KelvinBer2

Notations

Traditional name

Kelvin function of the first kind

Traditional notation

$\operatorname{ber}_\nu(z)$

Mathematica StandardForm notation

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KelvinBer[ν, z]
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Primary definition

03.18.02.0001.01

$$\operatorname{ber}_\nu(z) = \frac{1}{2} e^{-\frac{3}{4} i \pi \nu} z^\nu \left(\sqrt[4]{-1}\; z\right)^{-\nu} \left(e^{\frac{3 i \pi \nu}{2}} I_\nu\left(\sqrt[4]{-1}\; z\right) + J_\nu\left(\sqrt[4]{-1}\; z\right)\right)$$

Specific values

Specialized values

For fixed ν

03.18.03.0001.01

$$\operatorname{ber}_\nu(0) = 0 \;/;\; \nu \in \mathbb{N}^+ \bigvee \operatorname{Re}(\nu) > 0$$

03.18.03.0002.01

$$\operatorname{ber}_\nu(0) = \tilde{\infty} \;/;\; \operatorname{Re}(\nu) < 0$$

03.18.03.0003.01

$$\operatorname{ber}_\nu(0) = ¿ \;/;\; \operatorname{Re}(\nu) = 0 \land \nu \neq 0$$

For fixed z

Explicit rational ν

03.18.03.0004.01

$$\operatorname{ber}_0(z) = \operatorname{ber}(z)$$

03.18.03.0005.01

$$\operatorname{ber}_{-\frac{14}{3}}(z) = -\frac{i\left(\sqrt[4]{-1}\, z\right)^{14/3}}{162\, 3^{5/6}\, z^{26/3}\, ((1+i)\, z)^{2/3}} \left(144\sqrt{3}\left(9\, i\, z^2 + 110\right) \operatorname{Ai}\left(\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) ((1+i)\, z)^{4/3} + 144\sqrt{3}\, i\left(9\, z^2 + 110\, i\right) \operatorname{Ai}\left(-\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) ((1+i)\, z)^{4/3} - 144\, i\left(9\, z^2 - 110\, i\right) \operatorname{Bi}\left(\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) ((1+i)\, z)^{4/3} + 144\left(110 - 9\, i\, z^2\right) \operatorname{Bi}\left(-\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) ((1+i)\, z)^{4/3} + 3\sqrt[6]{3}\left(81\, z^4 - 4320\, i\, z^2 - 14080\right) \operatorname{Ai}'\left(\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) - 3\sqrt[6]{3}\left(81\, z^4 + 4320\, i\, z^2 - 14080\right) \operatorname{Ai}'\left(-\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) + 3^{2/3}\left(-81\, z^4 + 4320\, i\, z^2 + 14080\right) \operatorname{Bi}'\left(\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) + 3^{2/3}\left(81\, z^4 + 4320\, i\, z^2 - 14080\right) \operatorname{Bi}'\left(-\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right)\right)$$

03.18.03.0006.01

$$\operatorname{ber}_{-\frac{9}{2}}(z) = \frac{\sqrt[8]{-1}}{\sqrt{2\pi}\, z^{9/2}} \left(\sqrt[4]{-1}\left(z^4 + 45\, i\, z^2 - 105\right) \cos\left(\sqrt[4]{-1}\, z\right) - i\left(z^4 - 45\, i\, z^2 - 105\right) \cos\left((-1)^{3/4}\, z\right) - 5\, z\left(\left(2\, z^2 + 21\, i\right) \sin\left(\sqrt[4]{-1}\, z\right) + \sqrt[4]{-1}\left(2\, i\, z^2 + 21\right) \sin\left((-1)^{3/4}\, z\right)\right)\right)$$

03.18.03.0007.01

$$\operatorname{ber}_{-\frac{13}{3}}(z) = -\frac{\sqrt[4]{-1}}{54\, 2^{2/3}\, 3^{5/6}\, z^{13/3}} \left(-84\sqrt[6]{3}\left(9\, z^2 - 80\, i\right) \operatorname{Ai}'\left(\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) ((1+i)\, z)^{2/3} + 84\sqrt[6]{3}\left(80 - 9\, i\, z^2\right) \operatorname{Ai}'\left(-\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) ((1+i)\, z)^{2/3} - 28\, 3^{2/3}\left(9\, z^2 - 80\, i\right) \operatorname{Bi}'\left(\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) ((1+i)\, z)^{2/3} + 28\, 3^{2/3}\left(80 - 9\, i\, z^2\right) \operatorname{Bi}'\left(-\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) ((1+i)\, z)^{2/3} + \sqrt{3}\left(81\, i\, z^4 + 3024\, z^2 - 4480\, i\right) \operatorname{Ai}\left(\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) + \sqrt{3}\left(-81\, z^4 - 3024\, i\, z^2 + 4480\right) \operatorname{Ai}\left(-\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) + \left(81\, i\, z^4 + 3024\, z^2 - 4480\, i\right) \operatorname{Bi}\left(\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) + \left(-81\, z^4 - 3024\, i\, z^2 + 4480\right) \operatorname{Bi}\left(-\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right)\right)$$

03.18.03.0008.01

$$\operatorname{ber}_{-\frac{11}{3}}(z) = \frac{\left(\sqrt[4]{-1}\, z\right)^{8/3}}{108\, 3^{5/6}\, z^{17/3}\, ((1+i)\, z)^{2/3}} \left(9\sqrt{6}\, z\left(9\, i\, z^2 + 160\right) \operatorname{Ai}\left(\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) \sqrt[3]{(1+i)\, z} - 9\sqrt{6}\, z\left(9\, z^2 + 160\, i\right) \operatorname{Ai}\left(-\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) \sqrt[3]{(1+i)\, z} + 9\sqrt{2}\, z\left(-9\, i\, z^2 - 160\right) \operatorname{Bi}\left(\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) \sqrt[3]{(1+i)\, z} + 9\sqrt{2}\, z\left(9\, z^2 + 160\, i\right) \operatorname{Bi}\left(-\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) \sqrt[3]{(1+i)\, z} + 40\sqrt[4]{-1}\sqrt[6]{3}\left(96\, i - 27\, z^2\right) \operatorname{Ai}'\left(\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) + 120\sqrt[4]{-1}\sqrt[6]{3}\left(32 - 9\, i\, z^2\right) \operatorname{Ai}'\left(-\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) + 40\sqrt[4]{-1}\, 3^{2/3}\left(9\, z^2 - 32\, i\right) \operatorname{Bi}'\left(\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) + 40\, (-1)^{3/4}\, 3^{2/3}\left(9\, z^2 + 32\, i\right) \operatorname{Bi}'\left(-\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right)\right)$$

03.18.03.0009.01

$$\operatorname{ber}_{-\frac{7}{2}}(z) = -\frac{\sqrt[8]{-1}}{\sqrt{2\pi}\, z^{7/2}} \left(3\left(2 z^2 + 5 i\right)\cos\left(\sqrt[4]{-1}\, z\right) - 3\sqrt[4]{-1}\left(2 i z^2 + 5\right)\cos\left((-1)^{3/4} z\right) + \sqrt[4]{-1}\, z\left(z^2 + 15 i\right)\sin\left(\sqrt[4]{-1}\, z\right) + z\left(i z^2 + 15\right)\sin\left((-1)^{3/4} z\right)\right)$$

03.18.03.0010.01

$$\operatorname{ber}_{-\frac{10}{3}}(z) = \frac{i\left(\sqrt[4]{-1}\, z\right)^{10/3}}{18\, 3^{5/6} z^{16/3} ((1+i) z)^{2/3}} \left(\frac{8\sqrt{3}\left(14 - 9 i z^2\right)\operatorname{Ai}\left(-\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right)((1+i) z)^{4/3}}{z^2} + \frac{8\left(14 - 9 i z^2\right)\operatorname{Bi}\left(-\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right)((1+i) z)^{4/3}}{z^2} - \frac{8\sqrt{3}\left(9 i z^2 + 14\right)\operatorname{Ai}\left(\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right)((1+i) z)^{4/3}}{z^2} + \sqrt[6]{3}\left(336 i - 27 z^2\right)\operatorname{Ai}'\left(\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right) + 3\sqrt[6]{3}\left(9 z^2 + 112 i\right)\operatorname{Ai}'\left(-\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right) + \frac{16\left(9 z^2 - 14 i\right)\operatorname{Bi}\left(\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right)}{((1+i) z)^{2/3}} + 3^{2/3}\left(112 i - 9 z^2\right)\operatorname{Bi}'\left(\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right) + 3^{2/3}\left(9 z^2 + 112 i\right)\operatorname{Bi}'\left(-\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right)\right)$$

03.18.03.0011.01

$$\operatorname{ber}_{-\frac{8}{3}}(z) = \frac{\left(\sqrt[4]{-1}\, z\right)^{8/3}}{18\, 3^{5/6} z^{14/3} ((1+i) z)^{2/3}} \left(-45 i\sqrt{3}\operatorname{Ai}\left(\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right)((1+i) z)^{4/3} - 45\sqrt{3}\, i\operatorname{Ai}\left(-\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right)((1+i) z)^{4/3} + 45 i\operatorname{Bi}\left(\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right)((1+i) z)^{4/3} + 45 i\operatorname{Bi}\left(-\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right)((1+i) z)^{4/3} - 3\sqrt[6]{3}\left(9 z^2 - 40 i\right)\operatorname{Ai}'\left(\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right) + 3\sqrt[6]{3}\left(9 z^2 + 40 i\right)\operatorname{Ai}'\left(-\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right) + 3^{2/3}\left(9 z^2 - 40 i\right)\operatorname{Bi}'\left(\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right) - 3^{2/3}\left(9 z^2 + 40 i\right)\operatorname{Bi}'\left(-\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right)\right)$$

03.18.03.0012.01

$$\operatorname{ber}_{-\frac{5}{2}}(z) = -\frac{(-1)^{7/8}}{\sqrt{2\pi}\, z^{5/2}} \left(\left(3 - i z^2\right)\cos\left(\sqrt[4]{-1}\, z\right) + \sqrt[4]{-1}\left(i z^2 + 3\right)\cos\left((-1)^{3/4} z\right) + 3\sqrt[4]{-1}\, z\sin\left(\sqrt[4]{-1}\, z\right) - 3 z\sin\left((-1)^{3/4} z\right)\right)$$

03.18.03.0013.01

$$\operatorname{ber}_{-\frac{7}{3}}(z) = -\frac{(-1)^{3/4}}{6\, 2^{2/3}\, 3^{5/6} z^{7/3}} \left(-24 i\sqrt[6]{3}\operatorname{Ai}'\left(\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right)((1+i) z)^{2/3} + 24\sqrt[6]{3}\operatorname{Ai}'\left(-\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right)((1+i) z)^{2/3} - 8 i\, 3^{2/3}\operatorname{Bi}'\left(\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right)((1+i) z)^{2/3} + 8\, 3^{2/3}\operatorname{Bi}'\left(-\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right)((1+i) z)^{2/3} + \sqrt{3}\left(16 i - 9 z^2\right)\operatorname{Ai}\left(\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right) + \sqrt{3}\left(16 - 9 i z^2\right)\operatorname{Ai}\left(-\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right) + \left(16 i - 9 z^2\right)\operatorname{Bi}\left(\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right) + \left(16 - 9 i z^2\right)\operatorname{Bi}\left(-\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right)\right)$$

03.18.03.0014.01

$$\operatorname{ber}_{-\frac{5}{3}}(z) = \frac{i\left(\sqrt[4]{-1}\, z\right)^{2/3}}{6\sqrt{2}\, 3^{5/6} z^{2/3} ((1+i) z)^{5/3}} \left(-9 i\sqrt{3}\, \operatorname{Ai}\left(\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right) ((1+i) z)^{4/3} + 9\sqrt{3}\, \operatorname{Ai}\left(-\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right) ((1+i) z)^{4/3} + 9 i \operatorname{Bi}\left(\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right) ((1+i) z)^{4/3} - 9 \operatorname{Bi}\left(-\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right) ((1+i) z)^{4/3} + 24\sqrt[6]{3}\, i \operatorname{Ai}'\left(\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right) - 24\sqrt[6]{3}\, \operatorname{Ai}'\left(-\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right) - 8 i\, 3^{2/3} \operatorname{Bi}'\left(\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right) + 8\, 3^{2/3} \operatorname{Bi}'\left(-\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right)\right)$$

03.18.03.0015.01

$$\operatorname{ber}_{-\frac{3}{2}}(z) = \frac{\sqrt[8]{-1}}{\sqrt{2\pi}\, z^{3/2}} \left(\cos\left(\sqrt[4]{-1}\, z\right) - (-1)^{3/4} \cos\left((-1)^{3/4} z\right) + \sqrt[4]{-1}\, z \sin\left(\sqrt[4]{-1}\, z\right) + i z \sin\left((-1)^{3/4} z\right)\right)$$

03.18.03.0016.01

$$\operatorname{ber}_{-\frac{3}{2}}(z) = \sqrt{\frac{2}{\pi}}\, z^{-3/2} \left(\cos\left(\frac{3\pi}{8}\right) \sin\left(\frac{z}{\sqrt{2}}\right) \left(z\cosh\left(\frac{z}{\sqrt{2}}\right) + \sinh\left(\frac{z}{\sqrt{2}}\right)\right) + \cos\left(\frac{z}{\sqrt{2}}\right) \sin\left(\frac{3\pi}{8}\right) \left(\cosh\left(\frac{z}{\sqrt{2}}\right) - z\sinh\left(\frac{z}{\sqrt{2}}\right)\right)\right)$$

03.18.03.0017.01

$$\operatorname{ber}_{-\frac{4}{3}}(z) = \frac{1}{2\, 2^{2/3}\, 3^{5/6} z^{4/3}} \left(-3\sqrt[6]{3}\, \operatorname{Ai}'\left(\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right) ((1+i) z)^{2/3} + 3\sqrt[6]{3}\, \operatorname{Ai}'\left(-\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right) ((1+i) z)^{2/3} - 3^{2/3} \operatorname{Bi}'\left(\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right) ((1+i) z)^{2/3} + 3^{2/3} \operatorname{Bi}'\left(-\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right) ((1+i) z)^{2/3} + 2\sqrt{3}\, \operatorname{Ai}\left(\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right) + 2\sqrt{3}\, \operatorname{Ai}\left(-\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right) + 2 \operatorname{Bi}\left(\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right) + 2 \operatorname{Bi}\left(-\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right)\right)$$

03.18.03.0018.01

$$\operatorname{ber}_{-\frac{2}{3}}(z) = \frac{i}{2\sqrt[3]{2}\, 3^{2/3} z^{2/3}} \left(3 \operatorname{Ai}'\left(\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right) - 3 \operatorname{Ai}'\left(-\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right) + \sqrt{3} \left(\operatorname{Bi}'\left(-\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right) - \operatorname{Bi}'\left(\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right)\right)\right)$$

03.18.03.0019.01

$$\operatorname{ber}_{-\frac{1}{2}}(z) = \frac{(-1)^{3/8}}{\sqrt{2\pi}\, \sqrt{z}} \left(\cos\left(\sqrt[4]{-1}\, z\right) - \sqrt[4]{-1}\, \cos\left((-1)^{3/4} z\right)\right)$$

03.18.03.0020.01

$$\operatorname{ber}_{-\frac{1}{2}}(z) = \frac{1}{\sqrt{z}} \sqrt{\frac{2}{\pi}} \left(\cos\left(\frac{3\pi}{8}\right) \cos\left(\frac{z}{\sqrt{2}}\right) \cosh\left(\frac{z}{\sqrt{2}}\right) + \sin\left(\frac{3\pi}{8}\right) \sin\left(\frac{z}{\sqrt{2}}\right) \sinh\left(\frac{z}{\sqrt{2}}\right)\right)$$

03.18.03.0021.01

$$\operatorname{ber}_{-\frac{1}{3}}(z) = \frac{(1+i)}{4\sqrt[3]{z}} \sqrt[6]{\frac{3}{2}} \left(-i\sqrt{3}\, \operatorname{Ai}\left(\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right) + \sqrt{3}\, \operatorname{Ai}\left(-\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right) - i \operatorname{Bi}\left(\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right) + \operatorname{Bi}\left(-\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right)\right)$$

03.18.03.0022.01

$$\operatorname{ber}_{\frac{1}{3}}(z) = -\frac{\sqrt[6]{3}\,\sqrt[3]{(1+i)\,z}}{2\,2^{5/6}\,z^{2/3}} \left(\sqrt{3}\,\operatorname{Ai}\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right) + \sqrt{3}\,i\,\operatorname{Ai}\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right) - \operatorname{Bi}\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right) - i\,\operatorname{Bi}\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)\right)$$

03.18.03.0023.01

$$\operatorname{ber}_{\frac{1}{2}}(z) = -\frac{(-1)^{3/8}}{\sqrt{2\pi}\,\sqrt{z}} \left(\sin\left(\sqrt[4]{-1}\,z\right) + \sqrt[4]{-1}\,\sin\left((-1)^{3/4}\,z\right)\right)$$

03.18.03.0024.01

$$\operatorname{ber}_{\frac{1}{2}}(z) = \frac{1}{\sqrt{z}}\sqrt{\frac{2}{\pi}} \left(\sin\left(\frac{3\pi}{8}\right)\sinh\left(\frac{z}{\sqrt{2}}\right)\cos\left(\frac{z}{\sqrt{2}}\right) - \cos\left(\frac{3\pi}{8}\right)\cosh\left(\frac{z}{\sqrt{2}}\right)\sin\left(\frac{z}{\sqrt{2}}\right)\right)$$

03.18.03.0025.01

$$\operatorname{ber}_{\frac{2}{3}}(z) = -\frac{((1+i)\,z)^{2/3}}{2\,6^{2/3}\,z^{4/3}} \left(-3\,\operatorname{Ai}'\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right) + 3\,\operatorname{Ai}'\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right) + \sqrt{3}\left(\operatorname{Bi}'\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right) - \operatorname{Bi}'\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)\right)\right)$$

03.18.03.0026.01

$$\operatorname{ber}_{\frac{4}{3}}(z) = -\frac{z^{4/3}}{2\,3^{5/6}\,((1+i)\,z)^{4/3}\left(\sqrt[4]{-1}\,z\right)^{4/3}} \left(-3\sqrt[6]{3}\,\operatorname{Ai}'\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)((1+i)\,z)^{2/3} + 3\sqrt[6]{3}\,\operatorname{Ai}'\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)((1+i)\,z)^{2/3} + 3^{2/3}\,\operatorname{Bi}'\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)((1+i)\,z)^{2/3} - 3^{2/3}\,\operatorname{Bi}'\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)((1+i)\,z)^{2/3} + 2\sqrt{3}\,\operatorname{Ai}\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right) + 2\sqrt{3}\,\operatorname{Ai}\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right) - 2\,\operatorname{Bi}\left(\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right) - 2\,\operatorname{Bi}\left(-\frac{1}{2}\,3^{2/3}\,((1+i)\,z)^{2/3}\right)\right)$$

03.18.03.0027.01

$$\operatorname{ber}_{\frac{3}{2}}(z) = \frac{\sqrt[8]{-1}}{\sqrt{2\pi}\,z^{3/2}} \left(-\sqrt[4]{-1}\,z\cos\left(\sqrt[4]{-1}\,z\right) + i\,z\cos\left((-1)^{3/4}\,z\right) + \sin\left(\sqrt[4]{-1}\,z\right) + (-1)^{3/4}\sin\left((-1)^{3/4}\,z\right)\right)$$

03.18.03.0028.01

$$\operatorname{ber}_{\frac{3}{2}}(z) = -\frac{1}{z^{3/2}}\sqrt{\frac{2}{\pi}} \left(\cos\left(\frac{z}{\sqrt{2}}\right)\sin\left(\frac{\pi}{8}\right)\left(z\cosh\left(\frac{z}{\sqrt{2}}\right) + \sinh\left(\frac{z}{\sqrt{2}}\right)\right) + \cos\left(\frac{\pi}{8}\right)\sin\left(\frac{z}{\sqrt{2}}\right)\left(z\sinh\left(\frac{z}{\sqrt{2}}\right) - \cosh\left(\frac{z}{\sqrt{2}}\right)\right)\right)$$

03.18.03.0029.01

$$\operatorname{ber}_{\frac{5}{3}}(z)=\frac{z^{11/3}}{3\sqrt{2}\,3^{5/6}\,((1+i)z)^{8/3}\left(\sqrt[4]{-1}\,z\right)^{8/3}}\left(9\sqrt{3}\,z\operatorname{Ai}\left(\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right)\sqrt[3]{(1+i)z}+9\sqrt{3}\,i\,z\operatorname{Ai}\left(-\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right)\sqrt[3]{(1+i)z}+9z\operatorname{Bi}\left(\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right)\sqrt[3]{(1+i)z}+9\,i\,z\operatorname{Bi}\left(-\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right)\sqrt[3]{(1+i)z}-(12-12i)\sqrt[6]{3}\operatorname{Ai}'\left(\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right)-(12+12i)\sqrt[6]{3}\operatorname{Ai}'\left(-\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right)-(4-4i)3^{2/3}\operatorname{Bi}'\left(\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right)-(4+4i)3^{2/3}\operatorname{Bi}'\left(-\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right)\right)$$

03.18.03.0030.01

$$\operatorname{ber}_{\frac{7}{3}}(z)=\frac{\sqrt[4]{-1}}{6\sqrt[3]{2}\,3^{5/6}\,z^{5/3}((1+i)z)^{2/3}}\left(24\sqrt[6]{3}\,i\operatorname{Ai}'\left(\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right)((1+i)z)^{2/3}-24\sqrt[6]{3}\operatorname{Ai}'\left(-\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right)((1+i)z)^{2/3}-8\,i\,3^{2/3}\operatorname{Bi}'\left(\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right)((1+i)z)^{2/3}+8\,3^{2/3}\operatorname{Bi}'\left(-\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right)((1+i)z)^{2/3}+\sqrt{3}\left(9z^2-16i\right)\operatorname{Ai}\left(\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right)+\sqrt{3}\left(9\,i\,z^2-16\right)\operatorname{Ai}\left(-\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right)+\left(16i-9z^2\right)\operatorname{Bi}\left(\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right)+\left(16-9\,i\,z^2\right)\operatorname{Bi}\left(-\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right)\right)$$

03.18.03.0031.01

$$\operatorname{ber}_{\frac{5}{2}}(z)=\frac{1}{\sqrt{2\pi}\,z^{5/2}}\left(\sqrt[8]{-1}\left(3z\cos\left(\sqrt[4]{-1}\,z\right)-3(-1)^{3/4}z\cos\left((-1)^{3/4}z\right)+\sqrt[4]{-1}\left(z^2+3i\right)\sin\left(\sqrt[4]{-1}\,z\right)+\left(i\,z^2+3\right)\sin\left((-1)^{3/4}z\right)\right)\right)$$

03.18.03.0032.01

$$\operatorname{ber}_{\frac{8}{3}}(z)=-\frac{z^{2/3}}{18\,3^{5/6}((1+i)z)^{2/3}\left(\sqrt[4]{-1}\,z\right)^{8/3}}\left(-45\,i\sqrt{3}\operatorname{Ai}\left(\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right)((1+i)z)^{4/3}-45\sqrt{3}\,i\operatorname{Ai}\left(-\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right)((1+i)z)^{4/3}-45\,i\operatorname{Bi}\left(\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right)((1+i)z)^{4/3}-45\,i\operatorname{Bi}\left(-\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right)((1+i)z)^{4/3}-3\sqrt[6]{3}\left(9z^2-40i\right)\operatorname{Ai}'\left(\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right)+3\sqrt[6]{3}\left(9z^2+40i\right)\operatorname{Ai}'\left(-\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right)+3^{2/3}\left(40i-9z^2\right)\operatorname{Bi}'\left(\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right)+3^{2/3}\left(9z^2+40i\right)\operatorname{Bi}'\left(-\frac{1}{2}3^{2/3}((1+i)z)^{2/3}\right)\right)$$

03.18.03.0033.01

$$\operatorname{ber}_{\frac{10}{3}}(z) = -\frac{i\, z^{4/3}}{18\, 3^{5/6}\, ((1+i)\, z)^{2/3} \left(\sqrt[4]{-1}\; z\right)^{10/3}} \left(\frac{8\sqrt{3}\, \left(9\, i\, z^2 + 14\right) \operatorname{Ai}\!\left(\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) ((1+i)\, z)^{4/3}}{z^2} + \frac{8\sqrt{3}\, \left(9\, i\, z^2 - 14\right) \operatorname{Ai}\!\left(-\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) ((1+i)\, z)^{4/3}}{z^2} + \frac{8\left(14 - 9\, i\, z^2\right) \operatorname{Bi}\!\left(-\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) ((1+i)\, z)^{4/3}}{z^2} + 3\sqrt[6]{3}\, \left(9\, z^2 - 112\, i\right) \operatorname{Ai}'\!\left(\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) - 3\sqrt[6]{3}\, \left(9\, z^2 + 112\, i\right) \operatorname{Ai}'\!\left(-\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) + \frac{16\left(9\, z^2 - 14\, i\right) \operatorname{Bi}\!\left(\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right)}{((1+i)\, z)^{2/3}} + 3^{2/3} \left(112\, i - 9\, z^2\right) \operatorname{Bi}'\!\left(\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) + 3^{2/3} \left(9\, z^2 + 112\, i\right) \operatorname{Bi}'\!\left(-\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right)\right)$$

03.18.03.0034.01

$$\operatorname{ber}_{\frac{7}{2}}(z) = \frac{\sqrt[8]{-1}}{\sqrt{2\pi}\; z^{7/2}} \left(\sqrt[4]{-1}\; z\left(z^2 + 15\, i\right) \cos\!\left(\sqrt[4]{-1}\; z\right) + z\left(-i\, z^2 - 15\right) \cos\!\left((-1)^{3/4}\, z\right) - 3\left(2\, z^2 + 5\, i\right) \sin\!\left(\sqrt[4]{-1}\; z\right) - 3\sqrt[4]{-1}\; \left(2\, i\, z^2 + 5\right) \sin\!\left((-1)^{3/4}\, z\right)\right)$$

03.18.03.0035.01

$$\operatorname{ber}_{\frac{11}{3}}(z) = -\frac{i\, z^{5/3}}{108\, 3^{5/6}\, ((1+i)\, z)^{2/3} \left(\sqrt[4]{-1}\; z\right)^{14/3}} \left(9\sqrt{6}\; z\left(-9\, i\, z^2 - 160\right) \operatorname{Ai}\!\left(\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) \sqrt[3]{(1+i)\, z} + 9\sqrt{6}\; z\left(9\, z^2 + 160\, i\right) \operatorname{Ai}\!\left(-\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) \sqrt[3]{(1+i)\, z} + 9\sqrt{2}\; z\left(-9\, i\, z^2 - 160\right) \operatorname{Bi}\!\left(\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) \sqrt[3]{(1+i)\, z} + 9\sqrt{2}\; z\left(9\, z^2 + 160\, i\right) \operatorname{Bi}\!\left(-\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) \sqrt[3]{(1+i)\, z} + 120\sqrt[4]{-1}\; \sqrt[6]{3}\; \left(9\, z^2 - 32\, i\right) \operatorname{Ai}'\!\left(\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) + 120\, (-1)^{3/4}\, \sqrt[6]{3}\; \left(9\, z^2 + 32\, i\right) \operatorname{Ai}'\!\left(-\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) + 40\sqrt[4]{-1}\; 3^{2/3} \left(9\, z^2 - 32\, i\right) \operatorname{Bi}'\!\left(\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) + 40\, (-1)^{3/4}\, 3^{2/3} \left(9\, z^2 + 32\, i\right) \operatorname{Bi}'\!\left(-\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right)\right)$$

03.18.03.0036.01

$$\operatorname{ber}_{\frac{13}{3}}(z) = \frac{z^{4/3}}{108\, 3^{5/6}\, ((1+i)\, z)^{2/3} \left(\sqrt[4]{-1}\, z\right)^{16/3}} \left(\sqrt{6}\, (81\, i\, z^4 + 3024\, z^2 - 4480\, i)\, \operatorname{Ai}\!\left(\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) \sqrt[3]{(1+i)\, z} + \sqrt{6}\, (-81\, z^4 - 3024\, i\, z^2 + 4480) \operatorname{Ai}\!\left(-\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) \sqrt[3]{(1+i)\, z} + \sqrt{2}\, (-81\, i\, z^4 - 3024\, z^2 + 4480\, i)\, \operatorname{Bi}\!\left(\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) \sqrt[3]{(1+i)\, z} + \sqrt{2}\, (81\, z^4 + 3024\, i\, z^2 - 4480)\, \operatorname{Bi}\!\left(-\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) \sqrt[3]{(1+i)\, z} - 168\, \sqrt[4]{-1}\, \sqrt[6]{3}\, z\, (9\, z^2 - 80\, i)\, \operatorname{Ai}'\!\left(\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) + 168\, \sqrt[4]{-1}\, \sqrt[6]{3}\, z\, (80 - 9\, i\, z^2)\, \operatorname{Ai}'\!\left(-\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) + 56\, \sqrt[4]{-1}\, 3^{2/3}\, z\, (9\, z^2 - 80\, i)\, \operatorname{Bi}'\!\left(\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) + 56\, (-1)^{3/4}\, 3^{2/3}\, z\, (9\, z^2 + 80\, i)\, \operatorname{Bi}'\!\left(-\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right)\right)$$

03.18.03.0037.01

$$\operatorname{ber}_{\frac{9}{2}}(z) = -\frac{\sqrt[8]{-1}}{\sqrt{2\pi}\, z^{9/2}} \left(5\, z\, (2\, z^2 + 21\, i) \cos\!\left(\sqrt[4]{-1}\, z\right) - 5\, \sqrt[4]{-1}\, z\, (2\, i\, z^2 + 21) \cos\!\left((-1)^{3/4}\, z\right) + \sqrt[4]{-1}\, (z^4 + 45\, i\, z^2 - 105) \sin\!\left(\sqrt[4]{-1}\, z\right) + i\, (z^4 - 45\, i\, z^2 - 105) \sin\!\left((-1)^{3/4}\, z\right)\right)$$

03.18.03.0038.01

$$\operatorname{ber}_{\frac{14}{3}}(z) = -\frac{i\, z^{2/3}}{162\, 3^{5/6}\, ((1+i)\, z)^{2/3} \left(\sqrt[4]{-1}\, z\right)^{14/3}} \left(144\, \sqrt{3}\, (9\, i\, z^2 + 110)\, \operatorname{Ai}\!\left(\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) ((1+i)\, z)^{4/3} + 144\, \sqrt{3}\, i\, (9\, z^2 + 110\, i)\, \operatorname{Ai}\!\left(-\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) ((1+i)\, z)^{4/3} + 144\, (9\, i\, z^2 + 110)\, \operatorname{Bi}\!\left(\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) ((1+i)\, z)^{4/3} + 144\, i\, (9\, z^2 + 110\, i)\, \operatorname{Bi}\!\left(-\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) ((1+i)\, z)^{4/3} + 3\, \sqrt[6]{3}\, (81\, z^4 - 4320\, i\, z^2 - 14\,080)\, \operatorname{Ai}'\!\left(\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) - 3\, \sqrt[6]{3}\, (81\, z^4 + 4320\, i\, z^2 - 14\,080)\, \operatorname{Ai}'\!\left(-\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) + 3^{2/3}\, (81\, z^4 - 4320\, i\, z^2 - 14\,080)\, \operatorname{Bi}'\!\left(\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right) - 3^{2/3}\, (81\, z^4 + 4320\, i\, z^2 - 14\,080)\, \operatorname{Bi}'\!\left(-\frac{1}{2}\, 3^{2/3}\, ((1+i)\, z)^{2/3}\right)\right)$$

Symbolic rational ν

03.18.03.0039.01

$$\operatorname{ber}_\nu(z) = \frac{(-1)^{7/8} e^{-i\pi\nu}}{\sqrt{2\pi}\sqrt{z}} \left(\sum_{k=0}^{\left\lfloor \frac{1}{4}(2|\nu|-3) \right\rfloor} \frac{\left(2k+|\nu|+\frac{1}{2}\right)! \left(2\sqrt[4]{-1}\, z\right)^{-2k-1}}{(2k+1)! \left(-2k+|\nu|-\frac{3}{2}\right)!} \left(e^{\frac{1}{4} i\pi(4\nu+1)} \cos\left(\frac{1}{2}\pi\left(\frac{1}{2}-\nu\right) - \frac{1}{\sqrt[4]{-1}} z\right) - (-1)^k \cos\left(\frac{1}{2}\pi\left(\nu-\frac{1}{2}\right) - \sqrt[4]{-1}\, z\right)\right) + \sum_{k=0}^{\left\lfloor \frac{1}{4}(2|\nu|-1) \right\rfloor} \frac{\left(2k+|\nu|-\frac{1}{2}\right)! \left(2\sqrt[4]{-1}\, z\right)^{-2k}}{(2k)! \left(-2k+|\nu|-\frac{1}{2}\right)!} \left((-1)^{3/4} e^{i\pi\nu} \sin\left(\frac{1}{2}\pi\left(\frac{1}{2}-\nu\right) - \frac{1}{\sqrt[4]{-1}} z\right) + (-1)^k \sin\left(\frac{1}{2}\pi\left(\nu-\frac{1}{2}\right) - \sqrt[4]{-1}\, z\right)\right)\right) /; \nu - \frac{1}{2} \in \mathbb{Z}$$

03.18.03.0040.01

$$\operatorname{ber}_\nu(z) = \frac{\Gamma\left(-\frac{1}{3}\right) e^{\frac{1}{4}(-3) i\pi\nu} z^\nu \left(\sqrt[4]{-1}\, z\right)^{-\nu}}{2\,\Gamma(1-|\nu|)} \left(\frac{2^{|\nu|-1} \left(\sqrt[4]{-1}\, z\right)^{-|\nu|}}{3^{5/6}} \sum_{k=0}^{|\nu|-\frac{1}{3}} \frac{4^{-k} \left(i z^2\right)^k \left(-k+|\nu|-\frac{1}{3}\right)!}{k! \left(-2k+|\nu|-\frac{1}{3}\right)! \left(\frac{1}{3}\right)_k (1-|\nu|)_k} \left(-i^{\left(|\nu|-\frac{1}{3}\right)(\operatorname{sgn}(\nu)+1)} \operatorname{sgn}(\nu) \left(\sqrt{3}\, \operatorname{sgn}(\nu) \operatorname{Ai}\left(-\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right) - \operatorname{Bi}\left(-\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right)\right) + (-1)^k e^{\frac{3 i\pi\nu}{2}} \left(\sqrt{3}\, \operatorname{sgn}(\nu) \operatorname{Ai}\left(\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right) - \operatorname{Bi}\left(\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right)\right)\right) + \frac{2^{|\nu|-\frac{5}{3}} \left(\sqrt[4]{-1}\, z\right)^{\frac{2}{3}-|\nu|}}{3^{2/3}} \sum_{k=0}^{|\nu|-\frac{4}{3}} \frac{4^{-k} \left(i z^2\right)^k \left(-k+|\nu|-\frac{4}{3}\right)!}{k! \left(-2k+|\nu|-\frac{4}{3}\right)! \left(\frac{4}{3}\right)_k (1-|\nu|)_k} \left(i^{\left(|\nu|-\frac{1}{3}\right)(\operatorname{sgn}(\nu)+1)} \operatorname{sgn}(\nu) \left(\sqrt{3}\, \operatorname{Bi}'\left(-\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right) - 3 \operatorname{sgn}(\nu) \operatorname{Ai}'\left(-\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right)\right) + (-1)^k e^{\frac{3 i\pi\nu}{2}} \left(\sqrt{3}\, \operatorname{Bi}'\left(\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right) - 3 \operatorname{sgn}(\nu) \operatorname{Ai}'\left(\frac{1}{2} 3^{2/3} ((1+i) z)^{2/3}\right)\right)\right)\right) /; |\nu| - \frac{1}{3} \in \mathbb{Z}$$

03.18.03.0041.01

$$\operatorname{ber}_\nu(z) = \frac{e^{-\frac{3\pi i \nu}{4}} z^\nu \left(\sqrt[4]{-1}\, z\right)^{-\nu} \Gamma\left(-\frac{2}{3}\right) \operatorname{sgn}(\nu)}{2\,\Gamma(1-|\nu|)} \left(2^{|\nu|-\frac{7}{3}} \sqrt[6]{3} \left(\sqrt[4]{-1}\, z\right)^{\frac{4}{3}-|\nu|} \sum_{k=0}^{|\nu|-\frac{5}{3}} \frac{4^{-k} \left(i\, z^2\right)^k \left(-k+|\nu|-\frac{5}{3}\right)!}{k!\left(-2k+|\nu|-\frac{5}{3}\right)! \left(\frac{5}{3}\right)_k (1-|\nu|)_k} \right.$$
$$\left(i^{\left(|\nu|-\frac{2}{3}\right)(\operatorname{sgn}(\nu)+1)} \left(\sqrt{3}\, \operatorname{Ai}\left(-\frac{1}{2} 3^{2/3} ((1+i)z)^{2/3}\right) + \operatorname{Bi}\left(-\frac{1}{2} 3^{2/3} ((1+i)z)^{2/3}\right) \operatorname{sgn}(\nu)\right) + \right.$$
$$\left. (-1)^k e^{\frac{3 i \pi \nu}{2}} \left(\sqrt{3}\, \operatorname{Ai}\left(\frac{1}{2} 3^{2/3} ((1+i)z)^{2/3}\right) + \operatorname{Bi}\left(\frac{1}{2} 3^{2/3} ((1+i)z)^{2/3}\right) \operatorname{sgn}(\nu)\right)\right) +$$
$$\frac{2^{|\nu|} \left(\sqrt[4]{-1}\, z\right)^{-|\nu|}}{3\, 3^{2/3}} \sum_{k=0}^{|\nu|-\frac{2}{3}} \frac{4^{-k} \left(i\, z^2\right)^k \left(-k+|\nu|-\frac{2}{3}\right)!}{k!\left(-2k+|\nu|-\frac{2}{3}\right)! \left(\frac{2}{3}\right)_k (1-|\nu|)_k}$$
$$\left(-i^{\left(|\nu|-\frac{2}{3}\right)(\operatorname{sgn}(\nu)+1)} \left(3\operatorname{Ai}'\left(-\frac{1}{2} 3^{2/3} ((1+i)z)^{2/3}\right) + \sqrt{3}\, \operatorname{Bi}'\left(-\frac{1}{2} 3^{2/3} ((1+i)z)^{2/3}\right) \operatorname{sgn}(\nu)\right) - \right.$$
$$\left.\left. (-1)^k e^{\frac{3 i \pi \nu}{2}} \left(3\operatorname{Ai}'\left(\frac{1}{2} 3^{2/3} ((1+i)z)^{2/3}\right) + \sqrt{3}\, \operatorname{Bi}'\left(\frac{1}{2} 3^{2/3} ((1+i)z)^{2/3}\right) \operatorname{sgn}(\nu)\right)\right)\right) /;\ |\nu| - \frac{2}{3} \in \mathbb{Z}$$

Values at fixed points

03.18.03.0042.01

$$\operatorname{ber}_0(0) = 1$$

Values at infinities

03.18.03.0043.01

$$\lim_{x\to\infty} \operatorname{ber}_\nu(x) = \tilde{\infty}$$

General characteristics

Domain and analyticity

$\operatorname{ber}_\nu(z)$ is an analytical function of ν and z, which is defined in $\mathbb{C}^2$.

03.18.04.0001.01

$$(\nu * z) \longrightarrow \operatorname{ber}_\nu(z) :: (\mathbb{C} \otimes \mathbb{C}) \longrightarrow \mathbb{C}$$

Symmetries and periodicities

Parity

03.18.04.0002.01

$$\operatorname{ber}_\nu(-z) = (-z)^\nu z^{-\nu} \operatorname{ber}_\nu(z)$$

03.18.04.0003.01

$$\operatorname{ber}_{-n}(z) = (-1)^n \operatorname{ber}_n(z) /;\ n \in \mathbb{Z}$$

Mirror symmetry

03.18.04.0004.01

$\text{ber}_{\bar{\nu}}(\bar{z}) = \overline{\text{ber}_{\nu}(z)} \;/;\; z \notin (-\infty, 0)$

Periodicity

No periodicity

Poles and essential singularities

With respect to *z*

For fixed ν, the function $\text{ber}_\nu(z)$ has an essential singularity at $z = \tilde{\infty}$. At the same time, the point $z = \tilde{\infty}$ is a branch point for generic ν.

03.18.04.0005.01

$\mathcal{S}ing_z(\text{ber}_\nu(z)) = \{\{\tilde{\infty}, \infty\}\}$

With respect to *ν*

For fixed z, the function $\text{ber}_\nu(z)$ has only one singular point at $\nu = \tilde{\infty}$. It is an essential singular point.

03.18.04.0006.01

$\mathcal{S}ing_\nu(\text{ber}_\nu(z)) = \{\{\tilde{\infty}, \infty\}\}$

Branch points

With respect to *z*

For fixed noninteger ν, the function $\text{ber}_\nu(z)$ has two branch points: $z = 0$, $z = \tilde{\infty}$. At the same time, the point $z = \tilde{\infty}$ is an essential singularity.

03.18.04.0007.01

$\mathcal{BP}_z(\text{ber}_\nu(z)) = \{0, \tilde{\infty}\} \;/;\; \nu \notin \mathbb{Z}$

03.18.04.0008.01

$\mathcal{BP}_z(\text{ber}_\nu(z)) = \{\} \;/;\; \nu \in \mathbb{Z}$

03.18.04.0009.01

$\mathcal{R}_z(\text{ber}_\nu(z), 0) = \log \;/;\; \nu \notin \mathbb{Q}$

03.18.04.0010.01

$\mathcal{R}_z\left(\text{ber}_{\frac{p}{q}}(z), 0\right) = q \;/;\; p \in \mathbb{Z} \land q - 1 \in \mathbb{N}^+ \land \gcd(p, q) = 1$

03.18.04.0011.01

$\mathcal{R}_z(\text{ber}_\nu(z), \tilde{\infty}) = \log \;/;\; \nu \notin \mathbb{Q}$

03.18.04.0012.01

$\mathcal{R}_z\left(\text{ber}_{\frac{p}{q}}(z), \tilde{\infty}\right) = q \;/;\; p \in \mathbb{Z} \land q - 1 \in \mathbb{N}^+ \land \gcd(p, q) = 1$

With respect to *ν*

For fixed z, the function $\text{ber}_\nu(z)$ does not have branch points.

03.18.04.0013.01

$\mathcal{BP}_\nu(\text{ber}_\nu(z)) = \{\}$

Branch cuts

With respect to z

When ν is an integer, $\text{ber}_\nu(z)$ is an entire function of z. For fixed noninteger ν, it has one infinitely long branch cut. For fixed noninteger ν, the function $\text{ber}_\nu(z)$ is a single-valued function on the z-plane cut along the interval $(-\infty, 0)$, where it is continuous from above.

03.18.04.0014.01

$$\mathcal{BC}_z(\text{ber}_\nu(z)) = \{\{(-\infty, 0), -i\}\} /; \nu \notin \mathbb{Z}$$

03.18.04.0015.01

$$\mathcal{BC}_z(\text{ber}_\nu(z)) = \{\} /; \nu \in \mathbb{Z}$$

03.18.04.0016.01

$$\lim_{\epsilon \to +0} \text{ber}_\nu(x + i\,\epsilon) = \text{ber}_\nu(x) /; x \in \mathbb{R} \land x < 0$$

03.18.04.0017.01

$$\lim_{\epsilon \to +0} \text{ber}_\nu(x - i\,\epsilon) = e^{-2\pi i \nu}\, \text{ber}_\nu(x) /; x \in \mathbb{R} \land x < 0$$

With respect to ν

For fixed z, the function $\text{ber}_\nu(z)$ is an entire function of ν and does not have branch cuts.

03.18.04.0018.01

$$\mathcal{BC}_\nu(\text{ber}_\nu(z)) = \{\}$$

Series representations

Generalized power series

Expansions at $\nu = \pm n$

03.18.06.0001.01

$$\text{ber}_\nu(z) \propto \text{ber}_n(z) + \left(-\frac{\pi}{2}\text{bei}_n(z) - \text{ker}_n(z) + \frac{1}{2}\, n! \sum_{k=0}^{n-1} \frac{\left(\cos\left(\frac{3}{4}(k-n)\pi\right)\text{ber}_k(z) - \sin\left(\frac{3}{4}(k-n)\pi\right)\text{bei}_k(z)\right)}{k!\,(n-k)} \left(\frac{z}{2}\right)^{k-n}\right)(\nu - n) + \ldots /; (\nu \to n) \land n \in \mathbb{N}$$

03.18.06.0002.01

$$\text{ber}_\nu(z) \propto (-1)^n\, \text{ber}_n(z) + \left(-\frac{\pi}{2}(-1)^n\, \text{bei}_n(z) + (-1)^{n-1}\, \text{ker}_n(z) - \frac{(-1)^n\, n!}{2} \sum_{k=0}^{n-1} \frac{\left(\cos\left(\frac{3}{4}(k-n)\pi\right)\text{ber}_k(z) - \sin\left(\frac{3}{4}(k-n)\pi\right)\text{bei}_k(z)\right)}{k!\,(n-k)} \left(\frac{z}{2}\right)^{k-n}\right)(n+\nu) + \ldots /; (\nu \to -n) \land n \in \mathbb{N}$$

Expansions at generic point $z = z_0$

03.18.06.0003.01

$$\operatorname{ber}_\nu(z) \propto \left(\frac{1}{z_0}\right)^{\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} z_0^{\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} \left(\operatorname{ber}_\nu(z_0) - \frac{2\,\nu\,\operatorname{ber}_\nu(z_0) + \sqrt{2}\,(\operatorname{bei}_{\nu-1}(z_0) + \operatorname{ber}_{\nu-1}(z_0))\,z_0}{2\,z_0}\,(z-z_0) + \frac{2\,\nu\,(\nu+1)\operatorname{ber}_\nu(z_0) + z_0\left(\sqrt{2}\,(\operatorname{bei}_{\nu-1}(z_0) + \operatorname{ber}_{\nu-1}(z_0)) - 2\operatorname{bei}_\nu(z_0)\,z_0\right)}{4\,z_0^2}\,(z-z_0)^2 + \ldots\right) /;\ (z \to z_0)$$

03.18.06.0004.01

$$\operatorname{ber}_\nu(z) \propto \left(\frac{1}{z_0}\right)^{\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} z_0^{\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} \left(\operatorname{ber}_\nu(z_0) - \frac{2\,\nu\,\operatorname{ber}_\nu(z_0) + \sqrt{2}\,(\operatorname{bei}_{\nu-1}(z_0) + \operatorname{ber}_{\nu-1}(z_0))\,z_0}{2\,z_0}\,(z-z_0) + \frac{2\,\nu\,(\nu+1)\operatorname{ber}_\nu(z_0) + z_0\left(\sqrt{2}\,(\operatorname{bei}_{\nu-1}(z_0) + \operatorname{ber}_{\nu-1}(z_0)) - 2\operatorname{bei}_\nu(z_0)\,z_0\right)}{4\,z_0^2}\,(z-z_0)^2 + O\left((z-z_0)^3\right)\right)$$

03.18.06.0005.01

$$\operatorname{ber}_\nu(z) = \left(\frac{1}{z_0}\right)^{\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} z_0^{\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} \sum_{k=0}^{\infty} \frac{\operatorname{ber}_\nu^{(0,k)}(z_0)\,(z-z_0)^k}{k!}$$

03.18.06.0006.01

$$\operatorname{ber}_\nu(z) = 2^{-2\nu-1}\,e^{-\frac{3 i \pi \nu}{4}}\,\sqrt{\pi}\,z_0^\nu\,\Gamma(\nu+1)\left(\frac{1}{z_0}\right)^{\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} z_0^{\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} \sum_{k=0}^{\infty} \frac{2^k\,z_0^{-k}}{k!} \left(e^{\frac{3 i \pi \nu}{2}}\,{}_2\tilde{F}_3\left(\frac{\nu+1}{2}, \frac{\nu+2}{2}; \frac{1-k+\nu}{2}, \frac{2-k+\nu}{2}, \nu+1; \frac{i\,z_0^2}{4}\right) + {}_2\tilde{F}_3\left(\frac{\nu+1}{2}, \frac{\nu+2}{2}; \frac{1-k+\nu}{2}, \frac{2-k+\nu}{2}, \nu+1; -\frac{i\,z_0^2}{4}\right)\right)(z-z_0)^k$$

03.18.06.0007.01

$$\operatorname{ber}_\nu(z) = \left(\frac{1}{z_0}\right)^{\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} z_0^{\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} \sum_{k=0}^{\infty} \frac{2^{-\frac{3k}{2}-1}\,(i-1)^k}{k!} \left(\sum_{j=0}^{\left\lfloor \frac{k}{2} \right\rfloor} \binom{k}{2j} \left(i\left(1-i^k\right)\operatorname{bei}_{4j-k+\nu}(z_0) + \left(1+i^k\right)\operatorname{ber}_{4j-k+\nu}(z_0)\right) + \sum_{j=0}^{\left\lfloor \frac{k-1}{2} \right\rfloor} \binom{k}{2j+1} \left(-i\left(1-i^k\right)\operatorname{bei}_{4j-k+\nu+2}(z_0) - \left(1+i^k\right)\operatorname{ber}_{4j-k+\nu+2}(z_0)\right)\right)(z-z_0)^k$$

03.18.06.0008.01

$$\operatorname{ber}_\nu(z) \propto \left(\frac{1}{z_0}\right)^{\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} z_0^{\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} \operatorname{ber}_\nu(z_0)\,(1 + O(z-z_0))$$

Expansions on branch cuts

03.18.06.0009.01

$$\operatorname{ber}_\nu(z) \propto e^{2\nu\pi i\left\lfloor \frac{\arg(z-x)}{2\pi}\right\rfloor}\left(\operatorname{ber}_\nu(x) - \frac{\sqrt{2}\, x\,(\operatorname{bei}_{\nu-1}(x)+\operatorname{ber}_{\nu-1}(x)) + 2\,\nu\,\operatorname{ber}_\nu(x)}{2\,x}(z-x) + \frac{x\left(\sqrt{2}\,(\operatorname{bei}_{\nu-1}(x)+\operatorname{ber}_{\nu-1}(x)) - 2\operatorname{bei}_\nu(x)\,x\right) + 2\,\nu\,(\nu+1)\operatorname{ber}_\nu(x)}{4\,x^2}(z-x)^2 + \ldots\right) /;\ (z\to x)\land x\in\mathbb{R}\land x<0$$

03.18.06.0010.01

$$\operatorname{ber}_\nu(z) \propto e^{2\nu\pi i\left\lfloor \frac{\arg(z-x)}{2\pi}\right\rfloor}\left(\operatorname{ber}_\nu(x) - \frac{\sqrt{2}\, x\,(\operatorname{bei}_{\nu-1}(x)+\operatorname{ber}_{\nu-1}(x)) + 2\,\nu\,\operatorname{ber}_\nu(x)}{2\,x}(z-x) + \frac{x\left(\sqrt{2}\,(\operatorname{bei}_{\nu-1}(x)+\operatorname{ber}_{\nu-1}(x)) - 2\operatorname{bei}_\nu(x)\,x\right) + 2\,\nu\,(\nu+1)\operatorname{ber}_\nu(x)}{4\,x^2}(z-x)^2 + O\left((z-x)^3\right)\right) /;\ x\in\mathbb{R}\land x<0$$

03.18.06.0011.01

$$\operatorname{ber}_\nu(z) = 2^{-2\nu-1}\,e^{-\frac{3i\pi\nu}{4}}\sqrt{\pi}\;x^\nu\,\Gamma(\nu+1)\,e^{2\nu\pi i\left\lfloor \frac{\arg(z-x)}{2\pi}\right\rfloor}\sum_{k=0}^{\infty}\frac{1}{k!}\left(2^k x^{-k}\right)\left(e^{\frac{3i\pi\nu}{2}}\,{}_2\tilde{F}_3\left(\frac{\nu+1}{2},\frac{\nu+2}{2};\frac{1}{2}(-k+\nu+1),\frac{1}{2}(-k+\nu+2),\nu+1;\frac{i\,x^2}{4}\right) + {}_2\tilde{F}_3\left(\frac{\nu+1}{2},\frac{\nu+2}{2};\frac{1}{2}(-k+\nu+1),\frac{1}{2}(-k+\nu+2),\nu+1;-\frac{1}{4}\left(i\,x^2\right)\right)\right)(z-x)^k /;\ x\in\mathbb{R}\land x<0$$

03.18.06.0012.01

$$\operatorname{ber}_\nu(z) = e^{2\nu\pi i\left\lfloor \frac{\arg(z-x)}{2\pi}\right\rfloor}\sum_{k=0}^{\infty}\frac{2^{-\frac{3k}{2}-1}(i-1)^k}{k!}\left(\sum_{j=0}^{\left\lfloor\frac{k}{2}\right\rfloor}\binom{k}{2j}\left(i\left(1-i^k\right)\operatorname{bei}_{4j-k+\nu}(x) + \left(1+i^k\right)\operatorname{ber}_{4j-k+\nu}(x)\right) + \sum_{j=0}^{\left\lfloor\frac{k-1}{2}\right\rfloor}\binom{k}{2j+1}\left(-i\left(1-i^k\right)\operatorname{bei}_{4j-k+\nu+2}(x) - \left(1+i^k\right)\operatorname{ber}_{4j-k+\nu+2}(x)\right)\right)(z-x)^k /;\ x\in\mathbb{R}\land x<0$$

03.18.06.0013.01

$$\operatorname{ber}_\nu(z) \propto e^{2\nu\pi i\left\lfloor \frac{\arg(z-x)}{2\pi}\right\rfloor}\operatorname{ber}_\nu(x)\,(1+O(z-x)) /;\ x\in\mathbb{R}\land x<0$$

Expansions at $z == 0$

For the function itself

General case

03.18.06.0014.01

$$\operatorname{ber}_\nu(z) \propto \frac{\cos\left(\frac{3\pi\nu}{4}\right)}{\Gamma(\nu+1)}\left(\frac{z}{2}\right)^\nu\left(1 - \frac{z^4}{32\,(\nu+1)\,(\nu+2)} + \frac{z^8}{6144\,(\nu+1)\,(\nu+2)\,(\nu+3)\,(\nu+4)} + \ldots\right) - \frac{\sin\left(\frac{3\pi\nu}{4}\right)}{\Gamma(\nu+2)}\left(\frac{z}{2}\right)^{\nu+2}\left(1 - \frac{z^4}{96\,(\nu+2)\,(\nu+3)} + \frac{z^8}{30\,720\,(\nu+2)\,(\nu+3)\,(\nu+4)\,(\nu+5)} + \ldots\right) /;\ (z\to 0)\bigwedge -\nu\notin\mathbb{N}^+$$

03.18.06.0015.01

$$\operatorname{ber}_\nu(z) \propto \frac{\cos\left(\frac{3\pi\nu}{4}\right)}{\Gamma(\nu+1)}\left(\frac{z}{2}\right)^\nu \left(1-\frac{z^4}{32(\nu+1)(\nu+2)}+\frac{z^8}{6144(\nu+1)(\nu+2)(\nu+3)(\nu+4)}+O\left(z^{12}\right)\right)-\frac{\sin\left(\frac{3\pi\nu}{4}\right)}{\Gamma(\nu+2)}\left(\frac{z}{2}\right)^{\nu+2}\left(1-\frac{z^4}{96(\nu+2)(\nu+3)}+\frac{z^8}{30\,720(\nu+2)(\nu+3)(\nu+4)(\nu+5)}+O\left(z^{12}\right)\right) /;\ -\nu \notin \mathbb{N}^+$$

03.18.06.0016.01

$$\operatorname{ber}_\nu(z) = \left(\frac{z}{2}\right)^\nu \sum_{k=0}^{\infty} \frac{1}{\Gamma(k+\nu+1)\,k!}\cos\left(\frac{\pi}{4}(2k+3\nu)\right)\left(\frac{z}{2}\right)^{2k}$$

03.18.06.0017.01

$$\operatorname{ber}_\nu(z) = \frac{\cos\left(\frac{3\pi\nu}{4}\right)}{\Gamma(\nu+1)}\left(\frac{z}{2}\right)^\nu \sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{z}{4}\right)^{4k}}{\left(\frac{\nu+1}{2}\right)_k\left(\frac{\nu}{2}+1\right)_k\left(\frac{1}{2}\right)_k k!}-\frac{\sin\left(\frac{3\pi\nu}{4}\right)}{\Gamma(\nu+2)}\left(\frac{z}{2}\right)^{\nu+2}\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{z}{4}\right)^{4k}}{\left(\frac{\nu}{2}+1\right)_k\left(\frac{\nu+3}{2}\right)_k\left(\frac{3}{2}\right)_k k!} /;\ -\nu\notin\mathbb{N}^+$$

03.18.06.0018.01

$$\operatorname{ber}_\nu(z) = \frac{2^{-\nu}z^\nu\cos\left(\frac{3\pi\nu}{4}\right)}{\Gamma(\nu+1)}\,{}_0F_3\left(;\frac{1}{2},\frac{\nu+1}{2},\frac{\nu}{2}+1;-\frac{z^4}{256}\right)-\frac{2^{-\nu-2}z^{\nu+2}\sin\left(\frac{3\pi\nu}{4}\right)}{\Gamma(\nu+2)}\,{}_0F_3\left(;\frac{3}{2},\frac{\nu}{2}+1,\frac{\nu+3}{2};-\frac{z^4}{256}\right) /;\ -\nu\notin\mathbb{N}^+$$

03.18.06.0019.01

$$\operatorname{ber}_\nu(z) = 4^{-\nu}\pi z^\nu\cos\left(\frac{3\pi\nu}{4}\right){}_0\tilde{F}_3\left(;\frac{1}{2},\frac{\nu+1}{2},\frac{\nu}{2}+1;-\frac{z^4}{256}\right)-2^{-2(\nu+2)}\pi z^{\nu+2}\sin\left(\frac{3\pi\nu}{4}\right){}_0\tilde{F}_3\left(;\frac{3}{2},\frac{\nu}{2}+1,\frac{\nu+3}{2};-\frac{z^4}{256}\right) /;\ -\nu\notin\mathbb{N}^+$$

03.18.06.0020.01

$$\operatorname{ber}_\nu(z) \propto \frac{2^{-\nu}z^\nu\cos\left(\frac{3\pi\nu}{4}\right)}{\Gamma(\nu+1)}\left(1+O\left(z^2\right)\right) /;\ -\nu\notin\mathbb{N}^+$$

03.18.06.0021.01

$$\operatorname{ber}_\nu(z) \propto \begin{cases} \frac{(-1)^{\nu/4}2^\nu z^{-\nu}}{(-\nu)!}\left(1+O\left(z^2\right)\right) & \frac{\nu}{4}\in\mathbb{Z}\bigwedge\nu<0 \\ \frac{(-1)^{\frac{\nu-1}{4}}2^{\nu-\frac{1}{2}}z^{-\nu}}{(-\nu)!}\left(1+O\left(z^2\right)\right) & \frac{\nu-1}{4}\in\mathbb{Z}\bigwedge\nu<0 \\ \frac{(-1)^{\frac{\nu+2}{4}}2^{\nu-2}z^{2-\nu}}{(1-\nu)!}\left(1+O\left(z^2\right)\right) & \frac{\nu-2}{4}\in\mathbb{Z}\bigwedge\nu<0 \\ \frac{(-1)^{\frac{\nu+1}{4}}2^{\nu-\frac{1}{2}}z^{-\nu}}{(-\nu)!}\left(1+O\left(z^2\right)\right) & \frac{\nu-3}{4}\in\mathbb{Z}\bigwedge\nu<0 \\ \frac{2^{-\nu}z^\nu\cos\left(\frac{3\pi\nu}{4}\right)}{\Gamma(\nu+1)}\left(1+O\left(z^2\right)\right) & \text{True} \end{cases}$$

03.18.06.0022.01

$$\operatorname{ber}_\nu(z) = F_\infty(z,\nu) /; \left(\left(F_n(z,\nu) = \left(\frac{z}{2}\right)^\nu\sum_{k=0}^{n}\frac{\cos\left(\frac{1}{4}\pi(2k+3\nu)\right)\left(\frac{z}{2}\right)^{2k}}{\Gamma(k+\nu+1)\,k!} = \operatorname{ber}_\nu(z) - i(-i)^n 2^{-2n-\nu-3} e^{-\frac{3i\pi\nu}{4}} z^{2n+\nu+2}\left((-1)^n e^{\frac{3i\pi\nu}{2}}\,{}_1\tilde{F}_2\left(1;n+2,n+\nu+2;\frac{iz^2}{4}\right)-{}_1\tilde{F}_2\left(1;n+2,n+\nu+2;-\frac{1}{4}\left(iz^2\right)\right)\right)\right)\bigwedge n\in\mathbb{N}\right)$$

Summed form of the truncated series expansion.

Special cases

03.18.06.0023.01

$$\operatorname{ber}_{-2n}(z) \propto \frac{2^{-2n} z^{2n} \cos\left(\frac{n\pi}{2}\right)}{(2n)!} \left(1 - \frac{z^4}{64(n+1)(2n+1)} + \frac{z^8}{24576(n+1)(n+2)(2n+1)(2n+3)} + O(z^{12})\right) + \frac{2^{-2n-2} z^{2n+2} \sin\left(\frac{n\pi}{2}\right)}{(2n+1)!} \left(1 - \frac{z^4}{192(n+1)(2n+3)} + \frac{z^8}{122880(n+1)(n+2)(2n+3)(2n+5)} + O(z^{12})\right) /; n \in \mathbb{N}$$

03.18.06.0024.01

$$\operatorname{ber}_{-2n-1}(z) = \frac{(-1)^{\left\lfloor \frac{n+1}{2} \right\rfloor} 2^{-2n-\frac{3}{2}} z^{2n+1}}{(2n+1)!} \left(1 - \frac{z^4}{64(n+1)(2n+3)} + \frac{z^8}{24576(n+1)(n+2)(2n+3)(2n+5)} + O(z^{12})\right) + \frac{(-1)^{\left\lfloor \frac{n}{2} \right\rfloor} 2^{-2n-\frac{7}{2}} z^{2n+3}}{(2n+2)!} \left(1 - \frac{z^4}{192(n+2)(2n+3)} + \frac{z^8}{122880(n+2)(n+3)(2n+3)(2n+5)} + O(z^{12})\right) /; n \in \mathbb{N}$$

03.18.06.0025.01

$$\operatorname{ber}_\nu(z) == \sum_{k=0}^{\infty} \frac{\cos\left(\frac{1}{4}\pi(2k+\nu)\right)}{k!\,\Gamma(k-\nu+1)} \left(\frac{z}{2}\right)^{2k-\nu} /; -\nu \in \mathbb{N}^+$$

03.18.06.0026.01

$$\operatorname{ber}_\nu(z) = \sum_{k=0}^{\infty} \frac{\cos\left(\frac{1}{4}\pi(2k+2\nu+|\nu|)\right)}{\Gamma(k+|\nu|+1)\,k!} \left(\frac{z}{2}\right)^{2k+|\nu|} /; \nu \in \mathbb{Z}$$

03.18.06.0027.01

$$\operatorname{ber}_{-2n}(z) = \frac{2^{-2n} z^{2n} \cos\left(\frac{n\pi}{2}\right)}{(2n)!} \,{}_0F_3\left(; \frac{1}{2}, n+\frac{1}{2}, n+1; -\frac{z^4}{256}\right) + \frac{2^{-2n-2} z^{2n+2} \sin\left(\frac{n\pi}{2}\right)}{(2n+1)!} \,{}_0F_3\left(; \frac{3}{2}, n+1, n+\frac{3}{2}; -\frac{z^4}{256}\right) /; n \in \mathbb{N}$$

03.18.06.0028.01

$$\operatorname{ber}_{-2n-1}(z) = \frac{(-1)^{\left\lfloor \frac{n+1}{2} \right\rfloor} 2^{-2n-\frac{3}{2}} z^{2n+1}}{(2n+1)!} \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{z}{4}\right)^{4k}}{\left(\frac{1}{2}\right)_k \left(n+\frac{3}{2}\right)_k (n+1)_k\, k!} + \frac{(-1)^{\left\lfloor \frac{n}{2} \right\rfloor} 2^{-2n-\frac{7}{2}} z^{2n+3}}{(2n+2)!} \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{z}{4}\right)^{4k}}{\left(\frac{3}{2}\right)_k \left(n+\frac{3}{2}\right)_k (n+2)_k\, k!} /; n \in \mathbb{N}$$

03.18.06.0029.01

$$\operatorname{ber}_{-2n}(z) = \frac{2^{-2n} z^{2n} \cos\left(\frac{n\pi}{2}\right)}{(2n)!} \,{}_0F_3\left(; \frac{1}{2}, n+\frac{1}{2}, n+1; -\frac{z^4}{256}\right) + \frac{2^{-2n-2} z^{2n+2} \sin\left(\frac{n\pi}{2}\right)}{(2n+1)!} \,{}_0F_3\left(; \frac{3}{2}, n+1, n+\frac{3}{2}; -\frac{z^4}{256}\right) /; n \in \mathbb{N}$$

03.18.06.0030.01

$$\operatorname{ber}_{-2n-1}(z) = \frac{(-1)^{\left\lfloor \frac{n+1}{2} \right\rfloor} 2^{-2n-\frac{3}{2}} z^{2n+1}}{(2n+1)!} \,{}_0F_3\left(; \frac{1}{2}, n+1, n+\frac{3}{2}; -\frac{z^4}{256}\right) + \frac{(-1)^{\left\lfloor \frac{n}{2} \right\rfloor} 2^{-2n-\frac{7}{2}} z^{2n+3}}{(2n+2)!} \,{}_0F_3\left(; \frac{3}{2}, n+\frac{3}{2}, n+2; -\frac{z^4}{256}\right) /; n \in \mathbb{N}$$

03.18.06.0031.01

$$\operatorname{ber}_{-2n}(z) = 2^{-4n} \pi \cos\left(\frac{n\pi}{2}\right) z^{2n} \,{}_0\tilde{F}_3\left(; \frac{1}{2}, n+\frac{1}{2}, n+1; -\frac{z^4}{256}\right) + 16^{-n-1} \pi \sin\left(\frac{n\pi}{2}\right) z^{2n+2} \,{}_0\tilde{F}_3\left(; \frac{3}{2}, n+1, n+\frac{3}{2}; -\frac{z^4}{256}\right) /; n \in \mathbb{N}$$

03.18.06.0032.01

$$\operatorname{ber}_{-2n-1}(z) = (-1)^{\left\lfloor \frac{n+1}{2} \right\rfloor} 2^{-4n-\frac{5}{2}} \pi z^{2n+1} {}_0\tilde{F}_3\left(; \frac{1}{2}, n+1, n+\frac{3}{2}; -\frac{z^4}{256}\right) + (-1)^{\left\lfloor \frac{n}{2} \right\rfloor} 2^{-4n-\frac{13}{2}} \pi z^{2n+3} {}_0\tilde{F}_3\left(; \frac{3}{2}, n+\frac{3}{2}, n+2; -\frac{z^4}{256}\right) /; n \in \mathbb{N}$$

Asymptotic series expansions

Expansions inside Stokes sectors

Expansions containing $z \to \infty$

In exponential form ||| In exponential form

03.18.06.0033.01

$$\operatorname{ber}_\nu(z) \propto -\frac{1}{2\sqrt{2\pi}\sqrt{z}} \left(e^{-\frac{z}{\sqrt{2}}} \left(e^{-\frac{1}{8}(5 i \pi) + \frac{3 i \pi \nu}{2} - \frac{i z}{\sqrt{2}}} - e^{\frac{5 i \pi}{8} + \frac{i \pi \nu}{2} + \frac{i z}{\sqrt{2}}} \right) - e^{\frac{z}{\sqrt{2}}} \left(e^{\frac{i \pi}{8} - \frac{i \pi \nu}{2} - \frac{i z}{\sqrt{2}}} + e^{-\frac{1}{8}(i \pi) + \frac{i \pi \nu}{2} + \frac{i z}{\sqrt{2}}} \right) + \right.$$
$$\frac{1-4\nu^2}{8z} \left(e^{\frac{z}{\sqrt{2}}} \left(e^{-\frac{1}{8}(5 i \pi) - \frac{i \pi \nu}{2} - \frac{i z}{\sqrt{2}}} + e^{\frac{5 i \pi}{8} + \frac{i \pi \nu}{2} + \frac{i z}{\sqrt{2}}} \right) + e^{-\frac{z}{\sqrt{2}}} \left(e^{\frac{i \pi}{8} + \frac{3 i \pi \nu}{2} - \frac{i z}{\sqrt{2}}} - e^{-\frac{1}{8}(i \pi) + \frac{i \pi \nu}{2} + \frac{i z}{\sqrt{2}}} \right) \right) +$$
$$\frac{i\left(16\nu^4 - 40\nu^2 + 9\right)}{128 z^2} \left(e^{-\frac{z}{\sqrt{2}}} \left(-e^{-\frac{1}{8}(5 i \pi) + \frac{3 i \pi \nu}{2} - \frac{i z}{\sqrt{2}}} - e^{\frac{5 i \pi}{8} + \frac{i \pi \nu}{2} + \frac{i z}{\sqrt{2}}} \right) - e^{\frac{z}{\sqrt{2}}} \left(e^{\frac{i \pi}{8} - \frac{i \pi \nu}{2} - \frac{i z}{\sqrt{2}}} - e^{-\frac{1}{8}(i \pi) + \frac{i \pi \nu}{2} + \frac{i z}{\sqrt{2}}} \right) \right) -$$
$$\frac{i\left(64\nu^6 - 560\nu^4 + 1036\nu^2 - 225\right)}{3072 z^3} \left(e^{\frac{z}{\sqrt{2}}} \left(e^{-\frac{1}{8}(5 i \pi) - \frac{i \pi \nu}{2} - \frac{i z}{\sqrt{2}}} - e^{\frac{5 i \pi}{8} + \frac{i \pi \nu}{2} + \frac{i z}{\sqrt{2}}} \right) + \right.$$
$$\left. \left. e^{-\frac{z}{\sqrt{2}}} \left(-e^{\frac{i \pi}{8} + \frac{3 i \pi \nu}{2} - \frac{i z}{\sqrt{2}}} - e^{-\frac{1}{8}(i \pi) + \frac{i \pi \nu}{2} + \frac{i z}{\sqrt{2}}} \right) \right) + \ldots \right) /; -\frac{\pi}{2} < \arg(z) \leq \pi \bigwedge (|z| \to \infty)$$

03.18.06.0034.01

$$\operatorname{ber}_\nu(z) \propto -\frac{1}{2\sqrt{2\pi}\sqrt{z}} \left(\sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} \frac{\left(\frac{1}{2} - \nu\right)_{2k} \left(\nu + \frac{1}{2}\right)_{2k}}{(2k)!} \left(\frac{i}{4z^2}\right)^k \right.$$
$$\left(e^{-\frac{z}{\sqrt{2}}} \left((-1)^k e^{\frac{3 i \pi \nu}{2} - \frac{5 \pi i}{8} - \frac{i z}{\sqrt{2}}} - e^{\frac{i \pi \nu}{2} + \frac{5 \pi i}{8} + \frac{i z}{\sqrt{2}}} \right) - e^{\frac{z}{\sqrt{2}}} \left((-1)^k e^{\frac{i \pi \nu}{2} - \frac{\pi i}{8} + \frac{i z}{\sqrt{2}}} + e^{-\frac{1}{2}(i \pi \nu) + \frac{\pi i}{8} - \frac{i z}{\sqrt{2}}} \right) \right) +$$
$$\frac{1}{2z} \sum_{k=0}^{\left\lfloor \frac{n-1}{2} \right\rfloor} \frac{\left(\frac{1}{2} - \nu\right)_{2k+1} \left(\nu + \frac{1}{2}\right)_{2k+1}}{(2k+1)!} \left(\frac{i}{4z^2}\right)^k \left(e^{-\frac{z}{\sqrt{2}}} \left((-1)^k e^{\frac{3 i \pi \nu}{2} + \frac{\pi i}{8} - \frac{i z}{\sqrt{2}}} - e^{\frac{i \pi \nu}{2} - \frac{\pi i}{8} + \frac{i z}{\sqrt{2}}} \right) + \right.$$
$$\left. \left. e^{\frac{z}{\sqrt{2}}} \left((-1)^k e^{\frac{i \pi \nu}{2} + \frac{5 \pi i}{8} + \frac{i z}{\sqrt{2}}} + e^{-\frac{1}{2}(i \pi \nu) - \frac{5 \pi i}{8} - \frac{i z}{\sqrt{2}}} \right) \right) + \ldots \right) /; -\frac{\pi}{2} < \arg(z) \leq \pi \bigwedge (|z| \to \infty) \bigwedge n \in \mathbb{N}$$

03.18.06.0035.01

$$\text{ber}_\nu(z) \propto -\frac{1}{2\sqrt{2\pi}\,\sqrt{z}}\Bigg(e^{-\frac{z}{\sqrt{2}}}\Bigg(e^{\frac{3i\pi\nu}{2}-\frac{5\pi i}{8}-\frac{iz}{\sqrt{2}}}\,{}_4F_1\Big(\frac{1}{4}-\frac{\nu}{2},\frac{3}{4}-\frac{\nu}{2},\frac{\nu}{2}+\frac{1}{4},\frac{\nu}{2}+\frac{3}{4};\frac{1}{2};-\frac{i}{z^2}\Big)- e^{\frac{i\pi\nu}{2}+\frac{5\pi i}{8}+\frac{iz}{\sqrt{2}}}\,{}_4F_1\Big(\frac{1}{4}-\frac{\nu}{2},\frac{3}{4}-\frac{\nu}{2},\frac{\nu}{2}+\frac{1}{4},\frac{\nu}{2}+\frac{3}{4};\frac{1}{2};\frac{i}{z^2}\Big)\Bigg)- e^{\frac{z}{\sqrt{2}}}\Bigg(e^{-\frac{1}{2}(i\pi\nu)+\frac{\pi i}{8}-\frac{iz}{\sqrt{2}}}\,{}_4F_1\Big(\frac{1}{4}-\frac{\nu}{2},\frac{3}{4}-\frac{\nu}{2},\frac{\nu}{2}+\frac{1}{4},\frac{\nu}{2}+\frac{3}{4};\frac{1}{2};\frac{i}{z^2}\Big)+e^{\frac{i\pi\nu}{2}-\frac{\pi i}{8}+\frac{iz}{\sqrt{2}}}\,{}_4F_1\Big(\frac{1}{4}-\frac{\nu}{2},\frac{3}{4}-\frac{\nu}{2},\frac{\nu}{2}+\frac{1}{4},\frac{\nu}{2}+\frac{3}{4};\frac{1}{2};-\frac{i}{z^2}\Big)\Bigg)+\frac{1-4\nu^2}{8z}\Bigg(e^{\frac{z}{\sqrt{2}}}\Bigg(e^{-\frac{1}{2}(i\pi\nu)-\frac{5\pi i}{8}-\frac{iz}{\sqrt{2}}}\,{}_4F_1\Big(\frac{3}{4}-\frac{\nu}{2},\frac{5}{4}-\frac{\nu}{2},\frac{\nu}{2}+\frac{3}{4},\frac{\nu}{2}+\frac{5}{4};\frac{3}{2};\frac{i}{z^2}\Big)+e^{\frac{i\pi\nu}{2}+\frac{5\pi i}{8}+\frac{iz}{\sqrt{2}}}\,{}_4F_1\Big(\frac{3}{4}-\frac{\nu}{2},\frac{5}{4}-\frac{\nu}{2},\frac{\nu}{2}+\frac{3}{4},\frac{\nu}{2}+\frac{5}{4};\frac{3}{2};-\frac{i}{z^2}\Big)\Bigg)+e^{-\frac{z}{\sqrt{2}}}\Bigg(e^{\frac{3i\pi\nu}{2}+\frac{\pi i}{8}-\frac{iz}{\sqrt{2}}}\,{}_4F_1\Big(\frac{3}{4}-\frac{\nu}{2},\frac{5}{4}-\frac{\nu}{2},\frac{\nu}{2}+\frac{3}{4},\frac{\nu}{2}+\frac{5}{4};\frac{3}{2};-\frac{i}{z^2}\Big)- e^{\frac{i\pi\nu}{2}-\frac{\pi i}{8}+\frac{iz}{\sqrt{2}}}\,{}_4F_1\Big(\frac{3}{4}-\frac{\nu}{2},\frac{5}{4}-\frac{\nu}{2},\frac{\nu}{2}+\frac{3}{4},\frac{\nu}{2}+\frac{5}{4};\frac{3}{2};\frac{i}{z^2}\Big)\Bigg)\Bigg)\Bigg)\,/;\,-\frac{\pi}{2}<\arg(z)\leq\pi\bigwedge(|z|\to\infty)$$

03.18.06.0036.01

$$\text{ber}_\nu(z) \propto -\frac{1}{2\sqrt{2\pi}\,\sqrt{z}}\Bigg(-e^{\frac{z}{\sqrt{2}}}\Bigg(e^{\frac{i\pi\nu}{2}-\frac{\pi i}{8}+\frac{iz}{\sqrt{2}}}\Big(1+O\Big(\frac{1}{z^2}\Big)\Big)+e^{-\frac{i\pi\nu}{2}+\frac{\pi i}{8}-\frac{iz}{\sqrt{2}}}\Big(1+O\Big(\frac{1}{z^2}\Big)\Big)\Bigg)+e^{-\frac{z}{\sqrt{2}}}\Bigg(e^{\frac{3i\pi\nu}{2}-\frac{5\pi i}{8}-\frac{iz}{\sqrt{2}}}\Big(1+O\Big(\frac{1}{z^2}\Big)\Big)-e^{\frac{i\pi\nu}{2}+\frac{5\pi i}{8}+\frac{iz}{\sqrt{2}}}\Big(1+O\Big(\frac{1}{z^2}\Big)\Big)\Bigg)\Bigg)\,/;\,-\frac{\pi}{2}<\arg(z)\leq\pi\bigwedge(|z|\to\infty)$$

In trigonometric form ||| In trigonometric form

03.18.06.0037.01

$$\text{ber}_\nu(z) \propto \frac{1}{\sqrt{2\pi}\,\sqrt{z}}\Bigg(e^{\frac{z}{\sqrt{2}}}\cos\Big(\frac{1}{8}\big(\pi(1-4\nu)-4\sqrt{2}\,z\big)\Big)+e^{i\pi\nu-\frac{z}{\sqrt{2}}}\,i\sin\Big(\frac{1}{8}\big(\pi(4\nu+3)-4\sqrt{2}\,z\big)\Big)+\frac{1-4\nu^2}{8z}\Bigg(e^{\frac{z}{\sqrt{2}}}\sin\Big(\frac{1}{8}\big(4\sqrt{2}\,z+\pi(4\nu+1)\big)\Big)+e^{i\pi\nu-\frac{z}{\sqrt{2}}}\,i\sin\Big(\frac{1}{8}\big(4\sqrt{2}\,z-\pi(4\nu+1)\big)\Big)\Bigg)-\frac{16\nu^4-40\nu^2+9}{128z^2}\Bigg(i\,e^{i\pi\nu-\frac{z}{\sqrt{2}}}\cos\Big(\frac{1}{8}\big(4\sqrt{2}\,z-\pi(4\nu+3)\big)\Big)-e^{\frac{z}{\sqrt{2}}}\sin\Big(\frac{1}{8}\big(4\sqrt{2}\,z-\pi(1-4\nu)\big)\Big)\Bigg)-\frac{-64\nu^6+560\nu^4-1036\nu^2+225}{3072z^3}\Bigg(e^{\frac{z}{\sqrt{2}}}\cos\Big(\frac{1}{8}\big(-4\sqrt{2}\,z-\pi(4\nu+1)\big)\Big)-i\,e^{i\pi\nu-\frac{z}{\sqrt{2}}}\cos\Big(\frac{1}{8}\big(\pi(4\nu+1)-4\sqrt{2}\,z\big)\Big)\Bigg)+\ldots\Bigg)\,/;\,-\frac{\pi}{2}<\arg(z)\leq\pi\bigwedge(|z|\to\infty)$$

03.18.06.0038.01

$$\operatorname{ber}_\nu(z) \propto \frac{1}{\sqrt{2\pi}\sqrt{z}}\left(\sum_{k=0}^{\lfloor \frac{n}{2} \rfloor} \frac{\left(\frac{1}{2}-\nu\right)_{2k}\left(\nu+\frac{1}{2}\right)_{2k}}{(2k)!}\left(-\frac{1}{4z^2}\right)^k \left((-1)^k e^{\frac{z}{\sqrt{2}}}\cos\left(\frac{\pi k}{2}+\frac{1}{8}\left(\pi(1-4\nu)-4\sqrt{2}\,z\right)\right)+e^{i\pi\nu-\frac{z}{\sqrt{2}}}\,i\sin\left(\frac{\pi k}{2}+\frac{1}{8}\left(\pi(4\nu+3)-4\sqrt{2}\,z\right)\right)\right)+\frac{1}{2z}\sum_{k=0}^{\lfloor \frac{n-1}{2} \rfloor}\frac{\left(\frac{1}{2}-\nu\right)_{2k+1}\left(\nu+\frac{1}{2}\right)_{2k+1}}{(2k+1)!}\left(-\frac{1}{4z^2}\right)^k\left((-1)^k e^{i\pi\nu-\frac{z}{\sqrt{2}}}\,i\sin\left(\frac{\pi k}{2}+\frac{1}{8}\left(4\sqrt{2}\,z-\pi(4\nu+1)\right)\right)+e^{\frac{z}{\sqrt{2}}}\sin\left(\frac{\pi k}{2}+\frac{1}{8}\left(4\sqrt{2}\,z+\pi(4\nu+1)\right)\right)\right)+\ldots\right)\,/;\,-\frac{\pi}{2}<\arg(z)\leq\pi\bigwedge(|z|\to\infty)\bigwedge n\in\mathbb{N}$$

03.18.06.0039.01

$$\operatorname{ber}_\nu(z) \propto \frac{1}{\sqrt{2\pi}\sqrt{z}}\left(\left(e^{\frac{z}{\sqrt{2}}}\cos\left(\frac{1}{8}\left(\pi(1-4\nu)-4\sqrt{2}\,z\right)\right)+e^{i\pi\nu-\frac{z}{\sqrt{2}}}\,i\sin\left(\frac{1}{8}\left(\pi(4\nu+3)-4\sqrt{2}\,z\right)\right)\right){}_8F_3\left(\frac{1}{8}(1-2\nu),\frac{1}{8}(3-2\nu),\frac{1}{8}(5-2\nu),\frac{1}{8}(7-2\nu),\frac{1}{8}(2\nu+1),\frac{1}{8}(2\nu+3),\frac{1}{8}(2\nu+5),\frac{1}{8}(2\nu+7);\frac{1}{4},\frac{1}{2},\frac{3}{4};-\frac{16}{z^4}\right)+\frac{1-4\nu^2}{8z}\,{}_8F_3\left(\frac{1}{8}(3-2\nu),\frac{1}{8}(5-2\nu),\frac{1}{8}(7-2\nu),\frac{1}{8}(9-2\nu),\frac{1}{8}(2\nu+3),\frac{1}{8}(2\nu+5),\frac{1}{8}(2\nu+7),\frac{1}{8}(2\nu+9);\frac{1}{2},\frac{3}{4},\frac{5}{4};-\frac{16}{z^4}\right)\left(e^{i\pi\nu-\frac{z}{\sqrt{2}}}\,i\sin\left(\frac{1}{8}\left(4\sqrt{2}\,z-\pi(4\nu+1)\right)\right)+e^{\frac{z}{\sqrt{2}}}\sin\left(\frac{1}{8}\left(4\sqrt{2}\,z+\pi(4\nu+1)\right)\right)\right)-\frac{16\nu^4-40\nu^2+9}{128z^2}\left(e^{i\pi\nu-\frac{z}{\sqrt{2}}}\,i\cos\left(\frac{1}{8}\left(\pi(4\nu+3)-4\sqrt{2}\,z\right)\right)+e^{\frac{z}{\sqrt{2}}}\sin\left(\frac{1}{8}\left(\pi(1-4\nu)-4\sqrt{2}\,z\right)\right)\right){}_8F_3\left(\frac{1}{8}(5-2\nu),\frac{1}{8}(7-2\nu),\frac{1}{8}(9-2\nu),\frac{1}{8}(11-2\nu),\frac{1}{8}(2\nu+5),\frac{1}{8}(2\nu+7),\frac{1}{8}(2\nu+9),\frac{1}{8}(2\nu+11);\frac{3}{4},\frac{5}{4},\frac{3}{2};-\frac{16}{z^4}\right)-\frac{-64\nu^6+560\nu^4-1036\nu^2+225}{3072z^3}\left(e^{\frac{z}{\sqrt{2}}}\cos\left(\frac{1}{8}\left(4\sqrt{2}\,z+\pi(4\nu+1)\right)\right)-i\,e^{i\pi\nu-\frac{z}{\sqrt{2}}}\cos\left(\frac{1}{8}\left(4\sqrt{2}\,z-\pi(4\nu+1)\right)\right)\right){}_8F_3\left(\frac{1}{8}(7-2\nu),\frac{1}{8}(9-2\nu),\frac{1}{8}(11-2\nu),\frac{1}{8}(13-2\nu),\frac{1}{8}(2\nu+7),\frac{1}{8}(2\nu+9),\frac{1}{8}(2\nu+11),\frac{1}{8}(2\nu+13);\frac{5}{4},\frac{3}{2},\frac{7}{4};-\frac{16}{z^4}\right)\right)\,/;\,-\frac{\pi}{2}<\arg(z)\leq\pi\bigwedge(|z|\to\infty)$$

03.18.06.0040.01

$$\operatorname{ber}_\nu(z) \propto \frac{1}{\sqrt{2\pi}\sqrt{z}}\left(e^{\frac{z}{\sqrt{2}}}\cos\left(\frac{1}{8}\left(\pi(1-4\nu)-4\sqrt{2}\,z\right)\right)+e^{i\pi\nu-\frac{z}{\sqrt{2}}}\,i\sin\left(\frac{1}{8}\left(\pi(4\nu+3)-4\sqrt{2}\,z\right)\right)\right)\left(1+O\left(\frac{1}{z^4}\right)\right)\,/;\,-\frac{\pi}{2}<\arg(z)\leq\pi\bigwedge(|z|\to\infty)$$

Expansions containing $z \to -\infty$

In exponential form ||| In exponential form

03.18.06.0041.01

$$\begin{aligned}\operatorname{ber}_{\nu}(z) \propto \frac{1}{2\sqrt{2\pi}\,\sqrt{-z}} \Bigg(& e^{-\frac{z}{\sqrt{2}}} \left(e^{\frac{i\pi}{8}+\frac{iz}{\sqrt{2}}+\frac{i\pi\nu}{2}} + e^{-\frac{1}{8}(i\pi)-\frac{iz}{\sqrt{2}}+\frac{3i\pi\nu}{2}} \right) + e^{\frac{z}{\sqrt{2}}} \left(-e^{\frac{3i\pi}{8}-\frac{iz}{\sqrt{2}}+\frac{3i\pi\nu}{2}} + e^{\frac{3i\pi}{8}+\frac{iz}{\sqrt{2}}+\frac{5i\pi\nu}{2}} \right) - \\ & \frac{1-4\nu^2}{8z} \left(e^{-\frac{z}{\sqrt{2}}} \left(e^{\frac{3i\pi}{8}+\frac{iz}{\sqrt{2}}+\frac{i\pi\nu}{2}} + e^{-\frac{1}{8}(3i\pi)-\frac{iz}{\sqrt{2}}+\frac{3i\pi\nu}{2}} \right) + e^{\frac{z}{\sqrt{2}}} \left(e^{-\frac{1}{8}(i\pi)-\frac{iz}{\sqrt{2}}+\frac{3i\pi\nu}{2}} - e^{\frac{i\pi}{8}+\frac{iz}{\sqrt{2}}+\frac{5i\pi\nu}{2}} \right) \right) + \\ & \frac{i\left(16\nu^4-40\nu^2+9\right)}{128z^2} \left(e^{-\frac{z}{\sqrt{2}}} \left(e^{\frac{i\pi}{8}+\frac{iz}{\sqrt{2}}+\frac{i\pi\nu}{2}} - e^{-\frac{1}{8}(i\pi)-\frac{iz}{\sqrt{2}}+\frac{3i\pi\nu}{2}} \right) + e^{\frac{z}{\sqrt{2}}} \left(-e^{\frac{3i\pi}{8}-\frac{iz}{\sqrt{2}}+\frac{3i\pi\nu}{2}} - e^{\frac{3i\pi}{8}+\frac{iz}{\sqrt{2}}+\frac{5i\pi\nu}{2}} \right) \right) + \\ & \frac{i\left(64\nu^6-560\nu^4+1036\nu^2-225\right)}{3072z^3} \left(e^{-\frac{z}{\sqrt{2}}} \left(e^{\frac{3i\pi}{8}+\frac{iz}{\sqrt{2}}+\frac{i\pi\nu}{2}} - e^{-\frac{1}{8}(3i\pi)-\frac{iz}{\sqrt{2}}+\frac{3i\pi\nu}{2}} \right) + \right. \\ & \left. e^{\frac{z}{\sqrt{2}}} \left(e^{-\frac{1}{8}(i\pi)-\frac{iz}{\sqrt{2}}+\frac{3i\pi\nu}{2}} + e^{\frac{i\pi}{8}+\frac{iz}{\sqrt{2}}+\frac{5i\pi\nu}{2}} \right) \right) + \ldots \Bigg) /;\ \frac{\pi}{2} < \arg(z) \leq \pi \bigwedge (|z| \to \infty)\end{aligned}$$

03.18.06.0042.01

$$\begin{aligned}\operatorname{ber}_{\nu}(z) \propto \frac{1}{2\sqrt{2\pi}\,\sqrt{-z}} \Bigg(& \sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} \frac{\left(\frac{1}{2}-\nu\right)_{2k} \left(\nu+\frac{1}{2}\right)_{2k} \left(\frac{i}{4z^2}\right)^k}{(2k)!} \\ & \left(e^{-\frac{z}{\sqrt{2}}} \left(e^{\frac{i\pi\nu}{2}+\frac{\pi i}{8}} e^{\frac{iz}{\sqrt{2}}} + (-1)^k e^{\frac{3i\pi\nu}{2}-\frac{\pi i}{8}} e^{-\frac{iz}{\sqrt{2}}} \right) + e^{\frac{z}{\sqrt{2}}} \left((-1)^k e^{\frac{5i\pi\nu}{2}+\frac{3\pi i}{8}} e^{\frac{iz}{\sqrt{2}}} - e^{\frac{3i\pi\nu}{2}+\frac{3\pi i}{8}} e^{-\frac{iz}{\sqrt{2}}} \right) \right) - \\ & \frac{1}{2z} \sum_{k=0}^{\left\lfloor \frac{n-1}{2} \right\rfloor} \frac{\left(\frac{1}{2}-\nu\right)_{2k+1} \left(\nu+\frac{1}{2}\right)_{2k+1} \left(\frac{i}{4z^2}\right)^k}{(2k+1)!} \left(e^{\frac{z}{\sqrt{2}}} \left(-(-1)^k e^{\frac{5i\pi\nu}{2}+\frac{\pi i}{8}} e^{\frac{iz}{\sqrt{2}}} + e^{\frac{3i\pi\nu}{2}-\frac{\pi i}{8}} e^{-\frac{iz}{\sqrt{2}}} \right) + \right. \\ & \left. e^{-\frac{z}{\sqrt{2}}} \left(e^{\frac{i\pi\nu}{2}+\frac{3\pi i}{8}} e^{\frac{iz}{\sqrt{2}}} + (-1)^k e^{\frac{3i\pi\nu}{2}-\frac{3\pi i}{8}} e^{-\frac{iz}{\sqrt{2}}} \right) \right) + \ldots \Bigg) /;\ \frac{\pi}{2} < \arg(z) \leq \pi \bigwedge (|z| \to \infty) \bigwedge n \in \mathbb{N}\end{aligned}$$

03.18.06.0043.01

$$\operatorname{ber}_\nu(z) \propto \frac{1}{2\sqrt{2\pi}\sqrt{-z}}\left(e^{\frac{z}{\sqrt{2}}}\left(e^{\frac{5i\pi\nu}{2}+\frac{3\pi i}{8}} e^{\frac{iz}{\sqrt{2}}}\,{}_4F_1\left(\frac{1}{4}-\frac{\nu}{2},\frac{3}{4}-\frac{\nu}{2},\frac{\nu}{2}+\frac{1}{4},\frac{\nu}{2}+\frac{3}{4};\frac{1}{2};-\frac{i}{z^2}\right)- e^{\frac{3i\pi\nu}{2}+\frac{3\pi i}{8}} e^{-\frac{iz}{\sqrt{2}}}\,{}_4F_1\left(\frac{1}{4}-\frac{\nu}{2},\frac{3}{4}-\frac{\nu}{2},\frac{\nu}{2}+\frac{1}{4},\frac{\nu}{2}+\frac{3}{4};\frac{1}{2};\frac{i}{z^2}\right)\right)+e^{-\frac{z}{\sqrt{2}}}\left(e^{\frac{iz}{\sqrt{2}}} e^{\frac{i\pi\nu}{2}+\frac{\pi i}{8}}\,{}_4F_1\left(\frac{1}{4}-\frac{\nu}{2},\frac{3}{4}-\frac{\nu}{2},\frac{\nu}{2}+\frac{1}{4},\frac{\nu}{2}+\frac{3}{4};\frac{1}{2};\frac{i}{z^2}\right)+e^{-\frac{iz}{\sqrt{2}}} e^{\frac{3i\pi\nu}{2}-\frac{\pi i}{8}}\,{}_4F_1\left(\frac{1}{4}-\frac{\nu}{2},\frac{3}{4}-\frac{\nu}{2},\frac{\nu}{2}+\frac{1}{4},\frac{\nu}{2}+\frac{3}{4};\frac{1}{2};-\frac{i}{z^2}\right)\right)-\frac{1-\nu^2}{8z}\left(e^{\frac{z}{\sqrt{2}}}\left(e^{\frac{3i\pi\nu}{2}-\frac{\pi i}{8}} e^{-\frac{iz}{\sqrt{2}}}\,{}_4F_1\left(\frac{3}{4}-\frac{\nu}{2},\frac{5}{4}-\frac{\nu}{2},\frac{\nu}{2}+\frac{3}{4},\frac{\nu}{2}+\frac{5}{4};\frac{3}{2};\frac{i}{z^2}\right)- e^{\frac{5i\pi\nu}{2}+\frac{\pi i}{8}} e^{\frac{iz}{\sqrt{2}}}\,{}_4F_1\left(\frac{3}{4}-\frac{\nu}{2},\frac{5}{4}-\frac{\nu}{2},\frac{\nu}{2}+\frac{3}{4},\frac{\nu}{2}+\frac{5}{4};\frac{3}{2};-\frac{i}{z^2}\right)\right)+ e^{-\frac{z}{\sqrt{2}}}\left(e^{\frac{iz}{\sqrt{2}}} e^{\frac{i\pi\nu}{2}+\frac{3\pi i}{8}}\,{}_4F_1\left(\frac{3}{4}-\frac{\nu}{2},\frac{5}{4}-\frac{\nu}{2},\frac{\nu}{2}+\frac{3}{4},\frac{\nu}{2}+\frac{5}{4};\frac{3}{2};\frac{i}{z^2}\right)+e^{-\frac{iz}{\sqrt{2}}} e^{\frac{3i\pi\nu}{2}-\frac{3\pi i}{8}}\,{}_4F_1\left(\frac{3}{4}-\frac{\nu}{2},\frac{5}{4}-\frac{\nu}{2},\frac{\nu}{2}+\frac{3}{4},\frac{\nu}{2}+\frac{5}{4};\frac{3}{2};-\frac{i}{z^2}\right)\right)\right)\right)\;/;\;\frac{\pi}{2}<\arg(z)\leq\pi\bigwedge(|z|\to\infty)$$

03.18.06.0044.01

$$\operatorname{ber}_\nu(z) \propto \frac{(-1)^{3/8} e^{\frac{i\pi\nu}{2}}}{2\sqrt{2\pi}\sqrt{-z}}\left(-e^{-\frac{z}{\sqrt{2}}}\left((-1)^{3/4} e^{\frac{iz}{\sqrt{2}}}+i\,e^{i\pi(k+\nu)-\frac{iz}{\sqrt{2}}}\right)\left(1+O\left(\frac{1}{z}\right)\right)+e^{\frac{z}{\sqrt{2}}}\left(\sqrt[4]{-1}\,e^{i\pi\nu-\frac{iz}{\sqrt{2}}}+e^{\frac{iz}{\sqrt{2}}+i\pi(k+2\nu)}\right)\left(1+O\left(\frac{1}{z}\right)\right)\right)\;/;\;(z\to-\infty)$$

03.18.06.0045.01

$$\operatorname{ber}_\nu(z) \propto \frac{1}{2\sqrt{2\pi}\sqrt{-z}}\left(e^{\frac{z}{\sqrt{2}}}\left(e^{\frac{5i\pi\nu}{2}+\frac{3\pi i}{8}} e^{\frac{iz}{\sqrt{2}}}\left(1+O\left(\frac{1}{z^2}\right)\right)-e^{\frac{3i\pi\nu}{2}+\frac{3\pi i}{8}} e^{-\frac{iz}{\sqrt{2}}}\left(1+O\left(\frac{1}{z^2}\right)\right)\right)+ e^{-\frac{z}{\sqrt{2}}}\left(e^{\frac{iz}{\sqrt{2}}} e^{\frac{i\pi\nu}{2}+\frac{\pi i}{8}}\left(1+O\left(\frac{1}{z^2}\right)\right)+e^{-\frac{iz}{\sqrt{2}}} e^{\frac{3i\pi\nu}{2}-\frac{\pi i}{8}}\left(1+O\left(\frac{1}{z^2}\right)\right)\right)\right)\;/;\;\frac{\pi}{2}<\arg(z)\leq\pi\bigwedge(|z|\to\infty)$$

In trigonometric form ||| In trigonometric form

03.18.06.0046.01

$$\operatorname{ber}_{\nu}(z) \propto \frac{e^{i\pi\nu}}{\sqrt{2\pi}\sqrt{-z}} \Bigg(\Bigg(e^{\frac{z}{\sqrt{2}}+i\pi\nu} i \cos\left(\frac{1}{8}\left(\pi(1-4\nu)-4\sqrt{2}\,z\right)\right) + e^{-\frac{z}{\sqrt{2}}} \cos\left(\frac{1}{8}\left(4\sqrt{2}\,z+\pi(1-4\nu)\right)\right)\Bigg) + \frac{1-4\nu^2}{8z}\left(e^{\frac{z}{\sqrt{2}}+i\pi\nu} i\sin\left(\frac{1}{8}\left(4\sqrt{2}\,z+\pi(4\nu+1)\right)\right)+e^{-\frac{z}{\sqrt{2}}}\sin\left(\frac{1}{8}\left(4\sqrt{2}\,z-\pi(4\nu+1)\right)\right)\right) + \frac{16\nu^4-40\nu^2+9}{128z^2}\left(e^{-\frac{z}{\sqrt{2}}}\sin\left(\frac{1}{8}\left(-4\sqrt{2}\,z-\pi(1-4\nu)\right)\right)+e^{\frac{z}{\sqrt{2}}+i\pi\nu} i\sin\left(\frac{1}{8}\left(4\sqrt{2}\,z-\pi(1-4\nu)\right)\right)\right) + \frac{-64\nu^6+560\nu^4-1036\nu^2+225}{3072z^3}\left(e^{-\frac{z}{\sqrt{2}}}\cos\left(\frac{1}{8}\left(\pi(4\nu+1)-4\sqrt{2}\,z\right)\right)-i\,e^{\frac{z}{\sqrt{2}}+i\pi\nu}\cos\left(\frac{1}{8}\left(-4\sqrt{2}\,z-\pi(4\nu+1)\right)\right)\right) + \ldots\Bigg) /; (z\to-\infty)$$

03.18.06.0047.01

$$\operatorname{ber}_{\nu}(z) \propto \frac{e^{i\pi\nu}}{\sqrt{2\pi}\sqrt{-z}} \Bigg(\sum_{k=0}^{\lfloor\frac{n}{2}\rfloor} \frac{\left(\frac{1}{2}-\nu\right)_{2k}\left(\nu+\frac{1}{2}\right)_{2k}}{(2k)!}\left(\frac{1}{4z^2}\right)^k \left(e^{-\frac{z}{\sqrt{2}}}\cos\left(\frac{\pi k}{2}+\frac{1}{8}\left(4\sqrt{2}\,z+\pi(1-4\nu)\right)\right)+e^{\frac{z}{\sqrt{2}}+i\pi\nu} i\cos\left(\frac{\pi k}{2}+\frac{1}{8}\left(\pi(1-4\nu)-4\sqrt{2}\,z\right)\right)\right) + \frac{1}{2z}\sum_{k=0}^{\lfloor\frac{n-1}{2}\rfloor}\frac{\left(\frac{1}{2}-\nu\right)_{2k+1}\left(\nu+\frac{1}{2}\right)_{2k+1}}{(2k+1)!}\left(\frac{1}{4z^2}\right)^k\Bigg(e^{-\frac{z}{\sqrt{2}}}\sin\left(\frac{\pi k}{2}+\frac{1}{8}\left(4\sqrt{2}\,z-\pi(4\nu+1)\right)\right) + (-1)^k e^{\frac{z}{\sqrt{2}}+i\pi\nu} i\sin\left(\frac{\pi k}{2}+\frac{1}{8}\left(4\sqrt{2}\,z+\pi(4\nu+1)\right)\right)\Bigg)+\ldots\Bigg) /; (z\to-\infty)\land n\in\mathbb{N}$$

03.18.06.0048.01

$$\operatorname{ber}_\nu(z) \propto \frac{e^{i\pi\nu}}{\sqrt{2\pi}\sqrt{-z}}\left(\left(e^{-\frac{z}{\sqrt{2}}}\cos\left(\frac{1}{8}\left(4\sqrt{2}\,z+\pi(1-4\nu)\right)\right)+e^{\frac{z}{\sqrt{2}}+i\pi\nu}\,i\cos\left(\frac{1}{8}\left(\pi(1-4\nu)-4\sqrt{2}\,z\right)\right)\right){}_8F_3\left(\frac{1}{8}(1-2\nu),\frac{1}{8}(3-2\nu),\frac{1}{8}(5-2\nu),\frac{1}{8}(7-2\nu),\frac{1}{8}(2\nu+1),\frac{1}{8}(2\nu+3),\frac{1}{8}(2\nu+5),\frac{1}{8}(2\nu+7);\frac{1}{4},\frac{1}{2},\frac{3}{4};-\frac{16}{z^4}\right)+\frac{1-4\nu^2}{8z}\,{}_8F_3\left(\frac{1}{8}(3-2\nu),\frac{1}{8}(5-2\nu),\frac{1}{8}(7-2\nu),\frac{1}{8}(9-2\nu),\frac{1}{8}(2\nu+3),\frac{1}{8}(2\nu+5),\frac{1}{8}(2\nu+7),\frac{1}{8}(2\nu+9);\frac{1}{2},\frac{3}{4},\frac{5}{4};-\frac{16}{z^4}\right)\left(e^{-\frac{z}{\sqrt{2}}}\sin\left(\frac{1}{8}\left(4\sqrt{2}\,z-\pi(4\nu+1)\right)\right)+e^{\frac{z}{\sqrt{2}}+i\pi\nu}\,i\sin\left(\frac{1}{8}\left(4\sqrt{2}\,z+\pi(4\nu+1)\right)\right)\right)-\frac{16\nu^4-40\nu^2+9}{128z^2}\left(e^{-\frac{z}{\sqrt{2}}}\sin\left(\frac{1}{8}\left(4\sqrt{2}\,z+\pi(1-4\nu)\right)\right)+e^{\frac{z}{\sqrt{2}}+i\pi\nu}\,i\sin\left(\frac{1}{8}\left(\pi(1-4\nu)-4\sqrt{2}\,z\right)\right)\right){}_8F_3\left(\frac{1}{8}(5-2\nu),\frac{1}{8}(7-2\nu),\frac{1}{8}(9-2\nu),\frac{1}{8}(11-2\nu),\frac{1}{8}(2\nu+5),\frac{1}{8}(2\nu+7),\frac{1}{8}(2\nu+9),\frac{1}{8}(2\nu+11);\frac{3}{4},\frac{5}{4},\frac{3}{2};-\frac{16}{z^4}\right)+\frac{-64\nu^6+560\nu^4-1036\nu^2+225}{3072z^3}\,{}_8F_3\left(\frac{1}{8}(7-2\nu),\frac{1}{8}(9-2\nu),\frac{1}{8}(11-2\nu),\frac{1}{8}(13-2\nu),\frac{1}{8}(2\nu+7),\frac{1}{8}(2\nu+9),\frac{1}{8}(2\nu+11),\frac{1}{8}(2\nu+13);\frac{5}{4},\frac{3}{2},\frac{7}{4};-\frac{16}{z^4}\right)\left(e^{-\frac{z}{\sqrt{2}}}\sin\left(\frac{1}{8}\left(4\sqrt{2}\,z-\pi(4\nu+1)\right)+\frac{\pi}{2}\right)-i\,e^{\frac{z}{\sqrt{2}}+i\pi\nu}\sin\left(\frac{1}{8}\left(4\sqrt{2}\,z+\pi(4\nu+1)\right)+\frac{\pi}{2}\right)\right)\right)\;/;\;(z\to-\infty)$$

03.18.06.0049.01

$$\operatorname{ber}_\nu(z) \propto \frac{e^{i\pi\nu}}{\sqrt{2\pi}\sqrt{-z}}\left(e^{-\frac{z}{\sqrt{2}}}\cos\left(\frac{1}{8}\left(4\sqrt{2}\,z+\pi(1-4\nu)\right)\right)+e^{\frac{z}{\sqrt{2}}+i\pi\nu}\,i\cos\left(\frac{1}{8}\left(\pi(1-4\nu)-4\sqrt{2}\,z\right)\right)\right)\left(1+O\left(\frac{1}{z^4}\right)\right)\;/;\;(z\to-\infty)$$

Expansions for any z in exponential form

Using exponential function with branch cut-free arguments

03.18.06.0050.01

$$
\begin{aligned}
\operatorname{ber}_\nu(z) \propto{} & \frac{\sqrt[4]{-1}\, e^{-\frac{1}{4} i \pi \nu} z^\nu}{2\sqrt{2\pi}} \Bigg(e^{-\frac{z}{\sqrt{2}}} \bigg(e^{\frac{i z}{\sqrt{2}}} \left(-\sqrt[4]{-1}\, z\right)^{-\nu-\frac{1}{2}} + e^{\frac{3 i \pi \nu}{2}-\frac{i z}{\sqrt{2}}} \left((-1)^{3/4} z\right)^{-\nu-\frac{1}{2}} \left(\frac{\sqrt[4]{-1}\, \sqrt{i z^2}\, \cos(\pi \nu)}{z} - \sin(\pi \nu) \right) \bigg) + \\
& e^{\frac{z}{\sqrt{2}}} \bigg(e^{\frac{i z}{\sqrt{2}}+\frac{3 i \pi \nu}{2}} \left((-1)^{3/4} z\right)^{-\nu-\frac{1}{2}} - e^{-\frac{i z}{\sqrt{2}}} \left(-\sqrt[4]{-1}\, z\right)^{-\nu-\frac{1}{2}} \left(\frac{(-1)^{3/4} \sqrt{-i z^2}\, \cos(\pi \nu)}{z} + \sin(\pi \nu) \right) \bigg) \Bigg) + \\
& \frac{(-1)^{3/4} \left(1-4 \nu^2\right)}{8 z} \Bigg(e^{-\frac{z}{\sqrt{2}}} \bigg(e^{\frac{i z}{\sqrt{2}}}\, i \left(-\sqrt[4]{-1}\, z\right)^{-\nu-\frac{1}{2}} + e^{\frac{3 i \pi \nu}{2}-\frac{i z}{\sqrt{2}}} \left((-1)^{3/4} z\right)^{-\nu-\frac{1}{2}} \left(\frac{\sqrt[4]{-1}\, \sqrt{i z^2}\, \cos(\pi \nu)}{z} - \sin(\pi \nu) \right) \bigg) + \\
& e^{\frac{z}{\sqrt{2}}} \bigg(i\, e^{-\frac{i z}{\sqrt{2}}} \left(-\sqrt[4]{-1}\, z\right)^{-\nu-\frac{1}{2}} \left(\frac{(-1)^{3/4} \sqrt{-i z^2}\, \cos(\pi \nu)}{z} + \sin(\pi \nu) \right) - e^{\frac{i z}{\sqrt{2}}+\frac{3 i \pi \nu}{2}} \left((-1)^{3/4} z\right)^{-\nu-\frac{1}{2}} \bigg) \Bigg) + \\
& \frac{i \left(16 \nu^4-40 \nu^2+9\right)}{128 z^2} \Bigg(e^{-\frac{z}{\sqrt{2}}} \bigg(e^{\frac{i z}{\sqrt{2}}} \left(-\sqrt[4]{-1}\, z\right)^{-\nu-\frac{1}{2}} - e^{\frac{3 i \pi \nu}{2}-\frac{i z}{\sqrt{2}}} \left((-1)^{3/4} z\right)^{-\nu-\frac{1}{2}} \left(\frac{\sqrt[4]{-1}\, \sqrt{i z^2}\, \cos(\pi \nu)}{z} - \sin(\pi \nu) \right) \bigg) + \\
& e^{\frac{z}{\sqrt{2}}} \bigg(-e^{-\frac{i z}{\sqrt{2}}} \left(\frac{(-1)^{3/4} \sqrt{-i z^2}\, \cos(\pi \nu)}{z} + \sin(\pi \nu) \right) \left(-\sqrt[4]{-1}\, z\right)^{-\nu-\frac{1}{2}} - e^{\frac{i z}{\sqrt{2}}+\frac{3 i \pi \nu}{2}} \left((-1)^{3/4} z\right)^{-\nu-\frac{1}{2}} \bigg) \Bigg) + \\
& \frac{\sqrt[4]{-1} \left(64 \nu^6-560 \nu^4+1036 \nu^2-225\right)}{3072 z^3} \\
& \Bigg(e^{-\frac{z}{\sqrt{2}}} \bigg(i\, e^{\frac{i z}{\sqrt{2}}} \left(-\sqrt[4]{-1}\, z\right)^{-\nu-\frac{1}{2}} - e^{\frac{3 i \pi \nu}{2}-\frac{i z}{\sqrt{2}}} \left((-1)^{3/4} z\right)^{-\nu-\frac{1}{2}} \left(\frac{\sqrt[4]{-1}\, \sqrt{i z^2}\, \cos(\pi \nu)}{z} - \sin(\pi \nu) \right) \bigg) + e^{\frac{z}{\sqrt{2}}} \\
& \bigg(e^{-\frac{i z}{\sqrt{2}}}\, i \left(\frac{(-1)^{3/4} \sqrt{-i z^2}\, \cos(\pi \nu)}{z} + \sin(\pi \nu) \right) \left(-\sqrt[4]{-1}\, z\right)^{-\nu-\frac{1}{2}} + e^{\frac{i z}{\sqrt{2}}+\frac{3 i \pi \nu}{2}} \left((-1)^{3/4} z\right)^{-\nu-\frac{1}{2}} \bigg) \Bigg) + \ldots \Bigg) \;/;\; (|z| \to \infty)
\end{aligned}
$$

03.18.06.0051.01

$\operatorname{ber}_\nu(z) \propto$

$$\frac{\sqrt[4]{-1}\, e^{-\frac{i\pi\nu}{4}} z^\nu}{2\sqrt{2\pi}} \Bigg(\Bigg(e^{-\frac{z}{\sqrt{2}}} \Bigg(e^{\frac{iz}{\sqrt{2}}} \left(-\sqrt[4]{-1}\, z\right)^{-\nu-\frac{1}{2}} \left(\sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} \frac{2^{-2k} \left(\frac{1}{2}-\nu\right)_{2k} \left(\nu+\frac{1}{2}\right)_{2k}}{(2k)!} \left(\frac{i}{z^2}\right)^k + O\left(\frac{1}{z^{2\left\lfloor \frac{n}{2} \right\rfloor+2}}\right)\right) + e^{\frac{3i\pi\nu}{2}-\frac{iz}{\sqrt{2}}} \left((-1)^{3/4} z\right)^{-\nu-\frac{1}{2}}$$

$$\left(\frac{\sqrt[4]{-1}\,\sqrt{i z^2}}{z} \cos(\pi\nu) - \sin(\pi\nu)\right) \left(\sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} \frac{2^{-2k} \left(\frac{1}{2}-\nu\right)_{2k} \left(\nu+\frac{1}{2}\right)_{2k}}{(2k)!} \left(-\frac{i}{z^2}\right)^k + O\left(\frac{1}{z^{2\left\lfloor \frac{n}{2} \right\rfloor+2}}\right)\right)\Bigg)\Bigg) +$$

$$e^{\frac{z}{\sqrt{2}}} \Bigg(e^{\frac{iz}{\sqrt{2}}+\frac{3i\pi\nu}{2}} \left((-1)^{3/4} z\right)^{-\nu-\frac{1}{2}} \left(\sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} \frac{2^{-2k} \left(\frac{1}{2}-\nu\right)_{2k} \left(\nu+\frac{1}{2}\right)_{2k}}{(2k)!} \left(-\frac{i}{z^2}\right)^k + O\left(\frac{1}{z^{2\left\lfloor \frac{n}{2} \right\rfloor+2}}\right)\right) - e^{-\frac{iz}{\sqrt{2}}}$$

$$\left(-\sqrt[4]{-1}\, z\right)^{-\nu-\frac{1}{2}} \left(\frac{(-1)^{3/4}\sqrt{-i z^2}}{z} \cos(\pi\nu) + \sin(\pi\nu)\right) \left(\sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} \frac{2^{-2k} \left(\frac{1}{2}-\nu\right)_{2k} \left(\nu+\frac{1}{2}\right)_{2k}}{(2k)!} \left(\frac{i}{z^2}\right)^k + O\left(\frac{1}{z^{2\left\lfloor \frac{n}{2} \right\rfloor+2}}\right)\right)\Bigg)\Bigg) +$$

$$\frac{(-1)^{3/4}}{z} \Bigg(e^{-\frac{z}{\sqrt{2}}} \Bigg(e^{\frac{iz}{\sqrt{2}}}\, i \left(-\sqrt[4]{-1}\, z\right)^{-\nu-\frac{1}{2}} \left(\sum_{k=0}^{\left\lfloor \frac{n-1}{2} \right\rfloor} \frac{\left(2^{-2k-1} \left(\frac{1}{2}-\nu\right)_{2k+1} \left(\nu+\frac{1}{2}\right)_{2k+1}\right)}{(2k+1)!} \left(\frac{i}{z^2}\right)^k + O\left(\frac{1}{z^{2\left\lfloor \frac{n-1}{2} \right\rfloor+2}}\right)\right) +$$

$$e^{\frac{3i\pi\nu}{2}-\frac{iz}{\sqrt{2}}} \left((-1)^{3/4} z\right)^{-\nu-\frac{1}{2}} \left(\frac{\sqrt[4]{-1}\,\sqrt{i z^2}}{z} \cos(\pi\nu) - \sin(\pi\nu)\right)$$

$$\left(\sum_{k=0}^{\left\lfloor \frac{n-1}{2} \right\rfloor} \frac{\left(2^{-2k-1} \left(\frac{1}{2}-\nu\right)_{2k+1} \left(\nu+\frac{1}{2}\right)_{2k+1}\right)}{(2k+1)!} \left(-\frac{i}{z^2}\right)^k + O\left(\frac{1}{z^{2\left\lfloor \frac{n-1}{2} \right\rfloor+2}}\right)\right)\Bigg)\Bigg) +$$

$$e^{\frac{z}{\sqrt{2}}} \Bigg(i\, e^{-\frac{iz}{\sqrt{2}}} \left(-\sqrt[4]{-1}\, z\right)^{-\nu-\frac{1}{2}} \left(\frac{(-1)^{3/4}\sqrt{-i z^2}}{z} \cos(\pi\nu) + \sin(\pi\nu)\right)$$

$$\left(\sum_{k=0}^{\left\lfloor \frac{n-1}{2} \right\rfloor} \frac{\left(2^{-2k-1} \left(\frac{1}{2}-\nu\right)_{2k+1} \left(\nu+\frac{1}{2}\right)_{2k+1}\right)}{(2k+1)!} \left(\frac{i}{z^2}\right)^k + O\left(\frac{1}{z^{2\left\lfloor \frac{n-1}{2} \right\rfloor+2}}\right)\right) - e^{\frac{iz}{\sqrt{2}}+\frac{3i\pi\nu}{2}} \left((-1)^{3/4} z\right)^{-\nu-\frac{1}{2}}$$

$$\left(\sum_{k=0}^{\left\lfloor \frac{n-1}{2} \right\rfloor} \frac{\left(2^{-2k-1} \left(\frac{1}{2}-\nu\right)_{2k+1} \left(\nu+\frac{1}{2}\right)_{2k+1}\right)}{(2k+1)!} \left(-\frac{i}{z^2}\right)^k + O\left(\frac{1}{z^{2\left\lfloor \frac{n-1}{2} \right\rfloor+2}}\right)\right)\Bigg)\Bigg)\Bigg) \;/;\; (|z| \to \infty) \land n \in \mathbb{N}$$

03.18.06.0052.01

$$\mathrm{ber}_{\nu}(z) \propto \frac{(1+i)\, e^{-\frac{1}{4} i \pi \nu} z^{\nu}}{4\sqrt{\pi}} \Bigg(\Bigg(e^{\frac{3 i \pi \nu}{2}} \left((-1)^{3/4} z\right)^{-\nu-\frac{1}{2}} \left(e^{-\sqrt[4]{-1}\, z} \left(\frac{\sqrt[4]{-1}\, \sqrt{i z^2}\, \cos(\pi \nu)}{z} - \sin(\pi \nu) \right) + e^{\sqrt[4]{-1}\, z} \right)$$

$$\left(\sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} \frac{\left(\frac{1}{2}-\nu\right)_{2k} \left(\nu+\frac{1}{2}\right)_{2k}}{(2k)!} \left(-\frac{i}{4 z^2}\right)^k + O\left(\frac{1}{z^{2 \left\lfloor \frac{n}{2} \right\rfloor + 2}} \right) \right) + \left(-\sqrt[4]{-1}\, z\right)^{-\nu-\frac{1}{2}}$$

$$\left(e^{(-1)^{3/4} z} - e^{-(-1)^{3/4} z} \left(\frac{(-1)^{3/4} \sqrt{-i z^2}\, \cos(\pi \nu)}{z} + \sin(\pi \nu) \right) \right) \left(\sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} \frac{\left(\frac{1}{2}-\nu\right)_{2k} \left(\nu+\frac{1}{2}\right)_{2k}}{(2k)!} \left(\frac{i}{4 z^2}\right)^k + O\left(\frac{1}{z^{2 \left\lfloor \frac{n}{2} \right\rfloor + 2}} \right) \right) \Bigg) +$$

$$\frac{(-1)^{3/4}}{2 z} \Bigg(e^{\frac{3 i \pi \nu}{2}} \left((-1)^{3/4} z\right)^{-\nu-\frac{1}{2}} \left(e^{-\sqrt[4]{-1}\, z} \left(\frac{\sqrt[4]{-1}\, \sqrt{i z^2}\, \cos(\pi \nu)}{z} - \sin(\pi \nu) \right) - e^{\sqrt[4]{-1}\, z} \right)$$

$$\left(\sum_{k=0}^{\left\lfloor \frac{n-1}{2} \right\rfloor} \frac{\left(\frac{1}{2}-\nu\right)_{2k+1} \left(\nu+\frac{1}{2}\right)_{2k+1}}{(2k+1)!} \left(-\frac{i}{4 z^2}\right)^k + O\left(\frac{1}{z^{2 \left\lfloor \frac{n-1}{2} \right\rfloor + 2}} \right) \right) +$$

$$\left(-\sqrt[4]{-1}\, z\right)^{-\nu-\frac{1}{2}} \left(i\, e^{(-1)^{3/4} z} + e^{-(-1)^{3/4} z} \left(i \sin(\pi \nu) - \frac{\sqrt[4]{-1}\, \sqrt{-i z^2}\, \cos(\pi \nu)}{z} \right) \right)$$

$$\left(\sum_{k=0}^{\left\lfloor \frac{n-1}{2} \right\rfloor} \frac{\left(\frac{1}{2}-\nu\right)_{2k+1} \left(\nu+\frac{1}{2}\right)_{2k+1}}{(2k+1)!} \left(\frac{i}{4 z^2}\right)^k + O\left(\frac{1}{z^{2 \left\lfloor \frac{n-1}{2} \right\rfloor + 2}} \right) \right) \Bigg) \Bigg) \;/;\; (|z| \to \infty) \land n \in \mathbb{N}$$

03.18.06.0053.01

$$\operatorname{ber}_\nu(z) \propto \frac{\sqrt[4]{-1}\, e^{-\frac{1}{4}(i\pi\nu)} z^\nu}{2\sqrt{2\pi}} \Bigg(\Bigg(\Bigg(e^{\frac{z}{\sqrt{2}}}\Bigg(e^{\frac{iz}{\sqrt{2}}+\frac{3i\pi\nu}{2}} \left((-1)^{3/4} z\right)^{-\nu-\frac{1}{2}} {}_4F_1\left(\frac{1}{4}-\frac{\nu}{2}, \frac{3}{4}-\frac{\nu}{2}, \frac{\nu}{2}+\frac{1}{4}, \frac{\nu}{2}+\frac{3}{4}; \frac{1}{2}; -\frac{i}{z^2}\right) - e^{-\frac{iz}{\sqrt{2}}} \left(-\sqrt[4]{-1}\, z\right)^{-\nu-\frac{1}{2}}$$

$$\left(\frac{\left((-1)^{3/4}\sqrt{-i z^2}\right)\cos(\pi\nu)}{z} + \sin(\pi\nu)\right) {}_4F_1\left(\frac{1}{4}-\frac{\nu}{2}, \frac{3}{4}-\frac{\nu}{2}, \frac{\nu}{2}+\frac{1}{4}, \frac{\nu}{2}+\frac{3}{4}; \frac{1}{2}; \frac{i}{z^2}\right)\Bigg) +$$

$$e^{-\frac{z}{\sqrt{2}}}\Bigg(e^{\frac{iz}{\sqrt{2}}}\left(-\sqrt[4]{-1}\, z\right)^{-\nu-\frac{1}{2}} {}_4F_1\left(\frac{1}{4}-\frac{\nu}{2}, \frac{3}{4}-\frac{\nu}{2}, \frac{\nu}{2}+\frac{1}{4}, \frac{\nu}{2}+\frac{3}{4}; \frac{1}{2}; \frac{i}{z^2}\right) + e^{\frac{3i\pi\nu}{2}-\frac{iz}{\sqrt{2}}} \left((-1)^{3/4} z\right)^{-\nu-\frac{1}{2}}$$

$$\left(\frac{\sqrt[4]{-1}\sqrt{i z^2}}{z}\cos(\pi\nu) - \sin(\pi\nu)\right) {}_4F_1\left(\frac{1}{4}-\frac{\nu}{2}, \frac{3}{4}-\frac{\nu}{2}, \frac{\nu}{2}+\frac{1}{4}, \frac{\nu}{2}+\frac{3}{4}; \frac{1}{2}; -\frac{i}{z^2}\right)\Bigg)\Bigg) + \frac{(-1)^{3/4}\left(1-4\nu^2\right)}{8z}$$

$$\Bigg(e^{\frac{z}{\sqrt{2}}}\Bigg(i\, e^{-\frac{iz}{\sqrt{2}}}\left(-\sqrt[4]{-1}\, z\right)^{-\nu-\frac{1}{2}}\left(\frac{(-1)^{3/4}\sqrt{-i z^2}}{z}\cos(\pi\nu) + \sin(\pi\nu)\right) {}_4F_1\left(\frac{3}{4}-\frac{\nu}{2}, \frac{5}{4}-\frac{\nu}{2}, \frac{\nu}{2}+\frac{3}{4}, \frac{\nu}{2}+\frac{5}{4}; \frac{3}{2}; \frac{i}{z^2}\right) -$$

$$e^{\frac{iz}{\sqrt{2}}+\frac{3i\pi\nu}{2}}\left((-1)^{3/4} z\right)^{-\nu-\frac{1}{2}} {}_4F_1\left(\frac{3}{4}-\frac{\nu}{2}, \frac{5}{4}-\frac{\nu}{2}, \frac{\nu}{2}+\frac{3}{4}, \frac{\nu}{2}+\frac{5}{4}; \frac{3}{2}; -\frac{i}{z^2}\right)\Bigg) +$$

$$e^{-\frac{z}{\sqrt{2}}}\Bigg(e^{\frac{iz}{\sqrt{2}}}\, i\left(-\sqrt[4]{-1}\, z\right)^{-\nu-\frac{1}{2}} {}_4F_1\left(\frac{3}{4}-\frac{\nu}{2}, \frac{5}{4}-\frac{\nu}{2}, \frac{\nu}{2}+\frac{3}{4}, \frac{\nu}{2}+\frac{5}{4}; \frac{3}{2}; \frac{i}{z^2}\right) +$$

$$e^{\frac{3i\pi\nu}{2}-\frac{iz}{\sqrt{2}}}\left((-1)^{3/4} z\right)^{-\nu-\frac{1}{2}}\left(\frac{\sqrt[4]{-1}\sqrt{i z^2}}{z}\cos(\pi\nu) - \sin(\pi\nu)\right)$$

$${}_4F_1\left(\frac{3}{4}-\frac{\nu}{2}, \frac{5}{4}-\frac{\nu}{2}, \frac{\nu}{2}+\frac{3}{4}, \frac{\nu}{2}+\frac{5}{4}; \frac{3}{2}; -\frac{i}{z^2}\right)\Bigg)\Bigg)\Bigg) /; (|z| \to \infty)$$

03.18.06.0054.01

$$\operatorname{ber}_\nu(z) \propto \frac{\sqrt[4]{-1}\, e^{-\frac{1}{4}(i\pi\nu)} z^\nu}{2\sqrt{2\pi}} \Bigg(\Bigg(e^{\frac{z}{\sqrt{2}}}\Bigg(e^{\frac{iz}{\sqrt{2}}+\frac{3i\pi\nu}{2}} \left((-1)^{3/4} z\right)^{-\nu-\frac{1}{2}} \left(1+O\left(\frac{1}{z^2}\right)\right) - e^{-\frac{iz}{\sqrt{2}}} \left(-\sqrt[4]{-1}\, z\right)^{-\nu-\frac{1}{2}} \left(\frac{\left((-1)^{3/4}\sqrt{-i z^2}\right)\cos(\pi\nu)}{z} + \sin(\pi\nu)\right)$$

$$\left(1+O\left(\frac{1}{z^2}\right)\right)\Bigg) + e^{-\frac{z}{\sqrt{2}}}\Bigg(e^{\frac{iz}{\sqrt{2}}} \left(-\sqrt[4]{-1}\, z\right)^{-\nu-\frac{1}{2}} \left(1+O\left(\frac{1}{z^2}\right)\right) +$$

$$e^{\frac{3i\pi\nu}{2}-\frac{iz}{\sqrt{2}}} \left((-1)^{3/4} z\right)^{-\nu-\frac{1}{2}} \left(\frac{\sqrt[4]{-1}\,\sqrt{i z^2}}{z}\cos(\pi\nu) - \sin(\pi\nu)\right)\left(1+O\left(\frac{1}{z^2}\right)\right)\Bigg)\Bigg)\Bigg) \;/;\; (|z| \to \infty)$$

03.18.06.0055.01

$$\operatorname{ber}_\nu(z) \propto \begin{cases} \dfrac{\sqrt[8]{-1}\, e^{-\sqrt[4]{-1}\, z-\frac{i\pi\nu}{2}} \left(e^{\sqrt{2}\, z}+\sqrt[4]{-1}\, e^{2i\pi\nu}-(-1)^{3/4} e^{2\sqrt[4]{-1}\, z+i\pi\nu}+i\, e^{\sqrt{2}\, iz+i\pi\nu}\right)}{2\sqrt{2\pi}\,\sqrt{z}} & -\frac{1}{4} < \frac{\arg(z)}{\pi} \le \frac{1}{4} \\ \dfrac{\sqrt[8]{-1}\, e^{-\sqrt[4]{-1}\, z-\frac{i\pi\nu}{2}} \left(e^{\sqrt{2}\, z}+\sqrt[4]{-1}\, e^{2i\pi\nu}+i\, e^{\sqrt{2}\, iz+i\pi\nu}+(-1)^{3/4} e^{2\sqrt[4]{-1}\, z+3i\pi\nu}\right)}{2\sqrt{2\pi}\,\sqrt{z}} & \frac{1}{4} < \frac{\arg(z)}{\pi} \le \frac{3}{4} \\ \dfrac{\sqrt[8]{-1}\, e^{\frac{i\pi\nu}{2}-\sqrt[4]{-1}\, z} \left(i\, e^{i\sqrt{2}\, z}+\sqrt[4]{-1}\, e^{i\pi\nu}-e^{\sqrt{2}\, z+i\pi\nu}+(-1)^{3/4} e^{2\sqrt[4]{-1}\, z+2i\pi\nu}\right)}{2\sqrt{2\pi}\,\sqrt{z}} & \frac{\arg(z)}{\pi} > \frac{3}{4} \\ \dfrac{\sqrt[8]{-1}\, e^{-\sqrt[4]{-1}\, z-\frac{3i\pi\nu}{2}} \left(-i\, e^{i\sqrt{2}\, z}+\sqrt[4]{-1}\, e^{3i\pi\nu}+e^{\sqrt{2}\, z+i\pi\nu}-(-1)^{3/4} e^{2\sqrt[4]{-1}\, z+2i\pi\nu}\right)}{2\sqrt{2\pi}\,\sqrt{z}} & -\frac{3}{4} < \frac{\arg(z)}{\pi} \le -\frac{1}{4} \\ \dfrac{\sqrt[8]{-1}\, e^{-\sqrt[4]{-1}\, z-\frac{3i\pi\nu}{2}} \left(-i\, e^{i\sqrt{2}\, z}-\sqrt[4]{-1}\, e^{i\pi\nu}+e^{\sqrt{2}\, z+i\pi\nu}-(-1)^{3/4} e^{2\sqrt[4]{-1}\, z+2i\pi\nu}\right)}{2\sqrt{2\pi}\,\sqrt{z}} & \text{True} \end{cases} \;/;\; (|z| \to \infty)$$

Residue representations

03.18.06.0056.01

$$\operatorname{ber}_\nu(z) = \pi \sum_{j=0}^{\infty} \operatorname{res}_s\left(\frac{\left(\frac{z}{4}\right)^{-4s} \Gamma\left(s+\frac{\nu+2}{4}\right)}{\Gamma\left(s+\nu+\frac{1}{2}\right)\Gamma\left(-s-\nu+\frac{1}{2}\right)\Gamma\left(\frac{\nu+2}{4}-s\right)\Gamma\left(-s+\frac{\nu}{4}+1\right)}\,\Gamma\left(s+\frac{\nu}{4}\right)\right)\left(-j-\frac{\nu}{4}\right) +$$

$$\pi \sum_{j=0}^{\infty} \operatorname{res}_s\left(\frac{\left(\frac{z}{4}\right)^{-4s} \Gamma\left(s+\frac{\nu}{4}\right)}{\Gamma\left(s+\nu+\frac{1}{2}\right)\Gamma\left(-s-\nu+\frac{1}{2}\right)\Gamma\left(\frac{\nu+2}{4}-s\right)\Gamma\left(-s+\frac{\nu}{4}+1\right)}\,\Gamma\left(s+\frac{\nu+2}{4}\right)\right)\left(-j-\frac{\nu+2}{4}\right)$$

Integral representations

On the real axis

Of the direct function

03.18.07.0001.01

$$\operatorname{ber}_\nu(z) = \frac{1}{\Gamma\left(\nu+\frac{1}{2}\right)\sqrt{\pi}}\left(\frac{z}{2}\right)^\nu \int_0^\pi \left(\cos\left(\frac{3\pi\nu}{4}\right)\cos\left(\frac{z\cos(t)}{\sqrt{2}}\right)\cosh\left(\frac{z\cos(t)}{\sqrt{2}}\right) - \sin\left(\frac{3\pi\nu}{4}\right)\sin\left(\frac{z\cos(t)}{\sqrt{2}}\right)\sinh\left(\frac{z\cos(t)}{\sqrt{2}}\right)\right)\sin^{2\nu}(t)\,dt\ /;\ \operatorname{Re}(\nu) > -\frac{1}{2}$$

03.18.07.0002.01

$$\operatorname{ber}_\nu(z) = \frac{2^{1-\nu} z^\nu}{\sqrt{\pi}\,\Gamma\left(\nu+\frac{1}{2}\right)} \int_0^1 \left(1-t^2\right)^{\nu-\frac{1}{2}} \left(\cos\left(\frac{3\pi\nu}{4}\right)\cos\left(\frac{tz}{\sqrt{2}}\right)\cosh\left(\frac{tz}{\sqrt{2}}\right) - \sin\left(\frac{tz}{\sqrt{2}}\right)\sin\left(\frac{3\pi\nu}{4}\right)\sinh\left(\frac{tz}{\sqrt{2}}\right)\right)dt\ /;\ \operatorname{Re}(\nu) > -\frac{1}{2}$$

03.18.07.0003.01

$$\operatorname{ber}_\nu(z) = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\cos\left(\frac{3\pi\nu}{4}\right)\cos\left(\frac{z\sin(t)}{\sqrt{2}}\right)\cosh\left(\frac{z\sin(t)}{\sqrt{2}}\right) - \sin\left(\frac{3\pi\nu}{4}\right)\sin\left(\frac{z\sin(t)}{\sqrt{2}}\right)\sinh\left(\frac{z\sin(t)}{\sqrt{2}}\right)\right)\cos^{2\nu}(t)\,dt\ /;\ \operatorname{Re}(\nu) > -\frac{1}{2}$$

03.18.07.0004.01

$$\operatorname{ber}_n(z) = \frac{1}{\pi}\int_0^\pi e^{-\frac{z\cos(t)}{\sqrt{2}}}\left(\cos\left(\frac{n\pi}{2}\right)\cos\left(\frac{z\cos(t)}{\sqrt{2}}\right) - \sin\left(\frac{n\pi}{2}\right)\sin\left(\frac{z\cos(t)}{\sqrt{2}}\right)\right)\cos(nt)\,dt\ /;\ n\in\mathbb{N}^+$$

03.18.07.0005.01

$$\operatorname{ber}_n(z) = \frac{1}{\pi}\int_0^\pi \cos\left(nt + \frac{z\sin(t)}{\sqrt{2}}\right)\cosh\left(\frac{z\sin(t)}{\sqrt{2}}\right)dt\ /;\ n\in\mathbb{Z}$$

03.18.07.0006.01

$$\operatorname{ber}_\nu(z) = \frac{1}{2\pi i}\left(\frac{z}{2}\right)^\nu \int_{\gamma-i\infty}^{i\infty+\gamma} e^{\frac{z^2}{4\sqrt{2}\,t}+t}\cos\left(\frac{3\pi\nu}{4} - \frac{z^2}{4\sqrt{2}\,t}\right)t^{-\nu-1}\,dt\ /;\ \gamma > 0 \land \operatorname{Re}(\nu) > 0$$

Contour integral representations

03.18.07.0007.01

$$\operatorname{ber}_\nu(z) = \frac{1}{2i}\int_{\mathcal{L}} \frac{\Gamma\left(s+\frac{\nu}{4}\right)\Gamma\left(s+\frac{\nu+2}{4}\right)}{\Gamma\left(s+\nu+\frac{1}{2}\right)\Gamma\left(\frac{1}{2}-s-\nu\right)\Gamma\left(\frac{\nu+2}{4}-s\right)\Gamma\left(1-s+\frac{\nu}{4}\right)}\left(\frac{z}{4}\right)^{-4s} ds$$

Limit representations

03.18.09.0001.01

$$\operatorname{ber}_\nu(z) = 2^{-\nu-1} z^\nu \lim_{n\to\infty}\left(\frac{1}{n^\nu}\left(e^{-\frac{3i\pi\nu}{4}} P_n^{(\nu,b)}\left(\cos\left(\frac{(1+i)z}{\sqrt{2}\,n}\right)\right) + e^{\frac{3i\pi\nu}{4}} P_n^{(\nu,b)}\left(\cosh\left(\frac{(1+i)z}{\sqrt{2}\,n}\right)\right)\right)\right)$$

03.18.09.0002.01

$$\operatorname{ber}_\nu(z) = 2^{-\nu-1} z^\nu \left(\lim_{n\to\infty}\frac{1}{n^\nu}\left(e^{-\frac{3i\pi\nu}{4}} L_n^\nu\left(\frac{iz^2}{4n}\right) + e^{\frac{3i\pi\nu}{4}} L_n^\nu\left(-\frac{iz^2}{4n}\right)\right)\right)$$

03.18.09.0003.01

$$\operatorname{ber}_\nu(z) = \frac{1}{\Gamma(\nu+1)}\left(\frac{z}{2}\right)^\nu\left(\lim_{a\to\infty}\left(\cos\left(\frac{3\pi\nu}{4}\right)\left(\frac{z}{2}\right)^\nu {}_1F_3\left(a;\ \frac{1}{2},\ \frac{\nu+1}{2},\ \frac{\nu}{2}+1;\ -\frac{z^4}{256a}\right) - \frac{\sin\left(\frac{3\pi\nu}{4}\right)z^2}{4(\nu+1)}\,{}_1F_3\left(a;\ \frac{3}{2},\ \frac{\nu+3}{2},\ \frac{\nu}{2}+1;\ -\frac{z^4}{256a}\right)\right)\right)$$

Generating functions

03.18.11.0001.01

$$\sum_{k=-\infty}^{\infty} t^k \operatorname{ber}_k(x) = e^{-\frac{\left(t-\frac{1}{t}\right)x}{2\sqrt{2}}} \cos\left(\frac{\left(t-\frac{1}{t}\right)x}{2\sqrt{2}}\right)$$

Differential equations

Ordinary linear differential equations and wronskians

For the direct function itself

03.18.13.0001.01

$$w^{(4)}(z)\,z^4 + 2\,w^{(3)}(z)\,z^3 - \left(2\,\nu^2+1\right)w''(z)\,z^2 + \left(2\,\nu^2+1\right)w'(z)\,z + \left(z^4+\nu^4-4\,\nu^2\right)w(z) = 0 \,/;\, w(z) = \operatorname{ber}_\nu(z)\,c_1 + \operatorname{bei}_\nu(z)\,c_2 + \operatorname{ker}_\nu(z)\,c_3 + \operatorname{kei}_\nu(z)\,c_4$$

03.18.13.0002.01

$$W_z(\operatorname{ber}_\nu(z), \operatorname{bei}_\nu(z), \operatorname{ker}_\nu(z), \operatorname{kei}_\nu(z)) = -\frac{1}{z^2}$$

03.18.13.0003.01

$$g(z)^4\, g'(z)^3\, w^{(4)}(z) + 2\,g(z)^3\left(g'(z)^2 - 3\,g(z)\,g''(z)\right)g'(z)^2\,w^{(3)}(z) - g(z)^2\left(\left(2\,\nu^2+1\right)g'(z)^4 + 6\,g(z)\,g''(z)\,g'(z)^2 + 4\,g(z)^2\,g^{(3)}(z)\,g'(z) - 15\,g(z)^2\,g''(z)^2\right)g'(z)\,w''(z) + g(z)\left(\left(2\,\nu^2+1\right)g'(z)^6 + \left(2\,\nu^2+1\right)g(z)\,g''(z)\,g'(z)^4 - 2\,g(z)^2\,g^{(3)}(z)\,g'(z)^3 + g(z)^2\left(6\,g''(z)^2 - g(z)\,g^{(4)}(z)\right)g'(z)^2 + 10\,g(z)^3\,g''(z)\,g^{(3)}(z)\,g'(z) - 15\,g(z)^3\,g''(z)^3\right)w'(z) + \left(\nu^4 - 4\,\nu^2 + g(z)^4\right)g'(z)^7\,w(z) = 0 \,/;\, w(z) = c_1\operatorname{ber}_\nu(g(z)) + c_2\operatorname{bei}_\nu(g(z)) + c_3\operatorname{ker}_\nu(g(z)) + c_4\operatorname{kei}_\nu(g(z))$$

03.18.13.0004.01

$$W_z(\operatorname{ber}_\nu(g(z)), \operatorname{bei}_\nu(g(z)), \operatorname{ker}_\nu(g(z)), \operatorname{kei}_\nu(g(z))) = -\frac{g'(z)^6}{g(z)^2}$$

03.18.13.0005.01

$$g(z)^4\,g'(z)^3\,h(z)^4\,w^{(4)}(z) + 2\,g(z)^3\,g'(z)^2\left(h(z)\left(g'(z)^2 - 3\,g(z)\,g''(z)\right) - 2\,g(z)\,g'(z)\,h'(z)\right)h(z)^3\,w^{(3)}(z) + g(z)^2\,g'(z)\left(-\left(\left(2\,\nu^2+1\right)g'(z)^4 + 6\,g(z)\,g''(z)\,g'(z)^2 + 4\,g(z)^2\,g^{(3)}(z)\,g'(z) - 15\,g(z)^2\,g''(z)^2\right)h(z)^2 - 6\,g(z)\,g'(z)\left(h'(z)\,g'(z)^2 + g(z)\,h''(z)\,g'(z) - 3\,g(z)\,h'(z)\,g''(z)\right)h(z) + 12\,g(z)^2\,g'(z)^2\,h'(z)^2\right)h(z)^2\,w''(z) + g(z)\left(\left(\left(\left(2\,\nu^2+1\right)g'(z)^6 + \left(2\,\nu^2+1\right)g(z)\,g''(z)\,g'(z)^4 - 2\,g(z)^2\,g^{(3)}(z)\,g'(z)^3 + g(z)^2\left(6\,g''(z)^2 - g(z)\,g^{(4)}(z)\right)g'(z)^2 + 10\,g(z)^3\,g''(z)\,g^{(3)}(z)\,g'(z) - 15\,g(z)^3\,g''(z)^3\right)h(z)^3 + 2\,g(z)\,g'(z)\left(\left(2\,\nu^2+1\right)h'(z)\,g'(z)^4 - 3\,g(z)\,h''(z)\,g'(z)^3 - 2\,g(z)\left(g(z)\,h^{(3)}(z) - 3\,h'(z)\,g''(z)\right)g'(z)^2 + g(z)^2\left(9\,g''(z)\,h''(z) + 4\,h'(z)\,g^{(3)}(z)\right)g'(z) - 15\,g(z)^2\,h'(z)\,g''(z)^2\right)h(z)^2 + 12\,g(z)^2\,g'(z)^2\,h'(z)\left(h'(z)\,g'(z)^2 + 2\,g(z)\,h''(z)\,g'(z) - 3\,g(z)\,h'(z)\,g''(z)\right)h(z) - 24\,g(z)^3\,g'(z)^3\,h'(z)^3\right)h(z)\,w'(z) + \left(\left(\nu^4 - 4\,\nu^2 + g(z)^4\right)h(z)^4\,g'(z)^7 + g(z)^4\left(24\,h'(z)^4 - 36\,h(z)\,h''(z)\,h'(z)^2 + 8\,h(z)^2\,h^{(3)}(z)\,h'(z) + h(z)^2\left(6\,h''(z)^2 - h(z)\,h^{(4)}(z)\right)\right)g'(z)^3 - 2\,g(z)^3\,h(z)\left(g'(z)^2 - 3\,g(z)\,g''(z)\right)\left(6\,h'(z)^3 - 6\,h(z)\,h''(z)\,h'(z) + h(z)^2\,h^{(3)}(z)\right)g'(z)^2 + g(z)^2\,h(z)^2\left(h(z)\,h''(z) - 2\,h'(z)^2\right)\left(\left(2\,\nu^2+1\right)g'(z)^4 + 6\,g(z)\,g''(z)\,g'(z)^2 + 4\,g(z)^2\,g^{(3)}(z)\,g'(z) - 15\,g(z)^2\,g''(z)^2\right)g'(z) - g(z)\,h(z)^3\,h'(z)\left(\left(2\,\nu^2+1\right)g'(z)^6 + \left(2\,\nu^2+1\right)g(z)\,g''(z)\,g'(z)^4 - 2\,g(z)^2\,g^{(3)}(z)\,g'(z)^3 + g(z)^2\left(6\,g''(z)^2 - g(z)\,g^{(4)}(z)\right)g'(z)^2 + 10\,g(z)^3\,g''(z)\,g^{(3)}(z)\,g'(z) - 15\,g(z)^3\,g''(z)^3\right)\right)w(z) = 0 \,/;\, w(z) = c_1\,h(z)\operatorname{ber}_\nu(g(z)) + c_2\,h(z)\operatorname{bei}_\nu(g(z)) + c_3\,h(z)\operatorname{ker}_\nu(g(z)) + c_4\,h(z)\operatorname{kei}_\nu(g(z))$$

03.18.13.0006.01

$$W_z(h(z)\,\mathrm{ber}_\nu(g(z)),\, h(z)\,\mathrm{bei}_\nu(g(z)),\, h(z)\,\mathrm{ker}_\nu(g(z)),\, h(z)\,\mathrm{kei}_\nu(g(z))) = -\frac{h(z)^4\, g'(z)^6}{g(z)^2}$$

03.18.13.0007.01

$$z^4\, w^{(4)}(z) + (6-4r-4s)\, z^3\, w^{(3)}(z) + \left(7-2\left(\nu^2-2\right)r^2+12\,(s-1)\,r+6\,(s-2)\,s\right)z^2\, w''(z) + (2r+2s-1) \left(2r^2\nu^2-2\,(s-1)\,s+r\,(2-4s)-1\right)z\,w'(z) + \left(\left(a^4 z^{4r}+\nu^4-4\nu^2\right)r^4-4s\,\nu^2\,r^3-2s^2\left(\nu^2-2\right)r^2+4s^3\,r+s^4\right)w(z)=0\;/;\; w(z) = c_1\, z^s\,\mathrm{ber}_\nu(a\,z^r) + c_2\, z^s\,\mathrm{bei}_\nu(a\,z^r) + c_3\, z^s\,\mathrm{ker}_\nu(a\,z^r) + c_4\, z^s\,\mathrm{kei}_\nu(a\,z^r)$$

03.18.13.0008.01

$$W_z(z^s\,\mathrm{ber}_\nu(a\,z^r),\, z^s\,\mathrm{bei}_\nu(a\,z^r),\, z^s\,\mathrm{ker}_\nu(a\,z^r),\, z^s\,\mathrm{kei}_\nu(a\,z^r)) = -a^4\, r^6\, z^{4r+4s-6}$$

03.18.13.0009.01

$$w^{(4)}(z) - 4\,(\log(r)+\log(s))\, w^{(3)}(z) + 2\left(-\left(\nu^2-2\right)\log^2(r)+6\log(s)\log(r)+3\log^2(s)\right)w''(z) + 4\,(\log(r)+\log(s))\left(\nu^2\log^2(r)-2\log(s)\log(r)-\log^2(s)\right)w'(z) + \left(\left(a^4 r^{4z}+\nu^4-4\nu^2\right)\log^4(r)-4\nu^2\log(s)\log^3(r)-2\left(\nu^2-2\right)\log^2(s)\log^2(r)+4\log^3(s)\log(r)+\log^4(s)\right)w(z)=0\;/;\; w(z) = c_1\, s^z\,\mathrm{ber}_\nu(a\,r^z) + c_2\, s^z\,\mathrm{bei}_\nu(a\,r^z) + c_3\, s^z\,\mathrm{ker}_\nu(a\,r^z) + c_4\, s^z\,\mathrm{kei}_\nu(a\,r^z)$$

03.18.13.0010.01

$$W_z(s^z\,\mathrm{ber}_\nu(a\,r^z),\, s^z\,\mathrm{bei}_\nu(a\,r^z),\, s^z\,\mathrm{ker}_\nu(a\,r^z),\, s^z\,\mathrm{kei}_\nu(a\,r^z)) = -a^4\, r^{4z}\, s^{4z}\log^6(r)$$

Transformations

Transformations and argument simplifications

Argument involving basic arithmetic operations

03.18.16.0001.01

$$\mathrm{ber}_\nu(-z) = (-z)^\nu\, z^{-\nu}\,\mathrm{ber}_\nu(z)$$

03.18.16.0002.01

$$\mathrm{ber}_\nu(i\,z) = (i\,z)^\nu\, z^{-\nu}\left(\cos\left(\frac{3\pi\nu}{2}\right)\mathrm{ber}_\nu(z)+\sin\left(\frac{3\pi\nu}{2}\right)\mathrm{bei}_\nu(z)\right)$$

03.18.16.0003.01

$$\mathrm{ber}_\nu(-i\,z) = (-i\,z)^\nu\, z^{-\nu}\left(\cos\left(\frac{3\pi\nu}{2}\right)\mathrm{ber}_\nu(z)+\sin\left(\frac{3\pi\nu}{2}\right)\mathrm{bei}_\nu(z)\right)$$

03.18.16.0004.01

$$\mathrm{ber}_\nu\left(\frac{1}{\sqrt[4]{-1}}\,z\right) = \left(\sqrt[4]{-1}\,z\right)^{-\nu}\left(-(-1)^{3/4}\,z\right)^\nu\left(\cos\left(\frac{3\pi\nu}{2}\right)\mathrm{ber}_\nu\left(\sqrt[4]{-1}\,z\right)+\sin\left(\frac{3\pi\nu}{2}\right)\mathrm{bei}_\nu\left(\sqrt[4]{-1}\,z\right)\right)$$

03.18.16.0005.01

$$\mathrm{ber}_\nu\left((-1)^{-3/4}\,z\right) = \left((-1)^{-3/4}\,z\right)^\nu\left(\sqrt[4]{-1}\,z\right)^{-\nu}\mathrm{ber}_\nu\left(\sqrt[4]{-1}\,z\right)$$

03.18.16.0006.01

$$\mathrm{ber}_\nu\left((-1)^{3/4}\,z\right) = \left(\sqrt[4]{-1}\,z\right)^{-\nu}\left((-1)^{3/4}\,z\right)^\nu\left(\cos\left(\frac{3\pi\nu}{2}\right)\mathrm{ber}_\nu\left(\sqrt[4]{-1}\,z\right)+\sin\left(\frac{3\pi\nu}{2}\right)\mathrm{bei}_\nu\left(\sqrt[4]{-1}\,z\right)\right)$$

03.18.16.0007.01

$$\mathrm{ber}_\nu\left(\sqrt[4]{z^4}\right) = \frac{1}{2}\, z^{-\nu-2}\left(z^4\right)^{\nu/4}\left(\left(z^2-\sqrt{z^4}\right)\sin\left(\frac{3\pi\nu}{2}\right)\mathrm{bei}_\nu(z) + 2\left(z^2\cos^2\left(\frac{3\pi\nu}{4}\right)+\sqrt{z^4}\,\sin^2\left(\frac{3\pi\nu}{4}\right)\right)\mathrm{ber}_\nu(z)\right)$$

03.18.16.0008.01

$$\operatorname{ber}_{-\nu}(z) = \cos(\pi \nu) \operatorname{ber}_\nu(z) + \operatorname{bei}_\nu(z) \sin(\pi \nu) + \frac{2 \sin(\pi \nu)}{\pi} \operatorname{ker}_\nu(z)$$

Addition formulas

03.18.16.0009.01

$$\operatorname{ber}_\nu(z_1 - z_2) = \sum_{k=-\infty}^{\infty} (\operatorname{ber}_{k+\nu}(z_1) \operatorname{ber}_k(z_2) - \operatorname{bei}_{k+\nu}(z_1) \operatorname{bei}_k(z_2)) \;/;\; \left|\frac{z_2}{z_1}\right| < 1 \bigvee \nu \in \mathbb{Z}$$

03.18.16.0010.01

$$\operatorname{ber}_\nu(z_1 + z_2) = \sum_{k=-\infty}^{\infty} (\operatorname{ber}_{\nu-k}(z_1) \operatorname{ber}_k(z_2) - \operatorname{bei}_{\nu-k}(z_1) \operatorname{bei}_k(z_2)) \;/;\; \left|\frac{z_2}{z_1}\right| < 1 \bigvee \nu \in \mathbb{Z}$$

Multiple arguments

03.18.16.0011.01

$$\operatorname{ber}_\nu(z_1 z_2) = z_1^\nu \sum_{k=0}^{\infty} \frac{\left(1 - z_1^2\right)^k}{k!} \left(\frac{z_2}{2}\right)^k \left(\cos\left(\frac{3 k \pi}{4}\right) \operatorname{ber}_{k+\nu}(z_2) - \sin\left(\frac{3 k \pi}{4}\right) \operatorname{bei}_{k+\nu}(z_2)\right) /;\; \left|\frac{z_2}{z_1}\right| < 1 \bigvee \nu \in \mathbb{Z}$$

Related transformations

Involving $\operatorname{bei}_\nu(z)$

03.18.16.0012.01

$$\operatorname{ber}_\nu(z) + i \operatorname{bei}_\nu(z) = \frac{e^{\frac{3 i \pi \nu}{4}} z^\nu}{\left(\sqrt[4]{-1}\, z\right)^\nu} I_\nu\left(\sqrt[4]{-1}\, z\right)$$

03.18.16.0013.01

$$\operatorname{ber}_\nu(z) - i \operatorname{bei}_\nu(z) = \frac{e^{-\frac{3}{4} i \pi \nu} z^\nu}{\left((-1)^{3/4} z\right)^\nu} I_\nu\left((-1)^{3/4} z\right)$$

Identities

Recurrence identities

Consecutive neighbors

03.18.17.0001.01

$$\operatorname{ber}_\nu(z) = \frac{\sqrt{2}\, (\nu + 1)}{z} (\operatorname{bei}_{\nu+1}(z) - \operatorname{ber}_{\nu+1}(z)) - \operatorname{ber}_{\nu+2}(z)$$

03.18.17.0002.01

$$\operatorname{ber}_\nu(z) = \frac{\sqrt{2}\, (\nu - 1)}{z} (\operatorname{bei}_{\nu-1}(z) - \operatorname{ber}_{\nu-1}(z)) - \operatorname{ber}_{\nu-2}(z)$$

Distant neighbors

Increasing

03.18.17.0003.01

$$\text{ber}_{\nu}(z) = (\nu+1)_{n-1} \left((n+\nu) \sum_{k=0}^{\lfloor \frac{n}{2} \rfloor} \frac{(-1)^k (n-k)!\, 2^{n-2k} z^{2k-n}}{k!\,(n-2k)!\,(-n-\nu)_k (\nu+1)_k} \left(\cos\left(\frac{1}{4}(2k-3n)\pi\right) \text{ber}_{n+\nu}(z) - \sin\left(\frac{1}{4}(2k-3n)\pi\right) \text{bei}_{n+\nu}(z) \right) + \sum_{k=0}^{\lfloor \frac{n-1}{2} \rfloor} \frac{(-1)^k (-k+n-1)!\, 2^{-2k+n-1} z^{2k-n+1}}{k!\,(-2k+n-1)!\,(-n-\nu+1)_k (\nu+1)_k} \left(\cos\left(\frac{1}{4}(2k-3n-1)\pi\right) \text{ber}_{n+\nu+1}(z) - \sin\left(\frac{1}{4}(2k-3n-1)\pi\right) \text{bei}_{n+\nu+1}(z) \right) \right) /;\ n \in \mathbb{N}$$

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03.18.17.0004.01

$$\text{ber}_{\nu}(z) = -(2-\nu)_{n-2} (n-1) \left(\frac{2}{z}\right)^{n-2} {}_4F_7\left(\frac{1}{2}-\frac{n}{4}, \frac{3}{4}-\frac{n}{4}, 1-\frac{n}{4}, \frac{5}{4}-\frac{n}{4}; \frac{3}{2}, \frac{1}{2}, \frac{1}{2}-\frac{n}{2}, 1-\frac{n}{2}, 1-\frac{\nu}{2}, \frac{3}{2}-\frac{\nu}{2}, -\frac{n}{2}+\frac{\nu}{2}+\frac{1}{2}, -\frac{n}{2}+\frac{\nu}{2}+1; -\frac{z^4}{16}\right) \left(\cos\left(\frac{n\pi}{4}\right) \text{bei}_{\nu-n}(z) + \text{ber}_{\nu-n}(z) \sin\left(\frac{n\pi}{4}\right)\right) + (1-\nu)_{n-1} \left(\frac{2}{z}\right)^{n-1} {}_4F_7\left(\frac{1}{4}-\frac{n}{4}, \frac{1}{2}-\frac{n}{4}, \frac{3}{4}-\frac{n}{4}, 1-\frac{n}{4}; \frac{1}{2}, \frac{1}{2}-\frac{n}{2}, 1-\frac{n}{2}, \frac{1}{2}-\frac{\nu}{2}, 1-\frac{\nu}{2}, -\frac{n}{2}+\frac{\nu}{2}+\frac{1}{2}, -\frac{n}{2}+\frac{\nu}{2}+1; -\frac{z^4}{16}\right) \left(-\cos\left(\frac{1}{4}(n+1)\pi\right) \text{bei}_{-n+\nu-1}(z) - \text{ber}_{-n+\nu-1}(z) \sin\left(\frac{1}{4}(n+1)\pi\right)\right) + (1-\nu)_n \left(\frac{2}{z}\right)^{n} {}_4F_7\left(\frac{1}{4}-\frac{n}{4}, \frac{1}{2}-\frac{n}{4}, \frac{3}{4}-\frac{n}{4}, -\frac{n}{4}; \frac{1}{2}, \frac{1}{2}-\frac{n}{2}, -\frac{n}{2}, \frac{1}{2}-\frac{\nu}{2}, 1-\frac{\nu}{2}, \frac{\nu}{2}-\frac{n}{2}, -\frac{n}{2}+\frac{\nu}{2}+\frac{1}{2}; -\frac{z^4}{16}\right) \left(\text{ber}_{\nu-n}(z) \cos\left(\frac{n\pi}{4}\right) - \text{bei}_{\nu-n}(z) \sin\left(\frac{n\pi}{4}\right)\right) - \frac{(1-\nu)_{n-2} (n-2)}{1-\nu} \left(\frac{2}{z}\right)^{n-3} {}_4F_7\left(\frac{3}{4}-\frac{n}{4}, 1-\frac{n}{4}, \frac{5}{4}-\frac{n}{4}, \frac{3}{2}-\frac{n}{4}; \frac{3}{2}, 1-\frac{n}{2}, \frac{3}{2}-\frac{n}{2}, 1-\frac{\nu}{2}, \frac{3}{2}-\frac{\nu}{2}, -\frac{n}{2}+\frac{\nu}{2}+1, -\frac{n}{2}+\frac{\nu}{2}+\frac{3}{2}; -\frac{z^4}{16}\right) \left(\text{ber}_{-n+\nu-1}(z) \cos\left(\frac{1}{4}(n+1)\pi\right) - \text{bei}_{-n+\nu-1}(z) \sin\left(\frac{1}{4}(n+1)\pi\right)\right) /;\ n \in \mathbb{Z} \land n \geq 3$$

Brychkov Yu.A. (2005)

03.18.17.0005.01

$$\text{ber}_{\nu}(z) = -\frac{4(\nu+1)(\nu+2)\,\text{bei}_{\nu+2}(z)}{z^2} - \text{ber}_{\nu+2}(z) + \frac{\sqrt{2}\,(\nu+1)\,\text{ber}_{\nu+3}(z)}{z} - \frac{\sqrt{2}\,(\nu+1)\,\text{bei}_{\nu+3}(z)}{z}$$

03.18.17.0006.01

$$\operatorname{ber}_\nu(z) = \frac{2\sqrt{2}\,(\nu+2)\left(2(\nu+1)(\nu+3)-z^2\right)\operatorname{bei}_{\nu+3}(z)}{z^3} + \frac{4(\nu+1)(\nu+2)\operatorname{bei}_{\nu+4}(z)}{z^2} + \frac{2\sqrt{2}\,(\nu+2)\left(z^2+2(\nu+1)(\nu+3)\right)\operatorname{ber}_{\nu+3}(z)}{z^3} + \operatorname{ber}_{\nu+4}(z)$$

03.18.17.0007.01

$$\operatorname{ber}_\nu(z) = \frac{12(\nu+2)(\nu+3)\operatorname{bei}_{\nu+4}(z)}{z^2} + \frac{2\sqrt{2}\,(\nu+2)\left(z^2-2(\nu+1)(\nu+3)\right)\operatorname{bei}_{\nu+5}(z)}{z^3} + \frac{\left(z^4-16(\nu+1)(\nu+2)(\nu+3)(\nu+4)\right)\operatorname{ber}_{\nu+4}(z)}{z^4} - \frac{2\sqrt{2}\,(\nu+2)\left(z^2+2(\nu+1)(\nu+3)\right)\operatorname{ber}_{\nu+5}(z)}{z^3}$$

03.18.17.0008.01

$$\operatorname{ber}_\nu(z) = -\frac{\sqrt{2}\,(\nu+3)\left(-3z^4+16(\nu+2)(\nu+4)z^2+16(\nu+1)(\nu+2)(\nu+4)(\nu+5)\right)\operatorname{bei}_{\nu+5}(z)}{z^5} + \frac{\sqrt{2}\,(\nu+3)\left(-3z^4-16(\nu+2)(\nu+4)z^2+16(\nu+1)(\nu+2)(\nu+4)(\nu+5)\right)\operatorname{ber}_{\nu+5}(z)}{z^5} + \frac{\left(16(\nu+1)(\nu+2)(\nu+3)(\nu+4)-z^4\right)\operatorname{ber}_{\nu+6}(z)}{z^4} - \frac{12(\nu+2)(\nu+3)\operatorname{bei}_{\nu+6}(z)}{z^2}$$

Decreasing

03.18.17.0009.01

$$\operatorname{ber}_\nu(z) = (1-\nu)_{n-1} \left(\sum_{k=0}^{\left\lfloor \frac{n-1}{2} \right\rfloor} \frac{(-1)^k (-k+n-1)!\, 2^{-2k+n-1} z^{2k-n+1}}{k!\,(-2k+n-1)!\,(1-\nu)_k\,(-n+\nu+1)_k} \left(\sin\left(\frac{1}{4}(2k+n-1)\pi\right) \operatorname{bei}_{-n+\nu-1}(z) - \cos\left(\frac{1}{4}(2k+n-1)\pi\right) \operatorname{ber}_{-n+\nu-1}(z) \right) + (n-\nu) \sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} \frac{(-1)^k (n-k)!\, 2^{n-2k} z^{2k-n}}{k!\,(n-2k)!\,(1-\nu)_k\,(\nu-n)_k} \left(\cos\left(\frac{1}{4}(2k+n)\pi\right) \operatorname{ber}_{\nu-n}(z) - \sin\left(\frac{1}{4}(2k+n)\pi\right) \operatorname{bei}_{\nu-n}(z) \right) \right) /;\ n \in \mathbb{N}^+$$

03.18.17.0010.01

$$\operatorname{ber}_\nu(z) = -\frac{\sqrt{2}\,(\nu-1)\operatorname{bei}_{\nu-3}(z)}{z} + \frac{\sqrt{2}\,(\nu-1)\operatorname{ber}_{\nu-3}(z)}{z} - \operatorname{ber}_{\nu-2}(z) - \frac{4((\nu-3)\nu+2)\operatorname{bei}_{\nu-2}(z)}{z^2}$$

03.18.17.0011.01

$$\operatorname{ber}_\nu(z) = \frac{4(\nu-2)(\nu-1)\operatorname{bei}_{\nu-4}(z)}{z^2} + \operatorname{ber}_{\nu-4}(z) + \frac{2\sqrt{2}\left(z^2+2(\nu-3)(\nu-1)\right)(\nu-2)\operatorname{ber}_{\nu-3}(z)}{z^3} - \frac{2\sqrt{2}\left(z^2-2(\nu-3)(\nu-1)\right)(\nu-2)\operatorname{bei}_{\nu-3}(z)}{z^3}$$

03.18.17.0012.01

$$\operatorname{ber}_\nu(z) = \frac{2\sqrt{2}\left(z^2-2(\nu-3)(\nu-1)\right)(\nu-2)\operatorname{bei}_{\nu-5}(z)}{z^3} + \frac{12(\nu-3)(\nu-2)\operatorname{bei}_{\nu-4}(z)}{z^2} + \frac{\left(z^4-16(\nu-4)(\nu-3)(\nu-2)(\nu-1)\right)\operatorname{ber}_{\nu-4}(z)}{z^4} - \frac{2\sqrt{2}\left(z^2+2(\nu-3)(\nu-1)\right)(\nu-2)\operatorname{ber}_{\nu-5}(z)}{z^3}$$

03.18.17.0013.01

$$\operatorname{ber}_\nu(z) = -\frac{12(\nu-3)(\nu-2)\operatorname{bei}_{\nu-6}(z)}{z^2} + \frac{\sqrt{2}(\nu-3)\left(3z^4-16((\nu-6)\nu+8)z^2-16(\nu-5)(\nu-4)(\nu-2)(\nu-1)\right)\operatorname{bei}_{\nu-5}(z)}{z^5} + \frac{\left(16(\nu-4)(\nu-3)(\nu-2)(\nu-1)-z^4\right)\operatorname{ber}_{\nu-6}(z)}{z^4} + \frac{\sqrt{2}(\nu-3)\left(-3z^4-16((\nu-6)\nu+8)z^2+16(\nu-5)(\nu-4)(\nu-2)(\nu-1)\right)\operatorname{ber}_{\nu-5}(z)}{z^5}$$

Functional identities

Relations between contiguous functions

03.18.17.0014.01

$$\operatorname{ber}_\nu(z) = -\frac{z}{2\sqrt{2}\,\nu}\left(\operatorname{bei}_{\nu-1}(z)+\operatorname{bei}_{\nu+1}(z)+\operatorname{ber}_{\nu-1}(z)+\operatorname{ber}_{\nu+1}(z)\right)$$

Differentiation

Low-order differentiation

With respect to ν

03.18.20.0001.01

$$\operatorname{ber}_\nu^{(1,0)}(z) = -\left(\frac{z}{2}\right)^\nu \sum_{k=0}^\infty \frac{\cos\left(\frac{1}{4}\pi(2k+3\nu)\right)\psi(k+\nu+1)}{k!\,\Gamma(k+\nu+1)}\left(\frac{z}{2}\right)^{2k} - \frac{3\pi}{4}\operatorname{bei}_\nu(z) + \log\left(\frac{z}{2}\right)\operatorname{ber}_\nu(z)$$

03.18.20.0002.01

$$\operatorname{ber}_n^{(1,0)}(z) == -\frac{\pi}{2}\operatorname{bei}_n(z) - \operatorname{ker}_n(z) + \frac{n}{2}\sum_{k=0}^{n-1}\frac{1}{k!\,(n-k)}\left(\frac{z}{2}\right)^{k-n}\left(\cos\left(\frac{3}{4}(k-n)\pi\right)\operatorname{ber}_k(z) - \sin\left(\frac{3}{4}(k-n)\pi\right)\operatorname{bei}_k(z)\right) /;\ n\in\mathbb{N}$$

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03.18.20.0003.01

$$\operatorname{ber}_n^{(1,0)}(z) == 2^{n-1}\,n!\,(-z)^{-n}\sum_{k=0}^{n-1}\frac{1}{(n-k)\,k!}\left(-\frac{z}{2}\right)^k\left(\cos\left(\frac{1}{4}(k-n)\pi\right)\operatorname{ber}_k(z) + \sin\left(\frac{1}{4}(k-n)\pi\right)\operatorname{bei}_k(z)\right) - \frac{1}{2}\pi\operatorname{bei}_n(z) - \operatorname{ker}_n(z) + \left(\frac{1}{4}(i\pi+\log(4)) + \log(z) - \log((1+i)z)\right)\operatorname{ber}_n(z) /;\ n\in\mathbb{N}$$

03.18.20.0004.01

$$\operatorname{ber}_{-n}^{(1,0)}(z) = -\frac{1}{2}\pi(-1)^n \operatorname{bei}_n(z) + (-1)^{n-1}\operatorname{ker}_n(z) - \frac{(-1)^n n!}{2}\sum_{k=0}^{n-1}\frac{1}{k!\,(n-k)}\left(\frac{z}{2}\right)^{k-n}\left(\cos\left(\frac{3}{4}(k-n)\pi\right)\operatorname{ber}_k(z) - \sin\left(\frac{3}{4}(k-n)\pi\right)\operatorname{bei}_k(z)\right) /;\ n \in \mathbb{N}$$

03.18.20.0005.01

$$\operatorname{ber}_{-n}^{(1,0)}(z) + (-1)^n \operatorname{ber}_n^{(1,0)}(z) = (-1)^{n-1}(\pi \operatorname{bei}_n(z) + 2\operatorname{ker}_n(z)) /;\ n \in \mathbb{N}$$

03.18.20.0006.01

$$\operatorname{ber}_{n+\frac{1}{2}}^{(1,0)}(z) = -\frac{3\pi}{4}\operatorname{bei}_{n+\frac{1}{2}}(z) - \left(\log\left(\sqrt[4]{-1}\,z\right) - \log(z)\right)\operatorname{ber}_{n+\frac{1}{2}}(z) + \frac{(-1)^{3/8}\, 2^{-n-\frac{1}{2}}\, e^{\frac{3 i n\pi}{4}}\, z^{-n-\frac{1}{2}}}{n!\,\sqrt{\pi}}\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor} 2^{2k}\binom{n}{2k}(2n-2k)!\, i^k \left((-1)^{3/4} i^n\left(\cosh\left(\sqrt[4]{-1}\,z\right)\operatorname{Shi}\left(2\sqrt[4]{-1}\,z\right) - \left(\operatorname{Chi}\left(2\sqrt[4]{-1}\,z\right) - \psi\left(k+\frac{1}{2}\right) + \psi\left(k-n+\frac{1}{2}\right)\right)\sinh\left(\sqrt[4]{-1}\,z\right)\right) + (-1)^k\left(\cos\left(\sqrt[4]{-1}\,z\right)\operatorname{Si}\left(2\sqrt[4]{-1}\,z\right) - \left(\operatorname{Ci}\left(2\sqrt[4]{-1}\,z\right) - \psi\left(k+\frac{1}{2}\right) + \psi\left(k-n+\frac{1}{2}\right)\right)\sin\left(\sqrt[4]{-1}\,z\right)\right)\right) z^{2k} + \frac{(-1)^{5/8}\, 2^{\frac{1}{2}-n}\, e^{\frac{3 i n\pi}{4}}\, z^{\frac{1}{2}-n}}{n!\,\sqrt{\pi}}\sum_{k=0}^{\left\lfloor\frac{n-1}{2}\right\rfloor} 2^{2k}\binom{n}{2k+1}(-2k+2n-1)!\, i^k \left((-1)^{3/4} e^{\frac{i n\pi}{2}}\left(\cosh\left(\sqrt[4]{-1}\,z\right)\left(\operatorname{Chi}\left(2\sqrt[4]{-1}\,z\right) - \psi\left(k+\frac{3}{2}\right) + \psi\left(k-n+\frac{1}{2}\right)\right) - \sinh\left(\sqrt[4]{-1}\,z\right)\operatorname{Shi}\left(2\sqrt[4]{-1}\,z\right)\right) + (-1)^k\left(\cos\left(\sqrt[4]{-1}\,z\right)\left(\operatorname{Ci}\left(2\sqrt[4]{-1}\,z\right) - \psi\left(k+\frac{3}{2}\right) + \psi\left(k-n+\frac{1}{2}\right)\right) + \sin\left(\sqrt[4]{-1}\,z\right)\operatorname{Si}\left(2\sqrt[4]{-1}\,z\right)\right)\right) z^{2k} /;\ n \in \mathbb{N}$$

03.18.20.0007.01

$$\operatorname{ber}_{-n-\frac{1}{2}}^{(1,0)}(z) = -\frac{3\pi}{4}\operatorname{bei}_{-n-\frac{1}{2}}(z) + \left(\log(z) - \log\left(\sqrt[4]{-1}\,z\right)\right)\operatorname{ber}_{-n-\frac{1}{2}}(z) + \frac{(-1)^{3/8}\, 2^{-n-\frac{1}{2}}\, e^{\frac{7 i n\pi}{4}}\, z^{-n-\frac{1}{2}}}{\sqrt{\pi}\, n!}\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor} 2^{2k}\binom{n}{2k}(2n-2k)!\, i^k \left(e^{\frac{1}{4}(-3) i (2n+1)\pi}\left(\cosh\left(\sqrt[4]{-1}\,z\right)\operatorname{Chi}\left(2\sqrt[4]{-1}\,z\right) + \cosh\left(\sqrt[4]{-1}\,z\right)\left(\psi\left(k+\frac{1}{2}\right) - \psi\left(k-n+\frac{1}{2}\right)\right) - \sinh\left(\sqrt[4]{-1}\,z\right)\operatorname{Shi}\left(2\sqrt[4]{-1}\,z\right)\right) + (-1)^k\left(\cos\left(\sqrt[4]{-1}\,z\right)\operatorname{Ci}\left(2\sqrt[4]{-1}\,z\right) + \cos\left(\sqrt[4]{-1}\,z\right)\left(\psi\left(k+\frac{1}{2}\right) - \psi\left(k-n+\frac{1}{2}\right)\right) + \sin\left(\sqrt[4]{-1}\,z\right)\operatorname{Si}\left(2\sqrt[4]{-1}\,z\right)\right)\right) z^{2k} + \frac{(-1)^{5/8}\, 2^{\frac{1}{2}-n}\, e^{-\frac{1}{4}(i n\pi)}\, z^{\frac{1}{2}-n}}{\sqrt{\pi}\, n!}\sum_{k=0}^{\left\lfloor\frac{n-1}{2}\right\rfloor} 2^{2k}\binom{n}{2k+1}(-2k+2n-1)!\, i^k \left(e^{\frac{1}{4}(-3) i (2n+1)\pi}\left(-\operatorname{Chi}\left(2\sqrt[4]{-1}\,z\right)\sinh\left(\sqrt[4]{-1}\,z\right) - \left(\psi\left(k+\frac{3}{2}\right) - \psi\left(k-n+\frac{1}{2}\right)\right)\sinh\left(\sqrt[4]{-1}\,z\right) + \cosh\left(\sqrt[4]{-1}\,z\right)\operatorname{Shi}\left(2\sqrt[4]{-1}\,z\right)\right) + (-1)^k\left(\operatorname{Ci}\left(2\sqrt[4]{-1}\,z\right)\sin\left(\sqrt[4]{-1}\,z\right) + \left(\psi\left(k+\frac{3}{2}\right) - \psi\left(k-n+\frac{1}{2}\right)\right)\sin\left(\sqrt[4]{-1}\,z\right) - \cos\left(\sqrt[4]{-1}\,z\right)\operatorname{Si}\left(2\sqrt[4]{-1}\,z\right)\right)\right) z^{2k} /;\ n \in \mathbb{N}$$

With respect to z

03.18.20.0008.01

$$\frac{\partial \operatorname{ber}_\nu(z)}{\partial z} = -\frac{1}{\sqrt{2}\, z}\left(z \operatorname{bei}_{\nu-1}(z) + z \operatorname{ber}_{\nu-1}(z) + \sqrt{2}\,\nu \operatorname{ber}_\nu(z)\right)$$

03.18.20.0009.01

$$\frac{\partial \operatorname{ber}_\nu(z)}{\partial z} = \frac{1}{2\sqrt{2}} \left(-\operatorname{bei}_{\nu-1}(z) + \operatorname{bei}_{\nu+1}(z) - \operatorname{ber}_{\nu-1}(z) + \operatorname{ber}_{\nu+1}(z)\right)$$

03.18.20.0010.01

$$\frac{\partial (z^\nu \operatorname{ber}_\nu(z))}{\partial z} = -\frac{z^\nu}{\sqrt{2}} \left(\operatorname{bei}_{\nu-1}(z) + \operatorname{ber}_{\nu-1}(z)\right)$$

03.18.20.0011.01

$$\frac{\partial (z^{-\nu} \operatorname{ber}_\nu(z))}{\partial z} = \frac{z^{-\nu}}{\sqrt{2}} \left(\operatorname{bei}_{\nu+1}(z) + \operatorname{ber}_{\nu+1}(z)\right)$$

03.18.20.0012.01

$$\frac{\partial^2 \operatorname{ber}_\nu(z)}{\partial z^2} = \frac{1}{4} \left(\operatorname{bei}_{\nu-2}(z) - 2\operatorname{bei}_\nu(z) + \operatorname{bei}_{\nu+2}(z)\right)$$

03.18.20.0013.01

$$\frac{\partial^2 \operatorname{ber}_\nu(z)}{\partial z^2} = \frac{\operatorname{bei}_{\nu-1}(z)}{\sqrt{2}\, z} - \operatorname{bei}_\nu(z) + \frac{\operatorname{ber}_{\nu-1}(z)}{\sqrt{2}\, z} + \frac{(\nu(\nu+1)) \operatorname{ber}_\nu(z)}{z^2}$$

Symbolic differentiation

With respect to ν

03.18.20.0014.01

$$\operatorname{ber}_\nu^{(m,0)}(z) = \sum_{k=0}^{\infty} \frac{1}{k!} \left(\frac{z}{2}\right)^{2k} \frac{\partial^m \frac{\left(\frac{z}{2}\right)^\nu \cos\left(\frac{1}{4}\pi(2k+3\nu)\right)}{\Gamma(k+\nu+1)}}{\partial \nu^m} \;/;\; m \in \mathbb{N}$$

With respect to z

03.18.20.0015.01

$$\frac{\partial^n \operatorname{ber}_\nu(z)}{\partial z^n} =$$

$$z^{-n} \sum_{m=0}^{n} (-1)^{m+n} \binom{n}{m} (-\nu)_{n-m} \sum_{k=0}^{m} \frac{(-1)^k 2^{2k-m} (-m)_{2(m-k)} (\nu)_k}{(m-k)!} \left(\operatorname{ber}_\nu(z) \sum_{j=0}^{\left\lfloor \frac{k}{2} \right\rfloor} \frac{(-1)^j (k-2j)!}{(2j)!\,(k-4j)!\,(-k-\nu+1)_{2j}\,(\nu)_{2j}} \left(\frac{z}{2}\right)^{4j} + \right.$$

$$\frac{z}{2\sqrt{2}} \left(\operatorname{bei}_{\nu-1}(z) + \operatorname{ber}_{\nu-1}(z)\right) \sum_{j=0}^{\left\lfloor \frac{k-1}{2} \right\rfloor} \frac{(-1)^j (-2j+k-1)!}{(2j)!\,(-4j+k-1)!\,(-k-\nu+1)_{2j}\,(\nu)_{2j+1}} \left(\frac{z}{2}\right)^{4j} +$$

$$\frac{z^2}{4} \operatorname{bei}_\nu(z) \sum_{j=0}^{\left\lfloor \frac{k-1}{2} \right\rfloor} \frac{(-1)^j (-2j+k-1)!}{(2j+1)!\,(-4j+k-2)!\,(-k-\nu+1)_{2j+1}\,(\nu)_{2j+1}} \left(\frac{z}{2}\right)^{4j} +$$

$$\left. \frac{z^3}{8\sqrt{2}} \left(\operatorname{bei}_{\nu-1}(z) - \operatorname{ber}_{\nu-1}(z)\right) \sum_{j=0}^{\left\lfloor \frac{k-2}{2} \right\rfloor} \frac{(-1)^j (-2j+k-2)!}{(2j+1)!\,(-4j+k-3)!\,(-k-\nu+1)_{2j+1}\,(\nu)_{2j+2}} \left(\frac{z}{2}\right)^{4j} \right) /;\; n \in \mathbb{N}$$

03.18.20.0016.01

$$\frac{\partial^n \operatorname{ber}_\nu(z)}{\partial z^n} = 2^{n-2\nu-1} e^{\frac{1}{4}(-3)i\pi\nu} \sqrt{\pi}\, z^{\nu-n}\, \Gamma(\nu+1)\left(e^{\frac{3i\pi\nu}{2}}\, {}_2\tilde{F}_3\left(\frac{\nu+1}{2}, \frac{\nu+2}{2}; \frac{1}{2}(-n+\nu+1), \frac{1}{2}(-n+\nu+2), \nu+1; \frac{i z^2}{4}\right) + {}_2\tilde{F}_3\left(\frac{\nu+1}{2}, \frac{\nu+2}{2}; \frac{1}{2}(-n+\nu+1), \frac{1}{2}(-n+\nu+2), \nu+1; -\frac{i z^2}{4}\right)\right) /; n \in \mathbb{N}$$

03.18.20.0017.01

$$\frac{\partial^n \operatorname{ber}_\nu(z)}{\partial z^n} = 2^{-\frac{3n}{2}-1}(i-1)^n \left(\sum_{k=0}^{\left\lfloor \frac{n}{2}\right\rfloor} \binom{n}{2k} \left(i(1-i^n)\operatorname{bei}_{4k-n+\nu}(z) + (1+i^n)\operatorname{ber}_{4k-n+\nu}(z)\right) + \sum_{k=0}^{\left\lfloor \frac{n-1}{2}\right\rfloor} \binom{n}{2k+1} \left(-i(1-i^n)\operatorname{bei}_{4k-n+\nu+2}(z) - (1+i^n)\operatorname{ber}_{4k-n+\nu+2}(z)\right)\right) /; n \in \mathbb{N}$$

03.18.20.0018.01

$$\frac{\partial^n \operatorname{ber}_\nu(z)}{\partial z^n} = 2^{-\frac{3n}{2}-1}(i-1)^n \sum_{k=0}^{\left\lfloor \frac{n}{2}\right\rfloor} \left(\frac{n+1}{2k+1}\binom{n}{2k}\left(\left(i-i^{n+1}\right)\operatorname{bei}_{4k-n+\nu}(z) + (1+i^n)\operatorname{ber}_{4k-n+\nu}(z)\right) - \frac{(1+i)\sqrt{2}\,(4k-n+\nu+1)}{z}\binom{n}{2k+1}\left((-i+i^n)\operatorname{bei}_{4k-n+\nu+1}(z) + \left(-1+i^{n+1}\right)\operatorname{ber}_{4k-n+\nu+1}(z)\right)\right) /; n \in \mathbb{N}$$

03.18.20.0019.01

$$\frac{\partial^n \operatorname{ber}_\nu(z)}{\partial z^n} = \pi\, G_{5,9}^{2,4}\left(\frac{z}{4}, \frac{1}{4} \,\middle|\, \begin{array}{c} -\frac{n}{4}, \frac{1-n}{4}, \frac{2-n}{4}, \frac{3-n}{4}, \frac{1}{4}(-n+4\nu+2) \\ \frac{1}{4}(-n+\nu+2), \frac{\nu-n}{4}, \frac{1}{4}(-n-\nu+2), \frac{1}{4}(-n-\nu), \frac{1}{4}(-n+4\nu+2), 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4} \end{array}\right) /; n \in \mathbb{Z} \land n \geq 3$$

Fractional integro-differentiation

With respect to z

03.18.20.0020.01

$$\frac{\partial^\alpha \operatorname{ber}_\nu(z)}{\partial z^\alpha} = 2^{-\nu} z^{\nu-\alpha} \sum_{k=0}^{\infty} \frac{\cos\left(\frac{1}{4}\pi(2k+3\nu)\right)\Gamma(2k+\nu+1)}{\Gamma(k+\nu+1)\,\Gamma(2k-\alpha+\nu+1)\,k!}\left(\frac{z}{2}\right)^{2k}$$

03.18.20.0021.01

$$\frac{\partial^\alpha \operatorname{ber}_\nu(z)}{\partial z^\alpha} = \frac{2^{-\nu-1} z^{\nu-\alpha}}{\Gamma(\nu-\alpha+1)} \left(e^{\frac{3i\pi\nu}{4}}\, {}_2F_3\left(\frac{\nu+1}{2}, \frac{\nu}{2}+1; \frac{\nu-\alpha+1}{2}, \frac{\nu-\alpha}{2}+1, \nu+1; \frac{i z^2}{4}\right) + e^{-\frac{3i\pi\nu}{4}}\, {}_2F_3\left(\frac{\nu+1}{2}, \frac{\nu}{2}+1; \frac{\nu-\alpha+1}{2}, \frac{\nu-\alpha}{2}+1, \nu+1; -\frac{i z^2}{4}\right)\right)$$

Integration

Indefinite integration

03.18.21.0001.01

$$\int \operatorname{ber}_\nu(a z)\, dz = \frac{1}{4}\pi z\, G_{2,6}^{2,1}\left(\frac{a z}{4}, \frac{1}{4} \,\middle|\, \begin{array}{c} \frac{3}{4}, \nu+\frac{1}{2} \\ \frac{\nu}{4}, \frac{\nu+2}{4}, -\frac{1}{4}, \frac{2-\nu}{4}, -\frac{\nu}{4}, \nu+\frac{1}{2} \end{array}\right)$$

Definite integration

03.18.21.0002.01

$$\int_0^\infty t^{\alpha-1}\, e^{-p\,t}\,\mathrm{ber}_\nu(t)\,dt = \frac{2^{-\nu-2}\,p^{-\alpha-\nu}\,\Gamma(\alpha+\nu)}{\Gamma(\nu+1)}\left(4\cos\left(\frac{3\pi\nu}{4}\right){}_4F_3\left(\frac{\alpha}{4}+\frac{\nu}{4},\frac{\alpha}{4}+\frac{\nu}{4}+\frac{1}{4},\frac{\alpha}{4}+\frac{\nu}{4}+\frac{1}{2},\frac{\alpha}{4}+\frac{\nu}{4}+\frac{3}{4};\frac{1}{2},\frac{\nu}{2}+\frac{1}{2},\frac{\nu}{2}+1;-\frac{1}{p^4}\right)-\frac{(\alpha+\nu)(\alpha+\nu+1)\sin\left(\frac{3\pi\nu}{4}\right)}{p^2(\nu+1)}\,{}_4F_3\left(\frac{\alpha}{4}+\frac{\nu}{4}+\frac{1}{2},\frac{\alpha}{4}+\frac{\nu}{4}+\frac{3}{4},\frac{\alpha}{4}+\frac{\nu}{4}+1,\frac{\alpha}{4}+\frac{\nu}{4}+\frac{5}{4};\frac{3}{2},\frac{\nu}{2}+1,\frac{\nu}{2}+\frac{3}{2};-\frac{1}{p^4}\right)\right)\,/;\,\mathrm{Re}(\alpha+\nu)>0\bigwedge\mathrm{Re}(p)>\frac{1}{\sqrt{2}}$$

Integral transforms

Laplace transforms

03.18.22.0001.01

$$\mathcal{L}_t[\mathrm{ber}_\nu(t)](z) = 2^{-\nu-2}\,z^{-\nu-3}\left(4z^2\cos\left(\frac{3\pi\nu}{4}\right){}_4F_3\left(\frac{\nu}{4}+\frac{1}{4},\frac{\nu}{4}+\frac{1}{2},\frac{\nu}{4}+\frac{3}{4},\frac{\nu}{4}+1;\frac{1}{2},\frac{\nu}{2}+\frac{1}{2},\frac{\nu}{2}+1;-\frac{1}{z^4}\right)-(\nu+2)\sin\left(\frac{3\pi\nu}{4}\right){}_4F_3\left(\frac{\nu}{4}+\frac{3}{4},\frac{\nu}{4}+1,\frac{\nu}{4}+\frac{5}{4},\frac{\nu}{4}+\frac{3}{2};\frac{3}{2},\frac{\nu}{2}+1,\frac{\nu}{2}+\frac{3}{2};-\frac{1}{z^4}\right)\right)\,/;\,\mathrm{Re}(\nu)>-1\bigwedge\mathrm{Re}(z)>\frac{1}{\sqrt{2}}$$

Mellin transforms

03.18.22.0002.01

$$\mathcal{M}_t\left[e^{-p\,t}\,\mathrm{bei}_\nu(t)\right](z) = \frac{2^{-\nu-2}\,p^{-z-\nu}\,\Gamma(z+\nu)}{\Gamma(\nu+1)}\left(4\cos\left(\frac{3\pi\nu}{4}\right){}_4F_3\left(\frac{z}{4}+\frac{\nu}{4},\frac{z}{4}+\frac{\nu}{4}+\frac{1}{4},\frac{z}{4}+\frac{\nu}{4}+\frac{1}{2},\frac{z}{4}+\frac{\nu}{4}+\frac{3}{4};\frac{1}{2},\frac{\nu}{2}+\frac{1}{2},\frac{\nu}{2}+1;-\frac{1}{p^4}\right)-\frac{(z+\nu)(z+\nu+1)\sin\left(\frac{3\pi\nu}{4}\right)}{p^2(\nu+1)}\,{}_4F_3\left(\frac{z}{4}+\frac{\nu}{4}+\frac{1}{2},\frac{z}{4}+\frac{\nu}{4}+\frac{3}{4},\frac{z}{4}+\frac{\nu}{4}+1,\frac{z}{4}+\frac{\nu}{4}+\frac{5}{4};\frac{3}{2},\frac{\nu}{2}+1,\frac{\nu}{2}+\frac{3}{2};-\frac{1}{p^4}\right)\right)\,/;\,\mathrm{Re}(z+\nu)>0\bigwedge\mathrm{Re}(p)>\frac{1}{\sqrt{2}}$$

Representations through more general functions

Through hypergeometric functions

Involving ${}_p\tilde{F}_q$

03.18.26.0001.01

$$\mathrm{ber}_\nu(z) = 4^{-\nu}\,\pi\,z^\nu\cos\left(\frac{3\pi\nu}{4}\right){}_0\tilde{F}_3\left(;\frac{1}{2},\frac{\nu+1}{2},\frac{\nu}{2}+1;-\frac{z^4}{256}\right)-2^{-2(\nu+2)}\,\pi\,z^{\nu+2}\sin\left(\frac{3\pi\nu}{4}\right){}_0\tilde{F}_3\left(;\frac{3}{2},\frac{\nu+3}{2},\frac{\nu}{2}+1;-\frac{z^4}{256}\right)$$

Involving ${}_pF_q$

03.18.26.0002.01

$$\operatorname{ber}_\nu(z) = \frac{\cos\left(\frac{3\pi\nu}{4}\right)\left(\frac{z}{2}\right)^\nu}{\Gamma(\nu+1)}\,{}_0F_3\left(;\frac{1}{2},\frac{\nu+1}{2},\frac{\nu}{2}+1;-\frac{z^4}{256}\right) - \frac{\sin\left(\frac{3\pi\nu}{4}\right)\left(\frac{z}{2}\right)^{\nu+2}}{\Gamma(\nu+2)}\,{}_0F_3\left(;\frac{3}{2},\frac{\nu+3}{2},\frac{\nu}{2}+1;-\frac{z^4}{256}\right)\ /;\ -\nu \notin \mathbb{N}^+$$

Through Meijer G

Classical cases for the direct function itself

03.18.26.0003.01

$$\operatorname{ber}_\nu(z) = \pi\, G_{1,5}^{2,0}\left(\frac{z^4}{256}\,\middle|\, \begin{matrix}\frac{1}{2}(2\nu+1)\\ \frac{\nu}{4},\frac{\nu+2}{4},-\frac{\nu}{4},\frac{2-\nu}{4},\nu+\frac{1}{2}\end{matrix}\right)\ /;\ -\frac{\pi}{4} \le \arg(z) \le \frac{\pi}{4}$$

03.18.26.0004.01

$$\operatorname{ber}_{-\nu}(z) + \operatorname{ber}_\nu(z) = 2\pi\cos\left(\frac{\pi\nu}{2}\right) G_{3,7}^{4,0}\left(\frac{z^4}{256}\,\middle|\, \begin{matrix}0,\frac{1-\nu}{2},\frac{\nu+1}{2}\\ \frac{\nu}{4},-\frac{\nu}{4},\frac{\nu+2}{4},\frac{2-\nu}{4},0,\frac{1-\nu}{2},\frac{\nu+1}{2}\end{matrix}\right)\ /;\ -\frac{\pi}{4} \le \arg(z) \le \frac{\pi}{4}$$

Classical cases for powers of ber

03.18.26.0005.01

$$\operatorname{ber}_\nu\left(\sqrt[4]{z}\right)^2 = \frac{1}{2}\pi^{3/2} G_{1,5}^{1,0}\left(\frac{z}{64}\,\middle|\, \begin{matrix}\frac{\nu+1}{2}\\ \frac{\nu}{2},0,\frac{1}{2},-\frac{\nu}{2},\frac{\nu+1}{2}\end{matrix}\right) + \frac{\sqrt{\pi}}{2^{3/2}} G_{3,7}^{2,2}\left(\frac{z}{16}\,\middle|\, \begin{matrix}\frac{1}{4},\frac{3}{4},2\nu+\frac{1}{2}\\ \frac{\nu}{2},\frac{\nu+1}{2},0,\frac{1}{2},-\frac{\nu}{2},\frac{1-\nu}{2},2\nu+\frac{1}{2}\end{matrix}\right)$$

Brychkov Yu.A. (2006)

03.18.26.0006.01

$$\operatorname{ber}_\nu(z)^2 = \frac{1}{2}\pi^{3/2} G_{1,5}^{1,0}\left(\frac{z^4}{64}\,\middle|\, \begin{matrix}\frac{\nu+1}{2}\\ \frac{\nu}{2},0,\frac{1}{2},-\frac{\nu}{2},\frac{\nu+1}{2}\end{matrix}\right) + \frac{\sqrt{\pi}}{2^{3/2}} G_{3,7}^{2,2}\left(\frac{z^4}{16}\,\middle|\, \begin{matrix}\frac{1}{4},\frac{3}{4},2\nu+\frac{1}{2}\\ \frac{\nu}{2},\frac{\nu+1}{2},0,\frac{1}{2},-\frac{\nu}{2},\frac{1-\nu}{2},2\nu+\frac{1}{2}\end{matrix}\right)\ /;\ -\frac{\pi}{4} \le \arg(z) \le \frac{\pi}{4}$$

Brychkov Yu.A. (2006)

Classical cases for products of ber

03.18.26.0007.01

$$\operatorname{ber}_{-\nu}(z)\operatorname{ber}_\nu(z) = \frac{1}{4} e^{-\frac{1}{2}(3i\pi\nu)}\sqrt{\pi}\, G_{1,5}^{2,0}\left(-\frac{z^4}{64}\,\middle|\, \begin{matrix}\frac{1-\nu}{2}\\ 0,\frac{1}{2},-\frac{\nu}{2},\frac{\nu}{2},\frac{1-\nu}{2}\end{matrix}\right) +$$
$$\frac{1}{4} e^{\frac{3i\pi\nu}{2}}\sqrt{\pi}\, G_{1,5}^{2,0}\left(-\frac{z^4}{64}\,\middle|\, \begin{matrix}\frac{\nu+1}{2}\\ 0,\frac{1}{2},-\frac{\nu}{2},\frac{\nu}{2},\frac{\nu+1}{2}\end{matrix}\right) + \frac{\pi^{3/2}}{2\sqrt{2}} G_{3,7}^{1,2}\left(-\frac{z^4}{16}\,\middle|\, \begin{matrix}\frac{1}{4},\frac{3}{4},\frac{1}{2}\\ 0,\frac{1}{2},\frac{1}{2},-\frac{\nu}{2},\frac{\nu}{2},\frac{1-\nu}{2},\frac{\nu+1}{2}\end{matrix}\right)\ /;\ -\frac{\pi}{2} < \arg(z) \le 0$$

Classical cases involving powers of bei

03.18.26.0008.01

$$\operatorname{bei}_\nu\left(\sqrt[4]{z}\right)^2 + \operatorname{ber}_\nu\left(\sqrt[4]{z}\right)^2 = \pi^{3/2} G_{1,5}^{1,0}\left(\frac{z}{64}\,\middle|\, \begin{matrix}\frac{\nu+1}{2}\\ \frac{\nu}{2},0,\frac{1}{2},-\frac{\nu}{2},\frac{\nu+1}{2}\end{matrix}\right)$$

Brychkov Yu.A. (2006)

03.18.26.0009.01

$$\operatorname{bei}_\nu\left(\sqrt[4]{z}\right)^2-\operatorname{ber}_\nu\left(\sqrt[4]{z}\right)^2=-\sqrt{\frac{\pi}{2}}\,G_{3,7}^{2,2}\left(\frac{z}{16}\,\middle|\,\begin{array}{c}\frac{1}{4},\frac{3}{4},2\nu+\frac{1}{2}\\ \frac{\nu}{2},\frac{\nu+1}{2},0,\frac{1}{2},-\frac{\nu}{2},\frac{1-\nu}{2},2\nu+\frac{1}{2}\end{array}\right)$$

Brychkov Yu.A. (2006)

03.18.26.0010.01

$$\operatorname{bei}_\nu(z)^2+\operatorname{ber}_\nu(z)^2=\pi^{3/2}\,G_{1,5}^{1,0}\left(\frac{z^4}{64}\,\middle|\,\begin{array}{c}\frac{\nu+1}{2}\\ \frac{\nu}{2},0,\frac{1}{2},-\frac{\nu}{2},\frac{\nu+1}{2}\end{array}\right)/;-\frac{\pi}{4}\leq\arg(z)\leq\frac{\pi}{4}$$

Brychkov Yu.A. (2006)

03.18.26.0011.01

$$\operatorname{bei}_\nu(z)^2-\operatorname{ber}_\nu(z)^2=-\sqrt{\frac{\pi}{2}}\,G_{3,7}^{2,2}\left(\frac{z^4}{16}\,\middle|\,\begin{array}{c}\frac{1}{4},\frac{3}{4},2\nu+\frac{1}{2}\\ \frac{\nu}{2},\frac{\nu+1}{2},0,\frac{1}{2},-\frac{\nu}{2},\frac{1-\nu}{2},2\nu+\frac{1}{2}\end{array}\right)/;-\frac{\pi}{4}\leq\arg(z)\leq\frac{\pi}{4}$$

Brychkov Yu.A. (2006)

Classical cases involving bei

03.18.26.0012.01

$$\operatorname{bei}_\nu\left(\sqrt[4]{z}\right)\operatorname{ber}_\nu\left(\sqrt[4]{z}\right)=\frac{\sqrt{\pi}}{2^{3/2}}\,G_{3,7}^{2,2}\left(\frac{z}{16}\,\middle|\,\begin{array}{c}\frac{1}{4},\frac{3}{4},2\nu\\ \frac{\nu}{2},\frac{\nu+1}{2},0,\frac{1}{2},-\frac{\nu}{2},\frac{1-\nu}{2},2\nu\end{array}\right)$$

Brychkov Yu.A. (2006)

03.18.26.0013.01

$$\operatorname{bei}_{-\nu}\left(\sqrt[4]{z}\right)\operatorname{ber}_\nu\left(\sqrt[4]{z}\right)=-\frac{1}{4}\,e^{-\frac{1}{2}(3i\pi\nu)}\,i\sqrt{\pi}\,G_{1,5}^{2,0}\left(-\frac{z}{64}\,\middle|\,\begin{array}{c}\frac{1-\nu}{2}\\ 0,\frac{1}{2},-\frac{\nu}{2},\frac{\nu}{2},\frac{1-\nu}{2}\end{array}\right)+$$
$$\frac{1}{4}\,e^{\frac{3i\pi\nu}{2}}\,i\sqrt{\pi}\,G_{1,5}^{2,0}\left(-\frac{z}{64}\,\middle|\,\begin{array}{c}\frac{\nu+1}{2}\\ 0,\frac{1}{2},-\frac{\nu}{2},\frac{\nu}{2},\frac{\nu+1}{2}\end{array}\right)+\frac{i\pi^{3/2}}{2\sqrt{2}}\,G_{3,7}^{1,2}\left(-\frac{z}{16}\,\middle|\,\begin{array}{c}\frac{1}{4},\frac{3}{4},0\\ \frac{1}{2},0,0,-\frac{\nu}{2},\frac{\nu}{2},\frac{1-\nu}{2},\frac{\nu+1}{2}\end{array}\right)/;-\pi<\arg(z)\leq 0$$

03.18.26.0014.01

$$\operatorname{bei}_\nu\left(\sqrt[4]{z}\right)\operatorname{ber}_\mu\left(\sqrt[4]{z}\right)+\operatorname{bei}_\mu\left(\sqrt[4]{z}\right)\operatorname{ber}_\nu\left(\sqrt[4]{z}\right)=-2^{3/2}\,\pi^{5/2}\,G_{6,10}^{2,3}\left(\frac{z}{16}\,\middle|\,\begin{array}{c}\frac{1}{4},\frac{3}{4},\frac{1}{2},0,\frac{1}{3},\frac{2}{3}\\ \frac{\mu+\nu}{4},\frac{\mu+\nu+2}{4},\frac{1}{3},\frac{2}{3},-\frac{\mu+\nu}{4},\frac{\mu-\nu}{4},\frac{2-\mu-\nu+2}{4},\frac{\nu-\mu}{4},\frac{\mu-\nu+2}{4},\frac{2-\mu+\nu}{4}\end{array}\right)$$

Brychkov Yu.A. (2006)

03.18.26.0015.01

$$\operatorname{bei}_\nu\left(\sqrt[4]{z}\right)\operatorname{ber}_{-\nu}\left(\sqrt[4]{z}\right)+\operatorname{bei}_{-\nu}\left(\sqrt[4]{z}\right)\operatorname{ber}_\nu\left(\sqrt[4]{z}\right)=-2^{3/2}\,\pi^{5/2}\,G_{4,8}^{1,2}\left(\frac{z}{16}\,\middle|\,\begin{array}{c}\frac{1}{4},\frac{3}{4},\frac{1}{3},\frac{2}{3}\\ \frac{1}{2},0,\frac{1}{3},\frac{2}{3},-\frac{\nu}{2},\frac{\nu}{2},\frac{1-\nu}{2},\frac{\nu+1}{2}\end{array}\right)$$

Brychkov Yu.A. (2006)

03.18.26.0016.01

$$\operatorname{ber}_{\mu}\left(\sqrt[4]{z}\right)\operatorname{ber}_{\nu}\left(\sqrt[4]{z}\right)-\operatorname{bei}_{\nu}\left(\sqrt[4]{z}\right)\operatorname{bei}_{\mu}\left(\sqrt[4]{z}\right)=2^{3/2}\,\pi^{5/2}\,G_{6,10}^{2,3}\left(\frac{z}{16}\,\middle|\,\begin{matrix}0,\frac{1}{4},\frac{3}{4},\frac{1}{6},\frac{1}{2},\frac{5}{6}\\ \frac{\mu+\nu}{4},\frac{\mu+\nu+2}{4},\frac{1}{6},\frac{5}{6},-\frac{\mu+\nu}{4},\frac{\mu-\nu}{4},\frac{\nu-\mu}{4},\frac{2-\mu-\nu}{4},\frac{\mu-\nu+2}{4},\frac{2-\mu+\nu}{4}\end{matrix}\right)$$

Brychkov Yu.A. (2006)

03.18.26.0017.01

$$\operatorname{ber}_{-\nu}\left(\sqrt[4]{z}\right)\operatorname{ber}_{\nu}\left(\sqrt[4]{z}\right)-\operatorname{bei}_{-\nu}\left(\sqrt[4]{z}\right)\operatorname{bei}_{\nu}\left(\sqrt[4]{z}\right)=2^{3/2}\,\pi^{5/2}\,G_{5,9}^{2,2}\left(\frac{z}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4},\frac{1}{6},\frac{1}{2},\frac{5}{6}\\ 0,\frac{1}{2},\frac{1}{6},\frac{1}{2},\frac{5}{6},-\frac{\nu}{2},\frac{\nu}{2},\frac{1-\nu}{2},\frac{\nu+1}{2}\end{matrix}\right)$$

Brychkov Yu.A. (2006)

03.18.26.0018.01

$$\operatorname{bei}_{\nu}(z)\operatorname{ber}_{\nu}(z)=\frac{\sqrt{\pi}}{2^{3/2}}\,G_{3,7}^{2,2}\left(\frac{z^4}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4},2\nu\\ \frac{\nu}{2},\frac{\nu+1}{2},0,\frac{1}{2},-\frac{\nu}{2},\frac{1-\nu}{2},2\nu\end{matrix}\right)\;/;\;-\frac{\pi}{4}\leq\arg(z)\leq\frac{\pi}{4}$$

Brychkov Yu.A. (2006)

03.18.26.0019.01

$$\operatorname{bei}_{-\nu}(z)\operatorname{ber}_{\nu}(z)=-\frac{1}{4}\,e^{-\frac{1}{2}(3i\pi\nu)}\,i\sqrt{\pi}\,G_{1,5}^{2,0}\left(-\frac{z^4}{64}\,\middle|\,\begin{matrix}\frac{1-\nu}{2}\\ 0,\frac{1}{2},-\frac{\nu}{2},\frac{\nu}{2},\frac{1-\nu}{2}\end{matrix}\right)+$$

$$\frac{1}{4}\,e^{\frac{3i\pi\nu}{2}}\,i\sqrt{\pi}\,G_{1,5}^{2,0}\left(-\frac{z^4}{64}\,\middle|\,\begin{matrix}\frac{\nu+1}{2}\\ 0,\frac{1}{2},-\frac{\nu}{2},\frac{\nu}{2},\frac{\nu+1}{2}\end{matrix}\right)+\frac{i\,\pi^{3/2}}{2\sqrt{2}}\,G_{3,7}^{1,2}\left(-\frac{z^4}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4},0\\ \frac{1}{2},0,0,-\frac{\nu}{2},\frac{\nu}{2},\frac{1-\nu}{2},\frac{\nu+1}{2}\end{matrix}\right)\;/;\;-\frac{\pi}{2}<\arg(z)\leq 0$$

03.18.26.0020.01

$$\operatorname{bei}_{\nu}(z)\operatorname{ber}_{\mu}(z)+\operatorname{bei}_{\mu}(z)\operatorname{ber}_{\nu}(z)=$$

$$-2^{3/2}\,\pi^{5/2}\,G_{6,10}^{2,3}\left(\frac{z^4}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4},\frac{1}{2},0,\frac{1}{3},\frac{2}{3}\\ \frac{\mu+\nu}{4},\frac{\mu+\nu+2}{4},\frac{1}{3},\frac{2}{3},-\frac{\mu+\nu}{4},\frac{\mu-\nu}{4},\frac{2-\mu-\nu}{4},\frac{\nu-\mu}{4},\frac{\mu-\nu+2}{4},\frac{2-\mu+\nu}{4}\end{matrix}\right)\;/;\;-\frac{\pi}{4}\leq\arg(z)\leq\frac{\pi}{4}$$

Brychkov Yu.A. (2006)

03.18.26.0021.01

$$\operatorname{bei}_{\nu}(z)\operatorname{ber}_{-\nu}(z)+\operatorname{bei}_{-\nu}(z)\operatorname{ber}_{\nu}(z)=-2^{3/2}\,\pi^{5/2}\,G_{4,8}^{1,2}\left(\frac{z^4}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4},\frac{1}{3},\frac{2}{3}\\ \frac{1}{2},0,\frac{1}{3},\frac{2}{3},-\frac{\nu}{2},\frac{\nu}{2},\frac{1-\nu}{2},\frac{\nu+1}{2}\end{matrix}\right)\;/;\;-\frac{\pi}{4}\leq\arg(z)\leq\frac{\pi}{4}$$

Brychkov Yu.A. (2006)

03.18.26.0022.01

$$\operatorname{ber}_{\mu}(z)\operatorname{ber}_{\nu}(z)-\operatorname{bei}_{\nu}(z)\operatorname{bei}_{\mu}(z)=$$

$$2^{3/2}\,\pi^{5/2}\,G_{6,10}^{2,3}\left(\frac{z^4}{16}\,\middle|\,\begin{matrix}0,\frac{1}{4},\frac{3}{4},\frac{1}{6},\frac{1}{2},\frac{5}{6}\\ \frac{\mu+\nu}{4},\frac{\mu+\nu+2}{4},\frac{1}{6},\frac{5}{6},-\frac{\mu+\nu}{4},\frac{\mu-\nu}{4},\frac{\nu-\mu}{4},\frac{2-\mu-\nu}{4},\frac{\mu-\nu+2}{4},\frac{2-\mu+\nu}{4}\end{matrix}\right)\;/;\;-\frac{\pi}{4}\leq\arg(z)\leq\frac{\pi}{4}$$

Brychkov Yu.A. (2006)

03.18.26.0023.01

$$\operatorname{ber}_{-\nu}(z)\operatorname{ber}_\nu(z)-\operatorname{bei}_{-\nu}(z)\operatorname{bei}_\nu(z)=2^{3/2}\pi^{5/2}G_{5,9}^{2,2}\left(\frac{z^4}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4},\frac{1}{6},\frac{1}{2},\frac{5}{6}\\0,\frac{1}{2},\frac{1}{6},\frac{1}{2},\frac{5}{6},-\frac{\nu}{2},\frac{\nu}{2},\frac{1-\nu}{2},\frac{\nu+1}{2}\end{matrix}\right)\;/;\;-\frac{\pi}{4}\leq\arg(z)\leq\frac{\pi}{4}$$

Brychkov Yu.A. (2006)

Classical cases involving kei

03.18.26.0024.01

$$\operatorname{ber}_\nu\left(\sqrt[4]{z}\right)\operatorname{kei}_\nu\left(\sqrt[4]{z}\right)=-\frac{\sqrt{\pi}}{8}G_{1,5}^{3,0}\left(\frac{z}{64}\,\middle|\,\begin{matrix}\frac{3\nu}{2}\\0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2},\frac{3\nu}{2}\end{matrix}\right)-\frac{1}{2^{7/2}\sqrt{\pi}}G_{2,6}^{3,2}\left(\frac{z}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\\frac{1}{2},\frac{\nu}{2},\frac{\nu+1}{2},-\frac{\nu}{2},\frac{1-\nu}{2},0\end{matrix}\right)$$

Brychkov Yu.A. (2006)

03.18.26.0025.01

$$\operatorname{ber}_\nu\left(\sqrt[4]{z}\right)\operatorname{kei}_{-\nu}\left(\sqrt[4]{z}\right)=-\frac{1}{8}\sqrt{\pi}\,G_{0,4}^{2,0}\left(\frac{z}{64}\,\middle|\,0,\frac{1}{2},-\frac{\nu}{2},\frac{\nu}{2}\right)-\frac{1}{8\sqrt{2\pi}}G_{3,7}^{4,2}\left(\frac{z}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4},-\nu\\0,\frac{1}{2},\frac{\nu}{2},\frac{\nu+1}{2},\frac{1-\nu}{2},-\nu,-\frac{\nu}{2}\end{matrix}\right)$$

03.18.26.0026.01

$$\operatorname{ber}_\nu(z)\operatorname{kei}_\nu(z)=-\frac{\sqrt{\pi}}{8}G_{1,5}^{3,0}\left(\frac{z^4}{64}\,\middle|\,\begin{matrix}\frac{3\nu}{2}\\0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2},\frac{3\nu}{2}\end{matrix}\right)-\frac{1}{2^{7/2}\sqrt{\pi}}G_{2,6}^{3,2}\left(\frac{z^4}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\\frac{1}{2},\frac{\nu}{2},\frac{\nu+1}{2},-\frac{\nu}{2},\frac{1-\nu}{2},0\end{matrix}\right)\;/;\;-\frac{\pi}{4}\leq\arg(z)\leq\frac{\pi}{4}$$

Brychkov Yu.A. (2006)

03.18.26.0027.01

$$\operatorname{ber}_\nu(z)\operatorname{kei}_{-\nu}(z)=-\frac{1}{8}\sqrt{\pi}\,G_{0,4}^{2,0}\left(\frac{z^4}{64}\,\middle|\,0,\frac{1}{2},-\frac{\nu}{2},\frac{\nu}{2}\right)-\frac{1}{8\sqrt{2\pi}}G_{3,7}^{4,2}\left(\frac{z^4}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4},-\nu\\0,\frac{1}{2},\frac{\nu}{2},\frac{\nu+1}{2},\frac{1-\nu}{2},-\nu,-\frac{\nu}{2}\end{matrix}\right)\;/;\;-\frac{\pi}{4}\leq\arg(z)\leq\frac{\pi}{4}$$

Classical cases involving ker

03.18.26.0028.01

$$\operatorname{ber}_\nu\left(\sqrt[4]{z}\right)\operatorname{ker}_\nu\left(\sqrt[4]{z}\right)=\frac{1}{8}\sqrt{\pi}\,G_{1,5}^{3,0}\left(\frac{z}{64}\,\middle|\,\begin{matrix}\frac{1}{2}(3\nu+1)\\0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2},\frac{1}{2}(3\nu+1)\end{matrix}\right)+\frac{1}{2^{7/2}\sqrt{\pi}}G_{2,6}^{3,2}\left(\frac{z}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\0,\frac{\nu}{2},\frac{\nu+1}{2},\frac{1}{2},-\frac{\nu}{2},\frac{1-\nu}{2}\end{matrix}\right)$$

Brychkov Yu.A. (2006)

03.18.26.0029.01

$$\operatorname{ber}_\nu\left(\sqrt[4]{z}\right)\operatorname{ker}_{-\nu}\left(\sqrt[4]{z}\right)=\frac{1}{8}\sqrt{\pi}\,G_{1,5}^{3,0}\left(\frac{z}{64}\,\middle|\,\begin{matrix}\frac{\nu+1}{2}\\0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2},\frac{\nu+1}{2}\end{matrix}\right)+\frac{1}{8\sqrt{2\pi}}G_{3,7}^{4,2}\left(\frac{z}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4},\frac{1}{2}-\nu\\0,\frac{1}{2},\frac{\nu}{2},\frac{\nu+1}{2},\frac{1}{2}-\nu,\frac{1-\nu}{2},-\frac{\nu}{2}\end{matrix}\right)$$

03.18.26.0030.01

$$\operatorname{ber}_\nu(z)\operatorname{ker}_\nu(z)=\frac{1}{8}\sqrt{\pi}\,G_{1,5}^{3,0}\left(\frac{z^4}{64}\,\middle|\,\begin{matrix}\frac{1}{2}(3\nu+1)\\0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2},\frac{1}{2}(3\nu+1)\end{matrix}\right)+\frac{1}{2^{7/2}\sqrt{\pi}}G_{2,6}^{3,2}\left(\frac{z^4}{16}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\0,\frac{\nu}{2},\frac{\nu+1}{2},\frac{1}{2},-\frac{\nu}{2},\frac{1-\nu}{2}\end{matrix}\right)\;/;\;-\frac{\pi}{4}\leq\arg(z)\leq\frac{\pi}{4}$$

Brychkov Yu.A. (2006)

03.18.26.0031.01

$$\operatorname{ber}_\nu(z)\operatorname{ker}_{-\nu}(z) = \frac{1}{8}\sqrt{\pi}\, G_{1,5}^{3,0}\!\left(\frac{z^4}{64} \,\middle|\, \begin{matrix} \frac{\nu+1}{2} \\ 0, \frac{1}{2}, \frac{\nu}{2}, -\frac{\nu}{2}, \frac{\nu+1}{2} \end{matrix}\right) + \frac{1}{8\sqrt{2\pi}}\, G_{3,7}^{4,2}\!\left(\frac{z^4}{16} \,\middle|\, \begin{matrix} \frac{1}{4}, \frac{3}{4}, \frac{1}{2}-\nu \\ 0, \frac{1}{2}, \frac{\nu}{2}, \frac{\nu+1}{2}, \frac{1}{2}-\nu, \frac{1-\nu}{2}, -\frac{\nu}{2} \end{matrix}\right) /; -\frac{\pi}{4} \le \arg(z) \le \frac{\pi}{4}$$

Classical cases involving bei, ker and kei

03.18.26.0032.01

$$\operatorname{bei}_\nu\!\left(\sqrt[4]{z}\right)\operatorname{kei}_\nu\!\left(\sqrt[4]{z}\right) + \operatorname{ber}_\nu\!\left(\sqrt[4]{z}\right)\operatorname{ker}_\nu\!\left(\sqrt[4]{z}\right) = \frac{1}{4}\sqrt{\pi}\, G_{1,5}^{3,0}\!\left(\frac{z}{64} \,\middle|\, \begin{matrix} \frac{1}{2}(3\nu+1) \\ 0, \frac{1}{2}, \frac{\nu}{2}, -\frac{\nu}{2}, \frac{1}{2}(3\nu+1) \end{matrix}\right)$$

Brychkov Yu.A. (2006)

03.18.26.0033.01

$$\operatorname{bei}_\nu\!\left(\sqrt[4]{z}\right)\operatorname{kei}_\nu\!\left(\sqrt[4]{z}\right) - \operatorname{ber}_\nu\!\left(\sqrt[4]{z}\right)\operatorname{ker}_\nu\!\left(\sqrt[4]{z}\right) = -\frac{1}{2^{5/2}\sqrt{\pi}}\, G_{2,6}^{3,2}\!\left(\frac{z}{16} \,\middle|\, \begin{matrix} \frac{1}{4}, \frac{3}{4} \\ 0, \frac{\nu}{2}, \frac{\nu+1}{2}, \frac{1}{2}, -\frac{\nu}{2}, \frac{1-\nu}{2} \end{matrix}\right)$$

Brychkov Yu.A. (2006)

03.18.26.0034.01

$$\operatorname{ber}_\nu\!\left(\sqrt[4]{z}\right)\operatorname{kei}_\nu\!\left(\sqrt[4]{z}\right) + \operatorname{bei}_\nu\!\left(\sqrt[4]{z}\right)\operatorname{ker}_\nu\!\left(\sqrt[4]{z}\right) = -\frac{1}{2^{5/2}\sqrt{\pi}}\, G_{2,6}^{3,2}\!\left(\frac{z}{16} \,\middle|\, \begin{matrix} \frac{1}{4}, \frac{3}{4} \\ \frac{1}{2}, \frac{\nu}{2}, \frac{\nu+1}{2}, -\frac{\nu}{2}, \frac{1-\nu}{2}, 0 \end{matrix}\right)$$

Brychkov Yu.A. (2006)

03.18.26.0035.01

$$\operatorname{bei}_\nu\!\left(\sqrt[4]{z}\right)\operatorname{ker}_\nu\!\left(\sqrt[4]{z}\right) - \operatorname{ber}_\nu\!\left(\sqrt[4]{z}\right)\operatorname{kei}_\nu\!\left(\sqrt[4]{z}\right) = \frac{1}{4}\sqrt{\pi}\, G_{1,5}^{3,0}\!\left(\frac{z}{64} \,\middle|\, \begin{matrix} \frac{3\nu}{2} \\ 0, \frac{1}{2}, \frac{\nu}{2}, -\frac{\nu}{2}, \frac{3\nu}{2} \end{matrix}\right)$$

Brychkov Yu.A. (2006)

03.18.26.0036.01

$$\operatorname{bei}_\nu(z)\operatorname{kei}_\nu(z) + \operatorname{ber}_\nu(z)\operatorname{ker}_\nu(z) = \frac{1}{4}\sqrt{\pi}\, G_{1,5}^{3,0}\!\left(\frac{z^4}{64} \,\middle|\, \begin{matrix} \frac{1}{2}(3\nu+1) \\ 0, \frac{1}{2}, \frac{\nu}{2}, -\frac{\nu}{2}, \frac{1}{2}(3\nu+1) \end{matrix}\right) /; -\frac{\pi}{4} \le \arg(z) \le \frac{\pi}{4}$$

Brychkov Yu.A. (2006)

03.18.26.0037.01

$$\operatorname{bei}_\nu(z)\operatorname{kei}_\nu(z) - \operatorname{ber}_\nu(z)\operatorname{ker}_\nu(z) = -\frac{1}{2^{5/2}\sqrt{\pi}}\, G_{2,6}^{3,2}\!\left(\frac{z^4}{16} \,\middle|\, \begin{matrix} \frac{1}{4}, \frac{3}{4} \\ 0, \frac{\nu}{2}, \frac{\nu+1}{2}, \frac{1}{2}, -\frac{\nu}{2}, \frac{1-\nu}{2} \end{matrix}\right) /; -\frac{\pi}{4} \le \arg(z) \le \frac{\pi}{4}$$

Brychkov Yu.A. (2006)

03.18.26.0038.01

$$\operatorname{ber}_\nu(z)\operatorname{kei}_\nu(z) + \operatorname{bei}_\nu(z)\operatorname{ker}_\nu(z) = -\frac{1}{2^{5/2}\sqrt{\pi}}\, G_{2,6}^{3,2}\!\left(\frac{z^4}{16} \,\middle|\, \begin{matrix} \frac{1}{4}, \frac{3}{4} \\ \frac{1}{2}, \frac{\nu}{2}, \frac{\nu+1}{2}, -\frac{\nu}{2}, \frac{1-\nu}{2}, 0 \end{matrix}\right) /; -\frac{\pi}{4} \le \arg(z) \le \frac{\pi}{4}$$

Brychkov Yu.A. (2006)

03.18.26.0039.01

$$\mathrm{bei}_\nu(z)\,\mathrm{ker}_\nu(z)-\mathrm{ber}_\nu(z)\,\mathrm{kei}_\nu(z)=\frac{1}{4}\sqrt{\pi}\,G_{1,5}^{3,0}\!\left(\frac{z^4}{64}\,\middle|\,\begin{matrix}\frac{3\nu}{2}\\ 0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2},\frac{3\nu}{2}\end{matrix}\right)/;-\frac{\pi}{4}\le\arg(z)\le\frac{\pi}{4}$$

Brychkov Yu.A. (2006)

Classical cases involving Bessel *J*

03.18.26.0040.01

$$J_\nu\!\left(\frac{1}{\sqrt[4]{-1}}\,z\right)\mathrm{ber}_\nu(z)=2^{-\frac{3\nu}{2}-1}\,e^{\frac{1}{4}(-3)i\pi\nu}\sqrt{\pi}\,z^\nu\left(\sqrt[4]{-1}\,z\right)^{-\nu}\left(\frac{1}{\sqrt[4]{-1}}\,z\right)^{\nu}\left(G_{0,4}^{1,0}\!\left(-\frac{z^4}{64}\,\middle|\,\frac{\nu}{4},-\frac{\nu}{4},\frac{2-\nu}{4},-\frac{1}{4}(3\nu)\right)+\right.$$
$$\left.2^{\frac{3\nu}{2}}\,e^{\frac{3i\pi\nu}{2}}\csc\!\left(\pi\left(\nu+\frac{3}{4}\right)\right)G_{2,4}^{1,1}\!\left(i\,z^2\,\middle|\,\begin{matrix}\frac{1-\nu}{2},\frac{1}{4}(1-2\nu)\\ \frac{\nu}{2},-\frac{\nu}{2},-\frac{1}{2}(3\nu),\frac{1}{4}(1-2\nu)\end{matrix}\right)\right)/;-\frac{\pi}{2}<\arg(z)\le 0$$

03.18.26.0041.01

$$J_{-\nu}\!\left(\frac{1}{\sqrt[4]{-1}}\,z\right)\mathrm{ber}_\nu(z)=\sqrt{\frac{\pi}{2}}\,z^\nu\left(\sqrt[4]{-1}\,z\right)^{-\nu}\left(\frac{1}{\sqrt[4]{-1}}\,z\right)^{-\nu}$$
$$\left(2^{\frac{1}{2}(3\nu-1)}\,e^{-\frac{1}{4}(3i\pi\nu)}\,G_{1,5}^{2,0}\!\left(-\frac{z^4}{64}\,\middle|\,\begin{matrix}\frac{2-\nu}{4}\\ \frac{\nu}{4},\frac{\nu+2}{4},\frac{3\nu}{4},-\frac{\nu}{4},\frac{2-\nu}{4}\end{matrix}\right)+e^{\frac{3i\pi\nu}{4}}\,G_{2,4}^{1,1}\!\left(i\,z^2\,\middle|\,\begin{matrix}\frac{\nu+1}{2},\frac{1}{4}(2\nu+1)\\ \frac{\nu}{2},-\frac{\nu}{2},\frac{3\nu}{2},\frac{1}{4}(2\nu+1)\end{matrix}\right)\right)/;-\frac{\pi}{2}<\arg(z)\le 0$$

Classical cases involving Bessel *I*

03.18.26.0042.01

$$I_\nu\!\left(\sqrt[4]{-1}\,z\right)\mathrm{ber}_\nu(z)=\frac{1}{2}\,e^{\frac{1}{4}(-3)i\pi\nu}\sqrt{\pi}\,z^\nu\left(\sqrt[4]{-1}\,z\right)^{-\nu}$$
$$\left(G_{0,4}^{1,0}\!\left(-\frac{z^4}{64}\,\middle|\,\frac{\nu}{2},0,\frac{1}{2},-\frac{\nu}{2}\right)+e^{\frac{3i\pi\nu}{2}}\csc\!\left(\pi\left(\nu+\frac{3}{4}\right)\right)G_{2,4}^{1,1}\!\left(i\,z^2\,\middle|\,\begin{matrix}\frac{1}{2},\frac{1}{4}\\ \nu,0,\frac{1}{4},-\nu\end{matrix}\right)\right)/;-\frac{\pi}{2}<\arg(z)\le 0$$

03.18.26.0043.01

$$I_{-\nu}\!\left(\sqrt[4]{-1}\,z\right)\mathrm{ber}_\nu(z)=\sqrt{\frac{\pi}{2}}\,z^\nu\left(\sqrt[4]{-1}\,z\right)^{-\nu}\left(\frac{e^{\frac{1}{4}(-3)i\pi\nu}}{\sqrt{2}}\,G_{1,5}^{2,0}\!\left(-\frac{z^4}{64}\,\middle|\,\begin{matrix}\frac{1-\nu}{2}\\ 0,\frac{1}{2},\frac{1-\nu}{2},-\frac{\nu}{2},\frac{\nu}{2}\end{matrix}\right)+e^{\frac{3i\pi\nu}{4}}\,G_{2,4}^{1,1}\!\left(i\,z^2\,\middle|\,\begin{matrix}\frac{1}{2},\frac{1}{4}\\ 0,\frac{1}{4},\nu,-\nu\end{matrix}\right)\right)/;$$
$$-\frac{\pi}{2}<\arg(z)\le 0$$

03.18.26.0044.01

$$\left(I_\nu\!\left(\sqrt[4]{-1}\,z\right)-I_{-\nu}\!\left(\sqrt[4]{-1}\,z\right)\right)\mathrm{ber}_\nu(z)=\frac{1}{2}\left(\sqrt{\pi}\,\sin(\pi\nu)\right)z^\nu\left(\sqrt[4]{-1}\,z\right)^{-\nu}$$
$$\left(\sqrt{2}\,\csc\!\left(\pi\left(\nu+\frac{3}{4}\right)\right)e^{\frac{3i\pi\nu}{4}}\,G_{3,5}^{2,1}\!\left(i\,z^2\,\middle|\,\begin{matrix}\frac{1}{2},\frac{1}{4},\nu-\frac{1}{4}\\ 0,\nu,-\nu,\frac{1}{4},\nu-\frac{1}{4}\end{matrix}\right)-\frac{e^{-\frac{1}{4}(3i\pi\nu)}}{2\pi^2}\,G_{0,4}^{3,0}\!\left(-\frac{z^4}{64}\,\middle|\,0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2}\right)\right)/;-\frac{\pi}{2}<\arg(z)\le 0$$

Classical cases involving Bessel *K*

03.18.26.0045.01

$$K_\nu\left(\sqrt[4]{-1}\ z\right)\mathrm{ber}_\nu(z) = \frac{1}{4}\left(-\pi^{3/2}\right) z^\nu \left(\sqrt[4]{-1}\ z\right)^{-\nu} \left(\sqrt{2}\ \csc\left(\pi\left(\nu+\frac{3}{4}\right)\right) e^{\frac{3 i \pi \nu}{4}} G_{3,5}^{2,1}\left(i z^2 \,\middle|\, \begin{matrix} \frac{1}{2}, \frac{1}{4}, \nu-\frac{1}{4} \\ 0, \nu, -\nu, \frac{1}{4}, \nu-\frac{1}{4} \end{matrix}\right) - \frac{e^{-\frac{1}{4}(3 i \pi \nu)}}{2\pi^2} G_{0,4}^{3,0}\left(-\frac{z^4}{64} \,\middle|\, 0, \frac{1}{2}, \frac{\nu}{2}, -\frac{\nu}{2}\right)\right) /; -\frac{\pi}{2} < \arg(z) \le 0$$

Classical cases involving ${}_0F_1$

03.18.26.0046.01

$${}_0F_1\left(; \nu+1; \frac{i\sqrt{z}}{4}\right)\mathrm{ber}_\nu\left(\sqrt[4]{z}\right) = \frac{1}{2\sqrt{2}}\sqrt{\pi}\ \Gamma(\nu+1)\left(2^{\frac{1-\nu}{2}} e^{-\frac{3 i \pi \nu}{4}} \pi\, G_{1,5}^{1,0}\left(\frac{z}{64} \,\middle|\, \begin{matrix} \frac{\nu+2}{4} \\ \frac{\nu}{4}, -\frac{\nu}{4}, \frac{2-\nu}{4}, -\frac{1}{4}(3\nu), \frac{\nu+2}{4} \end{matrix}\right) + e^{\frac{3 i \pi \nu}{4}} G_{2,6}^{1,2}\left(\frac{z}{16} \,\middle|\, \begin{matrix} \frac{3-\nu}{4}, \frac{1-\nu}{4} \\ \frac{\nu}{4}, -\frac{\nu}{4}, \frac{\nu+2}{4}, \frac{2-\nu}{4}, \frac{1}{4}(2-3\nu), -\frac{1}{4}(3\nu) \end{matrix}\right) + e^{\frac{3 i \pi \nu}{4}} i\, G_{2,6}^{1,2}\left(\frac{z}{16} \,\middle|\, \begin{matrix} \frac{3-\nu}{4}, \frac{1-\nu}{4} \\ \frac{\nu+2}{4}, -\frac{\nu}{4}, \frac{\nu}{4}, \frac{2-\nu}{4}, \frac{1}{4}(2-3\nu), -\frac{1}{4}(3\nu) \end{matrix}\right)\right)$$

03.18.26.0047.01

$${}_0F_1\left(; 1-\nu; \frac{i\sqrt{z}}{4}\right)\mathrm{ber}_\nu\left(\sqrt[4]{z}\right) = \sqrt{\pi}\ \Gamma(1-\nu)\left(2^{\frac{\nu}{2}-1} e^{-\frac{3 i \pi \nu}{4}} \pi\left(G_{1,5}^{1,0}\left(\frac{z}{64} \,\middle|\, \begin{matrix} \frac{1}{4}(3\nu+2) \\ \frac{\nu}{4}, \frac{3\nu}{4}, \frac{\nu+2}{4}, -\frac{\nu}{4}, \frac{1}{4}(3\nu+2) \end{matrix}\right) + i\tan\left(\frac{\pi\nu}{2}\right) G_{1,5}^{1,0}\left(\frac{z}{64} \,\middle|\, \begin{matrix} \frac{3\nu}{4}+1 \\ \frac{\nu+2}{4}, \frac{\nu}{4}, \frac{3\nu}{4}, -\frac{\nu}{4}, \frac{3\nu}{4}+1 \end{matrix}\right)\right) + \frac{e^{\frac{3 i \pi \nu}{4}}}{2\sqrt{2}}\left(G_{2,6}^{1,2}\left(\frac{z}{16} \,\middle|\, \begin{matrix} \frac{\nu+1}{4}, \frac{\nu+3}{4} \\ \frac{\nu}{4}, \frac{\nu+2}{4}, \frac{1}{4}(3\nu+2), \frac{3\nu}{4}, \frac{2-\nu}{4}, -\frac{\nu}{4} \end{matrix}\right) + i\, G_{2,6}^{1,2}\left(\frac{z}{16} \,\middle|\, \begin{matrix} \frac{\nu+1}{4}, \frac{\nu+3}{4} \\ \frac{\nu+2}{4}, \frac{\nu}{4}, \frac{1}{4}(3\nu+2), \frac{3\nu}{4}, \frac{2-\nu}{4}, -\frac{\nu}{4} \end{matrix}\right)\right)\right)$$

03.18.26.0048.01

$${}_0F_1\left(; \nu+1; \frac{i z^2}{4}\right)\mathrm{ber}_\nu(z) = \frac{1}{2\sqrt{2}}\sqrt{\pi}\ \Gamma(\nu+1)\left(2^{\frac{1-\nu}{2}} e^{-\frac{3 i \pi \nu}{4}} \pi\, G_{1,5}^{1,0}\left(\frac{z^4}{64} \,\middle|\, \begin{matrix} \frac{\nu+2}{4} \\ \frac{\nu}{4}, -\frac{\nu}{4}, \frac{2-\nu}{4}, -\frac{1}{4}(3\nu), \frac{\nu+2}{4} \end{matrix}\right) + e^{\frac{3 i \pi \nu}{4}} G_{2,6}^{1,2}\left(\frac{z^4}{16} \,\middle|\, \begin{matrix} \frac{3-\nu}{4}, \frac{1-\nu}{4} \\ \frac{\nu}{4}, -\frac{\nu}{4}, \frac{\nu+2}{4}, \frac{2-\nu}{4}, \frac{1}{4}(2-3\nu), -\frac{1}{4}(3\nu) \end{matrix}\right) + e^{\frac{3 i \pi \nu}{4}} i\, G_{2,6}^{1,2}\left(\frac{z^4}{16} \,\middle|\, \begin{matrix} \frac{3-\nu}{4}, \frac{1-\nu}{4} \\ \frac{\nu+2}{4}, -\frac{\nu}{4}, \frac{\nu}{4}, \frac{2-\nu}{4}, \frac{1}{4}(2-3\nu), -\frac{1}{4}(3\nu) \end{matrix}\right)\right) /; -\frac{\pi}{4} \le \arg(z) \le \frac{\pi}{4}$$

03.18.26.0049.01

$${}_0F_1\left(; 1-\nu; \frac{i z^2}{4}\right)\mathrm{ber}_\nu(z) = \sqrt{\pi}\ \Gamma(1-\nu)\left(2^{\frac{\nu}{2}-1} e^{-\frac{3 i \pi \nu}{4}} \pi\left(G_{1,5}^{1,0}\left(\frac{z^4}{64} \,\middle|\, \begin{matrix} \frac{1}{4}(3\nu+2) \\ \frac{\nu}{4}, \frac{3\nu}{4}, \frac{\nu+2}{4}, -\frac{\nu}{4}, \frac{1}{4}(3\nu+2) \end{matrix}\right) + i\tan\left(\frac{\pi\nu}{2}\right) G_{1,5}^{1,0}\left(\frac{z^4}{64} \,\middle|\, \begin{matrix} \frac{3\nu}{4}+1 \\ \frac{\nu+2}{4}, \frac{\nu}{4}, \frac{3\nu}{4}, -\frac{\nu}{4}, \frac{3\nu}{4}+1 \end{matrix}\right)\right) + \frac{e^{\frac{3 i \pi \nu}{4}}}{2\sqrt{2}}\left(G_{2,6}^{1,2}\left(\frac{z^4}{16} \,\middle|\, \begin{matrix} \frac{\nu+1}{4}, \frac{\nu+3}{4} \\ \frac{\nu}{4}, \frac{\nu+2}{4}, \frac{1}{4}(3\nu+2), \frac{3\nu}{4}, \frac{2-\nu}{4}, -\frac{\nu}{4} \end{matrix}\right) + i\, G_{2,6}^{1,2}\left(\frac{z^4}{16} \,\middle|\, \begin{matrix} \frac{\nu+1}{4}, \frac{\nu+3}{4} \\ \frac{\nu+2}{4}, \frac{\nu}{4}, \frac{1}{4}(3\nu+2), \frac{3\nu}{4}, \frac{2-\nu}{4}, -\frac{\nu}{4} \end{matrix}\right)\right)\right) /; -\frac{\pi}{4} \le \arg(z) \le \frac{\pi}{4}$$

03.18.26.0050.01

$${}_0F_1\left(;\nu+1;\frac{i z^2}{4}\right)\mathrm{ber}_\nu(z)=2^{-\frac{\nu}{2}-1}\,e^{\frac{1}{4}(-3)i\pi\nu}\sqrt{\pi}\,z^\nu\left(\sqrt[4]{-1}\,z\right)^{-\nu}\Gamma(\nu+1)\left(G_{0,4}^{1,0}\left(-\frac{z^4}{64}\,\middle|\,\frac{\nu}{4},-\frac{\nu}{4},\frac{2-\nu}{4},-\frac{1}{4}(3\nu)\right)+ 2^{\frac{3\nu}{2}}\,e^{\frac{3i\pi\nu}{2}}\csc\left(\pi\left(\nu+\frac{3}{4}\right)\right)G_{2,4}^{1,1}\left(i z^2\,\middle|\,\begin{matrix}\frac{1-\nu}{2},\frac{1}{4}(1-2\nu)\\ \frac{\nu}{2},-\frac{\nu}{2},-\frac{1}{2}(3\nu),\frac{1}{4}(1-2\nu)\end{matrix}\right)\right)/;-\frac{\pi}{2}<\arg(z)\le 0$$

03.18.26.0051.01

$${}_0F_1\left(;1-\nu;\frac{i z^2}{4}\right)\mathrm{ber}_\nu(z)=2^{-\nu-\frac{1}{2}}\,e^{\frac{1}{4}(-3)i\pi\nu}\sqrt{\pi}\,z^\nu\left(\sqrt[4]{-1}\,z\right)^{-\nu}\Gamma(1-\nu)\left(2^{\frac{1}{2}(3\nu-1)}G_{1,5}^{2,0}\left(-\frac{z^4}{64}\,\middle|\,\begin{matrix}\frac{2-\nu}{4}\\ \frac{\nu}{4},\frac{\nu+2}{4},\frac{3\nu}{4},-\frac{\nu}{4},\frac{2-\nu}{4}\end{matrix}\right)+e^{\frac{3i\pi\nu}{2}}G_{2,4}^{1,1}\left(i z^2\,\middle|\,\begin{matrix}\frac{\nu+1}{2},\frac{1}{4}(2\nu+1)\\ \frac{\nu}{2},-\frac{\nu}{2},\frac{3\nu}{2},\frac{1}{4}(2\nu+1)\end{matrix}\right)\right)/;-\frac{\pi}{2}<\arg(z)\le 0$$

Classical cases involving ${}_0\tilde{F}_1$

03.18.26.0052.01

$${}_0\tilde{F}_1\left(;\nu+1;\frac{i\sqrt{z}}{4}\right)\mathrm{ber}_\nu\left(\sqrt[4]{z}\right)=\frac{1}{2\sqrt{2}}\sqrt{\pi}\left(2^{\frac{1-\nu}{2}}e^{-\frac{3i\pi\nu}{4}}\pi\,G_{1,5}^{1,0}\left(\frac{z}{64}\,\middle|\,\begin{matrix}\frac{\nu+2}{4}\\ \frac{\nu}{4},-\frac{\nu}{4},\frac{2-\nu}{4},-\frac{1}{4}(3\nu),\frac{\nu+2}{4}\end{matrix}\right)+e^{\frac{3i\pi\nu}{4}}G_{2,6}^{1,2}\left(\frac{z}{16}\,\middle|\,\begin{matrix}\frac{3-\nu}{4},\frac{1-\nu}{4}\\ \frac{\nu}{4},-\frac{\nu}{4},\frac{\nu+2}{4},\frac{2-\nu}{4},\frac{1}{4}(2-3\nu),-\frac{1}{4}(3\nu)\end{matrix}\right)+ e^{\frac{3i\pi\nu}{4}}i\,G_{2,6}^{1,2}\left(\frac{z}{16}\,\middle|\,\begin{matrix}\frac{3-\nu}{4},\frac{1-\nu}{4}\\ \frac{\nu+2}{4},-\frac{\nu}{4},\frac{\nu}{4},\frac{2-\nu}{4},\frac{1}{4}(2-3\nu),-\frac{1}{4}(3\nu)\end{matrix}\right)\right)$$

03.18.26.0053.01

$${}_0\tilde{F}_1\left(;1-\nu;\frac{i\sqrt{z}}{4}\right)\mathrm{ber}_\nu\left(\sqrt[4]{z}\right)=\sqrt{\pi}\left(2^{\frac{\nu}{2}-1}e^{-\frac{3i\pi\nu}{4}}\pi\left(G_{1,5}^{1,0}\left(\frac{z}{64}\,\middle|\,\begin{matrix}\frac{1}{4}(3\nu+2)\\ \frac{\nu}{4},\frac{3\nu}{4},\frac{\nu+2}{4},-\frac{\nu}{4},\frac{1}{4}(3\nu+2)\end{matrix}\right)+i\tan\left(\frac{\pi\nu}{2}\right)G_{1,5}^{1,0}\left(\frac{z}{64}\,\middle|\,\begin{matrix}\frac{3\nu}{4}+1\\ \frac{\nu+2}{4},\frac{\nu}{4},\frac{3\nu}{4},-\frac{\nu}{4},\frac{3\nu}{4}+1\end{matrix}\right)\right)+ \frac{e^{\frac{3i\pi\nu}{4}}}{2\sqrt{2}}\left(G_{2,6}^{1,2}\left(\frac{z}{16}\,\middle|\,\begin{matrix}\frac{\nu+1}{4},\frac{\nu+3}{4}\\ \frac{\nu}{4},\frac{\nu+2}{4},\frac{1}{4}(3\nu+2),\frac{3\nu}{4},\frac{2-\nu}{4},-\frac{\nu}{4}\end{matrix}\right)+i\,G_{2,6}^{1,2}\left(\frac{z}{16}\,\middle|\,\begin{matrix}\frac{\nu+1}{4},\frac{\nu+3}{4}\\ \frac{\nu+2}{4},\frac{\nu}{4},\frac{1}{4}(3\nu+2),\frac{3\nu}{4},\frac{2-\nu}{4},-\frac{\nu}{4}\end{matrix}\right)\right)\right)$$

03.18.26.0054.01

$${}_0\tilde{F}_1\left(;\nu+1;\frac{i z^2}{4}\right)\mathrm{ber}_\nu(z)=\frac{1}{2\sqrt{2}}\sqrt{\pi}\left(2^{\frac{1-\nu}{2}}e^{-\frac{3i\pi\nu}{4}}\pi\,G_{1,5}^{1,0}\left(\frac{z^4}{64}\,\middle|\,\begin{matrix}\frac{\nu+2}{4}\\ \frac{\nu}{4},-\frac{\nu}{4},\frac{2-\nu}{4},-\frac{1}{4}(3\nu),\frac{\nu+2}{4}\end{matrix}\right)+e^{\frac{3i\pi\nu}{4}}G_{2,6}^{1,2}\left(\frac{z^4}{16}\,\middle|\,\begin{matrix}\frac{3-\nu}{4},\frac{1-\nu}{4}\\ \frac{\nu}{4},-\frac{\nu}{4},\frac{\nu+2}{4},\frac{2-\nu}{4},\frac{1}{4}(2-3\nu),-\frac{1}{4}(3\nu)\end{matrix}\right)+ e^{\frac{3i\pi\nu}{4}}i\,G_{2,6}^{1,2}\left(\frac{z^4}{16}\,\middle|\,\begin{matrix}\frac{3-\nu}{4},\frac{1-\nu}{4}\\ \frac{\nu+2}{4},-\frac{\nu}{4},\frac{\nu}{4},\frac{2-\nu}{4},\frac{1}{4}(2-3\nu),-\frac{1}{4}(3\nu)\end{matrix}\right)\right)/;-\frac{\pi}{4}\le\arg(z)\le\frac{\pi}{4}$$

03.18.26.0055.01

$${}_0\tilde{F}_1\left(;1-\nu;\frac{i z^2}{4}\right)\operatorname{ber}_\nu(z)=$$
$$\sqrt{\pi}\left(2^{\frac{\nu}{2}-1}e^{-\frac{3i\pi\nu}{4}}\pi\left(G_{1,5}^{1,0}\left(\frac{z^4}{64}\middle|\begin{array}{c}\frac{1}{4}(3\nu+2)\\ \frac{\nu}{4},\frac{3\nu}{4},\frac{\nu+2}{4},-\frac{\nu}{4},\frac{1}{4}(3\nu+2)\end{array}\right)+i\tan\left(\frac{\pi\nu}{2}\right)G_{1,5}^{1,0}\left(\frac{z^4}{64}\middle|\begin{array}{c}\frac{3\nu}{4}+1\\ \frac{\nu+2}{4},\frac{\nu}{4},\frac{3\nu}{4},-\frac{\nu}{4},\frac{3\nu}{4}+1\end{array}\right)\right)+\frac{e^{\frac{3i\pi\nu}{4}}}{2\sqrt{2}}\right.$$
$$\left.\left(G_{2,6}^{1,2}\left(\frac{z^4}{16}\middle|\begin{array}{c}\frac{\nu+1}{4},\frac{\nu+3}{4}\\ \frac{\nu}{4},\frac{\nu+2}{4},\frac{1}{4}(3\nu+2),\frac{3\nu}{4},\frac{2-\nu}{4},-\frac{\nu}{4}\end{array}\right)+i\,G_{2,6}^{1,2}\left(\frac{z^4}{16}\middle|\begin{array}{c}\frac{\nu+1}{4},\frac{\nu+3}{4}\\ \frac{\nu+2}{4},\frac{\nu}{4},\frac{1}{4}(3\nu+2),\frac{3\nu}{4},\frac{2-\nu}{4},-\frac{\nu}{4}\end{array}\right)\right)\right)\;/;\;-\frac{\pi}{4}\leq\arg(z)\leq\frac{\pi}{4}$$

03.18.26.0056.01

$${}_0\tilde{F}_1\left(;\nu+1;\frac{i z^2}{4}\right)\operatorname{ber}_\nu(z)=2^{-\frac{\nu}{2}-1}e^{\frac{1}{4}(-3)i\pi\nu}\sqrt{\pi}\,z^\nu\left(\sqrt[4]{-1}\,z\right)^{-\nu}\left(G_{0,4}^{1,0}\left(-\frac{z^4}{64}\middle|\frac{\nu}{4},-\frac{\nu}{4},\frac{2-\nu}{4},-\frac{1}{4}(3\nu)\right)+\right.$$
$$\left.2^{\frac{3\nu}{2}}e^{\frac{3i\pi\nu}{2}}\csc\left(\pi\left(\nu+\frac{3}{4}\right)\right)G_{2,4}^{1,1}\left(i z^2\middle|\begin{array}{c}\frac{1-\nu}{2},\frac{1}{4}(1-2\nu)\\ \frac{\nu}{2},-\frac{\nu}{2},-\frac{1}{2}(3\nu),\frac{1}{4}(1-2\nu)\end{array}\right)\right)\;/;\;-\frac{\pi}{2}<\arg(z)\leq 0$$

03.18.26.0057.01

$${}_0\tilde{F}_1\left(;1-\nu;\frac{i z^2}{4}\right)\operatorname{ber}_\nu(z)=2^{-\nu-\frac{1}{2}}e^{\frac{1}{4}(-3)i\pi\nu}\sqrt{\pi}\,z^\nu\left(\sqrt[4]{-1}\,z\right)^{-\nu}$$
$$\left(2^{\frac{1}{2}(3\nu-1)}G_{1,5}^{2,0}\left(-\frac{z^4}{64}\middle|\begin{array}{c}\frac{2-\nu}{4}\\ \frac{\nu}{4},\frac{\nu+2}{4},\frac{3\nu}{4},-\frac{\nu}{4},\frac{2-\nu}{4}\end{array}\right)+e^{\frac{3i\pi\nu}{2}}G_{2,4}^{1,1}\left(i z^2\middle|\begin{array}{c}\frac{\nu+1}{2},\frac{1}{4}(2\nu+1)\\ \frac{\nu}{2},-\frac{\nu}{2},\frac{3\nu}{2},\frac{1}{4}(2\nu+1)\end{array}\right)\right)\;/;\;-\frac{\pi}{2}<\arg(z)\leq 0$$

Generalized cases for the direct function itself

03.18.26.0058.01

$$\operatorname{ber}_\nu(z)=\pi\,G_{1,5}^{2,0}\left(\frac{z}{4},\frac{1}{4}\middle|\begin{array}{c}\frac{1}{2}(2\nu+1)\\ \frac{\nu}{4},\frac{\nu+2}{4},-\frac{\nu}{4},\frac{2-\nu}{4},\nu+\frac{1}{2}\end{array}\right)$$

03.18.26.0059.01

$$\operatorname{ber}_{-\nu}(z)+\operatorname{ber}_\nu(z)=2\pi\cos\left(\frac{\pi\nu}{2}\right)G_{3,7}^{4,0}\left(\frac{z}{4},\frac{1}{4}\middle|\begin{array}{c}0,\frac{1-\nu}{2},\frac{\nu+1}{2}\\ \frac{\nu}{4},-\frac{\nu}{4},\frac{\nu+2}{4},\frac{2-\nu}{4},0,\frac{1-\nu}{2},\frac{\nu+1}{2}\end{array}\right)$$

Generalized cases for powers of ber

03.18.26.0060.01

$$\operatorname{ber}_\nu(z)^2=\frac{1}{2}\pi^{3/2}G_{1,5}^{1,0}\left(\frac{z}{2\sqrt{2}},\frac{1}{4}\middle|\begin{array}{c}\frac{\nu+1}{2}\\ \frac{\nu}{2},0,\frac{1}{2},-\frac{\nu}{2},\frac{\nu+1}{2}\end{array}\right)+\frac{\sqrt{\pi}}{2^{3/2}}G_{3,7}^{2,2}\left(\frac{z}{2},\frac{1}{4}\middle|\begin{array}{c}\frac{1}{4},\frac{3}{4},2\nu+\frac{1}{2}\\ \frac{\nu}{2},\frac{\nu+1}{2},0,\frac{1}{2},-\frac{\nu}{2},\frac{1-\nu}{2},2\nu+\frac{1}{2}\end{array}\right)$$

Brychkov Yu.A. (2006)

Generalized cases for products of ber

03.18.26.0061.01

$$\operatorname{ber}_{-\nu}(z)\operatorname{ber}_{\nu}(z)=\frac{1}{4}\sqrt{\pi}\left(e^{-\frac{1}{2}(3i\pi\nu)}G_{1,5}^{2,0}\left(\frac{\sqrt[4]{-1}\,z}{2\sqrt{2}},\frac{1}{4}\middle|\begin{array}{c}\frac{1-\nu}{2}\\0,\frac{1}{2},-\frac{\nu}{2},\frac{\nu}{2},\frac{1-\nu}{2}\end{array}\right)+e^{\frac{3i\pi\nu}{2}}G_{1,5}^{2,0}\left(\frac{\sqrt[4]{-1}\,z}{2\sqrt{2}},\frac{1}{4}\middle|\begin{array}{c}\frac{\nu+1}{2}\\0,\frac{1}{2},-\frac{\nu}{2},\frac{\nu}{2},\frac{\nu+1}{2}\end{array}\right)\right)+\frac{\pi^{3/2}}{2\sqrt{2}}G_{3,7}^{1,2}\left(\frac{1}{2}\sqrt[4]{-1}\,z,\frac{1}{4}\middle|\begin{array}{c}\frac{1}{4},\frac{3}{4},\frac{1}{2}\\0,\frac{1}{2},\frac{1}{2},-\frac{\nu}{2},\frac{\nu}{2},\frac{1-\nu}{2},\frac{\nu+1}{2}\end{array}\right)$$

Generalized cases involving powers of bei

03.18.26.0062.01

$$\operatorname{bei}_{\nu}(z)^2+\operatorname{ber}_{\nu}(z)^2=\pi^{3/2}G_{1,5}^{1,0}\left(\frac{z}{2\sqrt{2}},\frac{1}{4}\middle|\begin{array}{c}\frac{\nu+1}{2}\\\frac{\nu}{2},0,\frac{1}{2},-\frac{\nu}{2},\frac{\nu+1}{2}\end{array}\right)$$

Brychkov Yu.A. (2006)

03.18.26.0063.01

$$\operatorname{bei}_{\nu}(z)^2-\operatorname{ber}_{\nu}(z)^2=-\sqrt{\frac{\pi}{2}}\,G_{3,7}^{2,2}\left(\frac{z}{2},\frac{1}{4}\middle|\begin{array}{c}\frac{1}{4},\frac{3}{4},2\nu+\frac{1}{2}\\\frac{\nu}{2},\frac{\nu+1}{2},0,\frac{1}{2},-\frac{\nu}{2},\frac{1-\nu}{2},2\nu+\frac{1}{2}\end{array}\right)$$

Brychkov Yu.A. (2006)

Generalized cases involving bei

03.18.26.0064.01

$$\operatorname{bei}_{\nu}(z)\operatorname{ber}_{\nu}(z)=\frac{\sqrt{\pi}}{2\sqrt{2}}G_{3,7}^{2,2}\left(\frac{z}{2},\frac{1}{4}\middle|\begin{array}{c}\frac{1}{4},\frac{3}{4},2\nu\\\frac{\nu}{2},\frac{\nu+1}{2},0,\frac{1}{2},-\frac{\nu}{2},\frac{1-\nu}{2},2\nu\end{array}\right)$$

Brychkov Yu.A. (2006)

03.18.26.0065.01

$$\operatorname{bei}_{-\nu}(z)\operatorname{ber}_{\nu}(z)=-\frac{1}{4}e^{-\frac{3i\pi\nu}{2}}i\sqrt{\pi}\,G_{1,5}^{2,0}\left(\frac{\sqrt[4]{-1}\,z}{2\sqrt{2}},\frac{1}{4}\middle|\begin{array}{c}\frac{1-\nu}{2}\\0,\frac{1}{2},-\frac{\nu}{2},\frac{\nu}{2},\frac{1-\nu}{2}\end{array}\right)+\frac{1}{4}e^{\frac{3i\pi\nu}{2}}i\sqrt{\pi}\,G_{1,5}^{2,0}\left(\frac{\sqrt[4]{-1}\,z}{2\sqrt{2}},\frac{1}{4}\middle|\begin{array}{c}\frac{\nu+1}{2}\\0,\frac{1}{2},-\frac{\nu}{2},\frac{\nu}{2},\frac{\nu+1}{2}\end{array}\right)+\frac{i\pi^{3/2}}{2\sqrt{2}}G_{3,7}^{1,2}\left(\frac{1}{2}\sqrt[4]{-1}\,z,\frac{1}{4}\middle|\begin{array}{c}\frac{1}{4},\frac{3}{4},0\\\frac{1}{2},0,0,-\frac{\nu}{2},\frac{\nu}{2},\frac{1-\nu}{2},\frac{\nu+1}{2}\end{array}\right)$$

03.18.26.0066.01

$$\operatorname{bei}_{\nu}(z)\operatorname{ber}_{\mu}(z)+\operatorname{bei}_{\mu}(z)\operatorname{ber}_{\nu}(z)=-2^{3/2}\pi^{5/2}G_{6,10}^{2,3}\left(\frac{z}{2},\frac{1}{4}\middle|\begin{array}{c}\frac{1}{4},\frac{3}{4},\frac{1}{2},0,\frac{1}{3},\frac{2}{3}\\\frac{\mu+\nu}{4},\frac{\mu+\nu+2}{4},\frac{1}{3},\frac{2}{3},-\frac{\mu+\nu}{4},\frac{\mu-\nu}{4},\frac{2-\mu-\nu}{4},\frac{\nu-\mu}{4},\frac{\mu-\nu+2}{4},\frac{2-\mu+\nu}{4}\end{array}\right)$$

Brychkov Yu.A. (2006)

03.18.26.0067.01

$$\operatorname{bei}_{\nu}(z)\operatorname{ber}_{-\nu}(z)+\operatorname{bei}_{-\nu}(z)\operatorname{ber}_{\nu}(z)=-2^{3/2}\pi^{5/2}G_{4,8}^{1,2}\left(\frac{z}{2},\frac{1}{4}\middle|\begin{array}{c}\frac{1}{4},\frac{3}{4},\frac{1}{3},\frac{2}{3}\\\frac{1}{2},0,\frac{1}{3},\frac{2}{3},-\frac{\nu}{2},\frac{\nu}{2},\frac{1-\nu}{2},\frac{\nu+1}{2}\end{array}\right)$$

Brychkov Yu.A. (2006)

03.18.26.0068.01

$$\operatorname{ber}_\mu(z)\operatorname{ber}_\nu(z)-\operatorname{bei}_\nu(z)\operatorname{bei}_\mu(z)=2^{3/2}\,\pi^{5/2}\,G_{6,10}^{2,3}\!\left(\frac{z}{2},\frac{1}{4}\,\middle|\,\begin{matrix}0,\frac{1}{4},\frac{3}{4},\frac{1}{6},\frac{1}{2},\frac{5}{6}\\ \frac{\mu+\nu}{4},\frac{\mu+\nu+2}{4},\frac{1}{6},\frac{5}{6},-\frac{\mu+\nu}{4},\frac{\mu-\nu}{4},\frac{\nu-\mu}{4},\frac{2-\mu-\nu}{4},\frac{\mu-\nu+2}{4},\frac{2-\mu+\nu}{4}\end{matrix}\right)$$

Brychkov Yu.A. (2006)

03.18.26.0069.01

$$\operatorname{ber}_{-\nu}(z)\operatorname{ber}_\nu(z)-\operatorname{bei}_{-\nu}(z)\operatorname{bei}_\nu(z)=2^{3/2}\,\pi^{5/2}\,G_{5,9}^{2,2}\!\left(\frac{z}{2},\frac{1}{4}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4},\frac{1}{6},\frac{1}{2},\frac{5}{6}\\ 0,\frac{1}{2},\frac{1}{6},\frac{1}{2},\frac{5}{6},-\frac{\nu}{2},\frac{\nu}{2},\frac{1-\nu}{2},\frac{\nu+1}{2}\end{matrix}\right)$$

Brychkov Yu.A. (2006)

Generalized cases involving kei

03.18.26.0070.01

$$\operatorname{ber}_\nu(z)\operatorname{kei}_\nu(z)=\frac{1}{8}\left(-\sqrt{\pi}\right)G_{1,5}^{3,0}\!\left(\frac{z}{2\sqrt{2}},\frac{1}{4}\,\middle|\,\begin{matrix}\frac{3\nu}{2}\\ 0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2},\frac{3\nu}{2}\end{matrix}\right)-\frac{1}{2^{7/2}\sqrt{\pi}}\,G_{2,6}^{3,2}\!\left(\frac{z}{2},\frac{1}{4}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\ \frac{1}{2},\frac{\nu}{2},\frac{\nu+1}{2},-\frac{\nu}{2},\frac{1-\nu}{2},0\end{matrix}\right)$$

Brychkov Yu.A. (2006)

03.18.26.0071.01

$$\operatorname{ber}_\nu(z)\operatorname{kei}_{-\nu}(z)=-\frac{1}{8}\sqrt{\pi}\,G_{0,4}^{2,0}\!\left(\frac{z}{2\sqrt{2}},\frac{1}{4}\,\middle|\,0,\frac{1}{2},-\frac{\nu}{2},\frac{\nu}{2}\right)-\frac{1}{8\sqrt{2\pi}}\,G_{3,7}^{4,2}\!\left(\frac{z}{2},\frac{1}{4}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4},-\nu\\ 0,\frac{1}{2},\frac{\nu}{2},\frac{\nu+1}{2},\frac{1-\nu}{2},-\nu,-\frac{\nu}{2}\end{matrix}\right)$$

Generalized cases involving ker

03.18.26.0072.01

$$\operatorname{ber}_\nu(z)\operatorname{ker}_\nu(z)=\frac{1}{8}\sqrt{\pi}\,G_{1,5}^{3,0}\!\left(\frac{z}{2\sqrt{2}},\frac{1}{4}\,\middle|\,\begin{matrix}\frac{1}{2}(3\nu+1)\\ 0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2},\frac{1}{2}(3\nu+1)\end{matrix}\right)+\frac{1}{2^{7/2}\sqrt{\pi}}\,G_{2,6}^{3,2}\!\left(\frac{z}{2},\frac{1}{4}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\ 0,\frac{\nu}{2},\frac{\nu+1}{2},\frac{1}{2},-\frac{\nu}{2},\frac{1-\nu}{2}\end{matrix}\right)$$

Brychkov Yu.A. (2006)

03.18.26.0073.01

$$\operatorname{ber}_\nu(z)\operatorname{ker}_{-\nu}(z)=\frac{1}{8}\sqrt{\pi}\,G_{1,5}^{3,0}\!\left(\frac{z}{2\sqrt{2}},\frac{1}{4}\,\middle|\,\begin{matrix}\frac{\nu+1}{2}\\ 0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2},\frac{\nu+1}{2}\end{matrix}\right)+\frac{1}{8\sqrt{2\pi}}\,G_{3,7}^{4,2}\!\left(\frac{z}{2},\frac{1}{4}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4},\frac{1}{2}-\nu\\ 0,\frac{1}{2},\frac{\nu}{2},\frac{\nu+1}{2},\frac{1}{2}-\nu,\frac{1-\nu}{2},-\frac{\nu}{2}\end{matrix}\right)$$

Generalized cases involving bei, ker and kei

03.18.26.0074.01

$$\operatorname{bei}_\nu(z)\operatorname{kei}_\nu(z)+\operatorname{ber}_\nu(z)\operatorname{ker}_\nu(z)=\frac{1}{4}\sqrt{\pi}\,G_{1,5}^{3,0}\!\left(\frac{z}{2\sqrt{2}},\frac{1}{4}\,\middle|\,\begin{matrix}\frac{1}{2}(3\nu+1)\\ 0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2},\frac{1}{2}(3\nu+1)\end{matrix}\right)$$

Brychkov Yu.A. (2006)

03.18.26.0075.01

$$\operatorname{bei}_\nu(z)\operatorname{kei}_\nu(z)-\operatorname{ber}_\nu(z)\operatorname{ker}_\nu(z)=-\frac{1}{2^{5/2}\sqrt{\pi}}\,G_{2,6}^{3,2}\!\left(\frac{z}{2},\frac{1}{4}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\ 0,\frac{\nu}{2},\frac{\nu+1}{2},\frac{1}{2},-\frac{\nu}{2},\frac{1-\nu}{2}\end{matrix}\right)$$

Brychkov Yu.A. (2006)

03.18.26.0076.01

$$\operatorname{ber}_\nu(z)\operatorname{kei}_\nu(z)+\operatorname{bei}_\nu(z)\operatorname{ker}_\nu(z)=-\frac{1}{2^{5/2}\sqrt{\pi}}\,G_{2,6}^{3,2}\!\left(\frac{z}{2},\frac{1}{4}\,\middle|\,\begin{matrix}\frac{1}{4},\frac{3}{4}\\ \frac{1}{2},\frac{\nu}{2},\frac{\nu+1}{2},-\frac{\nu}{2},\frac{1-\nu}{2},0\end{matrix}\right)$$

Brychkov Yu.A. (2006)

03.18.26.0077.01

$$\operatorname{bei}_\nu(z)\operatorname{ker}_\nu(z)-\operatorname{ber}_\nu(z)\operatorname{kei}_\nu(z)=\frac{1}{4}\sqrt{\pi}\,G_{1,5}^{3,0}\!\left(\frac{z}{2\sqrt{2}},\frac{1}{4}\,\middle|\,\begin{matrix}\frac{3\nu}{2}\\ 0,\frac{1}{2},\frac{\nu}{2},-\frac{\nu}{2},\frac{3\nu}{2}\end{matrix}\right)$$

Brychkov Yu.A. (2006)

Generalized cases involving Bessel *J*

03.18.26.0078.01

$$J_\nu\!\left(\frac{1}{\sqrt[4]{-1}}z\right)\operatorname{ber}_\nu(z)=2^{-\frac{3\nu}{2}-1}e^{\frac{1}{4}(-3)i\pi\nu}\sqrt{\pi}\,z^\nu\left(\sqrt[4]{-1}\,z\right)^{-\nu}\left(\frac{1}{\sqrt[4]{-1}}z\right)^{\nu}$$
$$\left(G_{0,4}^{1,0}\!\left(\frac{\sqrt[4]{-1}\,z}{2\sqrt{2}},\frac{1}{4}\,\middle|\,\frac{\nu}{4},-\frac{\nu}{4},\frac{2-\nu}{4},-\frac{1}{4}(3\nu)\right)+2^{\frac{3\nu}{2}}e^{\frac{3i\pi\nu}{2}}\csc\!\left(\pi\left(\nu+\frac{3}{4}\right)\right)G_{2,4}^{1,1}\!\left(\sqrt[4]{-1}\,z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1-\nu}{2},\frac{1}{4}(1-2\nu)\\ \frac{\nu}{2},-\frac{\nu}{2},-\frac{1}{2}(3\nu),\frac{1}{4}(1-2\nu)\end{matrix}\right)\right)$$

03.18.26.0079.01

$$J_{-\nu}\!\left(\frac{1}{\sqrt[4]{-1}}z\right)\operatorname{ber}_\nu(z)=\sqrt{\frac{\pi}{2}}\,z^\nu\left(\sqrt[4]{-1}\,z\right)^{-\nu}\left(\frac{1}{\sqrt[4]{-1}}z\right)^{-\nu}$$
$$\left(2^{\frac{3\nu-1}{2}}e^{-\frac{1}{4}(3i\pi\nu)}G_{1,5}^{2,0}\!\left(\frac{\sqrt[4]{-1}\,z}{2\sqrt{2}},\frac{1}{4}\,\middle|\,\begin{matrix}\frac{2-\nu}{4}\\ \frac{\nu}{4},\frac{\nu+2}{4},\frac{3\nu}{4},-\frac{\nu}{4},\frac{2-\nu}{4}\end{matrix}\right)+e^{\frac{3i\pi\nu}{4}}G_{2,4}^{1,1}\!\left(\sqrt[4]{-1}\,z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{\nu+1}{2},\frac{1}{4}(2\nu+1)\\ \frac{\nu}{2},-\frac{\nu}{2},\frac{3\nu}{2},\frac{1}{4}(2\nu+1)\end{matrix}\right)\right)$$

Generalized cases involving Bessel *I*

03.18.26.0080.01

$$I_\nu\!\left(\sqrt[4]{-1}\,z\right)\operatorname{ber}_\nu(z)=$$
$$\frac{1}{2}e^{-\frac{3i\pi\nu}{4}}\sqrt{\pi}\,z^\nu\left(\sqrt[4]{-1}\,z\right)^{-\nu}\left(G_{0,4}^{1,0}\!\left(\frac{\sqrt[4]{-1}\,z}{2\sqrt{2}},\frac{1}{4}\,\middle|\,\frac{\nu}{2},0,\frac{1}{2},-\frac{\nu}{2}\right)+e^{\frac{3i\pi\nu}{2}}\csc\!\left(\pi\left(\nu+\frac{3}{4}\right)\right)G_{2,4}^{1,1}\!\left(\sqrt[4]{-1}\,z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1}{2},\frac{1}{4}\\ \nu,0,\frac{1}{4},-\nu\end{matrix}\right)\right)$$

03.18.26.0081.01

$$I_{-\nu}\!\left(\sqrt[4]{-1}\,z\right)\operatorname{ber}_\nu(z)=$$
$$\sqrt{\frac{\pi}{2}}\,z^\nu\left(\sqrt[4]{-1}\,z\right)^{-\nu}\left(\frac{e^{-\frac{3i\pi\nu}{4}}}{\sqrt{2}}G_{1,5}^{2,0}\!\left(\frac{\sqrt[4]{-1}\,z}{2\sqrt{2}},\frac{1}{4}\,\middle|\,\begin{matrix}\frac{1-\nu}{2}\\ 0,\frac{1}{2},\frac{1-\nu}{2},-\frac{\nu}{2},\frac{\nu}{2}\end{matrix}\right)+e^{\frac{3i\pi\nu}{4}}G_{2,4}^{1,1}\!\left(\sqrt[4]{-1}\,z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1}{2},\frac{1}{4}\\ 0,\frac{1}{4},\nu,-\nu\end{matrix}\right)\right)$$

03.18.26.0082.01

$$\left(I_\nu\left(\sqrt[4]{-1}\,z\right)-I_{-\nu}\left(\sqrt[4]{-1}\,z\right)\right)\operatorname{ber}_\nu(z)=\frac{\sqrt{\pi}\,\sin(\pi\nu)}{2}\,z^\nu\left(\sqrt[4]{-1}\,z\right)^{-\nu}\left(\sqrt{2}\,\csc\left(\pi\left(\nu+\frac{3}{4}\right)\right)e^{\frac{3i\pi\nu}{4}}\,G_{3,5}^{2,1}\left(\sqrt[4]{-1}\,z,\,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1}{2},\,\frac{1}{4},\,\nu-\frac{1}{4}\\0,\,\nu,\,-\nu,\,\frac{1}{4},\,\nu-\frac{1}{4}\end{matrix}\right)-\frac{e^{-\frac{3i\pi\nu}{4}}}{2\pi^2}\,G_{0,4}^{3,0}\left(\frac{\sqrt[4]{-1}\,z}{2\sqrt{2}},\,\frac{1}{4}\,\middle|\,0,\,\frac{1}{2},\,\frac{\nu}{2},\,-\frac{\nu}{2}\right)\right)$$

Generalized cases involving Bessel K

03.18.26.0083.01

$$K_\nu\left(\sqrt[4]{-1}\,z\right)\operatorname{ber}_\nu(z)=-\frac{\pi^{3/2}}{4}\,z^\nu\left(\sqrt[4]{-1}\,z\right)^{-\nu}\left(\sqrt{2}\,\csc\left(\pi\left(\nu+\frac{3}{4}\right)\right)e^{\frac{3i\pi\nu}{4}}\,G_{3,5}^{2,1}\left(\sqrt[4]{-1}\,z,\,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1}{2},\,\frac{1}{4},\,\nu-\frac{1}{4}\\0,\,\nu,\,-\nu,\,\frac{1}{4},\,\nu-\frac{1}{4}\end{matrix}\right)-\frac{e^{-\frac{3i\pi\nu}{4}}}{2\pi^2}\,G_{0,4}^{3,0}\left(\frac{\sqrt[4]{-1}\,z}{2\sqrt{2}},\,\frac{1}{4}\,\middle|\,0,\,\frac{1}{2},\,\frac{\nu}{2},\,-\frac{\nu}{2}\right)\right)$$

Generalized cases involving ${}_0F_1$

03.18.26.0084.01

$$ {}_0F_1\left(;\nu+1;\frac{i z^2}{4}\right)\operatorname{ber}_\nu(z)=\frac{\sqrt{\pi}\,\Gamma(\nu+1)}{2\sqrt{2}}\left(2^{\frac{1-\nu}{2}}\,e^{-\frac{3i\pi\nu}{4}}\,\pi\,G_{1,5}^{1,0}\left(\frac{z}{2\sqrt{2}},\,\frac{1}{4}\,\middle|\,\begin{matrix}\frac{\nu+2}{4}\\\frac{\nu}{4},\,-\frac{\nu}{4},\,\frac{2-\nu}{4},\,-\frac{1}{4}(3\nu),\,\frac{\nu+2}{4}\end{matrix}\right)+e^{\frac{3i\pi\nu}{4}}\,G_{2,6}^{1,2}\left(\frac{z}{2},\,\frac{1}{4}\,\middle|\,\begin{matrix}\frac{3-\nu}{4},\,\frac{1-\nu}{4}\\\frac{\nu}{4},\,-\frac{\nu}{4},\,\frac{\nu+2}{4},\,\frac{2-\nu}{4},\,\frac{1}{4}(2-3\nu),\,-\frac{1}{4}(3\nu)\end{matrix}\right)+e^{\frac{3i\pi\nu}{4}}\,i\,G_{2,6}^{1,2}\left(\frac{z}{2},\,\frac{1}{4}\,\middle|\,\begin{matrix}\frac{3-\nu}{4},\,\frac{1-\nu}{4}\\\frac{\nu+2}{4},\,-\frac{\nu}{4},\,\frac{\nu}{4},\,\frac{2-\nu}{4},\,\frac{1}{4}(2-3\nu),\,-\frac{1}{4}(3\nu)\end{matrix}\right)\right)$$

03.18.26.0085.01

$$ {}_0F_1\left(;1-\nu;\frac{i z^2}{4}\right)\operatorname{ber}_\nu(z)=\sqrt{\pi}\,\Gamma(1-\nu)\left(2^{\frac{\nu}{2}-1}\,e^{-\frac{3i\pi\nu}{4}}\,\pi\left(G_{1,5}^{1,0}\left(\frac{z}{2\sqrt{2}},\,\frac{1}{4}\,\middle|\,\begin{matrix}\frac{1}{4}(3\nu+2)\\\frac{\nu}{4},\,\frac{3\nu}{4},\,\frac{\nu+2}{4},\,-\frac{\nu}{4},\,\frac{1}{4}(3\nu+2)\end{matrix}\right)+i\tan\left(\frac{\pi\nu}{2}\right)G_{1,5}^{1,0}\left(\frac{z}{2\sqrt{2}},\,\frac{1}{4}\,\middle|\,\begin{matrix}\frac{3\nu}{4}+1\\\frac{\nu+2}{4},\,\frac{\nu}{4},\,\frac{3\nu}{4},\,-\frac{\nu}{4},\,\frac{3\nu}{4}+1\end{matrix}\right)\right)+\frac{e^{\frac{3i\pi\nu}{4}}}{2\sqrt{2}}\left(G_{2,6}^{1,2}\left(\frac{z}{2},\,\frac{1}{4}\,\middle|\,\begin{matrix}\frac{\nu+1}{4},\,\frac{\nu+3}{4}\\\frac{\nu}{4},\,\frac{\nu+2}{4},\,\frac{1}{4}(3\nu+2),\,\frac{3\nu}{4},\,\frac{2-\nu}{4},\,-\frac{\nu}{4}\end{matrix}\right)+i\,G_{2,6}^{1,2}\left(\frac{z}{2},\,\frac{1}{4}\,\middle|\,\begin{matrix}\frac{\nu+1}{4},\,\frac{\nu+3}{4}\\\frac{\nu+2}{4},\,\frac{\nu}{4},\,\frac{1}{4}(3\nu+2),\,\frac{3\nu}{4},\,\frac{2-\nu}{4},\,-\frac{\nu}{4}\end{matrix}\right)\right)\right)$$

03.18.26.0086.01

$$ {}_0F_1\left(;\nu+1;\frac{i z^2}{4}\right)\operatorname{ber}_\nu(z)=2^{-\frac{\nu}{2}-1}\,e^{-\frac{3i\pi\nu}{2}}\sqrt{\pi}\,z^\nu\left(\sqrt[4]{-1}\,z\right)^{-\nu}\Gamma(\nu+1)\left(G_{0,4}^{1,0}\left(\frac{\sqrt[4]{-1}\,z}{2\sqrt{2}},\,\frac{1}{4}\,\middle|\,\frac{\nu}{4},\,-\frac{\nu}{4},\,\frac{2-\nu}{4},\,-\frac{1}{4}(3\nu)\right)+2^{\frac{3\nu}{2}}\,e^{\frac{3i\pi\nu}{2}}\csc\left(\pi\left(\nu+\frac{3}{4}\right)\right)G_{2,4}^{1,1}\left(\sqrt[4]{-1}\,z,\,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1-\nu}{2},\,\frac{1}{4}(1-2\nu)\\\frac{\nu}{2},\,-\frac{\nu}{2},\,-\frac{1}{2}(3\nu),\,\frac{1}{4}(1-2\nu)\end{matrix}\right)\right)$$

03.18.26.0087.01

$$ {}_0F_1\left(;1-\nu;\frac{i z^2}{4}\right)\operatorname{ber}_\nu(z)=2^{-\nu-\frac{1}{2}}\,e^{\frac{1}{4}(-3)i\pi\nu}\sqrt{\pi}\,z^\nu\left(\sqrt[4]{-1}\,z\right)^{-\nu}\Gamma(1-\nu)\left(2^{\frac{3\nu-1}{2}}\,G_{1,5}^{2,0}\left(\frac{\sqrt[4]{-1}\,z}{2\sqrt{2}},\,\frac{1}{4}\,\middle|\,\begin{matrix}\frac{2-\nu}{4}\\\frac{\nu}{4},\,\frac{\nu+2}{4},\,\frac{3\nu}{4},\,-\frac{\nu}{4},\,\frac{2-\nu}{4}\end{matrix}\right)+e^{\frac{3i\pi\nu}{2}}\,G_{2,4}^{1,1}\left(\sqrt[4]{-1}\,z,\,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{\nu+1}{2},\,\frac{1}{4}(2\nu+1)\\\frac{\nu}{2},\,-\frac{\nu}{2},\,\frac{3\nu}{2},\,\frac{1}{4}(2\nu+1)\end{matrix}\right)\right)$$

Generalized cases involving ${}_0\tilde{F}_1$

03.18.26.0088.01

$$ {}_0\tilde{F}_1\left(;\nu+1;\frac{i z^2}{4}\right)\operatorname{ber}_\nu(z)=\frac{\sqrt{\pi}}{2\sqrt{2}}\left(2^{\frac{1-\nu}{2}}e^{-\frac{3i\pi\nu}{4}}\pi\, G_{1,5}^{1,0}\!\left(\frac{z}{2\sqrt{2}},\frac{1}{4}\,\middle|\,\begin{matrix}\frac{\nu+2}{4}\\ \frac{\nu}{4},-\frac{\nu}{4},\frac{2-\nu}{4},-\frac{1}{4}(3\nu),\frac{\nu+2}{4}\end{matrix}\right)+\right. $$
$$ \left. e^{\frac{3i\pi\nu}{4}}G_{2,6}^{1,2}\!\left(\frac{z}{2},\frac{1}{4}\,\middle|\,\begin{matrix}\frac{3-\nu}{4},\frac{1-\nu}{4}\\ \frac{\nu}{4},-\frac{\nu}{4},\frac{\nu+2}{4},\frac{2-\nu}{4},\frac{1}{4}(2-3\nu),-\frac{1}{4}(3\nu)\end{matrix}\right)+e^{\frac{3i\pi\nu}{4}}i\,G_{2,6}^{1,2}\!\left(\frac{z}{2},\frac{1}{4}\,\middle|\,\begin{matrix}\frac{3-\nu}{4},\frac{1-\nu}{4}\\ \frac{\nu+2}{4},-\frac{\nu}{4},\frac{\nu}{4},\frac{2-\nu}{4},\frac{1}{4}(2-3\nu),-\frac{1}{4}(3\nu)\end{matrix}\right)\right) $$

03.18.26.0089.01

$$ {}_0\tilde{F}_1\left(;1-\nu;\frac{i z^2}{4}\right)\operatorname{ber}_\nu(z)= $$
$$ \sqrt{\pi}\left(2^{\frac{\nu}{2}-1}e^{-\frac{3i\pi\nu}{4}}\pi\left(G_{1,5}^{1,0}\!\left(\frac{z}{2\sqrt{2}},\frac{1}{4}\,\middle|\,\begin{matrix}\frac{1}{4}(3\nu+2)\\ \frac{\nu}{4},\frac{3\nu}{4},\frac{\nu+2}{4},-\frac{\nu}{4},\frac{1}{4}(3\nu+2)\end{matrix}\right)+i\tan\!\left(\frac{\pi\nu}{2}\right)G_{1,5}^{1,0}\!\left(\frac{z}{2\sqrt{2}},\frac{1}{4}\,\middle|\,\begin{matrix}\frac{3\nu}{4}+1\\ \frac{\nu+2}{4},\frac{\nu}{4},\frac{3\nu}{4},-\frac{\nu}{4},\frac{3\nu}{4}+1\end{matrix}\right)\right)+\right. $$
$$ \left.\frac{e^{\frac{3i\pi\nu}{4}}}{2\sqrt{2}}\left(G_{2,6}^{1,2}\!\left(\frac{z}{2},\frac{1}{4}\,\middle|\,\begin{matrix}\frac{\nu+1}{4},\frac{\nu+3}{4}\\ \frac{\nu}{4},\frac{\nu+2}{4},\frac{1}{4}(3\nu+2),\frac{3\nu}{4},\frac{2-\nu}{4},-\frac{\nu}{4}\end{matrix}\right)+i\,G_{2,6}^{1,2}\!\left(\frac{z}{2},\frac{1}{4}\,\middle|\,\begin{matrix}\frac{\nu+1}{4},\frac{\nu+3}{4}\\ \frac{\nu+2}{4},\frac{\nu}{4},\frac{1}{4}(3\nu+2),\frac{3\nu}{4},\frac{2-\nu}{4},-\frac{\nu}{4}\end{matrix}\right)\right)\right) $$

03.18.26.0090.01

$$ {}_0\tilde{F}_1\left(;\nu+1;\frac{i z^2}{4}\right)\operatorname{ber}_\nu(z)=2^{-\frac{\nu}{2}-1}e^{-\frac{3i\pi\nu}{2}}\sqrt{\pi}\,z^\nu\left(\sqrt[4]{-1}\,z\right)^{-\nu} $$
$$ \left(G_{0,4}^{1,0}\!\left(\frac{\sqrt[4]{-1}\,z}{2\sqrt{2}},\frac{1}{4}\,\middle|\,\frac{\nu}{4},-\frac{\nu}{4},\frac{2-\nu}{4},-\frac{1}{4}(3\nu)\right)+2^{\frac{3\nu}{2}}e^{\frac{3i\pi\nu}{2}}\csc\!\left(\pi\left(\nu+\frac{3}{4}\right)\right)G_{2,4}^{1,1}\!\left(\sqrt[4]{-1}\,z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1-\nu}{2},\frac{1}{4}(1-2\nu)\\ \frac{\nu}{2},-\frac{\nu}{2},-\frac{1}{2}(3\nu),\frac{1}{4}(1-2\nu)\end{matrix}\right)\right) $$

03.18.26.0091.01

$$ {}_0\tilde{F}_1\left(;1-\nu;\frac{i z^2}{4}\right)\operatorname{ber}_\nu(z)=2^{-\nu-\frac{1}{2}}e^{\frac{1}{4}(-3)i\pi\nu}\sqrt{\pi}\,z^\nu\left(\sqrt[4]{-1}\,z\right)^{-\nu} $$
$$ \left(2^{\frac{3\nu-1}{2}}G_{1,5}^{2,0}\!\left(\frac{\sqrt[4]{-1}\,z}{2\sqrt{2}},\frac{1}{4}\,\middle|\,\begin{matrix}\frac{2-\nu}{4}\\ \frac{\nu}{4},\frac{\nu+2}{4},\frac{3\nu}{4},-\frac{\nu}{4},\frac{2-\nu}{4}\end{matrix}\right)+e^{\frac{3i\pi\nu}{2}}G_{2,4}^{1,1}\!\left(\sqrt[4]{-1}\,z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{\nu+1}{2},\frac{1}{4}(2\nu+1)\\ \frac{\nu}{2},-\frac{\nu}{2},\frac{3\nu}{2},\frac{1}{4}(2\nu+1)\end{matrix}\right)\right) $$

Through other functions

03.18.26.0092.01

$$ \operatorname{ber}_\nu(z)=\frac{\sqrt[8]{-1}\sqrt{z}}{2^{3/4}\sqrt{(1+i)z}}\left(e^{\frac{i\pi\nu}{2}}\boldsymbol{L}_{-\nu}\!\left(\sqrt[4]{-1}\,z\right)-i\,\boldsymbol{H}_{-\nu}\!\left(\sqrt[4]{-1}\,z\right)\right)/;\ \nu-\frac{1}{2}\in\mathbb{N} $$

Representations through equivalent functions

With related functions

03.18.27.0001.01

$$ \operatorname{ber}_\nu(z)=-\csc(\pi\nu)\operatorname{bei}_{-\nu}(z)+\cot(\pi\nu)\operatorname{bei}_\nu(z)+\frac{2}{\pi}\operatorname{kei}_\nu(z)\ /;\ \nu\notin\mathbb{Z} $$

03.18.27.0002.01

$$\operatorname{ber}_\nu(z) = \frac{1}{2} z^\nu \left(-z^4\right)^{-\frac{1}{4}(2+\nu)} \left(J_\nu\left(\sqrt[4]{-z^4}\right)\left(\sin\left(\frac{3\pi\nu}{4}\right) z^2 + \sqrt{-z^4} \cos\left(\frac{3\pi\nu}{4}\right)\right) + I_\nu\left(\sqrt[4]{-z^4}\right)\left(\sqrt{-z^4} \cos\left(\frac{3\pi\nu}{4}\right) - z^2 \sin\left(\frac{3\pi\nu}{4}\right)\right)\right)$$

03.18.27.0003.01

$$\operatorname{ber}_\nu(z) = \frac{1}{2} e^{-\frac{3}{4} i\pi\nu} z^\nu \left(\sqrt[4]{-1}\, z\right)^{-\nu} \left(e^{\frac{3 i\pi\nu}{2}} I_\nu\left(\sqrt[4]{-1}\, z\right) + J_\nu\left(\sqrt[4]{-1}\, z\right)\right)$$

03.18.27.0004.01

$$\operatorname{ber}_\nu(z) = \frac{1}{2}\left(e^{\frac{i\pi\nu}{2}} I_\nu\left(\sqrt[4]{-1}\, z\right) + e^{-i\pi\nu} J_\nu\left(\sqrt[4]{-1}\, z\right)\right) /;\ \nu \in \mathbb{Z}$$

03.18.27.0005.01

$$\operatorname{ber}_\nu(z) = \begin{cases} \frac{1}{2} e^{\frac{5 i\pi\nu}{2}} I_\nu\left(\sqrt[4]{-1}\, z\right) + \frac{1}{2} e^{i\pi\nu} J_\nu\left(\sqrt[4]{-1}\, z\right) & \frac{3\pi}{4} < \arg(z) \le \pi \\ \frac{1}{2} e^{\frac{i\pi\nu}{2}} I_\nu\left(\sqrt[4]{-1}\, z\right) + \frac{1}{2} e^{-i\pi\nu} J_\nu\left(\sqrt[4]{-1}\, z\right) & \text{True} \end{cases}$$

03.18.27.0006.01

$$\operatorname{ber}_\nu(z) + i \operatorname{bei}_\nu(z) = e^{\frac{3 i\pi\nu}{4}} z^\nu \left(\sqrt[4]{-1}\, z\right)^{-\nu} I_\nu\left(\sqrt[4]{-1}\, z\right)$$

03.18.27.0007.01

$$\operatorname{ber}_\nu(z) + i \operatorname{bei}_\nu(z) = \begin{cases} e^{\frac{5 i\pi\nu}{2}} I_\nu\left(\sqrt[4]{-1}\, z\right) & \frac{3\pi}{4} < \arg(z) \le \pi \\ e^{\frac{i\pi\nu}{2}} I_\nu\left(\sqrt[4]{-1}\, z\right) & \text{True} \end{cases}$$

03.18.27.0008.01

$$\operatorname{ber}_\nu(z) - i \operatorname{bei}_\nu(z) = e^{-\frac{3 i\pi\nu}{4}} z^\nu \left(\sqrt[4]{-1}\, z\right)^{-\nu} J_\nu\left(\sqrt[4]{-1}\, z\right)$$

03.18.27.0009.01

$$\operatorname{ber}_\nu(z) - i \operatorname{bei}_\nu(z) = \begin{cases} e^{i\pi\nu} J_\nu\left(\sqrt[4]{-1}\, z\right) & \frac{3\pi}{4} < \arg(z) \le \pi \\ e^{-i\pi\nu} J_\nu\left(\sqrt[4]{-1}\, z\right) & \text{True} \end{cases}$$

Theorems

History

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