EllipticThetaPrime1

Notations

Traditional name

Derivative of the Jacobi theta function ϑ_1

Traditional notation

$\vartheta_1'(z, q)$

Mathematica StandardForm notation

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EllipticThetaPrime[1, z, q]
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Primary definition

09.05.02.0001.01

$$\vartheta_1'(z, q) = 2\sqrt[4]{q}\sum_{k=0}^{\infty}(-1)^k q^{k(k+1)}(2k+1)\cos((2k+1)z)\;/;\;|q|<1$$

Specific values

Specialized values

For fixed *z*

09.05.03.0001.01

$$\vartheta_1'(z, 0) = 0$$

For fixed *q*

09.05.03.0004.02

$$\vartheta_1'(0, q) = 2\,\eta\left(-\frac{i\log(q)}{\pi}\right)^3$$

09.05.03.0005.01

$$\vartheta_1'(0, q)^8 = \left(\frac{2\,\omega_1}{\pi}\right)^{12}(e_2-e_3)^2(e_1-e_2)^2(e_1-e_3)^2\;/;$$

$$\{\omega_1, \omega_2, \omega_3\} = \{\omega(g_2, g_3), -\omega(g_2, g_3)-\omega'(g_2, g_3), \omega'(g_2, g_3)\}\bigwedge\tau = \frac{\omega_3}{\omega_1}\bigwedge q = e^{\tau\pi i}\bigwedge e_\alpha = \wp(\omega_\alpha; g_2, g_3)$$

09.05.03.0006.01

$$\vartheta_1'\left(0, e^{-\frac{i\pi}{\tau}}\right) = -\sqrt{i}\,\tau^{3/2}\,\vartheta_1'\left(0, e^{i\pi\tau}\right)$$

09.05.03.0007.01

$$\vartheta_1'(0, e^{-\pi}) = \frac{1}{4\pi^{9/4}} \Gamma\left(\frac{1}{4}\right)^3$$

09.05.03.0008.01

$$\vartheta_1'\left(-\frac{\pi}{2}, q\right) = 0$$

09.05.03.0002.01

$$\vartheta_1'\left(\frac{\pi}{2}, q\right) = 0$$

09.05.03.0009.01

$$\vartheta_1'(\pi m, q) = 2(-1)^m \eta\left(-\frac{i \log(q)}{\pi}\right)^3 /; m \in \mathbb{Z}$$

09.05.03.0003.02

$$\vartheta_1'\left(\pi m + \frac{\pi}{2}, q\right) = 0 /; m \in \mathbb{Z}$$

09.05.03.0010.01

$$\vartheta_1'(-i \log(q), q) = -\frac{2}{q} \eta\left(-\frac{i \log(q)}{\pi}\right)^3$$

09.05.03.0011.01

$$\vartheta_1'(m\pi - i n \log(q), q) = 2(-1)^{m+n} q^{-n^2} \eta\left(-\frac{i \log(q)}{\pi}\right)^3 /; m \in \mathbb{Z} \land n \in \mathbb{Z}$$

09.05.03.0012.01

$$\vartheta_1'\left(-\frac{1}{2}(i \log(q)), q\right) = -\frac{1}{\sqrt[4]{q}} \sqrt{\frac{2}{\pi}} \sqrt[4]{q^{-1}(q)} \sqrt{K\left(q^{-1}(q)\right)}$$

09.05.03.0013.01

$$\vartheta_1'\left(\frac{1}{2}(\pi - i \log(q)), q\right) = -i \frac{1}{\sqrt[4]{q}} \sqrt{\frac{2}{\pi}} \sqrt{K\left(q^{-1}(q)\right)}$$

General characteristics

Domain and analyticity

$\vartheta_1'(z, q)$ is an analytic function of z and q for $z, q \in \mathbb{C}$ and $|q| < 1$.

09.05.04.0001.01

$$(1 * z * q) \longrightarrow \vartheta_1'(z, q) :: (\{1\} \otimes \mathbb{C} \otimes \mathbb{C}) \longrightarrow \mathbb{C}$$

Symmetries and periodicities

Parity

$\vartheta_1'(z, q)$ is an even function with respect to z.

09.05.04.0002.01

$$\vartheta_1'(-z, q) = \vartheta_1'(z, q)$$

09.05.04.0003.01

$$\vartheta_1'(z, -q) = \exp\left(-\frac{i \pi}{4} \operatorname{sgn}(\operatorname{Im}(q))\right) \vartheta_1'(z, q)$$

Mirror symmetry

09.05.04.0004.01

$$\vartheta_1'(\bar{z}, \bar{q}) = \overline{\vartheta_1'(z, q)}$$

Periodicity

$\vartheta_1'(z, q)$, considered as a function of z, has a period of 2π.

09.05.04.0005.01

$$\vartheta_1'(z + \pi, q) = -\vartheta_1'(z, q)$$

09.05.04.0019.01

$$\vartheta_1'(z + 2\pi, q) = \vartheta_1'(z, q)$$

09.05.04.0006.01

$$\vartheta_1'(z + m\pi, q) = (-1)^m \vartheta_1'(z, q) \;/;\; m \in \mathbb{Z}$$

Poles and essential singularities

With respect to *q*

The function $\vartheta_1'(z, q)$ does not have poles and essential singularities inside of the unit circle $|q| < 1$.

09.05.04.0009.01

$$\mathcal{S}ing_q(\vartheta_1'(z, q)) = \{\}$$

With respect to *z*

09.05.04.0010.01

$$\mathcal{S}ing_z(\vartheta_1'(z, q)) = \{\}$$

Branch points

With respect to *q*

For fixed z, the function $\vartheta_1'(z, q)$ has one branch point: $q = 0$. (The point $q = -1$ is the branch cut endpoint.)

09.05.04.0011.01

$$\mathcal{BP}_q(\vartheta_1'(z, q)) = \{0\}$$

09.05.04.0012.01

$$\mathcal{R}_q(\vartheta_1'(z, q), 0) = 4$$

With respect to *z*

For fixed q, the function $\vartheta_1'(z, q)$ does not have branch points.

09.05.04.0013.01

$$\mathcal{BP}_z(\vartheta_1'(z, q)) = \{\}$$

Branch cuts

With respect to q

For fixed z, the function $\vartheta_1'(z, q)$ is a single-valued function inside the unit circle of the complex q-plane, cut along the interval $(-1, 0)$, where it is continuous from above.

09.05.04.0014.01

$$\mathcal{BC}_q(\vartheta_1'(z, q)) = \{\{(-1, 0), -i\}\}$$

09.05.04.0015.01

$$\lim_{\epsilon \to +0} \vartheta_1'(z, q + i\,\epsilon) = \vartheta_1'(z, q) \,/;\, -1 < q < 0$$

09.05.04.0016.01

$$\lim_{\epsilon \to +0} \vartheta_1'(z, q - i\,\epsilon) = -i\,\vartheta_1'(z, q) \,/;\, -1 < q < 0$$

With respect to z

For fixed q, the function $\vartheta_1'(z, q)$ does not have branch cuts.

09.05.04.0017.01

$$\mathcal{BC}_z(\vartheta_1'(z, q)) = \{\}$$

Natural boundary of analyticity

The unit circle $|q| = 1$ is the natural boundary of the region of analyticity.

09.05.04.0018.01

$$\mathcal{AB}_z(\vartheta_1'(z, q)) = \left\{e^{i\,(-\pi,\pi)}\right\}$$

Branch cut endpoints

The function $\vartheta_1'(z, q)$ has one branch cut endpoint: $q = -1$.

Series representations

q-series

Expansions at generic point $q = q_0$

09.05.06.0007.01

$$\vartheta_1'(z, q) \propto \vartheta_1'(z, q_0) + \vartheta_1^{(1,1)}(z, q_0)\,(q - q_0) + \frac{\vartheta_1^{(1,2)}(z, q_0)}{2}\,(q - q_0)^2 + \frac{\vartheta_1^{(1,3)}(z, q_0)}{6}\,(q - q_0)^3 + O\left((q - q_0)^4\right)$$

09.05.06.0008.01

$$\vartheta_1'(z, q) = \sum_{k=0}^{\infty} \frac{\vartheta_1^{(1,k)}(z, q_0)}{k!}\,(q - q_0)^k$$

09.05.06.0009.01

$$\vartheta_1'(z, q) \propto \vartheta_1'(z, q_0)\,(1 + O(q - q_0))$$

Expansions on branch cuts

09.05.06.0010.01

$$\vartheta_1'(z, q) \propto e^{\frac{\pi i}{2} \left\lfloor \frac{\arg(q-x)}{2\pi} \right\rfloor} \left(\vartheta_1'(z, x) + \vartheta_1^{(1,1)}(z, x)(q-x) + \frac{\vartheta_1^{(1,2)}(z, x)}{2} (q-x)^2 + \frac{\vartheta_1^{(1,3)}(z, x)}{6} (q-x)^3 + O\left((q-x)^4\right) \right) /; x \in \mathbb{R} \land -1 < x < 0$$

09.05.06.0011.01

$$\vartheta_1'(z, q) = e^{\frac{\pi i}{2} \left\lfloor \frac{\arg(q-x)}{2\pi} \right\rfloor} \sum_{k=0}^{\infty} \frac{\vartheta_1^{(1,k)}(z, x)}{k!} (q-x)^k /; x \in \mathbb{R} \land -1 < x < 0$$

09.05.06.0012.01

$$\vartheta_1'(z, q) \propto e^{\frac{\pi i}{2} \left\lfloor \frac{\arg(q-x)}{2\pi} \right\rfloor} \vartheta_1'(z, x)(1 + O(q-x)) /; x \in \mathbb{R} \land -1 < x < 0$$

Expansions at $q = 0$

09.05.06.0013.01

$$\vartheta_1'(z, q) \propto 2\sqrt[4]{q} \left(\cos(z) - 3\cos(3z) q^2 + 5\cos(5z) q^6 - 7\cos(7z) q^{12} + \ldots\right) /; (q \to 0)$$

09.05.06.0001.01

$$\vartheta_1'(z, q) = 2\sqrt[4]{q} \sum_{k=0}^{\infty} (-1)^k q^{k(k+1)} (2k+1) \cos((2k+1)z) /; |q| < 1$$

09.05.06.0002.01

$$\vartheta_1'(z, q) = \sqrt[4]{q} \sum_{k=-\infty}^{\infty} (-1)^k (2k+1) q^{k(k+1)} e^{(2k+1) i z}$$

09.05.06.0004.01

$$\vartheta_1'(0, q) = 2\sqrt[4]{q} \sum_{k=0}^{\infty} (-1)^k (2k+1) q^{k(k+1)}$$

09.05.06.0014.01

$$\vartheta_1'(z, q) \propto 2\sqrt[4]{q} (\cos(z) + O(q)) /; (q \to 0)$$

Expansions at $q = 1$

09.05.06.0015.01

$$\vartheta_1'(z, q) \propto \frac{2\sqrt{\pi}\, i}{(q-1)^{3/2}} e^{-i\pi \left\lfloor -\frac{\arg(q-1)}{2\pi} \right\rfloor} \left(1 + \frac{3(q-1)}{4} - \frac{1}{32}(q-1)^2 + \frac{3}{128}(q-1)^3 + \ldots\right) e^{\frac{4z^2+\pi^2}{4\log(q)}} \left(\pi \cosh\left(\frac{\pi z}{\log(q)}\right) + 2z \sinh\left(\frac{\pi z}{\log(q)}\right) - e^{\frac{2\pi^2}{\log(q)}} \left(3\pi \cosh\left(\frac{3\pi z}{\log(q)}\right) + 2z \sinh\left(\frac{3\pi z}{\log(q)}\right)\right) + \ldots\right) /; (q \to 1) \bigwedge |q| < 1$$

09.05.06.0016.01

$$\vartheta_1'(z, q) = \frac{6\sqrt{\pi}\, i}{(q-1)^{3/2}} e^{-i\pi \left\lfloor -\frac{\arg(q-1)}{2\pi} \right\rfloor} \sum_{k=0}^{\infty} \binom{k+\frac{3}{2}}{k} \sum_{j=0}^{k} \frac{(-1)^j}{2j+3} \binom{k}{j} p_{j,k} (q-1)^k e^{\frac{4z^2}{4\log(q)}} \sum_{m=0}^{\infty} (-1)^m e^{\frac{m(m+1)\pi^2}{\log(q)}} \left((2m+1)\pi \cosh\left(\frac{(2m+1)\pi z}{\log(q)}\right) + 2z \sinh\left(\frac{(2m+1)\pi z}{\log(q)}\right)\right) /;$$

$$(|q| < 1 \land |q-1| < 1) \bigwedge c_k = \frac{(-1)^{k-1}}{k+1} \bigwedge p_{j,0} = 1 \bigwedge p_{j,k} = -\frac{1}{k} \sum_{m=1}^{k} (jm - k + m) c_m p_{j,k-m} \bigwedge k \in \mathbb{N}^+$$

09.05.06.0017.01

$$\vartheta_1'(z, q) \propto \frac{2\sqrt{\pi}\, i}{(q-1)^{3/2}}\, e^{-i\pi\left\lfloor -\frac{\arg(q-1)}{2\pi}\right\rfloor} (1 + O(q-1))\, e^{\frac{4z^2+\pi^2}{4\log(q)}} \left(\pi \cosh\left(\frac{\pi z}{\log(q)}\right) + 2z \sinh\left(\frac{\pi z}{\log(q)}\right) + O\left(e^{\frac{2\pi^2}{\log(q)}}\left(3\pi\cosh\left(\frac{3\pi z}{\log(q)}\right) + 2z\sinh\left(\frac{3\pi z}{\log(q)}\right)\right)\right)\right) /; |q| < 1$$

Other q-series representations

09.05.06.0005.01

$$\frac{\vartheta_1'(z, q)}{\vartheta_1(z, q)} == \cot(z) + 4\sum_{k=1}^{\infty} \frac{q^{2k}}{1-q^{2k}} \sin(2kz)$$

Other series representations

09.05.06.0018.01

$$\vartheta_1'(z, q) = \frac{2\sqrt{\pi}}{(-\log(q))^{3/2}}\, e^{\frac{4z^2+\pi^2}{4\log(q)}} \left(\pi \sum_{k=0}^{\infty} (-1)^k e^{\frac{k(k+1)\pi^2}{\log(q)}} (2k+1)\cosh\left(\frac{(2k+1)\pi z}{\log(q)}\right) + 2z\sum_{k=0}^{\infty} (-1)^k e^{\frac{k(k+1)\pi^2}{\log(q)}} \sinh\left(\frac{(2k+1)\pi z}{\log(q)}\right)\right)$$

09.05.06.0006.01

$$\vartheta_1'(z, q) == -\frac{2\, i^{3/2}}{\tau^{3/2}} \sum_{n=-\infty}^{\infty} (-1)^n \left(\frac{z}{\pi} + n - \frac{1}{2}\right) \exp\left(-\frac{i\pi}{\tau}\left(\frac{z}{\pi} + n - \frac{1}{2}\right)^2\right) /; q == e^{i\pi\tau}$$

Product representations

09.05.08.0001.01

$$\vartheta_1'(0, q) == 2\sqrt[4]{q} \left(\prod_{n=1}^{\infty}\left(1 - q^{2n}\right)\right)^3$$

Differential equations

Partial differential equations

The elliptic theta functions satisfy the one-dimensional heat equation:

09.05.13.0001.01

$$\frac{\partial \vartheta_1'(z, q)}{\partial \tau} == -\frac{\pi i}{4} \frac{\partial^2 \vartheta_1'(z, q)}{\partial z^2} /; q == e^{i\pi\tau}$$

09.05.13.0002.01

$$4q\frac{\partial \vartheta_1'(z, q)}{\partial q} + \frac{\partial^2 \vartheta_1'(z, q)}{\partial z^2} == 0$$

Transformations

Transformations and argument simplifications

Argument involving basic arithmetic operations

09.05.16.0001.01

$$\vartheta_1'(z,q)=\frac{e^{\frac{4z^2+\pi^2}{4\log(q)}}\sqrt{\pi}}{\sqrt[4]{e^{\frac{\pi^2}{\log(q)}}}\,(-\log(q))^{3/2}}\left(\pi\,\vartheta_1'\left(\frac{i\pi z}{\log(q)},\,e^{\frac{\pi^2}{\log(q)}}\right)-2\,i\,z\,\vartheta_1\left(\frac{i\pi z}{\log(q)},\,e^{\frac{\pi^2}{\log(q)}}\right)\right)$$

Identities

Functional identities

09.05.17.0001.01

$$\vartheta_1'\left(z+\pi\tau,\,e^{i\pi\tau}\right)=2\,e^{-i(2z+\pi\tau)}\,i\,\vartheta_1\left(z,\,e^{i\pi\tau}\right)-e^{-i(2z+\pi\tau)}\,\vartheta_1'\left(z,\,e^{i\pi\tau}\right)\,/;\,\mathrm{Im}(\tau)>0$$

Differentiation

Low-order differentiation

With respect to *z*

09.05.20.0009.01

$$\frac{\partial\vartheta_1'(z,q)}{\partial z}=\frac{\vartheta_1'(z,q)^2}{\vartheta_1(z,q)}-\vartheta_3(0,q)^2\,\vartheta_4(0,q)^2\,\frac{\vartheta_2(z,q)^2}{\vartheta_1(z,q)}-\frac{4}{\pi^2}\,\vartheta_1(z,q)\left(\frac{\pi^2}{12}\left(\vartheta_3(0,q)^4+\vartheta_4(0,q)^4\right)+\zeta\left(1;\,g_2\left(1,\,\frac{\log(q)}{\pi i}\right),\,g_3\left(1,\,\frac{\log(q)}{\pi i}\right)\right)\right)$$

09.05.20.0001.01

$$\frac{\partial\vartheta_1'(z,q)}{\partial z}=-2\sqrt[4]{q}\sum_{k=0}^{\infty}(-1)^k\,q^{k(k+1)}\,(2k+1)^2\sin((2k+1)z)\,/;\,|q|<1$$

09.05.20.0002.01

$$\frac{\partial^2\vartheta_1'(z,q)}{\partial z^2}=2\sqrt[4]{q}\sum_{k=0}^{\infty}(-1)^{k-1}\,q^{k(k+1)}\,(2k+1)^3\cos((2k+1)z)\,/;\,|q|<1$$

With respect to *q*

09.05.20.0010.01

$$\frac{\partial\vartheta_1'(z,q)}{\partial q}=\frac{\vartheta_1'(z,q)^3}{4\,q\,\vartheta_1(z,q)^2}+\frac{3}{\pi^2 q}\,\vartheta_1'(z,q)\left(\frac{\pi^2}{4}\,\vartheta_3(0,q)^2\,\vartheta_4(0,q)^2\,\frac{\vartheta_2(z,q)^2}{\vartheta_1(z,q)^2}+\frac{\pi^2}{12}\left(\vartheta_3(0,q)^4+\vartheta_4(0,q)^4\right)+\zeta\left(1;\,g_2\left(1,\,\frac{\log(q)}{\pi i}\right),\,g_3\left(1,\,\frac{\log(q)}{\pi i}\right)\right)\right)-\frac{1}{2q}\,\vartheta_2(0,q)^2\,\vartheta_3(0,q)^2\,\vartheta_4(0,q)^2\,\frac{\vartheta_2(z,q)\,\vartheta_3(z,q)\,\vartheta_4(z,q)}{\vartheta_1(z,q)^2}$$

09.05.20.0003.01

$$\frac{\partial\vartheta_1'(z,q)}{\partial q}=2\,q^{-\frac{3}{4}}\sum_{k=1}^{\infty}(-1)^k\,q^{k(k+1)}\,k\,(k+1)\,(2k+1)\cos((2k+1)z)+\frac{\vartheta_1'(z,q)}{4q}\,/;\,|q|<1$$

09.05.20.0004.01

$$\frac{\partial^2 \vartheta_1'(z,q)}{\partial q^2} = 2\, q^{-\frac{7}{4}} \sum_{k=0}^{\infty} (-1)^k q^{k(k+1)} (2k+1) \left(k^2 + k - \frac{3}{4}\right) \left(k^2 + k + \frac{1}{4}\right) \cos((2k+1)z) \;/;\; |q| < 1$$

Symbolic differentiation

With respect to *z*

09.05.20.0005.01

$$\frac{\partial^n \vartheta_1'(z,q)}{\partial z^n} = 2 \sqrt[4]{q} \sum_{k=0}^{\infty} (-1)^k q^{k(k+1)} (2k+1)^{n+1} \cos\left(\frac{\pi n}{2} + (2k+1)z\right) \;/;\; |q| < 1 \land n \in \mathbb{N}^+$$

With respect to *q*

09.05.20.0006.01

$$\frac{\partial^n \vartheta_1(z,q)}{\partial q^n} = 2\, q^{\frac{1}{4}-n} \sum_{k=0}^{\infty} (-1)^k q^{k(k+1)} (2k+1) \left(k(k+1) - n + \frac{5}{4}\right)_n \cos((2k+1)z) \;/;\; |q| < 1 \land n \in \mathbb{N}^+$$

Fractional integro-differentiation

With respect to *z*

09.05.20.0007.01

$$\frac{\partial^\alpha \vartheta_1'(z,q)}{\partial z^\alpha} = 2^{\alpha+1} \sqrt{\pi} \sqrt[4]{q}\, z^{-\alpha} \sum_{k=0}^{\infty} (-1)^k q^{k(k+1)} (2k+1)\, {}_1\tilde{F}_2\left(1; \frac{1-\alpha}{2}, 1 - \frac{\alpha}{2}; -\frac{1}{4}(2k+1)^2 z^2\right) \;/;\; |q| < 1$$

With respect to *q*

09.05.20.0008.01

$$\frac{\partial^\alpha \vartheta_1'(z,q)}{\partial q^\alpha} = 2\, q^{\frac{1}{4}-\alpha} \sum_{k=0}^{\infty} \frac{(-1)^k q^{k(k+1)}\, \Gamma\left(k^2 + k + \frac{5}{4}\right) (2k+1) \cos((2k+1)z)}{\Gamma\left(k^2 + k - \alpha + \frac{5}{4}\right)} \;/;\; |q| < 1$$

Integration

Indefinite integration

Involving only one direct function

09.05.21.0001.01

$$\int \vartheta_1'(z,q)\, dz = \vartheta_1(z,q)$$

Involving only one direct function with respect to *q*

09.05.21.0002.01

$$\int \vartheta_1'(z,q)\, dq = 2 \sum_{k=0}^{\infty} \frac{(-1)^k q^{(k+1)k + \frac{5}{4}} (2k+1) \cos((2k+1)z)}{(k+1)k + \frac{5}{4}} \;/;\; |q| < 1$$

Representations through equivalent functions

With related functions

Involving theta functions

Involving $\vartheta_1(z, q)$

09.05.27.0001.02

$$\vartheta_1'\left(z, e^{i\pi\tau}\right) = 2\, e^{-i(2z-\pi\tau)}\, i\, \vartheta_1\left(z-\pi\tau, e^{i\pi\tau}\right) - e^{-i(2z-\pi\tau)}\, \vartheta_1'\left(z-\pi\tau, e^{i\pi\tau}\right) /; \operatorname{Im}(\tau) > 0$$

09.05.27.0004.01

$$\vartheta_1'(z, q) = -e^{2iz}\, q\, (2\, i\, \vartheta_1(z+\pi\tau, q) + \vartheta_1'(z+\pi\tau, q)) /; q = e^{i\pi\tau}$$

09.05.27.0005.01

$$\vartheta_1'(z, q) = (-1)^n\, e^{2inz}\, q^{n^2}\, (2\, i\, n\, \vartheta_1(z+n\pi\tau, q) + \vartheta_1'(z+n\pi\tau, q)) /; n \in \mathbb{Z} \bigwedge q = e^{i\pi\tau}$$

09.05.27.0006.01

$$\vartheta_1'(z, q) = (-1)^{m+n}\, e^{2inz}\, q^{n^2}\, (2\, i\, n\, \vartheta_1(z+\pi m+n\pi\tau, q) + \vartheta_1'(z+\pi m+n\pi\tau, q)) /; \{m, n\} \in \mathbb{Z} \bigwedge q = e^{i\pi\tau}$$

09.05.27.0007.01

$$\vartheta_1'(z, q) = -e^{2iz}\, q\, (\vartheta_1'(z-i\log(q), q) + 2\, i\, \vartheta_1(z-i\log(q), q))$$

09.05.27.0008.01

$$\vartheta_1'(z, q) = e^{-2-iz}\, q\, (2\, i\, \vartheta_1(z+i\log(q), q) - \vartheta_1'(z+i\log(q), q))$$

09.05.27.0009.01

$$\vartheta_1'(z, q) = (-1)^n\, e^{-2inz}\, q^{n^2}\, (\vartheta_1'(z+i\, n\log(q), q) - 2\, i\, n\, \vartheta_1(z+i\, n\log(q), q)) /; n \in \mathbb{Z}$$

09.05.27.0010.01

$$\vartheta_1'(z, q) = (-1)^{m+n}\, e^{-2inz}\, q^{n^2}\, (\vartheta_1'(z+\pi m+i\, n\log(q), q) - 2\, i\, n\, \vartheta_1(z+\pi m+i\, n\log(q), q)) /; \{m, n\} \in \mathbb{Z}$$

Involving $\vartheta_2'(z, q)$

09.05.27.0011.01

$$\vartheta_1'(z, q) = -\vartheta_2'\left(z+\frac{\pi}{2}, q\right)$$

09.05.27.0002.02

$$\vartheta_1'(z, q) = \vartheta_2'\left(z-\frac{\pi}{2}, q\right)$$

09.05.27.0003.02

$$\vartheta_1'(z, q) = (-1)^{m-1}\, \vartheta_2'\left(\frac{1}{2}\pi(2m+1)+z, q\right) /; m \in \mathbb{Z}$$

Involving $\vartheta_3(z, q)$

09.05.27.0012.01

$$\vartheta_1'(z, q) = e^{iz}\sqrt[4]{q}\, \vartheta_3\left(z+\frac{1}{2}\pi(\tau+1), q\right) - i\, e^{iz}\sqrt[4]{q}\, \vartheta_3'\left(z+\frac{1}{2}\pi(\tau+1), q\right) /; q = e^{i\pi\tau}$$

09.05.27.0013.01

$$\vartheta_1'(z, q) = (-1)^m\, e^{i(2m+1)z}\, q^{\left(m+\frac{1}{2}\right)^2}\left((2m+1)\, \vartheta_3\left(z+\frac{\pi}{2}(1+(2m+1)\tau), q\right) - i\, \vartheta_3'\left(z+\frac{\pi}{2}(1+(2m+1)\tau), q\right)\right) /; m \in \mathbb{Z} \bigwedge q = e^{i\pi\tau}$$

09.05.27.0014.01

$$\vartheta_1'(z,q) = e^{iz}\sqrt[4]{q}\left(\vartheta_3\left(z+\frac{1}{2}(\pi - i\log(q)),\, q\right) - i\,\vartheta_3'\left(z+\frac{1}{2}(\pi - i\log(q)),\, q\right)\right)$$

09.05.27.0015.01

$$\vartheta_1'(z,q) = (-1)^m e^{i(2m+1)z} q^{\left(m+\frac{1}{2}\right)^2}\left((2m+1)\,\vartheta_3\left(z+\frac{1}{2}(\pi - i(2m+1)\log(q)),\, q\right) - i\,\vartheta_3'\left(z+\frac{1}{2}(\pi - i(2m+1)\log(q)),\, q\right)\right) /;\, m \in \mathbb{Z}$$

Involving $\vartheta_4(z, q)$

09.05.27.0016.01

$$\vartheta_1'(z,q) = e^{-iz}\sqrt[4]{q}\left(\vartheta_4\left(z+\frac{\pi\tau}{2},\, q\right) + i\,\vartheta_4'\left(z+\frac{\pi\tau}{2},\, q\right)\right) /;\, q = e^{-i\pi\tau}$$

09.05.27.0017.01

$$\vartheta_1'(z,q) = (-1)^m e^{-i(2m+1)z} q^{\left(m+\frac{1}{2}\right)^2}\left((2m+1)\,\vartheta_4\left(z+m\pi\tau+\frac{\pi\tau}{2},\, q\right) + i\,\vartheta_4'\left(z+m\pi\tau+\frac{\pi\tau}{2},\, q\right)\right) /;\, m \in \mathbb{Z} \bigwedge q = e^{-i\pi\tau}$$

09.05.27.0018.01

$$\vartheta_1'(z,q) = e^{-iz}\sqrt[4]{q}\left(\vartheta_4\left(z+\frac{1}{2}i\log(q),\, q\right) + i\,\vartheta_4'\left(z+\frac{1}{2}i\log(q),\, q\right)\right)$$

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$$\vartheta_1'(z,q) = (-1)^m e^{-i(2m+1)z} q^{m^2+m+\frac{1}{4}}\left((2m+1)\,\vartheta_4\left(z+\frac{1}{2}i(2m+1)\log(q),\, q\right) + i\,\vartheta_4'\left(z+\frac{1}{2}i(2m+1)\log(q),\, q\right)\right) /;\, m \in \mathbb{Z}$$

Zeros

09.05.30.0002.01

$$\vartheta_1'(z, 0) = 0$$

09.05.30.0003.01

$$\vartheta_1'\left(\frac{\pi}{2},\, q\right) = 0$$

09.05.30.0001.02

$$\vartheta_1'\left(\frac{\pi}{2}+m\pi,\, q\right) = 0 /;\, m \in \mathbb{Z}$$

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