Hypergeometric0F1

Notations

Traditional name

Confluent hypergeometric function ${}_0F_1$

Traditional notation

${}_0F_1(;b;z)$

Mathematica StandardForm notation

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Hypergeometric0F1[b, z]
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Primary definition

07.17.02.0001.01

$${}_0F_1(;b;z) = \sum_{k=0}^{\infty} \frac{z^k}{(b)_k\, k!}$$

Specific values

Specialized values

For fixed *b*

07.17.03.0001.01

$${}_0F_1(;b;0) = 1$$

For fixed *z* and symbolic parameter

07.17.03.0002.01

$${}_0F_1(;b;z) = -\frac{\Gamma(b)}{\sqrt{\pi}} \exp\left(\frac{\pi i}{2}\left(\frac{3}{2}-b\right)\right) z^{\frac{1-2b}{4}} \left(\sinh\left(\frac{\pi i}{2}\left(\frac{3}{2}-b\right)-2\sqrt{z}\right) \sum_{k=0}^{\left\lfloor \frac{1}{4}(2|b-1|-1)\right\rfloor} \frac{\left(2k+|b-1|-\frac{1}{2}\right)!}{2^{4k}\,(2k)!\left(|b-1|-2k-\frac{1}{2}\right)!\, z^k} + \frac{1}{\sqrt{z}} \cosh\left(\frac{\pi i}{2}\left(\frac{3}{2}-b\right)-2\sqrt{z}\right) \sum_{k=0}^{\left\lfloor \frac{1}{4}(2|b-1|-3)\right\rfloor} \frac{\left(2k+|b-1|+\frac{1}{2}\right)!}{2^{4k+2}\,(2k+1)!\left(|b-1|-2k-\frac{3}{2}\right)!\, z^k}\right) /;\, b-\frac{1}{2} \in \mathbb{Z}$$

07.17.03.0003.01

$$ {}_0F_1(;b;z) = \frac{z^{\frac{1}{2}(-b-|b-1|+1)}\,\Gamma\!\left(-\frac{1}{3}\right)\Gamma(b)}{2\,3^{5/6}\,\Gamma(1-|b-1|)} \left(\sqrt[6]{3}\,\sqrt[3]{z}\left(\sqrt{3}\,\mathrm{Bi}'\!\left(3^{2/3}\sqrt[3]{z}\right) - 3\,\mathrm{sgn}(b-1)\,\mathrm{Ai}'\!\left(3^{2/3}\sqrt[3]{z}\right)\right)\sum_{k=0}^{|b-1|-\frac{4}{3}} \frac{\left(|b-1|-k-\frac{4}{3}\right)!\,(-z)^k}{k!\left(\frac{4}{3}\right)_k\left(|b-1|-2k-\frac{4}{3}\right)!\,(1-|b-1|)_k} + \left(\sqrt{3}\,\mathrm{sgn}(b-1)\,\mathrm{Ai}\!\left(3^{2/3}\sqrt[3]{z}\right) - \mathrm{Bi}\!\left(3^{2/3}\sqrt[3]{z}\right)\right)\sum_{k=0}^{|b-1|-\frac{1}{3}} \frac{\left(|b-1|-k-\frac{1}{3}\right)!\,(-z)^k}{k!\left(|b-1|-2k-\frac{1}{3}\right)!\left(\frac{1}{3}\right)_k(1-|b-1|)_k}\right) /;\ |b-1| + \frac{2}{3} \in \mathbb{Z} $$

07.17.03.0004.01

$$ {}_0F_1(;b;z) = \frac{z^{\frac{1}{2}(-b-|b-1|+1)}\,\Gamma\!\left(-\frac{2}{3}\right)\Gamma(b)\,\mathrm{sgn}(b-1)}{6\,3^{5/6}\,\Gamma(1-|b-1|)} \left(9\,z^{2/3}\left(\sqrt{3}\,\mathrm{Ai}\!\left(3^{2/3}\sqrt[3]{z}\right) + \mathrm{sgn}(b-1)\,\mathrm{Bi}\!\left(3^{2/3}\sqrt[3]{z}\right)\right)\sum_{k=0}^{|b-1|-\frac{5}{3}} \frac{\left(|b-1|-k-\frac{5}{3}\right)!\,(-1)^k z^k}{k!\left(|b-1|-2k-\frac{5}{3}\right)!\left(\frac{5}{3}\right)_k(1-|b-1|)_k} - 2\left(3\sqrt[6]{3}\,\mathrm{Ai}'\!\left(3^{2/3}\sqrt[3]{z}\right) + 3^{2/3}\,\mathrm{sgn}(b-1)\,\mathrm{Bi}'\!\left(3^{2/3}\sqrt[3]{z}\right)\right)\sum_{k=0}^{|b-1|-\frac{2}{3}} \frac{\left(|b-1|-k-\frac{2}{3}\right)!\,(-1)^k z^k}{k!\left(|b-1|-2k-\frac{2}{3}\right)!\left(\frac{2}{3}\right)_k(1-|b-1|)_k}\right) /;\ |b-1| - \frac{2}{3} \in \mathbb{Z} $$

07.17.03.0005.01

$$ {}_0F_1\!\left(;n+\frac{2}{3};z\right) = \frac{\sqrt[6]{3}}{2}\,\Gamma\!\left(n+\frac{2}{3}\right)\frac{\partial^n\left(\sqrt{3}\,\mathrm{Ai}\!\left(3^{2/3}\sqrt[3]{z}\right) + \mathrm{Bi}\!\left(3^{2/3}\sqrt[3]{z}\right)\right)}{\partial z^n} /;\ n \in \mathbb{N} $$

07.17.03.0006.01

$$ {}_0F_1\!\left(;n+\frac{4}{3};z\right) = \frac{\sqrt[6]{3}}{2}\,\Gamma\!\left(n+\frac{4}{3}\right)\frac{\partial^n\left(\frac{1}{\sqrt[3]{z}}\left(\mathrm{Bi}\!\left(3^{2/3}\sqrt[3]{z}\right) - \sqrt{3}\,\mathrm{Ai}\!\left(3^{2/3}\sqrt[3]{z}\right)\right)\right)}{\partial z^n} /;\ n \in \mathbb{N} $$

07.17.03.0007.01

$$ {}_0F_1\!\left(;\frac{2}{3}-n;z\right) = \frac{(-1)^n\,\pi}{\sqrt[3]{3}\,\Gamma\!\left(n+\frac{1}{3}\right)}\,z^{n+\frac{1}{3}}\,\frac{\partial^n\left(\frac{1}{\sqrt[3]{z}}\left(\sqrt{3}\,\mathrm{Ai}\!\left(3^{2/3}\sqrt[3]{z}\right) + \mathrm{Bi}\!\left(3^{2/3}\sqrt[3]{z}\right)\right)\right)}{\partial z^n} /;\ n \in \mathbb{N} $$

07.17.03.0008.01

$$ {}_0F_1\!\left(;\frac{4}{3}-n;z\right) = \frac{(-1)^{n-1}\,\pi\,z^{n-\frac{1}{3}}}{\sqrt[3]{3}\,\Gamma\!\left(n-\frac{1}{3}\right)}\,\frac{\partial^n\left(\mathrm{Bi}\!\left(3^{2/3}\sqrt[3]{z}\right) - \sqrt{3}\,\mathrm{Ai}\!\left(3^{2/3}\sqrt[3]{z}\right)\right)}{\partial z^n} /;\ n \in \mathbb{N} $$

For fixed z

For fixed z and $b = \frac{m}{2}$

07.17.03.0028.01

$$ {}_0F_1\left(;-\frac{11}{2};z\right)=\frac{(4z(8z(2z+105)+4725)+10395)\cosh\left(2\sqrt{z}\right)-42\sqrt{z}\,(16z(z+15)+495)\sinh\left(2\sqrt{z}\right)}{10395} $$

07.17.03.0029.01

$$ {}_0F_1\left(;-\frac{11}{2};-z\right)=\left(1-\frac{4z(8z(2z-105)+4725)}{10395}\right)\cos\left(2\sqrt{z}\right)+\frac{2}{495}\sqrt{z}\,(16(z-15)z+495)\sin\left(2\sqrt{z}\right) $$

07.17.03.0030.01

$$ {}_0F_1\left(;-\frac{9}{2};z\right)=\frac{1}{945}\left(15(16z(z+7)+63)\cosh\left(2\sqrt{z}\right)-2\sqrt{z}\,(4z(4z+105)+945)\sinh\left(2\sqrt{z}\right)\right) $$

07.17.03.0031.01

$$ {}_0F_1\left(;-\frac{9}{2};-z\right)=\frac{1}{63}(16(z-7)z+63)\cos\left(2\sqrt{z}\right)+\frac{2}{945}\sqrt{z}\,(4z(4z-105)+945)\sin\left(2\sqrt{z}\right) $$

07.17.03.0032.01

$$ {}_0F_1\left(;-\frac{7}{2};z\right)=\frac{1}{105}\left((4z(4z+45)+105)\cosh\left(2\sqrt{z}\right)-10\sqrt{z}\,(8z+21)\sinh\left(2\sqrt{z}\right)\right) $$

07.17.03.0033.01

$$ {}_0F_1\left(;-\frac{7}{2};-z\right)=\left(\frac{4}{105}z(4z-45)+1\right)\cos\left(2\sqrt{z}\right)+\frac{2}{21}(21-8z)\sqrt{z}\,\sin\left(2\sqrt{z}\right) $$

07.17.03.0009.01

$$ {}_0F_1\left(;-\frac{5}{2};z\right)=\left(\frac{8z}{5}+1\right)\cosh\left(2\sqrt{z}\right)-\frac{2}{15}\sqrt{z}\,(4z+15)\sinh\left(2\sqrt{z}\right) $$

07.17.03.0034.01

$$ {}_0F_1\left(;-\frac{5}{2};-z\right)=\left(1-\frac{8z}{5}\right)\cos\left(2\sqrt{z}\right)+\frac{2}{15}(15-4z)\sqrt{z}\,\sin\left(2\sqrt{z}\right) $$

07.17.03.0010.01

$$ {}_0F_1\left(;-\frac{3}{2};z\right)=\left(\frac{4z}{3}+1\right)\cosh\left(2\sqrt{z}\right)-2\sqrt{z}\,\sinh\left(2\sqrt{z}\right) $$

07.17.03.0035.01

$$ {}_0F_1\left(;-\frac{3}{2};-z\right)=\left(1-\frac{4z}{3}\right)\cos\left(2\sqrt{z}\right)+2\sqrt{z}\,\sin\left(2\sqrt{z}\right) $$

07.17.03.0011.01

$$ {}_0F_1\left(;-\frac{1}{2};z\right)=\cosh\left(2\sqrt{z}\right)-2\sqrt{z}\,\sinh\left(2\sqrt{z}\right) $$

07.17.03.0036.01

$$ {}_0F_1\left(;-\frac{1}{2};-z\right)=\cos\left(2\sqrt{z}\right)+2\sqrt{z}\,\sin\left(2\sqrt{z}\right) $$

07.17.03.0012.01

$$ {}_0F_1\left(;\frac{1}{2};z\right)=\cosh\left(2\sqrt{z}\right) $$

07.17.03.0037.01

$$ {}_0F_1\left(;\frac{1}{2};-z\right)=\cos\left(2\sqrt{z}\right) $$

07.17.03.0013.01

$$ {}_0F_1\left(;\frac{3}{2};z\right)=\frac{\sinh\left(2\sqrt{z}\right)}{2\sqrt{z}} $$

07.17.03.0038.01

$$ {}_0F_1\left(;\frac{3}{2};-z\right)=\frac{\sin\left(2\sqrt{z}\right)}{2\sqrt{z}} $$

07.17.03.0014.01

$$ {}_0F_1\left(;\frac{5}{2};z\right)=\frac{3}{8z}\left(2\cosh\left(2\sqrt{z}\right)-\frac{\sinh\left(2\sqrt{z}\right)}{\sqrt{z}}\right) $$

07.17.03.0039.01

$$ {}_0F_1\left(;\frac{5}{2};-z\right)=\frac{3\left(\sin\left(2\sqrt{z}\right)-2\sqrt{z}\cos\left(2\sqrt{z}\right)\right)}{8z^{3/2}} $$

07.17.03.0015.01

$$ {}_0F_1\left(;\frac{7}{2};z\right)=\frac{15}{32z^{5/2}}\left((4z+3)\sinh\left(2\sqrt{z}\right)-6\sqrt{z}\cosh\left(2\sqrt{z}\right)\right) $$

07.17.03.0040.01

$$ {}_0F_1\left(;\frac{7}{2};-z\right)=-\frac{15\left(6\sqrt{z}\cos\left(2\sqrt{z}\right)+(4z-3)\sin\left(2\sqrt{z}\right)\right)}{32z^{5/2}} $$

07.17.03.0016.01

$$ {}_0F_1\left(;\frac{9}{2};z\right)=\frac{105}{128z^{7/2}}\left(2\sqrt{z}(4z+15)\cosh\left(2\sqrt{z}\right)-3(8z+5)\sinh\left(2\sqrt{z}\right)\right) $$

07.17.03.0041.01

$$ {}_0F_1\left(;\frac{9}{2};-z\right)=\frac{105\left(2\sqrt{z}(4z-15)\cos\left(2\sqrt{z}\right)+3(5-8z)\sin\left(2\sqrt{z}\right)\right)}{128z^{7/2}} $$

07.17.03.0042.01

$$ {}_0F_1\left(;\frac{11}{2};z\right)=\frac{945\left((4z(4z+45)+105)\sinh\left(2\sqrt{z}\right)-10\sqrt{z}(8z+21)\cosh\left(2\sqrt{z}\right)\right)}{512z^{9/2}} $$

07.17.03.0043.01

$$ {}_0F_1\left(;\frac{11}{2};-z\right)=\frac{945\left(10\sqrt{z}(8z-21)\cos\left(2\sqrt{z}\right)+(4z(4z-45)+105)\sin\left(2\sqrt{z}\right)\right)}{512z^{9/2}} $$

For fixed z and $b=\frac{m}{3}$

07.17.03.0044.01

$$ {}_0F_1\left(;-\frac{17}{3};z\right)=-\frac{1}{628320\,3^{5/6}}\left(\left(99\sqrt{3}\,z^{2/3}\left(243z^2+4032z+9520\right)\operatorname{Ai}\left(3^{2/3}\sqrt[3]{z}\right)-3\sqrt[6]{3}\left(729z^3+42768z^2+277200z+209440\right)\operatorname{Ai}'\left(3^{2/3}\sqrt[3]{z}\right)-99\,z^{2/3}\left(243z^2+4032z+9520\right)\operatorname{Bi}\left(3^{2/3}\sqrt[3]{z}\right)+3^{2/3}\left(729z^3+42768z^2+277200z+209440\right)\operatorname{Bi}'\left(3^{2/3}\sqrt[3]{z}\right)\right)\Gamma\left(-\frac{2}{3}\right)\right) $$

07.17.03.0045.01

$$ {}_0F_1\left(;-\frac{17}{3};-z\right)=\frac{1}{1256640\,3^{5/6}}\left(\left(99\,(-1)^{2/3}\left(3\,i+\sqrt{3}\right)z^{2/3}\left(243\,z^2-4032\,z+9520\right)\text{Ai}\left(-3^{2/3}\sqrt[3]{z}\right)-6\sqrt[6]{3}\left(729\,z^3-42768\,z^2+277200\,z-209440\right)\text{Ai}'\left(-3^{2/3}\sqrt[3]{z}\right)-99\sqrt[6]{-1}\left(i-\sqrt{3}\right)z^{2/3}\left(243\,z^2-4032\,z+9520\right)\text{Bi}\left(-3^{2/3}\sqrt[3]{z}\right)+2\,3^{2/3}\left(729\,z^3-42768\,z^2+277200\,z-209440\right)\text{Bi}'\left(-3^{2/3}\sqrt[3]{z}\right)\right)\Gamma\left(-\frac{2}{3}\right)\right) $$

07.17.03.0046.01

$$ {}_0F_1\left(;-\frac{16}{3};z\right)=\frac{1}{116480\,3^{5/6}}\left(\left(-\sqrt{3}\left(729\,z^3+34020\,z^2+163800\,z+58240\right)\text{Ai}\left(3^{2/3}\sqrt[3]{z}\right)+30\sqrt[6]{3}\sqrt[3]{z}\left(243\,z^2+3276\,z+5824\right)\text{Ai}'\left(3^{2/3}\sqrt[3]{z}\right)+\left(-729\,z^3-34020\,z^2-163800\,z-58240\right)\text{Bi}\left(3^{2/3}\sqrt[3]{z}\right)+10\,3^{2/3}\sqrt[3]{z}\left(243\,z^2+3276\,z+5824\right)\text{Bi}'\left(3^{2/3}\sqrt[3]{z}\right)\right)\Gamma\left(-\frac{1}{3}\right)\right) $$

07.17.03.0047.01

$$ {}_0F_1\left(;-\frac{16}{3};-z\right)=\frac{1}{116480\,3^{5/6}}\left(\left(\sqrt{3}\left(729\,z^3-34020\,z^2+163800\,z-58240\right)\text{Ai}\left(-3^{2/3}\sqrt[3]{z}\right)+15\sqrt[3]{-1}\sqrt[6]{3}\left(-1+\sqrt[6]{-3}\right)\left(1+\sqrt[6]{-3}+\sqrt[3]{-3}\right)\sqrt[3]{z}\left(243\,z^2-3276\,z+5824\right)\text{Ai}'\left(-3^{2/3}\sqrt[3]{z}\right)+\left(729\,z^3-34020\,z^2+163800\,z-58240\right)\text{Bi}\left(-3^{2/3}\sqrt[3]{z}\right)+5\sqrt[3]{-1}\sqrt[6]{3}\left(3\,i-\sqrt{3}\right)\sqrt[3]{z}\left(243\,z^2-3276\,z+5824\right)\text{Bi}'\left(-3^{2/3}\sqrt[3]{z}\right)\right)\Gamma\left(-\frac{1}{3}\right)\right) $$

07.17.03.0048.01

$$ {}_0F_1\left(;-\frac{14}{3};z\right)=-\frac{1}{36960\,3^{5/6}}\left(\left(9\sqrt{3}\,z^{2/3}\left(81\,z^2+2376\,z+6160\right)\text{Ai}\left(3^{2/3}\sqrt[3]{z}\right)-24\sqrt[6]{3}\left(243\,z^2+1980\,z+1540\right)\text{Ai}'\left(3^{2/3}\sqrt[3]{z}\right)-9\,z^{2/3}\left(81\,z^2+2376\,z+6160\right)\text{Bi}\left(3^{2/3}\sqrt[3]{z}\right)+8\,3^{2/3}\left(243\,z^2+1980\,z+1540\right)\text{Bi}'\left(3^{2/3}\sqrt[3]{z}\right)\right)\Gamma\left(-\frac{2}{3}\right)\right) $$

07.17.03.0049.01

$$ {}_0F_1\left(;-\frac{14}{3};-z\right)=-\frac{1}{73920\,3^{5/6}}\left(\left(-9\,(-1)^{2/3}\left(3\,i+\sqrt{3}\right)z^{2/3}\left(81\,z^2-2376\,z+6160\right)\text{Ai}\left(-3^{2/3}\sqrt[3]{z}\right)-48\sqrt[6]{3}\left(243\,z^2-1980\,z+1540\right)\text{Ai}'\left(-3^{2/3}\sqrt[3]{z}\right)+9\sqrt[6]{-1}\left(i-\sqrt{3}\right)z^{2/3}\left(81\,z^2-2376\,z+6160\right)\text{Bi}\left(-3^{2/3}\sqrt[3]{z}\right)+16\,3^{2/3}\left(243\,z^2-1980\,z+1540\right)\text{Bi}'\left(-3^{2/3}\sqrt[3]{z}\right)\right)\Gamma\left(-\frac{2}{3}\right)\right) $$

07.17.03.0050.01

$$ {}_0F_1\left(;-\frac{13}{3};z\right)= \frac{1}{7280\, 3^{5/6}}\left(\left(-7\sqrt{3}\,\left(243 z^2+1440 z+520\right) \mathrm{Ai}\left(3^{2/3}\sqrt[3]{z}\right)+3\sqrt[6]{3}\,\sqrt[3]{z}\,\left(81 z^2+1890 z+3640\right) \mathrm{Ai}'\left(3^{2/3}\sqrt[3]{z}\right)-7\left(243 z^2+1440 z+520\right) \mathrm{Bi}\left(3^{2/3}\sqrt[3]{z}\right)+3^{2/3}\sqrt[3]{z}\,\left(81 z^2+1890 z+3640\right) \mathrm{Bi}'\left(3^{2/3}\sqrt[3]{z}\right)\right)\Gamma\left(-\frac{1}{3}\right)\right) $$

07.17.03.0051.01

$$ {}_0F_1\left(;-\frac{13}{3};-z\right)= \frac{1}{14\,560\, 3^{5/6}}\left(\left(-14\sqrt{3}\,\left(243 z^2-1440 z+520\right) \mathrm{Ai}\left(-3^{2/3}\sqrt[3]{z}\right)+3\sqrt[3]{-1}\,\sqrt[6]{3}\,\left(-1+\sqrt[6]{-3}\right)\left(1+\sqrt[6]{-3}+\sqrt[3]{-3}\right)\sqrt[3]{z}\,\left(81 z^2-1890 z+3640\right) \mathrm{Ai}'\left(-3^{2/3}\sqrt[3]{z}\right)-14\left(243 z^2-1440 z+520\right) \mathrm{Bi}\left(-3^{2/3}\sqrt[3]{z}\right)+\sqrt[3]{-1}\,\sqrt[6]{3}\,\left(3 i-\sqrt{3}\right)\sqrt[3]{z}\,\left(81 z^2-1890 z+3640\right) \mathrm{Bi}'\left(-3^{2/3}\sqrt[3]{z}\right)\right)\Gamma\left(-\frac{1}{3}\right)\right) $$

07.17.03.0052.01

$$ {}_0F_1\left(;-\frac{11}{3};z\right)= -\frac{1}{2640\, 3^{5/6}}\left(\left(72\sqrt{3}\,z^{2/3}\,(18 z+55)\,\mathrm{Ai}\left(3^{2/3}\sqrt[3]{z}\right)-3\sqrt[6]{3}\,\left(81 z^2+1080 z+880\right) \mathrm{Ai}'\left(3^{2/3}\sqrt[3]{z}\right)-72 z^{2/3}\,(18 z+55)\,\mathrm{Bi}\left(3^{2/3}\sqrt[3]{z}\right)+3^{2/3}\left(81 z^2+1080 z+880\right) \mathrm{Bi}'\left(3^{2/3}\sqrt[3]{z}\right)\right)\Gamma\left(-\frac{2}{3}\right)\right) $$

07.17.03.0053.01

$$ {}_0F_1\left(;-\frac{11}{3};-z\right)= -\frac{1}{2640\, 3^{5/6}}\left(\left(36\,(-1)^{2/3}\left(3 i+\sqrt{3}\right) z^{2/3}\,(18 z-55)\,\mathrm{Ai}\left(-3^{2/3}\sqrt[3]{z}\right)-3\sqrt[6]{3}\,\left(81 z^2-1080 z+880\right) \mathrm{Ai}'\left(-3^{2/3}\sqrt[3]{z}\right)-36\sqrt[6]{-1}\,\left(i-\sqrt{3}\right) z^{2/3}\,(18 z-55)\,\mathrm{Bi}\left(-3^{2/3}\sqrt[3]{z}\right)+3^{2/3}\left(81 z^2-1080 z+880\right) \mathrm{Bi}'\left(-3^{2/3}\sqrt[3]{z}\right)\right)\Gamma\left(-\frac{2}{3}\right)\right) $$

07.17.03.0054.01

$$ {}_0F_1\left(;-\frac{10}{3};z\right)= \frac{1}{560\, 3^{5/6}}\left(\left(-\sqrt{3}\,\left(81 z^2+756 z+280\right) \mathrm{Ai}\left(3^{2/3}\sqrt[3]{z}\right)+42\sqrt[6]{3}\,\sqrt[3]{z}\,(9 z+20)\,\mathrm{Ai}'\left(3^{2/3}\sqrt[3]{z}\right)+\left(-81 z^2-756 z-280\right) \mathrm{Bi}\left(3^{2/3}\sqrt[3]{z}\right)+14\, 3^{2/3}\sqrt[3]{z}\,(9 z+20)\,\mathrm{Bi}'\left(3^{2/3}\sqrt[3]{z}\right)\right)\Gamma\left(-\frac{1}{3}\right)\right) $$

07.17.03.0055.01

$$ {}_0F_1\left(;-\frac{10}{3};-z\right)= -\frac{1}{560\, 3^{5/6}}\left(\left(\sqrt{3}\,\left(81 z^2-756 z+280\right) \mathrm{Ai}\left(-3^{2/3}\sqrt[3]{z}\right)+21\sqrt[3]{-1}\,\sqrt[6]{3}\,\left(-1+\sqrt[6]{-3}\right)\left(1+\sqrt[6]{-3}+\sqrt[3]{-3}\right)\sqrt[3]{z}\,(9 z-20)\,\mathrm{Ai}'\left(-3^{2/3}\sqrt[3]{z}\right)+\left(81 z^2-756 z+280\right) \mathrm{Bi}\left(-3^{2/3}\sqrt[3]{z}\right)+7\sqrt[3]{-1}\,\sqrt[6]{3}\,\left(3 i-\sqrt{3}\right)\sqrt[3]{z}\,(9 z-20)\,\mathrm{Bi}'\left(-3^{2/3}\sqrt[3]{z}\right)\right)\Gamma\left(-\frac{1}{3}\right)\right) $$

07.17.03.0056.01

$$ {}_0F_1\left(;-\frac{8}{3};z\right)=-\frac{1}{240\,3^{5/6}}\left(\left(9\sqrt{3}\,z^{2/3}\,(9z+40)\,\mathrm{Ai}\left(3^{2/3}\sqrt[3]{z}\right)-30\sqrt[6]{3}\,(9z+8)\,\mathrm{Ai}'\left(3^{2/3}\sqrt[3]{z}\right)-9z^{2/3}\,(9z+40)\,\mathrm{Bi}\left(3^{2/3}\sqrt[3]{z}\right)+10\,3^{2/3}\,(9z+8)\,\mathrm{Bi}'\left(3^{2/3}\sqrt[3]{z}\right)\right)\Gamma\left(-\frac{2}{3}\right)\right) $$

07.17.03.0057.01

$$ {}_0F_1\left(;-\frac{8}{3};-z\right)=\frac{1}{480\,3^{5/6}}\left(\left(-9\,(-1)^{2/3}\left(3\,i+\sqrt{3}\right)z^{2/3}\,(9z-40)\,\mathrm{Ai}\left(-3^{2/3}\sqrt[3]{z}\right)-60\sqrt[6]{3}\,(9z-8)\,\mathrm{Ai}'\left(-3^{2/3}\sqrt[3]{z}\right)+9\sqrt[6]{-1}\left(i-\sqrt{3}\right)z^{2/3}\,(9z-40)\,\mathrm{Bi}\left(-3^{2/3}\sqrt[3]{z}\right)+20\,3^{2/3}\,(9z-8)\,\mathrm{Bi}'\left(-3^{2/3}\sqrt[3]{z}\right)\right)\Gamma\left(-\frac{2}{3}\right)\right) $$

07.17.03.0058.01

$$ {}_0F_1\left(;-\frac{7}{3};z\right)=-\frac{1}{56}\sqrt[6]{3}\,\Gamma\left(\frac{2}{3}\right)\left(3\sqrt[6]{3}\,(9z+28)\,\mathrm{Ai}'\left(3^{2/3}\sqrt[3]{z}\right)\sqrt[3]{z}+3^{2/3}\,(9z+28)\,\mathrm{Bi}'\left(3^{2/3}\sqrt[3]{z}\right)\sqrt[3]{z}-4\sqrt{3}\,(18z+7)\,\mathrm{Ai}\left(3^{2/3}\sqrt[3]{z}\right)-4\,(18z+7)\,\mathrm{Bi}\left(3^{2/3}\sqrt[3]{z}\right)\right) $$

07.17.03.0059.01

$$ {}_0F_1\left(;-\frac{7}{3};-z\right)=-\frac{1}{112\,3^{5/6}}\left(\left(-8\sqrt{3}\,(18z-7)\,\mathrm{Ai}\left(-3^{2/3}\sqrt[3]{z}\right)+3\sqrt[3]{-1}\sqrt[6]{3}\left(-1+\sqrt[6]{-3}\right)\left(1+\sqrt[6]{-3}+\sqrt[3]{-3}\right)\sqrt[3]{z}\,(9z-28)\,\mathrm{Ai}'\left(-3^{2/3}\sqrt[3]{z}\right)-8\,(18z-7)\,\mathrm{Bi}\left(-3^{2/3}\sqrt[3]{z}\right)+\sqrt[3]{-1}\sqrt[6]{3}\left(3\,i-\sqrt{3}\right)\sqrt[3]{z}\,(9z-28)\,\mathrm{Bi}'\left(-3^{2/3}\sqrt[3]{z}\right)\right)\Gamma\left(-\frac{1}{3}\right)\right) $$

07.17.03.0060.01

$$ {}_0F_1\left(;-\frac{5}{3};z\right)=\frac{1}{20}\sqrt[6]{3}\,\Gamma\left(\frac{4}{3}\right)\left(45\sqrt{3}\,z^{2/3}\,\mathrm{Ai}\left(3^{2/3}\sqrt[3]{z}\right)-45\,z^{2/3}\,\mathrm{Bi}\left(3^{2/3}\sqrt[3]{z}\right)-3\sqrt[6]{3}\,(9z+10)\,\mathrm{Ai}'\left(3^{2/3}\sqrt[3]{z}\right)+3^{2/3}\,(9z+10)\,\mathrm{Bi}'\left(3^{2/3}\sqrt[3]{z}\right)\right) $$

07.17.03.0061.01

$$ {}_0F_1\left(;-\frac{5}{3};-z\right)=\frac{1}{60\,3^{5/6}}\left(\left(45\,(-1)^{2/3}\left(3\,i+\sqrt{3}\right)z^{2/3}\,\mathrm{Ai}\left(-3^{2/3}\sqrt[3]{z}\right)-6\sqrt[6]{3}\,(9z-10)\,\mathrm{Ai}'\left(-3^{2/3}\sqrt[3]{z}\right)-45\sqrt[6]{-1}\left(i-\sqrt{3}\right)z^{2/3}\,\mathrm{Bi}\left(-3^{2/3}\sqrt[3]{z}\right)+2\,3^{2/3}\,(9z-10)\,\mathrm{Bi}'\left(-3^{2/3}\sqrt[3]{z}\right)\right)\Gamma\left(-\frac{2}{3}\right)\right) $$

07.17.03.0062.01

$$ {}_0F_1\left(;-\frac{4}{3};z\right)=-\frac{1}{8}\sqrt[6]{3}\,\Gamma\left(\frac{2}{3}\right)\left(12\sqrt[6]{3}\,\mathrm{Ai}'\left(3^{2/3}\sqrt[3]{z}\right)\sqrt[3]{z}+4\,3^{2/3}\,\mathrm{Bi}'\left(3^{2/3}\sqrt[3]{z}\right)\sqrt[3]{z}-\sqrt{3}\,(9z+4)\,\mathrm{Ai}\left(3^{2/3}\sqrt[3]{z}\right)+(-9z-4)\,\mathrm{Bi}\left(3^{2/3}\sqrt[3]{z}\right)\right) $$

07.17.03.0063.01

$$ {}_0F_1\left(;-\frac{4}{3};-z\right)= \frac{1}{8\,3^{5/6}}\left(\left(\sqrt{3}\,(9z-4)\,\mathrm{Ai}\left(-3^{2/3}\sqrt[3]{z}\right)+6\sqrt[3]{-1}\,\sqrt[6]{3}\left(-1+\sqrt[6]{-3}\right)\left(1+\sqrt[6]{-3}+\sqrt[3]{-3}\right)\sqrt[3]{z}\,\mathrm{Ai}'\left(-3^{2/3}\sqrt[3]{z}\right)+(9z-4)\,\mathrm{Bi}\left(-3^{2/3}\sqrt[3]{z}\right)+2\sqrt[3]{-1}\,\sqrt[6]{3}\left(3i-\sqrt{3}\right)\sqrt[3]{z}\,\mathrm{Bi}'\left(-3^{2/3}\sqrt[3]{z}\right)\right)\Gamma\left(-\frac{1}{3}\right)\right) $$

07.17.03.0020.01

$$ {}_0F_1\left(;-\frac{2}{3};z\right)=\frac{1}{4}\sqrt[6]{3}\,\Gamma\left(\frac{4}{3}\right)\left(9\sqrt{3}\,z^{2/3}\,\mathrm{Ai}\left(3^{2/3}\sqrt[3]{z}\right)-9z^{2/3}\,\mathrm{Bi}\left(3^{2/3}\sqrt[3]{z}\right)-6\sqrt[6]{3}\,\mathrm{Ai}'\left(3^{2/3}\sqrt[3]{z}\right)+2\,3^{2/3}\,\mathrm{Bi}'\left(3^{2/3}\sqrt[3]{z}\right)\right) $$

07.17.03.0064.01

$$ {}_0F_1\left(;-\frac{2}{3};-z\right)=\frac{1}{12\,3^{5/6}}\left(\left(9(-1)^{2/3}\left(3i+\sqrt{3}\right)z^{2/3}\,\mathrm{Ai}\left(-3^{2/3}\sqrt[3]{z}\right)+12\sqrt[6]{3}\,\mathrm{Ai}'\left(-3^{2/3}\sqrt[3]{z}\right)-9\sqrt[6]{-1}\left(i-\sqrt{3}\right)z^{2/3}\,\mathrm{Bi}\left(-3^{2/3}\sqrt[3]{z}\right)-4\,3^{2/3}\,\mathrm{Bi}'\left(-3^{2/3}\sqrt[3]{z}\right)\right)\Gamma\left(-\frac{2}{3}\right)\right) $$

07.17.03.0021.01

$$ {}_0F_1\left(;-\frac{1}{3};z\right)=-\frac{1}{2}\sqrt[6]{3}\,\Gamma\left(\frac{2}{3}\right)\left(3\sqrt[6]{3}\,\sqrt[3]{z}\,\mathrm{Ai}'\left(3^{2/3}\sqrt[3]{z}\right)+3^{2/3}\sqrt[3]{z}\,\mathrm{Bi}'\left(3^{2/3}\sqrt[3]{z}\right)-\sqrt{3}\,\mathrm{Ai}\left(3^{2/3}\sqrt[3]{z}\right)-\mathrm{Bi}\left(3^{2/3}\sqrt[3]{z}\right)\right) $$

07.17.03.0065.01

$$ {}_0F_1\left(;-\frac{1}{3};-z\right)=-\frac{1}{4}\sqrt[6]{3}\left(-2\sqrt{3}\,\mathrm{Ai}\left(-3^{2/3}\sqrt[3]{z}\right)+3\sqrt[3]{-1}\,\sqrt[6]{3}\left(-1+\sqrt[6]{-3}\right)\left(1+\sqrt[6]{-3}+\sqrt[3]{-3}\right)\sqrt[3]{z}\,\mathrm{Ai}'\left(-3^{2/3}\sqrt[3]{z}\right)-2\,\mathrm{Bi}\left(-3^{2/3}\sqrt[3]{z}\right)+\sqrt[3]{-1}\,\sqrt[6]{3}\left(3i-\sqrt{3}\right)\sqrt[3]{z}\,\mathrm{Bi}'\left(-3^{2/3}\sqrt[3]{z}\right)\right)\Gamma\left(\frac{2}{3}\right) $$

07.17.03.0022.01

$$ {}_0F_1\left(;\frac{1}{3};z\right)=\frac{1}{2}\sqrt[3]{3}\,\Gamma\left(\frac{4}{3}\right)\left(\sqrt{3}\,\mathrm{Bi}'\left(3^{2/3}\sqrt[3]{z}\right)-3\,\mathrm{Ai}'\left(3^{2/3}\sqrt[3]{z}\right)\right) $$

07.17.03.0066.01

$$ {}_0F_1\left(;\frac{1}{3};-z\right)=\frac{\left(\sqrt{3}\,\mathrm{Bi}'\left(-3^{2/3}\sqrt[3]{z}\right)-3\,\mathrm{Ai}'\left(-3^{2/3}\sqrt[3]{z}\right)\right)\Gamma\left(\frac{1}{3}\right)}{2\,3^{2/3}} $$

07.17.03.0023.01

$$ {}_0F_1\left(;\frac{2}{3};z\right)=\frac{\sqrt[6]{3}}{2}\,\Gamma\left(\frac{2}{3}\right)\left(\sqrt{3}\,\mathrm{Ai}\left(3^{2/3}\sqrt[3]{z}\right)+\mathrm{Bi}\left(3^{2/3}\sqrt[3]{z}\right)\right) $$

07.17.03.0067.01

$$ {}_0F_1\left(;\frac{2}{3};-z\right)=-\frac{\left(\sqrt{3}\,\mathrm{Ai}\left(-3^{2/3}\sqrt[3]{z}\right)+\mathrm{Bi}\left(-3^{2/3}\sqrt[3]{z}\right)\right)\Gamma\left(-\frac{1}{3}\right)}{2\,3^{5/6}} $$

07.17.03.0024.01

$$ {}_0F_1\left(;\frac{4}{3};z\right)=\frac{1}{2}\sqrt[6]{3}\,\Gamma\left(\frac{4}{3}\right)\left(\frac{1}{\sqrt[3]{z}}\left(\mathrm{Bi}\left(3^{2/3}\sqrt[3]{z}\right)-\sqrt{3}\,\mathrm{Ai}\left(3^{2/3}\sqrt[3]{z}\right)\right)\right) $$

07.17.03.0068.01

$$ {}_0F_1\left(;\frac{4}{3};-z\right)=\frac{\sqrt[6]{-1}\left(i\left(3\,i+\sqrt{3}\right)\mathrm{Ai}\left(-3^{2/3}\sqrt[3]{z}\right)+\left(-i+\sqrt{3}\right)\mathrm{Bi}\left(-3^{2/3}\sqrt[3]{z}\right)\right)\Gamma\left(-\frac{2}{3}\right)}{6\,3^{5/6}\sqrt[3]{z}} $$

07.17.03.0025.01

$$ {}_0F_1\left(;\frac{5}{3};z\right)=\frac{\Gamma\left(\frac{5}{3}\right)}{2\,3^{5/6}z^{2/3}}\left(3\sqrt[6]{3}\,\mathrm{Ai}'\left(3^{2/3}\sqrt[3]{z}\right)+3^{2/3}\,\mathrm{Bi}'\left(3^{2/3}\sqrt[3]{z}\right)\right) $$

07.17.03.0069.01

$$ {}_0F_1\left(;\frac{5}{3};-z\right)=\frac{(-1)^{5/6}\left(3\left(i+\sqrt{3}\right)\mathrm{Ai}'\left(-3^{2/3}\sqrt[3]{z}\right)+\left(3+i\sqrt{3}\right)\mathrm{Bi}'\left(-3^{2/3}\sqrt[3]{z}\right)\right)\Gamma\left(-\frac{1}{3}\right)}{18\,3^{2/3}z^{2/3}} $$

07.17.03.0026.01

$$ {}_0F_1\left(;\frac{7}{3};z\right)=\frac{\Gamma\left(\frac{7}{3}\right)}{2\,3^{5/6}z^{4/3}}\left(-3\sqrt[6]{3}\,\sqrt[3]{z}\,\mathrm{Ai}'\left(3^{2/3}\sqrt[3]{z}\right)+3^{2/3}\sqrt[3]{z}\,\mathrm{Bi}'\left(3^{2/3}\sqrt[3]{z}\right)+\sqrt{3}\,\mathrm{Ai}\left(3^{2/3}\sqrt[3]{z}\right)-\mathrm{Bi}\left(3^{2/3}\sqrt[3]{z}\right)\right) $$

07.17.03.0070.01

$$ {}_0F_1\left(;\frac{7}{3};-z\right)=\frac{1}{27\,3^{5/6}z^{4/3}}\left(2\left((-1)^{2/3}\left(3\,i+\sqrt{3}\right)\mathrm{Ai}\left(-3^{2/3}\sqrt[3]{z}\right)-\right.\right. $$
$$ \left.\left.6\sqrt[6]{3}\,\sqrt[3]{z}\,\mathrm{Ai}'\left(-3^{2/3}\sqrt[3]{z}\right)-\sqrt[6]{-1}\left(i-\sqrt{3}\right)\mathrm{Bi}\left(-3^{2/3}\sqrt[3]{z}\right)+2\,3^{2/3}\sqrt[3]{z}\,\mathrm{Bi}'\left(-3^{2/3}\sqrt[3]{z}\right)\right)\Gamma\left(-\frac{2}{3}\right)\right) $$

07.17.03.0071.01

$$ {}_0F_1\left(;\frac{8}{3};z\right)= $$
$$ -\frac{1}{81\,3^{5/6}z^{5/3}}\left(5\left(9\sqrt{3}\,z^{2/3}\,\mathrm{Ai}\left(3^{2/3}\sqrt[3]{z}\right)-6\sqrt[6]{3}\,\mathrm{Ai}'\left(3^{2/3}\sqrt[3]{z}\right)+9\,z^{2/3}\,\mathrm{Bi}\left(3^{2/3}\sqrt[3]{z}\right)-2\,3^{2/3}\,\mathrm{Bi}'\left(3^{2/3}\sqrt[3]{z}\right)\right)\Gamma\left(-\frac{1}{3}\right)\right) $$

07.17.03.0072.01

$$ {}_0F_1\left(;\frac{8}{3};-z\right)=\frac{1}{81\,3^{5/6}z^{5/3}} $$
$$ \left(5\left(9\sqrt{3}\,z^{2/3}\,\mathrm{Ai}\left(-3^{2/3}\sqrt[3]{z}\right)+3\sqrt[3]{-1}\,\sqrt[6]{3}\left(-1+\sqrt[6]{-3}\right)\left(1+\sqrt[6]{-3}+\sqrt[3]{-3}\right)\mathrm{Ai}'\left(-3^{2/3}\sqrt[3]{z}\right)+9\,z^{2/3}\,\mathrm{Bi}\left(-3^{2/3}\sqrt[3]{z}\right)+\right.\right. $$
$$ \left.\left.\sqrt[3]{-1}\,\sqrt[6]{3}\left(3\,i-\sqrt{3}\right)\mathrm{Bi}'\left(-3^{2/3}\sqrt[3]{z}\right)\right)\Gamma\left(-\frac{1}{3}\right)\right) $$

07.17.03.0073.01

$$ {}_0F_1\left(;\frac{10}{3};z\right)=\frac{1}{243\,3^{5/6}z^{7/3}} $$
$$ \left(28\left(\sqrt{3}\,(9\,z+4)\,\mathrm{Ai}\left(3^{2/3}\sqrt[3]{z}\right)-12\sqrt[6]{3}\,\sqrt[3]{z}\,\mathrm{Ai}'\left(3^{2/3}\sqrt[3]{z}\right)+(-9\,z-4)\,\mathrm{Bi}\left(3^{2/3}\sqrt[3]{z}\right)+4\,3^{2/3}\sqrt[3]{z}\,\mathrm{Bi}'\left(3^{2/3}\sqrt[3]{z}\right)\right)\Gamma\left(-\frac{2}{3}\right)\right) $$

07.17.03.0074.01

$$ {}_0F_1\left(;\frac{10}{3};-z\right)=\frac{1}{243\,3^{5/6}z^{7/3}}\left(14\left(-(-1)^{2/3}\left(3\,i+\sqrt{3}\right)(9\,z-4)\,\mathrm{Ai}\left(-3^{2/3}\sqrt[3]{z}\right)-\right.\right. $$
$$ \left.\left.24\sqrt[6]{3}\,\sqrt[3]{z}\,\mathrm{Ai}'\left(-3^{2/3}\sqrt[3]{z}\right)+\sqrt[6]{-1}\left(i-\sqrt{3}\right)(9\,z-4)\,\mathrm{Bi}\left(-3^{2/3}\sqrt[3]{z}\right)+8\,3^{2/3}\sqrt[3]{z}\,\mathrm{Bi}'\left(-3^{2/3}\sqrt[3]{z}\right)\right)\Gamma\left(-\frac{2}{3}\right)\right) $$

07.17.03.0075.01

$${}_0F_1\left(;\frac{11}{3};z\right)=-\frac{1}{729\,3^{5/6}\,z^{8/3}}\left(40\left(-45\sqrt{3}\,z^{2/3}\,\mathrm{Ai}\left(3^{2/3}\sqrt[3]{z}\right)+3\sqrt[6]{3}\,(9z+10)\,\mathrm{Ai}'\left(3^{2/3}\sqrt[3]{z}\right)-45\,z^{2/3}\,\mathrm{Bi}\left(3^{2/3}\sqrt[3]{z}\right)+3^{2/3}\,(9z+10)\,\mathrm{Bi}'\left(3^{2/3}\sqrt[3]{z}\right)\right)\Gamma\left(-\frac{1}{3}\right)\right)$$

07.17.03.0076.01

$${}_0F_1\left(;\frac{11}{3};-z\right)=-\frac{1}{729\,3^{5/6}\,z^{8/3}}\left(20\left(-90\sqrt{3}\,z^{2/3}\,\mathrm{Ai}\left(-3^{2/3}\sqrt[3]{z}\right)+3\sqrt[3]{-1}\,\sqrt[6]{3}\left(-1+\sqrt[6]{-3}\right)\left(1+\sqrt[6]{-3}+\sqrt[3]{-3}\right)(9z-10)\,\mathrm{Ai}'\left(-3^{2/3}\sqrt[3]{z}\right)-90\,z^{2/3}\,\mathrm{Bi}\left(-3^{2/3}\sqrt[3]{z}\right)+\sqrt[3]{-1}\,\sqrt[6]{3}\left(3\,i-\sqrt{3}\right)(9z-10)\,\mathrm{Bi}'\left(-3^{2/3}\sqrt[3]{z}\right)\right)\Gamma\left(-\frac{1}{3}\right)\right)$$

07.17.03.0077.01

$${}_0F_1\left(;\frac{13}{3};z\right)=-\frac{1}{2187\,3^{5/6}\,z^{10/3}}\left(280\left(4\sqrt{3}\,(18z+7)\,\mathrm{Ai}\left(3^{2/3}\sqrt[3]{z}\right)-3\sqrt[6]{3}\,\sqrt[3]{z}\,(9z+28)\,\mathrm{Ai}'\left(3^{2/3}\sqrt[3]{z}\right)-4\,(18z+7)\,\mathrm{Bi}\left(3^{2/3}\sqrt[3]{z}\right)+3^{2/3}\sqrt[3]{z}\,(9z+28)\,\mathrm{Bi}'\left(3^{2/3}\sqrt[3]{z}\right)\right)\Gamma\left(-\frac{2}{3}\right)\right)$$

07.17.03.0078.01

$${}_0F_1\left(;\frac{13}{3};-z\right)=-\frac{1}{2187\,3^{5/6}\,z^{10/3}}\left(280\left(2\,(-1)^{2/3}\left(3\,i+\sqrt{3}\right)(18z-7)\,\mathrm{Ai}\left(-3^{2/3}\sqrt[3]{z}\right)-3\sqrt[6]{3}\,\sqrt[3]{z}\,(9z-28)\,\mathrm{Ai}'\left(-3^{2/3}\sqrt[3]{z}\right)-2\sqrt[6]{-1}\left(i-\sqrt{3}\right)(18z-7)\,\mathrm{Bi}\left(-3^{2/3}\sqrt[3]{z}\right)+3^{2/3}\sqrt[3]{z}\,(9z-28)\,\mathrm{Bi}'\left(-3^{2/3}\sqrt[3]{z}\right)\right)\Gamma\left(-\frac{2}{3}\right)\right)$$

07.17.03.0079.01

$${}_0F_1\left(;\frac{14}{3};z\right)=\frac{1}{6561\,3^{5/6}\,z^{11/3}}\left(440\left(-9\sqrt{3}\,z^{2/3}\,(9z+40)\,\mathrm{Ai}\left(3^{2/3}\sqrt[3]{z}\right)+30\sqrt[6]{3}\,(9z+8)\,\mathrm{Ai}'\left(3^{2/3}\sqrt[3]{z}\right)-9\,z^{2/3}\,(9z+40)\,\mathrm{Bi}\left(3^{2/3}\sqrt[3]{z}\right)+10\,3^{2/3}\,(9z+8)\,\mathrm{Bi}'\left(3^{2/3}\sqrt[3]{z}\right)\right)\Gamma\left(-\frac{1}{3}\right)\right)$$

07.17.03.0080.01

$${}_0F_1\left(;\frac{14}{3};-z\right)=-\frac{1}{6561\,3^{5/6}\,z^{11/3}}\left(440\left(9\sqrt{3}\,z^{2/3}\,(9z-40)\,\mathrm{Ai}\left(-3^{2/3}\sqrt[3]{z}\right)+15\sqrt[3]{-1}\,\sqrt[6]{3}\left(-1+\sqrt[6]{-3}\right)\left(1+\sqrt[6]{-3}+\sqrt[3]{-3}\right)(9z-8)\,\mathrm{Ai}'\left(-3^{2/3}\sqrt[3]{z}\right)+9\,z^{2/3}\,(9z-40)\,\mathrm{Bi}\left(-3^{2/3}\sqrt[3]{z}\right)+5\sqrt[3]{-1}\,\sqrt[6]{3}\left(3\,i-\sqrt{3}\right)(9z-8)\,\mathrm{Bi}'\left(-3^{2/3}\sqrt[3]{z}\right)\right)\Gamma\left(-\frac{1}{3}\right)\right)$$

07.17.03.0081.01

$${}_0F_1\left(;\frac{16}{3};z\right)=\frac{1}{19683\,3^{5/6}\,z^{13/3}}\left(3640\left(\sqrt{3}\,\left(81\,z^2+756\,z+280\right)\mathrm{Ai}\left(3^{2/3}\,\sqrt[3]{z}\right)-42\,\sqrt[6]{3}\,\sqrt[3]{z}\,(9\,z+20)\,\mathrm{Ai}'\left(3^{2/3}\,\sqrt[3]{z}\right)+\left(-81\,z^2-756\,z-280\right)\mathrm{Bi}\left(3^{2/3}\,\sqrt[3]{z}\right)+14\,3^{2/3}\,\sqrt[3]{z}\,(9\,z+20)\,\mathrm{Bi}'\left(3^{2/3}\,\sqrt[3]{z}\right)\right)\Gamma\left(-\frac{2}{3}\right)\right)$$

07.17.03.0082.01

$${}_0F_1\left(;\frac{16}{3};-z\right)=-\frac{1}{19683\,3^{5/6}\,z^{13/3}}\left(1820\left(-(-1)^{2/3}\left(3\,i+\sqrt{3}\right)\left(81\,z^2-756\,z+280\right)\mathrm{Ai}\left(-3^{2/3}\,\sqrt[3]{z}\right)-84\,\sqrt[6]{3}\,\sqrt[3]{z}\,(9\,z-20)\,\mathrm{Ai}'\left(-3^{2/3}\,\sqrt[3]{z}\right)+\sqrt[6]{-1}\,\left(i-\sqrt{3}\right)\left(81\,z^2-756\,z+280\right)\mathrm{Bi}\left(-3^{2/3}\,\sqrt[3]{z}\right)+28\,3^{2/3}\,\sqrt[3]{z}\,(9\,z-20)\,\mathrm{Bi}'\left(-3^{2/3}\,\sqrt[3]{z}\right)\right)\Gamma\left(-\frac{2}{3}\right)\right)$$

07.17.03.0083.01

$${}_0F_1\left(;\frac{17}{3};z\right)=-\frac{1}{59049\,3^{5/6}\,z^{14/3}}\left(6160\left(-72\,\sqrt{3}\,z^{2/3}\,(18\,z+55)\,\mathrm{Ai}\left(3^{2/3}\,\sqrt[3]{z}\right)+3\,\sqrt[6]{3}\,\left(81\,z^2+1080\,z+880\right)\mathrm{Ai}'\left(3^{2/3}\,\sqrt[3]{z}\right)-72\,z^{2/3}\,(18\,z+55)\,\mathrm{Bi}\left(3^{2/3}\,\sqrt[3]{z}\right)+3^{2/3}\left(81\,z^2+1080\,z+880\right)\mathrm{Bi}'\left(3^{2/3}\,\sqrt[3]{z}\right)\right)\Gamma\left(-\frac{1}{3}\right)\right)$$

07.17.03.0084.01

$${}_0F_1\left(;\frac{17}{3};-z\right)=\frac{1}{59049\,3^{5/6}\,z^{14/3}}\left(3080\left(-144\,\sqrt{3}\,z^{2/3}\,(18\,z-55)\,\mathrm{Ai}\left(-3^{2/3}\,\sqrt[3]{z}\right)+3\,\sqrt[3]{-1}\,\sqrt[6]{3}\,\left(-1+\sqrt[6]{-3}\right)\left(1+\sqrt[6]{-3}+\sqrt[3]{-3}\right)\left(81\,z^2-1080\,z+880\right)\mathrm{Ai}'\left(-3^{2/3}\,\sqrt[3]{z}\right)-144\,z^{2/3}\,(18\,z-55)\,\mathrm{Bi}\left(-3^{2/3}\,\sqrt[3]{z}\right)+\sqrt[3]{-1}\,\sqrt[6]{3}\,\left(3\,i-\sqrt{3}\right)\left(81\,z^2-1080\,z+880\right)\mathrm{Bi}'\left(-3^{2/3}\,\sqrt[3]{z}\right)\right)\Gamma\left(-\frac{1}{3}\right)\right)$$

Values at infinities

07.17.03.0027.01

$${}_0F_1(;b;\infty)=\tilde{\infty}$$

General characteristics

Domain and analyticity

${}_0F_1(;b;z)$ is an analytical function of b and z which is defined in $\mathbb{C}^2$. For fixed b, it is an entire function of z.

07.17.04.0001.01

$$(b*z)\longrightarrow{}_0F_1(;b;z)::(\mathbb{C}\otimes\mathbb{C})\longrightarrow\mathbb{C}$$

Symmetries and periodicities

Mirror symmetry

07.17.04.0002.01

$${}_0F_1\left(;\overline{b};\overline{z}\right)=\overline{{}_0F_1(;b;z)}$$

Periodicity

No periodicity

Poles and essential singularities

With respect to *z*

For fixed b, the function ${}_0F_1(;b;z)$ has only one singular point at $z=\tilde{\infty}$. It is an essential singular point.

07.17.04.0003.01

$$\mathcal{Sing}_z({}_0F_1(;b;z))=\{\{\tilde{\infty},\infty\}\}$$

With respect to *b*

For fixed z, the function ${}_0F_1(;b;z)$ has an infinite set of singular points:

a) $b=-k\,/;\,k\in\mathbb{N}$, are the simple poles with residues $\frac{(-1)^k}{k!}\,{}_0\tilde{F}_1(;-k;z)$;

b) $b=\tilde{\infty}$ is the point of convergence of poles, which is an essential singular point.

07.17.04.0004.01

$$\mathcal{Sing}_b({}_0F_1(;b;z))=\{\{\{-k,1\}\,/;\,k\in\mathbb{N}\},\{\tilde{\infty},\infty\}\}$$

07.17.04.0005.01

$$\operatorname{res}_b({}_0F_1(;b;z))(-k)=\frac{(-1)^k}{k!}\,{}_0\tilde{F}_1(;-k;z)\,/;\,k\in\mathbb{N}$$

Branch points

With respect to *z*

The function ${}_0F_1(;b;z)$ does not have branch points with respect to z.

07.17.04.0006.01

$$\mathcal{BP}_z({}_0F_1(;b;z))=\{\}$$

With respect to *b*

The function ${}_0F_1(;b;z)$ does not have branch points with respect to b.

07.17.04.0007.01

$$\mathcal{BP}_b({}_0F_1(;b;z))=\{\}$$

Branch cuts

With respect to *z*

The function ${}_0F_1(;b;z)$ does not have branch cuts with respect to z.

07.17.04.0008.01

$$\mathcal{BC}_z({}_0F_1(;b;z))=\{\}$$

With respect to b

The function ${}_0F_1(;b;z)$ does not have branch cuts with respect to b.

07.17.04.0009.01

$$\mathcal{BC}_b({}_0F_1(;b;z)) == \{\}$$

Series representations

Generalized power series

Expansions at generic point $z == z_0$

For the function itself

07.17.06.0017.01

$${}_0F_1(;b;z) \propto {}_0F_1(;b;z_0) + \frac{1}{b}\,{}_0F_1(;b+1;z_0)(z-z_0) + \frac{1}{2\,b\,(b+1)}\,{}_0F_1(;b+2;z_0)(z-z_0)^2 + \ldots \,/;\, (z \to z_0)$$

07.17.06.0018.01

$${}_0F_1(;b;z) \propto {}_0F_1(;b;z_0) + \frac{1}{b}\,{}_0F_1(;b+1;z_0)(z-z_0) + \frac{1}{2\,b\,(b+1)}\,{}_0F_1(;b+2;z_0)(z-z_0)^2 + O\big((z-z_0)^3\big)$$

07.17.06.0019.01

$${}_0F_1(;b;z) == \sum_{k=0}^{\infty} \frac{1}{k!\,(b)_k}\,{}_0F_1(;b+k;z_0)(z-z_0)^k$$

07.17.06.0020.01

$${}_0F_1(;b;z) = F_{1\times 0\times 0}^{0\times 0\times 0}\left(\begin{matrix} ;\,;; \\ b;;; \end{matrix}\; z_0,\, z-z_0\right)$$

07.17.06.0021.01

$${}_0F_1(;b;z) \propto {}_0F_1(;b;z_0)\,(1 + O(z-z_0))$$

Expansions at $z == 0$

For the function itself

07.17.06.0001.02

$${}_0F_1(;b;z) \propto 1 + \frac{z}{b} + \frac{z^2}{2\,b\,(1+b)} + \ldots \,/;\, (z \to 0)$$

07.17.06.0022.01

$${}_0F_1(;b;z) \propto 1 + \frac{z}{b} + \frac{z^2}{2\,b\,(1+b)} + O\big(z^3\big)$$

07.17.06.0002.01

$${}_0F_1(;b;z) == \sum_{k=0}^{\infty} \frac{z^k}{(b)_k\,k!}$$

07.17.06.0003.02

$${}_0F_1(;b;z) \propto 1 + O(z)$$

07.17.06.0023.01

$$ {}_0F_1(;b;z) == F_\infty(z,b) \;/;\; \left(\left(F_m(z,b) == \sum_{k=0}^{n} \frac{z^k}{(b)_k\, k!} == {}_0F_1(;b;z) - \frac{z^{n+1}}{(n+1)!\,(b)_{n+1}}\, {}_1F_2(1; n+2, b+n+1; z)\right) \bigwedge n \in \mathbb{N}\right) $$

Summed form of the truncated series expansion.

Asymptotic series expansions

Expansions for $|\mathrm{Arg}(z)| < \pi$

07.17.06.0004.01

$$ {}_0F_1(;b;z) \propto \frac{\Gamma(b)}{2\sqrt{\pi}}\, z^{\frac{1-2b}{4}}\, e^{2\sqrt{z}}\, {}_2F_0\!\left(b-\frac{1}{2}, \frac{3}{2}-b;; \frac{1}{4\sqrt{z}}\right) /; |\arg(z)| < \pi \bigwedge (|z| \to \infty) $$

07.17.06.0005.01

$$ {}_0F_1(;b;z) \propto \frac{\Gamma(b)}{2\sqrt{\pi}}\, z^{\frac{1-2b}{4}}\, e^{2\sqrt{z}} \left(1 + O\!\left(\frac{1}{\sqrt{z}}\right)\right) /; |\arg(z)| < \pi \bigwedge (|z| \to \infty) $$

The general formulas

07.17.06.0006.01

$$ {}_0F_1(;b;z) \propto \Gamma(b)\, \mathcal{A}_{\tilde{F}}\!\left(\begin{matrix} ; \\ b; \end{matrix}\; \{z, \tilde{\infty}, \infty\}\right) /; (|z| \to \infty) $$

07.17.06.0007.01

$$ {}_0F_1(;b;z) \propto \Gamma(b)\, \mathcal{A}_{\tilde{F}}^{(\mathrm{trig})}\!\left(\begin{matrix} ; \\ b; \end{matrix}\; \{z, \tilde{\infty}, \infty\}\right) /; (|z| \to \infty) $$

Expansions for any z in exponential form

07.17.06.0008.01

$$ {}_0F_1(;b;z) \propto \frac{\Gamma(b)}{2\sqrt{\pi}} (-z)^{\frac{1-2b}{4}} \left(\exp\!\left(i\left(2\sqrt{-z} + \frac{1}{4}(1-2b)\pi\right)\right)\left(1 + \frac{i(2b-3)(2b-1)}{16\sqrt{-z}} + \frac{(2b-5)(2b-3)(2b-1)(2b+1)}{512z} + \ldots\right) + \exp\!\left(-i\left(2\sqrt{-z} + \frac{1}{4}(1-2b)\pi\right)\right)\left(1 - \frac{i(2b-3)(2b-1)}{16\sqrt{-z}} + \frac{(2b-5)(2b-3)(2b-1)(2b+1)}{512z} + \ldots\right)\right) /; (|z| \to \infty) $$

07.17.06.0009.01

$$ {}_0F_1(;b;z) \propto \frac{\Gamma(b)}{2\sqrt{\pi}} (-z)^{\frac{1-2b}{4}} \left(\exp\!\left(i\left(2\sqrt{-z} + \frac{1}{4}(1-2b)\pi\right)\right) {}_2F_0\!\left(b-\frac{1}{2}, \frac{3}{2}-b;; \frac{1}{4i\sqrt{-z}}\right) + \exp\!\left(-i\left(2\sqrt{-z} + \frac{1}{4}(1-2b)\pi\right)\right) {}_2F_0\!\left(b-\frac{1}{2}, \frac{3}{2}-b;; -\frac{1}{4i\sqrt{-z}}\right)\right) /; (|z| \to \infty) $$

07.17.06.0010.01

$$ {}_0F_1(;b;z) \propto \frac{\Gamma(b)}{2\sqrt{\pi}} (-z)^{\frac{1-2b}{4}} \left(\exp\!\left(i\left(2\sqrt{-z} + \frac{1}{4}(1-2b)\pi\right)\right)\left(1 + O\!\left(\frac{1}{\sqrt{z}}\right)\right) + \exp\!\left(-i\left(2\sqrt{-z} + \frac{1}{4}(1-2b)\pi\right)\right)\left(1 + O\!\left(\frac{1}{\sqrt{z}}\right)\right)\right) /; (|z| \to \infty) $$

07.17.06.0011.01

$$ {}_0F_1(;b;z)\propto\frac{\Gamma(b)}{2\sqrt{\pi}}\left(e^{-2i\sqrt{-z}}\left(-i\sqrt{-z}\right)^{\frac{1}{2}-b}{}_2F_0\!\left(b-\frac{1}{2},\frac{3}{2}-b;;-\frac{1}{4i\sqrt{-z}}\right)+e^{2i\sqrt{-z}}\left(i\sqrt{-z}\right)^{\frac{1}{2}-b}{}_2F_0\!\left(b-\frac{1}{2},\frac{3}{2}-b;;\frac{1}{4i\sqrt{-z}}\right)\right)/;(|z|\to\infty) $$

07.17.06.0012.01

$$ {}_0F_1(;b;z)\propto\frac{\Gamma(b)}{2\sqrt{\pi}}\left(e^{-2i\sqrt{-z}}\left(-i\sqrt{-z}\right)^{\frac{1}{2}-b}\left(1+O\!\left(\frac{1}{\sqrt{z}}\right)\right)+e^{2i\sqrt{-z}}\left(i\sqrt{-z}\right)^{\frac{1}{2}-b}\left(1+O\!\left(\frac{1}{\sqrt{z}}\right)\right)\right)/;(|z|\to\infty) $$

Expansions for any z in trigonometric form

07.17.06.0013.01

$$ {}_0F_1(;b;z)\propto\frac{\Gamma(b)(-z)^{\frac{1}{4}(1-2b)}}{\sqrt{\pi}}\left(\left(1+\frac{(3-2b)(5-2b)(-1+2b)(1+2b)}{512z}+\frac{1}{1572864z^2}((-9+2b)(-7+2b)(-5+2b)(-3+2b)(-1+2b)(1+2b)(3+2b)(5+2b))+\ldots\right)\cos\!\left(\frac{(2b-1)\pi}{4}-2\sqrt{-z}\right)+\frac{(2b-3)(2b-1)}{16\sqrt{-z}}\left(1+\frac{(5-2b)(7-2b)(1+2b)(3+2b)}{1536z}+\frac{1}{7864320z^2}((-11+2b)(-9+2b)(-7+2b)(-5+2b)(1+2b)(3+2b)(5+2b)(7+2b))+\ldots\right)\sin\!\left(\frac{(2b-1)\pi}{4}-2\sqrt{-z}\right)\right)/;(|z|\to\infty) $$

07.17.06.0024.01

$$ {}_0F_1(;b;z)\propto\frac{\Gamma(b)}{\sqrt{\pi}}(-z)^{\frac{1}{4}(1-2b)}\left(\cos\!\left(\frac{1}{4}(2b-1)\pi-2\sqrt{-z}\right)\left(\sum_{k=0}^{n}\frac{\left(\frac{1}{4}(3-2b)\right)_k\left(\frac{1}{4}(5-2b)\right)_k\left(\frac{1}{4}(2b-1)\right)_k\left(\frac{1}{4}(2b+1)\right)_k}{\left(\frac{1}{2}\right)_k}\left(\frac{1}{4z}\right)^k+O\!\left(\frac{1}{z^{n+1}}\right)\right)+\frac{(2b-1)(2b-3)}{16\sqrt{-z}}\sin\!\left(\frac{1}{4}(2b-1)\pi-2\sqrt{-z}\right)\left(\sum_{k=0}^{n}\frac{\left(\frac{1}{4}(5-2b)\right)_k\left(\frac{1}{4}(7-2b)\right)_k\left(\frac{1}{4}(2b+1)\right)_k\left(\frac{1}{4}(2b+3)\right)_k}{\left(\frac{3}{2}\right)_k}\left(\frac{1}{4z}\right)^k+O\!\left(\frac{1}{z^{n+1}}\right)\right)\right)/;(|z|\to\infty) $$

07.17.06.0014.01

$$ {}_0F_1(;b;z)\propto\frac{\Gamma(b)}{\sqrt{\pi}}(-z)^{\frac{1-2b}{4}}\left(\cos\!\left(\frac{2b-1}{4}\pi-2\sqrt{-z}\right){}_4F_1\!\left(\frac{3-2b}{4},\frac{5-2b}{4},\frac{2b-1}{4},\frac{2b+1}{4};\frac{1}{2};\frac{1}{4z}\right)+\frac{(2b-1)(2b-3)}{16\sqrt{-z}}\sin\!\left(\frac{2b-1}{4}\pi-2\sqrt{-z}\right){}_4F_1\!\left(\frac{5-2b}{4},\frac{7-2b}{4},\frac{2b+1}{4},\frac{2b+3}{4};\frac{3}{2};\frac{1}{4z}\right)\right)/;(|z|\to\infty) $$

07.17.06.0015.01

$$ {}_0F_1(;b;z) \propto \frac{\Gamma(b)}{\sqrt{\pi}}(-z)^{\frac{1-2b}{4}}\left(\cos\left(\frac{2b-1}{4}\pi-2\sqrt{-z}\right)\left(1+O\left(\frac{1}{z}\right)\right)+\frac{(2b-1)(2b-3)}{16\sqrt{-z}}\sin\left(\frac{2b-1}{4}\pi-2\sqrt{-z}\right)\left(1+O\left(\frac{1}{z}\right)\right)\right)\text{ /; } (|z|\to\infty) $$

Residue representations

07.17.06.0016.01

$$ {}_0F_1(;b;z) = \Gamma(b)\sum_{j=0}^{\infty}\operatorname{res}_s\left(\frac{(-z)^{-s}}{\Gamma(b-s)}\Gamma(s)\right)(-j) $$

Limit representations

07.17.09.0001.01

$$ {}_0F_1(;b;z) = \Gamma(b)\, z^{\frac{1-b}{2}} \lim_{\lambda\to\infty} \lambda^{b-1}\, P_{\lambda}^{1-b}\left(\cosh\left(\frac{2\sqrt{z}}{\lambda}\right)\right) $$

07.17.09.0002.01

$$ {}_0F_1(;b;z) = \Gamma(b)\lim_{n\to\infty}\frac{1}{n^{b-1}}L_n^{b-1}\left(-\frac{z}{n}\right) $$

07.17.09.0003.01

$$ {}_0F_1(;b;z) = \lim_{a\to\infty} {}_1F_1\left(a;b;\frac{z}{a}\right) $$

07.17.09.0004.01

$$ {}_0F_1(;b;z) = \lim_{n\to\infty}\lim_{m\to\infty} {}_2F_1\left(m,n;b;\frac{z}{m\,n}\right) $$

Continued fraction representations

Involving the function

07.17.10.0001.01

$$ {}_0F_1(;b;z) = 1+\cfrac{z/b}{1+\cfrac{-\frac{z}{2(1+b)}}{1+\frac{z}{2(1+b)}+\cfrac{-\frac{z}{3(2+b)}}{1+\frac{z}{3(2+b)}+\cfrac{-\frac{z}{4(3+b)}}{1+\frac{z}{4(3+b)}+\cfrac{-\frac{z}{5(4+b)}}{1+\frac{z}{5(4+b)}+\ldots}}}}} $$

07.17.10.0002.01

$$ {}_0F_1(;b;z) = 1+\frac{z}{b\left(1+\mathrm{K}_k\left(-\frac{z}{(k+1)(b+k)},\frac{z}{(k+1)(b+k)}+1\right)_1^{\infty}\right)} $$

Differential equations

Ordinary linear differential equations and wronskians

For the direct function itself

07.17.13.0005.01

$$z\,w''(z)+b\,w'(z)-w(z)=0\;/;\;w(z)=c_1\,{}_0\tilde{F}_1(;b;z)\;+c_2\,z^{\frac{1-b}{2}}\,K_{1-b}\left(2\sqrt{z}\right)$$

07.17.13.0006.01

$$W_z\left({}_0\tilde{F}_1(;b;z),\,z^{\frac{1-b}{2}}\,K_{1-b}\left(2\sqrt{z}\right)\right)=-\frac{z^{-b}}{2}$$

07.17.13.0001.01

$$z\,w''(z)+b\,w'(z)-w(z)=0\;/;\;w(z)=c_1\,{}_0F_1(;b;z)\;+c_2\,z^{1-b}\,{}_0F_1(;2-b;z)\bigwedge b\notin\mathbb{Z}$$

07.17.13.0002.02

$$W_z\left({}_0F_1(;b;z),\,z^{1-b}\,{}_0F_1(;2-b;z)\right)=(1-b)\,z^{-b}$$

07.17.13.0003.01

$$z\,w''(z)+b\,w'(z)-w(z)=0\;/;\;w(z)=c_1\,{}_0F_1(;b;z)\;+c_2\,z^{\frac{1-b}{2}}\,K_{1-b}\left(2\sqrt{z}\right)\bigwedge -b\notin\mathbb{N}$$

07.17.13.0004.02

$$W_z\left({}_0F_1(;b;z),\,z^{\frac{1-b}{2}}\,K_{1-b}\left(2\sqrt{z}\right)\right)=-\frac{\Gamma(b)}{2}\,z^{-b}$$

07.17.13.0007.01

$$w''(z)+\left(\frac{b\,g'(z)}{g(z)}-\frac{g''(z)}{g'(z)}\right)w'(z)-\frac{g'(z)^2}{g(z)}\,w(z)=0\;/;\;w(z)=c_1\,{}_0F_1(;b;g(z))\;+c_2\,g(z)^{1-b}\,{}_0F_1(;2-b;g(z))\bigwedge b\notin\mathbb{Z}$$

07.17.13.0008.01

$$W_z\left({}_0F_1(;b;g(z)),\,g(z)^{1-b}\,{}_0F_1(;2-b;g(z))\right)=(1-b)\,g'(z)\,g(z)^{-b}$$

07.17.13.0009.01

$$h(z)^2\,w''(z)\;+h(z)^2\left(\frac{b\,g'(z)}{g(z)}-\frac{2\,h'(z)}{h(z)}-\frac{g''(z)}{g'(z)}\right)w'(z)\;+$$
$$\left(2\,h'(z)^2+\frac{h(z)\,g''(z)\,h'(z)}{g'(z)}-h(z)\,h''(z)-\frac{h(z)\,g'(z)\,(h(z)\,g'(z)+b\,h'(z))}{g(z)}\right)w(z)\;=0\;/;$$
$$w(z)=c_1\,h(z)\,{}_0F_1(;b;g(z))\;+c_2\,h(z)\,g(z)^{1-b}\,{}_0F_1(;2-b;g(z))\bigwedge b\notin\mathbb{Z}$$

07.17.13.0010.01

$$W_z\left(h(z)\,{}_0F_1(;b;g(z)),\,h(z)\,g(z)^{1-b}\,{}_0F_1(;2-b;g(z))\right)=(1-b)\,h(z)^2\,g'(z)\,g(z)^{-b}$$

07.17.13.0011.01

$$z^2\,w''(z)\;+((b-1)\,r-2\,s+1)\,z\,w'(z)+\left(s\,(-b\,r+r+s)-a\,r^2\,z^r\right)w(z)=0\;/;$$
$$w(z)=c_1\,z^s\,{}_0F_1(;b;a\,z^r)\;+c_2\,z^{-b\,r+r+s}\,{}_0F_1(;2-b;a\,z^r)\bigwedge b\notin\mathbb{Z}$$

07.17.13.0012.01

$$W_z\left(z^s\,{}_0F_1(;b;a\,z^r),\,z^{-b\,r+r+s}\,{}_0F_1(;2-b;a\,z^r)\right)=(1-b)\,r\,z^{-b\,r+r+2\,s-1}$$

07.17.13.0013.01

$$z^2\,w''(z)\;+((b-1)\,r-2\,s+1)\,z\,w'(z)+\left(s\,(-b\,r+r+s)-a\,r^2\,z^r\right)w(z)=0\;/;$$
$$w(z)=c_1\,z^s\,{}_0F_1(;b;a\,z^r)\;+c_2\,z^{r+s}\,(a\,z^r)^{-b}\,{}_0F_1(;2-b;a\,z^r)\bigwedge b\notin\mathbb{Z}$$

07.17.13.0014.01

$$W_z\left(z^s\,{}_0F_1(;b;a\,z^r),\,a^b\,z^{r+s}\,(a\,z^r)^{-b}\,{}_0F_1(;2-b;a\,z^r)\right)=a^b\,(1-b)\,r\,z^{r+2\,s-1}\,(a\,z^r)^{-b}$$

07.17.13.0015.01

$$w''(z)+((b-1)\log(r)-2\log(s))\,w'(z)+\left(-a\log^2(r)\,r^z+\log^2(s)-(b-1)\log(r)\log(s)\right)w(z)=0\;/;\; w(z)=c_1\,s^z\,{}_0F_1(;b;a\,r^z)+c_2\,s^z\,r^{(1-b)\,z}\,{}_0F_1(;2-b;a\,r^z)\bigwedge b\notin\mathbb{Z}$$

07.17.13.0016.01

$$W_z\left(s^z\,{}_0F_1(;b;a\,r^z),\,s^z\,r^{(1-b)\,z}\,{}_0F_1(;2-b;a\,r^z)\right)=(1-b)\,r^{z-b\,z}\,s^{2\,z}\log(r)$$

07.17.13.0017.01

$$w''(z)+\left(\frac{b\,g'(z)}{g(z)}-\frac{g''(z)}{g'(z)}\right)w'(z)-\frac{g'(z)^2\,w(z)}{g(z)}=0\;/;\;w(z)=c_1\,{}_0\tilde{F}_1(;b;g(z))+c_2\,g(z)^{\frac{1-b}{2}}\,K_{1-b}\left(2\sqrt{g(z)}\right)$$

07.17.13.0018.01

$$W_z\left({}_0\tilde{F}_1(;b;g(z)),\,g(z)^{\frac{1-b}{2}}\,K_{1-b}\left(2\sqrt{g(z)}\right)\right)=-\frac{1}{2}\,g'(z)\,g(z)^{-b}$$

07.17.13.0019.01

$$h(z)^2\,w''(z)+h(z)^2\left(\frac{b\,g'(z)}{g(z)}-\frac{2\,h'(z)}{h(z)}-\frac{g''(z)}{g'(z)}\right)w'(z)+\left(2\,h'(z)^2+\frac{h(z)\,g''(z)\,h'(z)}{g'(z)}-h(z)\,h''(z)-\frac{h(z)\,g'(z)\,(h(z)\,g'(z)+b\,h'(z))}{g(z)}\right)w(z)=0\;/;\; w(z)=c_1\,h(z)\,{}_0\tilde{F}_1(;b;g(z))+c_2\,h(z)\,g(z)^{\frac{1-b}{2}}\,K_{1-b}\left(2\sqrt{g(z)}\right)$$

07.17.13.0020.01

$$W_z\left(h(z)\,{}_0\tilde{F}_1(;b;g(z)),\,h(z)\,g(z)^{\frac{1-b}{2}}\,K_{1-b}\left(2\sqrt{g(z)}\right)\right)=-\frac{1}{2}\,h(z)^2\,g'(z)\,g(z)^{-b}$$

07.17.13.0021.01

$$z^2\,w''(z)+((b-1)\,r-2\,s+1)\,z\,w'(z)+\left(s\,(-b\,r+r+s)-a\,r^2\,z^r\right)w(z)=0\;/;\; w(z)=c_1\,z^s\,{}_0\tilde{F}_1(;b;a\,z^r)+c_2\,z^s\,(a\,z^r)^{\frac{1-b}{2}}\,K_{1-b}\left(2\sqrt{a\,z^r}\right)$$

07.17.13.0022.01

$$W_z\left(z^s\,{}_0\tilde{F}_1(;b;a\,z^r),\,z^s\,(a\,z^r)^{\frac{1-b}{2}}\,K_{1-b}\left(2\sqrt{a\,z^r}\right)\right)=-\frac{1}{2}\,a\,r\,z^{r+2\,s-1}\,(a\,z^r)^{-b}$$

07.17.13.0023.01

$$w''(z)+((b-1)\log(r)-2\log(s))\,w'(z)+\left(-a\log^2(r)\,r^z+\log^2(s)-(b-1)\log(r)\log(s)\right)w(z)=0\;/;\; w(z)=c_1\,s^z\,{}_0\tilde{F}_1(;b;a\,r^z)+c_2\,s^z\,(a\,r^z)^{\frac{1-b}{2}}\,K_{1-b}\left(2\sqrt{a\,r^z}\right)$$

07.17.13.0024.01

$$W_z\left(s^z\,{}_0\tilde{F}_1(;b;a\,r^z),\,s^z\,(a\,r^z)^{\frac{1-b}{2}}\,K_{1-b}\left(2\sqrt{a\,r^z}\right)\right)=-\frac{1}{2}\,a\,r^z\,(a\,r^z)^{-b}\,s^{2\,z}\log(r)$$

Transformations

Products, sums, and powers of the direct function

Products of the direct function

07.17.16.0001.01

$$ {}_0F_1(;b;z)\,{}_0F_1(;c;z) == {}_2F_3\left(\frac{b+c}{2}, \frac{b+c-1}{2}; b, c, b+c-1; 4z\right) $$

07.17.16.0002.01

$$ {}_0F_1(;b;c\,z)\,{}_0F_1(;\beta;d\,z) == \sum_{k=0}^{\infty} c_k\,z^k \;/;\; c_k == \frac{d^k}{k!\,(\beta)_k}\,{}_2F_1\left(-k, 1-k-\beta; b; \frac{c}{d}\right) \bigvee c_k == \frac{c^k}{k!\,(b)_k}\,{}_2F_1\left(-k, 1-b-k; \beta; \frac{d}{c}\right) $$

07.17.16.0003.01

$$ {}_0F_1(;b;c\,z)\,{}_0F_1(;\beta;d\,z) == \sum_{k=0}^{\infty}\sum_{m=0}^{k} \frac{c^m\,d^{k-m}\,z^k}{(b)_m\,m!\,(\beta)_{k-m}\,(k-m)!} $$

07.17.16.0004.01

$$ {}_0F_1(;b;c\,z)\,{}_0F_1(;\beta;d\,z) == F_{0:1;1}^{0:0;0}\left(\begin{matrix} :;; \\ :b;\beta; \end{matrix}\; c\,z, d\,z\right) $$

Related transformations

07.17.16.0005.01

$$ {}_0F_1(;b;z) == \frac{2}{\Gamma(1-b)}\,z^{\frac{1-b}{2}}\,K_{b-1}\left(2\sqrt{z}\right) - \frac{\Gamma(b-1)}{\Gamma(1-b)}\,z^{1-b}\,{}_0F_1(;2-b;z) \;/;\; b \notin \mathbb{Z} $$

Identities

Recurrence identities

Consecutive neighbors

07.17.17.0001.01

$$ {}_0F_1(;b;z) == {}_0F_1(;b+1;z) + \frac{z}{b\,(b+1)}\,{}_0F_1(;b+2;z) $$

07.17.17.0002.01

$$ {}_0F_1(;b;z) == \frac{(b-2)\,(b-1)}{z}\left({}_0F_1(;b-2;z) - {}_0F_1(;b-1;z)\right) $$

Distant neighbors

Increasing

07.17.17.0003.01

$$ {}_0F_1(;b;z) == z\sum_{k=0}^{n-1} \frac{(n-k-1)!\,(-z)^k}{k!\,(b)_k\,(-b-n)_{k+2}\,(n-2k-1)!}\,{}_0F_1(;b+n+1;z) + \sum_{k=0}^{n-1} \frac{(n-k)!\,(-z)^k}{k!\,(b)_k\,(1-b-n)_k\,(n-2k)!}\,{}_0F_1(;b+n;z) \;/;\; n \in \mathbb{N}^+ $$

07.17.17.0010.01

$$ {}_0F_1(;b;z) == C_n(b,z)\,{}_0F_1(;b+n;z) + \frac{z}{(b+n-1)\,(b+n)}\,C_{n-1}(b,z)\,{}_0F_1(;b+n+1;z) \;/;\; C_0(b,z) == 1 \bigwedge C_1(b,z) == 1 \bigwedge C_2(b,z) == \frac{z}{b\,(b+1)} + 1 \bigwedge C_n(b,z) == C_{n-1}(b,z) + \frac{z}{(b+n-2)\,(b+n-1)}\,C_{n-2}(b,z) \bigwedge n \in \mathbb{N}^+ $$

07.17.17.0011.01

$$ {}_0F_1(;b;z) = C_n(b,z)\, {}_0F_1(;b+n;z) + \frac{z}{(b+n-1)(b+n)} C_{n-1}(b,z)\, {}_0F_1(;b+n+1;z) \ /; $$
$$ C_n(\nu,z) = {}_2F_3\left(\frac{1-n}{2}, -\frac{n}{2}; b, -n, -b-n+1; 4z\right) \bigwedge n \in \mathbb{N}^+ $$

Decreasing

07.17.17.0004.01

$$ {}_0F_1(;b;z) = (-1)^{n-1}(1-b)_n (2-b)_n $$
$$ z^{-n}\left(\sum_{k=0}^{n-1} \frac{(n-k-1)!\,(-z)^k}{k!\,(n-2k-1)!\,(2-b)_k (b-n)_k}\, {}_0F_1(;b-n-1;z) - \sum_{k=0}^{n-1} \frac{(n-k)!\,(-z)^k}{k!\,(n-2k)!\,(2-b)_k (b-n-1)_k}\, {}_0F_1(;b-n;z)\right) /; n \in \mathbb{N}^+ $$

07.17.17.0012.01

$$ {}_0F_1(;b;z) = C_n(b,z)\, {}_0F_1(;b-n;z) + \frac{(b-n-1)(b-n)}{z} C_{n-1}(b,z)\, {}_0F_1(;b-n-1;z) \ /; C_0(b,z) = 1 \bigwedge $$
$$ C_1(b,z) = -\frac{(b-2)(b-1)}{z} \bigwedge C_n(b,z) = -\frac{(b-n-1)(b-n)}{z} C_{n-1}(b,z) + \frac{(b-n)(b-n+1)}{z} C_{n-2}(b,z) \bigwedge n \in \mathbb{N}^+ $$

07.17.17.0013.01

$$ {}_0F_1(;b;z) = C_n(b,z)\, {}_0F_1(;b-n;z) + \frac{(b-n-1)(b-n)}{z} C_{n-1}(b,z)\, {}_0F_1(;b-n-1;z) \ /; $$
$$ C_n(\nu,z) = z^{-n}(-1)^n (1-b)_n (2-b)_n\, {}_2F_3\left(\frac{1-n}{2}, -\frac{n}{2}; 2-b, -n, b-n-1; 4z\right) \bigwedge n \in \mathbb{N}^+ $$

Functional identities

Relations between contiguous functions

07.17.17.0005.01

$$ (1-b)\, b\, {}_0F_1(;b-1;z) + z\, {}_0F_1(;b+1;z) + (b-1)\, b\, {}_0F_1(;b;z) = 0 $$

Relations of special kind

07.17.17.0006.01

$$ z\, {}_0F_1(;b+1;z)\, {}_0F_1(;2-b;z) + (b-1)\, b\, {}_0F_1(;1-b;z)\, {}_0F_1(;b;z) = b(b-1) $$

Division on even and odd parts and generalization

07.17.17.0007.01

$$ {}_0F_1(;b;z) = A^+(z) + A^-(z) \ /; A^+(z) = \frac{1}{2}\left({}_0F_1(;b;z) + {}_0F_1(;b;-z)\right) \bigwedge A^-(z) = \frac{1}{2}\left({}_0F_1(;b;z) - {}_0F_1(;b;-z)\right) $$

07.17.17.0008.01

$$ {}_0F_1(;b;z) = A^+(z) + A^-(z) \ /; A^+(z) = {}_0F_3\left(; \frac{1}{2}, \frac{b}{2}, \frac{b+1}{2}; \frac{z^2}{16}\right) \bigwedge A^-(z) = \frac{z}{b}\, {}_0F_3\left(; \frac{3}{2}, \frac{b+1}{2}, \frac{b+2}{2}; \frac{z^2}{16}\right) $$

07.17.17.0009.01

$$ {}_0F_1(;b;z) = \sum_{k=0}^{n-1} \frac{z^k}{(b)_k\, k!}\, {}_1F_{2n}\left(1; \frac{k+1}{n}, \ldots, \frac{k+n}{n}, \frac{b+k}{n}, \ldots, \frac{b+k+n-1}{n}; n^{-2n} z^n\right) $$

Differentiation

Low-order differentiation

With respect to *b*

07.17.20.0001.01

$$ {}_0F_1^{(1,0)}(;b;z) \equiv \psi(b)\,{}_0F_1(;b;z) - \sum_{k=0}^{\infty} \frac{\psi(b+k)\,z^k}{k!\,(b)_k} $$

07.17.20.0002.01

$$ {}_0F_1^{(1,0)}(;b;z) \equiv -\frac{z}{b^2}\,F_{2\times0\times1}^{0\times1\times2}\left(\begin{matrix} ;1;1,b; \\ 2,b+1;;b+1; \end{matrix} z, z\right) $$

07.17.20.0003.01

$$ {}_0F_1^{(1,0)}(;n;z) \equiv (-1)^n (n-1)!\left(K_{n-1}\left(2\sqrt{z}\right) - \frac{(n-1)!}{2}\,z^{-\frac{n-1}{2}} \sum_{k=0}^{n-2} \frac{(-1)^k I_k\left(2\sqrt{z}\right) z^{k/2}}{(n-k-1)\,k!}\right) z^{-\frac{n-1}{2}} + \left(\psi(n) - \frac{\log(z)}{2}\right) {}_0F_1(;n;z) \;/;\; n \in \mathbb{N}^+ $$

07.17.20.0012.01

$$ {}_0F_1^{(1,0)}(;n;z) = (n-1)!\,z^{\frac{1-n}{2}}\left((-1)^n K_{n-1}\left(2\sqrt{z}\right) + I_{n-1}\left(2\sqrt{z}\right)\left(\psi(n) - \frac{\log(z)}{2}\right) - \frac{(-1)^n (n-1)!}{2}\,z^{\frac{1-n}{2}} \sum_{k=0}^{n-2} \frac{(-1)^k I_k\left(2\sqrt{z}\right) z^{k/2}}{(-k+n-1)\,k!}\right) /;\; n \in \mathbb{N}^+ $$

Brychkov Yu.A. (2007)

07.17.20.0013.01

$$ {}_0F_1^{(1,0)}(;n;-z) = (n-1)!\,z^{\frac{1-n}{2}}\left(\frac{\pi}{2}\,Y_{n-1}\left(2\sqrt{z}\right) + J_{n-1}\left(2\sqrt{z}\right)\left(\psi(n) - \frac{\log(z)}{2}\right) + \frac{(n-1)!}{2}\,z^{\frac{1-n}{2}} \sum_{k=0}^{n-2} \frac{J_k\left(2\sqrt{z}\right) z^{k/2}}{(-k+n-1)\,k!}\right) /;\; n \in \mathbb{N}^+ $$

Brychkov Yu.A. (2007)

07.17.20.0014.01

$$
{}_0F_1^{(1,0)}\left(;n+\frac{1}{2};z\right)=
\delta_n\left(\cosh\left(2\sqrt{z}\right)\text{Chi}\left(4\sqrt{z}\right)-\sinh\left(2\sqrt{z}\right)\text{Shi}\left(4\sqrt{z}\right)\right)+I_{n-\frac{1}{2}}\left(2\sqrt{z}\right)\Gamma\left(n+\frac{1}{2}\right)\left(\psi\left(n+\frac{1}{2}\right)-\frac{\log(z)}{2}\right)z^{\frac{1-2n}{4}}-\frac{(-1)^n\,2^{2-2n}}{(n-1)!}\left(\frac{1}{2}\right)_n
z^{\frac{1}{2}-n}\left(4\sqrt{z}\left(\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor-1}\binom{n-1}{2k+1}(-2k+2n-3)!\left(-\cosh\left(2\sqrt{z}\right)\text{Chi}\left(4\sqrt{z}\right)+\cosh\left(2\sqrt{z}\right)\left(\psi\left(k+\frac{3}{2}\right)-\psi\left(k-n+\frac{3}{2}\right)\right)+\sinh\left(2\sqrt{z}\right)\text{Shi}\left(4\sqrt{z}\right)\right)(16z)^k\right)+\sum_{k=0}^{\left\lfloor\frac{n-1}{2}\right\rfloor}\binom{n-1}{2k}(2n-2k-2)!\left(\text{Chi}\left(4\sqrt{z}\right)\sinh\left(2\sqrt{z}\right)+\left(\psi\left(k-n+\frac{3}{2}\right)-\psi\left(k+\frac{1}{2}\right)\right)\sinh\left(2\sqrt{z}\right)-\cosh\left(2\sqrt{z}\right)\text{Shi}\left(4\sqrt{z}\right)\right)(16z)^k\right)\text{ /; } n\in\mathbb{N}
$$

Brychkov Yu.A. (2007)

07.17.20.0015.01

$$
{}_0F_1^{(1,0)}\left(;n+\frac{1}{2};-z\right)=
\delta_n\left(\cos\left(2\sqrt{z}\right)\text{Ci}\left(4\sqrt{z}\right)+\sin\left(2\sqrt{z}\right)\text{Si}\left(4\sqrt{z}\right)\right)+J_{n-\frac{1}{2}}\left(2\sqrt{z}\right)\Gamma\left(n+\frac{1}{2}\right)\left(\psi\left(n+\frac{1}{2}\right)-\frac{\log(z)}{2}\right)z^{\frac{1-2n}{4}}-\frac{2^{2-2n}\left(\frac{1}{2}\right)_n}{(n-1)!}z^{\frac{1}{2}-n}
\left(4\sqrt{z}\left(\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor-1}\binom{n-1}{2k+1}(2n-2k-3)!\left(\cos\left(2\sqrt{z}\right)\left(\text{Ci}\left(4\sqrt{z}\right)-\psi\left(k+\frac{3}{2}\right)+\psi\left(k-n+\frac{3}{2}\right)\right)+\sin\left(2\sqrt{z}\right)\text{Si}\left(4\sqrt{z}\right)\right)(-16z)^k\right)+\sum_{k=0}^{\left\lfloor\frac{n-1}{2}\right\rfloor}\binom{n-1}{2k}(2n-2k-2)!\left(\left(-\text{Ci}\left(4\sqrt{z}\right)+\psi\left(k+\frac{1}{2}\right)-\psi\left(k-n+\frac{3}{2}\right)\right)\sin\left(2\sqrt{z}\right)+\cos\left(2\sqrt{z}\right)\text{Si}\left(4\sqrt{z}\right)\right)(-16z)^k\right)\text{ /; } n\in\mathbb{N}
$$

Brychkov Yu.A. (2007)

07.17.20.0016.01

$$ {}_0F_1^{(1,0)}\left(;\frac{1}{2}-n;z\right)=I_{-n-\frac{1}{2}}\left(2\sqrt{z}\right)\Gamma\left(\frac{1}{2}-n\right)\left(\psi\left(\frac{1}{2}-n\right)-\frac{\log(z)}{2}\right)z^{\frac{1+2n}{4}}+ \frac{1}{(2n)!}\left(4\sqrt{z}\sum_{k=0}^{\left\lfloor\frac{n-1}{2}\right\rfloor}\binom{n}{2k+1}(2n-2k-1)!\left(\left(-\mathrm{Chi}\left(4\sqrt{z}\right)+\psi\left(k-n+\frac{1}{2}\right)-\psi\left(k+\frac{3}{2}\right)\right)\sinh\left(2\sqrt{z}\right)+\cosh\left(2\sqrt{z}\right)\mathrm{Shi}\left(4\sqrt{z}\right)\right)(16z)^k+\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\binom{n}{2k}(2n-2k)!\left(\cosh\left(2\sqrt{z}\right)\mathrm{Chi}\left(4\sqrt{z}\right)+\cosh\left(2\sqrt{z}\right)\left(\psi\left(k+\frac{1}{2}\right)-\psi\left(k-n+\frac{1}{2}\right)\right)-\sinh\left(2\sqrt{z}\right)\mathrm{Shi}\left(4\sqrt{z}\right)\right)(16z)^k\right)\;/;\;n\in\mathbb{N} $$

Brychkov Yu.A. (2007)

07.17.20.0017.01

$$ {}_0F_1^{(1,0)}\left(;\frac{1}{2}-n;-z\right)=J_{-n-\frac{1}{2}}\left(2\sqrt{z}\right)\Gamma\left(\frac{1}{2}-n\right)\left(\psi\left(\frac{1}{2}-n\right)-\frac{\log(z)}{2}\right)z^{\frac{2n+1}{4}}+ \frac{1}{(2n)!}\left(\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\binom{n}{2k}(2n-2k)!\left(\cos\left(2\sqrt{z}\right)\left(\mathrm{Ci}\left(4\sqrt{z}\right)+\psi\left(k+\frac{1}{2}\right)-\psi\left(k-n+\frac{1}{2}\right)\right)+\sin\left(2\sqrt{z}\right)\mathrm{Si}\left(4\sqrt{z}\right)\right)(-16z)^k-4\sqrt{z}\sum_{k=0}^{\left\lfloor\frac{n-1}{2}\right\rfloor}\binom{n}{2k+1}(2n-2k-1)!\left(\left(-\mathrm{Ci}\left(4\sqrt{z}\right)+\psi\left(k-n+\frac{1}{2}\right)-\psi\left(k+\frac{3}{2}\right)\right)\sin\left(2\sqrt{z}\right)+\cos\left(2\sqrt{z}\right)\mathrm{Si}\left(4\sqrt{z}\right)\right)(-16z)^k\right)\;/;\;n\in\mathbb{N} $$

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With respect to *z*

07.17.20.0004.01

$$ \frac{\partial\,{}_0F_1(;b;z)}{\partial z}=\frac{1}{b}\,{}_0F_1(;b+1;z) $$

07.17.20.0005.01

$$ \frac{\partial^2\,{}_0F_1(;b;z)}{\partial z^2}=\frac{1}{b\,(b+1)}\,{}_0F_1(;b+2;z) $$

Symbolic differentiation

With respect to *b*

07.17.20.0006.02

$$ {}_0F_1^{(n,0)}(;b;z)=\sum_{k=0}^{\infty}\frac{1}{k!}\,\frac{\partial^n\frac{1}{(b)_k}}{\partial b^n}\,z^k\;/;\;n\in\mathbb{N} $$

With respect to z

07.17.20.0018.01

$$\frac{\partial^n {}_0F_1(;b;z)}{\partial z^n} = z^{-n}(-1)^{n-1}(b-1)_n \left(\sum_{k=0}^{n-1} \frac{(n-k-1)!\,(-z)^k}{k!\,(n-2k-1)!\,(2-b-n)_k\,(b)_k}\,{}_0F_1(;b-1;z) - \sum_{k=0}^{n} \frac{(n-k)!\,(-z)^k}{k!\,(n-2k)!\,(2-b-n)_k\,(b-1)_k}\,{}_0F_1(;b;z)\right) /; n \in \mathbb{N}$$

07.17.20.0007.02

$$\frac{\partial^n {}_0F_1(;b;z)}{\partial z^n} = \frac{1}{(b)_n}\,{}_0F_1(;b+n;z) /; n \in \mathbb{N}$$

07.17.20.0008.02

$$\frac{\partial^n {}_0F_1(;b;z)}{\partial z^n} = z^{-n}\,\Gamma(b)\,{}_1\tilde{F}_2(1;b,1-n;z) /; n \in \mathbb{N}$$

07.17.20.0009.02

$$\frac{\partial^n (z^\alpha\,{}_0F_1(;b;z))}{\partial z^n} = (-1)^n(-\alpha)_n\,z^{\alpha-n}\,{}_1F_2(\alpha+1;1-n+\alpha,b;z) /; n \in \mathbb{N}$$

07.17.20.0010.02

$$\frac{\partial^n \left(z^{b-1}\,{}_0F_1(;b;z)\right)}{\partial z^n} = (b-n)_n\,z^{b-n-1}\,{}_0F_1(;b-n;z) /; n \in \mathbb{N}$$

Fractional integro-differentiation

With respect to z

07.17.20.0011.01

$$\frac{\partial^\alpha {}_0F_1(;b;z)}{\partial z^\alpha} = z^{-\alpha}\,\Gamma(b)\,{}_1\tilde{F}_2(1;b,1-\alpha;z)$$

Integration

Indefinite integration

Involving only one direct function

07.17.21.0001.01

$$\int {}_0F_1(;b;z)\,dz = (b-1)\,{}_0F_1(;b-1;z)$$

Involving one direct function and elementary functions

Involving power function

07.17.21.0002.01

$$\int z^{\alpha-1}\,{}_0F_1(;b;a\,z)\,dz = z^\alpha\,\Gamma(b)\,\Gamma(\alpha)\,{}_1\tilde{F}_2(\alpha;b,\alpha+1;a\,z)$$

07.17.21.0003.01

$$\int z^{\alpha-1}\,{}_0F_1(;b;z)\,dz = z^\alpha\,\Gamma(b)\,\Gamma(\alpha)\,{}_1\tilde{F}_2(\alpha;b,\alpha+1;z)$$

Involving functions of the direct function and elementary functions

Involving elementary functions of the direct function and elementary functions

Involving powers of the direct function and a power function

07.17.21.0004.01

$$\int z^{b-\frac{3}{2}}\, {}_0F_1(;b;z)^2\, dz = \frac{2\, z^{b-\frac{1}{2}}}{2\,b-1}\, {}_2F_3\left(b-\frac{1}{2}, b-\frac{1}{2}; b, 2\,b-1, b+\frac{1}{2}; 4\,z\right)$$

Involving products of the direct function and a power function

07.17.21.0005.01

$$\int z^{\frac{b+c-3}{2}}\, {}_0F_1(;b;z)\, {}_0F_1(;c;z)\, dz = \frac{2}{b+c-1}\, z^{\frac{b+c-1}{2}}\, {}_3F_4\left(\frac{b+c-1}{2}, \frac{b+c-1}{2}, \frac{b+c}{2}; b, c, b+c-1, \frac{b+c+1}{2}; 4\,z\right)$$

Definite integration

For the direct function itself

07.17.21.0006.01

$$\int_0^1 t^{\alpha-1}\, {}_0F_1(;b;t)\, dt = \frac{1}{\alpha}\, {}_1F_2(\alpha; b, \alpha+1; 1) \;/;\; \mathrm{Re}(\alpha) > 0$$

07.17.21.0007.01

$$\int_0^\infty t^{\alpha-1}\, {}_0F_1(;b;-t)\, dt = \frac{\Gamma(b)\,\Gamma(\alpha)}{\Gamma(b-\alpha)} \;/;\; 0 < \mathrm{Re}(\alpha) < \frac{2\,\mathrm{Re}(b)+1}{4}$$

Involving the direct function

07.17.21.0008.01

$$\int_0^\infty t^{\alpha-1}\, e^{-a\,t}\, {}_0F_1(;b;t)\, dt = a^{-\alpha}\, \Gamma(\alpha)\, {}_1F_1\left(\alpha; b; \frac{1}{a}\right) \;/;\; \mathrm{Re}(a) > 0 \land \mathrm{Re}(\alpha) > 0$$

Integral transforms

Laplace transforms

07.17.22.0001.01

$$\mathcal{L}_t[{}_0F_1(;b;t)](z) = (b-1)\, e^{1/z}\, z^{b-2}\left(\Gamma(b-1) - \Gamma\left(b-1, \frac{1}{z}\right)\right) \;/;\; \mathrm{Re}(z) > 0$$

Operations

Limit operation

07.17.25.0001.01

$$\lim_{b\to\infty} {}_0F_1(;b;b\,z) = {}_0F_0(;;z)$$

07.17.25.0002.01

$$\lim_{b\to -n}\frac{{}_0F_1(;b;z)}{\Gamma(b)} == z^{n+1}\,{}_0\tilde{F}_1(;n+2;z)\,/;\,n\in\mathbb{N}$$

Representations through more general functions

Through hypergeometric functions

Involving ${}_p\tilde{F}_q$

07.17.26.0001.01

$${}_0F_1(;b;z) == \Gamma(b)\,{}_0\tilde{F}_1(;b;z)$$

Involving ${}_pF_q$

07.17.26.0002.01

$${}_0F_1(;b;z) == {}_pF_q(a_1, a_2, \ldots, a_p; b_1, b_2, \ldots, b_q; z)\,/;\,p == 0 \land q == 1 \land b_1 == b$$

07.17.26.0003.01

$${}_0F_1(;b;z) == {}_1F_2(a_1; b, a_1; z)$$

07.17.26.0004.01

$${}_0F_1(;b;z)\,{}_0F_1(;c;z) == {}_2F_3\left(\frac{b+c}{2}, \frac{b+c-1}{2}; b, c, b+c-1; 4z\right)$$

Involving ${}_1\tilde{F}_1$

07.17.26.0005.01

$${}_0F_1(;b;z) == \Gamma(2b-1)\,e^{-2\sqrt{z}}\,{}_1\tilde{F}_1\left(b-\frac{1}{2}; 2b-1; 4\sqrt{z}\right)$$

Involving ${}_1F_1$

07.17.26.0006.01

$${}_0F_1(;b;z) == e^{-2\sqrt{z}}\,{}_1F_1\left(b-\frac{1}{2}; 2b-1; 4\sqrt{z}\right)$$

Through Meijer *G*

Classical cases for the direct function itself

07.17.26.0007.01

$${}_0F_1(;b;z) == \Gamma(b)\,G_{0,2}^{1,0}(-z \mid 0, 1-b)$$

07.17.26.0008.01

$${}_0F_1(;b;z) == \pi\,\Gamma(b)\,G_{1,3}^{1,0}\left(z \,\middle|\, \begin{matrix} \frac{1}{2} \\ 0, 1-b, \frac{1}{2} \end{matrix}\right)$$

Classical cases involving exp

07.17.26.0009.01

$$e^{-2\sqrt{z}}\,{}_0F_1(;b;z) == \frac{4^{b-1}\,\Gamma(b)}{\sqrt{\pi}}\,G_{1,2}^{1,1}\left(4\sqrt{z} \,\middle|\, \begin{matrix} \frac{3}{2}-b \\ 0, 2-2b \end{matrix}\right)$$

07.17.26.0010.01

$$e^{2\sqrt{z}}\,{}_0F_1(;b;z)=4^{b-1}\sqrt{\pi}\,\csc(b\pi)\,\Gamma(b)\,G_{2,3}^{1,1}\!\left(4\sqrt{z}\,\middle|\,\begin{matrix}\frac{3}{2}-b,\,1-b\\0,\,2-2b,\,1-b\end{matrix}\right)$$

07.17.26.0011.01

$$e^{-z}\,{}_0F_1\!\left(;b;\frac{z^2}{4}\right)=\frac{4^{b-1}\,\Gamma(b)}{\sqrt{\pi}}\,G_{1,2}^{1,1}\!\left(2z\,\middle|\,\begin{matrix}\frac{3}{2}-b\\0,\,2-2b\end{matrix}\right)$$

07.17.26.0012.01

$$e^{z}\,{}_0F_1\!\left(;b;\frac{z^2}{4}\right)=4^{b-1}\sqrt{\pi}\,\csc(b\pi)\,\Gamma(b)\,G_{2,3}^{1,1}\!\left(2z\,\middle|\,\begin{matrix}\frac{3}{2}-b,\,1-b\\0,\,2-2b,\,1-b\end{matrix}\right)$$

Classical cases involving cos

07.17.26.0013.01

$$\cos\left(2\sqrt{z}\right){}_0F_1(;b;-z)=2^{b-\frac{3}{2}}\,\Gamma(b)\,G_{2,4}^{1,2}\!\left(4z\,\middle|\,\begin{matrix}\frac{3-2b}{4},\,\frac{5-2b}{4}\\0,\,\frac{1}{2},\,1-b,\,\frac{3}{2}-b\end{matrix}\right)$$

07.17.26.0015.01

$$\cos\left(a+2\sqrt{z}\right){}_0F_1(;b;-z)=2^{b-\frac{3}{2}}\,\Gamma(b)\,G_{3,5}^{2,2}\!\left(4z\,\middle|\,\begin{matrix}\frac{3-2b}{4},\,\frac{5-2b}{4},\,\frac{a}{\pi}+\frac{1}{2}\\0,\,\frac{1}{2},\,1-b,\,\frac{3}{2}-b,\,\frac{a}{\pi}+\frac{1}{2}\end{matrix}\right)$$

07.17.26.0014.01

$$\cos(z)\,{}_0F_1\!\left(;b;-\frac{z^2}{4}\right)=2^{b-\frac{3}{2}}\,\Gamma(b)\,G_{2,4}^{1,2}\!\left(z^2\,\middle|\,\begin{matrix}\frac{3-2b}{4},\,\frac{5-2b}{4}\\0,\,\frac{1}{2},\,1-b,\,\frac{3}{2}-b\end{matrix}\right)$$

07.17.26.0090.01

$$\cos(a+z)\,{}_0F_1\!\left(;b;-\frac{z^2}{4}\right)=2^{b-\frac{3}{2}}\,\Gamma(b)\,G_{3,5}^{2,2}\!\left(z^2\,\middle|\,\begin{matrix}\frac{1}{4}(3-2b),\,\frac{1}{4}(5-2b),\,\frac{a}{\pi}+\frac{1}{2}\\0,\,\frac{1}{2},\,1-b,\,\frac{3}{2}-b,\,\frac{a}{\pi}+\frac{1}{2}\end{matrix}\right)/;\,-\frac{\pi}{2}<\arg(z)\leq\frac{\pi}{2}$$

07.17.26.0091.01

$$\cos(a+z)\,{}_0F_1\!\left(;b;-\frac{z^2}{4}\right)=2^{b-\frac{3}{2}}\,\Gamma(b)\left(\cos(a)\,G_{2,4}^{1,2}\!\left(z^2\,\middle|\,\begin{matrix}\frac{1}{4}(3-2b),\,\frac{1}{4}(5-2b)\\0,\,\frac{1}{2},\,1-b,\,\frac{3}{2}-b\end{matrix}\right)-\frac{\sin(a)}{z}\,G_{2,4}^{1,2}\!\left(z^2\,\middle|\,\begin{matrix}\frac{5}{4}-\frac{b}{2},\,\frac{7}{4}-\frac{b}{2}\\1,\,\frac{1}{2},\,\frac{3}{2}-b,\,2-b\end{matrix}\right)\right)$$

Classical cases involving sin

07.17.26.0016.01

$$\sin\left(2\sqrt{z}\right){}_0F_1(;b;-z)=2^{b-\frac{3}{2}}\,\Gamma(b)\,G_{2,4}^{1,2}\!\left(4z\,\middle|\,\begin{matrix}\frac{3-2b}{4},\,\frac{5-2b}{4}\\\frac{1}{2},\,0,\,1-b,\,\frac{3}{2}-b\end{matrix}\right)$$

07.17.26.0017.01

$$\sin\left(a+2\sqrt{z}\right){}_0F_1(;b;-z)=-2^{b-\frac{3}{2}}\,\Gamma(b)\,G_{3,5}^{2,2}\!\left(4z\,\middle|\,\begin{matrix}\frac{3-2b}{4},\,\frac{5-2b}{4},\,\frac{a}{\pi}+1\\0,\,\frac{1}{2},\,1-b,\,\frac{3}{2}-b,\,\frac{a}{\pi}+1\end{matrix}\right)$$

07.17.26.0092.01

$$\sin(z)\,{}_0F_1\!\left(;b;-\frac{z^2}{4}\right)=2^{b-\frac{3}{2}}\,\Gamma(b)\,G_{2,4}^{1,2}\!\left(z^2\,\middle|\,\begin{matrix}\frac{3-2b}{4},\,\frac{5-2b}{4}\\\frac{1}{2},\,0,\,1-b,\,\frac{3}{2}-b\end{matrix}\right)/;\,-\frac{\pi}{2}<\arg(z)\leq\frac{\pi}{2}$$

07.17.26.0093.01

$$\sin(z)\,_0F_1\left(;b;-\frac{z^2}{4}\right)=\frac{2^{b-\frac{3}{2}}\,\Gamma(b)}{z}\,G_{2,4}^{1,2}\left(z^2\,\middle|\,\begin{array}{c}\frac{5}{4}-\frac{b}{2},\frac{7}{4}-\frac{b}{2}\\ 1,\frac{1}{2},\frac{3}{2}-b,2-b\end{array}\right)$$

07.17.26.0094.01

$$\sin(a+z)\,_0F_1\left(;b;-\frac{z^2}{4}\right)=-2^{b-\frac{3}{2}}\,\Gamma(b)\,G_{3,5}^{2,2}\left(z^2\,\middle|\,\begin{array}{c}\frac{3-2b}{4},\frac{5-2b}{4},\frac{a}{\pi}+1\\ 0,\frac{1}{2},1-b,\frac{3}{2}-b,\frac{a}{\pi}+1\end{array}\right)/;\,-\frac{\pi}{2}<\arg(z)\leq\frac{\pi}{2}$$

07.17.26.0095.01

$$\sin(a+z)\,_0F_1\left(;b;-\frac{z^2}{4}\right)=2^{b-\frac{3}{2}}\,\Gamma(b)\left(\frac{\cos(a)}{z}\,G_{2,4}^{1,2}\left(z^2\,\middle|\,\begin{array}{c}\frac{5}{4}-\frac{b}{2},\frac{7}{4}-\frac{b}{2}\\ 1,\frac{1}{2},\frac{3}{2}-b,2-b\end{array}\right)+\sin(a)\,G_{2,4}^{1,2}\left(z^2\,\middle|\,\begin{array}{c}\frac{1}{4}(3-2b),\frac{1}{4}(5-2b)\\ 0,\frac{1}{2},1-b,\frac{3}{2}-b\end{array}\right)\right)$$

Classical cases involving cosh

07.17.26.0018.01

$$\cosh\left(2\sqrt{z}\right)\,_0F_1(;b;z)=2^{b-\frac{3}{2}}\,\pi\sec\left(\frac{b\pi}{2}\right)\Gamma(b)\,G_{3,5}^{1,2}\left(4z\,\middle|\,\begin{array}{c}\frac{3-2b}{4},\frac{5-2b}{4},\frac{1-b}{2}\\ 0,\frac{1}{2},1-b,\frac{3}{2}-b,\frac{1-b}{2}\end{array}\right)$$

07.17.26.0096.01

$$\cosh\left(a+2\sqrt{z}\right)\,_0F_1(;b;z)=\frac{2^{2b-3}\,e^{-a}\,\Gamma(b)}{\sqrt{\pi}}\left(G_{1,2}^{1,1}\left(4\sqrt{z}\,\middle|\,\begin{array}{c}\frac{3}{2}-b\\ 0,2-2b\end{array}\right)+e^{2a}\,\pi\csc(b\pi)\,G_{2,3}^{1,1}\left(4\sqrt{z}\,\middle|\,\begin{array}{c}\frac{3}{2}-b,1-b\\ 0,2-2b,1-b\end{array}\right)\right)$$

07.17.26.0019.01

$$\cosh(z)\,_0F_1\left(;b;\frac{z^2}{4}\right)=2^{b-\frac{3}{2}}\,\pi\sec\left(\frac{b\pi}{2}\right)\Gamma(b)\,G_{3,5}^{1,2}\left(z^2\,\middle|\,\begin{array}{c}\frac{3-2b}{4},\frac{5-2b}{4},\frac{1-b}{2}\\ 0,\frac{1}{2},1-b,\frac{3}{2}-b,\frac{1-b}{2}\end{array}\right)$$

07.17.26.0097.01

$$\cosh(a+z)\,_0F_1\left(;b;\frac{z^2}{4}\right)=2^{b-\frac{3}{2}}\,\Gamma(b)\,G_{3,5}^{2,2}\left(-z^2\,\middle|\,\begin{array}{c}\frac{1}{4}(3-2b),\frac{1}{4}(5-2b),\frac{1}{2}+\frac{ia}{\pi}\\ 0,\frac{1}{2},1-b,\frac{3}{2}-b,\frac{1}{2}+\frac{ia}{\pi}\end{array}\right)/;\,-\pi<\arg(z)\leq 0$$

07.17.26.0083.01

$$\cosh(a+z)\,_0F_1\left(;b;\frac{z^2}{4}\right)=\frac{2^{2b-3}\,\Gamma(b)}{\sqrt{\pi}}\left(e^{-a}\,G_{1,2}^{1,1}\left(2z\,\middle|\,\begin{array}{c}\frac{3}{2}-b\\ 0,2-2b\end{array}\right)+e^{a}\,\pi\csc(b\pi)\,G_{2,3}^{1,1}\left(2z\,\middle|\,\begin{array}{c}\frac{3}{2}-b,1-b\\ 0,2-2b,1-b\end{array}\right)\right)$$

Classical cases involving sinh

07.17.26.0020.01

$$\sinh\left(2\sqrt{z}\right)\,_0F_1(;b;z)=-2^{b-\frac{3}{2}}\,\pi\csc\left(\frac{b\pi}{2}\right)\Gamma(b)\,G_{3,5}^{1,2}\left(4z\,\middle|\,\begin{array}{c}\frac{3-2b}{4},\frac{5-2b}{4},\frac{1-b}{2}\\ \frac{1}{2},0,\frac{1-b}{2},1-b,\frac{3}{2}-b\end{array}\right)$$

07.17.26.0098.01

$$\sinh\left(a+2\sqrt{z}\right)\,_0F_1(;b;z)=\frac{2^{2b-3}\,e^{-a}\,\Gamma(b)}{\sqrt{\pi}}\left(-G_{1,2}^{1,1}\left(4\sqrt{z}\,\middle|\,\begin{array}{c}\frac{3}{2}-b\\ 0,2-2b\end{array}\right)+e^{2a}\,\pi\csc(b\pi)\,G_{2,3}^{1,1}\left(4\sqrt{z}\,\middle|\,\begin{array}{c}\frac{3}{2}-b,1-b\\ 0,2-2b,1-b\end{array}\right)\right)$$

07.17.26.0099.01

$$\sinh(z)\,_0F_1\left(;b;\frac{z^2}{4}\right)=-2^{b-\frac{3}{2}}\,\pi\csc\left(\frac{b\pi}{2}\right)\Gamma(b)\,G_{3,5}^{1,2}\left(z^2\,\middle|\,\begin{array}{c}\frac{3-2b}{4},\frac{5-2b}{4},\frac{1-b}{2}\\ \frac{1}{2},0,\frac{1-b}{2},1-b,\frac{3}{2}-b\end{array}\right)/;\,-\frac{\pi}{2}<\arg(z)\leq\frac{\pi}{2}$$

07.17.26.0100.01

$$\sinh(z)\, {}_0F_1\left(;b;\frac{z^2}{4}\right) = -\frac{2^{b-\frac{3}{2}}\,\Gamma(b)}{z}\, G_{2,4}^{1,2}\left(-z^2 \,\middle|\, \begin{matrix} \frac{5}{4}-\frac{b}{2}, \frac{7}{4}-\frac{b}{2} \\ 1, \frac{1}{2}, \frac{3}{2}-b, 2-b \end{matrix}\right)$$

07.17.26.0084.01

$$\sinh(z)\, {}_0F_1\left(;b;\frac{z^2}{4}\right) == \frac{2^{2b-3}\,\Gamma(b)}{\sqrt{\pi}} \left(\pi \csc(b\,\pi)\, G_{2,3}^{1,1}\left(2z \,\middle|\, \begin{matrix} \frac{3}{2}-b, 1-b \\ 0, 2-2b, 1-b \end{matrix}\right) - G_{1,2}^{1,1}\left(2z \,\middle|\, \begin{matrix} \frac{3}{2}-b \\ 0, 2-2b \end{matrix}\right)\right)$$

07.17.26.0101.01

$$\sinh(a+z)\, {}_0F_1\left(;b;\frac{z^2}{4}\right) = 2^{b-\frac{3}{2}}\, i\, \Gamma(b)\, G_{3,5}^{2,2}\left(-z^2 \,\middle|\, \begin{matrix} \frac{1}{4}(3-2b), \frac{1}{4}(5-2b), \frac{i\,a}{\pi}+1 \\ 0, \frac{1}{2}, 1-b, \frac{3}{2}-b, \frac{i\,a}{\pi}+1 \end{matrix}\right) /; -\pi < \arg(z) \leq 0$$

07.17.26.0085.01

$$\sinh(a+z)\, {}_0F_1\left(;b;\frac{z^2}{4}\right) == \frac{2^{2b-3}\,\Gamma(b)}{\sqrt{\pi}} \left(e^a\, \pi \csc(b\,\pi)\, G_{2,3}^{1,1}\left(2z \,\middle|\, \begin{matrix} \frac{3}{2}-b, 1-b \\ 0, 2-2b, 1-b \end{matrix}\right) - e^{-a}\, G_{1,2}^{1,1}\left(2z \,\middle|\, \begin{matrix} \frac{3}{2}-b \\ 0, 2-2b \end{matrix}\right)\right)$$

Classical cases involving Ai

07.17.26.0086.01

$$\text{Ai}\left(3^{2/3}\sqrt[3]{z}\right) {}_0F_1(;b;z) == \frac{\Gamma(b)\, 2^{b-\frac{7}{3}}}{\sqrt[6]{3}\, \pi^{3/2}}\, G_{2,4}^{2,2}\left(4z \,\middle|\, \begin{matrix} \frac{1}{6}(4-3b), \frac{1}{6}(7-3b) \\ 0, \frac{1}{3}, 1-b, \frac{4}{3}-b \end{matrix}\right)$$

07.17.26.0102.01

$$\text{Ai}(z)\, {}_0F_1\left(;b;\frac{z^3}{9}\right) = \frac{2^{b-\frac{7}{3}}\,\Gamma(b)}{\sqrt[6]{3}\, \pi^{3/2}}\, G_{2,4}^{2,2}\left(\frac{4z^3}{9} \,\middle|\, \begin{matrix} \frac{1}{6}(4-3b), \frac{1}{6}(7-3b) \\ 0, \frac{1}{3}, 1-b, \frac{4}{3}-b \end{matrix}\right) /; -\frac{\pi}{3} < \arg(z) \leq \frac{\pi}{3}$$

Classical cases involving Ai′

07.17.26.0087.01

$$\text{Ai}'\left(3^{2/3}\sqrt[3]{z}\right) {}_0F_1(;b;z) == -\frac{2^{b-\frac{8}{3}}\,\sqrt[6]{3}\,\Gamma(b)}{\pi^{3/2}}\, G_{2,4}^{2,2}\left(4z \,\middle|\, \begin{matrix} \frac{1}{6}(5-3b), \frac{1}{6}(8-3b) \\ 0, \frac{2}{3}, 1-b, \frac{5}{3}-b \end{matrix}\right)$$

07.17.26.0103.01

$$\text{Ai}'(z)\, {}_0F_1\left(;b;\frac{z^3}{9}\right) = -\frac{2^{b-\frac{8}{3}}\,\sqrt[6]{3}\,\Gamma(b)}{\pi^{3/2}}\, G_{2,4}^{2,2}\left(\frac{4z^3}{9} \,\middle|\, \begin{matrix} \frac{1}{6}(5-3b), \frac{1}{6}(8-3b) \\ 0, \frac{2}{3}, 1-b, \frac{5}{3}-b \end{matrix}\right) /; -\frac{\pi}{3} < \arg(z) \leq \frac{\pi}{3}$$

Classical cases involving Bi

07.17.26.0088.01

$$\text{Bi}\left(3^{2/3}\sqrt[3]{z}\right) {}_0F_1(;b;z) == \frac{2^{b-\frac{1}{3}}\,\sqrt{\pi}\,\Gamma(b)}{\sqrt[6]{3}}\, G_{4,6}^{2,2}\left(4z \,\middle|\, \begin{matrix} \frac{1}{6}(4-3b), \frac{1}{6}(7-3b), \frac{1}{6}, \frac{2}{3} \\ 0, \frac{1}{3}, \frac{1}{6}, \frac{2}{3}, 1-b, \frac{4}{3}-b \end{matrix}\right)$$

07.17.26.0104.01

$$\text{Bi}(z)\, {}_0F_1\left(;b;\frac{z^3}{9}\right) = \frac{2^{b-\frac{1}{3}}\,\sqrt{\pi}\,\Gamma(b)}{\sqrt[6]{3}}\, G_{4,6}^{2,2}\left(\frac{4z^3}{9} \,\middle|\, \begin{matrix} \frac{1}{6}(4-3b), \frac{1}{6}(7-3b), \frac{1}{6}, \frac{2}{3} \\ 0, \frac{1}{3}, \frac{1}{6}, \frac{2}{3}, 1-b, \frac{4}{3}-b \end{matrix}\right) /; -\frac{\pi}{3} < \arg(z) \leq \frac{\pi}{3}$$

Classical cases involving Bi′

07.17.26.0089.01

$$\mathrm{Bi}'\left(3^{2/3}\sqrt[3]{z}\right)\,{}_0F_1(;b;z) = 2^{b-\frac{2}{3}}\sqrt[6]{3}\,\sqrt{\pi}\,\Gamma(b)\,G_{4,6}^{2,2}\left(4z\,\middle|\,\begin{array}{c}\frac{1}{6}(5-3b),\,\frac{1}{6}(8-3b),\,\frac{1}{3},\,\frac{5}{6}\\ 0,\,\frac{2}{3},\,\frac{1}{3},\,\frac{5}{6},\,1-b,\,\frac{5}{3}-b\end{array}\right)$$

07.17.26.0105.01

$$\mathrm{Bi}'(z)\,{}_0F_1\left(;b;\frac{z^3}{9}\right) = 2^{b-\frac{2}{3}}\sqrt[6]{3}\,\sqrt{\pi}\,\Gamma(b)\,G_{4,6}^{2,2}\left(\frac{4z^3}{9}\,\middle|\,\begin{array}{c}\frac{1}{6}(5-3b),\,\frac{1}{6}(8-3b),\,\frac{1}{3},\,\frac{5}{6}\\ 0,\,\frac{2}{3},\,\frac{1}{3},\,\frac{5}{6},\,1-b,\,\frac{5}{3}-b\end{array}\right)\;/;\;-\frac{\pi}{3}<\arg(z)\le\frac{\pi}{3}$$

Classical cases for powers of ${}_0F_1$

07.17.26.0021.01

$${}_0F_1(;b;z)^2 = \sqrt{\pi}\,2^{2b-2}\,\Gamma(b)^2\,G_{2,4}^{1,1}\left(4z\,\middle|\,\begin{array}{c}\frac{3}{2}-b,\,\frac{1}{2}\\ 0,\,1-b,\,2-2b,\,\frac{1}{2}\end{array}\right)$$

07.17.26.0022.01

$${}_0F_1(;b;z)^2 = \frac{2^{2b-2}\,\Gamma(b)^2}{\sqrt{\pi}}\,G_{1,3}^{1,1}\left(-4z\,\middle|\,\begin{array}{c}\frac{3}{2}-b\\ 0,\,1-b,\,2-2b\end{array}\right)$$

Classical cases for products of ${}_0F_1$

07.17.26.0023.01

$${}_0F_1(;b;z)\,{}_0F_1(;c;z) = \sqrt{\pi}\,2^{b+c-2}\,\Gamma(b)\,\Gamma(c)\,G_{3,5}^{1,2}\left(4z\,\middle|\,\begin{array}{c}1-\frac{b+c}{2},\,\frac{3-b-c}{2},\,\frac{1}{2}\\ 0,\,1-b,\,1-c,\,2-b-c,\,\frac{1}{2}\end{array}\right)\;/;\;1-b-c\notin\mathbb{N}$$

07.17.26.0024.01

$${}_0F_1(;b;z)\,{}_0F_1(;c;z) = \frac{2^{b+c-2}\,\Gamma(b)\,\Gamma(c)}{\sqrt{\pi}}\,G_{2,4}^{1,2}\left(-4z\,\middle|\,\begin{array}{c}1-\frac{b+c}{2},\,\frac{3-b-c}{2}\\ 0,\,1-b,\,1-c,\,2-b-c\end{array}\right)\;/;\;1-b-c\notin\mathbb{N}$$

07.17.26.0106.01

$${}_0F_1(;b;z)\,{}_0F_1(;-b-n+1;z) = \frac{\Gamma(b)\,\Gamma(-b-n+1)}{2^{n+1}\sqrt{\pi}}$$
$$\left(2\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{\left((-1)^{\left\lfloor\frac{n+1}{2}\right\rfloor}\Gamma\left(k-n+\left\lfloor\frac{n}{2}\right\rfloor+\frac{1}{2}\right)\right)\left(1-k+\left\lfloor\frac{n}{2}\right\rfloor\right)_{n-\left\lfloor\frac{n}{2}\right\rfloor}(4z)^k}{k!\,\Gamma(b+k)\,\Gamma(1-b+k-n)}-(-1)^n\pi\,G_{3,5}^{1,2}\left(4z\,\middle|\,\begin{array}{c}\frac{n+1}{2},\,\frac{n+2}{2},\,\frac{1}{2}\\ n+1,\,b+n,\,0,\,1-b,\,\frac{1}{2}\end{array}\right)\right)\;/;\;n\in\mathbb{N}$$

07.17.26.0107.01

$${}_0F_1(;b;z)\,{}_0F_1(;1-b;z) = 1-\frac{1}{2}\pi^{3/2}\csc(b\pi)\,G_{2,4}^{1,1}\left(4z\,\middle|\,\begin{array}{c}1,\,\frac{1}{2}\\ 1,\,b,\,0,\,1-b\end{array}\right)$$

07.17.26.0108.01

$${}_0F_1(;b;z)\,{}_0F_1(;-b;z) = \frac{1}{4}\sqrt{\pi}\,\Gamma(-b)\,\Gamma(b)\,G_{3,5}^{1,2}\left(4z\,\middle|\,\begin{array}{c}1,\,\frac{3}{2},\,\frac{1}{2}\\ 2,\,b+1,\,0,\,1-b,\,\frac{1}{2}\end{array}\right)+1$$

07.17.26.0025.01

$${}_0F_1(;b;z)\,{}_0F_1(;2-b;z)=(1-b)\,\pi^{3/2}\csc(b\,\pi)\,G_{2,4}^{1,1}\left(4\,z\,\middle|\,\begin{array}{c}\frac{1}{2},\frac{1}{2}\\0,1-b,b-1,\frac{1}{2}\end{array}\right)$$

07.17.26.0026.01

$${}_0F_1(;b;z)\,{}_0F_1(;2-b;z)=(1-b)\sqrt{\pi}\csc(b\,\pi)\,G_{1,3}^{1,1}\left(-4\,z\,\middle|\,\begin{array}{c}\frac{1}{2}\\0,1-b,b-1\end{array}\right)$$

07.17.26.0027.01

$${}_0F_1(;b;z)\,{}_0F_1(;b-1;z)=2^{2b-3}\sqrt{\pi}\,\Gamma(b)\,\Gamma(b-1)\,G_{2,4}^{1,1}\left(4\,z\,\middle|\,\begin{array}{c}\frac{3}{2}-b,\frac{1}{2}\\0,1-b,3-2b,\frac{1}{2}\end{array}\right)$$

07.17.26.0028.01

$${}_0F_1(;b;z)\,{}_0F_1(;b-1;z)=\frac{2^{2b-3}\,\Gamma(b)\,\Gamma(b-1)}{\sqrt{\pi}}\,G_{1,3}^{1,1}\left(-4\,z\,\middle|\,\begin{array}{c}\frac{3}{2}-b\\0,1-b,3-2b\end{array}\right)$$

07.17.26.0029.01

$${}_0F_1(;b;z)\,{}_0F_1(;b+1;z)=2^{2b-1}\sqrt{\pi}\,b\,\Gamma(b)^2\,G_{2,4}^{1,1}\left(4\,z\,\middle|\,\begin{array}{c}\frac{1}{2}-b,\frac{1}{2}\\0,-b,1-2b,\frac{1}{2}\end{array}\right)$$

07.17.26.0030.01

$${}_0F_1(;b;z)\,{}_0F_1(;b+1;z)=\frac{2^{2b-1}\,b\,\Gamma(b)^2}{\sqrt{\pi}}\,G_{1,3}^{1,1}\left(-4\,z\,\middle|\,\begin{array}{c}\frac{1}{2}-b\\0,-b,1-2b\end{array}\right)$$

07.17.26.0031.01

$${}_0F_1(;b;z)\,{}_0F_1(;b;-z)=\sqrt{\pi}\,2^{1-b}\,\Gamma(b)^2\,G_{0,4}^{1,0}\left(\frac{z^2}{4}\,\middle|\,0,\frac{1}{2}-\frac{b}{2},1-\frac{b}{2},1-b\right)$$

07.17.26.0032.01

$${}_0F_1(;b;z)\,{}_0F_1(;2-b;-z)=(1-b)\,\pi^{3/2}\csc(\pi\,b)\,G_{1,5}^{2,0}\left(\frac{z^2}{4}\,\middle|\,\begin{array}{c}1-\frac{b}{2}\\0,\frac{1}{2},1-\frac{b}{2},\frac{1-b}{2},\frac{b-1}{2}\end{array}\right)/;\operatorname{Re}(z)<0\bigvee\arg(z)=-\frac{\pi}{2}$$

Classical cases involving Bessel J

07.17.26.0033.01

$${}_0F_1(;b;-z)\,J_\nu\left(2\sqrt{z}\right)=\frac{\Gamma(b)\,2^{b-1}}{\sqrt{\pi}}\,G_{2,4}^{1,2}\left(4\,z\,\middle|\,\begin{array}{c}\frac{1-b}{2},1-\frac{b}{2}\\\frac{\nu}{2},-\frac{\nu}{2},1-b+\frac{\nu}{2},1-b-\frac{\nu}{2}\end{array}\right)/;1-b-\nu\notin\mathbb{N}$$

07.17.26.0109.01

$${}_0F_1(;b;-z)\,J_{-b-n}\left(2\sqrt{z}\right)=\frac{\Gamma(b)}{\sqrt{\pi}}\left(2^{-n}\,z^{-\frac{b+n}{2}}\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{\left((-1)^{k+\left\lfloor\frac{n+1}{2}\right\rfloor}\Gamma\left(k-n+\left\lfloor\frac{n}{2}\right\rfloor+\frac{1}{2}\right)\right)\left(1-k+\left\lfloor\frac{n}{2}\right\rfloor\right)_{n-\left\lfloor\frac{n}{2}\right\rfloor}(4\,z)^k}{k!\,\Gamma(b+k)\,\Gamma(1-b+k-n)}-\right.$$

$$\left.(-1)^n\,2^{b-1}\,G_{2,4}^{1,2}\left(4\,z\,\middle|\,\begin{array}{c}\frac{1-b}{2},1-\frac{b}{2}\\\frac{n-b}{2}+1,\frac{b+n}{2},-\frac{1}{2}(b+n),1-\frac{1}{2}(3\,b+n)\end{array}\right)\right)/;n\in\mathbb{N}$$

07.17.26.0110.01

$$ {}_0F_1(;b;-z)\,J_{-b}\left(2\sqrt{z}\right) = \frac{\Gamma(b)}{2\pi}\left(2\,z^{-\frac{b}{2}}\sin(b\,\pi) - 2^b\sqrt{\pi}\;G_{3,5}^{2,2}\left(4\,z\,\middle|\,\begin{matrix}\frac{1-b}{2},\,1-\frac{b}{2},\,\frac{b}{2}\\ 1-\frac{b}{2},\,\frac{b}{2},\,\frac{b}{2},\,-\frac{b}{2},\,1-\frac{3b}{2}\end{matrix}\right)\right) $$

07.17.26.0111.01

$$ {}_0F_1(;b;-z)\,J_{-b-1}\left(2\sqrt{z}\right) = \frac{z^{-\frac{b}{2}-\frac{1}{2}}}{\Gamma(-b)} + \frac{2^{b-1}\,\Gamma(b)}{\sqrt{\pi}}\,G_{2,4}^{1,2}\left(4\,z\,\middle|\,\begin{matrix}\frac{1-b}{2},\,1-\frac{b}{2}\\ \frac{3-b}{2},\,\frac{b+1}{2},\,\frac{1}{2}(-b-1),\,\frac{1}{2}(1-3\,b)\end{matrix}\right) $$

07.17.26.0034.01

$$ {}_0F_1(;b;-z)\,J_b\left(2\sqrt{z}\right) = \frac{2^{b-1}\,\Gamma(b)}{\sqrt{\pi}}\,G_{1,3}^{1,1}\left(4\,z\,\middle|\,\begin{matrix}\frac{1-b}{2}\\ \frac{b}{2},\,-\frac{b}{2},\,1-\frac{3b}{2}\end{matrix}\right) $$

07.17.26.0035.01

$$ {}_0F_1(;b;z)\,J_{b-1}\left(2\sqrt{z}\right) = 2^{\frac{1-b}{2}}\sqrt{\pi}\;z^{\frac{b-1}{2}}\left(z^2\right)^{\frac{1-b}{4}}\Gamma(b)\,G_{0,4}^{1,0}\left(\frac{z^2}{4}\,\middle|\,\frac{b-1}{4},\,\frac{1-b}{4},\,\frac{3-b}{4},\,\frac{3-3b}{4}\right) $$

07.17.26.0036.01

$$ {}_0F_1(;b;z)\,J_{1-b}\left(2\sqrt{z}\right) = 2^{\frac{1-b}{2}}\sqrt{\pi}\;\Gamma(b)\,G_{1,5}^{2,0}\left(\frac{z^2}{4}\,\middle|\,\begin{matrix}\frac{b+1}{4}\\ \frac{1-b}{4},\,\frac{3-b}{4},\,\frac{3-3b}{4},\,\frac{b-1}{4},\,\frac{b+1}{4}\end{matrix}\right)\;/;\;-\frac{\pi}{2}<\arg(z)\leq\frac{\pi}{2} $$

07.17.26.0112.01

$$ {}_0F_1\left(;b;-\frac{z^2}{4}\right)J_\nu(z) = \frac{\Gamma(b)\,2^{b-1}}{\sqrt{\pi}}\,G_{2,4}^{1,2}\left(z^2\,\middle|\,\begin{matrix}\frac{1-b}{2},\,1-\frac{b}{2}\\ \frac{\nu}{2},\,-\frac{\nu}{2},\,-b+\frac{\nu}{2}+1,\,-b-\frac{\nu}{2}+1\end{matrix}\right)/;\;-b-\nu\notin\mathbb{N}\bigwedge-\frac{\pi}{2}<\arg(z)\leq\frac{\pi}{2} $$

07.17.26.0113.01

$$ {}_0F_1\left(;b;-\frac{z^2}{4}\right)J_{-b-n}(z) = \frac{2^{b-1}\,\Gamma(b)}{\sqrt{\pi}}\left(2\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{\left((-1)^{k+\left\lfloor\frac{n+1}{2}\right\rfloor}z^{-b+2k-n}\,\Gamma\left(k-n+\left\lfloor\frac{n}{2}\right\rfloor+\frac{1}{2}\right)\right)\left(-k+\left\lfloor\frac{n}{2}\right\rfloor+1\right)_{n-\left\lfloor\frac{n}{2}\right\rfloor}}{k!\,\Gamma(b+k)\,\Gamma(-b+k-n+1)} - \right. $$

$$ \left.(-1)^n\,G_{2,4}^{1,2}\left(z^2\,\middle|\,\begin{matrix}\frac{1-b}{2},\,1-\frac{b}{2}\\ \frac{n-b}{2}+1,\,\frac{b+n}{2},\,-\frac{1}{2}(b+n),\,1-\frac{1}{2}(3\,b+n)\end{matrix}\right)\right)/;\;n\in\mathbb{N}\bigwedge-\frac{\pi}{2}<\arg(z)\leq\frac{\pi}{2} $$

07.17.26.0114.01

$$ {}_0F_1\left(;b;-\frac{z^2}{4}\right)J_{-b}(z) = \frac{\Gamma(b)}{2\pi}\left(2^{b+1}z^{-b}\sin(b\,\pi) - 2^b\sqrt{\pi}\;G_{2,4}^{1,2}\left(z^2\,\middle|\,\begin{matrix}\frac{1-b}{2},\,1-\frac{b}{2}\\ 1-\frac{b}{2},\,\frac{b}{2},\,-\frac{b}{2},\,1-\frac{3b}{2}\end{matrix}\right)\right)/;\;-\frac{\pi}{2}<\arg(z)\leq\frac{\pi}{2} $$

07.17.26.0115.01

$$ {}_0F_1\left(;b;-\frac{z^2}{4}\right)J_{-b-1}(z) = 2^{b-1}\left(\frac{4\,z^{-b-1}}{\Gamma(-b)} + \frac{\Gamma(b)}{\sqrt{\pi}}\,G_{2,4}^{1,2}\left(z^2\,\middle|\,\begin{matrix}\frac{1-b}{2},\,1-\frac{b}{2}\\ \frac{3-b}{2},\,\frac{b+1}{2},\,\frac{1}{2}(-b-1),\,\frac{1}{2}(1-3\,b)\end{matrix}\right)\right)/;\;-\frac{\pi}{2}<\arg(z)\leq\frac{\pi}{2} $$

07.17.26.0116.01

$$ {}_0F_1\left(;b;-\frac{z^2}{4}\right)J_b(z) = \frac{\Gamma(b)\,2^{b-1}}{\sqrt{\pi}}\,G_{1,3}^{1,1}\left(z^2\,\middle|\,\begin{matrix}\frac{1-b}{2}\\ \frac{b}{2},\,-\frac{b}{2},\,1-\frac{3b}{2}\end{matrix}\right)/;\;-\frac{\pi}{2}<\arg(z)\leq\frac{\pi}{2} $$

07.17.26.0117.01

$$ {}_0F_1\left(;b;-\frac{z^2}{4}\right)J_{b-1}(z)=\frac{\Gamma(b)\,2^{b-1}}{\sqrt{\pi}}\,G_{1,3}^{1,1}\left(z^2\,\middle|\,\begin{array}{c}1-\frac{b}{2}\\ \frac{b-1}{2},\,\frac{1-b}{2},\,\frac{3(1-b)}{2}\end{array}\right)/;\,-\frac{\pi}{2}<\arg(z)\leq\frac{\pi}{2} $$

07.17.26.0118.01

$$ {}_0F_1\left(;b;-\frac{z^2}{4}\right)J_{1-b}(z)=\frac{2^{b-1}\,\Gamma(b)}{\sqrt{\pi}}\,G_{1,3}^{1,1}\left(z^2\,\middle|\,\begin{array}{c}1-\frac{b}{2}\\ \frac{1-b}{2},\,\frac{b-1}{2},\,\frac{3(1-b)}{2}\end{array}\right)/;\,-\frac{\pi}{2}<\arg(z)\leq\frac{\pi}{2} $$

Classical cases involving Bessel *I*

07.17.26.0037.01

$$ {}_0F_1(;b;z)\,I_\nu\left(2\sqrt{z}\right)=\sqrt{\pi}\,\csc\left(\frac{1}{4}\pi\,(2\,b+2\,\nu+1)\right)\Gamma(b)\,2^{b-1}\,G_{3,5}^{1,2}\left(4\,z\,\middle|\,\begin{array}{c}\frac{1-b}{2},\,1-\frac{b}{2},\,\frac{3-2\,b}{4}\\ \frac{\nu}{2},\,-\frac{\nu}{2},\,1-b+\frac{\nu}{2},\,1-b-\frac{\nu}{2},\,\frac{3-2\,b}{4}\end{array}\right)/;\,-b-\nu\notin\mathbb{N} $$

07.17.26.0119.01

$$ {}_0F_1(;b;z)\,I_{-b-n}\left(2\sqrt{z}\right)=\frac{\Gamma(b)}{\sqrt{\pi}}\left(2^{-n}\,z^{-\frac{1}{2}(b+n)}\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{(-1)^{\left\lfloor\frac{n+1}{2}\right\rfloor}\,\Gamma\left(k-n+\left\lfloor\frac{n}{2}\right\rfloor+\frac{1}{2}\right)\left(1-k+\left\lfloor\frac{n}{2}\right\rfloor\right)_{n-\left\lfloor\frac{n}{2}\right\rfloor}(4\,z)^k}{k!\,\Gamma(b+k)\,\Gamma(1-b+k-n)}-\right. $$

$$ \left.(-1)^{\left\lfloor\frac{n}{2}\right\rfloor}\,2^{b-\frac{1}{2}}\,\pi\,G_{3,5}^{1,2}\left(4\,z\,\middle|\,\begin{array}{c}\frac{1-b}{2},\,1-\frac{b}{2},\,\frac{1}{4}(3-2\,b)\\ \frac{n-b}{2}+1,\,\frac{b+n}{2},\,-\frac{1}{2}(b+n),\,1-\frac{1}{2}(3\,b+n),\,\frac{1}{4}(3-2\,b)\end{array}\right)\right)/;\,n\in\mathbb{N} $$

07.17.26.0120.01

$$ {}_0F_1(;b;z)\,I_{-b}\left(2\sqrt{z}\right)=\frac{\Gamma(b)}{\sqrt{\pi}}\left(\frac{z^{-\frac{b}{2}}\sin(b\,\pi)}{\sqrt{\pi}}-2^{b-\frac{1}{2}}\,\pi\,G_{3,5}^{1,2}\left(4\,z\,\middle|\,\begin{array}{c}\frac{1-b}{2},\,1-\frac{b}{2},\,\frac{1}{4}(3-2\,b)\\ 1-\frac{b}{2},\,\frac{b}{2},\,-\frac{b}{2},\,1-\frac{3\,b}{2},\,\frac{1}{4}(3-2\,b)\end{array}\right)\right) $$

07.17.26.0121.01

$$ {}_0F_1(;b;z)\,I_{-b-1}\left(2\sqrt{z}\right)=\frac{\Gamma(b)}{\sqrt{\pi}}\left(-\frac{b\sin(b\,\pi)\,z^{-\frac{1}{2}(b+1)}}{\sqrt{\pi}}-2^{b-\frac{1}{2}}\,\pi\,G_{3,5}^{1,2}\left(4\,z\,\middle|\,\begin{array}{c}\frac{1-b}{2},\,1-\frac{b}{2},\,\frac{1}{4}(3-2\,b)\\ \frac{3-b}{2},\,\frac{b+1}{2},\,\frac{1}{2}(-b-1),\,\frac{1}{2}(1-3\,b),\,\frac{1}{4}(3-2\,b)\end{array}\right)\right) $$

07.17.26.0122.01

$$ {}_0F_1(;b;z)\,I_b\left(2\sqrt{z}\right)=2^{b-1}\,\sqrt{\pi}\,\csc\left(\left(b+\frac{1}{4}\right)\pi\right)\Gamma(b)\,G_{2,4}^{1,1}\left(4\,z\,\middle|\,\begin{array}{c}\frac{1-b}{2},\,\frac{1}{4}(3-2\,b)\\ \frac{b}{2},\,-\frac{b}{2},\,1-\frac{3\,b}{2},\,\frac{1}{4}(3-2\,b)\end{array}\right) $$

07.17.26.0038.01

$$ {}_0F_1(;b;-z)\,I_{b-1}\left(2\sqrt{z}\right)=2^{1-b}\,\sqrt{\pi}\,z^{\frac{b-1}{2}}\,\Gamma(b)\,G_{0,4}^{1,0}\left(\frac{z^2}{4}\,\middle|\,0,\,\frac{1}{2}-\frac{b}{2},\,1-\frac{b}{2},\,1-b\right) $$

07.17.26.0039.01

$$ {}_0F_1(;b;-z)\,I_{1-b}\left(2\sqrt{z}\right)=2^{\frac{1-b}{2}}\,\sqrt{\pi}\,\Gamma(b)\,G_{1,5}^{2,0}\left(\frac{z^2}{4}\,\middle|\,\begin{array}{c}\frac{5-3\,b}{4}\\ \frac{3-b}{4},\,\frac{1-b}{4},\,\frac{5-3\,b}{4},\,\frac{3-3\,b}{4},\,\frac{b-1}{4}\end{array}\right)/;\,-\frac{\pi}{2}<\arg(z)\leq\frac{\pi}{2} $$

07.17.26.0123.01

$$ {}_0F_1\left(;b;\frac{z^2}{4}\right)I_\nu(z)=\sqrt{\pi}\,\csc\left(\frac{1}{4}\pi(2b+2\nu+1)\right)\Gamma(b)\,2^{b-1}\,G_{3,5}^{1,2}\left(z^2\,\middle|\,\begin{matrix}\frac{1-b}{2},1-\frac{b}{2},\frac{1}{4}(3-2b)\\ \frac{\nu}{2},-\frac{\nu}{2},-b+\frac{\nu}{2}+1,-b-\frac{\nu}{2}+1,\frac{1}{4}(3-2b)\end{matrix}\right)/;$$
$$-b-\nu\notin\mathbb{N}\bigwedge-\frac{\pi}{2}<\arg(z)\le\frac{\pi}{2}$$

07.17.26.0124.01

$$ {}_0F_1\left(;b;\frac{z^2}{4}\right)I_{-b-n}(z)=\frac{2^{b-1}\,\Gamma(b)}{\sqrt{\pi}}\left(2\sum_{k=0}^{\lfloor\frac{n}{2}\rfloor}\frac{(-1)^{\lfloor\frac{n+1}{2}\rfloor}z^{-b+2k-n}\,\Gamma\left(k-n+\lfloor\frac{n}{2}\rfloor+\frac{1}{2}\right)\left(1-k+\lfloor\frac{n}{2}\rfloor\right)_{n-\lfloor\frac{n}{2}\rfloor}}{k!\,\Gamma(b+k)\,\Gamma(1-b+k-n)}-\right.$$
$$\left.(-1)^{\lfloor\frac{n}{2}\rfloor}\sqrt{2}\,\pi\,G_{3,5}^{1,2}\left(z^2\,\middle|\,\begin{matrix}\frac{1-b}{2},1-\frac{b}{2},\frac{1}{4}(3-2b)\\ \frac{n-b}{2}+1,\frac{b+n}{2},-\frac{1}{2}(b+n),1-\frac{1}{2}(3b+n),\frac{1}{4}(3-2b)\end{matrix}\right)\right)/;n\in\mathbb{N}\bigwedge-\frac{\pi}{2}<\arg(z)\le\frac{\pi}{2}$$

07.17.26.0125.01

$$ {}_0F_1\left(;b;\frac{z^2}{4}\right)I_{-b}(z)=\frac{2^{b-1}\,\Gamma(b)}{\sqrt{\pi}}\left(\frac{2z^{-b}\sin(b\pi)}{\sqrt{\pi}}-\sqrt{2}\,\pi\,G_{3,5}^{1,2}\left(z^2\,\middle|\,\begin{matrix}\frac{1-b}{2},1-\frac{b}{2},\frac{1}{4}(3-2b)\\ 1-\frac{b}{2},\frac{b}{2},-\frac{b}{2},1-\frac{3b}{2},\frac{1}{4}(3-2b)\end{matrix}\right)\right)/;-\frac{\pi}{2}<\arg(z)\le\frac{\pi}{2}$$

07.17.26.0126.01

$$ {}_0F_1\left(;b;\frac{z^2}{4}\right)I_{-b-1}(z)=-\frac{2^{b-1}\,\Gamma(b)}{\sqrt{\pi}}\left(\frac{4b\sin(b\pi)\,z^{-b-1}}{\sqrt{\pi}}+\sqrt{2}\,\pi\,G_{3,5}^{1,2}\left(z^2\,\middle|\,\begin{matrix}\frac{1-b}{2},1-\frac{b}{2},\frac{1}{4}(3-2b)\\ \frac{1-b}{2}+1,\frac{b+1}{2},\frac{1}{2}(-b-1),\frac{1}{2}(1-3b),\frac{1}{4}(3-2b)\end{matrix}\right)\right)/;$$
$$-\frac{\pi}{2}<\arg(z)\le\frac{\pi}{2}$$

07.17.26.0127.01

$$ {}_0F_1\left(;b;\frac{z^2}{4}\right)I_b(z)=2^{b-1}\sqrt{\pi}\,\csc\left(\left(b+\frac{1}{4}\right)\pi\right)\Gamma(b)\,G_{2,4}^{1,1}\left(z^2\,\middle|\,\begin{matrix}\frac{1-b}{2},\frac{1}{4}(3-2b)\\ \frac{b}{2},-\frac{b}{2},1-\frac{3b}{2},\frac{1}{4}(3-2b)\end{matrix}\right)/;-\frac{\pi}{2}<\arg(z)\le\frac{\pi}{2}$$

07.17.26.0128.01

$$ {}_0F_1\left(;b;\frac{z^2}{4}\right)I_{b-1}(z)=\frac{\Gamma(b)\,2^{b-1}\,e^{\frac{1}{2}\pi i(1-b)}}{\sqrt{\pi}}\,G_{1,3}^{1,1}\left(-z^2\,\middle|\,\begin{matrix}1-\frac{b}{2}\\ \frac{b-1}{2},\frac{1-b}{2},\frac{3(1-b)}{2}\end{matrix}\right)/;-\pi<\arg(z)\le 0$$

07.17.26.0129.01

$$ {}_0F_1\left(;b;\frac{z^2}{4}\right)I_{1-b}(z)=\frac{\left(2^{b-1}\,\Gamma(b)\right)e^{\frac{1}{2}\pi i(b-1)}}{\sqrt{\pi}}\,G_{1,3}^{1,1}\left(-z^2\,\middle|\,\begin{matrix}1-\frac{b}{2}\\ \frac{1-b}{2},\frac{b-1}{2},\frac{3(1-b)}{2}\end{matrix}\right)/;-\pi<\arg(z)\le 0$$

Classical cases involving Bessel *Y*

07.17.26.0040.01

$$ {}_0F_1(;b;-z)\,Y_\nu\left(2\sqrt{z}\right)=-\frac{2^{b-1}\,\Gamma(b)}{\sqrt{\pi}}\,G_{3,5}^{2,2}\left(4z\,\middle|\,\begin{matrix}\frac{1-b}{2},1-\frac{b}{2},\frac{1-\nu}{2}\\ -\frac{\nu}{2},\frac{\nu}{2},\frac{1-\nu}{2},1-b-\frac{\nu}{2},1-b+\frac{\nu}{2}\end{matrix}\right)/;-b-\nu\notin\mathbb{N}\bigwedge-b+\nu\notin\mathbb{N}$$

07.17.26.0130.01

$$ {}_0F_1(;b;-z)\,Y_{b+n}\!\left(2\sqrt{z}\right)=\frac{2^{b-1}\,\Gamma(b)}{\sqrt{\pi}}\,G_{3,5}^{2,2}\!\left(4z\,\middle|\,\begin{array}{c}\frac{1-b}{2},1-\frac{b}{2},\frac{1}{2}(-b+n+1)\\ \frac{1}{2}(-b+n+2),\frac{b+n}{2},\frac{1}{2}(-3b-n+2),\frac{1}{2}(-b-n),\frac{1}{2}(-b+n+1)\end{array}\right)+ $$

$$ \frac{(-1)^{n+1}\,2^{-n}\,z^{\frac{1}{2}(-b-n)}\csc(b\pi)\,\Gamma(b)}{\sqrt{\pi}}\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{(-1)^{k+\left\lfloor\frac{n+1}{2}\right\rfloor}\,4^k\,z^k\,\Gamma\!\left(k-n+\left\lfloor\frac{n}{2}\right\rfloor+\frac{1}{2}\right)\left(1-k+\left\lfloor\frac{n}{2}\right\rfloor\right)_{n-\left\lfloor\frac{n}{2}\right\rfloor}}{k!\,\Gamma(b+k)\,\Gamma(1-b+k-n)}\;/;\;n\in\mathbb{N} $$

07.17.26.0131.01

$$ {}_0F_1(;b;-z)\,Y_{-b-n}\!\left(2\sqrt{z}\right)=\frac{(-1)^n\,2^{b-1}\,\Gamma(b)}{\sqrt{\pi}}\,G_{3,5}^{2,2}\!\left(4z\,\middle|\,\begin{array}{c}\frac{1-b}{2},1-\frac{b}{2},\frac{1}{2}(b+n+1)\\ \frac{1}{2}(-b+n+2),\frac{b+n}{2},\frac{1}{2}(-3b-n+2),\frac{1}{2}(-b-n),\frac{1}{2}(b+n+1)\end{array}\right)- $$

$$ \frac{2^{-n}\,z^{\frac{1}{2}(-b-n)}\cot(b\pi)\,\Gamma(b)}{\sqrt{\pi}}\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{(-1)^{k+\left\lfloor\frac{n+1}{2}\right\rfloor}\,4^k\,z^k\,\Gamma\!\left(k-n+\left\lfloor\frac{n}{2}\right\rfloor+\frac{1}{2}\right)\left(1-k+\left\lfloor\frac{n}{2}\right\rfloor\right)_{n-\left\lfloor\frac{n}{2}\right\rfloor}}{k!\,\Gamma(b+k)\,\Gamma(1-b+k-n)}\;/;\;n\in\mathbb{N} $$

07.17.26.0132.01

$$ {}_0F_1(;b;-z)\,Y_b\!\left(2\sqrt{z}\right)=\frac{2^{b-1}\,\Gamma(b)}{\sqrt{\pi}}\,G_{2,4}^{2,1}\!\left(4z\,\middle|\,\begin{array}{c}1-\frac{b}{2},\frac{1-b}{2}\\ 1-\frac{b}{2},\frac{b}{2},1-\frac{3b}{2},-\frac{b}{2}\end{array}\right)-\frac{z^{-\frac{b}{2}}\csc(b\pi)}{\Gamma(1-b)} $$

07.17.26.0133.01

$$ {}_0F_1(;b;-z)\,Y_{b+1}\!\left(2\sqrt{z}\right)=\frac{\csc(b\pi)\,z^{\frac{1}{2}(-b-1)}}{\Gamma(-b)}+\frac{2^{b-1}\,\Gamma(b)}{\sqrt{\pi}}\,G_{2,4}^{2,1}\!\left(4z\,\middle|\,\begin{array}{c}\frac{1-b}{2},1-\frac{b}{2}\\ \frac{3-b}{2},\frac{b+1}{2},\frac{1}{2}(1-3b),\frac{1}{2}(-b-1)\end{array}\right) $$

07.17.26.0134.01

$$ {}_0F_1(;b;-z)\,Y_{-b}\!\left(2\sqrt{z}\right)=-\frac{\cos(b\pi)\,\Gamma(b)\,z^{-\frac{b}{2}}}{2\pi}-\frac{2^{b-1}\,\Gamma(b)}{\sqrt{\pi}}\,G_{2,4}^{2,1}\!\left(4z\,\middle|\,\begin{array}{c}\frac{1-b}{2},\frac{b+1}{2}\\ -\frac{b}{2},\frac{b}{2},1-\frac{3b}{2},\frac{b+1}{2}\end{array}\right) $$

07.17.26.0135.01

$$ {}_0F_1(;b;-z)\,Y_{-b}\!\left(2\sqrt{z}\right)=\frac{2^{b-1}\,\Gamma(b)}{\sqrt{\pi}}\,G_{3,5}^{2,2}\!\left(4z\,\middle|\,\begin{array}{c}\frac{1-b}{2},1-\frac{b}{2},\frac{b+1}{2}\\ 1-\frac{b}{2},\frac{b}{2},1-\frac{3b}{2},-\frac{b}{2},\frac{b+1}{2}\end{array}\right)-\frac{z^{-\frac{b}{2}}\cot(b\pi)}{\Gamma(1-b)} $$

07.17.26.0136.01

$$ {}_0F_1(;b;-z)\,Y_{-b-1}\!\left(2\sqrt{z}\right)=-\frac{\cot(b\pi)\,z^{\frac{1}{2}(-b-1)}}{\Gamma(-b)}-\frac{2^{b-1}\,\Gamma(b)}{\sqrt{\pi}}\,G_{3,5}^{2,2}\!\left(4z\,\middle|\,\begin{array}{c}\frac{1-b}{2},1-\frac{b}{2},\frac{b+2}{2}\\ \frac{3-b}{2},\frac{b+1}{2},\frac{1}{2}(1-3b),\frac{1}{2}(-b-1),\frac{b+2}{2}\end{array}\right) $$

07.17.26.0041.01

$$ {}_0F_1(;b;-z)\,Y_{b-1}\left(2\sqrt{z}\right)=-\frac{2^{b-1}\,\Gamma(b)}{\sqrt{\pi}}\,G_{1,3}^{2,0}\!\left(4z\,\middle|\,\begin{array}{c}1-\frac{b}{2}\\ \frac{1-b}{2},\frac{b-1}{2},\frac{3-3b}{2}\end{array}\right) $$

07.17.26.0042.01

$$ {}_0F_1(;b;-z)\,Y_{b-2}\left(2\sqrt{z}\right)=\frac{2^{b-1}\,\Gamma(b)}{\sqrt{\pi}}\,G_{2,4}^{2,1}\!\left(4z\,\middle|\,\begin{array}{c}1-\frac{b}{2},\frac{1-b}{2}\\ 1-\frac{b}{2},\frac{b}{2}-1,2-\frac{3b}{2},-\frac{b}{2}\end{array}\right) $$

07.17.26.0043.01

$$ {}_0F_1(;b;-z)\,Y_{b-3}\left(2\sqrt{z}\right) = \frac{2^{b-1}\,\Gamma(b)}{\sqrt{\pi}}\,G_{2,4}^{2,1}\left(4z\,\middle|\,\begin{matrix}\frac{1-b}{2},1-\frac{b}{2}\\ \frac{3-b}{2},\frac{b-3}{2},\frac{5-3b}{2},-\frac{b+1}{2}\end{matrix}\right) $$

07.17.26.0045.01

$$ {}_0F_1(;b;z)\,Y_{b-1}\left(2\sqrt{z}\right) = -2^{\frac{1-b}{2}}\sqrt{\pi}\,\Gamma(b)\,G_{2,6}^{3,0}\left(\frac{z^2}{4}\,\middle|\,\begin{matrix}1-\frac{b}{4},\frac{2-b}{4}\\ \frac{3-b}{4},\frac{1-b}{4},\frac{b-1}{4},1-\frac{b}{4},\frac{2-b}{4},\frac{3-3b}{4}\end{matrix}\right)\,/;\,-\frac{\pi}{2}<\arg(z)\le\frac{\pi}{2} $$

07.17.26.0044.01

$$ {}_0F_1(;b;z)\,Y_{1-b}\left(2\sqrt{z}\right) = 2^{\frac{1-b}{2}}\sqrt{\pi}\,\Gamma(b)\,G_{2,6}^{3,0}\left(\frac{z^2}{4}\,\middle|\,\begin{matrix}\frac{b-2}{4},\frac{b}{4}\\ \frac{3-b}{4},\frac{1-b}{4},\frac{b-1}{4},\frac{b-2}{4},\frac{3-3b}{4},\frac{b}{4}\end{matrix}\right)\,/;\,-\frac{\pi}{2}<\arg(z)\le\frac{\pi}{2} $$

07.17.26.0137.01

$$ {}_0F_1\left(;b;-\frac{z^2}{4}\right)Y_\nu(z) = -\frac{2^{b-1}\,\Gamma(b)}{\sqrt{\pi}}\,G_{3,5}^{2,2}\left(z^2\,\middle|\,\begin{matrix}\frac{1-b}{2},1-\frac{b}{2},\frac{1-\nu}{2}\\ -\frac{\nu}{2},\frac{\nu}{2},\frac{1-\nu}{2},-b-\frac{\nu}{2}+1,-b+\frac{\nu}{2}+1\end{matrix}\right)/; $$

$$ -b-\nu\notin\mathbb{N}\bigwedge\nu-b\notin\mathbb{N}\bigwedge-\frac{\pi}{2}<\arg(z)\le\frac{\pi}{2} $$

07.17.26.0138.01

$$ {}_0F_1\left(;b;-\frac{z^2}{4}\right)Y_{b+n}(z) = \frac{2^{b-1}\,\Gamma(b)}{\sqrt{\pi}}\,G_{3,5}^{2,2}\left(z^2\,\middle|\,\begin{matrix}\frac{1-b}{2},1-\frac{b}{2},\frac{1}{2}(-b+n+1)\\ \frac{1}{2}(-b+n+2),\frac{b+n}{2},\frac{1}{2}(-3b-n+2),\frac{1}{2}(-b-n),\frac{1}{2}(-b+n+1)\end{matrix}\right)+ $$

$$ \frac{(-1)^{n+1}\,2^b\,z^{-b-n}\csc(b\pi)\,\Gamma(b)}{\sqrt{\pi}}\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{(-1)^{k+\left\lfloor\frac{n+1}{2}\right\rfloor}\,z^{2k}\,\Gamma\left(k-n+\left\lfloor\frac{n}{2}\right\rfloor+\frac{1}{2}\right)\left(1-k+\left\lfloor\frac{n}{2}\right\rfloor\right)_{n-\left\lfloor\frac{n}{2}\right\rfloor}}{k!\,\Gamma(b+k)\,\Gamma(1-b+k-n)}\,/;\,n\in\mathbb{N}\bigwedge-\frac{\pi}{2}<\arg(z)\le\frac{\pi}{2} $$

07.17.26.0139.01

$$ {}_0F_1\left(;b;-\frac{z^2}{4}\right)Y_{-b-n}(z) = \frac{(-1)^n\,2^{b-1}\,\Gamma(b)}{\sqrt{\pi}}\,G_{3,5}^{2,2}\left(z^2\,\middle|\,\begin{matrix}\frac{1-b}{2},1-\frac{b}{2},\frac{1}{2}(b+n+1)\\ \frac{1}{2}(-b+n+2),\frac{b+n}{2},\frac{1}{2}(-3b-n+2),\frac{1}{2}(-b-n),\frac{1}{2}(b+n+1)\end{matrix}\right)- $$

$$ \frac{2^b\,z^{-b-n}\cot(b\pi)\,\Gamma(b)}{\sqrt{\pi}}\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{(-1)^{k+\left\lfloor\frac{n+1}{2}\right\rfloor}\,z^{2k}\,\Gamma\left(k-n+\left\lfloor\frac{n}{2}\right\rfloor+\frac{1}{2}\right)\left(1-k+\left\lfloor\frac{n}{2}\right\rfloor\right)_{n-\left\lfloor\frac{n}{2}\right\rfloor}}{k!\,\Gamma(b+k)\,\Gamma(-b+k-n+1)}\,/;\,n\in\mathbb{N}\bigwedge-\frac{\pi}{2}<\arg(z)\le\frac{\pi}{2} $$

07.17.26.0140.01

$$ {}_0F_1\left(;b;-\frac{z^2}{4}\right)Y_b(z) = \frac{2^{b-1}\,\Gamma(b)}{\sqrt{\pi}}\,G_{2,4}^{2,1}\left(z^2\,\middle|\,\begin{matrix}1-\frac{b}{2},\frac{1-b}{2}\\ 1-\frac{b}{2},\frac{b}{2},1-\frac{3b}{2},-\frac{b}{2}\end{matrix}\right)-\frac{2^b\,z^{-b}\csc(b\pi)}{\Gamma(1-b)}\,/;\,-\frac{\pi}{2}<\arg(z)\le\frac{\pi}{2} $$

07.17.26.0141.01

$$ {}_0F_1\left(;b;-\frac{z^2}{4}\right)Y_{b+1}(z) = \frac{2^{b+1}\csc(b\pi)\,z^{-b-1}}{\Gamma(-b)}+\frac{2^{b-1}\,\Gamma(b)}{\sqrt{\pi}}\,G_{2,4}^{2,1}\left(z^2\,\middle|\,\begin{matrix}\frac{1-b}{2},1-\frac{b}{2}\\ \frac{3-b}{2},\frac{b+1}{2},\frac{1}{2}(1-3b),\frac{1}{2}(-b-1)\end{matrix}\right)/;\,-\frac{\pi}{2}<\arg(z)\le\frac{\pi}{2} $$

07.17.26.0142.01

$$ {}_0F_1\left(;b;-\frac{z^2}{4}\right)Y_{-b}(z) = -\frac{2^{b-1}\cos(b\pi)\,\Gamma(b)\,z^{-b}}{\pi}-\frac{2^{b-1}\,\Gamma(b)}{\sqrt{\pi}}\,G_{2,4}^{2,1}\left(z^2\,\middle|\,\begin{matrix}\frac{1-b}{2},\frac{b+1}{2}\\ -\frac{b}{2},\frac{b}{2},1-\frac{3b}{2},\frac{b+1}{2}\end{matrix}\right)/;\,-\frac{\pi}{2}<\arg(z)\le\frac{\pi}{2} $$

07.17.26.0143.01

$${}_0F_1\left(;b;-\frac{z^2}{4}\right)Y_{-b}(z)=\frac{2^{b-1}\,\Gamma(b)}{\sqrt{\pi}}\,G_{3,5}^{2,2}\left(z^2\,\middle|\,\begin{matrix}\frac{1-b}{2},1-\frac{b}{2},\frac{b+1}{2}\\1-\frac{b}{2},\frac{b}{2},1-\frac{3b}{2},-\frac{b}{2},\frac{b+1}{2}\end{matrix}\right)-\frac{2^b\,z^{-b}\cot(b\,\pi)}{\Gamma(1-b)}\;/;\;-\frac{\pi}{2}<\arg(z)\le\frac{\pi}{2}$$

07.17.26.0144.01

$${}_0F_1\left(;b;-\frac{z^2}{4}\right)Y_{-b-1}(z)=-\frac{2^{b+1}\cot(b\,\pi)\,z^{-b-1}}{\Gamma(-b)}-\frac{2^{b-1}\,\Gamma(b)}{\sqrt{\pi}}\,G_{3,5}^{2,2}\left(z^2\,\middle|\,\begin{matrix}\frac{1-b}{2},1-\frac{b}{2},\frac{b+2}{2}\\\frac{3-b}{2},\frac{b+1}{2},\frac{1}{2}(1-3\,b),\frac{1}{2}(-b-1),\frac{b+2}{2}\end{matrix}\right)/;\;-\frac{\pi}{2}<\arg(z)\le\frac{\pi}{2}$$

07.17.26.0046.01

$${}_0F_1(;b;\sqrt{z}\,)\,Y_{1-b}\left(2\sqrt[4]{z}\right)=2^{\frac{1-b}{2}}\sqrt{\pi}\;\Gamma(b)\,G_{2,6}^{3,0}\left(\frac{z}{4}\,\middle|\,\begin{matrix}\frac{b-2}{4},\frac{b}{4}\\\frac{3-b}{4},\frac{1-b}{4},\frac{b-1}{4},\frac{b-2}{4},\frac{3-3\,b}{4},\frac{b}{4}\end{matrix}\right)$$

07.17.26.0047.01

$${}_0F_1(;b;\sqrt{z}\,)\,Y_{b-1}\left(2\sqrt[4]{z}\right)=-2^{\frac{1-b}{2}}\sqrt{\pi}\;\Gamma(b)\,G_{2,6}^{3,0}\left(\frac{z}{4}\,\middle|\,\begin{matrix}1-\frac{b}{4},\frac{2-b}{4}\\\frac{3-b}{4},\frac{1-b}{4},\frac{b-1}{4},1-\frac{b}{4},\frac{2-b}{4},\frac{3-3\,b}{4}\end{matrix}\right)$$

Classical cases involving Bessel *K*

07.17.26.0048.01

$${}_0F_1(;b;z)\,K_\nu\left(2\sqrt{z}\,\right)=\frac{2^{b-2}\,\Gamma(b)}{\sqrt{\pi}}\,G_{2,4}^{2,2}\left(4\,z\,\middle|\,\begin{matrix}\frac{1-b}{2},1-\frac{b}{2}\\-\frac{\nu}{2},\frac{\nu}{2},1-b-\frac{\nu}{2},1-b+\frac{\nu}{2}\end{matrix}\right)/;\;-b-\nu\notin\mathbb{N}\bigwedge-b+\nu\notin\mathbb{N}$$

07.17.26.0145.01

$${}_0F_1(;b;z)\,K_{b+n}\left(2\sqrt{z}\,\right)=\pi\,(-1)^{n-1}\,2^{b-\frac{3}{2}}\,\Gamma(b)$$
$$\left(\sqrt{\pi}\,\csc\left(\frac{1}{4}\left(4\,b+(-1)^n\right)\pi\right)G_{4,6}^{2,2}\left(4\,z\,\middle|\,\begin{matrix}\frac{1-b}{2},1-\frac{b}{2},\frac{1}{4}(2\,b+1),\frac{1}{4}(3-2\,b)\\1-\frac{b-n}{2},\frac{b+n}{2},\frac{1}{4}(2\,b+1),\frac{1}{4}(3-2\,b),-\frac{1}{2}(b+n),1-\frac{1}{2}(3\,b+n)\end{matrix}\right)-\right.$$
$$\left.\frac{\csc(b\,\pi)\,2^{-b-n+\frac{1}{2}}\,z^{-\frac{b+n}{2}}}{\sqrt{\pi}}\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{(-1)^{\left\lfloor\frac{n+1}{2}\right\rfloor}\,4^k\,z^k\,\Gamma\left(k-n+\left\lfloor\frac{n}{2}\right\rfloor+\frac{1}{2}\right)\left(1-k+\left\lfloor\frac{n}{2}\right\rfloor\right)_{n-\left\lfloor\frac{n}{2}\right\rfloor}}{k!\,\Gamma(b+k)\,\Gamma(1-b+k-n)}\right)/;\;n\in\mathbb{N}$$

07.17.26.0146.01

$${}_0F_1(;b;z)\,K_{-b-n}\left(2\sqrt{z}\,\right)=\pi\,(-1)^{n-1}\,2^{b-\frac{3}{2}}\,\Gamma(b)$$
$$\left(\sqrt{\pi}\,\csc\left(\frac{1}{4}\left(4\,b+(-1)^n\right)\pi\right)G_{4,6}^{2,2}\left(4\,z\,\middle|\,\begin{matrix}\frac{1-b}{2},1-\frac{b}{2},\frac{1}{4}(2\,b+1),\frac{1}{4}(3-2\,b)\\1-\frac{b-n}{2},\frac{b+n}{2},\frac{1}{4}(2\,b+1),\frac{1}{4}(3-2\,b),-\frac{1}{2}(b+n),1-\frac{1}{2}(3\,b+n)\end{matrix}\right)-\right.$$
$$\left.\frac{\csc(b\,\pi)\,2^{-b-n+\frac{1}{2}}\,z^{-\frac{b+n}{2}}}{\sqrt{\pi}}\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{(-1)^{\left\lfloor\frac{n+1}{2}\right\rfloor}\,4^k\,z^k\,\Gamma\left(k-n+\left\lfloor\frac{n}{2}\right\rfloor+\frac{1}{2}\right)\left(1-k+\left\lfloor\frac{n}{2}\right\rfloor\right)_{n-\left\lfloor\frac{n}{2}\right\rfloor}}{k!\,\Gamma(b+k)\,\Gamma(1-b+k-n)}\right)/;\;n\in\mathbb{N}$$

07.17.26.0147.01

$${}_0F_1(;b;z)\,K_b\left(2\sqrt{z}\right)=\frac{1}{2}\,\Gamma(b)\left(z^{-\frac{b}{2}}-2^{b-\frac{1}{2}}\,\pi^{3/2}\csc\left(\frac{1}{4}\,(4\,b+1)\,\pi\right)G_{4,6}^{2,2}\left(4\,z\,\middle|\,\begin{array}{c}\frac{1-b}{2},\,1-\frac{b}{2},\,\frac{1}{4}\,(3-2\,b),\,\frac{1}{4}\,(2\,b+1)\\ 1-\frac{b}{2},\,\frac{b}{2},\,\frac{1}{4}\,(3-2\,b),\,1-\frac{3\,b}{2},\,-\frac{b}{2},\,\frac{1}{4}\,(2\,b+1)\end{array}\right)\right)$$

07.17.26.0148.01

$${}_0F_1(;b;z)\,K_{b+1}\left(2\sqrt{z}\right)=$$
$$\frac{1}{2}\,\Gamma(b)\left(b\,z^{-\frac{1}{2}(b+1)}+2^{b-\frac{1}{2}}\,\pi^{3/2}\csc\left(\frac{1}{4}\,(4\,b-1)\,\pi\right)G_{4,6}^{2,2}\left(4\,z\,\middle|\,\begin{array}{c}\frac{1-b}{2},\,1-\frac{b}{2},\,\frac{1}{4}\,(3-2\,b),\,\frac{1}{4}\,(2\,b+1)\\ \frac{3-b}{2},\,\frac{b+1}{2},\,\frac{1}{2}\,(1-3\,b),\,\frac{1}{4}\,(3-2\,b),\,-\frac{1}{2}\,(b+1),\,\frac{1}{4}\,(2\,b+1)\end{array}\right)\right)$$

07.17.26.0149.01

$${}_0F_1(;b;z)\,K_{-b}\left(2\sqrt{z}\right)=\frac{1}{2}\,\Gamma(b)\left(z^{-\frac{b}{2}}-2^{b-\frac{1}{2}}\,\pi^{3/2}\csc\left(\frac{1}{4}\,(4\,b+1)\,\pi\right)G_{4,6}^{2,2}\left(4\,z\,\middle|\,\begin{array}{c}\frac{1-b}{2},\,1-\frac{b}{2},\,\frac{1}{4}\,(3-2\,b),\,\frac{1}{4}\,(2\,b+1)\\ 1-\frac{b}{2},\,\frac{b}{2},\,\frac{1}{4}\,(3-2\,b),\,1-\frac{3\,b}{2},\,-\frac{b}{2},\,\frac{1}{4}\,(2\,b+1)\end{array}\right)\right)$$

07.17.26.0150.01

$${}_0F_1(;b;z)\,K_{-b-1}\left(2\sqrt{z}\right)=$$
$$\frac{1}{2}\,\Gamma(b)\left(b\,z^{-\frac{1}{2}(b+1)}+2^{b-\frac{1}{2}}\,\pi^{3/2}\csc\left(\frac{1}{4}\,(4\,b-1)\,\pi\right)G_{4,6}^{2,2}\left(4\,z\,\middle|\,\begin{array}{c}\frac{1-b}{2},\,1-\frac{b}{2},\,\frac{1}{4}\,(3-2\,b),\,\frac{1}{4}\,(2\,b+1)\\ \frac{3-b}{2},\,\frac{b+1}{2},\,\frac{1}{2}\,(1-3\,b),\,\frac{1}{4}\,(3-2\,b),\,-\frac{1}{2}\,(b+1),\,\frac{1}{4}\,(2\,b+1)\end{array}\right)\right)$$

07.17.26.0049.01

$${}_0F_1(;b;-z)\,K_{b-1}\left(2\sqrt{z}\right)=\frac{2^{-\frac{b+3}{2}}\,\Gamma(b)}{\sqrt{\pi}}\,G_{0,4}^{3,0}\left(\frac{z^2}{4}\,\middle|\,\frac{3-b}{4},\,\frac{1-b}{4},\,\frac{b-1}{4},\,\frac{3-3\,b}{4}\right)/;\,-\frac{\pi}{2}<\arg(z)\leq\frac{\pi}{2}$$

07.17.26.0050.01

$${}_0F_1(;b;-z)\,K_{1-b}\left(2\sqrt{z}\right)=\frac{2^{-\frac{b+3}{2}}\,\Gamma(b)}{\sqrt{\pi}}\,G_{0,4}^{3,0}\left(\frac{z^2}{4}\,\middle|\,\frac{3-b}{4},\,\frac{1-b}{4},\,\frac{b-1}{4},\,\frac{3-3\,b}{4}\right)/;\,-\frac{\pi}{2}<\arg(z)\leq\frac{\pi}{2}$$

07.17.26.0151.01

$${}_0F_1\left(;b;\frac{z^2}{4}\right)K_\nu(z)=\frac{2^{b-2}\,\Gamma(b)}{\sqrt{\pi}}\,G_{2,4}^{2,2}\left(z^2\,\middle|\,\begin{array}{c}\frac{1-b}{2},\,1-\frac{b}{2}\\ -\frac{\nu}{2},\,\frac{\nu}{2},\,-b-\frac{\nu}{2}+1,\,-b+\frac{\nu}{2}+1\end{array}\right)/;\,-b-\nu\notin\mathbb{N}\bigwedge\nu-b\notin\mathbb{N}\bigwedge-\frac{\pi}{2}<\arg(z)\leq\frac{\pi}{2}$$

07.17.26.0152.01

$${}_0F_1\left(;b;\frac{z^2}{4}\right)K_{b+n}(z)=\pi\,(-1)^{n-1}\,2^{b-\frac{3}{2}}\,\Gamma(b)$$
$$\left(\sqrt{\pi}\,\csc\left(\frac{1}{4}\,(4\,b+(-1)^n)\,\pi\right)G_{4,6}^{2,2}\left(z^2\,\middle|\,\begin{array}{c}\frac{1-b}{2},\,1-\frac{b}{2},\,\frac{1}{4}\,(2\,b+1),\,\frac{1}{4}\,(3-2\,b)\\ 1-\frac{b-n}{2},\,\frac{b+n}{2},\,\frac{1}{4}\,(2\,b+1),\,\frac{1}{4}\,(3-2\,b),\,-\frac{1}{2}\,(b+n),\,1-\frac{1}{2}\,(3\,b+n)\end{array}\right)-\right.$$
$$\left.\frac{\csc(b\,\pi)\sqrt{2}}{\sqrt{\pi}}\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{(-1)^{\left\lfloor\frac{n+1}{2}\right\rfloor}\,z^{-b+2k-n}\,\Gamma\left(k-n+\left\lfloor\frac{n}{2}\right\rfloor+\frac{1}{2}\right)\left(1-k+\left\lfloor\frac{n}{2}\right\rfloor\right)_{n-\left\lfloor\frac{n}{2}\right\rfloor}}{k!\,\Gamma(b+k)\,\Gamma(1-b+k-n)}\right)/;\,n\in\mathbb{N}\bigwedge-\frac{\pi}{2}<\arg(z)\leq\frac{\pi}{2}$$

07.17.26.0153.01

$$ {}_0F_1\left(;b;\frac{z^2}{4}\right)K_{-b-n}(z)=\pi\,(-1)^{n-1}\,2^{b-\frac{3}{2}}\,\Gamma(b)\left(\sqrt{\pi}\,\csc\left(\frac{1}{4}\,(4\,b+(-1)^n)\,\pi\right)G_{4,6}^{2,2}\left(z^2\,\middle|\,\begin{array}{c}\frac{1-b}{2},\,1-\frac{b}{2},\,\frac{1}{4}\,(2\,b+1),\,\frac{1}{4}\,(3-2\,b)\\ 1-\frac{b-n}{2},\,\frac{b+n}{2},\,\frac{1}{4}\,(2\,b+1),\,\frac{1}{4}\,(3-2\,b),\,-\frac{1}{2}\,(b+n),\,1-\frac{1}{2}\,(3\,b+n)\end{array}\right)-\frac{\csc(b\,\pi)\,\sqrt{2}}{\sqrt{\pi}}\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{(-1)^{\left\lfloor\frac{n+1}{2}\right\rfloor}\,z^{-b+2\,k-n}\,\Gamma\left(k-n+\left\lfloor\frac{n}{2}\right\rfloor+\frac{1}{2}\right)\left(1-k+\left\lfloor\frac{n}{2}\right\rfloor\right)_{n-\left\lfloor\frac{n}{2}\right\rfloor}}{k!\,\Gamma(b+k)\,\Gamma(1-b+k-n)}\right)\;/;\;n\in\mathbb{N}\bigwedge-\frac{\pi}{2}<\arg(z)\leq\frac{\pi}{2} $$

07.17.26.0154.01

$$ {}_0F_1\left(;b;\frac{z^2}{4}\right)K_b(z)=2^{b-1}\,\Gamma(b)\left(z^{-b}-\frac{\pi^{3/2}\csc\left(\frac{1}{4}\,(4\,b+1)\,\pi\right)}{\sqrt{2}}\,G_{4,6}^{2,2}\left(z^2\,\middle|\,\begin{array}{c}\frac{1-b}{2},\,1-\frac{b}{2},\,\frac{1}{4}\,(3-2\,b),\,\frac{1}{4}\,(2\,b+1)\\ 1-\frac{b}{2},\,\frac{b}{2},\,\frac{1}{4}\,(3-2\,b),\,1-\frac{3\,b}{2},\,-\frac{b}{2},\,\frac{1}{4}\,(2\,b+1)\end{array}\right)\right)/;\;-\frac{\pi}{2}<\arg(z)\leq\frac{\pi}{2} $$

07.17.26.0155.01

$$ {}_0F_1\left(;b;\frac{z^2}{4}\right)K_{b+1}(z)=2^{b-1}\,\Gamma(b)\left(2\,b\,z^{-b-1}+\sqrt{\frac{\pi}{2}}\,\pi\csc\left(\frac{1}{4}\,(4\,b-1)\,\pi\right)G_{4,6}^{2,2}\left(z^2\,\middle|\,\begin{array}{c}\frac{1-b}{2},\,1-\frac{b}{2},\,\frac{1}{4}\,(3-2\,b),\,\frac{1}{4}\,(2\,b+1)\\ \frac{3-b}{2},\,\frac{b+1}{2},\,\frac{1}{2}\,(1-3\,b),\,\frac{1}{4}\,(3-2\,b),\,-\frac{1}{2}\,(b+1),\,\frac{1}{4}\,(2\,b+1)\end{array}\right)\right)/;\;-\frac{\pi}{2}<\arg(z)\leq\frac{\pi}{2} $$

07.17.26.0156.01

$$ {}_0F_1\left(;b;\frac{z^2}{4}\right)K_{-b}(z)=2^{b-1}\,\Gamma(b)\left(z^{-b}-\frac{\pi^{3/2}\csc\left(\frac{1}{4}\,(4\,b+1)\,\pi\right)}{\sqrt{2}}\,G_{4,6}^{2,2}\left(z^2\,\middle|\,\begin{array}{c}\frac{1-b}{2},\,1-\frac{b}{2},\,\frac{1}{4}\,(3-2\,b),\,\frac{1}{4}\,(2\,b+1)\\ 1-\frac{b}{2},\,\frac{b}{2},\,\frac{1}{4}\,(3-2\,b),\,1-\frac{3\,b}{2},\,-\frac{b}{2},\,\frac{1}{4}\,(2\,b+1)\end{array}\right)\right)/;\;-\frac{\pi}{2}<\arg(z)\leq\frac{\pi}{2} $$

07.17.26.0157.01

$$ {}_0F_1\left(;b;\frac{z^2}{4}\right)K_{-b-1}(z)=2^{b-1}\,\Gamma(b)\left(2\,b\,z^{-b-1}+\sqrt{\frac{\pi}{2}}\,\pi\csc\left(\frac{1}{4}\,(4\,b-1)\,\pi\right)G_{4,6}^{2,2}\left(z^2\,\middle|\,\begin{array}{c}\frac{1-b}{2},\,1-\frac{b}{2},\,\frac{1}{4}\,(3-2\,b),\,\frac{1}{4}\,(2\,b+1)\\ \frac{3-b}{2},\,\frac{b+1}{2},\,\frac{1}{2}\,(1-3\,b),\,\frac{1}{4}\,(3-2\,b),\,-\frac{1}{2}\,(b+1),\,\frac{1}{4}\,(2\,b+1)\end{array}\right)\right)/;\;-\frac{\pi}{2}<\arg(z)\leq\frac{\pi}{2} $$

07.17.26.0051.01

$$ {}_0F_1\left(;b;-\sqrt{z}\right)K_{b-1}\left(2\sqrt[4]{z}\right)=\frac{2^{-\frac{b+3}{2}}\,\Gamma(b)}{\sqrt{\pi}}\,G_{0,4}^{3,0}\left(\frac{z}{4}\,\middle|\,\frac{3-b}{4},\,\frac{1-b}{4},\,\frac{b-1}{4},\,\frac{3-3\,b}{4}\right) $$

07.17.26.0052.01

$$ {}_0F_1\left(;b;-\sqrt{z}\right)K_{1-b}\left(2\sqrt[4]{z}\right)=\frac{2^{-\frac{b+3}{2}}\,\Gamma(b)}{\sqrt{\pi}}\,G_{0,4}^{3,0}\left(\frac{z}{4}\,\middle|\,\frac{3-b}{4},\frac{1-b}{4},\frac{b-1}{4},\frac{3-3b}{4}\right) $$

Classical cases involving ${}_0\tilde{F}_1$

07.17.26.0053.01

$$ {}_0F_1(;b;z)\,{}_0\tilde{F}_1(;c;z)=\sqrt{\pi}\,2^{b+c-2}\,\Gamma(b)\,G_{3,5}^{1,2}\left(4z\,\middle|\,\begin{matrix}1-\frac{b+c}{2},\frac{3-b-c}{2},\frac{1}{2}\\ 0,1-b,1-c,-b-c+2,\frac{1}{2}\end{matrix}\right)/;1-b-c\notin\mathbb{N} $$

07.17.26.0054.01

$$ {}_0F_1(;b;z)\,{}_0\tilde{F}_1(;c;z)=\frac{2^{b+c-2}\,\Gamma(b)}{\sqrt{\pi}}\,G_{2,4}^{1,2}\left(-4z\,\middle|\,\begin{matrix}1-\frac{b+c}{2},\frac{3-b-c}{2}\\ 0,1-b,1-c,2-b-c\end{matrix}\right)/;1-b-c\notin\mathbb{N} $$

07.17.26.0158.01

$$ {}_0F_1(;b;z)\,{}_0\tilde{F}_1(;-b-n+1;z)=\frac{\Gamma(b)}{2^{n+1}\sqrt{\pi}}\left(2\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{\left((-1)^{\left\lfloor\frac{n+1}{2}\right\rfloor}\Gamma\left(k-n+\left\lfloor\frac{n}{2}\right\rfloor+\frac{1}{2}\right)\right)\left(1-k+\left\lfloor\frac{n}{2}\right\rfloor\right)_{n-\left\lfloor\frac{n}{2}\right\rfloor}(4z)^k}{k!\,\Gamma(b+k)\,\Gamma(1-b+k-n)}-(-1)^n\pi\,G_{3,5}^{1,2}\left(4z\,\middle|\,\begin{matrix}\frac{n+1}{2},\frac{n+2}{2},\frac{1}{2}\\ n+1,b+n,0,1-b,\frac{1}{2}\end{matrix}\right)\right)/;n\in\mathbb{N} $$

07.17.26.0159.01

$$ {}_0F_1(;b;z)\,{}_0\tilde{F}_1(;1-b;z)=\frac{1}{\Gamma(1-b)}-\frac{\pi^{3/2}\csc(b\pi)}{2\,\Gamma(1-b)}\,G_{2,4}^{1,1}\left(4z\,\middle|\,\begin{matrix}1,\frac{1}{2}\\ 1,b,0,1-b\end{matrix}\right) $$

07.17.26.0160.01

$$ {}_0F_1(;b;z)\,{}_0\tilde{F}_1(;-b;z)=\frac{1}{4}\sqrt{\pi}\,\Gamma(b)\,G_{3,5}^{1,2}\left(4z\,\middle|\,\begin{matrix}1,\frac{3}{2},\frac{1}{2}\\ 2,b+1,0,1-b,\frac{1}{2}\end{matrix}\right)+\frac{1}{\Gamma(-b)} $$

07.17.26.0055.01

$$ {}_0F_1(;b;z)\,{}_0\tilde{F}_1(;2-b;z)=\sqrt{\pi}\,\Gamma(b)\,G_{2,4}^{1,1}\left(4z\,\middle|\,\begin{matrix}\frac{1}{2},\frac{1}{2}\\ 0,1-b,b-1,\frac{1}{2}\end{matrix}\right) $$

07.17.26.0056.01

$$ {}_0F_1(;b;z)\,{}_0\tilde{F}_1(;2-b;z)=\frac{\Gamma(b)}{\sqrt{\pi}}\,G_{1,3}^{1,1}\left(-4z\,\middle|\,\begin{matrix}\frac{1}{2}\\ 0,1-b,b-1\end{matrix}\right) $$

07.17.26.0057.01

$$ {}_0F_1(;b;z)\,{}_0\tilde{F}_1(;b-1;z)=2^{2b-3}\sqrt{\pi}\,\Gamma(b)\,G_{2,4}^{1,1}\left(4z\,\middle|\,\begin{matrix}\frac{3}{2}-b,\frac{1}{2}\\ 0,1-b,3-2b,\frac{1}{2}\end{matrix}\right) $$

07.17.26.0058.01

$$ {}_0F_1(;b;z)\,{}_0\tilde{F}_1(;b-1;z)=\frac{2^{2b-3}\,\Gamma(b)}{\sqrt{\pi}}\,G_{1,3}^{1,1}\left(-4z\,\middle|\,\begin{matrix}\frac{3}{2}-b\\ 0,1-b,3-2b\end{matrix}\right) $$

07.17.26.0059.01

$${}_0F_1(;b;z)\,{}_0\tilde{F}_1(;b;z) = 2^{2b-2}\sqrt{\pi}\,\Gamma(b)\,G_{2,4}^{1,1}\!\left(4z\,\middle|\,\begin{matrix}\frac{3}{2}-b,\frac{1}{2}\\ 0,1-b,2-2b,\frac{1}{2}\end{matrix}\right)$$

07.17.26.0060.01

$${}_0F_1(;b;z)\,{}_0\tilde{F}_1(;b;z) = \frac{2^{2b-2}\,\Gamma(b)}{\sqrt{\pi}}\,G_{1,3}^{1,1}\!\left(-4z\,\middle|\,\begin{matrix}\frac{3}{2}-b\\ 0,1-b,2-2b\end{matrix}\right)$$

07.17.26.0061.01

$${}_0F_1(;b;z)\,{}_0\tilde{F}_1(;b+1;z) = 2^{2b-1}\sqrt{\pi}\,\Gamma(b)\,G_{2,4}^{1,1}\!\left(4z\,\middle|\,\begin{matrix}\frac{1}{2}-b,\frac{1}{2}\\ 0,-b,1-2b,\frac{1}{2}\end{matrix}\right)$$

07.17.26.0062.01

$${}_0F_1(;b;z)\,{}_0\tilde{F}_1(;b+1;z) = \frac{2^{2b-1}\,b\,\Gamma(b)}{b\sqrt{\pi}}\,G_{1,3}^{1,1}\!\left(-4z\,\middle|\,\begin{matrix}\frac{1}{2}-b\\ 0,-b,1-2b\end{matrix}\right)$$

07.17.26.0063.01

$${}_0F_1(;b;z)\,{}_0\tilde{F}_1(;b;-z) = \sqrt{\pi}\;2^{1-b}\,\Gamma(b)\,G_{0,4}^{1,0}\!\left(\frac{z^2}{4}\,\middle|\,0,\frac{1-b}{2},1-\frac{b}{2},1-b\right)$$

07.17.26.0064.01

$${}_0F_1(;b;z)\,{}_0\tilde{F}_1(;2-b;-z) = \sqrt{\pi}\,\Gamma(b)\,G_{1,5}^{2,0}\!\left(\frac{z^2}{4}\,\middle|\,\begin{matrix}1-\frac{b}{2}\\ 0,\frac{1}{2},1-\frac{b}{2},\frac{1-b}{2},\frac{b-1}{2}\end{matrix}\right)\;/;\;\mathrm{Re}(z)<0\bigvee\arg(z)=-\frac{\pi}{2}$$

Generalized cases involving cos

07.17.26.0161.01

$$\cos(z)\,{}_0F_1\!\left(;b;-\frac{z^2}{4}\right) = 2^{b-\frac{3}{2}}\,\Gamma(b)\,G_{3,5}^{2,2}\!\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1}{4}(3-2b),\frac{1}{4}(5-2b),\frac{1}{2}\\ 0,\frac{1}{2},1-b,\frac{3}{2}-b,\frac{1}{2}\end{matrix}\right)$$

07.17.26.0065.01

$$\cos(a+z)\,{}_0F_1\!\left(;b;-\frac{z^2}{4}\right) = 2^{b-\frac{3}{2}}\,\Gamma(b)\,G_{3,5}^{2,2}\!\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{3-2b}{4},\frac{5-2b}{4},\frac{a}{\pi}+\frac{1}{2}\\ 0,\frac{1}{2},1-b,\frac{3}{2}-b,\frac{a}{\pi}+\frac{1}{2}\end{matrix}\right)$$

Generalized cases involving sin

07.17.26.0066.01

$$\sin(z)\,{}_0F_1\!\left(;b;-\frac{z^2}{4}\right) = 2^{b-\frac{3}{2}}\,\Gamma(b)\,G_{2,4}^{1,2}\!\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{3-2b}{4},\frac{5-2b}{4}\\ \frac{1}{2},0,1-b,\frac{3}{2}-b\end{matrix}\right)$$

07.17.26.0067.01

$$\sin(a+z)\,{}_0F_1\!\left(;b;-\frac{z^2}{4}\right) = 2^{b-\frac{3}{2}}\,\Gamma(b)\,G_{3,5}^{2,2}\!\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{3}{4}-\frac{b}{2},\frac{5}{4}-\frac{b}{2},\frac{a}{\pi}\\ 0,\frac{1}{2},1-b,\frac{3}{2}-b,\frac{a}{\pi}\end{matrix}\right)$$

Generalized cases involving cosh

07.17.26.0162.01

$$\cosh(z)\,{}_0F_1\left(;b;\frac{z^2}{4}\right)=2^{b-\frac{3}{2}}\pi\sec\left(\frac{b\pi}{2}\right)\Gamma(b)\,G_{3,5}^{1,2}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1}{4}(3-2b),\,\frac{1}{4}(5-2b),\,\frac{1-b}{2}\\ 0,\,\frac{1}{2},\,1-b,\,\frac{3}{2}-b,\,\frac{1-b}{2}\end{matrix}\right)$$

07.17.26.0163.01

$$\cosh(a+z)\,{}_0F_1\left(;b;\frac{z^2}{4}\right)=2^{b-\frac{3}{2}}\Gamma(b)\,G_{3,5}^{2,2}\left(i\,z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1}{4}(3-2b),\,\frac{1}{4}(5-2b),\,\frac{1}{2}+\frac{i\,a}{\pi}\\ 0,\,\frac{1}{2},\,1-b,\,\frac{3}{2}-b,\,\frac{1}{2}+\frac{i\,a}{\pi}\end{matrix}\right)$$

Generalized cases involving sinh

07.17.26.0068.01

$$\sinh(z)\,{}_0F_1\left(;b;\frac{z^2}{4}\right)=-2^{b-\frac{3}{2}}\pi\csc\left(\frac{b\pi}{2}\right)\Gamma(b)\,G_{3,5}^{1,2}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{3-2b}{4},\,\frac{5-2b}{4},\,\frac{1-b}{2}\\ \frac{1}{2},\,0,\,\frac{1-b}{2},\,1-b,\,\frac{3}{2}-b\end{matrix}\right)$$

07.17.26.0164.01

$$\sinh(z)\,{}_0F_1\left(;b;\frac{z^2}{4}\right)=-\frac{2^{b-\frac{3}{2}}\Gamma(b)}{z}\,G_{2,4}^{1,2}\left(i\,z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{5}{4}-\frac{b}{2},\,\frac{7}{4}-\frac{b}{2}\\ 1,\,\frac{1}{2},\,\frac{3}{2}-b,\,2-b\end{matrix}\right)$$

07.17.26.0165.01

$$\sinh(a+z)\,{}_0F_1\left(;b;\frac{z^2}{4}\right)=-2^{b-\frac{3}{2}}\,i\,\Gamma(b)\,G_{3,5}^{2,2}\left(i\,z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1}{4}(3-2b),\,\frac{1}{4}(5-2b),\,\frac{i\,a}{\pi}\\ 0,\,\frac{1}{2},\,1-b,\,\frac{3}{2}-b,\,\frac{i\,a}{\pi}\end{matrix}\right)$$

Generalized cases involving Ai

07.17.26.0069.01

$$\mathrm{Ai}\left(3^{2/3}\sqrt[3]{z}\right){}_0F_1(;b;z)=\frac{2^{b-\frac{7}{3}}\Gamma(b)}{\sqrt[6]{3}\,\pi^{3/2}}\,G_{2,4}^{2,2}\left(2^{2/3}\sqrt[3]{z},\frac{1}{3}\,\middle|\,\begin{matrix}\frac{1}{6}(4-3b),\frac{1}{6}(7-3b)\\ 0,\frac{1}{3},1-b,\frac{4}{3}-b\end{matrix}\right)$$

07.17.26.0166.01

$$\mathrm{Ai}(z)\,{}_0F_1\left(;b;\frac{z^3}{9}\right)=\frac{2^{b-\frac{7}{3}}\Gamma(b)}{\sqrt[6]{3}\,\pi^{3/2}}\,G_{2,4}^{2,2}\left(\left(\frac{2}{3}\right)^{2/3}z,\frac{1}{3}\,\middle|\,\begin{matrix}\frac{1}{6}(4-3b),\,\frac{1}{6}(7-3b)\\ 0,\,\frac{1}{3},\,1-b,\,\frac{4}{3}-b\end{matrix}\right)$$

Generalized cases involving Ai′

07.17.26.0070.01

$$\mathrm{Ai}'\left(3^{2/3}\sqrt[3]{z}\right){}_0F_1(;b;z)=-\frac{2^{b-\frac{8}{3}}\sqrt[6]{3}\,\Gamma(b)}{\pi^{3/2}}\,G_{2,4}^{2,2}\left(2^{2/3}\sqrt[3]{z},\frac{1}{3}\,\middle|\,\begin{matrix}\frac{1}{6}(5-3b),\,\frac{1}{6}(8-3b)\\ 0,\,\frac{2}{3},\,1-b,\,\frac{5}{3}-b\end{matrix}\right)$$

07.17.26.0167.01

$$\mathrm{Ai}'(z)\,{}_0F_1\left(;b;\frac{z^3}{9}\right)=-\frac{2^{b-\frac{8}{3}}\sqrt[6]{3}\,\Gamma(b)}{\pi^{3/2}}\,G_{2,4}^{2,2}\left(\left(\frac{2}{3}\right)^{2/3}z,\frac{1}{3}\,\middle|\,\begin{matrix}\frac{1}{6}(5-3b),\,\frac{1}{6}(8-3b)\\ 0,\,\frac{2}{3},\,1-b,\,\frac{5}{3}-b\end{matrix}\right)$$

Generalized cases involving Bi

07.17.26.0071.01

$$\operatorname{Bi}\left(3^{2/3}\sqrt[3]{z}\right){}_0F_1(;b;z) = \frac{2^{b-\frac{1}{3}}\sqrt{\pi}\,\Gamma(b)}{\sqrt[6]{3}} G_{4,6}^{2,2}\left(2^{2/3}\sqrt[3]{z}, \frac{1}{3} \middle| \begin{array}{c} \frac{1}{6}(4-3b), \frac{1}{6}(7-3b), \frac{1}{6}, \frac{2}{3} \\ 0, \frac{1}{3}, \frac{1}{6}, \frac{2}{3}, 1-b, \frac{4}{3}-b \end{array}\right)$$

07.17.26.0168.01

$$\operatorname{Bi}(z)\,{}_0F_1\left(;b;\frac{z^3}{9}\right) = \frac{2^{b-\frac{1}{3}}\sqrt{\pi}\,\Gamma(b)}{\sqrt[6]{3}} G_{4,6}^{2,2}\left(\left(\frac{2}{3}\right)^{2/3} z, \frac{1}{3} \middle| \begin{array}{c} \frac{1}{6}(4-3b), \frac{1}{6}(7-3b), \frac{1}{6}, \frac{2}{3} \\ 0, \frac{1}{3}, \frac{1}{6}, \frac{2}{3}, 1-b, \frac{4}{3}-b \end{array}\right)$$

Generalized cases involving Bi′

07.17.26.0072.01

$$\operatorname{Bi}'\left(3^{2/3}\sqrt[3]{z}\right){}_0F_1(;b;z) = 2^{b-\frac{2}{3}}\sqrt[6]{3}\sqrt{\pi}\,\Gamma(b)\, G_{4,6}^{2,2}\left(2^{2/3}\sqrt[3]{z}, \frac{1}{3} \middle| \begin{array}{c} \frac{1}{6}(5-3b), \frac{1}{6}(8-3b), \frac{1}{3}, \frac{5}{6} \\ 0, \frac{2}{3}, \frac{1}{3}, \frac{5}{6}, 1-b, \frac{5}{3}-b \end{array}\right)$$

07.17.26.0169.01

$$\operatorname{Bi}'(z)\,{}_0F_1\left(;b;\frac{z^3}{9}\right) = 2^{b-\frac{2}{3}}\sqrt[6]{3}\sqrt{\pi}\,\Gamma(b)\, G_{4,6}^{2,2}\left(\left(\frac{2}{3}\right)^{2/3} z, \frac{1}{3} \middle| \begin{array}{c} \frac{1}{6}(5-3b), \frac{1}{6}(8-3b), \frac{1}{3}, \frac{5}{6} \\ 0, \frac{2}{3}, \frac{1}{3}, \frac{5}{6}, 1-b, \frac{5}{3}-b \end{array}\right)$$

Classical cases for products of ${}_0F_1$

07.17.26.0170.01

$${}_0F_1(;b;z)\,{}_0F_1(;b;-z) = \sqrt{\pi}\,2^{1-b}\,\Gamma(b)^2\, G_{0,4}^{1,0}\left(\frac{z}{2}, \frac{1}{2} \middle| 0, \frac{1}{2}-\frac{b}{2}, 1-\frac{b}{2}, 1-b\right)$$

07.17.26.0171.01

$${}_0F_1(;b;z)\,{}_0F_1(;2-b;-z) = (1-b)\,\pi^{3/2}\csc(\pi b)\, G_{1,5}^{2,0}\left(-\frac{z}{2}, \frac{1}{2} \middle| \begin{array}{c} 1-\frac{b}{2} \\ 0, \frac{1}{2}, 1-\frac{b}{2}, \frac{1-b}{2}, \frac{b-1}{2} \end{array}\right)$$

Generalized cases involving Bessel J

07.17.26.0074.01

$${}_0F_1(;b;z)\,J_{b-1}\left(2\sqrt{z}\right) = 2^{\frac{1-b}{2}}\sqrt{\pi}\,\Gamma(b)\, G_{0,4}^{1,0}\left(\frac{z}{2}, \frac{1}{2} \middle| \frac{b-1}{4}, \frac{1-b}{4}, \frac{3-b}{4}, \frac{3-3b}{4}\right)$$

07.17.26.0075.01

$${}_0F_1(;b;z)\,J_{1-b}\left(2\sqrt{z}\right) = 2^{\frac{1-b}{2}}\sqrt{\pi}\,\Gamma(b)\, G_{1,5}^{2,0}\left(\frac{z}{2}, \frac{1}{2} \middle| \begin{array}{c} \frac{b+1}{4} \\ \frac{3-b}{4}, \frac{1-b}{4}, \frac{3-3b}{4}, \frac{b-1}{4}, \frac{b+1}{4} \end{array}\right)$$

07.17.26.0172.01

$${}_0F_1\left(;b;\frac{z^2}{4}\right) J_{b-1}(z) = 2^{\frac{1-b}{2}}\sqrt{\pi}\,\Gamma(b)\, G_{0,4}^{1,0}\left(\frac{z}{2\sqrt{2}}, \frac{1}{4} \middle| \frac{b-1}{4}, \frac{1-b}{4}, \frac{3-b}{4}, \frac{1}{4}(3-3b)\right)$$

07.17.26.0173.01

$${}_0F_1\left(;b;\frac{z^2}{4}\right) J_{1-b}(z) = 2^{\frac{1-b}{2}}\sqrt{\pi}\,\Gamma(b)\, G_{1,5}^{2,0}\left(\frac{z}{2\sqrt{2}}, \frac{1}{4} \middle| \begin{array}{c} \frac{b+1}{4} \\ \frac{1-b}{4}, \frac{3-b}{4}, \frac{1}{4}(3-3b), \frac{b-1}{4}, \frac{b+1}{4} \end{array}\right)$$

07.17.26.0073.01

$${}_0F_1\left(;b;-\frac{z^2}{4}\right) J_\nu(z) = \frac{\Gamma(b)\,2^{b-1}}{\sqrt{\pi}} G_{2,4}^{1,2}\left(z, \frac{1}{2} \middle| \begin{array}{c} \frac{1-b}{2}, 1-\frac{b}{2} \\ \frac{\nu}{2}, -\frac{\nu}{2}, 1-b+\frac{\nu}{2}, 1-b-\frac{\nu}{2} \end{array}\right) /; -b-\nu \notin \mathbb{N}$$

07.17.26.0174.01

$$ {}_0F_1\left(;b;-\frac{z^2}{4}\right)J_{-b-n}(z)=\frac{2^{b-1}\,\Gamma(b)}{\sqrt{\pi}}\left(2\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{\left((-1)^{k+\left\lfloor\frac{n+1}{2}\right\rfloor}z^{-b+2k-n}\,\Gamma\left(k-n+\left\lfloor\frac{n}{2}\right\rfloor+\frac{1}{2}\right)\right)\left(-k+\left\lfloor\frac{n}{2}\right\rfloor+1\right)_{n-\left\lfloor\frac{n}{2}\right\rfloor}}{k!\,\Gamma(b+k)\,\Gamma(-b+k-n+1)}-\right. $$

$$ \left.(-1)^n\,G_{2,4}^{1,2}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1-b}{2},1-\frac{b}{2}\\ \frac{n-b}{2}+1,\frac{b+n}{2},-\frac{1}{2}(b+n),1-\frac{1}{2}(3b+n)\end{matrix}\right)\right)/;\,n\in\mathbb{N} $$

07.17.26.0175.01

$$ {}_0F_1\left(;b;-\frac{z^2}{4}\right)J_{-b}(z)=\frac{\Gamma(b)}{2\pi}\left(2^{b+1}z^{-b}\sin(b\pi)-2^b\sqrt{\pi}\,G_{2,4}^{1,2}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1-b}{2},1-\frac{b}{2}\\ 1-\frac{b}{2},\frac{b}{2},-\frac{b}{2},1-\frac{3b}{2}\end{matrix}\right)\right) $$

07.17.26.0176.01

$$ {}_0F_1\left(;b;-\frac{z^2}{4}\right)J_{-b-1}(z)=2^{b-1}\left(\frac{4z^{-b-1}}{\Gamma(-b)}+\frac{\Gamma(b)}{\sqrt{\pi}}\,G_{2,4}^{1,2}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1-b}{2},1-\frac{b}{2}\\ \frac{3-b}{2},\frac{b+1}{2},\frac{1}{2}(-b-1),\frac{1}{2}(1-3b)\end{matrix}\right)\right) $$

07.17.26.0177.01

$$ {}_0F_1\left(;b;-\frac{z^2}{4}\right)J_b(z)=\frac{\Gamma(b)\,2^{b-1}}{\sqrt{\pi}}\,G_{1,3}^{1,1}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1-b}{2}\\ \frac{b}{2},-\frac{b}{2},1-\frac{3b}{2}\end{matrix}\right) $$

07.17.26.0178.01

$$ {}_0F_1\left(;b;-\frac{z^2}{4}\right)J_{b-1}(z)=\frac{\Gamma(b)\,2^{b-1}}{\sqrt{\pi}}\,G_{1,3}^{1,1}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}1-\frac{b}{2}\\ \frac{b-1}{2},\frac{1-b}{2},\frac{3(1-b)}{2}\end{matrix}\right) $$

07.17.26.0179.01

$$ {}_0F_1\left(;b;-\frac{z^2}{4}\right)J_{1-b}(z)=\frac{2^{b-1}\,\Gamma(b)}{\sqrt{\pi}}\,G_{1,3}^{1,1}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}1-\frac{b}{2}\\ \frac{1-b}{2},\frac{b-1}{2},\frac{3(1-b)}{2}\end{matrix}\right) $$

Generalized cases involving Bessel *I*

07.17.26.0076.01

$$ {}_0F_1(;b;-z)\,I_{b-1}\left(2\sqrt{z}\right)=2^{\frac{1-b}{2}}\sqrt{\pi}\,\Gamma(b)\,G_{0,4}^{1,0}\left(\frac{z}{2},\frac{1}{2}\,\middle|\,\frac{b-1}{4},\frac{1-b}{4},\frac{3-b}{4},\frac{3-3b}{4}\right) $$

07.17.26.0077.01

$$ {}_0F_1(;b;-z)\,I_{1-b}\left(2\sqrt{z}\right)=2^{\frac{1-b}{2}}\sqrt{\pi}\,\Gamma(b)\,G_{1,5}^{2,0}\left(\frac{z}{2},\frac{1}{2}\,\middle|\,\begin{matrix}\frac{5-3b}{4}\\ \frac{3-b}{4},\frac{1-b}{4},\frac{5-3b}{4},\frac{3-3b}{4},\frac{b-1}{4}\end{matrix}\right) $$

07.17.26.0180.01

$$ {}_0F_1\left(;b;-\frac{z^2}{4}\right)I_{b-1}(z)=2^{\frac{1-b}{2}}\sqrt{\pi}\,\Gamma(b)\,G_{0,4}^{1,0}\left(\frac{z}{2\sqrt{2}},\frac{1}{4}\,\middle|\,\frac{b-1}{4},\frac{1-b}{4},\frac{3-b}{4},\frac{1}{4}(3-3b)\right) $$

07.17.26.0181.01

$$ {}_0F_1\left(;b;-\frac{z^2}{4}\right)I_{1-b}(z)=2^{\frac{1-b}{2}}\sqrt{\pi}\,\Gamma(b)\,G_{1,5}^{2,0}\left(\frac{z}{2\sqrt{2}},\frac{1}{4}\,\middle|\,\begin{matrix}\frac{1}{4}(5-3b)\\ \frac{3-b}{4},\frac{1-b}{4},\frac{1}{4}(5-3b),\frac{1}{4}(3-3b),\frac{b-1}{4}\end{matrix}\right) $$

07.17.26.0182.01

$${}_0F_1\left(;b;\frac{z^2}{4}\right)I_\nu(z)=\sqrt{\pi}\,\csc\left(\frac{1}{4}\pi(2b+2\nu+1)\right)\Gamma(b)\,2^{b-1}\,G_{3,5}^{1,2}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1-b}{2},1-\frac{b}{2},\frac{1}{4}(3-2b)\\ \frac{\nu}{2},-\frac{\nu}{2},-b+\frac{\nu}{2}+1,-b-\frac{\nu}{2}+1,\frac{1}{4}(3-2b)\end{matrix}\right)\;/;\;-b-\nu\notin\mathbb{N}$$

07.17.26.0183.01

$${}_0F_1\left(;b;\frac{z^2}{4}\right)I_{-b-n}(z)=\frac{2^{b-1}\,\Gamma(b)}{\sqrt{\pi}}\left(2\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{(-1)^{\left\lfloor\frac{n+1}{2}\right\rfloor}z^{-b+2k-n}\,\Gamma\left(k-n+\left\lfloor\frac{n}{2}\right\rfloor+\frac{1}{2}\right)\left(1-k+\left\lfloor\frac{n}{2}\right\rfloor\right)_{n-\left\lfloor\frac{n}{2}\right\rfloor}}{k!\,\Gamma(b+k)\,\Gamma(-b+k-n+1)}-\right.$$
$$\left.(-1)^{\left\lfloor\frac{n}{2}\right\rfloor}\sqrt{2}\,\pi\,G_{3,5}^{1,2}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1-b}{2},1-\frac{b}{2},\frac{1}{4}(3-2b)\\ \frac{n-b}{2}+1,\frac{b+n}{2},-\frac{1}{2}(b+n),1-\frac{1}{2}(3b+n),\frac{1}{4}(3-2b)\end{matrix}\right)\right)\;/;\;n\in\mathbb{N}$$

07.17.26.0184.01

$${}_0F_1\left(;b;\frac{z^2}{4}\right)I_{-b}(z)=\frac{2^b\,z^{-b}}{\Gamma(1-b)}-2^{b-\frac{1}{2}}\sqrt{\pi}\,\Gamma(b)\,G_{4,6}^{2,2}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1-b}{2},1-\frac{b}{2},\frac{b}{2},\frac{1}{4}(3-2b)\\ 1-\frac{b}{2},\frac{b}{2},\frac{b}{2},-\frac{b}{2},1-\frac{3b}{2},\frac{1}{4}(3-2b)\end{matrix}\right)$$

07.17.26.0185.01

$${}_0F_1\left(;b;\frac{z^2}{4}\right)I_{-b-1}(z)=-\frac{2^{b-1}\,\Gamma(b)}{\sqrt{\pi}}\left(\frac{4b\sin(b\pi)\,z^{-b-1}}{\sqrt{\pi}}+\sqrt{2}\,\pi\,G_{3,5}^{1,2}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1-b}{2},1-\frac{b}{2},\frac{1}{4}(3-2b)\\ \frac{1-b}{2}+1,\frac{b+1}{2},\frac{1}{2}(-b-1),\frac{1}{2}(1-3b),\frac{1}{4}(3-2b)\end{matrix}\right)\right)$$

07.17.26.0186.01

$${}_0F_1\left(;b;\frac{z^2}{4}\right)I_b(z)=2^{b-1}\sqrt{\pi}\,\csc\left(\left(b+\frac{1}{4}\right)\pi\right)\Gamma(b)\,G_{2,4}^{1,1}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1-b}{2},\frac{1}{4}(3-2b)\\ \frac{b}{2},-\frac{b}{2},1-\frac{3b}{2},\frac{1}{4}(3-2b)\end{matrix}\right)$$

07.17.26.0187.01

$${}_0F_1\left(;b;\frac{z^2}{4}\right)I_{b-1}(z)=2^{b-1}\sqrt{\pi}\,\csc\left(\left(b-\frac{1}{4}\right)\pi\right)\Gamma(b)\,G_{3,5}^{1,2}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1-b}{2},1-\frac{b}{2},\frac{1}{4}(3-2b)\\ \frac{b-1}{2},\frac{1-b}{2},\frac{1-b}{2},\frac{1}{2}(-3)(b-1),\frac{1}{4}(3-2b)\end{matrix}\right)$$

07.17.26.0188.01

$$I_{1-b}(z)\,{}_0F_1\left(;b;\frac{z^2}{4}\right)=2^{b-\frac{1}{2}}\sqrt{\pi}\,\Gamma(b)\,G_{3,5}^{1,2}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1-b}{2},1-\frac{b}{2},\frac{1}{4}(3-2b)\\ \frac{1-b}{2},\frac{b-1}{2},\frac{1}{2}(-3)(b-1),\frac{1-b}{2},\frac{1}{4}(3-2b)\end{matrix}\right)$$

07.17.26.0189.01

$${}_0F_1\left(;b;-\frac{z^2}{4}\right)I_{b-1}(z)=2^{\frac{1}{2}-\frac{b}{2}}\sqrt{\pi}\,\Gamma(b)\,G_{0,4}^{1,0}\left(\frac{z}{2\sqrt{2}},\frac{1}{4}\,\middle|\,\frac{b-1}{4},\frac{1-b}{4},\frac{3-b}{4},\frac{3(1-b)}{4}\right)$$

07.17.26.0190.01

$${}_0F_1\left(;b;-\frac{z^2}{4}\right)I_{1-b}(z)=2^{\frac{1-b}{2}}\sqrt{\pi}\,\Gamma(b)\,G_{1,5}^{2,0}\left(\frac{z}{2^{3/2}},\frac{1}{4}\,\middle|\,\begin{matrix}\frac{1}{4}(5-3b)\\ \frac{3-b}{4},\frac{1-b}{4},\frac{1}{4}(5-3b),\frac{1}{4}(3-3b),\frac{b-1}{4}\end{matrix}\right)$$

Generalized cases involving Bessel Y

07.17.26.0078.01

$${}_0F_1(;b;z)\,Y_{b-1}\left(2\sqrt{z}\right)=-2^{\frac{1-b}{2}}\sqrt{\pi}\,\Gamma(b)\,G_{2,6}^{3,0}\left(\frac{z}{2},\frac{1}{2}\,\middle|\,\begin{matrix}1-\frac{b}{4},\frac{2-b}{4}\\ \frac{3-b}{4},\frac{1-b}{4},\frac{b-1}{4},1-\frac{b}{4},\frac{2-b}{4},\frac{3-3b}{4}\end{matrix}\right)$$

07.17.26.0079.01

$$ {}_0F_1(;b;z)\,Y_{1-b}\left(2\sqrt{z}\right) = 2^{\frac{1-b}{2}}\sqrt{\pi}\,\Gamma(b)\,G_{2,6}^{3,0}\left(\frac{z}{2},\frac{1}{2}\,\middle|\,\begin{matrix}\frac{b-2}{4},\frac{b}{4}\\ \frac{3-b}{4},\frac{1-b}{4},\frac{b-1}{4},\frac{b-2}{4},\frac{3-3b}{4},\frac{b}{4}\end{matrix}\right) $$

07.17.26.0191.01

$$ {}_0F_1\left(;b;\frac{z^2}{4}\right)Y_{b-1}(z) = -2^{\frac{1-b}{2}}\sqrt{\pi}\,\Gamma(b)\,G_{2,6}^{3,0}\left(\frac{z}{2\sqrt{2}},\frac{1}{4}\,\middle|\,\begin{matrix}1-\frac{b}{4},\frac{2-b}{4}\\ \frac{3-b}{4},\frac{1-b}{4},\frac{b-1}{4},1-\frac{b}{4},\frac{2-b}{4},\frac{1}{4}(3-3b)\end{matrix}\right) $$

07.17.26.0192.01

$$ {}_0F_1\left(;b;\frac{z^2}{4}\right)Y_{1-b}(z) = 2^{\frac{1-b}{2}}\sqrt{\pi}\,\Gamma(b)\,G_{2,6}^{3,0}\left(\frac{z}{2\sqrt{2}},\frac{1}{4}\,\middle|\,\begin{matrix}\frac{b-2}{4},\frac{b}{4}\\ \frac{3-b}{4},\frac{1-b}{4},\frac{b-1}{4},\frac{b-2}{4},\frac{1}{4}(3-3b),\frac{b}{4}\end{matrix}\right) $$

07.17.26.0193.01

$$ {}_0F_1\left(;b;-\frac{z^2}{4}\right)Y_\nu(z) = -\frac{2^{b-1}\,\Gamma(b)}{\sqrt{\pi}}\,G_{3,5}^{2,2}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1-b}{2},1-\frac{b}{2},\frac{1-\nu}{2}\\ -\frac{\nu}{2},\frac{\nu}{2},\frac{1-\nu}{2},-b-\frac{\nu}{2}+1,-b+\frac{\nu}{2}+1\end{matrix}\right)\,/;\,-b-\nu\notin\mathbb{N}\bigwedge\nu-b\notin\mathbb{N} $$

07.17.26.0194.01

$$ {}_0F_1\left(;b;-\frac{z^2}{4}\right)Y_{b+n}(z) = \frac{2^{b-1}\,\Gamma(b)}{\sqrt{\pi}}\,G_{3,5}^{2,2}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1-b}{2},1-\frac{b}{2},\frac{1}{2}(-b+n+1)\\ \frac{1}{2}(-b+n+2),\frac{b+n}{2},\frac{1}{2}(-3b-n+2),\frac{1}{2}(-b-n),\frac{1}{2}(-b+n+1)\end{matrix}\right)+ $$
$$ \frac{(-1)^{n+1}\,2^b\,z^{-b-n}\csc(b\pi)\,\Gamma(b)}{\sqrt{\pi}}\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{(-1)^{k+\left\lfloor\frac{n+1}{2}\right\rfloor}z^{2k}\,\Gamma\left(k-n+\left\lfloor\frac{n}{2}\right\rfloor+\frac{1}{2}\right)\left(1-k+\left\lfloor\frac{n}{2}\right\rfloor\right)_{n-\left\lfloor\frac{n}{2}\right\rfloor}}{k!\,\Gamma(b+k)\,\Gamma(1-b+k-n)}\,/;\,n\in\mathbb{N} $$

07.17.26.0195.01

$$ {}_0F_1\left(;b;-\frac{z^2}{4}\right)Y_{-b-n}(z) = \frac{(-1)^n\,2^{b-1}\,\Gamma(b)}{\sqrt{\pi}}\,G_{3,5}^{2,2}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1-b}{2},1-\frac{b}{2},\frac{1}{2}(b+n+1)\\ \frac{1}{2}(-b+n+2),\frac{b+n}{2},\frac{1}{2}(-3b-n+2),\frac{1}{2}(-b-n),\frac{1}{2}(b+n+1)\end{matrix}\right)- $$
$$ \frac{2^b\,z^{-b-n}\cot(b\pi)\,\Gamma(b)}{\sqrt{\pi}}\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{(-1)^{k+\left\lfloor\frac{n+1}{2}\right\rfloor}z^{2k}\,\Gamma\left(k-n+\left\lfloor\frac{n}{2}\right\rfloor+\frac{1}{2}\right)\left(1-k+\left\lfloor\frac{n}{2}\right\rfloor\right)_{n-\left\lfloor\frac{n}{2}\right\rfloor}}{k!\,\Gamma(b+k)\,\Gamma(-b+k-n+1)}\,/;\,n\in\mathbb{N} $$

07.17.26.0196.01

$$ {}_0F_1\left(;b;-\frac{z^2}{4}\right)Y_b(z) = \frac{2^{b-1}\,\Gamma(b)}{\sqrt{\pi}}\,G_{2,4}^{2,1}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}1-\frac{b}{2},\frac{1-b}{2}\\ 1-\frac{b}{2},\frac{b}{2},1-\frac{3b}{2},-\frac{b}{2}\end{matrix}\right)-\frac{2^b\,z^{-b}\csc(b\pi)}{\Gamma(1-b)} $$

07.17.26.0197.01

$$ {}_0F_1\left(;b;-\frac{z^2}{4}\right)Y_{b+1}(z) = \frac{2^{b+1}\csc(b\pi)\,z^{-b-1}}{\Gamma(-b)}+\frac{2^{b-1}\,\Gamma(b)}{\sqrt{\pi}}\,G_{2,4}^{2,1}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1-b}{2},1-\frac{b}{2}\\ \frac{3-b}{2},\frac{b+1}{2},\frac{1}{2}(1-3b),\frac{1}{2}(-b-1)\end{matrix}\right) $$

07.17.26.0198.01

$$ {}_0F_1\left(;b;-\frac{z^2}{4}\right)Y_{-b}(z) = -\frac{2^{b-1}\cos(b\pi)\,\Gamma(b)\,z^{-b}}{\pi}-\frac{2^{b-1}\,\Gamma(b)}{\sqrt{\pi}}\,G_{2,4}^{2,1}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1-b}{2},\frac{b+1}{2}\\ -\frac{b}{2},\frac{b}{2},1-\frac{3b}{2},\frac{b+1}{2}\end{matrix}\right) $$

07.17.26.0199.01

$$ {}_0F_1\left(;b;-\frac{z^2}{4}\right)Y_{-b}(z) = \frac{2^{b-1}\,\Gamma(b)}{\sqrt{\pi}}\,G_{3,5}^{2,2}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1-b}{2},1-\frac{b}{2},\frac{b+1}{2}\\ 1-\frac{b}{2},\frac{b}{2},1-\frac{3b}{2},-\frac{b}{2},\frac{b+1}{2}\end{matrix}\right)-\frac{2^b\,z^{-b}\cot(b\pi)}{\Gamma(1-b)} $$

07.17.26.0200.01

$$ {}_0F_1\left(;b;-\frac{z^2}{4}\right)Y_{-b-1}(z)=-\frac{2^{b+1}\cot(b\pi)\,z^{-b-1}}{\Gamma(-b)}-\frac{2^{b-1}\,\Gamma(b)}{\sqrt{\pi}}\,G_{3,5}^{2,2}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1-b}{2},1-\frac{b}{2},\frac{b+2}{2}\\ \frac{3-b}{2},\frac{b+1}{2},\frac{1}{2}(1-3b),\frac{1}{2}(-b-1),\frac{b+2}{2}\end{matrix}\right) $$

Generalized cases involving Bessel K

07.17.26.0080.01

$$ {}_0F_1(;b;-z)\,K_{b-1}\left(2\sqrt{z}\right)=\frac{2^{-\frac{b+3}{2}}\,\Gamma(b)}{\sqrt{\pi}}\,G_{0,4}^{3,0}\left(\frac{z}{2},\frac{1}{2}\,\middle|\,\frac{3-b}{4},\frac{1-b}{4},\frac{b-1}{4},\frac{3-3b}{4}\right) $$

07.17.26.0081.01

$$ {}_0F_1(;b;-z)\,K_{1-b}\left(2\sqrt{z}\right)=\frac{2^{-\frac{b+3}{2}}\,\Gamma(b)}{\sqrt{\pi}}\,G_{0,4}^{3,0}\left(\frac{z}{2},\frac{1}{2}\,\middle|\,\frac{3-b}{4},\frac{1-b}{4},\frac{b-1}{4},\frac{3-3b}{4}\right) $$

07.17.26.0201.01

$$ {}_0F_1\left(;b;-\frac{z^2}{4}\right)K_{b-1}(z)=\frac{2^{-\frac{1}{2}(b+3)}\,\Gamma(b)}{\sqrt{\pi}}\,G_{0,4}^{3,0}\left(\frac{z}{2\sqrt{2}},\frac{1}{4}\,\middle|\,\frac{3-b}{4},\frac{1-b}{4},\frac{b-1}{4},\frac{1}{4}(3-3b)\right) $$

07.17.26.0202.01

$$ {}_0F_1\left(;b;-\frac{z^2}{4}\right)K_{1-b}(z)=\frac{2^{-\frac{1}{2}(b+3)}\,\Gamma(b)}{\sqrt{\pi}}\,G_{0,4}^{3,0}\left(\frac{z}{2\sqrt{2}},\frac{1}{4}\,\middle|\,\frac{3-b}{4},\frac{1-b}{4},\frac{b-1}{4},\frac{1}{4}(3-3b)\right) $$

07.17.26.0203.01

$$ {}_0F_1\left(;b;\frac{z^2}{4}\right)K_{\nu}(z)=\frac{2^{b-2}\,\Gamma(b)}{\sqrt{\pi}}\,G_{2,4}^{2,2}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1-b}{2},1-\frac{b}{2}\\ -\frac{\nu}{2},\frac{\nu}{2},-b-\frac{\nu}{2}+1,-b+\frac{\nu}{2}+1\end{matrix}\right)/;-b-\nu\notin\mathbb{N}\bigwedge\nu-b\notin\mathbb{N} $$

07.17.26.0204.01

$$ {}_0F_1\left(;b;\frac{z^2}{4}\right)K_{b+n}(z)=\pi\,(-1)^{n-1}\,2^{b-\frac{3}{2}}\,\Gamma(b) $$

$$ \left(\sqrt{\pi}\,\csc\left(\frac{1}{4}(4b+(-1)^n)\pi\right)G_{4,6}^{2,2}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1-b}{2},1-\frac{b}{2},\frac{1}{4}(2b+1),\frac{1}{4}(3-2b)\\ 1-\frac{b-n}{2},\frac{b+n}{2},\frac{1}{4}(2b+1),\frac{1}{4}(3-2b),-\frac{1}{2}(b+n),1-\frac{1}{2}(3b+n)\end{matrix}\right)-\right. $$

$$ \left.\frac{\csc(b\pi)\sqrt{2}}{\sqrt{\pi}}\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{(-1)^{\left\lfloor\frac{n+1}{2}\right\rfloor}z^{-b+2k-n}\,\Gamma\left(k-n+\left\lfloor\frac{n}{2}\right\rfloor+\frac{1}{2}\right)\left(1-k+\left\lfloor\frac{n}{2}\right\rfloor\right)_{n-\left\lfloor\frac{n}{2}\right\rfloor}}{k!\,\Gamma(b+k)\,\Gamma(1-b+k-n)}\right)/;n\in\mathbb{N} $$

07.17.26.0205.01

$$ {}_0F_1\left(;b;\frac{z^2}{4}\right)K_{-b-n}(z)=\pi\,(-1)^{n-1}\,2^{b-\frac{3}{2}}\,\Gamma(b)\left(\sqrt{\pi}\,\csc\left(\frac{1}{4}\,(4\,b+(-1)^n)\,\pi\right)G_{4,6}^{2,2}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1-b}{2},1-\frac{b}{2},\frac{1}{4}\,(2\,b+1),\frac{1}{4}\,(3-2\,b)\\ 1-\frac{b-n}{2},\frac{b+n}{2},\frac{1}{4}\,(2\,b+1),\frac{1}{4}\,(3-2\,b),-\frac{1}{2}\,(b+n),1-\frac{1}{2}\,(3\,b+n)\end{matrix}\right)-\frac{\csc(b\,\pi)\sqrt{2}}{\sqrt{\pi}}\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{(-1)^{\left\lfloor\frac{n+1}{2}\right\rfloor}z^{-b+2k-n}\,\Gamma\left(k-n+\left\lfloor\frac{n}{2}\right\rfloor+\frac{1}{2}\right)\left(1-k+\left\lfloor\frac{n}{2}\right\rfloor\right)_{n-\left\lfloor\frac{n}{2}\right\rfloor}}{k!\,\Gamma(b+k)\,\Gamma(1-b+k-n)}\right)\,/;\,n\in\mathbb{N} $$

07.17.26.0206.01

$$ {}_0F_1\left(;b;\frac{z^2}{4}\right)K_b(z)=2^{b-1}\,\Gamma(b)\left(z^{-b}-\frac{\pi^{3/2}\csc\left(\frac{1}{4}\,(4\,b+1)\,\pi\right)}{\sqrt{2}}\,G_{4,6}^{2,2}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1-b}{2},1-\frac{b}{2},\frac{1}{4}\,(3-2\,b),\frac{1}{4}\,(2\,b+1)\\ 1-\frac{b}{2},\frac{b}{2},\frac{1}{4}\,(3-2\,b),1-\frac{3\,b}{2},-\frac{b}{2},\frac{1}{4}\,(2\,b+1)\end{matrix}\right)\right) $$

07.17.26.0207.01

$$ {}_0F_1\left(;b;\frac{z^2}{4}\right)K_{b+1}(z)=2^{b-1}\,\Gamma(b)\left(2\,b\,z^{-b-1}+\sqrt{\frac{\pi}{2}}\,\pi\csc\left(\frac{1}{4}\,(4\,b-1)\,\pi\right)G_{4,6}^{2,2}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1-b}{2},1-\frac{b}{2},\frac{1}{4}\,(3-2\,b),\frac{1}{4}\,(2\,b+1)\\ \frac{3-b}{2},\frac{b+1}{2},\frac{1}{2}\,(1-3\,b),\frac{1}{4}\,(3-2\,b),-\frac{1}{2}\,(b+1),\frac{1}{4}\,(2\,b+1)\end{matrix}\right)\right) $$

07.17.26.0208.01

$$ {}_0F_1\left(;b;\frac{z^2}{4}\right)K_{-b}(z)=2^{b-1}\,\Gamma(b)\left(z^{-b}-\frac{\pi^{3/2}\csc\left(\frac{1}{4}\,(4\,b+1)\,\pi\right)}{\sqrt{2}}\,G_{4,6}^{2,2}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1-b}{2},1-\frac{b}{2},\frac{1}{4}\,(3-2\,b),\frac{1}{4}\,(2\,b+1)\\ 1-\frac{b}{2},\frac{b}{2},\frac{1}{4}\,(3-2\,b),1-\frac{3\,b}{2},-\frac{b}{2},\frac{1}{4}\,(2\,b+1)\end{matrix}\right)\right) $$

07.17.26.0209.01

$$ {}_0F_1\left(;b;\frac{z^2}{4}\right)K_{-b-1}(z)=2^{b-1}\,\Gamma(b)\left(2\,b\,z^{-b-1}+\sqrt{\frac{\pi}{2}}\,\pi\csc\left(\frac{1}{4}\,(4\,b-1)\,\pi\right)G_{4,6}^{2,2}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1-b}{2},1-\frac{b}{2},\frac{1}{4}\,(3-2\,b),\frac{1}{4}\,(2\,b+1)\\ \frac{3-b}{2},\frac{b+1}{2},\frac{1}{2}\,(1-3\,b),\frac{1}{4}\,(3-2\,b),-\frac{1}{2}\,(b+1),\frac{1}{4}\,(2\,b+1)\end{matrix}\right)\right) $$

Generalized cases involving ${}_0\tilde{F}_1$

07.17.26.0210.01

$$ {}_0F_1(;b;z)\,{}_0\tilde{F}_1(;b;-z)=\sqrt{\pi}\,2^{1-b}\,\Gamma(b)\,G_{0,4}^{1,0}\left(\frac{z}{2},\frac{1}{2}\,\middle|\,0,\frac{1-b}{2},1-\frac{b}{2},1-b\right) $$

07.17.26.0211.01

$$ {}_0F_1(;b;z)\,{}_0\tilde{F}_1(;2-b;-z)=\frac{\pi^{3/2}\csc(b\,\pi)}{\Gamma(1-b)}\,G_{1,5}^{2,0}\left(-\frac{z}{2},\frac{1}{2}\,\middle|\,\begin{matrix}1-\frac{b}{2}\\ 0,\frac{1}{2},1-\frac{b}{2},\frac{1-b}{2},\frac{b-1}{2}\end{matrix}\right) $$

Through other functions

Involving some hypergeometric-type functions

07.17.26.0082.01

$${}_0F_1(;b;z) == z^{\frac{1-b}{2}}\,\Gamma(b)\,\boldsymbol{L}_{1-b}\left(2\sqrt{z}\right)\,/;\, b-\frac{3}{2}\in\mathbb{N}$$

Representations through equivalent functions

With related functions

07.17.27.0001.01

$${}_0F_1(;b;z) == \Gamma(b)\,{}_0\tilde{F}_1(;b;z)$$

07.17.27.0002.01

$${}_0F_1(;b;z) == \Gamma(b)\,(-z)^{\frac{1-b}{2}}\,J_{b-1}\left(2\sqrt{-z}\right)$$

07.17.27.0003.01

$${}_0F_1(;b;z) == \Gamma(b)\,z^{\frac{1-b}{2}}\,I_{b-1}\left(2\sqrt{z}\right)$$

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