KelvinBei

Notations

Traditional name

Kelvin function of the first kind

Traditional notation

$\operatorname{bei}(z)$

Mathematica StandardForm notation

`KelvinBei[z]`

Primary definition

03.13.02.0001.01

$$\operatorname{bei}(z) = -\frac{1}{2} i \left(I_0\left(\sqrt[4]{-1}\, z\right) - J_0\left(\sqrt[4]{-1}\, z\right)\right)$$

Specific values

Values at fixed points

03.13.03.0001.01

$$\operatorname{bei}(0) = 0$$

Values at infinities

03.13.03.0002.01

$$\lim_{x\to\infty} \operatorname{bei}(x) = \tilde{\infty}$$

General characteristics

Domain and analyticity

$\operatorname{bei}(z)$ is an entire, and so analytic, function of z, which is defined in the whole complex z-plane.

03.13.04.0001.01

$$z \longrightarrow \operatorname{bei}(z) :: \mathbb{C} \longrightarrow \mathbb{C}$$

Symmetries and periodicities

Parity

03.13.04.0002.01

$\operatorname{bei}(-z) = \operatorname{bei}(z)$

Mirror symmetry

03.13.04.0003.01

$\operatorname{bei}(\bar{z}) == \overline{\operatorname{bei}(z)}$

Periodicity

No periodicity

Poles and essential singularities

The function bei(z) has only one singular point at $z = \tilde{\infty}$. It is an essential singular point.

03.13.04.0004.01

$\mathcal{S}ing_z(\operatorname{bei}(z)) == \{\{\tilde{\infty}, \infty\}\}$

Branch points

The function bei(z) does not have branch points.

03.13.04.0005.01

$\mathcal{BP}_z(\operatorname{bei}(z)) == \{\}$

Branch cuts

The function bei(z) does not have branch cuts.

03.13.04.0006.01

$\mathcal{BC}_z(\operatorname{bei}(z)) == \{\}$

Series representations

Generalized power series

Expansions at generic point $z == z_0$

03.13.06.0001.01

$$\operatorname{bei}(z) \propto \operatorname{bei}(z_0) + \frac{\operatorname{bei}(z_0) - \operatorname{ber}_2(z_0)}{4} (z - z_0)^2 + \frac{\operatorname{bei}_1(z_0) - \operatorname{ber}_1(z_0)}{\sqrt{2}} (z - z_0) + \ldots /; (z \to z_0)$$

03.13.06.0002.01

$$\operatorname{bei}(z) \propto \operatorname{bei}(z_0) + \frac{\operatorname{bei}_1(z_0) - \operatorname{ber}_1(z_0)}{\sqrt{2}} (z - z_0) + \frac{\operatorname{bei}(z_0) - \operatorname{ber}_2(z_0)}{4} (z - z_0)^2 + O\left((z - z_0)^3\right)$$

03.13.06.0003.01

$$\operatorname{bei}(z) = \sum_{k=0}^{\infty} \frac{(-1+i)^k\, 2^{-\frac{3k}{2}-1}}{k!} \left(\sum_{j=0}^{\lfloor \frac{k}{2} \rfloor} \binom{k}{2j} \left(\left(1+i^k\right) \operatorname{bei}_{4j-k}(z_0) - i\left(1-i^k\right) \operatorname{ber}_{4j-k}(z_0) \right) + \sum_{j=0}^{\lfloor \frac{k-1}{2} \rfloor} \binom{k}{2j+1} \left(\left(-1-i^k\right) \operatorname{bei}_{4j-k+2}(z_0) + i\left(1-i^k\right) \operatorname{ber}_{4j-k+2}(z_0) \right) \right) (z-z_0)^k$$

03.13.06.0004.01

$$\operatorname{bei}(z) = \frac{i\sqrt{\pi}}{2} \sum_{k=0}^{\infty} \frac{2^k}{k!} \left({}_1\tilde{F}_2\left(\frac{1}{2}; \frac{1-k}{2}, \frac{2-k}{2}; -\frac{1}{4}\left(i z_0^2\right)\right) - {}_1\tilde{F}_2\left(\frac{1}{2}; \frac{1-k}{2}, \frac{2-k}{2}; \frac{i z_0^2}{4}\right) \right) z_0^{-k} (z-z_0)^k$$

03.13.06.0005.01

$$\operatorname{bei}(z) \propto \operatorname{bei}(z_0)\,(1+O(z-z_0))$$

Expansions at $z == 0$

For the function itself

03.13.06.0006.01

$$\operatorname{bei}(z) \propto \frac{1}{4} z^2 \left(1 - \frac{z^4}{576} + \frac{z^8}{3\,686\,400} - \frac{z^{12}}{104\,044\,953\,600} + O\left[z^{16}\right]\right) /; (z \to 0)$$

03.13.06.0007.01

$$\operatorname{bei}(z) \propto \frac{1}{4} z^2 \left(1 - \frac{z^4}{576} + \frac{z^8}{3\,686\,400} - \frac{z^{12}}{104\,044\,953\,600} + O\left(z^{16}\right)\right)$$

03.13.06.0008.01

$$\operatorname{bei}(z) == \frac{z^2}{4} \sum_{k=0}^{\infty} \frac{(-1)^k}{((2k+1)!)^2} \left(\frac{z}{2}\right)^{4k}$$

03.13.06.0009.01

$$\operatorname{bei}(z) = \frac{1}{4} z^2\, {}_0F_3\left(; 1, \frac{3}{2}, \frac{3}{2}; -\frac{z^4}{256}\right)$$

03.13.06.0010.01

$$\operatorname{bei}(z) \propto \frac{z^2}{4} + O\left(z^6\right)$$

03.13.06.0011.01

$$\operatorname{bei}(z) = F_\infty(z) /; \left(\left(F_n(z) = \frac{1}{4} z^2 \sum_{k=0}^{n} \frac{(-1)^k}{((a+2k)!)^2} \left(\frac{z}{2}\right)^{4k} = \operatorname{bei}(z) + \frac{(-1)^n\, 4^{-2n-3}\, z^{4n+6}}{\Gamma(2n+4)^2}\, {}_1F_4\left(1; n+2, n+2, n+\frac{5}{2}, n+\frac{5}{2}; -\frac{z^4}{256}\right) \right) \bigwedge n \in \mathbb{N} \right)$$

Summed form of the truncated series expansion.

For small integer powers of the function

03.13.06.0012.01

$$\operatorname{bei}(z)^2 \propto \frac{1}{16} z^4 \left(1 - \frac{z^4}{288} + \frac{59 z^8}{16\,588\,800} - \frac{z^{12}}{1\,040\,449\,536} + \ldots\right) /; (z \to 0)$$

03.13.06.0013.01

$$\operatorname{bei}(z)^2 \propto \frac{1}{16} z^4 \left(1 - \frac{z^4}{288} + \frac{59 z^8}{16\,588\,800} - \frac{z^{12}}{1\,040\,449\,536} + O\left(z^{16}\right)\right) /; (z \to 0)$$

03.13.06.0014.01

$$\operatorname{bei}(z)^2 = \frac{1}{2} \sum_{k=0}^{\infty} \frac{2^{-4k} z^{4k}}{(k!)^2 (2k)!} - \frac{1}{2} \sum_{k=0}^{\infty} \frac{16^{-k} (-1)^k \left(\frac{1}{4}\right)_k \left(\frac{3}{4}\right)_k z^{4k}}{\left(\frac{1}{2}\right)_k^{\,3} (k!)^3}$$

03.13.06.0015.01

$$\operatorname{bei}(z)^2 = \frac{1}{2} {}_0F_3\left(; 1, 1, \frac{1}{2}; \frac{z^4}{64}\right) - \frac{1}{2} {}_2F_5\left(\frac{1}{4}, \frac{3}{4}; \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, 1, 1; -\frac{z^4}{16}\right)$$

03.13.06.0016.01

$$\operatorname{bei}(z)^2 \propto \frac{z^4}{16} + O\left(z^8\right)$$

03.13.06.0017.01

$$\operatorname{bei}(z)^2 = F_\infty(z) /; \left(\left(F_n(z) = \frac{1}{2} \sum_{k=0}^{n} \frac{2^{-4k} z^{4k}}{(k!)^2 (2k)!} - \frac{1}{2} \sum_{k=0}^{n} \frac{16^{-k} (-1)^k \left(\frac{1}{4}\right)_k \left(\frac{3}{4}\right)_k z^{4k}}{\left(\frac{1}{2}\right)_k^{\,3} (k!)^3} = \operatorname{bei}(z)^2 - \frac{2^{-4n-5} z^{4(n+1)}}{\Gamma(n+2)^2 \Gamma(2n+3)} {}_1F_4\left(1; n + \frac{3}{2}, n+2, n+2, n+2; \frac{z^4}{64}\right) - \frac{(-1)^n z^{4n+4} \Gamma\left(2n + \frac{5}{2}\right)}{2\sqrt{\pi}\, \Gamma(2n+3)^3} {}_3F_6\left(1, n + \frac{5}{4}, n + \frac{7}{4}; n + \frac{3}{2}, n + \frac{3}{2}, n + \frac{3}{2}, n+2, n+2, n+2; -\frac{z^4}{16}\right)\right) \bigwedge n \in \mathbb{N}\right)$$

Summed form of the truncated series expansion.

Asymptotic series expansions

Expansions inside Stokes sectors

Expansions containing $z \to \infty$

In exponential form ||| In exponential form

03.13.06.0018.01

$$\operatorname{bei}(z) \propto -\frac{1}{2\sqrt{2\pi}\,\sqrt{z}} \Bigg(e^{-\frac{z}{\sqrt{2}}} \left(-e^{-\frac{1}{8}(i\pi)-\frac{iz}{\sqrt{2}}} + e^{\frac{i\pi}{8}+\frac{iz}{\sqrt{2}}} \right) + e^{\frac{z}{\sqrt{2}}} \left(e^{-\frac{1}{8}(3i\pi)-\frac{iz}{\sqrt{2}}} + e^{\frac{3i\pi}{8}+\frac{iz}{\sqrt{2}}} \right) + \frac{1}{8z} \left(e^{\frac{z}{\sqrt{2}}} \left(e^{-\frac{1}{8}(i\pi)-\frac{iz}{\sqrt{2}}} + e^{\frac{i\pi}{8}+\frac{iz}{\sqrt{2}}} \right) + e^{-\frac{z}{\sqrt{2}}} \left(e^{-\frac{1}{8}(3i\pi)-\frac{iz}{\sqrt{2}}} - e^{\frac{3i\pi}{8}+\frac{iz}{\sqrt{2}}} \right) \right) + \frac{9i}{128z^2} \left(e^{-\frac{z}{\sqrt{2}}} \left(e^{-\frac{1}{8}(i\pi)-\frac{iz}{\sqrt{2}}} + e^{\frac{i\pi}{8}+\frac{iz}{\sqrt{2}}} \right) + e^{\frac{z}{\sqrt{2}}} \left(e^{-\frac{1}{8}(3i\pi)-\frac{iz}{\sqrt{2}}} - e^{\frac{3i\pi}{8}+\frac{iz}{\sqrt{2}}} \right) \right) + \frac{75i}{1024z^3} \left(e^{\frac{z}{\sqrt{2}}} \left(e^{-\frac{1}{8}(i\pi)-\frac{iz}{\sqrt{2}}} - e^{\frac{i\pi}{8}+\frac{iz}{\sqrt{2}}} \right) + e^{-\frac{z}{\sqrt{2}}} \left(-e^{-\frac{1}{8}(3i\pi)-\frac{iz}{\sqrt{2}}} - e^{\frac{3i\pi}{8}+\frac{iz}{\sqrt{2}}} \right) \right) + \ldots \Bigg) /; -\frac{\pi}{2} < \arg(z) \leq \pi \bigwedge (|z| \to \infty)$$

03.13.06.0019.01

$$\operatorname{bei}(z) \propto -\frac{1}{2\sqrt{2\pi}\,\sqrt{z}} \Bigg(\sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} \frac{4^{-k}}{(2k)!} \left(e^{-\frac{z}{\sqrt{2}}} \left(-(-1)^k e^{-\frac{1}{8}(i\pi)-\frac{iz}{\sqrt{2}}} + e^{\frac{i\pi}{8}+\frac{iz}{\sqrt{2}}} \right) + e^{\frac{z}{\sqrt{2}}} \left(e^{-\frac{1}{8}(3i\pi)-\frac{iz}{\sqrt{2}}} + (-1)^k e^{\frac{3i\pi}{8}+\frac{iz}{\sqrt{2}}} \right) \right) \left(\frac{i}{z^2} \right)^k \left(\frac{1}{2} \right)_{2k}^2 + \frac{1}{2z} \sum_{k=0}^{\left\lfloor \frac{n-1}{2} \right\rfloor} \frac{4^{-k}}{(2k+1)!} \left(e^{\frac{z}{\sqrt{2}}} \left(e^{-\frac{1}{8}(i\pi)-\frac{iz}{\sqrt{2}}} + (-1)^k e^{\frac{i\pi}{8}+\frac{iz}{\sqrt{2}}} \right) + e^{-\frac{z}{\sqrt{2}}} \left((-1)^k e^{-\frac{1}{8}(3i\pi)-\frac{iz}{\sqrt{2}}} - e^{\frac{3i\pi}{8}+\frac{iz}{\sqrt{2}}} \right) \right) \left(\frac{i}{z^2} \right)^k \left(\frac{1}{2} \right)_{2k+1}^2 + \ldots \Bigg) /; -\frac{\pi}{2} < \arg(z) \leq \pi \bigwedge (|z| \to \infty) \bigwedge n \in \mathbb{N}$$

03.13.06.0020.01

$$\operatorname{bei}(z) \propto -\frac{1}{2\sqrt{2\pi}\,\sqrt{z}} \Bigg(e^{-\frac{z}{\sqrt{2}}} \left(e^{\frac{i\pi}{8}+\frac{iz}{\sqrt{2}}} {}_4F_1\!\left(\frac{1}{4}, \frac{1}{4}, \frac{3}{4}, \frac{3}{4}; \frac{1}{2}; \frac{i}{z^2} \right) - e^{-\frac{1}{8}(i\pi)-\frac{iz}{\sqrt{2}}} {}_4F_1\!\left(\frac{1}{4}, \frac{1}{4}, \frac{3}{4}, \frac{3}{4}; \frac{1}{2}; -\frac{i}{z^2} \right) \right) + e^{\frac{z}{\sqrt{2}}} \left(e^{-\frac{1}{8}(3i\pi)-\frac{iz}{\sqrt{2}}} {}_4F_1\!\left(\frac{1}{4}, \frac{1}{4}, \frac{3}{4}, \frac{3}{4}; \frac{1}{2}; \frac{i}{z^2} \right) + e^{\frac{3i\pi}{8}+\frac{iz}{\sqrt{2}}} {}_4F_1\!\left(\frac{1}{4}, \frac{1}{4}, \frac{3}{4}, \frac{3}{4}; \frac{1}{2}; -\frac{i}{z^2} \right) \right) + \frac{1}{8z} \left(e^{\frac{z}{\sqrt{2}}} \left(e^{-\frac{1}{8}(i\pi)-\frac{iz}{\sqrt{2}}} {}_4F_1\!\left(\frac{3}{4}, \frac{3}{4}, \frac{5}{4}, \frac{5}{4}; \frac{3}{2}; \frac{i}{z^2} \right) + e^{\frac{i\pi}{8}+\frac{iz}{\sqrt{2}}} {}_4F_1\!\left(\frac{3}{4}, \frac{3}{4}, \frac{5}{4}, \frac{5}{4}; \frac{3}{2}; -\frac{i}{z^2} \right) \right) + e^{-\frac{z}{\sqrt{2}}} \left(e^{-\frac{1}{8}(3i\pi)-\frac{iz}{\sqrt{2}}} {}_4F_1\!\left(\frac{3}{4}, \frac{3}{4}, \frac{5}{4}, \frac{5}{4}; \frac{3}{2}; -\frac{i}{z^2} \right) - e^{\frac{3i\pi}{8}+\frac{iz}{\sqrt{2}}} {}_4F_1\!\left(\frac{3}{4}, \frac{3}{4}, \frac{5}{4}, \frac{5}{4}; \frac{3}{2}; \frac{i}{z^2} \right) \right) \right) \Bigg) /; -\frac{\pi}{2} < \arg(z) \leq \pi \bigwedge (|z| \to \infty)$$

03.13.06.0021.01

$$\operatorname{bei}(z) \propto -\frac{1}{2\sqrt{2\pi}\,\sqrt{z}} \Bigg(e^{-\frac{z}{\sqrt{2}}} \left(e^{\frac{i\pi}{8}+\frac{iz}{\sqrt{2}}} \left(1 + O\!\left(\frac{1}{z^2} \right) \right) - e^{-\frac{1}{8}(i\pi)-\frac{iz}{\sqrt{2}}} \left(1 + O\!\left(\frac{1}{z^2} \right) \right) \right) + e^{\frac{z}{\sqrt{2}}} \left(e^{\frac{3i\pi}{8}+\frac{iz}{\sqrt{2}}} \left(1 + O\!\left(\frac{1}{z^2} \right) \right) + e^{-\frac{1}{8}(3i\pi)-\frac{iz}{\sqrt{2}}} \left(1 + O\!\left(\frac{1}{z^2} \right) \right) \right) \Bigg) /; -\frac{\pi}{2} < \arg(z) \leq \pi \bigwedge (|z| \to \infty)$$

In trigonometric form ||| In trigonometric form

03.13.06.0022.01

$$\operatorname{bei}(z) \propto -e^{\frac{z}{\sqrt{2}}} \sin\left(\frac{1}{8}\left(\pi - 4\sqrt{2}\, z\right)\right) - i\, e^{-\frac{z}{\sqrt{2}}} \cos\left(\frac{1}{8}\left(3\pi - 4\sqrt{2}\, z\right)\right) + \frac{1}{8z}\left(-e^{\frac{z}{\sqrt{2}}} \cos\left(\frac{1}{8}\left(4\sqrt{2}\, z + \pi\right)\right) + i\, e^{-\frac{z}{\sqrt{2}}} \cos\left(\frac{1}{8}\left(4\sqrt{2}\, z - \pi\right)\right)\right) + \frac{9}{128 z^2}\left(-e^{\frac{z}{\sqrt{2}}} \cos\left(\frac{1}{8}\left(4\sqrt{2}\, z - \pi\right)\right) + i\, e^{-\frac{z}{\sqrt{2}}} \sin\left(\frac{1}{8}\left(4\sqrt{2}\, z - 3\pi\right)\right)\right) + \frac{75}{1024 z^3}\left(e^{\frac{z}{\sqrt{2}}} \sin\left(\frac{1}{8}\left(-4\sqrt{2}\, z - \pi\right)\right) + i\, e^{-\frac{z}{\sqrt{2}}} \sin\left(\frac{1}{8}\left(\pi - 4\sqrt{2}\, z\right)\right)\right) + \ldots /; -\frac{\pi}{2} < \arg(z) \leq \pi \bigwedge (|z| \to \infty)$$

03.13.06.0023.01

$$\operatorname{bei}(z) \propto \frac{1}{\sqrt{2\pi}\, \sqrt{z}} \left(\frac{1}{2z} \sum_{k=0}^{\left\lfloor \frac{n-1}{2} \right\rfloor} \frac{4^{-k} \left(-\frac{1}{z^2}\right)^k \left(\frac{1}{2}\right)_{2k+1}^2}{(2k+1)!} \left(i(-1)^k e^{-\frac{z}{\sqrt{2}}} \cos\left(\frac{\pi k}{2} + \frac{1}{8}\left(4\sqrt{2}\, z - \pi\right)\right) - e^{\frac{z}{\sqrt{2}}} \cos\left(\frac{\pi k}{2} + \frac{1}{8}\left(4\sqrt{2}\, z + \pi\right)\right) \right) - \sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} \frac{4^{-k} \left(-\frac{1}{z^2}\right)^k \left(\frac{1}{2}\right)_{2k}^2}{(2k)!} \left(e^{-\frac{z}{\sqrt{2}}}\, i \cos\left(\frac{\pi k}{2} + \frac{1}{8}\left(3\pi - 4\sqrt{2}\, z\right)\right) + (-1)^k e^{\frac{z}{\sqrt{2}}} \sin\left(\frac{\pi k}{2} + \frac{1}{8}\left(\pi - 4\sqrt{2}\, z\right)\right) \right) + \ldots \right) /; -\frac{\pi}{2} < \arg(z) \leq \pi \bigwedge (|z| \to \infty) \bigwedge n \in \mathbb{N}$$

03.13.06.0024.01

$$\operatorname{bei}(z) \propto \frac{1}{\sqrt{2\pi}\, \sqrt{z}} \left({}_8F_3\left(\frac{1}{8}, \frac{1}{8}, \frac{3}{8}, \frac{3}{8}, \frac{5}{8}, \frac{5}{8}, \frac{7}{8}, \frac{7}{8}; \frac{1}{4}, \frac{1}{2}, \frac{3}{4}; -\frac{16}{z^4}\right) \left(e^{-\frac{z}{\sqrt{2}}} (-i) \cos\left(\frac{1}{8}\left(3\pi - 4\sqrt{2}\, z\right)\right) - e^{\frac{z}{\sqrt{2}}} \sin\left(\frac{1}{8}\left(\pi - 4\sqrt{2}\, z\right)\right) \right) + \frac{1}{8z} \left(i\, e^{-\frac{z}{\sqrt{2}}} \cos\left(\frac{1}{8}\left(4\sqrt{2}\, z - \pi\right)\right) - e^{\frac{z}{\sqrt{2}}} \cos\left(\frac{1}{8}\left(4\sqrt{2}\, z + \pi\right)\right) \right) {}_8F_3\left(\frac{3}{8}, \frac{3}{8}, \frac{5}{8}, \frac{5}{8}, \frac{7}{8}, \frac{7}{8}, \frac{9}{8}, \frac{9}{8}; \frac{1}{2}, \frac{3}{4}, \frac{5}{4}; -\frac{16}{z^4}\right) - \frac{9}{128 z^2}\, {}_8F_3\left(\frac{5}{8}, \frac{5}{8}, \frac{7}{8}, \frac{7}{8}, \frac{9}{8}, \frac{9}{8}, \frac{11}{8}, \frac{11}{8}; \frac{3}{4}, \frac{5}{4}, \frac{3}{2}; -\frac{16}{z^4}\right) \left(e^{\frac{z}{\sqrt{2}}} \cos\left(\frac{1}{8}\left(\pi - 4\sqrt{2}\, z\right)\right) + e^{-\frac{z}{\sqrt{2}}}\, i \sin\left(\frac{1}{8}\left(3\pi - 4\sqrt{2}\, z\right)\right) \right) + \frac{75}{1024 z^3} \left(e^{-\frac{z}{\sqrt{2}}} (-i) \sin\left(\frac{1}{8}\left(4\sqrt{2}\, z - \pi\right)\right) - e^{\frac{z}{\sqrt{2}}} \sin\left(\frac{1}{8}\left(4\sqrt{2}\, z + \pi\right)\right) \right) {}_8F_3\left(\frac{7}{8}, \frac{7}{8}, \frac{9}{8}, \frac{9}{8}, \frac{11}{8}, \frac{11}{8}, \frac{13}{8}, \frac{13}{8}; \frac{5}{4}, \frac{3}{2}, \frac{7}{4}; -\frac{16}{z^4}\right) \right) /; -\frac{\pi}{2} < \arg(z) \leq \pi \bigwedge (|z| \to \infty)$$

03.13.06.0025.01

$$\operatorname{bei}(z) \propto -\frac{1}{\sqrt{2\pi}\, \sqrt{z}} \left(e^{\frac{z}{\sqrt{2}}} \sin\left(\frac{1}{8}\left(\pi - 4\sqrt{2}\, z\right)\right) + e^{-\frac{z}{\sqrt{2}}}\, i \cos\left(\frac{1}{8}\left(3\pi - 4\sqrt{2}\, z\right)\right) \right) \left(1 + O\left(\frac{1}{z^4}\right)\right) /; -\frac{\pi}{2} < \arg(z) \leq \pi \bigwedge (|z| \to \infty)$$

Expansions containing $z \to -\infty$

In exponential form ||| In exponential form

03.13.06.0026.01

$$\operatorname{bei}(z) \propto -\frac{1}{2\sqrt{2\pi}\sqrt{-z}}\left(e^{\frac{z}{\sqrt{2}}}\left(e^{\frac{i\pi}{8}-\frac{iz}{\sqrt{2}}}-e^{-\frac{1}{8}(i\pi)+\frac{iz}{\sqrt{2}}}\right)+e^{-\frac{z}{\sqrt{2}}}\left(e^{\frac{3i\pi}{8}-\frac{iz}{\sqrt{2}}}+e^{-\frac{1}{8}(3i\pi)+\frac{iz}{\sqrt{2}}}\right)+\frac{1}{8z}\left(e^{-\frac{z}{\sqrt{2}}}\left(-e^{\frac{i\pi}{8}-\frac{iz}{\sqrt{2}}}-e^{-\frac{1}{8}(i\pi)+\frac{iz}{\sqrt{2}}}\right)+e^{\frac{z}{\sqrt{2}}}\left(e^{\frac{3i\pi}{8}-\frac{iz}{\sqrt{2}}}-e^{-\frac{1}{8}(3i\pi)+\frac{iz}{\sqrt{2}}}\right)\right)+\frac{9i}{128z^2}\left(e^{\frac{z}{\sqrt{2}}}\left(e^{\frac{i\pi}{8}-\frac{iz}{\sqrt{2}}}+e^{-\frac{1}{8}(i\pi)+\frac{iz}{\sqrt{2}}}\right)+e^{-\frac{z}{\sqrt{2}}}\left(-e^{\frac{3i\pi}{8}-\frac{iz}{\sqrt{2}}}+e^{-\frac{1}{8}(3i\pi)+\frac{iz}{\sqrt{2}}}\right)\right)+\frac{75i}{1024z^3}\left(e^{-\frac{z}{\sqrt{2}}}\left(e^{\frac{i\pi}{8}-\frac{iz}{\sqrt{2}}}-e^{-\frac{1}{8}(i\pi)+\frac{iz}{\sqrt{2}}}\right)+e^{\frac{z}{\sqrt{2}}}\left(e^{\frac{3i\pi}{8}-\frac{iz}{\sqrt{2}}}+e^{-\frac{1}{8}(3i\pi)+\frac{iz}{\sqrt{2}}}\right)\right)+\ldots\right)\,/;\,\frac{\pi}{2}<\arg(z)\leq\pi\bigwedge(|z|\to\infty)$$

03.13.06.0027.01

$$\operatorname{bei}(z) \propto -\frac{1}{2\sqrt{2\pi}\sqrt{-z}}\left(\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{4^{-k}}{(2k)!}\left(e^{\frac{z}{\sqrt{2}}}\left(e^{\frac{i\pi}{8}-\frac{iz}{\sqrt{2}}}-(-1)^k e^{-\frac{1}{8}(i\pi)+\frac{iz}{\sqrt{2}}}\right)+e^{-\frac{z}{\sqrt{2}}}\left((-1)^k e^{\frac{3i\pi}{8}-\frac{iz}{\sqrt{2}}}+e^{-\frac{1}{8}(3i\pi)+\frac{iz}{\sqrt{2}}}\right)\right)\left(\frac{i}{z^2}\right)^k\left(\frac{1}{2}\right)_{2k}^2+\frac{1}{2z}\sum_{k=0}^{\left\lfloor\frac{n-1}{2}\right\rfloor}\frac{4^{-k}}{(2k+1)!}\left(e^{-\frac{z}{\sqrt{2}}}\left(-(-1)^k e^{\frac{i\pi}{8}-\frac{iz}{\sqrt{2}}}-e^{-\frac{1}{8}(i\pi)+\frac{iz}{\sqrt{2}}}\right)+e^{\frac{z}{\sqrt{2}}}\left(e^{\frac{3i\pi}{8}-\frac{iz}{\sqrt{2}}}-(-1)^k e^{-\frac{1}{8}(3i\pi)+\frac{iz}{\sqrt{2}}}\right)\right)\left(\frac{i}{z^2}\right)^k\left(\frac{1}{2}\right)_{2k+1}^2+\ldots\right)\,/;\,\frac{\pi}{2}<\arg(z)\leq\pi\bigwedge(|z|\to\infty)\bigwedge n\in\mathbb{N}$$

03.13.06.0028.01

$$\operatorname{bei}(z) \propto -\frac{1}{2\sqrt{2\pi}\sqrt{-z}}\left(e^{-\frac{z}{\sqrt{2}}}\left(e^{-\frac{1}{8}(3i\pi)+\frac{iz}{\sqrt{2}}}\,{}_4F_1\left(\frac{1}{4},\frac{1}{4},\frac{3}{4},\frac{3}{4};\frac{1}{2};\frac{i}{z^2}\right)+e^{\frac{3i\pi}{8}-\frac{iz}{\sqrt{2}}}\,{}_4F_1\left(\frac{1}{4},\frac{1}{4},\frac{3}{4},\frac{3}{4};\frac{1}{2};-\frac{i}{z^2}\right)\right)+e^{\frac{z}{\sqrt{2}}}\left(e^{\frac{i\pi}{8}-\frac{iz}{\sqrt{2}}}\,{}_4F_1\left(\frac{1}{4},\frac{1}{4},\frac{3}{4},\frac{3}{4};\frac{1}{2};\frac{i}{z^2}\right)-e^{-\frac{1}{8}(i\pi)+\frac{iz}{\sqrt{2}}}\,{}_4F_1\left(\frac{1}{4},\frac{1}{4},\frac{3}{4},\frac{3}{4};\frac{1}{2};-\frac{i}{z^2}\right)\right)+\frac{1}{8z}\left(e^{\frac{z}{\sqrt{2}}}\left(e^{\frac{3i\pi}{8}-\frac{iz}{\sqrt{2}}}\,{}_4F_1\left(\frac{3}{4},\frac{3}{4},\frac{5}{4},\frac{5}{4};\frac{3}{2};\frac{i}{z^2}\right)-e^{-\frac{1}{8}(3i\pi)+\frac{iz}{\sqrt{2}}}\,{}_4F_1\left(\frac{3}{4},\frac{3}{4},\frac{5}{4},\frac{5}{4};\frac{3}{2};-\frac{i}{z^2}\right)\right)+e^{-\frac{z}{\sqrt{2}}}\left(-e^{-\frac{1}{8}(i\pi)+\frac{iz}{\sqrt{2}}}\,{}_4F_1\left(\frac{3}{4},\frac{3}{4},\frac{5}{4},\frac{5}{4};\frac{3}{2};\frac{i}{z^2}\right)-e^{\frac{i\pi}{8}-\frac{iz}{\sqrt{2}}}\,{}_4F_1\left(\frac{3}{4},\frac{3}{4},\frac{5}{4},\frac{5}{4};\frac{3}{2};-\frac{i}{z^2}\right)\right)\right)\right)\,/;\,\frac{\pi}{2}<\arg(z)\leq\pi\bigwedge(|z|\to\infty)$$

03.13.06.0029.01

$$\operatorname{bei}(z) \propto -\frac{1}{2\sqrt{2\pi}\sqrt{-z}} \left(e^{-\frac{z}{\sqrt{2}}} \left(e^{-\frac{1}{8}(3 i \pi)+\frac{i z}{\sqrt{2}}} \left(1+O\left(\frac{1}{z^2}\right)\right) + e^{\frac{3 i \pi}{8}-\frac{i z}{\sqrt{2}}} \left(1+O\left(\frac{1}{z^2}\right)\right)\right) + e^{\frac{z}{\sqrt{2}}} \left(e^{\frac{i\pi}{8}-\frac{i z}{\sqrt{2}}} \left(1+O\left(\frac{1}{z^2}\right)\right) - e^{-\frac{1}{8}(i \pi)+\frac{i z}{\sqrt{2}}} \left(1+O\left(\frac{1}{z^2}\right)\right)\right)\right) /; \frac{\pi}{2} < \arg(z) \leq \pi \bigwedge (|z| \to \infty)$$

In trigonometric form ||| In trigonometric form

03.13.06.0030.01

$$\operatorname{bei}(z) \propto -\frac{i}{\sqrt{2\pi}\sqrt{-z}} \left(e^{\frac{z}{\sqrt{2}}} \sin\left(\frac{1}{8}\left(\pi - 4\sqrt{2}\, z\right)\right) - i\, e^{-\frac{z}{\sqrt{2}}} \sin\left(\frac{1}{8}\left(4\sqrt{2}\, z + \pi\right)\right) + \frac{1}{8 z}\left(e^{-\frac{z}{\sqrt{2}}}\, i \cos\left(\frac{1}{8}\left(4\sqrt{2}\, z - \pi\right)\right) + e^{\frac{z}{\sqrt{2}}} \cos\left(\frac{1}{8}\left(4\sqrt{2}\, z + \pi\right)\right)\right) + \frac{9}{128 z^2}\left(e^{\frac{z}{\sqrt{2}}} \cos\left(\frac{1}{8}\left(4\sqrt{2}\, z - \pi\right)\right) - i\, e^{-\frac{z}{\sqrt{2}}} \cos\left(\frac{1}{8}\left(-4\sqrt{2}\, z - \pi\right)\right)\right) + \frac{75}{1024 z^3}\left(i\, e^{-\frac{z}{\sqrt{2}}} \sin\left(\frac{1}{8}\left(\pi - 4\sqrt{2}\, z\right)\right) - e^{\frac{z}{\sqrt{2}}} \sin\left(\frac{1}{8}\left(-4\sqrt{2}\, z - \pi\right)\right)\right) + \ldots \right) /; (z \to -\infty)$$

03.13.06.0031.01

$$\operatorname{bei}(z) \propto -\frac{i}{\sqrt{2\pi}\sqrt{-z}} \left(\frac{1}{2z} \sum_{k=0}^{\left\lfloor \frac{n-1}{2} \right\rfloor} \frac{4^{-k} \left(\frac{1}{z^2}\right)^k \left(\frac{1}{2}\right)_{2k+1}^2}{(2k+1)!} \left(e^{-\frac{z}{\sqrt{2}}}\, i \cos\left(\frac{\pi k}{2} + \frac{1}{8}\left(4\sqrt{2}\, z - \pi\right)\right) + (-1)^k e^{\frac{z}{\sqrt{2}}} \cos\left(\frac{\pi k}{2} + \frac{1}{8}\left(4\sqrt{2}\, z + \pi\right)\right)\right) + \sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} \frac{4^{-k} \left(\frac{1}{z^2}\right)^k \left(\frac{1}{2}\right)_{2k}^2}{(2k)!} \left(e^{\frac{z}{\sqrt{2}}} \sin\left(\frac{\pi k}{2} + \frac{1}{8}\left(\pi - 4\sqrt{2}\, z\right)\right) - i\, e^{-\frac{z}{\sqrt{2}}} \sin\left(\frac{\pi k}{2} + \frac{1}{8}\left(4\sqrt{2}\, z + \pi\right)\right)\right) + \ldots \right) /; (z \to -\infty) \land n \in \mathbb{N}$$

03.13.06.0032.01

$$\operatorname{bei}(z) \propto -\frac{i}{\sqrt{2\pi}\sqrt{-z}} \left(-\frac{1}{8z}\left(e^{-\frac{z}{\sqrt{2}}}(-i)\cos\left(\frac{1}{8}\left(4\sqrt{2}\,z-\pi\right)\right) - e^{\frac{z}{\sqrt{2}}}\cos\left(\frac{1}{8}\left(4\sqrt{2}\,z+\pi\right)\right)\right) {}_8F_3\left(\frac{3}{8},\frac{3}{8},\frac{5}{8},\frac{5}{8},\frac{7}{8},\frac{7}{8},\frac{9}{8},\frac{9}{8};\frac{1}{2},\frac{3}{4},\frac{5}{4};-\frac{16}{z^4}\right) + \frac{9}{128z^2}\left(e^{\frac{z}{\sqrt{2}}}\cos\left(\frac{1}{8}\left(\pi-4\sqrt{2}\,z\right)\right) - i\,e^{-\frac{z}{\sqrt{2}}}\cos\left(\frac{1}{8}\left(4\sqrt{2}\,z+\pi\right)\right)\right) {}_8F_3\left(\frac{5}{8},\frac{5}{8},\frac{7}{8},\frac{7}{8},\frac{9}{8},\frac{9}{8},\frac{11}{8},\frac{11}{8};\frac{3}{4},\frac{5}{4},\frac{3}{2};-\frac{16}{z^4}\right) + \frac{75}{1024z^3}\left(e^{\frac{z}{\sqrt{2}}}\sin\left(\frac{1}{8}\left(4\sqrt{2}\,z+\pi\right)\right) - i\,e^{-\frac{z}{\sqrt{2}}}\sin\left(\frac{1}{8}\left(4\sqrt{2}\,z-\pi\right)\right)\right) {}_8F_3\left(\frac{7}{8},\frac{7}{8},\frac{9}{8},\frac{9}{8},\frac{11}{8},\frac{11}{8},\frac{13}{8},\frac{13}{8};\frac{5}{4},\frac{3}{2},\frac{7}{4};-\frac{16}{z^4}\right) + \left(e^{\frac{z}{\sqrt{2}}}\sin\left(\frac{1}{8}\left(\pi-4\sqrt{2}\,z\right)\right) - i\,e^{-\frac{z}{\sqrt{2}}}\sin\left(\frac{1}{8}\left(4\sqrt{2}\,z+\pi\right)\right)\right) {}_8F_3\left(\frac{1}{8},\frac{1}{8},\frac{3}{8},\frac{3}{8},\frac{5}{8},\frac{5}{8},\frac{7}{8},\frac{7}{8};\frac{1}{4},\frac{1}{2},\frac{3}{4};-\frac{16}{z^4}\right)\right) /; (z \to -\infty)$$

03.13.06.0033.01

$$\operatorname{bei}(z) \propto -\frac{i}{\sqrt{2\pi}\sqrt{-z}}\left(e^{\frac{z}{\sqrt{2}}}\sin\left(\frac{1}{8}\left(\pi-4\sqrt{2}\,z\right)\right) - i\,e^{-\frac{z}{\sqrt{2}}}\sin\left(\frac{1}{8}\left(4\sqrt{2}\,z+\pi\right)\right)\right)\left(1+O\left(\frac{1}{z^4}\right)\right) /; (z \to -\infty)$$

Expansions for any z in exponential form

Using exponential function with branch cut-free arguments

03.13.06.0034.01

$$\operatorname{bei}(z) \propto -\frac{1}{2\sqrt{2\pi}}(-1)^{3/4}\left(e^{\frac{z}{\sqrt{2}}}\left(\frac{e^{\frac{iz}{\sqrt{2}}}}{\sqrt{(-1)^{3/4}z}} - \frac{e^{-\frac{iz}{\sqrt{2}}}}{\sqrt{\sqrt[4]{-1}\,z}}\right) + e^{-\frac{z}{\sqrt{2}}}\left(\frac{e^{-\frac{iz}{\sqrt{2}}}}{\sqrt{-(-1)^{3/4}z}} - \frac{e^{\frac{iz}{\sqrt{2}}}}{\sqrt{-\sqrt[4]{-1}\,z}}\right) - \frac{(-1)^{3/4}}{8z}\left(e^{\frac{z}{\sqrt{2}}}\left(\frac{e^{\frac{iz}{\sqrt{2}}}}{\sqrt{(-1)^{3/4}z}} - \frac{i\,e^{-\frac{iz}{\sqrt{2}}}}{\sqrt{\sqrt[4]{-1}\,z}}\right) + e^{-\frac{z}{\sqrt{2}}}\left(\frac{i\,e^{\frac{iz}{\sqrt{2}}}}{\sqrt{-\sqrt[4]{-1}\,z}} - \frac{e^{-\frac{iz}{\sqrt{2}}}}{\sqrt{-(-1)^{3/4}z}}\right)\right) - \frac{9i}{128z^2}\left(e^{\frac{z}{\sqrt{2}}}\left(\frac{e^{\frac{iz}{\sqrt{2}}}}{\sqrt{(-1)^{3/4}z}} + \frac{e^{-\frac{iz}{\sqrt{2}}}}{\sqrt{\sqrt[4]{-1}\,z}}\right) + e^{-\frac{z}{\sqrt{2}}}\left(\frac{e^{-\frac{iz}{\sqrt{2}}}}{\sqrt{-(-1)^{3/4}z}} + \frac{e^{\frac{iz}{\sqrt{2}}}}{\sqrt{-\sqrt[4]{-1}\,z}}\right)\right) + \frac{75\sqrt[4]{-1}}{1024z^3}\left(e^{-\frac{z}{\sqrt{2}}}\left(\frac{i\,e^{\frac{iz}{\sqrt{2}}}}{\sqrt{-\sqrt[4]{-1}\,z}} + \frac{e^{-\frac{iz}{\sqrt{2}}}}{\sqrt{-(-1)^{3/4}z}}\right) - e^{\frac{z}{\sqrt{2}}}\left(\frac{e^{\frac{iz}{\sqrt{2}}}}{\sqrt{(-1)^{3/4}z}} + \frac{i\,e^{-\frac{iz}{\sqrt{2}}}}{\sqrt{\sqrt[4]{-1}\,z}}\right)\right) + \ldots\right) /; (|z| \to \infty)$$

03.13.06.0035.01

$$\operatorname{bei}(z) \propto -\frac{(-1)^{3/4}}{2\sqrt{2\pi}} \left(e^{\frac{z}{\sqrt{2}}} \left(\frac{e^{\frac{iz}{\sqrt{2}}}}{\sqrt{(-1)^{3/4} z}} \left(\sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} \frac{\left(\frac{1}{2}\right)_{2k}^2}{(2k)!} \left(-\frac{i}{4z^2}\right)^k + O\left(\frac{1}{z^{2\left\lfloor \frac{n}{2} \right\rfloor+2}}\right) \right) - \frac{e^{-\frac{iz}{\sqrt{2}}}}{\sqrt{\sqrt[4]{-1}\, z}} \left(\sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} \frac{\left(\frac{1}{2}\right)_{2k}^2}{(2k)!} \left(\frac{i}{4z^2}\right)^k + O\left(\frac{1}{z^{2\left\lfloor \frac{n}{2} \right\rfloor+2}}\right) \right) \right) + e^{-\frac{z}{\sqrt{2}}} \left(\frac{e^{-\frac{iz}{\sqrt{2}}}}{\sqrt{-(-1)^{3/4} z}} \left(\sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} \frac{\left(\frac{1}{2}\right)_{2k}^2}{(2k)!} \left(-\frac{i}{4z^2}\right)^k + O\left(\frac{1}{z^{2\left\lfloor \frac{n}{2} \right\rfloor+2}}\right) \right) - \frac{e^{\frac{iz}{\sqrt{2}}}}{\sqrt{-\sqrt[4]{-1}\, z}} \left(\sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} \frac{\left(\frac{1}{2}\right)_{2k}^2}{(2k)!} \left(\frac{i}{4z^2}\right)^k + O\left(\frac{1}{z^{2\left\lfloor \frac{n}{2} \right\rfloor+2}}\right) \right) \right) \right) - \frac{(-1)^{3/4}}{2z} \left(e^{\frac{z}{\sqrt{2}}} \left(\frac{e^{\frac{iz}{\sqrt{2}}}}{\sqrt{(-1)^{3/4} z}} \left(\sum_{k=0}^{\left\lfloor \frac{n-1}{2} \right\rfloor} \frac{\left(\frac{1}{2}\right)_{2k+1}^2}{(2k+1)!} \left(-\frac{i}{4z^2}\right)^k + O\left(\frac{1}{z^{2\left\lfloor \frac{n-1}{2} \right\rfloor+2}}\right) \right) - \frac{i\, e^{-\frac{iz}{\sqrt{2}}}}{\sqrt{\sqrt[4]{-1}\, z}} \left(\sum_{k=0}^{\left\lfloor \frac{n-1}{2} \right\rfloor} \frac{\left(\frac{1}{2}\right)_{2k+1}^2}{(2k+1)!} \left(\frac{i}{4z^2}\right)^k + O\left(\frac{1}{z^{2\left\lfloor \frac{n-1}{2} \right\rfloor+2}}\right) \right) \right) + e^{-\frac{z}{\sqrt{2}}} \left(\frac{i\, e^{\frac{iz}{\sqrt{2}}}}{\sqrt{-\sqrt[4]{-1}\, z}} \left(\sum_{k=0}^{\left\lfloor \frac{n-1}{2} \right\rfloor} \frac{\left(\frac{1}{2}\right)_{2k+1}^2}{(2k+1)!} \left(\frac{i}{4z^2}\right)^k + O\left(\frac{1}{z^{2\left\lfloor \frac{n-1}{2} \right\rfloor+2}}\right) \right) - \frac{e^{-\frac{iz}{\sqrt{2}}}}{\sqrt{-(-1)^{3/4} z}} \left(\sum_{k=0}^{\left\lfloor \frac{n-1}{2} \right\rfloor} \frac{\left(\frac{1}{2}\right)_{2k+1}^2}{(2k+1)!} \left(-\frac{i}{4z^2}\right)^k + O\left(\frac{1}{z^{2\left\lfloor \frac{n-1}{2} \right\rfloor+2}}\right) \right) \right) \right) \;/;\; (|z| \to \infty) \land n \in \mathbb{N}$$

03.13.06.0036.01

$$\operatorname{bei}(z) \propto \frac{1-i}{4\sqrt{\pi}} \left(\frac{1}{\sqrt{(-1)^{3/4} z}} \left(\frac{\sqrt[4]{-1}\, e^{-\sqrt[4]{-1}\, z} \sqrt{i z^2}}{z} + e^{\sqrt[4]{-1}\, z} \right) \left(\sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} \frac{\left(\frac{1}{2}\right)_{2k}^2}{(2k)!} \left(-\frac{i}{4z^2}\right)^k + O\left(\frac{1}{z^{2\left\lfloor \frac{n}{2} \right\rfloor+2}}\right) \right) - \frac{1}{\sqrt{-\sqrt[4]{-1}\, z}} \left(e^{(-1)^{3/4} z} - \frac{(-1)^{3/4} e^{-(-1)^{3/4} z} \sqrt{-i z^2}}{z} \right) \left(\sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} \frac{\left(\frac{1}{2}\right)_{2k}^2}{(2k)!} \left(\frac{i}{4z^2}\right)^k + O\left(\frac{1}{z^{2\left\lfloor \frac{n}{2} \right\rfloor+2}}\right) \right) + \frac{(-1)^{3/4}}{2z} \left(\frac{1}{\sqrt{(-1)^{3/4} z}} \left(\frac{\sqrt[4]{-1}\, e^{-\sqrt[4]{-1}\, z} \sqrt{i z^2}}{z} - e^{\sqrt[4]{-1}\, z} \right) \left(\sum_{k=0}^{\left\lfloor \frac{n-1}{2} \right\rfloor} \frac{\left(\frac{1}{2}\right)_{2k+1}^2}{(2k+1)!} \left(-\frac{i}{4z^2}\right)^k + O\left(\frac{1}{z^{2\left\lfloor \frac{n-1}{2} \right\rfloor+2}}\right) \right) - \frac{1}{\sqrt{-\sqrt[4]{-1}\, z}} \left(i\, e^{(-1)^{3/4} z} - \frac{\sqrt[4]{-1}\, e^{-(-1)^{3/4} z} \sqrt{-i z^2}}{z} \right) \left(\sum_{k=0}^{\left\lfloor \frac{n-1}{2} \right\rfloor} \frac{\left(\frac{1}{2}\right)_{2k+1}^2}{(2k+1)!} \left(\frac{i}{4z^2}\right)^k + O\left(\frac{1}{z^{2\left\lfloor \frac{n-1}{2} \right\rfloor+2}}\right) \right) \right) \right) \;/;\; (|z| \to \infty) \land n \in \mathbb{N}$$

03.13.06.0037.01

$$\operatorname{bei}(z) \propto -\frac{(-1)^{3/4}}{2\sqrt{2\pi}} \left(e^{-\frac{z}{\sqrt{2}}} \left(\frac{e^{-\frac{i z}{\sqrt{2}}}}{\sqrt{-(-1)^{3/4} z}} \, {}_4F_1\!\left(\frac{1}{4}, \frac{1}{4}, \frac{3}{4}, \frac{3}{4}; \frac{1}{2}; -\frac{i}{z^2}\right) - \frac{e^{\frac{i z}{\sqrt{2}}}}{\sqrt{-\sqrt[4]{-1}\, z}} \, {}_4F_1\!\left(\frac{1}{4}, \frac{1}{4}, \frac{3}{4}, \frac{3}{4}; \frac{1}{2}; \frac{i}{z^2}\right) \right) + e^{\frac{z}{\sqrt{2}}} \left(\frac{e^{\frac{i z}{\sqrt{2}}}}{\sqrt{(-1)^{3/4} z}} \, {}_4F_1\!\left(\frac{1}{4}, \frac{1}{4}, \frac{3}{4}, \frac{3}{4}; \frac{1}{2}; -\frac{i}{z^2}\right) - \frac{e^{-\frac{i z}{\sqrt{2}}}}{\sqrt{\sqrt[4]{-1}\, z}} \, {}_4F_1\!\left(\frac{1}{4}, \frac{1}{4}, \frac{3}{4}, \frac{3}{4}; \frac{1}{2}; \frac{i}{z^2}\right) \right) - \frac{(-1)^{3/4}}{8 z} \left(e^{-\frac{z}{\sqrt{2}}} \left(\frac{i\, e^{\frac{i z}{\sqrt{2}}}}{\sqrt{-\sqrt[4]{-1}\, z}} \, {}_4F_1\!\left(\frac{3}{4}, \frac{3}{4}, \frac{5}{4}, \frac{5}{4}; \frac{3}{2}; \frac{i}{z^2}\right) - \frac{e^{-\frac{i z}{\sqrt{2}}}}{\sqrt{-(-1)^{3/4} z}} \, {}_4F_1\!\left(\frac{3}{4}, \frac{3}{4}, \frac{5}{4}, \frac{5}{4}; \frac{3}{2}; -\frac{i}{z^2}\right) \right) + e^{\frac{z}{\sqrt{2}}} \left(\frac{e^{\frac{i z}{\sqrt{2}}}}{\sqrt{(-1)^{3/4} z}} \, {}_4F_1\!\left(\frac{3}{4}, \frac{3}{4}, \frac{5}{4}, \frac{5}{4}; \frac{3}{2}; -\frac{i}{z^2}\right) - \frac{i\, e^{-\frac{i z}{\sqrt{2}}}}{\sqrt{\sqrt[4]{-1}\, z}} \, {}_4F_1\!\left(\frac{3}{4}, \frac{3}{4}, \frac{5}{4}, \frac{5}{4}; \frac{3}{2}; \frac{i}{z^2}\right) \right) \right) \right) /; (|z| \to \infty)$$

03.13.06.0038.01

$$\operatorname{bei}(z) \propto -\frac{(-1)^{3/4}}{2\sqrt{2\pi}} \left(e^{-\frac{z}{\sqrt{2}}} \left(\frac{e^{-\frac{i z}{\sqrt{2}}}}{\sqrt{-(-1)^{3/4} z}} \left(1 + O\!\left(\frac{1}{z^2}\right)\right) - \frac{e^{\frac{i z}{\sqrt{2}}}}{\sqrt{-\sqrt[4]{-1}\, z}} \left(1 + O\!\left(\frac{1}{z^2}\right)\right) \right) + e^{\frac{z}{\sqrt{2}}} \left(\frac{e^{\frac{i z}{\sqrt{2}}}}{\sqrt{(-1)^{3/4} z}} \left(1 + O\!\left(\frac{1}{z^2}\right)\right) - \frac{e^{-\frac{i z}{\sqrt{2}}}}{\sqrt{\sqrt[4]{-1}\, z}} \left(1 + O\!\left(\frac{1}{z^2}\right)\right) \right) \right) /; (|z| \to \infty)$$

03.13.06.0039.01

$$\operatorname{bei}(z) \propto \begin{cases} \dfrac{\sqrt[8]{-1}\, e^{-\sqrt[4]{-1}\, z} \left(-(-1)^{3/4} - \sqrt[4]{-1}\, e^{2\sqrt[4]{-1}\, z} - e^{i\sqrt{2}\, z} + i\, e^{\sqrt{2}\, z}\right)}{2\sqrt{2\pi}\, \sqrt{z}} & -\frac{1}{4} < \frac{\arg(z)}{\pi} \leq \frac{1}{4} \\ \dfrac{\sqrt[8]{-1}\, e^{-\sqrt[4]{-1}\, z} \left(-(-1)^{3/4} + \sqrt[4]{-1}\, e^{2\sqrt[4]{-1}\, z} - e^{i\sqrt{2}\, z} + i\, e^{\sqrt{2}\, z}\right)}{2\sqrt{2\pi}\, \sqrt{z}} & \frac{1}{4} < \frac{\arg(z)}{\pi} \leq \frac{3}{4} \\ \dfrac{\sqrt[8]{-1}\, e^{-\sqrt[4]{-1}\, z} \left(-(-1)^{3/4} + \sqrt[4]{-1}\, e^{2\sqrt[4]{-1}\, z} - e^{i\sqrt{2}\, z} - i\, e^{\sqrt{2}\, z}\right)}{2\sqrt{2\pi}\, \sqrt{z}} & \frac{\arg(z)}{\pi} > \frac{3}{4} \\ \dfrac{\sqrt[8]{-1}\, e^{-\sqrt[4]{-1}\, z} \left(-(-1)^{3/4} - \sqrt[4]{-1}\, e^{2\sqrt[4]{-1}\, z} + e^{i\sqrt{2}\, z} + i\, e^{\sqrt{2}\, z}\right)}{2\sqrt{2\pi}\, \sqrt{z}} & -\frac{3}{4} < \frac{\arg(z)}{\pi} \leq -\frac{1}{4} \\ \dfrac{\sqrt[8]{-1}\, e^{-\sqrt[4]{-1}\, z} \left((-1)^{3/4} - \sqrt[4]{-1}\, e^{2\sqrt[4]{-1}\, z} + e^{i\sqrt{2}\, z} + i\, e^{\sqrt{2}\, z}\right)}{2\sqrt{2\pi}\, \sqrt{z}} & \text{True} \end{cases} /; (|z| \to \infty)$$

Residue representations

03.13.06.0040.01

$$\operatorname{bei}(z) = \pi \sum_{j=0}^{\infty} \operatorname{res}_s\left(\frac{\left(\frac{z}{4}\right)^{-4s}}{\Gamma\left(\frac{1}{2}-s\right)\Gamma(1-s)^2}\,\Gamma\left(s+\frac{1}{2}\right)\right)\left(-j-\frac{1}{2}\right)$$

Integral representations

On the real axis

Of the direct function

03.13.07.0001.01

$$\operatorname{bei}(z) = \frac{1}{\pi}\int_0^{\pi} \sin\left(\frac{z\cos(t)}{\sqrt{2}}\right)\sinh\left(\frac{z\cos(t)}{\sqrt{2}}\right) dt$$

03.13.07.0002.01

$$\operatorname{bei}(z) = \frac{2}{\pi}\int_0^{1} \frac{\sin\left(\frac{tz}{\sqrt{2}}\right)\sinh\left(\frac{tz}{\sqrt{2}}\right)}{\sqrt{1-t^2}}\, dt$$

03.13.07.0003.01

$$\operatorname{bei}(z) = \frac{1}{\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin\left(\frac{z\sin(t)}{\sqrt{2}}\right)\sinh\left(\frac{z\sin(t)}{\sqrt{2}}\right) dt$$

03.13.07.0004.01

$$\operatorname{bei}(z) = \frac{1}{\pi}\int_0^{\pi} \sin\left(\frac{z\sin(t)}{\sqrt{2}}\right)\sinh\left(\frac{z\sin(t)}{\sqrt{2}}\right) dt$$

Contour integral representations

03.13.07.0005.01

$$\operatorname{bei}(z) == \frac{1}{2i}\int_{\mathcal{L}} \frac{\Gamma\left(s+\frac{1}{2}\right)}{\Gamma\left(\frac{1}{2}-s\right)\Gamma(1-s)^2}\left(\frac{z}{4}\right)^{-4s} ds$$

Limit representations

03.13.09.0001.01

$$\operatorname{bei}(z) = \frac{1}{2}\, i\left(\lim_{n\to\infty}\left(L_n\left(\frac{i z^2}{4n}\right) - L_n\left(-\frac{i z^2}{4n}\right)\right)\right)$$

03.13.09.0002.01

$$\operatorname{bei}(z) = \lim_{a\to\infty} \frac{1}{4} z^2\, {}_1F_3\left(a; 1, \frac{3}{2}, \frac{3}{2}; -\frac{z^4}{256\,a}\right)$$

Differential equations

Ordinary linear differential equations and wronskians

For the direct function itself

03.13.13.0001.01

$$w^{(4)}(z)\,z^4+2\,w^{(3)}(z)\,z^3-w''(z)\,z^2+w'(z)\,z+z^4\,w(z)=0\;/;\;w(z)=c_1\,\mathrm{ber}(z)+c_2\,\mathrm{bei}(z)+c_3\,\mathrm{ker}(z)+c_4\,\mathrm{kei}(z)$$

03.13.13.0002.01

$$W_z(\mathrm{ber}(z),\,\mathrm{bei}(z),\,\mathrm{ker}(z),\,\mathrm{kei}(z))=-\frac{1}{z^2}$$

03.13.13.0003.01

$$g(z)^4\,g'(z)^3\,w^{(4)}(z)+2\,g(z)^3\left(g'(z)^2-3\,g(z)\,g''(z)\right)g'(z)^2\,w^{(3)}(z)-g(z)^2\left(g'(z)^4+6\,g(z)\,g''(z)\,g'(z)^2+4\,g(z)^2\,g^{(3)}(z)\,g'(z)-15\,g(z)^2\,g''(z)^2\right)g'(z)\,w''(z)+g(z)\left(g'(z)^6+g(z)\,g''(z)\,g'(z)^4-2\,g(z)^2\,g^{(3)}(z)\,g'(z)^3+g(z)^2\left(6\,g''(z)^2-g(z)\,g^{(4)}(z)\right)g'(z)^2+10\,g(z)^3\,g''(z)\,g^{(3)}(z)\,g'(z)-15\,g(z)^3\,g''(z)^3\right)w'(z)+g(z)^4\,g'(z)^7\,w(z)=0\;/;\;w(z)=c_1\,\mathrm{ber}(g(z))+c_2\,\mathrm{bei}(g(z))+c_3\,\mathrm{ker}(g(z))+c_4\,\mathrm{kei}(g(z))$$

03.13.13.0004.01

$$W_z(\mathrm{ber}(g(z)),\,\mathrm{bei}(g(z)),\,\mathrm{ker}(g(z)),\,\mathrm{kei}(g(z)))=-\frac{g'(z)^6}{g(z)^2}$$

03.13.13.0005.01

$$g(z)^4\,g'(z)^3\,h(z)^4\,w^{(4)}(z)+2\,g(z)^3\,g'(z)^2\left(h(z)\left(g'(z)^2-3\,g(z)\,g''(z)\right)-2\,g(z)\,g'(z)\,h'(z)\right)h(z)^3\,w^{(3)}(z)+g(z)^2\,g'(z)\left(-\left(g'(z)^4+6\,g(z)\,g''(z)\,g'(z)^2+4\,g(z)^2\,g^{(3)}(z)\,g'(z)-15\,g(z)^2\,g''(z)^2\right)h(z)^2-6\,g(z)\,g'(z)\left(h'(z)\,g'(z)^2+g(z)\,h''(z)\,g'(z)-3\,g(z)\,h'(z)\,g''(z)\right)h(z)+12\,g(z)^2\,g'(z)^2\,h'(z)^2\right)h(z)^2\,w''(z)+g(z)\left(\left(g'(z)^6+g(z)\,g''(z)\,g'(z)^4-2\,g(z)^2\,g^{(3)}(z)\,g'(z)^3+g(z)^2\left(6\,g''(z)^2-g(z)\,g^{(4)}(z)\right)g'(z)^2+10\,g(z)^3\,g''(z)\,g^{(3)}(z)\,g'(z)-15\,g(z)^3\,g''(z)^3\right)h(z)^3+2\,g(z)\,g'(z)\left(h'(z)\,g'(z)^4-3\,g(z)\,h''(z)\,g'(z)^3-2\,g(z)\left(g(z)\,h^{(3)}(z)-3\,h'(z)\,g''(z)\right)g'(z)^2+g(z)^2\left(9\,g''(z)\,h''(z)+4\,h'(z)\,g^{(3)}(z)\right)g'(z)-15\,g(z)^2\,h'(z)\,g''(z)^2\right)h(z)^2+12\,g(z)^2\,g'(z)^2\,h'(z)\left(h'(z)\,g'(z)^2+2\,g(z)\,h''(z)\,g'(z)-3\,g(z)\,h'(z)\,g''(z)\right)h(z)-24\,g(z)^3\,g'(z)^3\,h'(z)^3\right)h(z)\,w'(z)+\left(g(z)^4\,h(z)^4\,g'(z)^7+g(z)^4\left(24\,h'(z)^4-36\,h(z)\,h''(z)\,h'(z)^2+8\,h(z)^2\,h^{(3)}(z)\,h'(z)+h(z)^2\left(6\,h''(z)^2-h(z)\,h^{(4)}(z)\right)\right)g'(z)^3-2\,g(z)^3\,h(z)\left(g'(z)^2-3\,g(z)\,g''(z)\right)\left(6\,h'(z)^3-6\,h(z)\,h''(z)\,h'(z)+h(z)^2\,h^{(3)}(z)\right)g'(z)^2+g(z)^2\,h(z)^2\left(h(z)\,h''(z)-2\,h'(z)^2\right)\left(g'(z)^4+6\,g(z)\,g''(z)\,g'(z)^2+4\,g(z)^2\,g^{(3)}(z)\,g'(z)-15\,g(z)^2\,g''(z)^2\right)g'(z)-g(z)\,h(z)^3\,h'(z)\left(g'(z)^6+g(z)\,g''(z)\,g'(z)^4-2\,g(z)^2\,g^{(3)}(z)\,g'(z)^3+g(z)^2\left(6\,g''(z)^2-g(z)\,g^{(4)}(z)\right)g'(z)^2+10\,g(z)^3\,g''(z)\,g^{(3)}(z)\,g'(z)-15\,g(z)^3\,g''(z)^3\right)\right)w(z)=0\;/;$$
$$w(z)=c_1\,h(z)\,\mathrm{ber}(g(z))+c_2\,h(z)\,\mathrm{bei}(g(z))+c_3\,h(z)\,\mathrm{ker}(g(z))+c_4\,h(z)\,\mathrm{kei}(g(z))$$

03.13.13.0006.01

$$W_z(h(z)\,\mathrm{ber}(g(z)),\,h(z)\,\mathrm{bei}(g(z)),\,h(z)\,\mathrm{ker}(g(z)),\,h(z)\,\mathrm{kei}(g(z)))=-\frac{h(z)^4\,g'(z)^6}{g(z)^2}$$

03.13.13.0007.01

$$z^4\,w^{(4)}(z)+(6-4\,r-4\,s)\,z^3\,w^{(3)}(z)+\left(4\,r^2+12\,(s-1)\,r+6\,(s-2)\,s+7\right)z^2\,w''(z)+(2\,r+2\,s-1)\,(-2\,(s-1)\,s+r\,(2-4\,s)-1)\,z\,w'(z)+\left(a^4\,r^4\,z^{4\,r}+s^4+4\,r\,s^3+4\,r^2\,s^2\right)w(z)=0\;/;$$
$$w(z)=c_1\,z^s\,\mathrm{ber}(a\,z^r)+c_2\,z^s\,\mathrm{bei}(a\,z^r)+c_3\,z^s\,\mathrm{ker}(a\,z^r)+c_4\,z^s\,\mathrm{kei}(a\,z^r)$$

03.13.13.0008.01

$$W_z(z^s\,\mathrm{ber}(a\,z^r),\,z^s\,\mathrm{bei}(a\,z^r),\,z^s\,\mathrm{ker}(a\,z^r),\,z^s\,\mathrm{kei}(a\,z^r))=-a^4\,r^6\,z^{4\,r+4\,s-6}$$

03.13.13.0009.01

$$w^{(4)}(z) - 4(\log(r) + \log(s))\, w^{(3)}(z) + 2\left(2\log^2(r) + 6\log(s)\log(r) + 3\log^2(s)\right) w''(z) + 4(\log(r) + \log(s))\left(-\log^2(s) - 2\log(r)\log(s)\right) w'(z) + \left(a^4 \log^4(r)\, r^{4z} + \log^4(s) + 4\log(r)\log^3(s) + 4\log^2(r)\log^2(s)\right) w(z) = 0 \;/;\; w(z) = c_1 s^z \operatorname{ber}(a r^z) + c_2 s^z \operatorname{bei}(a r^z) + c_3 s^z \operatorname{ker}(a r^z) + c_4 s^z \operatorname{kei}(a r^z)$$

03.13.13.0010.01

$$W_z(s^z \operatorname{ber}(a r^z), s^z \operatorname{bei}(a r^z), s^z \operatorname{ker}(a r^z), s^z \operatorname{kei}(a r^z)) = -a^4 r^{4z} s^{4z} \log^6(r)$$

Transformations

Transformations and argument simplifications

Argument involving basic arithmetic operations

03.13.16.0001.01

$$\operatorname{bei}(-z) = \operatorname{bei}(z)$$

03.13.16.0002.01

$$\operatorname{bei}(i z) = -\operatorname{bei}(z)$$

03.13.16.0003.01

$$\operatorname{bei}(-i z) = -\operatorname{bei}(z)$$

03.13.16.0004.01

$$\operatorname{bei}\left((-1)^{-1/4} z\right) = -\operatorname{bei}\left(\sqrt[4]{-1}\, z\right)$$

03.13.16.0005.01

$$\operatorname{bei}\left((-1)^{-3/4} z\right) = \operatorname{bei}\left(\sqrt[4]{-1}\, z\right)$$

03.13.16.0006.01

$$\operatorname{bei}\left((-1)^{3/4} z\right) = -\operatorname{bei}\left(\sqrt[4]{-1}\, z\right)$$

03.13.16.0007.01

$$\operatorname{bei}\left(\sqrt[4]{z^4}\right) = \frac{\sqrt{z^4}}{z^2} \operatorname{bei}(z)$$

Addition formulas

03.13.16.0008.01

$$\operatorname{bei}(z_1 - z_2) = \sum_{k=-\infty}^{\infty} (\operatorname{bei}_k(z_2) \operatorname{ber}_k(z_1) + \operatorname{bei}_k(z_1) \operatorname{ber}_k(z_2)) \;/;\; \left|\frac{z_2}{z_1}\right| < 1$$

03.13.16.0009.01

$$\operatorname{bei}(z_1 + z_2) = \sum_{k=-\infty}^{\infty} (-1)^k (\operatorname{bei}_k(z_2) \operatorname{ber}_k(z_1) + \operatorname{bei}_k(z_1) \operatorname{ber}_k(z_2)) \;/;\; \left|\frac{z_2}{z_1}\right| < 1$$

Multiple arguments

03.13.16.0010.01

$$\operatorname{bei}(z_1 z_2) = \sum_{k=0}^{\infty} \frac{\left(1 - z_1^2\right)^k}{k!} \left(\frac{z_2}{2}\right)^k \left(\cos\left(\frac{3 k \pi}{4}\right) \operatorname{bei}_k(z_2) + \operatorname{ber}_k(z_2) \sin\left(\frac{3 k \pi}{4}\right)\right) \;/;\; \left|\frac{z_2}{z_1}\right| < 1$$

Related transformations

Involving ber(z)

03.13.16.0011.01

$$\operatorname{bei}(z)+i\operatorname{ber}(z)=i\,J_0\left(\sqrt[4]{-1}\;z\right)$$

03.13.16.0012.01

$$\operatorname{bei}(z)-i\operatorname{ber}(z)=-i\,I_0\left(\sqrt[4]{-1}\;z\right)$$

Differentiation

Low-order differentiation

03.13.20.0001.01

$$\frac{\partial\operatorname{bei}(z)}{\partial z}=\frac{\operatorname{bei}_1(z)-\operatorname{ber}_1(z)}{\sqrt{2}}$$

03.13.20.0002.01

$$\frac{\partial^2\operatorname{bei}(z)}{\partial z^2}=\frac{1}{2}\left(\operatorname{ber}(z)-\operatorname{ber}_2(z)\right)$$

Symbolic differentiation

03.13.20.0003.01

$$\frac{\partial^n\operatorname{bei}(z)}{\partial z^n}=i\,2^{n-1}\sqrt{\pi}\;z^{-n}\left({}_1\tilde{F}_2\left(\frac{1}{2};\frac{1-n}{2},\frac{2-n}{2};-\frac{i z^2}{4}\right)-{}_1\tilde{F}_2\left(\frac{1}{2};\frac{1-n}{2},\frac{2-n}{2};\frac{i z^2}{4}\right)\right)/;\,n\in\mathbb{N}$$

03.13.20.0004.01

$$\frac{\partial^n\operatorname{bei}_0(z)}{\partial z^n}=2^{-\frac{3n}{2}-1}(i-1)^n$$

$$\left(\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\binom{n}{2k}\left((1+i^n)\operatorname{bei}_{4k-n}(z)-i\,(1-i^n)\operatorname{ber}_{4k-n}(z)\right)+\sum_{k=0}^{\left\lfloor\frac{n-1}{2}\right\rfloor}\binom{n}{2k+1}\left(i\,(1-i^n)\operatorname{ber}_{4k-n+2}(z)-(1+i^n)\operatorname{bei}_{4k-n+2}(z)\right)\right)/;\,n\in\mathbb{N}$$

03.13.20.0005.01

$$\frac{\partial^n\operatorname{bei}(z)}{\partial z^n}=2^{-\frac{3n}{2}-1}(i-1)^n\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\left(\frac{(n+1)\binom{n}{2k}}{2k+1}\left((1+i^n)\operatorname{bei}_{4k-n}(z)+\left(-i+i^{n+1}\right)\operatorname{ber}_{4k-n}(z)\right)+\right.$$

$$\left.\frac{1}{z}\left(\sqrt{2}\,(1+i)\,(4k-n+1)\binom{n}{2k+1}\left(\left(1-i^{n+1}\right)\operatorname{bei}_{4k-n+1}(z)+(-i+i^n)\operatorname{ber}_{4k-n+1}(z)\right)\right)\right)/;\,n\in\mathbb{N}$$

03.13.20.0006.01

$$\frac{\partial^n\operatorname{bei}(z)}{\partial z^n}=\pi\,G_{2,6}^{1,2}\left(\frac{z}{4},\frac{1}{4}\,\middle|\,\begin{array}{c}\frac{1-n}{4},\frac{3-n}{4}\\ \frac{2-n}{4},0,\frac{1}{4},\frac{1}{2},\frac{3}{4},-\frac{n}{4}\end{array}\right)/;\,n\in\mathbb{N}$$

Fractional integro-differentiation

03.13.20.0007.01

$$\frac{\partial^\alpha \operatorname{bei}(z)}{\partial z^\alpha} = z^{2-\alpha}\, 2^{2\alpha-\frac{11}{2}}\, \pi^2\, {}_2\tilde{F}_5\left(\frac{3}{4}, \frac{5}{4}; \frac{3}{2}, \frac{3-\alpha}{4}, 1-\frac{\alpha}{4}, \frac{5-\alpha}{4}, \frac{6-\alpha}{4}; -\frac{z^4}{256}\right)$$

Integration

Indefinite integration

03.13.21.0001.01

$$\int \operatorname{bei}(a\, z)\, dz = \frac{a^2\, z^3}{12}\, {}_1F_4\left(\frac{3}{4}; 1, \frac{3}{2}, \frac{3}{2}, \frac{7}{4}; -\frac{1}{256}\, a^4\, z^4\right)$$

Definite integration

03.13.21.0002.01

$$\int_0^\infty t^{\alpha-1}\, e^{-p\, t} \operatorname{bei}(t)\, dt = \frac{1}{4}\, p^{-\alpha-2}\, \Gamma(\alpha+2)\, {}_4F_3\left(\frac{\alpha+2}{4}, \frac{\alpha+3}{4}, \frac{\alpha}{4}+1, \frac{\alpha+5}{4}; 1, \frac{3}{2}, \frac{3}{2}; -\frac{1}{p^4}\right) /;$$
$$\operatorname{Re}(\alpha) > -2 \bigwedge \left(\operatorname{Re}(p) > \frac{1}{\sqrt{2}} \bigvee \left(\operatorname{Re}(p) = \frac{1}{\sqrt{2}} \bigwedge \operatorname{Re}(\alpha) < \frac{3}{2}\right)\right)$$

Integral transforms

Laplace transforms

03.13.22.0001.01

$$\mathcal{L}_t[\operatorname{bei}(t)]\,(z) = \frac{1}{\sqrt[4]{z^4+1}} \sin\left(\frac{1}{2} \tan^{-1}\left(\frac{1}{z^2}\right)\right) /; \operatorname{Re}(z) > \frac{1}{\sqrt{2}}$$

Mellin transforms

03.13.22.0002.01

$$\mathcal{M}_t\left[e^{-p\, t} \operatorname{bei}(t)\right](z) == \frac{1}{4}\, p^{-z-2}\, \Gamma(z+2)\, {}_4F_3\left(\frac{z+2}{4}, \frac{z+3}{4}, \frac{z}{4}+1, \frac{z+5}{4}; 1, \frac{3}{2}, \frac{3}{2}; -\frac{1}{p^4}\right) /; \operatorname{Re}(z) > -2 \bigwedge \operatorname{Re}(p) > \frac{1}{\sqrt{2}}$$

Representations through more general functions

Through hypergeometric functions

Involving ${}_p\tilde{F}_q$

03.13.26.0001.01

$$\operatorname{bei}(z) == \frac{1}{16}\, \pi\, z^2\, {}_0\tilde{F}_3\left(; \frac{3}{2}, \frac{3}{2}, 1; -\frac{z^4}{256}\right)$$

Involving ${}_pF_q$

03.13.26.0002.01

$$\operatorname{bei}(z) = \frac{1}{4} z^2 \, {}_0F_3\left(; \frac{3}{2}, \frac{3}{2}, 1; -\frac{z^4}{256}\right)$$

Through Meijer *G*

Classical cases for the direct function itself

03.13.26.0003.01

$$\operatorname{bei}(z) = \pi \, G_{0,4}^{1,0}\left(\frac{z^4}{256} \,\middle|\, \frac{1}{2}, 0, 0, \frac{1}{2}\right) /; -\frac{\pi}{4} \le \arg(z) \le \frac{\pi}{4} \bigvee \frac{3\pi}{4} < \arg(z) \le \pi \bigvee -\pi < \arg(z) < -\frac{3\pi}{4}$$

Classical cases for powers of bei

03.13.26.0004.01

$$\operatorname{bei}\left(\sqrt[4]{z}\right)^2 = \frac{1}{2} \pi^{3/2} \, G_{1,5}^{1,0}\left(\frac{z}{64} \,\middle|\, \begin{matrix} \frac{1}{2} \\ 0, 0, 0, \frac{1}{2}, \frac{1}{2} \end{matrix}\right) - \frac{1}{2} \sqrt{\frac{\pi}{2}} \, G_{2,6}^{1,2}\left(\frac{z}{16} \,\middle|\, \begin{matrix} \frac{1}{4}, \frac{3}{4} \\ 0, 0, 0, \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \end{matrix}\right)$$

Brychkov Yu.A. (2006)

03.13.26.0005.01

$$\operatorname{bei}(z)^2 = \frac{1}{2} \pi^{3/2} \, G_{1,5}^{1,0}\left(\frac{z^4}{64} \,\middle|\, \begin{matrix} \frac{1}{2} \\ 0, 0, 0, \frac{1}{2}, \frac{1}{2} \end{matrix}\right) - \frac{1}{2} \sqrt{\frac{\pi}{2}} \, G_{2,6}^{1,2}\left(\frac{z^4}{16} \,\middle|\, \begin{matrix} \frac{1}{4}, \frac{3}{4} \\ 0, 0, 0, \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \end{matrix}\right)$$

Brychkov Yu.A. (2006)

03.13.26.0006.01

$$\operatorname{bei}(z)^2 = \frac{1}{2} \sqrt{\pi} \, G_{0,4}^{1,0}\left(-\frac{z^4}{64} \,\middle|\, 0, 0, 0, \frac{1}{2}\right) - \frac{\pi^{3/2}}{2\sqrt{2}} \, G_{3,7}^{1,2}\left(-\frac{z^4}{16} \,\middle|\, \begin{matrix} \frac{1}{4}, \frac{3}{4}, \frac{1}{2} \\ 0, 0, 0, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \end{matrix}\right)$$

Classical cases involving powers of ber

03.13.26.0007.01

$$\operatorname{bei}\left(\sqrt[4]{z}\right)^2 + \operatorname{ber}\left(\sqrt[4]{z}\right)^2 = \pi^{3/2} \, G_{1,5}^{1,0}\left(\frac{z}{64} \,\middle|\, \begin{matrix} \frac{1}{2} \\ 0, 0, 0, \frac{1}{2}, \frac{1}{2} \end{matrix}\right)$$

Brychkov Yu.A. (2006)

03.13.26.0008.01

$$\operatorname{bei}\left(\sqrt[4]{z}\right)^2 - \operatorname{ber}\left(\sqrt[4]{z}\right)^2 = -\sqrt{\frac{\pi}{2}} \, G_{2,6}^{1,2}\left(\frac{z}{16} \,\middle|\, \begin{matrix} \frac{1}{4}, \frac{3}{4} \\ 0, 0, 0, \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \end{matrix}\right)$$

Brychkov Yu.A. (2006)

03.13.26.0009.01

$$\operatorname{bei}(z)^2 + \operatorname{ber}(z)^2 = \pi^{3/2} \, G_{1,5}^{1,0}\left(\frac{z^4}{64} \,\middle|\, \begin{matrix} \frac{1}{2} \\ 0, 0, 0, \frac{1}{2}, \frac{1}{2} \end{matrix}\right)$$

Brychkov Yu.A. (2006)

03.13.26.0010.01

$$\operatorname{bei}(z)^2-\operatorname{ber}(z)^2=-\sqrt{\frac{\pi}{2}}\,G_{2,6}^{1,2}\!\left(\frac{z^4}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\0,0,0,\frac{1}{2},\frac{1}{2},\frac{1}{2}\end{matrix}\right)$$

Brychkov Yu.A. (2006)

Classical cases involving ber

03.13.26.0011.01

$$\operatorname{bei}\!\left(\sqrt[4]{z}\right)\operatorname{ber}\!\left(\sqrt[4]{z}\right)=\frac{1}{2}\sqrt{\frac{\pi}{2}}\,G_{2,6}^{1,2}\!\left(\frac{z}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\\frac{1}{2},0,0,0,\frac{1}{2},\frac{1}{2}\end{matrix}\right)$$

Brychkov Yu.A. (2006)

03.13.26.0012.01

$$\operatorname{bei}\!\left(\sqrt[4]{z}\right)\operatorname{ber}\!\left(\sqrt[4]{z}\right)=\frac{i\,\pi^{3/2}}{2\sqrt{2}}\,G_{3,7}^{1,2}\!\left(-\frac{z}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4},0\\\frac{1}{2},0,0,0,0,\frac{1}{2},\frac{1}{2}\end{matrix}\right)/;\ -\pi<\arg(z)\le 0$$

03.13.26.0013.01

$$\operatorname{bei}(z)\operatorname{ber}(z)=\frac{1}{2}\sqrt{\frac{\pi}{2}}\,G_{2,6}^{1,2}\!\left(\frac{z^4}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\\frac{1}{2},0,0,0,\frac{1}{2},\frac{1}{2}\end{matrix}\right)/;\ -\frac{\pi}{4}\le\arg(z)\le\frac{\pi}{4}\bigwedge\frac{3\pi}{4}<\arg(z)\le\pi\bigwedge-\pi<\arg(z)<-\frac{3\pi}{4}$$

03.13.26.0014.01

$$\operatorname{bei}(z)\operatorname{ber}(z)=\frac{i\,\pi^{3/2}}{2\sqrt{2}}\,G_{3,7}^{1,2}\!\left(-\frac{z^4}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4},0\\\frac{1}{2},0,0,0,0,\frac{1}{2},\frac{1}{2}\end{matrix}\right)/;\ -\frac{\pi}{2}<\arg(z)\le 0\bigwedge\frac{\pi}{2}<\arg(z)\le\pi$$

Classical cases involving kei

03.13.26.0015.01

$$\operatorname{bei}\!\left(\sqrt[4]{z}\right)\operatorname{kei}\!\left(\sqrt[4]{z}\right)=\frac{1}{8}\sqrt{\pi}\,G_{0,4}^{2,0}\!\left(\frac{z}{64}\,\middle|\,0,0,0,\frac{1}{2}\right)-\frac{1}{8\sqrt{2\pi}}\,G_{2,6}^{3,2}\!\left(\frac{z}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\0,0,\frac{1}{2},0,\frac{1}{2},\frac{1}{2}\end{matrix}\right)$$

Brychkov Yu.A. (2006)

03.13.26.0016.01

$$\operatorname{bei}(z)\operatorname{kei}(z)=\frac{1}{8}\sqrt{\pi}\,G_{0,4}^{2,0}\!\left(\frac{z^4}{64}\,\middle|\,0,0,0,\frac{1}{2}\right)-\frac{1}{8\sqrt{2\pi}}\,G_{2,6}^{3,2}\!\left(\frac{z^4}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\0,0,\frac{1}{2},0,\frac{1}{2},\frac{1}{2}\end{matrix}\right)/;\ 0\le\arg(z)\le\frac{\pi}{4}$$

Brychkov Yu.A. (2006)

Classical cases involving ker

03.13.26.0017.01

$$\operatorname{bei}\!\left(\sqrt[4]{z}\right)\operatorname{ker}\!\left(\sqrt[4]{z}\right)=\frac{1}{8}\sqrt{\pi}\,G_{0,4}^{2,0}\!\left(\frac{z}{64}\,\middle|\,0,\frac{1}{2},0,0\right)-\frac{1}{8\sqrt{2\pi}}\,G_{2,6}^{3,2}\!\left(\frac{z}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\0,\frac{1}{2},\frac{1}{2},0,0,\frac{1}{2}\end{matrix}\right)$$

Brychkov Yu.A. (2006)

03.13.26.0018.01

$$\operatorname{bei}(z)\operatorname{ker}(z) = \frac{1}{8}\sqrt{\pi}\, G_{0,4}^{2,0}\left(\frac{z^4}{64} \,\middle|\, 0, \tfrac{1}{2}, 0, 0\right) - \frac{1}{8\sqrt{2\pi}} G_{2,6}^{3,2}\left(\frac{z^4}{16} \,\middle|\, \begin{matrix} \tfrac{1}{4}, \tfrac{3}{4} \\ 0, \tfrac{1}{2}, \tfrac{1}{2}, 0, 0, \tfrac{1}{2} \end{matrix}\right) /; 0 \leq \arg(z) \leq \frac{\pi}{4}$$

Brychkov Yu.A. (2006)

Classical cases involving ber, ker and kei

03.13.26.0019.01

$$\operatorname{bei}\left(\sqrt[4]{z}\right)\operatorname{kei}\left(\sqrt[4]{z}\right) + \operatorname{ber}\left(\sqrt[4]{z}\right)\operatorname{ker}\left(\sqrt[4]{z}\right) = \frac{1}{4}\sqrt{\pi}\, G_{0,4}^{2,0}\left(\frac{z}{64} \,\middle|\, 0, 0, 0, \tfrac{1}{2}\right)$$

Brychkov Yu.A. (2006)

03.13.26.0020.01

$$\operatorname{bei}\left(\sqrt[4]{z}\right)\operatorname{kei}\left(\sqrt[4]{z}\right) - \operatorname{ber}\left(\sqrt[4]{z}\right)\operatorname{ker}\left(\sqrt[4]{z}\right) = -\frac{1}{2^{5/2}\sqrt{\pi}} G_{2,6}^{3,2}\left(\frac{z}{16} \,\middle|\, \begin{matrix} \tfrac{1}{4}, \tfrac{3}{4} \\ 0, 0, \tfrac{1}{2}, 0, \tfrac{1}{2}, \tfrac{1}{2} \end{matrix}\right)$$

Brychkov Yu.A. (2006)

03.13.26.0021.01

$$\operatorname{ber}\left(\sqrt[4]{z}\right)\operatorname{kei}\left(\sqrt[4]{z}\right) + \operatorname{bei}\left(\sqrt[4]{z}\right)\operatorname{ker}\left(\sqrt[4]{z}\right) = -\frac{1}{2^{5/2}\sqrt{\pi}} G_{2,6}^{3,2}\left(\frac{z}{16} \,\middle|\, \begin{matrix} \tfrac{1}{4}, \tfrac{3}{4} \\ 0, \tfrac{1}{2}, \tfrac{1}{2}, 0, 0, \tfrac{1}{2} \end{matrix}\right)$$

Brychkov Yu.A. (2006)

03.13.26.0022.01

$$\operatorname{bei}\left(\sqrt[4]{z}\right)\operatorname{ker}\left(\sqrt[4]{z}\right) - \operatorname{ber}\left(\sqrt[4]{z}\right)\operatorname{kei}\left(\sqrt[4]{z}\right) = \frac{1}{4}\sqrt{\pi}\, G_{0,4}^{2,0}\left(\frac{z}{64} \,\middle|\, 0, \tfrac{1}{2}, 0, 0\right)$$

Brychkov Yu.A. (2006)

03.13.26.0023.01

$$\operatorname{bei}(z)\operatorname{kei}(z) + \operatorname{ber}(z)\operatorname{ker}(z) = \frac{1}{4}\sqrt{\pi}\, G_{0,4}^{2,0}\left(\frac{z^4}{64} \,\middle|\, 0, 0, 0, \tfrac{1}{2}\right) /; -\frac{\pi}{4} < \arg(z) \leq \frac{\pi}{4}$$

Brychkov Yu.A. (2006)

03.13.26.0024.01

$$\operatorname{bei}(z)\operatorname{kei}(z) - \operatorname{ber}(z)\operatorname{ker}(z) = -\frac{1}{2^{5/2}\sqrt{\pi}} G_{2,6}^{3,2}\left(\frac{z^4}{16} \,\middle|\, \begin{matrix} \tfrac{1}{4}, \tfrac{3}{4} \\ 0, 0, \tfrac{1}{2}, 0, \tfrac{1}{2}, \tfrac{1}{2} \end{matrix}\right) /; -\frac{\pi}{4} < \arg(z) \leq \frac{\pi}{4}$$

Brychkov Yu.A. (2006)

03.13.26.0025.01

$$\operatorname{ber}(z)\operatorname{kei}(z) + \operatorname{bei}(z)\operatorname{ker}(z) = -\frac{1}{2^{5/2}\sqrt{\pi}} G_{2,6}^{3,2}\left(\frac{z^4}{16} \,\middle|\, \begin{matrix} \tfrac{1}{4}, \tfrac{3}{4} \\ 0, \tfrac{1}{2}, \tfrac{1}{2}, 0, 0, \tfrac{1}{2} \end{matrix}\right) /; -\frac{\pi}{4} < \arg(z) \leq \frac{\pi}{4}$$

Brychkov Yu.A. (2006)

03.13.26.0026.01

$$\operatorname{bei}(z)\operatorname{ker}(z)-\operatorname{ber}(z)\operatorname{kei}(z)=\frac{1}{4}\sqrt{\pi}\,G_{0,4}^{2,0}\!\left(\frac{z^4}{64}\,\middle|\,0,\tfrac{1}{2},0,0\right)/;\;-\frac{\pi}{4}<\arg(z)\le\frac{\pi}{4}\bigvee\frac{3\pi}{4}<\arg(z)\le\pi\bigvee-\pi<\arg(z)\le-\frac{3\pi}{4}$$

Brychkov Yu.A. (2006)

Classical cases involving Bessel *J*

03.13.26.0027.01

$$J_0\!\left((-1)^{3/4}z\right)\operatorname{bei}(z)=-\frac{1}{2}i\sqrt{\pi}\left(\sqrt{2}\,G_{2,4}^{1,1}\!\left(i z^2\,\middle|\,\begin{matrix}\frac{1}{2},\frac{1}{4}\\0,0,0,\frac{1}{4}\end{matrix}\right)-G_{0,4}^{1,0}\!\left(-\frac{z^4}{64}\,\middle|\,0,0,0,\tfrac{1}{2}\right)\right)$$

Classical cases involving Bessel *I*

03.13.26.0028.01

$$I_0\!\left(\sqrt[4]{-1}\,z\right)\operatorname{bei}(z)=\frac{1}{2}i\sqrt{\pi}\left(G_{0,4}^{1,0}\!\left(-\frac{z^4}{64}\,\middle|\,0,0,0,\tfrac{1}{2}\right)-\sqrt{2}\,G_{2,4}^{1,1}\!\left(i z^2\,\middle|\,\begin{matrix}\frac{1}{2},\frac{1}{4}\\0,0,0,\frac{1}{4}\end{matrix}\right)\right)$$

Classical cases involving Bessel *K*

03.13.26.0029.01

$$K_0\!\left(\sqrt[4]{-1}\,z\right)\operatorname{bei}(z)=\frac{1}{4}i\pi^{3/2}\left(\frac{1}{2\pi^2}G_{0,4}^{3,0}\!\left(-\frac{z^4}{64}\,\middle|\,0,0,\tfrac{1}{2},0\right)+2\,G_{3,5}^{2,1}\!\left(i z^2\,\middle|\,\begin{matrix}\frac{1}{2},-\frac{1}{4},\frac{1}{4}\\0,0,-\frac{1}{4},0,\frac{1}{4}\end{matrix}\right)\right)/;\;-\frac{\pi}{2}<\arg(z)\le 0$$

Classical cases involving ${}_0F_1$

03.13.26.0030.01

$${}_0F_1\!\left(;1;\frac{i\sqrt{z}}{4}\right)\operatorname{bei}\!\left(\sqrt[4]{z}\right)=$$

$$-\frac{1}{2}i\sqrt{\frac{\pi}{2}}\left(-\sqrt{2}\,\pi\,G_{1,5}^{1,0}\!\left(\frac{z}{64}\,\middle|\,\begin{matrix}\frac{1}{2}\\0,0,0,\frac{1}{2},\frac{1}{2}\end{matrix}\right)+G_{2,6}^{1,2}\!\left(\frac{z}{16}\,\middle|\,\begin{matrix}\frac{3}{4},\frac{1}{4}\\0,0,0,\frac{1}{2},\frac{1}{2},\frac{1}{2}\end{matrix}\right)+i\,G_{2,6}^{1,2}\!\left(\frac{z}{16}\,\middle|\,\begin{matrix}\frac{3}{4},\frac{1}{4}\\\frac{1}{2},0,0,0,\frac{1}{2},\frac{1}{2}\end{matrix}\right)\right)$$

03.13.26.0031.01

$${}_0F_1\!\left(;1;\frac{i z^2}{4}\right)\operatorname{bei}(z)=$$

$$-\frac{1}{2}i\sqrt{\frac{\pi}{2}}\left(-\sqrt{2}\,\pi\,G_{1,5}^{1,0}\!\left(\frac{z^4}{64}\,\middle|\,\begin{matrix}\frac{1}{2}\\0,0,0,\frac{1}{2},\frac{1}{2}\end{matrix}\right)+G_{2,6}^{1,2}\!\left(\frac{z^4}{16}\,\middle|\,\begin{matrix}\frac{3}{4},\frac{1}{4}\\0,0,0,\frac{1}{2},\frac{1}{2},\frac{1}{2}\end{matrix}\right)+i\,G_{2,6}^{1,2}\!\left(\frac{z^4}{16}\,\middle|\,\begin{matrix}\frac{3}{4},\frac{1}{4}\\\frac{1}{2},0,0,0,\frac{1}{2},\frac{1}{2}\end{matrix}\right)\right)/;$$

$$-\frac{\pi}{4}<\arg(z)\le\frac{\pi}{4}\bigvee\frac{3\pi}{4}<\arg(z)\le\pi\bigvee-\pi<\arg(z)\le\frac{5\pi}{4}$$

03.13.26.0032.01

$${}_0F_1\!\left(;1;\frac{i z^2}{4}\right)\operatorname{bei}(z)=-\frac{1}{2}i\sqrt{\pi}\left(\sqrt{2}\,G_{2,4}^{1,1}\!\left(i z^2\,\middle|\,\begin{matrix}\frac{1}{2},\frac{1}{4}\\0,0,0,\frac{1}{4}\end{matrix}\right)-G_{0,4}^{1,0}\!\left(-\frac{z^4}{64}\,\middle|\,0,0,0,\tfrac{1}{2}\right)\right)$$

Generalized cases for the direct function itself

03.13.26.0033.01

$$\operatorname{bei}(z) = \pi\, G_{0,4}^{1,0}\!\left(\frac{z}{4}, \frac{1}{4} \,\middle|\, \tfrac{1}{2}, 0, 0, \tfrac{1}{2}\right)$$

Generalized cases for powers of bei

03.13.26.0034.01

$$\operatorname{bei}(z)^2 = \frac{1}{2}\pi^{3/2}\, G_{1,5}^{1,0}\!\left(\frac{z}{2\sqrt{2}}, \frac{1}{4} \,\middle|\, \begin{matrix} \tfrac{1}{2} \\ 0, 0, 0, \tfrac{1}{2}, \tfrac{1}{2} \end{matrix}\right) - \frac{1}{2}\sqrt{\frac{\pi}{2}}\, G_{2,6}^{1,2}\!\left(\frac{z}{2}, \frac{1}{4} \,\middle|\, \begin{matrix} \tfrac{1}{4}, \tfrac{3}{4} \\ 0, 0, 0, \tfrac{1}{2}, \tfrac{1}{2}, \tfrac{1}{2} \end{matrix}\right)$$

Brychkov Yu.A. (2006)

03.13.26.0035.01

$$\operatorname{bei}(z)^2 = \frac{1}{2}\sqrt{\pi}\, G_{0,4}^{1,0}\!\left(\frac{\sqrt[4]{-1}\, z}{2\sqrt{2}}, \frac{1}{4} \,\middle|\, 0, 0, 0, \tfrac{1}{2}\right) - \frac{\pi^{3/2}}{2\sqrt{2}}\, G_{3,7}^{1,2}\!\left(\frac{1}{2}\sqrt[4]{-1}\, z, \frac{1}{4} \,\middle|\, \begin{matrix} \tfrac{1}{4}, \tfrac{3}{4}, \tfrac{1}{2} \\ 0, 0, 0, \tfrac{1}{2}, \tfrac{1}{2}, \tfrac{1}{2}, \tfrac{1}{2} \end{matrix}\right)$$

Generalized cases involving powers of ber

03.13.26.0036.01

$$\operatorname{bei}(z)^2 + \operatorname{ber}(z)^2 = \pi^{3/2}\, G_{1,5}^{1,0}\!\left(\frac{z}{2\sqrt{2}}, \frac{1}{4} \,\middle|\, \begin{matrix} \tfrac{1}{2} \\ 0, 0, 0, \tfrac{1}{2}, \tfrac{1}{2} \end{matrix}\right)$$

Brychkov Yu.A. (2006)

03.13.26.0037.01

$$\operatorname{bei}(z)^2 - \operatorname{ber}(z)^2 = -\sqrt{\frac{\pi}{2}}\, G_{2,6}^{1,2}\!\left(\frac{z}{2}, \frac{1}{4} \,\middle|\, \begin{matrix} \tfrac{1}{4}, \tfrac{3}{4} \\ 0, 0, 0, \tfrac{1}{2}, \tfrac{1}{2}, \tfrac{1}{2} \end{matrix}\right)$$

Brychkov Yu.A. (2006)

Generalized cases involving ber

03.13.26.0038.01

$$\operatorname{bei}(z)\operatorname{ber}(z) = \frac{1}{2}\sqrt{\frac{\pi}{2}}\, G_{2,6}^{1,2}\!\left(\frac{z}{2}, \frac{1}{4} \,\middle|\, \begin{matrix} \tfrac{1}{4}, \tfrac{3}{4} \\ \tfrac{1}{2}, 0, 0, 0, \tfrac{1}{2}, \tfrac{1}{2} \end{matrix}\right)$$

Brychkov Yu.A. (2006)

03.13.26.0039.01

$$\operatorname{bei}(z)\operatorname{ber}(-z) = \frac{i\,\pi^{3/2}}{2\sqrt{2}}\, G_{3,7}^{1,2}\!\left(\frac{1}{2}\sqrt[4]{-1}\, z, \frac{1}{4} \,\middle|\, \begin{matrix} \tfrac{1}{4}, \tfrac{3}{4}, 0 \\ \tfrac{1}{2}, 0, 0, 0, 0, \tfrac{1}{2}, \tfrac{1}{2} \end{matrix}\right)$$

Generalized cases involving kei

03.13.26.0040.01

$$\operatorname{bei}(z)\operatorname{kei}(z) = \frac{1}{8}\sqrt{\pi}\, G_{0,4}^{2,0}\!\left(\frac{z}{2\sqrt{2}}, \frac{1}{4} \,\middle|\, 0, 0, 0, \tfrac{1}{2}\right) - \frac{1}{8\sqrt{2\pi}}\, G_{2,6}^{3,2}\!\left(\frac{z}{2}, \frac{1}{4} \,\middle|\, \begin{matrix} \tfrac{1}{4}, \tfrac{3}{4} \\ 0, 0, \tfrac{1}{2}, 0, \tfrac{1}{2}, \tfrac{1}{2} \end{matrix}\right)$$

Brychkov Yu.A. (2006)

Generalized cases involving ker

03.13.26.0041.01

$$\operatorname{bei}(z)\operatorname{ker}(z) = \frac{1}{8}\sqrt{\pi}\, G_{0,4}^{2,0}\left(\frac{z}{2\sqrt{2}}, \frac{1}{4} \,\middle|\, 0, \tfrac{1}{2}, 0, 0\right) - \frac{1}{8\sqrt{2\pi}} G_{2,6}^{3,2}\left(\frac{z}{2}, \frac{1}{4} \,\middle|\, \begin{matrix} \tfrac{1}{4}, \tfrac{3}{4} \\ 0, \tfrac{1}{2}, \tfrac{1}{2}, 0, 0, \tfrac{1}{2} \end{matrix}\right)$$

Brychkov Yu.A. (2006)

Generalized cases involving ber, ker and kei

03.13.26.0042.01

$$\operatorname{bei}(z)\operatorname{kei}(z) + \operatorname{ber}(z)\operatorname{ker}(z) = \frac{1}{4}\sqrt{\pi}\, G_{0,4}^{2,0}\left(\frac{z}{2\sqrt{2}}, \frac{1}{4} \,\middle|\, 0, 0, 0, \tfrac{1}{2}\right)$$

Brychkov Yu.A. (2006)

03.13.26.0043.01

$$\operatorname{bei}(z)\operatorname{kei}(z) - \operatorname{ber}(z)\operatorname{ker}(z) = -\frac{1}{4\sqrt{2\pi}} G_{2,6}^{3,2}\left(\frac{z}{2}, \frac{1}{4} \,\middle|\, \begin{matrix} \tfrac{1}{4}, \tfrac{3}{4} \\ 0, 0, \tfrac{1}{2}, 0, \tfrac{1}{2}, \tfrac{1}{2} \end{matrix}\right)$$

Brychkov Yu.A. (2006)

03.13.26.0044.01

$$\operatorname{bei}(z)\operatorname{ker}(z) + \operatorname{ber}(z)\operatorname{kei}(z) = -\frac{1}{4\sqrt{2\pi}} G_{2,6}^{3,2}\left(\frac{z}{2}, \frac{1}{4} \,\middle|\, \begin{matrix} \tfrac{1}{4}, \tfrac{3}{4} \\ 0, \tfrac{1}{2}, \tfrac{1}{2}, 0, 0, \tfrac{1}{2} \end{matrix}\right)$$

Brychkov Yu.A. (2006)

03.13.26.0045.01

$$\operatorname{bei}(z)\operatorname{ker}(z) - \operatorname{ber}(z)\operatorname{kei}(z) = \frac{1}{4}\sqrt{\pi}\, G_{0,4}^{2,0}\left(\frac{z}{2\sqrt{2}}, \frac{1}{4} \,\middle|\, 0, \tfrac{1}{2}, 0, 0\right)$$

Brychkov Yu.A. (2006)

Generalized cases involving Bessel *J*

03.13.26.0046.01

$$J_0\left((-1)^{3/4} z\right)\operatorname{bei}(z) = -\frac{1}{2}\, i\sqrt{\pi}\left(\sqrt{2}\, G_{2,4}^{1,1}\left(\sqrt[4]{-1}\, z, \frac{1}{2} \,\middle|\, \begin{matrix} \tfrac{1}{2}, \tfrac{1}{4} \\ 0, 0, 0, \tfrac{1}{4} \end{matrix}\right) - G_{0,4}^{1,0}\left(\frac{\sqrt[4]{-1}\, z}{2\sqrt{2}}, \frac{1}{4} \,\middle|\, 0, 0, 0, \tfrac{1}{2}\right)\right)$$

Generalized cases involving Bessel *I*

03.13.26.0047.01

$$I_0\left(\sqrt[4]{-1}\, z\right)\operatorname{bei}(z) = -\frac{1}{2}(-1)^{3/4} e^{-\frac{1}{4}(i\pi)}\sqrt{\pi}\left(\sqrt{2}\, G_{2,4}^{1,1}\left(\sqrt[4]{-1}\, z, \frac{1}{2} \,\middle|\, \begin{matrix} \tfrac{1}{2}, \tfrac{1}{4} \\ 0, 0, 0, \tfrac{1}{4} \end{matrix}\right) - G_{0,4}^{1,0}\left(\frac{\sqrt[4]{-1}\, z}{2\sqrt{2}}, \frac{1}{4} \,\middle|\, 0, 0, 0, \tfrac{1}{2}\right)\right)$$

Generalized cases involving Bessel *K*

03.13.26.0048.01

$$K_0\left(\sqrt[4]{-1}\, z\right) \operatorname{bei}(z) = \frac{1}{4}\, i\, \pi^{3/2} \left(\frac{1}{2\,\pi^2}\, G_{0,4}^{3,0}\left(\frac{\sqrt[4]{-1}\, z}{2\sqrt{2}}, \frac{1}{4} \,\middle|\, 0, 0, \tfrac{1}{2}, 0 \right) + 2\, G_{3,5}^{2,1}\left(\sqrt[4]{-1}\, z, \frac{1}{2} \,\middle|\, \begin{matrix} \frac{1}{2}, -\frac{1}{4}, \frac{1}{4} \\ 0, 0, -\frac{1}{4}, 0, \frac{1}{4} \end{matrix} \right) \right)$$

Generalized cases involving $_0F_1$

03.13.26.0049.01

$$_0F_1\left(; 1; \frac{i\, z^2}{4}\right) \operatorname{bei}(z) =$$

$$-\frac{1}{2}\, i \sqrt{\frac{\pi}{2}} \left(-\sqrt{2}\, \pi\, G_{1,5}^{1,0}\left(\frac{z}{2\sqrt{2}}, \frac{1}{4} \,\middle|\, \begin{matrix} \frac{1}{2} \\ 0, 0, 0, \frac{1}{2}, \frac{1}{2} \end{matrix} \right) + G_{2,6}^{1,2}\left(\frac{z}{2}, \frac{1}{4} \,\middle|\, \begin{matrix} \frac{3}{4}, \frac{1}{4} \\ 0, 0, 0, \frac{1}{2}, \frac{1}{2}, \frac{1}{2} \end{matrix} \right) + i\, G_{2,6}^{1,2}\left(\frac{z}{2}, \frac{1}{4} \,\middle|\, \begin{matrix} \frac{3}{4}, \frac{1}{4} \\ \frac{1}{2}, 0, 0, 0, \frac{1}{2}, \frac{1}{2} \end{matrix} \right) \right)$$

03.13.26.0050.01

$$_0F_1\left(; 1; \frac{i\, z^2}{4}\right) \operatorname{bei}(z) = -\frac{1}{2}\, i \sqrt{\pi} \left(\sqrt{2}\; G_{2,4}^{1,1}\left(\sqrt[4]{-1}\, z, \frac{1}{2} \,\middle|\, \begin{matrix} \frac{1}{2}, \frac{1}{4} \\ 0, 0, 0, \frac{1}{4} \end{matrix} \right) - G_{0,4}^{1,0}\left(\frac{\sqrt[4]{-1}\, z}{2\sqrt{2}}, \frac{1}{4} \,\middle|\, 0, 0, 0, \tfrac{1}{2} \right) \right)$$

Representations through equivalent functions

With related functions

03.13.27.0001.01

$$\operatorname{bei}(z) = \frac{z^2}{2\sqrt{-z^4}} \left(I_0\left(\sqrt[4]{-z^4}\right) - J_0\left(\sqrt[4]{-z^4}\right) \right)$$

03.13.27.0002.01

$$\operatorname{bei}(z) = -\frac{1}{2}\, i \left(I_0\left(\sqrt[4]{-1}\, z\right) - J_0\left(\sqrt[4]{-1}\, z\right) \right)$$

03.13.27.0003.01

$$\operatorname{bei}(z) + i \operatorname{ber}(z) = i\, J_0\left(\sqrt[4]{-1}\, z\right)$$

03.13.27.0004.01

$$\operatorname{bei}(z) - i \operatorname{ber}(z) = -i\, I_0\left(\sqrt[4]{-1}\, z\right)$$

Theorems

History

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