Erf

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Notations

Traditional name

Error function

Traditional notation

$\operatorname{erf}(z)$

Mathematica StandardForm notation

`Erf[z]`

Primary definition

06.25.02.0001.01

$$\operatorname{erf}(z) == \frac{2}{\sqrt{\pi}} \sum_{k=0}^{\infty} \frac{(-1)^k z^{2k+1}}{k! \, (2k+1)}$$

Specific values

Values at fixed points

06.25.03.0001.01

$$\operatorname{erf}(0) == 0$$

Values at infinities

06.25.03.0002.01

$$\operatorname{erf}(\infty) == 1$$

06.25.03.0003.01

$$\operatorname{erf}(-\infty) == -1$$

06.25.03.0004.01

$$\operatorname{erf}(i \infty) == i \infty$$

06.25.03.0005.01

$$\operatorname{erf}(-i \infty) == -i \infty$$

06.25.03.0006.01

$$\operatorname{erf}(\tilde{\infty}) == ¿$$

General characteristics

Domain and analyticity

erf(z) is an entire analytical function of z which is defined in the whole complex z-plane.

06.25.04.0001.01

$$z \longrightarrow \operatorname{erf}(z) :: \mathbb{C} \longrightarrow \mathbb{C}$$

Symmetries and periodicities

Parity

erf(z) is an odd function.

06.25.04.0002.01

$$\operatorname{erf}(-z) = -\operatorname{erf}(z)$$

Mirror symmetry

06.25.04.0003.01

$$\operatorname{erf}(\bar{z}) = \overline{\operatorname{erf}(z)}$$

Periodicity

No periodicity

Poles and essential singularities

The function erf(z) has only one singular point at $z = \tilde{\infty}$. It is an essential singular point.

06.25.04.0004.01

$$\mathcal{S}ing_z(\operatorname{erf}(z)) = \{\{\tilde{\infty}, \infty\}\}$$

Branch points

The function erf(z) does not have branch points.

06.25.04.0005.01

$$\mathcal{BP}_z(\operatorname{erf}(z)) = \{\}$$

Branch cuts

The function erf(z) does not have branch cuts.

06.25.04.0006.01

$$\mathcal{BC}_z(\operatorname{erf}(z)) = \{\}$$

Series representations

Generalized power series

Expansions at generic point $z = z_0$

For the function itself

06.25.06.0010.01

$$\operatorname{erf}(z) \propto \operatorname{erf}(z_0) + \frac{2\, e^{-z_0^2}}{\sqrt{\pi}} (z - z_0) - \frac{2\, e^{-z_0^2}\, z_0}{\sqrt{\pi}} (z - z_0)^2 + \ldots \;/;\; (z \to z_0)$$

06.25.06.0011.01

$$\operatorname{erf}(z) \propto \operatorname{erf}(z_0) + \frac{2\, e^{-z_0^2}}{\sqrt{\pi}} (z - z_0) - \frac{2\, e^{-z_0^2}\, z_0}{\sqrt{\pi}} (z - z_0)^2 + O\left((z - z_0)^3\right)$$

06.25.06.0012.01

$$\operatorname{erf}(z) = \operatorname{erf}(z_0) + \frac{2\, e^{-z_0^2}}{\sqrt{\pi}} \sum_{k=1}^{\infty} \sum_{j=0}^{k-1} \frac{(-1)^j\, (2\, j - k + 2)_{2(k-j-1)}}{k!\, (k - j - 1)!\, (2\, z_0)^{k-2\, j-1}} (z - z_0)^k$$

06.25.06.0013.01

$$\operatorname{erf}(z) = \sum_{k=0}^{\infty} \frac{2^k\, z_0^{1-k}}{k!}\, {}_2\tilde{F}_2\left(\frac{1}{2}, 1; 1 - \frac{k}{2}, \frac{3-k}{2}; -z_0^2\right) (z - z_0)^k$$

06.25.06.0014.01

$$\operatorname{erf}(z) \propto \operatorname{erf}(z_0)\, (1 + O(z - z_0))$$

Expansions at $z = 0$

For the function itself

06.25.06.0001.02

$$\operatorname{erf}(z) \propto \frac{2}{\sqrt{\pi}} \left(z - \frac{z^3}{3} + \frac{z^5}{10} - \ldots\right) /;\; (z \to 0)$$

06.25.06.0015.01

$$\operatorname{erf}(z) \propto \frac{2}{\sqrt{\pi}} \left(z - \frac{z^3}{3} + \frac{z^5}{10} - O\left(z^7\right)\right)$$

06.25.06.0002.01

$$\operatorname{erf}(z) = \frac{2}{\sqrt{\pi}} \sum_{k=0}^{\infty} \frac{(-1)^k\, z^{2\, k+1}}{k!\, (2\, k + 1)}$$

06.25.06.0003.01

$$\operatorname{erf}(z) = \frac{2\, z}{\sqrt{\pi}}\, {}_1F_1\left(\frac{1}{2}; \frac{3}{2}; -z^2\right)$$

06.25.06.0004.02

$$\operatorname{erf}(z) \propto \frac{2\, z}{\sqrt{\pi}} \left(1 + O\left(z^2\right)\right)$$

06.25.06.0016.01

$$\operatorname{erf}(z) = F_\infty(z) \;/;\; \left(\left(F_n(z) = \frac{2\, z}{\sqrt{\pi}} \sum_{k=0}^{n} \frac{(-1)^k\, z^{2\, k}}{(2\, k + 1)\, k!} = \operatorname{erf}(z) + \frac{(-1)^n\, 2\, z^{2\, n+3}}{\sqrt{\pi}\, (2\, n + 3)\, (n + 1)!}\, {}_2F_2\left(1, n + \frac{3}{2}; n + 2, n + \frac{5}{2}; -z^2\right)\right) \bigwedge n \in \mathbb{N}\right)$$

Summed form of the truncated series expansion.

Asymptotic series expansions

06.25.06.0005.01

$$\operatorname{erf}(z) \propto \frac{z}{\sqrt{z^2}} - \frac{1}{\sqrt{\pi}\, z} e^{-z^2} {}_2F_0\left(1, \frac{1}{2};; -\frac{1}{z^2}\right) /; (|z| \to \infty)$$

06.25.06.0006.02

$$\operatorname{erf}(z) \propto \frac{\sqrt{z^2}}{z} - \frac{1}{\sqrt{\pi}\, z} e^{-z^2} \left(1 + O\left(\frac{1}{z^2}\right)\right) /; (|z| \to \infty)$$

06.25.06.0017.01

$$\operatorname{erf}(z) \propto \begin{cases} 1 - \frac{e^{-z^2}}{\sqrt{\pi}\, z} & -\frac{\pi}{2} < \arg(z) \le \frac{\pi}{2} \\ -1 - \frac{e^{-z^2}}{\sqrt{\pi}\, z} & \text{True} \end{cases} /; (|z| \to \infty)$$

Residue representations

06.25.06.0007.01

$$\operatorname{erf}(z) = \frac{z}{\sqrt{\pi}} \sum_{j=0}^{\infty} \operatorname{res}_s \left(\frac{\Gamma\left(\frac{1}{2} - s\right) \left(z^2\right)^{-s}}{\Gamma\left(\frac{3}{2} - s\right)} \Gamma(s) \right)(-j)$$

06.25.06.0008.02

$$\operatorname{erf}(z) = \frac{1}{\sqrt{\pi}} \sum_{j=0}^{\infty} \operatorname{res}_s \left(\frac{\Gamma(-s)\, z^{-2s}}{\Gamma(1-s)} \Gamma\left(s + \frac{1}{2}\right)\right)\left(-\frac{1}{2} - j\right)$$

Other series representations

06.25.06.0009.01

$$\operatorname{erf}(z) = \frac{1}{\sqrt{\pi}} \sum_{k=0}^{\infty} \frac{(-1)^k H_{2k+1}(z)}{2^{3k+\frac{1}{2}}\, k!\, (2k+1)}$$

Limit representations

06.25.09.0001.01

$$\operatorname{erf}(z) = 1 - 2 \frac{B_{1/2 - z/\left(\sqrt{2}\,\sqrt{n}\right)}\left(\frac{n}{2}, \frac{n}{2}\right)}{B\left(\frac{n}{2}, \frac{n}{2}\right)}$$

Integral representations

On the real axis

Of the direct function

06.25.07.0001.01

$$\operatorname{erf}(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{-t^2} \, dt$$

06.25.07.0002.01

$$\operatorname{erf}(x) = \frac{2}{\pi} \int_0^\infty \frac{e^{-t^2} \sin(2\,x\,t)}{t} \, dt \,/;\, x \in \mathbb{R}$$

Contour integral representations

06.25.07.0003.01

$$\operatorname{erf}(z) = \frac{z}{\sqrt{\pi}\; 2\,\pi\, i} \int_{\mathcal{L}} \frac{\Gamma(s)\,\Gamma\left(\frac{1}{2}-s\right)}{\Gamma\left(\frac{3}{2}-s\right)} \left(z^2\right)^{-s} ds$$

06.25.07.0004.01

$$\operatorname{erf}(z) = \frac{1}{\sqrt{\pi}\; 2\,\pi\, i} \int_{\gamma-i\infty}^{\gamma+i\infty} \frac{\Gamma\left(s+\frac{1}{2}\right)\Gamma(-s)}{\Gamma(1-s)} z^{-2s} \, ds \,/;\, -\frac{1}{2} < \gamma \bigwedge |\arg(z)| < \frac{\pi}{2}$$

Continued fraction representations

Involving the function

06.25.10.0001.01

$$\operatorname{erf}(z) = 1 - \frac{e^{-z^2}}{\sqrt{\pi}} \cfrac{1}{z + \cfrac{1/2}{z + \cfrac{1}{z + \cfrac{3/2}{z + \cfrac{2}{z + \cfrac{5/2}{z + \cfrac{3}{z + \ldots}}}}}}} \,/;\, \operatorname{Re}(z) > 0$$

06.25.10.0002.01

$$\operatorname{erf}(z) = 1 - \frac{e^{-z^2}}{\sqrt{\pi}\left(z + K_k\left(\frac{k}{2}, z\right)_1^\infty\right)} \,/;\, \operatorname{Re}(z) > 0$$

06.25.10.0003.01

$$\operatorname{erf}(z) = \frac{2\,z}{\sqrt{\pi}} e^{-z^2} \cfrac{1}{1 - \cfrac{2\,z^2}{3 + \cfrac{4\,z^2}{5 - \cfrac{6\,z^2}{7 + \cfrac{8\,z^2}{9 - \cfrac{10\,z^2}{11 + \cfrac{12\,z^2}{13 - \ldots}}}}}}}$$

06.25.10.0004.01

$$\operatorname{erf}(z) == \frac{2\, z\, e^{-z^2}}{\sqrt{\pi}\, \left(1 + K_k\left((-1)^k\, 2\, k\, z^2,\, 2\, k + 1\right)_1^{\infty}\right)}$$

06.25.10.0005.01

$$\operatorname{erf}(z) == \frac{2\, z}{\sqrt{\pi}}\, e^{-z^2}\, \cfrac{1}{1 - 2\, z^2 + \cfrac{4\, z^2}{3 - 2\, z^2 + \cfrac{8\, z^2}{5 - 2\, z^2 + \cfrac{12\, z^2}{7 - 2\, z^2 + \cfrac{16\, z^2}{9 - 2\, z^2 + \cfrac{20\, z^2}{11 - 2\, z^2 + \cfrac{24\, z^2}{13 - 2\, z^2 + \ldots}}}}}}}$$

06.25.10.0006.01

$$\operatorname{erf}(z) == \frac{2\, z\, e^{-z^2}}{\sqrt{\pi}\, \left(1 - 2\, z^2 + K_k\left(4\, k\, z^2,\, -2\, z^2 + 2\, k + 1\right)_1^{\infty}\right)}$$

06.25.10.0007.01

$$\operatorname{erf}(z) == 1 - \frac{2}{\sqrt{\pi}}\, e^{-z^2}\, \cfrac{1}{2\, z + \cfrac{2}{2\, z + \cfrac{4}{2\, z + \cfrac{6}{2\, z + \cfrac{8}{2\, z + \cfrac{10}{2\, z + \cfrac{12}{2\, z + \ldots}}}}}}} \;/;\; \operatorname{Re}(z) > 0$$

06.25.10.0008.01

$$\operatorname{erf}(z) == 1 - \frac{2\, e^{-z^2}}{\sqrt{\pi}\, \left(2\, z + K_k(2\, k,\, 2\, z)_1^{\infty}\right)} \;/;\; \operatorname{Re}(z) > 0$$

06.25.10.0009.01

$$\operatorname{erf}(z) = 1 - \frac{2z}{\sqrt{\pi}} e^{-z^2} 1 \Bigg/ \left(1 + 2z^2 - \cfrac{2}{5 + 2z^2 - \cfrac{12}{9 + 2z^2 - \cfrac{30}{13 + 2z^2 - \cfrac{56}{17 + 2z^2 - \cfrac{90}{21 + 2z^2 - \cfrac{132}{2 \times 5 + 2z^2 - \ldots}}}}}} \right) /; \operatorname{Re}(z) > 0$$

06.25.10.0010.01

$$\operatorname{erf}(z) = 1 - \frac{2 z e^{-z^2}}{\sqrt{\pi} \left(1 + 2z^2 + \mathrm{K}_k\left(-2k(2k-1),\, 2z^2 + 4k + 1\right)_1^{\infty}\right)} /; \operatorname{Re}(z) > 0$$

Differential equations

Ordinary linear differential equations and wronskians

For the direct function itself

06.25.13.0001.01

$$w''(z) + 2 z w'(z) = 0 /; w(z) = \operatorname{erf}(z) \bigwedge w(0) = 0 \bigwedge w'(0) = \frac{2}{\sqrt{\pi}}$$

06.25.13.0002.01

$$w''(z) + 2 z w'(z) = 0 /; w(z) = c_1 \operatorname{erf}(z) + c_2$$

06.25.13.0003.01

$$W_z(1, \operatorname{erf}(z)) = \frac{2 e^{-z^2}}{\sqrt{\pi}}$$

06.25.13.0004.01

$$w''(z) + \left(2 g(z) g'(z) - \frac{g''(z)}{g'(z)}\right) w'(z) = 0 /; w(z) = c_1 \operatorname{erf}(g(z)) + c_2$$

06.25.13.0005.01

$$W_z(\operatorname{erf}(g(z)), 1) = -\frac{2 e^{-g(z)^2} g'(z)}{\sqrt{\pi}}$$

06.25.13.0006.01

$$w''(z) + \left(2 g(z) g'(z) - \frac{2 h'(z)}{h(z)} - \frac{g''(z)}{g'(z)}\right) w'(z) + \left(\frac{2 h'(z)^2}{h(z)^2} + \frac{g''(z) h'(z)}{h(z) g'(z)} - \frac{2 g(z) g'(z) h'(z)}{h(z)} - \frac{h''(z)}{h(z)}\right) w(z) = 0 /;$$

$$w(z) = c_1 h(z) \operatorname{erf}(g(z)) + c_2 h(z)$$

06.25.13.0007.01

$$W_z(h(z)\,\mathrm{erf}(g(z)),\,h(z)) == -\frac{2\,e^{-g(z)^2}\,h(z)^2\,g'(z)}{\sqrt{\pi}}$$

06.25.13.0008.01

$$z^2\,w''(z)\,+\left(2\,a^2\,r\,z^{2\,r}-r-2\,s+1\right)z\,w'(z)+s\left(-2\,a^2\,r\,z^{2\,r}+r+s\right)w(z)=0\;/;\;w(z)=c_1\,z^s\,\mathrm{erf}(a\,z^r)+c_2\,z^s$$

06.25.13.0009.01

$$W_z(z^s\,\mathrm{erf}(a\,z^r),\,z^s) == -\frac{2\,a\,e^{-a^2\,z^{2\,r}}\,r\,z^{r+2\,s-1}}{\sqrt{\pi}}$$

06.25.13.0010.01

$$w''(z)+\left(\left(2\,a^2\,r^{2\,z}-1\right)\log(r)-2\log(s)\right)w'(z)+\log(s)\left(-2\,a^2\log(r)\,r^{2\,z}+\log(r)+\log(s)\right)w(z)=0\;/;\;w(z)=c_1\,s^z\,\mathrm{erf}(a\,r^z)\,+c_2\,s^z$$

06.25.13.0011.01

$$W_z(s^z\,\mathrm{erf}(a\,r^z),\,s^z) == -\frac{2\,a\,e^{-a^2\,r^{2\,z}}\,r^z\,s^{2\,z}\log(r)}{\sqrt{\pi}}$$

Transformations

Transformations and argument simplifications

Argument involving basic arithmetic operations

06.25.16.0001.01

$$\mathrm{erf}(-z) == -\mathrm{erf}(z)$$

06.25.16.0002.01

$$\mathrm{erf}(i\,z) == i\,\mathrm{erfi}(z)$$

06.25.16.0003.01

$$\mathrm{erf}(-i\,z) == -i\,\mathrm{erfi}(z)$$

06.25.16.0004.01

$$\mathrm{erf}(a\,(b\,z^c)^m) == \frac{(b\,z^c)^m}{b^m\,z^{m\,c}}\,\mathrm{erf}(a\,b^m\,z^{m\,c})\;/;\;2\,m\in\mathbb{Z}$$

06.25.16.0005.01

$$\mathrm{erf}\left(\sqrt{z^2}\right) == \frac{\sqrt{z^2}}{z}\,\mathrm{erf}(z)$$

Complex characteristics

Real part

06.25.19.0001.01

$$\mathrm{Re}(\mathrm{erf}(x+i\,y)) == \frac{2\,x}{\sqrt{\pi}}\sum_{k=0}^{\infty}\frac{y^{2\,k}}{k!}\,{}_1F_1\left(k+\frac{1}{2};\,\frac{3}{2};\,-x^2\right)$$

06.25.19.0002.01

$$\operatorname{Re}(\operatorname{erf}(x + i\,y)) == \operatorname{erf}(x) + \frac{2}{\sqrt{\pi}}\, e^{-x^2} \sum_{k=0}^{\infty} \frac{(-1)^k\, y^{2k+2}}{(2k+2)!}\, H_{2k+1}(x)$$

06.25.19.0003.01

$$\operatorname{Re}(\operatorname{erf}(x + i\,y)) == \frac{1}{2}\left(\operatorname{erf}\left(x + x\sqrt{-\frac{y^2}{x^2}}\right) + \operatorname{erf}\left(x - x\sqrt{-\frac{y^2}{x^2}}\right)\right)$$

Imaginary part

06.25.19.0004.01

$$\operatorname{Im}(\operatorname{erf}(x + i\,y)) == \frac{2}{\sqrt{\pi}} \sum_{k=0}^{\infty} \frac{y^{2k+1}}{(2k+1)\,k!}\, {}_1F_1\left(k + \frac{1}{2}; \frac{1}{2}; -x^2\right)$$

06.25.19.0005.01

$$\operatorname{Im}(\operatorname{erf}(x + i\,y)) == \frac{2}{\sqrt{\pi}}\, e^{-x^2} \sum_{k=0}^{\infty} \frac{(-1)^k\, y^{2k+1}}{(2k+1)!}\, H_{2k}(x)$$

06.25.19.0006.01

$$\operatorname{Im}(\operatorname{erf}(x + i\,y)) == \frac{x}{2y}\sqrt{-\frac{y^2}{x^2}}\left(\operatorname{erf}\left(x - x\sqrt{-\frac{y^2}{x^2}}\right) - \operatorname{erf}\left(x + x\sqrt{-\frac{y^2}{x^2}}\right)\right)$$

Absolute value

06.25.19.0007.01

$$|\operatorname{erf}(x + i\,y)| == \sqrt{\operatorname{erf}\left(x - x\sqrt{-\frac{y^2}{x^2}}\right)\operatorname{erf}\left(x + x\sqrt{-\frac{y^2}{x^2}}\right)}$$

Argument

06.25.19.0008.01

$$\arg(\operatorname{erf}(x + i\,y)) == \tan^{-1}\left(\frac{1}{2}\left(\operatorname{erf}\left(x + x\sqrt{-\frac{y^2}{x^2}}\right) + \operatorname{erf}\left(x - x\sqrt{-\frac{y^2}{x^2}}\right)\right), \frac{x}{2y}\sqrt{-\frac{y^2}{x^2}}\left(\operatorname{erf}\left(x - x\sqrt{-\frac{y^2}{x^2}}\right) - \operatorname{erf}\left(x + x\sqrt{-\frac{y^2}{x^2}}\right)\right)\right)$$

Conjugate value

06.25.19.0009.01

$$\overline{\operatorname{erf}(x + i\,y)} == \frac{1}{2}\left(\operatorname{erf}\left(x + x\sqrt{-\frac{y^2}{x^2}}\right) + \operatorname{erf}\left(x - x\sqrt{-\frac{y^2}{x^2}}\right)\right) - \frac{i\,x}{2y}\sqrt{-\frac{y^2}{x^2}}\left(\operatorname{erf}\left(x - x\sqrt{-\frac{y^2}{x^2}}\right) - \operatorname{erf}\left(x + x\sqrt{-\frac{y^2}{x^2}}\right)\right)$$

Signum value

06.25.19.0010.01

$$\operatorname{sgn}(\operatorname{erf}(x+i\,y)) = \left(\frac{i}{y}\sqrt{-\frac{y^2}{x^2}}\,x\left(\operatorname{erf}\left(x-x\sqrt{-\frac{y^2}{x^2}}\right)-\operatorname{erf}\left(\sqrt{-\frac{y^2}{x^2}}\,x+x\right)\right)+\operatorname{erf}\left(\sqrt{-\frac{y^2}{x^2}}\,x+x\right)+\operatorname{erf}\left(x-x\sqrt{-\frac{y^2}{x^2}}\right)\right)\Big/ \left(2\sqrt{\operatorname{erf}\left(x-x\sqrt{-\frac{y^2}{x^2}}\right)\operatorname{erf}\left(\sqrt{-\frac{y^2}{x^2}}\,x+x\right)}\right)$$

Differentiation

Low-order differentiation

06.25.20.0001.01

$$\frac{\partial \operatorname{erf}(z)}{\partial z} = \frac{2\,e^{-z^2}}{\sqrt{\pi}}$$

06.25.20.0002.01

$$\frac{\partial^2 \operatorname{erf}(z)}{\partial z^2} = -\frac{4\,e^{-z^2}\,z}{\sqrt{\pi}}$$

Symbolic differentiation

06.25.20.0006.01

$$\frac{\partial^n \operatorname{erf}(z)}{\partial z^n} = \operatorname{erf}(z)\,\delta_n + \frac{2}{\sqrt{\pi}}\,e^{-z^2}\sum_{k=0}^{n-1}\frac{(-1)^k\,(2\,k-n+2)_{2\,(n-k-1)}}{(n-k-1)!\,(2\,z)^{n-2\,k-1}} \;/;\; n\in\mathbb{N}$$

06.25.20.0003.01

$$\frac{\partial^n \operatorname{erf}(z)}{\partial z^n} = \operatorname{erf}(z)\,\delta_n + \operatorname{boole}\left(n\neq 0,\ \frac{2^{-n}\,(n-1)!}{\sqrt{\pi}}\,e^{-z^2}\sum_{k=1}^{n}\frac{(-1)^{k-1}\,2^{2\,k}\,z^{2\,k-n-1}}{(2\,k-n-1)!\,(n-k)!}\right) /;\; n\in\mathbb{N}$$

06.25.20.0004.02

$$\frac{\partial^n \operatorname{erf}(z)}{\partial z^n} = 2^n\,z^{1-n}\,{}_2\tilde{F}_2\left(\frac{1}{2},\,1;\,1-\frac{n}{2},\,\frac{3-n}{2};\,-z^2\right) /;\; n\in\mathbb{N}$$

Fractional integro-differentiation

06.25.20.0005.01

$$\frac{\partial^\alpha \operatorname{erf}(z)}{\partial z^\alpha} = 2^\alpha\,z^{1-\alpha}\,{}_2\tilde{F}_2\left(\frac{1}{2},\,1;\,1-\frac{\alpha}{2},\,\frac{3-\alpha}{2};\,-z^2\right)$$

Integration

Indefinite integration

Involving only one direct function

06.25.21.0001.01

$$\int \operatorname{erf}(b + a\,z)\,dz = \frac{b\operatorname{erf}(b + a\,z)}{a} + z\operatorname{erf}(b + a\,z) + \frac{e^{-a^2 z^2 - 2\,a\,b\,z - b^2}}{a\sqrt{\pi}}$$

06.25.21.0002.01

$$\int \operatorname{erf}(a\,z)\,dz = z\operatorname{erf}(a\,z) + \frac{e^{-a^2 z^2}}{a\sqrt{\pi}}$$

06.25.21.0003.01

$$\int \operatorname{erf}(z)\,dz = z\operatorname{erf}(z) + \frac{e^{-z^2}}{\sqrt{\pi}}$$

Involving one direct function and elementary functions

Involving power function

Involving power

Linear argument

06.25.21.0004.01

$$\int z^{\alpha-1}\operatorname{erf}(a\,z)\,dz = \frac{z^{\alpha}}{\alpha}\left(\frac{a\,z\,(a^2 z^2)^{\frac{1}{2}(-\alpha-1)}}{\sqrt{\pi}}\,\Gamma\!\left(\frac{\alpha+1}{2}, a^2 z^2\right) + \operatorname{erf}(a\,z)\right)$$

06.25.21.0005.01

$$\int z^{\alpha-1}\operatorname{erf}(z)\,dz = \frac{z^{\alpha}}{\alpha}\left(\frac{z(z^2)^{\frac{1}{2}(-\alpha-1)}}{\sqrt{\pi}}\,\Gamma\!\left(\frac{\alpha+1}{2}, z^2\right) + \operatorname{erf}(z)\right)$$

06.25.21.0006.01

$$\int z\operatorname{erf}(a\,z)\,dz = \frac{1}{4}\left(\frac{2\,e^{-a^2 z^2}\,z}{a\sqrt{\pi}} + \left(2\,z^2 - \frac{1}{a^2}\right)\operatorname{erf}(a\,z)\right)$$

06.25.21.0007.01

$$\int \frac{\operatorname{erf}(a\,z)}{z}\,dz = \frac{2\,a\,z}{\sqrt{\pi}}\,{}_2F_2\!\left(\frac{1}{2}, \frac{1}{2}; \frac{3}{2}, \frac{3}{2}; -a^2 z^2\right)$$

06.25.21.0008.01

$$\int \frac{\operatorname{erf}(a\,z)}{z^2}\,dz = \frac{a\operatorname{Ei}\left(-a^2 z^2\right)}{\sqrt{\pi}} - \frac{\operatorname{erf}(a\,z)}{z}$$

Power arguments

06.25.21.0009.01

$$\int z^{\alpha-1}\operatorname{erf}(a\,z^r)\,dz = \frac{z^{\alpha}}{\alpha}\left(\frac{a\left(a^2 z^{2r}\right)^{-\frac{r+\alpha}{2r}} z^r}{\sqrt{\pi}}\,\Gamma\!\left(\frac{r+\alpha}{2\,r}, a^2 z^{2r}\right) + \operatorname{erf}(a\,z^r)\right)$$

Involving rational functions

06.25.21.0010.01

$$\int \frac{(z^2-b)\,\mathrm{erf}(a\,z)}{(z^2+b)^2}\,dz = \frac{a\,e^{a^2 b}\,\mathrm{Ei}\left(-a^2\left(z^2+b\right)\right)}{\sqrt{\pi}} - \frac{z\,\mathrm{erf}(a\,z)}{z^2+b}$$

Involving exponential function

Involving exp

06.25.21.0011.01

$$\int e^{b\,z}\,\mathrm{erf}(a\,z)\,dz = \frac{1}{b}\left(e^{b\,z}\,\mathrm{erf}(a\,z) + \exp\left(\frac{b^2}{4\,a^2}\right)\mathrm{erf}\left(\frac{b}{2\,a} - a\,z\right)\right)$$

06.25.21.0012.01

$$\int e^{b\,z^2}\,\mathrm{erf}(a\,z)\,dz = \frac{1}{\sqrt{\pi}\,b}\sum_{k=0}^{\infty}\frac{b^{-k}\,a^{2k+1}\,\Gamma\left(k+1,\,-b\,z^2\right)}{(2\,k+1)\,k!}$$

06.25.21.0013.01

$$\int e^{b\,z^2}\,\mathrm{erf}(a\,z)\,dz = \frac{\sqrt{\pi}\,\mathrm{erf}(a\,z)\,\mathrm{erfi}\left(\sqrt{b}\,z\right)}{2\sqrt{b}} + \frac{1}{a\sqrt{\pi}}\sum_{k=0}^{\infty}\frac{b^k\,a^{-2k}\,\Gamma\left(k+1,\,a^2\,z^2\right)}{(2\,k+1)\,k!}$$

06.25.21.0014.01

$$\int e^{-a^2 z^2}\,\mathrm{erf}(a\,z)\,dz = \frac{\sqrt{\pi}\,\mathrm{erf}(a\,z)^2}{4\,a}$$

Involving exponential function and a power function

Involving exp and power

06.25.21.0015.01

$$\int z^{\alpha-1}\,e^{b\,z}\,\mathrm{erf}(a\,z)\,dz = \frac{2\,a\,z^{\alpha}\,(-b\,z)^{-\alpha}}{b\sqrt{\pi}}\sum_{k=0}^{\infty}\frac{(-1)^k\,a^{2k}\,b^{-2k}\,\Gamma(2\,k+\alpha+1,\,-b\,z)}{(2\,k+1)\,k!}$$

06.25.21.0016.01

$$\int z^n\,e^{b\,z}\,\mathrm{erf}(a\,z)\,dz = -\frac{a\,n!\,(-b)^{-n-1}}{\sqrt{\pi}}\exp\left(\frac{b^2}{4\,a^2}\right)$$

$$\sum_{m=0}^{n}\frac{(-b)^m\left(-a^2\right)^{\frac{1}{2}(-m-1)}}{m!}\sum_{k=0}^{m}\binom{m}{k}\left(-\frac{b}{2\sqrt{-a^2}}\right)^{m-k}\left(\frac{b}{2\sqrt{-a^2}}+\sqrt{-a^2}\,z\right)^{k+1}\left(-\left(\frac{b}{2\sqrt{-a^2}}+\sqrt{-a^2}\,z\right)^2\right)^{\frac{1}{2}(-k-1)}$$

$$\Gamma\left(\frac{k+1}{2},\,-\left(\frac{b}{2\sqrt{-a^2}}+\sqrt{-a^2}\,z\right)^2\right) - (-b)^{-n-1}\,\mathrm{erf}(a\,z)\,\Gamma(n+1,\,-b\,z)\,/;\,n\in\mathbb{N}$$

06.25.21.0017.01

$$\int z\, e^{b z}\, \text{erf}(a z)\, dz = \frac{1}{2 a^2 b^2 \sqrt{\pi}} \left(e^{-a^2 z^2} \left(2 e^{z (z a^2+b)} \sqrt{\pi}\, (b z-1)\, \text{erf}(a z)\, a^2 + 2 b\, e^{b z} a - \left(2 a^2 - b^2\right) e^{\frac{b^2}{4 a^2}+a^2 z^2} \sqrt{\pi}\, \text{erf}\left(\frac{b}{2 a} - a z\right)\right)\right)$$

06.25.21.0018.01

$$\int z^2 e^{b z}\, \text{erf}(a z)\, dz = \frac{1}{4 a^4 b^3 \sqrt{\pi}} \left(e^{-a^2 z^2} \left(4 e^{z (z a^2+b)} \sqrt{\pi} \left(b^2 z^2 - 2 b z + 2\right) \text{erf}(a z)\, a^4 + 2 b\, e^{b z} \left(2 (b z-2) a^2 + b^2\right) a + \left(8 a^4 - 2 b^2 a^2 + b^4\right) e^{\frac{b^2}{4 a^2}+a^2 z^2} \sqrt{\pi}\, \text{erf}\left(\frac{b}{2 a} - a z\right)\right)\right)$$

06.25.21.0019.01

$$\int z^3 e^{b z}\, \text{erf}(a z)\, dz = \frac{1}{8 a^6 b^4 \sqrt{\pi}} \left(e^{-a^2 z^2} \left(8 e^{z (z a^2+b)} \sqrt{\pi} \left(b^3 z^3 - 3 b^2 z^2 + 6 b z - 6\right) \text{erf}(a z)\, a^6 + 2 b\, e^{b z} \left(4 \left(b^2 z^2 - 3 b z + 6\right) a^4 + 2 b^2 (b z-1) a^2 + b^4\right) a - \left(48 a^6 - 12 b^2 a^4 - b^6\right) e^{\frac{b^2}{4 a^2}+a^2 z^2} \sqrt{\pi}\, \text{erf}\left(\frac{b}{2 a} - a z\right)\right)\right)$$

06.25.21.0020.01

$$\int z^{\alpha-1} e^{b z^2}\, \text{erf}(a z)\, dz = -\frac{a z^{\alpha+1}}{\sqrt{\pi}\, \left(-b z^2\right)^{\frac{\alpha+1}{2}}} \sum_{k=0}^{\infty} \frac{b^{-k} a^{2 k}}{(2 k+1)\, k!} \Gamma\left(\frac{\alpha+1}{2} + k, -b z^2\right)$$

06.25.21.0021.01

$$\int z^{\alpha-1} e^{a^2 z^2}\, \text{erf}(a z)\, dz = \frac{a}{2} z^{\alpha+1}\, \Gamma\left(\frac{\alpha+1}{2}\right) {}_2\tilde{F}_2\left(1, \frac{\alpha+1}{2}; \frac{3}{2}, \frac{\alpha+3}{2}; a^2 z^2\right)$$

06.25.21.0022.01

$$\int z\, e^{a^2 z^2}\, \text{erf}(a z)\, dz = \frac{1}{2 a^2} \left(e^{a^2 z^2}\, \text{erf}(a z) - \frac{2 a z}{\sqrt{\pi}}\right)$$

06.25.21.0023.01

$$\int z\, e^{b z^2}\, \text{erf}(c + a z)\, dz = \frac{1}{2 b} \left(e^{b z^2}\, \text{erf}(c + a z) - \frac{a}{\sqrt{b - a^2}} \exp\left(\frac{b c^2}{a^2 - b}\right) \text{erfi}\left(\frac{-z a^2 - a c + b z}{\sqrt{b - a^2}}\right)\right)$$

06.25.21.0024.01

$$\int z\, e^{b z^2}\, \text{erf}(a z)\, dz = \frac{1}{2 b} \left(e^{b z^2}\, \text{erf}(a z) - \frac{a\, \text{erfi}\left(\sqrt{b - a^2}\, z\right)}{\sqrt{b - a^2}}\right)$$

06.25.21.0025.01

$$\int z^3 e^{b z^2} \operatorname{erf}(a z) \, dz = \frac{1}{2 b^2} \left(e^{b z^2} \left(b z^2 - 1\right) \operatorname{erf}(a z) + \left(a b z^3 \left(-\sqrt{\pi} \operatorname{erf}\left(\sqrt{\left(a^2 - b\right) z^2}\right) + \sqrt{\pi} + 2 e^{\left(b - a^2\right) z^2} \sqrt{\left(a^2 - b\right) z^2}\right)\right) \Big/ \left(2 \sqrt{\pi} \left(\left(a^2 - b\right) z^2\right)^{3/2}\right) + \frac{a \operatorname{erfi}\left(\sqrt{b - a^2} \, z\right)}{\sqrt{b - a^2}} \right)$$

06.25.21.0026.01

$$\int \frac{e^{b z^2} \operatorname{erf}(a z)}{z} \, dz = -\frac{a z}{\sqrt{-\pi b z^2}} \sum_{k=0}^{\infty} \frac{b^{-k} a^{2k} \Gamma\left(k + \frac{1}{2}, -b z^2\right)}{(2k+1) \, k!}$$

Involving trigonometric functions

Involving sin

06.25.21.0027.01

$$\int \sin(b z) \operatorname{erf}(a z) \, dz = \frac{1}{2 b} \left(\exp\left(-\frac{b^2}{4 a^2}\right) \left(\operatorname{erf}\left(\frac{2 z a^2 + i b}{2 a}\right) - i \operatorname{erfi}\left(\frac{b}{2 a} + i a z\right) \right) - 2 \cos(b z) \operatorname{erf}(a z) \right)$$

06.25.21.0028.01

$$\int \sin\left(b z^2\right) \operatorname{erf}(a z) \, dz = -\frac{1}{2 \sqrt{\pi} \, b} \left(\sum_{k=0}^{\infty} \frac{(i b)^{-k} a^{2k+1} \Gamma\left(k+1, -i b z^2\right)}{(2k+1) \, k!} + \sum_{k=0}^{\infty} \frac{(-i b)^{-k} a^{2k+1} \Gamma\left(k+1, i b z^2\right)}{(2k+1) \, k!} \right)$$

Involving cos

06.25.21.0029.01

$$\int \cos(b z) \operatorname{erf}(a z) \, dz = \frac{1}{2 b} \left(\exp\left(-\frac{b^2}{4 a^2}\right) \left(\operatorname{erfi}\left(\frac{b}{2 a} + i a z\right) - i \operatorname{erf}\left(\frac{2 z a^2 + i b}{2 a}\right) \right) + 2 \operatorname{erf}(a z) \sin(b z) \right)$$

06.25.21.0030.01

$$\int \cos\left(b z^2\right) \operatorname{erf}(a z) \, dz = \frac{i}{2 \sqrt{\pi} \, b} \left(\sum_{k=0}^{\infty} \frac{(-i b)^{-k} a^{2k+1} \Gamma\left(k+1, i b z^2\right)}{(2k+1) \, k!} - \sum_{k=0}^{\infty} \frac{(i b)^{-k} a^{2k+1} \Gamma\left(k+1, -i b z^2\right)}{(2k+1) \, k!} \right)$$

Involving trigonometric functions and a power function

Involving sin and power

06.25.21.0031.01

$$\int z^{\alpha-1} \sin(b z) \operatorname{erf}(a z) \, dz = -\frac{a z^{\alpha}}{b \sqrt{\pi}} \sum_{k=0}^{\infty} \frac{a^{2k} b^{-2k}}{(2k+1) \, k!} \left(\Gamma(2k + \alpha + 1, -i b z) (-i b z)^{-\alpha} + (i b z)^{-\alpha} \Gamma(2k + \alpha + 1, i b z) \right)$$

06.25.21.0032.01

$$\int z^n \sin(b z)\,\text{erf}(a z)\,dz = \frac{b^{-2n}}{2 b \sqrt{\pi}} \exp\left(-\frac{b^2}{4 a^2}\right) \left(-a\, n!\, (-i\, b)^n \sum_{m=0}^{n} \frac{1}{m!} (i\, b)^m \left(-a^2\right)^{\frac{1}{2}(-m-1)} \sum_{k=0}^{m} \binom{m}{k} \left(-\frac{-i\, b}{2 \sqrt{-a^2}}\right)^{m-k} \left(\sqrt{-a^2}\, z - \frac{i\, b}{2 \sqrt{-a^2}}\right)^{k+1} \left(-\left(\sqrt{-a^2}\, z - \frac{i\, b}{2 \sqrt{-a^2}}\right)^2\right)^{\frac{1}{2}(-k-1)} \Gamma\left(\frac{k+1}{2}, -\left(\sqrt{-a^2}\, z - \frac{i\, b}{2 \sqrt{-a^2}}\right)^2\right) + -(i\, b)^n \exp\left(\frac{b^2}{4 a^2}\right) \sqrt{\pi}\, \text{erf}(a z) \left(\Gamma(n+1, -i\, b z) + (-1)^n\, \Gamma(n+1, i\, b z)\right) - a\, (i\, b)^n\, n! \sum_{m=0}^{n} \frac{1}{m!} (-i\, b)^m \left(-a^2\right)^{\frac{1}{2}(-m-1)} \sum_{k=0}^{m} \binom{m}{k} \left(-\frac{i\, b}{2 \sqrt{-a^2}}\right)^{m-k} \left(\sqrt{-a^2}\, z + \frac{i\, b}{2 \sqrt{-a^2}}\right)^{k+1} \left(-\left(\sqrt{-a^2}\, z + \frac{i\, b}{2 \sqrt{-a^2}}\right)^2\right)^{\frac{1}{2}(-k-1)} \Gamma\left(\frac{k+1}{2}, -\left(\sqrt{-a^2}\, z + \frac{i\, b}{2 \sqrt{-a^2}}\right)^2\right)\right) /;\ n \in \mathbb{N}$$

06.25.21.0033.01

$$\int z \sin(b z)\,\text{erf}(a z)\,dz = \frac{1}{4 a^2 b^2 \sqrt{\pi}} \exp\left(-\frac{b^2}{4 a^2} - i\, z\, b - a^2 z^2\right) \left(-2 \exp\left(\frac{b^2}{4 a^2} + a^2 z^2\right) \sqrt{\pi} \left(-i + b z + e^{2 i b z} (i + b z)\right) \text{erf}(a z)\, a^2 + 2\, e^{z (z a^2 + b i)} \sqrt{\pi}\, \text{erfi}\left(\frac{b}{2 a} + a\, i\, z\right) a^2 - 2\, b\, e^{\frac{b^2}{4 a^2}}\, a - 2\, b\, e^{\frac{1}{4} b \left(\frac{b}{a^2} + 8 i z\right)}\, a - i \left(2 a^2 + b^2\right) e^{z (z a^2 + b i)} \sqrt{\pi}\, \text{erf}\left(\frac{2 z a^2 + b i}{2 a}\right) + b^2\, e^{z (z a^2 + b i)} \sqrt{\pi}\, \text{erfi}\left(\frac{b}{2 a} + a\, i\, z\right)\right)$$

06.25.21.0034.01

$$\int z^2 \sin(b z)\,\text{erf}(a z)\,dz = \frac{1}{8 a^4 b^3 \sqrt{\pi}} \left(e^{-\frac{b^2}{4 a^2} - i z b - a^2 z^2} \left(-4\, e^{\frac{b^2}{4 a^2} + a^2 z^2} \sqrt{\pi} \left(2 b \left(-1 + e^{2 i b z}\right) i\, z + \left(1 + e^{2 i b z}\right) \left(b^2 z^2 - 2\right)\right) \text{erf}(a z)\, a^4 + 8\, e^{z (z a^2 + b i)}\, i \sqrt{\pi}\, \text{erfi}\left(\frac{b}{2 a} + a\, i\, z\right) a^4 - 8\, i\, b\, e^{\frac{1}{4} b \left(\frac{b}{a^2} + 8 i z\right)}\, a^3 + 8\, b\, e^{\frac{b^2}{4 a^2}}\, i\, a^3 - 4\, b^2\, e^{\frac{b^2}{4 a^2}}\, z\, a^3 - 4\, b^2\, e^{\frac{1}{4} b \left(\frac{b}{a^2} + 8 i z\right)}\, z\, a^3 + 2\, b^2\, e^{z (z a^2 + b i)}\, i \sqrt{\pi}\, \text{erfi}\left(\frac{b}{2 a} + a\, i\, z\right) a^2 - 2\, i\, b^3\, e^{\frac{1}{4} b \left(\frac{b}{a^2} + 8 i z\right)}\, a + 2\, b^3\, e^{\frac{b^2}{4 a^2}}\, i\, a - \left(8 a^4 + 2 b^2 a^2 + b^4\right) e^{z (z a^2 + b i)} \sqrt{\pi}\, \text{erf}\left(\frac{2 z a^2 + b i}{2 a}\right) + b^4\, e^{z (z a^2 + b i)}\, i \sqrt{\pi}\, \text{erfi}\left(\frac{b}{2 a} + a\, i\, z\right)\right)\right)$$

06.25.21.0035.01

$$\int z^3 \sin(b z) \operatorname{erf}(a z) \, dz = \frac{1}{16 a^6 b^4 \sqrt{\pi}} \left(e^{-\frac{b^2}{4 a^2} - i z b - a^2 z^2} \left(-8 e^{\frac{b^2}{4 a^2} + a^2 z^2} \sqrt{\pi} \left(b \left(1 + e^{2 i b z} \right) z \left(b^2 z^2 - 6 \right) + 3 \left(-1 + e^{2 i b z} \right) i \left(b^2 z^2 - 2 \right) \right) \operatorname{erf}(a z) a^6 - 48 e^{z (z a^2 + b i)} \sqrt{\pi} \operatorname{erfi}\left(\frac{b}{2 a} + a i z \right) a^6 + 48 b e^{\frac{b^2}{4 a^2}} a^5 + 48 b e^{\frac{1}{4} b \left(\frac{b}{a^2} + 8 i z \right)} a^5 - 8 b^3 e^{\frac{b^2}{4 a^2}} z^2 a^5 - 8 b^3 e^{\frac{1}{4} b \left(\frac{b}{a^2} + 8 i z \right)} z^2 a^5 - 24 i b^2 e^{\frac{1}{4} b \left(\frac{b}{a^2} + 8 i z \right)} z a^5 + 24 b^2 e^{\frac{b^2}{4 a^2}} i z a^5 - 12 b^2 e^{z (z a^2 + b i)} \sqrt{\pi} \operatorname{erfi}\left(\frac{b}{2 a} + a i z \right) a^4 + 4 b^3 e^{\frac{b^2}{4 a^2}} a^3 + 4 b^3 e^{\frac{1}{4} b \left(\frac{b}{a^2} + 8 i z \right)} a^3 - 4 i b^4 e^{\frac{1}{4} b \left(\frac{b}{a^2} + 8 i z \right)} z a^3 + 4 b^4 e^{\frac{b^2}{4 a^2}} i z a^3 + 2 b^5 e^{\frac{b^2}{4 a^2}} a + 2 b^5 e^{\frac{1}{4} b \left(\frac{b}{a^2} + 8 i z \right)} a + \left(48 a^6 + 12 b^2 a^4 + b^6 \right) e^{z (z a^2 + b i)} i \sqrt{\pi} \operatorname{erf}\left(\frac{2 z a^2 + b i}{2 a} \right) - b^6 e^{z (z a^2 + b i)} \sqrt{\pi} \operatorname{erfi}\left(\frac{b}{2 a} + a i z \right) \right) \right)$$

06.25.21.0036.01

$$\int z^{\alpha-1} \sin\left(b z^2\right) \operatorname{erf}(a z) \, dz = \frac{i a z^{\alpha+1}}{2 \sqrt{\pi}} \left(b^2 z^4 \right)^{\frac{1}{2}(-\alpha-1)} \left(\left(i b z^2 \right)^{\frac{\alpha+1}{2}} \sum_{k=0}^{\infty} \frac{(i b)^{-k} a^{2k} \Gamma\left(\frac{\alpha+1}{2} + k, -i b z^2 \right)}{(2k+1) k!} - \left(-i b z^2 \right)^{\frac{\alpha+1}{2}} \sum_{k=0}^{\infty} \frac{(-i b)^{-k} a^{2k} \Gamma\left(\frac{\alpha+1}{2} + k, i b z^2 \right)}{(2k+1) k!} \right)$$

06.25.21.0037.01

$$\int z \sin\left(b z^2\right) \operatorname{erf}(c + a z) \, dz = \frac{1}{4 b \left(a^4 + b^2 \right)} e^{-c^2} \left(a \left(\sqrt{a^2 + b i} \left(a^2 - i b \right) \exp\left(\frac{a^2 c^2}{a^2 + b i} \right) \operatorname{erf}\left(\frac{z a^2 + c a + b i z}{\sqrt{a^2 + b i}} \right) + \left(b - i a^2 \right) \sqrt{a^2 - i b} \exp\left(\frac{a^2 c^2}{a^2 - i b} \right) \operatorname{erfi}\left(\frac{i z a^2 + c i a + b z}{\sqrt{a^2 - i b}} \right) \right) - 2 \left(a^4 + b^2 \right) e^{c^2} \cos\left(b z^2 \right) \operatorname{erf}(c + a z) \right)$$

06.25.21.0038.01

$$\int z \sin\left(b z^2\right) \operatorname{erf}(a z) \, dz = \frac{1}{4 b \left(a^4 + b^2 \right)} \left(a \left(\sqrt{a^2 + b i} \left(a^2 - i b \right) \operatorname{erf}\left(\sqrt{a^2 + b i} \, z \right) + \left(b - i a^2 \right) \sqrt{a^2 - i b} \operatorname{erfi}\left(\frac{\left(i a^2 + b \right) z}{\sqrt{a^2 - i b}} \right) \right) - 2 \left(a^4 + b^2 \right) \cos\left(b z^2 \right) \operatorname{erf}(a z) \right)$$

06.25.21.0039.01

$$\int z^3 \sin\left(b z^2\right) \operatorname{erf}(a z)\, dz = \frac{1}{4 b^2}\left(\frac{1}{2\sqrt{\pi}}\left(a b z^3 \left(\left(\sqrt{\pi}\, \operatorname{erf}\left(\sqrt{\left(a^2+b i\right) z^2}\right) - \sqrt{\pi} - 2 e^{-\left(a^2+b i\right) z^2} \sqrt{\left(a^2+b i\right) z^2}\right) \Big/ \left(\left(a^2+b i\right) z^2\right)^{3/2} + \left(\sqrt{\pi}\, \operatorname{erf}\left(\sqrt{\left(a^2-i b\right) z^2}\right) - \sqrt{\pi} - 2 e^{-\left(a^2-i b\right) z^2} \sqrt{\left(a^2-i b\right) z^2}\right) \Big/ \left(\left(a^2-i b\right) z^2\right)^{3/2}\right) + \frac{1}{a^4+b^2}\left(a\left(\left(-i a^2-b\right)\sqrt{a^2+b i}\, \operatorname{erf}\left(\sqrt{a^2+b i}\; z\right) + \left(a^2+b i\right)\sqrt{a^2-i b}\, \operatorname{erfi}\left(\frac{\left(i a^2+b\right) z}{\sqrt{a^2-i b}}\right)\right)\right)\right) - 2 \operatorname{erf}(a z)\left(b z^2 \cos\left(b z^2\right) - \sin\left(b z^2\right)\right)\right)$$

06.25.21.0040.01

$$\int \frac{\sin\left(b z^2\right) \operatorname{erf}(a z)}{z}\, dz = \frac{i a z}{2\sqrt{-\pi i b z^2}} \sum_{k=0}^{\infty} \frac{(i b)^{-k} a^{2k}\, \Gamma\left(k+\frac{1}{2}, -i b z^2\right)}{(2k+1)\, k!} - \frac{i a z}{2\sqrt{\pi i b z^2}} \sum_{k=0}^{\infty} \frac{(-i b)^{-k} a^{2k}\, \Gamma\left(k+\frac{1}{2}, i b z^2\right)}{(2k+1)\, k!}$$

Involving cos and power

06.25.21.0041.01

$$\int z^{\alpha-1} \cos(b z) \operatorname{erf}(a z)\, dz = \frac{i a z^{\alpha}}{b\sqrt{\pi}} \sum_{k=0}^{\infty} \frac{a^{2k} b^{-2k}}{(2k+1)\, k!} \left((i b z)^{-\alpha}\, \Gamma(2k+\alpha+1, i b z) - (-i b z)^{-\alpha}\, \Gamma(2k+\alpha+1, -i b z)\right)$$

06.25.21.0042.01

$$\int z^n \cos(b z) \operatorname{erf}(a z)\, dz = \frac{i b^{-2n}}{2 b\sqrt{\pi}} \exp\left(-\frac{b^2}{4a^2}\right)\left(a\, n!\, (-i b)^n \sum_{m=0}^{n} \frac{(i b)^m \left(-a^2\right)^{\frac{1}{2}(-m-1)}}{m!} \sum_{k=0}^{m} \binom{m}{k} \left(-\frac{-i b}{2\sqrt{-a^2}}\right)^{m-k} \left(\sqrt{-a^2}\; z - \frac{i b}{2\sqrt{-a^2}}\right)^{k+1} \left(-\left(\sqrt{-a^2}\; z - \frac{i b}{2\sqrt{-a^2}}\right)^2\right)^{\frac{1}{2}(-k-1)} \Gamma\left(\frac{k+1}{2}, -\left(\sqrt{-a^2}\; z - \frac{i b}{2\sqrt{-a^2}}\right)^2\right) - a\, n!\, (i b)^n \sum_{m=0}^{n} \frac{(-i b)^m \left(-a^2\right)^{\frac{1}{2}(-m-1)}}{m!} \sum_{k=0}^{m} \binom{m}{k} \left(-\frac{i b}{2\sqrt{-a^2}}\right)^{m-k} \left(\sqrt{-a^2}\; z + \frac{i b}{2\sqrt{-a^2}}\right)^{k+1} \left(-\left(\sqrt{-a^2}\; z + \frac{i b}{2\sqrt{-a^2}}\right)^2\right)^{\frac{1}{2}(-k-1)} \Gamma\left(\frac{k+1}{2}, -\left(\sqrt{-a^2}\; z + \frac{i b}{2\sqrt{-a^2}}\right)^2\right)\right) - \sqrt{\pi}\, (i b)^n e^{\frac{b^2}{4a^2}} \operatorname{erf}(a z) \left(\Gamma(n+1, -i b z) - (-1)^n\, \Gamma(n+1, i b z)\right) /;\ n \in \mathbb{N}$$

06.25.21.0043.01

$$\int z\cos(b\,z)\,\mathrm{erf}(a\,z)\,dz = \frac{1}{4\,a^2\,b^2\sqrt{\pi}}\exp\left(-\frac{b^2}{4\,a^2}-i\,z\,b-a^2\,z^2\right)\left(2\,e^{\frac{b^2}{4a^2}+a^2z^2}\sqrt{\pi}\,\mathrm{erf}(a\,z)\left(2\,b\,e^{i\,b\,z}\,z\sin(b\,z)+e^{2\,i\,b\,z}+1\right)a^2+ i\left(-2\,a\,b\,e^{\frac{b^2}{4a^2}}\left(-1+e^{2\,i\,b\,z}\right)+\left(2\,a^2+b^2\right)e^{z\left(z\,a^2+b\,i\right)}\,i\sqrt{\pi}\,\mathrm{erf}\left(\frac{2\,z\,a^2+b\,i}{2\,a}\right)+\left(2\,a^2+b^2\right)e^{z\left(z\,a^2+b\,i\right)}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{b}{2\,a}+a\,i\,z\right)\right)\right)$$

06.25.21.0044.01

$$\int z^2\cos(b\,z)\,\mathrm{erf}(a\,z)\,dz = \frac{1}{8\,a^4\,b^3\sqrt{\pi}} \left(e^{-\frac{b^2}{4a^2}-i\,z\,b-a^2\,z^2}\left(-8\,e^{z\left(z\,a^2+b\,i\right)}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{b}{2\,a}+a\,i\,z\right)a^4+8\,e^{\frac{b^2}{4a^2}+a^2z^2}\sqrt{\pi}\,\mathrm{erf}(a\,z)\left(b\left(1+e^{2\,i\,b\,z}\right)z+e^{i\,b\,z}\left(b^2\,z^2-2\right)\sin(b\,z)\right)\right.\right.$$
$$a^4+8\,b\,e^{\frac{b^2}{4a^2}}\,a^3+8\,b\,e^{\frac{1}{4}b\left(\frac{b}{a^2}+8\,i\,z\right)}\,a^3-4\,i\,b^2\,e^{\frac{1}{4}b\left(\frac{b}{a^2}+8\,i\,z\right)}\,z\,a^3+$$
$$4\,b^2\,e^{\frac{b^2}{4a^2}}\,i\,z\,a^3-2\,b^2\,e^{z\left(z\,a^2+b\,i\right)}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{b}{2\,a}+a\,i\,z\right)a^2+2\,b^3\,e^{\frac{b^2}{4a^2}}\,a+2\,b^3\,e^{\frac{1}{4}b\left(\frac{b}{a^2}+8\,i\,z\right)}\,a+$$
$$\left.\left.\left(8\,a^4+2\,b^2\,a^2+b^4\right)e^{z\left(z\,a^2+b\,i\right)}\,i\sqrt{\pi}\,\mathrm{erf}\left(\frac{2\,z\,a^2+b\,i}{2\,a}\right)-b^4\,e^{z\left(z\,a^2+b\,i\right)}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{b}{2\,a}+a\,i\,z\right)\right)\right)$$

06.25.21.0045.01

$$\int z^3\cos(b\,z)\,\mathrm{erf}(a\,z)\,dz = \frac{1}{16\,a^6\,b^4\sqrt{\pi}}\left(e^{-\frac{b^2}{4a^2}-i\,z\,b-a^2\,z^2}\left(-48\,i\,e^{z\left(z\,a^2+b\,i\right)}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{b}{2\,a}+a\,i\,z\right)a^6+\right.\right.$$
$$8\,e^{\frac{b^2}{4a^2}+a^2z^2}\sqrt{\pi}\,\mathrm{erf}(a\,z)\left(2\,b^3\,e^{i\,b\,z}\sin(b\,z)\,z^3+3\left(b^2\,z^2-2\,i\,b\,z+e^{2\,i\,b\,z}\left(b^2\,z^2+2\,b\,i\,z-2\right)-2\right)\right)a^6-$$
$$8\,i\,b^3\,e^{\frac{1}{4}b\left(\frac{b}{a^2}+8\,i\,z\right)}\,z^2\,a^5+8\,b^3\,e^{\frac{b^2}{4a^2}}\,i\,z^2\,a^5-48\,b\,e^{\frac{b^2}{4a^2}}\,i\,a^5+48\,b\,e^{\frac{1}{4}b\left(\frac{b}{a^2}+8\,i\,z\right)}\,i\,a^5+24\,b^2\,e^{\frac{b^2}{4a^2}}\,z\,a^5+$$
$$24\,b^2\,e^{\frac{1}{4}b\left(\frac{b}{a^2}+8\,i\,z\right)}\,z\,a^5-12\,i\,b^2\,e^{z\left(z\,a^2+b\,i\right)}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{b}{2\,a}+a\,i\,z\right)a^4-4\,i\,b^3\,e^{\frac{b^2}{4a^2}}\,a^3+$$
$$4\,b^3\,e^{\frac{1}{4}b\left(\frac{b}{a^2}+8\,i\,z\right)}\,i\,a^3+4\,b^4\,e^{\frac{b^2}{4a^2}}\,z\,a^3+4\,b^4\,e^{\frac{1}{4}b\left(\frac{b}{a^2}+8\,i\,z\right)}\,z\,a^3-2\,i\,b^5\,e^{\frac{b^2}{4a^2}}\,a+2\,b^5\,e^{\frac{1}{4}b\left(\frac{b}{a^2}+8\,i\,z\right)}\,i\,a+$$
$$\left.\left.\left(48\,a^6+12\,b^2\,a^4+b^6\right)e^{z\left(z\,a^2+b\,i\right)}\sqrt{\pi}\,\mathrm{erf}\left(\frac{2\,z\,a^2+b\,i}{2\,a}\right)-i\,b^6\,e^{z\left(z\,a^2+b\,i\right)}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{b}{2\,a}+a\,i\,z\right)\right)\right)$$

06.25.21.0046.01

$$\int z^{\alpha-1}\cos\left(b\,z^2\right)\mathrm{erf}(a\,z)\,dz =$$
$$\frac{a\,z^{\alpha+1}}{2\sqrt{\pi}}\left(b^2\,z^4\right)^{\frac{1}{2}(-\alpha-1)}\left(-\left(i\,b\,z^2\right)^{\frac{\alpha+1}{2}}\sum_{k=0}^{\infty}\frac{(i\,b)^{-k}\,a^{2k}\,\Gamma\left(\frac{\alpha+1}{2}+k,\,-i\,b\,z^2\right)}{(2\,k+1)\,k!}-\left(-i\,b\,z^2\right)^{\frac{\alpha+1}{2}}\sum_{k=0}^{\infty}\frac{(-i\,b)^{-k}\,a^{2k}\,\Gamma\left(\frac{\alpha+1}{2}+k,\,i\,b\,z^2\right)}{(2\,k+1)\,k!}\right)$$

06.25.21.0047.01

$$\int z\cos\left(b z^2\right)\operatorname{erf}(c+a z)\,dz = \frac{1}{4\sqrt{2}\,b\left(a^4+b^2\right)}\exp\left(-\frac{i b c^2}{a^2+b i}\right)\left((1+i)\,a\left(a^2+b i\right)\sqrt{i a^2+b}\,\exp\left(\frac{2 i a^2 b c^2}{a^4+b^2}\right)\operatorname{erf}\left(\frac{(1+i)\left(z a^2+c a-i b z\right)}{\sqrt{2}\sqrt{i a^2+b}}\right)-\sqrt[4]{-1}\,a\left(a^2-i b\right)\sqrt{b-i a^2}\,\operatorname{erfi}\left(\frac{(1+i)\left(z a^2+c a+b i z\right)}{\sqrt{2}\sqrt{b-i a^2}}\right)+2\left(a^4+b^2\right)\exp\left(\frac{i b c^2}{a^2+b i}\right)\operatorname{erf}(c+a z)\sin\left(b z^2\right)\right)$$

06.25.21.0048.01

$$\int z\cos\left(b z^2\right)\operatorname{erf}(a z)\,dz = \frac{1}{4 b\left(a^4+b^2\right)}\left(-\sqrt[4]{-1}\,a\sqrt{b-i a^2}\left(a^2-i b\right)\operatorname{erfi}\left((-1)^{3/4}\sqrt{b-i a^2}\,z\right)+\sqrt[4]{-1}\,a\left(b-i a^2\right)\sqrt{i a^2+b}\,\operatorname{erfi}\left(\sqrt[4]{-1}\sqrt{i a^2+b}\,z\right)+2\left(a^4+b^2\right)\operatorname{erf}(a z)\sin\left(b z^2\right)\right)$$

06.25.21.0049.01

$$\int z^3\cos\left(b z^2\right)\operatorname{erf}(a z)\,dz = \frac{1}{2 b^2}\left(\frac{i a b z^3}{4\sqrt{\pi}}\left(\left(-\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{\left(a^2+b i\right)z^2}\right)+\sqrt{\pi}+2 e^{-\left(a^2+b i\right)z^2}\sqrt{\left(a^2+b i\right)z^2}\right)\Big/\left(\left(a^2+b i\right)z^2\right)^{3/2}+\left(\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{\left(a^2-i b\right)z^2}\right)-\sqrt{\pi}-2 e^{-\left(a^2-i b\right)z^2}\sqrt{\left(a^2-i b\right)z^2}\right)\Big/\left(\left(a^2-i b\right)z^2\right)^{3/2}\right)+\frac{\sqrt[4]{-1}\,a}{2\left(a^4+b^2\right)}\left(\sqrt{b-i a^2}\left(i a^2+b\right)\operatorname{erfi}\left((-1)^{3/4}\sqrt{b-i a^2}\,z\right)+\sqrt{i a^2+b}\left(a^2+b i\right)\operatorname{erfi}\left(\sqrt[4]{-1}\sqrt{i a^2+b}\,z\right)\right)+\operatorname{erf}(a z)\left(b\sin\left(b z^2\right)z^2+\cos\left(b z^2\right)\right)\right)$$

06.25.21.0050.01

$$\int\frac{\cos\left(b z^2\right)\operatorname{erf}(a z)}{z}\,dz = -\frac{a z}{2\sqrt{-\pi i b z^2}}\sum_{k=0}^{\infty}\frac{(i b)^{-k}a^{2k}\,\Gamma\left(k+\frac{1}{2},-i b z^2\right)}{(2k+1)\,k!}-\frac{a z}{2\sqrt{\pi i b z^2}}\sum_{k=0}^{\infty}\frac{(-i b)^{-k}a^{2k}\,\Gamma\left(k+\frac{1}{2},i b z^2\right)}{(2k+1)\,k!}$$

Involving exponential function and trigonometric functions

Involving exp and sin

06.25.21.0051.01

$$\int e^{b z}\sin(c z)\operatorname{erf}(a z)\,dz = \frac{1}{2\left(b^2+c^2\right)}\left(e^{\frac{(b-i c)^2}{4 a^2}}\,i\left((b+c i)\operatorname{erf}\left(\frac{-2 z a^2+b-i c}{2 a}\right)-(b-i c)\,e^{\frac{i b c}{a^2}}\operatorname{erf}\left(\frac{-2 z a^2+b+c i}{2 a}\right)\right)+2 e^{b z}\operatorname{erf}(a z)\left(b\sin(c z)-c\cos(c z)\right)\right)$$

06.25.21.0052.01

$$\int e^{b z^2} \sin(c z^2) \operatorname{erf}(a z) \, dz = \frac{i}{2\sqrt{\pi}\,(b - i c)} \sum_{k=0}^{\infty} \frac{(b - i c)^{-k} a^{2k+1} \Gamma(k+1, -(b - i c) z^2)}{(2k+1)\, k!} - \frac{i}{2\sqrt{\pi}\,(b + c i)} \sum_{k=0}^{\infty} \frac{(b + c i)^{-k} a^{2k+1} \Gamma(k+1, -(b + c i) z^2)}{(2k+1)\, k!}$$

Involving exp and cos

06.25.21.0053.01

$$\int e^{b z} \cos(c z) \operatorname{erf}(a z) \, dz = \frac{1}{2(b^2 + c^2)} \left(e^{\frac{(b - i c)^2}{4 a^2}} \left((b - i c) e^{\frac{i b c}{a^2}} \operatorname{erf}\left(\frac{-2 z a^2 + b + c i}{2a} \right) + (b + c i) \operatorname{erf}\left(\frac{-2 z a^2 + b - i c}{2a} \right) \right) + 2 e^{b z} \operatorname{erf}(a z) (b \cos(c z) + c \sin(c z)) \right)$$

06.25.21.0054.01

$$\int e^{b z^2} \cos(c z^2) \operatorname{erf}(a z) \, dz = \frac{1}{2\sqrt{\pi}\,(b + c i)} \sum_{k=0}^{\infty} \frac{(b + c i)^{-k} a^{2k+1} \Gamma(k+1, -(b + c i) z^2)}{(2k+1)\, k!} + \frac{1}{2\sqrt{\pi}\,(b - i c)} \sum_{k=0}^{\infty} \frac{(b - i c)^{-k} a^{2k+1} \Gamma(k+1, -(b - i c) z^2)}{(2k+1)\, k!}$$

Involving power, exponential and trigonometric functions

Involving power, exp and sin

06.25.21.0055.01

$$\int z^{\alpha-1} e^{b z} \sin(c z) \operatorname{erf}(a z) \, dz = \frac{i a z^{\alpha} (-(b - i c) z)^{-\alpha}}{(b - i c)\sqrt{\pi}} \sum_{k=0}^{\infty} \frac{(-1)^k a^{2k} (b - i c)^{-2k}}{(2k+1)\, k!} \Gamma(2k + \alpha + 1, -(b - i c) z) - \frac{i a z^{\alpha} (-(b + c i) z)^{-\alpha}}{(b + c i)\sqrt{\pi}} \sum_{k=0}^{\infty} \frac{(-1)^k a^{2k} (b + c i)^{-2k}}{(2k+1)\, k!} \Gamma(2k + \alpha + 1, -(b + c i) z)$$

06.25.21.0056.01

$$\int z^n e^{b z} \sin(c z) \operatorname{erf}(a z) \, dz = -\frac{1}{2} i \operatorname{erf}(a z) \Gamma(n+1, (i c - b) z) (i c - b)^{-n-1} + \frac{1}{2} (-b - i c)^{-n-1} i \operatorname{erf}(a z) \Gamma(n+1, (-b - i c) z) + \frac{1}{2\sqrt{\pi}} \left(i a (-b - i c)^{-n-1} e^{\frac{(b+c i)^2}{4 a^2}} n! \sum_{m=0}^{n} \frac{1}{m!} \left((-(b + c i))^m \left(-a^2\right)^{\frac{1}{2}(-m-1)} \sum_{k=0}^{m} \binom{m}{k} \left(-\frac{b + c i}{2\sqrt{-a^2}} \right)^{m-k} \left(\frac{b + c i}{2\sqrt{-a^2}} + \sqrt{-a^2}\, z \right)^{k+1} \left(-\left(\frac{b + c i}{2\sqrt{-a^2}} + \sqrt{-a^2}\, z \right)^2 \right)^{\frac{1}{2}(-k-1)} \Gamma\left(\frac{k+1}{2}, -\left(\frac{b + c i}{2\sqrt{-a^2}} + \sqrt{-a^2}\, z \right)^2 \right) \right) \right) - \frac{1}{2\sqrt{\pi}} \left(i a (i c - b)^{-n-1} e^{\frac{(b - i c)^2}{4 a^2}} n! \sum_{m=0}^{n} \frac{1}{m!} \left((-(b - i c))^m \left(-a^2\right)^{\frac{1}{2}(-m-1)} \sum_{k=0}^{m} \binom{m}{k} \left(-\frac{b - i c}{2\sqrt{-a^2}} \right)^{m-k} \left(\frac{b - i c}{2\sqrt{-a^2}} + \sqrt{-a^2}\, z \right)^{k+1} \left(-\left(\frac{b - i c}{2\sqrt{-a^2}} + \sqrt{-a^2}\, z \right)^2 \right)^{\frac{1}{2}(-k-1)} \Gamma\left(\frac{k+1}{2}, -\left(\frac{b - i c}{2\sqrt{-a^2}} + \sqrt{-a^2}\, z \right)^2 \right) \right) \right) /; n \in \mathbb{N}$$

06.25.21.0057.01

$$\int z e^{b z} \sin(c z) \operatorname{erf}(a z) \, dz = \frac{i}{4 a^2 \sqrt{\pi}} e^{-a^2 z^2} \left(\frac{1}{(b - i c)^2} \left(2 e^{z\left(z a^2 + b - i c\right)} \sqrt{\pi} \, (b z - i c z - 1) \operatorname{erf}(a z) a^2 + 2 (b - i c) e^{(b - i c) z} a - \left(2 a^2 - (b - i c)^2\right) \exp\left(\frac{(b - i c)^2}{4 a^2} + a^2 z^2 \right) \sqrt{\pi} \operatorname{erf}\left(\frac{b - i c}{2 a} - a z \right) \right) - \frac{1}{(b + c i)^2} \left(2 e^{z\left(z a^2 + b + c i\right)} \sqrt{\pi} \, (b z + c i z - 1) \operatorname{erf}(a z) a^2 + 2 (b + c i) e^{(b + c i) z} a - \left(2 a^2 - (b + c i)^2\right) \exp\left(\frac{(b + c i)^2}{4 a^2} + a^2 z^2 \right) \sqrt{\pi} \operatorname{erf}\left(\frac{b + c i}{2 a} - a z \right) \right) \right)$$

06.25.21.0058.01

$$\int z^2 e^{bz}\sin(cz)\,\text{erf}(az)\,dz = \frac{i}{8a^4\sqrt{\pi}}e^{-a^2z^2}\Bigg(\frac{1}{(b-ic)^3}\Big(4e^{z(za^2+b-ic)}\sqrt{\pi}\left((b-ic)^2z^2-2(b-ic)z+2\right)\text{erf}(az)a^4+2(b-ic)e^{(b-ic)z}\left(2(bz-icz-2)a^2+(b-ic)^2\right)a+\left(8a^4-2(b-ic)^2a^2+(b-ic)^4\right)\exp\left(\frac{(b-ic)^2}{4a^2}+a^2z^2\right)\sqrt{\pi}\,\text{erf}\left(\frac{b-ic}{2a}-az\right)\Big)-\frac{1}{(b+ci)^3}\Big(4e^{z(za^2+b+ci)}\sqrt{\pi}\left((b+ci)^2z^2-2(b+ci)z+2\right)\text{erf}(az)a^4+2(b+ci)e^{(b+ci)z}\left(2(bz+ciz-2)a^2+(b+ci)^2\right)a+\left(8a^4-2(b+ci)^2a^2+(b+ci)^4\right)\exp\left(\frac{(b+ci)^2}{4a^2}+a^2z^2\right)\sqrt{\pi}\,\text{erf}\left(\frac{b+ci}{2a}-az\right)\Big)\Bigg)$$

06.25.21.0059.01

$$\int z^3 e^{bz}\sin(cz)\,\text{erf}(az)\,dz = \frac{i}{16a^6\sqrt{\pi}}e^{-a^2z^2}\Bigg(\frac{1}{(b-ic)^4}\Big(8e^{z(za^2+b-ic)}\sqrt{\pi}\left((b-ic)^3z^3-3(b-ic)^2z^2+6(b-ic)z-6\right)\text{erf}(az)a^6+2(b-ic)e^{(b-ic)z}\left(4\left((b-ic)^2z^2-3(b-ic)z+6\right)a^4+2(b-ic)^2(bz-icz-1)a^2+(b-ic)^4\right)a-\left(48a^6-12(b-ic)^2a^4-(b-ic)^6\right)\exp\left(\frac{(b-ic)^2}{4a^2}+a^2z^2\right)\sqrt{\pi}\,\text{erf}\left(\frac{b-ic}{2a}-az\right)\Big)-\frac{1}{(b+ci)^4}\Big(8e^{z(za^2+b+ci)}\sqrt{\pi}\left((b+ci)^3z^3-3(b+ci)^2z^2+6(b+ci)z-6\right)\text{erf}(az)a^6+2(b+ci)e^{(b+ci)z}\left(4\left((b+ci)^2z^2-3(b+ci)z+6\right)a^4+2(b+ci)^2(bz+ciz-1)a^2+(b+ci)^4\right)a-\left(48a^6-12(b+ci)^2a^4-(b+ci)^6\right)\exp\left(\frac{(b+ci)^2}{4a^2}+a^2z^2\right)\sqrt{\pi}\,\text{erf}\left(\frac{b+ci}{2a}-az\right)\Big)\Bigg)$$

06.25.21.0060.01

$$\int z^{\alpha-1}e^{bz^2}\sin\left(cz^2\right)\text{erf}(az)\,dz = \frac{i}{2\sqrt{\pi}}az^{\alpha+1}\left(\left(-(b+ci)z^2\right)^{\frac{1}{2}(-\alpha-1)}\sum_{k=0}^{\infty}\frac{(b+ci)^{-k}a^{2k}\,\Gamma\left(\frac{\alpha+1}{2}+k,-(b+ci)z^2\right)}{(2k+1)k!}-\left(-(b-ic)z^2\right)^{\frac{1}{2}(-\alpha-1)}\sum_{k=0}^{\infty}\frac{(b-ic)^{-k}a^{2k}\,\Gamma\left(\frac{\alpha+1}{2}+k,-(b-ic)z^2\right)}{(2k+1)k!}\right)$$

06.25.21.0061.01

$$\int z\, e^{b z^2} \sin(c z^2) \operatorname{erf}(a z)\, dz = \Big(a z \Big(-\sqrt{(a^2 - b + c\, i) z^2}\; b + \sqrt{(a^2 - b - i\, c) z^2}\; b - (b + c\, i) \sqrt{(a^2 - b - i\, c) z^2}\; \operatorname{erf}\Big(\sqrt{(a^2 - b + c\, i) z^2}\Big) + (b - i\, c) \sqrt{(a^2 - b + c\, i) z^2}\; \operatorname{erf}\Big(\sqrt{(a^2 - b - i\, c) z^2}\Big) + c\, i \sqrt{(a^2 - b + c\, i) z^2} + c\, i \sqrt{(a^2 - b - i\, c) z^2}\Big) + 2 e^{b z^2}\, i \sqrt{(a^2 - b + c\, i) z^2}\; \sqrt{(a^2 - b - i\, c) z^2}\; \operatorname{erf}(a z) \left(c \cos(c z^2) - b \sin(c z^2)\right)\Big) \Big/ \left(4 (b - i\, c)(c - i\, b) \sqrt{(a^2 - b - i\, c) z^2}\; \sqrt{(a^2 - b + c\, i) z^2}\right)$$

06.25.21.0062.01

$$\int \frac{e^{b z^2} \sin(c z^2) \operatorname{erf}(a z)}{z}\, dz = -\frac{i\, a z}{2 \sqrt{\pi}} \left(\frac{1}{\sqrt{-(b - i\, c) z^2}} \sum_{k=0}^{\infty} \frac{(b - i\, c)^{-k} a^{2k}\, \Gamma\!\left(k + \frac{1}{2}, -(b - i\, c) z^2\right)}{(2k + 1)\, k!} - \frac{1}{\sqrt{-(b + c\, i) z^2}} \sum_{k=0}^{\infty} \frac{(b + c\, i)^{-k} a^{2k}\, \Gamma\!\left(k + \frac{1}{2}, -(b + c\, i) z^2\right)}{(2k + 1)\, k!}\right)$$

Involving power, exp and cos

06.25.21.0063.01

$$\int z^{\alpha - 1} e^{b z} \cos(c z) \operatorname{erf}(a z)\, dz = \frac{a z^{\alpha} (-(b + c\, i) z)^{-\alpha}}{(b + c\, i) \sqrt{\pi}} \sum_{k=0}^{\infty} \frac{(-1)^k a^{2k} (b + c\, i)^{-2k}}{(2k + 1)\, k!}\, \Gamma(2k + \alpha + 1, -(b + c\, i) z) + \frac{a z^{\alpha} (-(b - i\, c) z)^{-\alpha}}{(b - i\, c) \sqrt{\pi}} \sum_{k=0}^{\infty} \frac{(-1)^k a^{2k} (b - i\, c)^{-2k}}{(2k + 1)\, k!}\, \Gamma(2k + \alpha + 1, -(b - i\, c) z)$$

06.25.21.0064.01

$$\int z^n e^{b z} \cos(c z) \operatorname{erf}(a z) \, dz = -\frac{1}{2} \operatorname{erf}(a z) \Gamma(n+1, (i c - b) z) (i c - b)^{-n-1} - \frac{1}{2} (-b - i c)^{-n-1} \operatorname{erf}(a z) \Gamma(n+1, (-b - i c) z) - \frac{1}{2\sqrt{\pi}} \left(a (-b - i c)^{-n-1} e^{\frac{(b+c i)^2}{4 a^2}} n! \sum_{m=0}^{n} \frac{1}{m!} \left((-(b + c i))^m \left(-a^2\right)^{\frac{1}{2}(-m-1)} \sum_{k=0}^{m} \binom{m}{k} \left(-\frac{b + c i}{2\sqrt{-a^2}} \right)^{m-k} \left(\frac{b + c i}{2\sqrt{-a^2}} + \sqrt{-a^2}\, z \right)^{k+1} \left(-\left(\frac{b + c i}{2\sqrt{-a^2}} + \sqrt{-a^2}\, z \right)^2 \right)^{\frac{1}{2}(-k-1)} \Gamma\left(\frac{k+1}{2}, -\left(\frac{b + c i}{2\sqrt{-a^2}} + \sqrt{-a^2}\, z \right)^2 \right) \right) \right) - \frac{1}{2\sqrt{\pi}} \left(a (i c - b)^{-n-1} e^{\frac{(b-i c)^2}{4 a^2}} n! \sum_{m=0}^{n} \frac{1}{m!} \left((-(b - i c))^m \left(-a^2\right)^{\frac{1}{2}(-m-1)} \sum_{k=0}^{m} \binom{m}{k} \left(-\frac{b - i c}{2\sqrt{-a^2}} \right)^{m-k} \left(\frac{b - i c}{2\sqrt{-a^2}} + \sqrt{-a^2}\, z \right)^{k+1} \left(-\left(\frac{b - i c}{2\sqrt{-a^2}} + \sqrt{-a^2}\, z \right)^2 \right)^{\frac{1}{2}(-k-1)} \Gamma\left(\frac{k+1}{2}, -\left(\frac{b - i c}{2\sqrt{-a^2}} + \sqrt{-a^2}\, z \right)^2 \right) \right) \right) /; n \in \mathbb{N}$$

06.25.21.0065.01

$$\int z e^{b z} \cos(c z) \operatorname{erf}(a z) \, dz = \frac{1}{4 a^2 \sqrt{\pi}} e^{-a^2 z^2} \left(\frac{1}{(b - i c)^2} \left(2 e^{z\left(z a^2 + b - i c\right)} \sqrt{\pi} (b z - i c z - 1) \operatorname{erf}(a z) a^2 + 2 (b - i c) e^{(b - i c) z} a - \left(2 a^2 - (b - i c)^2\right) \exp\left(\frac{(b - i c)^2}{4 a^2} + a^2 z^2 \right) \sqrt{\pi} \operatorname{erf}\left(\frac{b - i c}{2 a} - a z \right) \right) + \frac{1}{(b + c i)^2} \left(2 e^{z\left(z a^2 + b + c i\right)} \sqrt{\pi} (b z + c i z - 1) \operatorname{erf}(a z) a^2 + 2 (b + c i) e^{(b + c i) z} a - \left(2 a^2 - (b + c i)^2\right) \exp\left(\frac{(b + c i)^2}{4 a^2} + a^2 z^2 \right) \sqrt{\pi} \operatorname{erf}\left(\frac{b + c i}{2 a} - a z \right) \right) \right)$$

06.25.21.0066.01

$$\int z^2 e^{b z} \cos(c z) \operatorname{erf}(a z) \, dz = \frac{1}{8 a^4 \sqrt{\pi}} e^{-a^2 z^2} \left(\frac{1}{(b - i c)^3} \left(4 e^{z\left(z a^2 + b - i c\right)} \sqrt{\pi} \left((b - i c)^2 z^2 - 2 (b - i c) z + 2\right) \operatorname{erf}(a z) a^4 + 2 (b - i c) e^{(b - i c) z} \left(2 (b z - i c z - 2) a^2 + (b - i c)^2\right) a + \left(8 a^4 - 2 (b - i c)^2 a^2 + (b - i c)^4\right) \exp\left(\frac{(b - i c)^2}{4 a^2} + a^2 z^2 \right) \sqrt{\pi} \operatorname{erf}\left(\frac{b - i c}{2 a} - a z \right) \right) + \frac{1}{(b + c i)^3} \left(4 e^{z\left(z a^2 + b + c i\right)} \sqrt{\pi} \left((b + c i)^2 z^2 - 2 (b + c i) z + 2\right) \operatorname{erf}(a z) a^4 + 2 (b + c i) e^{(b + c i) z} \left(2 (b z + c i z - 2) a^2 + (b + c i)^2\right) a + \left(8 a^4 - 2 (b + c i)^2 a^2 + (b + c i)^4\right) \exp\left(\frac{(b + c i)^2}{4 a^2} + a^2 z^2 \right) \sqrt{\pi} \operatorname{erf}\left(\frac{b + c i}{2 a} - a z \right) \right) \right)$$

06.25.21.0067.01

$$\int z^3 e^{bz}\cos(cz)\,\text{erf}(az)\,dz = \frac{1}{16a^6\sqrt{\pi}} e^{-a^2z^2}\left(\frac{1}{(b-ic)^4}\left(8e^{z(za^2+b-ic)}\sqrt{\pi}\left((b-ic)^3z^3-3(b-ic)^2z^2+6(b-ic)z-6\right)\text{erf}(az)a^6+ 2(b-ic)e^{(b-ic)z}\left(4\left((b-ic)^2z^2-3(b-ic)z+6\right)a^4+2(b-ic)^2(bz-icz-1)a^2+(b-ic)^4\right)a- \left(48a^6-12(b-ic)^2a^4-(b-ic)^6\right)\exp\left(\frac{(b-ic)^2}{4a^2}+a^2z^2\right)\sqrt{\pi}\,\text{erf}\left(\frac{b-ic}{2a}-az\right)\right)+ \frac{1}{(b+ci)^4}\left(8e^{z(za^2+b+ci)}\sqrt{\pi}\left((b+ci)^3z^3-3(b+ci)^2z^2+6(b+ci)z-6\right)\text{erf}(az)a^6+ 2(b+ci)e^{(b+ci)z}\left(4\left((b+ci)^2z^2-3(b+ci)z+6\right)a^4+2(b+ci)^2(bz+ciz-1)a^2+(b+ci)^4\right)a- \left(48a^6-12(b+ci)^2a^4-(b+ci)^6\right)\exp\left(\frac{(b+ci)^2}{4a^2}+a^2z^2\right)\sqrt{\pi}\,\text{erf}\left(\frac{b+ci}{2a}-az\right)\right)\right)$$

06.25.21.0068.01

$$\int z^{\alpha-1}e^{bz^2}\cos(cz^2)\,\text{erf}(az)\,dz = -\frac{1}{2\sqrt{\pi}}az^{\alpha+1}\left(\sum_{k=0}^{\infty}\frac{(b+ci)^{-k}a^{2k}\,\Gamma\left(\frac{\alpha+1}{2}+k,-(b+ci)z^2\right)}{(2k+1)k!}\left(-(b+ci)z^2\right)^{\frac{1}{2}(-\alpha-1)}+ \left(-(b-ic)z^2\right)^{\frac{1}{2}(-\alpha-1)}\sum_{k=0}^{\infty}\frac{(b-ic)^{-k}a^{2k}\,\Gamma\left(\frac{\alpha+1}{2}+k,-(b-ic)z^2\right)}{(2k+1)k!}\right)$$

06.25.21.0069.01

$$\int ze^{bz^2}\cos(cz^2)\,\text{erf}(az)\,dz = \left(az\left(\sqrt{(a^2-b+ci)z^2}\,b+\sqrt{(a^2-b-ic)z^2}\,b-(b+ci)\sqrt{(a^2-b-ic)z^2}\,\text{erf}\left(\sqrt{(a^2-b+ci)z^2}\right)- (b-ic)\sqrt{(a^2-b+ci)z^2}\,\text{erf}\left(\sqrt{(a^2-b-ic)z^2}\right)-ic\sqrt{(a^2-b+ci)z^2}+ci\sqrt{(a^2-b-ic)z^2}\right)+ 2e^{bz^2}\sqrt{(a^2-b+ci)z^2}\sqrt{(a^2-b-ic)z^2}\,\text{erf}(az)\left(b\cos(cz^2)+c\sin(cz^2)\right)\right)\Big/ \left(4\left(b^2+c^2\right)\sqrt{(a^2-b-ic)z^2}\sqrt{(a^2-b+ci)z^2}\right)$$

06.25.21.0070.01

$$\int\frac{e^{bz^2}\cos(cz^2)\,\text{erf}(az)}{z}\,dz = -\frac{az}{2\sqrt{\pi}} \left(\frac{1}{\sqrt{-(b-ic)z^2}}\sum_{k=0}^{\infty}\frac{(b-ic)^{-k}a^{2k}\,\Gamma\left(k+\frac{1}{2},-(b-ic)z^2\right)}{(2k+1)k!}+\frac{1}{\sqrt{-(b+ci)z^2}}\sum_{k=0}^{\infty}\frac{(b+ci)^{-k}a^{2k}\,\Gamma\left(k+\frac{1}{2},-(b+ci)z^2\right)}{(2k+1)k!}\right)$$

Involving hyperbolic functions

Involving sinh

06.25.21.0071.01

$$\int \sinh(b\,z)\,\mathrm{erf}(a\,z)\,dz = \frac{1}{2\,b}\left(2\cosh(b\,z)\,\mathrm{erf}(a\,z) + \exp\left(\frac{b^2}{4\,a^2}\right)\left(\mathrm{erf}\left(\frac{b}{2\,a} - a\,z\right) - \mathrm{erf}\left(\frac{b}{2\,a} + a\,z\right)\right)\right)$$

06.25.21.0072.01

$$\int \sinh\left(b\,z^2\right)\mathrm{erf}(a\,z)\,dz = \frac{1}{2\sqrt{\pi}\,b}\left(\sum_{k=0}^{\infty}\frac{(-b)^{-k}\,a^{2k+1}\,\Gamma\left(k+1,\,b\,z^2\right)}{(2\,k+1)\,k!} + \sum_{k=0}^{\infty}\frac{b^{-k}\,a^{2k+1}\,\Gamma\left(k+1,\,-b\,z^2\right)}{(2\,k+1)\,k!}\right)$$

Involving cosh

06.25.21.0073.01

$$\int \cosh(b\,z)\,\mathrm{erf}(a\,z)\,dz = \frac{1}{2\,b}\left(e^{\frac{b^2}{4a^2}}\left(\mathrm{erf}\left(\frac{b}{2\,a} - a\,z\right) + \mathrm{erf}\left(\frac{b}{2\,a} + a\,z\right)\right) + 2\,\mathrm{erf}(a\,z)\sinh(b\,z)\right)$$

06.25.21.0074.01

$$\int \cosh\left(b\,z^2\right)\mathrm{erf}(a\,z)\,dz = \frac{1}{2\sqrt{\pi}\,b}\left(\sum_{k=0}^{\infty}\frac{b^{-k}\,a^{2k+1}\,\Gamma\left(k+1,\,-b\,z^2\right)}{(2\,k+1)\,k!} - \sum_{k=0}^{\infty}\frac{(-b)^{-k}\,a^{2k+1}\,\Gamma\left(k+1,\,b\,z^2\right)}{(2\,k+1)\,k!}\right)$$

Involving hyperbolic functions and a power function

Involving sinh and power

06.25.21.0075.01

$$\int z^{\alpha-1}\sinh(b\,z)\,\mathrm{erf}(a\,z)\,dz = \frac{a\,z^{\alpha}}{b\sqrt{\pi}}\sum_{k=0}^{\infty}\frac{(-1)^k\,a^{2k}\,b^{-2k}}{(2\,k+1)\,k!}\left((b\,z)^{-\alpha}\,\Gamma(2\,k+\alpha+1,\,b\,z) + (-b\,z)^{-\alpha}\,\Gamma(2\,k+\alpha+1,\,-b\,z)\right)$$

06.25.21.0076.01

$$\int z^n \sinh(b z) \operatorname{erf}(a z) \, dz = \frac{(-1)^{n-1} b^{-2n}}{2 b \sqrt{\pi}} \exp\left(\frac{b^2}{4 a^2}\right) \left(-a n! \, b^n \sum_{m=0}^{n} \frac{1}{m!} (-b)^m \left(-a^2\right)^{\frac{1}{2}(-m-1)} \sum_{k=0}^{m} \binom{m}{k} \left(-\frac{b}{2\sqrt{-a^2}}\right)^{m-k} \left(\sqrt{-a^2} \, z + \frac{b}{2\sqrt{-a^2}}\right)^{k+1} \left(-\left(\sqrt{-a^2} \, z + \frac{b}{2\sqrt{-a^2}}\right)^2\right)^{\frac{1}{2}(-k-1)} \Gamma\left(\frac{k+1}{2}, -\left(\sqrt{-a^2} \, z + \frac{b}{2\sqrt{-a^2}}\right)^2\right) + -(-b)^n \exp\left(-\frac{b^2}{4 a^2}\right) \sqrt{\pi} \operatorname{erf}(a z) \left(\Gamma(n+1, b z) + (-1)^n \Gamma(n+1, -b z)\right) - a (-b)^n n! \sum_{m=0}^{n} \frac{1}{m!} b^m \left(-a^2\right)^{\frac{1}{2}(-m-1)} \sum_{k=0}^{m} \binom{m}{k} \left(\frac{b}{2\sqrt{-a^2}}\right)^{m-k} \left(\sqrt{-a^2} \, z - \frac{b}{2\sqrt{-a^2}}\right)^{k+1} \left(-\left(\sqrt{-a^2} \, z - \frac{b}{2\sqrt{-a^2}}\right)^2\right)^{\frac{1}{2}(-k-1)} \Gamma\left(\frac{k+1}{2}, -\left(\sqrt{-a^2} \, z - \frac{b}{2\sqrt{-a^2}}\right)^2\right)\right) /; n \in \mathbb{N}$$

06.25.21.0077.01

$$\int z \sinh(b z) \operatorname{erf}(a z) \, dz = \frac{1}{4 a^2 b^2 \sqrt{\pi}} e^{-z (z a^2 + b)} \left(2 e^{a^2 z^2} \sqrt{\pi} \left(b z + e^{2 b z} (b z - 1) + 1\right) \operatorname{erf}(a z) a^2 - 2 e^{\frac{(2 z a^2 + b)^2}{4 a^2}} \sqrt{\pi} \operatorname{erf}\left(\frac{b}{2 a} + a z\right) a^2 + 2 b e^{2 b z} a + 2 b a + b^2 e^{\frac{(2 z a^2 + b)^2}{4 a^2}} \sqrt{\pi} \operatorname{erf}\left(\frac{b}{2 a} + a z\right) - \left(2 a^2 - b^2\right) e^{\frac{(2 z a^2 + b)^2}{4 a^2}} \sqrt{\pi} \operatorname{erf}\left(\frac{b}{2 a} - a z\right)\right)$$

06.25.21.0078.01

$$\int z^2 \sinh(b z) \operatorname{erf}(a z) \, dz = \frac{1}{8 a^4 b^3 \sqrt{\pi}} e^{-z (z a^2 + b)} \left(4 e^{a^2 z^2} \sqrt{\pi} \left(b^2 z^2 + 2 b z + e^{2 b z} \left(b^2 z^2 - 2 b z + 2\right) + 2\right) \operatorname{erf}(a z) a^4 - 8 e^{\frac{(2 z a^2 + b)^2}{4 a^2}} \sqrt{\pi} \operatorname{erf}\left(\frac{b}{2 a} + a z\right) a^4 - 8 b e^{2 b z} a^3 + 8 b a^3 + 4 b^2 e^{2 b z} z a^3 + 4 b^2 z a^3 + 2 b^2 e^{\frac{(2 z a^2 + b)^2}{4 a^2}} \sqrt{\pi} \operatorname{erf}\left(\frac{b}{2 a} + a z\right) a^2 + 2 b^3 e^{2 b z} a - 2 b^3 a - b^4 e^{\frac{(2 z a^2 + b)^2}{4 a^2}} \sqrt{\pi} \operatorname{erf}\left(\frac{b}{2 a} + a z\right) + \left(8 a^4 - 2 b^2 a^2 + b^4\right) e^{\frac{(2 z a^2 + b)^2}{4 a^2}} \sqrt{\pi} \operatorname{erf}\left(\frac{b}{2 a} - a z\right)\right)$$

06.25.21.0079.01

$$\int z^3 \sinh(b\,z)\,\mathrm{erf}(a\,z)\,dz = \frac{1}{16\,a^6\,b^4\,\sqrt{\pi}}\,e^{-z\left(z\,a^2+b\right)}\left(8\,e^{a^2 z^2}\,\sqrt{\pi}\,\left(b^3 z^3+3\,b^2 z^2+6\,b\,z+e^{2\,b\,z}\left(b^3 z^3-3\,b^2 z^2+6\,b\,z-6\right)+6\right)\mathrm{erf}(a\,z)\,a^6 - 48\,e^{\frac{\left(2\,z\,a^2+b\right)^2}{4\,a^2}}\,\sqrt{\pi}\,\mathrm{erf}\!\left(\frac{b}{2\,a}+a\,z\right)a^6+48\,b\,e^{2\,b\,z}\,a^5+8\,b^3\,e^{2\,b\,z}\,z^2\,a^5+8\,b^3\,z^2\,a^5+48\,b\,a^5-24\,b^2\,e^{2\,b\,z}\,z\,a^5+24\,b^2\,z\,a^5+12\,b^2\,e^{\frac{\left(2\,z\,a^2+b\right)^2}{4\,a^2}}\,\sqrt{\pi}\,\mathrm{erf}\!\left(\frac{b}{2\,a}+a\,z\right)a^4-4\,b^3\,e^{2\,b\,z}\,a^3-4\,b^3\,a^3+4\,b^4\,e^{2\,b\,z}\,z\,a^3-4\,b^4\,z\,a^3+2\,b^5\,e^{2\,b\,z}\,a+2\,b^5\,a+b^6\,e^{\frac{\left(2\,z\,a^2+b\right)^2}{4\,a^2}}\,\sqrt{\pi}\,\mathrm{erf}\!\left(\frac{b}{2\,a}+a\,z\right)-\left(48\,a^6-12\,b^2\,a^4-b^6\right)e^{\frac{\left(2\,z\,a^2+b\right)^2}{4\,a^2}}\,\sqrt{\pi}\,\mathrm{erf}\!\left(\frac{b}{2\,a}-a\,z\right)\right)$$

06.25.21.0080.01

$$\int z^{\alpha-1} \sinh\left(b\,z^2\right)\mathrm{erf}(a\,z)\,dz = \frac{a\,z^{\alpha+1}}{2\,\sqrt{\pi}}\left(-b^2\,z^4\right)^{\frac{1}{2}(-\alpha-1)}\left(\left(-b\,z^2\right)^{\frac{\alpha+1}{2}}\sum_{k=0}^{\infty}\frac{(-b)^{-k}\,a^{2\,k}\,\Gamma\!\left(\frac{\alpha+1}{2}+k,\,b\,z^2\right)}{(2\,k+1)\,k!}-\left(b\,z^2\right)^{\frac{\alpha+1}{2}}\sum_{k=0}^{\infty}\frac{b^{-k}\,a^{2\,k}\,\Gamma\!\left(\frac{\alpha+1}{2}+k,\,-b\,z^2\right)}{(2\,k+1)\,k!}\right)$$

06.25.21.0081.01

$$\int z \sinh\left(b\,z^2\right)\mathrm{erf}(c+a\,z)\,dz = \frac{1}{4\left(b^3-a^4\,b\right)}\,e^{-c^2}\left(a\left(\sqrt{a^2-b}\,\left(a^2+b\right)\exp\!\left(\frac{a^2\,c^2}{a^2-b}\right)\mathrm{erf}\!\left(\frac{z\,a^2+c\,a-b\,z}{\sqrt{a^2-b}}\right)+\left(a^2-b\right)\sqrt{a^2+b}\,\exp\!\left(\frac{a^2\,c^2}{a^2+b}\right)\mathrm{erf}\!\left(\frac{z\,a^2+c\,a+b\,z}{\sqrt{a^2+b}}\right)\right)-2\left(a^4-b^2\right)e^{c^2}\cosh\left(b\,z^2\right)\mathrm{erf}(c+a\,z)\right)$$

06.25.21.0082.01

$$\int z \sinh\left(b\,z^2\right)\mathrm{erf}(a\,z)\,dz = \frac{1}{4\,b\left(b^2-a^4\right)}\left(a\left(\sqrt{a^2-b}\,\left(a^2+b\right)\mathrm{erf}\!\left(\sqrt{a^2-b}\,z\right)+\left(a^2-b\right)\sqrt{a^2+b}\,\mathrm{erf}\!\left(\sqrt{a^2+b}\,z\right)\right)-2\left(a^4-b^2\right)\cosh\left(b\,z^2\right)\mathrm{erf}(a\,z)\right)$$

06.25.21.0083.01

$$\int z^3 \sinh\left(b\,z^2\right)\mathrm{erf}(a\,z)\,dz = \frac{1}{4\,b^2}\left(-\frac{a\,b\,z^3}{2\,\sqrt{\pi}}\left(\left(\sqrt{\pi}\,\mathrm{erf}\!\left(\sqrt{\left(a^2-b\right)z^2}\right)-\sqrt{\pi}-2\,e^{\left(b-a^2\right)z^2}\,\sqrt{\left(a^2-b\right)z^2}\right)\Big/\left(\left(a^2-b\right)z^2\right)^{3/2}+\left(\sqrt{\pi}\,\mathrm{erf}\!\left(\sqrt{\left(a^2+b\right)z^2}\right)-\sqrt{\pi}-2\,e^{-\left(a^2+b\right)z^2}\,\sqrt{\left(a^2+b\right)z^2}\right)\Big/\left(\left(a^2+b\right)z^2\right)^{3/2}\right)+\frac{a}{a^4-b^2}\left(\sqrt{a^2-b}\,\left(a^2+b\right)\mathrm{erf}\!\left(\sqrt{a^2-b}\,z\right)+\left(b-a^2\right)\sqrt{a^2+b}\,\mathrm{erf}\!\left(\sqrt{a^2+b}\,z\right)\right)+2\,\mathrm{erf}(a\,z)\left(b\,z^2\cosh\left(b\,z^2\right)-\sinh\left(b\,z^2\right)\right)\right)$$

06.25.21.0084.01

$$\int \frac{\sinh(b z^2)\,\text{erf}(a z)}{z}\,dz = \frac{a z}{2\sqrt{\pi b z^2}} \sum_{k=0}^{\infty} \frac{(-b)^{-k} a^{2k}\,\Gamma\!\left(k+\frac{1}{2}, b z^2\right)}{(2k+1)\,k!} - \frac{a z}{2\sqrt{-\pi b z^2}} \sum_{k=0}^{\infty} \frac{b^{-k} a^{2k}\,\Gamma\!\left(k+\frac{1}{2}, -b z^2\right)}{(2k+1)\,k!}$$

Involving cosh and power

06.25.21.0085.01

$$\int z^{\alpha-1}\cosh(b z)\,\text{erf}(a z)\,dz = \frac{a z^{\alpha}}{b\sqrt{\pi}} \sum_{k=0}^{\infty} \frac{(-1)^k a^{2k} b^{-2k}}{(2k+1)\,k!} \left((-b z)^{-\alpha}\,\Gamma(2k+\alpha+1, -b z) - (b z)^{-\alpha}\,\Gamma(2k+\alpha+1, b z)\right)$$

06.25.21.0086.01

$$\int z^n \cosh(b z)\,\text{erf}(a z)\,dz =$$

$$\frac{(-1)^n b^{-2n}}{2 b\sqrt{\pi}} \exp\!\left(\frac{b^2}{4a^2}\right) \Bigg(a\,n!\,b^n \sum_{m=0}^{n} \frac{(-b)^m \left(-a^2\right)^{\frac{1}{2}(-m-1)}}{m!} \sum_{k=0}^{m} \binom{m}{k} \left(-\frac{b}{2\sqrt{-a^2}}\right)^{m-k} \left(\sqrt{-a^2}\,z + \frac{b}{2\sqrt{-a^2}}\right)^{k+1}$$

$$\left(-\left(\sqrt{-a^2}\,z + \frac{b}{2\sqrt{-a^2}}\right)^2\right)^{\frac{1}{2}(-k-1)} \Gamma\!\left(\frac{k+1}{2}, -\left(\sqrt{-a^2}\,z + \frac{b}{2\sqrt{-a^2}}\right)^2\right) -$$

$$a\,n!\,(-b)^n \sum_{m=0}^{n} \frac{b^m \left(-a^2\right)^{\frac{1}{2}(-m-1)}}{m!} \sum_{k=0}^{m} \binom{m}{k} \left(\frac{b}{2\sqrt{-a^2}}\right)^{m-k} \left(\sqrt{-a^2}\,z - \frac{b}{2\sqrt{-a^2}}\right)^{k+1}$$

$$\left(-\left(\sqrt{-a^2}\,z - \frac{b}{2\sqrt{-a^2}}\right)^2\right)^{\frac{1}{2}(-k-1)} \Gamma\!\left(\frac{k+1}{2}, -\left(\sqrt{-a^2}\,z - \frac{b}{2\sqrt{-a^2}}\right)^2\right)\Bigg) -$$

$$\sqrt{\pi}\,(-b)^n e^{-\frac{b^2}{4a^2}}\,\text{erf}(a z)\,(\Gamma(n+1, b z) - (-1)^n\,\Gamma(n+1, -b z)) \;/;\; n \in \mathbb{N}$$

06.25.21.0087.01

$$\int z\cosh(b z)\,\text{erf}(a z)\,dz =$$

$$\frac{1}{4a^2 b^2\sqrt{\pi}} e^{-z(z a^2+b)} \Bigg(-2 e^{a^2 z^2}\sqrt{\pi}\left(b z + e^{2 b z}(1 - b z) + 1\right)\text{erf}(a z)\,a^2 + 2 e^{\frac{(2 z a^2+b)^2}{4a^2}}\sqrt{\pi}\,\text{erf}\!\left(\frac{b}{2a} + a z\right) a^2 +$$

$$2 b e^{2 b z} a - 2 b a - b^2 e^{\frac{(2 z a^2+b)^2}{4a^2}}\sqrt{\pi}\,\text{erf}\!\left(\frac{b}{2a} + a z\right) - \left(2a^2 - b^2\right) e^{\frac{(2 z a^2+b)^2}{4a^2}}\sqrt{\pi}\,\text{erf}\!\left(\frac{b}{2a} - a z\right)\Bigg)$$

06.25.21.0088.01

$$\int z^2 \cosh(b\,z)\,\mathrm{erf}(a\,z)\,dz = \frac{1}{8\,a^4\,b^3\,\sqrt{\pi}}\,e^{-z\,(z\,a^2+b)} \left(8\exp\left(\frac{(2\,z\,a^2+b)^2}{4\,a^2}\right)\sqrt{\pi}\,\mathrm{erf}\left(\frac{b}{2\,a}+a\,z\right)a^4 - 8\,e^{a^2\,z^2}\,\sqrt{\pi}\,\mathrm{erf}(a\,z)\left(-b^2\,e^{b\,z}\sinh(b\,z)\,z^2+b\,z+e^{2\,b\,z}\,(b\,z-1)+1\right)a^4 - 8\,b\,e^{2\,b\,z}\,a^3 - 8\,b\,a^3 + 4\,b^2\,e^{2\,b\,z}\,z\,a^3 - 4\,b^2\,z\,a^3 - 2\,b^2\,e^{\frac{(2\,z\,a^2+b)^2}{4\,a^2}}\,\sqrt{\pi}\,\mathrm{erf}\left(\frac{b}{2\,a}+a\,z\right)a^2 + 2\,b^3\,e^{2\,b\,z}\,a + 2\,b^3\,a + b^4\exp\left(\frac{(2\,z\,a^2+b)^2}{4\,a^2}\right)\sqrt{\pi}\,\mathrm{erf}\left(\frac{b}{2\,a}+a\,z\right) + \left(8\,a^4-2\,b^2\,a^2+b^4\right)e^{\frac{(2\,z\,a^2+b)^2}{4\,a^2}}\,\sqrt{\pi}\,\mathrm{erf}\left(\frac{b}{2\,a}-a\,z\right)\right)$$

06.25.21.0089.01

$$\int z^3 \cosh(b\,z)\,\mathrm{erf}(a\,z)\,dz = \frac{1}{16\,a^6\,b^4\,\sqrt{\pi}}\,e^{-z\,(z\,a^2+b)} \left(-8\,e^{a^2\,z^2}\,\sqrt{\pi}\left(b^3\,z^3+3\,b^2\,z^2+6\,b\,z+e^{2\,b\,z}\left(-b^3\,z^3+3\,b^2\,z^2-6\,b\,z+6\right)+6\right)\mathrm{erf}(a\,z)\,a^6 + 48\exp\left(\frac{(2\,z\,a^2+b)^2}{4\,a^2}\right)\sqrt{\pi}\,\mathrm{erf}\left(\frac{b}{2\,a}+a\,z\right)a^6 + 48\,b\,e^{2\,b\,z}\,a^5 + 8\,b^3\,e^{2\,b\,z}\,z^2\,a^5 - 8\,b^3\,z^2\,a^5 - 48\,b\,a^5 - 24\,b^2\,e^{2\,b\,z}\,z\,a^5 - 24\,b^2\,z\,a^5 - 12\,b^2\exp\left(\frac{(2\,z\,a^2+b)^2}{4\,a^2}\right)\sqrt{\pi}\,\mathrm{erf}\left(\frac{b}{2\,a}+a\,z\right)a^4 - 4\,b^3\,e^{2\,b\,z}\,a^3 + 4\,b^3\,a^3 + 4\,b^4\,e^{2\,b\,z}\,z\,a^3 + 4\,b^4\,z\,a^3 + 2\,b^5\,e^{2\,b\,z}\,a - 2\,b^5\,a - b^6\,e^{\frac{(2\,z\,a^2+b)^2}{4\,a^2}}\,\sqrt{\pi}\,\mathrm{erf}\left(\frac{b}{2\,a}+a\,z\right) - \left(48\,a^6-12\,b^2\,a^4-b^6\right)\exp\left(\frac{(2\,z\,a^2+b)^2}{4\,a^2}\right)\sqrt{\pi}\,\mathrm{erf}\left(\frac{b}{2\,a}-a\,z\right)\right)$$

06.25.21.0090.01

$$\int z^{\alpha-1}\cosh\left(b\,z^2\right)\mathrm{erf}(a\,z)\,dz = \frac{a\,z^{\alpha+1}}{2\,\sqrt{\pi}}\left(-b^2\,z^4\right)^{\frac{1}{2}(-\alpha-1)}\left(-\left(-b\,z^2\right)^{\frac{\alpha+1}{2}}\sum_{k=0}^{\infty}\frac{(-b)^{-k}\,a^{2\,k}\,\Gamma\left(\frac{\alpha+1}{2}+k,\,b\,z^2\right)}{(2\,k+1)\,k!} - \left(b\,z^2\right)^{\frac{\alpha+1}{2}}\sum_{k=0}^{\infty}\frac{b^{-k}\,a^{2\,k}\,\Gamma\left(\frac{\alpha+1}{2}+k,\,-b\,z^2\right)}{(2\,k+1)\,k!}\right)$$

06.25.21.0091.01

$$\int z\cosh\left(b\,z^2\right)\mathrm{erf}(c+a\,z)\,dz = -\frac{1}{4\left(b^3-a^4\,b\right)}\left(a\left(a^2-b\right)\sqrt{a^2+b}\,\exp\left(-\frac{b\,c^2}{a^2+b}\right)\mathrm{erf}\left(\frac{z\,a^2+c\,a+b\,z}{\sqrt{a^2+b}}\right) + \left(a^2+b\right)\left(a\,\sqrt{b-a^2}\,\exp\left(\frac{b\,c^2}{a^2-b}\right)\mathrm{erfi}\left(\frac{-z\,a^2-a\,c+b\,z}{\sqrt{b-a^2}}\right)+2\left(a^2-b\right)\mathrm{erf}(c+a\,z)\sinh\left(b\,z^2\right)\right)\right)$$

06.25.21.0092.01

$$\int z\cosh(b\,z^2)\,\mathrm{erf}(a\,z)\,dz = \frac{1}{4\,b\,(a^4-b^2)}\left(a\,(a^2-b)\sqrt{a^2+b}\;\mathrm{erf}\left(\sqrt{a^2+b}\;z\right)+(a^2+b)\left(a\sqrt{b-a^2}\;\mathrm{erfi}\left(\sqrt{b-a^2}\;z\right)+2\,(a^2-b)\,\mathrm{erf}(a\,z)\sinh(b\,z^2)\right)\right)$$

06.25.21.0093.01

$$\int z^3\cosh(b\,z^2)\,\mathrm{erf}(a\,z)\,dz = \frac{1}{4\,b^2}\left(-\frac{a\,b\,z^3}{2\sqrt{\pi}}\left(\left(\sqrt{\pi}\;\mathrm{erf}\left(\sqrt{(a^2-b)\,z^2}\right)-\sqrt{\pi}\;-2\,e^{(b-a^2)\,z^2}\sqrt{(a^2-b)\,z^2}\right)\Big/\left((a^2-b)\,z^2\right)^{3/2}+\left(-\sqrt{\pi}\;\mathrm{erf}\left(\sqrt{(a^2+b)\,z^2}\right)+\sqrt{\pi}\;+2\,e^{-(a^2+b)\,z^2}\sqrt{(a^2+b)\,z^2}\right)\Big/\left((a^2+b)\,z^2\right)^{3/2}\right)+\frac{a}{a^4-b^2}\left((a^2-b)\sqrt{a^2+b}\;\mathrm{erf}\left(\sqrt{a^2+b}\;z\right)-\sqrt{b-a^2}\;(a^2+b)\,\mathrm{erfi}\left(\sqrt{b-a^2}\;z\right)\right)+2\,\mathrm{erf}(a\,z)\left(b\,z^2\sinh(b\,z^2)-\cosh(b\,z^2)\right)\right)$$

06.25.21.0094.01

$$\int\frac{\cosh(b\,z^2)\,\mathrm{erf}(a\,z)}{z}\,dz = -\frac{a\,z}{2\sqrt{\pi\,b\,z^2}}\sum_{k=0}^{\infty}\frac{(-b)^{-k}\,a^{2k}\,\Gamma\left(k+\frac{1}{2},\,b\,z^2\right)}{(2\,k+1)\,k!}-\frac{a\,z}{2\sqrt{-\pi\,b\,z^2}}\sum_{k=0}^{\infty}\frac{b^{-k}\,a^{2k}\,\Gamma\left(k+\frac{1}{2},\,-b\,z^2\right)}{(2\,k+1)\,k!}$$

Involving exponential function and hyperbolic functions

Involving exp and sinh

06.25.21.0095.01

$$\int e^{b\,z}\sinh(c\,z)\,\mathrm{erf}(a\,z)\,dz = \frac{1}{2\,(b^2-c^2)}\left(\exp\left(\frac{(b-c)^2}{4\,a^2}\right)\left((b-c)\,e^{\frac{b\,c}{a^2}}\;\mathrm{erf}\left(\frac{-2\,z\,a^2+b+c}{2\,a}\right)-(b+c)\,\mathrm{erf}\left(\frac{-2\,z\,a^2+b-c}{2\,a}\right)\right)+2\,e^{b\,z}\,\mathrm{erf}(a\,z)\,(b\sinh(c\,z)-c\cosh(c\,z))\right)$$

06.25.21.0096.01

$$\int e^{b\,z^2}\sinh(c\,z^2)\,\mathrm{erf}(a\,z)\,dz = \frac{1}{2\sqrt{\pi}\;(b+c)}\sum_{k=0}^{\infty}\frac{(b+c)^{-k}\,a^{2k+1}\,\Gamma\left(k+1,\,-(b+c)\,z^2\right)}{(2\,k+1)\,k!}-\frac{1}{2\sqrt{\pi}\;(b-c)}\sum_{k=0}^{\infty}\frac{(b-c)^{-k}\,a^{2k+1}\,\Gamma\left(k+1,\,-(b-c)\,z^2\right)}{(2\,k+1)\,k!}$$

Involving exp and cosh

06.25.21.0097.01

$$\int e^{b\,z}\cosh(c\,z)\,\mathrm{erf}(a\,z)\,dz = \frac{1}{2\,(b^2-c^2)}\left(e^{\frac{(b-c)^2}{4\,a^2}}\left((b+c)\,\mathrm{erf}\left(\frac{-2\,z\,a^2+b-c}{2\,a}\right)+(b-c)\,e^{\frac{b\,c}{a^2}}\;\mathrm{erf}\left(\frac{-2\,z\,a^2+b+c}{2\,a}\right)\right)+2\,e^{b\,z}\,\mathrm{erf}(a\,z)\,(b\cosh(c\,z)-c\sinh(c\,z))\right)$$

06.25.21.0098.01

$$\int e^{b z^2} \cosh(c z^2) \operatorname{erf}(a z) \, dz = \frac{1}{2\sqrt{\pi}\,(b-c)} \sum_{k=0}^{\infty} \frac{(b-c)^{-k} a^{2k+1} \Gamma(k+1, -(b-c) z^2)}{(2k+1)\, k!} + \frac{1}{2\sqrt{\pi}\,(b+c)} \sum_{k=0}^{\infty} \frac{(b+c)^{-k} a^{2k+1} \Gamma(k+1, -(b+c) z^2)}{(2k+1)\, k!}$$

Involving power, exponential and hyperbolic functions

Involving power, exp and sinh

06.25.21.0099.01

$$\int z^{\alpha-1} e^{b z} \sinh(c z) \operatorname{erf}(a z) \, dz = \frac{a z^{\alpha} (-(b+c) z)^{-\alpha}}{(b+c)\sqrt{\pi}} \sum_{k=0}^{\infty} \frac{(-1)^k a^{2k} (b+c)^{-2k}}{(2k+1)\, k!} \Gamma(2k+\alpha+1, -(b+c) z) - \frac{a z^{\alpha} (-(b-c) z)^{-\alpha}}{(b-c)\sqrt{\pi}} \sum_{k=0}^{\infty} \frac{(-1)^k a^{2k} (b-c)^{-2k}}{(2k+1)\, k!} \Gamma(2k+\alpha+1, -(b-c) z)$$

06.25.21.0100.01

$$\int z^n e^{b z} \sinh(c z) \operatorname{erf}(a z) \, dz = -\frac{1}{2} (-c-b)^{-n-1} \operatorname{erf}(a z) \Gamma(n+1, (-c-b) z) + \frac{1}{2} (-b+c)^{-n-1} \operatorname{erf}(a z) \Gamma(n+1, (-b+c) z) + \frac{1}{2\sqrt{\pi}} \left(a (-b+c)^{-n-1} e^{\frac{(b-c)^2}{4a^2}} n! \sum_{m=0}^{n} \frac{1}{m!} \left((-(b-c))^m (-a^2)^{\frac{1}{2}(-m-1)} \sum_{k=0}^{m} \binom{m}{k} \left(-\frac{b-c}{2\sqrt{-a^2}} \right)^{m-k} \left(\frac{b-c}{2\sqrt{-a^2}} + \sqrt{-a^2}\, z \right)^{k+1} \left(-\left(\frac{b-c}{2\sqrt{-a^2}} + \sqrt{-a^2}\, z \right)^2 \right)^{\frac{1}{2}(-k-1)} \Gamma\left(\frac{k+1}{2}, -\left(\frac{b-c}{2\sqrt{-a^2}} + \sqrt{-a^2}\, z \right)^2 \right) \right) \right) - \frac{1}{2\sqrt{\pi}} \left(a (-c-b)^{-n-1} e^{\frac{(b+c)^2}{4a^2}} n! \sum_{m=0}^{n} \frac{1}{m!} \left((-(b+c))^m (-a^2)^{\frac{1}{2}(-m-1)} \sum_{k=0}^{m} \binom{m}{k} \left(-\frac{b+c}{2\sqrt{-a^2}} \right)^{m-k} \left(\frac{b+c}{2\sqrt{-a^2}} + \sqrt{-a^2}\, z \right)^{k+1} \left(-\left(\frac{b+c}{2\sqrt{-a^2}} + \sqrt{-a^2}\, z \right)^2 \right)^{\frac{1}{2}(-k-1)} \Gamma\left(\frac{k+1}{2}, -\left(\frac{b+c}{2\sqrt{-a^2}} + \sqrt{-a^2}\, z \right)^2 \right) \right) \right) /; n \in \mathbb{N}$$

06.25.21.0101.01

$$\int z\, e^{b z} \sinh(c z) \operatorname{erf}(a z)\, dz = \frac{1}{4 a^2 \sqrt{\pi}} e^{-a^2 z^2} \left(\frac{1}{(b+c)^2} \left(2 e^{z (z a^2+b+c)} \sqrt{\pi} (b z + c z - 1) \operatorname{erf}(a z) a^2 + 2 (b+c) e^{(b+c) z} a - \left(2 a^2 - (b+c)^2\right) e^{\frac{(b+c)^2}{4 a^2} + a^2 z^2} \sqrt{\pi} \operatorname{erf}\left(\frac{b+c}{2 a} - a z\right)\right) - \frac{1}{(b-c)^2} \left(2 e^{z (z a^2+b-c)} \sqrt{\pi} (b z - c z - 1) \operatorname{erf}(a z) a^2 + 2 (b-c) e^{(b-c) z} a - \left(2 a^2 - (b-c)^2\right) e^{\frac{(b-c)^2}{4 a^2} + a^2 z^2} \sqrt{\pi} \operatorname{erf}\left(\frac{b-c}{2 a} - a z\right)\right)\right)$$

06.25.21.0102.01

$$\int z^2 e^{b z} \sinh(c z) \operatorname{erf}(a z)\, dz = \frac{1}{8 a^4 \sqrt{\pi}} e^{-a^2 z^2} \left(\frac{1}{(b+c)^3} \left(4 e^{z (z a^2+b+c)} \sqrt{\pi} \left((b+c)^2 z^2 - 2 (b+c) z + 2\right) \operatorname{erf}(a z) a^4 + 2 (b+c) e^{(b+c) z} \left(2 (b z + c z - 2) a^2 + (b+c)^2\right) a + \left(8 a^4 - 2 (b+c)^2 a^2 + (b+c)^4\right) \exp\left(\frac{(b+c)^2}{4 a^2} + a^2 z^2\right) \sqrt{\pi} \operatorname{erf}\left(\frac{b+c}{2 a} - a z\right)\right) - \frac{1}{(b-c)^3} \left(4 e^{z (z a^2+b-c)} \sqrt{\pi} \left((b-c)^2 z^2 - 2 (b-c) z + 2\right) \operatorname{erf}(a z) a^4 + 2 (b-c) e^{(b-c) z} \left(2 (b z - c z - 2) a^2 + (b-c)^2\right) a + \left(8 a^4 - 2 (b-c)^2 a^2 + (b-c)^4\right) \exp\left(\frac{(b-c)^2}{4 a^2} + a^2 z^2\right) \sqrt{\pi} \operatorname{erf}\left(\frac{b-c}{2 a} - a z\right)\right)\right)$$

06.25.21.0103.01

$$\int z^3 e^{b z} \sinh(c z) \operatorname{erf}(a z)\, dz = \frac{1}{16 a^6 \sqrt{\pi}} e^{-a^2 z^2} \left(\frac{1}{(b+c)^4} \left(8 e^{z (z a^2+b+c)} \sqrt{\pi} \left((b+c)^3 z^3 - 3 (b+c)^2 z^2 + 6 (b+c) z - 6\right) \operatorname{erf}(a z) a^6 + 2 (b+c) e^{(b+c) z} \left(4 \left((b+c)^2 z^2 - 3 (b+c) z + 6\right) a^4 + 2 (b+c)^2 (b z + c z - 1) a^2 + (b+c)^4\right) a - \left(48 a^6 - 12 (b+c)^2 a^4 - (b+c)^6\right) \exp\left(\frac{(b+c)^2}{4 a^2} + a^2 z^2\right) \sqrt{\pi} \operatorname{erf}\left(\frac{b+c}{2 a} - a z\right)\right) - \frac{1}{(b-c)^4} \left(8 e^{z (z a^2+b-c)} \sqrt{\pi} \left((b-c)^3 z^3 - 3 (b-c)^2 z^2 + 6 (b-c) z - 6\right) \operatorname{erf}(a z) a^6 + 2 (b-c) e^{(b-c) z} \left(4 \left((b-c)^2 z^2 - 3 (b-c) z + 6\right) a^4 + 2 (b-c)^2 (b z - c z - 1) a^2 + (b-c)^4\right) a - \left(48 a^6 - 12 (b-c)^2 a^4 - (b-c)^6\right) \exp\left(\frac{(b-c)^2}{4 a^2} + a^2 z^2\right) \sqrt{\pi} \operatorname{erf}\left(\frac{b-c}{2 a} - a z\right)\right)\right)$$

06.25.21.0104.01

$$\int z^{\alpha-1} e^{b z^2} \sinh(c z^2) \operatorname{erf}(a z) \, dz = \frac{1}{2\sqrt{\pi}} a z^{\alpha+1} \left(\left(-(b-c) z^2\right)^{\frac{1}{2}(-\alpha-1)} \sum_{k=0}^{\infty} \frac{(b-c)^{-k} a^{2k} \Gamma\left(\frac{\alpha+1}{2}+k, -(b-c) z^2\right)}{(2k+1) k!} - \left(-(b+c) z^2\right)^{\frac{1}{2}(-\alpha-1)} \sum_{k=0}^{\infty} \frac{(b+c)^{-k} a^{2k} \Gamma\left(\frac{\alpha+1}{2}+k, -(b+c) z^2\right)}{(2k+1) k!} \right)$$

06.25.21.0105.01

$$\int z e^{b z^2} \sinh(c z^2) \operatorname{erf}(a z) \, dz = \left(a z \left(\sqrt{\left(a^2-b+c\right) z^2}\, b + (b+c) \sqrt{\left(a^2-b-c\right) z^2} \operatorname{erf}\left(\sqrt{\left(a^2-b+c\right) z^2}\right) + (c-b) \sqrt{\left(a^2-b+c\right) z^2} \operatorname{erf}\left(\sqrt{-\left(-a^2+b+c\right) z^2}\right) - b \sqrt{\left(a^2-b-c\right) z^2} - c \sqrt{\left(a^2-b-c\right) z^2} - c \sqrt{\left(a^2-b+c\right) z^2} \right) + 2 e^{b z^2} \sqrt{\left(a^2-b-c\right) z^2} \sqrt{\left(a^2-b+c\right) z^2} \operatorname{erf}(a z) \left(b \sinh(c z^2) - c \cosh(c z^2)\right) \right) \Big/ \left(4 \left(b^2-c^2\right) \sqrt{\left(a^2-b+c\right) z^2} \sqrt{-\left(-a^2+b+c\right) z^2} \right)$$

06.25.21.0106.01

$$\int \frac{e^{b z^2} \sinh(c z^2) \operatorname{erf}(a z)}{z} \, dz = -\frac{a z}{2\sqrt{\pi}} \left(\frac{1}{\sqrt{-(b+c) z^2}} \sum_{k=0}^{\infty} \frac{(b+c)^{-k} a^{2k} \Gamma\left(k+\frac{1}{2}, -(b+c) z^2\right)}{(2k+1) k!} - \frac{1}{\sqrt{-(b-c) z^2}} \sum_{k=0}^{\infty} \frac{(b-c)^{-k} a^{2k} \Gamma\left(k+\frac{1}{2}, -(b-c) z^2\right)}{(2k+1) k!} \right)$$

Involving power, exp and cosh

06.25.21.0107.01

$$\int z^{\alpha-1} e^{b z} \cosh(c z) \operatorname{erf}(a z) \, dz = \frac{a z^{\alpha} (-(b-c) z)^{-\alpha}}{(b-c)\sqrt{\pi}} \sum_{k=0}^{\infty} \frac{(-1)^k a^{2k} (b-c)^{-2k}}{(2k+1) k!} \Gamma(2k+\alpha+1, -(b-c) z) + \frac{a z^{\alpha} (-(b+c) z)^{-\alpha}}{(b+c)\sqrt{\pi}} \sum_{k=0}^{\infty} \frac{(-1)^k a^{2k} (b+c)^{-2k}}{(2k+1) k!} \Gamma(2k+\alpha+1, -(b+c) z)$$

06.25.21.0108.01

$$\int z^n e^{bz}\cosh(cz)\operatorname{erf}(az)\,dz = -\frac{1}{2}\operatorname{erf}(az)\,\Gamma(n+1,(-c-b)z)\,(-c-b)^{-n-1} - \frac{1}{2}(-b+c)^{-n-1}\operatorname{erf}(az)\,\Gamma(n+1,(-b+c)z) - \frac{1}{2\sqrt{\pi}}\left(a(-b+c)^{-n-1}e^{\frac{(b-c)^2}{4a^2}}\,n!\sum_{m=0}^{n}\frac{1}{m!}\left((-(b-c))^m\left(-a^2\right)^{\frac{1}{2}(-m-1)}\sum_{k=0}^{m}\binom{m}{k}\left(-\frac{b-c}{2\sqrt{-a^2}}\right)^{m-k}\left(\frac{b-c}{2\sqrt{-a^2}}+\sqrt{-a^2}\,z\right)^{k+1}\left(-\left(\frac{b-c}{2\sqrt{-a^2}}+\sqrt{-a^2}\,z\right)^2\right)^{\frac{1}{2}(-k-1)}\Gamma\left(\frac{k+1}{2},-\left(\frac{b-c}{2\sqrt{-a^2}}+\sqrt{-a^2}\,z\right)^2\right)\right)\right) - \frac{1}{2\sqrt{\pi}}\left(a(-c-b)^{-n-1}e^{\frac{(b+c)^2}{4a^2}}\,n!\sum_{m=0}^{n}\frac{1}{m!}\left((-(b+c))^m\left(-a^2\right)^{\frac{1}{2}(-m-1)}\sum_{k=0}^{m}\binom{m}{k}\left(-\frac{b+c}{2\sqrt{-a^2}}\right)^{m-k}\left(\frac{b+c}{2\sqrt{-a^2}}+\sqrt{-a^2}\,z\right)^{k+1}\left(-\left(\frac{b+c}{2\sqrt{-a^2}}+\sqrt{-a^2}\,z\right)^2\right)^{\frac{1}{2}(-k-1)}\Gamma\left(\frac{k+1}{2},-\left(\frac{b+c}{2\sqrt{-a^2}}+\sqrt{-a^2}\,z\right)^2\right)\right)\right)\,/;\,n\in\mathbb{N}$$

06.25.21.0109.01

$$\int z\,e^{bz}\cosh(cz)\operatorname{erf}(az)\,dz = \frac{1}{4a^2\sqrt{\pi}}e^{-a^2z^2}\left(\frac{1}{(b-c)^2}\left(2e^{z(za^2+b-c)}\sqrt{\pi}\,(bz-cz-1)\operatorname{erf}(az)\,a^2+2(b-c)\,e^{(b-c)z}\,a-\left(2a^2-(b-c)^2\right)e^{\frac{(b-c)^2}{4a^2}+a^2z^2}\sqrt{\pi}\operatorname{erf}\left(\frac{b-c}{2a}-az\right)\right)+\frac{1}{(b+c)^2}\left(2e^{z(za^2+b+c)}\sqrt{\pi}\,(bz+cz-1)\operatorname{erf}(az)\,a^2+2(b+c)\,e^{(b+c)z}\,a-\left(2a^2-(b+c)^2\right)e^{\frac{(b+c)^2}{4a^2}+a^2z^2}\sqrt{\pi}\operatorname{erf}\left(\frac{b+c}{2a}-az\right)\right)\right)$$

06.25.21.0110.01

$$\int z^2 e^{bz}\cosh(cz)\operatorname{erf}(az)\,dz = \frac{1}{8a^4\sqrt{\pi}}e^{-a^2z^2}\left(\frac{1}{(b-c)^3}\left(4e^{z(za^2+b-c)}\sqrt{\pi}\left((b-c)^2z^2-2(b-c)z+2\right)\operatorname{erf}(az)\,a^4+2(b-c)\,e^{(b-c)z}\left(2(bz-cz-2)a^2+(b-c)^2\right)a+\left(8a^4-2(b-c)^2a^2+(b-c)^4\right)e^{\frac{(b-c)^2}{4a^2}+a^2z^2}\sqrt{\pi}\operatorname{erf}\left(\frac{b-c}{2a}-az\right)\right)+\frac{1}{(b+c)^3}\left(4e^{z(za^2+b+c)}\sqrt{\pi}\left((b+c)^2z^2-2(b+c)z+2\right)\operatorname{erf}(az)\,a^4+2(b+c)\,e^{(b+c)z}\left(2(bz+cz-2)a^2+(b+c)^2\right)a+\left(8a^4-2(b+c)^2a^2+(b+c)^4\right)e^{\frac{(b+c)^2}{4a^2}+a^2z^2}\sqrt{\pi}\operatorname{erf}\left(\frac{b+c}{2a}-az\right)\right)\right)$$

06.25.21.0111.01

$$\int z^3 e^{b z} \cosh(c z)\operatorname{erf}(a z)\, dz = \frac{1}{16 a^6 \sqrt{\pi}} e^{-a^2 z^2} \left(\frac{1}{(b-c)^4}\left(8 e^{z\left(z a^2+b-c\right)} \sqrt{\pi} \left((b-c)^3 z^3 - 3 (b-c)^2 z^2 + 6 (b-c) z - 6\right) \operatorname{erf}(a z)\, a^6 + 2 (b-c) e^{(b-c) z} \left(4\left((b-c)^2 z^2 - 3 (b-c) z + 6\right) a^4 + 2 (b-c)^2 (b z - c z - 1) a^2 + (b-c)^4\right) a - \left(48 a^6 - 12 (b-c)^2 a^4 - (b-c)^6\right) e^{\frac{(b-c)^2}{4 a^2}+a^2 z^2} \sqrt{\pi} \operatorname{erf}\left(\frac{b-c}{2 a} - a z\right)\right) + \frac{1}{(b+c)^4}\left(8 e^{z\left(z a^2+b+c\right)} \sqrt{\pi} \left((b+c)^3 z^3 - 3 (b+c)^2 z^2 + 6 (b+c) z - 6\right) \operatorname{erf}(a z)\, a^6 + 2 (b+c) e^{(b+c) z} \left(4\left((b+c)^2 z^2 - 3 (b+c) z + 6\right) a^4 + 2 (b+c)^2 (b z + c z - 1) a^2 + (b+c)^4\right) a - \left(48 a^6 - 12 (b+c)^2 a^4 - (b+c)^6\right) e^{\frac{(b+c)^2}{4 a^2}+a^2 z^2} \sqrt{\pi} \operatorname{erf}\left(\frac{b+c}{2 a} - a z\right)\right)\right)$$

06.25.21.0112.01

$$\int z^{\alpha-1} e^{b z^2} \cosh\left(c z^2\right) \operatorname{erf}(a z)\, dz = -\frac{a z^{\alpha+1}}{2\sqrt{\pi}} \left(\left((-b-c) z^2\right)^{-\frac{\alpha+1}{2}} \sum_{k=0}^{\infty} \frac{(b+c)^{-k} a^{2k}\, \Gamma\left(\frac{\alpha+1}{2}+k,\, -(b+c) z^2\right)}{(2k+1)\, k!} + \left((c-b) z^2\right)^{-\frac{\alpha+1}{2}} \sum_{k=0}^{\infty} \frac{(b-c)^{-k} a^{2k}\, \Gamma\left(\frac{\alpha+1}{2}+k,\, -(b-c) z^2\right)}{(2k+1)\, k!}\right)$$

06.25.21.0113.01

$$\int z e^{b z^2} \cosh\left(c z^2\right) \operatorname{erf}(a z)\, dz = \left(a z \left(\sqrt{\left(a^2-b-c\right) z^2}\, b + \sqrt{\left(a^2-b+c\right) z^2}\, b - (b+c) \sqrt{\left(a^2-b-c\right) z^2} \operatorname{erf}\left(\sqrt{\left(a^2-b+c\right) z^2}\right) + (c-b) \sqrt{\left(a^2-b+c\right) z^2} \operatorname{erf}\left(\sqrt{-\left(-a^2+b+c\right) z^2}\right) + c \sqrt{\left(a^2-b-c\right) z^2} - c \sqrt{\left(a^2-b+c\right) z^2}\right) + 2 e^{b z^2} \sqrt{\left(a^2-b-c\right) z^2} \sqrt{\left(a^2-b+c\right) z^2} \operatorname{erf}(a z) \left(b \cosh\left(c z^2\right) - c \sinh\left(c z^2\right)\right)\right) \Big/ \left(4 \left(b^2-c^2\right) \sqrt{\left(a^2-b+c\right) z^2} \sqrt{-\left(-a^2+b+c\right) z^2}\right)$$

06.25.21.0114.01

$$\int \frac{e^{b z^2} \cosh\left(c z^2\right) \operatorname{erf}(a z)}{z}\, dz = -\frac{a z}{2\sqrt{\pi}} \left(\frac{1}{\sqrt{(c-b) z^2}} \sum_{k=0}^{\infty} \frac{(b-c)^{-k} a^{2k}\, \Gamma\left(k+\frac{1}{2},\, -(b-c) z^2\right)}{(2k+1)\, k!} + \frac{1}{\sqrt{(-b-c) z^2}} \sum_{k=0}^{\infty} \frac{(b+c)^{-k} a^{2k}\, \Gamma\left(k+\frac{1}{2},\, -(b+c) z^2\right)}{(2k+1)\, k!}\right)$$

Involving logarithm

Involving log

06.25.21.0115.01

$$\int \log(b\,z)\,\text{erf}(a\,z)\,dz = \frac{1}{2\,a\sqrt{\pi}}\,e^{-a^2 z^2}\left(-e^{a^2 z^2}\,\text{Ei}\left(-a^2 z^2\right) + 2\,a\,e^{a^2 z^2}\sqrt{\pi}\,z\,\text{erf}(a\,z)\,(\log(b\,z) - 1) + 2\log(b\,z) - 2\right)$$

Involving logarithm and a power function

Involving log and power

06.25.21.0116.01

$$\int z^{\alpha-1}\log(b\,z)\,\text{erf}(a\,z)\,dz = \frac{1}{\sqrt{\pi}\,\alpha^2\,(\alpha+1)^2}\,z^{\alpha}\left(a^2 z^2\right)^{\frac{1}{2}(-\alpha-1)}\left(2\,a\,z\,\alpha\,{}_2F_2\left(\frac{\alpha}{2}+\frac{1}{2},\frac{\alpha}{2}+\frac{1}{2};\frac{\alpha}{2}+\frac{3}{2},\frac{\alpha}{2}+\frac{3}{2};-a^2 z^2\right)\left(a^2 z^2\right)^{\frac{\alpha+1}{2}} + (\alpha+1)^2\left(\sqrt{\pi}\,\text{erf}(a\,z)\,(\alpha\log(b\,z)-1)\left(a^2 z^2\right)^{\frac{\alpha+1}{2}} + a\,z\left(\Gamma\left(\frac{\alpha+1}{2},a^2 z^2\right)(\alpha\log(b\,z)-1) - \alpha\,\Gamma\left(\frac{\alpha+1}{2}\right)\log(z)\right)\right)\right)$$

06.25.21.0117.01

$$\int z\log(b\,z)\,\text{erf}(a\,z)\,dz = \frac{a\,z^3}{36\sqrt{\pi}\left(a^2 z^2\right)^{3/2}}\left(a\,z\sqrt{a^2 z^2}\left(4\,a\,z\,{}_2F_2\left(\frac{3}{2},\frac{3}{2};\frac{5}{2},\frac{5}{2};-a^2 z^2\right) + 9\sqrt{\pi}\,\text{erf}(a\,z)\,(2\log(b\,z)-1)\right) - 9\left(\sqrt{\pi}\,\log(z) + \Gamma\left(\frac{3}{2},a^2 z^2\right)(1-2\log(b\,z))\right)\right)$$

06.25.21.0118.01

$$\int z^2\log(b\,z)\,\text{erf}(a\,z)\,dz = \frac{1}{18\,a^3\sqrt{\pi}}\,e^{-a^2 z^2}\left(2\,a^3\,e^{a^2 z^2}\sqrt{\pi}\,\text{erf}(a\,z)\,(3\log(b\,z)-1)\,z^3 - 2\,a^2 z^2 + 6\,a^2\log(b\,z)\,z^2 - 3\,e^{a^2 z^2}\,\text{Ei}\left(-a^2 z^2\right) + 6\log(b\,z) + 1\right)$$

06.25.21.0119.01

$$\int z^3\log(b\,z)\,\text{erf}(a\,z)\,dz = \frac{z}{400\,a^3\sqrt{\pi}\sqrt{a^2 z^2}}\left(a\,z\left(a^2 z^2\right)^{3/2}\left(8\,a\,z\,{}_2F_2\left(\frac{5}{2},\frac{5}{2};\frac{7}{2},\frac{7}{2};-a^2 z^2\right) + 25\sqrt{\pi}\,\text{erf}(a\,z)\,(4\log(b\,z)-1)\right) - 25\left(3\sqrt{\pi}\,\log(z) + \Gamma\left(\frac{5}{2},a^2 z^2\right)(1-4\log(b\,z))\right)\right)$$

Involving functions of the direct function

Involving elementary functions of the direct function

Involving powers of the direct function

06.25.21.0120.01

$$\int \text{erf}(a\,z)^2\,dz = \frac{1}{a}\left(a\,z\,\text{erf}(a\,z)^2 + \frac{2\,e^{-a^2 z^2}}{\sqrt{\pi}}\,\text{erf}(a\,z) - \sqrt{\frac{2}{\pi}}\,\text{erf}\left(\sqrt{2}\,a\,z\right)\right)$$

Involving products of the direct function

06.25.21.0121.01

$$\int \operatorname{erf}(a z) \operatorname{erf}(b z) \, dz = \operatorname{erf}(a z) \left(z \operatorname{erf}(b z) + \frac{e^{-b^2 z^2}}{b \sqrt{\pi}} \right) + \left(e^{-a^2 z^2} \left(b \sqrt{a^2 + b^2} \operatorname{erf}(b z) - (a^2 + b^2) e^{a^2 z^2} \operatorname{erf}\left(\sqrt{a^2 + b^2} \, z \right) \right) \right) \Big/ \left(a b \sqrt{a^2 + b^2} \sqrt{\pi} \right)$$

Involving functions of the direct function and elementary functions

Involving elementary functions of the direct function and elementary functions

Involving powers of the direct function and a power function

06.25.21.0122.01

$$\int z^{\alpha-1} \operatorname{erf}(a z)^2 \, dz = \frac{4 a z^{\alpha+1} (a^2 z^2)^{-\frac{\alpha}{2}}}{\alpha \pi} \sum_{k=0}^{\infty} \frac{(-1)^k \Gamma\left(k + \frac{\alpha}{2} + 1, a^2 z^2\right)}{(2k+1) k!} + \frac{z^\alpha \operatorname{erf}(a z)^2}{\alpha}$$

06.25.21.0123.01

$$\int z \operatorname{erf}(a z)^2 \, dz = \frac{1}{4 a^2 \pi} \left(\pi (2 a^2 z^2 - 1) \operatorname{erf}(a z)^2 + 4 a e^{-a^2 z^2} \sqrt{\pi} \, z \operatorname{erf}(a z) + 2 e^{-2 a^2 z^2} \right)$$

06.25.21.0124.01

$$\int z^2 \operatorname{erf}(a z)^2 \, dz = \frac{1}{12} \left(4 \operatorname{erf}(a z)^2 z^3 + \frac{8 e^{-a^2 z^2} (a^2 z^2 + 1) \operatorname{erf}(a z)}{a^3 \sqrt{\pi}} + \frac{4 a e^{-2 a^2 z^2} z - 5 \sqrt{2 \pi} \operatorname{erf}\left(\sqrt{2} \, a z\right)}{a^3 \pi} \right)$$

06.25.21.0125.01

$$\int z^3 \operatorname{erf}(a z)^2 \, dz = \frac{1}{16 a^4 \pi} \left(e^{-2 a^2 z^2} \left(4 a^2 z^2 + 4 a e^{a^2 z^2} \sqrt{\pi} (2 a^2 z^2 + 3) \operatorname{erf}(a z) z + e^{2 a^2 z^2} \pi (4 a^4 z^4 - 3) \operatorname{erf}(a z)^2 + 8 \right) \right)$$

Involving products of the direct function and a power function

06.25.21.0126.01

$$\int z^{\alpha-1} \operatorname{erf}(a z) \operatorname{erf}(b z) \, dz = \frac{z^\alpha \operatorname{erf}(a z) \operatorname{erf}(b z)}{\alpha} + \frac{2 b z^\alpha (a^2 z^2)^{-\frac{\alpha}{2}}}{\pi \alpha a} \sum_{k=0}^{\infty} \frac{(-a^2)^{-k} b^{2k} \Gamma\left(k + \frac{\alpha}{2} + 1, a^2 z^2\right)}{(2k+1) k!} + \frac{2 a z^\alpha (b^2 z^2)^{-\frac{\alpha}{2}}}{\pi \alpha b} \sum_{k=0}^{\infty} \frac{a^{2k} (-b^2)^{-k} \Gamma\left(k + \frac{\alpha}{2} + 1, b^2 z^2\right)}{(2k+1) k!}$$

06.25.21.0127.01

$$\int z^2 \operatorname{erf}(a z) \operatorname{erf}(b z) \, dz = \frac{1}{6}\left(-\frac{2}{b^3 \sqrt{\pi}}\left(-e^{-b^2 z^2}\left(b^2 z^2+1\right) \operatorname{erf}(a z)+\frac{a \operatorname{erf}\left(\sqrt{a^2+b^2}\, z\right)}{\sqrt{a^2+b^2}}-\frac{a b^2 z^3}{2 \sqrt{\pi}\left(\left(a^2+b^2\right) z^2\right)^{3/2}}\left(-\sqrt{\pi} \operatorname{erf}\left(\sqrt{\left(a^2+b^2\right) z^2}\right)+\sqrt{\pi}+2 e^{-\left(a^2+b^2\right) z^2} \sqrt{\left(a^2+b^2\right) z^2}\right)\right)+2\left(\operatorname{erf}(a z) z^3+\frac{e^{-a^2 z^2}\left(a^2 z^2+1\right)}{a^3 \sqrt{\pi}}\right) \operatorname{erf}(b z)+\frac{b\, e^{-\left(a^2+b^2\right) z^2}}{a^3\left(a^2+b^2\right)^{3/2} \pi}\left(2 a^2 \sqrt{a^2+b^2}\, z-\left(3 a^2+2 b^2\right) e^{\left(a^2+b^2\right) z^2} \sqrt{\pi} \operatorname{erf}\left(\sqrt{a^2+b^2}\, z\right)\right)\right)$$

Involving power of the direct function and exponential function

06.25.21.0128.01

$$\int \frac{e^{-a^2 z^2}}{\operatorname{erf}(a z)} \, dz = \frac{\sqrt{\pi} \log(\operatorname{erf}(a z))}{2 a}$$

06.25.21.0129.01

$$\int e^{-a^2 z^2} \operatorname{erf}(a z)^r \, dz = \frac{\sqrt{\pi} \operatorname{erf}(a z)^{r+1}}{2 a (r+1)}$$

Definite integration

For the direct function itself

06.25.21.0130.01

$$\int_0^{\infty} t^{\alpha-1} \operatorname{erf}(t) \, dt = -\frac{1}{\sqrt{\pi}\, \alpha} \Gamma\left(\frac{\alpha+1}{2}\right) /; -1 < \operatorname{Re}(\alpha) < 0$$

Involving the direct function

06.25.21.0131.01

$$\int_0^{\infty} t^{\alpha-1} e^{-z t} \operatorname{erf}(t) \, dt = z^{-\alpha} \Gamma(\alpha)-\frac{1}{\sqrt{\pi}}\left(\frac{1}{\alpha} \Gamma\left(\frac{\alpha+1}{2}\right) {}_2F_2\left(\frac{\alpha+1}{2}, \frac{\alpha}{2}; \frac{1}{2}, \frac{\alpha}{2}+1; \frac{z^2}{4}\right)-\frac{z}{\alpha+1} \Gamma\left(\frac{\alpha}{2}+1\right) {}_2F_2\left(\frac{\alpha+1}{2}, \frac{\alpha}{2}+1; \frac{3}{2}, \frac{\alpha+3}{2}; \frac{z^2}{4}\right)\right) /; \operatorname{Re}(z)>0 \land \operatorname{Re}(\alpha)>-1$$

Integral transforms

Laplace transforms

06.25.22.0001.01

$$\mathcal{L}_t[\operatorname{erf}(t)](z) = \frac{1}{z} e^{\frac{z^2}{4}} \operatorname{erfc}\left(\frac{z}{2}\right) /; \operatorname{Re}(z) > 0$$

Mellin transforms

06.25.22.0002.01

$$\mathcal{M}_t[\operatorname{erf}(t)](z) = -\frac{1}{\sqrt{\pi}\, z} \Gamma\left(\frac{z+1}{2}\right) /; -1 < \operatorname{Re}(z) < 0$$

Representations through more general functions

Through hypergeometric functions

Involving $_1F_1$

06.25.26.0001.01

$$\operatorname{erf}(z) = \frac{2z}{\sqrt{\pi}}\, {}_1F_1\left(\frac{1}{2}; \frac{3}{2}; -z^2\right)$$

Involving $_pF_q$

06.25.26.0018.01

$$\operatorname{erf}(z) = \frac{2z}{\sqrt{\pi}}\, {}_1F_2\left(\frac{1}{4}; \frac{1}{2}, \frac{5}{4}; \frac{z^4}{4}\right) - \frac{2z^3}{3\sqrt{\pi}}\, {}_1F_2\left(\frac{3}{4}; \frac{3}{2}, \frac{7}{4}; \frac{z^4}{4}\right)$$

Involving hypergeometric U

06.25.26.0002.01

$$\operatorname{erf}(z) = \frac{z}{\sqrt{z^2}}\left(1 - \frac{1}{\sqrt{\pi}} e^{-z^2} U\left(\frac{1}{2}, \frac{1}{2}, z^2\right)\right)$$

Through Meijer G

Classical cases for the direct function itself

06.25.26.0003.01

$$\operatorname{erf}(z) = \frac{z}{\sqrt{\pi}} G_{1,2}^{1,1}\left(z^2 \,\middle|\, \begin{matrix} \frac{1}{2} \\ 0, -\frac{1}{2} \end{matrix}\right)$$

06.25.26.0004.01

$$\operatorname{erf}(z) = \frac{\sqrt{z^2}}{\sqrt{\pi}\, z} G_{1,2}^{1,1}\left(z^2 \,\middle|\, \begin{matrix} 1 \\ \frac{1}{2}, 0 \end{matrix}\right)$$

06.25.26.0005.01

$$\operatorname{erf}(z) = \frac{1}{\sqrt{\pi}} G_{1,2}^{1,1}\left(z^2 \,\middle|\, \begin{matrix} 1 \\ \frac{1}{2}, 0 \end{matrix}\right) /; \operatorname{Re}(z) > 0$$

06.25.26.0006.01

$$\operatorname{erf}\left(\sqrt{z}\right) = \frac{1}{\sqrt{\pi}} G_{1,2}^{1,1}\left(z \,\middle|\, \begin{matrix} 1 \\ \frac{1}{2}, 0 \end{matrix}\right)$$

Classical cases involving exp

06.25.26.0019.01

$$e^{z^2} \operatorname{erf}(z) = -\pi\, G_{2,3}^{1,1}\left(z^2 \,\middle|\, \begin{matrix} \frac{1}{2}, 0 \\ \frac{1}{2}, 0, 0 \end{matrix}\right) /; -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

06.25.26.0020.01

$$e^{z} \operatorname{erf}\left(\sqrt{z}\right) = -\pi\, G_{2,3}^{1,1}\left(z \,\middle|\, \begin{matrix} \frac{1}{2}, 0 \\ \frac{1}{2}, 0, 0 \end{matrix}\right)$$

Classical cases involving erfi

06.25.26.0007.01

$$\operatorname{erf}\left(\sqrt[4]{z}\right) \operatorname{erfi}\left(\sqrt[4]{z}\right) = -\pi \sqrt{2}\, G_{3,5}^{1,2}\left(\frac{z}{4} \,\middle|\, \begin{matrix} \frac{1}{2}, 1, 0 \\ \frac{1}{2}, \frac{1}{4}, \frac{3}{4}, 0, 0 \end{matrix}\right)$$

Generalized cases for the direct function itself

06.25.26.0008.01

$$\operatorname{erf}(z) = \frac{1}{\sqrt{\pi}} G_{1,2}^{1,1}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} 1 \\ \frac{1}{2}, 0 \end{matrix}\right)$$

06.25.26.0009.01

$$\operatorname{erf}(z) = 1 - \frac{1}{\sqrt{\pi}} G_{1,2}^{2,0}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} 1 \\ 0, \frac{1}{2} \end{matrix}\right)$$

06.25.26.0021.01

$$\operatorname{erf}(z) = \pi\, G_{1,3}^{1,0}\left(\frac{z}{\sqrt{2}}, \frac{1}{4} \,\middle|\, \begin{matrix} 1 \\ \frac{1}{4}, 0, \frac{3}{4} \end{matrix}\right) - \pi\, G_{1,3}^{1,0}\left(\frac{z}{\sqrt{2}}, \frac{1}{4} \,\middle|\, \begin{matrix} 1 \\ \frac{3}{4}, 0, \frac{1}{4} \end{matrix}\right)$$

Generalized cases involving exp

06.25.26.0022.01

$$e^{z^2} \operatorname{erf}(z) = -\pi\, G_{2,3}^{1,1}\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} \frac{1}{2}, 0 \\ \frac{1}{2}, 0, 0 \end{matrix}\right)$$

06.25.26.0017.01

$$e^{z} \operatorname{erf}\left(\sqrt{z}\right) = (1 + i) \sqrt{2\pi}\, G_{1,3}^{1,1}\left(-\frac{i\, z}{2}, \frac{1}{2} \,\middle|\, \begin{matrix} \frac{1}{4} \\ \frac{1}{4}, 0, \frac{1}{2} \end{matrix}\right) - G_{1,2}^{1,1}\left(z \,\middle|\, \begin{matrix} \frac{1}{2} \\ \frac{1}{2}, 0 \end{matrix}\right) /; -\frac{\pi}{2} < \arg(z) \leq \pi$$

Generalized cases involving erfi

06.25.26.0010.01

$$\operatorname{erf}(z) \operatorname{erfi}(z) = -\pi \sqrt{2}\, G_{3,5}^{1,2}\left(\frac{z}{\sqrt{2}}, \frac{1}{4} \,\middle|\, \begin{matrix} \frac{1}{2}, 1, 0 \\ \frac{1}{2}, \frac{1}{4}, \frac{3}{4}, 0, 0 \end{matrix}\right)$$

06.25.26.0011.01

$$\operatorname{erf}(z)+\operatorname{erfi}(z)=2\pi\, G_{1,3}^{1,0}\!\left(\frac{z}{\sqrt{2}},\frac{1}{4}\,\middle|\,\begin{array}{c}1\\ \frac{1}{4},0,\frac{3}{4}\end{array}\right)$$

06.25.26.0012.01

$$\operatorname{erf}(z)-\operatorname{erfi}(z)=-2\pi\, G_{1,3}^{1,0}\!\left(\frac{z}{\sqrt{2}},\frac{1}{4}\,\middle|\,\begin{array}{c}1\\ \frac{3}{4},0,\frac{1}{4}\end{array}\right)$$

Through other functions

06.25.26.0013.01

$$\operatorname{erf}(z)=\operatorname{erf}(0,z)$$

06.25.26.0014.01

$$\operatorname{erf}(z)=\frac{\sqrt{z^2}}{z}\left(1-\frac{1}{\sqrt{\pi}}\,\Gamma\!\left(\frac{1}{2},z^2\right)\right)$$

06.25.26.0015.01

$$\operatorname{erf}(z)=\frac{\sqrt{z^2}}{z}\left(1-Q\!\left(\frac{1}{2},z^2\right)\right)$$

06.25.26.0016.01

$$\operatorname{erf}(z)=\frac{\sqrt{z^2}}{z}-\frac{z}{\sqrt{\pi}}\,E_{\frac{1}{2}}\!\left(z^2\right)$$

Representations through equivalent functions

With inverse function

06.25.27.0001.01

$$\operatorname{erf}\left(\operatorname{erf}^{-1}(z)\right)=z$$

06.25.27.0005.01

$$\operatorname{erf}\left(\operatorname{erfc}^{-1}(1-z)\right)=z$$

06.25.27.0006.01

$$\operatorname{erf}\left(\operatorname{erf}^{-1}(0,z)\right)=z$$

With related functions

06.25.27.0002.01

$$\operatorname{erf}(z)=1-\operatorname{erfc}(z)$$

06.25.27.0003.01

$$\operatorname{erf}(z)=-i\operatorname{erfi}(i\,z)$$

06.25.27.0004.01

$$\operatorname{erf}(z)=(1+i)\left(C\!\left(\frac{(1-i)\,z}{\sqrt{\pi}}\right)-i\,S\!\left(\frac{(1-i)\,z}{\sqrt{\pi}}\right)\right)$$

Zeros

06.25.30.0001.01

$\text{erf}(z) == 0 /; z == 0$

Theorems

Limit theorem of de Moivre-Laplace

In a large number of Bernoulli trials, the probability that a random variable X with a standard binomial distribution $P(k) = \binom{n}{k} p^k (1-p)^{n-k}$ takes on a value $a \leq x \leq b$ is given by

$$\lim_{n \to \infty} P\left(a \leq \frac{X - n\,p}{\sqrt{n\,p\,(1-p)}} \leq b\right) == \frac{1}{2}\left(\text{erf}\left(\frac{b}{\sqrt{2}}\right) - \text{erf}\left(\frac{a}{\sqrt{2}}\right)\right).$$

The smoothing function occuring in hyperasymptotic expansions across Stokes lines

The smoothing function $s(\xi)$ occuring in hyperasymptotic expansions across Stokes lines is of the universal form $s(\xi) == \frac{1}{2}(1 + \text{erf}(\xi))$.

History

– A. de Moivre (1718,1733)
– P.-S. Laplace (1774)
– A.de Moivre (1788);
– P.-S. Laplace (1812) derived an asymptotic expansion

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