\sin^{-1}\left(c\,z^{-r}\sqrt{-1+\frac{z^{2r}}{c^2}}\right)$

Involving $\cos^{-1}\left(c\,z^{-r}\sqrt{-1+\frac{z^{2r}}{c^2}}\right)$ and $\sin^{-1}\left(\frac{c}{z^r}\right)$

01.13.27.0392.01

$$\cos^{-1}\left(c\,z^{-r}\sqrt{\frac{z^{2r}}{c^2}-1}\right)=\frac{\pi}{2}-\sqrt{c^2\,z^{-2r}}\,\sqrt{\frac{z^{2r}}{c^2}}\left(\frac{\pi\,z^r\sqrt{c^2\,z^{-2r}}}{2\,c}-\sin^{-1}\left(\frac{c}{z^r}\right)\right)$$

Involving $\tan^{-1}$

Involving $\cos^{-1}(z)$

Involving $\cos^{-1}(z)$ and $\tan^{-1}\left(\frac{\sqrt{1-z^2}}{z}\right)$

01.13.27.0393.01

$$\cos^{-1}(z)=\tan^{-1}\left(\frac{\sqrt{1-z^2}}{z}\right)\;/;\;-\frac{\pi}{2}\leq\arg(z)<\frac{\pi}{2}$$

01.13.27.0394.01

$$\cos^{-1}(z)=\tan^{-1}\left(\frac{\sqrt{1-z^2}}{z}\right)+\pi\;/;\;\frac{\pi}{2}\leq\arg(z)\leq\pi\bigvee-\pi<\arg(z)<-\frac{\pi}{2}$$

01.13.27.0395.01

$$\cos^{-1}(z)=\tan^{-1}\left(\frac{\sqrt{1-z^2}}{z}\right)+\frac{\pi}{2}\left(1-\sqrt{\frac{1}{z^2}}\;z\right)$$

Involving $\cos^{-1}(z)$ and $\tan^{-1}\left(\frac{\sqrt{1-z^2}}{\sqrt{z^2}}\right)$

01.13.27.0396.01

$$\cos^{-1}(z)=\tan^{-1}\left(\frac{\sqrt{1-z^2}}{\sqrt{z^2}}\right)\;/;\;\mathrm{Re}(z)>0$$

01.13.27.0397.01

$$\cos^{-1}(z)=\pi-\tan^{-1}\left(\frac{\sqrt{1-z^2}}{\sqrt{z^2}}\right)\;/;\;\frac{\pi}{2}<\arg(z)\leq\pi\bigvee-\pi<\arg(z)<-\frac{\pi}{2}$$

01.13.27.0398.01

$$\cos^{-1}(z)=\pi+\tan^{-1}\left(\frac{\sqrt{1-z^2}}{\sqrt{z^2}}\right)\;/;\;(i\,z\in\mathbb{R}\land i\,z<0)$$

01.13.27.0399.01

$$\cos^{-1}(z)=-\tan^{-1}\left(\frac{\sqrt{1-z^2}}{\sqrt{z^2}}\right)\;/;\;(i\,z\in\mathbb{R}\land i\,z>0)$$

01.13.27.0400.01

$$\cos^{-1}(z) = \frac{1}{2}\pi\left(1 - \sqrt{\frac{1}{z^2}}\, z\right) + \frac{\sqrt{z^2}}{z}\tan^{-1}\left(\frac{\sqrt{1-z^2}}{\sqrt{z^2}}\right)$$

Involving $\cos^{-1}(z)$ and $\tan^{-1}\left(\frac{\sqrt{z^2-1}}{\sqrt{-z^2}}\right)$

01.13.27.0401.01

$$\cos^{-1}(z) = -\tan^{-1}\left(\frac{\sqrt{z^2-1}}{\sqrt{-z^2}}\right) /; -\frac{\pi}{2} < \arg(z) < 0 \bigvee 0 < \arg(z) < \frac{\pi}{2} \bigvee (z \in \mathbb{R} \land z > 1)$$

01.13.27.0402.01

$$\cos^{-1}(z) = \tan^{-1}\left(\frac{\sqrt{z^2-1}}{\sqrt{-z^2}}\right) + \pi /; \frac{\pi}{2} < \arg(z) < \pi \bigvee -\pi < \arg(z) < -\frac{\pi}{2} \bigvee (z \in \mathbb{R} \land z < -1)$$

01.13.27.0403.01

$$\cos^{-1}(z) = \pi - \tan^{-1}\left(\frac{\sqrt{z^2-1}}{\sqrt{-z^2}}\right) /; (i\, z \in \mathbb{R} \land i\, z < 0) \lor (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.0404.01

$$\cos^{-1}(z) = \tan^{-1}\left(\frac{\sqrt{z^2-1}}{\sqrt{-z^2}}\right) /; (i\, z \in \mathbb{R} \land i\, z > 0) \lor (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.0405.01

$$\cos^{-1}(z) = \frac{\pi}{2}\left(1 - \sqrt{z^{-2}}\, z\right) + \frac{\sqrt{-z^2}\,\sqrt{1-z^2}}{z\sqrt{z^2-1}}\tan^{-1}\left(\frac{\sqrt{z^2-1}}{\sqrt{-z^2}}\right)$$

Involving $\cos^{-1}(z)$ and $\tan^{-1}\left(\sqrt{\frac{1-z^2}{z^2}}\right)$

01.13.27.0406.01

$$\cos^{-1}(z) = \tan^{-1}\left(\sqrt{\frac{1-z^2}{z^2}}\right) /; -\frac{\pi}{2} \leq \arg(z) < \frac{\pi}{2}$$

01.13.27.0407.01

$$\cos^{-1}(z) = \pi - \tan^{-1}\left(\sqrt{\frac{1-z^2}{z^2}}\right) /; \frac{\pi}{2} \leq \arg(z) \leq \pi \bigvee -\pi < \arg(z) < -\frac{\pi}{2}$$

01.13.27.0408.01

$$\cos^{-1}(z) == \frac{\pi}{2}\left(1-\sqrt{\frac{1}{z^2}}\, z\right)+\frac{z}{\sqrt{1-z^2}}\sqrt{\frac{1-z^2}{z^2}}\,\tan^{-1}\left(\sqrt{\frac{1-z^2}{z^2}}\right)$$

Involving $\cos^{-1}(z)$ and $\tan^{-1}\left(\frac{z}{\sqrt{1-z^2}}\right)$

01.13.27.0409.01

$$\cos^{-1}(z) == \frac{\pi}{2}-\tan^{-1}\left(\frac{z}{\sqrt{1-z^2}}\right) /; z \notin (-\infty, -1) \bigwedge z \notin (1, \infty)$$

01.13.27.0410.01

$$\cos^{-1}(z) == -\frac{\pi}{2}-\tan^{-1}\left(\frac{z}{\sqrt{1-z^2}}\right) /; (z \in \mathbb{R} \land z > 1)$$

01.13.27.0411.01

$$\cos^{-1}(z) == \frac{3\pi}{2}-\tan^{-1}\left(\frac{z}{\sqrt{1-z^2}}\right) /; (z \in \mathbb{R} \land z < -1)$$

01.13.27.0412.01

$$\cos^{-1}(z) == \frac{\pi}{2}\left(\sqrt{\frac{1}{1-z}}\sqrt{1-z}-\sqrt{\frac{1}{z+1}}\sqrt{z+1}+1\right)-\tan^{-1}\left(\frac{z}{\sqrt{1-z^2}}\right)$$

Involving $\cos^{-1}(z)$ and $\tan^{-1}\left(\frac{\sqrt{z^2}}{\sqrt{1-z^2}}\right)$

01.13.27.0413.01

$$\cos^{-1}(z) == \frac{\pi}{2}-\tan^{-1}\left(\frac{\sqrt{z^2}}{\sqrt{1-z^2}}\right) /; -\frac{\pi}{2} < \arg(z) \le \frac{\pi}{2} \bigwedge z \notin (1, \infty)$$

01.13.27.0414.01

$$\cos^{-1}(z) == \frac{\pi}{2}+\tan^{-1}\left(\frac{\sqrt{z^2}}{\sqrt{1-z^2}}\right) /; \frac{\pi}{2} < \arg(z) < \pi \bigvee -\pi < \arg(z) \le -\frac{\pi}{2} \bigvee (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.0415.01

$$\cos^{-1}(z) == \frac{3\pi}{2}+\tan^{-1}\left(\frac{\sqrt{z^2}}{\sqrt{1-z^2}}\right) /; (z \in \mathbb{R} \land z < -1)$$

01.13.27.0416.01

$$\cos^{-1}(z) == \frac{1}{2}\pi\left(-\sqrt{\frac{1}{z+1}}\sqrt{z+1}+\sqrt{\frac{1}{1-z}}\sqrt{1-z}+1\right)-\frac{\sqrt{z^2}}{z}\tan^{-1}\left(\frac{\sqrt{z^2}}{\sqrt{1-z^2}}\right)$$

Involving $\cos^{-1}(z)$ and $\tan^{-1}\left(\frac{\sqrt{-z^2}}{\sqrt{z^2-1}}\right)$

01.13.27.0417.01

$$\cos^{-1}(z) = \frac{\pi}{2} + \tan^{-1}\left(\frac{\sqrt{-z^2}}{\sqrt{z^2-1}}\right) /; 0 < \arg(z) \leq \frac{\pi}{2} \bigvee -\frac{\pi}{2} < \arg(z) < 0 \bigvee (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.0418.01

$$\cos^{-1}(z) = \frac{\pi}{2} - \tan^{-1}\left(\frac{\sqrt{-z^2}}{\sqrt{z^2-1}}\right) /; \frac{\pi}{2} < \arg(z) < \pi \bigvee -\pi < \arg(z) \leq -\frac{\pi}{2} \bigvee (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.0419.01

$$\cos^{-1}(z) = \tan^{-1}\left(\frac{\sqrt{-z^2}}{\sqrt{z^2-1}}\right) - \frac{\pi}{2} /; (z \in \mathbb{R} \land z > 1)$$

01.13.27.0420.01

$$\cos^{-1}(z) = \frac{3\pi}{2} - \tan^{-1}\left(\frac{\sqrt{-z^2}}{\sqrt{z^2-1}}\right) /; (z \in \mathbb{R} \land z < -1)$$

01.13.27.0421.01

$$\cos^{-1}(z) = \frac{\sqrt{-z^2}\sqrt{z^2-1}}{z\sqrt{1-z^2}} \tan^{-1}\left(\frac{\sqrt{-z^2}}{\sqrt{z^2-1}}\right) + \frac{\pi}{2}\left(1 + \sqrt{\frac{1}{1-z}}\sqrt{1-z} - \sqrt{\frac{1}{z+1}}\sqrt{z+1}\right)$$

Involving $\cos^{-1}(z)$ and $\tan^{-1}\left(\sqrt{\frac{z^2}{1-z^2}}\right)$

01.13.27.0422.01

$$\cos^{-1}(z) = \frac{\pi}{2} - \tan^{-1}\left(\sqrt{\frac{z^2}{1-z^2}}\right) /; -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2} \bigwedge z \notin (1, \infty)$$

01.13.27.0423.01

$$\cos^{-1}(z) = \frac{\pi}{2} + \tan^{-1}\left(\sqrt{\frac{z^2}{1-z^2}}\right) /; \frac{\pi}{2} < \arg(z) < \pi \bigvee -\pi < \arg(z) \leq -\frac{\pi}{2} \bigvee (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.0424.01

$$\cos^{-1}(z) = \frac{3\pi}{2} - \tan^{-1}\left(\sqrt{\frac{z^2}{1-z^2}}\right) /; (z \in \mathbb{R} \land z < -1)$$

01.13.27.0425.01

$$\cos^{-1}(z) = \frac{\pi}{2}\left(-\sqrt{\frac{1}{z+1}}\sqrt{z+1}+\sqrt{\frac{1}{1-z}}\sqrt{1-z}+1\right)-\frac{\sqrt{1-z^2}}{z}\sqrt{\frac{z^2}{1-z^2}}\tan^{-1}\left(\sqrt{\frac{z^2}{1-z^2}}\right)$$

Involving $\cos^{-1}(z)$ and $\tan^{-1}\left(\frac{2z\sqrt{1-z^2}}{1-2z^2}\right)$

01.13.27.0426.01

$$\cos^{-1}(z) = \frac{\pi}{2}-\frac{1}{2}\tan^{-1}\left(\frac{2z\sqrt{1-z^2}}{1-2z^2}\right)\;/;\;\frac{\pi}{4}\leq|\arg(z)|\leq\frac{3\pi}{4}$$

01.13.27.0427.01

$$\cos^{-1}(z) = \frac{1}{4}\pi\left(-\frac{\sqrt{z^2-1}\,z}{\sqrt{z^4-z^2}}-\sqrt{\frac{1}{z}}\sqrt{\frac{1}{\sqrt{2}\,z-1}}\sqrt{\sqrt{2}\,z-1}\sqrt{z}+\sqrt{-\frac{1}{z}}\sqrt{-z}\sqrt{-\frac{1}{\sqrt{2}\,z+1}}\sqrt{-\sqrt{2}\,z-1}+2-\frac{\sqrt{z^2}}{z}\right)-\frac{1}{2}\tan^{-1}\left(\frac{2z\sqrt{1-z^2}}{1-2z^2}\right)$$

Involving $\cos^{-1}(z)$ and $\tan^{-1}\left(\frac{1-2z^2}{2z\sqrt{1-z^2}}\right)$

01.13.27.0428.01

$$\cos^{-1}(z) = \frac{\pi}{4}+\frac{1}{2}\tan^{-1}\left(\frac{1-2z^2}{2z\sqrt{1-z^2}}\right)\;/;\;-\frac{\pi}{2}\leq\arg(z)<0\bigvee 0<\arg(z)<\frac{\pi}{2}\bigvee(z\in\mathbb{R}\land 0<z<1)$$

01.13.27.0429.01

$$\cos^{-1}(z) = \frac{3\pi}{4}+\frac{1}{2}\tan^{-1}\left(\frac{1-2z^2}{2z\sqrt{1-z^2}}\right)\;/;\;\frac{\pi}{2}\leq\arg(z)<\pi\bigvee -\pi<\arg(z)<-\frac{\pi}{2}\bigvee(z\in\mathbb{R}\land -1<z<0)$$

01.13.27.0430.01

$$\cos^{-1}(z) = \frac{1}{2}\tan^{-1}\left(\frac{1-2z^2}{2z\sqrt{1-z^2}}\right)+\frac{5\pi}{4}\;/;\;(z\in\mathbb{R}\land z<-1)$$

01.13.27.0431.01

$$\cos^{-1}(z) = \frac{1}{2}\tan^{-1}\left(\frac{1-2z^2}{2z\sqrt{1-z^2}}\right)-\frac{\pi}{4}\;/;\;(z\in\mathbb{R}\land z>1)$$

01.13.27.0432.01

$$\cos^{-1}(z) = \frac{1}{4}\pi\left(\sqrt{\frac{1}{1-z}}\sqrt{1-z}-\sqrt{z+1}\sqrt{\frac{1}{z+1}}-\sqrt{i\,z}\sqrt{-\frac{i}{z}}+\sqrt{\frac{i}{z}}\sqrt{-i\,z}-\frac{\sqrt{z^2}}{z}+2\right)+\frac{1}{2}\tan^{-1}\left(\frac{1-2z^2}{2z\sqrt{1-z^2}}\right)$$

Involving $\cos^{-1}(z)$ and $\tan^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{1+z}}\right)$

01.13.27.0433.01

$$\cos^{-1}(z) = 2\tan^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{1+z}}\right) /;\ z \notin (-\infty, -1)$$

01.13.27.0434.01

$$\cos^{-1}(z) = 2\pi + 2\tan^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{1+z}}\right) /;\ (z \in \mathbb{R} \land z < -1)$$

01.13.27.0435.01

$$\cos^{-1}(z) = 2\tan^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{z+1}}\right) - \left(\sqrt{\frac{1}{z+1}}\ \sqrt{z+1} - 1\right)\pi$$

Involving $\cos^{-1}(z)$ and $\tan^{-1}\left(\frac{\sqrt{z-1}}{\sqrt{-z-1}}\right)$

01.13.27.0436.01

$$\cos^{-1}(z) = -2\tan^{-1}\left(\frac{\sqrt{z-1}}{\sqrt{-z-1}}\right) /;\ z \notin (-\infty, 1)$$

01.13.27.0437.01

$$\cos^{-1}(z) = 2\tan^{-1}\left(\frac{\sqrt{z-1}}{\sqrt{-z-1}}\right) /;\ (z \in \mathbb{R} \land -1 < z < 1)$$

01.13.27.0438.01

$$\cos^{-1}(z) = 2\pi - 2\tan^{-1}\left(\frac{\sqrt{z-1}}{\sqrt{-z-1}}\right) /;\ (z \in \mathbb{R} \land z < -1)$$

01.13.27.0439.01

$$\cos^{-1}(z) = \pi\left(1 - \sqrt{z+1}\ \sqrt{\frac{1}{z+1}}\right) + \frac{2\sqrt{z-1}\ \sqrt{z+1}}{\sqrt{-z-1}\ \sqrt{1-z}}\tan^{-1}\left(\frac{\sqrt{z-1}}{\sqrt{-z-1}}\right)$$

Involving $\cos^{-1}(z)$ and $\tan^{-1}\left(\sqrt{\frac{1-z}{1+z}}\right)$

01.13.27.0018.01

$$\cos^{-1}(z) = 2\tan^{-1}\left(\sqrt{\frac{1-z}{z+1}}\right) /;\ z \notin (-\infty, -1)$$

01.13.27.0440.01

$$\cos^{-1}(z) = 2\pi - 2\tan^{-1}\left(\sqrt{\frac{1-z}{z+1}}\right) /;\ (z \in \mathbb{R} \land z < -1)$$

01.13.27.0441.01

$$\cos^{-1}(z) = -\frac{2\sqrt{-z-1}\sqrt{-z}}{\sqrt{z+1}}\sqrt{\frac{1}{z}}\tan^{-1}\left(\sqrt{\frac{1-z}{z+1}}\right) + \frac{\pi\sqrt{-z-1}\sqrt{-z}}{\sqrt{z+1}}\sqrt{\frac{1}{z}} + \pi$$

Involving $\cos^{-1}(z)$ and $\tan^{-1}\left(\frac{\sqrt{1+z}}{\sqrt{1-z}}\right)$

01.13.27.0442.01

$$\cos^{-1}(z) = \pi - 2\tan^{-1}\left(\frac{\sqrt{z+1}}{\sqrt{1-z}}\right) /; z \notin (1, \infty)$$

01.13.27.0443.01

$$\cos^{-1}(z) = -\pi - 2\tan^{-1}\left(\frac{\sqrt{1+z}}{\sqrt{1-z}}\right) /; (z \in \mathbb{R} \land z > 1)$$

01.13.27.0444.01

$$\cos^{-1}(z) = -2\tan^{-1}\left(\frac{\sqrt{z+1}}{\sqrt{1-z}}\right) - \frac{\pi\sqrt{z-1}\sqrt{z}}{\sqrt{1-z}}\sqrt{-\frac{1}{z}}$$

Involving $\cos^{-1}(z)$ and $\tan^{-1}\left(\frac{\sqrt{-z-1}}{\sqrt{z-1}}\right)$

01.13.27.0445.01

$$\cos^{-1}(z) = \pi + 2\tan^{-1}\left(\frac{\sqrt{-z-1}}{\sqrt{z-1}}\right) /; z \notin (-1, \infty)$$

01.13.27.0446.01

$$\cos^{-1}(z) = \pi - 2\tan^{-1}\left(\frac{\sqrt{-z-1}}{\sqrt{z-1}}\right) /; (z \in \mathbb{R} \land -1 < z < 1)$$

01.13.27.0447.01

$$\cos^{-1}(z) = 2\tan^{-1}\left(\frac{\sqrt{-z-1}}{\sqrt{z-1}}\right) - \pi /; (z \in \mathbb{R} \land z > 1)$$

01.13.27.0448.01

$$\cos^{-1}(z) = \pi\sqrt{1-z}\sqrt{\frac{1}{1-z}} - \frac{2\sqrt{z-1}\sqrt{z+1}}{\sqrt{-z-1}\sqrt{1-z}}\tan^{-1}\left(\frac{\sqrt{-z-1}}{\sqrt{z-1}}\right)$$

Involving $\cos^{-1}(z)$ and $\tan^{-1}\left(\sqrt{\frac{1+z}{1-z}}\right)$

01.13.27.0449.01

$$\cos^{-1}(z) = \pi - 2\tan^{-1}\left(\sqrt{\frac{1+z}{1-z}}\right) /; z \notin (1, \infty)$$

01.13.27.0450.01

$$\cos^{-1}(z) == -\pi + 2\tan^{-1}\left(\sqrt{\frac{z+1}{1-z}}\right) /; (z \in \mathbb{R} \land z > 1)$$

01.13.27.0451.01

$$\cos^{-1}(z) == \frac{2\sqrt{z-1}\sqrt{z}}{\sqrt{1-z}}\sqrt{-\frac{1}{z}}\tan^{-1}\left(\sqrt{\frac{1+z}{1-z}}\right) - \frac{\pi\sqrt{z-1}\sqrt{z}}{\sqrt{1-z}}\sqrt{-\frac{1}{z}}$$

Involving $\cos^{-1}(z)$ and $\tan^{-1}\left(\frac{\sqrt{1-z^2}+1}{z}\right)$

01.13.27.0452.01

$$\cos^{-1}(z) == -\frac{\pi}{2} + 2\tan^{-1}\left(\frac{\sqrt{1-z^2}+1}{z}\right) /; -\frac{\pi}{2} \le \arg(z) < \frac{\pi}{2}$$

01.13.27.0453.01

$$\cos^{-1}(z) == \frac{3\pi}{2} + 2\tan^{-1}\left(\frac{\sqrt{1-z^2}+1}{z}\right) /; \frac{\pi}{2} \le \arg(z) \le \pi \bigvee -\pi < \arg(z) < -\frac{\pi}{2}$$

01.13.27.0454.01

$$\cos^{-1}(z) == \frac{\pi}{2} - \pi\sqrt{\frac{1}{z^2}}\, z + 2\tan^{-1}\left(\frac{\sqrt{1-z^2}+1}{z}\right)$$

Involving $\cos^{-1}(z)$ and $\tan^{-1}\left(\frac{1-\sqrt{1-z^2}}{z}\right)$

01.13.27.0455.01

$$\cos^{-1}(z) == \frac{\pi}{2} - 2\tan^{-1}\left(\frac{1-\sqrt{1-z^2}}{z}\right)$$

Involving $\cos^{-1}(z)$ and $\tan^{-1}\left(\frac{z}{1+\sqrt{1-z^2}}\right)$

01.13.27.0456.01

$$\cos^{-1}(z) == \frac{\pi}{2} - 2\tan^{-1}\left(\frac{z}{\sqrt{1-z^2}+1}\right)$$

Involving $\cos^{-1}(z)$ and $\tan^{-1}\left(\frac{z}{1-\sqrt{1-z^2}}\right)$

01.13.27.0457.01

$$\cos^{-1}(z) = -\frac{\pi}{2} + 2\tan^{-1}\left(\frac{z}{1-\sqrt{1-z^2}}\right) /; -\frac{\pi}{2} \leq \arg(z) < \frac{\pi}{2}$$

01.13.27.0458.01

$$\cos^{-1}(z) = \frac{3\pi}{2} + 2\tan^{-1}\left(\frac{z}{1-\sqrt{1-z^2}}\right) /; \frac{\pi}{2} \leq \arg(z) \leq \pi \bigvee -\pi < \arg(z) < -\frac{\pi}{2}$$

01.13.27.0459.01

$$\cos^{-1}(z) = \frac{\pi}{2} - \pi\sqrt{\frac{1}{z^2}}\, z + 2\tan^{-1}\left(\frac{z}{1-\sqrt{1-z^2}}\right)$$

Involving $\cos^{-1}\left(\frac{2z}{z^2+1}\right)$

Involving $\cos^{-1}\left(\frac{2z}{z^2+1}\right)$ and $\tan^{-1}(z)$

01.13.27.0019.01

$$\cos^{-1}\left(\frac{2z}{z^2+1}\right) = \frac{\pi}{2} - 2\tan^{-1}(z) /; |z| < 1$$

01.13.27.0460.01

$$\cos^{-1}\left(\frac{2z}{z^2+1}\right) = 2\tan^{-1}(z) - \frac{\pi}{2} /; |z| > 1 \bigwedge -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

01.13.27.0461.01

$$\cos^{-1}\left(\frac{2z}{z^2+1}\right) = \frac{3\pi}{2} + 2\tan^{-1}(z) /; |z| > 1 \bigwedge \left(\frac{\pi}{2} < \arg(z) \leq \pi \bigvee -\pi < \arg(z) \leq -\frac{\pi}{2}\right)$$

01.13.27.0462.01

$$\cos^{-1}\left(\frac{2z}{z^2+1}\right) = \frac{\pi}{2} - \frac{\pi\sqrt{z^2}}{z} + 2\tan^{-1}(z) /; |z| > 1$$

01.13.27.0463.01

$$\cos^{-1}\left(\frac{2z}{z^2+1}\right) = \frac{\pi}{2} - \frac{\pi\sqrt{z^2}}{2z}\left(1 - \frac{1-z}{1+z}\sqrt{\left(\frac{z+1}{z-1}\right)^2}\right) - \frac{2\,(1-z)}{1+z}\sqrt{\left(\frac{z+1}{z-1}\right)^2}\,\tan^{-1}(z)$$

Involving $\cos^{-1}\left(\frac{2z}{z^2+1}\right)$ and $\tan^{-1}\left(\frac{1}{z}\right)$

01.13.27.0464.01

$$\cos^{-1}\left(\frac{2z}{z^2+1}\right) = 2\tan^{-1}\left(\frac{1}{z}\right) - \frac{\pi}{2} /; |z| < 1 \bigwedge -\frac{\pi}{2} \leq \arg(z) < \frac{\pi}{2}$$

01.13.27.0465.01

$$\cos^{-1}\left(\frac{2z}{z^2+1}\right) = \frac{3\pi}{2} + 2\tan^{-1}\left(\frac{1}{z}\right) /; |z| < 1 \bigwedge \left(\frac{\pi}{2} \leq \arg(z) \leq \pi \bigvee -\pi < \arg(z) < -\frac{\pi}{2}\right)$$

01.13.27.0466.01

$$\cos^{-1}\left(\frac{2z}{z^2+1}\right) = \frac{\pi}{2} - \pi z\sqrt{\frac{1}{z^2}} + 2\tan^{-1}\left(\frac{1}{z}\right) /; |z| < 1$$

01.13.27.0467.01

$$\cos^{-1}\left(\frac{2z}{z^2+1}\right) = \frac{\pi}{2} - 2\tan^{-1}\left(\frac{1}{z}\right) /; |z| > 1$$

01.13.27.0468.01

$$\cos^{-1}\left(\frac{2z}{z^2+1}\right) = \frac{\pi}{2} - \frac{\pi z}{2}\sqrt{\frac{1}{z^2}}\left(\frac{1-z}{1+z}\sqrt{\left(\frac{z+1}{z-1}\right)^2} + 1\right) + \frac{2(1-z)}{1+z}\sqrt{\left(\frac{z+1}{z-1}\right)^2}\tan^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\frac{2z}{z^2+1}\right)$ and $\tan^{-1}(z^r)$

01.13.27.0469.01

$$\cos^{-1}\left(\frac{2z}{z^2+1}\right) = \frac{\pi}{2} - 2\tan^{-1}\left(z^{\frac{(1-z)\sqrt{\left(\frac{z+1}{1-z}\right)^2}}{z+1}}\right)$$

Involving $\cos^{-1}\left(\frac{1-z}{1+z}\right)$

Involving $\cos^{-1}\left(\frac{1-z}{1+z}\right)$ and $\tan^{-1}\left(\sqrt{z}\right)$

01.13.27.0470.01

$$\cos^{-1}\left(\frac{1-z}{1+z}\right) = 2\tan^{-1}\left(\sqrt{z}\right) /; z \notin (-\infty, -1)$$

01.13.27.0471.01

$$\cos^{-1}\left(\frac{1-z}{z+1}\right) = 2\pi - 2\tan^{-1}\left(\sqrt{z}\right) /; (z \in \mathbb{R} \land z < -1)$$

01.13.27.0472.01

$$\cos^{-1}\left(\frac{1-z}{1+z}\right) = \pi\left(1 - \sqrt{z+1}\sqrt{\frac{1}{z+1}}\right) + 2\sqrt{\frac{1}{z+1}}\sqrt{z+1}\tan^{-1}\left(\sqrt{z}\right)$$

Involving $\cos^{-1}\left(\frac{1-z}{1+z}\right)$ and $\tan^{-1}\left(\frac{1}{\sqrt{z}}\right)$

01.13.27.0473.01

$$\cos^{-1}\left(\frac{1-z}{1+z}\right) = \pi - 2\tan^{-1}\left(\frac{1}{\sqrt{z}}\right) /; |\arg(z)| < \pi$$

01.13.27.0474.01

$$\cos^{-1}\left(\frac{1-z}{1+z}\right) = \pi + 2\tan^{-1}\left(\frac{1}{\sqrt{z}}\right) /; (z \in \mathbb{R} \land z < -1)$$

01.13.27.0475.01

$$\cos^{-1}\left(\frac{1-z}{1+z}\right) = \pi\left(\sqrt{\frac{1}{z}}\sqrt{z} - \sqrt{\frac{1}{z+1}}\sqrt{z+1} + 1\right) - 2\sqrt{z+1}\sqrt{\frac{1}{z+1}}\tan^{-1}\left(\frac{1}{\sqrt{z}}\right)$$

Involving $\cos^{-1}\left(\frac{1-z}{1+z}\right)$ and $\tan^{-1}\left(\sqrt{\frac{1}{z}}\right)$

01.13.27.0476.01

$$\cos^{-1}\left(\frac{1-z}{1+z}\right) = \pi - 2\tan^{-1}\left(\sqrt{\frac{1}{z}}\right) /; z \notin (-1, 0)$$

01.13.27.0477.01

$$\cos^{-1}\left(\frac{1-z}{z+1}\right) = 2\tan^{-1}\left(\sqrt{\frac{1}{z}}\right) - \pi /; (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.0478.01

$$\cos^{-1}\left(\frac{1-z}{1+z}\right) = \pi\left(1 + \sqrt{\frac{1}{z}}\sqrt{z} - \sqrt{\frac{1}{z+1}}\sqrt{z+1}\right) - 2\sqrt{\frac{z}{z+1}}\sqrt{\frac{z+1}{z}}\tan^{-1}\left(\sqrt{\frac{1}{z}}\right)$$

Involving $\cos^{-1}\left(\frac{z-1}{z+1}\right)$

Involving $\cos^{-1}\left(\frac{z-1}{z+1}\right)$ and $\tan^{-1}\left(\sqrt{z}\right)$

01.13.27.0479.01

$$\cos^{-1}\left(\frac{z-1}{z+1}\right) = \pi - 2\tan^{-1}\left(\sqrt{z}\right) /; z \notin (-\infty, -1)$$

01.13.27.0480.01

$$\cos^{-1}\left(\frac{z-1}{z+1}\right) = 2\tan^{-1}\left(\sqrt{z}\right) - \pi /; (z \in \mathbb{R} \land z < -1)$$

01.13.27.0481.01

$$\cos^{-1}\left(\frac{z-1}{z+1}\right) = \sqrt{z+1}\sqrt{\frac{1}{z+1}}\,\pi - 2\sqrt{z+1}\sqrt{\frac{1}{z+1}}\tan^{-1}\left(\sqrt{z}\right)$$

Involving $\cos^{-1}\left(\frac{z-1}{z+1}\right)$ and $\tan^{-1}\left(\frac{1}{\sqrt{z}}\right)$

01.13.27.0482.01

$$\cos^{-1}\left(\frac{z-1}{z+1}\right) = 2\tan^{-1}\left(\frac{1}{\sqrt{z}}\right) /; |\arg(z)| < \pi$$

01.13.27.0483.01

$$\cos^{-1}\left(\frac{z-1}{z+1}\right) = -2\tan^{-1}\left(\frac{1}{\sqrt{z}}\right) /; (z \in \mathbb{R} \land z < -1)$$

01.13.27.0484.01

$$\cos^{-1}\left(\frac{z-1}{z+1}\right) = \pi\left(-\sqrt{\frac{1}{z}}\sqrt{z}+\sqrt{\frac{1}{1+z}}\sqrt{1+z}\right)+2\sqrt{\frac{1}{z+1}}\sqrt{z+1}\tan^{-1}\left(\frac{1}{\sqrt{z}}\right)$$

Involving $\cos^{-1}\left(\frac{z-1}{z+1}\right)$ and $\tan^{-1}\left(\sqrt{\frac{1}{z}}\right)$

01.13.27.0485.01

$$\cos^{-1}\left(\frac{z-1}{z+1}\right) = 2\tan^{-1}\left(\sqrt{\frac{1}{z}}\right)\, /;\, z \notin (-1, 0)$$

01.13.27.0486.01

$$\cos^{-1}\left(\frac{z-1}{z+1}\right) = 2\pi - 2\tan^{-1}\left(\sqrt{\frac{1}{z}}\right)\, /;\, (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.0487.01

$$\cos^{-1}\left(\frac{z-1}{z+1}\right) = \pi\left(-\sqrt{\frac{1}{z}}\sqrt{z}+\sqrt{\frac{1}{1+z}}\sqrt{1+z}\right)+2\sqrt{\frac{z}{z+1}}\sqrt{\frac{z+1}{z}}\tan^{-1}\left(\sqrt{\frac{1}{z}}\right)$$

Involving $\cos^{-1}\left(\frac{1+z}{1-z}\right)$

Involving $\cos^{-1}\left(\frac{1+z}{1-z}\right)$ and $\tan^{-1}\left(\sqrt{-z}\right)$

01.13.27.0488.01

$$\cos^{-1}\left(\frac{1+z}{1-z}\right) = 2\tan^{-1}\left(\sqrt{-z}\right)\, /;\, z \notin (1, \infty)$$

01.13.27.0489.01

$$\cos^{-1}\left(\frac{1+z}{1-z}\right) = 2\pi - 2\tan^{-1}\left(\sqrt{-z}\right)\, /;\, (z \in \mathbb{R} \land z > 1)$$

01.13.27.0490.01

$$\cos^{-1}\left(\frac{1+z}{1-z}\right) = \left(1-\sqrt{1-z}\sqrt{\frac{1}{1-z}}\right)\pi + 2\sqrt{1-z}\sqrt{\frac{1}{1-z}}\tan^{-1}\left(\sqrt{-z}\right)$$

Involving $\cos^{-1}\left(\frac{1+z}{1-z}\right)$ and $\tan^{-1}\left(\frac{1}{\sqrt{-z}}\right)$

01.13.27.0491.01

$$\cos^{-1}\left(\frac{1+z}{1-z}\right) = \pi - 2\tan^{-1}\left(\frac{1}{\sqrt{-z}}\right)\, /;\, z \notin (0, \infty)$$

01.13.27.0492.01

$$\cos^{-1}\left(\frac{1+z}{1-z}\right) = \pi + 2\tan^{-1}\left(\frac{1}{\sqrt{-z}}\right)\, /;\, (z \in \mathbb{R} \land z > 1)$$

01.13.27.0493.01

$$\cos^{-1}\left(\frac{1+z}{1-z}\right) = \pi\left(1 - \sqrt{\frac{1}{1-z}}\sqrt{1-z} + \sqrt{-\frac{1}{z}}\sqrt{-z}\right) - 2\sqrt{\frac{1}{1-z}}\sqrt{1-z}\tan^{-1}\left(\frac{1}{\sqrt{-z}}\right)$$

Involving $\cos^{-1}\left(\frac{1+z}{1-z}\right)$ and $\tan^{-1}\left(\sqrt{-\frac{1}{z}}\right)$

01.13.27.0494.01

$$\cos^{-1}\left(\frac{1+z}{1-z}\right) = \pi - 2\tan^{-1}\left(\sqrt{-\frac{1}{z}}\right) /; z \notin (0, 1)$$

01.13.27.0495.01

$$\cos^{-1}\left(\frac{1+z}{1-z}\right) = 2\tan^{-1}\left(\sqrt{-\frac{1}{z}}\right) - \pi /; (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.0496.01

$$\cos^{-1}\left(\frac{1+z}{1-z}\right) = \pi\left(1 + \sqrt{-\frac{1}{z}}\sqrt{-z} - \sqrt{\frac{1}{1-z}}\sqrt{1-z}\right) - 2\sqrt{\frac{z}{z-1}}\sqrt{\frac{z-1}{z}}\tan^{-1}\left(\sqrt{-\frac{1}{z}}\right)$$

Involving $\cos^{-1}\left(\frac{z+1}{z-1}\right)$

Involving $\cos^{-1}\left(\frac{z+1}{z-1}\right)$ and $\tan^{-1}\left(\sqrt{-z}\right)$

01.13.27.0497.01

$$\cos^{-1}\left(\frac{z+1}{z-1}\right) = \pi - 2\tan^{-1}\left(\sqrt{-z}\right) /; z \notin (1, \infty)$$

01.13.27.0498.01

$$\cos^{-1}\left(\frac{z+1}{z-1}\right) = 2\tan^{-1}\left(\sqrt{-z}\right) - \pi /; (z \in \mathbb{R} \land z > 1)$$

01.13.27.0499.01

$$\cos^{-1}\left(\frac{z+1}{z-1}\right) = \sqrt{1-z}\sqrt{\frac{1}{1-z}}\pi - 2\sqrt{1-z}\sqrt{\frac{1}{1-z}}\tan^{-1}\left(\sqrt{-z}\right)$$

Involving $\cos^{-1}\left(\frac{z+1}{z-1}\right)$ and $\tan^{-1}\left(\frac{1}{\sqrt{-z}}\right)$

01.13.27.0500.01

$$\cos^{-1}\left(\frac{z+1}{z-1}\right) = 2\tan^{-1}\left(\frac{1}{\sqrt{-z}}\right) /; z \notin (0, \infty)$$

01.13.27.0501.01

$$\cos^{-1}\left(\frac{z+1}{z-1}\right) = -2\tan^{-1}\left(\frac{1}{\sqrt{-z}}\right) /; (z \in \mathbb{R} \land z > 1)$$

01.13.27.0502.01

$$\cos^{-1}\left(\frac{z+1}{z-1}\right) = \pi\left(-\sqrt{-\frac{1}{z}}\sqrt{-z} + \sqrt{\frac{1}{1-z}}\sqrt{1-z}\right) + 2\sqrt{\frac{1}{1-z}}\sqrt{1-z}\tan^{-1}\left(\frac{1}{\sqrt{-z}}\right)$$

Involving $\cos^{-1}\left(\frac{z+1}{z-1}\right)$ and $\tan^{-1}\left(\sqrt{-\frac{1}{z}}\right)$

01.13.27.0503.01

$$\cos^{-1}\left(\frac{z+1}{z-1}\right) = 2\tan^{-1}\left(\sqrt{-\frac{1}{z}}\right)\ /;\ z \notin (0,\ 1)$$

01.13.27.0504.01

$$\cos^{-1}\left(\frac{z+1}{z-1}\right) = 2\pi - 2\tan^{-1}\left(\sqrt{-\frac{1}{z}}\right)\ /;\ (z \in \mathbb{R} \wedge 0 < z < 1)$$

01.13.27.0505.01

$$\cos^{-1}\left(\frac{z+1}{z-1}\right) = \pi\left(-\sqrt{-\frac{1}{z}}\sqrt{-z} + \sqrt{\frac{1}{1-z}}\sqrt{1-z}\right) + 2\sqrt{\frac{z}{z-1}}\sqrt{\frac{z-1}{z}}\tan^{-1}\left(\sqrt{-\frac{1}{z}}\right)$$

Involving $\cos^{-1}\left(\frac{1-z^2}{1+z^2}\right)$

Involving $\cos^{-1}\left(\frac{1-z^2}{1+z^2}\right)$ and $\tan^{-1}(z)$

01.13.27.0506.01

$$\cos^{-1}\left(\frac{1-z^2}{1+z^2}\right) = 2\tan^{-1}(z)\ /;\ \mathrm{Re}(z) > 0 \vee (-i\,z \in \mathbb{R} \wedge 0 < -i\,z < 1)$$

01.13.27.0507.01

$$\cos^{-1}\left(\frac{1-z^2}{1+z^2}\right) = -2\tan^{-1}(z)\ /;\ \mathrm{Re}(z) < 0 \vee (i\,z \in \mathbb{R} \wedge 0 < i\,z < 1)$$

01.13.27.0508.01

$$\cos^{-1}\left(\frac{1-z^2}{z^2+1}\right) = 2\pi + 2\tan^{-1}(z)\ /;\ (i\,z \in \mathbb{R} \wedge i\,z > 1)$$

01.13.27.0509.01

$$\cos^{-1}\left(\frac{1-z^2}{z^2+1}\right) = 2\pi - 2\tan^{-1}(z)\ /;\ (i\,z \in \mathbb{R} \wedge i\,z < -1)$$

01.13.27.0020.01

$$\cos^{-1}\left(\frac{1-z^2}{1+z^2}\right) = \frac{2\sqrt{z^2}}{z}\tan^{-1}(z)\ /;\ i\,z \notin (-\infty,\ -1) \bigwedge i\,z \notin (1,\ \infty)$$

01.13.27.0510.01

$$\cos^{-1}\left(\frac{1-z^2}{1+z^2}\right) = \pi\left(2-\sqrt{\frac{1}{i z+1}}\,\sqrt{i z+1}-\sqrt{\frac{1}{1-i z}}\,\sqrt{1-i z}\right)+\frac{2\sqrt{z^2}}{z}\sqrt{\frac{i+z}{i-z}}\,\sqrt{\frac{i-z}{i+z}}\,\tan^{-1}(z)$$

Involving $\cos^{-1}\left(\frac{1-z^2}{1+z^2}\right)$ and $\tan^{-1}\left(\frac{1}{z}\right)$

01.13.27.0511.01

$$\cos^{-1}\left(\frac{1-z^2}{1+z^2}\right) = \pi-2\tan^{-1}\left(\frac{1}{z}\right) /; \operatorname{Re}(z)>0 \lor (i z\in\mathbb{R}\land i z>1)$$

01.13.27.0512.01

$$\cos^{-1}\left(\frac{1-z^2}{z^2+1}\right) = \pi+2\tan^{-1}\left(\frac{1}{z}\right) /; \operatorname{Re}(z)<0 \lor (i z\in\mathbb{R}\land i z<-1)$$

01.13.27.0513.01

$$\cos^{-1}\left(\frac{1-z^2}{z^2+1}\right) = -\pi-2\tan^{-1}\left(\frac{1}{z}\right) /; (-i z\in\mathbb{R}\land 0<-i z<1)$$

01.13.27.0514.01

$$\cos^{-1}\left(\frac{1-z^2}{z^2+1}\right) = -\pi+2\tan^{-1}\left(\frac{1}{z}\right) /; (i z\in\mathbb{R}\land 0<i z<1)$$

01.13.27.0515.01

$$\cos^{-1}\left(\frac{1-z^2}{1+z^2}\right) = \pi\left(\sqrt{-\frac{i}{z}}\,\sqrt{i z}\,\sqrt{\frac{1}{i z+1}}\,\sqrt{i z+1}+\sqrt{\frac{i}{z}}\,\sqrt{-i z}\,\sqrt{\frac{1}{1-i z}}\,\sqrt{1-i z}-1\right)-\frac{2\sqrt{z^2}}{z}\sqrt{\frac{i-z}{i+z}}\,\sqrt{\frac{i+z}{i-z}}\,\tan^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\frac{z^2-1}{z^2+1}\right)$

Involving $\cos^{-1}\left(\frac{z^2-1}{z^2+1}\right)$ and $\tan^{-1}(z)$

01.13.27.0516.01

$$\cos^{-1}\left(\frac{z^2-1}{z^2+1}\right) = \pi-2\tan^{-1}(z) /; \operatorname{Re}(z)>0 \lor (-i z\in\mathbb{R}\land 0<-i z<1)$$

01.13.27.0517.01

$$\cos^{-1}\left(\frac{z^2-1}{z^2+1}\right) = \pi+2\tan^{-1}(z) /; \operatorname{Re}(z)<0 \lor (i z\in\mathbb{R}\land 0<i z<1)$$

01.13.27.0518.01

$$\cos^{-1}\left(\frac{z^2-1}{z^2+1}\right) = -2\tan^{-1}(z)-\pi /; (i z\in\mathbb{R}\land i z>1)$$

01.13.27.0519.01

$$\cos^{-1}\left(\frac{z^2-1}{z^2+1}\right) = 2\tan^{-1}(z) - \pi \;/;\; (i\,z \in \mathbb{R} \land i\,z < -1)$$

01.13.27.0520.01

$$\cos^{-1}\left(\frac{z^2-1}{z^2+1}\right) = \pi - \frac{2\sqrt{z^2}}{z}\tan^{-1}(z) \;/;\; i\,z \notin (-\infty, -1) \bigwedge i\,z \notin (1, \infty)$$

01.13.27.0521.01

$$\cos^{-1}\left(\frac{z^2-1}{z^2+1}\right) = \pi\left(-1 + \sqrt{\frac{1}{1-i\,z}}\sqrt{1-i\,z} + \sqrt{\frac{1}{1+i\,z}}\sqrt{1+i\,z}\right) - \frac{2\sqrt{z^2}}{z}\sqrt{\frac{i-z}{i+z}}\sqrt{\frac{i+z}{i-z}}\tan^{-1}(z)$$

Involving $\cos^{-1}\left(\frac{z^2-1}{z^2+1}\right)$ and $\tan^{-1}\left(\frac{1}{z}\right)$

01.13.27.0522.01

$$\cos^{-1}\left(\frac{z^2-1}{z^2+1}\right) = 2\tan^{-1}\left(\frac{1}{z}\right) \;/;\; \mathrm{Re}(z) > 0 \lor (i\,z \in \mathbb{R} \land i\,z > 1)$$

01.13.27.0523.01

$$\cos^{-1}\left(\frac{z^2-1}{z^2+1}\right) = -2\tan^{-1}\left(\frac{1}{z}\right) \;/;\; \mathrm{Re}(z) < 0 \lor (i\,z \in \mathbb{R} \land i\,z < -1)$$

01.13.27.0524.01

$$\cos^{-1}\left(\frac{z^2-1}{z^2+1}\right) = 2\pi + 2\tan^{-1}\left(\frac{1}{z}\right) \;/;\; (-i\,z \in \mathbb{R} \land 0 < -i\,z < 1)$$

01.13.27.0525.01

$$\cos^{-1}\left(\frac{z^2-1}{z^2+1}\right) = 2\pi - 2\tan^{-1}\left(\frac{1}{z}\right) \;/;\; (i\,z \in \mathbb{R} \land 0 < i\,z < 1)$$

01.13.27.0526.01

$$\cos^{-1}\left(\frac{z^2-1}{z^2+1}\right) = \frac{2\sqrt{z^2}}{z}\sqrt{\frac{i-z}{i+z}}\sqrt{\frac{i+z}{i-z}}\tan^{-1}\left(\frac{1}{z}\right) + \pi\left(2 - \sqrt{\frac{1}{1-i\,z}}\sqrt{1-i\,z}\sqrt{\frac{i}{z}}\sqrt{-i\,z} - \sqrt{\frac{1}{1+i\,z}}\sqrt{1+i\,z}\sqrt{-\frac{i}{z}}\sqrt{i\,z}\right)$$

Involving $\cos^{-1}\left(\frac{1}{\sqrt{z+1}}\right)$

Involving $\cos^{-1}\left(\frac{1}{\sqrt{z+1}}\right)$ and $\tan^{-1}\left(\sqrt{z}\right)$

01.13.27.0021.01

$$\cos^{-1}\left(\frac{1}{\sqrt{z+1}}\right) = \tan^{-1}\left(\sqrt{z}\right)$$

Involving $\cos^{-1}\left(\frac{1}{\sqrt{z+1}}\right)$ and $\tan^{-1}\left(\frac{1}{\sqrt{z}}\right)$

01.13.27.0527.01

$$\cos^{-1}\left(\frac{1}{\sqrt{z+1}}\right) = \frac{\pi}{2} - \tan^{-1}\left(\frac{1}{\sqrt{z}}\right) /; z \notin (-1, 0)$$

01.13.27.0528.01

$$\cos^{-1}\left(\frac{1}{\sqrt{z+1}}\right) = -\frac{\pi}{2} - \tan^{-1}\left(\frac{1}{\sqrt{z}}\right) /; (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.0529.01

$$\cos^{-1}\left(\frac{1}{\sqrt{z+1}}\right) = \frac{1}{2} \sqrt{z} \sqrt{\frac{1}{z}} \sqrt{z+1} \sqrt{\frac{1}{z+1}} \pi - \tan^{-1}\left(\frac{1}{\sqrt{z}}\right)$$

Involving $\cos^{-1}\left(\frac{1}{\sqrt{z+1}}\right)$ and $\tan^{-1}\left(\sqrt{\frac{1}{z}}\right)$

01.13.27.0530.01

$$\cos^{-1}\left(\frac{1}{\sqrt{z+1}}\right) = \frac{\pi}{2} - \tan^{-1}\left(\sqrt{\frac{1}{z}}\right) /; |\arg(z)| < \pi$$

01.13.27.0531.01

$$\cos^{-1}\left(\frac{1}{\sqrt{z+1}}\right) = -\frac{\pi}{2} + \tan^{-1}\left(\sqrt{\frac{1}{z}}\right) /; (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.0532.01

$$\cos^{-1}\left(\frac{1}{\sqrt{z+1}}\right) = \frac{\pi}{2} + \tan^{-1}\left(\sqrt{\frac{1}{z}}\right) /; (z \in \mathbb{R} \land z < -1)$$

01.13.27.0533.01

$$\cos^{-1}\left(\frac{1}{\sqrt{z+1}}\right) = \frac{1}{2} \sqrt{z} \sqrt{\frac{1}{z}} \sqrt{z+1} \sqrt{\frac{1}{z+1}} \pi - \sqrt{z} \sqrt{\frac{1}{z}} \tan^{-1}\left(\sqrt{\frac{1}{z}}\right)$$

Involving $\cos^{-1}\left(\sqrt{\frac{1}{z+1}}\right)$

Involving $\cos^{-1}\left(\sqrt{\frac{1}{z+1}}\right)$ and $\tan^{-1}\left(\sqrt{z}\right)$

01.13.27.0534.01

$$\cos^{-1}\left(\sqrt{\frac{1}{z+1}}\right) = \tan^{-1}\left(\sqrt{z}\right) /; z \notin (-\infty, -1)$$

01.13.27.0535.01

$$\cos^{-1}\left(\sqrt{\frac{1}{z+1}}\right) = \pi - \tan^{-1}\left(\sqrt{z}\right) /; (z \in \mathbb{R} \land z < -1)$$

01.13.27.0536.01

$$\cos^{-1}\left(\sqrt{\frac{1}{z+1}}\right) = \frac{\pi}{2} - \sqrt{\frac{1}{z+1}} \sqrt{z+1} \left(\frac{\pi}{2} - \tan^{-1}\left(\sqrt{z}\right)\right)$$

Involving $\cos^{-1}\left(\sqrt{\frac{1}{z+1}}\right)$ and $\tan^{-1}\left(\frac{1}{\sqrt{z}}\right)$

01.13.27.0537.01

$$\cos^{-1}\left(\sqrt{\frac{1}{z+1}}\right) = \frac{\pi}{2} - \tan^{-1}\left(\frac{1}{\sqrt{z}}\right) /; |\arg(z)| < \pi$$

01.13.27.0538.01

$$\cos^{-1}\left(\sqrt{\frac{1}{z+1}}\right) = -\tan^{-1}\left(\frac{1}{\sqrt{z}}\right) - \frac{\pi}{2} /; (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.0539.01

$$\cos^{-1}\left(\sqrt{\frac{1}{z+1}}\right) = \frac{\pi}{2} + \tan^{-1}\left(\frac{1}{\sqrt{z}}\right) /; (z \in \mathbb{R} \land z < -1)$$

01.13.27.0540.01

$$\cos^{-1}\left(\sqrt{\frac{1}{z+1}}\right) = \frac{\pi}{2} + \sqrt{\frac{1}{z+1}} \sqrt{z+1} \left(-\tan^{-1}\left(\frac{1}{\sqrt{z}}\right) + \frac{\pi}{2}\left(\sqrt{z} \sqrt{\frac{1}{z}} \sqrt{z+1} \sqrt{\frac{1}{z+1}} - 1\right)\right)$$

Involving $\cos^{-1}\left(\sqrt{\frac{1}{z+1}}\right)$ and $\tan^{-1}\left(\sqrt{\frac{1}{z}}\right)$

01.13.27.0541.01

$$\cos^{-1}\left(\sqrt{\frac{1}{z+1}}\right) = \frac{\pi}{2} - \tan^{-1}\left(\sqrt{\frac{1}{z}}\right) /; z \notin (-1, 0)$$

01.13.27.0542.01

$$\cos^{-1}\left(\sqrt{\frac{1}{z+1}}\right) = \tan^{-1}\left(\sqrt{\frac{1}{z}}\right) - \frac{\pi}{2} /; (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.0543.01

$$\cos^{-1}\left(\sqrt{\frac{1}{z+1}}\right) = \sqrt{\frac{z}{z+1}} \sqrt{\frac{z+1}{z}} \left(\frac{\pi}{2} - \tan^{-1}\left(\sqrt{\frac{1}{z}}\right)\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{z}}{\sqrt{a+z}}\right)$

Involving $\cos^{-1}\left(\frac{\sqrt{z}}{\sqrt{a+z}}\right)$ and $\tan^{-1}\left(\sqrt{\frac{z}{a}}\right)$

01.13.27.0544.01

$$\cos^{-1}\left(\frac{\sqrt{z}}{\sqrt{z+1}}\right) = \frac{\pi}{2} - \tan^{-1}\left(\sqrt{z}\right) /; z \notin (-\infty, -1)$$

01.13.27.0545.01

$$\cos^{-1}\left(\frac{\sqrt{z}}{\sqrt{z+1}}\right) = -\frac{\pi}{2} + \tan^{-1}\left(\sqrt{z}\right) /; (z \in \mathbb{R} \land z < -1)$$

01.13.27.0026.01

$$\cos^{-1}\left(\frac{\sqrt{z}}{\sqrt{z+1}}\right) = \frac{1}{2}\sqrt{\frac{1}{z+1}}\sqrt{z+1}\left(\pi - 2\tan^{-1}\left(\sqrt{z}\right)\right)$$

01.13.27.0546.01

$$\cos^{-1}\left(\frac{\sqrt{z}}{\sqrt{a+z}}\right) = \frac{\pi}{2} - \frac{1}{2\sqrt{z}}\left(\sqrt{\frac{z}{a+z}}\sqrt{a+z}\right)\left(\pi - \sqrt{\frac{a}{a+z}}\sqrt{\frac{z}{a}+1}\left(\pi - 2\tan^{-1}\left(\sqrt{\frac{z}{a}}\right)\right)\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{z}}{\sqrt{z+1}}\right)$ and $\tan^{-1}\left(\frac{1}{\sqrt{z}}\right)$

01.13.27.0547.01

$$\cos^{-1}\left(\frac{\sqrt{z}}{\sqrt{z+1}}\right) = \tan^{-1}\left(\frac{1}{\sqrt{z}}\right) /; |\arg(z)| < \pi$$

01.13.27.0548.01

$$\cos^{-1}\left(\frac{\sqrt{z}}{\sqrt{z+1}}\right) = \pi + \tan^{-1}\left(\frac{1}{\sqrt{z}}\right) /; (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.0549.01

$$\cos^{-1}\left(\frac{\sqrt{z}}{\sqrt{z+1}}\right) = -\tan^{-1}\left(\frac{1}{\sqrt{z}}\right) /; (z \in \mathbb{R} \land z < -1)$$

01.13.27.0550.01

$$\cos^{-1}\left(\frac{\sqrt{z}}{\sqrt{z+1}}\right) = \frac{1}{2}\pi\left(1 - \sqrt{z}\sqrt{\frac{1}{z}}\sqrt{z+1}\sqrt{\frac{1}{z+1}}\right) + \sqrt{\frac{1}{z+1}}\sqrt{z+1}\tan^{-1}\left(\frac{1}{\sqrt{z}}\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{z}}{\sqrt{z+1}}\right)$ and $\tan^{-1}\left(\sqrt{\frac{1}{z}}\right)$

01.13.27.0551.01

$$\cos^{-1}\left(\frac{\sqrt{z}}{\sqrt{z+1}}\right) = \tan^{-1}\left(\sqrt{\frac{1}{z}}\right) /; z \notin (-1, 0)$$

01.13.27.0552.01

$$\cos^{-1}\left(\frac{\sqrt{z}}{\sqrt{z+1}}\right) = \pi - \tan^{-1}\left(\sqrt{\frac{1}{z}}\right) /; (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.0553.01

$$\cos^{-1}\left(\frac{\sqrt{z}}{\sqrt{z+1}}\right) = \frac{\pi}{2}\left(1 - \sqrt{\frac{z+1}{z}}\sqrt{\frac{z}{z+1}}\right) + \sqrt{\frac{z}{z+1}}\sqrt{\frac{z+1}{z}}\tan^{-1}\left(\sqrt{\frac{1}{z}}\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{-z}}{\sqrt{-1-z}}\right)$

Involving $\cos^{-1}\left(\frac{\sqrt{-z}}{\sqrt{-1-z}}\right)$ and $\tan^{-1}(\sqrt{z})$

01.13.27.0554.01

$$\cos^{-1}\left(\frac{\sqrt{-z}}{\sqrt{-1-z}}\right) = \frac{\pi}{2} - \tan^{-1}(\sqrt{z}) /; |\arg(z)| < \pi$$

01.13.27.0555.01

$$\cos^{-1}\left(\frac{\sqrt{-z}}{\sqrt{-1-z}}\right) = \frac{\pi}{2} + \tan^{-1}(\sqrt{z}) /; (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.0556.01

$$\cos^{-1}\left(\frac{\sqrt{-z}}{\sqrt{-1-z}}\right) = -\frac{\pi}{2} + \tan^{-1}(\sqrt{z}) /; (z \in \mathbb{R} \land z < -1)$$

01.13.27.0557.01

$$\cos^{-1}\left(\frac{\sqrt{-z}}{\sqrt{-z-1}}\right) = \frac{1}{2}\left(\sqrt{\frac{1}{z}}\sqrt{z}\left(\pi - 2\tan^{-1}(\sqrt{z})\right) - \pi\sqrt{1+\frac{1}{z}}\sqrt{\frac{z}{z+1}} + \pi\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{-z}}{\sqrt{-1-z}}\right)$ and $\tan^{-1}\left(\frac{1}{\sqrt{z}}\right)$

01.13.27.0558.01

$$\cos^{-1}\left(\frac{\sqrt{-z}}{\sqrt{-1-z}}\right) = \tan^{-1}\left(\frac{1}{\sqrt{z}}\right) /; |\arg(z)| < \pi$$

01.13.27.0559.01

$$\cos^{-1}\left(\frac{\sqrt{-z}}{\sqrt{-z-1}}\right) = -\tan^{-1}\left(\frac{1}{\sqrt{z}}\right) /; (z \in \mathbb{R} \land z < 0)$$

01.13.27.0560.01

$$\cos^{-1}\left(\frac{\sqrt{-z}}{\sqrt{-z-1}}\right) = \sqrt{\frac{z+1}{z}}\sqrt{\frac{z}{(z+1)^2}}\sqrt{z+1}\tan^{-1}\left(\frac{1}{\sqrt{z}}\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{-z}}{\sqrt{-1-z}}\right)$ and $\tan^{-1}\left(\sqrt{\frac{1}{z}}\right)$

01.13.27.0561.01

$$\cos^{-1}\left(\frac{\sqrt{-z}}{\sqrt{-1-z}}\right) = \tan^{-1}\left(\sqrt{\frac{1}{z}}\right)$$

Involving $\cos^{-1}\left(\sqrt{\frac{z}{z+a}}\right)$

Involving $\cos^{-1}\left(\sqrt{\frac{z}{z+a}}\right)$ and $\tan^{-1}\left(\sqrt{\frac{z}{a}}\right)$

01.13.27.0562.01

$$\cos^{-1}\left(\sqrt{\frac{z}{z+1}}\right) = \frac{\pi}{2} - \tan^{-1}\left(\sqrt{z}\right) /; z \notin (-\infty, -1)$$

01.13.27.0563.01

$$\cos^{-1}\left(\sqrt{\frac{z}{z+1}}\right) = -\frac{\pi}{2} + \tan^{-1}\left(\sqrt{z}\right) /; (z \in \mathbb{R} \land z < -1)$$

01.13.27.0027.01

$$\cos^{-1}\left(\sqrt{\frac{z}{z+1}}\right) = \frac{1}{2} \sqrt{\frac{1}{z+1}} \sqrt{z+1} \left(\pi - 2\tan^{-1}\left(\sqrt{z}\right)\right)$$

01.13.27.0025.01

$$\cos^{-1}\left(\frac{\sqrt{z}}{\sqrt{a+z}}\right) = \frac{\pi}{2} - \frac{\sqrt{a+z}}{2\sqrt{z}} \sqrt{\frac{z}{a+z}} \left(\pi - \sqrt{\frac{a}{a+z}} \sqrt{\frac{z}{a}+1} \left(\pi - 2\tan^{-1}\left(\sqrt{\frac{z}{a}}\right)\right)\right)$$

Involving $\cos^{-1}\left(\sqrt{\frac{z}{z+1}}\right)$ and $\tan^{-1}\left(\frac{1}{\sqrt{z}}\right)$

01.13.27.0564.01

$$\cos^{-1}\left(\sqrt{\frac{z}{z+1}}\right) = \tan^{-1}\left(\frac{1}{\sqrt{z}}\right) /; |\arg(z)| < \pi$$

01.13.27.0565.01

$$\cos^{-1}\left(\sqrt{\frac{z}{z+1}}\right) = -\tan^{-1}\left(\frac{1}{\sqrt{z}}\right) /; (z \in \mathbb{R} \land z < -1)$$

01.13.27.0566.01

$$\cos^{-1}\left(\sqrt{\frac{z}{z+1}}\right) = \pi + \tan^{-1}\left(\frac{1}{\sqrt{z}}\right) /; (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.0567.01

$$\cos^{-1}\left(\sqrt{\frac{z}{z+1}}\right) == \frac{1}{2}\pi\left(1-\sqrt{1+\frac{1}{z}}\sqrt{\frac{z}{z+1}}\right)+\sqrt{\frac{1}{z+1}}\sqrt{z+1}\tan^{-1}\left(\frac{1}{\sqrt{z}}\right)$$

Involving $\cos^{-1}\left(\sqrt{\frac{z}{z+1}}\right)$ and $\tan^{-1}\left(\sqrt{\frac{1}{z}}\right)$

01.13.27.0568.01

$$\cos^{-1}\left(\sqrt{\frac{z}{z+1}}\right) == \tan^{-1}\left(\sqrt{\frac{1}{z}}\right) /; z \notin (-1, 0)$$

01.13.27.0569.01

$$\cos^{-1}\left(\sqrt{\frac{z}{z+1}}\right) == \pi-\tan^{-1}\left(\sqrt{\frac{1}{z}}\right) /; (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.0570.01

$$\cos^{-1}\left(\sqrt{\frac{z}{z+1}}\right) == \frac{\pi}{2}\left(1-\sqrt{\frac{z+1}{z}}\sqrt{\frac{z}{z+1}}\right)+\sqrt{\frac{z}{z+1}}\sqrt{\frac{z+1}{z}}\tan^{-1}\left(\sqrt{\frac{1}{z}}\right)$$

Involving $\cos^{-1}\left(\frac{1}{\sqrt{z^2+1}}\right)$

Involving $\cos^{-1}\left(\frac{1}{\sqrt{z^2+1}}\right)$ and $\tan^{-1}(z)$

01.13.27.0571.01

$$\cos^{-1}\left(\frac{1}{\sqrt{z^2+1}}\right) == \tan^{-1}(z) /; -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

01.13.27.0572.01

$$\cos^{-1}\left(\frac{1}{\sqrt{z^2+1}}\right) == -\tan^{-1}(z) /; \frac{\pi}{2} < \arg(z) \leq \pi \bigvee -\pi < \arg(z) \leq -\frac{\pi}{2}$$

01.13.27.0022.01

$$\cos^{-1}\left(\frac{1}{\sqrt{z^2+1}}\right) == \frac{\sqrt{z^2}}{z}\tan^{-1}(z)$$

Involving $\cos^{-1}\left(\frac{1}{\sqrt{z^2+1}}\right)$ and $\tan^{-1}\left(\frac{1}{z}\right)$

01.13.27.0573.01

$$\cos^{-1}\left(\frac{1}{\sqrt{z^2+1}}\right) = \frac{\pi}{2} + \tan^{-1}\left(\frac{1}{z}\right) /; \operatorname{Re}(z) < 0 \lor (i\,z \in \mathbb{R} \land i\,z > 1)$$

01.13.27.0574.01

$$\cos^{-1}\left(\frac{1}{\sqrt{z^2+1}}\right) = \frac{\pi}{2} - \tan^{-1}\left(\frac{1}{z}\right) /; \operatorname{Re}(z) > 0 \lor (i\,z \in \mathbb{R} \land i\,z < -1)$$

01.13.27.0575.01

$$\cos^{-1}\left(\frac{1}{\sqrt{z^2+1}}\right) = -\frac{\pi}{2} + \tan^{-1}\left(\frac{1}{z}\right) /; (i\,z \in \mathbb{R} \land 0 < i\,z < 1)$$

01.13.27.0576.01

$$\cos^{-1}\left(\frac{1}{\sqrt{z^2+1}}\right) = -\frac{\pi}{2} - \tan^{-1}\left(\frac{1}{z}\right) /; (-i\,z \in \mathbb{R} \land 0 < -i\,z < 1)$$

01.13.27.0577.01

$$\cos^{-1}\left(\frac{1}{\sqrt{z^2+1}}\right) = \frac{\pi}{2} - \frac{\sqrt{z^2}}{z}\tan^{-1}\left(\frac{1}{z}\right) + \frac{\pi\,i}{2}\left(\frac{\sqrt{-i\,z-1}}{\sqrt{i\,z+1}} + \frac{\sqrt{i\,z-1}}{\sqrt{1-i\,z}}\right)$$

Involving $\cos^{-1}\left(\sqrt{\frac{1}{z^2+1}}\right)$

Involving $\cos^{-1}\left(\sqrt{\frac{1}{z^2+1}}\right)$ and $\tan^{-1}(z)$

01.13.27.0578.01

$$\cos^{-1}\left(\sqrt{\frac{1}{z^2+1}}\right) = \tan^{-1}(z) /; \operatorname{Re}(z) > 0 \lor (i\,z \in \mathbb{R} \land -1 < i\,z < 0)$$

01.13.27.0579.01

$$\cos^{-1}\left(\sqrt{\frac{1}{z^2+1}}\right) = -\tan^{-1}(z) /; \operatorname{Re}(z) < 0 \lor (i\,z \in \mathbb{R} \land 0 < i\,z < 1)$$

01.13.27.0580.01

$$\cos^{-1}\left(\sqrt{\frac{1}{z^2+1}}\right) = \pi - \tan^{-1}(z) /; (i\,z \in \mathbb{R} \land i\,z < -1)$$

01.13.27.0581.01

$$\cos^{-1}\left(\sqrt{\frac{1}{z^2+1}}\right) = \pi + \tan^{-1}(z) /; (i\,z \in \mathbb{R} \land i\,z > 1)$$

01.13.27.0582.01

$$\cos^{-1}\left(\sqrt{\frac{1}{z^2+1}}\right) = \frac{\pi}{2} - \sqrt{\frac{1}{z^2+1}}\,\sqrt{z^2+1}\left(\frac{\pi}{2} - \frac{\sqrt{z^2}}{z}\tan^{-1}(z)\right)$$

Involving $\cos^{-1}\left(\sqrt{\frac{1}{z^2+1}}\right)$ and $\tan^{-1}\left(\frac{1}{z}\right)$

01.13.27.0583.01

$$\cos^{-1}\left(\sqrt{\frac{1}{z^2+1}}\right) = \frac{\pi}{2} - \tan^{-1}\left(\frac{1}{z}\right) /;\ \operatorname{Re}(z) > 0 \lor (i\,z \in \mathbb{R} \land i\,z > 1)$$

01.13.27.0584.01

$$\cos^{-1}\left(\sqrt{\frac{1}{z^2+1}}\right) = \frac{\pi}{2} + \tan^{-1}\left(\frac{1}{z}\right) /;\ \operatorname{Re}(z) < 0 \lor (i\,z \in \mathbb{R} \land i\,z < -1)$$

01.13.27.0585.01

$$\cos^{-1}\left(\sqrt{\frac{1}{z^2+1}}\right) = \tan^{-1}\left(\frac{1}{z}\right) - \frac{\pi}{2} /;\ (i\,z \in \mathbb{R} \land 0 < i\,z < 1)$$

01.13.27.0586.01

$$\cos^{-1}\left(\sqrt{\frac{1}{z^2+1}}\right) = -\tan^{-1}\left(\frac{1}{z}\right) - \frac{\pi}{2} /;\ (-i\,z \in \mathbb{R} \land 0 < -i\,z < 1)$$

01.13.27.0587.01

$$\cos^{-1}\left(\sqrt{\frac{1}{z^2+1}}\right) = \frac{\pi}{2} - \sqrt{\frac{1}{z^2+1}}\,\sqrt{z^2+1}\left(\frac{\sqrt{z^2}}{z}\tan^{-1}\left(\frac{1}{z}\right) - \frac{\pi\,i}{2}\left(\frac{\sqrt{-i\,z-1}}{\sqrt{i\,z+1}} + \frac{\sqrt{i\,z-1}}{\sqrt{1-i\,z}}\right)\right)$$

Involving $\cos^{-1}\left(\frac{z}{\sqrt{z^2+1}}\right)$

Involving $\cos^{-1}\left(\frac{z}{\sqrt{z^2+1}}\right)$ and $\tan^{-1}(z)$

01.13.27.0023.01

$$\cos^{-1}\left(\frac{z}{\sqrt{z^2+1}}\right) = \frac{\pi}{2} - \tan^{-1}(z) /;\ i\,z \notin (-\infty, -1) \bigwedge i\,z \notin (1, \infty)$$

01.13.27.0588.01

$$\cos^{-1}\left(\frac{z}{\sqrt{z^2+1}}\right) = -\frac{\pi}{2} + \tan^{-1}(z) /;\ (i\,z \in \mathbb{R} \land i\,z < -1)$$

01.13.27.0589.01

$$\cos^{-1}\left(\frac{z}{\sqrt{z^2+1}}\right) = \frac{3\pi}{2} + \tan^{-1}(z) /; (i z \in \mathbb{R} \land i z > 1)$$

01.13.27.0590.01

$$\cos^{-1}\left(\frac{z}{\sqrt{z^2+1}}\right) = \frac{1}{2}\pi\left(\sqrt{\frac{1}{i z+1}} \sqrt{i z+1} - \sqrt{\frac{1}{1-i z}} \sqrt{1-i z} + 1\right) - \sqrt{\frac{1}{z^2+1}} \sqrt{z^2+1} \tan^{-1}(z)$$

Involving $\cos^{-1}\left(\frac{z}{\sqrt{z^2+1}}\right)$ and $\tan^{-1}\left(\frac{1}{z}\right)$

01.13.27.0591.01

$$\cos^{-1}\left(\frac{z}{\sqrt{z^2+1}}\right) = \tan^{-1}\left(\frac{1}{z}\right) /; \mathrm{Re}(z) > 0 \lor (i z \in \mathbb{R} \land 0 < i z < 1)$$

01.13.27.0592.01

$$\cos^{-1}\left(\frac{z}{\sqrt{z^2+1}}\right) = \pi + \tan^{-1}\left(\frac{1}{z}\right) /; \mathrm{Re}(z) < 0 \lor (-i z \in \mathbb{R} \land 0 < -i z < 1)$$

01.13.27.0593.01

$$\cos^{-1}\left(\frac{z}{\sqrt{z^2+1}}\right) = \pi - \tan^{-1}\left(\frac{1}{z}\right) /; (i z \in \mathbb{R} \land i z > 1)$$

01.13.27.0594.01

$$\cos^{-1}\left(\frac{z}{\sqrt{z^2+1}}\right) = -\tan^{-1}\left(\frac{1}{z}\right) /; (i z \in \mathbb{R} \land i z < -1)$$

01.13.27.0595.01

$$\cos^{-1}\left(\frac{z}{\sqrt{z^2+1}}\right) = \frac{\pi}{2}\left(1 - \frac{z}{\sqrt{z^2+1}} \sqrt{1+\frac{1}{z^2}}\right) + \sqrt{\frac{1}{z^2+1}} \sqrt{z^2+1} \tan^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{z^2}}{\sqrt{z^2+1}}\right)$

Involving $\cos^{-1}\left(\frac{\sqrt{z^2}}{\sqrt{z^2+1}}\right)$ and $\tan^{-1}(z)$

01.13.27.0596.01

$$\cos^{-1}\left(\frac{\sqrt{z^2}}{\sqrt{z^2+1}}\right) = \frac{\pi}{2} - \tan^{-1}(z) /; \mathrm{Re}(z) > 0 \lor (-i z \in \mathbb{R} \land 0 < -i z < 1)$$

01.13.27.0597.01

$$\cos^{-1}\left(\frac{\sqrt{z^2}}{\sqrt{z^2+1}}\right) = \frac{\pi}{2} + \tan^{-1}(z) \,/;\, \operatorname{Re}(z) < 0 \lor (i\,z \in \mathbb{R} \land 0 < i\,z < 1)$$

01.13.27.0598.01

$$\cos^{-1}\left(\frac{\sqrt{z^2}}{\sqrt{z^2+1}}\right) = -\frac{\pi}{2} + \tan^{-1}(z) \,/;\, (i\,z \in \mathbb{R} \land i\,z < -1)$$

01.13.27.0599.01

$$\cos^{-1}\left(\frac{\sqrt{z^2}}{\sqrt{z^2+1}}\right) = -\frac{\pi}{2} - \tan^{-1}(z) \,/;\, (i\,z \in \mathbb{R} \land i\,z > 1)$$

01.13.27.0600.01

$$\cos^{-1}\left(\frac{\sqrt{z^2}}{\sqrt{z^2+1}}\right) = \frac{\pi}{2}\sqrt{\frac{1}{z^2+1}}\sqrt{z^2+1} - \frac{\sqrt{z^2}\sqrt{z^2+1}}{z}\sqrt{\frac{1}{z^2+1}}\tan^{-1}(z)$$

Involving $\cos^{-1}\left(\frac{\sqrt{z^2}}{\sqrt{z^2+1}}\right)$ and $\tan^{-1}\left(\frac{1}{z}\right)$

01.13.27.0601.01

$$\cos^{-1}\left(\frac{\sqrt{z^2}}{\sqrt{z^2+1}}\right) = \tan^{-1}\left(\frac{1}{z}\right) \,/;\, \operatorname{Re}(z) > 0 \lor (i\,z \in \mathbb{R} \land i\,z > 1)$$

01.13.27.0602.01

$$\cos^{-1}\left(\frac{\sqrt{z^2}}{\sqrt{z^2+1}}\right) = -\tan^{-1}\left(\frac{1}{z}\right) \,/;\, \operatorname{Re}(z) < 0 \lor (i\,z \in \mathbb{R} \land i\,z < -1)$$

01.13.27.0603.01

$$\cos^{-1}\left(\frac{\sqrt{z^2}}{\sqrt{z^2+1}}\right) = \pi + \tan^{-1}\left(\frac{1}{z}\right) \,/;\, (-i\,z \in \mathbb{R} \land 0 < -i\,z < 1)$$

01.13.27.0604.01

$$\cos^{-1}\left(\frac{\sqrt{z^2}}{\sqrt{z^2+1}}\right) = \pi - \tan^{-1}\left(\frac{1}{z}\right) \,/;\, (i\,z \in \mathbb{R} \land 0 < i\,z < 1)$$

01.13.27.0605.01

$$\cos^{-1}\left(\frac{\sqrt{z^2}}{\sqrt{z^2+1}}\right) = \frac{\sqrt{z^2}}{\sqrt{z^2+1}}\sqrt{\frac{z^2+1}{z^2}}\left(z\sqrt{\frac{1}{z^2}}\tan^{-1}\left(\frac{1}{z}\right) - \frac{\pi}{2}\right) + \frac{\pi}{2}$$

Involving $\cos^{-1}\left(\frac{\sqrt{-z^2}}{\sqrt{-z^2-1}}\right)$

Involving $\cos^{-1}\left(\frac{\sqrt{-z^2}}{\sqrt{-z^2-1}}\right)$ and $\tan^{-1}(z)$

01.13.27.0606.01

$$\cos^{-1}\left(\frac{\sqrt{-z^2}}{\sqrt{-z^2-1}}\right) = \frac{\pi}{2} + \tan^{-1}(z) \; /; \; \operatorname{Re}(z) < 0 \lor (i\, z \in \mathbb{R} \land -1 < i\, z < 0)$$

01.13.27.0607.01

$$\cos^{-1}\left(\frac{\sqrt{-z^2}}{\sqrt{-z^2-1}}\right) = \frac{\pi}{2} - \tan^{-1}(z) \; /; \; \operatorname{Re}(z) > 0 \lor (i\, z \in \mathbb{R} \land 0 < i\, z < 1)$$

01.13.27.0608.01

$$\cos^{-1}\left(\frac{\sqrt{-z^2}}{\sqrt{-z^2-1}}\right) = -\frac{\pi}{2} - \tan^{-1}(z) \; /; \; (i\, z \in \mathbb{R} \land i\, z > 1)$$

01.13.27.0609.01

$$\cos^{-1}\left(\frac{\sqrt{-z^2}}{\sqrt{-z^2-1}}\right) = -\frac{\pi}{2} + \tan^{-1}(z) \; /; \; (i\, z \in \mathbb{R} \land i\, z < -1)$$

01.13.27.0610.01

$$\cos^{-1}\left(\frac{\sqrt{-z^2}}{\sqrt{-z^2-1}}\right) = \frac{\pi}{2}\left(\sqrt{\frac{1}{i\, z+1}}\, \sqrt{i\, z+1} + \sqrt{\frac{1}{1-i\, z}}\, \sqrt{1-i\, z} - 1\right) - z \sqrt{\frac{1}{z^2}}\, \tan^{-1}(z)$$

Involving $\cos^{-1}\left(\frac{\sqrt{-z^2}}{\sqrt{-z^2-1}}\right)$ and $\tan^{-1}\left(\frac{1}{z}\right)$

01.13.27.0611.01

$$\cos^{-1}\left(\frac{\sqrt{-z^2}}{\sqrt{-z^2-1}}\right) = -\tan^{-1}\left(\frac{1}{z}\right) /; \; \frac{\pi}{2} \leq \arg(z) \leq \pi \bigvee -\pi < \arg(z) < -\frac{\pi}{2}$$

01.13.27.0612.01

$$\cos^{-1}\left(\frac{\sqrt{-z^2}}{\sqrt{-z^2-1}}\right) = \tan^{-1}\left(\frac{1}{z}\right) /; \; -\frac{\pi}{2} \leq \arg(z) < \frac{\pi}{2}$$

01.13.27.0613.01

$$\cos^{-1}\left(\frac{\sqrt{-z^2}}{\sqrt{-z^2-1}}\right) = z \sqrt{\frac{1}{z^2}}\, \tan^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\sqrt{\frac{z^2}{z^2+1}}\right)$

Involving $\cos^{-1}\left(\sqrt{\frac{z^2}{z^2+1}}\right)$ and $\tan^{-1}(z)$

01.13.27.0614.01

$$\cos^{-1}\left(\sqrt{\frac{z^2}{z^2+1}}\right) = \frac{\pi}{2} + \tan^{-1}(z) \;/;\; \operatorname{Re}(z) < 0 \lor (i\,z \in \mathbb{R} \land 0 < i\,z < 1)$$

01.13.27.0615.01

$$\cos^{-1}\left(\sqrt{\frac{z^2}{z^2+1}}\right) = \frac{\pi}{2} - \tan^{-1}(z) \;/;\; \operatorname{Re}(z) > 0 \lor (-i\,z \in \mathbb{R} \land 0 < -i\,z < 1)$$

01.13.27.0616.01

$$\cos^{-1}\left(\sqrt{\frac{z^2}{z^2+1}}\right) = -\frac{\pi}{2} + \tan^{-1}(z) \;/;\; (i\,z \in \mathbb{R} \land i\,z < -1)$$

01.13.27.0617.01

$$\cos^{-1}\left(\sqrt{\frac{z^2}{z^2+1}}\right) = -\frac{\pi}{2} - \tan^{-1}(z) \;/;\; (i\,z \in \mathbb{R} \land i\,z > 1)$$

01.13.27.0618.01

$$\cos^{-1}\left(\sqrt{\frac{z^2}{z^2+1}}\right) = \frac{\pi}{2} - \frac{\sqrt{z^2}\sqrt{-z^2-1}}{\sqrt{-z^2\left(z^2+1\right)}}\left(z\sqrt{\frac{1}{z^2}}\tan^{-1}(z) - \frac{1}{2}\pi\left(\sqrt{\frac{1}{i\,z+1}}\sqrt{i\,z+1} + \sqrt{\frac{1}{1-i\,z}}\sqrt{1-i\,z} - 2\right)\right)$$

Involving $\cos^{-1}\left(\sqrt{\frac{z^2}{z^2+1}}\right)$ and $\tan^{-1}\left(\frac{1}{z}\right)$

01.13.27.0619.01

$$\cos^{-1}\left(\sqrt{\frac{z^2}{z^2+1}}\right) = -\tan^{-1}\left(\frac{1}{z}\right) \;/;\; \operatorname{Re}(z) < 0 \lor (i\,z \in \mathbb{R} \land i\,z < -1)$$

01.13.27.0620.01

$$\cos^{-1}\left(\sqrt{\frac{z^2}{z^2+1}}\right) = \tan^{-1}\left(\frac{1}{z}\right) \;/;\; \operatorname{Re}(z) > 0 \lor (i\,z \in \mathbb{R} \land i\,z > 1)$$

01.13.27.0621.01

$$\cos^{-1}\left(\sqrt{\frac{z^2}{z^2+1}}\right) = \pi - \tan^{-1}\left(\frac{1}{z}\right) \;/;\; (i\,z \in \mathbb{R} \land 0 < i\,z < 1)$$

01.13.27.0622.01

$$\cos^{-1}\left(\sqrt{\frac{z^2}{z^2+1}}\right) = \pi + \tan^{-1}\left(\frac{1}{z}\right) /; (-i\, z \in \mathbb{R} \land 0 < -i\, z < 1)$$

01.13.27.0623.01

$$\cos^{-1}\left(\sqrt{\frac{z^2}{z^2+1}}\right) = \frac{\pi}{2} - \frac{\sqrt{z^2}\,\sqrt{-z^2-1}}{\sqrt{-z^2\,(z^2+1)}}\left(\frac{\pi}{2} - z\,\sqrt{\frac{1}{z^2}}\,\tan^{-1}\left(\frac{1}{z}\right)\right)$$

Involving $\cos^{-1}\left(\sqrt{\sqrt{1+z^2}-1} \Big/ \left(\sqrt{2}\,(1+z^2)^{1/4}\right)\right)$

Involving $\cos^{-1}\left(\sqrt{\sqrt{1+z^2}-1} \Big/ \left(\sqrt{2}\,(1+z^2)^{1/4}\right)\right)$ and $\tan^{-1}(z)$

01.13.27.0624.01

$$\cos^{-1}\left(\frac{1}{\sqrt{2}}\,\frac{\sqrt{\sqrt{1+z^2}-1}}{(1+z^2)^{1/4}}\right) = \frac{\pi}{2} - \frac{\tan^{-1}(z)}{2} /; -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

01.13.27.0625.01

$$\cos^{-1}\left(\frac{\sqrt{\sqrt{1+z^2}-1}}{\sqrt{2}\,(1+z^2)^{1/4}}\right) = \frac{\pi}{2} + \frac{1}{2}\tan^{-1}(z) /; \frac{\pi}{2} < \arg(z) \leq \pi \bigvee -\pi < \arg(z) \leq -\frac{\pi}{2}$$

01.13.27.0626.01

$$\cos^{-1}\left(\frac{1}{\sqrt{2}}\,\frac{\sqrt{\sqrt{1+z^2}-1}}{(1+z^2)^{1/4}}\right) = \frac{\pi}{2} - \frac{\sqrt{z^2}\,\tan^{-1}(z)}{2\,z}$$

Involving $\cos^{-1}\left(\sqrt{\sqrt{1+z^2}-1} \Big/ \left(\sqrt{2}\,(1+z^2)^{1/4}\right)\right)$ and $\tan^{-1}\left(\frac{1}{z}\right)$

01.13.27.0627.01

$$\cos^{-1}\left(\frac{\sqrt{\sqrt{1+z^2}-1}}{\sqrt{2}\,(1+z^2)^{1/4}}\right) = \frac{\pi}{4} + \frac{1}{2}\tan^{-1}\left(\frac{1}{z}\right) /; \mathrm{Re}(z) > 0 \lor (i\, z \in \mathbb{R} \land i\, z < -1)$$

01.13.27.0628.01

$$\cos^{-1}\left(\frac{\sqrt{\sqrt{1+z^2}-1}}{\sqrt{2}\,(1+z^2)^{1/4}}\right) = \frac{\pi}{4} - \frac{1}{2}\tan^{-1}\left(\frac{1}{z}\right) /; \operatorname{Re}(z) < 0 \lor (i\,z \in \mathbb{R} \land i\,z > 1)$$

01.13.27.0629.01

$$\cos^{-1}\left(\frac{\sqrt{\sqrt{z^2+1}-1}}{\sqrt{2}\,\sqrt[4]{z^2+1}}\right) = \frac{3\pi}{4} + \frac{1}{2}\tan^{-1}\left(\frac{1}{z}\right) /; (-i\,z \in \mathbb{R} \land 0 < -i\,z < 1)$$

01.13.27.0630.01

$$\cos^{-1}\left(\frac{\sqrt{\sqrt{z^2+1}-1}}{\sqrt{2}\,\sqrt[4]{z^2+1}}\right) = \frac{3\pi}{4} - \frac{1}{2}\tan^{-1}\left(\frac{1}{z}\right) /; (i\,z \in \mathbb{R} \land 0 < i\,z < 1)$$

01.13.27.0631.01

$$\cos^{-1}\left(\frac{\sqrt{\sqrt{z^2+1}-1}}{\sqrt{2}\,\sqrt[4]{z^2+1}}\right) = \frac{1}{4}\pi\left(2 - \sqrt{z^2}\,\sqrt{\frac{1}{z^2}}\,\sqrt{\frac{1}{z^2+1}}\,\sqrt{z^2+1}\right) + \frac{\sqrt{z^2}}{2\,z}\tan^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\sqrt{\sqrt{1+z^2}-z} \Big/ \left(\sqrt{2}\,(1+z^2)^{1/4}\right)\right)$

Involving $\cos^{-1}\left(\sqrt{\sqrt{1+z^2}-z} \Big/ \left(\sqrt{2}\,(1+z^2)^{1/4}\right)\right)$ and $\tan^{-1}(z)$

01.13.27.0632.01

$$\cos^{-1}\left(\frac{\sqrt{\sqrt{z^2+1}-z}}{\sqrt{2}\,\sqrt[4]{z^2+1}}\right) = \frac{1}{2}\tan^{-1}(z) + \frac{\pi}{4} /; i\,z \notin (1, \infty)$$

01.13.27.0633.01

$$\cos^{-1}\left(\frac{\sqrt{\sqrt{z^2+1}-z}}{\sqrt{2}\,\sqrt[4]{z^2+1}}\right) = -\frac{1}{2}\tan^{-1}(z) - \frac{\pi}{4} /; (i\,z \in \mathbb{R} \land i\,z > 1)$$

01.13.27.0634.01

$$\cos^{-1}\left(\frac{\sqrt{\sqrt{z^2+1}-z}}{\sqrt{2}\,\sqrt[4]{z^2+1}}\right) = \frac{1}{2}\tan^{-1}(z)\sqrt{\frac{1}{1-i\,z}}\,\sqrt{1-i\,z} + \frac{1}{4}\pi\sqrt{\frac{1}{1-i\,z}}\,\sqrt{1-i\,z}$$

Involving $\cos^{-1}\left(\sqrt{\sqrt{1+z^2}-z}\Big/\left(\sqrt{2}\,(1+z^2)^{1/4}\right)\right)$ and $\tan^{-1}\left(\frac{1}{z}\right)$

01.13.27.0635.01

$$\cos^{-1}\left(\frac{\sqrt{\sqrt{z^2+1}-z}}{\sqrt{2}\,\sqrt[4]{z^2+1}}\right) = \frac{\pi}{2} - \frac{1}{2}\tan^{-1}\left(\frac{1}{z}\right) /; \operatorname{Re}(z) > 0 \lor (i\,z \in \mathbb{R} \land 0 < i\,z < 1) \lor (i\,z \in \mathbb{R} \land i\,z < -1)$$

01.13.27.0636.01

$$\cos^{-1}\left(\frac{\sqrt{\sqrt{z^2+1}-z}}{\sqrt{2}\,\sqrt[4]{z^2+1}}\right) = -\frac{1}{2}\tan^{-1}\left(\frac{1}{z}\right) /; \operatorname{Re}(z) < 0 \lor (-i\,z \in \mathbb{R} \land 0 < -i\,z < 1)$$

01.13.27.0637.01

$$\cos^{-1}\left(\frac{\sqrt{\sqrt{z^2+1}-z}}{\sqrt{2}\,\sqrt[4]{z^2+1}}\right) = \frac{1}{4}\pi\left(\frac{\sqrt{z^2}}{z}+1\right) - \frac{1}{2}\tan^{-1}\left(\frac{1}{z}\right) /; i\,z \notin (-1, \infty)$$

01.13.27.0638.01

$$\cos^{-1}\left(\frac{\sqrt{\sqrt{z^2+1}-z}}{\sqrt{2}\,\sqrt[4]{z^2+1}}\right) = \frac{\pi}{4}\left(1 + \sqrt{\frac{z}{z-i}}\,\sqrt{\frac{z-i}{z}} - \sqrt{\frac{z}{z+i}}\,\sqrt{\frac{z+i}{z}} + \frac{\sqrt{z^2}}{z}\right) - \frac{1}{2}\sqrt{\frac{1}{1-i\,z}}\,\sqrt{1-i\,z}\,\tan^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\sqrt{\left(\sqrt{1+z^2}-1\right)\Big/\left(2\sqrt{1+z^2}\right)}\right)$

Involving $\cos^{-1}\left(\sqrt{\left(\sqrt{1+z^2}-1\right)\Big/\left(2\sqrt{1+z^2}\right)}\right)$ and $\tan^{-1}(z)$

01.13.27.0639.01

$$\cos^{-1}\left(\sqrt{\frac{\sqrt{1+z^2}-1}{2\sqrt{1+z^2}}}\right) = \frac{\pi}{2} - \frac{\tan^{-1}(z)}{2} \;/;\; -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

01.13.27.0640.01

$$\cos^{-1}\left(\sqrt{\frac{\sqrt{1+z^2}-1}{2\sqrt{1+z^2}}}\right) = \frac{\pi}{2} + \frac{1}{2}\tan^{-1}(z) \;/;\; \frac{\pi}{2} < \arg(z) \leq \pi \bigvee -\pi < \arg(z) \leq -\frac{\pi}{2}$$

01.13.27.0641.01

$$\cos^{-1}\left(\sqrt{\frac{\sqrt{1+z^2}-1}{2\sqrt{1+z^2}}}\right) = \frac{\pi}{2} - \frac{\sqrt{z^2}}{2z}\tan^{-1}(z)$$

Involving $\cos^{-1}\left(\sqrt{\left(\sqrt{z^2+1}-1\right)\Big/\left(2\sqrt{z^2+1}\right)}\right)$ and $\tan^{-1}\left(\frac{1}{z}\right)$

01.13.27.0642.01

$$\cos^{-1}\left(\sqrt{\frac{\sqrt{z^2+1}-1}{2\sqrt{z^2+1}}}\right) = \frac{\pi}{4} + \frac{1}{2}\tan^{-1}\left(\frac{1}{z}\right) \;/;\; \mathrm{Re}(z) > 0 \lor (i\,z \in \mathbb{R} \land i\,z < -1)$$

01.13.27.0643.01

$$\cos^{-1}\left(\sqrt{\frac{\sqrt{z^2+1}-1}{2\sqrt{z^2+1}}}\right) = -\frac{1}{2}\tan^{-1}\left(\frac{1}{z}\right) + \frac{\pi}{4} \;/;\; \mathrm{Re}(z) < 0 \lor (i\,z \in \mathbb{R} \land i\,z > 1)$$

01.13.27.0644.01

$$\cos^{-1}\left(\sqrt{\frac{\sqrt{z^2+1}-1}{2\sqrt{z^2+1}}}\right) = \frac{3\pi}{4} - \frac{1}{2}\tan^{-1}\left(\frac{1}{z}\right) \;/;\; (i\,z \in \mathbb{R} \land 0 < i\,z < 1)$$

01.13.27.0645.01

$$\cos^{-1}\left(\sqrt{\frac{\sqrt{z^2+1}-1}{2\sqrt{z^2+1}}}\right) = \frac{3\pi}{4} + \frac{1}{2}\tan^{-1}\left(\frac{1}{z}\right) \;/;\; (-i\,z \in \mathbb{R} \land 0 < -i\,z < 1)$$

01.13.27.0646.01

$$\cos^{-1}\left(\sqrt{\frac{\sqrt{z^2+1}-1}{2\sqrt{z^2+1}}}\right) = \frac{\pi}{4}\left(2 - \sqrt{z^2}\sqrt{\frac{1}{z^2}}\sqrt{\frac{1}{z^2+1}}\sqrt{z^2+1}\right) + \frac{\sqrt{z^2}}{2z}\tan^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\sqrt{\left(\sqrt{1+z^2}-z\right)\Big/\left(2\sqrt{1+z^2}\right)}\right)$

Involving $\cos^{-1}\left(\sqrt{\left(\sqrt{1+z^2}-z\right)\Big/\left(2\sqrt{1+z^2}\right)}\right)$ and $\tan^{-1}(z)$

01.13.27.0647.01

$$\cos^{-1}\left(\sqrt{\frac{\sqrt{z^2+1}-z}{2\sqrt{z^2+1}}}\right)=\frac{\pi}{4}+\frac{1}{2}\tan^{-1}(z)\;/;\;i\,z\notin(-\infty,-1)\bigwedge i\,z\notin(1,\infty)$$

01.13.27.0648.01

$$\cos^{-1}\left(\sqrt{\frac{\sqrt{1+z^2}-z}{2\sqrt{1+z^2}}}\right)=-\frac{1}{2}\tan^{-1}(z)-\frac{\pi}{4}\;/;\;(i\,z\in\mathbb{R}\land i\,z>1)$$

01.13.27.0649.01

$$\cos^{-1}\left(\sqrt{\frac{\sqrt{1+z^2}-z}{2\sqrt{1+z^2}}}\right)=\frac{3\pi}{4}-\frac{1}{2}\tan^{-1}(z)\;/;\;(i\,z\in\mathbb{R}\land i\,z<-1)$$

01.13.27.0650.01

$$\cos^{-1}\left(\sqrt{\frac{\sqrt{z^2+1}-z}{2\sqrt{z^2+1}}}\right)=-\frac{1}{4}\pi\sqrt{i\,z+1}\sqrt{\frac{1}{i\,z+1}}\left(2-\sqrt{1-i\,z}\sqrt{\frac{1}{1-i\,z}}\right)+\frac{1}{2}\tan^{-1}(z)\sqrt{\frac{1}{z^2+1}}\sqrt{z^2+1}+\frac{\pi}{2}$$

Involving $\cos^{-1}\left(\sqrt{\left(\sqrt{z^2+1}-z\right)\Big/\left(2\sqrt{z^2+1}\right)}\right)$ and $\tan^{-1}\left(\frac{1}{z}\right)$

01.13.27.0651.01

$$\cos^{-1}\left(\sqrt{\frac{\sqrt{z^2+1}-z}{2\sqrt{z^2+1}}}\right)=\frac{\pi}{2}-\frac{1}{2}\tan^{-1}\left(\frac{1}{z}\right)\;/;\;\mathrm{Re}(z)>0\lor(i\,z\in\mathbb{R}\land 0<i\,z<1)$$

01.13.27.0652.01

$$\cos^{-1}\left(\sqrt{\frac{\sqrt{z^2+1}-z}{2\sqrt{z^2+1}}}\right)=-\frac{1}{2}\tan^{-1}\left(\frac{1}{z}\right)\;/;\;\mathrm{Re}(z)<0\lor(-i\,z\in\mathbb{R}\land 0<-i\,z<1)$$

01.13.27.0653.01

$$\cos^{-1}\left(\sqrt{\frac{\sqrt{z^2+1}-z}{2\sqrt{z^2+1}}}\right)=\frac{1}{2}\tan^{-1}\left(\frac{1}{z}\right)\;/;\;(i\,z\in\mathbb{R}\land i\,z>1)$$

01.13.27.0654.01

$$\cos^{-1}\left(\sqrt{\frac{\sqrt{z^2+1}-z}{2\sqrt{z^2+1}}}\right) = \frac{\pi}{2} + \frac{1}{2}\tan^{-1}\left(\frac{1}{z}\right) /; (i\,z \in \mathbb{R} \land i\,z < -1)$$

01.13.27.0655.01

$$\cos^{-1}\left(\sqrt{\frac{\sqrt{z^2+1}-z}{2\sqrt{z^2+1}}}\right) = -\frac{1}{2}\tan^{-1}\left(\frac{1}{z}\right) + \frac{1}{4}\pi\left(\frac{\sqrt{z^2}}{z}+1\right) /; \operatorname{Re}(z) \neq 0$$

01.13.27.0656.01

$$\cos^{-1}\left(\sqrt{\frac{\sqrt{z^2+1}-z}{2\sqrt{z^2+1}}}\right) = -\frac{1}{2}\sqrt{z^2+1}\sqrt{\frac{1}{z^2+1}}\tan^{-1}\left(\frac{1}{z}\right) + \frac{1}{4}\pi\left(\sqrt{\frac{1}{z^2}}\,z - \sqrt{\frac{1}{i\,z+1}}\sqrt{i\,z+1} + \sqrt{\frac{1}{1-i\,z}}\sqrt{1-i\,z}+1\right)$$

Involving $\cos^{-1}\left(\frac{r\,z^c}{\sqrt{r^2\,z^{2c}+1}}\right)$

Involving $\cos^{-1}\left(\frac{a\,z^c}{\sqrt{a^2\,z^{2c}+1}}\right)$ and $\tan^{-1}(a\,z^c)$

01.13.27.0024.02

$$\cos^{-1}\left(\frac{a\,z^c}{\sqrt{a^2\,z^{2c}+1}}\right) = \frac{\pi}{2}\left(\sqrt{\frac{1}{a\,i\,z^c+1}}\sqrt{a\,i\,z^c+1} - \sqrt{\frac{1}{1-i\,a\,z^c}}\sqrt{1-i\,a\,z^c}+1\right) - \tan^{-1}(a\,z^c)\sqrt{\frac{1}{a^2\,z^{2c}+1}}\sqrt{a^2\,z^{2c}+1}$$

Involving $\cos^{-1}\left(\frac{1}{\sqrt{a\,z^c+1}}\right)$

Involving $\cos^{-1}\left(1\big/\sqrt{a\,z^c+1}\right)$ and $\tan^{-1}\left(\frac{1}{\sqrt{a}\,z^{c/2}}\right)$

01.13.27.0657.01

$$\cos^{-1}\left(\frac{1}{\sqrt{a\,z^c+1}}\right) = \frac{\pi}{2} - \frac{\sqrt{a}\,z^{c/2}}{\sqrt{a\,z^c}}\tan^{-1}\left(\frac{1}{\sqrt{a}\,z^{c/2}}\right) + \frac{\pi\,i}{2}\left(\frac{\sqrt{-i\sqrt{a}\sqrt{z^c}-1}}{\sqrt{i\sqrt{a}\sqrt{z^c}+1}} + \frac{\sqrt{i\sqrt{a}\sqrt{z^c}-1}}{\sqrt{1-i\sqrt{a}\sqrt{z^c}}}\right)$$

Involving $\cot^{-1}$

Involving $\cos^{-1}(z)$

Involving $\cos^{-1}(z)$ and $\cot^{-1}\left(\frac{z}{\sqrt{1-z^2}}\right)$

01.13.27.0658.01

$$\cos^{-1}(z) = \cot^{-1}\left(\frac{z}{\sqrt{1-z^2}}\right) /; -\frac{\pi}{2} \leq \arg(z) < \frac{\pi}{2}$$

01.13.27.0659.01

$$\cos^{-1}(z) = \pi + \cot^{-1}\left(\frac{z}{\sqrt{1-z^2}}\right) /; \frac{\pi}{2} \leq \arg(z) \leq \pi \bigvee -\pi < \arg(z) < -\frac{\pi}{2}$$

01.13.27.0660.01

$$\cos^{-1}(z) = \cot^{-1}\left(\frac{z}{\sqrt{1-z^2}}\right) + \frac{\pi}{2}\left(1 - \sqrt{\frac{1}{z^2}}\, z\right)$$

Involving $\cos^{-1}(z)$ and $\cot^{-1}\left(\frac{\sqrt{z^2}}{\sqrt{1-z^2}}\right)$

01.13.27.0661.01

$$\cos^{-1}(z) = \cot^{-1}\left(\frac{\sqrt{z^2}}{\sqrt{1-z^2}}\right) /; \operatorname{Re}(z) > 0$$

01.13.27.0662.01

$$\cos^{-1}(z) = \pi - \cot^{-1}\left(\frac{\sqrt{z^2}}{\sqrt{1-z^2}}\right) /; \frac{\pi}{2} < \arg(z) \leq \pi \bigvee -\pi < \arg(z) < -\frac{\pi}{2}$$

01.13.27.0663.01

$$\cos^{-1}(z) = \pi + \cot^{-1}\left(\frac{\sqrt{z^2}}{\sqrt{1-z^2}}\right) /; (i\, z \in \mathbb{R} \land i\, z < 0)$$

01.13.27.0664.01

$$\cos^{-1}(z) = -\cot^{-1}\left(\frac{\sqrt{z^2}}{\sqrt{1-z^2}}\right) /; (i\, z \in \mathbb{R} \land i\, z > 0)$$

01.13.27.0665.01

$$\cos^{-1}(z) = \frac{1}{2}\pi\left(1 - \sqrt{\frac{1}{z^2}}\, z\right) + \frac{\sqrt{z^2}}{z}\cot^{-1}\left(\frac{\sqrt{z^2}}{\sqrt{1-z^2}}\right)$$

Involving $\cos^{-1}(z)$ and $\cot^{-1}\left(\frac{\sqrt{-z^2}}{\sqrt{z^2-1}}\right)$

01.13.27.0666.01

$$\cos^{-1}(z) = -\cot^{-1}\left(\frac{\sqrt{-z^2}}{\sqrt{z^2-1}}\right) /; 0 < \arg(z) < \frac{\pi}{2} \bigvee -\frac{\pi}{2} < \arg(z) < 0 \bigvee (z \in \mathbb{R} \land z > 1)$$

01.13.27.0667.01

$$\cos^{-1}(z) = \pi + \cot^{-1}\left(\frac{\sqrt{-z^2}}{\sqrt{z^2-1}}\right) /; \frac{\pi}{2} < \arg(z) < \pi \bigvee -\pi < \arg(z) < -\frac{\pi}{2} \bigvee (z \in \mathbb{R} \land z < -1)$$

01.13.27.0668.01

$$\cos^{-1}(z) = \cot^{-1}\left(\frac{\sqrt{-z^2}}{\sqrt{z^2-1}}\right) /; (z \in \mathbb{R} \land 0 < z < 1) \lor (i z \in \mathbb{R} \land i z > 0)$$

01.13.27.0669.01

$$\cos^{-1}(z) = \pi - \cot^{-1}\left(\frac{\sqrt{-z^2}}{\sqrt{z^2-1}}\right) /; (z \in \mathbb{R} \land -1 < z < 0) \lor (i z \in \mathbb{R} \land i z < 0)$$

01.13.27.0670.01

$$\cos^{-1}(z) = \left(1 - z\sqrt{z^{-2}}\right)\frac{\pi}{2} - \frac{\sqrt{-z^2}\sqrt{z^2-1}}{z\sqrt{1-z^2}}\cot^{-1}\left(\frac{\sqrt{-z^2}}{\sqrt{z^2-1}}\right)$$

Involving $\cos^{-1}(z)$ and $\cot^{-1}\left(\sqrt{\frac{z^2}{1-z^2}}\right)$

01.13.27.0671.01

$$\cos^{-1}(z) = \cot^{-1}\left(\sqrt{\frac{z^2}{1-z^2}}\right) /; \operatorname{Re}(z) > 0 \bigwedge z \notin (1, \infty)$$

01.13.27.0672.01

$$\cos^{-1}(z) = \pi - \cot^{-1}\left(\sqrt{\frac{z^2}{1-z^2}}\right) /; \operatorname{Re}(z) < 0 \bigwedge z \notin (-\infty, -1)$$

01.13.27.0673.01

$$\cos^{-1}(z) = -\cot^{-1}\left(\sqrt{\frac{z^2}{1-z^2}}\right) /; (i z \in \mathbb{R} \land 0 < i z < 1)$$

01.13.27.0674.01

$$\cos^{-1}(z) = \pi + \cot^{-1}\left(\sqrt{\frac{z^2}{1-z^2}}\right) /; (-i z \in \mathbb{R} \land 0 < -i z < 1)$$

01.13.27.0675.01

$$\cos^{-1}(z) = \frac{\pi}{2}\left(1 - \sqrt{\frac{1}{z^2}}\, z\right) + \frac{\sqrt{1-z^2}}{z}\sqrt{\frac{z^2}{1-z^2}}\cot^{-1}\left(\sqrt{\frac{z^2}{1-z^2}}\right)$$

Involving $\cos^{-1}(z)$ and $\cot^{-1}\left(\frac{\sqrt{1-z^2}}{z}\right)$

01.13.27.0676.01

$$\cos^{-1}(z) = \frac{\pi}{2} - \cot^{-1}\left(\frac{\sqrt{1-z^2}}{z}\right) /; z \notin (-\infty, -1) \bigwedge z \notin (1, \infty)$$

01.13.27.0677.01

$$\cos^{-1}(z) = -\frac{\pi}{2} - \cot^{-1}\left(\frac{\sqrt{1-z^2}}{z}\right) /; (z \in \mathbb{R} \land z > 1)$$

01.13.27.0678.01

$$\cos^{-1}(z) = \frac{3\pi}{2} - \cot^{-1}\left(\frac{\sqrt{1-z^2}}{z}\right) /; (z \in \mathbb{R} \land z < -1)$$

01.13.27.0679.01

$$\cos^{-1}(z) = \frac{\pi}{2}\left(\sqrt{\frac{1}{1-z}}\sqrt{1-z} - \sqrt{\frac{1}{z+1}}\sqrt{z+1} + 1\right) - \cot^{-1}\left(\frac{\sqrt{1-z^2}}{z}\right)$$

Involving $\cos^{-1}(z)$ and $\cot^{-1}\left(\frac{\sqrt{1-z^2}}{\sqrt{z^2}}\right)$

01.13.27.0680.01

$$\cos^{-1}(z) = \frac{\pi}{2} - \cot^{-1}\left(\frac{\sqrt{1-z^2}}{\sqrt{z^2}}\right) /; -\frac{\pi}{2} < \arg(z) \le \frac{\pi}{2} \bigwedge z \notin (1, \infty)$$

01.13.27.0681.01

$$\cos^{-1}(z) = \frac{\pi}{2} + \cot^{-1}\left(\frac{\sqrt{1-z^2}}{\sqrt{z^2}}\right) /; \frac{\pi}{2} < \arg(z) < \pi \bigvee -\pi < \arg(z) \le -\frac{\pi}{2} \bigvee (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.0682.01

$$\cos^{-1}(z) = \frac{3\pi}{2} + \cot^{-1}\left(\frac{\sqrt{1-z^2}}{\sqrt{z^2}}\right) /; (z \in \mathbb{R} \land z < -1)$$

01.13.27.0683.01

$$\cos^{-1}(z) = \frac{1}{2}\pi\left(1 - \sqrt{\frac{1}{z+1}}\sqrt{z+1} + \sqrt{\frac{1}{1-z}}\sqrt{1-z}\right) - \frac{\sqrt{z^2}}{z}\cot^{-1}\left(\frac{\sqrt{1-z^2}}{\sqrt{z^2}}\right)$$

Involving $\cos^{-1}(z)$ and $\cot^{-1}\left(\frac{\sqrt{z^2-1}}{\sqrt{-z^2}}\right)$

01.13.27.0684.01

$$\cos^{-1}(z) = \frac{\pi}{2} - \cot^{-1}\left(\frac{\sqrt{z^2-1}}{\sqrt{-z^2}}\right) /; \frac{\pi}{2} < \arg(z) < \pi \bigvee -\pi < \arg(z) \leq -\frac{\pi}{2} \bigvee (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.0685.01

$$\cos^{-1}(z) = \frac{\pi}{2} + \cot^{-1}\left(\frac{\sqrt{z^2-1}}{\sqrt{-z^2}}\right) /; -\frac{\pi}{2} < \arg(z) < 0 \bigvee 0 < \arg(z) \leq \frac{\pi}{2} \bigvee (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.0686.01

$$\cos^{-1}(z) = \cot^{-1}\left(\frac{\sqrt{z^2-1}}{\sqrt{-z^2}}\right) - \frac{\pi}{2} /; (z \in \mathbb{R} \land z > 1)$$

01.13.27.0687.01

$$\cos^{-1}(z) = \frac{3\pi}{2} - \cot^{-1}\left(\frac{\sqrt{z^2-1}}{\sqrt{-z^2}}\right) /; (z \in \mathbb{R} \land z < -1)$$

01.13.27.0688.01

$$\cos^{-1}(z) = \frac{\sqrt{-z^2}\sqrt{z^2-1}}{z\sqrt{1-z^2}} \cot^{-1}\left(\frac{\sqrt{z^2-1}}{\sqrt{-z^2}}\right) + \frac{\pi}{2}\left(1 + \sqrt{\frac{1}{1-z}}\sqrt{1-z} - \sqrt{\frac{1}{z+1}}\sqrt{z+1}\right)$$

Involving $\cos^{-1}(z)$ and $\cot^{-1}\left(\sqrt{\frac{1-z^2}{z^2}}\right)$

01.13.27.0689.01

$$\cos^{-1}(z) = \frac{\pi}{2} - \cot^{-1}\left(\sqrt{\frac{1-z^2}{z^2}}\right) /; -\frac{\pi}{2} \leq \arg(z) < \frac{\pi}{2} \bigwedge z \notin (1, \infty)$$

01.13.27.0690.01

$$\cos^{-1}(z) = \frac{\pi}{2} + \cot^{-1}\left(\sqrt{\frac{1-z^2}{z^2}}\right) /; \frac{\pi}{2} \leq \arg(z) < \pi \bigvee -\pi < \arg(z) < -\frac{\pi}{2} \bigvee (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.0691.01

$$\cos^{-1}(z) = -\frac{\pi}{2} - \cot^{-1}\left(\sqrt{\frac{1-z^2}{z^2}}\right) /; (z \in \mathbb{R} \land z > 1)$$

01.13.27.0692.01

$$\cos^{-1}(z) = \frac{3\pi}{2} + \cot^{-1}\left(\sqrt{\frac{1-z^2}{z^2}}\right) /; (z \in \mathbb{R} \land z < -1)$$

01.13.27.0693.01

$$\cos^{-1}(z) = \frac{\pi}{2}\left(-\sqrt{\frac{1}{z+1}}\,\sqrt{z+1} + \sqrt{\frac{1}{1-z}}\,\sqrt{1-z} + 1\right) - z\,\sqrt{\frac{1}{z^2}}\,\cot^{-1}\left(\sqrt{\frac{1-z^2}{z^2}}\right)$$

Involving $\cos^{-1}(z)$ and $\cot^{-1}\left(\frac{1-2z^2}{2z\sqrt{1-z^2}}\right)$

01.13.27.0694.01

$$\cos^{-1}(z) = \frac{\pi}{2} - \frac{1}{2}\cot^{-1}\left(\frac{1-2z^2}{2z\sqrt{1-z^2}}\right) /;\ \frac{\pi}{4} \le |\arg(z)| < \frac{3\pi}{4}$$

01.13.27.0695.01

$$\cos^{-1}(z) = \frac{\pi}{4}\left(2 - \frac{\sqrt{z^2-1}\,z}{\sqrt{z^4-z^2}} - \sqrt{\frac{1}{z}}\,\sqrt{\frac{1}{\sqrt{2}\,z-1}}\,\sqrt{\sqrt{2}\,z-1}\,\sqrt{z} + \sqrt{-\frac{1}{z}}\,\sqrt{-z}\,\sqrt{-\sqrt{2}\,z-1}\,\sqrt{-\frac{1}{\sqrt{2}\,z+1}} - \frac{\sqrt{z^2}}{z}\right) - \frac{1}{2}\cot^{-1}\left(\frac{1-2z^2}{2z\sqrt{1-z^2}}\right)$$

Involving $\cos^{-1}(z)$ and $\cot^{-1}\left(\frac{2z\sqrt{1-z^2}}{1-2z^2}\right)$

01.13.27.0696.01

$$\cos^{-1}(z) = \frac{\pi}{4} + \frac{1}{2}\cot^{-1}\left(\frac{2z\sqrt{1-z^2}}{1-2z^2}\right) /;\ -\frac{\pi}{2} \le \arg(z) < 0 \bigvee 0 < \arg(z) < \frac{\pi}{2} \bigvee (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.0697.01

$$\cos^{-1}(z) = \frac{3\pi}{4} + \frac{1}{2}\cot^{-1}\left(\frac{2z\sqrt{1-z^2}}{1-2z^2}\right) /;\ \frac{\pi}{2} \le \arg(z) < \pi \bigvee -\pi < \arg(z) < -\frac{\pi}{2} \bigvee (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.0698.01

$$\cos^{-1}(z) = \frac{1}{2}\cot^{-1}\left(\frac{2z\sqrt{1-z^2}}{1-2z^2}\right) + \frac{5\pi}{4} /;\ (z \in \mathbb{R} \land z < -1)$$

01.13.27.0699.01

$$\cos^{-1}(z) = \frac{1}{2}\cot^{-1}\left(\frac{2z\sqrt{1-z^2}}{1-2z^2}\right) - \frac{\pi}{4} /;\ (z \in \mathbb{R} \land z > 1)$$

01.13.27.0700.01

$$\cos^{-1}(z) = \frac{\pi}{4}\left(2 + \sqrt{\frac{1}{1-z}}\,\sqrt{1-z} - \sqrt{\frac{1}{z+1}}\,\sqrt{z+1} + \sqrt{-i\,z}\,\sqrt{\frac{i}{z}} - \sqrt{-\frac{i}{z}}\,\sqrt{i\,z} - \frac{\sqrt{z^2}}{z}\right) + \frac{1}{2}\cot^{-1}\left(\frac{2z\sqrt{1-z^2}}{1-2z^2}\right)$$

Involving $\cos^{-1}(z)$ and $\cot^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{1+z}}\right)$

01.13.27.0701.01

$$\cos^{-1}(z) = \pi - 2\cot^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{z+1}}\right) /; z \notin (1, \infty)$$

01.13.27.0702.01

$$\cos^{-1}(z) = -\pi - 2\cot^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{z+1}}\right) /; (z \in \mathbb{R} \land z > 1)$$

01.13.27.0703.01

$$\cos^{-1}(z) = -2\cot^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{1+z}}\right) - \frac{\pi\sqrt{z-1}\sqrt{z}}{\sqrt{1-z}}\sqrt{-\frac{1}{z}}$$

Involving $\cos^{-1}(z)$ and $\cot^{-1}\left(\frac{\sqrt{z-1}}{\sqrt{-z-1}}\right)$

01.13.27.0704.01

$$\cos^{-1}(z) = \pi + 2\cot^{-1}\left(\frac{\sqrt{z-1}}{\sqrt{-z-1}}\right) /; z \notin (-1, \infty)$$

01.13.27.0705.01

$$\cos^{-1}(z) = \pi - 2\cot^{-1}\left(\frac{\sqrt{z-1}}{\sqrt{-z-1}}\right) /; (z \in \mathbb{R} \land -1 < z < 1)$$

01.13.27.0706.01

$$\cos^{-1}(z) = 2\cot^{-1}\left(\frac{\sqrt{z-1}}{\sqrt{-z-1}}\right) - \pi /; (z \in \mathbb{R} \land z > 1)$$

01.13.27.0707.01

$$\cos^{-1}(z) = \pi\sqrt{1-z}\sqrt{\frac{1}{1-z}} - \frac{2\sqrt{z-1}\sqrt{z+1}}{\sqrt{-z-1}\sqrt{1-z}}\cot^{-1}\left(\frac{\sqrt{z-1}}{\sqrt{-z-1}}\right)$$

Involving $\cos^{-1}(z)$ and $\cot^{-1}\left(\sqrt{\frac{1-z}{1+z}}\right)$

01.13.27.0708.01

$$\cos^{-1}(z) = \pi - 2\cot^{-1}\left(\sqrt{\frac{1-z}{z+1}}\right) /; z \notin (-\infty, -1) \bigwedge z \notin (1, \infty)$$

01.13.27.0709.01

$$\cos^{-1}(z) = -\pi - 2\cot^{-1}\left(\sqrt{\frac{1-z}{z+1}}\right) /; (z \in \mathbb{R} \land z > 1)$$

01.13.27.0710.01

$$\cos^{-1}(z) == \pi + 2\cot^{-1}\left(\sqrt{\frac{1-z}{z+1}}\right) /; (z \in \mathbb{R} \land z < -1)$$

01.13.27.0711.01

$$\cos^{-1}(z) == \pi\sqrt{1-z}\sqrt{\frac{1}{1-z}} - 2\sqrt{\frac{1}{z+1}}\sqrt{z+1}\cot^{-1}\left(\sqrt{\frac{1-z}{z+1}}\right)$$

Involving $\cos^{-1}(z)$ and $\cot^{-1}\left(\frac{\sqrt{1+z}}{\sqrt{1-z}}\right)$

01.13.27.0712.01

$$\cos^{-1}(z) == 2\cot^{-1}\left(\frac{\sqrt{z+1}}{\sqrt{1-z}}\right) /; z \notin (-\infty, -1)$$

01.13.27.0713.01

$$\cos^{-1}(z) == 2\pi + 2\cot^{-1}\left(\frac{\sqrt{z+1}}{\sqrt{1-z}}\right) /; (z \in \mathbb{R} \land z < -1)$$

01.13.27.0714.01

$$\cos^{-1}(z) == 2\cot^{-1}\left(\frac{\sqrt{1+z}}{\sqrt{1-z}}\right) - \left(\sqrt{\frac{1}{z+1}}\sqrt{z+1} - 1\right)\pi$$

Involving $\cos^{-1}(z)$ and $\cot^{-1}\left(\frac{\sqrt{-z-1}}{\sqrt{z-1}}\right)$

01.13.27.0715.01

$$\cos^{-1}(z) == -2\cot^{-1}\left(\frac{\sqrt{-z-1}}{\sqrt{z-1}}\right) /; z \notin (-\infty, 1)$$

01.13.27.0716.01

$$\cos^{-1}(z) == 2\cot^{-1}\left(\frac{\sqrt{-z-1}}{\sqrt{z-1}}\right) /; (z \in \mathbb{R} \land -1 < z < 1)$$

01.13.27.0717.01

$$\cos^{-1}(z) == 2\pi - 2\cot^{-1}\left(\frac{\sqrt{-z-1}}{\sqrt{z-1}}\right) /; (z \in \mathbb{R} \land z < -1)$$

01.13.27.0718.01

$$\cos^{-1}(z) == \pi\left(1 - \sqrt{z+1}\sqrt{\frac{1}{z+1}}\right) + \frac{2\sqrt{z-1}\sqrt{z+1}}{\sqrt{-z-1}\sqrt{1-z}}\cot^{-1}\left(\frac{\sqrt{-z-1}}{\sqrt{z-1}}\right)$$

Involving $\cos^{-1}(z)$ and $\cot^{-1}\left(\sqrt{\frac{1+z}{1-z}}\right)$

01.13.27.0719.01

$$\cos^{-1}(z) = 2\cot^{-1}\left(\sqrt{\frac{z+1}{1-z}}\right) /; z \notin (-\infty, -1) \bigwedge z \notin (1, \infty)$$

01.13.27.0720.01

$$\cos^{-1}(z) = -2\cot^{-1}\left(\sqrt{\frac{1+z}{1-z}}\right) /; (z \in \mathbb{R} \land z > 1)$$

01.13.27.0721.01

$$\cos^{-1}(z) = 2\pi + 2\cot^{-1}\left(\sqrt{\frac{z+1}{1-z}}\right) /; (z \in \mathbb{R} \land z < -1)$$

01.13.27.0722.01

$$\cos^{-1}(z) = \pi\left(1 - \sqrt{z+1}\sqrt{\frac{1}{z+1}}\right) + 2\sqrt{\frac{1}{1-z}}\sqrt{1-z}\cot^{-1}\left(\sqrt{\frac{z+1}{1-z}}\right)$$

Involving $\cos^{-1}(z)$ and $\cot^{-1}\left(\frac{1+\sqrt{1-z^2}}{z}\right)$

01.13.27.0723.01

$$\cos^{-1}(z) = \frac{\pi}{2} - 2\cot^{-1}\left(\frac{\sqrt{1-z^2}+1}{z}\right)$$

Involving $\cos^{-1}(z)$ and $\cot^{-1}\left(\frac{1-\sqrt{1-z^2}}{z}\right)$

01.13.27.0724.01

$$\cos^{-1}(z) = -\frac{\pi}{2} + 2\cot^{-1}\left(\frac{1-\sqrt{1-z^2}}{z}\right) /; -\frac{\pi}{2} \leq \arg(z) < \frac{\pi}{2}$$

01.13.27.0725.01

$$\cos^{-1}(z) = \frac{3\pi}{2} + 2\cot^{-1}\left(\frac{1-\sqrt{1-z^2}}{z}\right) /; \frac{\pi}{2} \leq \arg(z) \leq \pi \bigvee -\pi < \arg(z) < -\frac{\pi}{2}$$

01.13.27.0726.01

$$\cos^{-1}(z) = -\pi\sqrt{\frac{1}{z^2}}\, z + 2\cot^{-1}\left(\frac{1-\sqrt{1-z^2}}{z}\right) + \frac{\pi}{2}$$

Involving $\cos^{-1}(z)$ and $\cot^{-1}\left(\frac{z}{1+\sqrt{1-z^2}}\right)$

01.13.27.0727.01

$$\cos^{-1}(z) = -\frac{\pi}{2} + 2\cot^{-1}\left(\frac{z}{1+\sqrt{1-z^2}}\right) /; -\frac{\pi}{2} \le \arg(z) < \frac{\pi}{2}$$

01.13.27.0728.01

$$\cos^{-1}(z) = \frac{3\pi}{2} + 2\cot^{-1}\left(\frac{z}{1+\sqrt{1-z^2}}\right) /; \frac{\pi}{2} \le \arg(z) \le \pi \bigvee -\pi < \arg(z) < -\frac{\pi}{2}$$

01.13.27.0729.01

$$\cos^{-1}(z) = -\pi\sqrt{\frac{1}{z^2}}\, z + 2\cot^{-1}\left(\frac{z}{\sqrt{1-z^2}+1}\right) + \frac{\pi}{2}$$

Involving $\cos^{-1}(z)$ and $\cot^{-1}\left(\frac{z}{1-\sqrt{1-z^2}}\right)$

01.13.27.0730.01

$$\cos^{-1}(z) = \frac{\pi}{2} - 2\cot^{-1}\left(\frac{z}{1-\sqrt{1-z^2}}\right)$$

Involving $\cos^{-1}\left(\frac{2z}{z^2+1}\right)$

Involving $\cos^{-1}\left(\frac{2z}{z^2+1}\right)$ and $\cot^{-1}(z)$

01.13.27.0731.01

$$\cos^{-1}\left(\frac{2z}{z^2+1}\right) = 2\cot^{-1}(z) - \frac{\pi}{2} /; |z| < 1 \bigwedge -\frac{\pi}{2} \le \arg(z) < \frac{\pi}{2}$$

01.13.27.0732.01

$$\cos^{-1}\left(\frac{2z}{z^2+1}\right) = \frac{3\pi}{2} + 2\cot^{-1}(z) /; |z| < 1 \bigwedge \left(\frac{\pi}{2} \le \arg(z) \le \pi \bigvee -\pi < \arg(z) < -\frac{\pi}{2}\right)$$

01.13.27.0733.01

$$\cos^{-1}\left(\frac{2z}{z^2+1}\right) = \frac{\pi}{2} - \pi z\sqrt{\frac{1}{z^2}} + 2\cot^{-1}(z) /; |z| < 1$$

01.13.27.0029.01

$$\cos^{-1}\left(\frac{2z}{z^2+1}\right) = \frac{\pi}{2} - 2\cot^{-1}(z) /; |z| > 1$$

01.13.27.0734.01

$$\cos^{-1}\left(\frac{2z}{z^2+1}\right) = \frac{\pi}{2} - \frac{\pi z}{2}\sqrt{\frac{1}{z^2}}\left(\frac{1-z}{1+z}\sqrt{\left(\frac{z+1}{z-1}\right)^2} + 1\right) + \frac{2(1-z)}{1+z}\sqrt{\left(\frac{z+1}{z-1}\right)^2}\cot^{-1}(z)$$

Involving $\cos^{-1}\left(\frac{2z}{z^2+1}\right)$ and $\cot^{-1}\left(\frac{1}{z}\right)$

01.13.27.0735.01

$$\cos^{-1}\left(\frac{2z}{z^2+1}\right) = \frac{\pi}{2} - 2\cot^{-1}\left(\frac{1}{z}\right) /; |z| < 1$$

01.13.27.0736.01

$$\cos^{-1}\left(\frac{2z}{z^2+1}\right) = 2\cot^{-1}\left(\frac{1}{z}\right) - \frac{\pi}{2} /; |z| > 1 \bigwedge -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

01.13.27.0737.01

$$\cos^{-1}\left(\frac{2z}{z^2+1}\right) = \frac{3\pi}{2} + 2\cot^{-1}\left(\frac{1}{z}\right) /; |z| > 1 \bigwedge \left(\frac{\pi}{2} < \arg(z) \leq \pi \bigvee -\pi < \arg(z) \leq -\frac{\pi}{2}\right)$$

01.13.27.0738.01

$$\cos^{-1}\left(\frac{2z}{z^2+1}\right) = \frac{\pi}{2} - \frac{\pi\sqrt{z^2}}{z} + 2\cot^{-1}\left(\frac{1}{z}\right) /; |z| > 1$$

01.13.27.0739.01

$$\cos^{-1}\left(\frac{2z}{z^2+1}\right) = \frac{\pi}{2} - \frac{\pi\sqrt{z^2}}{2z}\left(1 - \frac{1-z}{1+z}\sqrt{\left(\frac{z+1}{z-1}\right)^2}\right) - \frac{2\,(1-z)}{1+z}\sqrt{\left(\frac{z+1}{z-1}\right)^2}\cot^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\frac{2z}{z^2+1}\right)$ and $\cot^{-1}(z^r)$

01.13.27.0740.01

$$\cos^{-1}\left(\frac{2z}{z^2+1}\right) = \frac{\pi}{2} - 2\cot^{-1}\left(z^{\frac{z-1}{z+1}\sqrt{\left(\frac{z+1}{z-1}\right)^2}}\right)$$

Involving $\cos^{-1}\left(\frac{1-z}{1+z}\right)$

Involving $\cos^{-1}\left(\frac{1-z}{1+z}\right)$ and $\cot^{-1}\left(\sqrt{z}\right)$

01.13.27.0741.01

$$\cos^{-1}\left(\frac{1-z}{1+z}\right) = \pi - 2\cot^{-1}\left(\sqrt{z}\right) /; |\arg(z)| < \pi$$

01.13.27.0742.01

$$\cos^{-1}\left(\frac{1-z}{1+z}\right) = \pi + 2\cot^{-1}\left(\sqrt{z}\right) /; (z \in \mathbb{R} \land z < -1)$$

01.13.27.0743.01

$$\cos^{-1}\left(\frac{1-z}{1+z}\right) = -\pi - 2\cot^{-1}\left(\sqrt{z}\right) /;\ (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.0744.01

$$\cos^{-1}\left(\frac{1-z}{1+z}\right) = \pi\left(\sqrt{\frac{1}{z}}\,\sqrt{z} - \sqrt{\frac{1}{z+1}}\,\sqrt{z+1} + 1\right) - 2\sqrt{z+1}\,\sqrt{\frac{1}{z+1}}\,\cot^{-1}\left(\sqrt{z}\right)$$

Involving $\cos^{-1}\left(\frac{1-z}{1+z}\right)$ and $\cot^{-1}\left(\frac{1}{\sqrt{z}}\right)$

01.13.27.0745.01

$$\cos^{-1}\left(\frac{1-z}{1+z}\right) = 2\cot^{-1}\left(\frac{1}{\sqrt{z}}\right) /;\ z \notin (-\infty, -1)$$

01.13.27.0746.01

$$\cos^{-1}\left(\frac{1-z}{z+1}\right) = 2\pi - 2\cot^{-1}\left(\frac{1}{\sqrt{z}}\right) /;\ (z \in \mathbb{R} \land z < -1)$$

01.13.27.0747.01

$$\cos^{-1}\left(\frac{1-z}{1+z}\right) = \pi\left(1 - \sqrt{z+1}\,\sqrt{\frac{1}{z+1}}\right) + 2\sqrt{\frac{1}{z+1}}\,\sqrt{z+1}\,\cot^{-1}\left(\frac{1}{\sqrt{z}}\right)$$

Involving $\cos^{-1}\left(\frac{z-1}{z+1}\right)$

Involving $\cos^{-1}\left(\frac{z-1}{z+1}\right)$ and $\cot^{-1}\left(\frac{1}{\sqrt{z}}\right)$

01.13.27.0748.01

$$\cos^{-1}\left(\frac{z-1}{z+1}\right) = \pi - 2\cot^{-1}\left(\frac{1}{\sqrt{z}}\right) /;\ z \notin (-\infty, -1)$$

01.13.27.0749.01

$$\cos^{-1}\left(\frac{z-1}{z+1}\right) = 2\cot^{-1}\left(\frac{1}{\sqrt{z}}\right) - \pi /;\ (z \in \mathbb{R} \land z < -1)$$

01.13.27.0750.01

$$\cos^{-1}\left(\frac{z-1}{z+1}\right) = \sqrt{z+1}\,\sqrt{\frac{1}{z+1}}\,\pi - 2\sqrt{z+1}\,\sqrt{\frac{1}{z+1}}\,\cot^{-1}\left(\frac{1}{\sqrt{z}}\right)$$

Involving $\cos^{-1}\left(\frac{z-1}{z+1}\right)$ and $\cot^{-1}\left(\sqrt{z}\right)$

01.13.27.0751.01

$$\cos^{-1}\left(\frac{z-1}{z+1}\right) = 2\cot^{-1}\left(\sqrt{z}\right) /;\ |\arg(z)| < \pi$$

01.13.27.0752.01

$$\cos^{-1}\left(\frac{z-1}{z+1}\right) = -2\cot^{-1}\left(\sqrt{z}\right) /;\ (z \in \mathbb{R} \land z < -1)$$

01.13.27.0753.01

$$\cos^{-1}\left(\frac{z-1}{z+1}\right) = \pi\left(-\sqrt{\frac{1}{z}}\,\sqrt{z} + \sqrt{\frac{1}{1+z}}\,\sqrt{1+z}\right) + 2\sqrt{\frac{1}{z+1}}\,\sqrt{z+1}\,\cot^{-1}\left(\sqrt{z}\right)$$

Involving $\cos^{-1}\left(\frac{z-1}{z+1}\right)$ and $\cot^{-1}\left(\sqrt{\frac{1}{z}}\right)$

01.13.27.0754.01

$$\cos^{-1}\left(\frac{z-1}{z+1}\right) = \pi - 2\cot^{-1}\left(\sqrt{\frac{1}{z}}\right) /;\ |\arg(z)| < \pi$$

01.13.27.0755.01

$$\cos^{-1}\left(\frac{z-1}{z+1}\right) = \pi + 2\cot^{-1}\left(\sqrt{\frac{1}{z}}\right) /;\ (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.0756.01

$$\cos^{-1}\left(\frac{z-1}{z+1}\right) = -2\cot^{-1}\left(\sqrt{\frac{1}{z}}\right) - \pi /;\ (z \in \mathbb{R} \land z < -1)$$

01.13.27.0757.01

$$\cos^{-1}\left(\frac{z-1}{z+1}\right) = \sqrt{z+1}\,\sqrt{\frac{1}{z+1}}\,\pi - 2\sqrt{\frac{z+1}{z}}\,\sqrt{\frac{z}{z+1}}\,\cot^{-1}\left(\sqrt{\frac{1}{z}}\right)$$

Involving $\cos^{-1}\left(\frac{1+z}{1-z}\right)$

Involving $\cos^{-1}\left(\frac{1+z}{1-z}\right)$ and $\cot^{-1}\left(\sqrt{-z}\right)$

01.13.27.0758.01

$$\cos^{-1}\left(\frac{1+z}{1-z}\right) = \pi - 2\cot^{-1}\left(\sqrt{-z}\right) /;\ z \notin (0, \infty)$$

01.13.27.0759.01

$$\cos^{-1}\left(\frac{1+z}{1-z}\right) = \pi + 2\cot^{-1}\left(\sqrt{-z}\right) /;\ (z \in \mathbb{R} \land z > 1)$$

01.13.27.0760.01

$$\cos^{-1}\left(\frac{1+z}{1-z}\right) = \pi\left(1 - \sqrt{\frac{1}{1-z}}\,\sqrt{1-z} + \sqrt{-\frac{1}{z}}\,\sqrt{-z}\right) - 2\sqrt{\frac{1}{1-z}}\,\sqrt{1-z}\,\cot^{-1}\left(\sqrt{-z}\right)$$

Involving $\cos^{-1}\left(\frac{1+z}{1-z}\right)$ and $\cot^{-1}\left(\frac{1}{\sqrt{-z}}\right)$

01.13.27.0761.01

$$\cos^{-1}\left(\frac{1+z}{1-z}\right) = 2\cot^{-1}\left(\frac{1}{\sqrt{-z}}\right) /;\ z \notin (1, \infty)$$

01.13.27.0762.01

$$\cos^{-1}\left(\frac{1+z}{1-z}\right) = 2\pi - 2\cot^{-1}\left(\frac{1}{\sqrt{-z}}\right) /; (z \in \mathbb{R} \land z > 1)$$

01.13.27.0763.01

$$\cos^{-1}\left(\frac{1+z}{1-z}\right) = \left(1 - \sqrt{1-z}\sqrt{\frac{1}{1-z}}\right)\pi + 2\sqrt{1-z}\sqrt{\frac{1}{1-z}}\cot^{-1}\left(\frac{1}{\sqrt{-z}}\right)$$

Involving $\cos^{-1}\left(\frac{1+z}{1-z}\right)$ and $\cot^{-1}\left(\sqrt{-\frac{1}{z}}\right)$

01.13.27.0764.01

$$\cos^{-1}\left(\frac{1+z}{1-z}\right) = 2\cot^{-1}\left(\sqrt{-\frac{1}{z}}\right) /; z \notin (0, \infty)$$

01.13.27.0765.01

$$\cos^{-1}\left(\frac{1+z}{1-z}\right) = -2\cot^{-1}\left(\sqrt{-\frac{1}{z}}\right) /; (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.0766.01

$$\cos^{-1}\left(\frac{1+z}{1-z}\right) = 2\pi + 2\cot^{-1}\left(\sqrt{-\frac{1}{z}}\right) /; (z \in \mathbb{R} \land z > 1)$$

01.13.27.0767.01

$$\cos^{-1}\left(\frac{1+z}{1-z}\right) = \left(1 - \sqrt{1-z}\sqrt{\frac{1}{1-z}}\right)\pi + 2\sqrt{1-z}\sqrt{\frac{1}{1-z}}\sqrt{-z}\sqrt{-\frac{1}{z}}\cot^{-1}\left(\sqrt{-\frac{1}{z}}\right)$$

Involving $\cos^{-1}\left(\frac{z+1}{z-1}\right)$

Involving $\cos^{-1}\left(\frac{z+1}{z-1}\right)$ and $\cot^{-1}\left(\sqrt{-z}\right)$

01.13.27.0768.01

$$\cos^{-1}\left(\frac{z+1}{z-1}\right) = 2\cot^{-1}\left(\sqrt{-z}\right) /; z \notin (0, \infty)$$

01.13.27.0769.01

$$\cos^{-1}\left(\frac{z+1}{z-1}\right) = -2\cot^{-1}\left(\sqrt{-z}\right) /; (z \in \mathbb{R} \land z > 1)$$

01.13.27.0770.01

$$\cos^{-1}\left(\frac{z+1}{z-1}\right) = \pi\left(\sqrt{\frac{1}{1-z}}\sqrt{1-z} - \sqrt{-\frac{1}{z}}\sqrt{-z}\right) + 2\sqrt{\frac{1}{1-z}}\sqrt{1-z}\cot^{-1}\left(\sqrt{-z}\right)$$

Involving $\cos^{-1}\left(\frac{z+1}{z-1}\right)$ and $\cot^{-1}\left(\frac{1}{\sqrt{-z}}\right)$

01.13.27.0771.01

$$\cos^{-1}\left(\frac{z+1}{z-1}\right) = \pi - 2\cot^{-1}\left(\frac{1}{\sqrt{-z}}\right) /; z \notin (1, \infty)$$

01.13.27.0772.01

$$\cos^{-1}\left(\frac{z+1}{z-1}\right) = 2\cot^{-1}\left(\frac{1}{\sqrt{-z}}\right) - \pi /; (z \in \mathbb{R} \land z > 1)$$

01.13.27.0773.01

$$\cos^{-1}\left(\frac{z+1}{z-1}\right) = \sqrt{1-z}\sqrt{\frac{1}{1-z}}\,\pi - 2\sqrt{1-z}\sqrt{\frac{1}{1-z}}\cot^{-1}\left(\frac{1}{\sqrt{-z}}\right)$$

Involving $\cos^{-1}\left(\frac{z+1}{z-1}\right)$ and $\cot^{-1}\left(\sqrt{-\frac{1}{z}}\right)$

01.13.27.0774.01

$$\cos^{-1}\left(\frac{z+1}{z-1}\right) = \pi - 2\cot^{-1}\left(\sqrt{-\frac{1}{z}}\right) /; z \notin (0, \infty)$$

01.13.27.0775.01

$$\cos^{-1}\left(\frac{z+1}{z-1}\right) = \pi + 2\cot^{-1}\left(\sqrt{-\frac{1}{z}}\right) /; (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.0776.01

$$\cos^{-1}\left(\frac{z+1}{z-1}\right) = -2\cot^{-1}\left(\sqrt{-\frac{1}{z}}\right) - \pi /; (z \in \mathbb{R} \land z > 1)$$

01.13.27.0777.01

$$\cos^{-1}\left(\frac{z+1}{z-1}\right) = \sqrt{1-z}\sqrt{\frac{1}{1-z}}\,\pi - 2\sqrt{1-z}\sqrt{\frac{1}{1-z}}\sqrt{-z}\sqrt{-\frac{1}{z}}\cot^{-1}\left(\sqrt{-\frac{1}{z}}\right)$$

Involving $\cos^{-1}\left(\frac{1-z^2}{1+z^2}\right)$

Involving $\cos^{-1}\left(\frac{1-z^2}{1+z^2}\right)$ and $\cot^{-1}(z)$

01.13.27.0778.01

$$\cos^{-1}\left(\frac{1-z^2}{1+z^2}\right) = \pi - 2\cot^{-1}(z) /; \operatorname{Re}(z) > 0 \lor (i z \in \mathbb{R} \land i z > 1)$$

01.13.27.0779.01

$$\cos^{-1}\left(\frac{1-z^2}{z^2+1}\right) = \pi + 2\cot^{-1}(z) /; \operatorname{Re}(z) < 0 \lor (i z \in \mathbb{R} \land i z < -1)$$

01.13.27.0780.01

$$\cos^{-1}\left(\frac{1-z^2}{z^2+1}\right) = -\pi - 2\cot^{-1}(z) /; (-i z \in \mathbb{R} \land 0 < -i z < 1)$$

01.13.27.0781.01

$$\cos^{-1}\left(\frac{1-z^2}{z^2+1}\right) = -\pi + 2\cot^{-1}(z) /; (i z \in \mathbb{R} \wedge 0 < i z < 1)$$

01.13.27.0782.01

$$\cos^{-1}\left(\frac{1-z^2}{1+z^2}\right) = \pi\left(\sqrt{-\frac{i}{z}}\sqrt{i z}\sqrt{\frac{1}{i z+1}}\sqrt{i z+1} + \sqrt{\frac{i}{z}}\sqrt{-i z}\sqrt{\frac{1}{1-i z}}\sqrt{1-i z} - 1\right) - \frac{2\sqrt{z^2}}{z}\sqrt{\frac{i-z}{i+z}}\sqrt{\frac{i+z}{i-z}}\cot^{-1}(z)$$

Involving $\cos^{-1}\left(\frac{1-z^2}{1+z^2}\right)$ and $\cot^{-1}\left(\frac{1}{z}\right)$

01.13.27.0783.01

$$\cos^{-1}\left(\frac{1-z^2}{1+z^2}\right) = 2\cot^{-1}\left(\frac{1}{z}\right) /; \operatorname{Re}(z) > 0 \vee (-i z \in \mathbb{R} \wedge 0 < -i z < 1)$$

01.13.27.0784.01

$$\cos^{-1}\left(\frac{1-z^2}{1+z^2}\right) = -2\cot^{-1}\left(\frac{1}{z}\right) /; \operatorname{Re}(z) < 0 \vee (i z \in \mathbb{R} \wedge 0 < i z < 1)$$

01.13.27.0785.01

$$\cos^{-1}\left(\frac{1-z^2}{z^2+1}\right) = 2\pi + 2\cot^{-1}\left(\frac{1}{z}\right) /; (i z \in \mathbb{R} \wedge i z > 1)$$

01.13.27.0786.01

$$\cos^{-1}\left(\frac{1-z^2}{z^2+1}\right) = 2\pi - 2\cot^{-1}\left(\frac{1}{z}\right) /; (i z \in \mathbb{R} \wedge i z < -1)$$

01.13.27.2518.01

$$\cos^{-1}\left(\frac{1-z^2}{1+z^2}\right) = \frac{2\sqrt{z^2}}{z}\cot^{-1}\left(\frac{1}{z}\right) /; i z \notin (-\infty, -1) \bigwedge i z \notin (1, \infty)$$

01.13.27.0787.01

$$\cos^{-1}\left(\frac{1-z^2}{1+z^2}\right) = \pi\left(2 - \sqrt{\frac{1}{i z+1}}\sqrt{i z+1} - \sqrt{\frac{1}{1-i z}}\sqrt{1-i z}\right) + \frac{2\sqrt{z^2}}{z}\sqrt{\frac{i+z}{i-z}}\sqrt{\frac{i-z}{i+z}}\cot^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\frac{z^2-1}{z^2+1}\right)$

Involving $\cos^{-1}\left(\frac{z^2-1}{z^2+1}\right)$ and $\cot^{-1}(z)$

01.13.27.0788.01

$$\cos^{-1}\left(\frac{z^2-1}{z^2+1}\right) = 2\cot^{-1}(z) /; \operatorname{Re}(z) > 0 \vee (i z \in \mathbb{R} \wedge i z > 1)$$

01.13.27.0789.01

$$\cos^{-1}\left(\frac{z^2-1}{z^2+1}\right) = -2\cot^{-1}(z) \;/;\; \mathrm{Re}(z) < 0 \lor (i\,z \in \mathbb{R} \land i\,z < -1)$$

01.13.27.0790.01

$$\cos^{-1}\left(\frac{z^2-1}{z^2+1}\right) = 2\,\pi + 2\cot^{-1}(z) \;/;\; (-i\,z \in \mathbb{R} \land 0 < -i\,z < 1)$$

01.13.27.0791.01

$$\cos^{-1}\left(\frac{z^2-1}{z^2+1}\right) = 2\,\pi - 2\cot^{-1}(z) \;/;\; (i\,z \in \mathbb{R} \land 0 < i\,z < 1)$$

01.13.27.0792.01

$$\cos^{-1}\left(\frac{z^2-1}{z^2+1}\right) = \frac{2\sqrt{z^2}}{z}\sqrt{\frac{i-z}{i+z}}\sqrt{\frac{i+z}{i-z}}\cot^{-1}(z) + \pi\left(2 - \sqrt{\frac{1}{1-i\,z}}\sqrt{1-i\,z}\sqrt{\frac{i}{z}}\sqrt{-i\,z} - \sqrt{\frac{1}{1+i\,z}}\sqrt{1+i\,z}\sqrt{-\frac{i}{z}}\sqrt{i\,z}\right)$$

Involving $\cos^{-1}\left(\frac{z^2-1}{z^2+1}\right)$ and $\cot^{-1}\left(\frac{1}{z}\right)$

01.13.27.0793.01

$$\cos^{-1}\left(\frac{z^2-1}{z^2+1}\right) = \pi - 2\cot^{-1}\left(\frac{1}{z}\right) \;/;\; \mathrm{Re}(z) > 0 \lor (-i\,z \in \mathbb{R} \land 0 < -i\,z < 1)$$

01.13.27.0794.01

$$\cos^{-1}\left(\frac{z^2-1}{z^2+1}\right) = \pi + 2\cot^{-1}\left(\frac{1}{z}\right) \;/;\; \mathrm{Re}(z) < 0 \lor (i\,z \in \mathbb{R} \land 0 < i\,z < 1)$$

01.13.27.0795.01

$$\cos^{-1}\left(\frac{z^2-1}{z^2+1}\right) = -2\cot^{-1}\left(\frac{1}{z}\right) - \pi \;/;\; (i\,z \in \mathbb{R} \land i\,z > 1)$$

01.13.27.0796.01

$$\cos^{-1}\left(\frac{z^2-1}{z^2+1}\right) = 2\cot^{-1}\left(\frac{1}{z}\right) - \pi \;/;\; (i\,z \in \mathbb{R} \land i\,z < -1)$$

01.13.27.0797.01

$$\cos^{-1}\left(\frac{z^2-1}{z^2+1}\right) = \pi - \frac{2\sqrt{z^2}}{z}\cot^{-1}\left(\frac{1}{z}\right) \;/;\; i\,z \notin (-\infty, -1) \bigwedge i\,z \notin (1, \infty)$$

01.13.27.0798.01

$$\cos^{-1}\left(\frac{z^2-1}{z^2+1}\right) = \pi\left(\sqrt{\frac{1}{i\,z+1}}\sqrt{i\,z+1} + \sqrt{\frac{1}{1-i\,z}}\sqrt{1-i\,z} - 1\right) - \frac{2\sqrt{z^2}}{z}\sqrt{\frac{i-z}{i+z}}\sqrt{\frac{i+z}{i-z}}\cot^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\frac{1}{\sqrt{z+1}}\right)$

Involving $\cos^{-1}\left(\frac{1}{\sqrt{z+1}}\right)$ and $\cot^{-1}\left(\sqrt{z}\right)$

01.13.27.0799.01

$$\cos^{-1}\left(\frac{1}{\sqrt{z+1}}\right) = \frac{\pi}{2} - \cot^{-1}\left(\sqrt{z}\right) /; z \notin (-1, 0)$$

01.13.27.0800.01

$$\cos^{-1}\left(\frac{1}{\sqrt{z+1}}\right) = -\frac{\pi}{2} - \cot^{-1}\left(\sqrt{z}\right) /; (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.0801.01

$$\cos^{-1}\left(\frac{1}{\sqrt{z+1}}\right) = \frac{1}{2} \sqrt{z} \sqrt{\frac{1}{z}} \sqrt{z+1} \sqrt{\frac{1}{z+1}} \pi - \cot^{-1}\left(\sqrt{z}\right)$$

Involving $\cos^{-1}\left(\frac{1}{\sqrt{z+1}}\right)$ and $\cot^{-1}\left(\frac{1}{\sqrt{z}}\right)$

01.13.27.0802.01

$$\cos^{-1}\left(\frac{1}{\sqrt{z+1}}\right) = \cot^{-1}\left(\frac{1}{\sqrt{z}}\right)$$

Involving $\cos^{-1}\left(\frac{1}{\sqrt{z+1}}\right)$ and $\cot^{-1}\left(\sqrt{\frac{1}{z}}\right)$

01.13.27.0803.01

$$\cos^{-1}\left(\frac{1}{\sqrt{z+1}}\right) = \cot^{-1}\left(\sqrt{\frac{1}{z}}\right) /; |\arg(z)| < \pi$$

01.13.27.0804.01

$$\cos^{-1}\left(\frac{1}{\sqrt{z+1}}\right) = -\cot^{-1}\left(\sqrt{\frac{1}{z}}\right) /; (z \in \mathbb{R} \land z < 0)$$

01.13.27.0805.01

$$\cos^{-1}\left(\frac{1}{\sqrt{z+1}}\right) = \sqrt{z} \sqrt{\frac{1}{z}} \cot^{-1}\left(\sqrt{\frac{1}{z}}\right)$$

Involving $\cos^{-1}\left(\sqrt{\frac{1}{z+1}}\right)$

Involving $\cos^{-1}\left(\sqrt{\frac{1}{z+1}}\right)$ and $\cot^{-1}\left(\sqrt{z}\right)$

01.13.27.0806.01

$$\cos^{-1}\left(\sqrt{\frac{1}{z+1}}\right) = \frac{\pi}{2} - \cot^{-1}\left(\sqrt{z}\right) /; |\arg(z)| < \pi$$

01.13.27.0807.01

$$\cos^{-1}\left(\sqrt{\frac{1}{z+1}}\right) = \frac{\pi}{2} + \cot^{-1}\left(\sqrt{z}\right) /; (z \in \mathbb{R} \land z < -1)$$

01.13.27.0808.01

$$\cos^{-1}\left(\sqrt{\frac{1}{z+1}}\right) = -\cot^{-1}\left(\sqrt{z}\right) - \frac{\pi}{2} /; (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.0809.01

$$\cos^{-1}\left(\sqrt{\frac{1}{z+1}}\right) = \frac{\pi}{2} - \sqrt{\frac{1}{z+1}}\sqrt{z+1}\left(\cot^{-1}\left(\sqrt{z}\right) - \frac{\pi}{2}\left(\sqrt{z}\sqrt{\frac{1}{z}}\sqrt{z+1}\sqrt{\frac{1}{z+1}} - 1\right)\right)$$

Involving $\cos^{-1}\left(\sqrt{\frac{1}{z+1}}\right)$ and $\cot^{-1}\left(\frac{1}{\sqrt{z}}\right)$

01.13.27.0810.01

$$\cos^{-1}\left(\sqrt{\frac{1}{z+1}}\right) = \cot^{-1}\left(\frac{1}{\sqrt{z}}\right) /; z \notin (-\infty, -1)$$

01.13.27.0811.01

$$\cos^{-1}\left(\sqrt{\frac{1}{z+1}}\right) = \pi - \cot^{-1}\left(\frac{1}{\sqrt{z}}\right) /; (z \in \mathbb{R} \land z < -1)$$

01.13.27.0812.01

$$\cos^{-1}\left(\sqrt{\frac{1}{z+1}}\right) = \frac{\pi}{2} - \sqrt{\frac{1}{z+1}}\sqrt{z+1}\left(\frac{\pi}{2} - \cot^{-1}\left(\frac{1}{\sqrt{z}}\right)\right)$$

Involving $\cos^{-1}\left(\sqrt{\frac{1}{z+1}}\right)$ and $\cot^{-1}\left(\sqrt{\frac{1}{z}}\right)$

01.13.27.0813.01

$$\cos^{-1}\left(\sqrt{\frac{1}{z+1}}\right) = \cot^{-1}\left(\sqrt{\frac{1}{z}}\right) /; |\arg(z)| < \pi$$

01.13.27.0814.01

$$\cos^{-1}\left(\sqrt{\frac{1}{z+1}}\right) = -\cot^{-1}\left(\sqrt{\frac{1}{z}}\right) /; (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.0815.01

$$\cos^{-1}\left(\sqrt{\frac{1}{z+1}}\right) = \pi + \cot^{-1}\left(\sqrt{\frac{1}{z}}\right) /; (z \in \mathbb{R} \land z < -1)$$

01.13.27.0816.01

$$\cos^{-1}\left(\sqrt{\frac{1}{z+1}}\right) == \frac{\pi}{2} - \sqrt{\frac{1}{z+1}}\,\sqrt{z+1}\left(\frac{\pi}{2} - \sqrt{z}\,\sqrt{\frac{1}{z}}\,\cot^{-1}\left(\sqrt{\frac{1}{z}}\right)\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{z}}{\sqrt{a+z}}\right)$

Involving $\cos^{-1}\left(\frac{\sqrt{z}}{\sqrt{z+1}}\right)$ and $\cot^{-1}\left(\sqrt{z}\right)$

01.13.27.0817.01

$$\cos^{-1}\left(\frac{\sqrt{z}}{\sqrt{z+1}}\right) == \cot^{-1}\left(\sqrt{z}\right) /;\ |\arg(z)| < \pi$$

01.13.27.0818.01

$$\cos^{-1}\left(\frac{\sqrt{z}}{\sqrt{z+1}}\right) == \pi + \cot^{-1}\left(\sqrt{z}\right) /;\ (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.0819.01

$$\cos^{-1}\left(\frac{\sqrt{z}}{\sqrt{z+1}}\right) == -\cot^{-1}\left(\sqrt{z}\right) /;\ (z \in \mathbb{R} \land z < -1)$$

01.13.27.0028.02

$$\cos^{-1}\left(\frac{\sqrt{z}}{\sqrt{z+1}}\right) == \frac{1}{2}\pi\left(1 - \sqrt{z}\,\sqrt{\frac{1}{z}}\,\sqrt{z+1}\,\sqrt{\frac{1}{z+1}}\right) + \sqrt{\frac{1}{z+1}}\,\sqrt{z+1}\,\cot^{-1}\left(\sqrt{z}\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{z}}{\sqrt{a+z}}\right)$ and $\cot^{-1}\left(\frac{1}{\sqrt{z}}\right)$

01.13.27.0820.01

$$\cos^{-1}\left(\frac{\sqrt{z}}{\sqrt{z+1}}\right) == \frac{\pi}{2} - \cot^{-1}\left(\frac{1}{\sqrt{z}}\right) /;\ z \notin (-\infty, -1)$$

01.13.27.0821.01

$$\cos^{-1}\left(\frac{\sqrt{z}}{\sqrt{z+1}}\right) == -\frac{\pi}{2} + \cot^{-1}\left(\frac{1}{\sqrt{z}}\right) /;\ (z \in \mathbb{R} \land z < -1)$$

01.13.27.0822.01

$$\cos^{-1}\left(\frac{\sqrt{z}}{\sqrt{z+1}}\right) == \frac{1}{2}\sqrt{\frac{1}{z+1}}\,\sqrt{z+1}\left(\pi - 2\cot^{-1}\left(\frac{1}{\sqrt{z}}\right)\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{z}}{\sqrt{a+z}}\right)$ and $\cot^{-1}\left(\sqrt{\frac{1}{z}}\right)$

01.13.27.0823.01

$$\cos^{-1}\left(\frac{\sqrt{z}}{\sqrt{z+1}}\right) = \frac{\pi}{2} - \cot^{-1}\left(\sqrt{\frac{1}{z}}\right) /; |\arg(z)| < \pi$$

01.13.27.0824.01

$$\cos^{-1}\left(\frac{\sqrt{z}}{\sqrt{z+1}}\right) = \frac{\pi}{2} + \cot^{-1}\left(\sqrt{\frac{1}{z}}\right) /; (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.0825.01

$$\cos^{-1}\left(\frac{\sqrt{z}}{\sqrt{z+1}}\right) = -\frac{\pi}{2} - \cot^{-1}\left(\sqrt{\frac{1}{z}}\right) /; (z \in \mathbb{R} \land z < -1)$$

01.13.27.0826.01

$$\cos^{-1}\left(\frac{\sqrt{z}}{\sqrt{z+1}}\right) = \frac{1}{2} \sqrt{\frac{1}{z+1}} \sqrt{z+1} \left(\pi - 2\sqrt{z} \sqrt{\frac{1}{z}} \cot^{-1}\left(\sqrt{\frac{1}{z}}\right)\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{-z}}{\sqrt{-1-z}}\right)$

Involving $\cos^{-1}\left(\frac{\sqrt{-z}}{\sqrt{-z-1}}\right)$ and $\cot^{-1}\left(\sqrt{z}\right)$

01.13.27.0827.01

$$\cos^{-1}\left(\frac{\sqrt{-z}}{\sqrt{-z-1}}\right) = \cot^{-1}\left(\sqrt{z}\right) /; |\arg(z)| < \pi$$

01.13.27.0828.01

$$\cos^{-1}\left(\frac{\sqrt{-z}}{\sqrt{-z-1}}\right) = -\cot^{-1}\left(\sqrt{z}\right) /; (z \in \mathbb{R} \land z < 0)$$

01.13.27.0829.01

$$\cos^{-1}\left(\frac{\sqrt{-z}}{\sqrt{-z-1}}\right) = \sqrt{\frac{z+1}{z}} \sqrt{\frac{z}{(z+1)^2}} \sqrt{z+1} \cot^{-1}\left(\sqrt{z}\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{-z}}{\sqrt{-z-1}}\right)$ and $\cot^{-1}\left(\frac{1}{\sqrt{z}}\right)$

01.13.27.0830.01

$$\cos^{-1}\left(\frac{\sqrt{-z}}{\sqrt{-z-1}}\right) = \frac{\pi}{2} - \cot^{-1}\left(\frac{1}{\sqrt{z}}\right) /; |\arg(z)| < \pi$$

01.13.27.0831.01

$$\cos^{-1}\left(\frac{\sqrt{-z}}{\sqrt{-z-1}}\right) = \frac{\pi}{2} + \cot^{-1}\left(\frac{1}{\sqrt{z}}\right) /; (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.0832.01

$$\cos^{-1}\left(\frac{\sqrt{-z}}{\sqrt{-z-1}}\right) = -\frac{\pi}{2} + \cot^{-1}\left(\frac{1}{\sqrt{z}}\right) /; (z \in \mathbb{R} \land z < -1)$$

01.13.27.0833.01

$$\cos^{-1}\left(\frac{\sqrt{-z}}{\sqrt{-z-1}}\right) = \frac{1}{2}\left(\sqrt{\frac{1}{z}}\sqrt{z}\left(\pi - 2\cot^{-1}\left(\frac{1}{\sqrt{z}}\right)\right) - \pi\sqrt{1+\frac{1}{z}}\sqrt{\frac{z}{z+1}} + \pi\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{-z}}{\sqrt{-z-1}}\right)$ and $\cot^{-1}\left(\sqrt{\frac{1}{z}}\right)$

01.13.27.0834.01

$$\cos^{-1}\left(\frac{\sqrt{-z}}{\sqrt{-z-1}}\right) = \frac{\pi}{2} - \cot^{-1}\left(\sqrt{\frac{1}{z}}\right) /; z \notin (-\infty, -1)$$

01.13.27.0835.01

$$\cos^{-1}\left(\frac{\sqrt{-z}}{\sqrt{-z-1}}\right) = -\frac{\pi}{2} - \cot^{-1}\left(\sqrt{\frac{1}{z}}\right) /; (z \in \mathbb{R} \land z < -1)$$

01.13.27.0836.01

$$\cos^{-1}\left(\frac{\sqrt{-z}}{\sqrt{-z-1}}\right) = \frac{1}{2}\sqrt{z+1}\sqrt{\frac{1}{z+1}}\,\pi - \cot^{-1}\left(\sqrt{\frac{1}{z}}\right)$$

Involving $\cos^{-1}\left(\sqrt{\frac{z}{z+1}}\right)$

Involving $\cos^{-1}\left(\sqrt{\frac{z}{z+1}}\right)$ and $\cot^{-1}\left(\sqrt{z}\right)$

01.13.27.0837.01

$$\cos^{-1}\left(\sqrt{\frac{z}{z+1}}\right) = \cot^{-1}\left(\sqrt{z}\right) /; |\arg(z)| < \pi$$

01.13.27.0838.01

$$\cos^{-1}\left(\sqrt{\frac{z}{z+1}}\right) = -\cot^{-1}\left(\sqrt{z}\right) /; (z \in \mathbb{R} \land z < -1)$$

01.13.27.0839.01

$$\cos^{-1}\left(\sqrt{\frac{z}{z+1}}\right) = \pi + \cot^{-1}\left(\sqrt{z}\right) /; (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.0840.01

$$\cos^{-1}\left(\sqrt{\frac{z}{z+1}}\right) = \frac{1}{2}\pi\left(1 - \sqrt{1+\frac{1}{z}}\sqrt{\frac{z}{z+1}}\right) + \sqrt{\frac{1}{z+1}}\sqrt{z+1}\cot^{-1}\left(\sqrt{z}\right)$$

Involving $\cos^{-1}\left(\sqrt{\frac{z}{z+1}}\right)$ and $\cot^{-1}\left(\frac{1}{\sqrt{z}}\right)$

01.13.27.0841.01

$$\cos^{-1}\left(\sqrt{\frac{z}{z+1}}\right) == \frac{\pi}{2} - \cot^{-1}\left(\frac{1}{\sqrt{z}}\right) /;\ z \notin (-\infty, -1)$$

01.13.27.0842.01

$$\cos^{-1}\left(\sqrt{\frac{z}{z+1}}\right) == -\frac{\pi}{2} + \cot^{-1}\left(\frac{1}{\sqrt{z}}\right) /;\ (z \in \mathbb{R} \land z < -1)$$

01.13.27.0843.01

$$\cos^{-1}\left(\sqrt{\frac{z}{z+1}}\right) == \frac{1}{2} \sqrt{\frac{1}{z+1}} \sqrt{z+1} \left(\pi - 2\cot^{-1}\left(\frac{1}{\sqrt{z}}\right)\right)$$

Involving $\cos^{-1}\left(\sqrt{\frac{z}{z+1}}\right)$ and $\cot^{-1}\left(\sqrt{\frac{1}{z}}\right)$

01.13.27.0844.01

$$\cos^{-1}\left(\sqrt{\frac{z}{z+1}}\right) == \frac{\pi}{2} - \cot^{-1}\left(\sqrt{\frac{1}{z}}\right) /;\ |\arg(z)| < \pi$$

01.13.27.0845.01

$$\cos^{-1}\left(\sqrt{\frac{z}{z+1}}\right) == \frac{\pi}{2} + \cot^{-1}\left(\sqrt{\frac{1}{z}}\right) /;\ (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.0846.01

$$\cos^{-1}\left(\sqrt{\frac{z}{z+1}}\right) == -\frac{\pi}{2} - \cot^{-1}\left(\sqrt{\frac{1}{z}}\right) /;\ (z \in \mathbb{R} \land z < -1)$$

01.13.27.0847.01

$$\cos^{-1}\left(\sqrt{\frac{z}{z+1}}\right) == \frac{1}{2} \sqrt{\frac{1}{z+1}} \sqrt{z+1} \left(\pi - 2\sqrt{z} \sqrt{\frac{1}{z}} \cot^{-1}\left(\sqrt{\frac{1}{z}}\right)\right)$$

Involving $\cos^{-1}\left(\frac{1}{\sqrt{z^2+1}}\right)$

Involving $\cos^{-1}\left(\frac{1}{\sqrt{z^2+1}}\right)$ and $\cot^{-1}(z)$

01.13.27.0848.01

$$\cos^{-1}\left(\frac{1}{\sqrt{z^2+1}}\right) == \frac{\pi}{2} - \cot^{-1}(z) /;\ \mathrm{Re}(z) > 0 \lor (i\,z \in \mathbb{R} \land i\,z < -1)$$

01.13.27.0849.01

$$\cos^{-1}\left(\frac{1}{\sqrt{z^2+1}}\right) == \frac{\pi}{2} + \cot^{-1}(z) /;\ \mathrm{Re}(z) < 0 \lor (i\,z \in \mathbb{R} \land i\,z > 1)$$

01.13.27.0850.01

$$\cos^{-1}\left(\frac{1}{\sqrt{z^2+1}}\right) = -\frac{\pi}{2} + \cot^{-1}(z) \;/;\; (i\,z \in \mathbb{R} \land 0 < i\,z < 1)$$

01.13.27.0851.01

$$\cos^{-1}\left(\frac{1}{\sqrt{z^2+1}}\right) = -\frac{\pi}{2} - \cot^{-1}(z) \;/;\; (-i\,z \in \mathbb{R} \land 0 < -i\,z < 1)$$

01.13.27.0852.01

$$\cos^{-1}\left(\frac{1}{\sqrt{z^2+1}}\right) = \frac{\pi}{2} - \frac{\sqrt{z^2}}{z} \cot^{-1}(z) \;/;\; i\,z \notin (-1,\, 1)$$

01.13.27.0853.01

$$\cos^{-1}\left(\frac{1}{\sqrt{z^2+1}}\right) = \frac{\pi}{2} - \frac{\sqrt{z^2}}{z} \cot^{-1}(z) + \frac{\pi\,i}{2}\left(\frac{\sqrt{-i\,z-1}}{\sqrt{i\,z+1}} + \frac{\sqrt{i\,z-1}}{\sqrt{1-i\,z}}\right)$$

Involving $\cos^{-1}\left(\frac{1}{\sqrt{z^2+1}}\right)$ and $\cot^{-1}\left(\frac{1}{z}\right)$

01.13.27.0854.01

$$\cos^{-1}\left(\frac{1}{\sqrt{z^2+1}}\right) = \cot^{-1}\left(\frac{1}{z}\right) \;/;\; -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

01.13.27.0855.01

$$\cos^{-1}\left(\frac{1}{\sqrt{z^2+1}}\right) = -\cot^{-1}\left(\frac{1}{z}\right) \;/;\; \frac{\pi}{2} < \arg(z) \leq \pi \bigvee -\pi < \arg(z) \leq -\frac{\pi}{2}$$

01.13.27.0856.01

$$\cos^{-1}\left(\frac{1}{\sqrt{z^2+1}}\right) = \frac{\sqrt{z^2}}{z} \cot^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\sqrt{\frac{1}{z^2+1}}\right)$

Involving $\cos^{-1}\left(\sqrt{\frac{1}{z^2+1}}\right)$ and $\cot^{-1}(z)$

01.13.27.0857.01

$$\cos^{-1}\left(\sqrt{\frac{1}{z^2+1}}\right) = \frac{\pi}{2} - \cot^{-1}(z) \;/;\; \mathrm{Re}(z) > 0 \lor (i\,z \in \mathbb{R} \land i\,z > 1)$$

01.13.27.0858.01

$$\cos^{-1}\left(\sqrt{\frac{1}{z^2+1}}\right) = \frac{\pi}{2} + \cot^{-1}(z) /; \operatorname{Re}(z) < 0 \lor (i\,z \in \mathbb{R} \land i\,z < -1)$$

01.13.27.0859.01

$$\cos^{-1}\left(\sqrt{\frac{1}{z^2+1}}\right) = \cot^{-1}(z) - \frac{\pi}{2} /; (i\,z \in \mathbb{R} \land 0 < i\,z < 1)$$

01.13.27.0860.01

$$\cos^{-1}\left(\sqrt{\frac{1}{z^2+1}}\right) = -\cot^{-1}(z) - \frac{\pi}{2} /; (-i\,z \in \mathbb{R} \land 0 < -i\,z < 1)$$

01.13.27.0861.01

$$\cos^{-1}\left(\sqrt{\frac{1}{z^2+1}}\right) = \frac{\pi}{2} - \frac{\sqrt{z^2}}{z} \cot^{-1}(z) /; \operatorname{Re}(z) \neq o$$

01.13.27.0862.01

$$\cos^{-1}\left(\sqrt{\frac{1}{z^2+1}}\right) = \frac{\pi}{2} - \sqrt{\frac{1}{z^2+1}} \sqrt{z^2+1} \left(\frac{\sqrt{z^2}}{z} \cot^{-1}(z) - \frac{\pi\,i}{2}\left(\frac{\sqrt{-i\,z-1}}{\sqrt{i\,z+1}} + \frac{\sqrt{i\,z-1}}{\sqrt{1-i\,z}}\right)\right)$$

Involving $\cos^{-1}\left(\sqrt{\frac{1}{z^2+1}}\right)$ and $\cot^{-1}\left(\frac{1}{z}\right)$

01.13.27.0863.01

$$\cos^{-1}\left(\sqrt{\frac{1}{z^2+1}}\right) = \cot^{-1}\left(\frac{1}{z}\right) /; \operatorname{Re}(z) > 0 \lor (i\,z \in \mathbb{R} \land -1 < i\,z < 0)$$

01.13.27.0864.01

$$\cos^{-1}\left(\sqrt{\frac{1}{z^2+1}}\right) = -\cot^{-1}\left(\frac{1}{z}\right) /; \operatorname{Re}(z) < 0 \lor (i\,z \in \mathbb{R} \land 0 < i\,z < 1)$$

01.13.27.0865.01

$$\cos^{-1}\left(\sqrt{\frac{1}{z^2+1}}\right) = \pi - \cot^{-1}\left(\frac{1}{z}\right) /; (i\,z \in \mathbb{R} \land i\,z < -1)$$

01.13.27.0866.01

$$\cos^{-1}\left(\sqrt{\frac{1}{z^2+1}}\right) = \pi + \cot^{-1}\left(\frac{1}{z}\right) /; (i\,z \in \mathbb{R} \land i\,z > 1)$$

01.13.27.0867.01

$$\cos^{-1}\left(\sqrt{\frac{1}{z^2+1}}\right) = \frac{\sqrt{z^2}}{z} \cot^{-1}\left(\frac{1}{z}\right) /; i\,z \notin (-\infty, -1) \bigwedge i\,z \notin (1, \infty)$$

01.13.27.0868.01

$$\cos^{-1}\left(\sqrt{\frac{1}{z^2+1}}\right) = \frac{\pi}{2} - \sqrt{\frac{1}{z^2+1}}\,\sqrt{z^2+1}\left(\frac{\pi}{2} - \frac{\sqrt{z^2}}{z}\cot^{-1}\left(\frac{1}{z}\right)\right)$$

Involving $\cos^{-1}\left(\frac{z}{\sqrt{z^2+1}}\right)$

Involving $\cos^{-1}\left(\frac{z}{\sqrt{z^2+1}}\right)$ and $\cot^{-1}(z)$

01.13.27.0869.01

$$\cos^{-1}\left(\frac{z}{\sqrt{z^2+1}}\right) = \cot^{-1}(z) \;/;\; \operatorname{Re}(z) > 0 \lor (i\,z \in \mathbb{R} \land 0 < i\,z < 1)$$

01.13.27.0870.01

$$\cos^{-1}\left(\frac{z}{\sqrt{z^2+1}}\right) = \pi + \cot^{-1}(z) \;/;\; \operatorname{Re}(z) < 0 \lor (-i\,z \in \mathbb{R} \land 0 < -i\,z < 1)$$

01.13.27.0871.01

$$\cos^{-1}\left(\frac{z}{\sqrt{z^2+1}}\right) = \pi - \cot^{-1}(z) \;/;\; (i\,z \in \mathbb{R} \land i\,z > 1)$$

01.13.27.0872.01

$$\cos^{-1}\left(\frac{z}{\sqrt{z^2+1}}\right) = -\cot^{-1}(z) \;/;\; (i\,z \in \mathbb{R} \land i\,z < -1)$$

01.13.27.0030.02

$$\cos^{-1}\left(\frac{z}{\sqrt{z^2+1}}\right) = \frac{\pi}{2}\left(1 - \frac{z}{\sqrt{z^2+1}}\sqrt{1+\frac{1}{z^2}}\right) + \sqrt{\frac{1}{z^2+1}}\,\sqrt{z^2+1}\,\cot^{-1}(z)$$

Involving $\cos^{-1}\left(\frac{z}{\sqrt{z^2+1}}\right)$ and $\cot^{-1}\left(\frac{1}{z}\right)$

01.13.27.0873.01

$$\cos^{-1}\left(\frac{z}{\sqrt{z^2+1}}\right) = \frac{\pi}{2} - \cot^{-1}\left(\frac{1}{z}\right) \;/;\; i\,z \notin (-\infty, -1) \bigwedge i\,z \notin (1, \infty)$$

01.13.27.0874.01

$$\cos^{-1}\left(\frac{z}{\sqrt{z^2+1}}\right) = -\frac{\pi}{2} + \cot^{-1}\left(\frac{1}{z}\right) \;/;\; (i\,z \in \mathbb{R} \land i\,z < -1)$$

01.13.27.0875.01

$$\cos^{-1}\left(\frac{z}{\sqrt{z^2+1}}\right) = \frac{3\pi}{2} + \cot^{-1}\left(\frac{1}{z}\right) /; (i\,z \in \mathbb{R} \land i\,z > 1)$$

01.13.27.0876.01

$$\cos^{-1}\left(\frac{z}{\sqrt{z^2+1}}\right) = \frac{\pi}{2}\left(\sqrt{\frac{1}{i\,z+1}}\,\sqrt{i\,z+1} - \sqrt{\frac{1}{1-i\,z}}\,\sqrt{1-i\,z} + 1\right) - \sqrt{\frac{1}{z^2+1}}\,\sqrt{z^2+1}\,\cot^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{z^2}}{\sqrt{z^2+1}}\right)$

Involving $\cos^{-1}\left(\frac{\sqrt{z^2}}{\sqrt{z^2+1}}\right)$ and $\cot^{-1}(z)$

01.13.27.0877.01

$$\cos^{-1}\left(\frac{\sqrt{z^2}}{\sqrt{z^2+1}}\right) = \cot^{-1}(z) /; \operatorname{Re}(z) > 0 \lor (i\,z \in \mathbb{R} \land i\,z > 1)$$

01.13.27.0878.01

$$\cos^{-1}\left(\frac{\sqrt{z^2}}{\sqrt{z^2+1}}\right) = -\cot^{-1}(z) /; \operatorname{Re}(z) < 0 \lor (i\,z \in \mathbb{R} \land i\,z < -1)$$

01.13.27.0879.01

$$\cos^{-1}\left(\frac{\sqrt{z^2}}{\sqrt{z^2+1}}\right) = \pi + \cot^{-1}(z) /; (-i\,z \in \mathbb{R} \land 0 < -i\,z < 1)$$

01.13.27.0880.01

$$\cos^{-1}\left(\frac{\sqrt{z^2}}{\sqrt{z^2+1}}\right) = \pi - \cot^{-1}(z) /; (i\,z \in \mathbb{R} \land 0 < i\,z < 1)$$

01.13.27.0881.01

$$\cos^{-1}\left(\frac{\sqrt{z^2}}{\sqrt{z^2+1}}\right) = \frac{\sqrt{z^2}}{\sqrt{z^2+1}}\sqrt{\frac{z^2+1}{z^2}}\left(z\sqrt{\frac{1}{z^2}}\cot^{-1}(z) - \frac{\pi}{2}\right) + \frac{\pi}{2}$$

Involving $\cos^{-1}\left(\frac{\sqrt{z^2}}{\sqrt{z^2+1}}\right)$ and $\cot^{-1}\left(\frac{1}{z}\right)$

01.13.27.0882.01

$$\cos^{-1}\left(\frac{\sqrt{z^2}}{\sqrt{z^2+1}}\right) = \frac{\pi}{2} - \cot^{-1}\left(\frac{1}{z}\right) /; \operatorname{Re}(z) > 0 \lor (-i\,z \in \mathbb{R} \land 0 < -i\,z < 1)$$

01.13.27.0883.01

$$\cos^{-1}\left(\frac{\sqrt{z^2}}{\sqrt{z^2+1}}\right) = \frac{\pi}{2} + \cot^{-1}\left(\frac{1}{z}\right) /; \operatorname{Re}(z) < 0 \lor (i\, z \in \mathbb{R} \land 0 < i\, z < 1)$$

01.13.27.0884.01

$$\cos^{-1}\left(\frac{\sqrt{z^2}}{\sqrt{z^2+1}}\right) = -\frac{\pi}{2} + \cot^{-1}\left(\frac{1}{z}\right) /; (i\, z \in \mathbb{R} \land i\, z < -1)$$

01.13.27.0885.01

$$\cos^{-1}\left(\frac{\sqrt{z^2}}{\sqrt{z^2+1}}\right) = -\frac{\pi}{2} - \cot^{-1}\left(\frac{1}{z}\right) /; (i\, z \in \mathbb{R} \land i\, z > 1)$$

01.13.27.0886.01

$$\cos^{-1}\left(\frac{\sqrt{z^2}}{\sqrt{z^2+1}}\right) = \frac{1}{2}\pi \sqrt{\frac{1}{z^2+1}} \sqrt{z^2+1} - \frac{z\sqrt{z^2}\sqrt{-z^2-1}}{\sqrt{-z^2\left(z^2+1\right)}} \sqrt{\frac{1}{z^2}} \cot^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{-z^2}}{\sqrt{-z^2-1}}\right)$

Involving $\cos^{-1}\left(\frac{\sqrt{-z^2}}{\sqrt{-z^2-1}}\right)$ and $\cot^{-1}(z)$

01.13.27.0887.01

$$\cos^{-1}\left(\frac{\sqrt{-z^2}}{\sqrt{-z^2-1}}\right) = -\cot^{-1}(z) /; \frac{\pi}{2} \leq \arg(z) \leq \pi \bigvee -\pi < \arg(z) < -\frac{\pi}{2}$$

01.13.27.0888.01

$$\cos^{-1}\left(\frac{\sqrt{-z^2}}{\sqrt{-z^2-1}}\right) = \cot^{-1}(z) /; -\frac{\pi}{2} \leq \arg(z) < \frac{\pi}{2}$$

01.13.27.0889.01

$$\cos^{-1}\left(\frac{\sqrt{-z^2}}{\sqrt{-z^2-1}}\right) = z \sqrt{\frac{1}{z^2}} \cot^{-1}(z)$$

Involving $\cos^{-1}\left(\frac{\sqrt{-z^2}}{\sqrt{-z^2-1}}\right)$ and $\cot^{-1}\left(\frac{1}{z}\right)$

01.13.27.0890.01

$$\cos^{-1}\left(\frac{\sqrt{-z^2}}{\sqrt{-z^2-1}}\right) = \frac{\pi}{2} + \cot^{-1}\left(\frac{1}{z}\right) /; \operatorname{Re}(z) < 0 \lor (-i\, z \in \mathbb{R} \land 0 < -i\, z < 1)$$

01.13.27.0891.01

$$\cos^{-1}\left(\frac{\sqrt{-z^2}}{\sqrt{-z^2-1}}\right) = \frac{\pi}{2} - \cot^{-1}\left(\frac{1}{z}\right) /; \operatorname{Re}(z) > 0 \lor (i z \in \mathbb{R} \land 0 < i z < 1)$$

01.13.27.0892.01

$$\cos^{-1}\left(\frac{\sqrt{-z^2}}{\sqrt{-z^2-1}}\right) = -\frac{\pi}{2} - \cot^{-1}\left(\frac{1}{z}\right) /; (i z \in \mathbb{R} \land i z > 1)$$

01.13.27.0893.01

$$\cos^{-1}\left(\frac{\sqrt{-z^2}}{\sqrt{-z^2-1}}\right) = -\frac{\pi}{2} + \cot^{-1}\left(\frac{1}{z}\right) /; (i z \in \mathbb{R} \land i z < -1)$$

01.13.27.0894.01

$$\cos^{-1}\left(\frac{\sqrt{-z^2}}{\sqrt{-z^2-1}}\right) = \frac{\pi}{2}\left(\sqrt{\frac{1}{i z+1}}\sqrt{i z+1} + \sqrt{\frac{1}{1-i z}}\sqrt{1-i z} - 1\right) - z\sqrt{\frac{1}{z^2}}\cot^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\sqrt{\frac{z^2}{z^2+1}}\right)$

Involving $\cos^{-1}\left(\sqrt{\frac{z^2}{z^2+1}}\right)$ and $\cot^{-1}(z)$

01.13.27.0895.01

$$\cos^{-1}\left(\sqrt{\frac{z^2}{z^2+1}}\right) = -\cot^{-1}(z) /; \operatorname{Re}(z) < 0 \lor (i z \in \mathbb{R} \land i z < -1)$$

01.13.27.0896.01

$$\cos^{-1}\left(\sqrt{\frac{z^2}{z^2+1}}\right) = \cot^{-1}(z) /; \operatorname{Re}(z) > 0 \lor (i z \in \mathbb{R} \land i z > 1)$$

01.13.27.0897.01

$$\cos^{-1}\left(\sqrt{\frac{z^2}{z^2+1}}\right) = \pi - \cot^{-1}(z) /; (i z \in \mathbb{R} \land 0 < i z < 1)$$

01.13.27.0898.01

$$\cos^{-1}\left(\sqrt{\frac{z^2}{z^2+1}}\right) = \pi + \cot^{-1}(z) /; (-i z \in \mathbb{R} \land 0 < -i z < 1)$$

01.13.27.0899.01

$$\cos^{-1}\left(\sqrt{\frac{z^2}{z^2+1}}\right) = \frac{\pi}{2} - \frac{\sqrt{z^2}\,\sqrt{-z^2-1}}{\sqrt{-z^2\,(z^2+1)}}\left(\frac{\pi}{2} - z\,\sqrt{\frac{1}{z^2}}\,\cot^{-1}(z)\right)$$

Involving $\cos^{-1}\left(\sqrt{\frac{z^2}{z^2+1}}\right)$ and $\cot^{-1}\left(\frac{1}{z}\right)$

01.13.27.0900.01

$$\cos^{-1}\left(\sqrt{\frac{z^2}{z^2+1}}\right) = \frac{\pi}{2} + \cot^{-1}\left(\frac{1}{z}\right) /; \operatorname{Re}(z) < 0 \lor (i\,z \in \mathbb{R} \land 0 < i\,z < 1)$$

01.13.27.0901.01

$$\cos^{-1}\left(\sqrt{\frac{z^2}{z^2+1}}\right) = \frac{\pi}{2} - \cot^{-1}\left(\frac{1}{z}\right) /; \operatorname{Re}(z) > 0 \lor (-i\,z \in \mathbb{R} \land 0 < -i\,z < 1)$$

01.13.27.0902.01

$$\cos^{-1}\left(\sqrt{\frac{z^2}{z^2+1}}\right) = -\frac{\pi}{2} + \cot^{-1}\left(\frac{1}{z}\right) /; (i\,z \in \mathbb{R} \land i\,z < -1)$$

01.13.27.0903.01

$$\cos^{-1}\left(\sqrt{\frac{z^2}{z^2+1}}\right) = -\frac{\pi}{2} - \cot^{-1}\left(\frac{1}{z}\right) /; (i\,z \in \mathbb{R} \land i\,z > 1)$$

01.13.27.0904.01

$$\cos^{-1}\left(\sqrt{\frac{z^2}{z^2+1}}\right) = \frac{\pi}{2} - \frac{\sqrt{z^2}\,\sqrt{-z^2-1}}{\sqrt{-z^2\,(z^2+1)}}\left(z\,\sqrt{\frac{1}{z^2}}\,\cot^{-1}\left(\frac{1}{z}\right) - \frac{1}{2}\,\pi\left(\sqrt{\frac{1}{i\,z+1}}\,\sqrt{i\,z+1} + \sqrt{\frac{1}{1-i\,z}}\,\sqrt{1-i\,z} - 2\right)\right)$$

Involving $\cos^{-1}\left(\sqrt{\sqrt{1+z^2}-1}\Big/\left(\sqrt{2}\,(1+z^2)^{1/4}\right)\right)$

Involving $\cos^{-1}\left(\sqrt{\sqrt{1+z^2}-1}\Big/\left(\sqrt{2}\,(1+z^2)^{1/4}\right)\right)$ and $\cot^{-1}(z)$

01.13.27.0905.01

$$\cos^{-1}\left(\frac{\sqrt{\sqrt{1+z^2}-1}}{\sqrt{2}\,(1+z^2)^{1/4}}\right) = \frac{\pi}{4} + \frac{1}{2}\cot^{-1}(z) /; \operatorname{Re}(z) > 0 \lor (i\,z \in \mathbb{R} \land i\,z < -1)$$

01.13.27.0906.01

$$\cos^{-1}\left(\frac{\sqrt{\sqrt{1+z^2}-1}}{\sqrt{2}\,(1+z^2)^{1/4}}\right) = \frac{\pi}{4} - \frac{1}{2}\cot^{-1}(z) \;/;\; \operatorname{Re}(z) < 0 \lor (i\,z \in \mathbb{R} \land i\,z > 1)$$

01.13.27.0907.01

$$\cos^{-1}\left(\frac{\sqrt{\sqrt{z^2+1}-1}}{\sqrt{2}\,\sqrt[4]{z^2+1}}\right) = \frac{3\pi}{4} + \frac{1}{2}\cot^{-1}(z) \;/;\; (-i\,z \in \mathbb{R} \land 0 < -i\,z < 1)$$

01.13.27.0908.01

$$\cos^{-1}\left(\frac{\sqrt{\sqrt{z^2+1}-1}}{\sqrt{2}\,\sqrt[4]{z^2+1}}\right) = \frac{3\pi}{4} - \frac{1}{2}\cot^{-1}(z) \;/;\; (i\,z \in \mathbb{R} \land 0 < i\,z < 1)$$

01.13.27.0909.01

$$\cos^{-1}\left(\frac{\sqrt{\sqrt{z^2+1}-1}}{\sqrt{2}\,\sqrt[4]{z^2+1}}\right) = \frac{1}{4}\pi\left(2 - \sqrt{z^2}\sqrt{\frac{1}{z^2}}\sqrt{\frac{1}{z^2+1}}\sqrt{z^2+1}\right) + \frac{\sqrt{z^2}}{2\,z}\cot^{-1}(z)$$

Involving $\cos^{-1}\left(\sqrt{\sqrt{1+z^2}-1}\Big/\left(\sqrt{2}\,(1+z^2)^{1/4}\right)\right)$ and $\cot^{-1}\left(\frac{1}{z}\right)$

01.13.27.0910.01

$$\cos^{-1}\left(\frac{\sqrt{\sqrt{z^2+1}-1}}{\sqrt{2}\,\sqrt[4]{z^2+1}}\right) = \frac{\pi}{2} - \frac{1}{2}\cot^{-1}\left(\frac{1}{z}\right) \;/;\; -\frac{\pi}{2} < \arg(z) \le \frac{\pi}{2}$$

01.13.27.0911.01

$$\cos^{-1}\left(\frac{\sqrt{\sqrt{z^2+1}-1}}{\sqrt{2}\,\sqrt[4]{z^2+1}}\right) = \frac{1}{2}\cot^{-1}\left(\frac{1}{z}\right) + \frac{\pi}{2} \;/;\; \frac{\pi}{2} < \arg(z) \le \pi \bigvee -\pi < \arg(z) \le -\frac{\pi}{2}$$

01.13.27.0912.01

$$\cos^{-1}\left(\frac{\sqrt{\sqrt{z^2+1}-1}}{\sqrt{2}\,\sqrt[4]{z^2+1}}\right) = \frac{\pi}{2} - \frac{\sqrt{z^2}}{2\,z}\cot^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\sqrt{\sqrt{1+z^2}-z}\Big/\left(\sqrt{2}\,(1+z^2)^{1/4}\right)\right)$

Involving $\cos^{-1}\left(\sqrt{\sqrt{1+z^2}-z}\Big/\left(\sqrt{2}\,(1+z^2)^{1/4}\right)\right)$ and $\cot^{-1}(z)$

01.13.27.0913.01

$$\cos^{-1}\left(\frac{\sqrt{\sqrt{z^2+1}-z}}{\sqrt{2}\,\sqrt[4]{z^2+1}}\right) = \frac{\pi}{2} - \frac{1}{2}\cot^{-1}(z) \;/;\; \mathrm{Re}(z) > 0 \lor (i\,z \in \mathbb{R} \land 0 < i\,z < 1) \lor (i\,z \in \mathbb{R} \land i\,z < -1)$$

01.13.27.0914.01

$$\cos^{-1}\left(\frac{\sqrt{\sqrt{z^2+1}-z}}{\sqrt{2}\,\sqrt[4]{z^2+1}}\right) = -\frac{1}{2}\cot^{-1}(z) \;/;\; \mathrm{Re}(z) < 0 \lor (-i\,z \in \mathbb{R} \land 0 < -i\,z < 1)$$

01.13.27.0915.01

$$\cos^{-1}\left(\frac{\sqrt{\sqrt{z^2+1}-z}}{\sqrt{2}\,\sqrt[4]{z^2+1}}\right) = -\frac{1}{2}\cot^{-1}(z) + \frac{\pi}{4}\left(\frac{\sqrt{z^2}}{z}+1\right) \;/;\; i\,z \notin (-1, \infty)$$

01.13.27.0916.01

$$\cos^{-1}\left(\frac{\sqrt{\sqrt{z^2+1}-z}}{\sqrt{2}\,\sqrt[4]{z^2+1}}\right) = \frac{\pi}{4}\left(1 + \sqrt{\frac{z}{z-i}}\,\sqrt{\frac{z-i}{z}} - \sqrt{\frac{z}{z+i}}\,\sqrt{\frac{z+i}{z}} + \frac{\sqrt{z^2}}{z}\right) - \frac{1}{2}\sqrt{\frac{1}{1-i\,z}}\,\sqrt{1-i\,z}\,\cot^{-1}(z)$$

Involving $\cos^{-1}\left(\sqrt{\sqrt{1+z^2}-z}\Big/\left(\sqrt{2}\,(1+z^2)^{1/4}\right)\right)$ and $\cot^{-1}\left(\frac{1}{z}\right)$

01.13.27.0917.01

$$\cos^{-1}\left(\frac{\sqrt{\sqrt{z^2+1}-z}}{\sqrt{2}\,\sqrt[4]{z^2+1}}\right) = \frac{1}{2}\cot^{-1}\left(\frac{1}{z}\right) + \frac{\pi}{4} \;/;\; i\,z \notin (1, \infty)$$

01.13.27.0918.01

$$\cos^{-1}\left(\frac{\sqrt{\sqrt{z^2+1}-z}}{\sqrt{2}\sqrt[4]{z^2+1}}\right) = -\frac{1}{2}\cot^{-1}\left(\frac{1}{z}\right)-\frac{\pi}{4}\ /;\ (i\,z\in\mathbb{R}\land i\,z>1)$$

01.13.27.0919.01

$$\cos^{-1}\left(\frac{\sqrt{\sqrt{z^2+1}-z}}{\sqrt{2}\sqrt[4]{z^2+1}}\right) = \frac{\sqrt{1-i\,z}}{2}\sqrt{\frac{1}{1-i\,z}}\cot^{-1}\left(\frac{1}{z}\right)+\frac{\pi}{4}\sqrt{\frac{1}{1-i\,z}}\sqrt{1-i\,z}$$

Involving $\cos^{-1}\left(\sqrt{\left(\sqrt{1+z^2}-1\right)\Big/\left(2\sqrt{1+z^2}\right)}\right)$

Involving $\cos^{-1}\left(\sqrt{\left(\sqrt{z^2+1}-1\right)\Big/\left(2\sqrt{z^2+1}\right)}\right)$ and $\cot^{-1}(z)$

01.13.27.0920.01

$$\cos^{-1}\left(\sqrt{\frac{\sqrt{z^2+1}-1}{2\sqrt{z^2+1}}}\right) = \frac{\pi}{4}+\frac{1}{2}\cot^{-1}(z)\ /;\ \mathrm{Re}(z)>0\lor(i\,z\in\mathbb{R}\land i\,z<-1)$$

01.13.27.0921.01

$$\cos^{-1}\left(\sqrt{\frac{\sqrt{z^2+1}-1}{2\sqrt{z^2+1}}}\right) = -\frac{1}{2}\cot^{-1}(z)+\frac{\pi}{4}\ /;\ \mathrm{Re}(z)<0\lor(i\,z\in\mathbb{R}\land i\,z>1)$$

01.13.27.0922.01

$$\cos^{-1}\left(\sqrt{\frac{\sqrt{z^2+1}-1}{2\sqrt{z^2+1}}}\right) = \frac{3\pi}{4}-\frac{1}{2}\cot^{-1}(z)\ /;\ (i\,z\in\mathbb{R}\land 0<i\,z<1)$$

01.13.27.0923.01

$$\cos^{-1}\left(\sqrt{\frac{\sqrt{z^2+1}-1}{2\sqrt{z^2+1}}}\right) = \frac{3\pi}{4}+\frac{1}{2}\cot^{-1}(z)\ /;\ (-i\,z\in\mathbb{R}\land 0<-i\,z<1)$$

01.13.27.0924.01

$$\cos^{-1}\left(\sqrt{\frac{\sqrt{z^2+1}-1}{2\sqrt{z^2+1}}}\right) = \frac{\pi}{4}\left(2-\sqrt{z^2}\sqrt{\frac{1}{z^2}}\sqrt{\frac{1}{z^2+1}}\sqrt{z^2+1}\right)+\frac{\sqrt{z^2}}{2\,z}\cot^{-1}(z)$$

Involving $\cos^{-1}\left(\sqrt{\left(\sqrt{z^2+1}-1\right)\Big/\left(2\sqrt{z^2+1}\right)}\right)$ and $\cot^{-1}\left(\frac{1}{z}\right)$

01.13.27.0925.01

$$\cos^{-1}\left(\sqrt{\frac{\sqrt{1+z^2}-1}{2\sqrt{1+z^2}}}\right) = \frac{\pi}{2} - \frac{1}{2}\cot^{-1}\left(\frac{1}{z}\right) /; -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

01.13.27.0926.01

$$\cos^{-1}\left(\sqrt{\frac{\sqrt{z^2+1}-1}{2\sqrt{z^2+1}}}\right) = \frac{1}{2}\cot^{-1}\left(\frac{1}{z}\right) + \frac{\pi}{2} /; \frac{\pi}{2} < \arg(z) \leq \pi \bigvee -\pi < \arg(z) \leq -\frac{\pi}{2}$$

01.13.27.0927.01

$$\cos^{-1}\left(\sqrt{\frac{\sqrt{1+z^2}-1}{2\sqrt{1+z^2}}}\right) = \frac{\pi}{2} - \frac{\sqrt{z^2}}{2z}\cot^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\sqrt{\left(\sqrt{1+z^2}-z\right)\Big/\left(2\sqrt{1+z^2}\right)}\right)$

Involving $\cos^{-1}\left(\sqrt{\left(\sqrt{z^2+1}-z\right)\Big/\left(2\sqrt{z^2+1}\right)}\right)$ and $\cot^{-1}(z)$

01.13.27.0928.01

$$\cos^{-1}\left(\sqrt{\frac{\sqrt{z^2+1}-z}{2\sqrt{z^2+1}}}\right) = \frac{\pi}{2} - \frac{1}{2}\cot^{-1}(z) /; \mathrm{Re}(z) > 0 \lor (i z \in \mathbb{R} \land 0 < i z < 1)$$

01.13.27.0929.01

$$\cos^{-1}\left(\sqrt{\frac{\sqrt{z^2+1}-z}{2\sqrt{z^2+1}}}\right) = -\frac{1}{2}\cot^{-1}(z) /; \mathrm{Re}(z) < 0 \lor (-i z \in \mathbb{R} \land 0 < -i z < 1)$$

01.13.27.0930.01

$$\cos^{-1}\left(\sqrt{\frac{\sqrt{z^2+1}-z}{2\sqrt{z^2+1}}}\right) = \frac{1}{2}\cot^{-1}(z) /; (i z \in \mathbb{R} \land i z > 1)$$

01.13.27.0931.01

$$\cos^{-1}\left(\sqrt{\frac{\sqrt{z^2+1}-z}{2\sqrt{z^2+1}}}\right) = \frac{\pi}{2} + \frac{1}{2}\cot^{-1}(z) /; (i z \in \mathbb{R} \land i z < -1)$$

01.13.27.0932.01

$$\cos^{-1}\left(\sqrt{\frac{\sqrt{z^2+1}-z}{2\sqrt{z^2+1}}}\right) = -\frac{1}{2}\cot^{-1}(z) + \frac{1}{4}\pi\left(\frac{\sqrt{z^2}}{z}+1\right) /; \operatorname{Re}(z) \neq 0$$

01.13.27.0933.01

$$\cos^{-1}\left(\sqrt{\frac{\sqrt{z^2+1}-z}{2\sqrt{z^2+1}}}\right) = -\frac{1}{2}\sqrt{z^2+1}\sqrt{\frac{1}{z^2+1}}\cot^{-1}(z) + \frac{1}{4}\pi\left(\sqrt{\frac{1}{z^2}}\,z - \sqrt{\frac{1}{i z+1}}\sqrt{i z+1} + \sqrt{\frac{1}{1-i z}}\sqrt{1-i z}+1\right)$$

Involving $\cos^{-1}\left(\sqrt{\left(\sqrt{z^2+1}-z\right)\Big/\left(2\sqrt{z^2+1}\right)}\right)$ and $\cot^{-1}\left(\frac{1}{z}\right)$

01.13.27.0934.01

$$\cos^{-1}\left(\sqrt{\frac{\sqrt{z^2+1}-z}{2\sqrt{z^2+1}}}\right) = \frac{1}{2}\cot^{-1}\left(\frac{1}{z}\right) + \frac{\pi}{4} /; i z \notin (-\infty, -1) \bigwedge i z \notin (1, \infty)$$

01.13.27.0935.01

$$\cos^{-1}\left(\sqrt{\frac{\sqrt{z^2+1}-z}{2\sqrt{z^2+1}}}\right) = -\frac{1}{2}\cot^{-1}\left(\frac{1}{z}\right) - \frac{\pi}{4} /; (i z \in \mathbb{R} \land i z > 1)$$

01.13.27.0936.01

$$\cos^{-1}\left(\sqrt{\frac{\sqrt{z^2+1}-z}{2\sqrt{z^2+1}}}\right) = \frac{3\pi}{4} - \frac{1}{2}\cot^{-1}\left(\frac{1}{z}\right) /; (i z \in \mathbb{R} \land i z < -1)$$

01.13.27.0937.01

$$\cos^{-1}\left(\sqrt{\frac{\sqrt{z^2+1}-z}{2\sqrt{z^2+1}}}\right) = -\frac{1}{4}\pi\sqrt{i z+1}\sqrt{\frac{1}{i z+1}}\left(2-\sqrt{1-i z}\sqrt{\frac{1}{1-i z}}\right) + \frac{1}{2}\cot^{-1}\left(\frac{1}{z}\right)\sqrt{\frac{1}{z^2+1}}\sqrt{z^2+1} + \frac{\pi}{2}$$

Involving $\cos^{-1}\left(\frac{1}{\sqrt{a z^c+1}}\right)$

Involving $\cos^{-1}\left(1\big/\sqrt{a z^c+1}\right)$ and $\cot^{-1}\left(\sqrt{a}\,z^{c/2}\right)$

01.13.27.0031.01

$$\cos^{-1}\left(\frac{1}{\sqrt{a z^c+1}}\right) = -\frac{\sqrt{a}\,z^{c/2}}{\sqrt{a z^c}}\cot^{-1}\left(\sqrt{a}\,z^{c/2}\right) + \frac{\pi i}{2}\left(\frac{\sqrt{-i\sqrt{a}\sqrt{z^c}-1}}{\sqrt{i\sqrt{a}\sqrt{z^c}+1}} + \frac{\sqrt{i\sqrt{a}\sqrt{z^c}-1}}{\sqrt{1-i\sqrt{a}\sqrt{z^c}}}\right) + \frac{\pi}{2}$$

Involving $\csc^{-1}$

Involving $\cos^{-1}(z)$

Involving $\cos^{-1}(z)$ and $\csc^{-1}\left(\frac{1}{z}\right)$

01.13.27.0938.01

$$\cos^{-1}(z) = \frac{\pi}{2} - \csc^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}(z)$ and $\csc^{-1}\left(\frac{1}{1-2z^2}\right)$

01.13.27.0939.01

$$\cos^{-1}(z) = \frac{1}{2}\csc^{-1}\left(\frac{1}{1-2z^2}\right) + \frac{\pi}{4} \,/;\, -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

01.13.27.0940.01

$$\cos^{-1}(z) = -\frac{1}{2}\csc^{-1}\left(\frac{1}{1-2z^2}\right) + \frac{3\pi}{4} \,/;\, \frac{\pi}{2} < \arg(z) \leq \pi \bigvee -\pi < \arg(z) \leq -\frac{\pi}{2}$$

01.13.27.0941.01

$$\cos^{-1}(z) = \frac{\pi}{2}\left(1 - \frac{\sqrt{z^2}}{2z}\right) + \frac{\sqrt{z^2}}{2z}\csc^{-1}\left(\frac{1}{1-2z^2}\right)$$

Involving $\cos^{-1}(z)$ and $\csc^{-1}\left(\frac{1}{2z^2-1}\right)$

01.13.27.0942.01

$$\cos^{-1}(z) = \frac{\pi}{4} - \frac{1}{2}\csc^{-1}\left(\frac{1}{2z^2-1}\right) \,/;\, -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

01.13.27.0943.01

$$\cos^{-1}(z) = \frac{3\pi}{4} + \frac{1}{2}\csc^{-1}\left(\frac{1}{2z^2-1}\right) \,/;\, \frac{\pi}{2} < \arg(z) \leq \pi \bigvee -\pi < \arg(z) \leq -\frac{\pi}{2}$$

01.13.27.0944.01

$$\cos^{-1}(z) = \frac{1}{2}\pi\left(1 - \frac{\sqrt{z^2}}{2z}\right) - \frac{\sqrt{z^2}}{2z}\csc^{-1}\left(\frac{1}{2z^2-1}\right)$$

Involving $\cos^{-1}(z)$ and $\csc^{-1}\left(\frac{\sqrt{2}}{\sqrt{1+z}}\right)$

01.13.27.0945.01

$$\cos^{-1}(z) = \pi - 2\csc^{-1}\left(\frac{\sqrt{2}}{\sqrt{1+z}}\right)$$

Involving $\cos^{-1}(z)$ and $\csc^{-1}\left(\sqrt{\frac{2}{1+z}}\right)$

01.13.27.0946.01

$$\cos^{-1}(z) = \pi - 2\csc^{-1}\left(\sqrt{\frac{2}{1+z}}\right) /; z \notin (-\infty, -1)$$

01.13.27.0947.01

$$\cos^{-1}(z) = 2\csc^{-1}\left(\sqrt{\frac{2}{1+z}}\right) + \pi /; (z \in \mathbb{R} \land z < -1)$$

01.13.27.0948.01

$$\cos^{-1}(z) = \pi - 2\sqrt{z+1}\,\sqrt{\frac{1}{z+1}}\,\csc^{-1}\left(\sqrt{\frac{2}{z+1}}\right)$$

Involving $\cos^{-1}(z)$ and $\csc^{-1}\left(\frac{\sqrt{2}}{\sqrt{1-z}}\right)$

01.13.27.0949.01

$$\cos^{-1}(z) = 2\csc^{-1}\left(\frac{\sqrt{2}}{\sqrt{1-z}}\right)$$

Involving $\cos^{-1}(z)$ and $\csc^{-1}\left(\sqrt{\frac{2}{1-z}}\right)$

01.13.27.0950.01

$$\cos^{-1}(z) = 2\csc^{-1}\left(\sqrt{\frac{2}{1-z}}\right) /; z \notin (1, \infty)$$

01.13.27.0951.01

$$\cos^{-1}(z) = -2\csc^{-1}\left(\sqrt{\frac{2}{1-z}}\right) /; (z \in \mathbb{R} \land z > 1)$$

01.13.27.0952.01

$$\cos^{-1}(z) = 2\sqrt{1-z}\,\sqrt{\frac{1}{1-z}}\,\csc^{-1}\left(\sqrt{\frac{2}{1-z}}\right)$$

Involving $\cos^{-1}(z)$ and $\csc^{-1}\left(\frac{1}{\sqrt{z^2}}\right)$

01.13.27.0953.01

$$\cos^{-1}(z) = \frac{\pi}{2} - \csc^{-1}\left(\frac{1}{\sqrt{z^2}}\right) /; -\frac{\pi}{2} < \arg(z) \le \frac{\pi}{2}$$

01.13.27.0954.01

$$\cos^{-1}(z) = \frac{\pi}{2} + \csc^{-1}\left(\frac{1}{\sqrt{z^2}}\right) /; \frac{\pi}{2} < \arg(z) \leq \pi \bigvee -\pi < \arg(z) \leq -\frac{\pi}{2}$$

01.13.27.0955.01

$$\cos^{-1}(z) = \frac{\pi}{2} - \frac{\sqrt{z^2}}{z} \csc^{-1}\left(\frac{1}{\sqrt{z^2}}\right)$$

Involving $\cos^{-1}(z)$ and $\csc^{-1}\left(\sqrt{\frac{1}{z^2}}\right)$

01.13.27.0956.01

$$\cos^{-1}(z) = \frac{\pi}{2} - \csc^{-1}\left(\sqrt{\frac{1}{z^2}}\right) /; -\frac{\pi}{2} \leq \arg(z) < \frac{\pi}{2}$$

01.13.27.0957.01

$$\cos^{-1}(z) = \frac{\pi}{2} + \csc^{-1}\left(\sqrt{\frac{1}{z^2}}\right) /; \frac{\pi}{2} \leq \arg(z) \leq \pi \bigvee -\pi < \arg(z) < -\frac{\pi}{2}$$

01.13.27.0958.01

$$\cos^{-1}(z) = \frac{\pi}{2} - z \sqrt{\frac{1}{z^2}} \csc^{-1}\left(\sqrt{\frac{1}{z^2}}\right)$$

Involving $\cos^{-1}(z)$ and $\csc^{-1}\left(\frac{1}{\sqrt{1-z^2}}\right)$

01.13.27.0959.01

$$\cos^{-1}(z) = \csc^{-1}\left(\frac{1}{\sqrt{1-z^2}}\right) /; -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

01.13.27.0960.01

$$\cos^{-1}(z) = \pi - \csc^{-1}\left(\frac{1}{\sqrt{1-z^2}}\right) /; \frac{\pi}{2} < \arg(z) \leq \pi \bigvee -\pi < \arg(z) \leq -\frac{\pi}{2}$$

01.13.27.0961.01

$$\cos^{-1}(z) = \frac{1}{2} \pi \left(1 - \frac{\sqrt{z^2}}{z}\right) + \frac{\sqrt{z^2}}{z} \csc^{-1}\left(\frac{1}{\sqrt{1-z^2}}\right)$$

Involving $\cos^{-1}(z)$ and $\csc^{-1}\left(\sqrt{\frac{1}{1-z^2}}\right)$

01.13.27.0962.01

$$\cos^{-1}(z) = \csc^{-1}\left(\sqrt{\frac{1}{1-z^2}}\right) /; 0 < \arg(z) \leq \frac{\pi}{2} \bigvee -\frac{\pi}{2} < \arg(z) < 0 \bigvee (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.0963.01

$$\cos^{-1}(z) = \pi - \csc^{-1}\left(\sqrt{\frac{1}{1-z^2}}\right) /; \frac{\pi}{2} < \arg(z) < \pi \bigvee -\pi < \arg(z) < -\frac{\pi}{2} \bigvee (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.0964.01

$$\cos^{-1}(z) = -\csc^{-1}\left(\sqrt{\frac{1}{1-z^2}}\right) /; (z \in \mathbb{R} \land z > 1)$$

01.13.27.0965.01

$$\cos^{-1}(z) = \pi + \csc^{-1}\left(\sqrt{\frac{1}{1-z^2}}\right) /; (z \in \mathbb{R} \land z < -1)$$

01.13.27.0966.01

$$\cos^{-1}(z) = \frac{1}{2}\left(\sqrt{\frac{1}{1-z}}\sqrt{1-z} - \sqrt{z+1}\sqrt{\frac{1}{z+1}} + 1\right)\pi - \frac{\sqrt{1-z^2}}{z}\sqrt{\frac{z^2}{1-z^2}}\sec^{-1}\left(\sqrt{\frac{1}{1-z^2}}\right)$$

Involving $\cos^{-1}(z)$ and $\csc^{-1}\left(\frac{1}{2z\sqrt{1-z^2}}\right)$

01.13.27.0967.01

$$\cos^{-1}(z) = \frac{\pi}{2} - \frac{1}{2}\csc^{-1}\left(\frac{1}{2z\sqrt{1-z^2}}\right) /; \frac{\pi}{4} \leq |\arg(z)| \leq \frac{3\pi}{4}$$

01.13.27.0968.01

$$\cos^{-1}(z) = \frac{\pi}{4}\left(2 - \frac{\sqrt{z^2}}{z} + \sqrt{\frac{1}{z}}\sqrt{z}\sqrt{\frac{1}{\sqrt{2}\,z+1}}\sqrt{\sqrt{2}\,z+1} - \sqrt{-\frac{1}{z}}\sqrt{-z}\sqrt{\frac{1}{1-\sqrt{2}\,z}}\sqrt{1-\sqrt{2}\,z} - \frac{\sqrt{z^4-z^2}}{z\sqrt{z^2-1}}\right) + \frac{\sqrt{-z^2}\sqrt{z^2-1}\sqrt{2z^2-1}}{2\sqrt{1-2z^2}\sqrt{z^4-z^2}}\csc^{-1}\left(\frac{1}{2z\sqrt{1-z^2}}\right)$$

Involving $\cos^{-1}(-z)$

Involving $\cos^{-1}(-z)$ and $\csc^{-1}\left(\frac{1}{z}\right)$

01.13.27.0969.01

$$\cos^{-1}(-z) = \frac{\pi}{2} + \csc^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\sqrt{z}\right)$

Involving $\cos^{-1}\left(\sqrt{z}\right)$ and $\csc^{-1}\left(\frac{1}{\sqrt{z}}\right)$

01.13.27.0970.01

$$\cos^{-1}\left(\sqrt{z}\right) = \frac{\pi}{2} - \csc^{-1}\left(\frac{1}{\sqrt{z}}\right)$$

Involving $\cos^{-1}\left(\sqrt{z}\right)$ and $\csc^{-1}\left(\sqrt{\frac{1}{z}}\right)$

01.13.27.0971.01

$$\cos^{-1}\left(\sqrt{z}\right) = \frac{\pi}{2} - \csc^{-1}\left(\sqrt{\frac{1}{z}}\right) /; z \notin (-\infty, 0)$$

01.13.27.0972.01

$$\cos^{-1}\left(\sqrt{z}\right) = \csc^{-1}\left(\sqrt{\frac{1}{z}}\right) + \frac{\pi}{2} /; (z \in \mathbb{R} \land z < 0)$$

01.13.27.0973.01

$$\cos^{-1}\left(\sqrt{z}\right) = \frac{\pi}{2} - \sqrt{z} \sqrt{\frac{1}{z}} \csc^{-1}\left(\sqrt{\frac{1}{z}}\right)$$

Involving $\cos^{-1}\left(\frac{1}{\sqrt{z}}\right)$

Involving $\cos^{-1}\left(\frac{1}{\sqrt{z}}\right)$ and $\csc^{-1}\left(\sqrt{z}\right)$

01.13.27.0974.01

$$\cos^{-1}\left(\frac{1}{\sqrt{z}}\right) = \frac{\pi}{2} - \csc^{-1}\left(\sqrt{z}\right)$$

Involving $\cos^{-1}\left(\frac{1}{\sqrt{z}}\right)$ and $\csc^{-1}\left(1 \Big/ \sqrt{\frac{1}{z}}\right)$

01.13.27.0975.01

$$\cos^{-1}\left(\frac{1}{\sqrt{z}}\right) = \frac{\pi}{2} - \csc^{-1}\left(1 \Big/ \sqrt{\frac{1}{z}}\right) /; |\arg(z)| < \pi$$

01.13.27.0976.01

$$\cos^{-1}\left(\frac{1}{\sqrt{z}}\right) = \csc^{-1}\left(1 \Big/ \sqrt{\frac{1}{z}}\right) + \frac{\pi}{2} /; (z \in \mathbb{R} \land z < 0)$$

01.13.27.0977.01

$$\cos^{-1}\left(\frac{1}{\sqrt{z}}\right) = \frac{\pi}{2} - \sqrt{z} \sqrt{\frac{1}{z}} \csc^{-1}\left(1 \Big/ \sqrt{\frac{1}{z}}\right)$$

Involving $\cos^{-1}\left(\sqrt{z^2}\right)$

Involving $\cos^{-1}\left(\sqrt{z^2}\right)$ and $\csc^{-1}\left(\frac{1}{z}\right)$

01.13.27.0978.01

$$\cos^{-1}\left(\sqrt{z^2}\right) = \frac{\pi}{2} - \csc^{-1}\left(\frac{1}{z}\right) /; -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

01.13.27.0979.01

$$\cos^{-1}\left(\sqrt{z^2}\right) = \frac{\pi}{2} + \csc^{-1}\left(\frac{1}{z}\right) /; \frac{\pi}{2} < \arg(z) \leq \pi \bigvee -\pi < \arg(z) \leq -\frac{\pi}{2}$$

01.13.27.0980.01

$$\cos^{-1}\left(\sqrt{z^2}\right) = \frac{\pi}{2} - \frac{\sqrt{z^2}}{z} \csc^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(a\left(b z^c\right)^m\right)$

Involving $\cos^{-1}(a (b z^c)^m)$ and $\csc^{-1}\left(\frac{1}{a b^m z^{m c}}\right)$

01.13.27.0981.01

$$\cos^{-1}(a (b z^c)^m) = \frac{\pi}{2} - \frac{(b z^c)^m}{b^m z^{m c}} \csc^{-1}\left(\frac{1}{a} b^{-m} z^{-m c}\right) /; 2 m \in \mathbb{Z}$$

Involving $\cos^{-1}\left(1 - 2 z^2\right)$

Involving $\cos^{-1}\left(1 - 2 z^2\right)$ and $\csc^{-1}\left(\frac{1}{z}\right)$

01.13.27.0982.01

$$\cos^{-1}\left(1 - 2 z^2\right) = 2 \csc^{-1}\left(\frac{1}{z}\right) /; -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

01.13.27.0983.01

$$\cos^{-1}\left(1 - 2 z^2\right) = -2 \csc^{-1}\left(\frac{1}{z}\right) /; \frac{\pi}{2} < \arg(z) \leq \pi \bigvee -\pi < \arg(z) \leq -\frac{\pi}{2}$$

01.13.27.0984.01

$$\cos^{-1}\left(1 - 2 z^2\right) = \frac{2 \sqrt{z^2}}{z} \csc^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}(2 z^2 - 1)$

Involving $\cos^{-1}(2 z^2 - 1)$ and $\csc^{-1}\left(\frac{1}{z}\right)$

01.13.27.0985.01

$$\cos^{-1}(2 z^2 - 1) = \pi - 2 \csc^{-1}\left(\frac{1}{z}\right) /; -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

01.13.27.0986.01

$$\cos^{-1}(2 z^2 - 1) = \pi + 2 \csc^{-1}\left(\frac{1}{z}\right) /; \frac{\pi}{2} < \arg(z) \leq \pi \bigvee -\pi < \arg(z) \leq -\frac{\pi}{2}$$

01.13.27.0987.01

$$\cos^{-1}(2 z^2 - 1) = \pi - \frac{2 \sqrt{z^2}}{z} \csc^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\frac{z^2-2}{z^2}\right)$

Involving $\cos^{-1}\left(\frac{z^2-2}{z^2}\right)$ and $\csc^{-1}(z)$

01.13.27.0988.01

$$\cos^{-1}\left(\frac{z^2 - 2}{z^2}\right) = 2 \csc^{-1}(z) /; -\frac{\pi}{2} \leq \arg(z) < \frac{\pi}{2}$$

01.13.27.0989.01

$$\cos^{-1}\left(\frac{z^2 - 2}{z^2}\right) = -2 \csc^{-1}(z) /; \frac{\pi}{2} \leq \arg(z) \leq \pi \bigvee -\pi < \arg(z) < -\frac{\pi}{2}$$

01.13.27.0990.01

$$\cos^{-1}\left(\frac{z^2 - 2}{z^2}\right) = 2 \sqrt{\frac{1}{z^2}} \, z \csc^{-1}(z)$$

Involving $\cos^{-1}\left(\frac{2-z^2}{z^2}\right)$

Involving $\cos^{-1}\left(\frac{2-z^2}{z^2}\right)$ and $\csc^{-1}(z)$

01.13.27.0991.01

$$\cos^{-1}\left(\frac{2 - z^2}{z^2}\right) = \pi - 2 \csc^{-1}(z) /; -\frac{\pi}{2} \leq \arg(z) < \frac{\pi}{2}$$

01.13.27.0992.01

$$\cos^{-1}\left(\frac{2 - z^2}{z^2}\right) = \pi + 2 \csc^{-1}(z) /; \frac{\pi}{2} \leq \arg(z) \leq \pi \bigvee -\pi < \arg(z) < -\frac{\pi}{2}$$

01.13.27.0993.01

$$\cos^{-1}\left(\frac{2-z^2}{z^2}\right) = \pi - 2\sqrt{\frac{1}{z^2}}\, z\csc^{-1}(z)$$

Involving $\cos^{-1}\left(\sqrt{1-z}\right)$

Involving $\cos^{-1}\left(\sqrt{1-z}\right)$ and $\csc^{-1}\left(\frac{1}{\sqrt{z}}\right)$

01.13.27.0994.01

$$\cos^{-1}\left(\sqrt{1-z}\right) = \csc^{-1}\left(\frac{1}{\sqrt{z}}\right)$$

Involving $\cos^{-1}\left(\sqrt{1-z}\right)$ and $\csc^{-1}\left(\sqrt{\frac{1}{z}}\right)$

01.13.27.0995.01

$$\cos^{-1}\left(\sqrt{1-z}\right) = \csc^{-1}\left(\sqrt{\frac{1}{z}}\right) /; |\arg(z)| < \pi$$

01.13.27.0996.01

$$\cos^{-1}\left(\sqrt{1-z}\right) = -\csc^{-1}\left(\sqrt{\frac{1}{z}}\right) /; (z \in \mathbb{R} \land z < 0)$$

01.13.27.0997.01

$$\cos^{-1}\left(\sqrt{1-z}\right) = \sqrt{z}\sqrt{\frac{1}{z}}\csc^{-1}\left(\sqrt{\frac{1}{z}}\right)$$

Involving $\cos^{-1}\left(\sqrt{\frac{1+c\,z}{2}}\right)$

Involving $\cos^{-1}\left(\sqrt{\frac{1+z}{2}}\right)$ and $\csc^{-1}\left(\frac{1}{z}\right)$

01.13.27.0998.01

$$\cos^{-1}\left(\sqrt{\frac{z+1}{2}}\right) = \frac{\pi}{4} - \frac{1}{2}\csc^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\sqrt{\frac{1-z}{2}}\right)$ and $\csc^{-1}\left(\frac{1}{z}\right)$

01.13.27.0999.01

$$\cos^{-1}\left(\sqrt{\frac{1-z}{2}}\right) = \frac{1}{2}\csc^{-1}\left(\frac{1}{z}\right) + \frac{\pi}{4}$$

Involving $\cos^{-1}\left(\frac{\sqrt{z-1}}{\sqrt{z}}\right)$

Involving $\cos^{-1}\left(\frac{\sqrt{z-a}}{\sqrt{z}}\right)$ and $\csc^{-1}\left(\sqrt{\frac{z}{a}}\right)$

01.13.27.1000.01

$$\cos^{-1}\left(\frac{\sqrt{z-1}}{\sqrt{z}}\right) = \csc^{-1}\left(\sqrt{z}\right) /; |\arg(z)| < \pi$$

01.13.27.1001.01

$$\cos^{-1}\left(\frac{\sqrt{z-1}}{\sqrt{z}}\right) = -\csc^{-1}\left(\sqrt{z}\right) /; (z \in \mathbb{R} \land z < 0)$$

01.13.27.0033.01

$$\cos^{-1}\left(\frac{\sqrt{z-1}}{\sqrt{z}}\right) = \sqrt{z}\sqrt{\frac{1}{z}}\csc^{-1}\left(\sqrt{z}\right)$$

01.13.27.0039.01

$$\cos^{-1}\left(\frac{\sqrt{z-a}}{\sqrt{z}}\right) = \frac{\pi}{2} - \frac{\sqrt{z-a}}{2\sqrt{1-\frac{a}{z}}\sqrt{z}}\left(\pi - 2\sqrt{\frac{a}{z}}\sqrt{\frac{z}{a}}\csc^{-1}\left(\sqrt{\frac{z}{a}}\right)\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{-z}}\right)$

Involving $\cos^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{-z}}\right)$ and $\csc^{-1}\left(\sqrt{z}\right)$

01.13.27.1002.01

$$\cos^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{-z}}\right) = \csc^{-1}\left(\sqrt{z}\right) /; z \notin (-\infty, 1)$$

01.13.27.1003.01

$$\cos^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{-z}}\right) = -\csc^{-1}\left(\sqrt{z}\right) /; (z \in \mathbb{R} \land z < 0)$$

01.13.27.1004.01

$$\cos^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{-z}}\right) = \pi - \csc^{-1}\left(\sqrt{z}\right) /; (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.1005.01

$$\cos^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{-z}}\right) = \frac{\pi}{2}\left(1-\sqrt{\frac{z-1}{z}}\sqrt{\frac{z}{z-1}}\right)+\sqrt{\frac{-1+z}{z^2}}\sqrt{z}\sqrt{\frac{z}{-1+z}}\csc^{-1}\left(\sqrt{z}\right)$$

Involving $\cos^{-1}\left(\sqrt{\frac{z-1}{z}}\right)$

Involving $\cos^{-1}\left(\sqrt{\frac{z-1}{z}}\right)$ and $\csc^{-1}\left(\sqrt{z}\right)$

01.13.27.1006.01

$$\cos^{-1}\left(\sqrt{\frac{z-1}{z}}\right) = \csc^{-1}\left(\sqrt{z}\right) /; |\arg(z)| < \pi$$

01.13.27.1007.01

$$\cos^{-1}\left(\sqrt{\frac{z-1}{z}}\right) = -\csc^{-1}\left(\sqrt{z}\right) /; (z \in \mathbb{R} \land z < 0)$$

01.13.27.0032.01

$$\cos^{-1}\left(\sqrt{\frac{z-1}{z}}\right) = \sqrt{z}\sqrt{\frac{1}{z}}\csc^{-1}\left(\sqrt{z}\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{z+a}}{\sqrt{2z}}\right)$

Involving $\cos^{-1}\left(\frac{\sqrt{z-1}}{\sqrt{2z}}\right)$ and $\csc^{-1}(z)$

01.13.27.0035.01

$$\cos^{-1}\left(\frac{\sqrt{z-1}}{\sqrt{2z}}\right) = \frac{1}{2}\csc^{-1}(z)+\frac{\pi}{4}$$

Involving $\cos^{-1}\left(\frac{\sqrt{z+1}}{\sqrt{2z}}\right)$ and $\csc^{-1}(z)$

01.13.27.1008.01

$$\cos^{-1}\left(\frac{\sqrt{z+1}}{\sqrt{2z}}\right) = \frac{\pi}{4}-\frac{1}{2}\csc^{-1}(z) /; z \notin (-1, 0)$$

01.13.27.1009.01

$$\cos^{-1}\left(\frac{\sqrt{z+1}}{\sqrt{2z}}\right) = \frac{1}{2}\left(\csc^{-1}(z)+\frac{3\pi}{2}\right) /; (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.1010.01

$$\cos^{-1}\left(\frac{\sqrt{z+1}}{\sqrt{2z}}\right) = \frac{1}{2}\pi\left(1-\frac{1}{2}\sqrt{1+\frac{1}{z}}\sqrt{\frac{z}{z+1}}\right)-\frac{1}{2}\sqrt{1+\frac{1}{z}}\sqrt{\frac{z}{z+1}}\csc^{-1}(z)$$

Involving $\cos^{-1}\left(\frac{\sqrt{a-z}}{\sqrt{-2z}}\right)$

Involving $\cos^{-1}\left(\frac{\sqrt{-1-z}}{\sqrt{-2z}}\right)$ and $\csc^{-1}(z)$

01.13.27.1011.01

$$\cos^{-1}\left(\frac{\sqrt{-z-1}}{\sqrt{-2z}}\right) = -\frac{1}{2}\csc^{-1}(z)+\frac{\pi}{4}$$

Involving $\cos^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{-2z}}\right)$ and $\csc^{-1}(z)$

01.13.27.1012.01

$$\cos^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{-2z}}\right) = \frac{\pi}{4}+\frac{1}{2}\csc^{-1}(z) \text{ /; } z \notin (0,1)$$

01.13.27.1013.01

$$\cos^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{-2z}}\right) = \frac{3\pi}{4}-\frac{1}{2}\csc^{-1}(z) \text{ /; } (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.1014.01

$$\cos^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{-2z}}\right) = \frac{1}{4}\pi\left(2-\sqrt{\frac{z-1}{z}}\sqrt{\frac{z}{z-1}}\right)+\frac{1}{2}\sqrt{\frac{z}{z-1}}\sqrt{\frac{z-1}{z}}\csc^{-1}(z)$$

Involving $\cos^{-1}\left(\sqrt{\frac{z+a}{2z}}\right)$

Involving $\cos^{-1}\left(\sqrt{\frac{z-1}{2z}}\right)$ and $\csc^{-1}(z)$

01.13.27.0034.01

$$\cos^{-1}\left(\sqrt{\frac{z-1}{2z}}\right) = \frac{1}{2}\csc^{-1}(z)+\frac{\pi}{4}$$

Involving $\cos^{-1}\left(\sqrt{\frac{z+1}{2z}}\right)$ and $\csc^{-1}(z)$

01.13.27.1015.01

$$\cos^{-1}\left(\sqrt{\frac{z+1}{2z}}\right) = -\frac{1}{2}\csc^{-1}(z) + \frac{\pi}{4}$$

Involving $\cos^{-1}\left(\sqrt{1-z^2}\right)$

Involving $\cos^{-1}\left(\sqrt{1-z^2}\right)$ and $\csc^{-1}\left(\frac{1}{z}\right)$

01.13.27.1016.01

$$\cos^{-1}\left(\sqrt{1-z^2}\right) = \csc^{-1}\left(\frac{1}{z}\right) /; -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

01.13.27.1017.01

$$\cos^{-1}\left(\sqrt{1-z^2}\right) = -\csc^{-1}\left(\frac{1}{z}\right) /; \frac{\pi}{2} < \arg(z) \leq \pi \bigvee -\pi < \arg(z) \leq -\frac{\pi}{2}$$

01.13.27.1018.01

$$\cos^{-1}\left(\sqrt{1-z^2}\right) = \frac{\sqrt{z^2}}{z}\csc^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{z^2-1}}{z}\right)$

Involving $\cos^{-1}\left(\frac{\sqrt{z^2-1}}{z}\right)$ and $\csc^{-1}(z)$

01.13.27.1019.01

$$\cos^{-1}\left(\frac{\sqrt{z^2-1}}{z}\right) = \csc^{-1}(z) /; \operatorname{Re}(z) > 0$$

01.13.27.1020.01

$$\cos^{-1}\left(\frac{\sqrt{z^2-1}}{z}\right) = \pi + \csc^{-1}(z) /; \frac{\pi}{2} < \arg(z) \leq \pi \bigvee -\pi < \arg(z) < -\frac{\pi}{2}$$

01.13.27.1021.01

$$\cos^{-1}\left(\frac{\sqrt{z^2-1}}{z}\right) = -\csc^{-1}(z) /; (i\,z \in \mathbb{R} \land i\,z < 0)$$

01.13.27.1022.01

$$\cos^{-1}\left(\frac{\sqrt{z^2-1}}{z}\right) = \pi - \csc^{-1}(z) /; (i\,z \in \mathbb{R} \land i\,z > 0)$$

01.13.27.0036.01

$$\cos^{-1}\left(\frac{\sqrt{z^2-1}}{z}\right) = \frac{\pi}{2}\left(1-\frac{\sqrt{z^2}}{z}\right) + \csc^{-1}(z) \;/;\; \mathrm{Re}(z) \neq 0$$

01.13.27.0037.01

$$\cos^{-1}\left(\frac{\sqrt{z^2-1}}{z}\right) = \frac{\pi}{2}\left(1-\frac{\sqrt{z^2}}{z}\right) + \sqrt{z^2}\,\sqrt{\frac{1}{z^2}}\,\csc^{-1}(z)$$

Involving $\cos^{-1}\left(\frac{\sqrt{z^2-1}}{\sqrt{z^2}}\right)$

Involving $\cos^{-1}\left(\frac{\sqrt{z^2-1}}{\sqrt{z^2}}\right)$ and $\csc^{-1}(z)$

01.13.27.1023.01

$$\cos^{-1}\left(\frac{\sqrt{z^2-1}}{\sqrt{z^2}}\right) = \csc^{-1}(z) \;/;\; -\frac{\pi}{2} \leq \arg(z) < \frac{\pi}{2}$$

01.13.27.1024.01

$$\cos^{-1}\left(\frac{\sqrt{z^2-1}}{\sqrt{z^2}}\right) = -\csc^{-1}(z) \;/;\; \frac{\pi}{2} < \arg(z) \leq \pi \bigvee -\pi < \arg(z) < -\frac{\pi}{2}$$

01.13.27.1025.01

$$\cos^{-1}\left(\frac{\sqrt{z^2-1}}{\sqrt{z^2}}\right) = z\sqrt{\frac{1}{z^2}}\,\csc^{-1}(z)$$

Involving $\cos^{-1}\left(\frac{\sqrt{1-z^2}}{\sqrt{-z^2}}\right)$

Involving $\cos^{-1}\left(\frac{\sqrt{1-z^2}}{\sqrt{-z^2}}\right)$ and $\csc^{-1}(z)$

01.13.27.1026.01

$$\cos^{-1}\left(\frac{\sqrt{1-z^2}}{\sqrt{-z^2}}\right) = \csc^{-1}(z) \;/;\; -\frac{\pi}{2} \leq \arg(z) < \frac{\pi}{2} \bigwedge z \notin (0,\, 1)$$

01.13.27.1027.01

$$\cos^{-1}\left(\frac{\sqrt{z^2-1}}{\sqrt{z^2}}\right) = -\csc^{-1}(z) \;/;\; \frac{\pi}{2} \leq \arg(z) \leq \pi \bigvee -\pi < \arg(z) < -\frac{\pi}{2}$$

01.13.27.1028.01

$$\cos^{-1}\left(\frac{\sqrt{1-z^2}}{\sqrt{-z^2}}\right) = \pi - \csc^{-1}(z) \;/;\; (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.1029.01

$$\cos^{-1}\left(\frac{\sqrt{1-z^2}}{\sqrt{-z^2}}\right) = \frac{1}{2}\pi\left(1 - \sqrt{1-\frac{1}{z^2}}\sqrt{\frac{z^2}{z^2-1}}\right) + z\sqrt{\frac{z^2-1}{z^4}}\sqrt{\frac{z^2}{z^2-1}}\csc^{-1}(z)$$

Involving $\cos^{-1}\left(\sqrt{\frac{z^2-1}{z^2}}\right)$

Involving $\cos^{-1}\left(\sqrt{\frac{z^2-1}{z^2}}\right)$ and $\csc^{-1}(z)$

01.13.27.1030.01

$$\cos^{-1}\left(\sqrt{\frac{z^2-1}{z^2}}\right) = \csc^{-1}(z) \;/;\; -\frac{\pi}{2} \le \arg(z) < \frac{\pi}{2}$$

01.13.27.1031.01

$$\cos^{-1}\left(\sqrt{\frac{z^2-1}{z^2}}\right) = -\csc^{-1}(z) \;/;\; \frac{\pi}{2} \le \arg(z) \le \pi \bigvee -\pi < \arg(z) < -\frac{\pi}{2}$$

01.13.27.1032.01

$$\cos^{-1}\left(\sqrt{\frac{z^2-1}{z^2}}\right) = z\sqrt{\frac{1}{z^2}}\csc^{-1}(z)$$

Involving $\cos^{-1}\left(2z\sqrt{1-z^2}\right)$

Involving $\cos^{-1}\left(2z\sqrt{1-z^2}\right)$ and $\csc^{-1}\left(\frac{1}{z}\right)$

01.13.27.1033.01

$$\cos^{-1}\left(2z\sqrt{1-z^2}\right) = \frac{\pi}{2} - 2\csc^{-1}\left(\frac{1}{z}\right) \;/;\; \frac{\pi}{4} \le |\arg(z)| \le \frac{3\pi}{4}$$

01.13.27.1034.01

$$\cos^{-1}\left(2 z \sqrt{1-z^2}\right) = -\frac{\pi \sqrt{1-2 z^2} \sqrt{z^4-z^2}}{2 \sqrt{-z^2} \sqrt{z^2-1} \sqrt{2 z^2-1}} \left(\frac{\sqrt{z^2}}{z} - \sqrt{\frac{1}{z}} \sqrt{z} \sqrt{\frac{1}{\sqrt{2} z+1}} \sqrt{\sqrt{2} z+1} + \sqrt{-\frac{1}{z}} \sqrt{-z} \sqrt{\frac{1}{1-\sqrt{2} z}} \sqrt{1-\sqrt{2} z} + \frac{\sqrt{z^4-z^2}}{z \sqrt{z^2-1}}\right) + \frac{2 \sqrt{1-2 z^2} \sqrt{z^4-z^2}}{\sqrt{-z^2} \sqrt{z^2-1} \sqrt{2 z^2-1}} \csc^{-1}\left(\frac{1}{z}\right) + \frac{\pi}{2}$$

Involving $\cos^{-1}\left(\frac{2\sqrt{z^2-1}}{z^2}\right)$

Involving $\cos^{-1}\left(\frac{2\sqrt{z^2-1}}{z^2}\right)$ and $\csc^{-1}(z)$

01.13.27.1035.01

$$\cos^{-1}\left(\frac{2\sqrt{z^2-1}}{z^2}\right) = \frac{\pi}{2} - 2\csc^{-1}(z) \;/;\; \frac{\pi}{4} \le |\arg(z)| < \frac{\pi}{2} \bigvee |z| \ge \sqrt{2} \bigwedge \operatorname{Re}(z) > 0$$

01.13.27.1036.01

$$\cos^{-1}\left(\frac{2\sqrt{z^2-1}}{z^2}\right) = \frac{\pi}{2} - \frac{2\sqrt{z^2}}{z}\csc^{-1}(z) \;/;\; \frac{\pi}{4} \le |\arg(z)| \le \frac{3\pi}{4} \bigvee |z| \ge \sqrt{2}$$

01.13.27.1037.01

$$\cos^{-1}\left(\frac{2\sqrt{z^2-1}}{z^2}\right) = \frac{\pi}{2} + \frac{z^3 \sqrt{z^2-2} \sqrt{z^2-1}}{2\sqrt{1-z}\,(z+1)\sqrt{-z^4+3 z^2-2}} \sqrt{\frac{1-z^2}{z^4}} \sqrt{\frac{1}{z}} \sqrt{-\frac{z+1}{z}} \left(\pi\left(\frac{z^3}{1-z^2}\sqrt{\frac{1-z^2}{z^2}}\sqrt{\frac{1-z^2}{z^4}} + \sqrt{\frac{1}{z^2}}\, z - \sqrt{\frac{1}{z}} \sqrt{\frac{z}{z+\sqrt{2}}} \sqrt{\frac{z+\sqrt{2}}{z}} \sqrt{z} + \sqrt{1-\frac{\sqrt{2}}{z}} \sqrt{-\frac{1}{z}} \sqrt{-z} \sqrt{\frac{z}{z-\sqrt{2}}}\right) - 4\csc^{-1}(z)\right)$$

Involving $\cos^{-1}\left(\sqrt{\left(1-\sqrt{1+c z^2}\right)/2}\right)$

Involving $\cos^{-1}\left(\sqrt{\left(1-\sqrt{1+z^2}\right)/2}\right)$ and $\csc^{-1}\left(\frac{i}{z}\right)$

01.13.27.1038.01

$$\cos^{-1}\left(\sqrt{\frac{1-\sqrt{1+z^2}}{2}}\right) = \frac{\pi}{2} + \frac{1}{2}\csc^{-1}\left(\frac{i}{z}\right) /; -\pi < \arg(z) \leq 0$$

01.13.27.1039.01

$$\cos^{-1}\left(\sqrt{\frac{1}{2}\left(1-\sqrt{z^2+1}\right)}\right) = \frac{\pi}{2} - \frac{1}{2}\csc^{-1}\left(\frac{i}{z}\right) /; 0 < \arg(z) \leq \pi$$

Involving $\cos^{-1}\left(\sqrt{\left(1-\sqrt{1-z^2}\right)/2}\right)$ and $\csc^{-1}\left(\frac{1}{z}\right)$

01.13.27.1040.01

$$\cos^{-1}\left(\sqrt{\frac{1-\sqrt{1-z^2}}{2}}\right) = \frac{\pi}{2} - \frac{1}{2}\csc^{-1}\left(\frac{1}{z}\right) /; -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

01.13.27.1041.01

$$\cos^{-1}\left(\sqrt{\frac{1}{2}\left(1-\sqrt{1-z^2}\right)}\right) = \frac{\pi}{2} + \frac{1}{2}\csc^{-1}\left(\frac{1}{z}\right) /; \frac{\pi}{2} < \arg(z) \leq \pi \bigvee -\pi < \arg(z) \leq -\frac{\pi}{2}$$

01.13.27.1042.01

$$\cos^{-1}\left(\sqrt{\frac{1-\sqrt{1-z^2}}{2}}\right) = \frac{\pi}{2} - \frac{\sqrt{z^2}}{2\,z}\csc^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(z\sqrt{1-\sqrt{1-z^2}}\Big/\sqrt{2\,z^2}\right)$

Involving $\cos^{-1}\left(z\sqrt{1-\sqrt{1-z^2}}\Big/\sqrt{2\,z^2}\right)$ and $\csc^{-1}\left(\frac{1}{z}\right)$

01.13.27.1043.01

$$\cos^{-1}\left(\frac{z\sqrt{1-\sqrt{1-z^2}}}{\sqrt{2}\sqrt{z^2}}\right) = \frac{\pi}{2} - \frac{1}{2}\csc^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(z\sqrt{\left(1-\sqrt{1-z^2}\right)/(2z^2)}\right)$

Involving $\cos^{-1}\left(z\sqrt{\left(1-\sqrt{1-z^2}\right)(2z^2)}\right)$ and $\csc^{-1}\left(\frac{1}{z}\right)$

01.13.27.1044.01

$$\cos^{-1}\left(z\sqrt{\frac{1-\sqrt{1-z^2}}{2z^2}}\right) = \frac{\pi}{2}-\frac{1}{2}\csc^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\sqrt{z-\sqrt{z^2-1}}\Big/\sqrt{2z}\right)$

Involving $\cos^{-1}\left(\sqrt{z-\sqrt{z^2-1}}\Big/\sqrt{2z}\right)$ and $\csc^{-1}(z)$

01.13.27.1045.01

$$\cos^{-1}\left(\frac{\sqrt{z-\sqrt{z^2-1}}}{\sqrt{2z}}\right) = \frac{\pi}{2}-\frac{1}{2}\csc^{-1}(z)\;/;\; -\frac{\pi}{2}<\arg(z)\leq\frac{\pi}{2}$$

01.13.27.1046.01

$$\cos^{-1}\left(\frac{\sqrt{z-\sqrt{z^2-1}}}{\sqrt{2z}}\right) = -\frac{1}{2}\csc^{-1}(z)\;/;\; \frac{\pi}{2}<\arg(z)<\pi \bigvee -\pi<\arg(z)<-\frac{\pi}{2} \bigvee (z\in\mathbb{R}\land z<-1)$$

01.13.27.1047.01

$$\cos^{-1}\left(\frac{\sqrt{z-\sqrt{z^2-1}}}{\sqrt{2z}}\right) = \pi+\frac{1}{2}\csc^{-1}(z)\;/;\; (z\in\mathbb{R}\land -1<z<0)$$

01.13.27.1048.01

$$\cos^{-1}\left(\frac{\sqrt{z-\sqrt{z^2-1}}}{\sqrt{2z}}\right) = \frac{1}{2}\csc^{-1}(z)\;/;\; (i\,z\in\mathbb{R}\land i\,z>0)$$

01.13.27.1049.01

$$\cos^{-1}\left(\frac{\sqrt{z-\sqrt{z^2-1}}}{\sqrt{2z}}\right) = \frac{1}{4}\pi\left(\frac{1}{\sqrt{\frac{z}{z+1}}}\sqrt{\frac{1}{z}}\sqrt{\frac{1}{z+1}}\left(\sqrt{z^2}-z\right)+2\right)-\frac{1}{2}\sqrt{\frac{i}{z}}\sqrt{\frac{1}{z}}\sqrt{-iz}\sqrt{z}\sqrt{\frac{1}{z+1}}\sqrt{z+1}\csc^{-1}(z)$$

Involving $\cos^{-1}\left(\sqrt{\left(z-\sqrt{z^2-1}\right)/(2z)}\right)$

Involving $\cos^{-1}\left(\sqrt{\left(z-\sqrt{z^2-1}\right)/(2z)}\right)$ and $\csc^{-1}(z)$

01.13.27.1050.01

$$\cos^{-1}\left(\sqrt{\frac{z-\sqrt{z^2-1}}{2z}}\right) = \frac{\pi}{2}-\frac{1}{2}\csc^{-1}(z)\ /;\ \mathrm{Re}(z)>0$$

01.13.27.1051.01

$$\cos^{-1}\left(\sqrt{\frac{z-\sqrt{z^2-1}}{2z}}\right) = -\frac{1}{2}\csc^{-1}(z)\ /;\ \mathrm{Re}(z)<0$$

01.13.27.1052.01

$$\cos^{-1}\left(\sqrt{\frac{z-\sqrt{z^2-1}}{2z}}\right) = \frac{\pi}{2}+\frac{1}{2}\csc^{-1}(z)\ /;\ (i\,z\in\mathbb{R}\land i\,z<0)$$

01.13.27.1053.01

$$\cos^{-1}\left(\sqrt{\frac{z-\sqrt{z^2-1}}{2z}}\right) = \frac{1}{2}\csc^{-1}(z)\ /;\ (i\,z\in\mathbb{R}\land i\,z>0)$$

01.13.27.1054.01

$$\cos^{-1}\left(\sqrt{\frac{z-\sqrt{z^2-1}}{2z}}\right) = -\frac{1}{2}\csc^{-1}(z)+\frac{\pi}{4}\left(\frac{\sqrt{z^2}}{z}+1\right)\ /;\ \mathrm{Re}(z)\neq 0$$

01.13.27.1055.01

$$\cos^{-1}\left(\sqrt{\frac{z-\sqrt{z^2-1}}{2z}}\right) = -\frac{1}{2}\sqrt{z^2}\sqrt{\frac{1}{z^2}}\csc^{-1}(z)+\frac{1}{4}\pi\left(\frac{\sqrt{z^2}}{z}+1\right)$$

Involving $\cos^{-1}\left(c\,z^{-r}\sqrt{-1+\frac{z^{2r}}{c^2}}\right)$

Involving $\cos^{-1}\left(c\,z^{-r}\sqrt{-1+\frac{z^{2r}}{c^2}}\right)$ and $\csc^{-1}\left(\frac{z^r}{c}\right)$

01.13.27.0038.02

$$\cos^{-1}\left(c\,z^{-r}\sqrt{\frac{z^{2c}}{c^2}-1}\right) = \frac{\pi}{2}-\sqrt{c^2 z^{-2c}}\sqrt{\frac{z^{2c}}{c^2}}\left(\frac{\pi\,z^c\sqrt{c^2 z^{-2c}}}{2c}-\csc^{-1}\left(\frac{z^c}{c}\right)\right)$$

Involving $\sec^{-1}$

Involving $\cos^{-1}(z)$

Involving $\cos^{-1}(z)$ and $\sec^{-1}\left(\frac{1}{z}\right)$

01.13.27.0041.01

$$\cos^{-1}(z) = \sec^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}(z)$ and $\sec^{-1}\left(-\frac{1}{z}\right)$

01.13.27.1056.01

$$\cos^{-1}(z) = \pi - \sec^{-1}\left(-\frac{1}{z}\right)$$

Involving $\cos^{-1}(z)$ and $\sec^{-1}\left(\frac{1}{1-2z^2}\right)$

01.13.27.1057.01

$$\cos^{-1}(z) = \frac{\pi}{2}-\frac{1}{2}\sec^{-1}\left(\frac{1}{1-2z^2}\right)\;/;\; -\frac{\pi}{2}<\arg(z)\leq\frac{\pi}{2}$$

01.13.27.1058.01

$$\cos^{-1}(z) = \frac{\pi}{2}+\frac{1}{2}\sec^{-1}\left(\frac{1}{1-2z^2}\right)\;/;\; \frac{\pi}{2}<\arg(z)\leq\pi \bigvee -\pi<\arg(z)\leq-\frac{\pi}{2}$$

01.13.27.1059.01

$$\cos^{-1}(z) == \frac{\pi}{2} - \frac{\sqrt{z^2}}{2\,z} \sec^{-1}\left(\frac{1}{1-2\,z^2}\right)$$

Involving $\cos^{-1}(z)$ and $\sec^{-1}\left(\frac{1}{2\,z^2-1}\right)$

01.13.27.1060.01

$$\cos^{-1}(z) == \frac{1}{2} \sec^{-1}\left(\frac{1}{2\,z^2-1}\right) /;\ -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

01.13.27.1061.01

$$\cos^{-1}(z) == \pi - \frac{1}{2} \sec^{-1}\left(\frac{1}{2\,z^2-1}\right) /;\ \frac{\pi}{2} < \arg(z) \leq \pi \bigvee -\pi < \arg(z) \leq -\frac{\pi}{2}$$

01.13.27.1062.01

$$\cos^{-1}(z) == \frac{\pi}{2}\left(1 - \frac{\sqrt{z^2}}{z}\right) + \frac{\sqrt{z^2}}{2\,z} \sec^{-1}\left(\frac{1}{2\,z^2-1}\right)$$

Involving $\cos^{-1}(z)$ and $\sec^{-1}\left(\frac{\sqrt{2}}{\sqrt{1+z}}\right)$

01.13.27.1063.01

$$\cos^{-1}(z) == 2 \sec^{-1}\left(\frac{\sqrt{2}}{\sqrt{1+z}}\right)$$

Involving $\cos^{-1}(z)$ and $\sec^{-1}\left(\sqrt{\frac{2}{1+z}}\right)$

01.13.27.1064.01

$$\cos^{-1}(z) == 2 \sec^{-1}\left(\sqrt{\frac{2}{1+z}}\right) /;\ z \notin (-\infty, -1)$$

01.13.27.1065.01

$$\cos^{-1}(z) == 2\,\pi - 2 \sec^{-1}\left(\sqrt{\frac{2}{1+z}}\right) /;\ (z \in \mathbb{R} \land z < -1)$$

01.13.27.1066.01

$$\cos^{-1}(z) == \pi\left(1 - \sqrt{z+1}\,\sqrt{\frac{1}{z+1}}\right) + 2\,\sqrt{z+1}\,\sqrt{\frac{1}{z+1}}\,\sec^{-1}\left(\sqrt{\frac{2}{z+1}}\right)$$

Involving $\cos^{-1}(z)$ and $\sec^{-1}\left(\frac{\sqrt{2}}{\sqrt{1-z}}\right)$

01.13.27.1067.01

$$\cos^{-1}(z) = \pi - 2\sec^{-1}\left(\frac{\sqrt{2}}{\sqrt{1-z}}\right)$$

Involving $\cos^{-1}(z)$ and $\sec^{-1}\left(\sqrt{\frac{2}{1-z}}\right)$

01.13.27.1068.01

$$\cos^{-1}(z) = \pi - 2\sec^{-1}\left(\sqrt{\frac{2}{1-z}}\right) /; z \notin (1, \infty)$$

01.13.27.1069.01

$$\cos^{-1}(z) = -\pi + 2\sec^{-1}\left(\sqrt{\frac{2}{1-z}}\right) /; (z \in \mathbb{R} \land z > 1)$$

01.13.27.1070.01

$$\cos^{-1}(z) = -2\sqrt{1-z}\sqrt{\frac{1}{1-z}}\sec^{-1}\left(\sqrt{\frac{2}{1-z}}\right) + \pi\sqrt{1-z}\sqrt{\frac{1}{1-z}}$$

Involving $\cos^{-1}(z)$ and $\sec^{-1}\left(\frac{1}{\sqrt{z^2}}\right)$

01.13.27.1071.01

$$\cos^{-1}(z) = \sec^{-1}\left(\frac{1}{\sqrt{z^2}}\right) /; -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

01.13.27.1072.01

$$\cos^{-1}(z) = \pi - \sec^{-1}\left(\frac{1}{\sqrt{z^2}}\right) /; \frac{\pi}{2} < \arg(z) \leq \pi \bigvee -\pi < \arg(z) \leq -\frac{\pi}{2}$$

01.13.27.1073.01

$$\cos^{-1}(z) = \frac{\pi}{2}\left(1 - \frac{\sqrt{z^2}}{z}\right) + \frac{\sqrt{z^2}}{z}\sec^{-1}\left(\frac{1}{\sqrt{z^2}}\right)$$

Involving $\cos^{-1}(z)$ and $\sec^{-1}\left(\sqrt{\frac{1}{z^2}}\right)$

01.13.27.1074.01

$$\cos^{-1}(z) = \sec^{-1}\left(\sqrt{\frac{1}{z^2}}\right) /; -\frac{\pi}{2} \leq \arg(z) < \frac{\pi}{2}$$

01.13.27.1075.01

$$\cos^{-1}(z) = \pi - \sec^{-1}\left(\sqrt{\frac{1}{z^2}}\right) /; \frac{\pi}{2} \leq \arg(z) \leq \pi \bigvee -\pi < \arg(z) < -\frac{\pi}{2}$$

01.13.27.1076.01

$$\cos^{-1}(z) = \frac{\pi}{2} - z\sqrt{\frac{1}{z^2}}\left(\frac{\pi}{2} - \sec^{-1}\left(\sqrt{\frac{1}{z^2}}\right)\right)$$

Involving $\cos^{-1}(z)$ and $\sec^{-1}\left(\frac{1}{\sqrt{1-z^2}}\right)$

01.13.27.1077.01

$$\cos^{-1}(z) = \frac{\pi}{2} - \sec^{-1}\left(\frac{1}{\sqrt{1-z^2}}\right) /; -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

01.13.27.1078.01

$$\cos^{-1}(z) = \frac{\pi}{2} + \sec^{-1}\left(\frac{1}{\sqrt{1-z^2}}\right) /; \frac{\pi}{2} < \arg(z) \leq \pi \bigvee -\pi < \arg(z) \leq -\frac{\pi}{2}$$

01.13.27.1079.01

$$\cos^{-1}(z) = \frac{\pi}{2} - \frac{\sqrt{z^2}}{z}\sec^{-1}\left(\frac{1}{\sqrt{1-z^2}}\right)$$

Involving $\cos^{-1}(z)$ and $\sec^{-1}\left(\sqrt{\frac{1}{1-z^2}}\right)$

01.13.27.1080.01

$$\cos^{-1}(z) = \frac{\pi}{2} - \sec^{-1}\left(\sqrt{\frac{1}{1-z^2}}\right) /; 0 < \arg(z) \leq \frac{\pi}{2} \bigvee -\frac{\pi}{2} < \arg(z) < 0 \bigvee (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.1081.01

$$\cos^{-1}(z) = \sec^{-1}\left(\sqrt{\frac{1}{1-z^2}}\right) + \frac{\pi}{2} /; \frac{\pi}{2} < \arg(z) < \pi \bigvee -\pi < \arg(z) < -\frac{\pi}{2} \bigvee (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.1082.01

$$\cos^{-1}(z) = -\frac{\pi}{2} + \sec^{-1}\left(\sqrt{\frac{1}{1-z^2}}\right) /; (z \in \mathbb{R} \land z > 1)$$

01.13.27.1083.01

$$\cos^{-1}(z) = \frac{3\pi}{2} - \sec^{-1}\left(\sqrt{\frac{1}{1-z^2}}\right) /; (z \in \mathbb{R} \land z < -1)$$

01.13.27.1084.01

$$\cos^{-1}(z) = \frac{1}{2}\left(\sqrt{\frac{1}{1-z}}\,\sqrt{1-z} - \sqrt{z+1}\,\sqrt{\frac{1}{z+1}} + 1\right)\pi - \frac{\sqrt{1-z^2}}{z}\,\sqrt{\frac{z^2}{1-z^2}}\,\sec^{-1}\left(\sqrt{\frac{1}{1-z^2}}\right)$$

Involving $\cos^{-1}(z)$ and $\sec^{-1}\left(\frac{1}{2z\sqrt{1-z^2}}\right)$

01.13.27.1085.01

$$\cos^{-1}(z) = \frac{\pi}{4} + \frac{1}{2}\sec^{-1}\left(\frac{1}{2z\sqrt{1-z^2}}\right) /; \frac{\pi}{4} \leq |\arg(z)| \leq \frac{3\pi}{4}$$

01.13.27.1086.01

$$\cos^{-1}(z) = \frac{\pi}{2} - \frac{\pi}{4}\left(\frac{\sqrt{z^2}}{z} - \sqrt{\frac{1}{z}}\,\sqrt{z}\,\sqrt{\frac{1}{\sqrt{2}\,z+1}}\,\sqrt{\sqrt{2}\,z+1} + \sqrt{-\frac{1}{z}}\,\sqrt{-z}\,\sqrt{\frac{1}{1-\sqrt{2}\,z}}\,\sqrt{1-\sqrt{2}\,z} + \frac{\sqrt{z^4-z^2}}{z\sqrt{z^2-1}}\right) + \frac{\sqrt{-z^2}\,\sqrt{z^2-1}\,\sqrt{2z^2-1}}{2\sqrt{1-2z^2}\,\sqrt{z^4-z^2}}\left(\frac{\pi}{2} - \sec^{-1}\left(\frac{1}{2z\sqrt{1-z^2}}\right)\right)$$

Involving $\cos^{-1}(-z)$

Involving $\cos^{-1}(-z)$ and $\sec^{-1}\left(\frac{1}{z}\right)$

01.13.27.1087.01

$$\cos^{-1}(-z) = \pi - \sec^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\sqrt{z}\right)$

Involving $\cos^{-1}\left(\sqrt{z}\right)$ and $\sec^{-1}\left(\frac{1}{\sqrt{z}}\right)$

01.13.27.1088.01

$$\cos^{-1}\left(\sqrt{z}\right) = \sec^{-1}\left(\frac{1}{\sqrt{z}}\right)$$

Involving $\cos^{-1}\left(\sqrt{z}\right)$ and $\sec^{-1}\left(\sqrt{\frac{1}{z}}\right)$

01.13.27.1089.01

$$\cos^{-1}\left(\sqrt{z}\right) = \sec^{-1}\left(\sqrt{\frac{1}{z}}\right) /; |\arg(z)| < \pi$$

01.13.27.1090.01

$$\cos^{-1}\left(\sqrt{z}\right) = \pi - \sec^{-1}\left(\sqrt{\frac{1}{z}}\right) /; (z \in \mathbb{R} \land z < 0)$$

01.13.27.1091.01

$$\cos^{-1}\left(\sqrt{z}\right) = \frac{\pi}{2}\left(1 - \sqrt{z}\sqrt{\frac{1}{z}}\right) + \sqrt{\frac{1}{z}}\sqrt{z}\sec^{-1}\left(\sqrt{\frac{1}{z}}\right)$$

Involving $\cos^{-1}\left(\frac{1}{\sqrt{z}}\right)$

Involving $\cos^{-1}\left(\frac{1}{\sqrt{z}}\right)$ and $\sec^{-1}\left(\sqrt{z}\right)$

01.13.27.1092.01

$$\cos^{-1}\left(\frac{1}{\sqrt{z}}\right) = \sec^{-1}\left(\sqrt{z}\right)$$

Involving $\cos^{-1}\left(\frac{1}{\sqrt{z}}\right)$ and $\sec^{-1}\left(1 \Big/ \sqrt{\frac{1}{z}}\right)$

01.13.27.1093.01

$$\cos^{-1}\left(\frac{1}{\sqrt{z}}\right) = \sec^{-1}\left(1 \Big/ \sqrt{\frac{1}{z}}\right) /; |\arg(z)| < \pi$$

01.13.27.1094.01

$$\cos^{-1}\left(\frac{1}{\sqrt{z}}\right) = \pi - \sec^{-1}\left(1 \Big/ \sqrt{\frac{1}{z}}\right) /; (z \in \mathbb{R} \land z < 0)$$

01.13.27.1095.01

$$\cos^{-1}\left(\frac{1}{\sqrt{z}}\right) = \frac{\pi}{2}\left(1 - \sqrt{z}\sqrt{\frac{1}{z}}\right) + \sqrt{z}\sqrt{\frac{1}{z}}\sec^{-1}\left(1 \Big/ \sqrt{\frac{1}{z}}\right)$$

Involving $\cos^{-1}\left(\sqrt{z^2}\right)$

Involving $\cos^{-1}\left(\sqrt{z^2}\right)$ and $\sec^{-1}\left(\frac{1}{z}\right)$

01.13.27.1096.01

$$\cos^{-1}\left(\sqrt{z^2}\right) = \sec^{-1}\left(\frac{1}{z}\right) /; -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

01.13.27.1097.01

$$\cos^{-1}\left(\sqrt{z^2}\right) = \pi - \sec^{-1}\left(\frac{1}{z}\right) /; \frac{\pi}{2} < \arg(z) \leq \pi \bigvee -\pi < \arg(z) \leq -\frac{\pi}{2}$$

01.13.27.1098.01

$$\cos^{-1}\left(\sqrt{z^2}\right) = \frac{\pi}{2} - \frac{\sqrt{z^2}}{z}\left(\frac{\pi}{2} - \sec^{-1}\left(\frac{1}{z}\right)\right)$$

Involving $\cos^{-1}\left(a\left(b z^c\right)^m\right)$

Involving $\cos^{-1}(a (b z^c)^m)$ and $\sec^{-1}\left(\frac{1}{a b^m z^{m c}}\right)$

01.13.27.1099.01

$$\cos^{-1}(a (b z^c)^m) = \frac{\pi}{2} - \frac{(b z^c)^m}{b^m z^{m c}}\left(\frac{\pi}{2} - \sec^{-1}\left(\frac{1}{a} b^{-m} z^{-m c}\right)\right) /; 2 m \in \mathbb{Z}$$

Involving $\cos^{-1}\left(1 - 2 z^2\right)$

Involving $\cos^{-1}\left(1 - 2 z^2\right)$ and $\sec^{-1}\left(\frac{1}{z}\right)$

01.13.27.1100.01

$$\cos^{-1}\left(1 - 2 z^2\right) = \pi - 2 \sec^{-1}\left(\frac{1}{z}\right) /; -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

01.13.27.1101.01

$$\cos^{-1}\left(1 - 2 z^2\right) = -\pi + 2 \sec^{-1}\left(\frac{1}{z}\right) /; \frac{\pi}{2} < \arg(z) \leq \pi \bigvee -\pi < \arg(z) \leq -\frac{\pi}{2}$$

01.13.27.1102.01

$$\cos^{-1}\left(1 - 2 z^2\right) = \frac{2\sqrt{z^2}}{z}\left(\frac{\pi}{2} - \sec^{-1}\left(\frac{1}{z}\right)\right)$$

Involving $\cos^{-1}\left(2 z^2 - 1\right)$

Involving $\cos^{-1}\left(2 z^2 - 1\right)$ and $\sec^{-1}\left(\frac{1}{z}\right)$

01.13.27.1103.01

$$\cos^{-1}\left(2 z^2 - 1\right) = 2 \sec^{-1}\left(\frac{1}{z}\right) /; -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

01.13.27.1104.01

$$\cos^{-1}\left(2 z^2 - 1\right) = 2 \pi - 2 \sec^{-1}\left(\frac{1}{z}\right) /; \frac{\pi}{2} < \arg(z) \leq \pi \bigvee -\pi < \arg(z) \leq -\frac{\pi}{2}$$

01.13.27.1105.01

$$\cos^{-1}\left(2z^2-1\right) = \pi\left(1-\frac{\sqrt{z^2}}{z}\right)+\frac{2\sqrt{z^2}}{z}\sec^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\frac{z^2-2}{z^2}\right)$

Involving $\cos^{-1}\left(\frac{z^2-2}{z^2}\right)$ and $\sec^{-1}(z)$

01.13.27.1106.01

$$\cos^{-1}\left(\frac{z^2-2}{z^2}\right) = \pi-2\sec^{-1}(z)\ /;\ -\frac{\pi}{2}\leq\arg(z)<\frac{\pi}{2}$$

01.13.27.1107.01

$$\cos^{-1}\left(\frac{z^2-2}{z^2}\right) = -\pi+2\sec^{-1}(z)\ /;\ \frac{\pi}{2}\leq\arg(z)\leq\pi\bigvee-\pi<\arg(z)<-\frac{\pi}{2}$$

01.13.27.1108.01

$$\cos^{-1}\left(\frac{z^2-2}{z^2}\right) = \sqrt{\frac{1}{z^2}}\,z\left(\pi-2\sec^{-1}(z)\right)$$

Involving $\cos^{-1}\left(\frac{2-z^2}{z^2}\right)$

Involving $\cos^{-1}\left(\frac{2-z^2}{z^2}\right)$ and $\sec^{-1}(z)$

01.13.27.1109.01

$$\cos^{-1}\left(\frac{2-z^2}{z^2}\right) = 2\sec^{-1}(z)\ /;\ -\frac{\pi}{2}\leq\arg(z)<\frac{\pi}{2}$$

01.13.27.1110.01

$$\cos^{-1}\left(\frac{2-z^2}{z^2}\right) = 2\pi-2\sec^{-1}(z)\ /;\ \frac{\pi}{2}\leq\arg(z)\leq\pi\bigvee-\pi<\arg(z)<-\frac{\pi}{2}$$

01.13.27.1111.01

$$\cos^{-1}\left(\frac{2-z^2}{z^2}\right) = \pi\left(1-\sqrt{\frac{1}{z^2}}\,z\right)+2z\sqrt{\frac{1}{z^2}}\sec^{-1}(z)$$

Involving $\cos^{-1}\left(\sqrt{1-z}\right)$

Involving $\cos^{-1}\left(\sqrt{1-z}\right)$ and $\sec^{-1}\left(\frac{1}{\sqrt{z}}\right)$

01.13.27.1112.01

$$\cos^{-1}\left(\sqrt{1-z}\right) = \frac{\pi}{2} - \sec^{-1}\left(\frac{1}{\sqrt{z}}\right)$$

Involving $\cos^{-1}\left(\sqrt{1-z}\right)$ and $\sec^{-1}\left(\sqrt{\frac{1}{z}}\right)$

01.13.27.1113.01

$$\cos^{-1}\left(\sqrt{1-z}\right) = \frac{\pi}{2} - \sec^{-1}\left(\sqrt{\frac{1}{z}}\right) /; |\arg(z)| < \pi$$

01.13.27.1114.01

$$\cos^{-1}\left(\sqrt{1-z}\right) = -\frac{\pi}{2} + \sec^{-1}\left(\sqrt{\frac{1}{z}}\right) /; (z \in \mathbb{R} \land z < 0)$$

01.13.27.1115.01

$$\cos^{-1}\left(\sqrt{1-z}\right) = \sqrt{z} \sqrt{\frac{1}{z}} \left(\frac{\pi}{2} - \sec^{-1}\left(\sqrt{\frac{1}{z}}\right)\right)$$

Involving $\cos^{-1}\left(\sqrt{\frac{1+c\,z}{2}}\right)$

Involving $\cos^{-1}\left(\sqrt{\frac{1+z}{2}}\right)$ and $\sec^{-1}\left(\frac{1}{z}\right)$

01.13.27.1116.01

$$\cos^{-1}\left(\sqrt{\frac{z+1}{2}}\right) = \frac{1}{2} \sec^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\sqrt{\frac{1-z}{2}}\right)$ and $\sec^{-1}\left(\frac{1}{z}\right)$

01.13.27.1117.01

$$\cos^{-1}\left(\sqrt{\frac{1-z}{2}}\right) = \frac{\pi}{2} - \frac{1}{2} \sec^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{z-1}}{\sqrt{z}}\right)$

Involving $\cos^{-1}\left(\frac{\sqrt{z-1}}{\sqrt{z}}\right)$ and $\sec^{-1}\left(\sqrt{z}\right)$

01.13.27.1118.01

$$\cos^{-1}\left(\frac{\sqrt{z-1}}{\sqrt{z}}\right) = \frac{\pi}{2} - \sec^{-1}\left(\sqrt{z}\right) /; |\arg(z)| < \pi$$

01.13.27.1119.01

$$\cos^{-1}\left(\frac{\sqrt{z-1}}{\sqrt{z}}\right) = -\frac{\pi}{2} + \sec^{-1}\left(\sqrt{z}\right) /; (z \in \mathbb{R} \land z < 0)$$

01.13.27.1120.01

$$\cos^{-1}\left(\frac{\sqrt{z-1}}{\sqrt{z}}\right) = \sqrt{\frac{1}{z}} \sqrt{z} \left(\frac{\pi}{2} - \sec^{-1}\left(\sqrt{z}\right)\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{-z}}\right)$

Involving $\cos^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{-z}}\right)$ and $\sec^{-1}\left(\sqrt{z}\right)$

01.13.27.1121.01

$$\cos^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{-z}}\right) = \frac{\pi}{2} - \sec^{-1}\left(\sqrt{z}\right) /; z \notin (-\infty, 1)$$

01.13.27.1122.01

$$\cos^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{-z}}\right) = \frac{\pi}{2} + \sec^{-1}\left(\sqrt{z}\right) /; (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.1123.01

$$\cos^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{-z}}\right) = -\frac{\pi}{2} + \sec^{-1}\left(\sqrt{z}\right) /; (z \in \mathbb{R} \land z < 0)$$

01.13.27.1124.01

$$\cos^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{-z}}\right) = \frac{\pi}{2} - \sqrt{\frac{z-1}{z}} \sqrt{\frac{z}{z-1}} \left(\frac{1}{2}\pi\left(1 - \sqrt{\frac{1}{z}} \sqrt{z}\right) + \sqrt{\frac{1}{z}} \sqrt{z} \sec^{-1}\left(\sqrt{z}\right)\right)$$

Involving $\cos^{-1}\left(\sqrt{\frac{z-1}{z}}\right)$

Involving $\cos^{-1}\left(\sqrt{\frac{z-1}{z}}\right)$ and $\sec^{-1}\left(\sqrt{z}\right)$

01.13.27.1125.01

$$\cos^{-1}\left(\sqrt{\frac{z-1}{z}}\right) = \frac{\pi}{2} - \sec^{-1}\left(\sqrt{z}\right) /; |\arg(z)| < \pi$$

01.13.27.1126.01

$$\cos^{-1}\left(\sqrt{\frac{z-1}{z}}\right) = -\frac{\pi}{2} + \sec^{-1}\left(\sqrt{z}\right) /; (z \in \mathbb{R} \land z < 0)$$

01.13.27.1127.01

$$\cos^{-1}\left(\sqrt{\frac{z-1}{z}}\right) = \sqrt{\frac{1}{z}} \sqrt{z} \left(\frac{\pi}{2} - \sec^{-1}\left(\sqrt{z}\right)\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{z+a}}{\sqrt{2z}}\right)$

Involving $\cos^{-1}\left(\frac{\sqrt{z-1}}{\sqrt{2z}}\right)$ and $\sec^{-1}(z)$

01.13.27.1128.01

$$\cos^{-1}\left(\frac{\sqrt{z-1}}{\sqrt{2z}}\right) = \frac{\pi}{2} - \frac{1}{2}\sec^{-1}(z)$$

Involving $\cos^{-1}\left(\frac{\sqrt{z+1}}{\sqrt{2z}}\right)$ and $\sec^{-1}(z)$

01.13.27.0043.01

$$\cos^{-1}\left(\frac{\sqrt{z+1}}{\sqrt{2z}}\right) = \frac{1}{2}\sec^{-1}(z) /; z \notin (-1, 0)$$

01.13.27.1129.01

$$\cos^{-1}\left(\frac{\sqrt{z+1}}{\sqrt{2z}}\right) = \pi - \frac{1}{2}\sec^{-1}(z) /; (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.1130.01

$$\cos^{-1}\left(\frac{\sqrt{z+1}}{\sqrt{2z}}\right) = \frac{\pi}{2}\left(1 - \sqrt{\frac{z+1}{z}} \sqrt{\frac{z}{z+1}}\right) + \frac{1}{2}\sqrt{\frac{z}{z+1}} \sqrt{\frac{z+1}{z}} \sec^{-1}(z)$$

Involving $\cos^{-1}\left(\frac{\sqrt{a-z}}{\sqrt{-2z}}\right)$

Involving $\cos^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{-2z}}\right)$ and $\sec^{-1}(z)$

01.13.27.1131.01

$$\cos^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{-2z}}\right) = \frac{\pi}{2} - \frac{1}{2}\sec^{-1}(z) /; z \notin (0, 1)$$

01.13.27.1132.01

$$\cos^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{-2z}}\right) = \frac{\pi}{2} + \frac{1}{2}\sec^{-1}(z) /; (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.1133.01

$$\cos^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{-2z}}\right) = \frac{\pi}{2} - \frac{1}{2}\sqrt{\frac{z-1}{z}}\sqrt{\frac{z}{z-1}}\sec^{-1}(z)$$

Involving $\cos^{-1}\left(\frac{\sqrt{-z-1}}{\sqrt{-2z}}\right)$ and $\sec^{-1}(z)$

01.13.27.1134.01

$$\cos^{-1}\left(\frac{\sqrt{-z-1}}{\sqrt{-2z}}\right) = \frac{1}{2}\sec^{-1}(z)$$

Involving $\cos^{-1}\left(\sqrt{\frac{z+a}{2z}}\right)$

Involving $\cos^{-1}\left(\sqrt{\frac{z-1}{2z}}\right)$ and $\sec^{-1}(z)$

01.13.27.1135.01

$$\cos^{-1}\left(\sqrt{\frac{z-1}{2z}}\right) = \frac{\pi}{2} - \frac{1}{2}\sec^{-1}(z)$$

Involving $\cos^{-1}\left(\sqrt{\frac{z+1}{2z}}\right)$ and $\sec^{-1}(z)$

01.13.27.0042.01

$$\cos^{-1}\left(\sqrt{\frac{z+1}{2z}}\right) = \frac{1}{2}\sec^{-1}(z)$$

Involving $\cos^{-1}\left(\sqrt{1-z^2}\right)$

Involving $\cos^{-1}\left(\sqrt{1-z^2}\right)$ and $\sec^{-1}\left(\frac{1}{z}\right)$

01.13.27.1136.01

$$\cos^{-1}\left(\sqrt{1-z^2}\right) = \frac{\pi}{2} - \sec^{-1}\left(\frac{1}{z}\right) /; -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

01.13.27.1137.01

$$\cos^{-1}\left(\sqrt{1-z^2}\right) = -\frac{\pi}{2} + \sec^{-1}\left(\frac{1}{z}\right) /; \frac{\pi}{2} < \arg(z) \leq \pi \bigvee -\pi < \arg(z) \leq -\frac{\pi}{2}$$

01.13.27.1138.01

$$\cos^{-1}\left(\sqrt{1-z^2}\right) = \frac{\sqrt{z^2}}{z}\left(\frac{\pi}{2} - \sec^{-1}\left(\frac{1}{z}\right)\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{z^2-1}}{z}\right)$

Involving $\cos^{-1}\left(\frac{\sqrt{z^2-1}}{z}\right)$ and $\sec^{-1}(z)$

01.13.27.1139.01

$$\cos^{-1}\left(\frac{\sqrt{z^2-1}}{z}\right) = \frac{\pi}{2} - \sec^{-1}(z) \;/;\; \mathrm{Re}(z) > 0$$

01.13.27.1140.01

$$\cos^{-1}\left(\frac{\sqrt{z^2-1}}{z}\right) = \frac{3\pi}{2} - \sec^{-1}(z) \;/;\; \mathrm{Re}(z) < 0$$

01.13.27.1141.01

$$\cos^{-1}\left(\frac{\sqrt{z^2-1}}{z}\right) = -\frac{\pi}{2} + \sec^{-1}(z) \;/;\; (i\,z \in \mathbb{R} \land i\,z < 0)$$

01.13.27.1142.01

$$\cos^{-1}\left(\frac{\sqrt{z^2-1}}{z}\right) = \frac{\pi}{2} + \sec^{-1}(z) \;/;\; (i\,z \in \mathbb{R} \land i\,z > 0)$$

01.13.27.1143.01

$$\cos^{-1}\left(\frac{\sqrt{z^2-1}}{z}\right) = \frac{\pi}{2}\left(2 - \frac{\sqrt{z^2}}{z}\right) - \sec^{-1}(z) \;/;\; \mathrm{Re}(z) \neq 0$$

01.13.27.1144.01

$$\cos^{-1}\left(\frac{\sqrt{z^2-1}}{z}\right) = \frac{\pi}{2}\left(\sqrt{\frac{1}{z^2}}\,\sqrt{z^2} - \frac{\sqrt{z^2}}{z} + 1\right) - \sqrt{z^2}\,\sqrt{\frac{1}{z^2}}\,\sec^{-1}(z)$$

Involving $\cos^{-1}\left(\frac{\sqrt{z^2-1}}{\sqrt{z^2}}\right)$

Involving $\cos^{-1}\left(\frac{\sqrt{z^2-1}}{\sqrt{z^2}}\right)$ and $\sec^{-1}(z)$

01.13.27.1145.01

$$\cos^{-1}\left(\frac{\sqrt{z^2-1}}{\sqrt{z^2}}\right) = \frac{\pi}{2} - \sec^{-1}(z) \;/;\; -\frac{\pi}{2} \leq \arg(z) < \frac{\pi}{2}$$

01.13.27.1146.01

$$\cos^{-1}\left(\frac{\sqrt{z^2-1}}{\sqrt{z^2}}\right) = -\frac{\pi}{2} + \sec^{-1}(z) \,/;\, \frac{\pi}{2} \le \arg(z) \le \pi \bigvee -\pi < \arg(z) < -\frac{\pi}{2}$$

01.13.27.1147.01

$$\cos^{-1}\left(\frac{\sqrt{z^2-1}}{\sqrt{z^2}}\right) = z\sqrt{\frac{1}{z^2}}\left(\frac{\pi}{2} - \sec^{-1}(z)\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{1-z^2}}{\sqrt{-z^2}}\right)$

Involving $\cos^{-1}\left(\frac{\sqrt{1-z^2}}{\sqrt{-z^2}}\right)$ and $\sec^{-1}(z)$

01.13.27.1148.01

$$\cos^{-1}\left(\frac{\sqrt{1-z^2}}{\sqrt{-z^2}}\right) = \frac{\pi}{2} - \sec^{-1}(z) \,/;\, -\frac{\pi}{2} \le \arg(z) < 0 \bigvee 0 < \arg(z) < \frac{\pi}{2} \bigvee (z \in \mathbb{R} \land z > 1)$$

01.13.27.1149.01

$$\cos^{-1}\left(\frac{\sqrt{1-z^2}}{\sqrt{-z^2}}\right) = -\frac{\pi}{2} + \sec^{-1}(z) \,/;\, \frac{\pi}{2} \le \arg(z) < \pi \bigvee -\pi < \arg(z) < -\frac{\pi}{2} \bigvee (z \in \mathbb{R} \land z < -1)$$

01.13.27.1150.01

$$\cos^{-1}\left(\frac{\sqrt{1-z^2}}{\sqrt{-z^2}}\right) = \frac{\pi}{2} + \sec^{-1}(z) \,/;\, (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.1151.01

$$\cos^{-1}\left(\frac{\sqrt{1-z^2}}{\sqrt{-z^2}}\right) = \frac{3\pi}{2} - \sec^{-1}(z) \,/;\, (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.1152.01

$$\cos^{-1}\left(\frac{\sqrt{1-z^2}}{\sqrt{-z^2}}\right) = \frac{\pi}{2} - \sqrt{1-\frac{1}{z^2}}\sqrt{\frac{z^2}{z^2-1}}\left(\frac{1}{2}\pi\left(1-\sqrt{\frac{1}{z^2}}\,z\right) + z\sqrt{\frac{1}{z^2}}\sec^{-1}(z)\right)$$

Involving $\cos^{-1}\left(\sqrt{\frac{z^2-1}{z^2}}\right)$

Involving $\cos^{-1}\left(\sqrt{\frac{z^2-1}{z^2}}\right)$ and $\sec^{-1}(z)$

01.13.27.1153.01

$$\cos^{-1}\left(\sqrt{\frac{z^2-1}{z^2}}\right) = \frac{\pi}{2} - \sec^{-1}(z) \text{ /; } -\frac{\pi}{2} \le \arg(z) < \frac{\pi}{2}$$

01.13.27.1154.01

$$\cos^{-1}\left(\sqrt{\frac{z^2-1}{z^2}}\right) = -\frac{\pi}{2} + \sec^{-1}(z) \text{ /; } \frac{\pi}{2} \le \arg(z) \le \pi \bigvee -\pi < \arg(z) < -\frac{\pi}{2}$$

01.13.27.1155.01

$$\cos^{-1}\left(\sqrt{\frac{z^2-1}{z^2}}\right) = z\sqrt{\frac{1}{z^2}}\left(\frac{\pi}{2} - \sec^{-1}(z)\right)$$

Involving $\cos^{-1}\left(2z\sqrt{1-z^2}\right)$

Involving $\cos^{-1}\left(2z\sqrt{1-z^2}\right)$ and $\sec^{-1}\left(\frac{1}{z}\right)$

01.13.27.1156.01

$$\cos^{-1}\left(2z\sqrt{1-z^2}\right) = 2\sec^{-1}\left(\frac{1}{z}\right) - \frac{\pi}{2} \text{ /; } \frac{\pi}{4} \le |\arg(z)| \le \frac{3\pi}{4}$$

01.13.27.1157.01

$$\cos^{-1}\left(2z\sqrt{1-z^2}\right) = \frac{\pi}{2} + \frac{\pi\sqrt{1-2z^2}\sqrt{z^4-z^2}}{2\sqrt{-z^2}\sqrt{z^2-1}\sqrt{2z^2-1}} \left(-\frac{\sqrt{z^2}}{z} + \sqrt{\frac{1}{z}}\sqrt{z}\sqrt{\frac{1}{\sqrt{2}\,z+1}}\sqrt{\sqrt{2}\,z+1} - \sqrt{-\frac{1}{z}}\sqrt{-z}\sqrt{\frac{1}{1-\sqrt{2}\,z}}\sqrt{1-\sqrt{2}\,z} - \frac{\sqrt{z^4-z^2}}{z\sqrt{z^2-1}} + 2\right) - \frac{2\sqrt{1-2z^2}\sqrt{z^4-z^2}}{\sqrt{-z^2}\sqrt{z^2-1}\sqrt{2z^2-1}}\sec^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\frac{2\sqrt{z^2-1}}{z^2}\right)$

Involving $\cos^{-1}\left(\frac{2\sqrt{z^2-1}}{z^2}\right)$ and $\sec^{-1}(z)$

01.13.27.1158.01

$$\cos^{-1}\left(\frac{2\sqrt{z^2-1}}{z^2}\right) = -\frac{\pi}{2} + 2\sec^{-1}(z) \text{ /; } \frac{\pi}{4} \le |\arg(z)| < \frac{\pi}{2} \bigvee |z| \ge \sqrt{2} \bigwedge \operatorname{Re}(z) > 0$$

01.13.27.1159.01

$$\cos^{-1}\left(\frac{2\sqrt{z^2-1}}{z^2}\right) = \frac{\pi}{2} - \frac{\sqrt{z^2}}{z}\left(\pi - 2\sec^{-1}(z)\right) /; \frac{\pi}{4} \leq |\arg(z)| \leq \frac{3\pi}{4} \bigvee |z| \geq \sqrt{2}$$

01.13.27.1160.01

$$\cos^{-1}\left(\frac{2\sqrt{z^2-1}}{z^2}\right) = \frac{\pi}{2} + \frac{z^3\sqrt{z^2-2}\sqrt{z^2-1}}{2\sqrt{1-z}\,(z+1)\sqrt{-z^4+3z^2-2}}\sqrt{\frac{1-z^2}{z^4}}\sqrt{\frac{1}{z}}\sqrt{-\frac{z+1}{z}}\left(\pi\left(\frac{z^3}{1-z^2}\sqrt{\frac{1-z^2}{z^2}}\sqrt{\frac{1-z^2}{z^4}} + \sqrt{\frac{1}{z^2}}\,z - \sqrt{\frac{1}{z}}\sqrt{\frac{z}{z+\sqrt{2}}}\sqrt{\frac{z+\sqrt{2}}{z}}\sqrt{z} + \sqrt{1-\frac{\sqrt{2}}{z}}\sqrt{-\frac{1}{z}}\sqrt{-z}\sqrt{\frac{z}{z-\sqrt{2}}} - 2\right) + 4\sec^{-1}(z)\right)$$

Involving $\cos^{-1}\left(\sqrt{\left(1-\sqrt{1+c\,z^2}\right)/2}\right)$

Involving $\cos^{-1}\left(\sqrt{\left(1-\sqrt{1+z^2}\right)/2}\right)$ and $\sec^{-1}\left(-\frac{i}{z}\right)$

01.13.27.1161.01

$$\cos^{-1}\left(\sqrt{\frac{1-\sqrt{1+z^2}}{2}}\right) = \frac{\pi}{4} + \frac{1}{2}\sec^{-1}\left(-\frac{i}{z}\right) /; -\pi < \arg(z) \leq 0$$

01.13.27.1162.01

$$\cos^{-1}\left(\sqrt{\frac{1}{2}\left(1-\sqrt{z^2+1}\right)}\right) = \frac{3\pi}{4} - \frac{1}{2}\sec^{-1}\left(-\frac{i}{z}\right) /; 0 < \arg(z) \leq \pi$$

Involving $\cos^{-1}\left(\sqrt{\left(1-\sqrt{1-z^2}\right)/2}\right)$ and $\sec^{-1}\left(\frac{1}{z}\right)$

01.13.27.1163.01

$$\cos^{-1}\left(\sqrt{\frac{1-\sqrt{1-z^2}}{2}}\right) = \frac{\pi}{4} + \frac{1}{2}\sec^{-1}\left(\frac{1}{z}\right) /; -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

01.13.27.1164.01

$$\cos^{-1}\left(\sqrt{\frac{1}{2}\left(1-\sqrt{1-z^2}\right)}\right) = \frac{3\pi}{4} - \frac{1}{2}\sec^{-1}\left(\frac{1}{z}\right) /; \frac{\pi}{2} < \arg(z) \leq \pi \bigvee -\pi < \arg(z) \leq -\frac{\pi}{2}$$

01.13.27.1165.01

$$\cos^{-1}\left(\sqrt{\frac{1-\sqrt{1-z^2}}{2}}\right) = \frac{\pi}{2}\left(1-\frac{\sqrt{z^2}}{2z}\right)+\frac{\sqrt{z^2}}{2z}\sec^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(z\sqrt{1-\sqrt{1-z^2}}\Big/\sqrt{2z^2}\right)$

Involving $\cos^{-1}\left(z\sqrt{1-\sqrt{1-z^2}}\Big/\sqrt{2z^2}\right)$ and $\sec^{-1}\left(\frac{1}{z}\right)$

01.13.27.1166.01

$$\cos^{-1}\left(\frac{z\sqrt{1-\sqrt{1-z^2}}}{\sqrt{2}\sqrt{z^2}}\right) = \frac{\pi}{4}+\frac{1}{2}\sec^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(z\sqrt{\left(1-\sqrt{1-z^2}\right)\Big/\left(2z^2\right)}\right)$

Involving $\cos^{-1}\left(z\sqrt{\left(1-\sqrt{1-z^2}\right)\Big/\left(2z^2\right)}\right)$ and $\sec^{-1}\left(\frac{1}{z}\right)$

01.13.27.1167.01

$$\cos^{-1}\left(z\sqrt{\frac{1-\sqrt{1-z^2}}{2z^2}}\right) = \frac{\pi}{4}+\frac{1}{2}\sec^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\sqrt{z-\sqrt{z^2-1}}\Big/\sqrt{2z}\right)$

Involving $\cos^{-1}\left(\sqrt{z-\sqrt{z^2-1}}\Big/\sqrt{2z}\right)$ and $\sec^{-1}(z)$

01.13.27.1168.01

$$\cos^{-1}\left(\frac{\sqrt{z-\sqrt{z^2-1}}}{\sqrt{2z}}\right) = \frac{1}{2}\left(\frac{\pi}{2}+\sec^{-1}(z)\right) /; -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

01.13.27.1169.01

$$\cos^{-1}\left(\frac{\sqrt{z-\sqrt{z^2-1}}}{\sqrt{2z}}\right) = -\frac{\pi}{4}+\frac{1}{2}\sec^{-1}(z) /; \frac{\pi}{2} < \arg(z) < \pi \bigvee -\pi < \arg(z) < -\frac{\pi}{2} \bigvee (z \in \mathbb{R} \land z < -1)$$

01.13.27.1170.01

$$\cos^{-1}\left(\frac{\sqrt{z-\sqrt{z^2-1}}}{\sqrt{2z}}\right) = \frac{5\pi}{4}-\frac{1}{2}\sec^{-1}(z) /; (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.1171.01

$$\cos^{-1}\left(\frac{\sqrt{z-\sqrt{z^2-1}}}{\sqrt{2z}}\right) = -\frac{1}{2}\sec^{-1}(z)+\frac{\pi}{4} /; (i z \in \mathbb{R} \land i z > 0)$$

01.13.27.1172.01

$$\cos^{-1}\left(\frac{\sqrt{z-\sqrt{z^2-1}}}{\sqrt{2z}}\right) = \frac{1}{4}\pi\left(\frac{\sqrt{z^2}-z}{\sqrt{\frac{z}{z+1}}}\sqrt{\frac{1}{z}}\sqrt{\frac{1}{z+1}} - \sqrt{\frac{i}{z}}\sqrt{\frac{1}{z}}\sqrt{-i z}\sqrt{z}\sqrt{\frac{1}{z+1}}\sqrt{z+1}+2\right)+$$
$$\frac{1}{2}\sqrt{\frac{1}{z}}\sqrt{\frac{i}{z}}\sqrt{z}\sqrt{-i z}\sqrt{\frac{1}{z+1}}\sqrt{z+1}\sec^{-1}(z)$$

Involving $\cos^{-1}\left(\sqrt{\left(z-\sqrt{z^2-1}\right)/(2z)}\right)$

Involving $\cos^{-1}\left(\sqrt{\left(z-\sqrt{z^2-1}\right)/(2z)}\right)$ and $\sec^{-1}(z)$

01.13.27.1173.01

$$\cos^{-1}\left(\sqrt{\frac{z-\sqrt{z^2-1}}{2z}}\right) = \frac{1}{2}\left(\frac{\pi}{2}+\sec^{-1}(z)\right) /; \operatorname{Re}(z) > 0$$

01.13.27.1174.01

$$\cos^{-1}\left(\sqrt{\frac{z-\sqrt{z^2-1}}{2\,z}}\right) = -\frac{\pi}{4}+\frac{1}{2}\sec^{-1}(z)\;/;\;\mathrm{Re}(z)<0$$

01.13.27.1175.01

$$\cos^{-1}\left(\sqrt{\frac{z-\sqrt{z^2-1}}{2\,z}}\right) = \frac{3\,\pi}{4}-\frac{1}{2}\sec^{-1}(z)\;/;\;(i\,z\in\mathbb{R}\land i\,z<0)$$

01.13.27.1176.01

$$\cos^{-1}\left(\sqrt{\frac{z-\sqrt{z^2-1}}{2\,z}}\right) = -\frac{1}{2}\sec^{-1}(z)+\frac{\pi}{4}\;/;\;(i\,z\in\mathbb{R}\land i\,z>0)$$

01.13.27.1177.01

$$\cos^{-1}\left(\sqrt{\frac{z-\sqrt{z^2-1}}{2\,z}}\right) = \frac{\pi\,z}{4\sqrt{z^2}}+\frac{1}{2}\sec^{-1}(z)\;/;\;\mathrm{Re}(z)\neq 0$$

01.13.27.1178.01

$$\cos^{-1}\left(\sqrt{\frac{z-\sqrt{z^2-1}}{2\,z}}\right) = \frac{\pi}{4}\left(1-\sqrt{\frac{1}{z^2}}\sqrt{z^2}+\frac{\sqrt{z^2}}{z}\right)+\frac{1}{2}\sqrt{z^2}\sqrt{\frac{1}{z^2}}\sec^{-1}(z)$$

Involving $\cos^{-1}\left(c\,z^{-r}\sqrt{-1+\frac{z^{2r}}{c^2}}\right)$

Involving $\cos^{-1}\left(c\,z^{-r}\sqrt{-1+\frac{z^{2r}}{c^2}}\right)$ and $\sec^{-1}\left(\frac{z^r}{c}\right)$

01.13.27.1179.01

$$\cos^{-1}\left(c\,z^{-r}\sqrt{\frac{z^{2r}}{c^2}-1}\right) = \frac{\pi}{2}-\sqrt{c^2\,z^{-2r}}\sqrt{\frac{z^{2r}}{c^2}}\left(\frac{\pi\,z^r\sqrt{c^2\,z^{-2r}}}{2\,c}+\sec^{-1}\left(\frac{z^r}{c}\right)-\frac{\pi}{2}\right)$$

Involving $\sinh^{-1}$

Involving $\cos^{-1}(z)$

Involving $\cos^{-1}(z)$ and $\sinh^{-1}(i\,z)$

01.13.27.0044.01

$$\cos^{-1}(z) = i \sinh^{-1}(i z) + \frac{\pi}{2}$$

Involving $\cos^{-1}(z)$ and $\sinh^{-1}(i(2 z^2 - 1))$

01.13.27.1180.01

$$\cos^{-1}(z) = \frac{1}{2}\left(\frac{\pi}{2} + i \sinh^{-1}\left(i\left(2 z^2 - 1\right)\right)\right) /; -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

01.13.27.1181.01

$$\cos^{-1}(z) = \frac{1}{2}\left(\frac{3\pi}{2} - i \sinh^{-1}\left(i\left(2 z^2 - 1\right)\right)\right) /; \frac{\pi}{2} < \arg(z) \leq \pi \bigvee -\pi < \arg(z) \leq -\frac{\pi}{2}$$

01.13.27.1182.01

$$\cos^{-1}(z) = \frac{\pi}{2}\left(1 - \frac{\sqrt{z^2}}{2 z}\right) + i \frac{\sqrt{z^2}}{2 z} \sinh^{-1}\left(i\left(2 z^2 - 1\right)\right)$$

Involving $\cos^{-1}(z)$ and $\sinh^{-1}\left(\sqrt{\frac{z-1}{2}}\right)$

01.13.27.1183.01

$$\cos^{-1}(z) = -2 i \sinh^{-1}\left(\sqrt{\frac{z-1}{2}}\right) /; 0 < \arg(z) \leq \pi \bigvee (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.1184.01

$$\cos^{-1}(z) = 2 i \sinh^{-1}\left(\sqrt{\frac{z-1}{2}}\right) /; -\pi < \arg(z) < 0 \bigvee (z \in \mathbb{R} \land z > 1)$$

01.13.27.1185.01

$$\cos^{-1}(z) = -\frac{2\sqrt{z-1}}{\sqrt{1-z}} \sinh^{-1}\left(\sqrt{\frac{z-1}{2}}\right)$$

Involving $\cos^{-1}(z)$ and $\sinh^{-1}\left(\sqrt{\frac{-z-1}{2}}\right)$

01.13.27.1186.01

$$\cos^{-1}(z) = \pi - 2 i \sinh^{-1}\left(\sqrt{\frac{-z-1}{2}}\right) /; 0 < \arg(z) < \pi \bigvee (z \in \mathbb{R} \land z < -1)$$

01.13.27.1187.01

$$\cos^{-1}(z) = \pi + 2 i \sinh^{-1}\left(\sqrt{\frac{-z-1}{2}}\right) /; -\pi < \arg(z) \leq 0 \bigvee (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.1188.01

$$\cos^{-1}(z) = \pi + \frac{2\sqrt{-z-1}}{\sqrt{z+1}} \sinh^{-1}\left(\sqrt{\frac{1}{2}(-z-1)}\right)$$

Involving $\cos^{-1}(z)$ and $\sinh^{-1}\left(\sqrt{-z^2}\right)$

01.13.27.1189.01

$$\cos^{-1}(z) = \frac{\pi}{2} - i \sinh^{-1}\left(\sqrt{-z^2}\right) /; 0 < \arg(z) \leq \pi$$

01.13.27.1190.01

$$\cos^{-1}(z) = \frac{\pi}{2} + i \sinh^{-1}\left(\sqrt{-z^2}\right) /; -\pi < \arg(z) \leq 0$$

01.13.27.1191.01

$$\cos^{-1}(z) = \frac{\sqrt{-z^2}}{z} \sinh^{-1}\left(\sqrt{-z^2}\right) + \frac{\pi}{2}$$

Involving $\cos^{-1}(z)$ and $\sinh^{-1}\left(\sqrt{z^2-1}\right)$

01.13.27.1192.01

$$\cos^{-1}(z) = -i \sinh^{-1}\left(\sqrt{z^2-1}\right) /; 0 < \arg(z) \leq \frac{\pi}{2} \bigvee (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.1193.01

$$\cos^{-1}(z) = i \sinh^{-1}\left(\sqrt{z^2-1}\right) /; -\frac{\pi}{2} < \arg(z) < 0 \bigvee (z \in \mathbb{R} \land z > 1)$$

01.13.27.1194.01

$$\cos^{-1}(z) = \pi - i \sinh^{-1}\left(\sqrt{z^2-1}\right) /; \frac{\pi}{2} < \arg(z) < \pi$$

01.13.27.1195.01

$$\cos^{-1}(z) = \pi + i \sinh^{-1}\left(\sqrt{z^2-1}\right) /; -\pi < \arg(z) \leq -\frac{\pi}{2} \bigvee (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.1196.01

$$\cos^{-1}(z) = \frac{\pi}{2}\left(1 - \frac{\sqrt{z^2}}{z}\right) + \frac{\sqrt{-z^2}}{z} \sqrt{\frac{z-1}{z+1}} \sqrt{\frac{z+1}{z-1}} \sinh^{-1}\left(\sqrt{z^2-1}\right)$$

Involving $\cos^{-1}(z)$ and $\sinh^{-1}\left(2 z \sqrt{z^2-1}\right)$

01.13.27.1197.01

$$\cos^{-1}(z) = \frac{\pi}{2} - \frac{\sqrt{-z^4}}{2 z^2} \sinh^{-1}\left(2 z \sqrt{z^2-1}\right) /; \frac{\pi}{4} \leq |\arg(z)| \leq \frac{3\pi}{4}$$

01.13.27.1198.01

$$\cos^{-1}(z) = \frac{1}{4}\pi\left(2 - \frac{\sqrt{z^2}}{z} + \sqrt{\frac{1}{z}}\sqrt{z}\sqrt{\frac{1}{\sqrt{2}\,z+1}}\sqrt{\sqrt{2}\,z+1} - \sqrt{-\frac{1}{z}}\sqrt{-z}\sqrt{\frac{1}{1-\sqrt{2}\,z}}\sqrt{1-\sqrt{2}\,z} - \frac{\sqrt{z^4-z^2}}{z\sqrt{z^2-1}}\right) - \frac{\sqrt{z^2(z^2-1)}\sqrt{2z^2-1}}{2z^2\sqrt{1-2z^2}}\sqrt{\frac{z^2}{z^2-1}}\sinh^{-1}\left(2z\sqrt{z^2-1}\right)$$

Involving $\cos^{-1}(c\,z)$

Involving $\cos^{-1}(i\,z)$ and $\sinh^{-1}(z)$

01.13.27.1199.01

$$\cos^{-1}(i\,z) = \frac{\pi}{2} - i\sinh^{-1}(z)$$

Involving $\cos^{-1}(-i\,z)$ and $\sinh^{-1}(z)$

01.13.27.1200.01

$$\cos^{-1}(-i\,z) = \frac{\pi}{2} + i\sinh^{-1}(z)$$

Involving $\cos^{-1}\left(\sqrt{c\,z}\right)$

Involving $\cos^{-1}\left(\sqrt{z}\right)$ and $\sinh^{-1}\left(\sqrt{-z}\right)$

01.13.27.1201.01

$$\cos^{-1}\left(\sqrt{z}\right) = \frac{\pi}{2} - i\sinh^{-1}\left(\sqrt{-z}\right) /; 0 < \arg(z) \leq \pi$$

01.13.27.1202.01

$$\cos^{-1}\left(\sqrt{z}\right) = \frac{\pi}{2} + i\sinh^{-1}\left(\sqrt{-z}\right) /; -\pi < \arg(z) \leq 0$$

01.13.27.1203.01

$$\cos^{-1}\left(\sqrt{z}\right) = \frac{\pi}{2} + \frac{\sqrt{-z^2}}{z}\sinh^{-1}\left(\sqrt{-z}\right)$$

Involving $\cos^{-1}\left(\sqrt{-z}\right)$ and $\sinh^{-1}\left(\sqrt{z}\right)$

01.13.27.1204.01

$$\cos^{-1}\left(\sqrt{-z}\right) = \frac{\pi}{2} + i\sinh^{-1}\left(\sqrt{z}\right) /; 0 < \arg(z) \leq \pi$$

01.13.27.1205.01

$$\cos^{-1}\left(\sqrt{-z}\right) = \frac{\pi}{2} - i\sinh^{-1}\left(\sqrt{z}\right) /; -\pi < \arg(z) \leq 0$$

01.13.27.0045.01

$$\cos^{-1}\left(\sqrt{-z}\right) = \frac{\pi}{2} - \frac{\sqrt{-z^2}}{z}\sinh^{-1}\left(\sqrt{z}\right)$$

Involving $\cos^{-1}\left(\frac{1}{\sqrt{c\,z}}\right)$

Involving $\cos^{-1}\left(\frac{1}{\sqrt{z}}\right)$ and $\sinh^{-1}\left(\frac{1}{\sqrt{-z}}\right)$

01.13.27.1206.01

$$\cos^{-1}\left(\frac{1}{\sqrt{z}}\right) = \frac{\pi}{2} + i\sinh^{-1}\left(\frac{1}{\sqrt{-z}}\right) /; 0 < \arg(z) \leq \pi$$

01.13.27.1207.01

$$\cos^{-1}\left(\frac{1}{\sqrt{z}}\right) = \frac{\pi}{2} - i\sinh^{-1}\left(\frac{1}{\sqrt{-z}}\right) /; -\pi < \arg(z) \leq 0$$

01.13.27.1208.01

$$\cos^{-1}\left(\frac{1}{\sqrt{z}}\right) = \frac{\pi}{2} - \frac{\sqrt{-z^2}}{z}\sinh^{-1}\left(\frac{1}{\sqrt{-z}}\right)$$

Involving $\cos^{-1}\left(\frac{1}{\sqrt{z}}\right)$ and $\sinh^{-1}\left(\sqrt{-\frac{1}{z}}\right)$

01.13.27.1209.01

$$\cos^{-1}\left(\frac{1}{\sqrt{z}}\right) = \frac{\pi}{2} + i\sinh^{-1}\left(\sqrt{-\frac{1}{z}}\right) /; \operatorname{Im}(z) \geq 0$$

01.13.27.1210.01

$$\cos^{-1}\left(\frac{1}{\sqrt{z}}\right) = \frac{\pi}{2} - i\sinh^{-1}\left(\sqrt{-\frac{1}{z}}\right) /; \operatorname{Im}(z) < 0$$

01.13.27.1211.01

$$\cos^{-1}\left(\frac{1}{\sqrt{z}}\right) = \frac{\pi}{2} + \sqrt{-\frac{1}{z}}\,\sqrt{z}\,\sinh^{-1}\left(\sqrt{-\frac{1}{z}}\right)$$

Involving $\cos^{-1}\left(\frac{1}{\sqrt{-z}}\right)$ and $\sinh^{-1}\left(\frac{1}{\sqrt{z}}\right)$

01.13.27.1212.01

$$\cos^{-1}\left(\frac{1}{\sqrt{-z}}\right) = \frac{\pi}{2} - i\sinh^{-1}\left(\frac{1}{\sqrt{z}}\right) /; 0 < \arg(z) \leq \pi$$

01.13.27.1213.01

$$\cos^{-1}\left(\frac{1}{\sqrt{-z}}\right) = i\sinh^{-1}\left(\frac{1}{\sqrt{z}}\right) + \frac{\pi}{2} /; -\pi < \arg(z) \leq 0$$

01.13.27.1214.01

$$\cos^{-1}\left(\frac{1}{\sqrt{-z}}\right) = \frac{\pi}{2} + \frac{\sqrt{-z^2}}{z} \sinh^{-1}\left(\frac{1}{\sqrt{z}}\right)$$

Involving $\cos^{-1}\left(\frac{1}{\sqrt{-z}}\right)$ and $\sinh^{-1}\left(\sqrt{\frac{1}{z}}\right)$

01.13.27.1215.01

$$\cos^{-1}\left(\frac{1}{\sqrt{-z}}\right) = \frac{\pi}{2} - i\sinh^{-1}\left(\sqrt{\frac{1}{z}}\right) /; 0 < \arg(z) < \pi$$

01.13.27.1216.01

$$\cos^{-1}\left(\frac{1}{\sqrt{-z}}\right) = i\sinh^{-1}\left(\sqrt{\frac{1}{z}}\right) + \frac{\pi}{2} /; \operatorname{Im}(z) \leq 0$$

01.13.27.1217.01

$$\cos^{-1}\left(\frac{1}{\sqrt{-z}}\right) = \frac{\pi}{2} + \sqrt{\frac{1}{z}}\sqrt{-z}\sinh^{-1}\left(\sqrt{\frac{1}{z}}\right)$$

Involving $\cos^{-1}\left(\sqrt{c\,z^2}\right)$

Involving $\cos^{-1}\left(\sqrt{z^2}\right)$ and $\sinh^{-1}(i\,z)$

01.13.27.1218.01

$$\cos^{-1}\left(\sqrt{z^2}\right) = \frac{\pi}{2} + i\sinh^{-1}(i\,z) /; -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

01.13.27.1219.01

$$\cos^{-1}\left(\sqrt{z^2}\right) = \frac{\pi}{2} - i\sinh^{-1}(i\,z) /; \frac{\pi}{2} < \arg(z) \leq \pi \bigvee -\pi < \arg(z) \leq -\frac{\pi}{2}$$

01.13.27.1220.01

$$\cos^{-1}\left(\sqrt{z^2}\right) = \frac{\pi}{2} + \frac{i\sqrt{z^2}}{z}\sinh^{-1}(i\,z)$$

Involving $\cos^{-1}\left(\sqrt{-z^2}\right)$ and $\sinh^{-1}(z)$

01.13.27.2519.01

$$\cos^{-1}\left(\sqrt{-z^2}\right) = \frac{\pi}{2} - i\sinh^{-1}(z) /; -\pi < \arg(z) \leq 0$$

01.13.27.1221.01

$$\cos^{-1}\left(\sqrt{-z^2}\right) = \frac{\pi}{2} + i \sinh^{-1}(z) \;/;\; 0 < \arg(z) \leq \pi$$

01.13.27.1222.01

$$\cos^{-1}\left(\sqrt{-z^2}\right) = \frac{\pi}{2} - \frac{\sqrt{-z^2}}{z} \sinh^{-1}(z)$$

Involving $\cos^{-1}\left(a\,(b\,z^c)^m\right)$

Involving $\cos^{-1}(a\,(b\,z^c)^m)$ and $\sinh^{-1}(i\,a\,b^m\,z^{m\,c})$

01.13.27.1223.01

$$\cos^{-1}(a\,(b\,z^c)^m) = \frac{\pi}{2} + \frac{i\,(b\,z^c)^m}{b^m\,z^{m\,c}} \sinh^{-1}(i\,a\,b^m\,z^{m\,c}) \;/;\; 2\,m \in \mathbb{Z}$$

Involving $\cos^{-1}\left(1 + 2\,c\,z^2\right)$

Involving $\cos^{-1}\left(1 + 2\,z^2\right)$ and $\sinh^{-1}(z)$

01.13.27.1224.01

$$\cos^{-1}\left(1 + 2\,z^2\right) = 2\,i \sinh^{-1}(z) \;/;\; -\pi < \arg(z) \leq 0$$

01.13.27.1225.01

$$\cos^{-1}\left(2\,z^2 + 1\right) = -2\,i \sinh^{-1}(z) \;/;\; 0 < \arg(z) \leq \pi$$

01.13.27.1226.01

$$\cos^{-1}\left(2\,z^2 + 1\right) = \frac{2\sqrt{-z^2}}{z} \sinh^{-1}(z)$$

Involving $\cos^{-1}\left(\frac{z^2+2}{z^2}\right)$

Involving $\cos^{-1}\left(\frac{z^2+2}{z^2}\right)$ and $\sinh^{-1}\left(\frac{1}{z}\right)$

01.13.27.1227.01

$$\cos^{-1}\left(\frac{z^2+2}{z^2}\right) = 2\,i \sinh^{-1}\left(\frac{1}{z}\right) \;/;\; 0 \leq \arg(z) < \pi$$

01.13.27.1228.01

$$\cos^{-1}\left(\frac{z^2+2}{z^2}\right) = -2\,i \sinh^{-1}\left(\frac{1}{z}\right) \;/;\; \operatorname{Im}(z) < 0 \lor (z \in \mathbb{R} \land z < 0)$$

01.13.27.1229.01

$$\cos^{-1}\left(\frac{z^2+2}{z^2}\right) == 2\,z\,\sqrt{-\frac{1}{z^2}}\,\sinh^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\sqrt{1+z}\right)$

Involving $\cos^{-1}\left(\sqrt{z+1}\right)$ and $\sinh^{-1}\left(\sqrt{z}\right)$

01.13.27.1230.01

$$\cos^{-1}\left(\sqrt{z+1}\right) == i\sinh^{-1}\left(\sqrt{z}\right) /; -\pi < \arg(z) \leq 0$$

01.13.27.1231.01

$$\cos^{-1}\left(\sqrt{z+1}\right) == -i\sinh^{-1}\left(\sqrt{z}\right) /; 0 < \arg(z) \leq \pi$$

01.13.27.0046.01

$$\cos^{-1}\left(\sqrt{z+1}\right) == \frac{\sqrt{-z^2}}{z}\sinh^{-1}\left(\sqrt{z}\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{1+c\,z}}{\sqrt{2}}\right)$

Involving $\cos^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{2}}\right)$ and $\sinh^{-1}(i\,z)$

01.13.27.1232.01

$$\cos^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{2}}\right) == \frac{1}{2}\left(\frac{\pi}{2} - i\sinh^{-1}(i\,z)\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{1+z}}{\sqrt{2}}\right)$ and $\sinh^{-1}(i\,z)$

01.13.27.1233.01

$$\cos^{-1}\left(\frac{\sqrt{1+z}}{\sqrt{2}}\right) == \frac{1}{2}\left(\frac{\pi}{2} + i\sinh^{-1}(i\,z)\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{z+1}}{\sqrt{z}}\right)$

Involving $\cos^{-1}\left(\frac{\sqrt{z+1}}{\sqrt{z}}\right)$ and $\sinh^{-1}\left(\frac{1}{\sqrt{z}}\right)$

01.13.27.1234.01

$$\cos^{-1}\left(\frac{\sqrt{z+1}}{\sqrt{z}}\right) == i\,\sinh^{-1}\left(\frac{1}{\sqrt{z}}\right) /; 0 \leq \arg(z) < \pi \bigvee (z \in \mathbb{R} \land z < -1)$$

01.13.27.1235.01

$$\cos^{-1}\left(\frac{\sqrt{z+1}}{\sqrt{z}}\right) = -i\sinh^{-1}\left(\frac{1}{\sqrt{z}}\right) /; \operatorname{Im}(z) < 0$$

01.13.27.1236.01

$$\cos^{-1}\left(\frac{\sqrt{z+1}}{\sqrt{z}}\right) = \pi - i\sinh^{-1}\left(\frac{1}{\sqrt{z}}\right) /; (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.1237.01

$$\cos^{-1}\left(\frac{\sqrt{z+1}}{\sqrt{z}}\right) = \frac{1}{2}\pi\left(1 - \frac{\sqrt{-z}\sqrt{z+1}}{\sqrt{-z-1}\sqrt{z}}\right) + \frac{\sqrt{-z(z+1)}}{\sqrt{-z-1}}\sqrt{-\frac{1}{z}}\sinh^{-1}\left(\frac{1}{\sqrt{z}}\right)$$

01.13.27.1238.01

$$\cos^{-1}\left(\frac{\sqrt{z+1}}{\sqrt{z}}\right) = \frac{1}{2}\pi\left(1 - \sqrt{\frac{z}{z+1}}\sqrt{\frac{z+1}{z}}\right) + \frac{z}{z+1}\sqrt{-\frac{(z+1)^2}{z^2}}\sinh^{-1}\left(\frac{1}{\sqrt{z}}\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{z+1}}{\sqrt{z}}\right)$ and $\sinh^{-1}\left(\sqrt{\frac{1}{z}}\right)$

01.13.27.1239.01

$$\cos^{-1}\left(\frac{\sqrt{z+1}}{\sqrt{z}}\right) = i\sinh^{-1}\left(\sqrt{\frac{1}{z}}\right) /; 0 \leq \arg(z) < \pi$$

01.13.27.1240.01

$$\cos^{-1}\left(\frac{\sqrt{z+1}}{\sqrt{z}}\right) = -i\sinh^{-1}\left(\sqrt{\frac{1}{z}}\right) /; \operatorname{Im}(z) < 0 \lor (z \in \mathbb{R} \land z < -1)$$

01.13.27.1241.01

$$\cos^{-1}\left(\frac{\sqrt{z+1}}{\sqrt{z}}\right) = \pi + i\sinh^{-1}\left(\sqrt{\frac{1}{z}}\right) /; (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.1242.01

$$\cos^{-1}\left(\frac{\sqrt{z+1}}{\sqrt{z}}\right) = \frac{1}{2}\pi\left(1 - \frac{\sqrt{-z}\sqrt{z+1}}{\sqrt{-z-1}\sqrt{z}}\right) + \frac{\sqrt{-z^2}\sqrt{1+z}}{\sqrt{-1-z}}\sqrt{-\frac{1}{z^2}}\sinh^{-1}\left(\sqrt{\frac{1}{z}}\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{-1-z}}{\sqrt{-z}}\right)$

Involving $\cos^{-1}\left(\frac{\sqrt{-1-z}}{\sqrt{-z}}\right)$ and $\sinh^{-1}\left(\frac{1}{\sqrt{z}}\right)$

01.13.27.1243.01

$$\cos^{-1}\left(\frac{\sqrt{-z-1}}{\sqrt{-z}}\right) = i\sinh^{-1}\left(\frac{1}{\sqrt{z}}\right) /; \operatorname{Im}(z) \geq 0$$

01.13.27.1244.01

$$\cos^{-1}\left(\frac{\sqrt{-z-1}}{\sqrt{-z}}\right) = -i \sinh^{-1}\left(\frac{1}{\sqrt{z}}\right) /; \operatorname{Im}(z) < 0$$

01.13.27.1245.01

$$\cos^{-1}\left(\frac{\sqrt{-z-1}}{\sqrt{-z}}\right) = \sqrt{-\frac{1}{z}} \sqrt{z} \sinh^{-1}\left(\frac{1}{\sqrt{z}}\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{-1-z}}{\sqrt{-z}}\right)$ and $\sinh^{-1}\left(\sqrt{\frac{1}{z}}\right)$

01.13.27.1246.01

$$\cos^{-1}\left(\frac{\sqrt{-z-1}}{\sqrt{-z}}\right) = i \sinh^{-1}\left(\sqrt{\frac{1}{z}}\right) /; 0 \leq \arg(z) < \pi$$

01.13.27.1247.01

$$\cos^{-1}\left(\frac{\sqrt{-z-1}}{\sqrt{-z}}\right) = -i \sinh^{-1}\left(\sqrt{\frac{1}{z}}\right) /; \operatorname{Im}(z) < 0 \lor (z \in \mathbb{R} \land z < 0)$$

01.13.27.1248.01

$$\cos^{-1}\left(\frac{\sqrt{-z-1}}{\sqrt{-z}}\right) = z \sqrt{-\frac{1}{z^2}} \sinh^{-1}\left(\sqrt{\frac{1}{z}}\right)$$

Involving $\cos^{-1}\left(\sqrt{\frac{z+1}{z}}\right)$

Involving $\cos^{-1}\left(\sqrt{\frac{z+1}{z}}\right)$ and $\sinh^{-1}\left(\frac{1}{\sqrt{z}}\right)$

01.13.27.1249.01

$$\cos^{-1}\left(\sqrt{\frac{z+1}{z}}\right) = i \sinh^{-1}\left(\frac{1}{\sqrt{z}}\right) /; \operatorname{Im}(z) \geq 0$$

01.13.27.1250.01

$$\cos^{-1}\left(\sqrt{\frac{z+1}{z}}\right) = -i \sinh^{-1}\left(\frac{1}{\sqrt{z}}\right) /; \operatorname{Im}(z) < 0$$

01.13.27.1251.01

$$\cos^{-1}\left(\sqrt{\frac{z+1}{z}}\right) = \sqrt{-\frac{1}{z}} \sqrt{z} \sinh^{-1}\left(\frac{1}{\sqrt{z}}\right)$$

Involving $\cos^{-1}\left(\sqrt{\frac{z+1}{z}}\right)$ and $\sinh^{-1}\left(\sqrt{\frac{1}{z}}\right)$

01.13.27.1252.01

$$\cos^{-1}\left(\sqrt{\frac{z+1}{z}}\right) = i \sinh^{-1}\left(\sqrt{\frac{1}{z}}\right) /; 0 \leq \arg(z) < \pi$$

01.13.27.1253.01

$$\cos^{-1}\left(\sqrt{\frac{z+1}{z}}\right) = -i \sinh^{-1}\left(\sqrt{\frac{1}{z}}\right) /; \operatorname{Im}(z) < 0 \lor (z \in \mathbb{R} \land z < 0)$$

01.13.27.1254.01

$$\cos^{-1}\left(\sqrt{\frac{z+1}{z}}\right) = z \sqrt{-\frac{1}{z^2}} \sinh^{-1}\left(\sqrt{\frac{1}{z}}\right)$$

Involving $\cos^{-1}\left(\sqrt{a z^c + 1}\right)$

Involving $\cos^{-1}\left(\sqrt{a z^c + 1}\right)$ and $\sinh^{-1}\left(\sqrt{a}\, z^{c/2}\right)$

01.13.27.1255.01

$$\cos^{-1}\left(\sqrt{z^2 + 1}\right) = i \sinh^{-1}(z) /; -\pi < \arg(z) \leq 0$$

01.13.27.1256.01

$$\cos^{-1}\left(\sqrt{z^2 + 1}\right) = -i \sinh^{-1}(z) /; 0 < \arg(z) \leq \pi$$

01.13.27.0047.01

$$\cos^{-1}\left(\sqrt{1 + z^2}\right) = \frac{\sqrt{-z^2}}{z} \sinh^{-1}(z)$$

01.13.27.0048.01

$$\cos^{-1}\left(\sqrt{a z^c + 1}\right) = -\frac{\sqrt{a}\, z^{c/2}}{\sqrt{-a z^c}} \sinh^{-1}\left(\sqrt{a}\, z^{c/2}\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{z^2+1}}{z}\right)$

Involving $\cos^{-1}\left(\frac{\sqrt{z^2+1}}{z}\right)$ and $\sinh^{-1}\left(\frac{1}{z}\right)$

01.13.27.1257.01

$$\cos^{-1}\left(\frac{\sqrt{z^2+1}}{z}\right) = i\sinh^{-1}\left(\frac{1}{z}\right) /;\ 0 \le \arg(z) < \frac{\pi}{2} \bigvee (i\,z \in \mathbb{R} \land i\,z < -1)$$

01.13.27.1258.01

$$\cos^{-1}\left(\frac{\sqrt{z^2+1}}{z}\right) = -i\sinh^{-1}\left(\frac{1}{z}\right) /;\ -\frac{\pi}{2} < \arg(z) < 0 \bigvee (i\,z \in \mathbb{R} \land 0 < i\,z < 1)$$

01.13.27.1259.01

$$\cos^{-1}\left(\frac{\sqrt{z^2+1}}{z}\right) = \pi - i\sinh^{-1}\left(\frac{1}{z}\right) /;\ \frac{\pi}{2} < \arg(z) < \pi \bigvee (-i\,z \in \mathbb{R} \land 0 < -i\,z < 1)$$

01.13.27.1260.01

$$\cos^{-1}\left(\frac{\sqrt{z^2+1}}{z}\right) = \pi + i\sinh^{-1}\left(\frac{1}{z}\right) /;\ -\pi < \arg(z) < -\frac{\pi}{2} \bigvee (z \in \mathbb{R} \land z < 0) \bigvee (i\,z \in \mathbb{R} \land i\,z > 1)$$

01.13.27.1261.01

$$\cos^{-1}\left(\frac{\sqrt{z^2+1}}{z}\right) = \frac{\pi}{2}\left(1 - \frac{\sqrt{z^2}}{z}\right) + \sqrt{-\frac{1}{z^2}}\,\sqrt{z^2}\,\sinh^{-1}\left(\frac{1}{z}\right) /;\ i\,z \notin (-1,\,1)$$

01.13.27.1262.01

$$\cos^{-1}\left(\frac{\sqrt{z^2+1}}{z}\right) = \frac{\pi}{2} - \frac{\sqrt{z^2+1}}{2\,z^2\sqrt{-\frac{z^2+1}{z^4}}}\left(\pi\sqrt{-\frac{1}{z^2}}\,z + 2\sinh^{-1}\left(\frac{1}{z}\right)\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{1+z^2}}{\sqrt{z^2}}\right)$

Involving $\cos^{-1}\left(\frac{\sqrt{1+z^2}}{\sqrt{z^2}}\right)$ and $\sinh^{-1}\left(\frac{1}{z}\right)$

01.13.27.1263.01

$$\cos^{-1}\left(\frac{\sqrt{z^2+1}}{\sqrt{z^2}}\right) = i\sinh^{-1}\left(\frac{1}{z}\right) /;\ 0 \le \arg(z) < \frac{\pi}{2} \bigvee \frac{\pi}{2} < \arg(z) < \pi \bigvee (i\,z \in \mathbb{R} \land i\,z < -1)$$

01.13.27.1264.01

$$\cos^{-1}\left(\frac{\sqrt{z^2+1}}{\sqrt{z^2}}\right) = -i\sinh^{-1}\left(\frac{1}{z}\right) /;\ -\pi < \arg(z) < -\frac{\pi}{2} \bigvee -\frac{\pi}{2} < \arg(z) < 0 \bigvee (z \in \mathbb{R} \land z < 0) \bigvee (i\,z \in \mathbb{R} \land i\,z > 1)$$

01.13.27.1265.01

$$\cos^{-1}\left(\frac{\sqrt{z^2+1}}{\sqrt{z^2}}\right) = \pi - i\sinh^{-1}\left(\frac{1}{z}\right) /; (-i z \in \mathbb{R} \land 0 < -i z < 1)$$

01.13.27.1266.01

$$\cos^{-1}\left(\frac{\sqrt{z^2+1}}{\sqrt{z^2}}\right) = \pi + i\sinh^{-1}\left(\frac{1}{z}\right) /; (i z \in \mathbb{R} \land 0 < i z < 1)$$

01.13.27.1267.01

$$\cos^{-1}\left(\frac{\sqrt{z^2+1}}{\sqrt{z^2}}\right) = \frac{\pi}{2} - \frac{\sqrt{-z^2\left(z^2+1\right)}}{\sqrt{z^2}\sqrt{-z^2-1}}\left(\frac{\pi}{2} - \sqrt{-\frac{1}{z^2}}\, z\sinh^{-1}\left(\frac{1}{z}\right)\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{-1-z^2}}{\sqrt{-z^2}}\right)$

Involving $\cos^{-1}\left(\frac{\sqrt{-1-z^2}}{\sqrt{-z^2}}\right)$ and $\sinh^{-1}\left(\frac{1}{z}\right)$

01.13.27.1268.01

$$\cos^{-1}\left(\frac{\sqrt{-z^2-1}}{\sqrt{-z^2}}\right) = i\sinh^{-1}\left(\frac{1}{z}\right) /; 0 \leq \arg(z) < \pi$$

01.13.27.1269.01

$$\cos^{-1}\left(\frac{\sqrt{-z^2-1}}{\sqrt{-z^2}}\right) = -i\sinh^{-1}\left(\frac{1}{z}\right) /; \operatorname{Im}(z) < 0 \lor (z \in \mathbb{R} \land z < 0)$$

01.13.27.1270.01

$$\cos^{-1}\left(\frac{\sqrt{-z^2-1}}{\sqrt{-z^2}}\right) = \sqrt{-\frac{1}{z^2}}\, z\sinh^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\sqrt{\frac{z^2+1}{z^2}}\right)$

Involving $\cos^{-1}\left(\sqrt{\frac{z^2+1}{z^2}}\right)$ and $\sinh^{-1}\left(\frac{1}{z}\right)$

01.13.27.1271.01

$$\cos^{-1}\left(\sqrt{\frac{z^2+1}{z^2}}\right) = i\sinh^{-1}\left(\frac{1}{z}\right) /; 0 \leq \arg(z) < \pi$$

01.13.27.1272.01

$$\cos^{-1}\left(\sqrt{\frac{z^2+1}{z^2}}\right) = -i \sinh^{-1}\left(\frac{1}{z}\right) /; \operatorname{Im}(z) < 0 \lor (z \in \mathbb{R} \land z < 0)$$

01.13.27.1273.01

$$\cos^{-1}\left(\sqrt{\frac{z^2+1}{z^2}}\right) = \sqrt{-\frac{1}{z^2}}\, z \sinh^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(2 z \sqrt{-1-z^2}\right)$

Involving $\cos^{-1}\left(2 z \sqrt{-1-z^2}\right)$ and $\sinh^{-1}(z)$

01.13.27.1274.01

$$\cos^{-1}\left(2 z \sqrt{-z^2-1}\right) = \frac{\pi}{2} - \frac{2\sqrt{-z^4}}{z^2} \sinh^{-1}(z) /; |\arg(z)| \leq \frac{\pi}{4} \bigvee \frac{3\pi}{4} \leq |\arg(z)| \leq \pi$$

01.13.27.1275.01

$$\cos^{-1}\left(2 z \sqrt{-z^2-1}\right) = \frac{\pi}{2} - \frac{\pi\sqrt{2z^2+1}\sqrt{z^4+z^2}}{2\sqrt{z^2}\sqrt{-2z^2-1}\sqrt{z^2+1}} \left(-\frac{\sqrt{-z^2}}{z} - i\sqrt{\frac{i}{z}}\sqrt{-i z}\sqrt{\frac{1}{1-i\sqrt{2}\, z}}\sqrt{1-i\sqrt{2}\, z} + i\sqrt{-\frac{i}{z}}\sqrt{i z}\sqrt{\frac{1}{\sqrt{2}\, i z+1}}\sqrt{\sqrt{2}\, i z+1} - \frac{\sqrt{z^4+z^2}}{z\sqrt{-z^2-1}}\right) + \frac{2\sqrt{2z^2+1}\sqrt{z^4+z^2}}{\sqrt{z^2}\sqrt{-2z^2-1}\sqrt{z^2+1}} \sinh^{-1}(z)$$

Involving $\cos^{-1}\left(\frac{2\sqrt{-1-z^2}}{z^2}\right)$

Involving $\cos^{-1}\left(\frac{2\sqrt{-1-z^2}}{z^2}\right)$ and $\sinh^{-1}\left(\frac{1}{z}\right)$

01.13.27.1276.01

$$\cos^{-1}\left(\frac{2\sqrt{-1-z^2}}{z^2}\right) = \frac{\pi}{2} - \frac{2\sqrt{-z^2}}{z} \sinh^{-1}\left(\frac{1}{z}\right) /; |\arg(z)| \leq \frac{\pi}{4} \bigvee \frac{3\pi}{4} \leq |\arg(z)| \leq \pi \bigvee |z| \geq \sqrt{2}$$

01.13.27.1277.01

$$\cos^{-1}\left(\frac{2\sqrt{-z^2-1}}{z^2}\right) = \frac{\pi}{2} - \frac{z^3\sqrt{-z^2-2}\sqrt{-z^2-1}}{2\sqrt{1-i z}\,(i z+1)\sqrt{-z^4-3z^2-2}}\sqrt{\frac{z^2+1}{z^4}}\sqrt{\frac{i-z}{z}}\sqrt{-\frac{i}{z}}\left(\pi\left(-\frac{z^3}{z^2+1}\sqrt{-\frac{z^2+1}{z^2}}\sqrt{\frac{z^2+1}{z^4}}+\sqrt{-\frac{1}{z^2}}\,z + i\sqrt{\frac{z-i\sqrt{2}}{z}}\sqrt{-\frac{i}{z}}\sqrt{i z}\sqrt{\frac{z}{-i\sqrt{2}+z}} - i\sqrt{\frac{z+i\sqrt{2}}{z}}\sqrt{\frac{i}{z}}\sqrt{-i z}\sqrt{\frac{z}{i\sqrt{2}+z}}\right)+4\sinh^{-1}\left(\frac{1}{z}\right)\right)$$

Involving $\cos^{-1}\left(\sqrt{\left(1-\sqrt{1+z^2}\right)/2}\right)$

Involving $\cos^{-1}\left(\sqrt{\left(1-\sqrt{1+z^2}\right)/2}\right)$ and $\sinh^{-1}(z)$

01.13.27.1278.01

$$\cos^{-1}\left(\sqrt{\frac{1-\sqrt{1+z^2}}{2}}\right) = \frac{\pi}{2} - \frac{i}{2}\sinh^{-1}(z) \;/;\; -\pi < \arg(z) \leq 0$$

01.13.27.1279.01

$$\cos^{-1}\left(\sqrt{\frac{1}{2}\left(1-\sqrt{z^2+1}\right)}\right) = \frac{\pi}{2} + \frac{1}{2}\, i \sinh^{-1}(z) \;/;\; 0 < \arg(z) \leq \pi$$

01.13.27.1280.01

$$\cos^{-1}\left(\sqrt{\frac{1-\sqrt{1+z^2}}{2}}\right) = \frac{\pi}{2} - \frac{\sqrt{-z^2}}{2z}\sinh^{-1}(z)$$

Involving $\cos^{-1}\left(z\sqrt{1-\sqrt{1+z^2}}\Big/\sqrt{2z^2}\right)$

Involving $\cos^{-1}\left(z\sqrt{1-\sqrt{1+z^2}}\Big/\sqrt{2z^2}\right)$ and $\sinh^{-1}(z)$

01.13.27.1281.01

$$\cos^{-1}\left(\frac{z\sqrt{1-\sqrt{z^2+1}}}{\sqrt{2}\sqrt{z^2}}\right) = \frac{\pi}{2} - \frac{1}{2} i \sinh^{-1}(z) /; \frac{\pi}{2} < \arg(z) \leq \pi \bigvee -\frac{\pi}{2} < \arg(z) \leq 0$$

01.13.27.1282.01

$$\cos^{-1}\left(\frac{z\sqrt{1-\sqrt{z^2+1}}}{\sqrt{2}\sqrt{z^2}}\right) = \frac{\pi}{2} + \frac{1}{2} i \sinh^{-1}(z) /; 0 < \arg(z) \leq \frac{\pi}{2} \bigvee -\pi < \arg(z) \leq -\frac{\pi}{2}$$

01.13.27.1283.01

$$\cos^{-1}\left(\frac{z\sqrt{1-\sqrt{z^2+1}}}{\sqrt{2}\sqrt{z^2}}\right) = \frac{\pi}{2} - \frac{\sqrt{-z^4}}{2 z^2} \sinh^{-1}(z)$$

Involving $\cos^{-1}\left(z\sqrt{\left(1-\sqrt{1+z^2}\right)/\left(2 z^2\right)}\right)$

Involving $\cos^{-1}\left(z\sqrt{\left(1-\sqrt{1+z^2}\right)/\left(2 z^2\right)}\right)$ and $\sinh^{-1}(z)$

01.13.27.1284.01

$$\cos^{-1}\left(z\sqrt{\frac{1-\sqrt{z^2+1}}{2 z^2}}\right) = \frac{\pi}{2} - \frac{1}{2} i \sinh^{-1}(z) /; 0 \leq \arg(z) \leq \frac{\pi}{2} \bigvee -\pi < \arg(z) \leq -\frac{\pi}{2} \bigvee (z \in \mathbb{R} \land z < 0)$$

01.13.27.1285.01

$$\cos^{-1}\left(z\sqrt{\frac{1-\sqrt{z^2+1}}{2 z^2}}\right) = \frac{\pi}{2} + \frac{1}{2} i \sinh^{-1}(z) /; \frac{\pi}{2} < \arg(z) < \pi \bigvee -\frac{\pi}{2} < \arg(z) < 0$$

01.13.27.1286.01

$$\cos^{-1}\left(z\sqrt{\frac{1-\sqrt{z^2+1}}{2 z^2}}\right) = \frac{\pi}{2} - \frac{1}{2}\sqrt{-\frac{1}{z^2}}\sqrt{z^2} \sinh^{-1}(z)$$

Involving $\cos^{-1}\left(\sqrt{z-\sqrt{z^2+1}}\Big/\sqrt{2 z}\right)$

Involving $\cos^{-1}\left(\sqrt{z-\sqrt{z^2+1}} \Big/ \sqrt{2\,z}\right)$ and $\sinh^{-1}\left(\frac{1}{z}\right)$

01.13.27.1287.01

$$\cos^{-1}\left(\frac{\sqrt{z-\sqrt{z^2+1}}}{\sqrt{2\,z}}\right) = \frac{\pi}{2} - \frac{1}{2}\,i\sinh^{-1}\left(\frac{1}{z}\right) /;\ 0 < \arg(z) < \frac{\pi}{2} \bigvee (i\,z \in \mathbb{R} \land i\,z < -1)$$

01.13.27.1288.01

$$\cos^{-1}\left(\frac{\sqrt{z-\sqrt{z^2+1}}}{\sqrt{2\,z}}\right) = \frac{\pi}{2} + \frac{1}{2}\,i\sinh^{-1}\left(\frac{1}{z}\right) /;\ -\frac{\pi}{2} < \arg(z) < 0 \bigvee (i\,z \in \mathbb{R} \land 0 < i\,z < 1)$$

01.13.27.1289.01

$$\cos^{-1}\left(\frac{\sqrt{z-\sqrt{z^2+1}}}{\sqrt{2\,z}}\right) = \frac{1}{2}\,i\sinh^{-1}\left(\frac{1}{z}\right) /;\ \frac{\pi}{2} < \arg(z) < \pi \bigvee (-i\,z \in \mathbb{R} \land 0 < -i\,z < 1)$$

01.13.27.1290.01

$$\cos^{-1}\left(\frac{\sqrt{z-\sqrt{z^2+1}}}{\sqrt{2\,z}}\right) = -\frac{1}{2}\,i\sinh^{-1}\left(\frac{1}{z}\right) /;\ -\pi < \arg(z) < -\frac{\pi}{2} \bigvee (i\,z \in \mathbb{R} \land i\,z > 1) \bigvee (z \in \mathbb{R} \land z < 0)$$

01.13.27.1291.01

$$\cos^{-1}\left(\frac{\sqrt{z-\sqrt{z^2+1}}}{\sqrt{2\,z}}\right) = -\frac{z^2}{2}\sqrt{-\frac{1}{z^4}}\sqrt{\frac{1}{z^2+1}}\sqrt{z^2+1}\,\sinh^{-1}\left(\frac{1}{z}\right) + \frac{1}{4}\,\pi\left(1 + z\sqrt{\frac{1}{z^2}}\sqrt{\frac{1}{z^2+1}}\sqrt{z^2+1}\right)$$

Involving $\cos^{-1}\left(\sqrt{\left(z-\sqrt{1+z^2}\right) \big/ (2\,z)}\right)$

Involving $\cos^{-1}\left(\sqrt{\left(z-\sqrt{1+z^2}\right) \big/ (2\,z)}\right)$ and $\sinh^{-1}\left(\frac{1}{z}\right)$

01.13.27.1292.01

$$\cos^{-1}\left(\sqrt{\frac{z-\sqrt{1+z^2}}{2z}}\right) = \frac{\pi}{2} - \frac{1}{2}\, i \sinh^{-1}\left(\frac{1}{z}\right) /;\ 0 < \arg(z) < \frac{\pi}{2} \bigvee (i z \in \mathbb{R} \land i z < -1)$$

01.13.27.1293.01

$$\cos^{-1}\left(\sqrt{\frac{z-\sqrt{1+z^2}}{2z}}\right) = \frac{\pi}{2} + \frac{1}{2}\, i \sinh^{-1}\left(\frac{1}{z}\right) /;\ -\frac{\pi}{2} < \arg(z) < 0 \bigvee (i z \in \mathbb{R} \land 0 < i z < 1)$$

01.13.27.1294.01

$$\cos^{-1}\left(\sqrt{\frac{z-\sqrt{1+z^2}}{2z}}\right) = \frac{1}{2}\, i \sinh^{-1}\left(\frac{1}{z}\right) /;\ \frac{\pi}{2} < \arg(z) < \pi \bigvee (-i z \in \mathbb{R} \land 0 < -i z < 1)$$

01.13.27.1295.01

$$\cos^{-1}\left(\sqrt{\frac{z-\sqrt{1+z^2}}{2z}}\right) = -\frac{1}{2}\, i \sinh^{-1}\left(\frac{1}{z}\right) /;\ -\pi < \arg(z) < -\frac{\pi}{2} \bigvee (i z \in \mathbb{R} \land i z > 1) \bigvee (z \in \mathbb{R} \land z < 0)$$

01.13.27.1296.01

$$\cos^{-1}\left(\sqrt{\frac{z-\sqrt{z^2+1}}{2z}}\right) = -\frac{z^2}{2} \sqrt{-\frac{1}{z^4}} \sqrt{\frac{1}{z^2+1}} \sqrt{z^2+1}\, \sinh^{-1}\left(\frac{1}{z}\right) + \frac{\pi}{4}\left(1 + z\sqrt{\frac{1}{z^2}} \sqrt{\frac{1}{z^2+1}} \sqrt{z^2+1}\right)$$

Involving $\cosh^{-1}$

Involving $\cos^{-1}(z)$

Involving $\cos^{-1}(z)$ and $\cosh^{-1}(z)$

01.13.27.0051.02

$$\cos^{-1}(z) = -i \cosh^{-1}(z) /;\ \operatorname{Im}(z) > 0 \lor 0 < z < 1$$

01.13.27.0052.02

$$\cos^{-1}(z) = i \cosh^{-1}(z) /;\ \operatorname{Im}(z) < 0 \lor z > 1$$

01.13.27.0049.01

$$\cos^{-1}(z) = \frac{\sqrt{1-z}}{\sqrt{z-1}} \cosh^{-1}(z)$$

Involving $\cos^{-1}(z)$ and $\cosh^{-1}(-z)$

01.13.27.1297.01

$$\cos^{-1}(z) = \pi - i \cosh^{-1}(-z) /;\ \operatorname{Im}(z) > 0 \lor (z \in \mathbb{R} \land z < -1)$$

01.13.27.1298.01

$$\cos^{-1}(z) = \pi + i \cosh^{-1}(-z) /;\ \operatorname{Im}(z) < 0 \lor (z \in \mathbb{R} \land z > -1)$$

01.13.27.1299.01

$$\cos^{-1}(z) = \pi - \frac{\sqrt{1+z}}{\sqrt{-1-z}} \cosh^{-1}(-z)$$

Involving $\cos^{-1}(z)$ and $\cosh^{-1}(1-2z^2)$

01.13.27.1300.01

$$\cos^{-1}(z) = \frac{\pi}{2} + \frac{i}{2} \cosh^{-1}(1-2z^2) \;/;\; -\pi < \arg(z) \leq 0$$

01.13.27.1301.01

$$\cos^{-1}(z) = \frac{\pi}{2} - \frac{1}{2} i \cosh^{-1}(1-2z^2) \;/;\; 0 < \arg(z) \leq \pi$$

01.13.27.1302.01

$$\cos^{-1}(z) = \frac{\pi}{2} + \frac{\sqrt{-z^2}}{2z} \cosh^{-1}(1-2z^2)$$

Involving $\cos^{-1}(z)$ and $\cosh^{-1}(2z^2-1)$

01.13.27.1303.01

$$\cos^{-1}(z) = -\frac{1}{2} i \cosh^{-1}(2z^2-1) \;/;\; 0 < \arg(z) \leq \frac{\pi}{2} \bigvee (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.1304.01

$$\cos^{-1}(z) = \frac{1}{2} i \cosh^{-1}(2z^2-1) \;/;\; -\frac{\pi}{2} < \arg(z) < 0 \bigvee (z \in \mathbb{R} \land z > 1)$$

01.13.27.1305.01

$$\cos^{-1}(z) = \pi - \frac{1}{2} i \cosh^{-1}(2z^2-1) \;/;\; \frac{\pi}{2} < \arg(z) < \pi \bigvee (z \in \mathbb{R} \land z < -1)$$

01.13.27.1306.01

$$\cos^{-1}(z) = \pi + \frac{1}{2} i \cosh^{-1}(2z^2-1) \;/;\; -\pi < \arg(z) \leq -\frac{\pi}{2} \bigvee (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.1307.01

$$\cos^{-1}(z) = -\frac{z\sqrt{z^2-1}}{2\sqrt{z^2-z^4}} \cosh^{-1}(2z^2-1) + \frac{\pi}{2}\left(1 - \frac{\sqrt{z^2}}{z}\right)$$

Involving $\cos^{-1}(z)$ and $\cosh^{-1}\left(\sqrt{\frac{1+z}{2}}\right)$

01.13.27.1308.01

$$\cos^{-1}(z) = -2 i \cosh^{-1}\left(\sqrt{\frac{z+1}{2}}\right) \;/;\; 0 < \arg(z) \leq \pi \lor (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.1309.01

$$\cos^{-1}(z) = 2 i \cosh^{-1}\left(\sqrt{\frac{1+z}{2}}\right) \;/;\; \operatorname{Im}(z) < 0 \lor (z \in \mathbb{R} \land z > 1)$$

01.13.27.1310.01

$$\cos^{-1}(z) = 2\sqrt{-z^2}\sqrt{\frac{1}{z-1}}\sqrt{\frac{z-1}{z^2}}\cosh^{-1}\left(\sqrt{\frac{z+1}{2}}\right)$$

Involving $\cos^{-1}(z)$ and $\cosh^{-1}\left(\sqrt{\frac{1-z}{2}}\right)$

01.13.27.1311.01

$$\cos^{-1}(z) = \pi + 2\,i\cosh^{-1}\left(\sqrt{\frac{1-z}{2}}\right) /; -\pi < \arg(z) \leq 0 \bigvee (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.1312.01

$$\cos^{-1}(z) = \pi - 2\,i\cosh^{-1}\left(\sqrt{\frac{1-z}{2}}\right) /; \operatorname{Im}(z) > 0 \lor (z \in \mathbb{R} \land z < -1)$$

01.13.27.1313.01

$$\cos^{-1}(z) = \pi - 2\sqrt{-z^2}\sqrt{-\frac{1}{z+1}}\sqrt{-\frac{z+1}{z^2}}\cosh^{-1}\left(\sqrt{\frac{1-z}{2}}\right)$$

Involving $\cos^{-1}(z)$ and $\cosh^{-1}\left(\sqrt{z^2}\right)$

01.13.27.1314.01

$$\cos^{-1}(z) = -i\cosh^{-1}\left(\sqrt{z^2}\right) /; 0 < \arg(z) \leq \frac{\pi}{2} \bigvee (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.1315.01

$$\cos^{-1}(z) = i\cosh^{-1}\left(\sqrt{z^2}\right) /; -\frac{\pi}{2} < \arg(z) < 0 \bigvee (z \in \mathbb{R} \land z > 1)$$

01.13.27.1316.01

$$\cos^{-1}(z) = \pi - i\cosh^{-1}\left(\sqrt{z^2}\right) /; \frac{\pi}{2} < \arg(z) < \pi \bigvee (z \in \mathbb{R} \land z < -1)$$

01.13.27.1317.01

$$\cos^{-1}(z) = \pi + i\cosh^{-1}\left(\sqrt{z^2}\right) /; -\pi < \arg(z) \leq -\frac{\pi}{2} \bigvee (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.1318.01

$$\cos^{-1}(z) = \frac{1}{2}\pi\left(1 - \frac{\sqrt{z^2}}{z}\right) + \frac{\sqrt{-z^2}}{z}\sqrt{\frac{z-1}{z+1}}\sqrt{\frac{z+1}{z-1}}\cosh^{-1}\left(\sqrt{z^2}\right)$$

Involving $\cos^{-1}(z)$ and $\cosh^{-1}\left(\sqrt{1-z^2}\right)$

01.13.27.1319.01

$$\cos^{-1}(z) = \frac{\pi}{2} + i\cosh^{-1}\left(\sqrt{1-z^2}\right) /; -\pi < \arg(z) \leq 0$$

01.13.27.1320.01

$$\cos^{-1}(z) = \frac{\pi}{2} - i \cosh^{-1}\left(\sqrt{1-z^2}\right) /; 0 < \arg(z) \leq \pi$$

01.13.27.1321.01

$$\cos^{-1}(z) = \frac{\pi}{2} + \frac{\sqrt{-z^2}}{z} \cosh^{-1}\left(\sqrt{1-z^2}\right)$$

Involving $\cos^{-1}(z)$ and $\cosh^{-1}\left(2 z \sqrt{1-z^2}\right)$

01.13.27.1322.01

$$\cos^{-1}(z) = \frac{\pi}{4} + \frac{1}{2} i \cosh^{-1}\left(2 z \sqrt{1-z^2}\right) /; \operatorname{Re}(z) > -\frac{1}{\sqrt{2}} \bigwedge \operatorname{Im}(z) < 0$$

01.13.27.1323.01

$$\cos^{-1}(z) = \frac{\pi}{4} - \frac{1}{2} i \cosh^{-1}\left(2 z \sqrt{1-z^2}\right) /; \operatorname{Re}(z) > -\frac{1}{\sqrt{2}} \bigwedge \operatorname{Im}(z) > 0$$

01.13.27.1324.01

$$\cos^{-1}(z) = \frac{\pi}{4} + \frac{\sqrt{-z^2}}{2z} \cosh^{-1}\left(2 z \sqrt{1-z^2}\right) /; \operatorname{Re}(z) > -\frac{1}{\sqrt{2}} \bigwedge \operatorname{Im}(z) \neq 0$$

01.13.27.1325.01

$$\cos^{-1}(z) = \frac{\pi}{2} - \frac{1}{4}\pi\left(\frac{\sqrt{z^2}}{z} - \sqrt{\frac{1}{z}} \sqrt{z} \sqrt{\frac{1}{\sqrt{2}\, z+1}} \sqrt{\sqrt{2}\, z+1} + \sqrt{-\frac{1}{z}} \sqrt{-z} \sqrt{\frac{1}{1-\sqrt{2}\, z}} \sqrt{1-\sqrt{2}\, z} + \frac{\sqrt{z^4-z^2}}{z\sqrt{z^2-1}}\right) + \frac{\sqrt{-z^2} \sqrt{z^2-1} \sqrt{2z^2-1}}{2\sqrt{1-2z^2} \sqrt{z^4-z^2}} \left(\frac{\pi}{2} - \frac{1}{\sqrt{2z\sqrt{1-z^2}-1}} \sqrt{1-2z\sqrt{1-z^2}} \cosh^{-1}\left(2z\sqrt{1-z^2}\right)\right)$$

Involving $\cos^{-1}(z)$ and $\cosh^{-1}\left(2 \sqrt{-1+\frac{1}{z^2}}\, z^2\right)$

01.13.27.1326.01

$$\cos^{-1}(z) = \frac{\pi}{2} - \frac{1}{4} \sqrt{1-2z^2} \sqrt{\frac{1}{1-2z^2}} \left(\pi\left(\sqrt{\frac{1}{z^2}}\, z - \sqrt{\frac{1}{\sqrt{2}\, z+1}} \sqrt{\sqrt{2}\, z+1} + \sqrt{\frac{1}{1-\sqrt{2}\, z}} \sqrt{1-\sqrt{2}\, z}\right) + 2z\sqrt{-\frac{1}{z^2}} \cosh^{-1}\left(2\sqrt{\frac{1}{z^2}-1}\, z^2\right)\right)$$

Involving $\cos^{-1}(-z)$

Involving $\cos^{-1}(-z)$ and $\cosh^{-1}(z)$

01.13.27.1327.01

$$\cos^{-1}(-z) = i\cosh^{-1}(z) + \pi \;/;\; \operatorname{Im}(z) > 0 \lor (z \in \mathbb{R} \land z < 1)$$

01.13.27.1328.01

$$\cos^{-1}(-z) = \pi - i\cosh^{-1}(z) \;/;\; \operatorname{Im}(z) < 0 \lor (z \in \mathbb{R} \land z > 1)$$

01.13.27.1329.01

$$\cos^{-1}(-z) = \pi - \frac{\sqrt{1-z}}{\sqrt{z-1}} \cosh^{-1}(z)$$

Involving $\cos^{-1}\left(\sqrt{c\,z}\right)$

Involving $\cos^{-1}\left(\sqrt{z}\right)$ and $\cosh^{-1}\left(\sqrt{z}\right)$

01.13.27.1330.01

$$\cos^{-1}\left(\sqrt{z}\right) = -i\cosh^{-1}\left(\sqrt{z}\right) \;/;\; \operatorname{Im}(z) > 0 \lor (z \in \mathbb{R} \land z < 1)$$

01.13.27.1331.01

$$\cos^{-1}\left(\sqrt{z}\right) = i\cosh^{-1}\left(\sqrt{z}\right) \;/;\; \operatorname{Im}(z) < 0 \lor (z \in \mathbb{R} \land z > 1)$$

01.13.27.0050.01

$$\cos^{-1}\left(\sqrt{z}\right) = \frac{\sqrt{1-z}}{\sqrt{z-1}} \cosh^{-1}\left(\sqrt{z}\right)$$

Involving $\cos^{-1}\left(\sqrt{z}\right)$ and $\cosh^{-1}\left(1\Big/\sqrt{\frac{1}{z}}\right)$

01.13.27.1332.01

$$\cos^{-1}\left(\sqrt{z}\right) = -i\cosh^{-1}\left(1\Big/\sqrt{\frac{1}{z}}\right) \;/;\; \operatorname{Im}(z) > 0 \lor (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.1333.01

$$\cos^{-1}\left(\sqrt{z}\right) = i\cosh^{-1}\left(1\Big/\sqrt{\frac{1}{z}}\right) \;/;\; \operatorname{Im}(z) < 0 \lor (z \in \mathbb{R} \land z > 1)$$

01.13.27.1334.01

$$\cos^{-1}\left(\sqrt{z}\right) = \pi - i\cosh^{-1}\left(1\Big/\sqrt{\frac{1}{z}}\right) \;/;\; (z \in \mathbb{R} \land z < 0)$$

01.13.27.1335.01

$$\cos^{-1}\left(\sqrt{z}\right) = \frac{\pi}{2}\left(1 - \sqrt{z}\sqrt{\frac{1}{z}}\right) + \frac{\sqrt{1-z}}{\sqrt{z-1}} \cosh^{-1}\left(1\Big/\sqrt{\frac{1}{z}}\right)$$

Involving $\cos^{-1}\left(\sqrt{-z}\right)$ and $\cosh^{-1}\left(\sqrt{1+z}\right)$

01.13.27.1336.01

$$\cos^{-1}\left(\sqrt{-z}\right) = \frac{\pi}{2} - i \cosh^{-1}\left(\sqrt{z+1}\right) /; -\pi < \arg(z) \leq 0$$

01.13.27.1337.01

$$\cos^{-1}\left(\sqrt{-z}\right) = \frac{\pi}{2} + i \cosh^{-1}\left(\sqrt{z+1}\right) /; 0 < \arg(z) \leq \pi$$

01.13.27.1338.01

$$\cos^{-1}\left(\sqrt{-z}\right) = \frac{\pi}{2} - \frac{\sqrt{-z^2}}{z} \cosh^{-1}\left(\sqrt{1+z}\right)$$

Involving $\cos^{-1}\left(\sqrt{-z}\right)$ and $\cosh^{-1}(1+2z)$

01.13.27.1339.01

$$\cos^{-1}\left(\sqrt{-z}\right) = \frac{\pi}{2} - \frac{1}{2} i \cosh^{-1}(2z+1) /; -\pi < \arg(z) \leq 0$$

01.13.27.1340.01

$$\cos^{-1}\left(\sqrt{-z}\right) = \frac{\pi}{2} + \frac{1}{2} i \cosh^{-1}(2z+1) /; 0 < \arg(z) \leq \pi$$

01.13.27.1341.01

$$\cos^{-1}\left(\sqrt{-z}\right) = \frac{\pi}{2} - \frac{\sqrt{-z^2}}{2z} \cosh^{-1}(2z+1)$$

Involving $\cos^{-1}\left(\frac{1}{\sqrt{c\,z}}\right)$

Involving $\cos^{-1}\left(\frac{1}{\sqrt{z}}\right)$ and $\cosh^{-1}\left(\frac{1}{\sqrt{z}}\right)$

01.13.27.1342.01

$$\cos^{-1}\left(\frac{1}{\sqrt{z}}\right) = i \cosh^{-1}\left(\frac{1}{\sqrt{z}}\right) /; \operatorname{Im}(z) > 0 \lor (z \in \mathbb{R} \land z < 1)$$

01.13.27.1343.01

$$\cos^{-1}\left(\frac{1}{\sqrt{z}}\right) = -i \cosh^{-1}\left(\frac{1}{\sqrt{z}}\right) /; \operatorname{Im}(z) < 0 \lor (z \in \mathbb{R} \land z > 1)$$

01.13.27.1344.01

$$\cos^{-1}\left(\frac{1}{\sqrt{z}}\right) = \frac{\sqrt{z-1}}{\sqrt{1-z}} \cosh^{-1}\left(\frac{1}{\sqrt{z}}\right)$$

Involving $\cos^{-1}\left(\frac{1}{\sqrt{z}}\right)$ and $\cosh^{-1}\left(\sqrt{\frac{1}{z}}\right)$

01.13.27.1345.01

$$\cos^{-1}\left(\frac{1}{\sqrt{z}}\right) = i \cosh^{-1}\left(\sqrt{\frac{1}{z}}\right) /; \operatorname{Im}(z) > 0 \lor (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.1346.01

$$\cos^{-1}\left(\frac{1}{\sqrt{z}}\right) = -i \cosh^{-1}\left(\sqrt{\frac{1}{z}}\right) /; \operatorname{Im}(z) < 0 \lor (z \in \mathbb{R} \land z > 1)$$

01.13.27.1347.01

$$\cos^{-1}\left(\frac{1}{\sqrt{z}}\right) = \pi + i \cosh^{-1}\left(\sqrt{\frac{1}{z}}\right) /; (z \in \mathbb{R} \land z < 0)$$

01.13.27.1348.01

$$\cos^{-1}\left(\frac{1}{\sqrt{z}}\right) = \frac{\pi}{2}\left(1 - \sqrt{z}\sqrt{\frac{1}{z}}\right) + \frac{\sqrt{z-1}}{\sqrt{1-z}} \cosh^{-1}\left(\sqrt{\frac{1}{z}}\right)$$

Involving $\cos^{-1}\left(\frac{1}{\sqrt{-z}}\right)$ and $\cosh^{-1}\left(\sqrt{\frac{z+1}{z}}\right)$

01.13.27.1349.01

$$\cos^{-1}\left(\frac{1}{\sqrt{-z}}\right) = \frac{\pi}{2} - i \cosh^{-1}\left(\sqrt{\frac{z+1}{z}}\right) /; \operatorname{Im}(z) > 0$$

01.13.27.1350.01

$$\cos^{-1}\left(\frac{1}{\sqrt{-z}}\right) = \frac{\pi}{2} + i \cosh^{-1}\left(\sqrt{\frac{z+1}{z}}\right) /; \operatorname{Im}(z) \leq 0$$

01.13.27.1351.01

$$\cos^{-1}\left(\frac{1}{\sqrt{-z}}\right) = \frac{\pi}{2} + \sqrt{-z}\sqrt{\frac{1}{z}} \cosh^{-1}\left(\sqrt{\frac{z+1}{z}}\right)$$

Involving $\cos^{-1}\left(\frac{1}{\sqrt{-z}}\right)$ and $\cosh^{-1}\left(\frac{z+2}{z}\right)$

01.13.27.1352.01

$$\cos^{-1}\left(\frac{1}{\sqrt{-z}}\right) = \frac{\pi}{2} + \frac{i}{2} \cosh^{-1}\left(\frac{z+2}{z}\right) /; \operatorname{Im}(z) \leq 0$$

01.13.27.1353.01

$$\cos^{-1}\left(\frac{1}{\sqrt{-z}}\right) = \frac{\pi}{2} - \frac{1}{2} i \cosh^{-1}\left(\frac{z+2}{z}\right) /; 0 < \arg(z) < \pi$$

01.13.27.1354.01

$$\cos^{-1}\left(\frac{1}{\sqrt{-z}}\right) = \frac{\pi}{2} + \frac{\sqrt{-z}}{2}\sqrt{\frac{1}{z}} \cosh^{-1}\left(\frac{z+2}{z}\right)$$

Involving $\cos^{-1}\left(\sqrt{z^2}\right)$

Involving $\cos^{-1}\left(\sqrt{z^2}\right)$ and $\cosh^{-1}(z)$

01.13.27.1355.01

$$\cos^{-1}\left(\sqrt{z^2}\right) = -i\cosh^{-1}(z) /; 0 < \arg(z) \le \frac{\pi}{2} \bigvee (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.1356.01

$$\cos^{-1}\left(\sqrt{z^2}\right) = i\cosh^{-1}(z) /; -\frac{\pi}{2} < \arg(z) < 0 \bigvee (z \in \mathbb{R} \land z > 1)$$

01.13.27.1357.01

$$\cos^{-1}\left(\sqrt{z^2}\right) = \pi + i\cosh^{-1}(z) /; \frac{\pi}{2} < \arg(z) \le \pi$$

01.13.27.1358.01

$$\cos^{-1}\left(\sqrt{z^2}\right) = \pi - i\cosh^{-1}(z) /; -\pi < \arg(z) \le -\frac{\pi}{2}$$

01.13.27.1359.01

$$\cos^{-1}\left(\sqrt{z^2}\right) = \frac{\sqrt{1-z}}{\sqrt{z-1}}\cosh^{-1}(z) /; -\frac{\pi}{2} < \arg(z) \le \frac{\pi}{2}$$

01.13.27.1360.01

$$\cos^{-1}\left(\sqrt{z^2}\right) = \frac{\pi}{2}\left(1 - \frac{\sqrt{z^2}}{z}\right) + \frac{\sqrt{z^2}}{z}\frac{\sqrt{1-z}}{\sqrt{z-1}}\cosh^{-1}(z)$$

Involving $\cos^{-1}\left(a\left(b z^c\right)^m\right)$

Involving $\cos^{-1}(a(b z^c)^m)$ and $\cosh^{-1}(a b^m z^{m c})$

01.13.27.1361.01

$$\cos^{-1}(a(b z^c)^m) = \frac{\pi}{2} - \frac{(b z^c)^m}{b^m z^{m c}}\left(\frac{\pi}{2} - \frac{\sqrt{1 - a b^m z^{m c}}}{\sqrt{a b^m z^{m c} - 1}}\cosh^{-1}(a b^m z^{m c})\right) /; 2m \in \mathbb{Z}$$

Involving $\cos^{-1}\left(1 - 2z^2\right)$

Involving $\cos^{-1}\left(1 - 2z^2\right)$ and $\cosh^{-1}(z)$

01.13.27.1362.01

$$\cos^{-1}\left(1 - 2z^2\right) = \pi + 2i\cosh^{-1}(z) /; 0 < \arg(z) \le \frac{\pi}{2} \bigvee (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.1363.01

$$\cos^{-1}(1-2z^2) = \pi - 2 i \cosh^{-1}(z) \;/;\; -\frac{\pi}{2} < \arg(z) < 0 \bigvee (z \in \mathbb{R} \land z > 1)$$

01.13.27.1364.01

$$\cos^{-1}(1-2z^2) = -\pi - 2 i \cosh^{-1}(z) \;/;\; \frac{\pi}{2} < \arg(z) \leq \pi$$

01.13.27.1365.01

$$\cos^{-1}(1-2z^2) = -\pi + 2 i \cosh^{-1}(z) \;/;\; -\pi < \arg(z) \leq -\frac{\pi}{2}$$

01.13.27.1366.01

$$\cos^{-1}(1-2z^2) = \frac{\pi\sqrt{z^2}}{z} - \frac{2\sqrt{z^2}\sqrt{1-z}}{z\sqrt{z-1}} \cosh^{-1}(z)$$

Involving $\cos^{-1}(2z^2-1)$

Involving $\cos^{-1}(2z^2-1)$ and $\cosh^{-1}(z)$

01.13.27.1367.01

$$\cos^{-1}(2z^2-1) = -2 i \cosh^{-1}(z) \;/;\; 0 < \arg(z) \leq \frac{\pi}{2} \bigvee (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.1368.01

$$\cos^{-1}(2z^2-1) = 2 i \cosh^{-1}(z) \;/;\; -\frac{\pi}{2} < \arg(z) < 0 \bigvee (z \in \mathbb{R} \land z > 1)$$

01.13.27.1369.01

$$\cos^{-1}(2z^2-1) = 2\pi + 2 i \cosh^{-1}(z) \;/;\; \frac{\pi}{2} < \arg(z) \leq \pi$$

01.13.27.1370.01

$$\cos^{-1}(2z^2-1) = 2\pi - 2 i \cosh^{-1}(z) \;/;\; -\pi < \arg(z) \leq -\frac{\pi}{2}$$

01.13.27.1371.01

$$\cos^{-1}(2z^2-1) = \pi\left(1 - \frac{\sqrt{z^2}}{z}\right) + 2\frac{\sqrt{1-z}\sqrt{z^2}}{\sqrt{z-1}\, z} \cosh^{-1}(z)$$

Involving $\cos^{-1}\left(\frac{z^2-2}{z^2}\right)$

Involving $\cos^{-1}\left(\frac{z^2-2}{z^2}\right)$ and $\cosh^{-1}\left(\frac{1}{z}\right)$

01.13.27.1372.01

$$\cos^{-1}\left(\frac{z^2-2}{z^2}\right) = \pi - 2 i \cosh^{-1}\left(\frac{1}{z}\right) \;/;\; 0 < \arg(z) < \frac{\pi}{2} \bigvee (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.1373.01

$$\cos^{-1}\left(\frac{z^2-2}{z^2}\right) = \pi + 2\,i\cosh^{-1}\left(\frac{1}{z}\right) /; -\frac{\pi}{2} \leq \arg(z) < 0 \bigvee (z \in \mathbb{R} \land z > 1)$$

01.13.27.1374.01

$$\cos^{-1}\left(\frac{z^2-2}{z^2}\right) = -\pi + 2\,i\cosh^{-1}\left(\frac{1}{z}\right) /; \frac{\pi}{2} \leq \arg(z) < \pi$$

01.13.27.1375.01

$$\cos^{-1}\left(\frac{z^2-2}{z^2}\right) = -\pi - 2\,i\cosh^{-1}\left(\frac{1}{z}\right) /; -\pi < \arg(z) < -\frac{\pi}{2} \bigvee (z \in \mathbb{R} \land z < 0)$$

01.13.27.1376.01

$$\cos^{-1}\left(\frac{z^2-2}{z^2}\right) = \pi \sqrt{\frac{1}{z^2}}\, z - \frac{2\,z}{\sqrt{\frac{1}{z}-1}} \sqrt{1-\frac{1}{z}} \sqrt{\frac{1}{z^2}} \cosh^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\frac{2-z^2}{z^2}\right)$

Involving $\cos^{-1}\left(\frac{2-z^2}{z^2}\right)$ and $\cosh^{-1}\left(\frac{1}{z}\right)$

01.13.27.1377.01

$$\cos^{-1}\left(\frac{2-z^2}{z^2}\right) = 2\,i\cosh^{-1}\left(\frac{1}{z}\right) /; 0 < \arg(z) < \frac{\pi}{2} \bigvee (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.1378.01

$$\cos^{-1}\left(\frac{2-z^2}{z^2}\right) = -2\,i\cosh^{-1}\left(\frac{1}{z}\right) /; -\frac{\pi}{2} \leq \arg(z) < 0 \bigvee (z \in \mathbb{R} \land z > 1)$$

01.13.27.1379.01

$$\cos^{-1}\left(\frac{2-z^2}{z^2}\right) = 2\,\pi - 2\,i\cosh^{-1}\left(\frac{1}{z}\right) /; \frac{\pi}{2} \leq \arg(z) < \pi$$

01.13.27.1380.01

$$\cos^{-1}\left(\frac{2-z^2}{z^2}\right) = 2\,\pi + 2\,i\cosh^{-1}\left(\frac{1}{z}\right) /; -\pi < \arg(z) < -\frac{\pi}{2} \bigvee (z \in \mathbb{R} \land z < 0)$$

01.13.27.1381.01

$$\cos^{-1}\left(\frac{2-z^2}{z^2}\right) = \pi\left(1 - \sqrt{\frac{1}{z^2}}\, z\right) + \frac{2\,z}{\sqrt{\frac{1}{z}-1}} \sqrt{1-\frac{1}{z}} \sqrt{\frac{1}{z^2}} \cosh^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\sqrt{1-z}\right)$

Involving $\cos^{-1}\left(\sqrt{1-z}\right)$ and $\cosh^{-1}\left(\sqrt{z}\right)$

01.13.27.1382.01

$$\cos^{-1}\left(\sqrt{1-z}\right) = \frac{\pi}{2} - i\cosh^{-1}\left(\sqrt{z}\right) /; -\pi < \arg(z) < 0 \bigvee (z \in \mathbb{R} \land z > 1)$$

01.13.27.1383.01

$$\cos^{-1}\left(\sqrt{1-z}\right) = \frac{\pi}{2} + i\cosh^{-1}\left(\sqrt{z}\right) /; 0 < \arg(z) \leq \pi \bigvee (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.1384.01

$$\cos^{-1}\left(\sqrt{1-z}\right) = \frac{\pi}{2} - \frac{\sqrt{1-z}}{\sqrt{z-1}}\cosh^{-1}\left(\sqrt{z}\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{1+c z}}{\sqrt{2}}\right)$

Involving $\cos^{-1}\left(\sqrt{\frac{1+z}{2}}\right)$ and $\cosh^{-1}(z)$

01.13.27.1385.01

$$\cos^{-1}\left(\sqrt{\frac{z+1}{2}}\right) = \frac{i}{2}\cosh^{-1}(z) /; -\pi < \arg(z) < 0 \bigvee (z \in \mathbb{R} \land z > 1)$$

01.13.27.1386.01

$$\cos^{-1}\left(\sqrt{\frac{1+z}{2}}\right) = -\frac{i}{2}\cosh^{-1}(z) /; 0 < \arg(z) \leq \pi \bigvee (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.1387.01

$$\cos^{-1}\left(\sqrt{\frac{1+z}{2}}\right) = \frac{\sqrt{1-z}}{2\sqrt{z-1}}\cosh^{-1}(z)$$

Involving $\cos^{-1}\left(\sqrt{\frac{1-z}{2}}\right)$ and $\cosh^{-1}(z)$

01.13.27.1388.01

$$\cos^{-1}\left(\sqrt{\frac{1-z}{2}}\right) = \frac{\pi}{2} - \frac{1}{2} i\cosh^{-1}(z) /; -\pi < \arg(z) < 0 \bigvee (z \in \mathbb{R} \land z > 1)$$

01.13.27.1389.01

$$\cos^{-1}\left(\sqrt{\frac{1-z}{2}}\right) = \frac{\pi}{2} + \frac{i}{2}\cosh^{-1}(z) /; 0 < \arg(z) \leq \pi \bigvee (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.1390.01

$$\cos^{-1}\left(\sqrt{\frac{1-z}{2}}\right) = \frac{\pi}{2} - \frac{1}{2}\frac{\sqrt{1-z}}{\sqrt{z-1}}\cosh^{-1}(z)$$

Involving $\cos^{-1}\left(\frac{\sqrt{z+a}}{\sqrt{z}}\right)$

Involving $\cos^{-1}\left(\frac{\sqrt{z-1}}{\sqrt{z}}\right)$ and $\cosh^{-1}\left(\frac{1}{\sqrt{z}}\right)$

01.13.27.1391.01

$$\cos^{-1}\left(\frac{\sqrt{z-1}}{\sqrt{z}}\right) = \frac{\pi}{2} - i\cosh^{-1}\left(\frac{1}{\sqrt{z}}\right) /; 0 < \arg(z) < \pi \bigvee (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.1392.01

$$\cos^{-1}\left(\frac{\sqrt{z-1}}{\sqrt{z}}\right) = \frac{\pi}{2} + i\cosh^{-1}\left(\frac{1}{\sqrt{z}}\right) /; -\pi < \arg(z) < 0 \bigvee (z \in \mathbb{R} \land z > 1)$$

01.13.27.1393.01

$$\cos^{-1}\left(\frac{\sqrt{z-1}}{\sqrt{z}}\right) = \frac{\pi}{2} - \frac{\sqrt{z-1}}{\sqrt{1-z}}\cosh^{-1}\left(\frac{1}{\sqrt{z}}\right) /; |\arg(z)| < \pi$$

01.13.27.1394.01

$$\cos^{-1}\left(\frac{\sqrt{z-1}}{\sqrt{z}}\right) = \frac{\pi}{2}\sqrt{\frac{1}{z}}\sqrt{z} - \frac{\sqrt{z-1}\sqrt{z}}{\sqrt{1-z}}\sqrt{\frac{1}{z}}\cosh^{-1}\left(\frac{1}{\sqrt{z}}\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{z-1}}{\sqrt{z}}\right)$ and $\cosh^{-1}\left(\sqrt{\frac{1}{z}}\right)$

01.13.27.1395.01

$$\cos^{-1}\left(\frac{\sqrt{z-1}}{\sqrt{z}}\right) = \frac{\pi}{2} - i\cosh^{-1}\left(\sqrt{\frac{1}{z}}\right) /; 0 < \arg(z) < \pi \bigvee (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.1396.01

$$\cos^{-1}\left(\frac{\sqrt{z-1}}{\sqrt{z}}\right) = \frac{\pi}{2} + i\cosh^{-1}\left(\sqrt{\frac{1}{z}}\right) /; -\pi < \arg(z) < 0 \bigvee (z \in \mathbb{R} \land z > 1) \bigvee (z \in \mathbb{R} \land z < 0)$$

01.13.27.1397.01

$$\cos^{-1}\left(\frac{\sqrt{z-1}}{\sqrt{z}}\right) = \frac{\pi}{2} - \frac{\sqrt{z-1}}{\sqrt{1-z}}\cosh^{-1}\left(\sqrt{\frac{1}{z}}\right) /; |\arg(z)| < \pi$$

01.13.27.1398.01

$$\cos^{-1}\left(\frac{\sqrt{z-1}}{\sqrt{z}}\right) = \frac{\pi}{2} - \frac{\sqrt{z-1}\sqrt{z}}{\sqrt{1-z}}\sqrt{\frac{1}{z}}\cosh^{-1}\left(\sqrt{\frac{1}{z}}\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{z+1}}{\sqrt{z}}\right)$ and $\cosh^{-1}\left(1 + \frac{2}{z}\right)$

01.13.27.1399.01

$$\cos^{-1}\left(\frac{\sqrt{z+1}}{\sqrt{z}}\right) = \frac{\pi}{2} - \frac{\sqrt{-z}\sqrt{z+1}}{2\sqrt{-z-1}\sqrt{z}}\left(\pi - z\sqrt{-\frac{1}{z^2}}\cosh^{-1}\left(1 + \frac{2}{z}\right)\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{z+1}}{\sqrt{z}}\right)$ and $\cosh^{-1}\left(\sqrt{\frac{z+1}{z}}\right)$

01.13.27.1400.01

$$\cos^{-1}\left(\frac{\sqrt{z+1}}{\sqrt{z}}\right) = \frac{\pi}{2} - \sqrt{\frac{z+1}{z}}\,\sqrt{\frac{z}{z+1}}\left(\frac{\pi}{2} - z\sqrt{-\frac{1}{z^2}}\,\cosh^{-1}\left(\sqrt{\frac{z+1}{z}}\right)\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{-z}}\right)$

Involving $\cos^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{-z}}\right)$ and $\cosh^{-1}\left(\frac{1}{\sqrt{z}}\right)$

01.13.27.1401.01

$$\cos^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{-z}}\right) = \frac{\pi}{2} - i\cosh^{-1}\left(\frac{1}{\sqrt{z}}\right) /; \operatorname{Im}(z) > 0$$

01.13.27.1402.01

$$\cos^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{-z}}\right) = \frac{\pi}{2} + i\cosh^{-1}\left(\frac{1}{\sqrt{z}}\right) /; -\pi < \arg(z) \leq 0$$

01.13.27.1403.01

$$\cos^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{-z}}\right) = \frac{1}{2}\pi\left(\sqrt{\frac{z}{z-1}}\,\sqrt{\frac{z-1}{z}}\left(\sqrt{\frac{1}{z}}\,\sqrt{z} - 1\right) + 1\right) + \frac{\sqrt{-z^2}}{\sqrt{z-1}}\,\sqrt{\frac{z-1}{z^2}}\,\cosh^{-1}\left(\frac{1}{\sqrt{z}}\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{-z}}\right)$ and $\cosh^{-1}\left(\sqrt{\frac{1}{z}}\right)$

01.13.27.1404.01

$$\cos^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{-z}}\right) = \frac{\pi}{2} - i\cosh^{-1}\left(\sqrt{\frac{1}{z}}\right) /; \operatorname{Im}(z) > 0$$

01.13.27.1405.01

$$\cos^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{-z}}\right) = \frac{\pi}{2} + i\cosh^{-1}\left(\sqrt{\frac{1}{z}}\right) /; \operatorname{Im}(z) \leq 0$$

01.13.27.1406.01

$$\cos^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{-z}}\right) = \sqrt{\frac{1}{z}}\,\sqrt{-z}\,\cosh^{-1}\left(\sqrt{\frac{1}{z}}\right) + \frac{\pi}{2}$$

Involving $\cos^{-1}\left(\sqrt{\frac{z+a}{z}}\right)$

Involving $\cos^{-1}\left(\sqrt{\frac{z-1}{z}}\right)$ and $\cosh^{-1}\left(\frac{1}{\sqrt{z}}\right)$

01.13.27.1407.01

$$\cos^{-1}\left(\sqrt{\frac{z-1}{z}}\right) = \frac{\pi}{2} - i\cosh^{-1}\left(\frac{1}{\sqrt{z}}\right) /; \operatorname{Im}(z) > 0 \lor (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.1408.01

$$\cos^{-1}\left(\sqrt{\frac{z-1}{z}}\right) = \frac{\pi}{2} + i\cosh^{-1}\left(\frac{1}{\sqrt{z}}\right) /; \operatorname{Im}(z) < 0 \lor (z \in \mathbb{R} \land z > 1)$$

01.13.27.1409.01

$$\cos^{-1}\left(\sqrt{\frac{z-1}{z}}\right) = \frac{\pi}{2} - \frac{\sqrt{z-1}}{\sqrt{1-z}}\cosh^{-1}\left(\frac{1}{\sqrt{z}}\right) /; |\arg(z)| < \pi$$

01.13.27.1410.01

$$\cos^{-1}\left(\sqrt{\frac{z-1}{z}}\right) = -\sqrt{\frac{1}{z}}\sqrt{z}\frac{\sqrt{z-1}}{\sqrt{1-z}}\cosh^{-1}\left(\frac{1}{\sqrt{z}}\right) + \frac{\pi}{2}\sqrt{\frac{1}{z}}\sqrt{z}$$

Involving $\cos^{-1}\left(\sqrt{\frac{z-1}{z}}\right)$ and $\cosh^{-1}\left(\sqrt{\frac{1}{z}}\right)$

01.13.27.1411.01

$$\cos^{-1}\left(\sqrt{\frac{z-1}{z}}\right) = \frac{\pi}{2} - i\cosh^{-1}\left(\sqrt{\frac{1}{z}}\right) /; \operatorname{Im}(z) > 0 \lor (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.1412.01

$$\cos^{-1}\left(\sqrt{\frac{z-1}{z}}\right) = \frac{\pi}{2} + i\cosh^{-1}\left(\sqrt{\frac{1}{z}}\right) /; \operatorname{Im}(z) < 0 \lor (z \in \mathbb{R} \land z > 1) \lor (z \in \mathbb{R} \land z < 0)$$

01.13.27.1413.01

$$\cos^{-1}\left(\sqrt{\frac{z-1}{z}}\right) = \frac{\pi}{2} - \frac{\sqrt{z-1}}{\sqrt{1-z}}\cosh^{-1}\left(\sqrt{\frac{1}{z}}\right) /; |\arg(z)| < \pi$$

01.13.27.1414.01

$$\cos^{-1}\left(\sqrt{\frac{z-1}{z}}\right) = -\sqrt{\frac{1}{z}}\sqrt{z}\frac{\sqrt{z-1}}{\sqrt{1-z}}\cosh^{-1}\left(\sqrt{\frac{1}{z}}\right) + \frac{\pi}{2}$$

Involving $\cos^{-1}\left(\sqrt{\frac{z+1}{z}}\right)$ and $\cosh^{-1}\left(\sqrt{\frac{z+1}{z}}\right)$

01.13.27.1415.01

$$\cos^{-1}\left(\sqrt{\frac{z+1}{z}}\right) = z\sqrt{-\frac{1}{z^2}}\cosh^{-1}\left(\sqrt{\frac{z+1}{z}}\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{z+a}}{\sqrt{2z}}\right)$

Involving $\cos^{-1}\left(\frac{\sqrt{z+1}}{\sqrt{2z}}\right)$ and $\cosh^{-1}\left(\frac{1}{z}\right)$

01.13.27.1416.01

$$\cos^{-1}\left(\frac{\sqrt{z+1}}{\sqrt{2z}}\right) = \frac{i}{2}\cosh^{-1}\left(\frac{1}{z}\right) /; \operatorname{Im}(z) > 0 \lor (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.1417.01

$$\cos^{-1}\left(\frac{\sqrt{z+1}}{\sqrt{2z}}\right) = -\frac{i}{2}\cosh^{-1}\left(\frac{1}{z}\right) /; \operatorname{Im}(z) < 0 \lor (z \in \mathbb{R} \land z < -1) \lor (z \in \mathbb{R} \land z > 1)$$

01.13.27.1418.01

$$\cos^{-1}\left(\frac{\sqrt{z+1}}{\sqrt{2z}}\right) = \frac{\sqrt{z-1}}{2\sqrt{1-z}}\cosh^{-1}\left(\frac{1}{z}\right) /; |\arg(z)| < \pi$$

01.13.27.1419.01

$$\cos^{-1}\left(\frac{\sqrt{z+1}}{\sqrt{2z}}\right) = \frac{\pi}{2} - \frac{1}{2}\sqrt{1+\frac{1}{z}}\sqrt{\frac{z}{z+1}}\left(\pi - \frac{1}{\sqrt{\frac{1}{z}-1}}\sqrt{1-\frac{1}{z}}\cosh^{-1}\left(\frac{1}{z}\right)\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{z-1}}{\sqrt{2z}}\right)$ and $\cosh^{-1}\left(\frac{1}{z}\right)$

01.13.27.1420.01

$$\cos^{-1}\left(\frac{\sqrt{z-1}}{\sqrt{2z}}\right) = \frac{\pi}{2} - \frac{1}{2} i \cosh^{-1}\left(\frac{1}{z}\right) /; \operatorname{Im}(z) > 0 \lor (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.1421.01

$$\cos^{-1}\left(\frac{\sqrt{z-1}}{\sqrt{2z}}\right) = \frac{\pi}{2} + \frac{i}{2}\cosh^{-1}\left(\frac{1}{z}\right) /; \operatorname{Im}(z) < 0 \lor (z \in \mathbb{R} \land z > 1) \lor (z \in \mathbb{R} \land z < 0)$$

01.13.27.1422.01

$$\cos^{-1}\left(\frac{\sqrt{z-1}}{\sqrt{2z}}\right) = \frac{\pi}{2} - \frac{1}{2\sqrt{\frac{1}{z}-1}}\sqrt{1-\frac{1}{z}}\cosh^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{a-z}}{\sqrt{-2z}}\right)$

Involving $\cos^{-1}\left(\frac{\sqrt{-z-1}}{\sqrt{-2z}}\right)$ and $\cosh^{-1}\left(\frac{1}{z}\right)$

01.13.27.1423.01

$$\cos^{-1}\left(\frac{\sqrt{-z-1}}{\sqrt{-2z}}\right) = \frac{i}{2}\cosh^{-1}\left(\frac{1}{z}\right) /; \operatorname{Im}(z) > 0 \lor (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.1424.01

$$\cos^{-1}\left(\frac{\sqrt{-z-1}}{\sqrt{-2z}}\right) = -\frac{i}{2}\cosh^{-1}\left(\frac{1}{z}\right) /; \operatorname{Im}(z) < 0 \lor (z \in \mathbb{R} \land z > 1) \lor (z \in \mathbb{R} \land z < 0)$$

01.13.27.1425.01

$$\cos^{-1}\left(\frac{\sqrt{-z-1}}{\sqrt{-2z}}\right) = \frac{1}{2\sqrt{\frac{1}{z}-1}}\sqrt{1-\frac{1}{z}}\cosh^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{-2z}}\right)$ and $\cosh^{-1}\left(\frac{1}{z}\right)$

01.13.27.1426.01

$$\cos^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{-2z}}\right) = \frac{\pi}{2} + \frac{1}{2} i \cosh^{-1}\left(\frac{1}{z}\right) /; \operatorname{Im}(z) \le 0$$

01.13.27.1427.01

$$\cos^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{-2z}}\right) = \frac{\pi}{2} - \frac{1}{2} i \cosh^{-1}\left(\frac{1}{z}\right) /; \operatorname{Im}(z) > 0$$

01.13.27.1428.01

$$\cos^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{-2z}}\right) = \frac{\pi}{2} + \frac{1}{2}\sqrt{\frac{1}{z}}\sqrt{-z}\cosh^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\sqrt{\frac{z+a}{2z}}\right)$

Involving $\cos^{-1}\left(\sqrt{\frac{z+1}{2z}}\right)$ and $\cosh^{-1}\left(\frac{1}{z}\right)$

01.13.27.1429.01

$$\cos^{-1}\left(\sqrt{\frac{z+1}{2z}}\right) = \frac{i}{2}\cosh^{-1}\left(\frac{1}{z}\right) /; \operatorname{Im}(z) > 0 \lor (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.1430.01

$$\cos^{-1}\left(\sqrt{\frac{z+1}{2z}}\right) = -\frac{i}{2}\cosh^{-1}\left(\frac{1}{z}\right) /; \operatorname{Im}(z) < 0 \lor (z \in \mathbb{R} \land z < 0) \lor (z \in \mathbb{R} \land z > 1)$$

01.13.27.2520.01

$$\cos^{-1}\left(\sqrt{\frac{z+1}{2z}}\right) = \frac{\sqrt{z-1}}{2\sqrt{1-z}}\cosh^{-1}\left(\frac{1}{z}\right) /; |\arg(z)| < \pi$$

01.13.27.2521.01

$$\cos^{-1}\left(\sqrt{\frac{z+1}{2z}}\right) = \frac{1}{2\sqrt{\frac{1}{z}-1}} \sqrt{1-\frac{1}{z}} \cosh^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\sqrt{\frac{z-1}{2z}}\right)$ and $\cosh^{-1}\left(\frac{1}{z}\right)$

01.13.27.1431.01

$$\cos^{-1}\left(\sqrt{\frac{z-1}{2z}}\right) = \frac{\pi}{2} - \frac{1}{2} i \cosh^{-1}\left(\frac{1}{z}\right) /; \operatorname{Im}(z) > 0 \lor (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.1432.01

$$\cos^{-1}\left(\sqrt{\frac{z-1}{2z}}\right) = \frac{\pi}{2} + \frac{1}{2} i \cosh^{-1}\left(\frac{1}{z}\right) /; \operatorname{Im}(z) < 0 \lor (z \in \mathbb{R} \land z < 0) \lor (z \in \mathbb{R} \land z > 1)$$

01.13.27.2522.01

$$\cos^{-1}\left(\sqrt{\frac{z-1}{2z}}\right) = \frac{\pi}{2} - \frac{1}{2\sqrt{\frac{1}{z}-1}} \sqrt{1-\frac{1}{z}} \cosh^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\sqrt{a z^2+1}\right)$

Involving $\cos^{-1}\left(\sqrt{z^2+1}\right)$ and $\cosh^{-1}(1+2z^2)$

01.13.27.1433.01

$$\cos^{-1}\left(\sqrt{z^2+1}\right) = \frac{1}{2} i \cosh^{-1}(2z^2+1) /; \frac{\pi}{2} < \arg(z) \le \pi \bigvee -\frac{\pi}{2} < \arg(z) \le 0$$

01.13.27.1434.01

$$\cos^{-1}\left(\sqrt{z^2+1}\right) = -\frac{1}{2} i \cosh^{-1}(2z^2+1) /; 0 < \arg(z) \le \frac{\pi}{2} \bigvee -\pi < \arg(z) \le -\frac{\pi}{2}$$

01.13.27.1435.01

$$\cos^{-1}\left(\sqrt{z^2+1}\right) = \frac{\sqrt{-z^4}}{2z^2} \cosh^{-1}(2z^2+1)$$

Involving $\cos^{-1}\left(\sqrt{z^2+1}\right)$ and $\cosh^{-1}\left(\sqrt{z^2+1}\right)$

01.13.27.1436.01

$$\cos^{-1}\left(\sqrt{z^2+1}\right) = i \cosh^{-1}\left(\sqrt{z^2+1}\right) /; \frac{\pi}{2} < \arg(z) \le \pi \bigvee -\frac{\pi}{2} < \arg(z) \le 0$$

01.13.27.1437.01

$$\cos^{-1}\left(\sqrt{z^2+1}\right) = -i\cosh^{-1}\left(\sqrt{z^2+1}\right) /; 0 < \arg(z) \leq \frac{\pi}{2} \bigvee -\pi < \arg(z) \leq -\frac{\pi}{2}$$

01.13.27.1438.01

$$\cos^{-1}\left(\sqrt{z^2+1}\right) = \frac{\sqrt{-z^4}}{z^2}\cosh^{-1}\left(\sqrt{z^2+1}\right)$$

Involving $\cos^{-1}\left(\sqrt{1-z^2}\right)$ and $\cosh^{-1}(z)$

01.13.27.1439.01

$$\cos^{-1}\left(\sqrt{1-z^2}\right) = \frac{\pi}{2} + i\cosh^{-1}(z) /; 0 < \arg(z) \leq \frac{\pi}{2} \bigvee (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.1440.01

$$\cos^{-1}\left(\sqrt{1-z^2}\right) = \frac{\pi}{2} - i\cosh^{-1}(z) /; -\frac{\pi}{2} < \arg(z) < 0 \bigvee (z \in \mathbb{R} \land z > 1)$$

01.13.27.1441.01

$$\cos^{-1}\left(\sqrt{1-z^2}\right) = -\frac{\pi}{2} - i\cosh^{-1}(z) /; \frac{\pi}{2} < \arg(z) \leq \pi$$

01.13.27.1442.01

$$\cos^{-1}\left(\sqrt{1-z^2}\right) = -\frac{\pi}{2} + i\cosh^{-1}(z) /; -\pi < \arg(z) \leq -\frac{\pi}{2}$$

01.13.27.1443.01

$$\cos^{-1}\left(\sqrt{1-z^2}\right) = \frac{\pi}{2} - \frac{\sqrt{1-z}}{\sqrt{z-1}}\cosh^{-1}(z) /; -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

01.13.27.1444.01

$$\cos^{-1}\left(\sqrt{1-z^2}\right) = -\frac{\sqrt{z^2}\sqrt{1-z}}{z\sqrt{z-1}}\cosh^{-1}(z) + \frac{\pi\sqrt{z^2}}{2z}$$

Involving $\cos^{-1}\left(\frac{\sqrt{z^2+a}}{z}\right)$

Involving $\cos^{-1}\left(\frac{\sqrt{z^2-1}}{z}\right)$ and $\cosh^{-1}\left(\frac{1}{z}\right)$

01.13.27.1445.01

$$\cos^{-1}\left(\frac{\sqrt{z^2-1}}{z}\right) = \frac{\pi}{2} - i\cosh^{-1}\left(\frac{1}{z}\right) /; 0 < \arg(z) < \frac{\pi}{2} \bigvee (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.1446.01

$$\cos^{-1}\left(\frac{\sqrt{z^2-1}}{z}\right) = \frac{\pi}{2} + i\cosh^{-1}\left(\frac{1}{z}\right) /; -\frac{\pi}{2} < \arg(z) < 0 \bigvee \boldsymbol{(z \in \mathbb{R} \land z > 1)}$$

01.13.27.1447.01

$$\cos^{-1}\left(\frac{\sqrt{z^2-1}}{z}\right) = \frac{3\pi}{2} - i\cosh^{-1}\left(\frac{1}{z}\right) /; \frac{\pi}{2} < \arg(z) < \pi$$

01.13.27.1448.01

$$\cos^{-1}\left(\frac{\sqrt{z^2-1}}{z}\right) = \frac{3\pi}{2} + i\cosh^{-1}\left(\frac{1}{z}\right) /; -\pi < \arg(z) < -\frac{\pi}{2} \bigvee (z \in \mathbb{R} \wedge z < 0)$$

01.13.27.1449.01

$$\cos^{-1}\left(\frac{\sqrt{z^2-1}}{z}\right) = \pi\left(1 - \frac{\sqrt{z^2}}{2z}\right) - \frac{\sqrt{z-1}}{\sqrt{1-z}}\cosh^{-1}\left(\frac{1}{z}\right) /; \operatorname{Re}(z) \neq 0 \bigwedge |\arg(z)| < \pi$$

01.13.27.1450.01

$$\cos^{-1}\left(\frac{\sqrt{z^2-1}}{z}\right) = \frac{\pi}{2}\left(1 - \frac{\sqrt{z^2}}{z} + \sqrt{z^2}\sqrt{\frac{1}{z^2}}\right) - \frac{\sqrt{z^2}}{\sqrt{\frac{1-z}{z}}}\sqrt{\frac{z-1}{z}}\sqrt{\frac{1}{z^2}}\cosh^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{z^2+1}}{z}\right)$ and $\cosh^{-1}(1+2z^{-2})$

01.13.27.1451.01

$$\cos^{-1}\left(\frac{\sqrt{z^2+1}}{z}\right) = \frac{i}{2}\cosh^{-1}\left(1+\frac{2}{z^2}\right) /; 0 \leq \arg(z) < \frac{\pi}{2}$$

01.13.27.1452.01

$$\cos^{-1}\left(\frac{\sqrt{z^2+1}}{z}\right) = -\frac{i}{2}\cosh^{-1}\left(1+\frac{2}{z^2}\right) /; -\frac{\pi}{2} < \arg(z) < 0 \bigvee (i z \in \mathbb{R} \wedge i z < -1) \bigvee (i z \in \mathbb{R} \wedge 0 < i z < 1)$$

01.13.27.1453.01

$$\cos^{-1}\left(\frac{\sqrt{z^2+1}}{z}\right) = \frac{1}{2}\left(i\cosh^{-1}\left(1+\frac{2}{z^2}\right) + 2\pi\right) /; \frac{\pi}{2} < \arg(z) < \pi \bigvee (-i z \in \mathbb{R} \wedge 0 < -i z < 1) \bigvee (i z \in \mathbb{R} \wedge i z > 1)$$

01.13.27.1454.01

$$\cos^{-1}\left(\frac{\sqrt{z^2+1}}{z}\right) = \frac{1}{2}\left(-i\cosh^{-1}\left(1+\frac{2}{z^2}\right) + 2\pi\right) /; -\pi < \arg(z) < -\frac{\pi}{2} \bigvee (z \in \mathbb{R} \wedge z < 0)$$

01.13.27.1455.01

$$\cos^{-1}\left(\frac{\sqrt{z^2+1}}{z}\right) = \frac{\pi}{2} - \frac{\sqrt{z^2+1}}{2z\sqrt{-\frac{z^2+1}{z^4}}}\left(\sqrt{\frac{1}{z^2}}\cosh^{-1}\left(1+\frac{2}{z^2}\right) + \pi\sqrt{-\frac{1}{z^2}}\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{z^2+1}}{z}\right)$ and $\cosh^{-1}\left(\sqrt{1+\frac{1}{z^2}}\right)$

01.13.27.1456.01

$$\cos^{-1}\left(\frac{\sqrt{z^2+1}}{z}\right) = i\cosh^{-1}\left(\sqrt{1+\frac{1}{z^2}}\right) /; 0 \leq \arg(z) < \frac{\pi}{2}$$

01.13.27.1457.01

$$\cos^{-1}\left(\frac{\sqrt{z^2+1}}{z}\right) = -i\cosh^{-1}\left(\sqrt{1+\frac{1}{z^2}}\right) /; -\frac{\pi}{2} < \arg(z) < 0 \bigvee (i\, z \in \mathbb{R} \wedge 0 < i\, z < 1)$$

01.13.27.1458.01

$$\cos^{-1}\left(\frac{\sqrt{z^2+1}}{z}\right) = \pi + i\cosh^{-1}\left(\sqrt{1+\frac{1}{z^2}}\right) /; \frac{\pi}{2} < \arg(z) < \pi \bigvee (-i\, z \in \mathbb{R} \wedge 0 < -i\, z < 1) \bigvee (i\, z \in \mathbb{R} \wedge i\, z > 1)$$

01.13.27.1459.01

$$\cos^{-1}\left(\frac{\sqrt{z^2+1}}{z}\right) = \pi - i\cosh^{-1}\left(\sqrt{1+\frac{1}{z^2}}\right) /; -\pi < \arg(z) < -\frac{\pi}{2} \bigvee (z \in \mathbb{R} \wedge z < 0)$$

01.13.27.1460.01

$$\cos^{-1}\left(\frac{\sqrt{z^2+1}}{z}\right) = \frac{\pi}{2} - \frac{\sqrt{z^2+1}}{2\, z\sqrt{-\frac{z^2+1}{z^4}}}\left(2\sqrt{\frac{1}{z^2}}\cosh^{-1}\left(\sqrt{1+\frac{1}{z^2}}\right) + \pi\sqrt{-\frac{1}{z^2}}\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{z^2-1}}{\sqrt{z^2}}\right)$

Involving $\cos^{-1}\left(\frac{\sqrt{z^2-1}}{\sqrt{z^2}}\right)$ and $\cosh^{-1}\left(\frac{1}{z}\right)$

01.13.27.1461.01

$$\cos^{-1}\left(\frac{\sqrt{z^2-1}}{\sqrt{z^2}}\right) = \frac{\pi}{2} - i\cosh^{-1}\left(\frac{1}{z}\right) /; 0 < \arg(z) < \frac{\pi}{2} \bigvee (z \in \mathbb{R} \wedge 0 < z < 1)$$

01.13.27.1462.01

$$\cos^{-1}\left(\frac{\sqrt{z^2-1}}{\sqrt{z^2}}\right) = \frac{\pi}{2} + i\cosh^{-1}\left(\frac{1}{z}\right) /; -\frac{\pi}{2} \leq \arg(z) < 0 \bigvee (z \in \mathbb{R} \wedge z > 1)$$

01.13.27.1463.01

$$\cos^{-1}\left(\frac{\sqrt{z^2-1}}{\sqrt{z^2}}\right) = -\frac{\pi}{2} + i\cosh^{-1}\left(\frac{1}{z}\right) /; \frac{\pi}{2} \leq \arg(z) < \pi$$

01.13.27.1464.01

$$\cos^{-1}\left(\frac{\sqrt{z^2-1}}{\sqrt{z^2}}\right) = -\frac{\pi}{2} - i\cosh^{-1}\left(\frac{1}{z}\right) /; -\pi < \arg(z) < -\frac{\pi}{2} \bigvee (z \in \mathbb{R} \land z < 0)$$

01.13.27.1465.01

$$\cos^{-1}\left(\frac{\sqrt{z^2-1}}{\sqrt{z^2}}\right) = \frac{\pi z}{2}\sqrt{\frac{1}{z^2}} - \frac{z}{\sqrt{\frac{1-z}{z}}}\sqrt{\frac{z-1}{z}}\sqrt{\frac{1}{z^2}}\cosh^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{a-z^2}}{\sqrt{-z^2}}\right)$

Involving $\cos^{-1}\left(\frac{\sqrt{1-z^2}}{\sqrt{-z^2}}\right)$ and $\cosh^{-1}\left(\frac{1}{z}\right)$

01.13.27.1466.01

$$\cos^{-1}\left(\frac{\sqrt{1-z^2}}{\sqrt{-z^2}}\right) = \frac{\pi}{2} - i\cosh^{-1}\left(\frac{1}{z}\right) /; 0 < \arg(z) < \frac{\pi}{2}$$

01.13.27.1467.01

$$\cos^{-1}\left(\frac{\sqrt{1-z^2}}{\sqrt{-z^2}}\right) = \frac{\pi}{2} + i\cosh^{-1}\left(\frac{1}{z}\right) /; -\frac{\pi}{2} \leq \arg(z) \leq 0$$

01.13.27.1468.01

$$\cos^{-1}\left(\frac{\sqrt{1-z^2}}{\sqrt{-z^2}}\right) = -\frac{\pi}{2} + i\cosh^{-1}\left(\frac{1}{z}\right) /; \frac{\pi}{2} \leq \arg(z) < \pi$$

01.13.27.1469.01

$$\cos^{-1}\left(\frac{\sqrt{1-z^2}}{\sqrt{-z^2}}\right) = -\frac{\pi}{2} - i\cosh^{-1}\left(\frac{1}{z}\right) /; -\pi < \arg(z) < -\frac{\pi}{2} \bigvee (z \in \mathbb{R} \land z < -1)$$

01.13.27.1470.01

$$\cos^{-1}\left(\frac{\sqrt{1-z^2}}{\sqrt{-z^2}}\right) = \frac{3\pi}{2} + i\cosh^{-1}\left(\frac{1}{z}\right) /; (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.1471.01

$$\cos^{-1}\left(\frac{\sqrt{1-z^2}}{\sqrt{-z^2}}\right) = \frac{\pi}{2} - \sqrt{\frac{z^2-1}{z^2}}\sqrt{\frac{z^2}{z^2-1}}\left(\frac{\pi}{2}\left(1-\sqrt{\frac{1}{z^2}}\,z\right) + \frac{\sqrt{z-1}\,\sqrt{z}}{\sqrt{\frac{1-z}{z}}}\sqrt{\frac{1}{z^2}}\cosh^{-1}\left(\frac{1}{z}\right)\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{-1-z^2}}{\sqrt{-z^2}}\right)$ and $\cosh^{-1}\left(\frac{z^2+2}{z^2}\right)$

01.13.27.1472.01

$$\cos^{-1}\left(\frac{\sqrt{-1-z^2}}{\sqrt{-z^2}}\right) = \frac{1}{2}\,i\cosh^{-1}\left(\frac{z^2+2}{z^2}\right) /;\ 0 \le \arg(z) < \frac{\pi}{2} \bigvee -\pi < \arg(z) < -\frac{\pi}{2} \bigvee (z \in \mathbb{R} \land z < 0)$$

01.13.27.1473.01

$$\cos^{-1}\left(\frac{\sqrt{-1-z^2}}{\sqrt{-z^2}}\right) = -\frac{1}{2}\,i\cosh^{-1}\left(\frac{z^2+2}{z^2}\right) /;\ \frac{\pi}{2} \le \arg(z) < \pi \bigvee -\frac{\pi}{2} \le \arg(z) < 0$$

01.13.27.1474.01

$$\cos^{-1}\left(\frac{\sqrt{-1-z^2}}{\sqrt{-z^2}}\right) = \frac{z^2}{2}\sqrt{-\frac{1}{z^4}}\cosh^{-1}\left(\frac{z^2+2}{z^2}\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{-1-z^2}}{\sqrt{-z^2}}\right)$ and $\cosh^{-1}\left(\sqrt{1+\frac{1}{z^2}}\right)$

01.13.27.1475.01

$$\cos^{-1}\left(\frac{\sqrt{-1-z^2}}{\sqrt{-z^2}}\right) = i\cosh^{-1}\left(\sqrt{1+\frac{1}{z^2}}\right) /;\ 0 \le \arg(z) < \frac{\pi}{2} \bigvee -\pi < \arg(z) < -\frac{\pi}{2} \bigvee (z \in \mathbb{R} \land z < 0)$$

01.13.27.1476.01

$$\cos^{-1}\left(\frac{\sqrt{-1-z^2}}{\sqrt{-z^2}}\right) = -i\cosh^{-1}\left(\sqrt{1+\frac{1}{z^2}}\right) /;\ \frac{\pi}{2} \le \arg(z) < \pi \bigvee -\frac{\pi}{2} \le \arg(z) < 0$$

01.13.27.1477.01

$$\cos^{-1}\left(\frac{\sqrt{-1-z^2}}{\sqrt{-z^2}}\right) = z^2\sqrt{-\frac{1}{z^4}}\cosh^{-1}\left(\sqrt{1+\frac{1}{z^2}}\right)$$

Involving $\cos^{-1}\left(\sqrt{\frac{z^2+c}{z^2}}\right)$

Involving $\cos^{-1}\left(\sqrt{\frac{z^2-1}{z^2}}\right)$ and $\cosh^{-1}\left(\frac{1}{z}\right)$

01.13.27.1478.01

$$\cos^{-1}\left(\sqrt{\frac{z^2-1}{z^2}}\right) = \frac{\pi}{2} - i\cosh^{-1}\left(\frac{1}{z}\right) /;\ 0 < \arg(z) < \frac{\pi}{2} \bigvee (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.1479.01

$$\cos^{-1}\left(\sqrt{\frac{z^2-1}{z^2}}\right) = \frac{\pi}{2} + i\cosh^{-1}\left(\frac{1}{z}\right) /;\ -\frac{\pi}{2} \leq \arg(z) < 0 \bigvee (z \in \mathbb{R} \land z > 1)$$

01.13.27.1480.01

$$\cos^{-1}\left(\sqrt{\frac{z^2-1}{z^2}}\right) = -\frac{\pi}{2} + i\cosh^{-1}\left(\frac{1}{z}\right) /;\ \frac{\pi}{2} \leq \arg(z) < \pi$$

01.13.27.1481.01

$$\cos^{-1}\left(\sqrt{\frac{z^2-1}{z^2}}\right) = -\frac{\pi}{2} - i\cosh^{-1}\left(\frac{1}{z}\right) /;\ -\pi < \arg(z) < -\frac{\pi}{2} \bigvee (z \in \mathbb{R} \land z < 0)$$

01.13.27.1482.01

$$\cos^{-1}\left(\sqrt{\frac{z^2-1}{z^2}}\right) = \frac{\pi z}{2}\sqrt{\frac{1}{z^2}} - \frac{\sqrt{z-1}\,\sqrt{z}}{\sqrt{\frac{1-z}{z}}}\sqrt{\frac{1}{z^2}}\cosh^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\sqrt{\frac{z^2+1}{z^2}}\right)$ and $\cosh^{-1}\left(1+2z^{-2}\right)$

01.13.27.1483.01

$$\cos^{-1}\left(\sqrt{\frac{z^2+1}{z^2}}\right) = \frac{i}{2}\cosh^{-1}\left(1+\frac{2}{z^2}\right) /;\ 0 \leq \arg(z) < \frac{\pi}{2} \bigvee -\pi < \arg(z) < -\frac{\pi}{2} \bigvee (z \in \mathbb{R} \land z < 0)$$

01.13.27.1484.01

$$\cos^{-1}\left(\sqrt{\frac{z^2+1}{z^2}}\right) = -\frac{i}{2}\cosh^{-1}\left(1+\frac{2}{z^2}\right) /;\ \frac{\pi}{2} \leq \arg(z) < \pi \bigvee -\frac{\pi}{2} \leq \arg(z) < 0$$

01.13.27.1485.01

$$\cos^{-1}\left(\sqrt{\frac{z^2+1}{z^2}}\right) = \frac{1}{2}\left(z^2\sqrt{-\frac{1}{z^4}}\right)\cosh^{-1}\left(1+\frac{2}{z^2}\right)$$

Involving $\cos^{-1}\left(\sqrt{\frac{z^2+1}{z^2}}\right)$ and $\cosh^{-1}\left(\sqrt{z^{-2}+1}\right)$

01.13.27.1486.01

$$\cos^{-1}\left(\sqrt{\frac{z^2+1}{z^2}}\right) = i \cosh^{-1}\left(\sqrt{1+\frac{1}{z^2}}\right) /; 0 \leq \arg(z) < \frac{\pi}{2} \bigvee -\pi < \arg(z) < -\frac{\pi}{2} \bigvee (z \in \mathbb{R} \land z < 0)$$

01.13.27.1487.01

$$\cos^{-1}\left(\sqrt{\frac{z^2+1}{z^2}}\right) = -i \cosh^{-1}\left(\sqrt{1+\frac{1}{z^2}}\right) /; \frac{\pi}{2} \leq \arg(z) < \pi \bigvee -\frac{\pi}{2} \leq \arg(z) < 0$$

01.13.27.1488.01

$$\cos^{-1}\left(\sqrt{\frac{z^2+1}{z^2}}\right) = z^2 \sqrt{-\frac{1}{z^4}} \cosh^{-1}\left(\sqrt{1+\frac{1}{z^2}}\right)$$

Involving $\cos^{-1}\left(2 z \sqrt{1-z^2}\right)$

Involving $\cos^{-1}\left(2 z \sqrt{1-z^2}\right)$ and $\cosh^{-1}(z)$

01.13.27.1489.01

$$\cos^{-1}\left(2 z \sqrt{1-z^2}\right) = -\frac{\pi}{2} - 2 i \cosh^{-1}(z) /; \frac{\pi}{4} \leq \arg(z) \leq \frac{3\pi}{4}$$

01.13.27.1490.01

$$\cos^{-1}\left(2 z \sqrt{1-z^2}\right) = -\frac{\pi}{2} + 2 i \cosh^{-1}(z) /; -\frac{3\pi}{4} \leq \arg(z) \leq -\frac{\pi}{4}$$

01.13.27.1491.01

$$\cos^{-1}\left(2 z \sqrt{1-z^2}\right) = -\frac{\pi}{2} + 2 \frac{\sqrt{1-z}}{\sqrt{z-1}} \cosh^{-1}(z) /; \frac{\pi}{4} \leq |\arg(z)| \leq \frac{3\pi}{4}$$

01.13.27.1492.01

$$\cos^{-1}\left(2 z \sqrt{1-z^2}\right) = \frac{\pi}{2} - \frac{\pi \sqrt{1-2z^2} \sqrt{z^4-z^2}}{2 \sqrt{-z^2} \sqrt{z^2-1} \sqrt{2z^2-1}}$$
$$\left(\frac{\sqrt{z^2}}{z} - \sqrt{\frac{1}{z}} \sqrt{z} \sqrt{\frac{1}{\sqrt{2}\, z+1}} \sqrt{\sqrt{2}\, z+1} + \sqrt{-\frac{1}{z}} \sqrt{-z} \sqrt{\frac{1}{1-\sqrt{2}\, z}} \sqrt{1-\sqrt{2}\, z} + \frac{\sqrt{z^4-z^2}}{z\sqrt{z^2-1}} - 2\right) -$$
$$\frac{2\sqrt{1-z} \sqrt{1-2z^2} \sqrt{z^4-z^2}}{\sqrt{z-1} \sqrt{-z^2} \sqrt{z^2-1} \sqrt{2z^2-1}} \cosh^{-1}(z)$$

Involving $\cos^{-1}\left(\frac{2\sqrt{-1+z^2}}{z^2}\right)$

Involving $\cos^{-1}\left(\frac{2\sqrt{z^2-1}}{z^2}\right)$ and $\cosh^{-1}\left(\frac{1}{z}\right)$

01.13.27.1493.01

$$\cos^{-1}\left(\frac{2\sqrt{z^2-1}}{z^2}\right) = \frac{\pi}{2}\left(1-\frac{2\sqrt{z^2}}{z}\right)+\frac{2\sqrt{z^2}\sqrt{z-1}}{z\sqrt{1-z}}\cosh^{-1}\left(\frac{1}{z}\right) /;\ |z| \geq \sqrt{2} \bigwedge \operatorname{Im}(z) \neq 0$$

01.13.27.1494.01

$$\cos^{-1}\left(\frac{2\sqrt{z^2-1}}{z^2}\right) = \frac{\pi}{2}+\frac{z^3\sqrt{z^2-2}\sqrt{z^2-1}}{2\sqrt{1-z}\,(z+1)\sqrt{-z^4+3z^2-2}}\sqrt{\frac{1-z^2}{z^4}}\sqrt{\frac{1}{z}}$$
$$\sqrt{-\frac{z+1}{z}}\left(\pi\left(\frac{z^3}{1-z^2}\sqrt{\frac{1-z^2}{z^2}}\sqrt{\frac{1-z^2}{z^4}}+\sqrt{\frac{1}{z^2}}\,z-\sqrt{\frac{1}{z}}\sqrt{\frac{z}{z+\sqrt{2}}}\sqrt{\frac{z+\sqrt{2}}{z}}\sqrt{z}+\right.\right.$$
$$\left.\left.\sqrt{1-\frac{\sqrt{2}}{z}}\sqrt{-\frac{1}{z}}\sqrt{-z}\sqrt{\frac{z}{z-\sqrt{2}}}-2\right)+\frac{4}{\sqrt{\frac{1}{z}-1}}\sqrt{1-\frac{1}{z}}\cosh^{-1}\left(\frac{1}{z}\right)\right)$$

Involving $\cos^{-1}\left(\sqrt{\left(1-\sqrt{1+c\,z^2}\right)/2}\right)$

Involving $\cos^{-1}\left(\sqrt{\left(1-\sqrt{1+z^2}\right)/2}\right)$ and $\cosh^{-1}(i\,z)$

01.13.27.1495.01

$$\cos^{-1}\left(\sqrt{\frac{1}{2}\left(1-\sqrt{z^2+1}\right)}\right) = \frac{3\pi}{4}+\frac{1}{2}\,i\cosh^{-1}(i\,z) /;\ 0 < \arg(z) \leq \frac{\pi}{2}$$

01.13.27.1496.01

$$\cos^{-1}\left(\sqrt{\frac{1}{2}\left(1-\sqrt{z^2+1}\right)}\right) = -\frac{1}{2}\,i\cosh^{-1}(i\,z)+\frac{\pi}{4} /;\ -\frac{\pi}{2} < \arg(z) \leq 0 \bigvee (i\,z \in \mathbb{R} \land 0 < i\,z < 1)$$

01.13.27.1497.01

$$\cos^{-1}\left(\sqrt{\frac{1}{2}\left(1-\sqrt{z^2+1}\right)}\right) = \frac{3\pi}{4}-\frac{1}{2}\,i\cosh^{-1}(i\,z) /;\ \frac{\pi}{2} < \arg(z) \leq \pi$$

01.13.27.1498.01

$$\cos^{-1}\left(\sqrt{\frac{1}{2}\left(1-\sqrt{z^2+1}\right)}\right) = \frac{\pi}{4}+\frac{1}{2}\,i\cosh^{-1}(i\,z)\;/;\;-\pi<\arg(z)<-\frac{\pi}{2}\bigvee(i\,z\in\mathbb{R}\land i\,z>1)$$

01.13.27.1499.01

$$\cos^{-1}\left(\sqrt{\frac{1}{2}\left(1-\sqrt{z^2+1}\right)}\right) = \frac{3\pi}{4}-\frac{\sqrt{1-i\,z}}{2\sqrt{i\,z-1}}\cosh^{-1}(i\,z)\;/;\;0<\arg(z)\leq\pi$$

01.13.27.1500.01

$$\cos^{-1}\left(\sqrt{\frac{1-\sqrt{1+z^2}}{2}}\right) = \frac{\pi}{4}+\frac{\sqrt{1-i\,z}}{2\sqrt{i\,z-1}}\cosh^{-1}(i\,z)\;/;\;-\pi<\arg(z)\leq 0$$

Involving $\cos^{-1}\left(\sqrt{\left(1-\sqrt{1+z^2}\right)/2}\right)$ and $\cosh^{-1}(i\,z)$

01.13.27.1501.01

$$\cos^{-1}\left(\sqrt{\frac{1}{2}\left(1-\sqrt{1-z^2}\right)}\right) = \frac{1}{2}\left(-i\cosh^{-1}(z)+\frac{\pi}{2}\right)\;/;\;0<\arg(z)\leq\frac{\pi}{2}\bigvee(z\in\mathbb{R}\land 0<z<1)$$

01.13.27.1502.01

$$\cos^{-1}\left(\sqrt{\frac{1}{2}\left(1-\sqrt{1-z^2}\right)}\right) = \frac{1}{2}\left(\frac{\pi}{2}+i\cosh^{-1}(z)\right)\;/;\;-\frac{\pi}{2}<\arg(z)<0\bigvee(z\in\mathbb{R}\land z>1)$$

01.13.27.1503.01

$$\cos^{-1}\left(\sqrt{\frac{1}{2}\left(1-\sqrt{1-z^2}\right)}\right) = \frac{1}{2}\left(i\cosh^{-1}(z)+\frac{3\pi}{2}\right)\;/;\;\frac{\pi}{2}<\arg(z)\leq\pi$$

01.13.27.1504.01

$$\cos^{-1}\left(\sqrt{\frac{1}{2}\left(1-\sqrt{1-z^2}\right)}\right) = \frac{1}{2}\left(-i\cosh^{-1}(z)+\frac{3\pi}{2}\right)\;/;\;-\pi<\arg(z)\leq-\frac{\pi}{2}$$

01.13.27.1505.01

$$\cos^{-1}\left(\sqrt{\frac{1-\sqrt{1-z^2}}{2}}\right) = \frac{1}{2}\left(\frac{\pi}{2}+\frac{\sqrt{1-z}}{\sqrt{z-1}}\cosh^{-1}(z)\right)\;/;\;-\frac{\pi}{2}<\arg(z)\leq\frac{\pi}{2}$$

01.13.27.1506.01

$$\cos^{-1}\left(\sqrt{\frac{1}{2}\left(1-\sqrt{1-z^2}\right)}\right) = \frac{1}{2}\left(-\frac{\sqrt{1-z}}{\sqrt{z-1}}\cosh^{-1}(z)+\frac{3\pi}{2}\right)\;/;\;\mathrm{Re}(z)<0\bigvee\arg(z)=-\frac{\pi}{2}$$

01.13.27.1507.01

$$\cos^{-1}\left(\sqrt{\frac{1-\sqrt{1-z^2}}{2}}\right) = \frac{1}{2}\pi\left(1-\frac{\sqrt{z^2}}{2\,z}\right)-\frac{\sqrt{z-1}\;z\cosh^{-1}(z)}{2\sqrt{1-z}\;\sqrt{z^2}}$$

Involving $\cos^{-1}\left(z\sqrt{1-\sqrt{1-z^2}}\Big/\sqrt{2z^2}\right)$

Involving $\cos^{-1}\left(z\sqrt{1-\sqrt{1-z^2}}\Big/\sqrt{2z^2}\right)$ and $\cosh^{-1}(z)$

01.13.27.1508.01

$$\cos^{-1}\left(\frac{z\sqrt{1-\sqrt{1-z^2}}}{\sqrt{2z^2}}\right) = \frac{1}{2}\left(-i\cosh^{-1}(z)+\frac{\pi}{2}\right) /;\ 0<\arg(z)\le\pi \bigvee (z\in\mathbb{R}\land 0<z<1)$$

01.13.27.1509.01

$$\cos^{-1}\left(\frac{z\sqrt{1-\sqrt{1-z^2}}}{\sqrt{2z^2}}\right) = \frac{1}{2}\left(\frac{\pi}{2}+i\cosh^{-1}(z)\right) /;\ -\pi<\arg(z)<0 \bigvee (z\in\mathbb{R}\land z>1)$$

01.13.27.1510.01

$$\cos^{-1}\left(\frac{z\sqrt{1-\sqrt{1-z^2}}}{\sqrt{2z^2}}\right) = \frac{1}{2}\left(\frac{\pi}{2}+\frac{\sqrt{1-z}}{\sqrt{z-1}}\cosh^{-1}(z)\right)$$

Involving $\cos^{-1}\left(z\sqrt{\left(1-\sqrt{1-z^2}\right)\Big/\left(2z^2\right)}\right)$

Involving $\cos^{-1}\left(z\sqrt{\left(1-\sqrt{1-z^2}\right)\Big/\left(2z^2\right)}\right)$ and $\cosh^{-1}(z)$

01.13.27.1511.01

$$\cos^{-1}\left(z\sqrt{\frac{1-\sqrt{1-z^2}}{2z^2}}\right) = \frac{1}{2}\left(-i\cosh^{-1}(z)+\frac{\pi}{2}\right) /;\ 0<\arg(z)\le\pi \bigvee (z\in\mathbb{R}\land 0<z<1)$$

01.13.27.1512.01

$$\cos^{-1}\left(z\sqrt{\frac{1-\sqrt{1-z^2}}{2z^2}}\right) = \frac{1}{2}\left(\frac{\pi}{2}+i\cosh^{-1}(z)\right) /;\ -\pi<\arg(z)<0 \bigvee (z\in\mathbb{R}\land z>1)$$

01.13.27.1513.01

$$\cos^{-1}\left(z\sqrt{\frac{1-\sqrt{1-z^2}}{2z^2}}\right) = \frac{1}{2}\left(\frac{\pi}{2}+\frac{\sqrt{1-z}}{\sqrt{z-1}}\cosh^{-1}(z)\right)$$

Involving $\cos^{-1}\left(\sqrt{z-\sqrt{z^2-1}}\Big/\sqrt{2z}\right)$

Involving $\cos^{-1}\left(\sqrt{z-\sqrt{z^2-1}}\Big/\sqrt{2z}\right)$ and $\cosh^{-1}\left(\frac{1}{z}\right)$

01.13.27.1514.01

$$\cos^{-1}\left(\frac{\sqrt{z-\sqrt{z^2-1}}}{\sqrt{2z}}\right) = \frac{1}{2}\left(\frac{\pi}{2}+i\cosh^{-1}\left(\frac{1}{z}\right)\right) /;\ 0<\arg(z)\leq\frac{\pi}{2}\bigvee(z\in\mathbb{R}\land 0<z<1)\bigvee\arg(z)==-\frac{\pi}{2}$$

01.13.27.1515.01

$$\cos^{-1}\left(\frac{\sqrt{z-\sqrt{z^2-1}}}{\sqrt{2z}}\right) = \frac{1}{2}\left(-i\cosh^{-1}\left(\frac{1}{z}\right)+\frac{\pi}{2}\right) /;\ -\frac{\pi}{2}<\arg(z)<0\bigvee(z\in\mathbb{R}\land z>1)$$

01.13.27.1516.01

$$\cos^{-1}\left(\frac{\sqrt{z-\sqrt{z^2-1}}}{\sqrt{2z}}\right) = \frac{1}{2}\left(-\frac{\pi}{2}+i\cosh^{-1}\left(\frac{1}{z}\right)\right) /;\ \frac{\pi}{2}<\arg(z)<\pi$$

01.13.27.1517.01

$$\cos^{-1}\left(\frac{\sqrt{z-\sqrt{z^2-1}}}{\sqrt{2z}}\right) = -\frac{1}{2}\left(i\cosh^{-1}\left(\frac{1}{z}\right)+\frac{\pi}{2}\right) /;\ -\pi<\arg(z)<-\frac{\pi}{2}\bigvee(z\in\mathbb{R}\land z<-1)$$

01.13.27.1518.01

$$\cos^{-1}\left(\frac{\sqrt{z-\sqrt{z^2-1}}}{\sqrt{2z}}\right) = \frac{1}{2}\left(i\cosh^{-1}\left(\frac{1}{z}\right)+\frac{5\pi}{2}\right) /;\ (z\in\mathbb{R}\land -1<z<0)$$

01.13.27.1519.01

$$\cos^{-1}\left(\frac{\sqrt{z-\sqrt{z^2-1}}}{\sqrt{2z}}\right) = \frac{\pi}{2} - \frac{\sqrt{z-1}\,\sqrt{-i z}\,\sqrt{z}}{2\sqrt{\frac{1}{z}-1}}\sqrt{\frac{i}{z}}\,\sqrt{-\frac{1}{z+1}}\,\sqrt{-\frac{z+1}{z^2}}\,\cosh^{-1}\left(\frac{1}{z}\right) + \frac{1}{4}\pi\sqrt{-\frac{1}{z+1}}\,\sqrt{-\frac{z+1}{z^2}}\left(\sqrt{\frac{i}{z}}\,\sqrt{-i z}\,z + z - \sqrt{z^2}\right)$$

Involving $\cos^{-1}\left(\sqrt{\left(z-\sqrt{z^2-1}\right)/(2z)}\right)$

Involving $\cos^{-1}\left(\sqrt{\left(z-\sqrt{z^2-1}\right)/(2z)}\right)$ and $\cosh^{-1}\left(\frac{1}{z}\right)$

01.13.27.1520.01

$$\cos^{-1}\left(\sqrt{\frac{z-\sqrt{z^2-1}}{2z}}\right) = \frac{1}{2}\left(\frac{\pi}{2} + i\cosh^{-1}\left(\frac{1}{z}\right)\right) /;\ 0 < \arg(z) < \frac{\pi}{2} \bigvee (z \in \mathbb{R} \land 0 < z < 1) \bigvee \arg(z) = -\frac{\pi}{2}$$

01.13.27.1521.01

$$\cos^{-1}\left(\sqrt{\frac{z-\sqrt{z^2-1}}{2z}}\right) = \frac{1}{2}\left(-i\cosh^{-1}\left(\frac{1}{z}\right) + \frac{\pi}{2}\right) /;\ -\frac{\pi}{2} < \arg(z) < 0 \bigvee (z \in \mathbb{R} \land z > 1)$$

01.13.27.1522.01

$$\cos^{-1}\left(\sqrt{\frac{z-\sqrt{z^2-1}}{2z}}\right) = \frac{1}{2}\left(-\frac{\pi}{2} + i\cosh^{-1}\left(\frac{1}{z}\right)\right) /;\ \frac{\pi}{2} < \arg(z) < \pi$$

01.13.27.1523.01

$$\cos^{-1}\left(\sqrt{\frac{z-\sqrt{z^2-1}}{2z}}\right) = -\frac{1}{2}\left(i\cosh^{-1}\left(\frac{1}{z}\right) + \frac{\pi}{2}\right) /;\ -\pi < \arg(z) < -\frac{\pi}{2} \bigvee (z \in \mathbb{R} \land z < 0)$$

01.13.27.1524.01

$$\cos^{-1}\left(\sqrt{\frac{z-\sqrt{z^2-1}}{2z}}\right) = \frac{1}{2}\left(\frac{3\pi}{2} - i\cosh^{-1}\left(\frac{1}{z}\right)\right) /;\ \arg(z) = \frac{\pi}{2}$$

01.13.27.1525.01

$$\cos^{-1}\left(\sqrt{\frac{z-\sqrt{z^2-1}}{2z}}\right) = \frac{\pi}{2} - \frac{z\sqrt{z^2}\sqrt{z-1}\sqrt{z+1}}{2\sqrt{1-z}}\sqrt{\frac{1}{z^2}}\sqrt{-\frac{1}{(z+1)^2}}\sqrt{-\frac{z+1}{z^2}}\cosh^{-1}\left(\frac{1}{z}\right) + \frac{1}{4}\pi\sqrt{\frac{1}{z}}\sqrt{z}\sqrt{-\frac{1}{(z+1)^2}}\sqrt{z+1}\sqrt{-\frac{z+1}{z^2}}\sqrt{i z}\sqrt{-\frac{i}{z}}\left(\sqrt{\frac{i}{z}}\sqrt{-i z}\,z+z-\sqrt{z^2}\right)$$

Involving $\tanh^{-1}$

Involving $\cos^{-1}(z)$

Involving $\cos^{-1}(z)$ and $\tanh^{-1}\left(\frac{\sqrt{z^2-1}}{z}\right)$

01.13.27.1526.01

$$\cos^{-1}(z) = -i\tanh^{-1}\left(\frac{\sqrt{z^2-1}}{z}\right) /; 0 < \arg(z) < \frac{\pi}{2} \bigvee (z \in \mathbb{R} \land 0 < z < 1) \bigvee \arg(z) = -\frac{\pi}{2}$$

01.13.27.1527.01

$$\cos^{-1}(z) = i\tanh^{-1}\left(\frac{\sqrt{z^2-1}}{z}\right) /; -\frac{\pi}{2} < \arg(z) < 0 \bigvee (z \in \mathbb{R} \land z > 1)$$

01.13.27.1528.01

$$\cos^{-1}(z) = \pi + i\tanh^{-1}\left(\frac{\sqrt{z^2-1}}{z}\right) /; \frac{\pi}{2} < \arg(z) < \pi \bigvee (z \in \mathbb{R} \land z < -1)$$

01.13.27.1529.01

$$\cos^{-1}(z) = \pi - i\tanh^{-1}\left(\frac{\sqrt{z^2-1}}{z}\right) /; -\pi < \arg(z) < -\frac{\pi}{2} \bigvee (z \in \mathbb{R} \land -1 < z < 0) \bigvee \arg(z) = \frac{\pi}{2}$$

01.13.27.1530.01

$$\cos^{-1}(z) = \frac{1}{2}\pi\left(1 - z\sqrt{\frac{1}{z^2}}\right) + \frac{\sqrt{1-z^2}}{\sqrt{z^2-1}}\tanh^{-1}\left(\frac{\sqrt{z^2-1}}{z}\right)$$

Involving $\cos^{-1}(z)$ and $\tanh^{-1}\left(\frac{\sqrt{z^2-1}}{\sqrt{z^2}}\right)$

01.13.27.1531.01

$$\cos^{-1}(z) = -i\tanh^{-1}\left(\frac{\sqrt{z^2-1}}{\sqrt{z^2}}\right) /; 0 < \arg(z) < \frac{\pi}{2} \bigvee (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.1532.01

$$\cos^{-1}(z) = i \tanh^{-1}\left(\frac{\sqrt{z^2-1}}{\sqrt{z^2}}\right) /; -\frac{\pi}{2} \leq \arg(z) < 0 \bigvee (z \in \mathbb{R} \land z > 1)$$

01.13.27.1533.01

$$\cos^{-1}(z) = \pi - i \tanh^{-1}\left(\frac{\sqrt{z^2-1}}{\sqrt{z^2}}\right) /; \frac{\pi}{2} \leq \arg(z) < \pi \bigvee (z \in \mathbb{R} \land z < -1)$$

01.13.27.1534.01

$$\cos^{-1}(z) = \pi + i \tanh^{-1}\left(\frac{\sqrt{z^2-1}}{\sqrt{z^2}}\right) /; -\pi < \arg(z) < -\frac{\pi}{2} \bigvee (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.1535.01

$$\cos^{-1}(z) = \frac{\pi}{2}\left(1 - z\sqrt{z^{-2}}\right) + \frac{\sqrt{z^2-z^4}}{z\sqrt{z^2-1}} \tanh^{-1}\left(\frac{\sqrt{z^2-1}}{\sqrt{z^2}}\right)$$

Involving $\cos^{-1}(z)$ and $\tanh^{-1}\left(\frac{\sqrt{1-z^2}}{\sqrt{-z^2}}\right)$

01.13.27.1536.01

$$\cos^{-1}(z) = -i \tanh^{-1}\left(\frac{\sqrt{1-z^2}}{\sqrt{-z^2}}\right) /; 0 < \arg(z) < \frac{\pi}{2}$$

01.13.27.1537.01

$$\cos^{-1}(z) = i \tanh^{-1}\left(\frac{\sqrt{1-z^2}}{\sqrt{-z^2}}\right) /; -\frac{\pi}{2} \leq \arg(z) \leq 0$$

01.13.27.1538.01

$$\cos^{-1}(z) = \pi - i \tanh^{-1}\left(\frac{\sqrt{1-z^2}}{\sqrt{-z^2}}\right) /; \frac{\pi}{2} \leq \arg(z) \leq \pi$$

01.13.27.1539.01

$$\cos^{-1}(z) = \pi + i \tanh^{-1}\left(\frac{\sqrt{1-z^2}}{\sqrt{-z^2}}\right) /; -\pi < \arg(z) < -\frac{\pi}{2}$$

01.13.27.1540.01

$$\cos^{-1}(z) = \frac{\pi}{2}\left(1 - \sqrt{z^{-2}}\, z\right) + \frac{\sqrt{-z}}{\sqrt{z}} \tanh^{-1}\left(\frac{\sqrt{1-z^2}}{\sqrt{-z^2}}\right)$$

Involving $\cos^{-1}(z)$ and $\tanh^{-1}\left(\sqrt{\frac{z^2-1}{z^2}}\right)$

01.13.27.1541.01

$$\cos^{-1}(z) = -i \tanh^{-1}\left(\sqrt{\frac{z^2-1}{z^2}}\right) /; 0 < \arg(z) < \frac{\pi}{2} \bigvee (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.1542.01

$$\cos^{-1}(z) = i \tanh^{-1}\left(\sqrt{\frac{z^2-1}{z^2}}\right) /; -\frac{\pi}{2} \leq \arg(z) < 0 \bigvee (z \in \mathbb{R} \land z > 1)$$

01.13.27.1543.01

$$\cos^{-1}(z) = \pi - i \tanh^{-1}\left(\sqrt{\frac{z^2-1}{z^2}}\right) /; \frac{\pi}{2} \leq \arg(z) < \pi \bigvee (z \in \mathbb{R} \land z < -1)$$

01.13.27.1544.01

$$\cos^{-1}(z) = \pi + i \tanh^{-1}\left(\sqrt{\frac{z^2-1}{z^2}}\right) /; -\pi < \arg(z) < -\frac{\pi}{2} \bigvee (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.1545.01

$$\cos^{-1}(z) = -\frac{z\sqrt{z^2-1}}{\sqrt{z^2-z^4}} \tanh^{-1}\left(\sqrt{\frac{z^2-1}{z^2}}\right) + \frac{\pi}{2}\left(1 - z\sqrt{z^{-2}}\right)$$

Involving $\cos^{-1}(z)$ and $\tanh^{-1}\left(\frac{z}{\sqrt{z^2-1}}\right)$

01.13.27.1546.01

$$\cos^{-1}(z) = \frac{\pi}{2} - i \tanh^{-1}\left(\frac{z}{\sqrt{z^2-1}}\right) /; 0 < \arg(z) \leq \frac{\pi}{2} \bigvee -\pi < \arg(z) \leq -\frac{\pi}{2} \bigvee (z \in \mathbb{R} \land -1 < z < 1)$$

01.13.27.1547.01

$$\cos^{-1}(z) = \frac{\pi}{2} + i \tanh^{-1}\left(\frac{z}{\sqrt{z^2-1}}\right) /; \frac{\pi}{2} < \arg(z) < \pi \bigvee -\frac{\pi}{2} < \arg(z) < 0$$

01.13.27.1548.01

$$\cos^{-1}(z) = i \tanh^{-1}\left(\frac{z}{\sqrt{z^2-1}}\right) + \frac{3\pi}{2} /; (z \in \mathbb{R} \land z < -1)$$

01.13.27.1549.01

$$\cos^{-1}(z) = i \tanh^{-1}\left(\frac{z}{\sqrt{z^2-1}}\right) - \frac{\pi}{2} /; (z \in \mathbb{R} \land z > 1)$$

01.13.27.1550.01

$$\cos^{-1}(z) = \frac{\pi}{2} - \frac{\sqrt{z^2-1}}{\sqrt{1-z^2}} \tanh^{-1}\left(\frac{z}{\sqrt{z^2-1}}\right) + \frac{\pi}{2\sqrt{1-z^2}} \left((1-z)\sqrt{\frac{1+z}{1-z}} - (1+z)\sqrt{\frac{1-z}{1+z}}\right)$$

Involving $\cos^{-1}(z)$ and $\tanh^{-1}\left(\frac{\sqrt{z^2}}{\sqrt{z^2-1}}\right)$

01.13.27.1551.01

$$\cos^{-1}(z) = \frac{\pi}{2} - i \tanh^{-1}\left(\frac{\sqrt{z^2}}{\sqrt{z^2-1}}\right) /; \operatorname{Im}(z) > 0 \lor (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.1552.01

$$\cos^{-1}(z) = \frac{\pi}{2} + i \tanh^{-1}\left(\frac{\sqrt{z^2}}{\sqrt{z^2-1}}\right) /; \operatorname{Im}(z) < 0 \lor (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.1553.01

$$\cos^{-1}(z) = -\frac{\pi}{2} + i \tanh^{-1}\left(\frac{\sqrt{z^2}}{\sqrt{z^2-1}}\right) /; (z \in \mathbb{R} \land z > 1)$$

01.13.27.1554.01

$$\cos^{-1}(z) = \frac{3\pi}{2} - i \tanh^{-1}\left(\frac{\sqrt{z^2}}{\sqrt{z^2-1}}\right) /; (z \in \mathbb{R} \land z < -1)$$

01.13.27.1555.01

$$\cos^{-1}(z) = -\frac{\sqrt{z^2}\sqrt{z^2-1}}{z\sqrt{1-z^2}} \tanh^{-1}\left(\frac{\sqrt{z^2}}{\sqrt{z^2-1}}\right) + \frac{\pi}{2}\left(1 + \sqrt{\frac{1}{1-z}}\sqrt{1-z} - \sqrt{\frac{1}{z+1}}\sqrt{z+1}\right)$$

Involving $\cos^{-1}(z)$ and $\tanh^{-1}\left(\frac{\sqrt{-z^2}}{\sqrt{1-z^2}}\right)$

01.13.27.1556.01

$$\cos^{-1}(z) = \frac{\pi}{2} - i \tanh^{-1}\left(\frac{\sqrt{-z^2}}{\sqrt{1-z^2}}\right) /; \operatorname{Im}(z) > 0 \lor (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.1557.01

$$\cos^{-1}(z) = \frac{\pi}{2} + i \tanh^{-1}\left(\frac{\sqrt{-z^2}}{\sqrt{1-z^2}}\right) /; \operatorname{Im}(z) < 0 \lor (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.1558.01

$$\cos^{-1}(z) = i \tanh^{-1}\left(\frac{\sqrt{-z^2}}{\sqrt{1-z^2}}\right) - \frac{\pi}{2} /; (z \in \mathbb{R} \land z > 1)$$

01.13.27.1559.01

$$\cos^{-1}(z) = \frac{3\pi}{2} - i\tanh^{-1}\left(\frac{\sqrt{-z^2}}{\sqrt{1-z^2}}\right) /; (z \in \mathbb{R} \land z < -1)$$

01.13.27.1560.01

$$\cos^{-1}(z) = \frac{\sqrt{-z^2}}{z}\tanh^{-1}\left(\frac{\sqrt{-z^2}}{\sqrt{1-z^2}}\right) + \frac{\pi}{2}\left(1 + \sqrt{\frac{1}{1-z}}\sqrt{1-z} - \sqrt{\frac{1}{z+1}}\sqrt{z+1}\right)$$

Involving $\cos^{-1}(z)$ and $\tanh^{-1}\left(\sqrt{\frac{z^2}{z^2-1}}\right)$

01.13.27.1561.01

$$\cos^{-1}(z) = \frac{\pi}{2} - i\tanh^{-1}\left(\sqrt{\frac{z^2}{z^2-1}}\right) /; \operatorname{Im}(z) > 0 \lor (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.1562.01

$$\cos^{-1}(z) = \frac{\pi}{2} + i\tanh^{-1}\left(\sqrt{\frac{z^2}{z^2-1}}\right) /; \operatorname{Im}(z) < 0 \lor (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.1563.01

$$\cos^{-1}(z) = -\frac{\pi}{2} + i\tanh^{-1}\left(\sqrt{\frac{z^2}{z^2-1}}\right) /; (z \in \mathbb{R} \land z > 1)$$

01.13.27.1564.01

$$\cos^{-1}(z) = \frac{3\pi}{2} - i\tanh^{-1}\left(\sqrt{\frac{z^2}{z^2-1}}\right) /; (z \in \mathbb{R} \land z < -1)$$

01.13.27.1565.01

$$\cos^{-1}(z) = -\frac{\sqrt{z}}{\sqrt{-z}}\tanh^{-1}\left(\sqrt{\frac{z^2}{z^2-1}}\right) + \frac{\pi}{2}\left(1 + \sqrt{\frac{1}{1-z}}\sqrt{1-z} - \sqrt{\frac{1}{z+1}}\sqrt{z+1}\right)$$

Involving $\cos^{-1}(z)$ and $\tanh^{-1}\left(\frac{2z\sqrt{z^2-1}}{1-2z^2}\right)$

01.13.27.1566.01

$$\cos^{-1}(z) = \frac{\pi}{2} - \frac{\sqrt{-z^4}}{2z^2}\tanh^{-1}\left(\frac{2z\sqrt{z^2-1}}{1-2z^2}\right) /; \frac{\pi}{4} \le |\arg(z)| \le \frac{3\pi}{4}$$

01.13.27.1567.01

$$\cos^{-1}(z) = \frac{1}{4}\pi\left(2-\frac{\sqrt{z^2-1}\,z}{\sqrt{z^4-z^2}}-\sqrt{\frac{1}{z}}\sqrt{\frac{1}{\sqrt{2}\,z-1}}\sqrt{\sqrt{2}\,z-1}\sqrt{z}+\sqrt{-\frac{1}{z}}\sqrt{-z}\sqrt{-\sqrt{2}\,z-1}\sqrt{-\frac{1}{\sqrt{2}\,z+1}}-\frac{\sqrt{z^2}}{z}\right)-\frac{\sqrt{1-z^2}}{2\sqrt{z^2-1}}\tanh^{-1}\left(\frac{2z\sqrt{z^2-1}}{1-2z^2}\right)$$

Involving $\cos^{-1}(z)$ and $\tanh^{-1}\left(\frac{1-2z^2}{2z\sqrt{z^2-1}}\right)$

01.13.27.1568.01

$$\cos^{-1}(z) = \frac{\pi}{4}+\frac{1}{2}i\tanh^{-1}\left(\frac{1-2z^2}{2z\sqrt{z^2-1}}\right)\;/;\;0<\arg(z)<\frac{\pi}{2}\bigvee(z\in\mathbb{R}\land 0<z<1)\bigvee(iz\in\mathbb{R}\land iz>0)$$

01.13.27.1569.01

$$\cos^{-1}(z) = \frac{\pi}{4}-\frac{1}{2}i\tanh^{-1}\left(\frac{1-2z^2}{2z\sqrt{z^2-1}}\right)\;/;\;-\frac{\pi}{2}<\arg(z)<0$$

01.13.27.1570.01

$$\cos^{-1}(z) = \frac{3\pi}{4}-\frac{1}{2}i\tanh^{-1}\left(\frac{1-2z^2}{2z\sqrt{z^2-1}}\right)\;/;\;\frac{\pi}{2}<\arg(z)<\pi$$

01.13.27.1571.01

$$\cos^{-1}(z) = \frac{3\pi}{4}+\frac{1}{2}i\tanh^{-1}\left(\frac{1-2z^2}{2z\sqrt{z^2-1}}\right)\;/;\;-\pi<\arg(z)<-\frac{\pi}{2}\bigvee(z\in\mathbb{R}\land -1<z<0)\bigvee(iz\in\mathbb{R}\land iz<0)$$

01.13.27.1572.01

$$\cos^{-1}(z) = \frac{5\pi}{4}-\frac{i}{2}\tanh^{-1}\left(\frac{1-2z^2}{2z\sqrt{z^2-1}}\right)\;/;\;(z\in\mathbb{R}\land z<-1)$$

01.13.27.1573.01

$$\cos^{-1}(z) = -\frac{\pi}{4}-\frac{i}{2}\tanh^{-1}\left(\frac{1-2z^2}{2z\sqrt{z^2-1}}\right)\;/;\;(z\in\mathbb{R}\land z>1)$$

01.13.27.1574.01

$$\cos^{-1}(z) = \frac{\pi}{4}\left(2+\sqrt{\frac{1}{1-z}}\sqrt{1-z}-\sqrt{\frac{1}{z+1}}\sqrt{z+1}+\sqrt{-iz}\sqrt{\frac{i}{z}}-\sqrt{-\frac{i}{z}}\sqrt{iz}-\frac{\sqrt{z^2}}{z}\right)+\frac{\sqrt{z^2-1}}{2\sqrt{1-z^2}}\tanh^{-1}\left(\frac{1-2z^2}{2z\sqrt{z^2-1}}\right)$$

Involving $\cos^{-1}(z)$ and $\tanh^{-1}\left(\frac{\sqrt{z-1}}{\sqrt{z+1}}\right)$

01.13.27.1575.01

$$\cos^{-1}(z) = -2\, i \tanh^{-1}\left(\frac{\sqrt{z-1}}{\sqrt{z+1}}\right) /;\ \mathrm{Im}(z) > 0 \lor (z \in \mathbb{R} \land -1 < z < 1)$$

01.13.27.1576.01

$$\cos^{-1}(z) = 2\, i \tanh^{-1}\left(\frac{\sqrt{z-1}}{\sqrt{z+1}}\right) /;\ \mathrm{Im}(z) < 0 \lor (z \in \mathbb{R} \land z > 1)$$

01.13.27.1577.01

$$\cos^{-1}(z) = 2\pi - 2\, i \tanh^{-1}\left(\frac{\sqrt{z-1}}{\sqrt{z+1}}\right) /;\ (z \in \mathbb{R} \land z < -1)$$

01.13.27.1578.01

$$\cos^{-1}(z) = -\frac{2\sqrt{z-1}}{\sqrt{1-z}} \tanh^{-1}\left(\frac{\sqrt{z-1}}{\sqrt{z+1}}\right) + \frac{\pi \sqrt{-z-1}\, \sqrt{-z}}{\sqrt{z+1}} \sqrt{\frac{1}{z}} + \pi$$

Involving $\cos^{-1}(z)$ and $\tanh^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{-1-z}}\right)$

01.13.27.1579.01

$$\cos^{-1}(z) = -2\, i \tanh^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{-1-z}}\right) /;\ \mathrm{Im}(z) > 0$$

01.13.27.1580.01

$$\cos^{-1}(z) = 2\, i \tanh^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{-1-z}}\right) /;\ -\pi < \arg(z) \le 0 \bigvee (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.1581.01

$$\cos^{-1}(z) = 2\pi - 2\, i \tanh^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{-1-z}}\right) /;\ (z \in \mathbb{R} \land z < -1)$$

01.13.27.1582.01

$$\cos^{-1}(z) = \pi\left(\frac{\sqrt{-z-1}\, \sqrt{-z}}{\sqrt{z+1}} \sqrt{\frac{1}{z}} + 1\right) - \frac{2\sqrt{z+1}}{\sqrt{-z-1}} \tanh^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{-z-1}}\right)$$

Involving $\cos^{-1}(z)$ and $\tanh^{-1}\left(\sqrt{\frac{z-1}{z+1}}\right)$

01.13.27.1583.01

$$\cos^{-1}(z) = -2\, i \tanh^{-1}\left(\sqrt{\frac{z-1}{z+1}}\right) /;\ \mathrm{Im}(z) > 0 \lor (z \in \mathbb{R} \land -1 < z < 1)$$

01.13.27.1584.01

$$\cos^{-1}(z) = 2\, i \tanh^{-1}\left(\sqrt{\frac{z-1}{z+1}}\right) /;\ \mathrm{Im}(z) < 0 \lor (z \in \mathbb{R} \land z > 1)$$

01.13.27.1585.01

$$\cos^{-1}(z) == 2\,\pi - 2\,i\tanh^{-1}\left(\sqrt{\frac{z-1}{z+1}}\right) /;\ (z \in \mathbb{R} \land z < -1)$$

01.13.27.1586.01

$$\cos^{-1}(z) == 2\sqrt{\frac{z+1}{z-1}}\ \sqrt{z-1}\ \sqrt{\frac{1}{z}}\ \sqrt{-\frac{z}{z+1}}\ \tanh^{-1}\left(\sqrt{\frac{z-1}{z+1}}\right) + \pi + \frac{\pi\sqrt{-z-1}\ \sqrt{-z}}{\sqrt{z+1}}\sqrt{\frac{1}{z}}$$

Involving $\cos^{-1}(z)$ and $\tanh^{-1}\left(\frac{\sqrt{z+1}}{\sqrt{z-1}}\right)$

01.13.27.1587.01

$$\cos^{-1}(z) == -2\,i\tanh^{-1}\left(\frac{\sqrt{z+1}}{\sqrt{z-1}}\right) + \pi /;\ \mathrm{Im}(z) > 0 \lor (z \in \mathbb{R} \land z < 1)$$

01.13.27.1588.01

$$\cos^{-1}(z) == 2\,i\tanh^{-1}\left(\frac{\sqrt{z+1}}{\sqrt{z-1}}\right) + \pi /;\ \mathrm{Im}(z) < 0$$

01.13.27.1589.01

$$\cos^{-1}(z) == -\pi + 2\,i\tanh^{-1}\left(\frac{\sqrt{z+1}}{\sqrt{z-1}}\right) /;\ (z \in \mathbb{R} \land z > 1)$$

01.13.27.1590.01

$$\cos^{-1}(z) == -\frac{2\sqrt{z-1}}{\sqrt{1-z}}\tanh^{-1}\left(\frac{\sqrt{z+1}}{\sqrt{z-1}}\right) - \frac{\pi\sqrt{z-1}\ \sqrt{z}}{\sqrt{1-z}}\sqrt{-\frac{1}{z}}$$

Involving $\cos^{-1}(z)$ and $\tanh^{-1}\left(\frac{\sqrt{-1-z}}{\sqrt{1-z}}\right)$

01.13.27.1591.01

$$\cos^{-1}(z) == \pi - 2\,i\tanh^{-1}\left(\frac{\sqrt{-1-z}}{\sqrt{1-z}}\right) /;\ \mathrm{Im}(z) > 0 \lor (z \in \mathbb{R} \land z < -1)$$

01.13.27.1592.01

$$\cos^{-1}(z) == \pi + 2\,i\tanh^{-1}\left(\frac{\sqrt{-1-z}}{\sqrt{1-z}}\right) /;\ \mathrm{Im}(z) < 0 \lor (z \in \mathbb{R} \land -1 < z < 1)$$

01.13.27.1593.01

$$\cos^{-1}(z) == 2\,i\tanh^{-1}\left(\frac{\sqrt{-1-z}}{\sqrt{1-z}}\right) - \pi /;\ (z \in \mathbb{R} \land z > 1)$$

01.13.27.1594.01

$$\cos^{-1}(z) == -\frac{2\sqrt{1+z}}{\sqrt{-1-z}}\tanh^{-1}\left(\frac{\sqrt{-1-z}}{\sqrt{1-z}}\right) - \pi\frac{\sqrt{-1+z}\ \sqrt{z}}{\sqrt{1-z}}\sqrt{-\frac{1}{z}}$$

Involving $\cos^{-1}(z)$ and $\tanh^{-1}\left(\sqrt{\frac{z+1}{z-1}}\right)$

01.13.27.1595.01

$$\cos^{-1}(z) = \pi - 2\,i\tanh^{-1}\left(\sqrt{\frac{z+1}{z-1}}\right) /;\ \operatorname{Im}(z) > 0 \lor (z \in \mathbb{R} \land z < -1)$$

01.13.27.1596.01

$$\cos^{-1}(z) = \pi + 2\,i\tanh^{-1}\left(\sqrt{\frac{z+1}{z-1}}\right) /;\ \operatorname{Im}(z) < 0 \lor (z \in \mathbb{R} \land -1 < z < 1)$$

01.13.27.1597.01

$$\cos^{-1}(z) = 2\,i\tanh^{-1}\left(\sqrt{\frac{z+1}{z-1}}\right) - \pi /;\ (z \in \mathbb{R} \land z > 1)$$

01.13.27.1598.01

$$\cos^{-1}(z) = -2\sqrt{-z-1}\sqrt{\frac{z}{1-z}}\sqrt{-\frac{1}{z}}\sqrt{\frac{z-1}{z+1}}\tanh^{-1}\left(\sqrt{\frac{z+1}{z-1}}\right) - \frac{\pi\sqrt{z-1}\sqrt{z}}{\sqrt{1-z}}\sqrt{-\frac{1}{z}}$$

Involving $\cos^{-1}\left(\frac{2z}{z^2+1}\right)$

Involving $\cos^{-1}\left(\frac{2z}{z^2+1}\right)$ and $\tanh^{-1}(i\,z)$

01.13.27.1599.01

$$\cos^{-1}\left(\frac{2z}{z^2+1}\right) = \frac{\pi}{2} + 2\,i\tanh^{-1}(i\,z) /;\ |z| < 1$$

01.13.27.1600.01

$$\cos^{-1}\left(\frac{2z}{z^2+1}\right) = -2\,i\tanh^{-1}(i\,z) - \frac{\pi}{2} /;\ |z| > 1 \bigwedge -\frac{\pi}{2} < \arg(z) \le \frac{\pi}{2}$$

01.13.27.1601.01

$$\cos^{-1}\left(\frac{2z}{z^2+1}\right) = \frac{3\pi}{2} - 2\,i\tanh^{-1}(i\,z) /;\ |z| > 1 \bigwedge \left(\frac{\pi}{2} < \arg(z) \le \pi \bigvee -\pi < \arg(z) \le -\frac{\pi}{2}\right)$$

01.13.27.1602.01

$$\cos^{-1}\left(\frac{2z}{z^2+1}\right) = \frac{\pi}{2} - \frac{\pi\sqrt{z^2}}{z} - 2\,i\tanh^{-1}(i\,z) /;\ |z| > 1$$

01.13.27.1603.01

$$\cos^{-1}\left(\frac{2z}{z^2+1}\right) = \frac{\pi}{2} - \frac{\pi\sqrt{z^2}}{2z}\left(1 - \frac{1-z}{1+z}\sqrt{\left(\frac{z+1}{z-1}\right)^2}\right) + \frac{2\,i\,(1-z)}{1+z}\sqrt{\left(\frac{z+1}{z-1}\right)^2}\tanh^{-1}(i\,z)$$

Involving $\cos^{-1}\left(\frac{2z}{z^2+1}\right)$ and $\tanh^{-1}\left(\frac{i}{z}\right)$

01.13.27.1604.01

$$\cos^{-1}\left(\frac{2z}{z^2+1}\right) = -2\,i\tanh^{-1}\left(\frac{i}{z}\right) - \frac{\pi}{2}\;/;\;|z|<1 \bigwedge -\frac{\pi}{2} \leq \arg(z) < \frac{\pi}{2}$$

01.13.27.1605.01

$$\cos^{-1}\left(\frac{2z}{z^2+1}\right) = \frac{3\pi}{2} - 2\,i\tanh^{-1}\left(\frac{i}{z}\right)\;/;\;|z|<1 \bigwedge \left(\frac{\pi}{2} \leq \arg(z) \leq \pi \bigvee -\pi < \arg(z) < -\frac{\pi}{2}\right)$$

01.13.27.1606.01

$$\cos^{-1}\left(\frac{2z}{z^2+1}\right) = \frac{\pi}{2} - \pi z\sqrt{\frac{1}{z^2}} - 2\,i\tanh^{-1}\left(\frac{i}{z}\right)\;/;\;|z|<1$$

01.13.27.1607.01

$$\cos^{-1}\left(\frac{2z}{z^2+1}\right) = \frac{\pi}{2} + 2\,i\tanh^{-1}\left(\frac{i}{z}\right)\;/;\;|z|>1$$

01.13.27.1608.01

$$\cos^{-1}\left(\frac{2z}{z^2+1}\right) = \frac{\pi}{2} - \frac{\pi z}{2}\sqrt{\frac{1}{z^2}}\left(\frac{1-z}{1+z}\sqrt{\left(\frac{z+1}{z-1}\right)^2} + 1\right) - \frac{2\,i\,(1-z)}{1+z}\sqrt{\left(\frac{z+1}{z-1}\right)^2}\tanh^{-1}\left(\frac{i}{z}\right)$$

Involving $\cos^{-1}\left(\frac{2z}{z^2+1}\right)$ and $\tanh^{-1}(i\,z^r)$

01.13.27.1609.01

$$\cos^{-1}\left(\frac{2z}{z^2+1}\right) = \frac{\pi}{2} + 2\,i\tanh^{-1}\left(i\,z^{\frac{(1-z)\sqrt{\left(\frac{z+1}{1-z}\right)^2}}{z+1}}\right)$$

Involving $\cos^{-1}\left(\frac{1-z}{1+z}\right)$

Involving $\cos^{-1}\left(\frac{1-z}{1+z}\right)$ and $\tanh^{-1}\left(\sqrt{-z}\right)$

01.13.27.1610.01

$$\cos^{-1}\left(\frac{1-z}{1+z}\right) = 2\,i\tanh^{-1}\left(\sqrt{-z}\right)\;/;\;\operatorname{Im}(z)>0 \lor (z\in\mathbb{R} \land -1<z<0)$$

01.13.27.1611.01

$$\cos^{-1}\left(\frac{1-z}{1+z}\right) = -2\,i\tanh^{-1}\left(\sqrt{-z}\right)\;/;\;-\pi < \arg(z) \leq 0$$

01.13.27.1612.01

$$\cos^{-1}\left(\frac{1-z}{1+z}\right) = 2\pi - 2\,i\tanh^{-1}\left(\sqrt{-z}\right)\;/;\;(z\in\mathbb{R} \land z<-1)$$

01.13.27.1613.01

$$\cos^{-1}\left(\frac{1-z}{1+z}\right) = \pi\left(1 - \sqrt{1+z}\sqrt{\frac{1}{1+z}}\right) - \frac{2\sqrt{-(1+z)\,z}}{\sqrt{z}}\sqrt{\frac{1}{1+z}}\tanh^{-1}\left(\sqrt{-z}\right)$$

Involving $\cos^{-1}\left(\frac{1-z}{1+z}\right)$ and $\tanh^{-1}\left(\frac{1}{\sqrt{-z}}\right)$

01.13.27.1614.01

$$\cos^{-1}\left(\frac{1-z}{1+z}\right) = \pi + 2\,i\tanh^{-1}\left(\frac{1}{\sqrt{-z}}\right) /; \operatorname{Im}(z) > 0$$

01.13.27.1615.01

$$\cos^{-1}\left(\frac{1-z}{1+z}\right) = \pi - 2\,i\tanh^{-1}\left(\frac{1}{\sqrt{-z}}\right) /; -\pi < \arg(z) \leq 0 \bigvee (z \in \mathbb{R} \land z < -1)$$

01.13.27.1616.01

$$\cos^{-1}\left(\frac{1-z}{1+z}\right) = 2\,i\tanh^{-1}\left(\frac{1}{\sqrt{-z}}\right) - \pi /; (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.1617.01

$$\cos^{-1}\left(\frac{1-z}{1+z}\right) = \pi\left(1 - \sqrt{\frac{1}{1+z}}\sqrt{1+z} + \sqrt{\frac{1}{z}}\sqrt{z}\right) - \frac{2\sqrt{-(1+z)\,z}}{\sqrt{z}}\sqrt{\frac{1}{1+z}}\tanh^{-1}\left(\frac{1}{\sqrt{-z}}\right)$$

Involving $\cos^{-1}\left(\frac{1-z}{1+z}\right)$ and $\tanh^{-1}\left(\sqrt{-\frac{1}{z}}\right)$

01.13.27.1618.01

$$\cos^{-1}\left(\frac{1-z}{1+z}\right) = \pi - 2\,i\tanh^{-1}\left(\sqrt{-\frac{1}{z}}\right) /; -\pi < \arg(z) < 0 \bigvee (z \in \mathbb{R} \land z < -1)$$

01.13.27.1619.01

$$\cos^{-1}\left(\frac{1-z}{1+z}\right) = \pi + 2\,i\tanh^{-1}\left(\sqrt{-\frac{1}{z}}\right) /; 0 \leq \arg(z) < \pi$$

01.13.27.1620.01

$$\cos^{-1}\left(\frac{1-z}{1+z}\right) = 2\,i\tanh^{-1}\left(\sqrt{-\frac{1}{z}}\right) - \pi /; (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.1621.01

$$\cos^{-1}\left(\frac{1-z}{1+z}\right) = \pi\left(1 - \sqrt{\frac{1}{1+z}}\sqrt{1+z} + \sqrt{\frac{1}{z}}\sqrt{z}\right) + 2\sqrt{\frac{-1-z}{z}}\sqrt{\frac{1}{1+z}}\sqrt{z}\tanh^{-1}\left(\sqrt{-\frac{1}{z}}\right)$$

Involving $\cos^{-1}\left(\frac{z-1}{z+1}\right)$

Involving $\cos^{-1}\left(\frac{z-1}{z+1}\right)$ and $\tanh^{-1}\left(\sqrt{-z}\right)$

01.13.27.1622.01

$$\cos^{-1}\left(\frac{z-1}{z+1}\right) = \pi - 2\,i\tanh^{-1}\left(\sqrt{-z}\right) /; \operatorname{Im}(z) > 0 \lor (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.1623.01

$$\cos^{-1}\left(\frac{z-1}{z+1}\right) = \pi + 2\,i\tanh^{-1}\left(\sqrt{-z}\right) /; -\pi < \arg(z) \leq 0$$

01.13.27.1624.01

$$\cos^{-1}\left(\frac{z-1}{z+1}\right) = 2\,i\tanh^{-1}\left(\sqrt{-z}\right) - \pi /; (z \in \mathbb{R} \land z < -1)$$

01.13.27.1625.01

$$\cos^{-1}\left(\frac{z-1}{z+1}\right) = \pi\sqrt{1+z}\sqrt{\frac{1}{1+z}} + \frac{2\sqrt{-(1+z)\,z}}{\sqrt{z}}\sqrt{\frac{1}{1+z}}\tanh^{-1}\left(\sqrt{-z}\right)$$

Involving $\cos^{-1}\left(\frac{z-1}{z+1}\right)$ and $\tanh^{-1}\left(\frac{1}{\sqrt{-z}}\right)$

01.13.27.1626.01

$$\cos^{-1}\left(\frac{z-1}{z+1}\right) = -2\,i\tanh^{-1}\left(\frac{1}{\sqrt{-z}}\right) /; \operatorname{Im}(z) > 0$$

01.13.27.1627.01

$$\cos^{-1}\left(\frac{z-1}{z+1}\right) = 2\,i\tanh^{-1}\left(\frac{1}{\sqrt{-z}}\right) /; -\pi < \arg(z) \leq 0 \lor (z \in \mathbb{R} \land z < -1)$$

01.13.27.1628.01

$$\cos^{-1}\left(\frac{z-1}{z+1}\right) = 2\pi - 2\,i\tanh^{-1}\left(\frac{1}{\sqrt{-z}}\right) /; (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.1629.01

$$\cos^{-1}\left(\frac{z-1}{z+1}\right) = \pi\left(\sqrt{\frac{1}{1+z}}\sqrt{1+z} - \sqrt{\frac{1}{z}}\sqrt{z}\right) + \frac{2\sqrt{-(1+z)\,z}}{\sqrt{z}}\sqrt{\frac{1}{1+z}}\tanh^{-1}\left(\frac{1}{\sqrt{-z}}\right)$$

Involving $\cos^{-1}\left(\frac{z-1}{z+1}\right)$ and $\tanh^{-1}\left(\sqrt{-\frac{1}{z}}\right)$

01.13.27.1630.01

$$\cos^{-1}\left(\frac{z-1}{z+1}\right) = 2\,i\tanh^{-1}\left(\sqrt{-\frac{1}{z}}\right) /; -\pi < \arg(z) < 0 \lor (z \in \mathbb{R} \land z < -1)$$

01.13.27.1631.01

$$\cos^{-1}\left(\frac{z-1}{z+1}\right) = -2\,i\tanh^{-1}\left(\sqrt{-\frac{1}{z}}\right) /; 0 \leq \arg(z) < \pi$$

01.13.27.1632.01

$$\cos^{-1}\left(\frac{z-1}{z+1}\right) = 2\pi - 2\,i\tanh^{-1}\left(\sqrt{-\frac{1}{z}}\right) /; (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.1633.01

$$\cos^{-1}\left(\frac{z-1}{z+1}\right) = \pi\left(\sqrt{\frac{1}{1+z}}\sqrt{1+z} - \sqrt{\frac{1}{z}}\sqrt{z}\right) - 2\sqrt{\frac{-1-z}{z}}\sqrt{\frac{1}{1+z}}\sqrt{z}\tanh^{-1}\left(\sqrt{-\frac{1}{z}}\right)$$

Involving $\cos^{-1}\left(\frac{1+z}{1-z}\right)$

Involving $\cos^{-1}\left(\frac{1+z}{1-z}\right)$ and $\tanh^{-1}\left(\sqrt{z}\right)$

01.13.27.1634.01

$$\cos^{-1}\left(\frac{z+1}{1-z}\right) = -2\,i\tanh^{-1}\left(\sqrt{z}\right) /; 0 < \arg(z) \leq \pi$$

01.13.27.1635.01

$$\cos^{-1}\left(\frac{z+1}{1-z}\right) = 2\,i\tanh^{-1}\left(\sqrt{z}\right) /; -\pi < \arg(z) < 0 \bigvee (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.1636.01

$$\cos^{-1}\left(\frac{z+1}{1-z}\right) = 2\pi - 2\,i\tanh^{-1}\left(\sqrt{z}\right) /; (z \in \mathbb{R} \land z > 1)$$

01.13.27.1637.01

$$\cos^{-1}\left(\frac{1+z}{1-z}\right) = -\pi\left(\sqrt{1-z}\sqrt{\frac{1}{1-z}} - 1\right) - \frac{2\sqrt{(1-z)\,z}}{\sqrt{-z}}\sqrt{\frac{1}{1-z}}\tanh^{-1}\left(\sqrt{z}\right)$$

Involving $\cos^{-1}\left(\frac{1+z}{1-z}\right)$ and $\tanh^{-1}\left(\frac{1}{\sqrt{z}}\right)$

01.13.27.1638.01

$$\cos^{-1}\left(\frac{z+1}{1-z}\right) = -2\,i\tanh^{-1}\left(\frac{1}{\sqrt{z}}\right) + \pi /; 0 < \arg(z) \leq \pi \bigvee (z \in \mathbb{R} \land z > 1)$$

01.13.27.1639.01

$$\cos^{-1}\left(\frac{z+1}{1-z}\right) = 2\,i\tanh^{-1}\left(\frac{1}{\sqrt{z}}\right) + \pi /; \operatorname{Im}(z) < 0$$

01.13.27.1640.01

$$\cos^{-1}\left(\frac{z+1}{1-z}\right) = -\pi + 2\,i\tanh^{-1}\left(\frac{1}{\sqrt{z}}\right) /; (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.1641.01

$$\cos^{-1}\left(\frac{1+z}{1-z}\right) = \pi\left(-\sqrt{\frac{1}{1-z}}\sqrt{1-z} + \sqrt{-\frac{1}{z}}\sqrt{-z} + 1\right) - \frac{2\sqrt{(1-z)\,z}}{\sqrt{-z}}\sqrt{\frac{1}{1-z}}\tanh^{-1}\left(\frac{1}{\sqrt{z}}\right)$$

Involving $\cos^{-1}\left(\frac{1+z}{1-z}\right)$ and $\tanh^{-1}\left(\frac{1}{\sqrt{z}}\right)$

01.13.27.1642.01

$$\cos^{-1}\left(\frac{1+z}{1-z}\right) = -2\,i\tanh^{-1}\left(\sqrt{\frac{1}{z}}\right) + \pi /; 0 < \arg(z) < \pi \bigvee (z \in \mathbb{R} \land z > 1)$$

01.13.27.1643.01

$$\cos^{-1}\left(\frac{1+z}{1-z}\right) = 2\,i\tanh^{-1}\left(\sqrt{\frac{1}{z}}\right) + \pi \;/;\; \operatorname{Im}(z) < 0 \lor (z \in \mathbb{R} \land z < 0)$$

01.13.27.1644.01

$$\cos^{-1}\left(\frac{1+z}{1-z}\right) = -\pi + 2\,i\tanh^{-1}\left(\sqrt{\frac{1}{z}}\right) \;/;\; (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.1645.01

$$\cos^{-1}\left(\frac{1+z}{1-z}\right) = \pi\left(-\sqrt{\frac{1}{1-z}}\sqrt{1-z} + \sqrt{-\frac{1}{z}}\sqrt{-z} + 1\right) + 2\sqrt{\frac{1-z}{z}}\sqrt{\frac{1}{1-z}}\sqrt{-z}\tanh^{-1}\left(\sqrt{\frac{1}{z}}\right)$$

Involving $\cos^{-1}\left(\frac{z+1}{z-1}\right)$

Involving $\cos^{-1}\left(\frac{z+1}{z-1}\right)$ and $\tanh^{-1}\left(\sqrt{z}\right)$

01.13.27.1646.01

$$\cos^{-1}\left(\frac{z+1}{z-1}\right) = \pi + 2\,i\tanh^{-1}\left(\sqrt{z}\right) \;/;\; 0 < \arg(z) \le \pi$$

01.13.27.1647.01

$$\cos^{-1}\left(\frac{z+1}{z-1}\right) = \pi - 2\,i\tanh^{-1}\left(\sqrt{z}\right) \;/;\; -\pi < \arg(z) < 0 \lor (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.1648.01

$$\cos^{-1}\left(\frac{z+1}{z-1}\right) = 2\,i\tanh^{-1}\left(\sqrt{z}\right) - \pi \;/;\; (z \in \mathbb{R} \land z > 1)$$

01.13.27.1649.01

$$\cos^{-1}\left(\frac{z+1}{z-1}\right) = \sqrt{-z+1}\sqrt{\frac{1}{-z+1}}\,\pi + \frac{2\sqrt{(1-z)\,z}}{\sqrt{-z}}\sqrt{\frac{1}{1-z}}\tanh^{-1}\left(\sqrt{z}\right)$$

Involving $\cos^{-1}\left(\frac{z+1}{z-1}\right)$ and $\tanh^{-1}\left(\frac{1}{\sqrt{z}}\right)$

01.13.27.1650.01

$$\cos^{-1}\left(\frac{z+1}{z-1}\right) = 2\,i\tanh^{-1}\left(\frac{1}{\sqrt{z}}\right) \;/;\; 0 < \arg(z) \le \pi \lor (z \in \mathbb{R} \land z > 1)$$

01.13.27.1651.01

$$\cos^{-1}\left(\frac{z+1}{z-1}\right) = -2\,i\tanh^{-1}\left(\frac{1}{\sqrt{z}}\right) \;/;\; \operatorname{Im}(z) < 0$$

01.13.27.1652.01

$$\cos^{-1}\left(\frac{z+1}{z-1}\right) = 2\pi - 2\,i\tanh^{-1}\left(\frac{1}{\sqrt{z}}\right) \;/;\; (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.1653.01

$$\cos^{-1}\left(\frac{z+1}{z-1}\right) == \pi\left(-\sqrt{-\frac{1}{z}}\ \sqrt{-z}+\sqrt{\frac{1}{1-z}}\ \sqrt{1-z}\right)+\frac{2\sqrt{(1-z)\,z}}{\sqrt{-z}}\sqrt{\frac{1}{1-z}}\ \tanh^{-1}\left(\frac{1}{\sqrt{z}}\right)$$

Involving $\cos^{-1}\left(\frac{z+1}{z-1}\right)$ and $\tanh^{-1}\left(\sqrt{\frac{1}{z}}\right)$

01.13.27.1654.01

$$\cos^{-1}\left(\frac{z+1}{z-1}\right) == 2\,i\tanh^{-1}\left(\sqrt{\frac{1}{z}}\right)\ /;\ 0<\arg(z)<\pi\bigvee(z\in\mathbb{R}\land z>1)$$

01.13.27.1655.01

$$\cos^{-1}\left(\frac{z+1}{z-1}\right) == -2\,i\tanh^{-1}\left(\sqrt{\frac{1}{z}}\right)\ /;\ \mathrm{Im}(z)<0\lor(z\in\mathbb{R}\land z<0)$$

01.13.27.1656.01

$$\cos^{-1}\left(\frac{z+1}{z-1}\right) == 2\,\pi-2\,i\tanh^{-1}\left(\sqrt{\frac{1}{z}}\right)\ /;\ (z\in\mathbb{R}\land 0<z<1)$$

01.13.27.1657.01

$$\cos^{-1}\left(\frac{z+1}{z-1}\right) == \pi\left(-\sqrt{-\frac{1}{z}}\ \sqrt{-z}+\sqrt{\frac{1}{1-z}}\ \sqrt{1-z}\right)-2\sqrt{\frac{1-z}{z}}\ \sqrt{\frac{1}{1-z}}\ \sqrt{-z}\ \tanh^{-1}\left(\sqrt{\frac{1}{z}}\right)$$

Involving $\cos^{-1}\left(\frac{1+z^2}{1-z^2}\right)$

Involving $\cos^{-1}\left(\frac{1+z^2}{1-z^2}\right)$ and $\tanh^{-1}(z)$

01.13.27.1658.01

$$\cos^{-1}\left(\frac{1+z^2}{1-z^2}\right) == -2\,i\tanh^{-1}(z)\ /;\ \mathrm{Im}(z)>0\lor(z\in\mathbb{R}\land -1<z<0)$$

01.13.27.1659.01

$$\cos^{-1}\left(\frac{z^2+1}{1-z^2}\right) == 2\,i\tanh^{-1}(z)\ /;\ \mathrm{Im}(z)<0\lor(z\in\mathbb{R}\land 0<z<1)$$

01.13.27.1660.01

$$\cos^{-1}\left(\frac{z^2+1}{1-z^2}\right) == 2\,\pi-2\,i\tanh^{-1}(z)\ /;\ (z\in\mathbb{R}\land z>1)$$

01.13.27.1661.01

$$\cos^{-1}\left(\frac{z^2+1}{1-z^2}\right) == 2\,\pi+2\,i\tanh^{-1}(z)\ /;\ (z\in\mathbb{R}\land z<-1)$$

01.13.27.0053.01

$$\cos^{-1}\left(\frac{z^2+1}{1-z^2}\right) == \frac{2\sqrt{-z^2}}{z}\tanh^{-1}(z)\ /;\ z\notin(-\infty,-1)\bigwedge z\notin(1,\infty)$$

01.13.27.1662.01

$$\cos^{-1}\left(\frac{1+z^2}{1-z^2}\right) = \pi\left(2 - \sqrt{\frac{1}{1-z}}\,\sqrt{1-z} - \sqrt{\frac{1}{z+1}}\,\sqrt{z+1}\right) + \frac{2\sqrt{-z^2}}{z}\sqrt{\frac{1-z}{z+1}}\,\sqrt{\frac{z+1}{1-z}}\,\tanh^{-1}(z)$$

Involving $\cos^{-1}\left(\frac{1+z^2}{1-z^2}\right)$ and $\tanh^{-1}\left(\frac{1}{z}\right)$

01.13.27.1663.01

$$\cos^{-1}\left(\frac{z^2+1}{1-z^2}\right) = \pi - 2\,i\tanh^{-1}\left(\frac{1}{z}\right) /;\ \mathrm{Im}(z) > 0 \lor (z \in \mathbb{R} \land z > 1)$$

01.13.27.1664.01

$$\cos^{-1}\left(\frac{z^2+1}{1-z^2}\right) = \pi + 2\,i\tanh^{-1}\left(\frac{1}{z}\right) /;\ \mathrm{Im}(z) < 0 \lor (z \in \mathbb{R} \land z < -1)$$

01.13.27.1665.01

$$\cos^{-1}\left(\frac{z^2+1}{1-z^2}\right) = -\pi + 2\,i\tanh^{-1}\left(\frac{1}{z}\right) /;\ (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.1666.01

$$\cos^{-1}\left(\frac{z^2+1}{1-z^2}\right) = -\pi - 2\,i\tanh^{-1}\left(\frac{1}{z}\right) /;\ (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.1667.01

$$\cos^{-1}\left(\frac{1+z^2}{1-z^2}\right) =$$

$$\pi\left(\sqrt{\frac{1}{1-z}}\,\sqrt{1-z}\,\sqrt{-\frac{1}{z}}\,\sqrt{-z} + \sqrt{\frac{1}{z}}\,\sqrt{z}\,\sqrt{\frac{1}{z+1}}\,\sqrt{z+1} - 1\right) + \frac{2\sqrt{-z^2}}{z}\sqrt{\frac{1-z}{z+1}}\,\sqrt{\frac{z+1}{1-z}}\,\tanh^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\frac{z^2+1}{z^2-1}\right)$

Involving $\cos^{-1}\left(\frac{z^2+1}{z^2-1}\right)$ and $\tanh^{-1}(z)$

01.13.27.1668.01

$$\cos^{-1}\left(\frac{z^2+1}{z^2-1}\right) = \pi + 2\,i\tanh^{-1}(z) /;\ \mathrm{Im}(z) > 0 \lor (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.1669.01

$$\cos^{-1}\left(\frac{z^2+1}{z^2-1}\right) = \pi - 2\,i\tanh^{-1}(z) /;\ \mathrm{Im}(z) < 0 \lor (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.1670.01

$$\cos^{-1}\left(\frac{z^2+1}{z^2-1}\right) = 2\,i\tanh^{-1}(z) - \pi /;\ (z \in \mathbb{R} \land z > 1)$$

01.13.27.1671.01

$$\cos^{-1}\left(\frac{z^2+1}{z^2-1}\right) = -2\,i\tanh^{-1}(z) - \pi \;/;\; (z \in \mathbb{R} \land z < -1)$$

01.13.27.1672.01

$$\cos^{-1}\left(\frac{z^2+1}{z^2-1}\right) = \pi - \frac{2\sqrt{-z^2}}{z}\tanh^{-1}(z) \;/;\; z \notin (-\infty, -1) \bigwedge z \notin (1, \infty)$$

01.13.27.1673.01

$$\cos^{-1}\left(\frac{z^2+1}{z^2-1}\right) = \pi\left(-1 + \sqrt{\frac{1}{1-z}}\sqrt{1-z} + \sqrt{\frac{1}{1+z}}\sqrt{1+z}\right) - \frac{2\sqrt{-z^2}}{z}\sqrt{\frac{1-z}{z+1}}\sqrt{\frac{z+1}{1-z}}\tanh^{-1}(z)$$

Involving $\cos^{-1}\left(\frac{z^2+1}{z^2-1}\right)$ and $\tanh^{-1}\left(\frac{1}{z}\right)$

01.13.27.1674.01

$$\cos^{-1}\left(\frac{z^2+1}{z^2-1}\right) = 2\,i\tanh^{-1}\left(\frac{1}{z}\right) \;/;\; \operatorname{Im}(z) > 0 \lor (z \in \mathbb{R} \land z > 1)$$

01.13.27.1675.01

$$\cos^{-1}\left(\frac{z^2+1}{z^2-1}\right) = -2\,i\tanh^{-1}\left(\frac{1}{z}\right) \;/;\; \operatorname{Im}(z) < 0 \lor (z \in \mathbb{R} \land z < -1)$$

01.13.27.1676.01

$$\cos^{-1}\left(\frac{z^2+1}{z^2-1}\right) = 2\pi - 2\,i\tanh^{-1}\left(\frac{1}{z}\right) \;/;\; (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.1677.01

$$\cos^{-1}\left(\frac{z^2+1}{z^2-1}\right) = 2\pi + 2\,i\tanh^{-1}\left(\frac{1}{z}\right) \;/;\; (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.1678.01

$$\cos^{-1}\left(\frac{z^2+1}{z^2-1}\right) =$$
$$\pi\left(2 - \sqrt{\frac{1}{1-z}}\sqrt{1-z}\sqrt{-\frac{1}{z}}\sqrt{-z} - \sqrt{\frac{1}{z}}\sqrt{z}\sqrt{\frac{1}{1+z}}\sqrt{1+z}\right) - \frac{2\sqrt{-z^2}}{z}\sqrt{\frac{1-z}{z+1}}\sqrt{\frac{z+1}{1-z}}\tanh^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\frac{1}{\sqrt{1-z}}\right)$

Involving $\cos^{-1}\left(\frac{1}{\sqrt{1-z}}\right)$ and $\tanh^{-1}\left(\sqrt{z}\right)$

01.13.27.1679.01

$$\cos^{-1}\left(\frac{1}{\sqrt{1-z}}\right) = -i\tanh^{-1}\left(\sqrt{z}\right) \;/;\; 0 < \arg(z) \le \pi$$

01.13.27.1680.01

$$\cos^{-1}\left(\frac{1}{\sqrt{1-z}}\right) = i\tanh^{-1}\left(\sqrt{z}\right)\; /;\; -\pi < \arg(z) \leq 0$$

01.13.27.0054.01

$$\cos^{-1}\left(\frac{1}{\sqrt{1-z}}\right) = \frac{\sqrt{-z^2}}{z}\tanh^{-1}\left(\sqrt{z}\right)$$

Involving $\cos^{-1}\left(\frac{1}{\sqrt{1-z}}\right)$ and $\tanh^{-1}\left(\frac{1}{\sqrt{z}}\right)$

01.13.27.1681.01

$$\cos^{-1}\left(\frac{1}{\sqrt{1-z}}\right) = \frac{\pi}{2} + i\tanh^{-1}\left(\frac{1}{\sqrt{z}}\right)\; /;\; -\pi < \arg(z) < 0 \bigvee (z \in \mathbb{R} \land z > 1)$$

01.13.27.1682.01

$$\cos^{-1}\left(\frac{1}{\sqrt{1-z}}\right) = \frac{\pi}{2} - i\tanh^{-1}\left(\frac{1}{\sqrt{z}}\right)\; /;\; 0 < \arg(z) \leq \pi$$

01.13.27.1683.01

$$\cos^{-1}\left(\frac{1}{\sqrt{1-z}}\right) = i\tanh^{-1}\left(\frac{1}{\sqrt{z}}\right) - \frac{\pi}{2}\; /;\; (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.1684.01

$$\cos^{-1}\left(\frac{1}{\sqrt{1-z}}\right) = \frac{1}{2}\sqrt{-z}\sqrt{-\frac{1}{z}}\sqrt{1-z}\sqrt{\frac{1}{1-z}}\,\pi - \frac{\sqrt{z}}{\sqrt{-z}}\tanh^{-1}\left(\frac{1}{\sqrt{z}}\right)$$

Involving $\cos^{-1}\left(\frac{1}{\sqrt{1-z}}\right)$ and $\tanh^{-1}\left(\sqrt{\frac{1}{z}}\right)$

01.13.27.1685.01

$$\cos^{-1}\left(\frac{1}{\sqrt{1-z}}\right) = \frac{\pi}{2} - i\tanh^{-1}\left(\sqrt{\frac{1}{z}}\right)\; /;\; 0 < \arg(z) < \pi$$

01.13.27.1686.01

$$\cos^{-1}\left(\frac{1}{\sqrt{1-z}}\right) = \frac{\pi}{2} + i\tanh^{-1}\left(\sqrt{\frac{1}{z}}\right)\; /;\; -\pi < \arg(z) < 0 \bigvee (z \in \mathbb{R} \land z > 1) \bigvee (z \in \mathbb{R} \land z < 0)$$

01.13.27.1687.01

$$\cos^{-1}\left(\frac{1}{\sqrt{1-z}}\right) = i\tanh^{-1}\left(\sqrt{\frac{1}{z}}\right) - \frac{\pi}{2}\; /;\; (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.1688.01

$$\cos^{-1}\left(\frac{1}{\sqrt{1-z}}\right) = \frac{1}{2}\sqrt{-z}\sqrt{-\frac{1}{z}}\sqrt{1-z}\sqrt{\frac{1}{1-z}}\,\pi + \sqrt{-z}\sqrt{\frac{1}{z}}\tanh^{-1}\left(\sqrt{\frac{1}{z}}\right)$$

Involving $\cos^{-1}\left(\sqrt{\frac{1}{1-z}}\right)$

Involving $\cos^{-1}\left(\sqrt{\frac{1}{1-z}}\right)$ and $\tanh^{-1}\left(\sqrt{z}\right)$

01.13.27.1689.01

$$\cos^{-1}\left(\sqrt{\frac{1}{1-z}}\right) = -i\tanh^{-1}\left(\sqrt{z}\right) /; 0 < \arg(z) \leq \pi$$

01.13.27.1690.01

$$\cos^{-1}\left(\sqrt{\frac{1}{1-z}}\right) = i\tanh^{-1}\left(\sqrt{z}\right) /; -\pi < \arg(z) < 0 \bigvee (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.1691.01

$$\cos^{-1}\left(\sqrt{\frac{1}{1-z}}\right) = \pi - i\tanh^{-1}\left(\sqrt{z}\right) /; (z \in \mathbb{R} \land z > 1)$$

01.13.27.1692.01

$$\cos^{-1}\left(\sqrt{\frac{1}{1-z}}\right) = \frac{\pi}{2} - \sqrt{\frac{1}{1-z}}\sqrt{1-z}\left(\frac{\pi}{2} - \frac{\sqrt{-z^2}}{z}\tanh^{-1}\left(\sqrt{z}\right)\right)$$

Involving $\cos^{-1}\left(\sqrt{\frac{1}{1-z}}\right)$ and $\tanh^{-1}\left(\frac{1}{\sqrt{z}}\right)$

01.13.27.1693.01

$$\cos^{-1}\left(\sqrt{\frac{1}{1-z}}\right) = \frac{\pi}{2} + i\tanh^{-1}\left(\frac{1}{\sqrt{z}}\right) /; -\pi < \arg(z) < 0$$

01.13.27.1694.01

$$\cos^{-1}\left(\sqrt{\frac{1}{1-z}}\right) = \frac{\pi}{2} - i\tanh^{-1}\left(\frac{1}{\sqrt{z}}\right) /; 0 < \arg(z) \leq \pi \bigvee (z \in \mathbb{R} \land z > 1)$$

01.13.27.1695.01

$$\cos^{-1}\left(\sqrt{\frac{1}{1-z}}\right) = i\tanh^{-1}\left(\frac{1}{\sqrt{z}}\right) - \frac{\pi}{2} /; (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.1696.01

$$\cos^{-1}\left(\sqrt{\frac{1}{1-z}}\right) = -\frac{\sqrt{1-z}}{\sqrt{-z}}\sqrt{\frac{z}{1-z}}\tanh^{-1}\left(\frac{1}{\sqrt{z}}\right) + \frac{\pi}{2}\left(1 + \sqrt{-z}\sqrt{-\frac{1}{z}} - \sqrt{\frac{1}{1-z}}\sqrt{1-z}\right)$$

Involving $\cos^{-1}\left(\sqrt{\frac{1}{1-z}}\right)$ and $\tanh^{-1}\left(\sqrt{\frac{1}{z}}\right)$

01.13.27.1697.01

$$\cos^{-1}\left(\sqrt{\frac{1}{1-z}}\right) = \frac{\pi}{2} - i \tanh^{-1}\left(\sqrt{\frac{1}{z}}\right) /; 0 < \arg(z) < \pi \bigvee (z \in \mathbb{R} \wedge z > 1)$$

01.13.27.1698.01

$$\cos^{-1}\left(\sqrt{\frac{1}{1-z}}\right) = \frac{\pi}{2} + i \tanh^{-1}\left(\sqrt{\frac{1}{z}}\right) /; -\pi < \arg(z) < 0 \bigvee (z \in \mathbb{R} \wedge z < 0)$$

01.13.27.1699.01

$$\cos^{-1}\left(\sqrt{\frac{1}{1-z}}\right) = i \tanh^{-1}\left(\sqrt{\frac{1}{z}}\right) - \frac{\pi}{2} /; (z \in \mathbb{R} \wedge 0 < z < 1)$$

01.13.27.1700.01

$$\cos^{-1}\left(\sqrt{\frac{1}{1-z}}\right) = \sqrt{\frac{1-z}{z}} \sqrt{\frac{1}{1-z}} \sqrt{-z} \tanh^{-1}\left(\sqrt{\frac{1}{z}}\right) + \frac{\pi}{2}\left(1 + \sqrt{-z} \sqrt{-\frac{1}{z}} - \sqrt{\frac{1}{1-z}} \sqrt{1-z}\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{z}}{\sqrt{z-a}}\right)$

Involving $\cos^{-1}\left(\frac{\sqrt{z}}{\sqrt{z-a}}\right)$ and $\tanh^{-1}\left(\sqrt{\frac{z}{a}}\right)$

01.13.27.1701.01

$$\cos^{-1}\left(\frac{\sqrt{z}}{\sqrt{z-1}}\right) = \frac{\pi}{2} + i \tanh^{-1}\left(\sqrt{z}\right) /; 0 < \arg(z) \leq \pi \bigvee (z \in \mathbb{R} \wedge 0 < z < 1)$$

01.13.27.1702.01

$$\cos^{-1}\left(\frac{\sqrt{z}}{\sqrt{z-1}}\right) = \frac{\pi}{2} - i \tanh^{-1}\left(\sqrt{z}\right) /; -\pi < \arg(z) < 0$$

01.13.27.1703.01

$$\cos^{-1}\left(\frac{\sqrt{z}}{\sqrt{z-1}}\right) = -\frac{\pi}{2} + i \tanh^{-1}\left(\sqrt{z}\right) /; (z \in \mathbb{R} \wedge z > 1)$$

01.13.27.1704.01

$$\cos^{-1}\left(\frac{\sqrt{z}}{\sqrt{z-1}}\right) = \frac{\pi}{2} \sqrt{1-z} \sqrt{\frac{1}{1-z}} + \sqrt{-\frac{1}{z}} \sqrt{z} \tanh^{-1}\left(\sqrt{z}\right)$$

01.13.27.1705.01

$$\cos^{-1}\left(\frac{\sqrt{z}}{\sqrt{z-a}}\right) = \sqrt{-\frac{1}{z}} \sqrt{z} \tanh^{-1}\left(\sqrt{\frac{z}{a}}\right) + \frac{1}{2} \pi \sqrt{\frac{1}{a-z}} \sqrt{a-z} /; a > 0$$

Involving $\cos^{-1}\left(\frac{\sqrt{z}}{\sqrt{z-1}}\right)$ and $\tanh^{-1}\left(\frac{1}{\sqrt{z}}\right)$

01.13.27.1706.01

$$\cos^{-1}\left(\frac{\sqrt{z}}{\sqrt{z-1}}\right) = i\tanh^{-1}\left(\frac{1}{\sqrt{z}}\right) /; \operatorname{Im}(z) \geq 0$$

01.13.27.1707.01

$$\cos^{-1}\left(\frac{\sqrt{z}}{\sqrt{z-1}}\right) = -i\tanh^{-1}\left(\frac{1}{\sqrt{z}}\right) /; \operatorname{Im}(z) < 0$$

01.13.27.1708.01

$$\cos^{-1}\left(\frac{\sqrt{z}}{\sqrt{z-1}}\right) = \sqrt{-\frac{1}{z}}\,\sqrt{z}\,\tanh^{-1}\left(\frac{1}{\sqrt{z}}\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{z}}{\sqrt{z-1}}\right)$ and $\tanh^{-1}\left(\sqrt{\frac{1}{z}}\right)$

01.13.27.1709.01

$$\cos^{-1}\left(\frac{\sqrt{z}}{\sqrt{z-1}}\right) = i\tanh^{-1}\left(\sqrt{\frac{1}{z}}\right) /; 0 \leq \arg(z) < \pi$$

01.13.27.1710.01

$$\cos^{-1}\left(\frac{\sqrt{z}}{\sqrt{z-1}}\right) = -i\tanh^{-1}\left(\sqrt{\frac{1}{z}}\right) /; \operatorname{Im}(z) < 0 \lor (z \in \mathbb{R} \land z < 0)$$

01.13.27.1711.01

$$\cos^{-1}\left(\frac{\sqrt{z}}{\sqrt{z-1}}\right) = z\sqrt{-\frac{1}{z^2}}\,\tanh^{-1}\left(\sqrt{\frac{1}{z}}\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{-z}}{\sqrt{1-z}}\right)$

Involving $\cos^{-1}\left(\frac{\sqrt{-z}}{\sqrt{1-z}}\right)$ and $\tanh^{-1}\left(\sqrt{z}\right)$

01.13.27.1712.01

$$\cos^{-1}\left(\frac{\sqrt{-z}}{\sqrt{1-z}}\right) = \frac{\pi}{2} - i\tanh^{-1}\left(\sqrt{z}\right) /; \operatorname{Im}(z) < 0 \lor (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.1713.01

$$\cos^{-1}\left(\frac{\sqrt{-z}}{\sqrt{1-z}}\right) = \frac{\pi}{2} + i\tanh^{-1}\left(\sqrt{z}\right) /; 0 < \arg(z) \leq \pi$$

01.13.27.1714.01

$$\cos^{-1}\left(\frac{\sqrt{-z}}{\sqrt{1-z}}\right) == i\tanh^{-1}\left(\sqrt{z}\right) - \frac{\pi}{2} \;/;\; (z \in \mathbb{R} \land z > 1)$$

01.13.27.1715.01

$$\cos^{-1}\left(\frac{\sqrt{-z}}{\sqrt{1-z}}\right) == \sqrt{\frac{1}{1-z}}\;\sqrt{1-z}\left(-\frac{\sqrt{-z}}{\sqrt{z}}\tanh^{-1}\left(\sqrt{z}\right) + \frac{\pi}{2}\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{-z}}{\sqrt{1-z}}\right)$ and $\tanh^{-1}\left(\frac{1}{\sqrt{z}}\right)$

01.13.27.1716.01

$$\cos^{-1}\left(\frac{\sqrt{-z}}{\sqrt{1-z}}\right) == i\tanh^{-1}\left(\frac{1}{\sqrt{z}}\right) \;/;\; \operatorname{Im}(z) > 0 \lor (z \in \mathbb{R} \land z > 1)$$

01.13.27.1717.01

$$\cos^{-1}\left(\frac{\sqrt{-z}}{\sqrt{1-z}}\right) == -i\tanh^{-1}\left(\frac{1}{\sqrt{z}}\right) \;/;\; \operatorname{Im}(z) < 0$$

01.13.27.1718.01

$$\cos^{-1}\left(\frac{\sqrt{-z}}{\sqrt{1-z}}\right) == \pi - i\tanh^{-1}\left(\frac{1}{\sqrt{z}}\right) \;/;\; (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.1719.01

$$\cos^{-1}\left(\frac{\sqrt{-z}}{\sqrt{1-z}}\right) == -\frac{\sqrt{-z^2}\;\sqrt{1-z}}{z}\sqrt{\frac{1}{1-z}}\tanh^{-1}\left(\frac{1}{\sqrt{z}}\right) + \frac{\pi}{2}\left(1 - \sqrt{\frac{z-1}{z}}\;\sqrt{\frac{z}{z-1}}\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{-z}}{\sqrt{1-z}}\right)$ and $\tanh^{-1}\left(\sqrt{\frac{1}{z}}\right)$

01.13.27.1720.01

$$\cos^{-1}\left(\frac{\sqrt{-z}}{\sqrt{1-z}}\right) == i\tanh^{-1}\left(\sqrt{\frac{1}{z}}\right) \;/;\; \operatorname{Im}(z) > 0 \lor (z \in \mathbb{R} \land z > 1)$$

01.13.27.1721.01

$$\cos^{-1}\left(\frac{\sqrt{-z}}{\sqrt{1-z}}\right) == -i\tanh^{-1}\left(\sqrt{\frac{1}{z}}\right) \;/;\; \operatorname{Im}(z) < 0 \lor (z \in \mathbb{R} \land z < 0)$$

01.13.27.1722.01

$$\cos^{-1}\left(\frac{\sqrt{-z}}{\sqrt{1-z}}\right) == \pi - i\tanh^{-1}\left(\sqrt{\frac{1}{z}}\right) \;/;\; (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.1723.01

$$\cos^{-1}\left(\frac{\sqrt{-z}}{\sqrt{1-z}}\right) == -\sqrt{\frac{1-z}{z}}\;\sqrt{\frac{1}{1-z}}\;\sqrt{-z}\tanh^{-1}\left(\sqrt{\frac{1}{z}}\right) + \frac{\pi}{2}\left(1 - \sqrt{\frac{z-1}{z}}\;\sqrt{\frac{z}{z-1}}\right)$$

Involving $\cos^{-1}\left(\sqrt{\frac{z}{z-a}}\right)$

Involving $\cos^{-1}\left(\sqrt{\frac{z}{z-1}}\right)$ and $\tanh^{-1}\left(\sqrt{z}\right)$

01.13.27.1724.01

$$\cos^{-1}\left(\sqrt{\frac{z}{z-1}}\right) = \frac{\pi}{2} - i\tanh^{-1}\left(\sqrt{z}\right) /; \operatorname{Im}(z) < 0 \lor (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.1725.01

$$\cos^{-1}\left(\sqrt{\frac{z}{z-1}}\right) = \frac{\pi}{2} + i\tanh^{-1}\left(\sqrt{z}\right) /; 0 < \arg(z) \leq \pi$$

01.13.27.1726.01

$$\cos^{-1}\left(\sqrt{\frac{z}{z-1}}\right) = i\tanh^{-1}\left(\sqrt{z}\right) - \frac{\pi}{2} /; (z \in \mathbb{R} \land z > 1)$$

01.13.27.0059.01

$$\cos^{-1}\left(\sqrt{\frac{z}{z-1}}\right) = \sqrt{\frac{1}{1-z}}\sqrt{1-z}\left(\frac{\pi}{2} - \frac{\sqrt{-z}}{\sqrt{z}}\tanh^{-1}\left(\sqrt{z}\right)\right)$$

Involving $\cos^{-1}\left(\sqrt{\frac{z}{z-a}}\right)$ and $\tanh^{-1}\left(\frac{1}{\sqrt{z}}\right)$

01.13.27.1727.01

$$\cos^{-1}\left(\sqrt{\frac{z}{z-1}}\right) = i\tanh^{-1}\left(\frac{1}{\sqrt{z}}\right) /; 0 < \arg(z) \leq \pi \lor (z \in \mathbb{R} \land z > 1)$$

01.13.27.1728.01

$$\cos^{-1}\left(\sqrt{\frac{z}{z-1}}\right) = -i\tanh^{-1}\left(\frac{1}{\sqrt{z}}\right) /; \operatorname{Im}(z) < 0$$

01.13.27.1729.01

$$\cos^{-1}\left(\sqrt{\frac{z}{z-1}}\right) = \pi - i\tanh^{-1}\left(\frac{1}{\sqrt{z}}\right) /; (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.1730.01

$$\cos^{-1}\left(\sqrt{\frac{z}{z-1}}\right) = -\frac{\sqrt{-z^2}\sqrt{1-z}}{z}\sqrt{\frac{1}{1-z}}\tanh^{-1}\left(\frac{1}{\sqrt{z}}\right) + \frac{\pi}{2}\left(1 - \sqrt{\frac{z-1}{z}}\sqrt{\frac{z}{z-1}}\right)$$

01.13.27.0058.01

$$\cos^{-1}\left(\frac{\sqrt{z}}{\sqrt{z-a}}\right) = \frac{\pi}{2} - \frac{\sqrt{z-a}}{2\sqrt{z}}\sqrt{\frac{z}{z-a}}\left(\frac{1}{z}\sqrt{1-\frac{z}{a}}\sqrt{\frac{a}{a-z}}\left(2a\sqrt{-\frac{z^2}{a^2}}\tanh^{-1}\left(\sqrt{\frac{z}{a}}\right) - \pi z\right) + \pi\right) /; a > 0$$

Involving $\cos^{-1}\left(\sqrt{\frac{z}{z-1}}\right)$ and $\tanh^{-1}\left(\sqrt{\frac{1}{z}}\right)$

01.13.27.1731.01

$$\cos^{-1}\left(\sqrt{\frac{z}{z-1}}\right) = i\tanh^{-1}\left(\sqrt{\frac{1}{z}}\right) /; 0 < \arg(z) < \pi \bigvee (z \in \mathbb{R} \land z > 1)$$

01.13.27.1732.01

$$\cos^{-1}\left(\sqrt{\frac{z}{z-1}}\right) = -i\tanh^{-1}\left(\sqrt{\frac{1}{z}}\right) /; \operatorname{Im}(z) < 0 \lor (z \in \mathbb{R} \land z < 0)$$

01.13.27.1733.01

$$\cos^{-1}\left(\sqrt{\frac{z}{z-1}}\right) = \pi - i\tanh^{-1}\left(\sqrt{\frac{1}{z}}\right) /; (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.1734.01

$$\cos^{-1}\left(\sqrt{\frac{z}{z-1}}\right) = -\sqrt{\frac{1-z}{z}}\sqrt{\frac{1}{1-z}}\sqrt{-z}\tanh^{-1}\left(\sqrt{\frac{1}{z}}\right) + \frac{\pi}{2}\left(1 - \sqrt{\frac{z-1}{z}}\sqrt{\frac{z}{z-1}}\right)$$

Involving $\cos^{-1}\left(\frac{1}{\sqrt{1-z^2}}\right)$

Involving $\cos^{-1}\left(\frac{1}{\sqrt{1-z^2}}\right)$ and $\tanh^{-1}(z)$

01.13.27.1735.01

$$\cos^{-1}\left(\frac{1}{\sqrt{1-z^2}}\right) = i\tanh^{-1}(z) /; -\pi < \arg(z) \leq 0$$

01.13.27.1736.01

$$\cos^{-1}\left(\frac{1}{\sqrt{1-z^2}}\right) = -i\tanh^{-1}(z) /; 0 < \arg(z) \leq \pi$$

01.13.27.1737.01

$$\cos^{-1}\left(\frac{1}{\sqrt{1-z^2}}\right) = \frac{\sqrt{-z^2}}{z}\tanh^{-1}(z)$$

Involving $\cos^{-1}\left(\frac{1}{\sqrt{1-z^2}}\right)$ and $\tanh^{-1}\left(\frac{1}{z}\right)$

01.13.27.1738.01

$$\cos^{-1}\left(\frac{1}{\sqrt{1-z^2}}\right) = \frac{\pi}{2} - i \tanh^{-1}\left(\frac{1}{z}\right) /; \operatorname{Im}(z) > 0 \lor (z \in \mathbb{R} \land z < -1)$$

01.13.27.1739.01

$$\cos^{-1}\left(\frac{1}{\sqrt{1-z^2}}\right) = \frac{\pi}{2} + i \tanh^{-1}\left(\frac{1}{z}\right) /; \operatorname{Im}(z) < 0 \lor (z \in \mathbb{R} \land z > 1)$$

01.13.27.1740.01

$$\cos^{-1}\left(\frac{1}{\sqrt{1-z^2}}\right) = -\frac{\pi}{2} - i \tanh^{-1}\left(\frac{1}{z}\right) /; (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.1741.01

$$\cos^{-1}\left(\frac{1}{\sqrt{1-z^2}}\right) = -\frac{\pi}{2} + i \tanh^{-1}\left(\frac{1}{z}\right) /; (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.1742.01

$$\cos^{-1}\left(\frac{1}{\sqrt{1-z^2}}\right) = \frac{\pi}{2} + \frac{\sqrt{-z^2}}{z} \tanh^{-1}\left(\frac{1}{z}\right) /; z \notin (-1, 1)$$

01.13.27.1743.01

$$\cos^{-1}\left(\frac{1}{\sqrt{1-z^2}}\right) = \frac{\sqrt{-z^2}}{z} \tanh^{-1}\left(\frac{1}{z}\right) + \frac{\pi i}{2}\left(\frac{\sqrt{-z-1}}{\sqrt{z+1}} + \frac{\sqrt{z-1}}{\sqrt{1-z}}\right) + \frac{\pi}{2}$$

Involving $\cos^{-1}\left(\sqrt{\frac{1}{1-z^2}}\right)$

Involving $\cos^{-1}\left(\sqrt{\frac{1}{1-z^2}}\right)$ and $\tanh^{-1}(z)$

01.13.27.1744.01

$$\cos^{-1}\left(\sqrt{\frac{1}{1-z^2}}\right) = -i \tanh^{-1}(z) /; \operatorname{Im}(z) > 0 \lor (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.1745.01

$$\cos^{-1}\left(\sqrt{\frac{1}{1-z^2}}\right) = i \tanh^{-1}(z) /; \operatorname{Im}(z) < 0 \lor (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.1746.01

$$\cos^{-1}\left(\sqrt{\frac{1}{1-z^2}}\right) = \pi - i \tanh^{-1}(z) /; (z \in \mathbb{R} \land z > 1)$$

01.13.27.1747.01

$$\cos^{-1}\left(\sqrt{\frac{1}{1-z^2}}\right) = \pi + i \tanh^{-1}(z) \;/;\; (z \in \mathbb{R} \land z < -1)$$

01.13.27.1748.01

$$\cos^{-1}\left(\sqrt{\frac{1}{1-z^2}}\right) = \frac{\pi}{2} - \sqrt{\frac{1}{1-z^2}} \sqrt{1-z^2} \left(\frac{\pi}{2} - \frac{\sqrt{-z^2}}{z} \tanh^{-1}(z)\right)$$

Involving $\cos^{-1}\left(\sqrt{\frac{1}{1-z^2}}\right)$ and $\tanh^{-1}\left(\frac{1}{z}\right)$

01.13.27.1749.01

$$\cos^{-1}\left(\sqrt{\frac{1}{1-z^2}}\right) = \frac{\pi}{2} - i \tanh^{-1}\left(\frac{1}{z}\right) \;/;\; \operatorname{Im}(z) > 0 \lor (z \in \mathbb{R} \land z > 1)$$

01.13.27.1750.01

$$\cos^{-1}\left(\sqrt{\frac{1}{1-z^2}}\right) = \frac{\pi}{2} + i \tanh^{-1}\left(\frac{1}{z}\right) \;/;\; \operatorname{Im}(z) < 0 \lor (z \in \mathbb{R} \land z < -1)$$

01.13.27.1751.01

$$\cos^{-1}\left(\sqrt{\frac{1}{1-z^2}}\right) = -i \tanh^{-1}\left(\frac{1}{z}\right) - \frac{\pi}{2} \;/;\; (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.1752.01

$$\cos^{-1}\left(\sqrt{\frac{1}{1-z^2}}\right) = i \tanh^{-1}\left(\frac{1}{z}\right) - \frac{\pi}{2} \;/;\; (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.1753.01

$$\cos^{-1}\left(\sqrt{\frac{1}{1-z^2}}\right) = \frac{\pi}{2} + \frac{\sqrt{-z^2}}{z} \tanh^{-1}\left(\frac{1}{z}\right) \;/;\; \operatorname{Im}(z) \neq 0$$

01.13.27.1754.01

$$\cos^{-1}\left(\sqrt{\frac{1}{1-z^2}}\right) = \frac{\pi}{2} + \sqrt{\frac{1}{1-z^2}} \sqrt{1-z^2} \left(\frac{\sqrt{-z^2}}{z} \tanh^{-1}\left(\frac{1}{z}\right) + \frac{\pi i}{2}\left(\frac{\sqrt{-z-1}}{\sqrt{z+1}} + \frac{\sqrt{z-1}}{\sqrt{1-z}}\right)\right)$$

Involving $\cos^{-1}\left(\frac{z}{\sqrt{z^2-1}}\right)$

Involving $\cos^{-1}\left(\frac{z}{\sqrt{z^2-1}}\right)$ and $\tanh^{-1}(z)$

01.13.27.1755.01

$$\cos^{-1}\left(\frac{z}{\sqrt{z^2-1}}\right) = \frac{\pi}{2} + i \tanh^{-1}(z) /; 0 < \arg(z) \leq \frac{\pi}{2} \bigvee -\pi < \arg(z) \leq -\frac{\pi}{2} \bigvee (z \in \mathbb{R} \land -1 < z < 1)$$

01.13.27.1756.01

$$\cos^{-1}\left(\frac{z}{\sqrt{z^2-1}}\right) = \frac{\pi}{2} - i \tanh^{-1}(z) /; \frac{\pi}{2} < \arg(z) < \pi \bigvee -\frac{\pi}{2} < \arg(z) < 0$$

01.13.27.1757.01

$$\cos^{-1}\left(\frac{z}{\sqrt{z^2-1}}\right) = i \tanh^{-1}(z) - \frac{\pi}{2} /; (z \in \mathbb{R} \land z > 1)$$

01.13.27.1758.01

$$\cos^{-1}\left(\frac{z}{\sqrt{z^2-1}}\right) = \frac{3\pi}{2} + i \tanh^{-1}(z) /; (z \in \mathbb{R} \land z < -1)$$

01.13.27.0055.01

$$\cos^{-1}\left(\frac{z}{\sqrt{z^2-1}}\right) = \sqrt{z^2-1} \sqrt{\frac{1}{1-z^2}} \tanh^{-1}(z) + \frac{\pi}{2} /; z \notin (-\infty, -1) \bigwedge z \notin (1, \infty)$$

01.13.27.0056.01

$$\cos^{-1}\left(\frac{z}{\sqrt{z^2-1}}\right) = \frac{\pi}{2}\left(-\sqrt{\frac{1}{z+1}} \sqrt{z+1} + \sqrt{\frac{1}{1-z}} \sqrt{1-z} + 1\right) + \sqrt{z^2-1} \sqrt{\frac{1}{1-z^2}} \tanh^{-1}(z)$$

Involving $\cos^{-1}\left(\frac{z}{\sqrt{z^2-1}}\right)$ and $\tanh^{-1}\left(\frac{1}{z}\right)$

01.13.27.1759.01

$$\cos^{-1}\left(\frac{z}{\sqrt{z^2-1}}\right) = i \tanh^{-1}\left(\frac{1}{z}\right) /; 0 \leq \arg(z) \leq \frac{\pi}{2}$$

01.13.27.1760.01

$$\cos^{-1}\left(\frac{z}{\sqrt{z^2-1}}\right) = -i \tanh^{-1}\left(\frac{1}{z}\right) /; -\frac{\pi}{2} < \arg(z) < 0$$

01.13.27.1761.01

$$\cos^{-1}\left(\frac{z}{\sqrt{z^2-1}}\right) = \pi - i \tanh^{-1}\left(\frac{1}{z}\right) /; \frac{\pi}{2} < \arg(z) < \pi$$

01.13.27.1762.01

$$\cos^{-1}\left(\frac{z}{\sqrt{z^2-1}}\right) = \pi + i \tanh^{-1}\left(\frac{1}{z}\right) /; -\pi < \arg(z) \leq -\frac{\pi}{2} \bigvee (z \in \mathbb{R} \land z < 0)$$

01.13.27.1763.01

$$\cos^{-1}\left(\frac{z}{\sqrt{z^2-1}}\right) = \frac{\pi}{2}\left(1-\frac{\sqrt{z^2}}{z}\right)+\sqrt{\frac{1}{1-z^2}}\,\sqrt{z^2-1}\,\tanh^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{z^2}}{\sqrt{z^2-1}}\right)$

Involving $\cos^{-1}\left(\frac{\sqrt{z^2}}{\sqrt{z^2-1}}\right)$ and $\tanh^{-1}(z)$

01.13.27.1764.01

$$\cos^{-1}\left(\frac{\sqrt{z^2}}{\sqrt{z^2-1}}\right) = \frac{\pi}{2}+i\tanh^{-1}(z) \;/;\; \operatorname{Im}(z)>0 \lor (z\in\mathbb{R}\land 0<z<1)$$

01.13.27.1765.01

$$\cos^{-1}\left(\frac{\sqrt{z^2}}{\sqrt{z^2-1}}\right) = \frac{\pi}{2}-i\tanh^{-1}(z) \;/;\; \operatorname{Im}(z)<0 \lor (z\in\mathbb{R}\land -1<z<0)$$

01.13.27.1766.01

$$\cos^{-1}\left(\frac{\sqrt{z^2}}{\sqrt{z^2-1}}\right) = i\tanh^{-1}(z)-\frac{\pi}{2} \;/;\; (z\in\mathbb{R}\land z>1)$$

01.13.27.1767.01

$$\cos^{-1}\left(\frac{\sqrt{z^2}}{\sqrt{z^2-1}}\right) = -\frac{\pi}{2}-i\tanh^{-1}(z) \;/;\; (z\in\mathbb{R}\land z<-1)$$

01.13.27.1768.01

$$\cos^{-1}\left(\frac{\sqrt{z^2}}{\sqrt{z^2-1}}\right) = \frac{1}{2}\pi\left(-1+\sqrt{\frac{1}{1-z}}\,\sqrt{1-z}+\sqrt{\frac{1}{z+1}}\,\sqrt{z+1}\right)+z\sqrt{-\frac{1}{z^2}}\,\tanh^{-1}(z)$$

Involving $\cos^{-1}\left(\frac{\sqrt{z^2}}{\sqrt{z^2-1}}\right)$ and $\tanh^{-1}\left(\frac{1}{z}\right)$

01.13.27.1769.01

$$\cos^{-1}\left(\frac{\sqrt{z^2}}{\sqrt{z^2-1}}\right) = -i\tanh^{-1}\left(\frac{1}{z}\right) \;/;\; -\pi<\arg(z)<0 \lor (z\in\mathbb{R}\land z<0)$$

01.13.27.1770.01

$$\cos^{-1}\left(\frac{\sqrt{z^2}}{\sqrt{z^2-1}}\right) = i\tanh^{-1}\left(\frac{1}{z}\right) \;/;\; 0\le\arg(z)<\pi$$

01.13.27.1771.01

$$\cos^{-1}\left(\frac{\sqrt{z^2}}{\sqrt{z^2-1}}\right) = z\sqrt{-\frac{1}{z^2}}\tanh^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{-z^2}}{\sqrt{1-z^2}}\right)$

Involving $\cos^{-1}\left(\frac{\sqrt{-z^2}}{\sqrt{1-z^2}}\right)$ and $\tanh^{-1}(z)$

01.13.27.1772.01

$$\cos^{-1}\left(\frac{\sqrt{-z^2}}{\sqrt{1-z^2}}\right) = \frac{\pi}{2} - i\tanh^{-1}(z)\ /;\ \operatorname{Im}(z) < 0 \lor (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.1773.01

$$\cos^{-1}\left(\frac{\sqrt{-z^2}}{\sqrt{1-z^2}}\right) = \frac{\pi}{2} + i\tanh^{-1}(z)\ /;\ \operatorname{Im}(z) > 0 \lor (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.1774.01

$$\cos^{-1}\left(\frac{\sqrt{-z^2}}{\sqrt{1-z^2}}\right) = i\tanh^{-1}(z) - \frac{\pi}{2}\ /;\ (z \in \mathbb{R} \land z > 1)$$

01.13.27.1775.01

$$\cos^{-1}\left(\frac{\sqrt{-z^2}}{\sqrt{1-z^2}}\right) = -\frac{\pi}{2} - i\tanh^{-1}(z)\ /;\ (z \in \mathbb{R} \land z < -1)$$

01.13.27.1776.01

$$\cos^{-1}\left(\frac{\sqrt{-z^2}}{\sqrt{1-z^2}}\right) = \frac{\pi}{2} - \frac{\sqrt{-z^2}\sqrt{z^2-1}}{\sqrt{z^2(1-z^2)}}\left(\frac{\pi}{2}\left(-\sqrt{\frac{1}{1-z}}\sqrt{1-z} - \sqrt{\frac{1}{z+1}}\sqrt{z+1} + 2\right) - \sqrt{-\frac{1}{z^2}}\, z\tanh^{-1}(z)\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{-z^2}}{\sqrt{1-z^2}}\right)$ and $\tanh^{-1}\left(\frac{1}{z}\right)$

01.13.27.1777.01

$$\cos^{-1}\left(\frac{\sqrt{-z^2}}{\sqrt{1-z^2}}\right) = -i\tanh^{-1}\left(\frac{1}{z}\right)\ /;\ -\pi < \arg(z) < 0 \bigvee (z \in \mathbb{R} \land z < -1)$$

01.13.27.1778.01

$$\cos^{-1}\left(\frac{\sqrt{-z^2}}{\sqrt{1-z^2}}\right) = i\tanh^{-1}\left(\frac{1}{z}\right)\ /;\ \operatorname{Im}(z) > 0 \lor (z \in \mathbb{R} \land z > 1)$$

01.13.27.1779.01

$$\cos^{-1}\left(\frac{\sqrt{-z^2}}{\sqrt{1-z^2}}\right) = \pi + i\tanh^{-1}\left(\frac{1}{z}\right) /; (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.1780.01

$$\cos^{-1}\left(\frac{\sqrt{-z^2}}{\sqrt{1-z^2}}\right) = \pi - i\tanh^{-1}\left(\frac{1}{z}\right) /; (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.1781.01

$$\cos^{-1}\left(\frac{\sqrt{-z^2}}{\sqrt{1-z^2}}\right) = \frac{\pi}{2} - \frac{\sqrt{-z^2}\sqrt{z^2-1}}{\sqrt{z^2(1-z^2)}}\left(\frac{\pi}{2} - \sqrt{-\frac{1}{z^2}}\, z\tanh^{-1}\left(\frac{1}{z}\right)\right)$$

Involving $\cos^{-1}\left(\sqrt{\frac{z^2}{z^2+1}}\right)$

Involving $\cos^{-1}\left(\sqrt{\frac{z^2}{z^2-1}}\right)$ and $\tanh^{-1}\left(\frac{1}{z}\right)$

01.13.27.1782.01

$$\cos^{-1}\left(\sqrt{\frac{z^2}{z^2-1}}\right) = \frac{\pi}{2} - i\tanh^{-1}(z) /; \operatorname{Im}(z) < 0 \lor (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.1783.01

$$\cos^{-1}\left(\sqrt{\frac{z^2}{z^2-1}}\right) = \frac{\pi}{2} + i\tanh^{-1}(z) /; \operatorname{Im}(z) > 0 \lor (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.1784.01

$$\cos^{-1}\left(\sqrt{\frac{z^2}{z^2-1}}\right) = -\frac{\pi}{2} + i\tanh^{-1}(z) /; (z \in \mathbb{R} \land z > 1)$$

01.13.27.1785.01

$$\cos^{-1}\left(\sqrt{\frac{z^2}{z^2-1}}\right) = -\frac{\pi}{2} - i\tanh^{-1}(z) /; (z \in \mathbb{R} \land z < -1)$$

01.13.27.1786.01

$$\cos^{-1}\left(\sqrt{\frac{z^2}{z^2-1}}\right) = \frac{\pi}{2} - \frac{\sqrt{-z^2}\sqrt{-1+z^2}}{\sqrt{z^2-z^4}}\left(\frac{1}{2}\pi\left(2 - \sqrt{\frac{1}{1-z}}\sqrt{1-z} - \sqrt{\frac{1}{z+1}}\sqrt{z+1}\right) - z\sqrt{-\frac{1}{z^2}}\tanh^{-1}(z)\right)$$

Involving $\cos^{-1}\left(\sqrt{\frac{z^2}{z^2-1}}\right)$ and $\tanh^{-1}\left(\frac{1}{z}\right)$

01.13.27.1787.01

$$\cos^{-1}\left(\sqrt{\frac{z^2}{z^2-1}}\right) = -i \tanh^{-1}\left(\frac{1}{z}\right) /; -\pi < \arg(z) < 0 \bigvee (z \in \mathbb{R} \wedge z < -1)$$

01.13.27.1788.01

$$\cos^{-1}\left(\sqrt{\frac{z^2}{z^2-1}}\right) = i \tanh^{-1}\left(\frac{1}{z}\right) /; \operatorname{Im}(z) > 0 \vee (z \in \mathbb{R} \wedge z > 1)$$

01.13.27.1789.01

$$\cos^{-1}\left(\sqrt{\frac{z^2}{z^2-1}}\right) = \pi + i \tanh^{-1}\left(\frac{1}{z}\right) /; (z \in \mathbb{R} \wedge -1 < z < 0)$$

01.13.27.1790.01

$$\cos^{-1}\left(\sqrt{\frac{z^2}{z^2-1}}\right) = \pi - i \tanh^{-1}\left(\frac{1}{z}\right) /; (z \in \mathbb{R} \wedge 0 < z < 1)$$

01.13.27.1791.01

$$\cos^{-1}\left(\sqrt{\frac{z^2}{z^2-1}}\right) = \frac{\pi}{2} - \frac{\sqrt{-z^2}\,\sqrt{z^2-1}}{\sqrt{z^2\left(1-z^2\right)}}\left(\frac{\pi}{2} - \sqrt{-\frac{1}{z^2}}\; z \tanh^{-1}\left(\frac{1}{z}\right)\right)$$

Involving $\cos^{-1}\left(\sqrt{\sqrt{1-z^2}-1} \Big/ \left(\sqrt{2}\left(1-z^2\right)^{1/4}\right)\right)$

Involving $\cos^{-1}\left(\sqrt{\sqrt{1-z^2}-1} \Big/ \left(\sqrt{2}\left(1-z^2\right)^{1/4}\right)\right)$ and $\tanh^{-1}(z)$

01.13.27.1792.01

$$\cos^{-1}\left(\frac{\sqrt{\sqrt{1-z^2}-1}}{\sqrt{2}\,\sqrt[4]{1-z^2}}\right) = \frac{\pi}{2} - \frac{1}{2}\, i \tanh^{-1}(z) /; -\pi < \arg(z) \leq 0$$

01.13.27.1793.01

$$\cos^{-1}\left(\frac{\sqrt{\sqrt{1-z^2}-1}}{\sqrt{2}\,(1-z^2)^{1/4}}\right) = \frac{\pi}{2} + \frac{i}{2}\tanh^{-1}(z) \;/;\; 0 < \arg(z) \le \pi$$

01.13.27.1794.01

$$\cos^{-1}\left(\frac{\sqrt{\sqrt{1-z^2}-1}}{\sqrt{2}\,(1-z^2)^{1/4}}\right) = \frac{\pi}{2} - \frac{\sqrt{-z^2}}{2\,z}\tanh^{-1}(z)$$

Involving $\cos^{-1}\left(\sqrt{\sqrt{1-z^2}-1} \Big/ \left(\sqrt{2}\,(1-z^2)^{1/4}\right)\right)$ and $\tanh^{-1}\left(\frac{1}{z}\right)$

01.13.27.1795.01

$$\cos^{-1}\left(\frac{\sqrt{\sqrt{1-z^2}-1}}{\sqrt{2}\,\sqrt[4]{1-z^2}}\right) = \frac{\pi}{4} + \frac{1}{2}\,i\tanh^{-1}\left(\frac{1}{z}\right) \;/;\; \operatorname{Im}(z) > 0 \lor (z \in \mathbb{R} \land z < -1)$$

01.13.27.1796.01

$$\cos^{-1}\left(\frac{\sqrt{\sqrt{1-z^2}-1}}{\sqrt{2}\,\sqrt[4]{1-z^2}}\right) = -\frac{1}{2}\,i\tanh^{-1}\left(\frac{1}{z}\right) + \frac{\pi}{4} \;/;\; \operatorname{Im}(z) < 0 \lor (z \in \mathbb{R} \land z > 1)$$

01.13.27.1797.01

$$\cos^{-1}\left(\frac{\sqrt{\sqrt{1-z^2}-1}}{\sqrt{2}\,\sqrt[4]{1-z^2}}\right) = \frac{3\pi}{4} + \frac{1}{2}\,i\tanh^{-1}\left(\frac{1}{z}\right) \;/;\; (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.1798.01

$$\cos^{-1}\left(\frac{\sqrt{\sqrt{1-z^2}-1}}{\sqrt{2}\,\sqrt[4]{1-z^2}}\right) = -\frac{1}{2}\,i\tanh^{-1}\left(\frac{1}{z}\right) + \frac{3\pi}{4} \;/;\; (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.1799.01

$$\cos^{-1}\left(\frac{\sqrt{\sqrt{1-z^2}-1}}{\sqrt{2}\,\sqrt[4]{1-z^2}}\right) = \frac{\pi}{4}\left(2 - \frac{\sqrt{-z^2}\,\sqrt{z^2-1}}{\sqrt{z^2-z^4}}\right) - \frac{\sqrt{-z^2}}{2\,z}\tanh^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\sqrt{\sqrt{z^2-1}-z} \Big/ \left(\sqrt{2}\,(z^2-1)^{1/4}\right)\right)$

Involving $\cos^{-1}\left(\sqrt{\sqrt{z^2-1}-z} \Big/ \left(\sqrt{2}\,(z^2-1)^{1/4}\right)\right)$ and $\tanh^{-1}(z)$

01.13.27.1800.01

$$\cos^{-1}\left(\frac{\sqrt{\sqrt{z^2-1}-z}}{\sqrt{2}\,\sqrt[4]{z^2-1}}\right) = -\frac{1}{2}\,i\tanh^{-1}(z)+\frac{\pi}{4} \;/;\; 0<\arg(z)\leq\frac{\pi}{2} \bigvee -\pi<\arg(z)\leq-\frac{\pi}{2} \bigvee (z\in\mathbb{R}\land -1<z<1)$$

01.13.27.1801.01

$$\cos^{-1}\left(\frac{\sqrt{\sqrt{z^2-1}-z}}{\sqrt{2}\,\sqrt[4]{z^2-1}}\right) = \frac{\pi}{4}+\frac{1}{2}\,i\tanh^{-1}(z) \;/;\; \frac{\pi}{2}<\arg(z)<\pi \bigvee -\frac{\pi}{2}<\arg(z)<0$$

01.13.27.1802.01

$$\cos^{-1}\left(\frac{\sqrt{\sqrt{z^2-1}-z}}{\sqrt{2}\,\sqrt[4]{z^2-1}}\right) = \frac{3\pi}{4}-\frac{1}{2}\,i\tanh^{-1}(z) \;/;\; (z\in\mathbb{R}\land z>1)$$

01.13.27.1803.01

$$\cos^{-1}\left(\frac{\sqrt{\sqrt{z^2-1}-z}}{\sqrt{2}\,\sqrt[4]{z^2-1}}\right) = -\frac{1}{2}\,i\tanh^{-1}(z)-\frac{\pi}{4} \;/;\; (z\in\mathbb{R}\land z<-1)$$

01.13.27.1804.01

$$\cos^{-1}\left(\frac{\sqrt{\sqrt{z^2-1}-z}}{\sqrt{2}\,\sqrt[4]{z^2-1}}\right) = \frac{\pi}{4}\left(-\sqrt{\frac{1}{1-z}}\,\sqrt{1-z}+\sqrt{z+1}\,\sqrt{\frac{1}{z+1}}+1\right)-\frac{1}{2}\sqrt{z^2}\,\sqrt{-\frac{1}{z^2}}\,\tanh^{-1}(z)$$

Involving $\cos^{-1}\left(\sqrt{\sqrt{z^2-1}-z} \Big/ \left(\sqrt{2}\,(z^2-1)^{1/4}\right)\right)$ and $\tanh^{-1}\left(\frac{1}{z}\right)$

01.13.27.1805.01

$$\cos^{-1}\left(\frac{\sqrt{\sqrt{z^2-1}-z}}{\sqrt{2}\sqrt[4]{z^2-1}}\right) = \frac{\pi}{2} - \frac{i}{2}\tanh^{-1}\left(\frac{1}{z}\right) /; 0 \le \arg(z) \le \frac{\pi}{2}$$

01.13.27.1806.01

$$\cos^{-1}\left(\frac{\sqrt{\sqrt{z^2-1}-z}}{\sqrt{2}\sqrt[4]{z^2-1}}\right) = \frac{\pi}{2} + \frac{i}{2}\tanh^{-1}\left(\frac{1}{z}\right) /; -\frac{\pi}{2} < \arg(z) < 0$$

01.13.27.1807.01

$$\cos^{-1}\left(\frac{\sqrt{\sqrt{z^2-1}-z}}{\sqrt{2}\sqrt[4]{z^2-1}}\right) = \frac{i}{2}\tanh^{-1}\left(\frac{1}{z}\right) /; \frac{\pi}{2} < \arg(z) < \pi$$

01.13.27.1808.01

$$\cos^{-1}\left(\frac{\sqrt{\sqrt{z^2-1}-z}}{\sqrt{2}\sqrt[4]{z^2-1}}\right) = -\frac{i}{2}\tanh^{-1}\left(\frac{1}{z}\right) /; -\pi < \arg(z) \le -\frac{\pi}{2} \bigvee (z \in \mathbb{R} \land z < 0)$$

01.13.27.1809.01

$$\cos^{-1}\left(\frac{\sqrt{\sqrt{z^2-1}-z}}{\sqrt{2}\sqrt[4]{z^2-1}}\right) = \frac{\pi}{4}\left(1 + \frac{\sqrt{z^2}}{z}\right) - \frac{1}{2}\sqrt{z^2}\sqrt{-\frac{1}{z^2}}\tanh^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\sqrt{\left(\sqrt{1-z^2}-1\right)\big/\left(2\sqrt{1-z^2}\right)}\right)$

Involving $\cos^{-1}\left(\sqrt{\left(\sqrt{1-z^2}-1\right)\big/\left(2\sqrt{1-z^2}\right)}\right)$ and $\tanh^{-1}(z)$

01.13.27.1810.01

$$\cos^{-1}\left(\frac{1}{\sqrt{2}}\sqrt{\frac{\sqrt{1-z^2}-1}{\sqrt{1-z^2}}}\right) = \frac{\pi}{2} - \frac{1}{2}i\tanh^{-1}(z) /; -\pi < \arg(z) \le 0$$

01.13.27.1811.01

$$\cos^{-1}\left(\frac{1}{\sqrt{2}}\sqrt{\frac{\sqrt{1-z^2}-1}{\sqrt{1-z^2}}}\right)=\frac{\pi}{2}+\frac{i}{2}\tanh^{-1}(z)\;/;\;0<\arg(z)\leq\pi$$

01.13.27.1812.01

$$\cos^{-1}\left(\frac{1}{\sqrt{2}}\sqrt{\frac{\sqrt{1-z^2}-1}{\sqrt{1-z^2}}}\right)=\frac{\pi}{2}-\frac{\sqrt{-z^2}}{2z}\tanh^{-1}(z)$$

Involving $\cos^{-1}\left(\sqrt{\left(\sqrt{1-z^2}-1\right)\Big/\left(2\sqrt{1-z^2}\right)}\right)$ and $\tanh^{-1}\left(\frac{1}{z}\right)$

01.13.27.1813.01

$$\cos^{-1}\left(\sqrt{\frac{\sqrt{1-z^2}-1}{2\sqrt{1-z^2}}}\right)=\frac{\pi}{4}+\frac{1}{2}i\tanh^{-1}\left(\frac{1}{z}\right)\;/;\;\operatorname{Im}(z)>0\lor(z\in\mathbb{R}\land z<-1)$$

01.13.27.1814.01

$$\cos^{-1}\left(\sqrt{\frac{\sqrt{1-z^2}-1}{2\sqrt{1-z^2}}}\right)=-\frac{1}{2}i\tanh^{-1}\left(\frac{1}{z}\right)+\frac{\pi}{4}\;/;\;\operatorname{Im}(z)<0\lor(z\in\mathbb{R}\land z>1)$$

01.13.27.1815.01

$$\cos^{-1}\left(\sqrt{\frac{\sqrt{1-z^2}-1}{2\sqrt{1-z^2}}}\right)=\frac{3\pi}{4}+\frac{1}{2}i\tanh^{-1}\left(\frac{1}{z}\right)\;/;\;(z\in\mathbb{R}\land -1<z<0)$$

01.13.27.1816.01

$$\cos^{-1}\left(\sqrt{\frac{\sqrt{1-z^2}-1}{2\sqrt{1-z^2}}}\right)=\frac{3\pi}{4}-\frac{1}{2}i\tanh^{-1}\left(\frac{1}{z}\right)\;/;\;(z\in\mathbb{R}\land 0<z<1)$$

01.13.27.1817.01

$$\cos^{-1}\left(\sqrt{\frac{\sqrt{1-z^2}-1}{2\sqrt{1-z^2}}}\right)=-\frac{\sqrt{-z^2}}{2z}\tanh^{-1}\left(\frac{1}{z}\right)+\left(2-\frac{\sqrt{-z^2}\sqrt{-1+z^2}}{\sqrt{z^2-z^4}}\right)\frac{\pi}{4}$$

Involving $\cos^{-1}\left(\sqrt{\left(\sqrt{z^2-1}-z\right)\Big/\left(2\sqrt{z^2-1}\right)}\right)$

Involving $\cos^{-1}\left(\sqrt{\left(\sqrt{z^2-1}-z\right)\Big/\left(2\sqrt{z^2-1}\right)}\right)$ and $\tanh^{-1}(z)$

01.13.27.1818.01

$$\cos^{-1}\left(\sqrt{\frac{\sqrt{z^2-1}-z}{2\sqrt{z^2-1}}}\right) = -\frac{1}{2}\,i\tanh^{-1}(z)+\frac{\pi}{4}\;/;\;0<\arg(z)\leq\frac{\pi}{2}\bigvee-\pi<\arg(z)\leq-\frac{\pi}{2}\bigvee(z\in\mathbb{R}\land-1<z<1)$$

01.13.27.1819.01

$$\cos^{-1}\left(\sqrt{\frac{\sqrt{z^2-1}-z}{2\sqrt{z^2-1}}}\right) = \frac{\pi}{4}+\frac{1}{2}\,i\tanh^{-1}(z)\;/;\;\frac{\pi}{2}<\arg(z)<\pi\bigvee-\frac{\pi}{2}<\arg(z)<0$$

01.13.27.1820.01

$$\cos^{-1}\left(\sqrt{\frac{\sqrt{z^2-1}-z}{2\sqrt{z^2-1}}}\right) = \frac{3\pi}{4}-\frac{1}{2}\,i\tanh^{-1}(z)\;/;\;(z\in\mathbb{R}\land z>1)$$

01.13.27.1821.01

$$\cos^{-1}\left(\sqrt{\frac{\sqrt{z^2-1}-z}{2\sqrt{z^2-1}}}\right) = -\frac{1}{2}\,i\tanh^{-1}(z)-\frac{\pi}{4}\;/;\;(z\in\mathbb{R}\land z<-1)$$

01.13.27.1822.01

$$\cos^{-1}\left(\sqrt{\frac{\sqrt{z^2-1}-z}{2\sqrt{z^2-1}}}\right) = \frac{\pi}{4}\left(-\sqrt{\frac{1}{1-z}}\sqrt{1-z}+\sqrt{z+1}\sqrt{\frac{1}{z+1}}+1\right)-\frac{1}{2}\sqrt{z^2}\sqrt{-\frac{1}{z^2}}\tanh^{-1}(z)$$

Involving $\cos^{-1}\left(\sqrt{\left(\sqrt{z^2-1}-z\right)\Big/\left(2\sqrt{z^2-1}\right)}\right)$ and $\tanh^{-1}\left(\frac{1}{z}\right)$

01.13.27.1823.01

$$\cos^{-1}\left(\sqrt{\frac{\sqrt{z^2-1}-z}{2\sqrt{z^2-1}}}\right) = \frac{\pi}{2}-\frac{i}{2}\tanh^{-1}\left(\frac{1}{z}\right)\;/;\;0\leq\arg(z)\leq\frac{\pi}{2}$$

01.13.27.1824.01

$$\cos^{-1}\left(\sqrt{\frac{\sqrt{z^2-1}-z}{2\sqrt{z^2-1}}}\right) = \frac{\pi}{2}+\frac{i}{2}\tanh^{-1}\left(\frac{1}{z}\right)\;/;\;-\frac{\pi}{2}<\arg(z)<0$$

01.13.27.1825.01

$$\cos^{-1}\left(\sqrt{\frac{\sqrt{z^2-1}-z}{2\sqrt{z^2-1}}}\right) = \frac{i}{2}\tanh^{-1}\left(\frac{1}{z}\right)\;/;\;\frac{\pi}{2}<\arg(z)<\pi$$

01.13.27.1826.01

$$\cos^{-1}\left(\sqrt{\frac{\sqrt{z^2-1}-z}{2\sqrt{z^2-1}}}\right) = -\frac{i}{2}\tanh^{-1}\left(\frac{1}{z}\right) /; -\pi < \arg(z) \le -\frac{\pi}{2} \bigvee (z \in \mathbb{R} \land z < 0)$$

01.13.27.1827.01

$$\cos^{-1}\left(\sqrt{\frac{\sqrt{z^2-1}-z}{2\sqrt{z^2-1}}}\right) = \frac{\pi}{4}\left(1+\frac{\sqrt{z^2}}{z}\right) - \frac{1}{2}\sqrt{z^2}\sqrt{-\frac{1}{z^2}}\tanh^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\frac{a z^c}{\sqrt{a^2 z^{2c}-1}}\right)$

Involving $\cos^{-1}\left(\frac{a z^c}{\sqrt{a^2 z^{2c}-1}}\right)$ and $\tanh^{-1}(a z^c)$

01.13.27.0057.02

$$\cos^{-1}\left(\frac{a z^c}{\sqrt{a^2 z^{2c}-1}}\right) = \frac{\pi}{2}\left(1-\sqrt{\frac{1}{a z^c+1}}\sqrt{a z^c+1}+\sqrt{\frac{1}{1-a z^c}}\sqrt{1-a z^c}\right)+\sqrt{\frac{1}{1-a^2 z^{2c}}}\sqrt{a^2 z^{2c}-1}\tanh^{-1}(a z^c)$$

Involving $\coth^{-1}$

Involving $\cos^{-1}(z)$

Involving $\cos^{-1}(z)$ and $\coth^{-1}\left(\frac{z}{\sqrt{z^2-1}}\right)$

01.13.27.1828.01

$$\cos^{-1}(z) = -i\coth^{-1}\left(\frac{z}{\sqrt{z^2-1}}\right) /; 0 < \arg(z) < \frac{\pi}{2} \bigvee (z \in \mathbb{R} \land 0 < z < 1) \bigvee \arg(z) = -\frac{\pi}{2}$$

01.13.27.1829.01

$$\cos^{-1}(z) = i\coth^{-1}\left(\frac{z}{\sqrt{z^2-1}}\right) /; -\frac{\pi}{2} < \arg(z) < 0 \bigvee (z \in \mathbb{R} \land z > 1)$$

01.13.27.1830.01

$$\cos^{-1}(z) = \pi + i\coth^{-1}\left(\frac{z}{\sqrt{z^2-1}}\right) /; \frac{\pi}{2} < \arg(z) < \pi \bigvee (z \in \mathbb{R} \land z < -1)$$

01.13.27.1831.01

$$\cos^{-1}(z) = \pi - i\coth^{-1}\left(\frac{z}{\sqrt{z^2-1}}\right) /; -\pi < \arg(z) < -\frac{\pi}{2} \bigvee (z \in \mathbb{R} \land -1 < z < 0) \bigvee \arg(z) = \frac{\pi}{2}$$

01.13.27.1832.01

$$\cos^{-1}(z) = \frac{\pi}{2}\left(1 - z\sqrt{z^{-2}}\right) - \frac{\sqrt{z^2-1}}{\sqrt{1-z^2}}\coth^{-1}\left(\frac{z}{\sqrt{z^2-1}}\right)$$

Involving $\cos^{-1}(z)$ and $\coth^{-1}\left(\frac{\sqrt{z^2}}{\sqrt{z^2-1}}\right)$

01.13.27.1833.01

$$\cos^{-1}(z) = -i\coth^{-1}\left(\frac{\sqrt{z^2}}{\sqrt{z^2-1}}\right) /; 0 < \arg(z) < \frac{\pi}{2} \bigvee (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.1834.01

$$\cos^{-1}(z) = i\coth^{-1}\left(\frac{\sqrt{z^2}}{\sqrt{z^2-1}}\right) /; -\frac{\pi}{2} \leq \arg(z) < 0 \bigvee (z \in \mathbb{R} \land z > 1)$$

01.13.27.1835.01

$$\cos^{-1}(z) = \pi - i\coth^{-1}\left(\frac{\sqrt{z^2}}{\sqrt{z^2-1}}\right) /; \frac{\pi}{2} \leq \arg(z) < \pi \bigvee (z \in \mathbb{R} \land z < -1)$$

01.13.27.1836.01

$$\cos^{-1}(z) = i\coth^{-1}\left(\frac{\sqrt{z^2}}{\sqrt{z^2-1}}\right) + \pi /; -\pi < \arg(z) < -\frac{\pi}{2} \bigvee (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.1837.01

$$\cos^{-1}(z) = \frac{\pi}{2}\left(1 - z\sqrt{z^{-2}}\right) - \frac{\sqrt{z^2}\sqrt{z^2-1}}{z\sqrt{1-z^2}}\coth^{-1}\left(\frac{\sqrt{z^2}}{\sqrt{z^2-1}}\right)$$

Involving $\cos^{-1}(z)$ and $\coth^{-1}\left(\frac{\sqrt{-z^2}}{\sqrt{1-z^2}}\right)$

01.13.27.1838.01

$$\cos^{-1}(z) = -i\coth^{-1}\left(\frac{\sqrt{-z^2}}{\sqrt{1-z^2}}\right) /; 0 < \arg(z) < \frac{\pi}{2}$$

01.13.27.1839.01

$$\cos^{-1}(z) = i\coth^{-1}\left(\frac{\sqrt{-z^2}}{\sqrt{1-z^2}}\right) /; -\frac{\pi}{2} \leq \arg(z) \leq 0$$

01.13.27.1840.01

$$\cos^{-1}(z) = \pi - i\coth^{-1}\left(\frac{\sqrt{-z^2}}{\sqrt{1-z^2}}\right) /; \frac{\pi}{2} \leq \arg(z) \leq \pi$$

01.13.27.1841.01

$$\cos^{-1}(z) = i\coth^{-1}\left(\frac{\sqrt{-z^2}}{\sqrt{1-z^2}}\right) + \pi \;/;\; -\pi < \arg(z) < -\frac{\pi}{2}$$

01.13.27.1842.01

$$\cos^{-1}(z) = \frac{\pi}{2}\left(1 - \sqrt{z^{-2}}\; z\right) + \frac{\sqrt{-z^2}}{z}\coth^{-1}\left(\frac{\sqrt{-z^2}}{\sqrt{1-z^2}}\right)$$

Involving $\sin^{-1}(z)$ and $\coth^{-1}\left(\sqrt{\frac{z^2}{z^2-1}}\right)$

01.13.27.1843.01

$$\cos^{-1}(z) = -i\coth^{-1}\left(\sqrt{\frac{z^2}{z^2-1}}\right) \;/;\; 0 < \arg(z) < \frac{\pi}{2}$$

01.13.27.1844.01

$$\cos^{-1}(z) = i\coth^{-1}\left(\sqrt{\frac{z^2}{z^2-1}}\right) \;/;\; -\frac{\pi}{2} \le \arg(z) \le 0$$

01.13.27.1845.01

$$\cos^{-1}(z) = \pi - i\coth^{-1}\left(\sqrt{\frac{z^2}{z^2-1}}\right) \;/;\; \frac{\pi}{2} \le \arg(z) \le \pi$$

01.13.27.1846.01

$$\cos^{-1}(z) = i\coth^{-1}\left(\sqrt{\frac{z^2}{z^2-1}}\right) + \pi \;/;\; -\pi < \arg(z) < -\frac{\pi}{2}$$

01.13.27.1847.01

$$\cos^{-1}(z) = \frac{\pi}{2}\left(1 - z\sqrt{z^{-2}}\right) + \frac{\sqrt{-z}}{\sqrt{z}}\coth^{-1}\left(\sqrt{\frac{z^2}{z^2-1}}\right)$$

Involving $\cos^{-1}(z)$ and $\coth^{-1}\left(\frac{\sqrt{z^2-1}}{z}\right)$

01.13.27.1848.01

$$\cos^{-1}(z) = \frac{\pi}{2} - i\coth^{-1}\left(\frac{\sqrt{z^2-1}}{z}\right) \;/;\; 0 < \arg(z) \le \frac{\pi}{2} \bigvee -\pi < \arg(z) \le -\frac{\pi}{2} \bigvee (z \in \mathbb{R} \land -1 < z < 1)$$

01.13.27.1849.01

$$\cos^{-1}(z) = \frac{\pi}{2} + i \coth^{-1}\left(\frac{\sqrt{z^2-1}}{z}\right) /; \frac{\pi}{2} < \arg(z) < \pi \bigvee -\frac{\pi}{2} < \arg(z) < 0$$

01.13.27.1850.01

$$\cos^{-1}(z) = \frac{3\pi}{2} + i \coth^{-1}\left(\frac{\sqrt{z^2-1}}{z}\right) /; (z \in \mathbb{R} \land z < -1)$$

01.13.27.1851.01

$$\cos^{-1}(z) = i \coth^{-1}\left(\frac{\sqrt{z^2-1}}{z}\right) - \frac{\pi}{2} /; (z \in \mathbb{R} \land z > 1)$$

01.13.27.1852.01

$$\cos^{-1}(z) = \frac{\pi}{2} - \frac{\sqrt{z^2-1}}{\sqrt{1-z^2}} \coth^{-1}\left(\frac{\sqrt{z^2-1}}{z}\right) + \frac{\pi}{2\sqrt{1-z^2}}\left((1-z)\sqrt{\frac{1+z}{1-z}} - (1+z)\sqrt{\frac{1-z}{1+z}}\right)$$

Involving $\cos^{-1}(z)$ and $\coth^{-1}\left(\frac{\sqrt{z^2-1}}{\sqrt{z^2}}\right)$

01.13.27.1853.01

$$\cos^{-1}(z) = \frac{\pi}{2} - i \coth^{-1}\left(\frac{\sqrt{z^2-1}}{\sqrt{z^2}}\right) /; \operatorname{Im}(z) > 0 \lor (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.1854.01

$$\cos^{-1}(z) = \frac{\pi}{2} + i \coth^{-1}\left(\frac{\sqrt{z^2-1}}{\sqrt{z^2}}\right) /; \operatorname{Im}(z) < 0 \lor (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.1855.01

$$\cos^{-1}(z) = -\frac{\pi}{2} + i \coth^{-1}\left(\frac{\sqrt{z^2-1}}{\sqrt{z^2}}\right) /; (z \in \mathbb{R} \land z > 1)$$

01.13.27.1856.01

$$\cos^{-1}(z) = \frac{3\pi}{2} - i \coth^{-1}\left(\frac{\sqrt{z^2-1}}{\sqrt{z^2}}\right) /; (z \in \mathbb{R} \land z < -1)$$

01.13.27.1857.01

$$\cos^{-1}(z) = \frac{\pi}{2}\left(1 + \sqrt{\frac{1}{1-z}}\sqrt{1-z} - \sqrt{\frac{1}{z+1}}\sqrt{z+1}\right) + \frac{\sqrt{z^2-z^4}}{z\sqrt{z^2-1}} \coth^{-1}\left(\frac{\sqrt{z^2-1}}{\sqrt{z^2}}\right)$$

Involving $\cos^{-1}(z)$ and $\coth^{-1}\left(\frac{\sqrt{1-z^2}}{\sqrt{-z^2}}\right)$

01.13.27.1858.01

$$\cos^{-1}(z) = \frac{\pi}{2} - i \coth^{-1}\left(\frac{\sqrt{1-z^2}}{\sqrt{-z^2}}\right) /; \operatorname{Im}(z) > 0 \lor (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.1859.01

$$\cos^{-1}(z) = \frac{\pi}{2} + i \coth^{-1}\left(\frac{\sqrt{1-z^2}}{\sqrt{-z^2}}\right) /; \operatorname{Im}(z) < 0 \lor (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.1860.01

$$\cos^{-1}(z) = i \coth^{-1}\left(\frac{\sqrt{1-z^2}}{\sqrt{-z^2}}\right) - \frac{\pi}{2} /; (z \in \mathbb{R} \land z > 1)$$

01.13.27.1861.01

$$\cos^{-1}(z) = \frac{3\pi}{2} - i \coth^{-1}\left(\frac{\sqrt{1-z^2}}{\sqrt{-z^2}}\right) /; (z \in \mathbb{R} \land z < -1)$$

01.13.27.1862.01

$$\cos^{-1}(z) = \frac{\pi}{2}\left(1 + \sqrt{\frac{1}{1-z}}\sqrt{1-z} - \sqrt{\frac{1}{z+1}}\sqrt{z+1}\right) + \frac{\sqrt{-z^2}}{z}\coth^{-1}\left(\frac{\sqrt{1-z^2}}{\sqrt{-z^2}}\right)$$

Involving $\cos^{-1}(z)$ and $\coth^{-1}\left(\sqrt{\frac{z^2-1}{z^2}}\right)$

01.13.27.1863.01

$$\cos^{-1}(z) = \frac{\pi}{2} - i \coth^{-1}\left(\sqrt{\frac{z^2-1}{z^2}}\right) /; \operatorname{Im}(z) > 0 \lor (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.1864.01

$$\cos^{-1}(z) = \frac{\pi}{2} + i \coth^{-1}\left(\sqrt{\frac{z^2-1}{z^2}}\right) /; \operatorname{Im}(z) < 0 \lor (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.1865.01

$$\cos^{-1}(z) = i \coth^{-1}\left(\sqrt{\frac{z^2-1}{z^2}}\right) - \frac{\pi}{2} /; (z \in \mathbb{R} \land z > 1)$$

01.13.27.1866.01

$$\cos^{-1}(z) = \frac{3\pi}{2} - i \coth^{-1}\left(\sqrt{\frac{z^2-1}{z^2}}\right) /; (z \in \mathbb{R} \land z < -1)$$

01.13.27.1867.01

$$\cos^{-1}(z) = -\frac{z\sqrt{z^2-1}}{\sqrt{z^2-z^4}}\coth^{-1}\left(\sqrt{\frac{z^2-1}{z^2}}\right)+\frac{\pi}{2}\left(1+\sqrt{\frac{1}{1-z}}\sqrt{1-z}-\sqrt{\frac{1}{z+1}}\sqrt{z+1}\right)$$

Involving $\cos^{-1}(z)$ and $\coth^{-1}\left(\frac{2z\sqrt{z^2-1}}{1-2z^2}\right)$

01.13.27.1868.01

$$\cos^{-1}(z) = \frac{\pi}{4}+\frac{1}{2}\,i\coth^{-1}\left(\frac{2z\sqrt{z^2-1}}{1-2z^2}\right)\,/;\,0<\arg(z)<\frac{\pi}{2}\bigvee(z\in\mathbb{R}\land 0<z<1)\bigvee(i\,z\in\mathbb{R}\land i\,z>0)$$

01.13.27.1869.01

$$\cos^{-1}(z) = \frac{\pi}{4}-\frac{1}{2}\,i\coth^{-1}\left(\frac{2z\sqrt{z^2-1}}{1-2z^2}\right)\,/;\,-\frac{\pi}{2}<\arg(z)<0$$

01.13.27.1870.01

$$\cos^{-1}(z) = \frac{3\pi}{4}-\frac{1}{2}\,i\coth^{-1}\left(\frac{2z\sqrt{z^2-1}}{1-2z^2}\right)\,/;\,\frac{\pi}{2}<\arg(z)<\pi$$

01.13.27.1871.01

$$\cos^{-1}(z) = \frac{3\pi}{4}+\frac{1}{2}\,i\coth^{-1}\left(\frac{2z\sqrt{z^2-1}}{1-2z^2}\right)\,/;\,-\pi<\arg(z)<-\frac{\pi}{2}\bigvee(z\in\mathbb{R}\land -1<z<0)\bigvee(i\,z\in\mathbb{R}\land i\,z<0)$$

01.13.27.1872.01

$$\cos^{-1}(z) = \frac{5\pi}{4}-\frac{i}{2}\coth^{-1}\left(\frac{2z\sqrt{z^2-1}}{1-2z^2}\right)\,/;\,(z\in\mathbb{R}\land z<-1)$$

01.13.27.1873.01

$$\cos^{-1}(z) = -\frac{\pi}{4}-\frac{i}{2}\coth^{-1}\left(\frac{2z\sqrt{z^2-1}}{1-2z^2}\right)\,/;\,(z\in\mathbb{R}\land z>1)$$

01.13.27.1874.01

$$\cos^{-1}(z) = \frac{\pi}{4}\left(2+\sqrt{\frac{1}{1-z}}\sqrt{1-z}-\sqrt{\frac{1}{z+1}}\sqrt{z+1}+\sqrt{-i\,z}\sqrt{\frac{i}{z}}-\sqrt{-\frac{i}{z}}\sqrt{i\,z}-\frac{\sqrt{z^2}}{z}\right)+\frac{\sqrt{z^2-1}}{2\sqrt{1-z^2}}\coth^{-1}\left(\frac{2z\sqrt{z^2-1}}{1-2z^2}\right)$$

Involving $\cos^{-1}(z)$ and $\coth^{-1}\left(\frac{1-2z^2}{2z\sqrt{z^2-1}}\right)$

01.13.27.1875.01

$$\cos^{-1}(z) == \frac{\pi}{2} - \frac{\sqrt{-z^4}}{2 z^2} \coth^{-1}\left(\frac{1-2 z^2}{2 z \sqrt{z^2-1}}\right) /; \frac{\pi}{4} \leq |\arg(z)| \leq \frac{3 \pi}{4}$$

01.13.27.1876.01

$$\cos^{-1}(z) == \frac{1}{4} \pi \left(2 - \frac{\sqrt{z^2-1}\, z}{\sqrt{z^4-z^2}} - \sqrt{\frac{1}{z}} \sqrt{\frac{1}{\sqrt{2}\, z-1}} \sqrt{\sqrt{2}\, z-1} \sqrt{z} + \sqrt{-\frac{1}{z}} \sqrt{-z} \sqrt{-\sqrt{2}\, z-1} \sqrt{-\frac{1}{\sqrt{2}\, z+1}} - \frac{\sqrt{z^2}}{z}\right) - \frac{\sqrt{1-z^2}}{2 \sqrt{z^2-1}} \coth^{-1}\left(\frac{1-2 z^2}{2 z \sqrt{z^2-1}}\right)$$

Involving $\cos^{-1}(z)$ and $\coth^{-1}\left(\frac{\sqrt{z-1}}{\sqrt{z+1}}\right)$

01.13.27.1877.01

$$\cos^{-1}(z) == \pi - 2 i \coth^{-1}\left(\frac{\sqrt{z-1}}{\sqrt{z+1}}\right) /; \operatorname{Im}(z) > 0 \lor (z \in \mathbb{R} \land z < 1)$$

01.13.27.1878.01

$$\cos^{-1}(z) == \pi + 2 i \coth^{-1}\left(\frac{\sqrt{z-1}}{\sqrt{z+1}}\right) /; \operatorname{Im}(z) < 0$$

01.13.27.1879.01

$$\cos^{-1}(z) == 2 i \coth^{-1}\left(\frac{\sqrt{z-1}}{\sqrt{z+1}}\right) - \pi /; (z \in \mathbb{R} \land z > 1)$$

01.13.27.1880.01

$$\cos^{-1}(z) == \frac{2 \sqrt{1-z}}{\sqrt{z-1}} \coth^{-1}\left(\frac{\sqrt{z-1}}{\sqrt{z+1}}\right) - \frac{\pi \sqrt{z-1} \sqrt{z}}{\sqrt{1-z}} \sqrt{-\frac{1}{z}}$$

Involving $\cos^{-1}(z)$ and $\coth^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{-1-z}}\right)$

01.13.27.1881.01

$$\cos^{-1}(z) == \pi - 2 i \coth^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{-1-z}}\right) /; \operatorname{Im}(z) > 0 \lor (z \in \mathbb{R} \land z < -1)$$

01.13.27.1882.01

$$\cos^{-1}(z) == \pi + 2 i \coth^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{-1-z}}\right) /; \operatorname{Im}(z) < 0 \lor (z \in \mathbb{R} \land -1 < z < 1)$$

01.13.27.1883.01

$$\cos^{-1}(z) == 2 i \coth^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{-1-z}}\right) - \pi /; (z \in \mathbb{R} \land z > 1)$$

01.13.27.1884.01

$$\cos^{-1}(z) = -\frac{2\sqrt{1+z}}{\sqrt{-1-z}}\coth^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{-1-z}}\right) - \pi\frac{\sqrt{-1+z}\sqrt{z}}{\sqrt{1-z}}\sqrt{-\frac{1}{z}}$$

Involving $\cos^{-1}(z)$ and $\coth^{-1}\left(\sqrt{\frac{z-1}{z+1}}\right)$

01.13.27.1885.01

$$\cos^{-1}(z) = \pi - 2\,i\coth^{-1}\left(\sqrt{\frac{z-1}{z+1}}\right) /;\ 0 < \arg(z) \leq \pi \vee (z \in \mathbb{R} \wedge 0 < z < 1)$$

01.13.27.1886.01

$$\cos^{-1}(z) = \pi + 2\,i\coth^{-1}\left(\sqrt{\frac{z-1}{z+1}}\right) /;\ \operatorname{Im}(z) < 0$$

01.13.27.1887.01

$$\cos^{-1}(z) = 2\,i\coth^{-1}\left(\sqrt{\frac{z-1}{z+1}}\right) - \pi /;\ (z \in \mathbb{R} \wedge z > 1)$$

01.13.27.1888.01

$$\cos^{-1}(z) = -2\sqrt{\frac{z}{1-z}}\sqrt{1-z}\sqrt{-\frac{1}{z}}\coth^{-1}\left(\sqrt{\frac{z-1}{z+1}}\right) - \frac{\pi\sqrt{z-1}\sqrt{z}}{\sqrt{1-z}}\sqrt{-\frac{1}{z}}$$

Involving $\cos^{-1}(z)$ and $\coth^{-1}\left(\frac{\sqrt{z+1}}{\sqrt{z-1}}\right)$

01.13.27.1889.01

$$\cos^{-1}(z) = -2\,i\coth^{-1}\left(\frac{\sqrt{z+1}}{\sqrt{z-1}}\right) /;\ \operatorname{Im}(z) > 0 \vee (z \in \mathbb{R} \wedge -1 < z < 1)$$

01.13.27.1890.01

$$\cos^{-1}(z) = 2\,i\coth^{-1}\left(\frac{\sqrt{z+1}}{\sqrt{z-1}}\right) /;\ \operatorname{Im}(z) < 0 \vee (z \in \mathbb{R} \wedge z > 1)$$

01.13.27.1891.01

$$\cos^{-1}(z) = 2\pi - 2\,i\coth^{-1}\left(\frac{\sqrt{z+1}}{\sqrt{z-1}}\right) /;\ (z \in \mathbb{R} \wedge z < -1)$$

01.13.27.1892.01

$$\cos^{-1}(z) = -\frac{2\sqrt{z-1}}{\sqrt{1-z}}\coth^{-1}\left(\frac{\sqrt{z+1}}{\sqrt{z-1}}\right) + \frac{\pi\sqrt{-z-1}\sqrt{-z}}{\sqrt{z+1}}\sqrt{\frac{1}{z}} + \pi$$

Involving $\cos^{-1}(z)$ and $\coth^{-1}\left(\frac{\sqrt{-1-z}}{\sqrt{1-z}}\right)$

01.13.27.1893.01

$$\cos^{-1}(z) = -2\, i \coth^{-1}\left(\frac{\sqrt{-1-z}}{\sqrt{1-z}}\right) /;\ \mathrm{Im}(z) > 0$$

01.13.27.1894.01

$$\cos^{-1}(z) = 2\, i \coth^{-1}\left(\frac{\sqrt{-1-z}}{\sqrt{1-z}}\right) /;\ -\pi < \arg(z) \leq 0 \bigvee (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.1895.01

$$\cos^{-1}(z) = 2\pi - 2\, i \coth^{-1}\left(\frac{\sqrt{-1-z}}{\sqrt{1-z}}\right) /;\ (z \in \mathbb{R} \land z < -1)$$

01.13.27.1896.01

$$\cos^{-1}(z) = \pi\left(\frac{\sqrt{-z-1}\,\sqrt{-z}}{\sqrt{z+1}}\sqrt{\frac{1}{z}} + 1\right) - \frac{2\sqrt{z+1}}{\sqrt{-z-1}} \coth^{-1}\left(\frac{\sqrt{-z-1}}{\sqrt{1-z}}\right)$$

Involving $\cos^{-1}(z)$ and $\coth^{-1}\left(\sqrt{\frac{z+1}{z-1}}\right)$

01.13.27.1897.01

$$\cos^{-1}(z) = 2\, i \coth^{-1}\left(\sqrt{\frac{z+1}{z-1}}\right) /;\ -\pi < \arg(z) \leq 0 \bigvee (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.1898.01

$$\cos^{-1}(z) = -2\, i \coth^{-1}\left(\sqrt{\frac{z+1}{z-1}}\right) /;\ \mathrm{Im}(z) > 0$$

01.13.27.1899.01

$$\cos^{-1}(z) = 2\pi - 2\, i \coth^{-1}\left(\sqrt{\frac{z+1}{z-1}}\right) /;\ (z \in \mathbb{R} \land z < -1)$$

01.13.27.1900.01

$$\cos^{-1}(z) = 2\sqrt{\frac{1}{z}}\sqrt{-\frac{z}{z+1}}\sqrt{z+1} \coth^{-1}\left(\sqrt{\frac{z+1}{z-1}}\right) + \pi + \frac{\pi\sqrt{-z-1}\,\sqrt{-z}}{\sqrt{z+1}}\sqrt{\frac{1}{z}}$$

Involving $\cos^{-1}\left(\frac{2z}{z^2+1}\right)$

Involving $\cos^{-1}\left(\frac{2z}{z^2+1}\right)$ and $\coth^{-1}(i z)$

01.13.27.1901.01

$$\cos^{-1}\left(\frac{2z}{z^2+1}\right) = 2\, i \coth^{-1}(i z) - \frac{\pi}{2} /;\ |z| < 1 \bigwedge -\frac{\pi}{2} \leq \arg(z) < \frac{\pi}{2}$$

01.13.27.1902.01

$$\cos^{-1}\left(\frac{2z}{z^2+1}\right) = \frac{3\pi}{2} + 2\, i \coth^{-1}(i z) /;\ |z| < 1 \bigwedge \left(\frac{\pi}{2} \leq \arg(z) \leq \pi \bigvee -\pi < \arg(z) < -\frac{\pi}{2}\right)$$

01.13.27.1903.01

$$\cos^{-1}\left(\frac{2z}{z^2+1}\right) = \frac{\pi}{2} - \pi z \sqrt{\frac{1}{z^2}} + 2i\coth^{-1}(iz) \;/;\; |z| < 1$$

01.13.27.1904.01

$$\cos^{-1}\left(\frac{2z}{z^2+1}\right) = \frac{\pi}{2} - 2i\coth^{-1}(iz) \;/;\; |z| > 1$$

01.13.27.1905.01

$$\cos^{-1}\left(\frac{2z}{z^2+1}\right) = \frac{\pi}{2} - \frac{\pi z}{2}\sqrt{\frac{1}{z^2}}\left(\frac{1-z}{1+z}\sqrt{\left(\frac{z+1}{z-1}\right)^2} + 1\right) + \frac{2i(1-z)}{1+z}\sqrt{\left(\frac{z+1}{z-1}\right)^2}\coth^{-1}(iz)$$

Involving $\cos^{-1}\left(\frac{2z}{z^2+1}\right)$ and $\coth^{-1}\left(\frac{i}{z}\right)$

01.13.27.1906.01

$$\cos^{-1}\left(\frac{2z}{z^2+1}\right) = \frac{\pi}{2} - 2i\coth^{-1}\left(\frac{i}{z}\right) \;/;\; |z| < 1$$

01.13.27.1907.01

$$\cos^{-1}\left(\frac{2z}{z^2+1}\right) = 2i\coth^{-1}\left(\frac{i}{z}\right) - \frac{\pi}{2} \;/;\; |z| > 1 \bigwedge -\frac{\pi}{2} < \arg(z) \le \frac{\pi}{2}$$

01.13.27.1908.01

$$\cos^{-1}\left(\frac{2z}{z^2+1}\right) = \frac{3\pi}{2} + 2i\coth^{-1}\left(\frac{i}{z}\right) \;/;\; |z| > 1 \bigwedge \left(\frac{\pi}{2} < \arg(z) \le \pi \bigvee -\pi < \arg(z) \le -\frac{\pi}{2}\right)$$

01.13.27.1909.01

$$\cos^{-1}\left(\frac{2z}{z^2+1}\right) = \frac{\pi}{2} - \frac{\pi\sqrt{z^2}}{z} + 2i\coth^{-1}\left(\frac{i}{z}\right) \;/;\; |z| > 1$$

01.13.27.1910.01

$$\cos^{-1}\left(\frac{2z}{z^2+1}\right) = \frac{\pi}{2} - \frac{\pi\sqrt{z^2}}{2z}\left(1 - \frac{1-z}{1+z}\sqrt{\left(\frac{z+1}{z-1}\right)^2}\right) - \frac{2i(1-z)}{1+z}\sqrt{\left(\frac{z+1}{z-1}\right)^2}\coth^{-1}\left(\frac{i}{z}\right)$$

Involving $\cos^{-1}\left(\frac{2z}{z^2+1}\right)$ and $\coth^{-1}(i z^r)$

01.13.27.1911.01

$$\cos^{-1}\left(\frac{2z}{z^2+1}\right) = \frac{\pi}{2} - 2i\coth^{-1}\left(i z^{\frac{z-1}{z+1}\sqrt{\left(\frac{z+1}{z-1}\right)^2}}\right)$$

Involving $\cos^{-1}\left(\frac{1-z}{1+z}\right)$

Involving $\cos^{-1}\left(\frac{1-z}{1+z}\right)$ and $\coth^{-1}\left(\sqrt{-z}\right)$

01.13.27.1912.01

$$\cos^{-1}\left(\frac{1-z}{1+z}\right) = \pi + 2\,i\coth^{-1}\left(\sqrt{-z}\right) /;\ \operatorname{Im}(z) > 0$$

01.13.27.1913.01

$$\cos^{-1}\left(\frac{1-z}{1+z}\right) = \pi - 2\,i\coth^{-1}\left(\sqrt{-z}\right) /;\ -\pi < \arg(z) \le 0 \bigvee (z \in \mathbb{R} \land z < -1)$$

01.13.27.1914.01

$$\cos^{-1}\left(\frac{1-z}{1+z}\right) = 2\,i\coth^{-1}\left(\sqrt{-z}\right) - \pi /;\ (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.1915.01

$$\cos^{-1}\left(\frac{1-z}{1+z}\right) = \pi\left(1 - \sqrt{\frac{1}{1+z}}\sqrt{1+z} + \sqrt{\frac{1}{z}}\sqrt{z}\right) - \frac{2\sqrt{-(1+z)\,z}}{\sqrt{z}}\sqrt{\frac{1}{1+z}}\coth^{-1}\left(\sqrt{-z}\right)$$

Involving $\cos^{-1}\left(\frac{1-z}{1+z}\right)$ and $\coth^{-1}\left(\frac{1}{\sqrt{-z}}\right)$

01.13.27.1916.01

$$\cos^{-1}\left(\frac{1-z}{1+z}\right) = 2\,i\coth^{-1}\left(\frac{1}{\sqrt{-z}}\right) /;\ \operatorname{Im}(z) > 0 \lor (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.1917.01

$$\cos^{-1}\left(\frac{1-z}{1+z}\right) = -2\,i\coth^{-1}\left(\frac{1}{\sqrt{-z}}\right) /;\ -\pi < \arg(z) \le 0$$

01.13.27.1918.01

$$\cos^{-1}\left(\frac{1-z}{1+z}\right) = 2\pi - 2\,i\coth^{-1}\left(\frac{1}{\sqrt{-z}}\right) /;\ (z \in \mathbb{R} \land z < -1)$$

01.13.27.1919.01

$$\cos^{-1}\left(\frac{1-z}{1+z}\right) = \pi\left(1 - \sqrt{1+z}\sqrt{\frac{1}{1+z}}\right) - \frac{2\sqrt{-(1+z)\,z}}{\sqrt{z}}\sqrt{\frac{1}{1+z}}\coth^{-1}\left(\frac{1}{\sqrt{-z}}\right)$$

Involving $\cos^{-1}\left(\frac{1-z}{1+z}\right)$ and $\coth^{-1}\left(\sqrt{-\frac{1}{z}}\right)$

01.13.27.1920.01

$$\cos^{-1}\left(\frac{1-z}{1+z}\right) = 2\,i\coth^{-1}\left(\sqrt{-\frac{1}{z}}\right) /;\ 0 \le \arg(z) < \pi \bigvee (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.1921.01

$$\cos^{-1}\left(\frac{1-z}{1+z}\right) = -2\,i\coth^{-1}\left(\sqrt{-\frac{1}{z}}\right) /;\ -\pi < \arg(z) < 0$$

01.13.27.1922.01

$$\cos^{-1}\left(\frac{1-z}{1+z}\right) = 2\pi - 2\,i\coth^{-1}\left(\sqrt{-\frac{1}{z}}\right) /;\ (z \in \mathbb{R} \land z < -1)$$

01.13.27.1923.01

$$\cos^{-1}\left(\frac{1-z}{1+z}\right) = \left(1-\sqrt{z+1}\sqrt{\frac{1}{z+1}}\right)\pi + 2\sqrt{z}\sqrt{\frac{1}{z+1}}\sqrt{-\frac{z+1}{z}}\coth^{-1}\left(\sqrt{-\frac{1}{z}}\right)$$

Involving $\cos^{-1}\left(\frac{z-1}{z+1}\right)$

Involving $\cos^{-1}\left(\frac{z-1}{z+1}\right)$ and $\coth^{-1}\left(\sqrt{-z}\right)$

01.13.27.1924.01

$$\cos^{-1}\left(\frac{z-1}{z+1}\right) = -2\,i\coth^{-1}\left(\sqrt{-z}\right)\;/;\;\operatorname{Im}(z)>0$$

01.13.27.1925.01

$$\cos^{-1}\left(\frac{z-1}{z+1}\right) = 2\,i\coth^{-1}\left(\sqrt{-z}\right)\;/;\;-\pi<\arg(z)\leq 0\,\bigvee\,(z\in\mathbb{R}\land z<-1)$$

01.13.27.1926.01

$$\cos^{-1}\left(\frac{z-1}{z+1}\right) = 2\pi-2\,i\coth^{-1}\left(\sqrt{-z}\right)\;/;\;(z\in\mathbb{R}\land -1<z<0)$$

01.13.27.1927.01

$$\cos^{-1}\left(\frac{z-1}{z+1}\right) = \pi\left(\sqrt{\frac{1}{1+z}}\sqrt{1+z}-\sqrt{\frac{1}{z}}\sqrt{z}\right)+\frac{2\sqrt{-(1+z)\,z}}{\sqrt{z}}\sqrt{\frac{1}{1+z}}\coth^{-1}\left(\sqrt{-z}\right)$$

Involving $\cos^{-1}\left(\frac{z-1}{z+1}\right)$ and $\coth^{-1}\left(\frac{1}{\sqrt{-z}}\right)$

01.13.27.1928.01

$$\cos^{-1}\left(\frac{z-1}{z+1}\right) = \pi-2\,i\coth^{-1}\left(\frac{1}{\sqrt{-z}}\right)\;/;\;\operatorname{Im}(z)>0\lor(z\in\mathbb{R}\land -1<z<0)$$

01.13.27.1929.01

$$\cos^{-1}\left(\frac{z-1}{z+1}\right) = \pi+2\,i\coth^{-1}\left(\frac{1}{\sqrt{-z}}\right)\;/;\;-\pi<\arg(z)\leq 0$$

01.13.27.1930.01

$$\cos^{-1}\left(\frac{z-1}{z+1}\right) = 2\,i\coth^{-1}\left(\frac{1}{\sqrt{-z}}\right)-\pi\;/;\;(z\in\mathbb{R}\land z<-1)$$

01.13.27.1931.01

$$\cos^{-1}\left(\frac{z-1}{z+1}\right) = \pi\sqrt{1+z}\sqrt{\frac{1}{1+z}}+\frac{2\sqrt{-(1+z)\,z}}{\sqrt{z}}\sqrt{\frac{1}{1+z}}\coth^{-1}\left(\frac{1}{\sqrt{-z}}\right)$$

Involving $\cos^{-1}\left(\frac{z-1}{z+1}\right)$ and $\coth^{-1}\left(\sqrt{-\frac{1}{z}}\right)$

01.13.27.1932.01

$$\cos^{-1}\left(\frac{z-1}{z+1}\right) = \pi - 2\,i\coth^{-1}\left(\sqrt{-\frac{1}{z}}\right) /;\ 0 \leq \arg(z) < \pi \lor (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.1933.01

$$\cos^{-1}\left(\frac{z-1}{z+1}\right) = \pi + 2\,i\coth^{-1}\left(\sqrt{-\frac{1}{z}}\right) /;\ -\pi < \arg(z) < 0$$

01.13.27.1934.01

$$\cos^{-1}\left(\frac{z-1}{z+1}\right) = 2\,i\coth^{-1}\left(\sqrt{-\frac{1}{z}}\right) - \pi /;\ (z \in \mathbb{R} \land z < -1)$$

01.13.27.1935.01

$$\cos^{-1}\left(\frac{z-1}{z+1}\right) = \sqrt{z+1}\,\sqrt{\frac{1}{z+1}}\,\pi - 2\sqrt{z}\,\sqrt{\frac{1}{z+1}}\,\sqrt{-\frac{z+1}{z}}\,\coth^{-1}\left(\sqrt{-\frac{1}{z}}\right)$$

Involving $\cos^{-1}\left(\frac{1+z}{1-z}\right)$

Involving $\cos^{-1}\left(\frac{1+z}{1-z}\right)$ and $\coth^{-1}\left(\sqrt{z}\right)$

01.13.27.1936.01

$$\cos^{-1}\left(\frac{z+1}{1-z}\right) = \pi - 2\,i\coth^{-1}\left(\sqrt{z}\right) /;\ 0 < \arg(z) \leq \pi \lor (z \in \mathbb{R} \land z > 1)$$

01.13.27.1937.01

$$\cos^{-1}\left(\frac{z+1}{1-z}\right) = \pi + 2\,i\coth^{-1}\left(\sqrt{z}\right) /;\ \operatorname{Im}(z) < 0$$

01.13.27.1938.01

$$\cos^{-1}\left(\frac{z+1}{1-z}\right) = 2\,i\coth^{-1}\left(\sqrt{z}\right) - \pi /;\ (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.1939.01

$$\cos^{-1}\left(\frac{1+z}{1-z}\right) = \pi\left(-\sqrt{\frac{1}{1-z}}\,\sqrt{1-z} + \sqrt{-\frac{1}{z}}\,\sqrt{-z} + 1\right) - \frac{2\sqrt{(1-z)\,z}}{\sqrt{-z}}\,\sqrt{\frac{1}{1-z}}\,\coth^{-1}\left(\sqrt{z}\right)$$

Involving $\cos^{-1}\left(\frac{1+z}{1-z}\right)$ and $\coth^{-1}\left(\frac{1}{\sqrt{z}}\right)$

01.13.27.1940.01

$$\cos^{-1}\left(\frac{1+z}{1-z}\right) = -2\,i\coth^{-1}\left(\frac{1}{\sqrt{z}}\right) /;\ 0 < \arg(z) \leq \pi$$

01.13.27.1941.01

$$\cos^{-1}\left(\frac{1+z}{1-z}\right) = 2\,i\coth^{-1}\left(\frac{1}{\sqrt{z}}\right) /;\ -\pi < \arg(z) < 0 \lor (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.1942.01

$$\cos^{-1}\left(\frac{1+z}{1-z}\right) = 2\pi - 2i\coth^{-1}\left(\frac{1}{\sqrt{z}}\right) /; (z \in \mathbb{R} \land z > 1)$$

01.13.27.1943.01

$$\cos^{-1}\left(\frac{1+z}{1-z}\right) = \pi\left(-\sqrt{1-z}\sqrt{\frac{1}{1-z}} + 1\right) - \frac{2\sqrt{(1-z)z}}{\sqrt{-z}}\sqrt{\frac{1}{1-z}}\coth^{-1}\left(\frac{1}{\sqrt{z}}\right)$$

Involving $\cos^{-1}\left(\frac{1+z}{1-z}\right)$ and $\coth^{-1}\left(\sqrt{\frac{1}{z}}\right)$

01.13.27.1944.01

$$\cos^{-1}\left(\frac{1+z}{1-z}\right) = -2i\coth^{-1}\left(\sqrt{\frac{1}{z}}\right) /; \operatorname{Im}(z) > 0$$

01.13.27.1945.01

$$\cos^{-1}\left(\frac{1+z}{1-z}\right) = 2i\coth^{-1}\left(\sqrt{\frac{1}{z}}\right) /; \operatorname{Im}(z) < 0 \lor (z \in \mathbb{R} \land z < 1)$$

01.13.27.1946.01

$$\cos^{-1}\left(\frac{1+z}{1-z}\right) = 2\pi - 2i\coth^{-1}\left(\sqrt{\frac{1}{z}}\right) /; (z \in \mathbb{R} \land z > 1)$$

01.13.27.1947.01

$$\cos^{-1}\left(\frac{1+z}{1-z}\right) = \pi\left(1 - \sqrt{1-z}\sqrt{\frac{1}{1-z}}\right) + 2\sqrt{\frac{1-z}{z}}\sqrt{\frac{1}{1-z}}\sqrt{-z}\coth^{-1}\left(\sqrt{\frac{1}{z}}\right)$$

Involving $\cos^{-1}\left(\frac{z+1}{z-1}\right)$

Involving $\cos^{-1}\left(\frac{z+1}{z-1}\right)$ and $\coth^{-1}\left(\sqrt{z}\right)$

01.13.27.1948.01

$$\cos^{-1}\left(\frac{z+1}{z-1}\right) = 2i\coth^{-1}\left(\sqrt{z}\right) /; 0 < \arg(z) \leq \pi \bigvee (z \in \mathbb{R} \land z > 1)$$

01.13.27.1949.01

$$\cos^{-1}\left(\frac{z+1}{z-1}\right) = -2i\coth^{-1}\left(\sqrt{z}\right) /; \operatorname{Im}(z) < 0$$

01.13.27.1950.01

$$\cos^{-1}\left(\frac{z+1}{z-1}\right) = 2\pi - 2i\coth^{-1}\left(\sqrt{z}\right) /; (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.1951.01

$$\cos^{-1}\left(\frac{z+1}{z-1}\right) = \pi\left(-\sqrt{-\frac{1}{z}}\sqrt{-z} + \sqrt{\frac{1}{1-z}}\sqrt{1-z}\right) + \frac{2\sqrt{(1-z)z}}{\sqrt{-z}}\sqrt{\frac{1}{1-z}}\coth^{-1}\left(\sqrt{z}\right)$$

Involving $\cos^{-1}\left(\frac{z+1}{z-1}\right)$ and $\coth^{-1}\left(\frac{1}{\sqrt{z}}\right)$

01.13.27.1952.01

$$\cos^{-1}\left(\frac{z+1}{z-1}\right) = \pi + 2\,i\coth^{-1}\left(\frac{1}{\sqrt{z}}\right) /;\ 0 < \arg(z) \leq \pi$$

01.13.27.1953.01

$$\cos^{-1}\left(\frac{z+1}{z-1}\right) = \pi - 2\,i\coth^{-1}\left(\frac{1}{\sqrt{z}}\right) /;\ -\pi < \arg(z) < 0 \bigvee (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.1954.01

$$\cos^{-1}\left(\frac{z+1}{z-1}\right) = 2\,i\coth^{-1}\left(\frac{1}{\sqrt{z}}\right) - \pi /;\ (z \in \mathbb{R} \land z > 1)$$

01.13.27.1955.01

$$\cos^{-1}\left(\frac{z+1}{z-1}\right) = \sqrt{-z+1}\,\sqrt{\frac{1}{-z+1}}\,\pi + \frac{2\sqrt{(1-z)\,z}}{\sqrt{-z}}\,\sqrt{\frac{1}{1-z}}\,\coth^{-1}\left(\frac{1}{\sqrt{z}}\right)$$

Involving $\cos^{-1}\left(\frac{z+1}{z-1}\right)$ and $\coth^{-1}\left(\sqrt{\frac{1}{z}}\right)$

01.13.27.1956.01

$$\cos^{-1}\left(\frac{z+1}{z-1}\right) = \pi + 2\,i\coth^{-1}\left(\sqrt{\frac{1}{z}}\right) /;\ \operatorname{Im}(z) > 0$$

01.13.27.1957.01

$$\cos^{-1}\left(\frac{z+1}{z-1}\right) = \pi - 2\,i\coth^{-1}\left(\sqrt{\frac{1}{z}}\right) /;\ \operatorname{Im}(z) < 0 \lor (z \in \mathbb{R} \land z < 1)$$

01.13.27.1958.01

$$\cos^{-1}\left(\frac{z+1}{z-1}\right) = 2\,i\coth^{-1}\left(\sqrt{\frac{1}{z}}\right) - \pi /;\ (z \in \mathbb{R} \land z > 1)$$

01.13.27.1959.01

$$\cos^{-1}\left(\frac{z+1}{z-1}\right) = \sqrt{-z+1}\,\sqrt{\frac{1}{-z+1}}\,\pi - 2\sqrt{\frac{1-z}{z}}\,\sqrt{\frac{1}{1-z}}\,\sqrt{-z}\,\coth^{-1}\left(\sqrt{\frac{1}{z}}\right)$$

Involving $\cos^{-1}\left(\frac{1+z^2}{1-z^2}\right)$

Involving $\cos^{-1}\left(\frac{1+z^2}{1-z^2}\right)$ and $\coth^{-1}(z)$

01.13.27.1960.01

$$\cos^{-1}\left(\frac{1+z^2}{1-z^2}\right) = \pi - 2\,i\coth^{-1}(z) /;\ \operatorname{Im}(z) > 0 \lor (z \in \mathbb{R} \land z > 1)$$

01.13.27.1961.01

$$\cos^{-1}\left(\frac{1+z^2}{1-z^2}\right) = \pi + 2\,i\,\coth^{-1}(z)\;/;\;\operatorname{Im}(z) < 0 \lor (z \in \mathbb{R} \land z < -1)$$

01.13.27.1962.01

$$\cos^{-1}\left(\frac{1+z^2}{1-z^2}\right) = 2\,i\,\coth^{-1}(z) - \pi\;/;\;(z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.1963.01

$$\cos^{-1}\left(\frac{1+z^2}{1-z^2}\right) = -2\,i\,\coth^{-1}(z) - \pi\;/;\;(z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.1964.01

$$\cos^{-1}\left(\frac{1+z^2}{1-z^2}\right) = \pi\left(-1 + \sqrt{\frac{1}{1-z}}\,\sqrt{1-z}\,\sqrt{-\frac{1}{z}}\,\sqrt{-z} + \sqrt{\frac{1}{z}}\,\sqrt{z}\,\sqrt{\frac{1}{z+1}}\,\sqrt{z+1}\right) + \frac{2\sqrt{-z^2}}{z}\sqrt{\frac{1-z}{z+1}}\,\sqrt{\frac{z+1}{1-z}}\,\coth^{-1}(z)$$

Involving $\cos^{-1}\left(\frac{1+z^2}{1-z^2}\right)$ and $\coth^{-1}\left(\frac{1}{z}\right)$

01.13.27.1965.01

$$\cos^{-1}\left(\frac{1+z^2}{1-z^2}\right) = -2\,i\,\coth^{-1}\left(\frac{1}{z}\right)\;/;\;\operatorname{Im}(z) > 0 \lor (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.1966.01

$$\cos^{-1}\left(\frac{1+z^2}{1-z^2}\right) = 2\,i\,\coth^{-1}\left(\frac{1}{z}\right)\;/;\;\operatorname{Im}(z) < 0 \lor (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.1967.01

$$\cos^{-1}\left(\frac{1+z^2}{1-z^2}\right) = 2\,\pi - 2\,i\,\coth^{-1}\left(\frac{1}{z}\right)\;/;\;(z \in \mathbb{R} \land z > 1)$$

01.13.27.1968.01

$$\cos^{-1}\left(\frac{1+z^2}{1-z^2}\right) = 2\,\pi + 2\,i\,\coth^{-1}\left(\frac{1}{z}\right)\;/;\;(z \in \mathbb{R} \land z < -1)$$

01.13.27.1969.01

$$\cos^{-1}\left(\frac{1+z^2}{1-z^2}\right) = \frac{2\sqrt{-z^2}}{z}\coth^{-1}\left(\frac{1}{z}\right)\;/;\;z \notin (-\infty, -1) \bigwedge z \notin (1, \infty)$$

01.13.27.1970.01

$$\cos^{-1}\left(\frac{1+z^2}{1-z^2}\right) = \pi\left(2 - \sqrt{\frac{1}{1-z}}\,\sqrt{1-z} - \sqrt{\frac{1}{z+1}}\,\sqrt{z+1}\right) + \frac{2\sqrt{-z^2}}{z}\sqrt{\frac{1-z}{z+1}}\,\sqrt{\frac{z+1}{1-z}}\,\coth^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\frac{z^2+1}{z^2-1}\right)$

Involving $\cos^{-1}\left(\frac{z^2+1}{z^2-1}\right)$ and $\coth^{-1}(z)$

01.13.27.1971.01

$$\cos^{-1}\left(\frac{z^2+1}{z^2-1}\right) = 2\,i\coth^{-1}(z)\;/;\;\mathrm{Im}(z) > 0 \lor (z \in \mathbb{R} \land z > 1)$$

01.13.27.1972.01

$$\cos^{-1}\left(\frac{z^2+1}{z^2-1}\right) = -2\,i\coth^{-1}(z)\;/;\;\mathrm{Im}(z) < 0 \lor (z \in \mathbb{R} \land z < -1)$$

01.13.27.1973.01

$$\cos^{-1}\left(\frac{z^2+1}{z^2-1}\right) = 2\pi - 2\,i\coth^{-1}(z)\;/;\;(z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.1974.01

$$\cos^{-1}\left(\frac{z^2+1}{z^2-1}\right) = 2\pi + 2\,i\coth^{-1}(z)\;/;\;(z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.1975.01

$$\cos^{-1}\left(\frac{z^2+1}{z^2-1}\right) = \pi\left(1 - \sqrt{\frac{1}{z^2}}\sqrt{z^2}\sqrt{z^2-1}\sqrt{\frac{1}{z^2-1}}\right) - \frac{2\sqrt{-z^2}}{z}\sqrt{\frac{1-z}{z+1}}\sqrt{\frac{z+1}{1-z}}\coth^{-1}(z)$$

Involving $\cos^{-1}\left(\frac{z^2+1}{z^2-1}\right)$ and $\coth^{-1}\left(\frac{1}{z}\right)$

01.13.27.1976.01

$$\cos^{-1}\left(\frac{z^2+1}{z^2-1}\right) = \pi + 2\,i\coth^{-1}\left(\frac{1}{z}\right)\;/;\;\mathrm{Im}(z) > 0 \lor (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.1977.01

$$\cos^{-1}\left(\frac{z^2+1}{z^2-1}\right) = \pi - 2\,i\coth^{-1}\left(\frac{1}{z}\right)\;/;\;\mathrm{Im}(z) < 0 \lor (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.1978.01

$$\cos^{-1}\left(\frac{z^2+1}{z^2-1}\right) = 2\,i\coth^{-1}\left(\frac{1}{z}\right) - \pi\;/;\;(z \in \mathbb{R} \land z > 1)$$

01.13.27.1979.01

$$\cos^{-1}\left(\frac{z^2+1}{z^2-1}\right) = -2\,i\coth^{-1}\left(\frac{1}{z}\right) - \pi\;/;\;(z \in \mathbb{R} \land z < -1)$$

01.13.27.1980.01

$$\cos^{-1}\left(\frac{z^2+1}{z^2-1}\right) = \pi - \frac{2\sqrt{-z^2}}{z}\coth^{-1}\left(\frac{1}{z}\right)\;/;\;z \notin (-\infty, -1) \bigwedge z \notin (1, \infty)$$

01.13.27.1981.01

$$\cos^{-1}\left(\frac{z^2+1}{z^2-1}\right) = \pi\sqrt{\frac{1}{1-z^2}}\sqrt{1-z^2} - \frac{2\sqrt{-z^2}}{z}\sqrt{\frac{1-z}{z+1}}\sqrt{\frac{z+1}{1-z}}\coth^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\frac{1}{\sqrt{1-z}}\right)$

Involving $\cos^{-1}\left(\frac{1}{\sqrt{1-z}}\right)$ and $\coth^{-1}\left(\sqrt{z}\right)$

01.13.27.1982.01

$$\cos^{-1}\left(\frac{1}{\sqrt{1-z}}\right) = \frac{\pi}{2} + i\coth^{-1}\left(\sqrt{z}\right) /; -\pi < \arg(z) < 0 \lor (z \in \mathbb{R} \land z > 1)$$

01.13.27.1983.01

$$\cos^{-1}\left(\frac{1}{\sqrt{1-z}}\right) = \frac{\pi}{2} - i\coth^{-1}\left(\sqrt{z}\right) /; 0 < \arg(z) \leq \pi$$

01.13.27.1984.01

$$\cos^{-1}\left(\frac{1}{\sqrt{1-z}}\right) = i\coth^{-1}\left(\sqrt{z}\right) - \frac{\pi}{2} /; (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.1985.01

$$\cos^{-1}\left(\frac{1}{\sqrt{1-z}}\right) = -\frac{\sqrt{z}}{\sqrt{-z}}\coth^{-1}\left(\sqrt{z}\right) + \frac{\pi}{2}\sqrt{-z}\sqrt{-\frac{1}{z}}\sqrt{1-z}\sqrt{\frac{1}{1-z}}$$

Involving $\cos^{-1}\left(\frac{1}{\sqrt{1-z}}\right)$ and $\coth^{-1}\left(\frac{1}{\sqrt{z}}\right)$

01.13.27.1986.01

$$\cos^{-1}\left(\frac{1}{\sqrt{1-z}}\right) = -i\coth^{-1}\left(\frac{1}{\sqrt{z}}\right) /; 0 < \arg(z) \leq \pi$$

01.13.27.1987.01

$$\cos^{-1}\left(\frac{1}{\sqrt{1-z}}\right) = i\coth^{-1}\left(\frac{1}{\sqrt{z}}\right) /; -\pi < \arg(z) \leq 0$$

01.13.27.1988.01

$$\cos^{-1}\left(\frac{1}{\sqrt{1-z}}\right) = \frac{\sqrt{-z^2}}{z}\coth^{-1}\left(\frac{1}{\sqrt{z}}\right)$$

Involving $\cos^{-1}\left(\frac{1}{\sqrt{1-z}}\right)$ and $\coth^{-1}\left(\sqrt{\frac{1}{z}}\right)$

01.13.27.1989.01

$$\cos^{-1}\left(\frac{1}{\sqrt{1-z}}\right) = -i\coth^{-1}\left(\sqrt{\frac{1}{z}}\right) /; 0 < \arg(z) < \pi$$

01.13.27.1990.01

$$\cos^{-1}\left(\frac{1}{\sqrt{1-z}}\right) = i\coth^{-1}\left(\sqrt{\frac{1}{z}}\right) /; \operatorname{Im}(z) \leq 0$$

01.13.27.1991.01

$$\cos^{-1}\left(\frac{1}{\sqrt{1-z}}\right) = \sqrt{\frac{1}{z}}\sqrt{-z}\coth^{-1}\left(\sqrt{\frac{1}{z}}\right)$$

Involving $\cos^{-1}\left(\sqrt{\frac{1}{1-z}}\right)$

Involving $\cos^{-1}\left(\sqrt{\frac{1}{1-z}}\right)$ and $\coth^{-1}\left(\sqrt{z}\right)$

01.13.27.1992.01

$$\cos^{-1}\left(\sqrt{\frac{1}{1-z}}\right) = \frac{\pi}{2} + i\coth^{-1}\left(\sqrt{z}\right) /; \operatorname{Im}(z) < 0$$

01.13.27.1993.01

$$\cos^{-1}\left(\sqrt{\frac{1}{1-z}}\right) = \frac{\pi}{2} - i\coth^{-1}\left(\sqrt{z}\right) /; 0 < \arg(z) \leq \pi \bigvee (z \in \mathbb{R} \wedge z > 1)$$

01.13.27.1994.01

$$\cos^{-1}\left(\sqrt{\frac{1}{1-z}}\right) = i\coth^{-1}\left(\sqrt{z}\right) - \frac{\pi}{2} /; (z \in \mathbb{R} \wedge 0 < z < 1)$$

01.13.27.1995.01

$$\cos^{-1}\left(\sqrt{\frac{1}{1-z}}\right) = -\frac{\sqrt{1-z}}{\sqrt{-z}}\sqrt{\frac{z}{1-z}}\coth^{-1}\left(\sqrt{z}\right) + \frac{\pi}{2}\left(1 + \sqrt{-z}\sqrt{-\frac{1}{z}} - \sqrt{1-z}\sqrt{\frac{1}{1-z}}\right)$$

Involving $\cos^{-1}\left(\sqrt{\frac{1}{1-z}}\right)$ and $\coth^{-1}\left(\frac{1}{\sqrt{z}}\right)$

01.13.27.1996.01

$$\cos^{-1}\left(\sqrt{\frac{1}{1-z}}\right) = -i\coth^{-1}\left(\frac{1}{\sqrt{z}}\right) /; 0 < \arg(z) \leq \pi$$

01.13.27.1997.01

$$\cos^{-1}\left(\sqrt{\frac{1}{1-z}}\right) = i\coth^{-1}\left(\frac{1}{\sqrt{z}}\right) /; -\pi < \arg(z) < 0 \bigvee (z \in \mathbb{R} \wedge 0 < z < 1)$$

01.13.27.1998.01

$$\cos^{-1}\left(\sqrt{\frac{1}{1-z}}\right) = \pi - i\coth^{-1}\left(\frac{1}{\sqrt{z}}\right) /; (z \in \mathbb{R} \wedge z > 1)$$

01.13.27.1999.01

$$\cos^{-1}\left(\sqrt{\frac{1}{1-z}}\right) = \frac{\pi}{2} - \sqrt{\frac{1}{1-z}}\sqrt{1-z}\left(\frac{\pi}{2} - \frac{\sqrt{-z^2}}{z}\coth^{-1}\left(\frac{1}{\sqrt{z}}\right)\right)$$

Involving $\cos^{-1}\left(\sqrt{\frac{1}{1-z}}\right)$ and $\coth^{-1}\left(\sqrt{\frac{1}{z}}\right)$

01.13.27.2000.01

$$\cos^{-1}\left(\sqrt{\frac{1}{1-z}}\right) = -i \coth^{-1}\left(\sqrt{\frac{1}{z}}\right) /; 0 < \arg(z) < \pi$$

01.13.27.2001.01

$$\cos^{-1}\left(\sqrt{\frac{1}{1-z}}\right) = i \coth^{-1}\left(\sqrt{\frac{1}{z}}\right) /; -\pi < \arg(z) < 0 \bigvee (z \in \mathbb{R} \land z < 1)$$

01.13.27.2002.01

$$\cos^{-1}\left(\sqrt{\frac{1}{1-z}}\right) = \pi - i \coth^{-1}\left(\sqrt{\frac{1}{z}}\right) /; (z \in \mathbb{R} \land z > 1)$$

01.13.27.2003.01

$$\cos^{-1}\left(\sqrt{\frac{1}{1-z}}\right) = \frac{\pi}{2} - \sqrt{\frac{1}{1-z}} \sqrt{1-z} \left(\frac{\pi}{2} - \sqrt{\frac{1}{z}} \sqrt{-z} \coth^{-1}\left(\sqrt{\frac{1}{z}}\right)\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{z}}{\sqrt{z-1}}\right)$

Involving $\cos^{-1}\left(\frac{\sqrt{z}}{\sqrt{z-1}}\right)$ and $\coth^{-1}\left(\sqrt{z}\right)$

01.13.27.2004.01

$$\cos^{-1}\left(\frac{\sqrt{z}}{\sqrt{z-1}}\right) = i \coth^{-1}\left(\sqrt{z}\right) /; \operatorname{Im}(z) \geq 0$$

01.13.27.2005.01

$$\cos^{-1}\left(\frac{\sqrt{z}}{\sqrt{z-1}}\right) = -i \coth^{-1}\left(\sqrt{z}\right) /; \operatorname{Im}(z) < 0$$

01.13.27.0062.02

$$\cos^{-1}\left(\frac{\sqrt{z}}{\sqrt{z-1}}\right) = \sqrt{-\frac{1}{z}} \sqrt{z} \coth^{-1}\left(\sqrt{z}\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{z}}{\sqrt{z-1}}\right)$ and $\coth^{-1}\left(\frac{1}{\sqrt{z}}\right)$

01.13.27.2006.01

$$\cos^{-1}\left(\frac{\sqrt{z}}{\sqrt{z-1}}\right) = \frac{\pi}{2} + i \coth^{-1}\left(\frac{1}{\sqrt{z}}\right) /; 0 < \arg(z) \leq \pi \bigvee (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.2007.01

$$\cos^{-1}\left(\frac{\sqrt{z}}{\sqrt{z-1}}\right) = \frac{\pi}{2} - i\coth^{-1}\left(\frac{1}{\sqrt{z}}\right) /; -\pi < \arg(z) < 0$$

01.13.27.2008.01

$$\cos^{-1}\left(\frac{\sqrt{z}}{\sqrt{z-1}}\right) = i\coth^{-1}\left(\frac{1}{\sqrt{z}}\right) - \frac{\pi}{2} /; (z \in \mathbb{R} \land z > 1)$$

01.13.27.2009.01

$$\cos^{-1}\left(\frac{\sqrt{z}}{\sqrt{z-1}}\right) = \frac{\pi}{2} - \frac{\sqrt{(1-z)z}}{\sqrt{z-1}\sqrt{-z}}\left(\sqrt{\frac{1}{1-z}}\sqrt{1-z}\left(\frac{\sqrt{-z}}{\sqrt{z}}\coth^{-1}\left(\frac{1}{\sqrt{z}}\right) - \frac{\pi}{2}\right) + \frac{\pi}{2}\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{z}}{\sqrt{z-1}}\right)$ and $\coth^{-1}\left(\sqrt{\frac{1}{z}}\right)$

01.13.27.2010.01

$$\cos^{-1}\left(\frac{\sqrt{z}}{\sqrt{z-1}}\right) = \frac{\pi}{2} + i\coth^{-1}\left(\sqrt{\frac{1}{z}}\right) /; 0 < \arg(z) < \pi \bigvee (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.2011.01

$$\cos^{-1}\left(\frac{\sqrt{z}}{\sqrt{z-1}}\right) = \frac{\pi}{2} - i\coth^{-1}\left(\sqrt{\frac{1}{z}}\right) /; -\pi < \arg(z) < 0 \bigvee (z \in \mathbb{R} \land z < 0)$$

01.13.27.2012.01

$$\cos^{-1}\left(\frac{\sqrt{z}}{\sqrt{z-1}}\right) = i\coth^{-1}\left(\sqrt{\frac{1}{z}}\right) - \frac{\pi}{2} /; (z \in \mathbb{R} \land z > 1)$$

01.13.27.2013.01

$$\cos^{-1}\left(\frac{\sqrt{z}}{\sqrt{z-1}}\right) = \frac{\pi}{2} - \frac{\sqrt{(1-z)z}}{\sqrt{z-1}\sqrt{-z}}\left(\sqrt{\frac{1}{1-z}}\sqrt{1-z}\left(\sqrt{-z}\sqrt{\frac{1}{z}}\coth^{-1}\left(\sqrt{\frac{1}{z}}\right) - \frac{\pi}{2}\right) + \frac{\pi}{2}\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{-z}}{\sqrt{1-z}}\right)$

Involving $\cos^{-1}\left(\frac{\sqrt{-z}}{\sqrt{1-z}}\right)$ and $\coth^{-1}\left(\sqrt{z}\right)$

01.13.27.2014.01

$$\cos^{-1}\left(\frac{\sqrt{-z}}{\sqrt{1-z}}\right) = i\coth^{-1}\left(\sqrt{z}\right) /; \operatorname{Im}(z) > 0 \lor (z \in \mathbb{R} \land z > 1)$$

01.13.27.2015.01

$$\cos^{-1}\left(\frac{\sqrt{-z}}{\sqrt{1-z}}\right) = -i\coth^{-1}\left(\sqrt{z}\right) /; \operatorname{Im}(z) < 0$$

01.13.27.2016.01

$$\cos^{-1}\left(\frac{\sqrt{-z}}{\sqrt{1-z}}\right) = \pi - i \coth^{-1}\left(\sqrt{z}\right) /; (z \in \mathbb{R} \wedge 0 < z < 1)$$

01.13.27.2017.01

$$\cos^{-1}\left(\frac{\sqrt{-z}}{\sqrt{1-z}}\right) = \frac{\pi}{2}\left(1 - \sqrt{\frac{z-1}{z}}\sqrt{\frac{z}{z-1}}\right) - \frac{\sqrt{-z^2}\sqrt{1-z}}{z}\sqrt{\frac{1}{1-z}}\coth^{-1}\left(\sqrt{z}\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{-z}}{\sqrt{1-z}}\right)$ and $\coth^{-1}\left(\frac{1}{\sqrt{z}}\right)$

01.13.27.2018.01

$$\cos^{-1}\left(\frac{\sqrt{-z}}{\sqrt{1-z}}\right) = \frac{\pi}{2} - i \coth^{-1}\left(\frac{1}{\sqrt{z}}\right) /; \operatorname{Im}(z) < 0 \vee (z \in \mathbb{R} \wedge 0 < z < 1)$$

01.13.27.2019.01

$$\cos^{-1}\left(\frac{\sqrt{-z}}{\sqrt{1-z}}\right) = \frac{\pi}{2} + i \coth^{-1}\left(\frac{1}{\sqrt{z}}\right) /; 0 < \arg(z) \leq \pi$$

01.13.27.2020.01

$$\cos^{-1}\left(\frac{\sqrt{-z}}{\sqrt{1-z}}\right) = i \coth^{-1}\left(\frac{1}{\sqrt{z}}\right) - \frac{\pi}{2} /; (z \in \mathbb{R} \wedge z > 1)$$

01.13.27.2021.01

$$\cos^{-1}\left(\frac{\sqrt{-z}}{\sqrt{1-z}}\right) = -\sqrt{\frac{1}{1-z}}\sqrt{1-z}\left(\frac{\sqrt{-z}}{\sqrt{z}}\coth^{-1}\left(\frac{1}{\sqrt{z}}\right) - \frac{\pi}{2}\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{-z}}{\sqrt{1-z}}\right)$ and $\coth^{-1}\left(\sqrt{\frac{1}{z}}\right)$

01.13.27.2022.01

$$\cos^{-1}\left(\frac{\sqrt{-z}}{\sqrt{1-z}}\right) = \frac{\pi}{2} + i \coth^{-1}\left(\sqrt{\frac{1}{z}}\right) /; 0 < \arg(z) < \pi$$

01.13.27.2023.01

$$\cos^{-1}\left(\frac{\sqrt{-z}}{\sqrt{1-z}}\right) = \frac{\pi}{2} - i \coth^{-1}\left(\sqrt{\frac{1}{z}}\right) /; \operatorname{Im}(z) < 0 \vee (z \in \mathbb{R} \wedge z < 1)$$

01.13.27.2024.01

$$\cos^{-1}\left(\frac{\sqrt{-z}}{\sqrt{1-z}}\right) = i \coth^{-1}\left(\sqrt{\frac{1}{z}}\right) - \frac{\pi}{2} /; (z \in \mathbb{R} \wedge z > 1)$$

01.13.27.2025.01

$$\cos^{-1}\left(\frac{\sqrt{-z}}{\sqrt{1-z}}\right) = -\sqrt{\frac{1}{1-z}}\sqrt{1-z}\left(\sqrt{-z}\sqrt{\frac{1}{z}}\coth^{-1}\left(\sqrt{\frac{1}{z}}\right) - \frac{\pi}{2}\right)$$

Involving $\cos^{-1}\left(\sqrt{\frac{z}{z-1}}\right)$

Involving $\cos^{-1}\left(\sqrt{\frac{z}{z-1}}\right)$ and $\coth^{-1}\left(\sqrt{z}\right)$

01.13.27.2026.01

$$\cos^{-1}\left(\sqrt{\frac{z}{z-1}}\right) = i \coth^{-1}\left(\sqrt{z}\right) /; 0 < \arg(z) \leq \pi \bigvee (z \in \mathbb{R} \land z > 1)$$

01.13.27.2027.01

$$\cos^{-1}\left(\sqrt{\frac{z}{z-1}}\right) = -i \coth^{-1}\left(\sqrt{z}\right) /; \operatorname{Im}(z) < 0$$

01.13.27.2028.01

$$\cos^{-1}\left(\sqrt{\frac{z}{z-1}}\right) = \pi - i \coth^{-1}\left(\sqrt{z}\right) /; (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.2029.01

$$\cos^{-1}\left(\sqrt{\frac{z}{z-1}}\right) = \frac{\pi}{2}\left(1 - \sqrt{\frac{z-1}{z}} \sqrt{\frac{z}{z-1}}\right) - \frac{\sqrt{-z^2} \sqrt{1-z}}{z} \sqrt{\frac{1}{1-z}} \coth^{-1}\left(\sqrt{z}\right)$$

Involving $\cos^{-1}\left(\sqrt{\frac{z}{z-1}}\right)$ and $\coth^{-1}\left(\frac{1}{\sqrt{z}}\right)$

01.13.27.2030.01

$$\cos^{-1}\left(\sqrt{\frac{z}{z-1}}\right) = \frac{\pi}{2} - i \coth^{-1}\left(\frac{1}{\sqrt{z}}\right) /; \operatorname{Im}(z) < 0 \lor (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.2031.01

$$\cos^{-1}\left(\sqrt{\frac{z}{z-1}}\right) = \frac{\pi}{2} + i \coth^{-1}\left(\frac{1}{\sqrt{z}}\right) /; 0 < \arg(z) \leq \pi$$

01.13.27.2032.01

$$\cos^{-1}\left(\sqrt{\frac{z}{z-1}}\right) = i \coth^{-1}\left(\frac{1}{\sqrt{z}}\right) - \frac{\pi}{2} /; (z \in \mathbb{R} \land z > 1)$$

01.13.27.2033.01

$$\cos^{-1}\left(\sqrt{\frac{z}{z-1}}\right) = -\sqrt{\frac{1}{1-z}} \sqrt{1-z} \left(\frac{\sqrt{-z}}{\sqrt{z}} \coth^{-1}\left(\frac{1}{\sqrt{z}}\right) - \frac{\pi}{2}\right)$$

Involving $\cos^{-1}\left(\sqrt{\frac{z}{z-1}}\right)$ and $\coth^{-1}\left(\sqrt{\frac{1}{z}}\right)$

01.13.27.2034.01

$$\cos^{-1}\left(\sqrt{\frac{z}{z-1}}\right) = \frac{\pi}{2} + i \coth^{-1}\left(\sqrt{\frac{1}{z}}\right) /; 0 < \arg(z) < \pi$$

01.13.27.2035.01

$$\cos^{-1}\left(\sqrt{\frac{z}{z-1}}\right) = \frac{\pi}{2} - i \coth^{-1}\left(\sqrt{\frac{1}{z}}\right) /; \operatorname{Im}(z) < 0 \lor (z \in \mathbb{R} \land z < 1)$$

01.13.27.2036.01

$$\cos^{-1}\left(\sqrt{\frac{z}{z-1}}\right) = i \coth^{-1}\left(\sqrt{\frac{1}{z}}\right) - \frac{\pi}{2} /; (z \in \mathbb{R} \land z > 1)$$

01.13.27.2037.01

$$\cos^{-1}\left(\sqrt{\frac{z}{z-1}}\right) = -\sqrt{\frac{1}{1-z}} \sqrt{1-z} \left(\sqrt{-z} \sqrt{\frac{1}{z}} \coth^{-1}\left(\sqrt{\frac{1}{z}}\right) - \frac{\pi}{2}\right)$$

Involving $\cos^{-1}\left(\frac{1}{\sqrt{1-z^2}}\right)$

Involving $\cos^{-1}\left(\frac{1}{\sqrt{1-z^2}}\right)$ and $\coth^{-1}(z)$

01.13.27.2038.01

$$\cos^{-1}\left(\frac{1}{\sqrt{1-z^2}}\right) = \frac{\pi}{2} - i \coth^{-1}(z) /; \operatorname{Im}(z) > 0 \lor (z \in \mathbb{R} \land z < -1)$$

01.13.27.2039.01

$$\cos^{-1}\left(\frac{1}{\sqrt{1-z^2}}\right) = \frac{\pi}{2} + i \coth^{-1}(z) /; \operatorname{Im}(z) < 0 \lor (z \in \mathbb{R} \land z > 1)$$

01.13.27.2040.01

$$\cos^{-1}\left(\frac{1}{\sqrt{1-z^2}}\right) = -i \coth^{-1}(z) - \frac{\pi}{2} /; (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.2041.01

$$\cos^{-1}\left(\frac{1}{\sqrt{1-z^2}}\right) = i \coth^{-1}(z) - \frac{\pi}{2} /; (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.2042.01

$$\cos^{-1}\left(\frac{1}{\sqrt{1-z^2}}\right) = \frac{\pi}{2} + \frac{\sqrt{-z^2}}{z} \coth^{-1}(z) /; z \notin (-1, 1)$$

01.13.27.2043.01

$$\cos^{-1}\left(\frac{1}{\sqrt{1-z^2}}\right) = \frac{\sqrt{-z^2}}{z} \coth^{-1}(z) + \frac{\pi i}{2}\left(\frac{\sqrt{-z-1}}{\sqrt{z+1}} + \frac{\sqrt{z-1}}{\sqrt{1-z}}\right) + \frac{\pi}{2}$$

Involving $\cos^{-1}\left(\frac{1}{\sqrt{1-z^2}}\right)$ and $\coth^{-1}\left(\frac{1}{z}\right)$

01.13.27.2044.01

$$\cos^{-1}\left(\frac{1}{\sqrt{1-z^2}}\right) = i \coth^{-1}\left(\frac{1}{z}\right) /; -\pi < \arg(z) \leq 0$$

01.13.27.2045.01

$$\cos^{-1}\left(\frac{1}{\sqrt{1-z^2}}\right) = -i \coth^{-1}\left(\frac{1}{z}\right) /; 0 < \arg(z) \leq \pi$$

01.13.27.2046.01

$$\cos^{-1}\left(\frac{1}{\sqrt{1-z^2}}\right) = \frac{\sqrt{-z^2}}{z} \coth^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\sqrt{\frac{1}{1-z^2}}\right)$

Involving $\cos^{-1}\left(\sqrt{\frac{1}{1-z^2}}\right)$ and $\coth^{-1}(z)$

01.13.27.2047.01

$$\cos^{-1}\left(\sqrt{\frac{1}{1-z^2}}\right) = \frac{\pi}{2} - i \coth^{-1}(z) /; \operatorname{Im}(z) > 0 \lor (z \in \mathbb{R} \land z > 1)$$

01.13.27.2048.01

$$\cos^{-1}\left(\sqrt{\frac{1}{1-z^2}}\right) = \frac{\pi}{2} + i \coth^{-1}(z) /; \operatorname{Im}(z) < 0 \lor (z \in \mathbb{R} \land z < -1)$$

01.13.27.2049.01

$$\cos^{-1}\left(\sqrt{\frac{1}{1-z^2}}\right) = -i \coth^{-1}(z) - \frac{\pi}{2} /; (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.2050.01

$$\cos^{-1}\left(\sqrt{\frac{1}{1-z^2}}\right) = i \coth^{-1}(z) - \frac{\pi}{2} /; (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.2051.01

$$\cos^{-1}\left(\sqrt{\frac{1}{1-z^2}}\right) = \frac{\sqrt{-z^2}}{z} \coth^{-1}(z) + \frac{\pi}{2} /; \operatorname{Im}(z) \neq 0$$

01.13.27.2052.01

$$\cos^{-1}\left(\sqrt{\frac{1}{1-z^2}}\right) = \frac{\pi}{2} + \sqrt{\frac{1}{1-z^2}}\sqrt{1-z^2}\left(\frac{\pi i}{2}\left(\frac{\sqrt{-z-1}}{\sqrt{z+1}} + \frac{\sqrt{z-1}}{\sqrt{1-z}}\right) + \frac{\sqrt{-z^2}}{z}\coth^{-1}(z)\right)$$

Involving $\cos^{-1}\left(\sqrt{\frac{1}{1-z^2}}\right)$ and $\coth^{-1}\left(\frac{1}{z}\right)$

01.13.27.2053.01

$$\cos^{-1}\left(\sqrt{\frac{1}{1-z^2}}\right) = i\coth^{-1}\left(\frac{1}{z}\right) /; \operatorname{Im}(z) < 0 \lor (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.2054.01

$$\cos^{-1}\left(\sqrt{\frac{1}{1-z^2}}\right) = -i\coth^{-1}\left(\frac{1}{z}\right) /; \operatorname{Im}(z) > 0 \lor (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.2055.01

$$\cos^{-1}\left(\sqrt{\frac{1}{1-z^2}}\right) = \pi - i\coth^{-1}\left(\frac{1}{z}\right) /; (z \in \mathbb{R} \land z > 1)$$

01.13.27.2056.01

$$\cos^{-1}\left(\sqrt{\frac{1}{1-z^2}}\right) = \pi + i\coth^{-1}\left(\frac{1}{z}\right) /; (z \in \mathbb{R} \land z < -1)$$

01.13.27.2057.01

$$\cos^{-1}\left(\sqrt{\frac{1}{1-z^2}}\right) = \frac{\pi}{2} - \sqrt{\frac{1}{1-z^2}}\sqrt{1-z^2}\left(\frac{\pi}{2} - \frac{\sqrt{-z^2}}{z}\coth^{-1}\left(\frac{1}{z}\right)\right)$$

Involving $\cos^{-1}\left(\frac{z}{\sqrt{z^2-1}}\right)$

Involving $\cos^{-1}\left(\frac{z}{\sqrt{z^2-1}}\right)$ and $\coth^{-1}(z)$

01.13.27.2058.01

$$\cos^{-1}\left(\frac{z}{\sqrt{z^2-1}}\right) = i\coth^{-1}(z) /; 0 < \arg(z) \le \frac{\pi}{2}$$

01.13.27.2059.01

$$\cos^{-1}\left(\frac{z}{\sqrt{z^2-1}}\right) = -i\coth^{-1}(z) /; -\frac{\pi}{2} < \arg(z) < 0$$

01.13.27.2060.01

$$\cos^{-1}\left(\frac{z}{\sqrt{z^2-1}}\right) = \pi - i\coth^{-1}(z) /; \frac{\pi}{2} < \arg(z) < \pi$$

01.13.27.2061.01

$$\cos^{-1}\left(\frac{z}{\sqrt{z^2-1}}\right) = i\coth^{-1}(z) + \pi /; -\pi < \arg(z) \leq -\frac{\pi}{2} \bigvee (z \in \mathbb{R} \land z < 0)$$

01.13.27.0060.01

$$\cos^{-1}\left(\frac{z}{\sqrt{z^2-1}}\right) = \frac{\pi}{2}\left(1 - \frac{\sqrt{z^2}}{z}\right) + \sqrt{z^2-1}\sqrt{\frac{1}{1-z^2}}\coth^{-1}(z)$$

01.13.27.0061.01

$$\cos^{-1}\left(\frac{z}{\sqrt{z^2-1}}\right) = \frac{\pi}{2} - \frac{z}{\sqrt{z^2-1}}\sqrt{1-\frac{1}{z^2}}\left(\frac{\pi}{2} - \sqrt{-\frac{1}{z^2}}\, z\coth^{-1}(z)\right)$$

Involving $\cos^{-1}\left(\frac{z}{\sqrt{z^2-1}}\right)$ and $\coth^{-1}\left(\frac{1}{z}\right)$

01.13.27.2062.01

$$\cos^{-1}\left(\frac{z}{\sqrt{z^2-1}}\right) = \frac{\pi}{2} + i\coth^{-1}\left(\frac{1}{z}\right) /; 0 < \arg(z) \leq \frac{\pi}{2} \bigvee -\pi < \arg(z) \leq -\frac{\pi}{2} \bigvee (z \in \mathbb{R} \land -1 < z < 1)$$

01.13.27.2063.01

$$\cos^{-1}\left(\frac{z}{\sqrt{z^2-1}}\right) = \frac{\pi}{2} - i\coth^{-1}\left(\frac{1}{z}\right) /; \frac{\pi}{2} < \arg(z) < \pi \bigvee -\frac{\pi}{2} < \arg(z) < 0$$

01.13.27.2064.01

$$\cos^{-1}\left(\frac{z}{\sqrt{z^2-1}}\right) = i\coth^{-1}\left(\frac{1}{z}\right) - \frac{\pi}{2} /; (z \in \mathbb{R} \land z > 1)$$

01.13.27.2065.01

$$\cos^{-1}\left(\frac{z}{\sqrt{z^2-1}}\right) = \frac{3\pi}{2} + i\coth^{-1}\left(\frac{1}{z}\right) /; (z \in \mathbb{R} \land z < -1)$$

01.13.27.2066.01

$$\cos^{-1}\left(\frac{z}{\sqrt{z^2-1}}\right) = \frac{\pi}{2} + \sqrt{z^2-1}\sqrt{\frac{1}{1-z^2}}\coth^{-1}\left(\frac{1}{z}\right) /; z \notin (-\infty, -1) \bigwedge z \notin (1, \infty)$$

01.13.27.2067.01

$$\cos^{-1}\left(\frac{z}{\sqrt{z^2-1}}\right) = \frac{\pi}{2}\left(1 - \sqrt{\frac{1}{z+1}}\sqrt{z+1} + \sqrt{\frac{1}{1-z}}\sqrt{1-z}\right) + \sqrt{\frac{1}{1-z^2}}\sqrt{z^2-1}\coth^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{z^2}}{\sqrt{z^2-1}}\right)$

Involving $\cos^{-1}\left(\frac{\sqrt{z^2}}{\sqrt{z^2-1}}\right)$ and $\coth^{-1}(z)$

01.13.27.2068.01

$$\cos^{-1}\left(\frac{\sqrt{z^2}}{\sqrt{z^2-1}}\right) = -i\coth^{-1}(z) \;/;\; -\pi < \arg(z) < 0 \lor (z \in \mathbb{R} \land z < 0)$$

01.13.27.2069.01

$$\cos^{-1}\left(\frac{\sqrt{z^2}}{\sqrt{z^2-1}}\right) = i\coth^{-1}(z) \;/;\; 0 \leq \arg(z) < \pi$$

01.13.27.2070.01

$$\cos^{-1}\left(\frac{\sqrt{z^2}}{\sqrt{z^2-1}}\right) = \sqrt{-\frac{1}{z^2}}\, z\coth^{-1}(z)$$

Involving $\cos^{-1}\left(\frac{\sqrt{z^2}}{\sqrt{z^2-1}}\right)$ and $\coth^{-1}\left(\frac{1}{z}\right)$

01.13.27.2071.01

$$\cos^{-1}\left(\frac{\sqrt{z^2}}{\sqrt{z^2-1}}\right) = i\coth^{-1}\left(\frac{1}{z}\right) + \frac{\pi}{2} \;/;\; \operatorname{Im}(z) > 0 \lor (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.2072.01

$$\cos^{-1}\left(\frac{\sqrt{z^2}}{\sqrt{z^2-1}}\right) = \frac{\pi}{2} - i\coth^{-1}\left(\frac{1}{z}\right) \;/;\; \operatorname{Im}(z) < 0 \lor (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.2073.01

$$\cos^{-1}\left(\frac{\sqrt{z^2}}{\sqrt{z^2-1}}\right) = i\coth^{-1}\left(\frac{1}{z}\right) - \frac{\pi}{2} \;/;\; (z \in \mathbb{R} \land z > 1)$$

01.13.27.2074.01

$$\cos^{-1}\left(\frac{\sqrt{z^2}}{\sqrt{z^2-1}}\right) = -\frac{\pi}{2} - i\coth^{-1}\left(\frac{1}{z}\right) \;/;\; (z \in \mathbb{R} \land z < -1)$$

01.13.27.2075.01

$$\cos^{-1}\left(\frac{\sqrt{z^2}}{\sqrt{z^2-1}}\right) = \frac{1}{2}\pi\left(-1 + \sqrt{\frac{1}{1-z}}\sqrt{1-z} + \sqrt{\frac{1}{z+1}}\sqrt{z+1}\right) + \sqrt{-\frac{1}{z^2}}\, z\coth^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{-z^2}}{\sqrt{1-z^2}}\right)$

Involving $\cos^{-1}\left(\frac{\sqrt{-z^2}}{\sqrt{1-z^2}}\right)$ and $\coth^{-1}(z)$

01.13.27.2076.01

$$\cos^{-1}\left(\frac{\sqrt{-z^2}}{\sqrt{1-z^2}}\right) = -i\coth^{-1}(z) /; -\pi < \arg(z) < 0 \bigvee (z \in \mathbb{R} \land z < -1)$$

01.13.27.2077.01

$$\cos^{-1}\left(\frac{\sqrt{-z^2}}{\sqrt{1-z^2}}\right) = i\coth^{-1}(z) /; \operatorname{Im}(z) > 0 \lor (z \in \mathbb{R} \land z > 1)$$

01.13.27.2078.01

$$\cos^{-1}\left(\frac{\sqrt{-z^2}}{\sqrt{1-z^2}}\right) = \pi + i\coth^{-1}(z) /; (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.2079.01

$$\cos^{-1}\left(\frac{\sqrt{-z^2}}{\sqrt{1-z^2}}\right) = \pi - i\coth^{-1}(z) /; (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.2080.01

$$\cos^{-1}\left(\frac{\sqrt{-z^2}}{\sqrt{1-z^2}}\right) = \frac{\pi}{2} - \frac{\sqrt{-z^2}\sqrt{z^2-1}}{\sqrt{z^2(1-z^2)}}\left(\frac{\pi}{2} - \sqrt{-\frac{1}{z^2}}\, z\coth^{-1}(z)\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{-z^2}}{\sqrt{1-z^2}}\right)$ and $\coth^{-1}\left(\frac{1}{z}\right)$

01.13.27.2081.01

$$\cos^{-1}\left(\frac{\sqrt{-z^2}}{\sqrt{1-z^2}}\right) = \frac{\pi}{2} - i\coth^{-1}\left(\frac{1}{z}\right) /; \operatorname{Im}(z) < 0 \lor (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.2082.01

$$\cos^{-1}\left(\frac{\sqrt{-z^2}}{\sqrt{1-z^2}}\right) = \frac{\pi}{2} + i\coth^{-1}\left(\frac{1}{z}\right) /; \operatorname{Im}(z) > 0 \lor (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.2083.01

$$\cos^{-1}\left(\frac{\sqrt{-z^2}}{\sqrt{1-z^2}}\right) = i\coth^{-1}\left(\frac{1}{z}\right) - \frac{\pi}{2} /; (z \in \mathbb{R} \land z > 1)$$

01.13.27.2084.01

$$\cos^{-1}\left(\frac{\sqrt{-z^2}}{\sqrt{1-z^2}}\right) = -i \coth^{-1}\left(\frac{1}{z}\right) - \frac{\pi}{2} \;/;\; (z \in \mathbb{R} \land z < -1)$$

01.13.27.2085.01

$$\cos^{-1}\left(\frac{\sqrt{-z^2}}{\sqrt{1-z^2}}\right) = \frac{\pi}{2} - \frac{\sqrt{-z^2}\,\sqrt{z^2-1}}{\sqrt{z^2\,(1-z^2)}}\left(\frac{\pi}{2}\left(-\sqrt{\frac{1}{1-z}}\,\sqrt{1-z} - \sqrt{\frac{1}{z+1}}\,\sqrt{z+1} + 2\right) - \sqrt{-\frac{1}{z^2}}\; z \coth^{-1}\left(\frac{1}{z}\right)\right)$$

Involving $\cos^{-1}\left(\sqrt{\frac{z^2}{z^2-1}}\right)$

Involving $\cos^{-1}\left(\sqrt{\frac{z^2}{z^2-1}}\right)$ and $\coth^{-1}(z)$

01.13.27.2086.01

$$\cos^{-1}\left(\sqrt{\frac{z^2}{z^2-1}}\right) = -i \coth^{-1}(z) \;/;\; -\pi < \arg(z) < 0 \bigvee (z \in \mathbb{R} \land z < -1)$$

01.13.27.2087.01

$$\cos^{-1}\left(\sqrt{\frac{z^2}{z^2-1}}\right) = i \coth^{-1}(z) \;/;\; \mathrm{Im}(z) > 0 \lor (z \in \mathbb{R} \land z > 1)$$

01.13.27.2088.01

$$\cos^{-1}\left(\sqrt{\frac{z^2}{z^2-1}}\right) = \pi + i \coth^{-1}(z) \;/;\; (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.2089.01

$$\cos^{-1}\left(\sqrt{\frac{z^2}{z^2-1}}\right) = \pi - i \coth^{-1}(z) \;/;\; (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.2090.01

$$\cos^{-1}\left(\sqrt{\frac{z^2}{z^2-1}}\right) = \frac{\pi}{2} - \frac{\sqrt{-z^2}\,\sqrt{z^2-1}}{\sqrt{z^2\,(1-z^2)}}\left(\frac{\pi}{2} - \sqrt{-\frac{1}{z^2}}\; z \coth^{-1}(z)\right)$$

Involving $\cos^{-1}\left(\sqrt{\frac{z^2}{z^2-1}}\right)$ and $\coth^{-1}\left(\frac{1}{z}\right)$

01.13.27.2091.01

$$\cos^{-1}\left(\sqrt{\frac{z^2}{z^2-1}}\right) = \frac{\pi}{2} - i\coth^{-1}\left(\frac{1}{z}\right) /; \operatorname{Im}(z) < 0 \lor (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.2092.01

$$\cos^{-1}\left(\sqrt{\frac{z^2}{z^2-1}}\right) = \frac{\pi}{2} + i\coth^{-1}\left(\frac{1}{z}\right) /; \operatorname{Im}(z) > 0 \lor (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.2093.01

$$\cos^{-1}\left(\sqrt{\frac{z^2}{z^2-1}}\right) = i\coth^{-1}\left(\frac{1}{z}\right) - \frac{\pi}{2} /; (z \in \mathbb{R} \land z > 1)$$

01.13.27.2094.01

$$\cos^{-1}\left(\sqrt{\frac{z^2}{z^2-1}}\right) = -\frac{\pi}{2} - i\coth^{-1}\left(\frac{1}{z}\right) /; (z \in \mathbb{R} \land z < -1)$$

01.13.27.2095.01

$$\cos^{-1}\left(\sqrt{\frac{z^2}{z^2-1}}\right) = \frac{\pi}{2} - \frac{\sqrt{-z^2}\sqrt{z^2-1}}{\sqrt{z^2(1-z^2)}}\left(\frac{\pi}{2}\left(-\sqrt{\frac{1}{1-z}}\sqrt{1-z} - \sqrt{\frac{1}{z+1}}\sqrt{z+1} + 2\right) - \sqrt{-\frac{1}{z^2}}\, z\coth^{-1}\left(\frac{1}{z}\right)\right)$$

Involving $\cos^{-1}\left(\sqrt{\sqrt{1-z^2}-1} \Big/ \left(\sqrt{2}\,(1-z^2)^{1/4}\right)\right)$

Involving $\cos^{-1}\left(\sqrt{\sqrt{1-z^2}-1} \Big/ \left(\sqrt{2}\,(1-z^2)^{1/4}\right)\right)$ and $\coth^{-1}(z)$

01.13.27.2096.01

$$\cos^{-1}\left(\frac{\sqrt{\sqrt{1-z^2}-1}}{\sqrt{2}\sqrt[4]{1-z^2}}\right) = \frac{\pi}{4} + \frac{1}{2}i\coth^{-1}(z) /; \operatorname{Im}(z) > 0 \lor (z \in \mathbb{R} \land z < -1)$$

01.13.27.2097.01

$$\cos^{-1}\left(\frac{\sqrt{\sqrt{1-z^2}-1}}{\sqrt{2}\sqrt[4]{1-z^2}}\right) = -\frac{1}{2}i\coth^{-1}(z) + \frac{\pi}{4} /; \operatorname{Im}(z) < 0 \lor (z \in \mathbb{R} \land z > 1)$$

01.13.27.2098.01

$$\cos^{-1}\left(\frac{\sqrt{\sqrt{1-z^2}-1}}{\sqrt{2}\sqrt[4]{1-z^2}}\right) = \frac{3\pi}{4} + \frac{1}{2} i \coth^{-1}(z) /; (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.2099.01

$$\cos^{-1}\left(\frac{\sqrt{\sqrt{1-z^2}-1}}{\sqrt{2}\sqrt[4]{1-z^2}}\right) = -\frac{1}{2} i \coth^{-1}(z) + \frac{3\pi}{4} /; (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.2100.01

$$\cos^{-1}\left(\frac{\sqrt{\sqrt{1-z^2}-1}}{\sqrt{2}\sqrt[4]{1-z^2}}\right) = \frac{\pi}{4}\left(2 - \frac{\sqrt{-z^2}\sqrt{z^2-1}}{\sqrt{z^2-z^4}}\right) - \frac{\sqrt{-z^2}}{2z} \coth^{-1}(z)$$

Involving $\cos^{-1}\left(\sqrt{\sqrt{1-z^2}-1} \Big/ \left(\sqrt{2}\,(1-z^2)^{1/4}\right)\right)$ and $\coth^{-1}\left(\frac{1}{z}\right)$

01.13.27.2101.01

$$\cos^{-1}\left(\frac{\sqrt{\sqrt{1-z^2}-1}}{\sqrt{2}\sqrt[4]{1-z^2}}\right) = \frac{\pi}{2} - \frac{i}{2} \coth^{-1}\left(\frac{1}{z}\right) /; -\pi < \arg(z) \leq 0$$

01.13.27.2102.01

$$\cos^{-1}\left(\frac{1}{\sqrt{2}} \frac{\sqrt{\sqrt{1-z^2}-1}}{(1-z^2)^{1/4}}\right) = \frac{\pi}{2} + \frac{i}{2} \coth^{-1}\left(\frac{1}{z}\right) /; 0 < \arg(z) \leq \pi$$

01.13.27.2103.01

$$\cos^{-1}\left(\frac{1}{\sqrt{2}} \frac{\sqrt{\sqrt{1-z^2}-1}}{(1-z^2)^{1/4}}\right) = \frac{\pi}{2} - \frac{\sqrt{-z^2}}{2z} \coth^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\sqrt{\sqrt{z^2-1}-z} \Big/ \left(\sqrt{2}\,(z^2-1)^{1/4}\right)\right)$

Involving $\cos^{-1}\left(\sqrt{\sqrt{z^2-1}-z} \Big/ \left(\sqrt{2}\,(z^2-1)^{1/4}\right)\right)$ and $\coth^{-1}(z)$

01.13.27.2104.01

$$\cos^{-1}\left(\frac{\sqrt{\sqrt{z^2-1}-z}}{\sqrt{2}\,\sqrt[4]{z^2-1}}\right) = \frac{\pi}{2} - \frac{i}{2}\coth^{-1}(z) \;/;\; 0 \le \arg(z) \le \frac{\pi}{2}$$

01.13.27.2105.01

$$\cos^{-1}\left(\frac{\sqrt{\sqrt{z^2-1}-z}}{\sqrt{2}\,\sqrt[4]{z^2-1}}\right) = \frac{\pi}{2} + \frac{i}{2}\coth^{-1}(z) \;/;\; -\frac{\pi}{2} < \arg(z) < 0$$

01.13.27.2106.01

$$\cos^{-1}\left(\frac{\sqrt{\sqrt{z^2-1}-z}}{\sqrt{2}\,\sqrt[4]{z^2-1}}\right) = \frac{i}{2}\coth^{-1}(z) \;/;\; \frac{\pi}{2} < \arg(z) < \pi$$

01.13.27.2107.01

$$\cos^{-1}\left(\frac{\sqrt{\sqrt{z^2-1}-z}}{\sqrt{2}\,\sqrt[4]{z^2-1}}\right) = -\frac{i}{2}\coth^{-1}(z) \;/;\; -\pi < \arg(z) \le -\frac{\pi}{2} \bigvee (z \in \mathbb{R} \land z < 0)$$

01.13.27.2108.01

$$\cos^{-1}\left(\frac{\sqrt{\sqrt{z^2-1}-z}}{\sqrt{2}\,\sqrt[4]{z^2-1}}\right) = \frac{\pi}{4}\left(1 + \frac{\sqrt{z^2}}{z}\right) - \frac{1}{2}\sqrt{z^2}\,\sqrt{-\frac{1}{z^2}}\,\coth^{-1}(z)$$

Involving $\cos^{-1}\left(\sqrt{\sqrt{z^2-1}-z} \Big/ \left(\sqrt{2}\,(z^2-1)^{1/4}\right)\right)$ and $\coth^{-1}\left(\frac{1}{z}\right)$

01.13.27.2109.01

$$\cos^{-1}\left(\frac{\sqrt{\sqrt{z^2-1}-z}}{\sqrt{2}\,\sqrt[4]{z^2-1}}\right) = -\frac{1}{2}\,i\coth^{-1}\left(\frac{1}{z}\right) + \frac{\pi}{4} \;/;\; 0 < \arg(z) \le \frac{\pi}{2} \bigvee -\pi < \arg(z) \le -\frac{\pi}{2} \bigvee (z \in \mathbb{R} \land -1 < z < 1)$$

01.13.27.2110.01

$$\cos^{-1}\left(\frac{\sqrt{\sqrt{z^2-1}-z}}{\sqrt{2}\sqrt[4]{z^2-1}}\right) = \frac{\pi}{4}+\frac{1}{2}\,i\coth^{-1}\left(\frac{1}{z}\right)\,/;\,\frac{\pi}{2}<\arg(z)<\pi\bigvee -\frac{\pi}{2}<\arg(z)<0$$

01.13.27.2111.01

$$\cos^{-1}\left(\frac{\sqrt{\sqrt{z^2-1}-z}}{\sqrt{2}\sqrt[4]{z^2-1}}\right) = \frac{3\pi}{4}-\frac{1}{2}\,i\coth^{-1}\left(\frac{1}{z}\right)\,/;\,(z\in\mathbb{R}\land z>1)$$

01.13.27.2112.01

$$\cos^{-1}\left(\frac{\sqrt{\sqrt{z^2-1}-z}}{\sqrt{2}\sqrt[4]{z^2-1}}\right) = -\frac{1}{2}\,i\coth^{-1}\left(\frac{1}{z}\right)-\frac{\pi}{4}\,/;\,(z\in\mathbb{R}\land z<-1)$$

01.13.27.2113.01

$$\cos^{-1}\left(\frac{\sqrt{\sqrt{z^2-1}-z}}{\sqrt{2}\sqrt[4]{z^2-1}}\right) = \frac{\pi}{4}\left(-\sqrt{\frac{1}{1-z}}\sqrt{1-z}+\sqrt{z+1}\sqrt{\frac{1}{z+1}}+1\right)-\frac{1}{2}\sqrt{z^2}\sqrt{-\frac{1}{z^2}}\coth^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\sqrt{\left(\sqrt{1-z^2}-1\right)/\left(2\sqrt{1-z^2}\right)}\right)$

Involving $\cos^{-1}\left(\sqrt{\left(\sqrt{1-z^2}-1\right)/\left(2\sqrt{1-z^2}\right)}\right)$ and $\coth^{-1}(z)$

01.13.27.2114.01

$$\cos^{-1}\left(\sqrt{\frac{\sqrt{1-z^2}-1}{2\sqrt{1-z^2}}}\right) = \frac{\pi}{4}+\frac{1}{2}\,i\coth^{-1}(z)\,/;\,\mathrm{Im}(z)>0\lor(z\in\mathbb{R}\land z<-1)$$

01.13.27.2115.01

$$\cos^{-1}\left(\sqrt{\frac{\sqrt{1-z^2}-1}{2\sqrt{1-z^2}}}\right) = -\frac{1}{2}\,i\coth^{-1}(z)+\frac{\pi}{4}\,/;\,\mathrm{Im}(z)<0\lor(z\in\mathbb{R}\land z>1)$$

01.13.27.2116.01

$$\cos^{-1}\left(\sqrt{\frac{\sqrt{1-z^2}-1}{2\sqrt{1-z^2}}}\right) = \frac{3\pi}{4}+\frac{1}{2}\,i\coth^{-1}(z)\,/;\,(z\in\mathbb{R}\land -1<z<0)$$

01.13.27.2117.01

$$\cos^{-1}\left(\sqrt{\frac{\sqrt{1-z^2}-1}{2\sqrt{1-z^2}}}\right) = \frac{3\pi}{4} - \frac{1}{2}\, i \coth^{-1}(z) \;/;\; (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.2118.01

$$\cos^{-1}\left(\sqrt{\frac{\sqrt{1-z^2}-1}{2\sqrt{1-z^2}}}\right) = -\frac{\sqrt{-z^2}}{2z}\coth^{-1}(z) + \left(2 - \frac{\sqrt{-z^2}\sqrt{-1+z^2}}{\sqrt{z^2-z^4}}\right)\frac{\pi}{4}$$

Involving $\cos^{-1}\left(\sqrt{\left(\sqrt{1-z^2}-1\right)\Big/\left(2\sqrt{1-z^2}\right)}\right)$ and $\coth^{-1}\left(\frac{1}{z}\right)$

01.13.27.2119.01

$$\cos^{-1}\left(\frac{1}{\sqrt{2}}\sqrt{\frac{\sqrt{1-z^2}-1}{\sqrt{1-z^2}}}\right) = \frac{\pi}{2} - \frac{i}{2}\coth^{-1}\left(\frac{1}{z}\right) \;/;\; -\pi < \arg(z) \le 0$$

01.13.27.2120.01

$$\cos^{-1}\left(\frac{1}{\sqrt{2}}\sqrt{1-\frac{1}{\sqrt{1-z^2}}}\right) = \frac{\pi}{2} + \frac{i}{2}\coth^{-1}\left(\frac{1}{z}\right) \;/;\; 0 < \arg(z) \le \pi$$

01.13.27.2121.01

$$\cos^{-1}\left(\frac{1}{\sqrt{2}}\sqrt{1-\frac{1}{\sqrt{1-z^2}}}\right) = \frac{\pi}{2} - \frac{\sqrt{-z^2}}{2z}\coth^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\sqrt{\left(\sqrt{z^2-1}-z\right)\Big/\left(2\sqrt{z^2-1}\right)}\right)$

Involving $\cos^{-1}\left(\sqrt{\left(\sqrt{z^2-1}-z\right)\Big/\left(2\sqrt{z^2-1}\right)}\right)$ and $\coth^{-1}(z)$

01.13.27.2122.01

$$\cos^{-1}\left(\sqrt{\frac{\sqrt{z^2-1}-z}{2\sqrt{z^2-1}}}\right) = \frac{\pi}{2} - \frac{i}{2}\coth^{-1}(z) \;/;\; 0 \le \arg(z) \le \frac{\pi}{2}$$

01.13.27.2123.01

$$\cos^{-1}\left(\sqrt{\frac{\sqrt{z^2-1}-z}{2\sqrt{z^2-1}}}\right) = \frac{\pi}{2} + \frac{i}{2}\coth^{-1}(z) \;/;\; -\frac{\pi}{2} < \arg(z) < 0$$

01.13.27.2124.01

$$\cos^{-1}\left(\sqrt{\frac{\sqrt{z^2-1}-z}{2\sqrt{z^2-1}}}\right)=\frac{i}{2}\coth^{-1}(z)\ /;\ \frac{\pi}{2}<\arg(z)<\pi$$

01.13.27.2125.01

$$\cos^{-1}\left(\sqrt{\frac{\sqrt{z^2-1}-z}{2\sqrt{z^2-1}}}\right)=-\frac{i}{2}\coth^{-1}(z)\ /;\ -\pi<\arg(z)\leq-\frac{\pi}{2}\bigvee(z\in\mathbb{R}\land z<0)$$

01.13.27.2126.01

$$\cos^{-1}\left(\sqrt{\frac{\sqrt{z^2-1}-z}{2\sqrt{z^2-1}}}\right)=\frac{\pi}{4}\left(1+\frac{\sqrt{z^2}}{z}\right)-\frac{1}{2}\sqrt{z^2}\sqrt{-\frac{1}{z^2}}\coth^{-1}(z)$$

Involving $\cos^{-1}\left(\sqrt{\left(\sqrt{z^2-1}-z\right)/\left(2\sqrt{z^2-1}\right)}\right)$ and $\coth^{-1}\left(\frac{1}{z}\right)$

01.13.27.2127.01

$$\cos^{-1}\left(\sqrt{\frac{\sqrt{z^2-1}-z}{2\sqrt{z^2-1}}}\right)=-\frac{1}{2}i\coth^{-1}\left(\frac{1}{z}\right)+\frac{\pi}{4}\ /;\ 0<\arg(z)\leq\frac{\pi}{2}\bigvee-\pi<\arg(z)\leq-\frac{\pi}{2}\bigvee(z\in\mathbb{R}\land-1<z<1)$$

01.13.27.2128.01

$$\cos^{-1}\left(\sqrt{\frac{\sqrt{z^2-1}-z}{2\sqrt{z^2-1}}}\right)=\frac{\pi}{4}+\frac{1}{2}i\coth^{-1}\left(\frac{1}{z}\right)/;\ \frac{\pi}{2}<\arg(z)<\pi\bigvee-\frac{\pi}{2}<\arg(z)<0$$

01.13.27.2129.01

$$\cos^{-1}\left(\sqrt{\frac{\sqrt{z^2-1}-z}{2\sqrt{z^2-1}}}\right)=\frac{3\pi}{4}-\frac{1}{2}i\coth^{-1}\left(\frac{1}{z}\right)/;\ (z\in\mathbb{R}\land z>1)$$

01.13.27.2130.01

$$\cos^{-1}\left(\sqrt{\frac{\sqrt{z^2-1}-z}{2\sqrt{z^2-1}}}\right)=-\frac{1}{2}i\coth^{-1}\left(\frac{1}{z}\right)-\frac{\pi}{4}\ /;\ (z\in\mathbb{R}\land z<-1)$$

01.13.27.2131.01

$$\cos^{-1}\left(\sqrt{\frac{\sqrt{z^2-1}-z}{2\sqrt{z^2-1}}}\right)=\frac{\pi}{4}\left(-\sqrt{\frac{1}{1-z}}\sqrt{1-z}+\sqrt{z+1}\sqrt{\frac{1}{z+1}}+1\right)-\frac{1}{2}\sqrt{z^2}\sqrt{-\frac{1}{z^2}}\coth^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\frac{a z^c}{\sqrt{a^2 z^{2c}-1}}\right)$

Involving $\cos^{-1}\left(\frac{a z^c}{\sqrt{a^2 z^{2c}-1}}\right)$ and $\coth^{-1}\left(\frac{1}{a z^c}\right)$

01.13.27.2132.01

$$\cos^{-1}\left(\frac{a z^c}{\sqrt{a^2 z^{2c}-1}}\right) = \frac{1}{2}\pi\left(-\sqrt{\frac{1}{a z^c+1}}\sqrt{a z^c+1}+\sqrt{\frac{1}{1-a z^c}}\sqrt{1-a z^c}+1\right)+\sqrt{a^2 z^{2c}-1}\sqrt{\frac{1}{1-a^2 z^{2c}}}\coth^{-1}\left(\frac{1}{a z^c}\right)$$

Involving $\mathbf{csch^{-1}}$

Involving $\cos^{-1}(z)$

Involving $\cos^{-1}(z)$ and $\operatorname{csch}^{-1}\left(\frac{i}{z}\right)$

01.13.27.2133.01

$$\cos^{-1}(z) = \frac{\pi}{2} - i\operatorname{csch}^{-1}\left(\frac{i}{z}\right)$$

Involving $\cos^{-1}(z)$ and $\operatorname{csch}^{-1}\left(\frac{i}{1-2z^2}\right)$

01.13.27.2134.01

$$\cos^{-1}(z) = \frac{1}{2}\left(i\operatorname{csch}^{-1}\left(\frac{i}{1-2z^2}\right)+\frac{\pi}{2}\right) /; -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

01.13.27.2135.01

$$\cos^{-1}(z) = \frac{1}{2}\left(-i\operatorname{csch}^{-1}\left(\frac{i}{1-2z^2}\right)+\frac{3\pi}{2}\right) /; \frac{\pi}{2} < \arg(z) \leq \pi \bigvee -\pi < \arg(z) \leq -\frac{\pi}{2}$$

01.13.27.2136.01

$$\cos^{-1}(z) = \left(1-\frac{\sqrt{z^2}}{2z}\right)\frac{\pi}{2}+\frac{\sqrt{z^2}}{2z}\, i\operatorname{csch}^{-1}\left(\frac{i}{1-2z^2}\right)$$

Involving $\cos^{-1}(z)$ and $\operatorname{csch}^{-1}\left(\frac{i}{2z^2-1}\right)$

01.13.27.2137.01

$$\cos^{-1}(z) = \frac{1}{2}\left(-i\operatorname{csch}^{-1}\left(\frac{i}{2z^2-1}\right)+\frac{\pi}{2}\right) /; -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

01.13.27.2138.01

$$\cos^{-1}(z) = \frac{1}{2}\left(i\operatorname{csch}^{-1}\left(\frac{i}{2z^2-1}\right)+\frac{3\pi}{2}\right) /; \frac{\pi}{2} < \arg(z) \leq \pi \bigvee -\pi < \arg(z) \leq -\frac{\pi}{2}$$

01.13.27.2139.01

$$\cos^{-1}(z) = \frac{\pi}{2}\left(1 - \frac{\sqrt{z^2}}{2z}\right) - i\,\frac{\sqrt{z^2}}{2z}\,\operatorname{csch}^{-1}\left(\frac{i}{2z^2-1}\right)$$

Involving $\cos^{-1}(z)$ and $\operatorname{csch}^{-1}\left(\frac{\sqrt{2}}{\sqrt{-z-1}}\right)$

01.13.27.2140.01

$$\cos^{-1}(z) = \pi - 2\,i\operatorname{csch}^{-1}\left(\frac{\sqrt{2}}{\sqrt{-z-1}}\right) /;\ 0 < \arg(z) < \pi \bigvee (z \in \mathbb{R} \land z < -1)$$

01.13.27.2141.01

$$\cos^{-1}(z) = \pi + 2\,i\operatorname{csch}^{-1}\left(\frac{\sqrt{2}}{\sqrt{-z-1}}\right) /;\ -\pi < \arg(z) \le 0 \bigvee (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.2142.01

$$\cos^{-1}(z) = \pi - \frac{2\sqrt{1+z}}{\sqrt{-z-1}}\operatorname{csch}^{-1}\left(\frac{\sqrt{2}}{\sqrt{-z-1}}\right)$$

Involving $\cos^{-1}(z)$ and $\operatorname{csch}^{-1}\left(\sqrt{\frac{2}{-z-1}}\right)$

01.13.27.2143.01

$$\cos^{-1}(z) = \pi + 2\,i\operatorname{csch}^{-1}\left(\sqrt{\frac{2}{-z-1}}\right) /;\ \operatorname{Im}(z) < 0$$

01.13.27.2144.01

$$\cos^{-1}(z) = \pi - 2\,i\operatorname{csch}^{-1}\left(\sqrt{\frac{2}{-z-1}}\right) /;\ \operatorname{Im}(z) \ge 0$$

01.13.27.2145.01

$$\cos^{-1}(z) = \pi - 2\sqrt{1+z}\,\sqrt{-\frac{1}{z+1}}\,\operatorname{csch}^{-1}\left(\sqrt{\frac{2}{-z-1}}\right)$$

Involving $\cos^{-1}(z)$ and $\operatorname{csch}^{-1}\left(\frac{\sqrt{2}}{\sqrt{z-1}}\right)$

01.13.27.2146.01

$$\cos^{-1}(z) = -2\,i\operatorname{csch}^{-1}\left(\frac{\sqrt{2}}{\sqrt{z-1}}\right) /;\ 0 < \arg(z) \le \pi \bigvee (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.2147.01

$$\cos^{-1}(z) = 2\,i\operatorname{csch}^{-1}\left(\frac{\sqrt{2}}{\sqrt{z-1}}\right) /;\ -\pi < \arg(z) < 0 \bigvee (z \in \mathbb{R} \land z > 1)$$

01.13.27.2148.01

$$\cos^{-1}(z) = \frac{2\sqrt{1-z}}{\sqrt{z-1}} \operatorname{csch}^{-1}\left(\frac{\sqrt{2}}{\sqrt{z-1}}\right)$$

Involving $\cos^{-1}(z)$ and $\operatorname{csch}^{-1}\left(\sqrt{\frac{2}{z-1}}\right)$

01.13.27.2149.01

$$\cos^{-1}(z) = -2\,i \operatorname{csch}^{-1}\left(\sqrt{\frac{2}{z-1}}\right) /; \operatorname{Im}(z) > 0$$

01.13.27.2150.01

$$\cos^{-1}(z) = 2\,i \operatorname{csch}^{-1}\left(\sqrt{\frac{2}{z-1}}\right) /; \operatorname{Im}(z) \leq 0$$

01.13.27.2151.01

$$\cos^{-1}(z) = 2\sqrt{1-z}\,\sqrt{\frac{1}{z-1}} \operatorname{csch}^{-1}\left(\sqrt{\frac{2}{z-1}}\right)$$

Involving $\cos^{-1}(z)$ and $\operatorname{csch}^{-1}\left(\frac{1}{\sqrt{-z^2}}\right)$

01.13.27.2152.01

$$\cos^{-1}(z) = \frac{\pi}{2} - i \operatorname{csch}^{-1}\left(\frac{1}{\sqrt{-z^2}}\right) /; 0 < \arg(z) \leq \pi$$

01.13.27.2153.01

$$\cos^{-1}(z) = \frac{\pi}{2} + i \operatorname{csch}^{-1}\left(\frac{1}{\sqrt{-z^2}}\right) /; -\pi < \arg(z) \leq 0$$

01.13.27.2154.01

$$\cos^{-1}(z) = \frac{\sqrt{-z^2}}{z} \operatorname{csch}^{-1}\left(\frac{1}{\sqrt{-z^2}}\right) + \frac{\pi}{2}$$

Involving $\cos^{-1}(z)$ and $\operatorname{csch}^{-1}\left(\sqrt{-\frac{1}{z^2}}\right)$

01.13.27.2155.01

$$\cos^{-1}(z) = \frac{\pi}{2} - i \operatorname{csch}^{-1}\left(\sqrt{-\frac{1}{z^2}}\right) /; 0 \leq \arg(z) < \pi$$

01.13.27.2156.01

$$\cos^{-1}(z) = \frac{\pi}{2} + i\,\text{csch}^{-1}\left(\sqrt{-\frac{1}{z^2}}\right) /; -\pi < \arg(z) < 0 \bigvee (z \in \mathbb{R} \land z < 0)$$

01.13.27.2157.01

$$\cos^{-1}(z) = \frac{\pi}{2} - z\sqrt{-\frac{1}{z^2}}\,\text{csch}^{-1}\left(\sqrt{-\frac{1}{z^2}}\right)$$

Involving $\cos^{-1}(z)$ and $\text{csch}^{-1}\left(\frac{1}{\sqrt{-1+z^2}}\right)$

01.13.27.2158.01

$$\cos^{-1}(z) = -i\,\text{csch}^{-1}\left(\frac{1}{\sqrt{z^2-1}}\right) /; 0 < \arg(z) \leq \frac{\pi}{2} \bigvee (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.2159.01

$$\cos^{-1}(z) = i\,\text{csch}^{-1}\left(\frac{1}{\sqrt{z^2-1}}\right) /; -\frac{\pi}{2} < \arg(z) < 0 \bigvee (z \in \mathbb{R} \land z > 1)$$

01.13.27.2160.01

$$\cos^{-1}(z) = \pi - i\,\text{csch}^{-1}\left(\frac{1}{\sqrt{z^2-1}}\right) /; \frac{\pi}{2} < \arg(z) < \pi \bigvee (z \in \mathbb{R} \land z < -1)$$

01.13.27.2161.01

$$\cos^{-1}(z) = \pi + i\,\text{csch}^{-1}\left(\frac{1}{\sqrt{z^2-1}}\right) /; -\pi < \arg(z) \leq -\frac{\pi}{2} \bigvee (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.2162.01

$$\cos^{-1}(z) = \frac{\pi}{2}\left(1 - \frac{\sqrt{z^2}}{z}\right) + \frac{\sqrt{-z^2}}{z}\sqrt{\frac{z-1}{z+1}}\sqrt{\frac{z+1}{z-1}}\,\text{csch}^{-1}\left(\frac{1}{\sqrt{z^2-1}}\right)$$

Involving $\cos^{-1}(z)$ and $\text{csch}^{-1}\left(\sqrt{\frac{1}{z^2-1}}\right)$

01.13.27.2163.01

$$\cos^{-1}(z) = -i\,\text{csch}^{-1}\left(\sqrt{\frac{1}{z^2-1}}\right) /; 0 < \arg(z) < \frac{\pi}{2}$$

01.13.27.2164.01

$$\cos^{-1}(z) = i\,\text{csch}^{-1}\left(\sqrt{\frac{1}{z^2-1}}\right) /; -\frac{\pi}{2} < \arg(z) \leq 0 \bigvee (i\,z \in \mathbb{R} \land i\,z < 0)$$

01.13.27.2165.01

$$\cos^{-1}(z) = \pi - i\,\mathrm{csch}^{-1}\left(\sqrt{\frac{1}{z^2-1}}\right) /; \frac{\pi}{2} < \arg(z) \le \pi \bigvee (i\,z \in \mathbb{R} \land i\,z > 0)$$

01.13.27.2166.01

$$\cos^{-1}(z) = \pi + i\,\mathrm{csch}^{-1}\left(\sqrt{\frac{1}{z^2-1}}\right) /; -\pi < \arg(z) < -\frac{\pi}{2}$$

01.13.27.2167.01

$$\cos^{-1}(z) = \frac{\pi}{2}\left(1 - \frac{\sqrt{z^2}}{z}\right) + \frac{\sqrt{-z-1}\,\sqrt{z-1}\,z}{\sqrt{z^2-1}}\sqrt{\frac{1}{z^2}}\,\mathrm{csch}^{-1}\left(\sqrt{\frac{1}{z^2-1}}\right)$$

Involving $\cos^{-1}(z)$ and $\mathrm{csch}^{-1}\left(\frac{1}{2\,z\sqrt{z^2-1}}\right)$

01.13.27.2168.01

$$\cos^{-1}(z) = \frac{\pi}{2} - \frac{\sqrt{-z^4}}{2\,z^2}\,\mathrm{csch}^{-1}\left(\frac{1}{2\,z\sqrt{z^2-1}}\right) /; \frac{\pi}{4} \le |\arg(z)| \le \frac{3\,\pi}{4}$$

01.13.27.2169.01

$$\cos^{-1}(z) = \frac{1}{4}\pi\left(2 - \frac{\sqrt{z^2}}{z} + \sqrt{\frac{1}{z}}\,\sqrt{z}\,\sqrt{\frac{1}{\sqrt{2}\,z+1}}\,\sqrt{\sqrt{2}\,z+1} - \sqrt{-\frac{1}{z}}\,\sqrt{-z}\,\sqrt{\frac{1}{1-\sqrt{2}\,z}}\,\sqrt{1-\sqrt{2}\,z} - \frac{\sqrt{z^4-z^2}}{z\sqrt{z^2-1}}\right) - \frac{\sqrt{z^2\left(z^2-1\right)}\,\sqrt{2\,z^2-1}}{2\,z^2\sqrt{1-2\,z^2}}\sqrt{\frac{z^2}{z^2-1}}\,\mathrm{csch}^{-1}\left(\frac{1}{2\,z\sqrt{z^2-1}}\right)$$

Involving $\cos^{-1}(c\,z)$

Involving $\cos^{-1}(i\,z)$ and $\mathrm{csch}^{-1}\left(\frac{1}{z}\right)$

01.13.27.2170.01

$$\cos^{-1}(i\,z) = \frac{\pi}{2} - i\,\mathrm{csch}^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}(-i\,z)$ and $\mathrm{csch}^{-1}\left(\frac{1}{z}\right)$

01.13.27.2171.01

$$\cos^{-1}(-i\,z) = \frac{\pi}{2} + i\,\mathrm{csch}^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\sqrt{c\, z}\right)$

Involving $\cos^{-1}\left(\sqrt{z}\right)$ and $\operatorname{csch}^{-1}\left(\frac{1}{\sqrt{-z}}\right)$

01.13.27.2172.01

$$\cos^{-1}\left(\sqrt{z}\right) = \frac{\pi}{2} - i \operatorname{csch}^{-1}\left(\frac{1}{\sqrt{-z}}\right) /;\ 0 < \arg(z) \leq \pi$$

01.13.27.2173.01

$$\cos^{-1}\left(\sqrt{z}\right) = \frac{\pi}{2} + i \operatorname{csch}^{-1}\left(\frac{1}{\sqrt{-z}}\right) /;\ -\pi < \arg(z) \leq 0$$

01.13.27.2174.01

$$\cos^{-1}\left(\sqrt{z}\right) = \frac{\pi}{2} + \frac{\sqrt{-z^2}}{z} \operatorname{csch}^{-1}\left(\frac{1}{\sqrt{-z}}\right)$$

Involving $\cos^{-1}\left(\sqrt{z}\right)$ and $\operatorname{csch}^{-1}\left(\sqrt{-\frac{1}{z}}\right)$

01.13.27.2175.01

$$\cos^{-1}\left(\sqrt{z}\right) = \frac{\pi}{2} - i \operatorname{csch}^{-1}\left(\sqrt{-\frac{1}{z}}\right) /;\ \operatorname{Im}(z) \geq 0$$

01.13.27.2176.01

$$\cos^{-1}\left(\sqrt{z}\right) = \frac{\pi}{2} + i \operatorname{csch}^{-1}\left(\sqrt{-\frac{1}{z}}\right) /;\ \operatorname{Im}(z) < 0$$

01.13.27.2177.01

$$\cos^{-1}\left(\sqrt{z}\right) = \frac{\pi}{2} - \sqrt{-\frac{1}{z}}\ \sqrt{z}\ \operatorname{csch}^{-1}\left(\sqrt{-\frac{1}{z}}\right)$$

Involving $\cos^{-1}\left(\sqrt{-z}\right)$ and $\operatorname{csch}^{-1}\left(\frac{1}{\sqrt{z}}\right)$

01.13.27.2178.01

$$\cos^{-1}\left(\sqrt{-z}\right) = \frac{\pi}{2} + i \operatorname{csch}^{-1}\left(\frac{1}{\sqrt{z}}\right) /;\ 0 < \arg(z) \leq \pi$$

01.13.27.2179.01

$$\cos^{-1}\left(\sqrt{-z}\right) = \frac{\pi}{2} - i \operatorname{csch}^{-1}\left(\frac{1}{\sqrt{z}}\right) /;\ -\pi < \arg(z) \leq 0$$

01.13.27.2180.01

$$\cos^{-1}\left(\sqrt{-z}\right) = \frac{\pi}{2} - \frac{\sqrt{-z^2}}{z} \operatorname{csch}^{-1}\left(\frac{1}{\sqrt{z}}\right)$$

Involving $\cos^{-1}(\sqrt{-z})$ and $\operatorname{csch}^{-1}\left(\sqrt{\frac{1}{z}}\right)$

01.13.27.2181.01

$$\cos^{-1}(\sqrt{-z}) = \frac{\pi}{2} + i\operatorname{csch}^{-1}\left(\sqrt{\frac{1}{z}}\right) /; 0 < \arg(z) < \pi$$

01.13.27.2182.01

$$\cos^{-1}(\sqrt{-z}) = \frac{\pi}{2} - i\operatorname{csch}^{-1}\left(\sqrt{\frac{1}{z}}\right) /; \operatorname{Im}(z) \leq 0$$

01.13.27.2183.01

$$\cos^{-1}(\sqrt{-z}) = \frac{\pi}{2} - \sqrt{\frac{1}{z}}\sqrt{-z}\operatorname{csch}^{-1}\left(\sqrt{\frac{1}{z}}\right)$$

Involving $\cos^{-1}\left(\frac{1}{\sqrt{c z}}\right)$

Involving $\cos^{-1}\left(\frac{1}{\sqrt{z}}\right)$ and $\operatorname{csch}^{-1}(\sqrt{-z})$

01.13.27.2184.01

$$\cos^{-1}\left(\frac{1}{\sqrt{z}}\right) = \frac{\pi}{2} + i\operatorname{csch}^{-1}(\sqrt{-z}) /; 0 < \arg(z) \leq \pi$$

01.13.27.2185.01

$$\cos^{-1}\left(\frac{1}{\sqrt{z}}\right) = \frac{\pi}{2} - i\operatorname{csch}^{-1}(\sqrt{-z}) /; -\pi < \arg(z) \leq 0$$

01.13.27.2186.01

$$\cos^{-1}\left(\frac{1}{\sqrt{z}}\right) = \frac{\pi}{2} - \frac{\sqrt{-z^2}}{z}\operatorname{csch}^{-1}(\sqrt{-z})$$

Involving $\cos^{-1}\left(\frac{1}{\sqrt{-z}}\right)$ and $\operatorname{csch}^{-1}(\sqrt{z})$

01.13.27.2187.01

$$\cos^{-1}\left(\frac{1}{\sqrt{-z}}\right) = \frac{\pi}{2} - i\operatorname{csch}^{-1}(\sqrt{z}) /; 0 < \arg(z) \leq \pi$$

01.13.27.2188.01

$$\cos^{-1}\left(\frac{1}{\sqrt{-z}}\right) = \frac{\pi}{2} + i\operatorname{csch}^{-1}(\sqrt{z}) /; -\pi < \arg(z) \leq 0$$

01.13.27.2189.01

$$\cos^{-1}\left(\frac{1}{\sqrt{-z}}\right) = \frac{\pi}{2} - \frac{\sqrt{z}}{\sqrt{-z}}\operatorname{csch}^{-1}(\sqrt{z})$$

Involving $\cos^{-1}\left(\sqrt{c\, z^2}\right)$

Involving $\cos^{-1}\left(\sqrt{z^2}\right)$ and $\operatorname{csch}^{-1}\left(\frac{i}{z}\right)$

01.13.27.2190.01

$$\cos^{-1}\left(\sqrt{z^2}\right) = \frac{\pi}{2} - i \operatorname{csch}^{-1}\left(\frac{i}{z}\right) /; -\frac{\pi}{2} < \arg(z) \le \frac{\pi}{2}$$

01.13.27.2191.01

$$\cos^{-1}\left(\sqrt{z^2}\right) = \frac{\pi}{2} + i \operatorname{csch}^{-1}\left(\frac{i}{z}\right) /; \frac{\pi}{2} < \arg(z) \le \pi \bigvee -\pi < \arg(z) \le -\frac{\pi}{2}$$

01.13.27.2192.01

$$\cos^{-1}\left(\sqrt{z^2}\right) = \frac{\pi}{2} - \frac{i\sqrt{z^2}}{z} \operatorname{csch}^{-1}\left(\frac{i}{z}\right)$$

Involving $\cos^{-1}\left(\sqrt{-z^2}\right)$ and $\operatorname{csch}^{-1}\left(\frac{1}{z}\right)$

01.13.27.2193.01

$$\cos^{-1}\left(\sqrt{-z^2}\right) = \frac{\pi}{2} - i \operatorname{csch}^{-1}\left(\frac{1}{z}\right) /; -\pi < \arg(z) \le 0$$

01.13.27.2194.01

$$\cos^{-1}\left(\sqrt{-z^2}\right) = \frac{\pi}{2} + i \operatorname{csch}^{-1}\left(\frac{1}{z}\right) /; 0 < \arg(z) \le \pi$$

01.13.27.2195.01

$$\cos^{-1}\left(\sqrt{-z^2}\right) = \frac{\pi}{2} - \frac{\sqrt{-z^2}}{z} \operatorname{csch}^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(a\left(b\, z^c\right)^m\right)$

Involving $\cos^{-1}(a\,(b\, z^c)^m)$ and $\operatorname{csch}^{-1}\left(\frac{i}{a}\, b^{-m}\, z^{-m\,c}\right)$

01.13.27.2196.01

$$\cos^{-1}(a\,(b\, z^c)^m) = \frac{\pi}{2} - \frac{i\,(b\, z^c)^m}{b^m\, z^{m\,c}} \operatorname{csch}^{-1}\left(\frac{i}{a}\, b^{-m}\, z^{-m\,c}\right) /; 2\, m \in \mathbb{Z}$$

Involving $\cos^{-1}\left(1 + 2\, z^2\right)$

Involving $\cos^{-1}\left(1 + 2\, z^2\right)$ and $\operatorname{csch}^{-1}\left(\frac{1}{z}\right)$

01.13.27.2197.01

$$\cos^{-1}(1+2z^2) = 2\,i\,\text{csch}^{-1}\left(\frac{1}{z}\right)\,/;\,-\pi < \arg(z) \le 0$$

01.13.27.2198.01

$$\cos^{-1}(2z^2+1) = -2\,i\,\text{csch}^{-1}\left(\frac{1}{z}\right)\,/;\,0 < \arg(z) \le \pi$$

01.13.27.2199.01

$$\cos^{-1}(2z^2+1) = \frac{2\sqrt{-z^2}}{z}\,\text{csch}^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\frac{z^2+2}{z^2}\right)$

Involving $\cos^{-1}\left(\frac{z^2+2}{z^2}\right)$ and $\text{csch}^{-1}(z)$

01.13.27.2200.01

$$\cos^{-1}\left(\frac{z^2+2}{z^2}\right) = 2\,i\,\text{csch}^{-1}(z)\,/;\,0 \le \arg(z) < \pi$$

01.13.27.2201.01

$$\cos^{-1}\left(\frac{z^2+2}{z^2}\right) = -2\,i\,\text{csch}^{-1}(z)\,/;\,\text{Im}(z) < 0 \lor (z \in \mathbb{R} \land z < 0)$$

01.13.27.2202.01

$$\cos^{-1}\left(\frac{z^2+2}{z^2}\right) = 2\,z\,\sqrt{-\frac{1}{z^2}}\,\text{csch}^{-1}(z)$$

Involving $\cos^{-1}\left(\sqrt{1+z}\right)$

Involving $\cos^{-1}\left(\sqrt{z+1}\right)$ and $\text{csch}^{-1}\left(\frac{1}{\sqrt{z}}\right)$

01.13.27.2203.01

$$\cos^{-1}\left(\sqrt{z+1}\right) = i\,\text{csch}^{-1}\left(\frac{1}{\sqrt{z}}\right)\,/;\,-\pi < \arg(z) \le 0$$

01.13.27.2204.01

$$\cos^{-1}\left(\sqrt{z+1}\right) = -i\,\text{csch}^{-1}\left(\frac{1}{\sqrt{z}}\right)\,/;\,0 < \arg(z) \le \pi$$

01.13.27.2205.01

$$\cos^{-1}\left(\sqrt{1+z}\right) = \frac{\sqrt{-z^2}}{z}\,\text{csch}^{-1}\left(\frac{1}{\sqrt{z}}\right)$$

Involving $\cos^{-1}\left(\sqrt{z+1}\right)$ and $\operatorname{csch}^{-1}\left(\sqrt{\frac{1}{z}}\right)$

01.13.27.2206.01

$$\cos^{-1}\left(\sqrt{z+1}\right) = -i \operatorname{csch}^{-1}\left(\sqrt{\frac{1}{z}}\right) /; 0 < \arg(z) < \pi$$

01.13.27.2207.01

$$\cos^{-1}\left(\sqrt{z+1}\right) = i \operatorname{csch}^{-1}\left(\sqrt{\frac{1}{z}}\right) /; \operatorname{Im}(z) \leq 0$$

01.13.27.2208.01

$$\cos^{-1}\left(\sqrt{1+z}\right) = \sqrt{\frac{1}{z}} \sqrt{-z} \operatorname{csch}^{-1}\left(\sqrt{\frac{1}{z}}\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{1+c z}}{\sqrt{2}}\right)$

Involving $\cos^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{2}}\right)$ and $\operatorname{csch}^{-1}\left(\frac{i}{z}\right)$

01.13.27.2209.01

$$\cos^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{2}}\right) = \frac{1}{2}\left(\frac{\pi}{2} + i \operatorname{csch}^{-1}\left(\frac{i}{z}\right)\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{1+z}}{\sqrt{2}}\right)$ and $\operatorname{csch}^{-1}\left(\frac{i}{z}\right)$

01.13.27.2210.01

$$\cos^{-1}\left(\frac{\sqrt{1+z}}{\sqrt{2}}\right) = \frac{1}{2}\left(\frac{\pi}{2} - i \operatorname{csch}^{-1}\left(\frac{i}{z}\right)\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{z+c}}{\sqrt{z}}\right)$

Involving $\cos^{-1}\left(\frac{\sqrt{z+a}}{\sqrt{z}}\right)$ and $\operatorname{csch}^{-1}\left(\sqrt{\frac{z}{a}}\right)$

01.13.27.2211.01

$$\cos^{-1}\left(\frac{\sqrt{z+1}}{\sqrt{z}}\right) = i \operatorname{csch}^{-1}\left(\sqrt{z}\right) /; 0 \leq \arg(z) < \pi \bigvee (z \in \mathbb{R} \land z < -1)$$

01.13.27.2212.01

$$\cos^{-1}\left(\frac{\sqrt{z+1}}{\sqrt{z}}\right) = -i \operatorname{csch}^{-1}\left(\sqrt{z}\right) /; \operatorname{Im}(z) < 0$$

01.13.27.2213.01

$$\cos^{-1}\left(\frac{\sqrt{z+1}}{\sqrt{z}}\right) = \pi - i\,\text{csch}^{-1}\left(\sqrt{z}\right) /; (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.0071.01

$$\cos^{-1}\left(\frac{\sqrt{z+1}}{\sqrt{z}}\right) = \frac{\pi}{2} - \frac{\sqrt{-z}\,\sqrt{z+1}}{\sqrt{-z-1}\,\sqrt{z}}\left(\frac{\pi}{2} - \sqrt{-\frac{1}{z}}\,\sqrt{z}\,\text{csch}^{-1}\left(\sqrt{z}\right)\right)$$

01.13.27.0064.01

$$\cos^{-1}\left(\frac{\sqrt{z+1}}{\sqrt{z}}\right) = \frac{\pi}{2}\left(1 - \sqrt{1+\frac{1}{z}}\,\sqrt{\frac{z}{z+1}}\right) + \frac{z}{z+1}\sqrt{-\frac{(z+1)^2}{z^2}}\,\text{csch}^{-1}\left(\sqrt{z}\right)$$

01.13.27.0069.01

$$\cos^{-1}\left(\frac{\sqrt{a+z}}{\sqrt{z}}\right) = \frac{\pi}{2} - \frac{\sqrt{a+z}}{\sqrt{\frac{a}{z}+1}\,\sqrt{z}}\left(\frac{\pi}{2} - \sqrt{-\frac{a}{z}}\,\sqrt{\frac{z}{a}}\,\text{csch}^{-1}\left(\sqrt{\frac{z}{a}}\right)\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{-1-z}}{\sqrt{-z}}\right)$

Involving $\cos^{-1}\left(\frac{\sqrt{-1-z}}{\sqrt{-z}}\right)$ and $\text{csch}^{-1}\left(\sqrt{z}\right)$

01.13.27.2214.01

$$\cos^{-1}\left(\frac{\sqrt{-z-1}}{\sqrt{-z}}\right) = i\,\text{csch}^{-1}\left(\sqrt{z}\right) /; \text{Im}(z) \geq 0$$

01.13.27.2215.01

$$\cos^{-1}\left(\frac{\sqrt{-z-1}}{\sqrt{-z}}\right) = -i\,\text{csch}^{-1}\left(\sqrt{z}\right) /; \text{Im}(z) < 0$$

01.13.27.2216.01

$$\cos^{-1}\left(\frac{\sqrt{-z-1}}{\sqrt{-z}}\right) = \sqrt{-\frac{1}{z}}\,\sqrt{z}\,\text{csch}^{-1}\left(\sqrt{z}\right)$$

Involving $\cos^{-1}\left(\sqrt{\frac{z+1}{z}}\right)$

Involving $\cos^{-1}\left(\frac{\sqrt{-1-z}}{\sqrt{-z}}\right)$ and $\text{csch}^{-1}\left(\sqrt{z}\right)$

01.13.27.2217.01

$$\cos^{-1}\left(\sqrt{\frac{z+1}{z}}\right) = i\,\text{csch}^{-1}\left(\sqrt{z}\right) /; \text{Im}(z) \geq 0$$

01.13.27.2218.01

$$\cos^{-1}\left(\sqrt{\frac{z+1}{z}}\right) == -i \operatorname{csch}^{-1}\left(\sqrt{z}\right) /; \operatorname{Im}(z) < 0$$

01.13.27.0063.01

$$\cos^{-1}\left(\sqrt{\frac{z+1}{z}}\right) == \sqrt{z} \sqrt{-\frac{1}{z}} \operatorname{csch}^{-1}\left(\sqrt{z}\right)$$

Involving $\cos^{-1}\left(\sqrt{a z^c + 1}\right)$

Involving $\cos^{-1}\left(\sqrt{a z^c + 1}\right)$ and $\operatorname{csch}^{-1}\left(\frac{1}{\sqrt{a}\, z^{c/2}}\right)$

01.13.27.2219.01

$$\cos^{-1}\left(\sqrt{z^2+1}\right) == i \operatorname{csch}^{-1}\left(\frac{1}{z}\right) /; -\pi < \arg(z) \leq 0$$

01.13.27.2220.01

$$\cos^{-1}\left(\sqrt{z^2+1}\right) == -i \operatorname{csch}^{-1}\left(\frac{1}{z}\right) /; 0 < \arg(z) \leq \pi$$

01.13.27.2221.01

$$\cos^{-1}\left(\sqrt{z^2+1}\right) == \frac{\sqrt{-z^2}}{z} \operatorname{csch}^{-1}\left(\frac{1}{z}\right)$$

01.13.27.2222.01

$$\cos^{-1}\left(\sqrt{a z^c + 1}\right) == -\frac{\sqrt{a}\, z^{c/2}}{\sqrt{-a z^c}} \operatorname{csch}^{-1}\left(\frac{1}{\sqrt{a}\, z^{c/2}}\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{z^2+1}}{z}\right)$

Involving $\cos^{-1}\left(\frac{\sqrt{z^2+1}}{z}\right)$ and $\operatorname{csch}^{-1}(z)$

01.13.27.2223.01

$$\cos^{-1}\left(\frac{\sqrt{z^2+1}}{z}\right) == i \operatorname{csch}^{-1}(z) /; 0 \leq \arg(z) < \frac{\pi}{2} \bigvee (i z \in \mathbb{R} \land i z < -1)$$

01.13.27.2224.01

$$\cos^{-1}\left(\frac{\sqrt{z^2+1}}{z}\right) == -i \operatorname{csch}^{-1}(z) /; -\frac{\pi}{2} < \arg(z) < 0 \bigvee (i z \in \mathbb{R} \land 0 < i z < 1)$$

01.13.27.2225.01

$$\cos^{-1}\left(\frac{\sqrt{z^2+1}}{z}\right) = \pi - i\,\mathrm{csch}^{-1}(z) \;/;\; \frac{\pi}{2} < \arg(z) < \pi \bigvee (-i\,z \in \mathbb{R} \land 0 < -i\,z < 1)$$

01.13.27.2226.01

$$\cos^{-1}\left(\frac{\sqrt{z^2+1}}{z}\right) = \pi + i\,\mathrm{csch}^{-1}(z) \;/;\; -\pi < \arg(z) < -\frac{\pi}{2} \bigvee (z \in \mathbb{R} \land z < 0) \bigvee (i\,z \in \mathbb{R} \land i\,z > 1)$$

01.13.27.2227.01

$$\cos^{-1}\left(\frac{\sqrt{z^2+1}}{z}\right) = \frac{\pi}{2}\left(1 - \frac{\sqrt{z^2}}{z}\right) + \sqrt{-\frac{1}{z^2}}\,\sqrt{z^2}\,\mathrm{csch}^{-1}(z) \;/;\; i\,z \notin (-1,\,1)$$

01.13.27.0065.01

$$\cos^{-1}\left(\frac{\sqrt{z^2+1}}{z}\right) = \sqrt{-\frac{1}{z^2}}\,z\,\mathrm{csch}^{-1}(z) \;/;\; \mathrm{Re}(z) > 0$$

01.13.27.0066.01

$$\cos^{-1}\left(\frac{\sqrt{z^2+1}}{z}\right) = \frac{\pi}{2}\left(1 - \frac{\sqrt{z^2}}{z}\right) - \frac{\sqrt{-z^4}\,\mathrm{csch}^{-1}(z)}{z^2} \;/;\; \mathrm{Re}(z)\,\mathrm{Im}(z) \neq 0$$

01.13.27.0067.01

$$\cos^{-1}\left(\frac{\sqrt{z^2+1}}{z}\right) = \frac{\pi}{2}\left(1 - \frac{z}{\sqrt{z^2+1}}\sqrt{\frac{z^2+1}{z^2}}\right) - \frac{\sqrt{z^2+1}\,\mathrm{csch}^{-1}(z)}{z^2\sqrt{\frac{-z^2-1}{z^4}}}$$

Involving $\cos^{-1}\left(\frac{\sqrt{1+z^2}}{\sqrt{z^2}}\right)$

Involving $\cos^{-1}\left(\frac{\sqrt{1+z^2}}{\sqrt{z^2}}\right)$ and $\mathrm{csch}^{-1}(z)$

01.13.27.2228.01

$$\cos^{-1}\left(\frac{\sqrt{z^2+1}}{\sqrt{z^2}}\right) = i\,\mathrm{csch}^{-1}(z) \;/;\; 0 \leq \arg(z) < \frac{\pi}{2} \bigvee \frac{\pi}{2} < \arg(z) < \pi \bigvee (i\,z \in \mathbb{R} \land i\,z < -1)$$

01.13.27.2229.01

$$\cos^{-1}\left(\frac{\sqrt{z^2+1}}{\sqrt{z^2}}\right) = -i\,\mathrm{csch}^{-1}(z) \;/;\; -\pi < \arg(z) < -\frac{\pi}{2} \bigvee -\frac{\pi}{2} < \arg(z) < 0 \bigvee (z \in \mathbb{R} \land z < 0) \bigvee (i\,z \in \mathbb{R} \land i\,z > 1)$$

01.13.27.2230.01

$$\cos^{-1}\left(\frac{\sqrt{z^2+1}}{\sqrt{z^2}}\right) = \pi - i\operatorname{csch}^{-1}(z) /; (-i z \in \mathbb{R} \land 0 < -i z < 1)$$

01.13.27.2231.01

$$\cos^{-1}\left(\frac{\sqrt{z^2+1}}{\sqrt{z^2}}\right) = \pi + i\operatorname{csch}^{-1}(z) /; (i z \in \mathbb{R} \land 0 < i z < 1)$$

01.13.27.2232.01

$$\cos^{-1}\left(\frac{\sqrt{z^2+1}}{\sqrt{z^2}}\right) = \frac{\pi}{2} - \frac{\sqrt{z^2}\sqrt{z}}{\sqrt{-z}\sqrt{-z^2-1}}\sqrt{\frac{-z^2-1}{z^2}}\left(\operatorname{csch}^{-1}(z) + \frac{1}{2}(\pi z)\sqrt{-\frac{1}{z^2}}\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{-1-z^2}}{\sqrt{-z^2}}\right)$

Involving $\cos^{-1}\left(\frac{\sqrt{-1-z^2}}{\sqrt{-z^2}}\right)$ and $\operatorname{csch}^{-1}(z)$

01.13.27.2233.01

$$\cos^{-1}\left(\frac{\sqrt{-z^2-1}}{\sqrt{-z^2}}\right) = i\operatorname{csch}^{-1}(z) /; 0 \leq \arg(z) < \pi$$

01.13.27.2234.01

$$\cos^{-1}\left(\frac{\sqrt{-z^2-1}}{\sqrt{-z^2}}\right) = -i\operatorname{csch}^{-1}(z) /; \operatorname{Im}(z) < 0 \lor (z \in \mathbb{R} \land z < 0)$$

01.13.27.2235.01

$$\cos^{-1}\left(\frac{\sqrt{-1-z^2}}{\sqrt{-z^2}}\right) = \sqrt{-\frac{1}{z^2}}\, z \operatorname{csch}^{-1}(z)$$

Involving $\cos^{-1}\left(\sqrt{\frac{z^2+1}{z^2}}\right)$

Involving $\cos^{-1}\left(\sqrt{\frac{z^2+1}{z^2}}\right)$ and $\operatorname{csch}^{-1}(z)$

01.13.27.2236.01

$$\cos^{-1}\left(\sqrt{\frac{z^2+1}{z^2}}\right) = i\operatorname{csch}^{-1}(z) /; 0 \leq \arg(z) < \pi$$

01.13.27.2237.01

$$\cos^{-1}\left(\sqrt{\frac{z^2+1}{z^2}}\right) = -i\,\operatorname{csch}^{-1}(z)\ /;\ \operatorname{Im}(z) < 0 \lor (z \in \mathbb{R} \land z < 0)$$

01.13.27.2238.01

$$\cos^{-1}\left(\sqrt{\frac{z^2+1}{z^2}}\right) = \sqrt{-\frac{1}{z^2}}\ z\,\operatorname{csch}^{-1}(z)$$

Involving $\cos^{-1}\left(2\,z\,\sqrt{-1-z^2}\right)$

Involving $\cos^{-1}\left(2\,z\,\sqrt{-1-z^2}\right)$ and $\operatorname{csch}^{-1}\left(\frac{1}{z}\right)$

01.13.27.2523.01

$$\cos^{-1}\left(2\,z\,\sqrt{-z^2-1}\right) = \frac{\pi}{2} - \frac{2\sqrt{-z^4}}{z^2}\operatorname{csch}^{-1}\left(\frac{1}{z}\right)\ /;\ |\arg(z)| \le \frac{\pi}{4} \lor \frac{3\pi}{4} \le |\arg(z)| \le \pi$$

01.13.27.2239.01

$$\cos^{-1}\left(2\,z\,\sqrt{-z^2-1}\right) = \frac{\pi}{2} - \frac{\pi\sqrt{2\,z^2+1}\,\sqrt{z^4+z^2}}{2\sqrt{z^2}\,\sqrt{-2\,z^2-1}\,\sqrt{z^2+1}}\left(-\frac{\sqrt{-z^2}}{z} - i\sqrt{\frac{i}{z}}\,\sqrt{-i\,z}\,\sqrt{\frac{1}{1-i\sqrt{2}\,z}}\,\sqrt{1-i\sqrt{2}\,z} + i\sqrt{-\frac{i}{z}}\,\sqrt{i\,z}\,\sqrt{\frac{1}{\sqrt{2}\,i\,z+1}}\,\sqrt{\sqrt{2}\,i\,z+1} - \frac{\sqrt{z^4+z^2}}{z\sqrt{-z^2-1}}\right) + \frac{2\sqrt{2\,z^2+1}\,\sqrt{z^4+z^2}}{\sqrt{z^2}\,\sqrt{-2\,z^2-1}\,\sqrt{z^2+1}}\operatorname{csch}^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\frac{2\sqrt{-1-z^2}}{z^2}\right)$

Involving $\cos^{-1}\left(\frac{2\sqrt{-1-z^2}}{z^2}\right)$ and $\operatorname{csch}^{-1}(z)$

01.13.27.2240.01

$$\cos^{-1}\left(\frac{2\sqrt{-1-z^2}}{z^2}\right) = \frac{\pi}{2} - \frac{2\sqrt{-z^2}}{z}\operatorname{csch}^{-1}(z)\ /;\ |\arg(z)| \le \frac{\pi}{4} \lor \frac{3\pi}{4} \le |\arg(z)| \le \pi \lor |z| \ge \sqrt{2}$$

01.13.27.2241.01

$$\cos^{-1}\left(\frac{2\sqrt{-z^2-1}}{z^2}\right) = \frac{\pi}{2} - \frac{z^3\sqrt{-z^2-2}\sqrt{-z^2-1}}{2\sqrt{1-iz}\,(iz+1)\sqrt{-z^4-3z^2-2}}\sqrt{\frac{z^2+1}{z^4}}\sqrt{\frac{i-z}{z}}\sqrt{-\frac{i}{z}}\left(\pi\left(-\frac{z^3}{z^2+1}\sqrt{-\frac{z^2+1}{z^2}}\sqrt{\frac{z^2+1}{z^4}}+\sqrt{-\frac{1}{z^2}}\,z+ i\sqrt{\frac{z-i\sqrt{2}}{z}}\sqrt{-\frac{i}{z}}\sqrt{iz}\sqrt{\frac{z}{-i\sqrt{2}+z}} - i\sqrt{\frac{z+i\sqrt{2}}{z}}\sqrt{\frac{i}{z}}\sqrt{-iz}\sqrt{\frac{z}{i\sqrt{2}+z}}\right)+4\operatorname{csch}^{-1}(z)\right)$$

Involving $\cos^{-1}\left(\sqrt{\left(1-\sqrt{1+z^2}\right)/2}\right)$

Involving $\cos^{-1}\left(\sqrt{\left(1-\sqrt{1+z^2}\right)/2}\right)$ and $\operatorname{csch}^{-1}\left(\frac{1}{z}\right)$

01.13.27.2242.01

$$\cos^{-1}\left(\sqrt{\frac{1-\sqrt{z^2+1}}{2}}\right) = \frac{\pi}{2} - \frac{i}{2}\operatorname{csch}^{-1}\left(\frac{1}{z}\right) /; -\pi < \arg(z) \leq 0$$

01.13.27.2243.01

$$\cos^{-1}\left(\sqrt{\frac{1}{2}\left(1-\sqrt{z^2+1}\right)}\right) = \frac{\pi}{2} + \frac{i}{2}\operatorname{csch}^{-1}\left(\frac{1}{z}\right) /; 0 < \arg(z) \leq \pi$$

01.13.27.2244.01

$$\cos^{-1}\left(\sqrt{\frac{1-\sqrt{z^2+1}}{2}}\right) = \frac{\pi}{2} - \frac{\sqrt{-z^2}}{2z}\operatorname{csch}^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(z\sqrt{1-\sqrt{1+z^2}}\,/\sqrt{2z^2}\right)$

Involving $\cos^{-1}\left(z\sqrt{1-\sqrt{1+z^2}}\,/\sqrt{2z^2}\right)$ and $\operatorname{csch}^{-1}\left(\frac{1}{z}\right)$

01.13.27.2245.01

$$\cos^{-1}\left(\frac{z\sqrt{1-\sqrt{z^2+1}}}{\sqrt{2}\sqrt{z^2}}\right) = \frac{\pi}{2} - \frac{i}{2}\operatorname{csch}^{-1}\left(\frac{1}{z}\right) /; \frac{\pi}{2} < \arg(z) \leq \pi \bigvee -\frac{\pi}{2} < \arg(z) \leq 0$$

01.13.27.2246.01

$$\cos^{-1}\left(\frac{z\sqrt{1-\sqrt{z^2+1}}}{\sqrt{2}\sqrt{z^2}}\right) = \frac{\pi}{2} + \frac{i}{2}\operatorname{csch}^{-1}\left(\frac{1}{z}\right) /; 0 < \arg(z) \leq \frac{\pi}{2} \bigvee -\pi < \arg(z) \leq -\frac{\pi}{2}$$

01.13.27.2247.01

$$\cos^{-1}\left(\frac{z\sqrt{1-\sqrt{z^2+1}}}{\sqrt{2}\sqrt{z^2}}\right) = \frac{\pi}{2} - \frac{\sqrt{-z^4}}{2z^2}\operatorname{csch}^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(z\sqrt{\left(1-\sqrt{1+z^2}\right)/\left(2z^2\right)}\right)$

Involving $\cos^{-1}\left(z\sqrt{\left(1-\sqrt{1+z^2}\right)/\left(2z^2\right)}\right)$ and $\operatorname{csch}^{-1}\left(\frac{1}{z}\right)$

01.13.27.2248.01

$$\cos^{-1}\left(z\sqrt{\frac{1-\sqrt{z^2+1}}{2z^2}}\right) = \frac{\pi}{2} - \frac{i}{2}\operatorname{csch}^{-1}\left(\frac{1}{z}\right) /; 0 \leq \arg(z) \leq \frac{\pi}{2} \bigvee -\pi < \arg(z) \leq -\frac{\pi}{2} \bigvee (z \in \mathbb{R} \land z < 0)$$

01.13.27.2249.01

$$\cos^{-1}\left(z\sqrt{\frac{1-\sqrt{z^2+1}}{2z^2}}\right) = \frac{\pi}{2} + \frac{i}{2}\operatorname{csch}^{-1}\left(\frac{1}{z}\right) /; \frac{\pi}{2} < \arg(z) < \pi \bigvee -\frac{\pi}{2} < \arg(z) < 0$$

01.13.27.2250.01

$$\cos^{-1}\left(z\sqrt{\frac{1-\sqrt{z^2+1}}{2z^2}}\right) = \frac{\pi}{2} - \frac{1}{2}\sqrt{-\frac{1}{z^2}}\sqrt{z^2}\operatorname{csch}^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\sqrt{z-\sqrt{z^2+1}}\Big/\sqrt{2z}\right)$

Involving $\cos^{-1}\left(\sqrt{z-\sqrt{z^2+1}}\middle/\sqrt{2z}\right)$ and $\operatorname{csch}^{-1}(z)$

01.13.27.2251.01

$$\cos^{-1}\left(\frac{\sqrt{z-\sqrt{z^2+1}}}{\sqrt{2z}}\right) = \frac{\pi}{2}-\frac{1}{2}\,i\operatorname{csch}^{-1}(z)\;/;\;0<\arg(z)<\frac{\pi}{2}\bigvee(i\,z\in\mathbb{R}\wedge i\,z<-1)$$

01.13.27.2252.01

$$\cos^{-1}\left(\frac{\sqrt{z-\sqrt{z^2+1}}}{\sqrt{2z}}\right) = \frac{\pi}{2}+\frac{1}{2}\,i\operatorname{csch}^{-1}(z)\;/;\;-\frac{\pi}{2}<\arg(z)<0\bigvee(i\,z\in\mathbb{R}\wedge 0<i\,z<1)$$

01.13.27.2253.01

$$\cos^{-1}\left(\frac{\sqrt{z-\sqrt{z^2+1}}}{\sqrt{2z}}\right) = \frac{1}{2}\,i\operatorname{csch}^{-1}(z)\;/;\;\frac{\pi}{2}<\arg(z)<\pi\bigvee(-i\,z\in\mathbb{R}\wedge 0<-i\,z<1)$$

01.13.27.2254.01

$$\cos^{-1}\left(\frac{\sqrt{z-\sqrt{z^2+1}}}{\sqrt{2z}}\right) = -\frac{1}{2}\,i\operatorname{csch}^{-1}(z)\;/;\;-\pi<\arg(z)<-\frac{\pi}{2}\bigvee(i\,z\in\mathbb{R}\wedge i\,z>1)\bigvee(z\in\mathbb{R}\wedge z<0)$$

01.13.27.2255.01

$$\cos^{-1}\left(\frac{\sqrt{z-\sqrt{z^2+1}}}{\sqrt{2z}}\right) = -\frac{z^2}{2}\sqrt{-\frac{1}{z^4}}\sqrt{\frac{1}{z^2+1}}\sqrt{z^2+1}\operatorname{csch}^{-1}(z)+\frac{1}{4}\pi\left(1+z\sqrt{\frac{1}{z^2}}\sqrt{\frac{1}{z^2+1}}\sqrt{z^2+1}\right)$$

Involving $\cos^{-1}\left(\sqrt{\left(z-\sqrt{1+z^2}\right)\middle/(2z)}\right)$

Involving $\cos^{-1}\left(\sqrt{\left(z-\sqrt{1+z^2}\right)\middle/(2z)}\right)$ and $\operatorname{csch}^{-1}(z)$

01.13.27.2256.01

$$\cos^{-1}\left(\sqrt{\frac{z-\sqrt{z^2+1}}{2\,z}}\right) = \frac{\pi}{2} - \frac{1}{2}\,i\,\text{csch}^{-1}(z)\,/;\,0 \le \arg(z) < \frac{\pi}{2} \bigvee (i\,z \in \mathbb{R} \land i\,z < -1)$$

01.13.27.2257.01

$$\cos^{-1}\left(\sqrt{\frac{z-\sqrt{1+z^2}}{2\,z}}\right) = \frac{\pi}{2} + \frac{1}{2}\,i\,\text{csch}^{-1}(z)\,/;\,-\frac{\pi}{2} < \arg(z) < 0 \bigvee (i\,z \in \mathbb{R} \land 0 < i\,z < 1)$$

01.13.27.2258.01

$$\cos^{-1}\left(\sqrt{\frac{z-\sqrt{1+z^2}}{2\,z}}\right) = \frac{1}{2}\,i\,\text{csch}^{-1}(z)\,/;\,\frac{\pi}{2} < \arg(z) < \pi \bigvee (-i\,z \in \mathbb{R} \land 0 < -i\,z < 1)$$

01.13.27.2259.01

$$\cos^{-1}\left(\sqrt{\frac{z-\sqrt{1+z^2}}{2\,z}}\right) = -\frac{1}{2}\,i\,\text{csch}^{-1}(z)\,/;\,-\pi < \arg(z) < -\frac{\pi}{2} \bigvee (i\,z \in \mathbb{R} \land i\,z > 1) \bigvee (z \in \mathbb{R} \land z < 0)$$

01.13.27.2260.01

$$\cos^{-1}\left(\sqrt{\frac{z-\sqrt{z^2+1}}{2\,z}}\right) = -\frac{z^2}{2}\sqrt{-\frac{1}{z^4}}\sqrt{\frac{1}{z^2+1}}\sqrt{z^2+1}\,\text{csch}^{-1}(z) + \frac{\pi}{4}\left(1+z\sqrt{\frac{1}{z^2}}\sqrt{\frac{1}{z^2+1}}\sqrt{z^2+1}\right)$$

Involving $\cos^{-1}\left(a\,z^{-c}\sqrt{\frac{z^{2c}}{a^2}+1}\right)$

Involving $\cos^{-1}\left(a\,z^{-c}\sqrt{\frac{z^{2c}}{a^2}+1}\right)$ and $\text{csch}^{-1}\left(\frac{z^c}{a}\right)$

01.13.27.0068.02

$$\cos^{-1}\left(a\,z^{-c}\sqrt{\frac{z^{2c}}{a^2}+1}\right) = \frac{\pi}{2} - \frac{a}{2\,z^{2c}\sqrt{-a^2\,z^{-4c}\left(z^{2c}+a^2\right)}}\sqrt{\frac{z^{2c}}{a^2}+1}\left(\pi\sqrt{-\frac{a^2}{z^{2c}}}\,z^c + 2\,a\,\text{csch}^{-1}\left(\frac{z^c}{a}\right)\right)$$

Involving sech^{-1}

Involving $\cos^{-1}(z)$

Involving $\cos^{-1}(z)$ and $\text{sech}^{-1}\left(\frac{1}{z}\right)$

01.13.27.2261.01

$$\cos^{-1}(z) = -i \operatorname{sech}^{-1}\left(\frac{1}{z}\right) /; \operatorname{Im}(z) > 0 \lor (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.2262.01

$$\cos^{-1}(z) = i \operatorname{sech}^{-1}\left(\frac{1}{z}\right) /; \operatorname{Im}(z) < 0 \lor (z \in \mathbb{R} \land z > 1)$$

01.13.27.2263.01

$$\cos^{-1}(z) = \frac{\sqrt{1-z}}{\sqrt{z-1}} \operatorname{sech}^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}(z)$ and $\operatorname{sech}^{-1}\left(-\frac{1}{z}\right)$

01.13.27.2264.01

$$\cos^{-1}(z) = \pi - i \operatorname{sech}^{-1}\left(-\frac{1}{z}\right) /; \operatorname{Im}(z) > 0 \lor (z \in \mathbb{R} \land z < -1)$$

01.13.27.2265.01

$$\cos^{-1}(z) = \pi + i \operatorname{sech}^{-1}\left(-\frac{1}{z}\right) /; \operatorname{Im}(z) < 0 \lor (z \in \mathbb{R} \land z > -1)$$

01.13.27.2266.01

$$\cos^{-1}(z) = \pi - \frac{\sqrt{1+z}}{\sqrt{-1-z}} \operatorname{sech}^{-1}\left(-\frac{1}{z}\right)$$

Involving $\cos^{-1}(z)$ and $\operatorname{sech}^{-1}\left(\frac{1}{1-2z^2}\right)$

01.13.27.2267.01

$$\cos^{-1}(z) = \frac{\pi}{2} + \frac{i}{2} \operatorname{sech}^{-1}\left(\frac{1}{1-2z^2}\right) /; -\pi < \arg(z) \leq 0$$

01.13.27.2268.01

$$\cos^{-1}(z) = \frac{\pi}{2} - \frac{i}{2} \operatorname{sech}^{-1}\left(\frac{1}{1-2z^2}\right) /; 0 < \arg(z) \leq \pi$$

01.13.27.2269.01

$$\cos^{-1}(z) = \frac{\pi}{2} + \frac{\sqrt{-z^2}}{2z} \operatorname{sech}^{-1}\left(\frac{1}{1-2z^2}\right)$$

Involving $\cos^{-1}(z)$ and $\operatorname{sech}^{-1}\left(\frac{1}{2z^2-1}\right)$

01.13.27.2270.01

$$\cos^{-1}(z) = -\frac{1}{2} i \operatorname{sech}^{-1}\left(\frac{1}{2z^2-1}\right) /; 0 < \arg(z) \leq \frac{\pi}{2} \bigvee (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.2271.01

$$\cos^{-1}(z) = \frac{1}{2} i \operatorname{sech}^{-1}\left(\frac{1}{2z^2-1}\right) /; -\frac{\pi}{2} < \arg(z) < 0 \bigvee (z \in \mathbb{R} \land z > 1)$$

01.13.27.2272.01

$$\cos^{-1}(z) = \pi - \frac{1}{2} i \operatorname{sech}^{-1}\left(\frac{1}{2 z^2 - 1}\right) /; \frac{\pi}{2} < \arg(z) < \pi \bigvee (z \in \mathbb{R} \land z < -1)$$

01.13.27.2273.01

$$\cos^{-1}(z) = \pi + \frac{1}{2} i \operatorname{sech}^{-1}\left(\frac{1}{2 z^2 - 1}\right) /; -\pi < \arg(z) \leq -\frac{\pi}{2} \bigvee (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.2274.01

$$\cos^{-1}(z) = \frac{\pi}{2}\left(1 - \frac{\sqrt{z^2}}{z}\right) - \frac{z \sqrt{z^2 - 1}}{2 \sqrt{z^2 - z^4}} \operatorname{sech}^{-1}\left(\frac{1}{2 z^2 - 1}\right)$$

Involving $\cos^{-1}(z)$ and $\operatorname{sech}^{-1}\left(\frac{\sqrt{2}}{\sqrt{1+z}}\right)$

01.13.27.2275.01

$$\cos^{-1}(z) = -2 i \operatorname{sech}^{-1}\left(\frac{\sqrt{2}}{\sqrt{1 + z}}\right) /; 0 < \arg(z) \leq \pi \lor (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.2276.01

$$\cos^{-1}(z) = 2 i \operatorname{sech}^{-1}\left(\frac{\sqrt{2}}{\sqrt{1 + z}}\right) /; \operatorname{Im}(z) < 0 \lor (z \in \mathbb{R} \land z > 1)$$

01.13.27.2277.01

$$\cos^{-1}(z) = 2 \sqrt{-z^2} \sqrt{\frac{1}{z - 1}} \sqrt{\frac{z - 1}{z^2}} \operatorname{sech}^{-1}\left(\frac{\sqrt{2}}{\sqrt{1 + z}}\right)$$

Involving $\cos^{-1}(z)$ and $\operatorname{sech}^{-1}\left(\sqrt{\frac{2}{1+z}}\right)$

01.13.27.2278.01

$$\cos^{-1}(z) = -2 i \operatorname{sech}^{-1}\left(\sqrt{\frac{2}{1 + z}}\right) /; \operatorname{Im}(z) > 0 \lor (z \in \mathbb{R} \land -1 < z < 1)$$

01.13.27.2279.01

$$\cos^{-1}(z) = 2 i \operatorname{sech}^{-1}\left(\sqrt{\frac{2}{1 + z}}\right) /; \operatorname{Im}(z) < 0 \lor (z \in \mathbb{R} \land z > 1)$$

01.13.27.2280.01

$$\cos^{-1}(z) = 2 \pi - 2 i \operatorname{sech}^{-1}\left(\sqrt{\frac{2}{1 + z}}\right) /; (z \in \mathbb{R} \land z < -1)$$

01.13.27.2281.01

$$\cos^{-1}(z) = 2 \sqrt{-z^2} \sqrt{\frac{1}{z - 1}} \sqrt{\frac{z - 1}{z^2}} \operatorname{sech}^{-1}\left(\sqrt{\frac{2}{1 + z}}\right) + \left(1 - \sqrt{1 + z} \sqrt{\frac{1}{1 + z}}\right) \pi$$

Involving $\cos^{-1}(z)$ and $\operatorname{sech}^{-1}\left(\frac{\sqrt{2}}{\sqrt{1-z}}\right)$

01.13.27.2282.01

$$\cos^{-1}(z) = \pi + 2\,i\,\operatorname{sech}^{-1}\left(\frac{\sqrt{2}}{\sqrt{1-z}}\right) /;\ -\pi < \arg(z) \leq 0 \bigvee (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.2283.01

$$\cos^{-1}(z) = \pi - 2\,i\,\operatorname{sech}^{-1}\left(\frac{\sqrt{2}}{\sqrt{1-z}}\right) /;\ \operatorname{Im}(z) > 0 \lor (z \in \mathbb{R} \land z < -1)$$

01.13.27.2284.01

$$\cos^{-1}(z) = \pi - 2\sqrt{-z^2}\,\sqrt{-\frac{1}{z+1}}\,\sqrt{-\frac{z+1}{z^2}}\,\operatorname{sech}^{-1}\left(\frac{\sqrt{2}}{\sqrt{1-z}}\right)$$

Involving $\cos^{-1}(z)$ and $\operatorname{sech}^{-1}\left(\sqrt{\frac{2}{1-z}}\right)$

01.13.27.2285.01

$$\cos^{-1}(z) = \pi + 2\,i\,\operatorname{sech}^{-1}\left(\sqrt{\frac{2}{1-z}}\right) /;\ -\pi < \arg(z) < 0 \bigvee (z \in \mathbb{R} \land -1 < z < 1)$$

01.13.27.2286.01

$$\cos^{-1}(z) = \pi - 2\,i\,\operatorname{sech}^{-1}\left(\sqrt{\frac{2}{1-z}}\right) /;\ \operatorname{Im}(z) > 0 \lor (z \in \mathbb{R} \land z < -1)$$

01.13.27.2287.01

$$\cos^{-1}(z) = 2\,i\,\operatorname{sech}^{-1}\left(\sqrt{\frac{2}{1-z}}\right) - \pi /;\ (z \in \mathbb{R} \land z > 1)$$

01.13.27.2288.01

$$\cos^{-1}(z) = -2\sqrt{-z^2}\,\sqrt{-\frac{1}{z+1}}\,\sqrt{-\frac{z+1}{z^2}}\,\operatorname{sech}^{-1}\left(\sqrt{\frac{2}{1-z}}\right) + \sqrt{1-z}\,\sqrt{\frac{1}{1-z}}\,\pi$$

Involving $\cos^{-1}(z)$ and $\operatorname{sech}^{-1}\left(\frac{1}{\sqrt{z^2}}\right)$

01.13.27.2289.01

$$\cos^{-1}(z) = -i\,\operatorname{sech}^{-1}\left(\frac{1}{\sqrt{z^2}}\right) /;\ 0 < \arg(z) \leq \frac{\pi}{2} \bigvee (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.2290.01

$$\cos^{-1}(z) = i\,\operatorname{sech}^{-1}\left(\frac{1}{\sqrt{z^2}}\right) /;\ -\frac{\pi}{2} < \arg(z) < 0 \bigvee (z \in \mathbb{R} \land z > 1)$$

01.13.27.2291.01

$$\cos^{-1}(z) = \pi - i\,\text{sech}^{-1}\left(\frac{1}{\sqrt{z^2}}\right) /;\ \frac{\pi}{2} < \arg(z) < \pi \bigvee (z \in \mathbb{R} \land z < -1)$$

01.13.27.2292.01

$$\cos^{-1}(z) = \pi + i\,\text{sech}^{-1}\left(\frac{1}{\sqrt{z^2}}\right) /;\ -\pi < \arg(z) \leq -\frac{\pi}{2} \bigvee (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.2293.01

$$\cos^{-1}(z) = \frac{\pi}{2}\left(1 - \frac{\sqrt{z^2}}{z}\right) + \frac{\sqrt{-z^2}}{z}\sqrt{\frac{z-1}{z+1}}\sqrt{\frac{z+1}{z-1}}\ \text{sech}^{-1}\left(\frac{1}{\sqrt{z^2}}\right)$$

Involving $\cos^{-1}(z)$ and $\text{sech}^{-1}\left(\sqrt{\frac{1}{z^2}}\right)$

01.13.27.2294.01

$$\cos^{-1}(z) = -i\,\text{sech}^{-1}\left(\sqrt{\frac{1}{z^2}}\right) /;\ 0 < \arg(z) < \frac{\pi}{2} \bigvee (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.2295.01

$$\cos^{-1}(z) = i\,\text{sech}^{-1}\left(\sqrt{\frac{1}{z^2}}\right) /;\ -\frac{\pi}{2} \leq \arg(z) < 0 \bigvee (z \in \mathbb{R} \land z > 1)$$

01.13.27.2296.01

$$\cos^{-1}(z) = \pi - i\,\text{sech}^{-1}\left(\sqrt{\frac{1}{z^2}}\right) /;\ \frac{\pi}{2} \leq \arg(z) < \pi \bigvee (z \in \mathbb{R} \land z < -1)$$

01.13.27.2297.01

$$\cos^{-1}(z) = \pi + i\,\text{sech}^{-1}\left(\sqrt{\frac{1}{z^2}}\right) /;\ -\pi < \arg(z) < -\frac{\pi}{2} \bigvee (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.2298.01

$$\cos^{-1}(z) = \frac{\pi}{2}\left(1 - \sqrt{\frac{1}{z^2}}\ z\right) - z\sqrt{-\frac{1}{z^2}}\sqrt{\frac{1}{1-z^2}}\sqrt{1-z^2}\ \text{sech}^{-1}\left(\sqrt{\frac{1}{z^2}}\right)$$

Involving $\cos^{-1}(z)$ and $\text{sech}^{-1}\left(\frac{1}{\sqrt{1-z^2}}\right)$

01.13.27.2299.01

$$\cos^{-1}(z) = \frac{\pi}{2} + i\,\text{sech}^{-1}\left(\frac{1}{\sqrt{1-z^2}}\right) /;\ -\pi < \arg(z) \leq 0$$

01.13.27.2300.01

$$\cos^{-1}(z) = \frac{\pi}{2} - i\,\operatorname{sech}^{-1}\left(\frac{1}{\sqrt{1-z^2}}\right) /; 0 < \arg(z) \leq \pi$$

01.13.27.2301.01

$$\cos^{-1}(z) = \frac{\sqrt{-z^2}}{z}\operatorname{sech}^{-1}\left(\frac{1}{\sqrt{1-z^2}}\right) + \frac{\pi}{2}$$

Involving $\cos^{-1}(z)$ and $\operatorname{sech}^{-1}\left(\sqrt{\frac{1}{1-z^2}}\right)$

01.13.27.2302.01

$$\cos^{-1}(z) = \frac{\pi}{2} + i\,\operatorname{sech}^{-1}\left(\sqrt{\frac{1}{1-z^2}}\right) /; \operatorname{Im}(z) < 0 \lor (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.2303.01

$$\cos^{-1}(z) = \frac{\pi}{2} - i\,\operatorname{sech}^{-1}\left(\sqrt{\frac{1}{1-z^2}}\right) /; \operatorname{Im}(z) > 0 \lor (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.2304.01

$$\cos^{-1}(z) = i\,\operatorname{sech}^{-1}\left(\sqrt{\frac{1}{1-z^2}}\right) - \frac{\pi}{2} /; (z \in \mathbb{R} \land z > 1)$$

01.13.27.2305.01

$$\cos^{-1}(z) = \frac{3\pi}{2} - i\,\operatorname{sech}^{-1}\left(\sqrt{\frac{1}{1-z^2}}\right) /; (z \in \mathbb{R} \land z < -1)$$

01.13.27.2306.01

$$\cos^{-1}(z) = \frac{\pi}{2}\left(1 - \sqrt{\frac{1}{z+1}}\sqrt{z+1} + \sqrt{\frac{1}{1-z}}\sqrt{1-z}\right) + \frac{\sqrt{-z^2}}{z}\operatorname{sech}^{-1}\left(\sqrt{\frac{1}{1-z^2}}\right)$$

Involving $\cos^{-1}(z)$ and $\operatorname{sech}^{-1}\left(\frac{1}{2z\sqrt{1-z^2}}\right)$

01.13.27.2307.01

$$\cos^{-1}(z) = \frac{\pi}{4} + \frac{1}{2}i\,\operatorname{sech}^{-1}\left(\frac{1}{2z\sqrt{1-z^2}}\right) /; \operatorname{Re}(z) > -\frac{1}{\sqrt{2}} \bigwedge \operatorname{Im}(z) < 0$$

01.13.27.2308.01

$$\cos^{-1}(z) = \frac{\pi}{4} - \frac{1}{2}i\,\operatorname{sech}^{-1}\left(\frac{1}{2z\sqrt{1-z^2}}\right) /; \operatorname{Re}(z) > -\frac{1}{\sqrt{2}} \bigwedge \operatorname{Im}(z) > 0$$

01.13.27.2309.01

$$\cos^{-1}(z) = \frac{\pi}{4} + \frac{\sqrt{-z^2}}{2z}\,\mathrm{sech}^{-1}\left(\frac{1}{2z\sqrt{1-z^2}}\right) /;\ \mathrm{Re}(z) > -\frac{1}{\sqrt{2}} \bigwedge \mathrm{Im}(z) \neq 0$$

01.13.27.2310.01

$$\cos^{-1}(z) = \frac{1}{4}\pi\left(2 - \frac{\sqrt{z^2}}{z} + \sqrt{\frac{1}{z}}\,\sqrt{z}\,\sqrt{\frac{1}{\sqrt{2}\,z+1}}\,\sqrt{\sqrt{2}\,z+1} - \sqrt{-\frac{1}{z}}\,\sqrt{-z}\,\sqrt{\frac{1}{1-\sqrt{2}\,z}}\,\sqrt{1-\sqrt{2}\,z} - \frac{\sqrt{z^4-z^2}}{z\sqrt{z^2-1}}\right) + \frac{\sqrt{-z^2}\,\sqrt{z^2-1}\,\sqrt{2z^2-1}}{2\sqrt{1-2z^2}\,\sqrt{z^4-z^2}}\left(\frac{\pi}{2} - \frac{1}{\sqrt{2z\sqrt{1-z^2}-1}}\sqrt{1-2z\sqrt{1-z^2}}\,\mathrm{sech}^{-1}\left(\frac{1}{2z\sqrt{1-z^2}}\right)\right)$$

Involving $\cos^{-1}(-z)$

Involving $\cos^{-1}(-z)$ and $\mathrm{sech}^{-1}\left(\frac{1}{z}\right)$

01.13.27.2311.01

$$\cos^{-1}(-z) = \pi + i\,\mathrm{sech}^{-1}\left(\frac{1}{z}\right) /;\ \mathrm{Im}(z) > 0 \lor (z \in \mathbb{R} \land z < 1)$$

01.13.27.2312.01

$$\cos^{-1}(-z) = \pi - i\,\mathrm{sech}^{-1}\left(\frac{1}{z}\right) /;\ \mathrm{Im}(z) < 0 \lor (z \in \mathbb{R} \land z > 1)$$

01.13.27.2313.01

$$\cos^{-1}(-z) = \pi - \frac{\sqrt{1-z}}{\sqrt{z-1}}\,\mathrm{sech}^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\sqrt{c\,z}\right)$

Involving $\cos^{-1}\left(\sqrt{z}\right)$ and $\mathrm{sech}^{-1}\left(\frac{1}{\sqrt{z}}\right)$

01.13.27.2314.01

$$\cos^{-1}\left(\sqrt{z}\right) = i\,\mathrm{sech}^{-1}\left(\frac{1}{\sqrt{z}}\right) /;\ \mathrm{Im}(z) < 0 \lor (z \in \mathbb{R} \land z > 1)$$

01.13.27.2315.01

$$\cos^{-1}\left(\sqrt{z}\right) = -i\,\mathrm{sech}^{-1}\left(\frac{1}{\sqrt{z}}\right) /;\ \mathrm{Im}(z) > 0 \lor (z \in \mathbb{R} \land z < 1)$$

01.13.27.2316.01

$$\cos^{-1}\left(\sqrt{z}\right) = \frac{\sqrt{1-z}}{\sqrt{z-1}} \operatorname{sech}^{-1}\left(\frac{1}{\sqrt{z}}\right)$$

Involving $\cos^{-1}\left(\sqrt{z}\right)$ and $\operatorname{sech}^{-1}\left(\sqrt{\frac{1}{z}}\right)$

01.13.27.2317.01

$$\cos^{-1}\left(\sqrt{z}\right) = i \operatorname{sech}^{-1}\left(\sqrt{\frac{1}{z}}\right) /; \operatorname{Im}(z) < 0 \lor (z \in \mathbb{R} \land z > 1)$$

01.13.27.2318.01

$$\cos^{-1}\left(\sqrt{z}\right) = -i \operatorname{sech}^{-1}\left(\sqrt{\frac{1}{z}}\right) /; \operatorname{Im}(z) > 0 \lor (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.2319.01

$$\cos^{-1}\left(\sqrt{z}\right) = \pi - i \operatorname{sech}^{-1}\left(\sqrt{\frac{1}{z}}\right) /; (z \in \mathbb{R} \land z < 0)$$

01.13.27.2320.01

$$\cos^{-1}\left(\sqrt{z}\right) = \frac{\pi}{2}\left(1 - \sqrt{\frac{1}{z}}\sqrt{z}\right) + \frac{\sqrt{1-z}}{\sqrt{z-1}} \operatorname{sech}^{-1}\left(\sqrt{\frac{1}{z}}\right)$$

Involving $\cos^{-1}\left(\sqrt{-z}\right)$ and $\operatorname{sech}^{-1}\left(\frac{1}{\sqrt{1+z}}\right)$

01.13.27.2321.01

$$\cos^{-1}\left(\sqrt{-z}\right) = \frac{\pi}{2} - i \operatorname{sech}^{-1}\left(\frac{1}{\sqrt{1+z}}\right) /; -\pi < \arg(z) \leq 0$$

01.13.27.2322.01

$$\cos^{-1}\left(\sqrt{-z}\right) = \frac{\pi}{2} + i \operatorname{sech}^{-1}\left(\frac{1}{\sqrt{1+z}}\right) /; 0 < \arg(z) \leq \pi$$

01.13.27.2323.01

$$\cos^{-1}\left(\sqrt{-z}\right) = \frac{\pi}{2} - \frac{\sqrt{-z^2}}{z} \operatorname{sech}^{-1}\left(\frac{1}{\sqrt{1+z}}\right)$$

Involving $\cos^{-1}\left(\sqrt{-z}\right)$ and $\operatorname{sech}^{-1}\left(\frac{1}{1+2z}\right)$

01.13.27.2324.01

$$\cos^{-1}\left(\sqrt{-z}\right) = \frac{\pi}{2} - \frac{i}{2} \operatorname{sech}^{-1}\left(\frac{1}{1+2z}\right) /; -\pi < \arg(z) \leq 0$$

01.13.27.2325.01

$$\cos^{-1}\left(\sqrt{-z}\right) = \frac{\pi}{2} + \frac{i}{2} \operatorname{sech}^{-1}\left(\frac{1}{1+2z}\right) /; 0 < \arg(z) \leq \pi$$

01.13.27.2326.01

$$\cos^{-1}\left(\sqrt{-z}\right) = \frac{\pi}{2} - \frac{\sqrt{-z^2}}{2\,z}\,\text{sech}^{-1}\left(\frac{1}{1+2\,z}\right)$$

Involving $\cos^{-1}\left(\frac{1}{\sqrt{c\,z}}\right)$

Involving $\cos^{-1}\left(\frac{1}{\sqrt{z}}\right)$ and $\text{sech}^{-1}\left(\sqrt{z}\right)$

01.13.27.2327.01

$$\cos^{-1}\left(\frac{1}{\sqrt{z}}\right) = i\,\text{sech}^{-1}\left(\sqrt{z}\right)\,/;\, 0 < \arg(z) \le \pi \bigvee (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.2328.01

$$\cos^{-1}\left(\frac{1}{\sqrt{z}}\right) = -i\,\text{sech}^{-1}\left(\sqrt{z}\right)\,/;\, -\pi < \arg(z) < 0 \bigvee (z \in \mathbb{R} \land z > 1)$$

01.13.27.0072.01

$$\cos^{-1}\left(\frac{1}{\sqrt{z}}\right) = \frac{\sqrt{z-1}}{\sqrt{1-z}}\,\text{sech}^{-1}\left(\sqrt{z}\right)$$

Involving $\cos^{-1}\left(\frac{1}{\sqrt{z}}\right)$ and $\text{sech}^{-1}\left(1\Big/\sqrt{\frac{1}{z}}\right)$

01.13.27.2329.01

$$\cos^{-1}\left(\frac{1}{\sqrt{z}}\right) = i\,\text{sech}^{-1}\left(1\Big/\sqrt{\frac{1}{z}}\right)/;\, \text{Im}(z) > 0 \lor (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.2330.01

$$\cos^{-1}\left(\frac{1}{\sqrt{z}}\right) = -i\,\text{sech}^{-1}\left(1\Big/\sqrt{\frac{1}{z}}\right)/;\, \text{Im}(z) < 0 \lor (z \in \mathbb{R} \land z > 1)$$

01.13.27.2331.01

$$\cos^{-1}\left(\frac{1}{\sqrt{z}}\right) = \pi + i\,\text{sech}^{-1}\left(1\Big/\sqrt{\frac{1}{z}}\right)/;\, (z \in \mathbb{R} \land z < 0)$$

01.13.27.2332.01

$$\cos^{-1}\left(\frac{1}{\sqrt{z}}\right) = \frac{\pi}{2}\left(1 - \sqrt{z}\,\sqrt{\frac{1}{z}}\right) + \frac{\sqrt{z-1}}{\sqrt{1-z}}\,\text{sech}^{-1}\left(1\Big/\sqrt{\frac{1}{z}}\right)$$

Involving $\cos^{-1}\left(\frac{1}{\sqrt{-z}}\right)$ and $\text{sech}^{-1}\left(\sqrt{\frac{z}{z+1}}\right)$

01.13.27.2333.01

$$\cos^{-1}\left(\frac{1}{\sqrt{-z}}\right) = \frac{\pi}{2} - i\,\text{sech}^{-1}\left(\sqrt{\frac{z}{z+1}}\right)/;\, \text{Im}(z) > 0$$

01.13.27.2334.01

$$\cos^{-1}\left(\frac{1}{\sqrt{-z}}\right) = \frac{\pi}{2} + i \operatorname{sech}^{-1}\left(\sqrt{\frac{z}{z+1}}\right) /; \operatorname{Im}(z) < 0 \lor (z \in \mathbb{R} \land z > 0) \lor (z \in \mathbb{R} \land z < -1)$$

01.13.27.2335.01

$$\cos^{-1}\left(\frac{1}{\sqrt{-z}}\right) = i \operatorname{sech}^{-1}\left(\sqrt{\frac{z}{z+1}}\right) - \frac{\pi}{2} /; (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.2336.01

$$\cos^{-1}\left(\frac{1}{\sqrt{-z}}\right) = \frac{\pi}{2}\sqrt{\frac{z}{z+1}}\sqrt{\frac{z+1}{z}} + \sqrt{-z}\sqrt{\frac{1}{z}} \operatorname{sech}^{-1}\left(\sqrt{\frac{z}{z+1}}\right)$$

Involving $\cos^{-1}\left(\frac{1}{\sqrt{-z}}\right)$ and $\operatorname{sech}^{-1}\left(\frac{z}{z+2}\right)$

01.13.27.2337.01

$$\cos^{-1}\left(\frac{1}{\sqrt{-z}}\right) = \frac{\pi}{2} + \frac{i}{2} \operatorname{sech}^{-1}\left(\frac{z}{z+2}\right) /; \operatorname{Im}(z) \leq 0$$

01.13.27.2338.01

$$\cos^{-1}\left(\frac{1}{\sqrt{-z}}\right) = \frac{\pi}{2} - \frac{1}{2} i \operatorname{sech}^{-1}\left(\frac{z}{z+2}\right) /; 0 < \arg(z) < \pi$$

01.13.27.2339.01

$$\cos^{-1}\left(\frac{1}{\sqrt{-z}}\right) = \frac{\pi}{2} + \frac{\sqrt{-z}}{2}\sqrt{\frac{1}{z}} \operatorname{sech}^{-1}\left(\frac{z}{z+2}\right)$$

Involving $\cos^{-1}\left(\sqrt{z^2}\right)$

Involving $\cos^{-1}\left(\sqrt{z^2}\right)$ and $\operatorname{sech}^{-1}\left(\frac{1}{z}\right)$

01.13.27.2340.01

$$\cos^{-1}\left(\sqrt{z^2}\right) = -i \operatorname{sech}^{-1}\left(\frac{1}{z}\right) /; 0 < \arg(z) \leq \frac{\pi}{2} \bigvee (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.2341.01

$$\cos^{-1}\left(\sqrt{z^2}\right) = i \operatorname{sech}^{-1}\left(\frac{1}{z}\right) /; -\frac{\pi}{2} < \arg(z) < 0 \bigvee (z \in \mathbb{R} \land z > 1)$$

01.13.27.2342.01

$$\cos^{-1}\left(\sqrt{z^2}\right) = \pi + i \operatorname{sech}^{-1}\left(\frac{1}{z}\right) /; \frac{\pi}{2} < \arg(z) \leq \pi$$

01.13.27.2343.01

$$\cos^{-1}\left(\sqrt{z^2}\right) = \pi - i \operatorname{sech}^{-1}\left(\frac{1}{z}\right) /; -\pi < \arg(z) \leq -\frac{\pi}{2}$$

01.13.27.2344.01

$$\cos^{-1}\left(\sqrt{z^2}\right) = \frac{\sqrt{1-z}}{\sqrt{z-1}} \operatorname{sech}^{-1}\left(\frac{1}{z}\right) /; -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

01.13.27.2345.01

$$\cos^{-1}\left(\sqrt{z^2}\right) = \frac{\pi}{2}\left(1 - \frac{\sqrt{z^2}}{z}\right) + \frac{\sqrt{z^2}\,\sqrt{1-z}}{z\,\sqrt{z-1}} \operatorname{sech}^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(a\,(b\,z^c)^m\right)$

Involving $\cos^{-1}(a\,(b\,z^c)^m)$ and $\operatorname{sech}^{-1}\left(\frac{1}{a\,b^m\,z^{m\,c}}\right)$

01.13.27.2346.01

$$\cos^{-1}(a\,(b\,z^c)^m) = \frac{1}{2}\,\pi\left(1 - \frac{(b\,z^c)^m}{b^m\,z^{m\,c}}\right) + \frac{(b\,z^c)^m\,\sqrt{1-a\,b^m\,z^{m\,c}}}{b^m\,z^{m\,c}\,\sqrt{a\,b^m\,z^{m\,c}-1}} \operatorname{sech}^{-1}\left(\frac{1}{a\,b^m\,z^{m\,c}}\right) /; 2\,m \in \mathbb{Z}$$

Involving $\cos^{-1}\left(1-2\,z^2\right)$

Involving $\cos^{-1}\left(1-2\,z^2\right)$ and $\operatorname{sech}^{-1}\left(\frac{1}{z}\right)$

01.13.27.2347.01

$$\cos^{-1}\left(1-2\,z^2\right) = \pi + 2\,i\operatorname{sech}^{-1}\left(\frac{1}{z}\right) /; 0 < \arg(z) \leq \frac{\pi}{2} \bigvee (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.2348.01

$$\cos^{-1}\left(1-2\,z^2\right) = \pi - 2\,i\operatorname{sech}^{-1}\left(\frac{1}{z}\right) /; -\frac{\pi}{2} < \arg(z) < 0 \bigvee (z \in \mathbb{R} \land z > 1)$$

01.13.27.2349.01

$$\cos^{-1}\left(1-2\,z^2\right) = -\pi - 2\,i\operatorname{sech}^{-1}\left(\frac{1}{z}\right) /; \frac{\pi}{2} < \arg(z) \leq \pi$$

01.13.27.2350.01

$$\cos^{-1}\left(1-2\,z^2\right) = 2\,i\operatorname{sech}^{-1}\left(\frac{1}{z}\right) - \pi /; -\pi < \arg(z) \leq -\frac{\pi}{2}$$

01.13.27.2351.01

$$\cos^{-1}\left(1-2\,z^2\right) = \frac{\pi\sqrt{z^2}}{z} - \frac{2\sqrt{z^2}\,\sqrt{1-z}}{z\,\sqrt{z-1}} \operatorname{sech}^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(2\,z^2-1\right)$

Involving $\cos^{-1}\left(2\,z^2-1\right)$ and $\operatorname{sech}^{-1}\left(\frac{1}{z}\right)$

01.13.27.2352.01

$$\cos^{-1}(2z^2-1) = -2\,i\,\mathrm{sech}^{-1}\left(\frac{1}{z}\right) /;\ 0 < \arg(z) \leq \frac{\pi}{2} \bigvee (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.2353.01

$$\cos^{-1}(2z^2-1) = 2\,i\,\mathrm{sech}^{-1}\left(\frac{1}{z}\right) /;\ -\frac{\pi}{2} < \arg(z) < 0 \bigvee (z \in \mathbb{R} \land z > 1)$$

01.13.27.2354.01

$$\cos^{-1}(2z^2-1) = 2\pi + 2\,i\,\mathrm{sech}^{-1}\left(\frac{1}{z}\right) /;\ \frac{\pi}{2} < \arg(z) \leq \pi$$

01.13.27.2355.01

$$\cos^{-1}(2z^2-1) = 2\pi - 2\,i\,\mathrm{sech}^{-1}\left(\frac{1}{z}\right) /;\ -\pi < \arg(z) \leq -\frac{\pi}{2}$$

01.13.27.2356.01

$$\cos^{-1}(2z^2-1) = \pi\left(1 - \frac{\sqrt{z^2}}{z}\right) + \frac{2\sqrt{1-z}\,\sqrt{z^2}}{\sqrt{z-1}\,z}\,\mathrm{sech}^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\frac{z^2-2}{z^2}\right)$

Involving $\cos^{-1}\left(\frac{z^2-2}{z^2}\right)$ and $\mathrm{sech}^{-1}(z)$

01.13.27.2357.01

$$\cos^{-1}\left(\frac{z^2-2}{z^2}\right) = \pi - 2\,i\,\mathrm{sech}^{-1}(z) /;\ 0 < \arg(z) < \frac{\pi}{2} \bigvee (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.2358.01

$$\cos^{-1}\left(\frac{z^2-2}{z^2}\right) = \pi + 2\,i\,\mathrm{sech}^{-1}(z) /;\ -\frac{\pi}{2} \leq \arg(z) < 0 \bigvee (z \in \mathbb{R} \land z > 1)$$

01.13.27.2359.01

$$\cos^{-1}\left(\frac{z^2-2}{z^2}\right) = 2\,i\,\mathrm{sech}^{-1}(z) - \pi /;\ \frac{\pi}{2} \leq \arg(z) < \pi$$

01.13.27.2360.01

$$\cos^{-1}\left(\frac{z^2-2}{z^2}\right) = -2\,i\,\mathrm{sech}^{-1}(z) - \pi /;\ -\pi < \arg(z) < -\frac{\pi}{2} \bigvee (z \in \mathbb{R} \land z < 0)$$

01.13.27.2361.01

$$\cos^{-1}\left(\frac{z^2-2}{z^2}\right) = \pi\sqrt{\frac{1}{z^2}}\,z - \frac{2z}{\sqrt{\frac{1}{z}-1}}\sqrt{1-\frac{1}{z}}\,\sqrt{\frac{1}{z^2}}\,\mathrm{sech}^{-1}(z)$$

Involving $\cos^{-1}\left(\frac{2-z^2}{z^2}\right)$

Involving $\cos^{-1}\left(\frac{2-z^2}{z^2}\right)$ and $\text{sech}^{-1}(z)$

01.13.27.2362.01

$$\cos^{-1}\left(\frac{2-z^2}{z^2}\right) = 2\,i\,\text{sech}^{-1}(z)\ /;\ 0 < \arg(z) < \frac{\pi}{2} \bigvee (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.2363.01

$$\cos^{-1}\left(\frac{2-z^2}{z^2}\right) = -2\,i\,\text{sech}^{-1}(z)\ /;\ -\frac{\pi}{2} \leq \arg(z) < 0 \bigvee (z \in \mathbb{R} \land z > 1)$$

01.13.27.2364.01

$$\cos^{-1}\left(\frac{2-z^2}{z^2}\right) = 2\,\pi - 2\,i\,\text{sech}^{-1}(z)\ /;\ \frac{\pi}{2} \leq \arg(z) < \pi$$

01.13.27.2365.01

$$\cos^{-1}\left(\frac{2-z^2}{z^2}\right) = 2\,\pi + 2\,i\,\text{sech}^{-1}(z)\ /;\ -\pi < \arg(z) < -\frac{\pi}{2} \bigvee (z \in \mathbb{R} \land z < 0)$$

01.13.27.2366.01

$$\cos^{-1}\left(\frac{2-z^2}{z^2}\right) = \pi\left(1 - \sqrt{\frac{1}{z^2}}\ z\right) + \frac{2\,z}{\sqrt{\frac{1}{z}-1}} \sqrt{1-\frac{1}{z}}\ \sqrt{\frac{1}{z^2}}\ \text{sech}^{-1}(z)$$

Involving $\cos^{-1}\left(\sqrt{1+c\,z}\right)$

Involving $\cos^{-1}\left(\sqrt{1-z}\right)$ and $\text{sech}^{-1}\left(\frac{1}{\sqrt{z}}\right)$

01.13.27.2367.01

$$\cos^{-1}\left(\sqrt{1-z}\right) = \frac{\pi}{2} - i\,\text{sech}^{-1}\left(\frac{1}{\sqrt{z}}\right) /;\ -\pi < \arg(z) < 0 \bigvee (z \in \mathbb{R} \land z > 1)$$

01.13.27.2368.01

$$\cos^{-1}\left(\sqrt{1-z}\right) = \frac{\pi}{2} + i\,\text{sech}^{-1}\left(\frac{1}{\sqrt{z}}\right) /;\ 0 < \arg(z) \leq \pi \bigvee (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.2369.01

$$\cos^{-1}\left(\sqrt{1-z}\right) = \frac{\pi}{2} - \frac{\sqrt{1-z}}{\sqrt{z-1}}\ \text{sech}^{-1}\left(\frac{1}{\sqrt{z}}\right)$$

Involving $\cos^{-1}\left(\sqrt{1-z}\right)$ and $\text{sech}^{-1}\left(\sqrt{\frac{1}{z}}\right)$

01.13.27.2370.01

$$\cos^{-1}\left(\sqrt{1-z}\right) = \frac{\pi}{2} - i\,\text{sech}^{-1}\left(\sqrt{\frac{1}{z}}\right) /;\ -\pi < \arg(z) < 0 \bigvee (z \in \mathbb{R} \land z > 1)$$

01.13.27.2371.01

$$\cos^{-1}\left(\sqrt{1-z}\right) = \frac{\pi}{2} + i\,\text{sech}^{-1}\left(\sqrt{\frac{1}{z}}\right) /;\ 0 < \arg(z) < \pi \bigvee (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.2372.01

$$\cos^{-1}\left(\sqrt{1-z}\right) = -\frac{\pi}{2} + i\,\text{sech}^{-1}\left(\sqrt{\frac{1}{z}}\right) /;\ (z \in \mathbb{R} \land z < 0)$$

01.13.27.2373.01

$$\cos^{-1}\left(\sqrt{1-z}\right) = \sqrt{z}\,\sqrt{\frac{1}{z}}\,\frac{\pi}{2} - \frac{\sqrt{1-z}}{\sqrt{z-1}}\,\text{sech}^{-1}\left(\sqrt{\frac{1}{z}}\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{1+c\,z}}{\sqrt{2}}\right)$

Involving $\cos^{-1}\left(\sqrt{\frac{1+z}{2}}\right)$ and $\text{sech}^{-1}\left(\frac{1}{z}\right)$

01.13.27.2374.01

$$\cos^{-1}\left(\sqrt{\frac{z+1}{2}}\right) = \frac{i}{2}\,\text{sech}^{-1}\left(\frac{1}{z}\right) /;\ -\pi < \arg(z) < 0 \bigvee (z \in \mathbb{R} \land z > 1)$$

01.13.27.2375.01

$$\cos^{-1}\left(\sqrt{\frac{1+z}{2}}\right) = -\frac{i}{2}\,\text{sech}^{-1}\left(\frac{1}{z}\right) /;\ 0 < \arg(z) \le \pi \bigvee (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.2376.01

$$\cos^{-1}\left(\sqrt{\frac{1+z}{2}}\right) = \frac{\sqrt{1-z}}{2\sqrt{z-1}}\,\text{sech}^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\sqrt{\frac{1-z}{2}}\right)$ and $\text{sech}^{-1}\left(\frac{1}{z}\right)$

01.13.27.2377.01

$$\cos^{-1}\left(\sqrt{\frac{1-z}{2}}\right) = \frac{\pi}{2} - \frac{i}{2}\,\text{sech}^{-1}\left(\frac{1}{z}\right) /;\ -\pi < \arg(z) < 0 \bigvee (z \in \mathbb{R} \land z > 1)$$

01.13.27.2378.01

$$\cos^{-1}\left(\sqrt{\frac{1-z}{2}}\right) = \frac{\pi}{2} + \frac{i}{2}\,\text{sech}^{-1}\left(\frac{1}{z}\right) /;\ 0 < \arg(z) \le \pi \bigvee (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.2379.01

$$\cos^{-1}\left(\sqrt{\frac{1-z}{2}}\right) = \frac{\pi}{2} - \frac{1}{2}\,\frac{\sqrt{1-z}}{\sqrt{z-1}}\,\operatorname{sech}^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{z+a}}{\sqrt{z}}\right)$

Involving $\cos^{-1}\left(\frac{\sqrt{z-1}}{\sqrt{z}}\right)$ and $\operatorname{sech}^{-1}\left(\sqrt{z}\right)$

01.13.27.2380.01

$$\cos^{-1}\left(\frac{\sqrt{z-1}}{\sqrt{z}}\right) = \frac{\pi}{2} - i\,\operatorname{sech}^{-1}\left(\sqrt{z}\right) \;/;\; 0 < \arg(z) < \pi \lor (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.2381.01

$$\cos^{-1}\left(\frac{\sqrt{z-1}}{\sqrt{z}}\right) = \frac{\pi}{2} + i\,\operatorname{sech}^{-1}\left(\sqrt{z}\right) \;/;\; -\pi < \arg(z) < 0 \lor (z \in \mathbb{R} \land z > 1)$$

01.13.27.2382.01

$$\cos^{-1}\left(\frac{\sqrt{z-1}}{\sqrt{z}}\right) = \frac{\pi}{2} - \frac{\sqrt{z-1}}{\sqrt{1-z}}\,\operatorname{sech}^{-1}\left(\sqrt{z}\right) \;/;\; |\arg(z)| < \pi$$

01.13.27.2383.01

$$\cos^{-1}\left(\frac{\sqrt{z-1}}{\sqrt{z}}\right) = \frac{1}{2}\,\pi\,\sqrt{\frac{1}{z}}\,\sqrt{z} - \frac{\sqrt{z-1}\,\sqrt{z}}{\sqrt{1-z}}\,\sqrt{\frac{1}{z}}\,\operatorname{sech}^{-1}\left(\sqrt{z}\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{z+1}}{\sqrt{z}}\right)$ and $\operatorname{sech}^{-1}\left(\frac{z}{z+2}\right)$

01.13.27.2384.01

$$\cos^{-1}\left(\frac{\sqrt{z+1}}{\sqrt{z}}\right) = \frac{1}{2}\,i\,\operatorname{sech}^{-1}\left(\frac{z}{z+2}\right) \;/;\; 0 \le \arg(z) < \pi$$

01.13.27.2385.01

$$\cos^{-1}\left(\frac{\sqrt{z+1}}{\sqrt{z}}\right) = -\frac{i}{2}\,\operatorname{sech}^{-1}\left(\frac{z}{z+2}\right) \;/;\; -\pi < \arg(z) < 0 \lor (z \in \mathbb{R} \land z < -1)$$

01.13.27.2386.01

$$\cos^{-1}\left(\frac{\sqrt{z+1}}{\sqrt{z}}\right) = \frac{1}{2}\,i\,\operatorname{sech}^{-1}\left(\frac{z}{z+2}\right) + \pi \;/;\; (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.2387.01

$$\cos^{-1}\left(\frac{\sqrt{z+1}}{\sqrt{z}}\right) = \frac{\pi}{2} - \frac{\sqrt{-z}\,\sqrt{z+1}}{2\,\sqrt{-z-1}\,\sqrt{z}}\left(\pi - z\,\sqrt{-\frac{1}{z^2}}\,\operatorname{sech}^{-1}\left(\frac{z}{z+2}\right)\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{z+1}}{\sqrt{z}}\right)$ and $\operatorname{sech}^{-1}\left(\sqrt{\frac{z}{z+1}}\right)$

01.13.27.2388.01

$$\cos^{-1}\left(\frac{\sqrt{z+1}}{\sqrt{z}}\right) = i \operatorname{sech}^{-1}\left(\sqrt{\frac{z}{z+1}}\right) /; 0 < \arg(z) < \pi \bigvee (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.2389.01

$$\cos^{-1}\left(\frac{\sqrt{z+1}}{\sqrt{z}}\right) = -i \operatorname{sech}^{-1}\left(\sqrt{\frac{z}{z+1}}\right) /; -\pi < \arg(z) < 0 \bigvee (z \in \mathbb{R} \land z < -1)$$

01.13.27.2390.01

$$\cos^{-1}\left(\frac{\sqrt{z+1}}{\sqrt{z}}\right) = z \sqrt{-\frac{1}{z^2}} \sqrt{\frac{z+1}{z}} \sqrt{\frac{z}{z+1}} \operatorname{sech}^{-1}\left(\sqrt{\frac{z}{z+1}}\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{-z}}\right)$

Involving $\cos^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{-z}}\right)$ and $\operatorname{sech}^{-1}\left(\sqrt{z}\right)$

01.13.27.2391.01

$$\cos^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{-z}}\right) = \frac{\pi}{2} - i \operatorname{sech}^{-1}\left(\sqrt{z}\right) /; \operatorname{Im}(z) > 0$$

01.13.27.2392.01

$$\cos^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{-z}}\right) = \frac{\pi}{2} + i \operatorname{sech}^{-1}\left(\sqrt{z}\right) /; -\pi < \arg(z) \leq 0$$

01.13.27.2393.01

$$\cos^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{-z}}\right) = \frac{\pi}{2} - \frac{1}{2} \sqrt{\frac{z-1}{z}} \sqrt{\frac{z}{z-1}} \left(\frac{2\sqrt{z-1}\sqrt{z}}{\sqrt{1-z}} \sqrt{\frac{1}{z}} \operatorname{sech}^{-1}\left(\sqrt{z}\right) - \pi \sqrt{\frac{1}{z}} \sqrt{z} + \pi\right)$$

Involving $\cos^{-1}\left(\sqrt{\frac{z+c}{z}}\right)$

Involving $\cos^{-1}\left(\sqrt{\frac{z-1}{z}}\right)$ and $\operatorname{sech}^{-1}\left(\sqrt{z}\right)$

01.13.27.2394.01

$$\cos^{-1}\left(\sqrt{\frac{z-1}{z}}\right) = \frac{\pi}{2} - i \operatorname{sech}^{-1}\left(\sqrt{z}\right) /; \operatorname{Im}(z) > 0 \lor (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.2395.01

$$\cos^{-1}\left(\sqrt{\frac{z-1}{z}}\right) = \frac{\pi}{2} + i\,\text{sech}^{-1}\left(\sqrt{z}\right) /;\ \text{Im}(z) < 0 \lor (z \in \mathbb{R} \land z > 1)$$

01.13.27.2396.01

$$\cos^{-1}\left(\sqrt{\frac{z-1}{z}}\right) = \frac{\pi}{2} - \frac{\sqrt{z-1}}{\sqrt{1-z}}\,\text{sech}^{-1}\left(\sqrt{z}\right) /;\ |\arg(z)| < \pi$$

01.13.27.2397.01

$$\cos^{-1}\left(\sqrt{\frac{z-1}{z}}\right) = -\sqrt{\frac{1}{z}}\,\sqrt{z}\,\frac{\sqrt{z-1}}{\sqrt{1-z}}\,\text{sech}^{-1}\left(\sqrt{z}\right) + \frac{\pi}{2}\sqrt{\frac{1}{z}}\,\sqrt{z}$$

Involving $\cos^{-1}\left(\sqrt{\frac{z+1}{z}}\right)$ and $\text{sech}^{-1}\left(\sqrt{\frac{z}{z+1}}\right)$

01.13.27.2398.01

$$\cos^{-1}\left(\sqrt{\frac{z+1}{z}}\right) = i\,\text{sech}^{-1}\left(\sqrt{\frac{z}{z+1}}\right) /;\ 0 \le \arg(z) < \pi$$

01.13.27.2399.01

$$\cos^{-1}\left(\sqrt{\frac{z+1}{z}}\right) = -i\,\text{sech}^{-1}\left(\sqrt{\frac{z}{z+1}}\right) /;\ -\pi < \arg(z) < 0 \lor (z \in \mathbb{R} \land z < -1)$$

01.13.27.2400.01

$$\cos^{-1}\left(\sqrt{\frac{z+1}{z}}\right) = \pi - i\,\text{sech}^{-1}\left(\sqrt{\frac{z}{z+1}}\right) /;\ (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.2401.01

$$\cos^{-1}\left(\sqrt{\frac{z+1}{z}}\right) = \frac{\pi}{2}\left(1 - \sqrt{z}\,\sqrt{\frac{1}{z}}\,\sqrt{z+1}\,\sqrt{\frac{1}{z+1}}\right) + z\sqrt{-\frac{1}{z^2}}\,\text{sech}^{-1}\left(\sqrt{\frac{z}{z+1}}\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{z+a}}{\sqrt{2z}}\right)$

Involving $\cos^{-1}\left(\frac{\sqrt{z+1}}{\sqrt{2z}}\right)$ and $\text{sech}^{-1}(z)$

01.13.27.2402.01

$$\cos^{-1}\left(\frac{\sqrt{z+1}}{\sqrt{2z}}\right) = \frac{i}{2}\,\text{sech}^{-1}(z) /;\ \text{Im}(z) > 0 \lor (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.2403.01

$$\cos^{-1}\left(\frac{\sqrt{z+1}}{\sqrt{2z}}\right) = -\frac{i}{2}\,\text{sech}^{-1}(z) /;\ \text{Im}(z) < 0 \lor (z \in \mathbb{R} \land z < -1) \lor (z \in \mathbb{R} \land z > 1)$$

01.13.27.2404.01

$$\cos^{-1}\left(\frac{\sqrt{z+1}}{\sqrt{2\,z}}\right) = \frac{\sqrt{z-1}}{2\sqrt{1-z}}\,\mathrm{sech}^{-1}(z) \;/;\; |\arg(z)| < \pi$$

01.13.27.2405.01

$$\cos^{-1}\left(\frac{\sqrt{z+1}}{\sqrt{2\,z}}\right) = \frac{1}{2}\,\pi\left(1 - \sqrt{1+\frac{1}{z}}\,\sqrt{\frac{z}{z+1}}\right) + \frac{1}{2\sqrt{\frac{1}{z}-1}}\,\sqrt{1-\frac{1}{z^2}}\,\sqrt{\frac{z}{z+1}}\,\mathrm{sech}^{-1}(z)$$

Involving $\cos^{-1}\left(\frac{\sqrt{z-1}}{\sqrt{2\,z}}\right)$ and $\mathrm{sech}^{-1}(z)$

01.13.27.2406.01

$$\cos^{-1}\left(\frac{\sqrt{z-1}}{\sqrt{2\,z}}\right) = \frac{\pi}{2} - \frac{1}{2}\,i\,\mathrm{sech}^{-1}(z) \;/;\; \mathrm{Im}(z) > 0 \lor (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.2407.01

$$\cos^{-1}\left(\frac{\sqrt{z-1}}{\sqrt{2\,z}}\right) = \frac{\pi}{2} + \frac{i}{2}\,\mathrm{sech}^{-1}(z) \;/;\; \mathrm{Im}(z) < 0 \lor (z \in \mathbb{R} \land z > 1) \lor (z \in \mathbb{R} \land z < 0)$$

01.13.27.2408.01

$$\cos^{-1}\left(\frac{\sqrt{z-1}}{\sqrt{2\,z}}\right) = \frac{\pi}{2} - \frac{1}{2\sqrt{\frac{1}{z}-1}}\,\sqrt{1-\frac{1}{z}}\,\mathrm{sech}^{-1}(z)$$

Involving $\cos^{-1}\left(\frac{\sqrt{a-z}}{\sqrt{-2\,z}}\right)$

Involving $\cos^{-1}\left(\frac{\sqrt{-z-1}}{\sqrt{-2\,z}}\right)$ and $\mathrm{sech}^{-1}(z)$

01.13.27.2409.01

$$\cos^{-1}\left(\frac{\sqrt{-z-1}}{\sqrt{-2\,z}}\right) = \frac{i}{2}\,\mathrm{sech}^{-1}(z) \;/;\; \mathrm{Im}(z) > 0 \lor (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.2410.01

$$\cos^{-1}\left(\frac{\sqrt{-z-1}}{\sqrt{-2\,z}}\right) = -\frac{i}{2}\,\mathrm{sech}^{-1}(z) \;/;\; \mathrm{Im}(z) < 0 \lor (z \in \mathbb{R} \land z > 1) \lor (z \in \mathbb{R} \land z < 0)$$

01.13.27.2411.01

$$\cos^{-1}\left(\frac{\sqrt{-z-1}}{\sqrt{-2\,z}}\right) = \frac{1}{2\sqrt{\frac{1}{z}-1}}\,\sqrt{1-\frac{1}{z}}\,\mathrm{sech}^{-1}(z)$$

Involving $\cos^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{-2\,z}}\right)$ and $\mathrm{sech}^{-1}(z)$

01.13.27.2412.01

$$\cos^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{-2z}}\right) = \frac{\pi}{2} + \frac{i}{2}\operatorname{sech}^{-1}(z) \;/;\; \operatorname{Im}(z) \leq 0$$

01.13.27.2413.01

$$\cos^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{-2z}}\right) = \frac{\pi}{2} - \frac{i}{2}\operatorname{sech}^{-1}(z) \;/;\; \operatorname{Im}(z) > 0$$

01.13.27.2414.01

$$\cos^{-1}\left(\frac{\sqrt{1-z}}{\sqrt{-2z}}\right) = \frac{\pi}{2} + \frac{1}{2}\sqrt{\frac{1}{z}}\sqrt{-z}\operatorname{sech}^{-1}(z)$$

Involving $\cos^{-1}\left(\sqrt{\frac{z+a}{2z}}\right)$

Involving $\cos^{-1}\left(\sqrt{\frac{z+1}{2z}}\right)$ and $\operatorname{sech}^{-1}(z)$

01.13.27.2415.01

$$\cos^{-1}\left(\sqrt{\frac{z+1}{2z}}\right) = \frac{i}{2}\operatorname{sech}^{-1}(z) \;/;\; \operatorname{Im}(z) > 0 \lor (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.2416.01

$$\cos^{-1}\left(\sqrt{\frac{z+1}{2z}}\right) = -\frac{i}{2}\operatorname{sech}^{-1}(z) \;/;\; \operatorname{Im}(z) < 0 \lor (z \in \mathbb{R} \land z < 0) \lor (z \in \mathbb{R} \land z > 1)$$

01.13.27.2417.01

$$\cos^{-1}\left(\sqrt{\frac{z+1}{2z}}\right) = \frac{\sqrt{z-1}}{2\sqrt{1-z}}\operatorname{sech}^{-1}(z) \;/;\; |\arg(z)| < \pi$$

01.13.27.2418.01

$$\cos^{-1}\left(\sqrt{\frac{z+1}{2z}}\right) = \frac{1}{2\sqrt{\frac{1}{z}-1}}\sqrt{1-\frac{1}{z}}\operatorname{sech}^{-1}(z)$$

Involving $\cos^{-1}\left(\sqrt{\frac{z-1}{2z}}\right)$ and $\operatorname{sech}^{-1}(z)$

01.13.27.2419.01

$$\cos^{-1}\left(\sqrt{\frac{z-1}{2z}}\right) = \frac{\pi}{2} - \frac{i}{2}\operatorname{sech}^{-1}(z) \;/;\; \operatorname{Im}(z) > 0 \lor (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.2420.01

$$\cos^{-1}\left(\sqrt{\frac{z-1}{2z}}\right) = \frac{\pi}{2} + \frac{i}{2}\operatorname{sech}^{-1}(z) \;/;\; \operatorname{Im}(z) < 0 \lor (z \in \mathbb{R} \land z < 0) \lor (z \in \mathbb{R} \land z > 1)$$

01.13.27.2421.01

$$\cos^{-1}\left(\sqrt{\frac{z-1}{2z}}\right) = \frac{\pi}{2} - \frac{1}{2\sqrt{\frac{1}{z}-1}} \sqrt{1-\frac{1}{z}} \operatorname{sech}^{-1}(z)$$

Involving $\cos^{-1}\left(\sqrt{c z^2+1}\right)$

Involving $\cos^{-1}\left(\sqrt{z^2+1}\right)$ and $\operatorname{sech}^{-1}\left(\frac{1}{1+2z^2}\right)$

01.13.27.2422.01

$$\cos^{-1}\left(\sqrt{z^2+1}\right) = \frac{i}{2} \operatorname{sech}^{-1}\left(\frac{1}{1+2z^2}\right) /; \frac{\pi}{2} < \arg(z) \leq \pi \bigvee -\frac{\pi}{2} < \arg(z) \leq 0$$

01.13.27.2423.01

$$\cos^{-1}\left(\sqrt{z^2+1}\right) = -\frac{i}{2} \operatorname{sech}^{-1}\left(\frac{1}{1+2z^2}\right) /; 0 < \arg(z) \leq \frac{\pi}{2} \bigvee -\pi < \arg(z) \leq -\frac{\pi}{2}$$

01.13.27.2424.01

$$\cos^{-1}\left(\sqrt{z^2+1}\right) = \frac{\sqrt{-z^4}}{2z^2} \operatorname{sech}^{-1}\left(\frac{1}{1+2z^2}\right)$$

Involving $\cos^{-1}\left(\sqrt{z^2+1}\right)$ and $\operatorname{sech}^{-1}\left(\frac{1}{\sqrt{z^2+1}}\right)$

01.13.27.2425.01

$$\cos^{-1}\left(\sqrt{z^2+1}\right) = i \operatorname{sech}^{-1}\left(\frac{1}{\sqrt{z^2+1}}\right) /; \frac{\pi}{2} < \arg(z) \leq \pi \bigvee -\frac{\pi}{2} < \arg(z) \leq 0$$

01.13.27.2426.01

$$\cos^{-1}\left(\sqrt{z^2+1}\right) = -i \operatorname{sech}^{-1}\left(\frac{1}{\sqrt{z^2+1}}\right) /; 0 < \arg(z) \leq \frac{\pi}{2} \bigvee -\pi < \arg(z) \leq -\frac{\pi}{2}$$

01.13.27.2427.01

$$\cos^{-1}\left(\sqrt{z^2+1}\right) = \frac{\sqrt{-z^4}}{z^2} \operatorname{sech}^{-1}\left(\frac{1}{\sqrt{z^2+1}}\right)$$

Involving $\cos^{-1}\left(\sqrt{1-z^2}\right)$ and $\operatorname{sech}^{-1}\left(\frac{1}{z}\right)$

01.13.27.2428.01

$$\cos^{-1}\left(\sqrt{1-z^2}\right) = \frac{\pi}{2} + i \operatorname{sech}^{-1}\left(\frac{1}{z}\right) /; 0 < \arg(z) \leq \frac{\pi}{2} \bigvee (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.2429.01

$$\cos^{-1}\left(\sqrt{1-z^2}\right) = \frac{\pi}{2} - i\,\text{sech}^{-1}\left(\frac{1}{z}\right) /; -\frac{\pi}{2} < \arg(z) < 0 \bigvee (z \in \mathbb{R} \land z > 1)$$

01.13.27.2430.01

$$\cos^{-1}\left(\sqrt{1-z^2}\right) = -i\,\text{sech}^{-1}\left(\frac{1}{z}\right) - \frac{\pi}{2} /; \frac{\pi}{2} < \arg(z) \leq \pi$$

01.13.27.2431.01

$$\cos^{-1}\left(\sqrt{1-z^2}\right) = i\,\text{sech}^{-1}\left(\frac{1}{z}\right) - \frac{\pi}{2} /; -\pi < \arg(z) \leq -\frac{\pi}{2}$$

01.13.27.2432.01

$$\cos^{-1}\left(\sqrt{1-z^2}\right) = \frac{\pi}{2} - \frac{\sqrt{1-z}}{\sqrt{z-1}}\,\text{sech}^{-1}\left(\frac{1}{z}\right) /; -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

01.13.27.2433.01

$$\cos^{-1}\left(\sqrt{1-z^2}\right) = \frac{\pi\sqrt{z^2}}{2\,z} - \frac{\sqrt{z^2}\,\sqrt{1-z}}{z\,\sqrt{z-1}}\,\text{sech}^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{z^2+a}}{z}\right)$

Involving $\cos^{-1}\left(\frac{\sqrt{z^2-1}}{z}\right)$ and $\text{sech}^{-1}(z)$

01.13.27.2434.01

$$\cos^{-1}\left(\frac{\sqrt{z^2-1}}{z}\right) = \frac{\pi}{2} - i\,\text{sech}^{-1}(z) /; 0 < \arg(z) < \frac{\pi}{2} \bigvee (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.2435.01

$$\cos^{-1}\left(\frac{\sqrt{z^2-1}}{z}\right) = \frac{\pi}{2} + i\,\text{sech}^{-1}(z) /; -\frac{\pi}{2} < \arg(z) < 0 \bigvee (z \in \mathbb{R} \bigwedge z > 1)$$

01.13.27.2436.01

$$\cos^{-1}\left(\frac{\sqrt{z^2-1}}{z}\right) = \frac{3\pi}{2} - i\,\text{sech}^{-1}(z) /; \frac{\pi}{2} < \arg(z) < \pi$$

01.13.27.2437.01

$$\cos^{-1}\left(\frac{\sqrt{z^2-1}}{z}\right) = \frac{3\pi}{2} + i\,\text{sech}^{-1}(z) /; -\pi < \arg(z) < -\frac{\pi}{2} \bigvee (z \in \mathbb{R} \land z < 0)$$

01.13.27.2438.01

$$\cos^{-1}\left(\frac{\sqrt{z^2-1}}{z}\right) = \frac{\pi}{2}\left(2 - \frac{\sqrt{z^2}}{z}\right) - \frac{\sqrt{z-1}}{\sqrt{1-z}}\,\text{sech}^{-1}(z) /; \text{Re}(z) \neq 0 \bigwedge |\arg(z)| < \pi$$

01.13.27.2439.01

$$\cos^{-1}\left(\frac{\sqrt{z^2-1}}{z}\right) = \frac{\pi}{2z}\left(\sqrt{z^2}\sqrt{\frac{1}{z^2}}\, z + z - \sqrt{z^2}\right) - \frac{\sqrt{z-1}\sqrt{z}\sqrt{z^2}}{\sqrt{1-z}}\sqrt{\frac{1}{z^2}}\sqrt{\frac{1}{z}}\,\operatorname{sech}^{-1}(z)$$

Involving $\cos^{-1}\left(\frac{\sqrt{z^2+1}}{z}\right)$ and $\operatorname{sech}^{-1}\left(\frac{z^2}{z^2+2}\right)$

01.13.27.2440.01

$$\cos^{-1}\left(\frac{\sqrt{z^2+1}}{z}\right) = \frac{i}{2}\operatorname{sech}^{-1}\left(\frac{z^2}{z^2+2}\right) /;\ 0 \le \arg(z) < \frac{\pi}{2}$$

01.13.27.2441.01

$$\cos^{-1}\left(\frac{\sqrt{z^2+1}}{z}\right) = -\frac{i}{2}\operatorname{sech}^{-1}\left(\frac{z^2}{z^2+2}\right) /;\ -\frac{\pi}{2} < \arg(z) < 0 \bigvee (i\,z \in \mathbb{R} \land i\,z < -1) \bigvee (i\,z \in \mathbb{R} \land 0 < i\,z < 1)$$

01.13.27.2442.01

$$\cos^{-1}\left(\frac{\sqrt{z^2+1}}{z}\right) = \pi + \frac{i}{2}\operatorname{sech}^{-1}\left(\frac{z^2}{z^2+2}\right) /;\ \frac{\pi}{2} < \arg(z) < \pi \bigvee (-i\,z \in \mathbb{R} \land 0 < -i\,z < 1) \bigvee (i\,z \in \mathbb{R} \land i\,z > 1)$$

01.13.27.2443.01

$$\cos^{-1}\left(\frac{\sqrt{z^2+1}}{z}\right) = \pi - \frac{i}{2}\operatorname{sech}^{-1}\left(\frac{z^2}{z^2+2}\right) /;\ -\pi < \arg(z) < -\frac{\pi}{2} \bigvee (z \in \mathbb{R} \land z < 0)$$

01.13.27.2444.01

$$\cos^{-1}\left(\frac{\sqrt{z^2+1}}{z}\right) = \frac{\pi}{2} - \frac{\sqrt{z^2+1}}{2z\sqrt{-\frac{z^2+1}{z^4}}}\left(\sqrt{\frac{1}{z^2}}\operatorname{sech}^{-1}\left(\frac{z^2}{z^2+2}\right) + \pi\sqrt{-\frac{1}{z^2}}\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{z^2+a}}{\sqrt{z^2}}\right)$

Involving $\cos^{-1}\left(\frac{\sqrt{z^2-1}}{\sqrt{z^2}}\right)$ and $\operatorname{sech}^{-1}(z)$

01.13.27.2445.01

$$\cos^{-1}\left(\frac{\sqrt{z^2-1}}{\sqrt{z^2}}\right) = \frac{\pi}{2} - i\operatorname{sech}^{-1}(z) /;\ 0 < \arg(z) < \frac{\pi}{2} \bigvee (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.2446.01

$$\cos^{-1}\left(\frac{\sqrt{z^2-1}}{\sqrt{z^2}}\right) = \frac{\pi}{2} + i\operatorname{sech}^{-1}(z) /;\ -\frac{\pi}{2} \le \arg(z) < 0 \bigvee (z \in \mathbb{R} \land z > 1)$$

01.13.27.2447.01

$$\cos^{-1}\left(\frac{\sqrt{z^2-1}}{\sqrt{z^2}}\right) = -\frac{\pi}{2} + i\,\text{sech}^{-1}(z) \;/;\; \frac{\pi}{2} \leq \arg(z) < \pi$$

01.13.27.2448.01

$$\cos^{-1}\left(\frac{\sqrt{z^2-1}}{\sqrt{z^2}}\right) = -\frac{\pi}{2} - i\,\text{sech}^{-1}(z) \;/;\; -\pi < \arg(z) < -\frac{\pi}{2} \bigvee (z \in \mathbb{R} \land z < 0)$$

01.13.27.2449.01

$$\cos^{-1}\left(\frac{\sqrt{z^2-1}}{\sqrt{z^2}}\right) = \frac{\pi}{2}\sqrt{\frac{1}{z^2}}\; z - \frac{\sqrt{z-1}\,\sqrt{z}}{\sqrt{\frac{1-z}{z}}}\sqrt{\frac{1}{z^2}}\;\text{sech}^{-1}(z)$$

Involving $\cos^{-1}\left(\frac{\sqrt{1+z^2}}{\sqrt{z^2}}\right)$ and $\text{sech}^{-1}\left(\frac{z^2}{z^2+2}\right)$

01.13.27.2450.01

$$\cos^{-1}\left(\frac{\sqrt{z^2+1}}{\sqrt{z^2}}\right) = \frac{1}{2}\, i\,\text{sech}^{-1}\left(\frac{z^2}{z^2+2}\right) /;\; 0 < \arg(z) < \frac{\pi}{2} \bigvee -\pi < \arg(z) < -\frac{\pi}{2} \bigvee (z \in \mathbb{R} \land z < 0)$$

01.13.27.2451.01

$$\cos^{-1}\left(\frac{\sqrt{z^2+1}}{\sqrt{z^2}}\right) = -\frac{i}{2}\,\text{sech}^{-1}\left(\frac{z^2}{z^2+2}\right) /;\; \frac{\pi}{2} < \arg(z) < \pi \bigvee -\frac{\pi}{2} < \arg(z) < 0 \bigvee (i\,z \in \mathbb{R} \land i\,z < -1) \bigvee (i\,z \in \mathbb{R} \land i\,z > 1)$$

01.13.27.2452.01

$$\cos^{-1}\left(\frac{\sqrt{z^2+1}}{\sqrt{z^2}}\right) = \frac{1}{2}\, i\,\text{sech}^{-1}\left(\frac{z^2}{z^2+2}\right) + \pi \;/;\; (i\,z \in \mathbb{R} \land -1 < i\,z < 1)$$

01.13.27.2453.01

$$\cos^{-1}\left(\frac{\sqrt{z^2+1}}{\sqrt{z^2}}\right) = \frac{\pi}{2} - \frac{\sqrt{-z^2\left(z^2+1\right)}}{\sqrt{z^2}\,\sqrt{-z^2-1}}\left(\frac{\pi}{2} - \frac{1}{2} z^2 \sqrt{-\frac{1}{z^4}}\;\text{sech}^{-1}\left(\frac{z^2}{z^2+2}\right)\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{a-z^2}}{\sqrt{-z^2}}\right)$

Involving $\cos^{-1}\left(\frac{\sqrt{1-z^2}}{\sqrt{-z^2}}\right)$ and $\text{sech}^{-1}(z)$

01.13.27.2454.01

$$\cos^{-1}\left(\frac{\sqrt{1-z^2}}{\sqrt{-z^2}}\right) = \frac{\pi}{2} - i\,\text{sech}^{-1}(z) \;/;\; 0 < \arg(z) < \frac{\pi}{2}$$

01.13.27.2455.01

$$\cos^{-1}\left(\frac{\sqrt{1-z^2}}{\sqrt{-z^2}}\right) = \frac{\pi}{2} + i\,\text{sech}^{-1}(z) \;/;\; -\frac{\pi}{2} \leq \arg(z) \leq 0$$

01.13.27.2456.01

$$\cos^{-1}\left(\frac{\sqrt{1-z^2}}{\sqrt{-z^2}}\right) = i\,\text{sech}^{-1}(z) - \frac{\pi}{2} \;/;\; \frac{\pi}{2} \leq \arg(z) < \pi$$

01.13.27.2457.01

$$\cos^{-1}\left(\frac{\sqrt{1-z^2}}{\sqrt{-z^2}}\right) = -\frac{\pi}{2} - i\,\text{sech}^{-1}(z) \;/;\; -\pi < \arg(z) < -\frac{\pi}{2} \bigvee (z \in \mathbb{R} \land z < -1)$$

01.13.27.2458.01

$$\cos^{-1}\left(\frac{\sqrt{1-z^2}}{\sqrt{-z^2}}\right) = \frac{3\pi}{2} + i\,\text{sech}^{-1}(z) \;/;\; (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.2459.01

$$\cos^{-1}\left(\frac{\sqrt{1-z^2}}{\sqrt{-z^2}}\right) = \frac{\pi}{2} - \sqrt{\frac{z^2-1}{z^2}}\sqrt{\frac{z^2}{z^2-1}}\left(\frac{\pi}{2}\left(1-\sqrt{\frac{1}{z^2}}\,z\right) + \frac{\sqrt{z-1}\,\sqrt{z}}{\sqrt{\frac{1-z}{z}}}\sqrt{\frac{1}{z^2}}\,\text{sech}^{-1}(z)\right)$$

Involving $\cos^{-1}\left(\frac{\sqrt{-1-z^2}}{\sqrt{-z^2}}\right)$ and $\text{sech}^{-1}\left(\frac{z^2}{z^2+2}\right)$

01.13.27.2460.01

$$\cos^{-1}\left(\frac{\sqrt{-1-z^2}}{\sqrt{-z^2}}\right) = \frac{i}{2}\,\text{sech}^{-1}\left(\frac{z^2}{z^2+2}\right) \;/;\; 0 \leq \arg(z) < \frac{\pi}{2} \bigvee -\pi < \arg(z) < -\frac{\pi}{2} \bigvee (z \in \mathbb{R} \land z < 0)$$

01.13.27.2461.01

$$\cos^{-1}\left(\frac{\sqrt{-1-z^2}}{\sqrt{-z^2}}\right) = -\frac{i}{2}\,\text{sech}^{-1}\left(\frac{z^2}{z^2+2}\right) \;/;\; \frac{\pi}{2} \leq \arg(z) < \pi \bigvee -\frac{\pi}{2} \leq \arg(z) < 0$$

01.13.27.2462.01

$$\cos^{-1}\left(\frac{\sqrt{-1-z^2}}{\sqrt{-z^2}}\right) = \frac{1}{2}\,z^2\sqrt{-\frac{1}{z^4}}\;\text{sech}^{-1}\left(\frac{z^2}{z^2+2}\right)$$

Involving $\cos^{-1}\left(\sqrt{\frac{z^2+a}{z^2}}\right)$

Involving $\cos^{-1}\left(\sqrt{\frac{z^2-1}{z^2}}\right)$ and $\operatorname{sech}^{-1}(z)$

01.13.27.2463.01

$$\cos^{-1}\left(\sqrt{\frac{z^2-1}{z^2}}\right) = \frac{\pi}{2} - i\operatorname{sech}^{-1}(z) \,/;\, 0 < \arg(z) < \frac{\pi}{2} \bigvee (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.2464.01

$$\cos^{-1}\left(\sqrt{\frac{z^2-1}{z^2}}\right) = \frac{\pi}{2} + i\operatorname{sech}^{-1}(z) \,/;\, -\frac{\pi}{2} \leq \arg(z) < 0 \bigvee (z \in \mathbb{R} \land z > 1)$$

01.13.27.2465.01

$$\cos^{-1}\left(\sqrt{\frac{z^2-1}{z^2}}\right) = i\operatorname{sech}^{-1}(z) - \frac{\pi}{2} \,/;\, \frac{\pi}{2} \leq \arg(z) < \pi$$

01.13.27.2466.01

$$\cos^{-1}\left(\sqrt{\frac{z^2-1}{z^2}}\right) = -\frac{\pi}{2} - i\operatorname{sech}^{-1}(z) \,/;\, -\pi < \arg(z) < -\frac{\pi}{2} \bigvee (z \in \mathbb{R} \land z < 0)$$

01.13.27.2467.01

$$\cos^{-1}\left(\sqrt{\frac{z^2-1}{z^2}}\right) = \frac{\pi}{2}\sqrt{\frac{1}{z^2}}\, z - \frac{\sqrt{z-1}\,\sqrt{z}}{\sqrt{\frac{1-z}{z}}}\sqrt{\frac{1}{z^2}}\,\operatorname{sech}^{-1}(z)$$

Involving $\cos^{-1}\left(\sqrt{\frac{z^2+1}{z^2}}\right)$ and $\operatorname{sech}^{-1}\left(\frac{z^2}{z^2+2}\right)$

01.13.27.2468.01

$$\cos^{-1}\left(\sqrt{\frac{z^2+1}{z^2}}\right) = \frac{i}{2}\operatorname{sech}^{-1}\left(\frac{z^2}{z^2+2}\right) /;\, 0 \leq \arg(z) < \frac{\pi}{2} \bigvee -\pi < \arg(z) < -\frac{\pi}{2} \bigvee (z \in \mathbb{R} \land z < 0)$$

01.13.27.2469.01

$$\cos^{-1}\left(\sqrt{\frac{z^2+1}{z^2}}\right) = -\frac{i}{2}\operatorname{sech}^{-1}\left(\frac{z^2}{z^2+2}\right) /;\, \frac{\pi}{2} \leq \arg(z) < \pi \bigvee -\frac{\pi}{2} \leq \arg(z) < 0$$

01.13.27.2470.01

$$\cos^{-1}\left(\sqrt{\frac{z^2+1}{z^2}}\right) = \frac{1}{2}\left(z^2\sqrt{-\frac{1}{z^4}}\right)\operatorname{sech}^{-1}\left(\frac{z^2}{z^2+2}\right)$$

Involving $\cos^{-1}\left(2 z \sqrt{1-z^2}\right)$

Involving $\cos^{-1}\left(2 z \sqrt{1-z^2}\right)$ and $\operatorname{sech}^{-1}\left(\frac{1}{z}\right)$

01.13.27.2471.01

$$\cos^{-1}\left(2 z \sqrt{1-z^2}\right) = -2 i \operatorname{sech}^{-1}\left(\frac{1}{z}\right) - \frac{\pi}{2} /; \frac{\pi}{4} \leq \arg(z) \leq \frac{3\pi}{4}$$

01.13.27.2472.01

$$\cos^{-1}\left(2 z \sqrt{1-z^2}\right) = 2 i \operatorname{sech}^{-1}\left(\frac{1}{z}\right) - \frac{\pi}{2} /; -\frac{3\pi}{4} \leq \arg(z) \leq -\frac{\pi}{4}$$

01.13.27.2473.01

$$\cos^{-1}\left(2 z \sqrt{1-z^2}\right) = -\frac{\pi}{2} + 2 \frac{\sqrt{1-z}}{\sqrt{z-1}} \operatorname{sech}^{-1}\left(\frac{1}{z}\right) /; \frac{\pi}{4} \leq |\arg(z)| \leq \frac{3\pi}{4}$$

01.13.27.2474.01

$$\cos^{-1}\left(2 z \sqrt{1-z^2}\right) = \frac{\pi}{2} - \frac{\pi \sqrt{1-2z^2} \sqrt{z^4-z^2}}{2 \sqrt{-z^2} \sqrt{z^2-1} \sqrt{2z^2-1}} \left(\frac{\sqrt{z^2}}{z} - \sqrt{\frac{1}{z}} \sqrt{z} \sqrt{\frac{1}{\sqrt{2}\, z+1}} \sqrt{\sqrt{2}\, z+1} + \sqrt{-\frac{1}{z}} \sqrt{-z} \sqrt{\frac{1}{1-\sqrt{2}\, z}} \sqrt{1-\sqrt{2}\, z} + \frac{\sqrt{z^4-z^2}}{z \sqrt{z^2-1}} - 2\right) - \frac{2 \sqrt{1-z} \sqrt{1-2z^2} \sqrt{z^4-z^2}}{\sqrt{z-1} \sqrt{-z^2} \sqrt{z^2-1} \sqrt{2z^2-1}} \operatorname{sech}^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(\frac{2\sqrt{-1+z^2}}{z^2}\right)$

Involving $\cos^{-1}\left(\frac{2\sqrt{z^2-1}}{z^2}\right)$ and $\operatorname{sech}^{-1}(z)$

01.13.27.2475.01

$$\cos^{-1}\left(\frac{2\sqrt{z^2-1}}{z^2}\right) = \frac{\pi}{2}\left(1 - \frac{2\sqrt{z^2}}{z}\right) + \frac{2\sqrt{z^2}\sqrt{z-1}}{z\sqrt{1-z}} \operatorname{sech}^{-1}(z) /; |z| > \sqrt{2} \bigwedge \operatorname{Im}(z) \neq 0$$

01.13.27.2476.01

$$\cos^{-1}\left(\frac{2\sqrt{z^2-1}}{z^2}\right) = \frac{\pi}{2} + \frac{z^3\sqrt{z^2-2}\sqrt{z^2-1}}{2\sqrt{1-z}\,(z+1)\sqrt{-z^4+3z^2-2}}\sqrt{\frac{1-z^2}{z^4}}\sqrt{\frac{1}{z}}\sqrt{-\frac{z+1}{z}}\left(\pi\left(\frac{z^3}{1-z^2}\sqrt{\frac{1-z^2}{z^2}}\sqrt{\frac{1-z^2}{z^4}} + \sqrt{\frac{1}{z^2}}\,z - \sqrt{\frac{1}{z}}\sqrt{\frac{z}{z+\sqrt{2}}}\sqrt{\frac{z+\sqrt{2}}{z}}\sqrt{z} + \sqrt{1-\frac{\sqrt{2}}{z}}\sqrt{-\frac{1}{z}}\sqrt{-z}\sqrt{\frac{z}{z-\sqrt{2}}} - 2\right) + \frac{4}{\sqrt{\frac{1}{z}-1}}\sqrt{1-\frac{1}{z}}\operatorname{sech}^{-1}(z)\right)$$

Involving $\cos^{-1}\left(\sqrt{\left(1-\sqrt{1+c\,z^2}\right)/2}\right)$

Involving $\cos^{-1}\left(\sqrt{\left(1-\sqrt{1+z^2}\right)/2}\right)$ and $\operatorname{sech}^{-1}\left(-\frac{i}{z}\right)$

01.13.27.2477.01

$$\cos^{-1}\left(\sqrt{\frac{1}{2}\left(1-\sqrt{z^2+1}\right)}\right) = \frac{1}{2}\,i\operatorname{sech}^{-1}\left(-\frac{i}{z}\right) + \frac{3\pi}{4} \;/;\; 0 < \arg(z) \le \frac{\pi}{2}$$

01.13.27.2478.01

$$\cos^{-1}\left(\sqrt{\frac{1}{2}\left(1-\sqrt{z^2+1}\right)}\right) = \frac{\pi}{4} - \frac{i}{2}\operatorname{sech}^{-1}\left(-\frac{i}{z}\right) \;/;\; -\frac{\pi}{2} < \arg(z) \le 0 \bigvee (i\,z \in \mathbb{R} \land 0 < i\,z < 1)$$

01.13.27.2479.01

$$\cos^{-1}\left(\sqrt{\frac{1}{2}\left(1-\sqrt{z^2+1}\right)}\right) = \frac{3\pi}{4} - \frac{i}{2}\operatorname{sech}^{-1}\left(-\frac{i}{z}\right) \;/;\; \frac{\pi}{2} < \arg(z) \le \pi$$

01.13.27.2480.01

$$\cos^{-1}\left(\sqrt{\frac{1}{2}\left(1-\sqrt{z^2+1}\right)}\right) = \frac{\pi}{4} + \frac{i}{2}\operatorname{sech}^{-1}\left(-\frac{i}{z}\right) \;/;\; -\pi < \arg(z) < -\frac{\pi}{2} \bigvee (i\,z \in \mathbb{R} \land i\,z > 1)$$

01.13.27.2481.01

$$\cos^{-1}\left(\sqrt{\frac{1-\sqrt{1+z^2}}{2}}\right) = \frac{\pi}{4} + \frac{\sqrt{1-i\,z}}{2\sqrt{i\,z-1}}\operatorname{sech}^{-1}\left(-\frac{i}{z}\right) \;/;\; -\pi < \arg(z) \le 0$$

01.13.27.2482.01

$$\cos^{-1}\left(\sqrt{\frac{1}{2}\left(1-\sqrt{z^2+1}\right)}\right) = \frac{1}{2}\pi\left(1+\frac{i\sqrt{-z^2}}{2z}\right) - \frac{i\sqrt{-z^2}\sqrt{1-iz}}{2z\sqrt{iz-1}}\operatorname{sech}^{-1}\left(-\frac{i}{z}\right)$$

Involving $\cos^{-1}\left(\sqrt{\left(1-\sqrt{1-z^2}\right)/2}\right)$ and $\operatorname{sech}^{-1}\left(\frac{1}{z}\right)$

01.13.27.2483.01

$$\cos^{-1}\left(\sqrt{\frac{1}{2}\left(1-\sqrt{1-z^2}\right)}\right) = \frac{1}{2}\left(-i\operatorname{sech}^{-1}\left(\frac{1}{z}\right)+\frac{\pi}{2}\right) /;\ 0<\arg(z)\leq\frac{\pi}{2}\bigvee(z\in\mathbb{R}\land 0<z<1)$$

01.13.27.2484.01

$$\cos^{-1}\left(\sqrt{\frac{1}{2}\left(1-\sqrt{1-z^2}\right)}\right) = \frac{1}{2}\left(\frac{\pi}{2}+i\operatorname{sech}^{-1}\left(\frac{1}{z}\right)\right) /;\ -\frac{\pi}{2}<\arg(z)<0\bigvee(z\in\mathbb{R}\land z>1)$$

01.13.27.2485.01

$$\cos^{-1}\left(\sqrt{\frac{1}{2}\left(1-\sqrt{1-z^2}\right)}\right) = \frac{1}{2}\left(i\operatorname{sech}^{-1}\left(\frac{1}{z}\right)+\frac{3\pi}{2}\right) /;\ \frac{\pi}{2}<\arg(z)\leq\pi$$

01.13.27.2486.01

$$\cos^{-1}\left(\sqrt{\frac{1}{2}\left(1-\sqrt{1-z^2}\right)}\right) = \frac{1}{2}\left(\frac{3\pi}{2}-i\operatorname{sech}^{-1}\left(\frac{1}{z}\right)\right) /;\ -\pi<\arg(z)\leq-\frac{\pi}{2}$$

01.13.27.2487.01

$$\cos^{-1}\left(\sqrt{\frac{1-\sqrt{1-z^2}}{2}}\right) = \frac{1}{2}\left(\frac{\pi}{2}+\frac{\sqrt{1-z}}{\sqrt{z-1}}\operatorname{sech}^{-1}\left(\frac{1}{z}\right)\right) /;\ -\frac{\pi}{2}<\arg(z)\leq\frac{\pi}{2}$$

01.13.27.2488.01

$$\cos^{-1}\left(\sqrt{\frac{1}{2}\left(1-\sqrt{1-z^2}\right)}\right) = \frac{1}{2}\left(\frac{3\pi}{2}-\frac{\sqrt{1-z}}{\sqrt{z-1}}\operatorname{sech}^{-1}\left(\frac{1}{z}\right)\right) /;\ \operatorname{Re}(z)<0\bigvee\arg(z)=-\frac{\pi}{2}$$

01.13.27.2489.01

$$\cos^{-1}\left(\sqrt{\frac{1-\sqrt{1-z^2}}{2}}\right) = \frac{\pi}{2}\left(1-\frac{\sqrt{z^2}}{2z}\right)+\frac{\sqrt{z^2}\sqrt{1-z}}{2z\sqrt{z-1}}\operatorname{sech}^{-1}\left(\frac{1}{z}\right)$$

Involving $\cos^{-1}\left(z\sqrt{1-\sqrt{1-z^2}}\,\Big/\sqrt{2z^2}\right)$

Involving $\cos^{-1}\left(z\sqrt{1-\sqrt{1-z^2}}\,\Big/\sqrt{2z^2}\right)$ and $\operatorname{sech}^{-1}\left(\frac{1}{z}\right)$

01.13.27.2490.01

$$\cos^{-1}\left(\frac{z\sqrt{1-\sqrt{1-z^2}}}{\sqrt{2z^2}}\right) = \frac{1}{2}\left(-i\operatorname{sech}^{-1}\left(\frac{1}{z}\right)+\frac{\pi}{2}\right) /; 0 < \arg(z) \le \pi \bigvee (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.2491.01

$$\cos^{-1}\left(\frac{z\sqrt{1-\sqrt{1-z^2}}}{\sqrt{2z^2}}\right) = \frac{1}{2}\left(\frac{\pi}{2}+i\operatorname{sech}^{-1}\left(\frac{1}{z}\right)\right) /; -\pi < \arg(z) < 0 \bigvee (z \in \mathbb{R} \land z > 1)$$

01.13.27.2492.01

$$\cos^{-1}\left(\frac{z\sqrt{1-\sqrt{1-z^2}}}{\sqrt{2z^2}}\right) = \frac{1}{2}\left(\frac{\pi}{2}+\frac{\sqrt{1-z}}{\sqrt{z-1}}\operatorname{sech}^{-1}\left(\frac{1}{z}\right)\right)$$

Involving $\cos^{-1}\left(z\sqrt{\left(1-\sqrt{1-z^2}\right)/(2z^2)}\right)$

Involving $\cos^{-1}\left(z\sqrt{\left(1-\sqrt{1-z^2}\right)/(2z^2)}\right)$ and $\operatorname{sech}^{-1}\left(\frac{1}{z}\right)$

01.13.27.2493.01

$$\cos^{-1}\left(z\sqrt{\frac{1-\sqrt{1-z^2}}{2z^2}}\right) = \frac{1}{2}\left(-i\operatorname{sech}^{-1}\left(\frac{1}{z}\right)+\frac{\pi}{2}\right) /; 0 < \arg(z) \le \pi \bigvee (z \in \mathbb{R} \land 0 < z < 1)$$

01.13.27.2494.01

$$\cos^{-1}\left(z\sqrt{\frac{1-\sqrt{1-z^2}}{2z^2}}\right) = \frac{1}{2}\left(\frac{\pi}{2}+i\operatorname{sech}^{-1}\left(\frac{1}{z}\right)\right) /; -\pi < \arg(z) < 0 \bigvee (z \in \mathbb{R} \land z > 1)$$

01.13.27.2495.01

$$\cos^{-1}\left(z\sqrt{\frac{1-\sqrt{1-z^2}}{2z^2}}\right) = \frac{1}{2}\left(\frac{\pi}{2}+\frac{\sqrt{1-z}}{\sqrt{z-1}}\operatorname{sech}^{-1}\left(\frac{1}{z}\right)\right)$$

Involving $\cos^{-1}\left(\sqrt{z-\sqrt{z^2-1}}\Big/\sqrt{2z}\right)$

Involving $\cos^{-1}\left(\sqrt{z-\sqrt{z^2-1}} \,/\, \sqrt{2z}\right)$ and $\operatorname{sech}^{-1}(z)$

01.13.27.2496.01

$$\cos^{-1}\left(\frac{\sqrt{z-\sqrt{z^2-1}}}{\sqrt{2z}}\right) = \frac{1}{2}\left(\frac{\pi}{2} + i\operatorname{sech}^{-1}(z)\right) /;\ 0 < \arg(z) \le \frac{\pi}{2} \bigvee (z \in \mathbb{R} \land 0 < z < 1) \bigvee \arg(z) = -\frac{\pi}{2}$$

01.13.27.2497.01

$$\cos^{-1}\left(\frac{\sqrt{z-\sqrt{z^2-1}}}{\sqrt{2z}}\right) = \frac{1}{2}\left(-i\operatorname{sech}^{-1}(z) + \frac{\pi}{2}\right) /;\ -\frac{\pi}{2} < \arg(z) < 0 \bigvee (z \in \mathbb{R} \land z > 1)$$

01.13.27.2498.01

$$\cos^{-1}\left(\frac{\sqrt{z-\sqrt{z^2-1}}}{\sqrt{2z}}\right) = \frac{1}{2}\left(-\frac{\pi}{2} + i\operatorname{sech}^{-1}(z)\right) /;\ \frac{\pi}{2} < \arg(z) < \pi$$

01.13.27.2499.01

$$\cos^{-1}\left(\frac{\sqrt{z-\sqrt{z^2-1}}}{\sqrt{2z}}\right) = -\frac{1}{2}\left(i\operatorname{sech}^{-1}(z) + \frac{\pi}{2}\right) /;\ -\pi < \arg(z) < -\frac{\pi}{2} \bigvee (z \in \mathbb{R} \land z < -1)$$

01.13.27.2500.01

$$\cos^{-1}\left(\frac{\sqrt{z-\sqrt{z^2-1}}}{\sqrt{2z}}\right) = \frac{1}{2}\left(i\operatorname{sech}^{-1}(z) + \frac{5\pi}{2}\right) /;\ (z \in \mathbb{R} \land -1 < z < 0)$$

01.13.27.2501.01

$$\cos^{-1}\left(\frac{\sqrt{z-\sqrt{z^2-1}}}{\sqrt{2z}}\right) =$$

$$\frac{\pi}{2} - \frac{\sqrt{z-1}\,\sqrt{-iz}\,\sqrt{z}}{2\sqrt{\frac{1}{z}-1}}\sqrt{\frac{i}{z}}\sqrt{-\frac{1}{z+1}}\sqrt{-\frac{z+1}{z^2}}\operatorname{sech}^{-1}(z) + \frac{1}{4}\pi\sqrt{-\frac{1}{z+1}}\sqrt{-\frac{z+1}{z^2}}\left(\sqrt{\frac{i}{z}}\sqrt{-iz}\,z + z - \sqrt{z^2}\right)$$

Involving $\cos^{-1}\left(\sqrt{\left(z-\sqrt{z^2-1}\right)/(2z)}\right)$

Involving $\cos^{-1}\left(\sqrt{\left(z-\sqrt{z^2-1}\right)/(2z)}\right)$ and $\operatorname{sech}^{-1}(z)$

01.13.27.2502.01

$$\cos^{-1}\left(\sqrt{\frac{z-\sqrt{z^2-1}}{2z}}\right)=\frac{1}{2}\left(\frac{\pi}{2}+i\operatorname{sech}^{-1}(z)\right)/;\ 0<\arg(z)<\frac{\pi}{2}\bigvee(z\in\mathbb{R}\wedge 0<z<1)\bigvee\arg(z)=-\frac{\pi}{2}$$

01.13.27.2503.01

$$\cos^{-1}\left(\sqrt{\frac{z-\sqrt{z^2-1}}{2z}}\right)=\frac{1}{2}\left(-i\operatorname{sech}^{-1}(z)+\frac{\pi}{2}\right)/;\ -\frac{\pi}{2}<\arg(z)<0\bigvee(z\in\mathbb{R}\wedge z>1)$$

01.13.27.2504.01

$$\cos^{-1}\left(\sqrt{\frac{z-\sqrt{z^2-1}}{2z}}\right)=\frac{1}{2}\left(-\frac{\pi}{2}+i\operatorname{sech}^{-1}(z)\right)/;\ \frac{\pi}{2}<\arg(z)<\pi$$

01.13.27.2505.01

$$\cos^{-1}\left(\sqrt{\frac{z-\sqrt{z^2-1}}{2z}}\right)=-\frac{1}{2}\left(i\operatorname{sech}^{-1}(z)+\frac{\pi}{2}\right)/;\ -\pi<\arg(z)<-\frac{\pi}{2}\bigvee(z\in\mathbb{R}\wedge z<0)$$

01.13.27.2506.01

$$\cos^{-1}\left(\sqrt{\frac{z-\sqrt{z^2-1}}{2z}}\right)=\frac{1}{4}\pi\sqrt{\frac{1}{z}}\sqrt{-\frac{i}{z}}\sqrt{z}\sqrt{iz}\sqrt{-\frac{z+1}{z^2}}\sqrt{z+1}\sqrt{-\frac{1}{(z+1)^2}}\left(\sqrt{\frac{i}{z}}\sqrt{-iz}\,z+z-\sqrt{z^2}\right)-$$
$$\frac{1}{2\sqrt{1-z}}\left(\sqrt{z-1}\sqrt{\frac{1}{z^2}}\,z\sqrt{z^2}\sqrt{-\frac{1}{(z+1)^2}}\sqrt{z+1}\sqrt{-\frac{z+1}{z^2}}\right)\operatorname{sech}^{-1}(z)+\frac{\pi}{2}$$

Inequalities

01.13.29.0001.01

$$\cos^{-1}(x)\geq 0\ /;\ |x|\leq 1\wedge x\in\mathbb{R}$$

01.13.29.0002.01

$$\cos^{-1}(x)\leq\pi\ /;\ |x|\leq 1\wedge x\in\mathbb{R}$$

Zeros

01.13.30.0001.01

$\cos^{-1}(z) == 0 /; z == 1$

History

– J. Herschel (1813) introduced the notation $\cos^{-1}$

The function $\cos^{-1}$ is often encountered in mathematics and the natural sciences.

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