EllipticK

Notations

Traditional name

Complete elliptic integral of the first kind

Traditional notation

$K(z)$

Mathematica StandardForm notation

`EllipticK[z]`

Primary definition

08.02.02.0001.01

$$K(z) == F\left(\frac{\pi}{2} \,\middle|\, z\right)$$

Specific values

Values at fixed points

08.02.03.0001.01

$$K(0) == \frac{\pi}{2}$$

08.02.03.0002.01

$$K\left(\frac{1}{2}\right) == \frac{8\,\pi^{3/2}}{\Gamma\left(-\frac{1}{4}\right)^2}$$

08.02.03.0030.01

$$K\left(17 - 12\sqrt{2}\right) == \frac{2\left(2 + \sqrt{2}\right)\pi^{3/2}}{\Gamma\left(-\frac{1}{4}\right)^2}$$

08.02.03.0003.01

$$K(1) == \tilde{\infty}$$

08.02.03.0004.01

$$K(-1) == \frac{\Gamma\left(\frac{1}{4}\right)^2}{4\sqrt{2\pi}}$$

Values at infinities

08.02.03.0005.01

$K(\infty) == 0$

08.02.03.0006.01

$K(-\infty) == 0$

08.02.03.0007.01

$K(i\,\infty) == 0$

08.02.03.0008.01

$K(-i\,\infty) == 0$

08.02.03.0009.01

$K(\tilde{\infty}) == 0$

Singular values

08.02.03.0010.01

$$\frac{K(1-z^2)}{K(z^2)} == \sqrt{1} \;/;\; z == \frac{1}{\sqrt{2}}$$

08.02.03.0011.01

$$\frac{K(1-z^2)}{K(z^2)} == \sqrt{2} \;/;\; z == \sqrt{2} - 1$$

08.02.03.0012.01

$$\frac{K(1-z^2)}{K(z^2)} == \sqrt{3} \;/;\; z == \frac{\sqrt{3} - 1}{2\sqrt{2}}$$

08.02.03.0013.01

$$\frac{K(1-z^2)}{K(z^2)} == \sqrt{4} \;/;\; z == 3 - 2\sqrt{2}$$

08.02.03.0014.01

$$\frac{K(1-z^2)}{K(z^2)} == \sqrt{5} \;/;\; z == \sqrt{\frac{1}{2} - \sqrt{-2 + \sqrt{5}}}$$

08.02.03.0015.01

$$\frac{K(1-z^2)}{K(z^2)} == \sqrt{6} \;/;\; z == -3 - 2\sqrt{2} + 2\sqrt{3} + \sqrt{6}$$

08.02.03.0016.01

$$\frac{K(1-z^2)}{K(z^2)} == \sqrt{7} \;/;\; z == \frac{1}{4}\sqrt{8 - 3\sqrt{7}}$$

08.02.03.0017.01

$$\frac{K(1-z^2)}{K(z^2)} == \sqrt{8} \;/;\; z == 5 + 4\sqrt{2} - 2\sqrt{2\left(7 + 5\sqrt{2}\right)}$$

08.02.03.0018.01

$$\frac{K(1-z^2)}{K(z^2)} = \sqrt{9} \;/;\; z = \frac{1}{\sqrt{194+112\sqrt{3}+4\sqrt{4680+2702\sqrt{3}}}}$$

08.02.03.0019.01

$$\frac{K(1-z^2)}{K(z^2)} = \sqrt{10} \;/;\; z = -9+3\sqrt{10}-2\sqrt{38-12\sqrt{10}}$$

08.02.03.0020.01

$$\frac{K(1-z^2)}{K(z^2)} = \sqrt{11} \;/;\; z = \frac{1}{2}\sqrt{\left(\left(32\,3^{2/3}+3\left(9+7\sqrt{33}\right)^{1/3}-4\,3^{1/3}\left(9+7\sqrt{33}\right)^{2/3}\right)\Big/\left(6\left(9+7\sqrt{33}\right)^{1/3}+\sqrt{3\left(-32\,3^{2/3}\left(9+7\sqrt{33}\right)^{1/3}+9\left(9+7\sqrt{33}\right)^{2/3}+4\,3^{1/3}\left(9+7\sqrt{33}\right)\right)}\right)\right)}$$

08.02.03.0021.01

$$\frac{K(1-z^2)}{K(z^2)} = \sqrt{12} \;/;\; z = \sqrt{833-340\sqrt{6}-12\sqrt{9602-3920\sqrt{6}}}$$

08.02.03.0022.01

$$\frac{K(1-z^2)}{K(z^2)} = \sqrt{13} \;/;\; z = \frac{1}{\sqrt{2\left(649+180\sqrt{13}+6\sqrt{23\,382+6485\sqrt{13}}\right)}}$$

08.02.03.0023.01

$$\frac{K(1-z^2)}{K(z^2)} = \sqrt{14} \;/;\; z = \sqrt{\left(z;\,1-7960\,z-3364\,z^2-42\,152\,z^3+107\,206\,z^4-42\,152\,z^5-3364\,z^6-7960\,z^7+z^8\right)_1^{-1}}$$

08.02.03.0024.01

$$\frac{K(1-z^2)}{K(z^2)} = \sqrt{15} \;/;\; z = \frac{1}{2\sqrt{2\left(376+168\sqrt{5}+\sqrt{6\left(47\,067+21\,049\sqrt{5}\right)}\right)}}$$

08.02.03.0025.01

$$\frac{K(1-z^2)}{K(z^2)} = \sqrt{16} \;/;\; z = \sqrt{4481+3168\sqrt{2}-24\sqrt{69\,708+49\,291\sqrt{2}}}$$

08.02.03.0026.01

$$\frac{K(1-z^2)}{K(z^2)} = \sqrt{17} \;/;\; z = \frac{1}{\sqrt{\dfrac{2+4\sqrt{-412-100\sqrt{17}+5\sqrt{13\,598+3298\sqrt{17}}}}{1649+400\sqrt{17}-20\sqrt{13\,598+3298\sqrt{17}}}}}$$

08.02.03.0027.01

$$\frac{K(1-z^2)}{K(z^2)} = \sqrt{18} \;/;\; z = \sqrt{9603 - 6790\sqrt{2} - 56\sqrt{58\,803 - 41\,580\sqrt{2}}}$$

08.02.03.0028.01

$$\frac{K(1-z^2)}{K(z^2)} = \sqrt{19} \;/;\; z = \frac{1}{2}\sqrt{\frac{96 + \left(1+3\sqrt{57}\right)^{1/3} - 12\left(1+3\sqrt{57}\right)^{2/3}}{2\left(1+3\sqrt{57}\right)^{1/3} + \sqrt{3\left(4+12\sqrt{57} - 32\left(1+3\sqrt{57}\right)^{1/3} + \left(1+3\sqrt{57}\right)^{2/3}\right)}}}$$

08.02.03.0029.01

$$\frac{K(1-z^2)}{K(z^2)} = \sqrt{20} \;/;$$

$$z = \sqrt{\left(z;\, 1 - 78\,984\,z - 290\,020\,z^2 - 2\,454\,456\,z^3 + 6\,695\,494\,z^4 - 2\,454\,456\,z^5 - 290\,020\,z^6 - 78\,984\,z^7 + z^8\right)_1^{-1}}$$

General characteristics

Domain and analyticity

$K(z)$ is an analytical function of z which is defined over the whole complex z-plane.

08.02.04.0001.01

$$z \longrightarrow K(z) :: \mathbb{C} \longrightarrow \mathbb{C}$$

Symmetries and periodicities

Mirror symmetry

08.02.04.0002.01

$$K(\bar{z}) = \overline{K(z)} \;/;\; z \notin (1, \infty)$$

Periodicity

The function $K(z)$ is not periodic.

Poles and essential singularities

The function $K(z)$ does not have poles and essential singularities.

08.02.04.0003.01

$$\mathcal{Sing}_z(K(z)) = \{\}$$

Branch points

The function $K(z)$ has two branch points: $z = 1$, $z = \tilde{\infty}$.

08.02.04.0004.01

$$\mathcal{BP}_z(K(z)) = \{1, \tilde{\infty}\}$$

08.02.04.0005.01

$$\mathcal{R}_z(K(z), 1) = \log$$

08.02.04.0006.01

$$\mathcal{R}_z(K(z), \tilde{\infty}) == \log$$

Branch cuts

The function $K(z)$ is a single-valued function on the z-plane cut along the interval $(1, \infty)$.

The function $K(z)$ is continuous from below on the interval $(1, \infty)$.

08.02.04.0007.01

$$\mathcal{BC}_z(K(z)) == \{\{(1, \infty), i\}\}$$

08.02.04.0008.01

$$\lim_{\epsilon \to +0} K(x - i\,\epsilon) == K(x) \;/;\; x > 1$$

08.02.04.0009.01

$$\lim_{\epsilon \to +0} K(x + i\,\epsilon) == 2\,i\,K(1-x) + K(x) \;/;\; x > 1$$

Series representations

Generalized power series

Expansions at generic point $z == z_0$

For the function itself

08.02.06.0017.01

$$K(z) \propto K(z_0) - 2\,i\,K(1-z_0)\left\lfloor \frac{\arg(1-z_0)+\pi}{2\,\pi} \right\rfloor \left\lfloor \frac{\arg(z_0-z)}{2\,\pi} \right\rfloor + \frac{1}{2\,\pi}\left(G_{2,2}^{2,2}\!\left(1-z_0 \,\middle|\, \begin{matrix} -\frac{1}{2}, -\frac{1}{2} \\ 0, -1 \end{matrix}\right) + \frac{2\,i\,\pi}{(z_0-1)\,z_0}\left\lfloor \frac{\arg(1-z_0)+\pi}{2\,\pi} \right\rfloor \left\lfloor \frac{\arg(z_0-z)}{2\,\pi} \right\rfloor (K(1-z_0)\,z_0 - E(1-z_0))\right)(z-z_0) + \frac{1}{4\,\pi}\left(G_{2,2}^{2,2}\!\left(1-z_0 \,\middle|\, \begin{matrix} -\frac{3}{2}, -\frac{3}{2} \\ 0, -2 \end{matrix}\right) + \frac{i\,\pi}{(z_0-1)^2\,z_0^2}\left\lfloor \frac{\arg(1-z_0)+\pi}{2\,\pi} \right\rfloor \left\lfloor \frac{\arg(z_0-z)}{2\,\pi} \right\rfloor \left(-3\,K(1-z_0)\,z_0^2 + (4\,E(1-z_0)+K(1-z_0))\,z_0 - 2\,E(1-z_0)\right)\right)(z-z_0)^2 + \ldots \;/;\; (z \to z_0)$$

08.02.06.0018.01

$$K(z) \propto K(z_0) - 2\,i\,K(1-z_0)\left\lfloor \frac{\arg(1-z_0)+\pi}{2\,\pi} \right\rfloor \left\lfloor \frac{\arg(z_0-z)}{2\,\pi} \right\rfloor + \frac{1}{2\,\pi}\left(G_{2,2}^{2,2}\!\left(1-z_0 \,\middle|\, \begin{matrix} -\frac{1}{2}, -\frac{1}{2} \\ 0, -1 \end{matrix}\right) + \frac{2\,i\,\pi}{(z_0-1)\,z_0}\left\lfloor \frac{\arg(1-z_0)+\pi}{2\,\pi} \right\rfloor \left\lfloor \frac{\arg(z_0-z)}{2\,\pi} \right\rfloor (K(1-z_0)\,z_0 - E(1-z_0))\right)(z-z_0) + \frac{1}{4\,\pi}\left(G_{2,2}^{2,2}\!\left(1-z_0 \,\middle|\, \begin{matrix} -\frac{3}{2}, -\frac{3}{2} \\ 0, -2 \end{matrix}\right) + \frac{i\,\pi}{(z_0-1)^2\,z_0^2}\left\lfloor \frac{\arg(1-z_0)+\pi}{2\,\pi} \right\rfloor \left\lfloor \frac{\arg(z_0-z)}{2\,\pi} \right\rfloor \left(-3\,K(1-z_0)\,z_0^2 + (4\,E(1-z_0)+K(1-z_0))\,z_0 - 2\,E(1-z_0)\right)\right)(z-z_0)^2 + O\!\left((z-z_0)^3\right)$$

08.02.06.0019.01

$$K(z) = \frac{1}{2\pi} \sum_{k=0}^{\infty} \frac{1}{k!} \left(G_{2,2}^{2,2}\left(1 - z_0 \,\middle|\, \begin{matrix} \frac{1}{2} - k, \frac{1}{2} - k \\ 0, -k \end{matrix}\right) - 2\pi i (-1)^k \left\lfloor \frac{\arg(1 - z_0) + \pi}{2\pi} \right\rfloor \left\lfloor \frac{\arg(z_0 - z)}{2\pi} \right\rfloor \Gamma\left(k + \frac{1}{2}\right)^2 {}_2\tilde{F}_1\left(k + \frac{1}{2}, k + \frac{1}{2}; k + 1; 1 - z_0\right) \right) (z - z_0)^k$$

08.02.06.0020.01

$$K(z) = \frac{\pi}{2} \sum_{k=0}^{\infty} \frac{\left(\frac{1}{2}\right)_k^2}{(k!)^2} \, {}_2F_1\left(k + \frac{1}{2}, k + \frac{1}{2}; k + 1; z_0\right) (z - z_0)^k$$

08.02.06.0021.01

$$K(z) \propto K(z_0) - 2 i K(1 - z_0) \left\lfloor \frac{\arg(1 - z_0) + \pi}{2\pi} \right\rfloor \left\lfloor \frac{\arg(z_0 - z)}{2\pi} \right\rfloor + O(z - z_0)$$

Expansions on branch cuts

For the function itself

08.02.06.0022.01

$$K(z) \propto K(x) - 2 i K(1 - x) \left\lfloor \frac{\arg(x - z)}{2\pi} \right\rfloor + \frac{1}{2\pi} \left(\frac{2 i \pi}{(x - 1) x} (K(1 - x) x - E(1 - x)) \left\lfloor \frac{\arg(x - z)}{2\pi} \right\rfloor + G_{2,2}^{2,2}\left(1 - x \,\middle|\, \begin{matrix} -\frac{1}{2}, -\frac{1}{2} \\ 0, -1 \end{matrix}\right) \right) (z - x) + \frac{1}{4\pi} \left(\frac{i \pi}{(x - 1)^2 x^2} \left\lfloor \frac{\arg(x - z)}{2\pi} \right\rfloor \left(-3 K(1 - x) x^2 + (4 E(1 - x) + K(1 - x)) x - 2 E(1 - x)\right) + G_{2,2}^{2,2}\left(1 - x \,\middle|\, \begin{matrix} -\frac{3}{2}, -\frac{3}{2} \\ 0, -2 \end{matrix}\right) \right) (z - x)^2 + \ldots \;/;\; (z \to x) \land x \in \mathbb{R} \land x > 1$$

08.02.06.0023.01

$$K(z) \propto K(x) - 2 i K(1 - x) \left\lfloor \frac{\arg(x - z)}{2\pi} \right\rfloor + \frac{1}{2\pi} \left(\frac{2 i \pi}{(x - 1) x} (K(1 - x) x - E(1 - x)) \left\lfloor \frac{\arg(x - z)}{2\pi} \right\rfloor + G_{2,2}^{2,2}\left(1 - x \,\middle|\, \begin{matrix} -\frac{1}{2}, -\frac{1}{2} \\ 0, -1 \end{matrix}\right) \right) (z - x) + \frac{1}{4\pi} \left(\frac{i \pi}{(x - 1)^2 x^2} \left\lfloor \frac{\arg(x - z)}{2\pi} \right\rfloor \left(-3 K(1 - x) x^2 + (4 E(1 - x) + K(1 - x)) x - 2 E(1 - x)\right) + G_{2,2}^{2,2}\left(1 - x \,\middle|\, \begin{matrix} -\frac{3}{2}, -\frac{3}{2} \\ 0, -2 \end{matrix}\right) \right) (z - x)^2 + O\left((z - x)^3\right) \;/;\; x \in \mathbb{R} \land x > 1$$

08.02.06.0024.01

$$K(z) = \frac{1}{2\pi} \sum_{k=0}^{\infty} \frac{1}{k!} \left(G_{2,2}^{2,2}\left(1 - x \,\middle|\, \begin{matrix} \frac{1}{2} - k, \frac{1}{2} - k \\ 0, -k \end{matrix}\right) - 2\pi i (-1)^k \left\lfloor \frac{\arg(x - z)}{2\pi} \right\rfloor \Gamma\left(k + \frac{1}{2}\right)^2 {}_2\tilde{F}_1\left(k + \frac{1}{2}, k + \frac{1}{2}; k + 1; 1 - x\right) \right) (z - x)^k \;/;\; x \in \mathbb{R} \land x > 1$$

08.02.06.0025.01

$$K(z) \propto K(x) - 2 i K(1 - x) \left\lfloor \frac{\arg(x - z)}{2\pi} \right\rfloor + O(z - x) \;/;\; x \in \mathbb{R} \land x > 1$$

Expansions at $z == 0$

For the function itself

08.02.06.0001.02

$$K(z) \propto \frac{\pi}{2}\left(1+\frac{z}{4}+\frac{9z^2}{64}+\ldots\right) /; (z \to 0)$$

08.02.06.0026.01

$$K(z) \propto \frac{\pi}{2}\left(1+\frac{z}{4}+\frac{9z^2}{64}+O\left(z^3\right)\right)$$

08.02.06.0002.01

$$K(z) = \frac{\pi}{2}\sum_{k=0}^{\infty}\frac{\left(\frac{1}{2}\right)_k\left(\frac{1}{2}\right)_k z^k}{k!^2} /; |z|<1$$

08.02.06.0003.01

$$K(z) = \frac{\pi}{2}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2};1;z\right) /; |z|<1$$

08.02.06.0004.02

$$K(z) \propto \frac{\pi}{2}+O(z)$$

08.02.06.0027.01

$$K(z) = F_\infty(z) /; \left(\left(F_n(z) = \frac{\pi}{2}\sum_{k=0}^{n}\frac{\left(\frac{1}{2}\right)_k\left(\frac{1}{2}\right)_k z^k}{k!^2} = K(z) - \frac{z^{n+1}\,\Gamma\left(n+\frac{3}{2}\right)^2}{2\,(n+1)!^2}\,{}_3F_2\left(1,n+\frac{3}{2},n+\frac{3}{2};n+2,n+2;z\right)\right)\bigwedge n\in\mathbb{N}\right)$$

Summed form of the truncated series expansion.

Expansions at $z = 1$

For the function itself

08.02.06.0005.02

$$K(z) \propto -\frac{1}{2}\log(1-z)\left(1-\frac{z-1}{4}+\frac{9}{64}(z-1)^2+\ldots\right)+\log(4)+\frac{1}{4}(1-\log(4))(z-1)+\frac{3}{128}(6\log(4)-7)(z-1)^2+\ldots /; (z\to 1)$$

08.02.06.0028.01

$$K(z) \propto -\frac{1}{2}\log(1-z)\left(1-\frac{z-1}{4}+\frac{9}{64}(z-1)^2+O\left((z-1)^3\right)\right)+ \log(4)+\frac{1}{4}(1-\log(4))(z-1)+\frac{3}{128}(6\log(4)-7)(z-1)^2+O\left((z-1)^3\right)$$

08.02.06.0029.01

$$K(z) = -\frac{1}{2}\log(1-z)\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{1}{2}\right)_k^2}{(k!)^2}(z-1)^k+\frac{z-1}{4}+2\log(2)\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{1}{2}\right)_k^2}{(k!)^2}(z-1)^k+ \frac{9}{16}\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{5}{2}\right)_k^2}{((k+2)!)^2}\left(\frac{1}{k+1}+\frac{1}{k+2}-\frac{2}{2k+3}-\sum_{i=k+1}^{2k+1}\frac{2}{i}\right)(z-1)^{k+2} /; |z-1|<1$$

08.02.06.0006.02

$$K(z) = -\frac{1}{2}\log(1-z)\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{1}{2}\right)_k^2}{k!^2}(z-1)^k + \sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{1}{2}\right)_k^2}{k!^2}\left(\psi(k+1)-\psi\left(k+\frac{1}{2}\right)\right)(z-1)^k\ /;\ |z-1|<1$$

08.02.06.0007.01

$$K(z) = -\frac{\log(1-z)}{\pi}K(1-z) + \sum_{k=0}^{\infty}\frac{\left(\frac{1}{2}\right)_k^2}{k!^2}\left(\psi(k+1)-\psi\left(k+\frac{1}{2}\right)\right)(1-z)^k\ /;\ |z-1|<1$$

08.02.06.0008.02

$$K(z) \propto -\frac{1}{2}\log(1-z)\,(1+O(z-1)) + \log(4)\,(1+O(z-1))$$

08.02.06.0030.01

$$K(z) = F_\infty(z)\ /;$$

$$\left(\left(F_n(z) = \frac{1}{2}\sum_{k=0}^{n}\frac{\left(\frac{1}{2}\right)_k^2}{k!^2}\left(-\log(1-z)+2\,\psi(k+1)-2\,\psi\left(k+\frac{1}{2}\right)\right)(1-z)^k = K(z) - \frac{1}{2\pi}G_{4,4}^{2,4}\left(1-z\,\middle|\,\begin{matrix} n+1,\ n+1,\ \frac{1}{2},\ \frac{1}{2} \\ n+1,\ n+1,\ 0,\ 0\end{matrix}\right)\right)\bigwedge n\in\mathbb{N}\right)$$

Summed form of the truncated series expansion.

Expansions at $z = \infty$

For the function itself

08.02.06.0009.02

$$K(z) \propto \frac{\log(-z)}{2\sqrt{-z}}\left(1+\frac{1}{4z}+\frac{9}{64z^2}+\ldots\right) + \frac{1}{\sqrt{-z}}\left(\log(4)+\frac{\log(4)-1}{4z}+\frac{3(6\log(4)-7)}{128z^2}+\ldots\right)\ /;\ (|z|\to\infty)$$

08.02.06.0031.01

$$K(z) \propto \frac{\log(-z)}{2\sqrt{-z}}\left(1+\frac{1}{4z}+\frac{9}{64z^2}+O\left(\frac{1}{z^3}\right)\right) + \frac{1}{\sqrt{-z}}\left(\log(4)+\frac{\log(4)-1}{4z}+\frac{3(6\log(4)-7)}{128z^2}+O\left(\frac{1}{z^3}\right)\right)$$

08.02.06.0032.01

$$K(z) = \frac{\log(-z)}{2\sqrt{-z}}\sum_{k=0}^{\infty}\frac{\left(\frac{1}{2}\right)_k^2 z^{-k}}{(k!)^2} +$$

$$\frac{1}{\sqrt{-z}}\left(\frac{9}{16}\sum_{k=0}^{\infty}\frac{\left(\frac{5}{2}\right)_k^2}{((k+2)!)^2}\left(\frac{1}{k+1}+\frac{1}{k+2}-\frac{2}{2k+3}-2\sum_{i=k+1}^{2k+1}\frac{1}{i}\right)z^{-k-2} + 2\log(2)\sum_{k=0}^{\infty}\frac{\left(\frac{1}{2}\right)_k^2 z^{-k}}{(k!)^2} - \frac{1}{4z}\sum_{i=0}^{\infty}\delta_{i-1}\right)\ /;\ |z|>1$$

08.02.06.0010.02

$$K(z) = \frac{\log(-z)}{2\sqrt{-z}}\sum_{k=0}^{\infty}\frac{\left(\frac{1}{2}\right)_k^2 z^{-k}}{k!^2} + \frac{1}{\sqrt{-z}}\sum_{k=0}^{\infty}\frac{\left(\frac{1}{2}\right)_k^2}{k!^2}\left(\psi(k+1)-\psi\left(\frac{1}{2}+k\right)\right)z^{-k}\ /;\ |z|>1$$

08.02.06.0011.01

$$K(z) = \frac{\log(-z)}{\pi\sqrt{-z}} K\left(\frac{1}{z}\right) + \frac{1}{\sqrt{-z}} \sum_{k=0}^{\infty} \frac{\left(\frac{1}{2}\right)_k^2}{k!^2} \left(\psi(k+1) - \psi\left(\frac{1}{2}+k\right)\right) z^{-k} \;/;\; |z| > 1$$

08.02.06.0012.02

$$K(z) \propto \frac{\log(4)}{\sqrt{-z}} \left(1 + O\left(\frac{1}{z}\right)\right) + \frac{\log(-z)}{2\sqrt{-z}} \left(1 + O\left(\frac{1}{z}\right)\right)$$

08.02.06.0033.01

$$K(z) \propto \begin{cases} -\frac{i\log(z)}{2\sqrt{z}} & \arg(z) \leq 0 \\ \frac{i\log(z)}{2\sqrt{z}} & \text{True} \end{cases} \;/;\; (|z| \to \infty)$$

08.02.06.0034.01

$$K(z) = F_\infty(z) \;/;\; \left(\left(F_n(z) = \frac{1}{2\sqrt{-z}} \sum_{k=0}^{m} \frac{\left(\frac{1}{2}\right)_k^2}{k!^2} \left(\log(-z) + 2\psi(k+1) - \psi\left(\frac{1}{2}-k\right) - \psi\left(k+\frac{1}{2}\right)\right) z^{-k} = K(z) - \frac{1}{2} G_{4,4}^{3,2}\left(-z \,\middle|\, \begin{matrix} -m-\frac{1}{2}, -m-\frac{1}{2}, \frac{1}{2}, \frac{1}{2} \\ 0, -m-\frac{1}{2}, -m-\frac{1}{2}, 0 \end{matrix}\right)\right) \bigwedge m \in \mathbb{N}\right)$$

Summed form of the truncated series expansion.

Residue representations

08.02.06.0013.01

$$K(z) = \frac{1}{2} \sum_{j=0}^{\infty} \operatorname{res}_s\left(\frac{\Gamma\left(\frac{1}{2}-s\right)^2 (-z)^{-s}}{\Gamma(1-s)} \Gamma(s)\right)(-j) \;/;\; |z| < 1$$

08.02.06.0014.01

$$K(z) = -\frac{1}{2} \sum_{j=0}^{\infty} \operatorname{res}_s\left(\frac{\Gamma(s)(-z)^{-s}}{\Gamma(1-s)} \Gamma\left(\frac{1}{2}-s\right)^2\right)\left(j+\frac{1}{2}\right) \;/;\; |z| > 1$$

Other series representations

08.02.06.0015.01

$$K(z) = \frac{\pi}{2}\left(2\sum_{k=1}^{\infty} q(z)^{k^2} + 1\right)^2$$

08.02.06.0016.01

$$K(z) = \frac{\pi}{2}\left(1 + 4\sum_{k=1}^{\infty} \frac{q(z)^k}{q(z)^{2k}+1}\right)$$

Integral representations

On the real axis

Of the direct function

08.02.07.0001.01

$$K(z) = \int_0^{\frac{\pi}{2}} \frac{1}{\sqrt{1 - z\sin^2(t)}} \, dt \,/;\, |\arg(1 - z)| < \pi$$

08.02.07.0002.01

$$K(z) = \int_0^1 \frac{1}{\sqrt{1 - t^2}\,\sqrt{1 - z\,t^2}} \, dt \,/;\, |\arg(1 - z)| < \pi$$

08.02.07.0003.01

$$K(z) = \int_1^{\infty} \frac{1}{\sqrt{t^2 - 1}\,\sqrt{t^2 - z}} \, dt \,/;\, |\arg(1 - z)| < \pi$$

Contour integral representations

08.02.07.0004.01

$$K(z) = \frac{1}{4\pi i} \int_{\mathcal{L}} \frac{\Gamma(s)\,\Gamma\left(\frac{1}{2} - s\right)^2}{\Gamma(1 - s)} (-z)^{-s} \, ds$$

Differential equations

Ordinary linear differential equations and wronskians

For the direct function itself

08.02.13.0001.01

$$(1 - z)\, z\, w''(z) + (1 - 2z)\, w'(z) - \frac{1}{4} w(z) = 0 \,/;\, w(z) = c_1 K(z) + c_2 K(1 - z)$$

08.02.13.0002.02

$$W_z(K(z), K(1 - z)) = \frac{\pi}{4(z - 1)\, z}$$

08.02.13.0003.01

$$w''(z) + \left(\frac{(2g(z) - 1)\, g'(z)}{(g(z) - 1)\, g(z)} - \frac{g''(z)}{g'(z)}\right) w'(z) + \frac{g'(z)^2}{4(g(z) - 1)\, g(z)} w(z) = 0 \,/;\, w(z) = c_1 K(g(z)) + c_2 K(1 - g(z))$$

08.02.13.0004.01

$$W_z(K(g(z)), (1 - g(z))) = \frac{\pi\, g'(z)}{4(g(z) - 1)\, g(z)}$$

08.02.13.0005.01

$$w''(z) + \left(\frac{(2g(z) - 1)\, g'(z)}{(g(z) - 1)\, g(z)} - \frac{2h'(z)}{h(z)} - \frac{g''(z)}{g'(z)}\right) w'(z) + \left(\frac{g'(z)^2}{4(g(z) - 1)\, g(z)} + \frac{(1 - 2g(z))\, h'(z)\, g'(z)}{(g(z) - 1)\, g(z)\, h(z)} + \frac{2h'(z)^2}{h(z)^2} + \frac{h'(z)\, g''(z)}{h(z)\, g'(z)} - \frac{h''(z)}{h(z)}\right) w(z) = 0 \,/;\, w(z) = c_1 h(z) K(g(z)) + c_2 h(z) K(1 - g(z))$$

08.02.13.0006.01

$$W_z(h(z)\,K(g(z)),\,h(z)\,K(1-g\,(z))) \equiv \frac{\pi\,h(z)^2\,g'(z)}{4\,(g(z)-1)\,g(z)}$$

08.02.13.0007.01

$$4\,z^2(a\,z^r-1)\,w''(z)\;+4\,(a\,(r+1)\,z^r+s\,(2-2\,a\,z^r)-1)\,z\,w'(z)+\left(a\,r^2\,z^r-4\,a\,r\,s\,z^r+4\,s^2\,(a\,z^r-1)\right)w(z)=0\;/;\; w(z)=c_1\,z^s\,K(a\,z^r)\;+c_2\,z^s\,K(1-a\,z^r)$$

08.02.13.0008.01

$$W_z(z^s\,K(a\,z^r),\,z^s\,K(1-a\,z^r)) \equiv \frac{\pi\,r\,z^{2s-1}}{4\,(a\,z^r-1)}$$

08.02.13.0009.01

$$w''(z)+\left(\left(1+\frac{1}{a\,r^z-1}\right)\log(r)-2\log(s)\right)w'(z)+\frac{a\,r^z\,(\log(r)-2\log(s))^2-4\log^2(s)}{4\,(a\,r^z-1)}\,w(z)=0\;/;\; w(z)=c_1\,s^z\,K(a\,r^z)\;+c_2\,s^z\,K(1-a\,r^z)$$

08.02.13.0010.01

$$W_z(s^z\,K(a\,r^z),\,s^z\,K(1-a\,r^z)) \equiv \frac{\pi\,s^{2z}\log(r)}{4\,(a\,r^z-1)}$$

Identities

Functional identities

08.02.17.0001.01

$$K(z) \equiv \frac{1}{\sqrt{1-z}}\,K\!\left(\frac{z}{z-1}\right)\;/;\;|\arg(1-z)|<\pi$$

08.02.17.0004.01

$$K(z)=\frac{1}{\sqrt{1-z}}\,\overline{K\!\left(\frac{z}{z-1}\right)}\;/;\;z\in\mathbb{R}$$

08.02.17.0005.01

$$K(z)=\frac{1}{\sqrt{1-z}}\,K\!\left(\frac{z}{z-1}\right)+i\left(\frac{1}{\sqrt{1-z}}-\sqrt{\frac{1}{1-z}}\right)K\!\left(\frac{1}{1-z}\right)$$

08.02.17.0006.01

$$\mathrm{KK}(z)=\sqrt{\frac{1}{z}}\;K\!\left(\frac{1}{z}\right)+\sqrt{\frac{1}{1-z}}\;\sqrt{1-z}\;\sqrt{-\frac{1}{z}}\;K\!\left(1-\frac{1}{z}\right)$$

08.02.17.0007.01

$$\mathrm{KK}(z)=\frac{1}{\sqrt{z}}\,K\!\left(\frac{1}{z}\right)+\sqrt{\frac{1}{1-z}}\;\sqrt{z\,(1-z)}\;\sqrt{-\frac{1}{z}}\;K(1-z)$$

08.02.17.0002.02

$$K\!\left(\frac{1}{z}\right) \equiv \sqrt{z}\left(K(z)-\sqrt{-\frac{1}{z}}\;\sqrt{\frac{1}{1-z}}\;\sqrt{z\,(1-z)}\;K(1-z)\right)$$

08.02.17.0003.01

$$K(z) = \frac{2}{1+\sqrt{1-z}}\, K\left(\left(\frac{1-\sqrt{1-z}}{1+\sqrt{1-z}}\right)^2\right)$$

Complex characteristics

Real part

08.02.19.0001.01

$$\operatorname{Re}(K(x+i\,y)) = \frac{\pi}{2}\, F_{2\times 1\times 1}^{4\times 0\times 0}\left(\begin{matrix} \frac{3}{4}, \frac{3}{4}, \frac{5}{4}, \frac{5}{4};;; \\ 1, \frac{3}{2}; \frac{1}{2}; \frac{3}{2}; \end{matrix} -y^2, x^2\right) + \frac{\pi\,x}{8}\, F_{2\times 1\times 1}^{4\times 0\times 0}\left(\begin{matrix} \frac{1}{4}, \frac{1}{4}, \frac{3}{4}, \frac{3}{4};;; \\ 1, \frac{1}{2}; \frac{1}{2}; \frac{1}{2}; \end{matrix} -y^2, x^2\right) /;\, x \in \mathbb{R} \bigwedge y \in \mathbb{R}$$

08.02.19.0003.01

$$\operatorname{Re}\left(K\left(e^{i\,x}\right)\right) = \frac{1}{2}\cos\left(\frac{x}{4}\right) K\left(1-\sin^2\left(\frac{x}{4}\right)\right) + \frac{1}{2}\, K\left(\sin^2\left(\frac{x}{4}\right)\right)\sin\left(\frac{x}{4}\right) /;\, x \in \mathbb{R} \land 0 < x < \pi$$

Imaginary part

08.02.19.0002.01

$$\operatorname{Im}(K(x+i\,y)) = \frac{\pi\,y}{8}\, F_{2\times 1\times 1}^{4\times 0\times 0}\left(\begin{matrix} \frac{3}{4}, \frac{5}{4}, \frac{3}{4}, \frac{5}{4};;; \\ 1, \frac{3}{2}; \frac{3}{2}; \frac{1}{2}; \end{matrix} -y^2, x^2\right) + \frac{9\,\pi\,x\,y}{64}\, F_{2\times 1\times 1}^{4\times 0\times 0}\left(\begin{matrix} \frac{5}{4}, \frac{7}{4}, \frac{7}{4}, \frac{5}{4};;; \\ 2, \frac{3}{2}; \frac{3}{2}; \frac{3}{2}; \end{matrix} -y^2, x^2\right) /;\, x \in \mathbb{R} \bigwedge y \in \mathbb{R}$$

08.02.19.0004.01

$$\operatorname{Im}\left(K\left(e^{i\,x}\right)\right) = \frac{1}{2}\cos\left(\frac{x}{4}\right) K\left(\sin^2\left(\frac{x}{4}\right)\right) - \frac{1}{2}\, K\left(1-\sin^2\left(\frac{x}{4}\right)\right)\sin\left(\frac{x}{4}\right) /;\, x \in \mathbb{R} \land 0 < x < \pi$$

Differentiation

Low-order differentiation

08.02.20.0001.01

$$\frac{\partial K(z)}{\partial z} = \frac{E(z)-(1-z)\,K(z)}{2\,(1-z)\,z}$$

08.02.20.0002.01

$$\frac{\partial^2 K(z)}{\partial z^2} = \frac{2\,(2\,z-1)\,E(z)+\left(3\,z^2-5\,z+2\right)K(z)}{4\,(z-1)^2\,z^2}$$

Symbolic differentiation

08.02.20.0003.02

$$\frac{\partial^n K(z)}{\partial z^n} = \frac{\pi\left(\left(\frac{1}{2}\right)_n\right)^2}{2\,n!}\, {}_2F_1\left(n+\frac{1}{2}, n+\frac{1}{2}; n+1; z\right) /;\, n \in \mathbb{N}$$

08.02.20.0004.02

$$\frac{\partial^n K(z)}{\partial z^n} = \frac{\pi\,z^{-n}}{2}\, {}_2\tilde{F}_1\left(\frac{1}{2}, \frac{1}{2}; 1-n; z\right) /;\, n \in \mathbb{N}$$

Fractional integro-differentiation

08.02.20.0005.01

$$\frac{\partial^\alpha K(z)}{\partial z^\alpha} = \frac{\pi z^{-\alpha}}{2} \, {}_2\tilde{F}_1\left(\frac{1}{2}, \frac{1}{2}; 1-\alpha; z\right)$$

Integration

Indefinite integration

Involving only one direct function

08.02.21.0001.01

$$\int K(a z) \, dz = \frac{2 (E(a z) + (a z - 1) K(a z))}{a}$$

08.02.21.0002.01

$$\int K(z) \, dz = 2 (E(z) + (z - 1) K(z))$$

Involving one direct function and elementary functions

Involving power function

Involving power

Linear argument

08.02.21.0003.01

$$\int z^{\alpha-1} K(a z) \, dz = \frac{\pi}{2 \alpha} z^\alpha \, {}_3F_2\left(\frac{1}{2}, \frac{1}{2}, \alpha; 1, \alpha+1; a z\right)$$

08.02.21.0004.01

$$\int z^{\alpha-1} K(z) \, dz = \frac{\pi z^\alpha}{2 \alpha} \, {}_3F_2\left(\frac{1}{2}, \frac{1}{2}, \alpha; 1, \alpha+1; z\right)$$

08.02.21.0005.01

$$\int z K(a z) \, dz = \frac{2 \left((a z + 4) E(a z) + \left(3 a^2 z^2 + a z - 4\right) K(a z)\right)}{9 a^2}$$

08.02.21.0006.01

$$\int \frac{K(a z)}{\sqrt{z}} \, dz = \pi \sqrt{z} \, {}_3F_2\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}; 1, \frac{3}{2}; a z\right)$$

08.02.21.0007.01

$$\int \frac{K(a z)}{z} \, dz = \frac{1}{8} \pi \left(a z \, {}_4F_3\left(1, 1, \frac{3}{2}, \frac{3}{2}; 2, 2, 2; a z\right) - 4 \gamma^2 - 8 \log(4) + 4 \log(-a z) + 4\right)$$

08.02.21.0008.01

$$\int \frac{K(a z)}{z^2} \, dz = \frac{1}{128 z} \left(\pi \left(9 a^2 \, {}_4F_3\left(1, 1, \frac{5}{2}, \frac{5}{2}; 2, 3, 3; a z\right) z^2 + 16 (-2 a z (\log(4) - 1) + a z \log(-a z) - 4)\right)\right)$$

Power arguments

08.02.21.0009.01

$$\int z^{\alpha-1} K(a z^r)\, dz = \frac{\pi}{2\alpha} z^\alpha\, {}_3F_2\left(\frac{1}{2}, \frac{1}{2}, \frac{\alpha}{r}; 1, \frac{\alpha}{r}+1; a z^r\right)$$

08.02.21.0010.01

$$\int z K(a z^2)\, dz = \frac{E(a z^2) + (a z^2 - 1) K(a z^2)}{a}$$

08.02.21.0011.01

$$\int z^3 K(a z^2)\, dz = \frac{(a z^2 + 4) E(a z^2) + (3 a^2 z^4 + a z^2 - 4) K(a z^2)}{9 a^2}$$

08.02.21.0012.01

$$\int z^5 K(a z^2)\, dz = \frac{1}{225 a^3} \left((9 a^2 z^4 + 16 a z^2 + 64) E(a z^2) + (45 a^3 z^6 + 3 a^2 z^4 + 16 a z^2 - 64) K(a z^2)\right)$$

08.02.21.0013.01

$$\int K(a z^2)\, dz = \frac{1}{2} \pi z\, {}_3F_2\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}; 1, \frac{3}{2}; a z^2\right)$$

08.02.21.0014.01

$$\int \frac{K(a z^2)}{z}\, dz = \frac{1}{16} \pi \left(a\, {}_4F_3\left(1, 1, \frac{3}{2}, \frac{3}{2}; 2, 2, 2; a z^2\right) z^2 - 4\gamma^2 - 8\log(4) + 4\log(-a z^2) + 4\right)$$

08.02.21.0015.01

$$\int \frac{K(a z^2)}{z^2}\, dz = -\frac{E(a z^2)}{z}$$

08.02.21.0016.01

$$\int \frac{K(a z^2)}{z^3}\, dz = \frac{1}{256 z^2} \left(\pi \left(9 a^2\, {}_4F_3\left(1, 1, \frac{5}{2}, \frac{5}{2}; 2, 3, 3; a z^2\right) z^4 + 16 \left(-2 a (\log(4) - 1) z^2 + a \log(-a z^2) z^2 - 4\right)\right)\right)$$

Involving algebraic functions

08.02.21.0017.01

$$\int \frac{z E(z^2)}{(1 - z^2)^{3/2}}\, dz = \frac{E(z^2) + (z^2 - 1) K(z^2)}{\sqrt{1 - z^2}}$$

Definite integration

For the direct function itself

08.02.21.0018.01

$$\int_0^1 t^{\alpha-1} K(t)\, dt = \frac{\pi}{2\alpha}\, {}_3F_2\left(\frac{1}{2}, \frac{1}{2}, \alpha; 1, \alpha+1; 1\right) /; \operatorname{Re}(\alpha) > 0$$

08.02.21.0019.01

$$\int_0^\infty t^{\alpha-1} K(-t)\, dt = \frac{\Gamma\left(\frac{1}{2} - \alpha\right)^2 \Gamma(\alpha)}{2\, \Gamma(1 - \alpha)} /; 0 < \operatorname{Re}(\alpha) < \frac{1}{2}$$

08.02.21.0020.01

$$\int_0^{\infty}\sqrt{\frac{\sqrt{a^2+x^2}-a}{a^2+x^2}}\,\frac{1}{b+\sqrt{b^2+x^2}}\,K\left(\left(\frac{\sqrt{b^2+x^2}-b}{b+\sqrt{b^2+x^2}}\right)^2\right)dx = \frac{\sqrt{a-\sqrt{a^2-b^2}}}{b}\operatorname{sech}^2(\alpha)\,K\left(\operatorname{sech}^2(\alpha)\right)K\left(\tanh^2(\alpha)\right)\;/;$$

$$\operatorname{Re}(a)\geq\operatorname{Re}(b)>0\bigwedge\cosh^{-1}\left(\frac{\sqrt{b+\sqrt{2a^2-2a\sqrt{a^2-b^2}}}}{\sqrt{2b}}\right)$$

Representations through more general functions

Through hypergeometric functions

Involving ${}_2F_1$

08.02.26.0001.01

$$K(z) = \frac{\pi}{2}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2};1;z\right)$$

Through Meijer *G*

Classical cases for the direct function itself

08.02.26.0002.01

$$K(z) = \frac{1}{2}G_{2,2}^{1,2}\left(-z\,\middle|\,\begin{matrix}\frac{1}{2},\frac{1}{2}\\0,0\end{matrix}\right)$$

Classical cases involving algebraic functions

08.02.26.0142.01

$$K(1-z) = \frac{1}{2\pi}G_{2,2}^{2,2}\left(z\,\middle|\,\begin{matrix}\frac{1}{2},\frac{1}{2}\\0,0\end{matrix}\right)\;/;\;z\notin(-\infty,-1)$$

08.02.26.0003.01

$$\frac{1}{\sqrt{z+1}}K\left(\frac{1}{z+1}\right) = \frac{1}{2}G_{2,2}^{2,1}\left(z\,\middle|\,\begin{matrix}\frac{1}{2},\frac{1}{2}\\0,0\end{matrix}\right)\;/;\;z\notin(-1,0)$$

08.02.26.0004.01

$$\frac{1}{\sqrt{z+1}}K\left(\frac{z}{z+1}\right) = \frac{1}{2}G_{2,2}^{1,2}\left(z\,\middle|\,\begin{matrix}\frac{1}{2},\frac{1}{2}\\0,0\end{matrix}\right)\;/;\;z\notin(-\infty,-1)$$

08.02.26.0005.01

$$K\left(-\frac{\left(\sqrt{z}-1\right)^2}{4\sqrt{z}}\right) = \frac{1}{2\pi}G_{2,2}^{2,2}\left(z\,\middle|\,\begin{matrix}\frac{3}{4},\frac{3}{4}\\\frac{1}{4},\frac{1}{4}\end{matrix}\right)\;/;\;z\notin(-\infty,0)$$

08.02.26.0006.01

$$\frac{1}{\sqrt{z}+1} K\left(\frac{4\sqrt{z}}{\left(\sqrt{z}+1\right)^2}\right) = \frac{\pi}{2} G_{2,2}^{1,1}\left(z \,\middle|\, \begin{matrix} \frac{1}{2}, \frac{1}{2} \\ 0, 0 \end{matrix}\right)$$

08.02.26.0007.01

$$\frac{1}{\sqrt{z}+1} K\left(\left(\frac{1-\sqrt{z}}{1+\sqrt{z}}\right)^2\right) = \frac{1}{4\pi} G_{2,2}^{2,2}\left(z \,\middle|\, \begin{matrix} \frac{1}{2}, \frac{1}{2} \\ 0, 0 \end{matrix}\right)$$

08.02.26.0008.01

$$K\left(\frac{1}{2}\left(1-\sqrt{z+1}\right)\right) = \frac{\pi}{2\,\Gamma\left(\frac{1}{4}\right)^2} G_{2,2}^{1,2}\left(z \,\middle|\, \begin{matrix} \frac{3}{4}, \frac{3}{4} \\ 0, 0 \end{matrix}\right)$$

08.02.26.0009.01

$$\frac{1}{\sqrt{z+1}} K\left(\frac{1-\sqrt{z+1}}{2}\right) = \frac{\pi}{2\,\Gamma\left(\frac{3}{4}\right)^2} G_{2,2}^{1,2}\left(z \,\middle|\, \begin{matrix} \frac{1}{4}, \frac{1}{4} \\ 0, 0 \end{matrix}\right)$$

08.02.26.0010.01

$$\sqrt{\sqrt{z+1}-1}\, K\left(\frac{2\left(\sqrt{z+1}-1\right)}{z}\right) = \frac{\pi}{2\,\Gamma\left(\frac{1}{4}\right)^2} G_{2,2}^{2,1}\left(z \,\middle|\, \begin{matrix} \frac{5}{4}, \frac{5}{4} \\ \frac{1}{2}, \frac{1}{2} \end{matrix}\right) \Big/ z \notin (-1, 0)$$

08.02.26.0011.01

$$\frac{\sqrt{\sqrt{z+1}-1}}{\sqrt{z+1}} K\left(\frac{2\left(\sqrt{z+1}-1\right)}{z}\right) = \frac{\pi}{2\,\Gamma\left(\frac{3}{4}\right)^2} G_{2,2}^{2,1}\left(z \,\middle|\, \begin{matrix} \frac{3}{4}, \frac{3}{4} \\ \frac{1}{2}, \frac{1}{2} \end{matrix}\right) \Big/ z \notin (-1, 0)$$

08.02.26.0012.01

$$\frac{1}{\sqrt{\sqrt{z+1}-1}} K\left(-\frac{2\left(\sqrt{z+1}+1\right)}{z}\right) = \frac{\pi}{2\,\Gamma\left(\frac{1}{4}\right)^2} G_{2,2}^{2,1}\left(z \,\middle|\, \begin{matrix} \frac{3}{4}, \frac{3}{4} \\ 0, 0 \end{matrix}\right) /;\ z \notin (-1, 0)$$

08.02.26.0013.01

$$\frac{1}{\sqrt{z+1}\sqrt{\sqrt{z+1}-1}} K\left(-\frac{2\left(\sqrt{z+1}+1\right)}{z}\right) = \frac{\pi}{2\,\Gamma\left(\frac{3}{4}\right)^2} G_{2,2}^{2,1}\left(z \,\middle|\, \begin{matrix} \frac{1}{4}, \frac{1}{4} \\ 0, 0 \end{matrix}\right) /;\ z \notin (-1, 0)$$

08.02.26.0014.01

$$\frac{1}{\sqrt{\sqrt{z+1}-1}} K\left(\frac{2}{1-\sqrt{z+1}}\right) = \frac{\pi}{2\,\Gamma\left(\frac{1}{4}\right)^2} G_{2,2}^{2,1}\left(z \,\middle|\, \begin{matrix} \frac{3}{4}, \frac{3}{4} \\ 0, 0 \end{matrix}\right) /;\ z \notin (-1, 0)$$

08.02.26.0015.01

$$\frac{1}{\sqrt{z+1}\sqrt{\sqrt{z+1}-1}} K\left(\frac{2}{1-\sqrt{z+1}}\right) = \frac{\pi}{2\,\Gamma\left(\frac{3}{4}\right)^2} G_{2,2}^{2,1}\left(z \,\middle|\, \begin{matrix} \frac{1}{4}, \frac{1}{4} \\ 0, 0 \end{matrix}\right) /;\ z \notin (-1, 0)$$

08.02.26.0016.01

$$K\left(\frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right) = \frac{\pi}{2\,\Gamma\left(\frac{1}{4}\right)^2} G_{2,2}^{2,1}\left(z \,\middle|\, \begin{matrix} 1, 1 \\ \frac{1}{4}, \frac{1}{4} \end{matrix}\right) \Big/ z \notin (-1, 0)$$

08.02.26.0017.01

$$\frac{1}{\sqrt{z+1}} K\left(\frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right) = \frac{\pi}{2\,\Gamma\left(\frac{3}{4}\right)^2} G_{2,2}^{2,1}\left(z \middle| \begin{matrix} \frac{1}{2}, \frac{1}{2} \\ \frac{1}{4}, \frac{1}{4} \end{matrix}\right) /; z \notin (-1, 0)$$

08.02.26.0018.01

$$\frac{1}{\sqrt[4]{z+1}} K\left(\frac{\sqrt{z+1}-1}{2\sqrt{z+1}}\right) = \frac{1}{2\sqrt{2}} G_{2,2}^{1,2}\left(z \middle| \begin{matrix} \frac{1}{4}, \frac{3}{4} \\ 0, 0 \end{matrix}\right)$$

08.02.26.0019.01

$$\frac{1}{\sqrt[4]{z+1}} K\left(\frac{\sqrt{z+1}-\sqrt{z}}{2\sqrt{z+1}}\right) = \frac{1}{2\sqrt{2}} G_{2,2}^{2,1}\left(z \middle| \begin{matrix} \frac{3}{4}, \frac{3}{4} \\ 0, \frac{1}{2} \end{matrix}\right) /; z \notin (-1, 0)$$

08.02.26.0020.01

$$\frac{1}{\sqrt[4]{z+1}} K\left(\frac{1}{2\left(\sqrt{z}+\sqrt{z+1}\right)\sqrt{z+1}}\right) = \frac{1}{2\sqrt{2}} G_{2,2}^{2,1}\left(z \middle| \begin{matrix} \frac{3}{4}, \frac{3}{4} \\ 0, \frac{1}{2} \end{matrix}\right) /; z \notin (-1, 0)$$

08.02.26.0021.01

$$\frac{1}{\sqrt[4]{z+1}} K\left(\frac{z}{2\left(\sqrt{z+1}+1\right)\sqrt{z+1}}\right) = \frac{1}{2\sqrt{2}} G_{2,2}^{1,2}\left(z \middle| \begin{matrix} \frac{3}{4}, \frac{1}{4} \\ 0, 0 \end{matrix}\right)$$

08.02.26.0022.01

$$\frac{1}{\sqrt{\sqrt{z+1}+1}} K\left(\frac{\sqrt{z+1}-1}{\sqrt{z+1}+1}\right) = \frac{\pi}{2\sqrt{2}\,\Gamma\left(\frac{1}{4}\right)^2} G_{2,2}^{1,2}\left(z \middle| \begin{matrix} \frac{3}{4}, \frac{3}{4} \\ 0, 0 \end{matrix}\right)$$

08.02.26.0023.01

$$\frac{1}{\sqrt{z+1}\sqrt{\sqrt{z+1}+1}} K\left(\frac{\sqrt{z+1}-1}{\sqrt{z+1}+1}\right) = \frac{\pi}{2\sqrt{2}\,\Gamma\left(\frac{3}{4}\right)^2} G_{2,2}^{1,2}\left(z \middle| \begin{matrix} \frac{1}{4}, \frac{1}{4} \\ 0, 0 \end{matrix}\right)$$

08.02.26.0024.01

$$\frac{1}{\sqrt{\sqrt{z+1}+1}} K\left(\frac{1-\sqrt{z+1}}{1+\sqrt{z+1}}\right) = \frac{1}{4} G_{2,2}^{1,2}\left(z \middle| \begin{matrix} \frac{1}{4}, \frac{3}{4} \\ 0, 0 \end{matrix}\right)$$

08.02.26.0025.01

$$\frac{1}{\sqrt{\sqrt{z}+\sqrt{z+1}}} K\left(\frac{\sqrt{z+1}-\sqrt{z}}{\sqrt{z}+1+\sqrt{z}}\right) = \frac{\pi}{2\sqrt{2}\,\Gamma\left(\frac{1}{4}\right)^2} G_{2,2}^{2,1}\left(z \middle| \begin{matrix} \frac{3}{4}, \frac{3}{4} \\ 0, 0 \end{matrix}\right) /; z \notin (-1, 0)$$

08.02.26.0026.01

$$\frac{1}{\sqrt{z+1}\sqrt{\sqrt{z}+\sqrt{z+1}}} K\left(\frac{\sqrt{z+1}-\sqrt{z}}{\sqrt{z}+\sqrt{z+1}}\right) = \frac{\pi}{2\sqrt{2}\,\Gamma\left(\frac{3}{4}\right)^2} G_{2,2}^{2,1}\left(z \middle| \begin{matrix} \frac{1}{4}, \frac{1}{4} \\ 0, 0 \end{matrix}\right) /; z \notin (-1, 0)$$

08.02.26.0027.01

$$\frac{1}{\sqrt{\sqrt{z}+\sqrt{z+1}}} K\left(\frac{\sqrt{z}-\sqrt{z+1}}{\sqrt{z}+\sqrt{z+1}}\right) = \frac{1}{4} G_{2,2}^{2,1}\left(z \middle| \begin{matrix} \frac{3}{4}, \frac{3}{4} \\ 0, \frac{1}{2} \end{matrix}\right) /; z \notin (-1, 0)$$

08.02.26.0028.01

$$\sqrt{\sqrt{z+1}-\sqrt{z}}\ K\left(2\left(\sqrt{z^2+z}-z\right)\right)=\frac{\pi}{2\,\Gamma\left(\frac{1}{4}\right)^2}\,G_{2,2}^{1,2}\left(z\,\middle|\,\begin{array}{c}\frac{3}{4},\frac{3}{4}\\0,0\end{array}\right)\;/;\;\mathrm{Re}(z)\geq 0$$

08.02.26.0029.01

$$\frac{\sqrt{\sqrt{z+1}-\sqrt{z}}}{\sqrt{z+1}}\,K\left(2\left(\sqrt{z^2+z}-z\right)\right)=\frac{\pi}{2\,\Gamma\left(\frac{3}{4}\right)^2}\,G_{2,2}^{1,2}\left(z\,\middle|\,\begin{array}{c}\frac{1}{4},\frac{1}{4}\\0,0\end{array}\right)\;/;\;\mathrm{Re}(z)\geq 0$$

08.02.26.0030.01

$$\frac{1}{\sqrt{\sqrt{z+1}-\sqrt{z}}}\,K\left(-2\left(z+\sqrt{z^2+z}\right)\right)=\frac{\pi}{2\,\Gamma\left(\frac{1}{4}\right)^2}\,G_{2,2}^{1,2}\left(z\,\middle|\,\begin{array}{c}\frac{3}{4},\frac{3}{4}\\0,0\end{array}\right)\;/;\;\mathrm{Re}(z)\geq 0$$

08.02.26.0031.01

$$\frac{1}{\sqrt{z+1}\,\sqrt{\sqrt{z+1}-\sqrt{z}}}\,K\left(-2\left(z+\sqrt{z^2+z}\right)\right)=\frac{\pi}{2\,\Gamma\left(\frac{3}{4}\right)^2}\,G_{2,2}^{1,2}\left(z\,\middle|\,\begin{array}{c}\frac{1}{4},\frac{1}{4}\\0,0\end{array}\right)\;/;\;\mathrm{Re}(z)\geq 0$$

08.02.26.0032.01

$$\left(\sqrt{z+1}-1\right)K\left(\frac{4\sqrt{z+1}}{\left(\sqrt{z+1}+1\right)^2}\right)=\frac{1}{2}\,G_{2,2}^{2,1}\left(z\,\middle|\,\begin{array}{c}\frac{3}{2},\frac{3}{2}\\1,1\end{array}\right)\;/;\;\mathrm{Re}(z)\geq 0$$

08.02.26.0033.01

$$\left(\sqrt{z+1}+1\right)K\left(-\frac{4\sqrt{z+1}}{\left(1-\sqrt{z+1}\right)^2}\right)=\frac{1}{2}\,G_{2,2}^{2,1}\left(z\,\middle|\,\begin{array}{c}\frac{3}{2},\frac{3}{2}\\1,1\end{array}\right)\;/;\;\mathrm{Re}(z)\geq 0$$

08.02.26.0034.01

$$\left(\sqrt{z+1}-\sqrt{z}\right)K\left(\frac{4\sqrt{z^2+z}}{\left(\sqrt{z}+\sqrt{z+1}\right)^2}\right)=\frac{1}{2}\,G_{2,2}^{1,2}\left(z\,\middle|\,\begin{array}{c}\frac{1}{2},\frac{1}{2}\\0,0\end{array}\right)\;/;\;\mathrm{Re}(z)\geq 0$$

08.02.26.0035.01

$$\left(\sqrt{z}+\sqrt{z+1}\right)K\left(-\frac{4\sqrt{z^2+z}}{\left(\sqrt{z}-\sqrt{z+1}\right)^2}\right)=\frac{1}{2}\,G_{2,2}^{1,2}\left(z\,\middle|\,\begin{array}{c}\frac{1}{2},\frac{1}{2}\\0,0\end{array}\right)\;/;\;\mathrm{Re}(z)\geq 0$$

08.02.26.0036.01

$$\frac{1}{\sqrt[4]{z+1}}\,K\left(-\frac{\left(\sqrt{z+1}-1\right)^2}{4\sqrt{z+1}}\right)=\frac{1}{2}\,G_{2,2}^{1,2}\left(z\,\middle|\,\begin{array}{c}\frac{1}{2},\frac{1}{2}\\0,0\end{array}\right)$$

08.02.26.0037.01

$$\frac{1}{\sqrt[4]{z+1}}\,K\left(-\frac{\left(\sqrt{z+1}-\sqrt{z}\right)^2}{4\sqrt{z}\,\sqrt{z+1}}\right)=\frac{1}{2}\,G_{2,2}^{2,1}\left(z\,\middle|\,\begin{array}{c}\frac{3}{4},\frac{3}{4}\\\frac{1}{4},\frac{1}{4}\end{array}\right)\;/;\;z\notin(-1,0)$$

08.02.26.0038.01

$$\left(\sqrt{z+1}-\sqrt{z}\right)K\left(\left(\sqrt{z}-\sqrt{z+1}\right)^4\right)=\frac{1}{4}G_{2,2}^{2,1}\left(z\,\middle|\,\begin{matrix}\frac{1}{2},\frac{1}{2}\\0,0\end{matrix}\right)/;z\notin(-1,0)$$

08.02.26.0039.01

$$\frac{1}{\sqrt{z}+\sqrt{z+1}}K\left(\left(\sqrt{z}-\sqrt{z+1}\right)^4\right)=\frac{1}{4}G_{2,2}^{2,1}\left(z\,\middle|\,\begin{matrix}\frac{1}{2},\frac{1}{2}\\0,0\end{matrix}\right)/;z\notin(-1,0)$$

08.02.26.0040.01

$$\left(\sqrt{z+1}-\sqrt{z}\right)K\left(\frac{1}{\left(\sqrt{z}+\sqrt{z+1}\right)^4}\right)=\frac{1}{4}G_{2,2}^{2,1}\left(z\,\middle|\,\begin{matrix}\frac{1}{2},\frac{1}{2}\\0,0\end{matrix}\right)/;z\notin(-1,0)$$

08.02.26.0041.01

$$\frac{1}{\sqrt{z}+\sqrt{z+1}}K\left(\frac{1}{\left(\sqrt{z}+\sqrt{z+1}\right)^4}\right)=\frac{1}{4}G_{2,2}^{2,1}\left(z\,\middle|\,\begin{matrix}\frac{1}{2},\frac{1}{2}\\0,0\end{matrix}\right)/;z\notin(-1,0)$$

08.02.26.0042.01

$$\left(\sqrt{z+1}-1\right)K\left(\frac{\left(\sqrt{z+1}-1\right)^4}{z^2}\right)=\frac{1}{4}G_{2,2}^{1,2}\left(z\,\middle|\,\begin{matrix}\frac{3}{2},\frac{3}{2}\\1,1\end{matrix}\right)$$

08.02.26.0043.01

$$\frac{1}{\sqrt{z+1}+1}K\left(\frac{\left(1-\sqrt{z+1}\right)^4}{z^2}\right)=\frac{1}{4}G_{2,2}^{1,2}\left(z\,\middle|\,\begin{matrix}\frac{1}{2},\frac{1}{2}\\0,0\end{matrix}\right)$$

08.02.26.0044.01

$$\left(\sqrt{z+1}-1\right)K\left(\frac{z^2}{\left(\sqrt{z+1}+1\right)^4}\right)=\frac{1}{4}G_{2,2}^{1,2}\left(z\,\middle|\,\begin{matrix}\frac{3}{2},\frac{3}{2}\\1,1\end{matrix}\right)$$

08.02.26.0045.01

$$\frac{1}{\sqrt{z+1}+1}K\left(\frac{z^2}{\left(\sqrt{z+1}+1\right)^4}\right)=\frac{1}{4}G_{2,2}^{1,2}\left(z\,\middle|\,\begin{matrix}\frac{1}{2},\frac{1}{2}\\0,0\end{matrix}\right)$$

Classical cases involving unit step θ

08.02.26.0046.01

$$\theta(1-|z|)\,K(1-z)=\frac{\pi}{2}G_{2,2}^{2,0}\left(z\,\middle|\,\begin{matrix}\frac{1}{2},\frac{1}{2}\\0,0\end{matrix}\right)/;z\notin(-1,0)$$

08.02.26.0047.01

$$\theta(|z|-1)\,K(1-z)=\frac{\pi}{2}G_{2,2}^{0,2}\left(z\,\middle|\,\begin{matrix}\frac{1}{2},\frac{1}{2}\\0,0\end{matrix}\right)$$

08.02.26.0048.01

$$\theta(1-|z|)\,K\left(1-\frac{1}{z}\right)=\frac{\pi}{2}G_{2,2}^{2,0}\left(z\,\middle|\,\begin{matrix}1,1\\\frac{1}{2},\frac{1}{2}\end{matrix}\right)$$

08.02.26.0049.01

$$\theta(|z|-1)\,K\left(1-\frac{1}{z}\right)=\frac{\pi}{2}\,G_{2,2}^{0,2}\left(z\,\middle|\,\begin{matrix}1,\,1\\ \frac{1}{2},\,\frac{1}{2}\end{matrix}\right)/;\,z\notin(-\infty,\,-1)$$

08.02.26.0050.01

$$\theta(1-|z|)\,K\left(\frac{1-\sqrt{z}}{2}\right)=\frac{\pi}{2}\,G_{2,2}^{2,0}\left(z\,\middle|\,\begin{matrix}\frac{3}{4},\,\frac{3}{4}\\ 0,\,\frac{1}{2}\end{matrix}\right)/;\,z\notin(-1,\,0)$$

08.02.26.0051.01

$$\theta(|z|-1)\,K\left(\frac{1-\sqrt{z}}{2}\right)=\frac{\pi}{2}\,G_{2,2}^{0,2}\left(z\,\middle|\,\begin{matrix}\frac{3}{4},\,\frac{3}{4}\\ 0,\,\frac{1}{2}\end{matrix}\right)$$

08.02.26.0052.01

$$\theta(1-|z|)\,K\left(\frac{\sqrt{z}-1}{2\sqrt{z}}\right)=\frac{\pi}{2}\,G_{2,2}^{2,0}\left(z\,\middle|\,\begin{matrix}\frac{1}{2},\,1\\ \frac{1}{4},\,\frac{1}{4}\end{matrix}\right)/;\,z\notin(-1,\,0)$$

08.02.26.0053.01

$$\theta(|z|-1)\,K\left(\frac{\sqrt{z}-1}{2\sqrt{z}}\right)=\frac{\pi}{2}\,G_{2,2}^{0,2}\left(z\,\middle|\,\begin{matrix}\frac{1}{2},\,1\\ \frac{1}{4},\,\frac{1}{4}\end{matrix}\right)$$

08.02.26.0054.01

$$\frac{\theta(1-|z|)}{\sqrt{\sqrt{z}+1}}\,K\left(\frac{1-\sqrt{z}}{1+\sqrt{z}}\right)=\frac{\pi}{2\sqrt{2}}\,G_{2,2}^{2,0}\left(z\,\middle|\,\begin{matrix}\frac{1}{4},\,\frac{3}{4}\\ 0,\,0\end{matrix}\right)/;\,z\notin(-1,\,0)$$

08.02.26.0055.01

$$\frac{\theta(|z|-1)}{\sqrt{\sqrt{z}+1}}\,K\left(\frac{1-\sqrt{z}}{1+\sqrt{z}}\right)=\frac{\pi}{2\sqrt{2}}\,G_{2,2}^{0,2}\left(z\,\middle|\,\begin{matrix}\frac{1}{4},\,\frac{3}{4}\\ 0,\,0\end{matrix}\right)$$

08.02.26.0056.01

$$\frac{\theta(1-|z|)}{\sqrt{\sqrt{z}+1}}\,K\left(\frac{\sqrt{z}-1}{\sqrt{z}+1}\right)=\frac{\pi}{2\sqrt{2}}\,G_{2,2}^{2,0}\left(z\,\middle|\,\begin{matrix}\frac{3}{4},\,\frac{3}{4}\\ 0,\,\frac{1}{2}\end{matrix}\right)/;\,z\notin(-1,\,0)$$

08.02.26.0057.01

$$\frac{\theta(|z|-1)}{\sqrt{\sqrt{z}+1}}\,K\left(\frac{\sqrt{z}-1}{\sqrt{z}+1}\right)=\frac{\pi}{2\sqrt{2}}\,G_{2,2}^{0,2}\left(z\,\middle|\,\begin{matrix}\frac{3}{4},\,\frac{3}{4}\\ 0,\,\frac{1}{2}\end{matrix}\right)$$

08.02.26.0058.01

$$\theta(1-|z|)\,K\left(-\frac{\left(\sqrt{z}-1\right)^2}{4\sqrt{z}}\right)=\frac{\pi}{2}\,G_{2,2}^{2,0}\left(z\,\middle|\,\begin{matrix}\frac{3}{4},\,\frac{3}{4}\\ \frac{1}{4},\,\frac{1}{4}\end{matrix}\right)/;\,z\notin(-1,\,0)$$

08.02.26.0059.01

$$\theta(|z|-1)\,K\left(-\frac{\left(\sqrt{z}-1\right)^2}{4\sqrt{z}}\right)=\frac{\pi}{2}\,G_{2,2}^{0,2}\left(z\,\middle|\,\begin{matrix}\frac{3}{4},\,\frac{3}{4}\\ \frac{1}{4},\,\frac{1}{4}\end{matrix}\right)$$

08.02.26.0060.01

$$\frac{\theta(1-|z|)}{\sqrt{z}+1} K\left(\left(\frac{1-\sqrt{z}}{1+\sqrt{z}}\right)^2\right) = \frac{\pi}{4} G_{2,2}^{2,0}\left(z \left| \begin{matrix} \frac{1}{2}, \frac{1}{2} \\ 0, 0 \end{matrix} \right.\right) /; z \notin (-1, 0)$$

08.02.26.0061.01

$$\frac{\theta(|z|-1)}{\sqrt{z}+1} K\left(\left(\frac{1-\sqrt{z}}{1+\sqrt{z}}\right)^2\right) = \frac{\pi}{4} G_{2,2}^{0,2}\left(z \left| \begin{matrix} \frac{1}{2}, \frac{1}{2} \\ 0, 0 \end{matrix} \right.\right)$$

08.02.26.0062.01

$$\theta(1-|z|) \sqrt{1-\sqrt{1-z}}\; K\left(\frac{2\sqrt{1-z}}{\sqrt{1-z}+1}\right) = \frac{\pi}{2} G_{2,2}^{2,0}\left(z \left| \begin{matrix} \frac{3}{4}, \frac{5}{4} \\ \frac{1}{2}, \frac{1}{2} \end{matrix} \right.\right) /; z \notin (-1, 0)$$

08.02.26.0063.01

$$\frac{\theta(1-|z|)}{\sqrt{\sqrt{1-z}+1}} K\left(\frac{2\sqrt{1-z}}{\sqrt{1-z}+1}\right) = \frac{\pi}{2} G_{2,2}^{2,0}\left(z \left| \begin{matrix} \frac{1}{4}, \frac{3}{4} \\ 0, 0 \end{matrix} \right.\right) /; z \notin (-1, 0)$$

08.02.26.0064.01

$$\frac{\theta(|z|-1)}{\sqrt{\sqrt{1-z}+1}} K\left(\frac{2\sqrt{1-z}}{\sqrt{1-z}+1}\right) = \frac{\pi}{2} G_{2,2}^{0,2}\left(z \left| \begin{matrix} \frac{1}{4}, \frac{3}{4} \\ 0, 0 \end{matrix} \right.\right)$$

08.02.26.0065.01

$$\theta(|z|-1) \sqrt{\sqrt{1-z}+1}\; K\left(\frac{2\sqrt{1-z}}{\sqrt{1-z}-1}\right) = \frac{\pi}{2} G_{2,2}^{0,2}\left(z \left| \begin{matrix} \frac{3}{4}, \frac{5}{4} \\ \frac{1}{2}, \frac{1}{2} \end{matrix} \right.\right) /; z \notin (-\infty, -1)$$

08.02.26.0066.01

$$\frac{\theta(1-|z|)}{\sqrt{1-\sqrt{1-z}}} K\left(\frac{2\sqrt{1-z}}{\sqrt{1-z}-1}\right) = \frac{\pi}{2} G_{2,2}^{2,0}\left(z \left| \begin{matrix} \frac{1}{4}, \frac{3}{4} \\ 0, 0 \end{matrix} \right.\right) /; z \notin (-1, 0)$$

08.02.26.0067.01

$$\frac{\theta(|z|-1)}{\sqrt{1-\sqrt{1-z}}} K\left(\frac{2\sqrt{1-z}}{\sqrt{1-z}-1}\right) = \frac{\pi}{2} G_{2,2}^{0,2}\left(z \left| \begin{matrix} \frac{1}{4}, \frac{3}{4} \\ 0, 0 \end{matrix} \right.\right) /; z \notin (-\infty, -1)$$

08.02.26.0068.01

$$\frac{\theta(1-|z|)}{\sqrt{1-z}+1} K\left(\frac{4\sqrt{1-z}}{\left(\sqrt{1-z}+1\right)^2}\right) = \frac{\pi}{2} G_{2,2}^{2,0}\left(z \left| \begin{matrix} \frac{1}{2}, \frac{1}{2} \\ 0, 0 \end{matrix} \right.\right) /; \mathrm{Re}(z) > 0$$

08.02.26.0069.01

$$\frac{\theta(1-|z|)}{1-\sqrt{1-z}} K\left(-\frac{4\sqrt{1-z}}{\left(\sqrt{1-z}-1\right)^2}\right) = \frac{\pi}{2} G_{2,2}^{2,0}\left(z \left| \begin{matrix} \frac{1}{2}, \frac{1}{2} \\ 0, 0 \end{matrix} \right.\right) /; \mathrm{Re}(z) > 0$$

08.02.26.0070.01

$$\theta(1-|z|) \sqrt{1-\sqrt{1-z}}\; K\left(\frac{2\sqrt{1-z}\left(1-\sqrt{1-z}\right)}{z}\right) = \frac{\pi}{2} G_{2,2}^{2,0}\left(z \left| \begin{matrix} \frac{3}{4}, \frac{5}{4} \\ \frac{1}{2}, \frac{1}{2} \end{matrix} \right.\right) /; z \notin (-1, 0)$$

08.02.26.0071.01

$$\theta(|z|-1)\sqrt{\sqrt{z}-\sqrt{z-1}}\;K\left(2\left(\sqrt{z}-\sqrt{z-1}\right)\sqrt{z-1}\right)=\frac{\pi}{2}\,G_{2,2}^{0,2}\left(z\,\middle|\,\begin{array}{c}\frac{3}{4},\frac{3}{4}\\0,\frac{1}{2}\end{array}\right)/;\;z\notin(-\infty,-1)$$

08.02.26.0072.01

$$\theta(|z|-1)\sqrt{\sqrt{z}-\sqrt{z-1}}\;K\left(\frac{2\sqrt{z-1}}{\sqrt{z-1}+\sqrt{z}}\right)=\frac{\pi}{2}\,G_{2,2}^{0,2}\left(z\,\middle|\,\begin{array}{c}\frac{3}{4},\frac{3}{4}\\0,\frac{1}{2}\end{array}\right)/;\;z\notin(-\infty,-1)$$

08.02.26.0073.01

$$\frac{\theta(1-|z|)}{\sqrt{\sqrt{z-1}+\sqrt{z}}}\,K\left(\frac{2\sqrt{z-1}}{\sqrt{z-1}+\sqrt{z}}\right)=\frac{\pi}{2}\,G_{2,2}^{2,0}\left(z\,\middle|\,\begin{array}{c}\frac{3}{4},\frac{3}{4}\\0,\frac{1}{2}\end{array}\right)/;\;z\notin(-1,0)$$

08.02.26.0074.01

$$\frac{\theta(|z|-1)}{\sqrt{\sqrt{z-1}+\sqrt{z}}}\,K\left(\frac{2\sqrt{z-1}}{\sqrt{z-1}+\sqrt{z}}\right)=\frac{\pi}{2}\,G_{2,2}^{0,2}\left(z\,\middle|\,\begin{array}{c}\frac{3}{4},\frac{3}{4}\\0,\frac{1}{2}\end{array}\right)/;\;z\notin(-\infty,-1)$$

08.02.26.0075.01

$$\theta(1-|z|)\sqrt{\sqrt{z-1}+\sqrt{z}}\;K\left(\frac{2\sqrt{z-1}}{\sqrt{z-1}-\sqrt{z}}\right)=\frac{\pi}{2}\,G_{2,2}^{2,0}\left(z\,\middle|\,\begin{array}{c}\frac{3}{4},\frac{3}{4}\\0,\frac{1}{2}\end{array}\right)/;\;z\notin(-1,0)$$

08.02.26.0076.01

$$\frac{\theta(1-|z|)}{\sqrt{\sqrt{z}-\sqrt{z-1}}}\,K\left(\frac{2\sqrt{z-1}}{\sqrt{z-1}-\sqrt{z}}\right)=\frac{\pi}{2}\,G_{2,2}^{2,0}\left(z\,\middle|\,\begin{array}{c}\frac{3}{4},\frac{3}{4}\\0,\frac{1}{2}\end{array}\right)/;\;\notin(-1,0)$$

08.02.26.0077.01

$$\frac{\theta(|z|-1)}{\sqrt{\sqrt{z-1}+\sqrt{z}}}\,K\left(\frac{2\sqrt{z-1}}{\sqrt{z-1}+\sqrt{z}}\right)=\frac{\pi}{2}\,G_{2,2}^{0,2}\left(z\,\middle|\,\begin{array}{c}\frac{3}{4},\frac{3}{4}\\0,\frac{1}{2}\end{array}\right)$$

08.02.26.0078.01

$$\frac{\theta(|z|-1)}{\sqrt{z-1}+\sqrt{z}}\,K\left(\frac{4\sqrt{z-1}\sqrt{z}}{\left(\sqrt{z-1}+\sqrt{z}\right)^2}\right)=\frac{\pi}{2}\,G_{2,2}^{0,2}\left(z\,\middle|\,\begin{array}{c}\frac{1}{2},\frac{1}{2}\\0,0\end{array}\right)/;\;\mathrm{Re}(z)>0$$

08.02.26.0079.01

$$\frac{\theta(|z|-1)}{\sqrt{z}-\sqrt{z-1}}\,K\left(-\frac{4\sqrt{z-1}\sqrt{z}}{\left(\sqrt{z-1}-\sqrt{z}\right)^2}\right)=\frac{\pi}{2}\,G_{2,2}^{0,2}\left(z\,\middle|\,\begin{array}{c}\frac{1}{2},\frac{1}{2}\\0,0\end{array}\right)/;\;\mathrm{Re}(z)>0$$

Classical cases involving sgn

08.02.26.0080.01

$$\frac{\mathrm{sgn}(1-|z|)}{1-\sqrt{z}}\,K\left(-\frac{4\sqrt{z}}{\left(\sqrt{z}-1\right)^2}\right)=\frac{\pi}{2}\,G_{2,2}^{1,1}\left(z\,\middle|\,\begin{array}{c}\frac{1}{2},\frac{1}{2}\\0,0\end{array}\right)$$

Classical cases involving powers of complete elliptic integral *K*

08.02.26.0081.01

$$K\left(\left(\frac{1-\sqrt{z+1}}{2}\right)^2\right) = \frac{\sqrt{\pi}}{4} G_{3,3}^{1,3}\left(z \middle| \begin{matrix} \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \\ 0, 0, 0 \end{matrix}\right)$$

08.02.26.0082.01

$$K\left(\left(\frac{\sqrt{z}-\sqrt{z+1}}{2\sqrt{z}}\right)^2\right) = \frac{\sqrt{\pi}}{4} G_{3,3}^{3,1}\left(z \middle| \begin{matrix} 1, 1, 1 \\ \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \end{matrix}\right) /; z \notin (-1, 0)$$

08.02.26.0083.01

$$\frac{1}{\sqrt{z}+\sqrt{z+1}} K\left(\left(\sqrt{z+1}-\sqrt{z}\right)^2\right)^2 = \frac{\sqrt{\pi}}{8} G_{3,3}^{3,1}\left(z \middle| \begin{matrix} \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \\ 0, 0, 0 \end{matrix}\right) /; z \notin (-1, 0)$$

08.02.26.0084.01

$$\frac{1}{\sqrt{z+1}+1} K\left(\left(\frac{\left(\sqrt{z+1}-1\right)^2}{z}\right)^2\right) = \frac{\sqrt{\pi}}{8} G_{3,3}^{1,3}\left(z \middle| \begin{matrix} \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \\ 0, 0, 0 \end{matrix}\right) /; z \notin (-1, 0)$$

Generalized cases involving algebraic functions

08.02.26.0085.01

$$K\left(-\frac{(z-1)^2}{4z}\right) = \frac{1}{2\pi} G_{2,2}^{2,2}\left(z, \frac{1}{2} \middle| \begin{matrix} \frac{3}{4}, \frac{3}{4} \\ \frac{1}{4}, \frac{1}{4} \end{matrix}\right) /; z \notin (-\infty, 0)$$

08.02.26.0086.01

$$\frac{1}{z+1} K\left(\frac{4z}{(z+1)^2}\right) = \frac{\pi}{2} G_{2,2}^{1,1}\left(z, \frac{1}{2} \middle| \begin{matrix} \frac{1}{2}, \frac{1}{2} \\ 0, 0 \end{matrix}\right)$$

08.02.26.0087.01

$$\frac{1}{z+1} K\left(\frac{(1-z)^2}{(1+z)^2}\right) = \frac{1}{4\pi} G_{2,2}^{2,2}\left(z, \frac{1}{2} \middle| \begin{matrix} \frac{1}{2}, \frac{1}{2} \\ 0, 0 \end{matrix}\right)$$

08.02.26.0088.01

$$K\left(\frac{z-\sqrt{z^2+1}}{2z}\right) = \frac{\pi}{2\,\Gamma\left(\frac{1}{4}\right)^2} G_{2,2}^{2,1}\left(z, \frac{1}{2} \middle| \begin{matrix} 1, 1 \\ \frac{1}{4}, \frac{1}{4} \end{matrix}\right) /; \operatorname{Re}(z) > 0$$

08.02.26.0089.01

$$\frac{1}{\sqrt{z^2+1}} K\left(\frac{z-\sqrt{z^2+1}}{2z}\right) = \frac{\pi}{2\,\Gamma\left(\frac{3}{4}\right)^2} G_{2,2}^{2,1}\left(z, \frac{1}{2} \middle| \begin{matrix} \frac{1}{2}, \frac{1}{2} \\ \frac{1}{4}, \frac{1}{4} \end{matrix}\right) /; \operatorname{Re}(z) > 0$$

08.02.26.0090.01

$$\frac{1}{\sqrt[4]{z^2+1}} K\left(\frac{\sqrt{z^2+1}-z}{2\sqrt{z^2+1}}\right) = \frac{1}{2\sqrt{2}} G_{2,2}^{2,1}\left(z, \frac{1}{2} \middle| \begin{matrix} \frac{3}{4}, \frac{3}{4} \\ 0, \frac{1}{2} \end{matrix}\right) /; \operatorname{Re}(z) > 0$$

08.02.26.0091.01

$$\frac{1}{\sqrt[4]{z^2+1}} K\left(\frac{1}{2\left(z+\sqrt{z^2+1}\right)\sqrt{z^2+1}}\right) = \frac{1}{2\sqrt{2}} G_{2,2}^{2,1}\left(z, \frac{1}{2} \middle| \begin{matrix} \frac{3}{4}, \frac{3}{4} \\ 0, \frac{1}{2} \end{matrix}\right) /; \operatorname{Re}(z) > 0$$

08.02.26.0092.01

$$\frac{1}{\sqrt{z+\sqrt{z^2+1}}}\,K\left(\frac{\sqrt{z^2+1}-z}{\sqrt{z^2+1}+z}\right)=\frac{\pi}{2\sqrt{2}\,\Gamma\left(\frac{1}{4}\right)^2}\,G_{2,2}^{2,1}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{3}{4},\frac{3}{4}\\0,0\end{matrix}\right)/;\operatorname{Re}(z)>0$$

08.02.26.0093.01

$$\frac{1}{\sqrt{z^2+1}\,\sqrt{\sqrt{z^2+1}+z}}\,K\left(\frac{\sqrt{z^2+1}-z}{\sqrt{z^2+1}+z}\right)=\frac{\pi}{2\sqrt{2}\,\Gamma\left(\frac{3}{4}\right)^2}\,G_{2,2}^{2,1}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{1}{4}\\0,0\end{matrix}\right)/;\operatorname{Re}(z)>0$$

08.02.26.0094.01

$$\frac{1}{\sqrt{z+\sqrt{z^2+1}}}\,K\left(\frac{z-\sqrt{z^2+1}}{z+\sqrt{z^2+1}}\right)=\frac{1}{4}\,G_{2,2}^{2,1}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{3}{4},\frac{3}{4}\\0,\frac{1}{2}\end{matrix}\right)/;\operatorname{Re}(z)>0$$

08.02.26.0095.01

$$\sqrt{\sqrt{z^2+1}-z}\;K\left(2z\left(\sqrt{z^2+1}-z\right)\right)=\frac{\pi}{2\,\Gamma\left(\frac{1}{4}\right)^2}\,G_{2,2}^{1,2}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{3}{4},\frac{3}{4}\\0,0\end{matrix}\right)$$

08.02.26.0096.01

$$\frac{\sqrt{\sqrt{z^2+1}-z}}{\sqrt{z^2+1}}\,K\left(2z\left(\sqrt{z^2+1}-z\right)\right)=\frac{\pi}{2\,\Gamma\left(\frac{3}{4}\right)^2}\,G_{2,2}^{1,2}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{1}{4}\\0,0\end{matrix}\right)$$

08.02.26.0097.01

$$\frac{1}{\sqrt{\sqrt{z^2+1}-z}}\,K\left(-2z\left(z+\sqrt{z^2+1}\right)\right)=\frac{\pi}{2\,\Gamma\left(\frac{1}{4}\right)^2}\,G_{2,2}^{1,2}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{3}{4},\frac{3}{4}\\0,0\end{matrix}\right)$$

08.02.26.0098.01

$$\frac{1}{\sqrt{z^2+1}\,\sqrt{\sqrt{z^2+1}-z}}\,K\left(-2z\left(z+\sqrt{z^2+1}\right)\right)=\frac{\pi}{2\,\Gamma\left(\frac{3}{4}\right)^2}\,G_{2,2}^{1,2}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{1}{4}\\0,0\end{matrix}\right)$$

08.02.26.0099.01

$$\left(\sqrt{z^2+1}-z\right)K\left(\frac{4z\sqrt{z^2+1}}{\left(z+\sqrt{z^2+1}\right)^2}\right)=\frac{1}{2}\,G_{2,2}^{1,2}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1}{2},\frac{1}{2}\\0,0\end{matrix}\right)/;\,i\,z\notin(-\infty,-1)\bigwedge i\,z\notin(1,\infty)$$

08.02.26.0100.01

$$\left(z+\sqrt{z^2+1}\right)K\left(-\frac{4z\sqrt{z^2+1}}{\left(z-\sqrt{z^2+1}\right)^2}\right)=\frac{1}{2}\,G_{2,2}^{1,2}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1}{2},\frac{1}{2}\\0,0\end{matrix}\right)/;\,i\,z\notin(-\infty,-1)\bigwedge i\,z\notin(1,\infty)$$

08.02.26.0101.01

$$\frac{1}{\sqrt[4]{z^2+1}}\,K\left(-\frac{\left(\sqrt{z^2+1}-z\right)^2}{4\,z\,\sqrt{z^2+1}}\right)=\frac{1}{2}\,G_{2,2}^{2,1}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{3}{4},\frac{3}{4}\\\frac{1}{4},\frac{1}{4}\end{matrix}\right)\text{/; }\operatorname{Re}(z)>0$$

08.02.26.0102.01

$$\left(\sqrt{z^2+1}-z\right)K\left(\left(z-\sqrt{z^2+1}\right)^4\right)=\frac{1}{4}\,G_{2,2}^{2,1}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1}{2},\frac{1}{2}\\0,0\end{matrix}\right)\text{/; }\operatorname{Re}(z)>0$$

08.02.26.0103.01

$$\frac{1}{z+\sqrt{z^2+1}}\,K\left(\left(z-\sqrt{z^2+1}\right)^4\right)=\frac{1}{4}\,G_{2,2}^{2,1}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1}{2},\frac{1}{2}\\0,0\end{matrix}\right)\text{/; }\operatorname{Re}(z)>0$$

08.02.26.0104.01

$$\left(\sqrt{z^2+1}-z\right)K\left(\frac{1}{\left(z+\sqrt{z^2+1}\right)^4}\right)=\frac{1}{4}\,G_{2,2}^{2,1}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1}{2},\frac{1}{2}\\0,0\end{matrix}\right)\text{/; }\operatorname{Re}(z)>0$$

08.02.26.0105.01

$$\frac{1}{z+\sqrt{z^2+1}}\,K\left(\frac{1}{\left(z+\sqrt{z^2+1}\right)^4}\right)=\frac{1}{4}\,G_{2,2}^{2,1}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1}{2},\frac{1}{2}\\0,0\end{matrix}\right)\text{/; }\operatorname{Re}(z)>0$$

Generalized cases involving unit step θ

08.02.26.0106.01

$$\theta(1-|z|)\,K\left(\frac{1-z}{2}\right)=\frac{\pi}{2}\,G_{2,2}^{2,0}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{3}{4},\frac{3}{4}\\0,\frac{1}{2}\end{matrix}\right)$$

08.02.26.0107.01

$$\theta(1-|z|)\,K\left(\frac{z-1}{2\,z}\right)=\frac{\pi}{2}\,G_{2,2}^{2,0}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1}{2},1\\\frac{1}{4},\frac{1}{4}\end{matrix}\right)$$

08.02.26.0108.01

$$\theta(|z|-1)\,K\left(\frac{1-z}{2}\right)=\frac{\pi}{2}\,G_{2,2}^{0,2}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{3}{4},\frac{3}{4}\\0,\frac{1}{2}\end{matrix}\right)$$

08.02.26.0109.01

$$\theta(|z|-1)\,K\left(\frac{z-1}{2\,z}\right)=\frac{\pi}{2}\,G_{2,2}^{0,2}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1}{2},1\\\frac{1}{4},\frac{1}{4}\end{matrix}\right)$$

08.02.26.0110.01

$$\frac{\theta(1-|z|)}{\sqrt{z+1}}\,K\left(\frac{z-1}{z+1}\right)=\frac{\pi}{2\,\sqrt{2}}\,G_{2,2}^{2,0}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{3}{4},\frac{3}{4}\\0,\frac{1}{2}\end{matrix}\right)$$

08.02.26.0111.01

$$\frac{\theta(1-|z|)}{\sqrt{z+1}} K\left(\frac{1-z}{1+z}\right) = \frac{\pi}{2\sqrt{2}} G_{2,2}^{2,0}\left(z, \frac{1}{2} \middle| \begin{matrix} \frac{1}{4}, \frac{3}{4} \\ 0, 0 \end{matrix}\right) /; z \notin (-1, 0)$$

08.02.26.0112.01

$$\frac{\theta(|z|-1)}{\sqrt{z+1}} K\left(\frac{z-1}{z+1}\right) = \frac{\pi}{2\sqrt{2}} G_{2,2}^{0,2}\left(z, \frac{1}{2} \middle| \begin{matrix} \frac{3}{4}, \frac{3}{4} \\ 0, \frac{1}{2} \end{matrix}\right) /; z \notin (-\infty, -1)$$

08.02.26.0113.01

$$\frac{\theta(|z|-1)}{\sqrt{z+1}} K\left(\frac{1-z}{1+z}\right) = \frac{\pi}{2\sqrt{2}} G_{2,2}^{0,2}\left(z, \frac{1}{2} \middle| \begin{matrix} \frac{1}{4}, \frac{3}{4} \\ 0, 0 \end{matrix}\right)$$

08.02.26.0114.01

$$\theta(1-|z|) K\left(-\frac{(z-1)^2}{4z}\right) = \frac{\pi}{2} G_{2,2}^{2,0}\left(z, \frac{1}{2} \middle| \begin{matrix} \frac{3}{4}, \frac{3}{4} \\ \frac{1}{4}, \frac{1}{4} \end{matrix}\right)$$

08.02.26.0115.01

$$\theta(|z|-1) K\left(-\frac{(z-1)^2}{4z}\right) = \frac{\pi}{2} G_{2,2}^{0,2}\left(z, \frac{1}{2} \middle| \begin{matrix} \frac{3}{4}, \frac{3}{4} \\ \frac{1}{4}, \frac{1}{4} \end{matrix}\right)$$

08.02.26.0116.01

$$\frac{\theta(1-|z|)}{z+1} K\left(\frac{(1-z)^2}{(z+1)^2}\right) = \frac{\pi}{4} G_{2,2}^{2,0}\left(z, \frac{1}{2} \middle| \begin{matrix} \frac{1}{2}, \frac{1}{2} \\ 0, 0 \end{matrix}\right) /; z \notin (-1, 0)$$

08.02.26.0117.01

$$\frac{\theta(|z|-1)}{z+1} K\left(\frac{(1-z)^2}{(z+1)^2}\right) = \frac{\pi}{4} G_{2,2}^{0,2}\left(z, \frac{1}{2} \middle| \begin{matrix} \frac{1}{2}, \frac{1}{2} \\ 0, 0 \end{matrix}\right) /; z \notin (-\infty, -1)$$

08.02.26.0118.01

$$\theta(|z|-1) \sqrt{z-\sqrt{z^2-1}}\, K\left(2\left(z-\sqrt{z^2-1}\right)\sqrt{z^2-1}\right) = \frac{\pi}{2} G_{2,2}^{0,2}\left(z, \frac{1}{2} \middle| \begin{matrix} \frac{3}{4}, \frac{3}{4} \\ 0, \frac{1}{2} \end{matrix}\right) /; \operatorname{Re}(z) > 0$$

08.02.26.0119.01

$$\theta(|z|-1) \sqrt{z-\sqrt{z^2-1}}\, K\left(\frac{2\sqrt{z^2-1}}{z+\sqrt{z^2-1}}\right) = \frac{\pi}{2} G_{2,2}^{0,2}\left(z, \frac{1}{2} \middle| \begin{matrix} \frac{3}{4}, \frac{3}{4} \\ 0, \frac{1}{2} \end{matrix}\right) /; \operatorname{Re}(z) > 0$$

08.02.26.0120.01

$$\frac{\theta(1-|z|)}{\sqrt{z+\sqrt{z^2-1}}} K\left(\frac{2\sqrt{z^2-1}}{z+\sqrt{z^2-1}}\right) = \frac{\pi}{2} G_{2,2}^{2,0}\left(z, \frac{1}{2} \middle| \begin{matrix} \frac{3}{4}, \frac{3}{4} \\ 0, \frac{1}{2} \end{matrix}\right) /; \operatorname{Re}(z) > 0$$

08.02.26.0121.01

$$\frac{\theta(|z|-1)}{\sqrt{z+\sqrt{z^2-1}}} K\left(\frac{2\sqrt{z^2-1}}{z+\sqrt{z^2-1}}\right) = \frac{\pi}{2} G_{2,2}^{0,2}\left(z, \frac{1}{2} \middle| \begin{matrix} \frac{3}{4}, \frac{3}{4} \\ 0, \frac{1}{2} \end{matrix}\right) /; \operatorname{Re}(z) > 0$$

08.02.26.0122.01

$$\theta(1-|z|)\sqrt{z+\sqrt{z^2-1}}\;K\left(\frac{2\sqrt{z^2-1}}{\sqrt{z^2-1}-z}\right)=\frac{\pi}{2}\,G_{2,2}^{2,0}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{3}{4},\frac{3}{4}\\0,\frac{1}{2}\end{matrix}\right)/;\operatorname{Re}(z)>0$$

08.02.26.0123.01

$$\frac{\theta(1-|z|)}{\sqrt{z-\sqrt{z^2-1}}}\,K\left(\frac{2\sqrt{z^2-1}}{\sqrt{z^2-1}-z}\right)=\frac{\pi}{2}\,G_{2,2}^{2,0}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{3}{4},\frac{3}{4}\\0,\frac{1}{2}\end{matrix}\right)/;\operatorname{Re}(z)>0$$

08.02.26.0124.01

$$\frac{\theta(|z|-1)}{\sqrt{z-\sqrt{z^2-1}}}\,K\left(\frac{2\sqrt{z^2-1}}{\sqrt{z^2-1}-z}\right)=\frac{\pi}{2}\,G_{2,2}^{0,2}\left(\frac{1}{2}\,\middle|\,\begin{matrix}\frac{3}{4},\frac{3}{4}\\0,\frac{1}{2}\end{matrix}\right)/;\operatorname{Re}(z)>0$$

Generalized cases involving sgn

08.02.26.0125.01

$$\frac{\operatorname{sgn}(1-|z|)}{1-z}\,K\left(-\frac{4z}{(z-1)^2}\right)=\frac{\pi}{2}\,G_{2,2}^{1,1}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1}{2},\frac{1}{2}\\0,0\end{matrix}\right)$$

Generalized cases involving powers of complete elliptic integral *K*

08.02.26.0126.01

$$K\left(\frac{z-\sqrt{z^2+1}}{2z}\right)^2=\frac{\sqrt{\pi}}{4}\,G_{3,3}^{3,1}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}1,1,1\\\frac{1}{2},\frac{1}{2},\frac{1}{2}\end{matrix}\right)/;\operatorname{Re}(z)>0$$

08.02.26.0127.01

$$\frac{K\left(\left(\sqrt{z^2+1}-z\right)^2\right)^2}{z+\sqrt{z^2+1}}=\frac{\sqrt{\pi}}{8}\,G_{3,3}^{3,1}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1}{2},\frac{1}{2},\frac{1}{2}\\0,0,0\end{matrix}\right)/;\operatorname{Re}(z)>0$$

Through other functions

Involving incomplete elliptic integrals

08.02.26.0128.01

$$K(z)=\Pi\left(0;\frac{\pi}{2}\,\middle|\,z\right)$$

08.02.26.0129.01

$$K(z)=\Pi(0\mid z)$$

08.02.26.0130.01

$$K(z)=F\left(\frac{\pi}{2}\,\middle|\,z\right)$$

08.02.26.0131.01

$$K(z)=\frac{1}{\sqrt{z}}\,F\left(\sin^{-1}\left(\sqrt{z}\right)\,\middle|\,\frac{1}{z}\right)$$

Involving elliptic theta functions

08.02.26.0132.01

$$K(m) = \frac{\pi}{2}\,\vartheta_3(0, q(m))^2$$

Involving inverse Jacobi functions

08.02.26.0134.01

$$K(z) = \mathrm{sn}^{-1}(1 \mid z)$$

08.02.26.0135.01

$$K(z) = \mathrm{dn}^{-1}\left(\sqrt{1-z} \mid z\right)$$

08.02.26.0136.01

$$K(z) = \mathrm{cn}^{-1}(0 \mid z) \;/;\; z \in \mathbb{R} \land z < 1$$

Involving some elliptic-type functions

08.02.26.0133.01

$$K(z) = \frac{\pi}{2\,\mathrm{agm}\left(1, \sqrt{1-z}\right)}$$

08.02.26.0137.01

$$K\left(q^{-1}\left(\exp\left(\frac{i\,\pi\,\omega_2}{\omega_1}\right)\right)\right) = \sqrt{e_1 - e_3}\;\omega_1 \;/;$$
$$\{e_1, e_2, e_3\} = \{\wp(\omega_1; g_2, g_3), \wp(\omega_1 + \omega_2; g_2, g_3), \wp(\omega_2; g_2, g_3)\} \bigwedge \{g_2, g_3\} = \left\{g_2\,(\omega_1, \omega_2), g_3\,(\omega_1, \omega_2)\right\}$$

08.02.26.0138.01

$$\frac{K(1-z)}{K(z)} = -\frac{i\,\omega_2}{\omega_1} \;/;\; z = q^{-1}\left(\exp\left(\frac{i\,\pi\,\omega_2}{\omega_1}\right)\right) \bigwedge \{\omega_1, \omega_2\} = \left\{\omega_1\left(g_2, g_3\right), \omega_2\left(g_2, g_3\right)\right\}$$

Involving Legendre functions

08.02.26.0139.01

$$K(z) = \frac{\pi}{2}\,P_{-\frac{1}{2}}(1 - 2\,z)$$

08.02.26.0140.01

$$K(z) = Q_{-\frac{1}{2}}(2\,z - 1)$$

Involving some hypergeometric-type functions

08.02.26.0141.01

$$K(z) = F_1\left(\frac{1}{2}; \frac{1}{2}, \frac{1}{2}; \frac{3}{2}; 1, z\right)$$

Representations through equivalent functions

With inverse function

08.02.27.0001.01

$$\mathrm{am}(K(m) \mid m) = \frac{\pi}{2}$$

With related functions

08.02.27.0002.01

$$E(z)\,K(1-z)-K(z)\,K(1-z)+E(1-z)\,K(z)=\frac{\pi}{2}$$

08.02.27.0003.01

$$K(2)=\frac{\sqrt{2}\,\pi^{3/2}\,\Gamma\left(\frac{3}{4}\right)^2-\pi^2\,E(2)}{2\,\Gamma\left(\frac{3}{4}\right)^4}$$

Theorems

The period *T* of a mathematical pendulum in a gravitational field

The period T of a mathematical pendulum of length l in a gravitational field with acceleration g and maximal angle of excursion α is given by $T = 4\sqrt{\frac{l}{g}}\,K\left(\sin^2\left(\frac{\alpha}{2}\right)\right)$.

The partition function for a one-dimensional monatomic ideal classical gas

The partition function Z for a one-dimensional monatomic ideal classical gas of n atoms in a box of length l at temperature T is given by $Z = \frac{1}{2^n\,n!}\left(\sqrt{\frac{2}{\pi}\,K\left(q^{-1}\left(\exp\left(-\frac{3\,\lambda(T)^2}{8\,\pi\,l^2}\right)\right)\right)}-1\right)^n$, where $\lambda(T)$ is the thermal de Broglie wavelength.

The magnetic induction of an infiniely long selenoid

The magnetic induction $\boldsymbol{B}$ of an infinitely long solenoid formed by a wire (parametrized by ϕ) $\{R\cos(\phi),\, R\sin(\phi),\, R\,\phi\tan(\alpha)\}$ carrying the current i_0 is at the center line is given by

$$\boldsymbol{B}\propto\left\{0,\,-i_0(\cot(\alpha)\,K_0(\cot(\alpha))+K_1(\cot(\alpha))),\,i_0\,\frac{\cot(\alpha)}{2\,\pi\,R}\right\}.$$

The lattice Green function for the body-centered cubic lattice

The lattice Green function $G(\varepsilon) = \frac{1}{\pi^3}\int_0^\pi\int_0^\pi\int_0^\pi\left(1-\frac{\varepsilon}{3}\,(\cos(x)\cos(y)\cos(z))\right)^{-1}dx\,dy\,dz$ for the simple cubic lattice can be expressed as

$$\frac{4}{\pi^2}\sqrt{1-\frac{3}{4}\,\alpha^2}\,\frac{1}{1-\alpha}\,K\left(\frac{1}{2}+\frac{\beta}{4}\sqrt{4-\beta}-\frac{(2-\beta)}{4}\sqrt{1-\beta}\right)K\left(\frac{1}{2}-\frac{\beta}{4}\sqrt{4-\beta}-\frac{(2-\beta)}{4}\sqrt{1-\beta}\right)/;$$

$$\alpha=\frac{1}{2}+\frac{\varepsilon^2}{6}-\frac{1}{2}\sqrt{1-\varepsilon^2}\,\sqrt{1-\frac{\varepsilon^2}{9}}\bigwedge\beta=\frac{\alpha}{\alpha-1}.$$

The probability that a random walk in three dimensions will return to its origin

The probability p_0 that a random walk in three dimensions will return to its point of origin is given by

$$p_0 = 1 - \frac{\pi^2}{72}\left(6 + 2\sqrt{3} + \sqrt{6}\right) K\left(35 + 24\sqrt{2} - 20\sqrt{3} - 14\sqrt{6}\right)^{-2} \approx 0.34053732955099914283 \ldots .$$

History

– A. M. Legendre (1811, 1825)

– C. G. J. Jacobi (1829)

Copyright