Factorial

Notations

Traditional name

Factorial

Traditional notation

$n!$

Mathematica StandardForm notation

`Factorial[n]`

Specific values

Specialized values

06.01.03.0001.01

$$n! = \prod_{k=1}^{n} k \;/;\; n \in \mathbb{N}$$

06.01.03.0002.01

$$(-n)! = \tilde{\infty} \;/;\; n \in \mathbb{N}^+$$

06.01.03.0003.01

$$\left(n + \frac{p}{q}\right)! = \frac{1}{q^n} \frac{p}{q}! \prod_{k=1}^{n} (p + k\, q) \;/;\; n \in \mathbb{N} \land p \in \mathbb{N}^+ \land q \in \mathbb{N}^+ \land p < q$$

06.01.03.0004.01

$$\left(\frac{p}{q} - n\right)! = \frac{(-1)^n q^n}{\prod_{k=1}^{n} (-p + k\, q - q)} \frac{p}{q}! \;/;\; n \in \mathbb{N} \land p \in \mathbb{N}^+ \land q \in \mathbb{N}^+ \land p < q$$

Values at fixed points

06.01.03.0005.01

$$(-2)! = \tilde{\infty}$$

06.01.03.0006.01

$$(-1)! = \tilde{\infty}$$

06.01.03.0007.01

$$0! = 1$$

06.01.03.0008.01

$1! = 1$

06.01.03.0009.01

$2! = 2$

06.01.03.0010.01

$3! = 6$

06.01.03.0011.01

$4! = 24$

06.01.03.0012.01

$5! = 120$

06.01.03.0013.01

$6! = 720$

06.01.03.0014.01

$7! = 5040$

06.01.03.0015.01

$8! = 40320$

06.01.03.0016.01

$9! = 362880$

06.01.03.0017.01

$10! = 3628800$

Values at infinities

06.01.03.0018.01

$\infty! = \infty$

06.01.03.0019.01

$(-\infty)! = ¿$

06.01.03.0020.01

$(i \infty)! = 0$

06.01.03.0021.01

$(-i \infty)! = 0$

06.01.03.0022.01

$\tilde{\infty}! = ¿$

General characteristics

Domain and analyticity

$n!$ is an analytical function of n which is defined in the whole complex n-plane with the exception of countably many points $n = -k \,/;\, k \in \mathbb{N}^+$. $1/n!$ is an entire function.

06.01.04.0001.01

$n \longrightarrow n! :: \mathbb{C} \longrightarrow \mathbb{C}$

Symmetries and periodicities

Mirror symmetry

06.01.04.0002.01

$\overline{n}! == \overline{n!}$

Periodicity

No periodicity

Poles and essential singularities

The function $n!$ has an infinite set of singular points:

a) $n == -k$ /; $k \in \mathbb{N}^+$ are the simple poles with residues $\frac{(-1)^{k-1}}{(k-1)!}$;

b) $n == \tilde{\infty}$ is the point of convergence of poles, which is an essential singular point.

06.01.04.0003.01

$$\mathcal{Sing}_n(n!) == \{\{\{-k, 1\} /; k \in \mathbb{N}^+\}, \{\tilde{\infty}, \infty\}\}$$

06.01.04.0004.01

$$\text{res}_n(n!)(-k) == \frac{(-1)^{k-1}}{(k-1)!} /; k \in \mathbb{N}^+$$

Branch points

The function $n!$ does not have branch points.

06.01.04.0005.01

$$\mathcal{BP}_n(n!) == \{\}$$

Branch cuts

The function $n!$ does not have branch cuts.

06.01.04.0006.01

$$\mathcal{BC}_n(n!) == \{\}$$

Series representations

Generalized power series

Expansions at $n == n_0$ /; $n_0 \neq -m$

06.01.06.0001.02

$$n! \propto n_0!\left(1 + \psi(n_0+1)(n-n_0) + \frac{1}{2}\left(\psi(n_0+1)^2 + \psi^{(1)}(n_0+1)\right)(n-n_0)^2 + \frac{1}{6}\left(\psi(n_0+1)^3 + 3\,\psi^{(1)}(n_0+1)\,\psi(n_0+1) + \psi^{(2)}(n_0+1)\right)(n-n_0)^3 + \ldots\right) /; (n \to n_0) \bigwedge -n_0 \notin \mathbb{N}^+$$

06.01.06.0002.02

$$n! == \sum_{k=0}^{\infty} \frac{\Gamma^{(k)}(n_0+1)}{k!} (n-n_0)^k \,/;\, (n \to n_0) \bigwedge -n_0 \notin \mathbb{N}^+$$

06.01.06.0003.02

$$n! \propto n_0! \,(1+\psi(n_0+1)\,(n-n_0)) + O\big((n-n_0)^2\big) \,/;\, (n \to n_0) \bigwedge -n_0 \notin \mathbb{N}^+$$

Expansions at $n == -m$

06.01.06.0004.01

$$n! \propto \frac{(-1)^{m-1}}{(m-1)!\,(n+m)} (1+O(n+m)) \,/;\, (n \to -m) \bigwedge m \in \mathbb{N}^+$$

06.01.06.0005.01

$$n! \propto \frac{(-1)^{m-1}}{(m-1)!\,(n+m)} + \frac{(-1)^{m-1}\,\psi(m)}{(m-1)!} + O(n+m) \,/;\, (n \to -m) \bigwedge m \in \mathbb{N}^+$$

06.01.06.0006.01

$$n! \propto \frac{(-1)^{m-1}}{(m-1)!\,(n+m)} + \frac{(-1)^{m-1}}{(m-1)!}\Big(\psi(m) + \frac{1}{6}\big(3\,\psi(m)^2+\pi^2-3\,\psi^{(1)}(m)\big)(n+m) + \frac{1}{6}\big(\psi(m)^3+\big(\pi^2-3\,\psi^{(1)}(m)\big)\psi(m)+\psi^{(2)}(m)\big)(n+m)^2 + \frac{1}{360}\big(15\big(\psi(m)^4+2\big(\pi^2-3\,\psi^{(1)}(m)\big)\psi(m)^2+4\,\psi^{(2)}(m)\,\psi(m)+\big(3\,\psi^{(1)}(m)-2\,\pi^2\big)\psi^{(1)}(m)\big)+7\,\pi^4-15\,\psi^{(3)}(m)\big)(n+m)^3\Big) + O\big((n+m)^4\big) \,/;\, (n \to -m) \bigwedge m \in \mathbb{N}^+$$

Asymptotic series expansions

06.01.06.0007.01

$$n! \propto \sqrt{2\,\pi\,n}\; n^n\, e^{-n} \,/;\, (n \to \infty)$$

Stirling's formula

06.01.06.0008.01

$$n! \propto \sqrt{2\,\pi}\; n^{n+\frac{1}{2}}\, e^{-n}\Big(1+\frac{1}{12\,n}+\frac{1}{288\,n^2}-\frac{139}{51840\,n^3}-\frac{571}{2488320\,n^4}+\frac{163879}{209018880\,n^5}+\frac{5246819}{75246796800\,n^6}-\frac{534703531}{902961561600\,n^7}-\frac{4483131259}{86684309913600\,n^8}+\frac{432261921612371}{514904800886784000\,n^9}+O\Big(\frac{1}{n^{10}}\Big)\Big) \,/;\, |\arg(n)|<\pi \bigwedge (|n| \to \infty)$$

06.01.06.0009.01

$$n! \propto \sqrt{2\,\pi}\; n^{n+\frac{1}{2}}\, e^{-n}\Big(1+\sum_{k=1}^{\infty}\sum_{j=1}^{2k}\frac{(-1)^j\,P(2\,(j+k),\,j)\,n^{-k}}{2^{j+k}\,(j+k)!}\Big) \,/;$$

$$(|\arg(n)|<\pi \bigwedge (|n| \to \infty) \bigwedge P(m,\,j) == (m-1)\,((m-2)\,P(m-3,\,j-1)+P(m-1,\,j)) \bigwedge P(0,\,0) == 1 \bigwedge P(m,\,1) == (m-1)! \bigwedge P(m,\,j) == 0 \,/;\, m \le 3\,j-1)$$

06.01.06.0010.01

$$n! \propto \sqrt{2\,\pi}\; n^{n+\frac{1}{2}}\, e^{-n}\Big(1+O\Big(\frac{1}{n}\Big)\Big) \,/;\, |\arg(n)|<\pi \bigwedge (|n| \to \infty)$$

06.01.06.0011.01

$$\frac{(n+a)!}{(n+b)!} \propto n^{a-b} \sum_{k=0}^{\infty} \frac{(-1)^k (b-a)_k B(k, a-b+1, a+1) n^{-k}}{k!} /; |\arg(a+n+1)| < \pi \bigwedge (|n| \to \infty) \bigwedge B(n, \alpha, z) = n! \left([t^n] \frac{t^{\alpha} e^{t z}}{(e^t - 1)^{\alpha}} \right)$$

06.01.06.0012.01

$$\frac{(n+a)!}{(n+b)!} \propto n^{a-b} \left(1 + \frac{(a+b+1)(a-b)}{2n} + O\left(\frac{1}{n^2}\right) \right) /; |\arg(a+n+1)| < \pi \land (|n| \to \infty)$$

Product representations

06.01.08.0001.01

$$n! = \prod_{k=1}^{n} k /; n \in \mathbb{N}$$

06.01.08.0002.01

$$\frac{1}{n!} = (n+1) e^{(n+1)\gamma} \prod_{k=1}^{\infty} \left(1 + \frac{n+1}{k}\right) e^{-\frac{n+1}{k}}$$

06.01.08.0003.01

$$n! = \frac{1}{n+1} \prod_{k=1}^{\infty} \frac{\left(1 + \frac{1}{k}\right)^{n+1}}{\frac{n+1}{k} + 1} /; -n \notin \mathbb{N}^+$$

06.01.08.0004.01

$$n! = \frac{1}{e^{n\gamma}} \sqrt{\frac{\pi n}{\sin(\pi n)}} \prod_{k=1}^{\infty} \exp\left(-\frac{\zeta(2k+1) n^{2k+1}}{2k+1}\right)$$

06.01.08.0005.01

$$(n+1)! = e^{n(1-\gamma)} \prod_{k=2}^{\infty} \exp\left(\frac{(-1)^k (\zeta(k) - 1) n^k}{k}\right)$$

Limit representations

06.01.09.0001.01

$$n! = \lim_{x \to 1} \frac{(1-x)^{n-1}}{\prod_{k=2}^{n} \left(1 - x^{1/k}\right)} /; n \in \mathbb{N}^+$$

06.01.09.0002.01

$$n! = \lim_{m \to \infty} \frac{(1)_m m^n}{(n+1)_m}$$

06.01.09.0003.01

$$n! = \lim_{m \to \infty} m^{n+1} \mathrm{B}(n+1, m)$$

06.01.09.0004.01

$$n! = \lim_{w \to \infty} \frac{w^{n+1}}{n+1} \, {}_1F_1(n+1; n+2; -w)$$

06.01.09.0005.01

$$n! == \lim_{m\to\infty} \int_0^m \left(1 - \frac{t}{m}\right)^m t^n \, dt \;/;\; \operatorname{Re}(n) > -1$$

Transformations

Transformations and argument simplifications

Argument involving basic arithmetic operations

06.01.16.0001.01

$$(-n)! == \frac{\pi \csc(\pi n)}{(n-1)!}$$

06.01.16.0002.01

$$(n+1)! == (n+1)\, n!$$

06.01.16.0003.01

$$(n+m)! == (n+1)_m\, n!$$

06.01.16.0004.01

$$(n-1)! == \frac{n!}{n}$$

06.01.16.0005.01

$$(n-m)! == \frac{(-1)^m\, n!}{(-n)_m} \;/;\; m \in \mathbb{Z}$$

Multiple arguments

06.01.16.0006.01

$$(2n)! == \frac{2^{2n}\, n}{\sqrt{\pi}} (n-1)! \left(n - \frac{1}{2}\right)!$$

06.01.16.0007.01

$$(3n)! == \frac{3^{3n+\frac{1}{2}}\, n}{2\pi} (n-1)! \left(n - \frac{2}{3}\right)! \left(n - \frac{1}{3}\right)!$$

06.01.16.0008.01

$$(m\,n)! == n\, m^{m n+\frac{1}{2}} (2\pi)^{\frac{1-m}{2}} \prod_{k=0}^{m-1} \left(\frac{k}{m} + n - 1\right)! \;/;\; m \in \mathbb{N}^+$$

Products, sums, and powers of the direct function

Products of the direct function

06.01.16.0009.01

$$(n-1)!\,(-n)! == \frac{\pi}{\sin(\pi n)}$$

06.01.16.0010.01

$$n!\,(-n)! == n\,\pi \csc(n\,\pi)$$

06.01.16.0011.01

$$n!\,(1-n)! = (1-n)\,n\,\pi\,\csc(n\,\pi)$$

06.01.16.0012.01

$$(n+1)!\,(-n)! = (n+1)\,n\,\pi\,\csc(n\,\pi)$$

06.01.16.0013.01

$$n!\,m! = \frac{(m+n)!}{\binom{m+n}{n}}$$

06.01.16.0014.01

$$\frac{n!}{m!} = (m+1)_{n-m}$$

06.01.16.0015.01

$$\frac{n!}{m!} = (n-m)!\binom{n}{n-m}$$

06.01.16.0016.01

$$\frac{(m+n)!}{m!} = (m+1)_n$$

06.01.16.0017.01

$$\frac{m!\,n!}{(m+n+1)!} = \mathrm{B}(m+1,\,n+1)$$

06.01.16.0018.01

$$\frac{m!\,n!}{(m+n)!} = (m+n+1)\,\mathrm{B}(m+1,\,n+1)$$

Identities

Recurrence identities

Consecutive neighbors

06.01.17.0001.01

$$n! = \frac{1}{n+1}\,(n+1)!$$

06.01.17.0002.01

$$n! = n\,(n-1)!$$

Distant neighbors

06.01.17.0003.01

$$n! = \frac{(n+m)!}{(n+1)_m}$$

06.01.17.0004.01

$$n! = (-1)^m\,(-n)_m\,(n-m)!\;/;\;m\in\mathbb{Z}$$

Functional identities

Relations of special kind

06.01.17.0005.01

$$f(n) = n\, f(n-1) \;/;\; f(n) = n!\, g(n) \bigwedge g(n) = g(n-1) \bigwedge f(1) = 1$$

$n!$ is the unique nonzero solution of the functional equation $f(n) = n\, f(n-1)$ which is logarithmically convex for all real $n > 0$; that is, for which $\log(f(n))$ is a convex function for $n > 0$.

Differentiation

Low-order differentiation

06.01.20.0001.01

$$\frac{\partial n!}{\partial n} = \Gamma(n+1)\, \psi(n+1)$$

06.01.20.0002.01

$$\frac{\partial^2 n!}{\partial n^2} = \Gamma(n+1)\, \psi(n+1)^2 + \Gamma(n+1)\, \psi^{(1)}(n+1)$$

Symbolic differentiation

06.01.20.0003.01

$$\frac{\partial^m n!}{\partial z^m} = \Gamma(n+1)\, R(m, n+1) \;/;\; R(m, z) = \psi(z)\, R(m-1, z) + R^{(0,1)}(m-1, z) \bigwedge R(0, z) = 1 \bigwedge m \in \mathbb{N}^+$$

06.01.20.0004.01

$$\frac{\partial^m n!}{\partial n^m} = \int_1^\infty t^n \log^m(t)\, e^{-t}\, dt + \frac{(-m)_m}{(n+1)^{m+1}}\; {}_mF_m(z_1, z_2, \ldots, z_m; z_1+1, z_2+1, \ldots, z_m+1; -1) \;/;$$
$$z_1 = z_2 = \ldots = z_m = n+1 \bigwedge m \in \mathbb{N}^+$$

Fractional integro-differentiation

06.01.20.0005.01

$$\frac{\partial^\alpha n!}{\partial n^\alpha} = n^{-\alpha} \int_1^\infty t^n\, (n \log(t))^\alpha\, (1 - Q(-\alpha, n \log(t)))\, e^{-t}\, dt - n^{-\alpha} \sum_{k=1}^\infty \frac{(-1)^k}{k!}\; {}_2\tilde{F}_1\!\left(1, 1; 1-\alpha; -\frac{n}{k}\right)$$

Summation

Finite summation

06.01.23.0001.01

$$\sum_{k_{1,1}=0}^{o} \cdots \sum_{k_{m,n}=0}^{o} \prod_{i=1}^{m} \prod_{j=1}^{m} \frac{1}{k_{i,j}!} = \frac{p!}{\prod_{i=1}^m a_i!\, \prod_{j=1}^n b_j!} \;/;\; \sum_{j=1}^{n} k_{i,j} = a_i \bigwedge \sum_{i=1}^{m} k_{i,j} = b_i \bigwedge \sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j = p \bigwedge o = \operatorname{Max}(k_{1,1}, \ldots, k_{m,n})$$

06.01.23.0002.01

$$\sum_{k=n}^{\infty} \frac{k}{(k+1)!} = \frac{1}{n!}$$

H. J. Brothers

Infinite summation

Parameter-free sums

06.01.23.0003.01

$$\sum_{k=0}^{\infty} \frac{(k!)^2}{(k+1)!\,(k+2)!} == \frac{1}{6}\left(-6+\pi^2\right)$$

Troy Kessler

06.01.23.0004.01

$$\sum_{k=0}^{\infty} \frac{(k!)^2}{(k+2)!\,(k+4)!} == \frac{1}{27}\left(-29+3\,\pi^2\right)$$

Troy Kessler

06.01.23.0005.01

$$\sum_{k=0}^{\infty} \frac{(k!)^2}{(k+3)!\,(k+6)!} == -\frac{41\,383}{288\,000} + \frac{7\,\pi^2}{480}$$

Troy Kessler

06.01.23.0006.01

$$\sum_{k=0}^{\infty} \frac{(k!)^2}{(k+4)!\,(k+8)!} == -\frac{435\,179}{66\,679\,200} + \frac{\pi^2}{1512}$$

Troy Kessler

06.01.23.0007.01

$$\sum_{k=0}^{\infty} \frac{(k!)^2}{(k+5)!\,(k+10)!} == \frac{-1\,493\,750\,663 + 151\,351\,200\,\pi^2}{11\,061\,279\,129\,600}$$

Troy Kessler

06.01.23.0008.01

$$\sum_{k=0}^{\infty} \frac{(k!)^3}{((k+2)!)^3} == 10-\pi^2$$

Troy Kessler

06.01.23.0009.01

$$\sum_{k=0}^{\infty} \frac{(k!)^3}{((k+4)!)^3} == \frac{-6217+630\,\pi^2}{11\,664}$$

Troy Kessler

06.01.23.0010.01

$$\sum_{k=0}^{\infty} \frac{(k!)^3}{((k+6)!)^3} == -\frac{11\left(-323\,329+32\,760\,\pi^2\right)}{3\,110\,400\,000}$$

Troy Kessler

06.01.23.0011.01

$$\sum_{k=0}^{\infty} \frac{(k!)^3}{((k+8)!)^3} = \frac{-2\,680\,498\,753 + 271\,591\,320\,\pi^2}{8\,782\,450\,790\,400\,000}$$

Troy Kessler

06.01.23.0012.01

$$\sum_{k=0}^{\infty} \frac{(k!)^3}{((k+3)!)^3} = \frac{29}{32} - \frac{3\,\zeta(3)}{4}$$

Troy Kessler

06.01.23.0013.01

$$\sum_{k=0}^{\infty} \frac{(k!)^3}{((k+5)!)^3} = \frac{5\,(1728\,\zeta(3) - 2077)}{1\,327\,104}$$

Troy Kessler

06.01.23.0014.01

$$\sum_{k=0}^{\infty} \frac{(k!)^3}{((k+7)!)^3} = \frac{7\,(389\,467 - 324\,000\,\zeta(3))}{503\,884\,800\,000}$$

Troy Kessler

06.01.23.0015.01

$$\sum_{k=0}^{\infty} \frac{(k!)^3}{((k+9)!)^3} = \frac{19\,559\,232\,000\,\zeta(3) - 23\,511\,309\,071}{37\,000\,716\,107\,120\,640\,000}$$

Troy Kessler

06.01.23.0016.01

$$\sum_{k=0}^{\infty} \frac{(k!)^4}{((k+1)!)^4} = \frac{\pi^4}{90}$$

Troy Kessler

06.01.23.0017.01

$$\sum_{k=0}^{\infty} \frac{(k!)^4}{((k+2)!)^4} = \frac{1}{45}\left(-1575 + 150\,\pi^2 + \pi^4\right)$$

Troy Kessler

06.01.23.0018.01

$$\sum_{k=0}^{\infty} \frac{(k!)^2}{((k+1)!)^2\, 2^k} = 2\left(\frac{\pi^2}{12} - \frac{\log^2(2)}{2}\right)$$

Troy Kessler

06.01.23.0019.01

$$\sum_{k=0}^{\infty} \frac{(k!)^2}{((k+2)!)^2\, 2^k} == 2\left(3\left(\frac{\pi^2}{12} - \frac{\log^2(2)}{2}\right) + 2\log(2) - 3\right)$$

Troy Kessler

06.01.23.0020.01

$$\sum_{k=1}^{\infty} \frac{(-1)^k \left(5 - 32\,k + 56\,k^2\right) ((-1+k)!)^3}{4\,(-1+2\,k)^2\,(3\,k)!} == -\zeta(3)$$

Troy Kessler

06.01.23.0021.01

$$\sum_{k=2}^{\infty} \frac{(-1)^{-k} \left(35 - 88\,k + 56\,k^2\right) (3\,k)!\,((-2+2\,k)!)^6}{16\,(-1+k)^5\,((-1+k)!)^2\,k!\,((-3+2\,k)!)^3\,(-2+6\,k)!} == \frac{10-\pi^2}{4}$$

Troy Kessler

Parameter-containing sums

06.01.23.0022.01

$$\sum_{k=0}^{\infty} \frac{k!}{(k+n)!} == \frac{1}{(n-1)\,(n-1)!} \;/;\; n \in \mathbb{N}^+$$

Troy Kessler

06.01.23.0023.01

$$\sum_{k=0}^{\infty} \frac{(-1)^k\,k!}{(n+k)!} == \frac{2^{n-1}\log(2)}{(n-1)!} - \sum_{j=1}^{n-1} \frac{2^{j-1}\,(-j+n-1)!}{(n-j)!\,(n-1)!} \;/;\; n \in \mathbb{N}^+$$

Troy Kessler

06.01.23.0024.01

$$\sum_{k=0}^{\infty} \frac{(-1)^k\,(2\,k)!}{(4\,n+2\,k)!} == -\frac{\left(3\,2^{2\,n-1}\,(-1)^n\right)\left(\frac{5-\pi}{12} - \frac{\log(4)}{12}\right)}{(4\,n-1)!} - \sum_{j=0}^{n-2} \frac{(-1)^j\,2^{2\,j-1}\left(20\,n^2 - (40\,j+29)\,n + j\,(20\,j+29) + 10\right)}{(-n+j+1)\,(-2\,n+2\,j+1)\,(-4\,n+4\,j+3)\,(4\,n-1)!} \;/;\; n \in \mathbb{N}^+$$

Troy Kessler

06.01.23.0025.01

$$\sum_{k=0}^{\infty} \frac{(-1)^k\,(2\,k)!}{(4\,n+2\,k+1)!} == \frac{\left(4^{n-1}\,(-1)^n\right)\pi}{(4\,n)!} - \sum_{j=0}^{n-1} \frac{(-1)^j\,2^{2\,j+1}\left(20\,n^2 - (40\,j+19)\,n + j\,(20\,j+19) + 4\right)}{(-2\,n+2\,j+1)\,(-4\,n+4\,j+1)\,(-4\,n+4\,j+3)\,(4\,n)!} \;/;\; n \in \mathbb{N}^+$$

Troy Kessler

06.01.23.0026.01

$$\sum_{k=0}^{\infty} \frac{(-1)^k\,(2\,k)!}{(4\,n+2\,k+2)!} == \frac{(-4)^n\left(\frac{\pi}{4} - \frac{\log(4)}{4}\right)}{(4\,n+1)!} - \sum_{j=0}^{n-1} \frac{(-1)^j\,4^{j-1}\left(40\,j^2 + 18\,j + 40\,n^2 - 2\,n\,(40\,j+9) + 1\right)}{(j-n)\,(2\,j-2\,n+1)\,(4\,j-4\,n+1)\,(4\,n+1)!} \;/;\; n \in \mathbb{N}^+$$

Troy Kessler

06.01.23.0027.01

$$\sum_{k=0}^{\infty} \frac{(-1)^k (2k)!}{(4n+2k+3)!} = \frac{\left(2^{2n+1} (-1)^n\right)\left(\frac{1}{2} - \frac{\log(2)}{2}\right)}{(4n+2)!} + \sum_{j=0}^{n-1} \frac{(-1)^j 2^{2j-1} \left(40 j^2 - 2(40n+1) j + 40 n^2 + 2n - 1\right)}{(j-n)(4j-4n-1)(4j-4n+1)(4n+2)!(-1)} \;/;\; n \in \mathbb{N}^+$$

Troy Kessler

06.01.23.0028.01

$$\sum_{k=0}^{\infty} \frac{(-1)^k k!}{(n+k)!\, 2^k} = \frac{3^{n-1} (2\log(3) - 2\log(2))}{(n-1)!} + \sum_{j=0}^{n-2} \frac{2\, 3^j}{(j-n+1)(n-1)!} \;/;\; n \in \mathbb{N}^+$$

Troy Kessler

06.01.23.0029.01

$$\sum_{k=0}^{\infty} \frac{(-1)^k k!}{(n+k)!\, 3^k} = \frac{4^{n-1} (6\log(2) - 3\log(3))}{(n-1)!} + \sum_{j=0}^{n-2} \frac{3\, 4^j}{(j-n+1)(n-1)!} \;/;\; n \in \mathbb{N}^+$$

Troy Kessler

06.01.23.0030.01

$$\sum_{k=0}^{\infty} \frac{(-1)^k (2k)!}{(3n+2k)!\, 3^k} = -\sum_{j=0}^{n-2} \frac{(-8)^j (9j-9n+7)}{(j-n+1)(3j-3n+2)(3n-1)!} - \frac{\left((-1)^n 2^{3n-2}\right)\left(\frac{3}{2}(\log(3) - \log(4) + 1) - \frac{\pi}{2\sqrt{3}}\right)}{(3n-1)!} \;/;\; n \in \mathbb{N}^+$$

Troy Kessler

06.01.23.0031.01

$$\sum_{k=0}^{\infty} \frac{(-1)^k (2k)!}{(3n+2k+1)!\, 3^k} = \frac{(-8)^n \pi}{(3n(3n-1)!)\left(2\sqrt{3}\right)} - \sum_{j=0}^{n-1} \frac{3(-8)^j (9j-9n+4)}{(3j-3n+1)(3j-3n+2)(3n)!} \;/;\; n \in \mathbb{N}^+$$

Troy Kessler

06.01.23.0032.01

$$\sum_{k=0}^{\infty} \frac{(-1)^k (2k)!}{(3n+2k+2)!\, 3^k} = \sum_{j=0}^{n-2} \frac{(-8)^j (-9j+9n-1)}{(j-n)(3j-3n+1)(3n+1)!} - \frac{(3(-8)^n)\left(\frac{1}{6}(-3\log(3) + 3\log(4) + 1) - \frac{\pi}{6\sqrt{3}}\right)}{(3n+1)!} \;/;\; n \in \mathbb{N}^+$$

Troy Kessler

06.01.23.0033.01

$$\sum_{k=0}^{\infty} \frac{(n+k)!}{k!\, p^k} = n!\left(\frac{p}{p-1}\right)^{n+1} \;/;\; n \in \mathbb{N}^+ \land p \in \mathbb{N} \land p > 1$$

Troy Kessler

06.01.23.0034.01

$$\sum_{k=0}^{\infty} \frac{(-1)^k (k+n)!}{k!\, p^k} = n!\left(\frac{p}{p+1}\right)^{n+1} \;/;\; n \in \mathbb{N}^+ \land p \in \mathbb{N} \land p > 1$$

Troy Kessler

06.01.23.0035.01

$$\sum_{k=0}^{\infty} \frac{(-1)^k\, k!}{(k+n)!\, p^k} = \frac{\left(p \log\left(\frac{p+1}{p}\right)\right) (p+1)^{n-1}}{(n-1)!} + \sum_{j=0}^{n-2} \frac{p\, (p+1)^j}{(j-n+1)\, (n-1)!} \;/;\; n \in \mathbb{N}^+$$

Troy Kessler

06.01.23.0036.01

$$\sum_{k=0}^{\infty} \frac{(-1)^k\, (k!)^2}{((2\,n+k)!)^2} = \frac{\left((n!)^2\, (-1)\, (-16)^n\right) (3-4\log(2))}{4\, (2\,n-1)!\, ((2\,n)!)^2} + \sum_{j=0}^{n-2} \frac{(-16)^j\, (5\,j-5\,n+4)\, (-2\,j+2\,n-1)!\, ((n-1)!)^2}{(j-n+1)\, (2\,j-2\,n+1)^2\, ((2\,n-1)!)^3\, ((-j+n-1)!)^2} \;/;\; n \in \mathbb{N}^+$$

Troy Kessler

06.01.23.0037.01

$$\sum_{k=0}^{\infty} \frac{(-1)^k\, (k!)^2}{((2\,n+k+1)!)^2} = \sum_{j=0}^{n-2} \frac{(-1)^j\, (10\,j-10\,n+3)\, ((-j+n-1)!)^2}{4\, (2\,j-2\,n+1)\, (2\,n)!\, (2\,n-2\,j)!\, n^2\, ((n-1)!)^2} - \frac{2\, (-1)^{-n} \left(\frac{7}{16} - \frac{\pi^2}{24}\right)}{(n!)^2\, (2\,n)!} \;/;\; n \in \mathbb{N}^+$$

Troy Kessler

06.01.23.0038.01

$$\sum_{k=0}^{\infty} \frac{(k!)^2}{((n+k)!)^2} = \frac{(2\,n-2)!\, \pi^2}{6\, ((n-1)!)^4} - \sum_{j=0}^{n-2} \frac{3\, (n-j)^2\, (2\,n-2)!\, ((-j+n-1)!)^4}{(j-n+1)^2\, ((n-j)!)^2\, ((n-1)!)^4\, (-2\,j+2\,n-2)!} \;/;\; n \in \mathbb{N}^+$$

Troy Kessler

06.01.23.0039.01

$$\sum_{k=0}^{\infty} \frac{(2\,k)!}{(n+2\,k)!} = \frac{2^n \log(2)}{4\, (n-1)!} - \sum_{j=0}^{n-3} \frac{2^j}{(-j+n-2)\, (n-1)!} \;/;\; n \in \mathbb{N} \land n > 1$$

Troy Kessler

06.01.23.0040.01

$$\sum_{k=0}^{\infty} \frac{k!}{(n+k)!\, p^k} = -\frac{(p\, (\log(p) - \log(p-1)))\, (1-p)^n}{(n-1)!\, (p-1)} - \sum_{j=0}^{n-2} \frac{(1-p)^j\, p}{(j-n+1)\, (n-1)!} \;/;\; n \in \mathbb{N}^+$$

Troy Kessler

06.01.23.0041.01

$$\sum_{k=1}^{\infty} \frac{(-1)^k}{(2\,n+2\,k)!} = (-1)^n \cos(1) - \sum_{j=0}^{n} \frac{(-1)^j}{(2\,n-2\,j)!} \;/;\; n \in \mathbb{N}^+$$

Troy Kessler

06.01.23.0042.01

$$\sum_{k=0}^{\infty} \frac{p^k}{(2\,n+2\,k)!} = \cosh\left(\sqrt{p}\right) \left(\frac{1}{p}\right)^n + \sum_{j=0}^{n-1} - \frac{\left(\frac{1}{p}\right)^{j+1}}{(-2\,j+2\,n-2)!} \;/;\; n \in \mathbb{N}^+$$

Troy Kessler

06.01.23.0043.01

$$\sum_{k=0}^{\infty} \frac{(k!)^4}{((n+k)!)^4} == a_n \,/;\, n \in \mathbb{N} \bigwedge a_1 == \frac{\pi^4}{90} \bigwedge a_2 == \frac{1}{45}\left(-1575 + 150\,\pi^2 + \pi^4\right) \bigwedge \left(a_n == \frac{1}{(n-2)^3\,(n-1)^7}\left(2\,(2\,(n-2)+1)\left(3\,(n-2)^2 + 3\,(n-2)+1\right) a_{n-1}\,(n-2)^3 - \frac{5\left(15\,(n-2)^2 + 10\,(n-2)+2\right)(n-1)^3\,(n-2)}{((n-1)!)^4} + 4\,(4\,(n-2)-1)\,(4\,(n-2)+1)\,a(n-2)\right) /;\, n > 2\right)$$

Troy Kessler

06.01.23.0044.01

$$\sum_{k=0}^{\infty} \frac{(k!)^2}{((n+k)!)^2\, 2^k} == a_n \,/;\, n \in \mathbb{N} \bigwedge a_1 == 2\left(\frac{\pi^2}{12} - \frac{\log^2(2)}{2}\right) \bigwedge a_2 == 2\left(3\left(\frac{\pi^2}{12} - \frac{\log^2(2)}{2}\right) + 2\log(2) - 3\right) \bigwedge \left(a_n == \frac{\frac{2\,(2\,(n-2)-1)\,(n-1)}{((n-1)!)^2} + a(n-2) - 3\,(2\,(n-2)+1)\,(n-2)\,a(n-1)}{(2-n)\,(n-1)^3} \,/;\, n > 2\right)$$

Troy Kessler

06.01.23.0045.01

$$\sum_{k=0}^{\infty} \frac{(k!)^3}{((k+2\,n+1)!)^3} == \frac{(3\,n)!}{(n!)^3\,((2\,n)!)^3} \sum_{j=0}^{n-1} \frac{(-1)^j \left(56\,j^2 - 112\,n\,j + 32\,j + 56\,n^2 - 32\,n + 5\right)((-j+n-1)!)^3}{4\,(-2\,j+2\,n-1)^2\,(3\,n-3\,j)!} + \frac{((-1)^n\,(3\,n)!)\,\zeta(3)}{(n!)^3\,((2\,n)!)^3} \,/;\, n \in \mathbb{N}^+$$

Troy Kessler

06.01.23.0046.01

$$\sum_{k=0}^{\infty} \frac{(3\,k)!}{27^k\, k!\,(2\,k+n)!} = a_n \,/;\, a_0 = \frac{2\cos\left(\frac{\pi}{18}\right)}{\sqrt{3}} \bigwedge a_1 = 6\sin\left(\frac{\pi}{18}\right) \bigwedge a_n = \frac{-\frac{36\,(n-1)}{(n-1)!} + 27\,a_{n-2} + 9\,(2\,n-3)\,a_{n-1}}{(3\,n-4)\,(3\,n-2)} \,/;\, k \in \mathbb{N}$$

Troy Kessler

06.01.23.0047.01

$$\sum_{k=0}^{\infty} \frac{(k!)^3}{((k+2\,n)!)^3} == \frac{\left(10-\pi^2\right)\left((-1)^{n-1}\,((n-1)!)^2\, n!\,(6\,n-2)!\right)}{4\,(3\,n)!\,((2\,n-1)!)^6} + \sum_{j=0}^{n-2}\left((-1)^j\,(8\,(n-j)\,(-7\,j+7\,n-11)+35)\,((n-1)!)^2\, n!\,(6\,n-2)!\,(3\,n-3\,j)!\,((-2\,j+2\,n-1)!)^6\right) \Big/ \left(16\,(2\,j-2\,n+1)^6\,(-j+n-1)^5\,(3\,n)!\,((2\,n-1)!)^6\,((-2\,j+2\,n-3)!)^3\,(-6\,j+6\,n-2)!\,((-j+n-1)!)^2\,(n-j)!\right) /;\, n \in \mathbb{N}^+$$

Troy Kessler

06.01.23.0048.01

$$\sum_{k=0}^{\infty} \frac{(2k+2n)!}{(-4)^k ((k+n)!)^2 z^k} = \sum_{j=0}^{n-1} \frac{4^{1+j} (-z)^j z (-2+2n-2j)!}{((n-j-1)!)^2} + (-4z)^n \sqrt{\frac{z}{z+1}} \;/;\; n \in \mathbb{N}^+$$

Troy Kessler

06.01.23.0049.01

$$\sum_{k=0}^{\infty} \frac{((2k)!)^2}{((2k+2n)!)^2} = \frac{\pi^2 (4n-2)!}{12((2n-1)!)^4} + \frac{1}{2}\left(\frac{((n!)^2 (-1)(-16)^n)(3-4\log(2))}{4(2n-1)!((2n)!)^2} + \sum_{j=0}^{n-2} \frac{(-16)^j (5j-5n+4)((n-1)!)^2 (-2j+2n-1)!}{(j-n+1)(2j-2n+1)^2 ((2n-1)!)^3 ((-j+n-1)!)^2}\right) + \frac{1}{2} \sum_{j=0}^{2n-2} -\frac{3(4n-2)!((-j+2n-2)!)^2}{((2n-1)!)^4 (-2j+4n-2)!} \;/;\; n \in \mathbb{N}^+$$

Troy Kessler

06.01.23.0050.01

$$\sum_{k=0}^{\infty} \frac{((2k)!)^2}{((2k+2n)!)^2} = \sum_{k=0}^{\infty} \frac{((2k)!)^2}{((2k+2n+1)!)^2} = \frac{1}{2} \sum_{j=0}^{2n-1} -\frac{3(4n)!((2n-j-1)!)^2}{((2n)!)^4 (4n-2j)!} + \frac{(4n)!\pi^2}{12((2n)!)^4} + \frac{1}{2}\left(\sum_{j=0}^{n-2} \frac{(-1)^j (3-10n+10j)((n-j-1)!)^2}{4(1-2n+2j)(2n)!(2n-2j)!n^2((n-1)!)^2} - \frac{(2(-1)^{-n})\left(\frac{7}{16}-\frac{\pi^2}{24}\right)}{(n!)^2 (2n)!}\right) \;/;\; n \in \mathbb{N}^+$$

Troy Kessler

06.01.23.0051.01

$$\sum_{k=0}^{\infty} \frac{(2k)!}{(2k+n)! z^k} = \frac{\left(\sqrt{z} \log\left(\frac{\sqrt{z}}{\sqrt{z}-1}\right)\right)\left(1-\sqrt{z}\right)^{n-1}}{2(n-1)!} + \frac{\left(\sqrt{z}+1\right)^{n-1} \left(\sqrt{z} \log\left(\frac{\sqrt{z}+1}{\sqrt{z}}\right)\right)}{2(n-1)!} + \frac{1}{2} \sum_{j=0}^{n-2} \frac{\sqrt{z}\left(\sqrt{z}+1\right)^j - \left(1-\sqrt{z}\right)^j \sqrt{z}}{(j-n+1)(n-1)!} \;/;\; n \in \mathbb{N}^+$$

Troy Kessler

06.01.23.0052.01

$$\sum_{k=0}^{\infty} \frac{(4k)!}{(4k+4n+1)!} = \frac{1}{2}\left(\sum_{j=0}^{n-1} -\frac{(-1)^j 2^{2j+1}\left(20n^2-(40j+19)n+j(20j+19)+4\right)}{(2j-2n+1)(4j-4n+1)(4j-4n+3)(4n)!} + \frac{\left(4^{n-1}(-1)^n\right)\pi}{(4n)!}\right) + \frac{1}{2}\left(\frac{2^{4n-1}\log(2)}{(4n)!} + \sum_{j=0}^{4n-2} -\frac{2^j}{(-j+4n+1-2)(4n)!}\right) \;/;\; n \in \mathbb{N}^+$$

Troy Kessler

06.01.23.0053.01

$$\sum_{k=0}^{\infty}\frac{(4k)!}{(4k+4n)!}=\frac{1}{2}\left(\sum_{j=0}^{n-2}-\frac{(-1)^j\,2^{2j-1}\left(20n^2-(40j+29)n+j(20j+29)+10\right)}{(j-n+1)(2j-2n+1)(4j-4n+3)(4n-1)!}-\frac{\left(3\,2^{2n-1}(-1)^n\right)\left(\frac{5-\pi}{12}-\frac{\log(4)}{12}\right)}{(4n-1)!}\right)+\frac{1}{2}\left(\frac{2^{4n}\log(2)}{4(4n-1)!}+\sum_{j=0}^{4n-3}-\frac{2^j}{(-j+4n-2)(4n-1)!}\right)/;\, n\in\mathbb{N}^+$$

Troy Kessler

06.01.23.0054.01

$$\sum_{k=0}^{\infty}\frac{(4k)!}{(4k+4n+2)!}=\frac{1}{2}\left(\frac{(-4)^n\left(\frac{\pi}{4}-\frac{\log(4)}{4}\right)}{(4n+1)!}+\sum_{j=0}^{n-1}-\frac{(-1)^j\,4^{j-1}\left(40j^2+18j+40n^2-2n(40j+9)+1\right)}{(j-n)(2j-2n+1)(4j-4n+1)(4n+1)!}\right)+\frac{1}{2}\left(\frac{2^{4n+2}\log(2)}{4(4n+1)!}+\sum_{j=0}^{4n-1}-\frac{2^j}{(4n-j)(4n+1)!}\right)/;\, n\in\mathbb{N}^+$$

Troy Kessler

06.01.23.0055.01

$$\sum_{k=0}^{\infty}\frac{(4k)!}{(4k+4n+3)!}=\frac{1}{2}\left(\frac{\left(2^{2n+1}(-1)^n\right)\left(\frac{1}{2}-\frac{\log(2)}{2}\right)}{(4n+2)!}+\sum_{j=0}^{n-1}\frac{(-1)^j\,2^{2j-1}\left(40j^2-2(40n+1)j+40n^2+2n-1\right)}{(j-n)(4j-4n-1)(4j-4n+1)(4n+2)!\,(-1)}\right)+\frac{1}{2}\left(\frac{2^{4n+3}\log(2)}{4(4n+2)!}+\sum_{j=0}^{4n}-\frac{2^j}{(-j+4n+1)(4n+2)!}\right)/;\, n\in\mathbb{N}^+$$

Troy Kessler

06.01.23.0056.01

$$\sum_{k=0}^{\infty}\frac{(4k)!}{(4k+3n)!\,9^k}=\frac{\left(\sqrt{3}\,\log\left(\frac{\sqrt{3}}{\sqrt{3}-1}\right)\right)\left(1-\sqrt{3}\right)^{3n-1}}{4(3n-1)!}+\frac{1}{2}\left(\sum_{j=0}^{n-2}-\frac{(-8)^j(9j-9n+7)}{(j-n+1)(3j-3n+2)(3n-1)!}-\frac{\left((-1)^n\,2^{3n-2}\right)\left(\frac{3}{2}(\log(3)-\log(4)+1)-\frac{\pi}{2\sqrt{3}}\right)}{(3n-1)!}\right)+\frac{\left(\sqrt{3}+1\right)^{3n-1}\left(\sqrt{3}\,\log\left(\frac{\sqrt{3}+1}{\sqrt{3}}\right)\right)}{4(3n-1)!}+\frac{1}{4}\sum_{j=0}^{3n-2}\frac{\sqrt{3}\left(\sqrt{3}+1\right)^j-\left(1-\sqrt{3}\right)^j\sqrt{3}}{(j-3n+1)(3n-1)!}/;\, n\in\mathbb{N}^+$$

Troy Kessler

06.01.23.0057.01

$$\sum_{k=0}^{\infty} \frac{(4k)!}{(4k+3n+1)!\, 9^k} = \frac{\left(\sqrt{3} \log\left(\frac{\sqrt{3}+1}{\sqrt{3}}\right)\right)\left(\sqrt{3}+1\right)^{3n}}{4\,(3n)!} + \frac{1}{2}\left(\sum_{j=0}^{n-1} -\frac{3\,(-8)^j\,(9j-9n+4)}{(3j-3n+1)\,(3j-3n+2)\,(3n)!} + \frac{(-8)^n\,\pi}{(3n\,(3n-1)!)\left(2\sqrt{3}\right)}\right) + \frac{\left(1-\sqrt{3}\right)^{3n}\left(\sqrt{3} \log\left(\frac{\sqrt{3}}{\sqrt{3}-1}\right)\right)}{4\,(3n)!} + \frac{1}{4}\sum_{j=0}^{3n-1} \frac{\sqrt{3}\left(\sqrt{3}+1\right)^j - \left(1-\sqrt{3}\right)^j\sqrt{3}}{(j-3n)\,(3n)!} \;/;\; n \in \mathbb{N}^+$$

Troy Kessler

06.01.23.0058.01

$$\sum_{k=0}^{\infty} \frac{(4k)!}{(4k+3n+2)!\, 9^k} = \frac{\left(\sqrt{3} \log\left(\frac{\sqrt{3}+1}{\sqrt{3}}\right)\right)\left(\sqrt{3}+1\right)^{3n+1}}{4\,(3n+1)!} + \frac{1}{2}\left(\sum_{j=0}^{n-2} \frac{(-8)^j\,(-9j+9n-1)}{(j-n)\,(3j-3n+1)\,(3n+1)!} - \frac{(3\,(-8)^n)\left(\frac{1}{6}\,(-3\log(3)+3\log(4)+1) - \frac{\pi}{6\sqrt{3}}\right)}{(3n+1)!}\right) + \frac{\left(1-\sqrt{3}\right)^{3n+1}\left(\sqrt{3} \log\left(\frac{\sqrt{3}}{\sqrt{3}-1}\right)\right)}{4\,(3n+1)!} + \frac{1}{4}\sum_{j=0}^{3n} \frac{\sqrt{3}\left(\sqrt{3}+1\right)^j - \left(1-\sqrt{3}\right)^j\sqrt{3}}{(j-3n-1)\,(3n+1)!} \;/;\; n \in \mathbb{N}^+$$

Troy Kessler

06.01.23.0059.01

$$\sum_{k=0}^{\infty} \frac{(3k)!}{27^k\, k!\, (2k+n)!} = a_n \;/;\; a_0 = \frac{2\cos\left(\frac{\pi}{18}\right)}{\sqrt{3}} \bigwedge a_1 = 6\sin\left(\frac{\pi}{18}\right) \bigwedge a_n = \frac{-\frac{36\,(n-1)}{(n-1)!} + 27\,a_{n-2} + 9\,(2n-3)\,a_{n-1}}{(3n-4)\,(3n-2)} \;/;\; n \in \mathbb{N}$$

Troy Kessler

Operations

Limit operation

06.01.25.0001.01

$$\lim_{n\to\infty} \frac{(a+n)!\, n^{b-a}}{(b+n)!} = 1$$

Representations through more general functions

Through other functions

Involving some hypergeometric-type functions

06.01.26.0001.01

$n! == \Gamma(n+1, 0) /; \operatorname{Re}(n) > -1$

Representations through equivalent functions

With related functions

06.01.27.0001.01

$n! == \Gamma(n+1)$

06.01.27.0002.01

$n! == (1)_n$

06.01.27.0003.01

$n! == 2^{\frac{1}{4}(\cos(2 n \pi)-1)-n} \pi^{\frac{1}{2}\sin^2(n \pi)} (2 n)!!$

06.01.27.0004.01

$n! == (n-1)!! \, n!!$

Inequalities

06.01.29.0001.01

$k!^n n^{k n} \geq n!^k k^{k n} /; n \in \mathbb{N}^+ \bigwedge k \in \mathbb{N}^+ \bigwedge n \geq k$

06.01.29.0002.01

$n! \leq (n+1)^{n+1} e^{-n} /; n \in \mathbb{N}$

06.01.29.0003.01

$\prod_{k=1}^{n} (2 k)!^{(2 k)!} > (n (n+1)!)^{n^2 (n+1)!} /; n \in \mathbb{N}^+$

Zeros

06.01.30.0001.01

$n! \neq 0 /; \forall n$

Theorems

Taylor's formula

$$f(z) == \sum_{k=0}^{\infty} \frac{f^{(k)}(a) (z-a)^k}{k!}.$$

Derivative of composition (Faà di Bruno's formula)

$$\frac{\partial^n f(g(x))}{\partial x^n} = \sum_{m=1}^{n} \sum_{\substack{k_1+k_2+\ldots+k_n=m \\ k_1+2k_2+\ldots+n\,k_n=n}} f^{(m)}(g(x)) \frac{n!}{\prod_{j=1}^{n} j!^{k_j}\, k_j!} \prod_{j=1}^{n} g^{(j)}(x)^{k_j} =$$

$$\sum_{m=1}^{n} \frac{1}{m!} \left(\sum_{j=0}^{m-1} (-1)^j \binom{m}{j} g(x)^j \frac{\partial^n g(x)^{m-j}}{\partial x^n} \right) f^{(m)}(g(x)).$$

Composition of two series

$$\sum_{m=0}^{\infty} b_m \left(\sum_{k=1}^{\infty} a_k z^k \right)^m = \sum_{n=0}^{\infty} c_n z^n \;/;\; c_0 = b_0 \bigwedge c_1 = a_1 b_1 \bigwedge c_2 = a_1^2 b_2 + a_2 b_1 \bigwedge c_3 = a_3 b_1 + 2 a_1 a_2 b_2 + a_1^3 b_3 \bigwedge$$

$$c_4 = a_4 b_1 + 2 a_1 a_3 b_2 + a_2^2 b_2 + 3 a_1^2 a_2 b_3 + a_1^4 b_4 \bigwedge c_n = \sum_{\substack{k_1,k_2,\ldots,k_n=0 \\ k_1+2k_2+\ldots+n\,k_n=n}}^{n} m!\, b_m \prod_{j=1}^{n} \frac{a_j^{k_j}}{k_j!}. \bigwedge m = \sum_{j=1}^{n} k_j$$

Maxfield theorem and Castell conjecture

J. E. Maxfield proved that the base 10 digits of any positive integer occur in $m!$ as the first digits for some integer m (J. E. Maxfield. Math. Mag. 43, 64, (1970)).
Castell's conjecture states that the digits 1 to $b-1$ of the base b expansion of $n!$ are asymptotically equally distributed (S. P. Castell. Eureka, n36, 44 1973).

History

– J. Stirling (1730) found his famous asymptotic formula
– L. Euler (1751)
– C. Kramp (1808, 1816) introduced the notation $n!$

Copyright