ExpIntegralEi

Notations

Traditional name

Exponential integral Ei

Traditional notation

$\mathrm{Ei}(z)$

Mathematica StandardForm notation

`ExpIntegralEi[z]`

Primary definition

06.35.02.0001.01

$$\mathrm{Ei}(z) = \sum_{k=1}^{\infty} \frac{z^k}{k\,k!} + \gamma + \frac{1}{2}\left(\log(z) - \log\left(\frac{1}{z}\right)\right)$$

Specific values

Values at fixed points

06.35.03.0001.01

$$\mathrm{Ei}(0) = -\infty$$

Values at infinities

06.35.03.0002.01

$$\mathrm{Ei}(\infty) = \infty$$

06.35.03.0003.01

$$\mathrm{Ei}(-\infty) = 0$$

06.35.03.0004.01

$$\mathrm{Ei}(i\infty) = i\pi$$

06.35.03.0005.01

$$\mathrm{Ei}(-i\infty) = -i\pi$$

06.35.03.0006.01

$$\mathrm{Ei}(\tilde{\infty}) = ¿$$

General characteristics

Domain and analyticity

Ei(z) is an analytical function of z which is defined over the whole complex z-plane.

06.35.04.0001.01

$$z \longrightarrow \text{Ei}(z) :: \mathbb{C} \longrightarrow \mathbb{C}$$

Symmetries and periodicities

Mirror symmetry

06.35.04.0002.01

$$\text{Ei}(\bar{z}) = \overline{\text{Ei}(z)}$$

Periodicity

No periodicity

Poles and essential singularities

The function Ei(z) has an essential singularity at $z = \tilde{\infty}$. At the same time, the point $z = \tilde{\infty}$ is a branch point.

06.35.04.0003.01

$$\mathcal{S}ing_z(\text{Ei}(z)) = \{\{\tilde{\infty}, \infty\}\}$$

Branch points

The function Ei(z) has two branch points: $z = 0$, $z = \tilde{\infty}$. At the same time, the point $z = \tilde{\infty}$ is an essential singularity.

06.35.04.0004.01

$$\mathcal{BP}_z(\text{Ei}(z)) = \{0, \tilde{\infty}\}$$

06.35.04.0005.01

$$\mathcal{R}_z(\text{Ei}(z), 0) = \log$$

06.35.04.0006.01

$$\mathcal{R}_z(\text{Ei}(z), \tilde{\infty}) = \log$$

Branch cuts

The function Ei(z) is a single-valued function on the z-plane cut along the interval $(-\infty, 0)$ where it has discontinuities from both sides.

06.35.04.0007.01

$$\mathcal{BC}_z(\text{Ei}(z)) = \{\{(-\infty, 0), \{\}\}\}$$

06.35.04.0008.01

$$\lim_{\epsilon \to +0} \text{Ei}(x + i\,\epsilon) = \text{Ei}(x) + \pi\, i \;/;\; x < 0$$

06.35.04.0009.01

$$\lim_{\epsilon \to +0} \text{Ei}(x - i\,\epsilon) = \text{Ei}(x) - \pi\, i \;/;\; x < 0$$

Series representations

Generalized power series

Expansions at generic point $z == z_0$

For the function itself

06.35.06.0011.01

$$\mathrm{Ei}(z) \propto +\mathrm{Ei}(z_0) - i\,\pi \left\lfloor \frac{\arg(z)+\pi}{2\pi} \right\rfloor + \frac{1}{2}\left(\log\left(\frac{1}{z_0}\right) - \log(z_0)\right) + \log(z_0) + \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0)\right) + \frac{e^{z_0}}{z_0}(z-z_0) + \frac{e^{z_0}(z_0-1)}{2 z_0^2}(z-z_0)^2 + \ldots /; (z \to z_0)$$

06.35.06.0012.01

$$\mathrm{Ei}(z) \propto +\mathrm{Ei}(z_0) - i\,\pi \left\lfloor \frac{\arg(z)+\pi}{2\pi} \right\rfloor + \frac{1}{2}\left(\log\left(\frac{1}{z_0}\right) - \log(z_0)\right) + \log(z_0) + \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0)\right) + \frac{e^{z_0}}{z_0}(z-z_0) + \frac{e^{z_0}(z_0-1)}{2 z_0^2}(z-z_0)^2 + O\left((z-z_0)^3\right)$$

06.35.06.0013.01

$$\mathrm{Ei}(z) == \mathrm{Ei}(z_0) - i\,\pi \left\lfloor \frac{\arg(z)+\pi}{2\pi} \right\rfloor + \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0)\right) + \frac{1}{2}\left(\log\left(\frac{1}{z_0}\right) - \log(z_0)\right) + \log(z_0) - \sum_{k=1}^{\infty} \frac{(-1)^k z_0^{-k}}{k!} \Gamma(k, -z_0)(z-z_0)^k$$

06.35.06.0014.01

$$\mathrm{Ei}(z) == \mathrm{Ei}(z_0) - i\,\pi \left\lfloor \frac{\arg(z)+\pi}{2\pi} \right\rfloor + \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0)\right) + \frac{1}{2}\left(\log\left(\frac{1}{z_0}\right) - \log(z_0)\right) + \log(z_0) - e^{z_0} \sum_{k=1}^{\infty} \sum_{j=0}^{k-1} \frac{(-1)^{k-j} z_0^{j-k}}{k\, j!} (z-z_0)^k$$

06.35.06.0015.01

$$\mathrm{Ei}\,(z) == \mathrm{Ei}(z_0) - i\,\pi \left\lfloor \frac{\arg(z)+\pi}{2\pi} \right\rfloor + \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0)\right) + \frac{1}{2}\left(\log\left(\frac{1}{z_0}\right) - \log(z_0)\right) + \log(z_0) + e^{z_0} \sum_{k=1}^{\infty} \frac{1}{k!} \sum_{j=0}^{k-1} j!\,(-1)^j z_0^{-j-1} \binom{k-1}{j} (z-z_0)^k$$

06.35.06.0016.01

$$\mathrm{Ei}(z) == \mathrm{Ei}(z_0) - i\,\pi \left\lfloor \frac{\arg(z)+\pi}{2\pi} \right\rfloor + \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0)\right) + \frac{1}{2}\left(\log\left(\frac{1}{z_0}\right) - \log(z_0)\right) + \log(z_0) + \sum_{k=1}^{\infty} \left(\frac{(-1)^{k-1} z_0^{-k}}{k} + \frac{z_0^{1-k}}{k!}\, {}_2\tilde{F}_2(1, 1; 2, 2-k; z_0)\right)(z-z_0)^k$$

06.35.06.0017.01

$$\mathrm{Ei}(z) \propto +\mathrm{Ei}(z_0) - i\pi \left\lfloor \frac{\arg(z)+\pi}{2\pi} \right\rfloor + \frac{1}{2}\left(\log\left(\frac{1}{z_0}\right) - \log(z_0)\right) + \log(z_0) + \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor \left(\log\left(\frac{1}{z_0}\right) + \log(z_0)\right) + O(z-z_0)$$

Expansions on branch cuts

For the function itself

06.35.06.0018.01

$$\mathrm{Ei}(z) \propto \mathrm{Ei}(x) - i\pi \left\lfloor \frac{\arg(z)+\pi}{2\pi} \right\rfloor + 2i\pi \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor + i\pi + \frac{e^x}{x}(z-x) + \frac{e^x(x-1)}{2x^2}(z-x)^2 + \ldots \;/;\; (z \to x) \land x \in \mathbb{R} \land x < 0$$

06.35.06.0019.01

$$\mathrm{Ei}(z) \propto \mathrm{Ei}(x) - i\pi \left\lfloor \frac{\arg(z)+\pi}{2\pi} \right\rfloor + 2i\pi \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor + i\pi + \frac{e^x}{x}(z-x) + \frac{e^x(x-1)}{2x^2}(z-x)^2 + O\left((z-x)^3\right) \;/;\; x \in \mathbb{R} \land x < 0$$

06.35.06.0020.01

$$\mathrm{Ei}(z) = \mathrm{Ei}(x) + \pi i - i\pi \left\lfloor \frac{\arg(z)+\pi}{2\pi} \right\rfloor + 2i\pi \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor - \sum_{k=1}^{\infty} \frac{(-1)^k x^{-k}}{k!} \Gamma(k,-x)(z-x)^k \;/;\; x \in \mathbb{R} \land x < 0$$

06.35.06.0021.01

$$\mathrm{Ei}(z) = \mathrm{Ei}(x) + \pi i - i\pi \left\lfloor \frac{\arg(z)+\pi}{2\pi} \right\rfloor + 2i\pi \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor - e^x \sum_{k=1}^{\infty} \sum_{j=0}^{k-1} \frac{(-1)^{k-j} x^{j-k}}{k\, j!} (z-x)^k \;/;\; x \in \mathbb{R} \land x < 0$$

06.35.06.0022.01

$$\mathrm{Ei}(z) = \mathrm{Ei}(x) + \pi i - i\pi \left\lfloor \frac{\arg(z)+\pi}{2\pi} \right\rfloor + 2i\pi \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor + e^x \sum_{k=1}^{\infty} \frac{1}{k!} \sum_{j=0}^{k-1} j!\,(-1)^j x^{-j-1} \binom{k-1}{j} (z-x)^k \;/;\; x \in \mathbb{R} \land x < 0$$

06.35.06.0023.01

$$\mathrm{Ei}(z) = \mathrm{Ei}(x) + \pi i - i\pi \left\lfloor \frac{\arg(z)+\pi}{2\pi} \right\rfloor + 2i\pi \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor + \sum_{k=1}^{\infty} \left(\frac{(-1)^{k-1} x^{-k}}{k} + \frac{x^{1-k}}{k!}\, {}_2\tilde{F}_2(1,1;2,2-k;x) \right)(z-x)^k \;/;\; x \in \mathbb{R} \land x < 0$$

06.35.06.0024.01

$$\mathrm{Ei}(z) \propto \mathrm{Ei}(x) - i\pi \left\lfloor \frac{\arg(z)+\pi}{2\pi} \right\rfloor + 2i\pi \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor + i\pi + O(z-x) \;/;\; x \in \mathbb{R} \land x < 0$$

Expansions at $z = 0$

For the function itself

06.35.06.0001.01

$$\mathrm{Ei}(z) \propto \frac{1}{2}\left(\log(z) - \log\left(\frac{1}{z}\right)\right) + \gamma + z + \frac{z^2}{4} + \frac{z^3}{18} + \ldots \;/;\; (z \to 0)$$

06.35.06.0025.01

$$\mathrm{Ei}(z) \propto \frac{1}{2}\left(\log(z) - \log\left(\frac{1}{z}\right)\right) + \gamma + z + \frac{z^2}{4} + \frac{z^3}{18} + O\left(z^4\right)$$

06.35.06.0002.01

$$\mathrm{Ei}(z) = \frac{1}{2}\left(\log(z) - \log\left(\frac{1}{z}\right)\right) + \sum_{k=1}^{\infty} \frac{z^k}{k\, k!} + \gamma$$

06.35.06.0026.01

$$\mathrm{Ei}(z) = \log(z) + \sum_{k=1}^{\infty} \frac{z^k}{k\,k!} + \gamma - \begin{cases} \pi\,i & \arg(z) = \pi \\ 0 & \text{True} \end{cases}$$

06.35.06.0003.01

$$\mathrm{Ei}(z) == z\,{}_2F_2(1, 1; 2, 2; z) + \frac{1}{2}\left(\log(z) - \log\left(\frac{1}{z}\right)\right) + \gamma$$

06.35.06.0004.01

$$\mathrm{Ei}(z) \propto \frac{1}{2}\left(\log(z) - \log\left(\frac{1}{z}\right)\right) + \gamma + z\,(1 + O(z))\;/;\;(z \to 0)$$

06.35.06.0027.01

$$\mathrm{Ei}(z) == F_\infty(z)\;/;$$
$$\left(\left(F_n(z) == \frac{1}{2}\left(\log(z) - \log\left(\frac{1}{z}\right)\right) + \gamma + z\sum_{k=0}^{n} \frac{z^k}{(k+1)^2\,k!} == \mathrm{Ei}(z) - \frac{z^{n+2}}{(n+2)\,(n+2)!}\,{}_2F_2(1, n+2; n+3, n+3; z)\right) \bigwedge n \in \mathbb{N}\right)$$

Summed form of the truncated series expansion.

06.35.06.0005.01

$$\mathrm{Ei}(z) == -e^z \sum_{k=1}^{\infty}\sum_{j=1}^{k} \frac{(-1)^k z^k}{j\,k!} + \gamma + \frac{1}{2}\left(\log(z) - \log\left(\frac{1}{z}\right)\right)$$

Asymptotic series expansions

06.35.06.0028.01

$$\mathrm{Ei}(z) \propto \frac{1}{2}\left(-\log\left(\frac{1}{z}\right) + \log\left(-\frac{1}{z}\right) - \log(-z) + \log(z)\right) + e^z\left(1 + \frac{1}{z^2} + \frac{2}{z^3} + \frac{6}{z^4} + \ldots\right)/;\;(|z| \to \infty)$$

06.35.06.0006.01

$$\mathrm{Ei}(z) \propto \frac{1}{z}\,e^z\,{}_2F_0\left(1, 1; ; \frac{1}{z}\right) + \frac{1}{2}\left(-\log\left(\frac{1}{z}\right) + \log\left(-\frac{1}{z}\right) - \log(-z) + \log(z)\right)/;\;(|z| \to \infty)$$

06.35.06.0029.01

$$\mathrm{Ei}(z) \propto \frac{1}{2}\left(-\log\left(\frac{1}{z}\right) + \log\left(-\frac{1}{z}\right) - \log(-z) + \log(z)\right) + \frac{e^z}{z}\sum_{k=0}^{\infty} \frac{k!}{z^k}\;/;\;(|z| \to \infty)$$

06.35.06.0030.01

$$\mathrm{Ei}(z) \propto \frac{e^z}{z}\sum_{k=0}^{\infty} \frac{k!}{z^k} + \pi\,i\,\mathrm{sgn}(\mathrm{Im}(z))\;/;\;(|z| \to \infty)$$

06.35.06.0007.01

$$\mathrm{Ei}(z) \propto \frac{1}{2}\left(-\log\left(\frac{1}{z}\right) + \log\left(-\frac{1}{z}\right) - \log(-z) + \log(z)\right) + \frac{1}{z}\,e^z\left(1 + O\left(\frac{1}{z}\right)\right)/;\;(|z| \to \infty)$$

06.35.06.0031.01

$$\mathrm{Ei}(z) \propto \begin{cases} -i\,\pi + \frac{e^z}{z} & \arg(z) < 0 \\ \frac{e^z}{z} & \arg(z) = \pi \bigvee \arg(z) = 0 \\ i\,\pi + \frac{e^z}{z} & \text{True} \end{cases}\;/;\;(|z| \to \infty)$$

06.35.06.0032.01

$$\operatorname{Ei}(z) \propto \begin{cases} -\pi i & \arg(z) \leq -\frac{\pi}{2} \\ \pi i & \frac{\pi}{2} \leq \arg(z) < \pi \\ \frac{e^z}{z} & \text{True} \end{cases} /; (|z| \to \infty)$$

Residue representations

06.35.06.0008.01

$$\operatorname{Ei}(z) = \gamma - \frac{1}{2}\left(\log\left(\frac{1}{z}\right) - \log(z)\right) - \sum_{j=0}^{\infty} \operatorname{res}_s\left(\frac{\Gamma(-s)^2 (-z)^{-s}}{\Gamma(1-s)^2}\,\Gamma(s+1)\right)(-j-1)$$

06.35.06.0009.02

$$\operatorname{Ei}(z) = -\log(-z) - \frac{1}{2}\left(\log\left(\frac{1}{z}\right) - \log(z)\right) - \operatorname{res}_s\left((-z)^{-s}\,\frac{\Gamma(s)}{s}\right)(0) - \sum_{j=1}^{\infty} \operatorname{res}_s\left(\frac{(-z)^{-s}}{s}\,\Gamma(s)\right)(-j)$$

Other series representations

06.35.06.0010.01

$$\operatorname{Ei}(x) = -e^x \sum_{k=0}^{\infty} \frac{L_k(-x)}{k+1} /; x < 0$$

Integral representations

On the real axis

Of the direct function

06.35.07.0001.01

$$\operatorname{Ei}(z) = \int_0^z \frac{e^t - 1}{t}\,dt + \frac{1}{2}\left(\log(z) - \log\left(\frac{1}{z}\right)\right) + \gamma$$

06.35.07.0002.01

$$\operatorname{Ei}(x) = -\mathcal{P}\int_{-x}^{\infty} \frac{e^{-t}}{t}\,dt /; x \in \mathbb{R}$$

06.35.07.0003.01

$$\operatorname{Ei}(x) = \mathcal{P}\int_{-\infty}^{x} \frac{e^{t}}{t}\,dt /; x \in \mathbb{R}$$

Contour integral representations

06.35.07.0004.01

$$\operatorname{Ei}(z) = \gamma - \frac{1}{2}\left(\log\left(\frac{1}{z}\right) - \log(z)\right) - \frac{1}{2\pi i}\int_{\mathcal{L}} \frac{\Gamma(s+1)\,\Gamma(-s)^2}{\Gamma(1-s)^2}\,(-z)^{-s}\,ds$$

06.35.07.0005.01

$$\operatorname{Ei}(z) = \gamma - \frac{1}{2}\left(\log\left(\frac{1}{z}\right) - \log(z)\right) - \frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty} \frac{\Gamma(s+1)\,\Gamma(-s)^2}{\Gamma(1-s)^2}\,(-z)^{-s}\,ds /; -1 < \gamma < 0 \bigwedge |\arg(-z)| < \frac{\pi}{2}$$

06.35.07.0006.01

$$\mathrm{Ei}(z) = -\log(-z) - \frac{1}{2}\left(\log\left(\frac{1}{z}\right) - \log(z)\right) - \frac{1}{2\pi i}\int_{\mathcal{L}} \frac{\Gamma(s)^2}{\Gamma(s+1)} (-z)^{-s}\, ds$$

06.35.07.0007.01

$$\mathrm{Ei}(z) = -\log(-z) - \frac{1}{2}\left(\log\left(\frac{1}{z}\right) - \log(z)\right) - \frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty} \frac{\Gamma(s)^2}{\Gamma(s+1)} (-z)^{-s}\, ds \;/;\; 0 < \gamma \bigwedge |\arg(-z)| < \frac{\pi}{2}$$

Continued fraction representations

06.35.10.0001.01

$$\mathrm{Ei}(z) = i\pi\,\mathrm{sgn}(\mathrm{Im}(z)) - e^z \cfrac{1}{1-z+\cfrac{-1}{3-z+\cfrac{-4}{5-z+\cfrac{-9}{7-z+\cfrac{-16}{9-z+\cfrac{-25}{11-z+\cfrac{-36}{13-z+\ldots}}}}}}} \;/;\; |\arg(-z)| < \pi$$

06.35.10.0002.01

$$\mathrm{Ei}(z) = i\pi\,\mathrm{sgn}(\mathrm{Im}(z)) - \frac{e^z}{1-z+\mathrm{K}_k\left(-k^2,\, 2k-z+1\right)_1^\infty} \;/;\; |\arg(-z)| < \pi$$

06.35.10.0003.01

$$\mathrm{Ei}(z) = i\pi\,\mathrm{sgn}(\mathrm{Im}(z)) - e^z \cfrac{1}{-z+\cfrac{1}{1+\cfrac{1}{-z+\cfrac{2}{1+\cfrac{2}{-z+\cfrac{3}{1+\cfrac{3}{-z+\ldots}}}}}}} \;/;\; |\arg(-z)| < \pi$$

06.35.10.0004.01

$$\mathrm{Ei}(z) = i\pi\,\mathrm{sgn}(\mathrm{Im}(z)) - \frac{e^z}{-z+\mathrm{K}_k\left(\left\lfloor \frac{k+1}{2} \right\rfloor,\, (-z)^{\frac{1}{2}(1+(-1)^k)}\right)_1^\infty} \;/;\; |\arg(-z)| < \pi$$

Differential equations

Ordinary linear differential equations and wronskians

For the direct function itself

06.35.13.0001.01

$$z\, w^{(3)}(z) + 2\, w''(z) - z\, w'(z) = 0 \;/;\; w(z) = c_1\, \mathrm{Ei}(z) + c_2\, \mathrm{Ei}(-z) + c_3$$

06.35.13.0004.01

$$W_z(1,\, \mathrm{Ei}(z),\, \mathrm{Ei}(-z)) = -\frac{2}{z^2}$$

06.35.13.0002.01

$$z\, w^{(3)}(z) + 2\, w''(z) - z\, w'(z) = 0 \,/;\, w(z) = c_1\, \mathrm{Ei}(z) + c_2\, \mathrm{Shi}(z) + c_3$$

06.35.13.0005.01

$$W_z(1,\, \mathrm{Ei}(z),\, \mathrm{Shi}(z)) = \frac{1}{z^2}$$

06.35.13.0003.01

$$z\, w^{(3)}(z) + 2\, w''(z) - z\, w'(z) = 0 \,/;\, w(z) = c_1\, \mathrm{Ei}(z) + c_2\, \mathrm{Chi}(z) + c_3$$

06.35.13.0006.01

$$W_z(1,\, \mathrm{Ei}(z),\, \mathrm{Chi}(z)) = -\frac{1}{z^2}$$

06.35.13.0007.01

$$w^{(3)}(z) + \left(\frac{2\, g'(z)}{g(z)} - \frac{3\, g''(z)}{g'(z)}\right) w''(z) + \left(-g'(z)^2 + \frac{3\, g''(z)^2}{g'(z)^2} - \frac{2\, g''(z)}{g(z)} - \frac{g^{(3)}(z)}{g'(z)}\right) w'(z) = 0 \,/;\, w(z) = c_1\, \mathrm{Ei}(g(z)) + c_2\, \mathrm{Ei}(-g(z)) + c_3$$

06.35.13.0008.01

$$W_z(\mathrm{Ei}(g(z)),\, \mathrm{Ei}(-g(z)),\, 1) = -\frac{2\, g'(z)^3}{g(z)^2}$$

06.35.13.0009.01

$$w^{(3)}(z) + \left(\frac{2\, g'(z)}{g(z)} - \frac{3\, h'(z)}{h(z)} - \frac{3\, g''(z)}{g'(z)}\right) w''(z) +$$
$$\left(-g'(z)^2 - \frac{4\, h'(z)\, g'(z)}{g(z)\, h(z)} + \frac{6\, h'(z)^2}{h(z)^2} + \frac{3\, g''(z)^2}{g'(z)^2} + \frac{6\, h'(z)\, g''(z)}{h(z)\, g'(z)} - \frac{2\, g''(z)}{g(z)} - \frac{3\, h''(z)}{h(z)} - \frac{g^{(3)}(z)}{g'(z)}\right) w'(z) +$$
$$\left(-\frac{6\, h'(z)^3}{h(z)^3} + \frac{4\, g'(z)\, h'(z)^2}{g(z)\, h(z)^2} - \frac{6\, g''(z)\, h'(z)^2}{h(z)^2\, g'(z)} + \frac{6\, h''(z)\, h'(z)}{h(z)^2} - \frac{3\, g''(z)^2\, h'(z)}{h(z)\, g'(z)^2} + \frac{2\, h'(z)\, g''(z) - 2\, g'(z)\, h''(z)}{g(z)\, h(z)} + \right.$$
$$\left.\frac{3\, g''(z)\, h''(z) + h'(z)\, g^{(3)}(z)}{h(z)\, g'(z)} + \frac{g'(z)^2\, h'(z) - h^{(3)}(z)}{h(z)}\right) w(z) = 0 \,/;\, w(z) = c_1\, \mathrm{Ei}(g(z))\, h(z) + c_2\, \mathrm{Ei}(-g(z))\, h(z) + c_3\, h(z)$$

06.35.13.0010.01

$$W_z(h(z)\, \mathrm{Ei}(g(z)),\, h(z)\, \mathrm{Ei}(-g(z)),\, h(z)) = -\frac{2\, h(z)^3\, g'(z)^3}{g(z)^2}$$

06.35.13.0011.01

$$z^3\, w^{(3)}(z) - (r + 3\, s - 3)\, z^2\, w''(z) - \left(a^2\, r^2\, z^{2\, r} - 3\, s^2 + r - 2\, r\, s + 3\, s - 1\right) z\, w'(z) - s\left(-a^2\, r^2\, z^{2\, r} + s^2 + r\, s\right) w(z) = 0 \,/;$$
$$w(z) = c_1\, z^s\, \mathrm{Ei}(a\, z^r) + c_2\, z^s\, \mathrm{Ei}(-a\, z^r) + c_3\, z^s$$

06.35.13.0012.01

$$W_z(z^s\, \mathrm{Ei}(a\, z^r),\, z^s\, \mathrm{Ei}(-a\, z^r),\, z^s) = -2\, a\, r^3\, z^{-3+r+3\, s}$$

06.35.13.0013.01

$$w^{(3)}(z) - (\log(r) + 3\log(s))\, w''(z) + \left(-a^2 \log^2(r)\, r^{2\, z} + 3\log^2(s) + 2\log(r)\log(s)\right) w'(z) -$$
$$\log(s)\left(-a^2 \log^2(r)\, r^{2\, z} + \log^2(s) + \log(r)\log(s)\right) w(z) = 0 \,/;\, w(z) = c_1\, s^z\, \mathrm{Ei}(a\, r^z) + c_2\, s^z\, \mathrm{Ei}(-a\, r^z) + c_3\, s^z$$

06.35.13.0014.01

$$W_z(s^z \,\mathrm{Ei}(a\, r^z),\, s^z\, \mathrm{Ei}(-a\, r^z),\, s^z) == -2\, a\, r^z\, s^{3z} \log^3(r)$$

Transformations

Transformations and argument simplifications

Argument involving basic arithmetic operations

06.35.16.0001.01

$$\mathrm{Ei}(i\, z) == \mathrm{Ci}(z) + i\, \mathrm{Si}(z) - \log(z) - \frac{1}{2}\left(\log\left(-\frac{i}{z}\right) - \log(i\, z)\right)$$

06.35.16.0002.01

$$\mathrm{Ei}(-i\, z) == \mathrm{Ci}(z) - i\, \mathrm{Si}(z) - \log(z) - \frac{1}{2}\left(\log\left(\frac{i}{z}\right) - \log(-i\, z)\right)$$

06.35.16.0003.01

$$\mathrm{Ei}\left(\sqrt{z^2}\right) == \mathrm{Ei}(z) + \frac{1}{2}\left(\log\left(\frac{1}{z}\right) - \log(z) + \log(-i\, z) + \log(i\, z)\right) + \left(\frac{\sqrt{z^2}}{z} - 1\right) \mathrm{Shi}(z)$$

Related transformations

06.35.16.0004.01

$$\mathrm{Ei}(\log(z)) == \mathrm{li}(z)$$

Complex characteristics

Real part

06.35.19.0001.01

$$\mathrm{Re}(\mathrm{Ei}(x + i\, y)) == \mathrm{Chi}(x) - \log(x) + \frac{1}{2}\log\left(x^2 + y^2\right) - \sum_{j=0}^{\infty} \frac{(-1)^j\, y^{2j+2}}{2\,(j+1)\,(2\,j+2)!}\, {}_1F_2\left(j+1;\, \frac{1}{2},\, j+2;\, \frac{x^2}{4}\right) + x \sum_{j=0}^{\infty} \frac{(-1)^j\, y^{2j}}{(2\,j+1)\,(2\,j)!}\, {}_1F_2\left(j+\frac{1}{2};\, \frac{3}{2},\, j+\frac{3}{2};\, \frac{x^2}{4}\right)$$

06.35.19.0002.01

$$\mathrm{Re}(\mathrm{Ei}(x + i\, y)) == \frac{1}{2}\log\left(x^2 + y^2\right) + \sum_{k=1}^{\infty} \frac{x^k}{k\, k!} \left(\frac{y^2}{x^2} + 1\right)^{k/2} \cos\left(k \tan^{-1}\left(\frac{y}{x}\right)\right) + \gamma$$

06.35.19.0003.01

$$\mathrm{Re}(\mathrm{Ei}(x + i\, y)) == \frac{1}{2}\log\left(x^2 + y^2\right) + \sum_{k=1}^{\infty} \sum_{j=0}^{\left\lfloor \frac{k}{2} \right\rfloor} \frac{(-1)^j\, y^{2j}\, x^{k-2j}}{k\,(2\,j)!\,(k-2\,j)!} + \gamma$$

06.35.19.0004.01

$$\mathrm{Re}(\mathrm{Ei}(x + i\, y)) == \frac{1}{2}\left(\mathrm{Ei}\left(x + x\sqrt{-\frac{y^2}{x^2}}\right) + \mathrm{Ei}\left(x - x\sqrt{-\frac{y^2}{x^2}}\right)\right)$$

Imaginary part

06.35.19.0005.01

$$\operatorname{Im}(\operatorname{Ei}(x+i\,y)) = x\sum_{k=0}^{20}\frac{(-1)^k\,y^{2k+1}}{(2k+2)!}\,{}_1F_2\left(k+1;\,\frac{3}{2},\,k+2;\,\frac{x^2}{4}\right)+\sum_{k=0}^{20}\frac{(-1)^k\,y^{2k+1}}{(2k+1)\,(2k+1)!}\,{}_1F_2\left(k+\frac{1}{2};\,\frac{1}{2},\,k+\frac{3}{2};\,\frac{x^2}{4}\right)+\frac{1}{2}\left(\tan^{-1}(x,\,y)-\tan^{-1}(x,\,-y)\right)$$

06.35.19.0006.01

$$\operatorname{Im}(\operatorname{Ei}(x+i\,y)) = \sum_{k=1}^{\infty}\frac{x^k}{k\,k!}\left(1+\frac{y^2}{x^2}\right)^{k/2}\sin\left(k\tan^{-1}\left(\frac{y}{x}\right)\right)+\frac{1}{2}\left(\tan^{-1}(x,\,y)-\tan^{-1}(x,\,-y)\right)$$

06.35.19.0007.01

$$\operatorname{Im}(\operatorname{Ei}(x+i\,y)) = \sum_{k=1}^{\infty}\sum_{j=0}^{\left\lfloor\frac{k-1}{2}\right\rfloor}\frac{(-1)^j\,y^{2j+1}\,x^{k-2j-1}}{k\,(2j+1)!\,(k-2j-1)!}+\frac{1}{2}\left(\tan^{-1}(x,\,y)-\tan^{-1}(x,\,-y)\right)$$

06.35.19.0008.01

$$\operatorname{Im}(\operatorname{Ei}(x+i\,y)) = \frac{x}{2\,y}\sqrt{-\frac{y^2}{x^2}}\left(\operatorname{Ei}\left(x-x\sqrt{-\frac{y^2}{x^2}}\right)-\operatorname{Ei}\left(x+x\sqrt{-\frac{y^2}{x^2}}\right)\right)$$

Absolute value

06.35.19.0009.01

$$|\operatorname{Ei}(x+i\,y)| = \sqrt{\operatorname{Ei}\left(x-x\sqrt{-\frac{y^2}{x^2}}\right)\operatorname{Ei}\left(x+x\sqrt{-\frac{y^2}{x^2}}\right)}$$

Argument

06.35.19.0010.01

$$\arg(\operatorname{Ei}(x+i\,y)) = \tan^{-1}\left(\frac{1}{2}\left(\operatorname{Ei}\left(x+x\sqrt{-\frac{y^2}{x^2}}\right)+\operatorname{Ei}\left(x-x\sqrt{-\frac{y^2}{x^2}}\right)\right),\,\frac{x}{2\,y}\sqrt{-\frac{y^2}{x^2}}\left(\operatorname{Ei}\left(x-x\sqrt{-\frac{y^2}{x^2}}\right)-\operatorname{Ei}\left(x+x\sqrt{-\frac{y^2}{x^2}}\right)\right)\right)$$

Conjugate value

06.35.19.0011.01

$$\overline{\operatorname{Ei}(x+i\,y)} = \frac{1}{2}\left(\operatorname{Ei}\left(x+x\sqrt{-\frac{y^2}{x^2}}\right)+\operatorname{Ei}\left(x-x\sqrt{-\frac{y^2}{x^2}}\right)\right)-\frac{i\,x}{2\,y}\sqrt{-\frac{y^2}{x^2}}\left(\operatorname{Ei}\left(x-x\sqrt{-\frac{y^2}{x^2}}\right)-\operatorname{Ei}\left(x+x\sqrt{-\frac{y^2}{x^2}}\right)\right)$$

Signum value

06.35.19.0012.01

$$\operatorname{sgn}(\operatorname{Ei}(x+i\,y)) = \left(\frac{i}{y}\sqrt{-\frac{y^2}{x^2}}\;x\left(\operatorname{Ei}\left(x-x\sqrt{-\frac{y^2}{x^2}}\right)-\operatorname{Ei}\left(\sqrt{-\frac{y^2}{x^2}}\;x+x\right)\right)+\operatorname{Ei}\left(\sqrt{-\frac{y^2}{x^2}}\;x+x\right)+\operatorname{Ei}\left(x-x\sqrt{-\frac{y^2}{x^2}}\right)\right)\Big/ \left(2\sqrt{\operatorname{Ei}\left(x-x\sqrt{-\frac{y^2}{x^2}}\right)\operatorname{Ei}\left(\sqrt{-\frac{y^2}{x^2}}\;x+x\right)}\right)$$

Differentiation

Low-order differentiation

06.35.20.0001.01

$$\frac{\partial\operatorname{Ei}(z)}{\partial z} = \frac{e^z}{z}$$

06.35.20.0002.01

$$\frac{\partial^2\operatorname{Ei}(z)}{\partial z^2} = \frac{e^z\,(z-1)}{z^2}$$

Symbolic differentiation

06.35.20.0007.01

$$\frac{\partial^n\operatorname{Ei}(z)}{\partial z^n} = \delta_n\operatorname{Ei}(z) - e^z\sum_{k=0}^{n-1}\frac{(-1)^{n-k}\,z^{k-n}\,(n-1)!}{k!}\;/;\;n\in\mathbb{N}$$

06.35.20.0008.01

$$\frac{\partial^n\operatorname{Ei}(z)}{\partial z^n} = \delta_n\left(\frac{1}{2}\left(\log(z)-\log\left(\frac{1}{z}\right)\right)-\log(-z)\right)-e^z\,(-1)^n\,e^{-z}\,z^{-n}\,\Gamma(n,-z)\;/;\;n\in\mathbb{N}$$

06.35.20.0003.01

$$\frac{\partial^n\operatorname{Ei}(z)}{\partial z^n} = \delta_n\operatorname{Ei}(z)+\operatorname{Boole}\left(n\neq 0,\;e^z\sum_{k=0}^{n-1}k!\,(-1)^k\,z^{-k-1}\binom{n-1}{k}\right)\;/;\;n\in\mathbb{N}$$

06.35.20.0004.01

$$\frac{\partial^n\operatorname{Ei}(z)}{\partial z^n} = \delta_n\operatorname{Ei}(z)-\operatorname{Boole}(n\neq 0,\,(-1)^n\,z^{-n}\,\Gamma(n,-z))\;/;\;n\in\mathbb{N}$$

06.35.20.0005.01

$$\frac{\partial^n\operatorname{Ei}(z)}{\partial z^n} = {}_2\tilde{F}_2(1,1;2,2-n;z)\,z^{1-n}+(-1)^{n-1}\,(n-1)!\,z^{-n}\;/;\;n\in\mathbb{N}^+$$

Fractional integro-differentiation

06.35.20.0006.01

$$\frac{\partial^\alpha\operatorname{Ei}(z)}{\partial z^\alpha} = z^{1-\alpha}\,{}_2\tilde{F}_2(1,1;2,2-\alpha;z)+\mathcal{FC}_{\log}^{(\alpha)}(z)\,z^{-\alpha}+\frac{z^{-\alpha}}{\Gamma(1-\alpha)}\left(\gamma-\frac{1}{2}\left(\log\left(\frac{1}{z}\right)+\log(z)\right)\right)$$

Integration

Indefinite integration

Involving only one direct function

06.35.21.0001.01

$$\int \mathrm{Ei}(b + a\, z)\, dz = \frac{(b + a\, z)\, \mathrm{Ei}(b + a\, z) - e^{b+a\, z}}{a}$$

06.35.21.0002.01

$$\int \mathrm{Ei}(-a\, z)\, dz = z\, \mathrm{Ei}(-a\, z) + \frac{e^{-a\, z}}{a}$$

06.35.21.0003.01

$$\int \mathrm{Ei}(a\, z)\, dz = z\, \mathrm{Ei}(a\, z) - \frac{e^{a\, z}}{a}$$

06.35.21.0004.01

$$\int \mathrm{Ei}(z)\, dz = z\, \mathrm{Ei}(z) - e^{z}$$

Involving one direct function and elementary functions

Involving power function

Involving power

Linear argument

06.35.21.0005.01

$$\int z^{\alpha-1}\, \mathrm{Ei}(-a\, z)\, dz = \frac{z^{\alpha}\, (\Gamma(\alpha,\, a\, z)\, (a\, z)^{-\alpha} + \mathrm{Ei}(-a\, z))}{\alpha}$$

06.35.21.0006.01

$$\int z^{\alpha-1}\, \mathrm{Ei}(a\, z)\, dz = \frac{z^{\alpha}}{\alpha}\, (\Gamma(\alpha,\, -a\, z)\, (-a\, z)^{-\alpha} + \mathrm{Ei}(a\, z))$$

06.35.21.0007.01

$$\int z^{\alpha-1}\, \mathrm{Ei}(z)\, dz = \frac{z^{\alpha}}{\alpha}\, (\Gamma(\alpha,\, -z)\, (-z)^{-\alpha} + \mathrm{Ei}(z))$$

06.35.21.0008.01

$$\int z\, \mathrm{Ei}(a\, z)\, dz = \frac{a^2\, \mathrm{Ei}(a\, z)\, z^2 + e^{a\, z}\, (1 - a\, z)}{2\, a^2}$$

06.35.21.0009.01

$$\int \frac{\mathrm{Ei}(a\, z)}{z}\, dz = a\, z\, {}_3F_3(1,\, 1,\, 1;\, 2,\, 2,\, 2;\, a\, z) + \frac{1}{2}\, \log(z)\, (2\, \mathrm{Ei}(a\, z) + 2\, \Gamma(0,\, -a\, z) - \log(z) + 2\, \log(-a\, z) + 2\, \gamma)$$

06.35.21.0010.01

$$\int \frac{\mathrm{Ei}(a\, z)}{z^2}\, dz = \frac{(a\, z - 1)\, \mathrm{Ei}(a\, z) - e^{a\, z}}{z}$$

06.35.21.0011.01

$$\int \frac{\mathrm{Ei}(b+a\,z)}{z^2}\,dz = \frac{a\,e^b\,z\,\mathrm{Ei}(a\,z)-(b+a\,z)\,\mathrm{Ei}(b+a\,z)}{b\,z}$$

Power arguments

06.35.21.0012.01

$$\int z^{\alpha-1}\,\mathrm{Ei}(a\,z^r)\,dz = \frac{z^\alpha}{\alpha}\left(\Gamma\left(\frac{\alpha}{r},\,-a\,z^r\right)(-a\,z^r)^{-\frac{\alpha}{r}}+\mathrm{Ei}(a\,z^r)\right)$$

Involving exponential function

Involving exp

Linear argument

06.35.21.0013.01

$$\int e^{b\,z}\,\mathrm{Ei}(a\,z)\,dz = \frac{e^{b\,z}\,\mathrm{Ei}(a\,z)-\mathrm{Ei}((a+b)\,z)}{b}$$

06.35.21.0014.01

$$\int e^{-a\,z}\,\mathrm{Ei}(a\,z)\,dz = \frac{\log(a\,z)-e^{-a\,z}\,\mathrm{Ei}(a\,z)}{a}$$

Power arguments

06.35.21.0015.01

$$\int e^{-a\,z^r}\,\mathrm{Ei}(a\,z^r)\,dz = \frac{z}{2\,r}\left((a\,z^r)^{-\frac{1}{r}}\,\Gamma\left(\frac{1}{r},\,a\,z^r\right)\left(\log\left(\frac{z^{-r}}{a}\right)+2\log(-a\,z^r)-\log(a\,z^r)\right)-2\,G_{2,3}^{2,2}\left(-a\,z^r\,\middle|\,\begin{matrix}0,\,1-\frac{1}{r}\\0,\,0,\,-\frac{1}{r}\end{matrix}\right)\right)$$

Involving exponential function and a power function

Involving exp and power

Linear arguments

06.35.21.0016.01

$$\int z^n\,e^{b\,z}\,\mathrm{Ei}(a\,z)\,dz = \frac{1}{b}(-b)^{-n}\left(n!\sum_{k=1}^{n}\frac{b^k\,(a+b)^{-k}\,\Gamma(k,\,-(a+b)\,z)}{k!}+\mathrm{Ei}((a+b)\,z)\,(-n!)+\mathrm{Ei}(a\,z)\,\Gamma(n+1,\,-b\,z)\right)/;\,n\in\mathbb{N}$$

06.35.21.0017.01

$$\int z^n\,e^{-a\,z}\,\mathrm{Ei}(a\,z)\,dz = -a^{-n-1}\left(\mathrm{Ei}(a\,z)\,\Gamma(n+1,\,a\,z)-n!\log(z)-n!\sum_{k=1}^{n}\frac{(a\,z)^k}{k!\,k}\right)/;\,n\in\mathbb{N}$$

06.35.21.0018.01

$$\int z\, e^{b z}\, \text{Ei}(a z)\, dz = \frac{1}{b^2 (a+b)} \left(-b\, e^{(a+b) z} + (a+b)\, e^{b z}\, (b z-1)\, \text{Ei}(a z) + (a+b)\, \text{Ei}((a+b) z)\right)$$

06.35.21.0019.01

$$\int z^2\, e^{b z}\, \text{Ei}(a z)\, dz = \frac{1}{b^3} \left(e^{b z} \left(b^2 z^2 - 2 b z + 2\right) \text{Ei}(a z) - \frac{1}{(a+b)^2} \left(2\, \text{Ei}((a+b) z)\, (a+b)^2 + b\, e^{(a+b) z}\, (b (b z-3) + a (b z-2))\right)\right)$$

06.35.21.0020.01

$$\int z^3\, e^{b z}\, \text{Ei}(a z)\, dz = \frac{1}{b^4} \Bigg(e^{b z} \left(b^3 z^3 - 3 b^2 z^2 + 6 b z - 6\right) \text{Ei}(a z) + \\ \frac{1}{(a+b)^3} \left(6 (a+b)^3\, \text{Ei}((a+b) z) - b\, e^{(a+b) z} \left(\left(b^2 z^2 - 3 b z + 6\right) a^2 + b \left(2 b^2 z^2 - 8 b z + 15\right) a + b^2 \left(b^2 z^2 - 5 b z + 11\right)\right)\right)\Bigg)$$

06.35.21.0021.01

$$\int z^{\alpha-1}\, e^{-a z}\, \text{Ei}(a z)\, dz = -\frac{1}{2} z^\alpha\, \Gamma(\alpha, a z) \left(-2 \log(-a z) + \log(a z) - \log\left(\frac{1}{a z}\right)\right) (a z)^{-\alpha} - z^\alpha\, G_{2,3}^{2,2}\left(-a z \,\middle|\, \begin{array}{c} 0, 1-\alpha \\ 0, 0, -\alpha \end{array}\right)$$

06.35.21.0022.01

$$\int z^n\, e^{-a z}\, \text{Ei}(a z)\, dz = -a^{-n-1} \left(\text{Ei}(a z)\, \Gamma(n+1, a z) - n!\, \log(z) - n! \sum_{k=1}^{n} \frac{(a z)^k}{k\, k!}\right) /;\, n \in \mathbb{N}$$

06.35.21.0023.01

$$\int z\, e^{-a z}\, \text{Ei}(a z)\, dz = \frac{a z - e^{-a z}\, (a z+1)\, \text{Ei}(a z) + \log(z)}{a^2}$$

06.35.21.0024.01

$$\int z\, e^{c z}\, \text{Ei}(b + a z)\, dz = \\ \frac{1}{a c^2 (a+c)} \left(e^{-\frac{b c}{a}} \left(-a\, e^{\frac{(a+c)(b+a z)}{a}}\, c + (a+c) (a + b c)\, \text{Ei}\left(\frac{(a+c)(b+a z)}{a}\right)\right) + a (a+c)\, e^{c \left(\frac{b}{a}+z\right)}\, (c z-1)\, \text{Ei}(b + a z)\right)$$

06.35.21.0025.01

$$\int \frac{e^{a z}\, \text{Ei}(a z)}{z}\, dz = \frac{1}{2}\, \text{Ei}(a z)^2$$

Power arguments

06.35.21.0026.01

$$\int z^{\alpha-1}\, e^{-a z^r}\, \text{Ei}(a z^r)\, dz = \frac{z^\alpha}{2 r} \left((a z^r)^{-\frac{\alpha}{r}}\, \Gamma\left(\frac{\alpha}{r}, a z^r\right) \left(\log\left(\frac{z^{-r}}{a}\right) + 2 \log(-a z^r) - \log(a z^r)\right) - 2\, G_{2,3}^{2,2}\left(-a z^r \,\middle|\, \begin{array}{c} 0, 1-\frac{\alpha}{r} \\ 0, 0, -\frac{\alpha}{r} \end{array}\right)\right)$$

Involving trigonometric functions

Involving sin

06.35.21.0027.01

$$\int \sin(b z)\, \text{Ei}(a z)\, dz = \frac{-2 \cos(b z)\, \text{Ei}(a z) + \text{Ei}((a + b i) z) + \text{Ei}((a - i b) z)}{2 b}$$

Involving cos

06.35.21.0028.01

$$\int \cos(b\,z)\,\text{Ei}(a\,z)\,dz = -\frac{i\,(-\text{Ei}((a+b\,i)\,z)+\text{Ei}((a-i\,b)\,z)+2\,i\,\text{Ei}(a\,z)\sin(b\,z))}{2\,b}$$

Involving trigonometric functions and a power function

Involving sin and power

06.35.21.0029.01

$$\int z^n \sin(b\,z)\,\text{Ei}(a\,z)\,dz = -\frac{(i\,b)^{-n}}{2\,b}\left(-\text{Ei}((a-i\,b)\,z)\,n! + n!\sum_{k=1}^{n}\frac{b^k\,(b+a\,i)^{-k}\,\Gamma(k,\,(i\,b-a)\,z)}{k!} + \text{Ei}(a\,z)\,\Gamma(n+1,\,i\,b\,z) + (-1)^n\left(-\text{Ei}((a+b\,i)\,z)\,n! + n!\sum_{k=1}^{n}\frac{b^k\,(b-i\,a)^{-k}\,\Gamma(k,\,-(a+b\,i)\,z)}{k!} + \text{Ei}(a\,z)\,\Gamma(n+1,\,-i\,b\,z)\right)\right) /;\, n\in\mathbb{N}$$

06.35.21.0030.01

$$\int z\sin(b\,z)\,\text{Ei}(a\,z)\,dz = \frac{1}{2\,b^2}\left(i\,\text{Ei}((a+b\,i)\,z) - i\,\text{Ei}((a-i\,b)\,z) + 2\,\text{Ei}(a\,z)\,(\sin(b\,z)-b\,z\cos(b\,z)) + \frac{2\,b\,e^{a\,z}\,(a\cos(b\,z)+b\sin(b\,z))}{a^2+b^2}\right)$$

06.35.21.0031.01

$$\int z^2\sin(b\,z)\,\text{Ei}(a\,z)\,dz = -\frac{1}{b^3}\left(-\frac{e^{a\,z}\left(a^2+\left(a^2+b^2\right)z\,a+3\,b^2\right)\cos(b\,z)\,b^2}{\left(a^2+b^2\right)^2} - \frac{e^{a\,z}\left(b^2\left(a^2+b^2\right)z-2\,a\left(a^2+2\,b^2\right)\right)\sin(b\,z)\,b}{\left(a^2+b^2\right)^2} + \text{Ei}((a+b\,i)\,z) + \text{Ei}((a-i\,b)\,z) + \text{Ei}(a\,z)\left(\left(b^2\,z^2-2\right)\cos(b\,z)-2\,b\,z\sin(b\,z)\right)\right)$$

06.35.21.0032.01

$$\int z^3\sin(b\,z)\,\text{Ei}(a\,z)\,dz = \frac{1}{b^4}\left(\text{Ei}(a\,z)\left(b\,z\left(6-b^2\,z^2\right)\cos(b\,z)+3\left(b^2\,z^2-2\right)\sin(b\,z)\right) + \frac{1}{\left(a^2+b^2\right)^3}\left(3\,i\,(\text{Ei}((a-i\,b)\,z)-\text{Ei}((a+b\,i)\,z))\left(a^2+b^2\right)^3 + b\,e^{a\,z}\left(a\left(a^2+b^2\right)^2 z^2\,b^2+\left(a^2+b^2\right)\left(a^2+5\,b^2\right)z\,b^2-2\,a\left(3\,a^4+8\,b^2\,a^2+9\,b^4\right)\right)\cos(b\,z) + b^2\,e^{a\,z}\left(-3\,a^4-6\,b^2\,a^2-\left(a^2+b^2\right)\left(3\,a^2+7\,b^2\right)z\,a-11\,b^4+b^2\left(a^2+b^2\right)^2 z^2\right)\sin(b\,z)\right)\right)$$

Involving cos and power

06.35.21.0033.01

$$\int z^n \cos(b z)\,\mathrm{Ei}(a z)\,dz = \frac{1}{2 b} i (i b)^{-n} \left(-\mathrm{Ei}((a - i b) z)\, n! + n! \sum_{k=1}^{n} \frac{b^k (b + a i)^{-k} \Gamma(k, (i b - a) z)}{k!} + \mathrm{Ei}(a z)\,\Gamma(n+1, i b z) + (-1)^n \left(\mathrm{Ei}((a + b i) z)\, n! - n! \sum_{k=1}^{n} \frac{b^k (b - i a)^{-k} \Gamma(k, -(a + b i) z)}{k!} - \mathrm{Ei}(a z)\,\Gamma(n+1, -i b z)\right)\right) /;\ n \in \mathbb{N}$$

06.35.21.0034.01

$$\int z \cos(b z)\,\mathrm{Ei}(a z)\,dz = -\frac{1}{2 b^2} \left(\mathrm{Ei}((a + b i) z) + \mathrm{Ei}((a - i b) z) + \frac{2 b e^{a z} (a \sin(b z) - b \cos(b z))}{a^2 + b^2} - 2\,\mathrm{Ei}(a z) (\cos(b z) + b z \sin(b z))\right)$$

06.35.21.0035.01

$$\int z^2 \cos(b z)\,\mathrm{Ei}(a z)\,dz = \frac{1}{b^3} \left(-\frac{e^{a z} \left(a^2 + \left(a^2 + b^2\right) z a + 3 b^2\right) \sin(b z)\, b^2}{\left(a^2 + b^2\right)^2} + \frac{e^{a z} \left(b^2 \left(a^2 + b^2\right) z - 2 a \left(a^2 + 2 b^2\right)\right) \cos(b z)\, b}{\left(a^2 + b^2\right)^2} + i (\mathrm{Ei}((a - i b) z) - \mathrm{Ei}((a + b i) z)) + \mathrm{Ei}(a z) \left(2 b z \cos(b z) + \left(b^2 z^2 - 2\right) \sin(b z)\right)\right)$$

06.35.21.0036.01

$$\int z^3 \cos(b z)\,\mathrm{Ei}(a z)\,dz = \frac{1}{b^4} \left(\frac{1}{\left(a^2 + b^2\right)^3} \left(b^2 e^{a z} \left(-3 a^4 - 6 b^2 a^2 - \left(a^2 + b^2\right) \left(3 a^2 + 7 b^2\right) z a - 11 b^4 + b^2 \left(a^2 + b^2\right)^2 z^2\right) \cos(b z)\right) + 3 (\mathrm{Ei}((a + b i) z) + \mathrm{Ei}((a - i b) z)) - \frac{1}{\left(a^2 + b^2\right)^3} \left(b e^{a z} \left(a \left(a^2 + b^2\right)^2 z^2 b^2 + \left(a^2 + b^2\right) \left(a^2 + 5 b^2\right) z b^2 - 2 a \left(3 a^4 + 8 b^2 a^2 + 9 b^4\right)\right) \sin(b z)\right) + \mathrm{Ei}(a z) \left(3 \left(b^2 z^2 - 2\right) \cos(b z) + b z \left(b^2 z^2 - 6\right) \sin(b z)\right)\right)$$

Involving hyperbolic functions

Involving sinh

06.35.21.0037.01

$$\int \sinh(b z)\,\mathrm{Ei}(a z)\,dz = -\frac{-2 \cosh(b z)\,\mathrm{Ei}(a z) + \mathrm{Ei}((a - b) z) + \mathrm{Ei}((a + b) z)}{2 b}$$

06.35.21.0038.01

$$\int \sinh(a z)\,\mathrm{Ei}(a z)\,dz = -\frac{-(e^{-a z} + e^{a z})\,\mathrm{Ei}(a z) + \mathrm{Ei}(2 a z) + \log(a z)}{2 a}$$

Involving cosh

06.35.21.0039.01

$$\int \cosh(b z)\,\mathrm{Ei}(a z)\,dz = \frac{\mathrm{Ei}((a - b) z) - \mathrm{Ei}((a + b) z) + 2\,\mathrm{Ei}(a z) \sinh(b z)}{2 b}$$

06.35.21.0040.01

$$\int \cosh(a z)\,\text{Ei}(a z)\,dz = -\frac{\text{Chi}(2 a z) - \log(a z) - 2\,\text{Ei}(a z)\sinh(a z) + \text{Shi}(2 a z)}{2 a}$$

Involving hyperbolic functions and a power function

Involving sinh and power

06.35.21.0041.01

$$\int z^n \sinh(b z)\,\text{Ei}(a z)\,dz = \frac{1}{2} b^{-n-1} \left(-\text{Ei}((a-b) z)\, n! + n! \sum_{k=1}^{n} \frac{b^k (b-a)^{-k}\, \Gamma(k, (b-a) z)}{k!} + \text{Ei}(a z)\, \Gamma(n+1, b z) + (-1)^n \left(-\text{Ei}((a+b) z)\, n! + n! \sum_{k=1}^{n} \frac{b^k (a+b)^{-k}\, \Gamma(k, -(a+b) z)}{k!} + \text{Ei}(a z)\, \Gamma(n+1, -b z) \right) \right) /;\ n \in \mathbb{N}$$

06.35.21.0042.01

$$\int z^n \sinh(b z)\,\text{Ei}(a z)\,dz = \frac{1}{2} a^{-n-1} \left(\text{Ei}(a z) \left((-1)^n\, \Gamma(n+1, -a z) + \Gamma(n+1, a z) \right) - \left((-1)^n\, \text{Ei}(2 a z)\, n! + \log(z)\, n! \right) + (-1)^n\, n! \sum_{k=1}^{n} \frac{2^{-k}\, \Gamma(k, -2 a z)}{k!} - n! \sum_{k=1}^{n} \frac{(a z)^k}{k\, k!} \right) /;\ n \in \mathbb{N}$$

06.35.21.0043.01

$$\int z \sinh(b z)\,\text{Ei}(a z)\,dz = \frac{1}{2 b^2} \left(-\text{Ei}((a-b) z) + \text{Ei}((a+b) z) + 2\,\text{Ei}(a z) (b z \cosh(b z) - \sinh(b z)) + \frac{2 b\, e^{a z} (b \sinh(b z) - a \cosh(b z))}{a^2 - b^2} \right)$$

06.35.21.0044.01

$$\int z^2 \sinh(b z)\,\text{Ei}(a z)\,dz = -\frac{1}{b^3} \left(\frac{e^{a z} \left(z a^3 + a^2 - b^2 z a - 3 b^2 \right) \cosh(b z)\, b^2}{\left(a^2 - b^2\right)^2} + \frac{e^{a z} \left(z b^4 + 4 a b^2 - a^2 z b^2 - 2 a^3 \right) \sinh(b z)\, b}{\left(a^2 - b^2\right)^2} + \text{Ei}((a-b) z) + \text{Ei}((a+b) z) + \text{Ei}(a z) \left(2 b z \sinh(b z) - \left(b^2 z^2 + 2 \right) \cosh(b z) \right) \right)$$

06.35.21.0045.01

$$\int z^3 \sinh(b z)\,\text{Ei}(a z)\,dz = \frac{1}{b^4} \left(\frac{1}{(a-b)^3 (a+b)^3} \left(3 \left(\text{Ei}((a-b) z) - \text{Ei}((a+b) z) \right) \left(b^2 - a^2 \right)^3 - b\, e^{a z} \left(a \left(a^2 - b^2 \right)^2 z^2 b^2 + \left(a^4 - 6 b^2 a^2 + 5 b^4 \right) z b^2 + 2 a \left(3 a^4 - 8 b^2 a^2 + 9 b^4 \right) \right) \cosh(b z) + b^2 e^{a z} \left(3 a^4 - 6 b^2 a^2 + \left(3 a^4 - 10 b^2 a^2 + 7 b^4 \right) z a + 11 b^4 + b^2 \left(a^2 - b^2 \right)^2 z^2 \right) \sinh(b z) \right) + \text{Ei}(a z) \left(b z \left(b^2 z^2 + 6 \right) \cosh(b z) - 3 \left(b^2 z^2 + 2 \right) \sinh(b z) \right) \right)$$

Involving cosh and power

06.35.21.0046.01

$$\int z^n \cosh(b\,z)\,\mathrm{Ei}(a\,z)\,dz = \frac{1}{2}\,b^{-n-1}\left(\mathrm{Ei}((a-b)\,z)\,n! - n!\sum_{k=1}^{n}\frac{b^k\,(b-a)^{-k}\,\Gamma(k,\,(b-a)\,z)}{k!} - \mathrm{Ei}(a\,z)\,\Gamma(n+1,\,b\,z) + (-1)^n\left(-\mathrm{Ei}((a+b)\,z)\,n! + n!\sum_{k=1}^{n}\frac{b^k\,(a+b)^{-k}\,\Gamma(k,\,-(a+b)\,z)}{k!} + \mathrm{Ei}(a\,z)\,\Gamma(n+1,\,-b\,z)\right)\right)\;/;\;n\in\mathbb{N}$$

06.35.21.0047.01

$$\int z^n \cosh(b\,z)\,\mathrm{Ei}(a\,z)\,dz = \frac{1}{2}\,a^{-n-1}\left(\mathrm{Ei}(a\,z)\,((-1)^n\,\Gamma(n+1,\,-a\,z) - \Gamma(n+1,\,a\,z)) - ((-1)^n\,\mathrm{Ei}(2\,a\,z)\,n! - n!\log(z)) + (-1)^n\,n!\sum_{k=1}^{n}\frac{2^{-k}\,\Gamma(k,\,-2\,a\,z)}{k!} + n!\sum_{k=1}^{n}\frac{(a\,z)^k}{k\,k!}\right)\;/;\;n\in\mathbb{N}$$

06.35.21.0048.01

$$\int z\cosh(b\,z)\,\mathrm{Ei}(a\,z)\,dz = \frac{1}{2\,b^2}\left(\mathrm{Ei}((a-b)\,z) + \mathrm{Ei}((a+b)\,z) + 2\,\mathrm{Ei}(a\,z)\,(b\,z\sinh(b\,z) - \cosh(b\,z)) - \frac{2\,b\,e^{a\,z}\,(b\cosh(b\,z) - a\sinh(b\,z))}{b^2-a^2}\right)$$

06.35.21.0049.01

$$\int z^2\cosh(b\,z)\,\mathrm{Ei}(a\,z)\,dz = \frac{1}{b^3}\left(\frac{e^{a\,z}\left(3\,b^2 - a\left(z\,a^2 + a - b^2\,z\right)\right)\sinh(b\,z)\,b^2}{\left(a^2-b^2\right)^2} + \frac{e^{a\,z}\left(-z\,b^4 - 4\,a\,b^2 + a^2\,z\,b^2 + 2\,a^3\right)\cosh(b\,z)\,b}{\left(a^2-b^2\right)^2} + \mathrm{Ei}((a-b)\,z) - \mathrm{Ei}((a+b)\,z) + \mathrm{Ei}(a\,z)\left(\left(b^2\,z^2+2\right)\sinh(b\,z) - 2\,b\,z\cosh(b\,z)\right)\right)$$

06.35.21.0050.01

$$\int z^3\cosh(b\,z)\,\mathrm{Ei}(a\,z)\,dz = \frac{1}{b^4}\left(\mathrm{Ei}(a\,z)\left(b\,z\left(b^2\,z^2+6\right)\sinh(b\,z) - 3\left(b^2\,z^2+2\right)\cosh(b\,z)\right) + \frac{1}{(a-b)^3\,(a+b)^3}\left(3\,(\mathrm{Ei}((a-b)\,z) + \mathrm{Ei}((a+b)\,z))\left(a^2-b^2\right)^3 + b^2\,e^{a\,z}\left(3\,a^4 - 6\,b^2\,a^2 + \left(3\,a^4 - 10\,b^2\,a^2 + 7\,b^4\right)z\,a + 11\,b^4 + b^2\left(a^2-b^2\right)^2 z^2\right)\cosh(b\,z) - b\,e^{a\,z}\left(a\left(a^2-b^2\right)^2 z^2\,b^2 + \left(a^4 - 6\,b^2\,a^2 + 5\,b^4\right)z\,b^2 + 2\,a\left(3\,a^4 - 8\,b^2\,a^2 + 9\,b^4\right)\right)\sinh(b\,z)\right)\right)$$

Involving logarithm

Involving log

06.35.21.0051.01

$$\int \log(b\,z)\,\mathrm{Ei}(a\,z)\,dz = \frac{\mathrm{Ei}(a\,z)\,(-a\,z + a\log(b\,z)\,z + 1) - e^{a\,z}\,(\log(b\,z) - 1)}{a}$$

Involving logarithm and a power function

Involving log and power

06.35.21.0052.01

$$\int z^{\alpha-1}\log(b\,z)\,\mathrm{Ei}(a\,z)\,dz = \frac{1}{\alpha^3}\,(z^{\alpha}\,(-a\,z)^{-\alpha}\,({}_2F_2(\alpha,\,\alpha;\,\alpha+1,\,\alpha+1;\,a\,z)\,(-a\,z)^{\alpha} + \alpha\,(\mathrm{Ei}(a\,z)\,(\alpha\log(b\,z)-1)\,(-a\,z)^{\alpha} - \Gamma(\alpha+1)\log(z) + \Gamma(\alpha,\,-a\,z)\,(\alpha\log(b\,z)-1))))$$

06.35.21.0053.01

$$\int z\log(z)\,\mathrm{Ei}(a\,z)\,dz = \frac{1}{4\,a^2}\left(\mathrm{Ei}(a\,z)\left(-a^2\,z^2+2\,a^2\log(z)\,z^2-2\right)+e^{a\,z}\,(a\,z+(2-2\,a\,z)\log(z)+1)\right)$$

06.35.21.0054.01

$$\int z^2\log(z)\,\mathrm{Ei}(a\,z)\,dz = \frac{1}{9\,a^3}\left(\mathrm{Ei}(a\,z)\left(-a^3\,z^3+3\,a^3\log(z)\,z^3+6\right)+e^{a\,z}\left(a^2\,z^2+a\,z-3\left(a^2\,z^2-2\,a\,z+2\right)\log(z)-7\right)\right)$$

06.35.21.0055.01

$$\int z^3\log(z)\,\mathrm{Ei}(a\,z)\,dz = \frac{1}{16\,a^4}\left(\mathrm{Ei}(a\,z)\left(-a^4\,z^4+4\,a^4\log(z)\,z^4-24\right)+e^{a\,z}\left(a^3\,z^3+a^2\,z^2-14\,a\,z-4\left(a^3\,z^3-3\,a^2\,z^2+6\,a\,z-6\right)\log(z)+38\right)\right)$$

Involving functions of the direct function

Involving elementary functions of the direct function

Involving powers of the direct function

06.35.21.0056.01

$$\int \mathrm{Ei}(a\,z)^2\,dz = \frac{a\,z\,\mathrm{Ei}(a\,z)^2-2\,e^{a\,z}\,\mathrm{Ei}(a\,z)+2\,\mathrm{Ei}(2\,a\,z)}{a}$$

Involving products of the direct function

06.35.21.0057.01

$$\int \mathrm{Ei}(-a\,z)\,\mathrm{Ei}(a\,z)\,dz = \frac{e^{-a\,z}\,\mathrm{Ei}(a\,z)+\mathrm{Ei}(-a\,z)\,(a\,z\,\mathrm{Ei}(a\,z)-e^{a\,z})}{a}$$

06.35.21.0058.01

$$\int \mathrm{Ei}(a\,z)\,\mathrm{Ei}(b\,z)\,dz = \frac{1}{a\,b}\left(-b\,e^{a\,z}\,\mathrm{Ei}(b\,z)-a\,\mathrm{Ei}(a\,z)\left(e^{b\,z}-b\,z\,\mathrm{Ei}(b\,z)\right)+(a+b)\,\mathrm{Ei}((a+b)\,z)\right)$$

Involving functions of the direct function and elementary functions

Involving elementary functions of the direct function and elementary functions

Involving powers of the direct function and a power function

06.35.21.0059.01

$$\int z^n \operatorname{Ei}(a z)^2 \, dz = \frac{1}{n+1} \left(2 (-a)^{-n-1} \left(-\operatorname{Ei}(2 a z) \, n! + n! \sum_{k=1}^{n} \frac{2^{-k} \, \Gamma(k, -2 a z)}{k!} + \operatorname{Ei}(a z) \, \Gamma(n+1, -a z)\right) + z^{n+1} \operatorname{Ei}(a z)^2\right) /; n \in \mathbb{N}$$

06.35.21.0060.01

$$\int z \operatorname{Ei}(a z)^2 \, dz = \frac{a^2 z^2 \operatorname{Ei}(a z)^2 - 2 e^{a z} (a z - 1) \operatorname{Ei}(a z) + e^{2 a z} - 2 \operatorname{Ei}(2 a z)}{2 a^2}$$

06.35.21.0061.01

$$\int z^2 \operatorname{Ei}(a z)^2 \, dz = \frac{1}{6 a^3} \left(2 a^3 \operatorname{Ei}(a z)^2 z^3 + e^{2 a z} (2 a z - 5) - 4 e^{a z} \left(a^2 z^2 - 2 a z + 2\right) \operatorname{Ei}(a z) + 8 \operatorname{Ei}(2 a z)\right)$$

06.35.21.0062.01

$$\int z^3 \operatorname{Ei}(a z)^2 \, dz = \frac{1}{4 a^4} \left(a^4 \operatorname{Ei}(a z)^2 z^4 + e^{2 a z} \left(a^2 z^2 - 4 a z + 8\right) - 2 e^{a z} \left(a^3 z^3 - 3 a^2 z^2 + 6 a z - 6\right) \operatorname{Ei}(a z) - 12 \operatorname{Ei}(2 a z)\right)$$

Involving products of the direct function and a power function

06.35.21.0063.01

$$\int z^n \operatorname{Ei}(a z) \operatorname{Ei}(b z) \, dz = \frac{1}{n+1} \left(n! \, (-a)^{-n-1} \sum_{k=1}^{n} \frac{a^k (a+b)^{-k} \, \Gamma(k, -(a+b) z)}{k!} + \frac{\operatorname{Ei}((a+b) z) \, n! \, (-a)^{-n}}{a} + \operatorname{Ei}(b z) \left(\Gamma(n+1, -a z) (-a)^{-n-1} + z^{n+1} \operatorname{Ei}(a z)\right) + \frac{1}{b} (-b)^{-n} \left(\operatorname{Ei}((a+b) z) \, n! - n! \sum_{k=1}^{n} \frac{b^k (a+b)^{-k} \, \Gamma(k, -(a+b) z)}{k!} - \operatorname{Ei}(a z) \, \Gamma(n+1, -b z)\right)\right) /; n \in \mathbb{N}$$

06.35.21.0064.01

$$\int z^n \operatorname{Ei}(a z) \operatorname{Ei}(-a z) \, dz = \frac{(-a)^{-n}}{a (n+1)} \left((-1)^n a^{n+1} \operatorname{Ei}(-a z) \operatorname{Ei}(a z) z^{n+1} - \operatorname{Ei}(-a z) \, \Gamma(n+1, -a z) + (-1)^n \operatorname{Ei}(a z) \, \Gamma(n+1, a z) - (-1)^n n! \log(z) + n! \log(z) + n! \sum_{k=1}^{n} \frac{(-a z)^k}{k \, k!} - (-1)^n n! \sum_{k=1}^{n} \frac{(a z)^k}{k \, k!}\right) /; n \in \mathbb{N}$$

06.35.21.0065.01

$$\int z \operatorname{Ei}(a z) \operatorname{Ei}(b z) \, dz = -\frac{1}{2 a^2 b^2} \left(\operatorname{Ei}(a z) \left(e^{b z} (b z - 1) - b^2 z^2 \operatorname{Ei}(b z)\right) a^2 - a b \, e^{(a+b) z} + b^2 e^{a z} (a z - 1) \operatorname{Ei}(b z) + \left(a^2 + b^2\right) \operatorname{Ei}((a+b) z)\right)$$

06.35.21.0066.01

$$\int z \operatorname{Ei}(a z) \operatorname{Ei}(-a z) \, dz = \frac{1}{2 a^2} \left(e^{-a z} \left((a z + 1) \operatorname{Ei}(a z) + e^{a z} \operatorname{Ei}(-a z) \left(a^2 \operatorname{Ei}(a z) z^2 + e^{a z} (1 - a z)\right) - 2 e^{a z} \log(z)\right)\right)$$

06.35.21.0067.01

$$\int z^2 \operatorname{Ei}(a z) \operatorname{Ei}(b z) \, dz = \frac{1}{3 a^3 b^3 (a+b)} \left(a b \, e^{(a+b) z} \left(-2 a^2 - b a + b (a+b) z a - 2 b^2\right) + (a+b) \left(\operatorname{Ei}(a z) \left(b^3 z^3 \operatorname{Ei}(b z) - e^{b z} (b z (b z - 2) + 2)\right) a^3 - b^3 e^{a z} (a z (a z - 2) + 2) \operatorname{Ei}(b z) + 2 \left(a^3 + b^3\right) \operatorname{Ei}((a+b) z)\right)\right)$$

06.35.21.0068.01

$$\int z^2 \,\mathrm{Ei}(a\,z)\,\mathrm{Ei}(-a\,z)\,dz = \frac{1}{3\,a^3}\left(e^{-a\,z}\left(-4\,a\,e^{a\,z}\,z + \left(a^2\,z^2 + 2\,a\,z + 2\right)\mathrm{Ei}(a\,z) - e^{a\,z}\,\mathrm{Ei}(-a\,z)\left(e^{a\,z}\left(a^2\,z^2 - 2\,a\,z + 2\right) - a^3\,z^3\,\mathrm{Ei}(a\,z)\right)\right)\right)$$

06.35.21.0069.01

$$\int z^3 \,\mathrm{Ei}(a\,z)\,\mathrm{Ei}(b\,z)\,dz = \frac{1}{4\,a^4\,b^4}\Bigg(\mathrm{Ei}(a\,z)\left(b^4\,\mathrm{Ei}(b\,z)\,z^4 + e^{b\,z}\,(6 - b\,z\,(b\,z\,(b\,z-3)+6))\right)a^4 + \frac{1}{(a+b)^2}\left(a\,b\,e^{(a+b)\,z}\left((b\,z\,(b\,z-3)+6)\,a^4 + b\,(b\,z\,(2\,b\,z-5)+9)\,a^3 + b^2\,(b\,z\,(b\,z-5)+2)\,a^2 - 3\,b^3\,(b\,z-3)\,a + 6\,b^4\right)\right) - b^4\,e^{a\,z}\,(a\,z\,(a\,z\,(a\,z-3)+6)-6)\,\mathrm{Ei}(b\,z) - 6\left(a^4+b^4\right)\mathrm{Ei}((a+b)\,z)\Bigg)$$

06.35.21.0070.01

$$\int z^3 \,\mathrm{Ei}(a\,z)\,\mathrm{Ei}(-a\,z)\,dz = \frac{1}{4\,a^4}\left(e^{-a\,z}\left(\left(a^3\,z^3 + 3\,a^2\,z^2 + 6\,a\,z + 6\right)\mathrm{Ei}(a\,z) - e^{a\,z}\,\mathrm{Ei}(-a\,z)\left(e^{a\,z}\left(a^3\,z^3 - 3\,a^2\,z^2 + 6\,a\,z - 6\right) - a^4\,z^4\,\mathrm{Ei}(a\,z)\right) - 3\,e^{a\,z}\left(a^2\,z^2 + 4\log(z)\right)\right)\right)$$

Definite integration

Involving the direct function

06.35.21.0071.01

$$\int_0^\infty e^{-t\,z}\,\mathrm{Ei}(t)\,dt = -\frac{\log(z-1)}{z} \;/;\; \mathrm{Re}(z) > 1$$

Integral transforms

Laplace transforms

06.35.22.0001.01

$$\mathcal{L}_t[\mathrm{Ei}(t)]\,(z) = -\frac{\log(z-1)}{z} \;/;\; \mathrm{Re}(z) > 1$$

Operations

Limit operation

06.35.25.0001.01

$$\lim_{x\to\infty} \mathrm{Ei}(a + b\,x) = \begin{cases} \pi\,i & \left(0 < \arg(a) < \pi \bigwedge \frac{\pi}{2} \le \arg(b) \le \pi\right) \bigvee \left(\mathrm{Im}(a) \le 0 \bigwedge \frac{\pi}{2} \le \arg(b) < \pi\right) \\ -\pi\,i & \left(0 \le \arg(a) \le \pi \bigwedge \arg(b) \le -\frac{\pi}{2}\right) \bigvee \left(-\pi < \arg(a) < 0 \bigwedge \left(\arg(b) \le -\frac{\pi}{2} \bigvee \arg(b) = \pi\right)\right) \\ 0 & \mathrm{Im}(a) = 0 \land \arg(b) = \pi \\ \tilde{\infty} & \mathrm{True} \end{cases}$$

Representations through more general functions

Through hypergeometric functions

Involving ${}_pF_q$

06.35.26.0001.01

$$\text{Ei}(z) = z\,{}_2F_2(1,1;2,2;z) + \frac{1}{2}\left(\log(z) - \log\left(\frac{1}{z}\right)\right) + \gamma$$

Involving hypergeometric U

06.35.26.0009.01

$$\text{Ei}(z) = -e^z\,U(1,1,-z) - \log(-z) + \frac{1}{2}\left(\log(z) - \log\left(\frac{1}{z}\right)\right)$$

Through Meijer G

Classical cases for the direct function itself

06.35.26.0002.01

$$\text{Ei}(z) = -\frac{1}{2}\left(\log\left(\frac{1}{z}\right) - \log(z)\right) - G_{2,3}^{1,2}\left(-z \,\middle|\, \begin{matrix} 1,1 \\ 1,0,0 \end{matrix}\right) + \gamma$$

06.35.26.0003.01

$$\text{Ei}(z) = -\log(-z) - \frac{1}{2}\left(\log\left(\frac{1}{z}\right) - \log(z)\right) - G_{1,2}^{2,0}\left(-z \,\middle|\, \begin{matrix} 1 \\ 0,0 \end{matrix}\right)$$

Classical cases involving exp

06.35.26.0004.01

$$e^{-z}\,\text{Ei}(z) = -\frac{1}{2}\,e^{-z}\left(\log\left(\frac{1}{z}\right) + 2\log(-z) - \log(z)\right) - G_{1,2}^{2,1}\left(-z \,\middle|\, \begin{matrix} 0 \\ 0,0 \end{matrix}\right)$$

06.35.26.0010.01

$$e^{-z}\,\text{Ei}(z) = -\frac{1}{2}\,e^{-z}\left(\log\left(\frac{1}{z}\right) + \log(z)\right) - \pi\,G_{2,3}^{2,1}\left(z \,\middle|\, \begin{matrix} 0, \frac{1}{2} \\ 0,0,\frac{1}{2} \end{matrix}\right)$$

06.35.26.0005.01

$$e^{-z}\,\text{Ei}(z) = -\pi\,G_{2,3}^{2,1}\left(z \,\middle|\, \begin{matrix} 0, \frac{1}{2} \\ 0,0,\frac{1}{2} \end{matrix}\right) /; z \notin (-\infty, 0)$$

Classical cases for products of Ei

06.35.26.0006.01

$$\text{Ei}(-z)\,\text{Ei}(z) = \frac{\sqrt{\pi}}{2}\,G_{2,4}^{3,1}\left(\frac{z^2}{4} \,\middle|\, \begin{matrix} 0,1 \\ 0,0,0,\frac{1}{2} \end{matrix}\right) /; \text{Im}(z) = 0$$

Through other functions

06.35.26.0007.01

$$\text{Ei}(z) = -\Gamma(0,-z) + \frac{1}{2}\left(\log(z) - \log\left(\frac{1}{z}\right)\right) - \log(-z)$$

06.35.26.0008.01

$$\text{Ei}(z) = -E_1(-z) + \frac{1}{2}\left(\log(z) - \log\left(\frac{1}{z}\right)\right) - \log(-z)$$

Representations through equivalent functions

With related functions

06.35.27.0001.01

$$\text{Ei}(z) = \text{Chi}(z) + \text{Shi}(z) - \frac{1}{2}\left(\log\left(\frac{1}{z}\right) + \log(z)\right)$$

06.35.27.0002.01

$$\text{Ei}(z) = \text{Ci}(i z) - i \,\text{Si}(i z) - \frac{1}{2}\left(\log\left(\frac{1}{z}\right) - \log(z)\right) - \log(i z)$$

06.35.27.0003.01

$$\text{Ei}(z) = \text{li}(e^z) \;/;\; -\pi < \text{Im}(z) \leq \pi$$

Theorems

The real part of the Heisenberg-Euler Lagrangian of quantum electrodynamics

The real part of the Heisenberg–Euler Lagrangian of quantum electrodynamics $L_{\text{eff}}(\boldsymbol{E}, \boldsymbol{B})$ can be expressed as the following series.

$$\text{Re}(L_{\text{eff}}(\boldsymbol{E}, \boldsymbol{B})) = c_1 - \frac{\alpha}{\pi^2} |c_2| \sum_{k=1}^{\infty} \coth\left(\pi k \frac{\eta}{\xi}\right)\left(\text{Ci}\left(\frac{k}{\xi}\right)\cos\left(\frac{k}{\xi}\right) + \text{Si}\left(\frac{k}{\xi}\right)\sin\left(\frac{k}{\xi}\right)\right) - \coth\left(\pi k \frac{\xi}{\eta}\right)\left(\text{Ei}\left(-\frac{k}{\xi}\right)\exp\left(\frac{k}{\xi}\right) + \text{Ei}\left(\frac{k}{\xi}\right)\exp\left(-\frac{k}{\xi}\right)\right) /;$$

$$\xi = \frac{e}{\pi}\sqrt{-c_1 + \sqrt{c_1^2 + c_2^2}} \land \eta = \frac{e}{\pi}\sqrt{c_1 + \sqrt{c_1^2 + c_2^2}} \land c_1 = \frac{1}{2}\left(\boldsymbol{E}^2 - \boldsymbol{B}^2\right) \land c_2 = \boldsymbol{E} \cdot \boldsymbol{B},$$

where $\boldsymbol{E}$ is the electric field, $\boldsymbol{B}$ is the magnetic induction and α is the fine structure constant.

The Nambu-Goldstone modes for weakly interacting fermions in a magnetic field

The Nambu-Goldstone modes for weakly interacting fermions in a magnetic field are described by $\left(-\Delta + m^2 + V(\boldsymbol{r})\right)\psi(\boldsymbol{r}) = 0$, where in 3+1–dimensional ladder quantum electrodynamics the potential $V(\boldsymbol{r})$ takes the form $V(\boldsymbol{r}) \propto e^{\frac{r^2}{2 l^2}} \text{Ei}\left(-\frac{r^2}{2 l^2}\right)$, where l is the magnetic length.

History

– A. M. Legendre (1811)
– O. Schlomilch (1846)
– F. Arndt (1847)
– J. W. L. Glaisher (1870) introduced the notation Ei

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