LucasL

Notations

Traditional name

Lucas numbers

Traditional notation

L_ν

Mathematica StandardForm notation

LucasL[ν]

Primary definition

04.22.02.0001.01

$$L_\nu = \phi^\nu + \phi^{-\nu} \cos(\pi\,\nu)$$

Specific values

Specialized values

04.22.03.0001.01

$$L_n = \left(\frac{1+\sqrt{5}}{2}\right)^n + \left(\frac{1-\sqrt{5}}{2}\right)^n \;/;\; n \in \mathbb{Z}$$

04.22.03.0002.01

$$L_{-n} = (-1)^n \left(\frac{1+\sqrt{5}}{2}\right)^n + (-1)^n \left(\frac{1-\sqrt{5}}{2}\right)^n \;/;\; n \in \mathbb{Z}$$

04.22.03.0003.01

$$L_n = \lfloor \phi^n \rceil \;/;\; n \in \mathbb{Z} \land n > 1$$

04.22.03.0004.01

$$L_n = \phi^n + (1-\phi)^n \;/;\; n \in \mathbb{Z}$$

04.22.03.0005.01

$$L_{n+1} = \left\lfloor \frac{1}{2}\left(\left(1+\sqrt{5}\right) L_n + 1\right)\right\rfloor \;/;\; n \in \mathbb{Z} \land n > 3$$

04.22.03.0006.01

$$L_n = \frac{\phi^n + (1-\phi)^n}{\phi + (1-\phi)} \;/;\; n \in \mathbb{Z}$$

Values at fixed points

04.22.03.0007.01

$L_0 = 2$

04.22.03.0008.01

$L_1 = 1$

04.22.03.0009.01

$L_2 = 3$

04.22.03.0010.01

$L_3 = 4$

04.22.03.0011.01

$L_4 = 7$

04.22.03.0012.01

$L_5 = 11$

04.22.03.0013.01

$L_6 = 18$

04.22.03.0014.01

$L_7 = 29$

04.22.03.0015.01

$L_8 = 47$

04.22.03.0016.01

$L_9 = 76$

04.22.03.0017.01

$L_{10} = 123$

Values at infinities

04.22.03.0018.01

$L_\infty = \infty$

General characteristics

Domain and analyticity

L_ν is an entire analytical function of ν which is defined over the whole complex ν-plane.

04.22.04.0001.01

$\nu \longrightarrow L_\nu :: \mathbb{C} \longrightarrow \mathbb{C}$

Symmetries and periodicities

Parity

04.22.04.0002.01

$L_{-n} = (-1)^n L_n \;/;\; n \in \mathbb{Z}$

Mirror symmetry

04.22.04.0003.01

$$L_{\bar{\nu}} = \overline{L_\nu}$$

Periodicity

No periodicity

Poles and essential singularities

The function L_ν has only one singular point at $\nu = \tilde{\infty}$. It is an essential singular point.

04.22.04.0004.01

$$\mathcal{Sing}_\nu(L_\nu) = \{\{\tilde{\infty}, \infty\}\}$$

Branch points

The function L_ν does not have branch points.

04.22.04.0005.01

$$\mathcal{BP}_\nu(L_\nu) = \{\}$$

Branch cuts

The function L_ν does not have branch cuts.

04.22.04.0006.01

$$\mathcal{BC}_\nu(L_\nu) = \{\}$$

Series representations

Generalized power series

Expansions at generic point $\nu = \nu_0$

For the function itself

04.22.06.0001.01

$$L_\nu \propto L_{\nu_0} + \left(\phi^{\nu_0} \operatorname{csch}^{-1}(2) + \frac{1}{2} \phi^{-\nu_0} \left(e^{i \pi \nu_0} \left(i \pi - \operatorname{csch}^{-1}(2)\right) - e^{-i \pi \nu_0} \left(i \pi + \operatorname{csch}^{-1}(2)\right)\right)\right)(\nu - \nu_0) + \frac{1}{2}\left(\phi^{\nu_0} \operatorname{csch}^{-1}(2)^2 + \frac{1}{2} \phi^{-\nu_0} \left(e^{i \pi \nu_0} \left(i \pi - \operatorname{csch}^{-1}(2)\right)^2 + e^{-i \pi \nu_0} \left(i \pi + \operatorname{csch}^{-1}(2)\right)^2\right)\right)(\nu - \nu_0)^2 + \ldots /; (\nu \to \nu_0)$$

04.22.06.0002.01

$$L_\nu \propto L_{\nu_0} + \left(\phi^{\nu_0} \operatorname{csch}^{-1}(2) + \frac{1}{2} \phi^{-\nu_0} \left(e^{i \pi \nu_0} \left(i \pi - \operatorname{csch}^{-1}(2)\right) - e^{-i \pi \nu_0} \left(i \pi + \operatorname{csch}^{-1}(2)\right)\right)\right)(\nu - \nu_0) + \frac{1}{2}\left(\phi^{\nu_0} \operatorname{csch}^{-1}(2)^2 + \frac{1}{2} \phi^{-\nu_0} \left(e^{i \pi \nu_0} \left(i \pi - \operatorname{csch}^{-1}(2)\right)^2 + e^{-i \pi \nu_0} \left(i \pi + \operatorname{csch}^{-1}(2)\right)^2\right)\right)(\nu - \nu_0)^2 + O\left((\nu - \nu_0)^3\right)$$

04.22.06.0003.01

$$L_\nu = \sum_{k=0}^{\infty} \frac{\phi^{\nu_0} \operatorname{csch}^{-1}(2)^k + \frac{1}{2} \phi^{-\nu_0} \left(e^{\pi i \nu_0} \left(\pi i - \operatorname{csch}^{-1}(2)\right)^k + (-1)^k e^{-\pi i \nu_0} \left(\pi i + \operatorname{csch}^{-1}(2)\right)^k\right)}{k!} (\nu - \nu_0)^k$$

04.22.06.0004.01

$$L_\nu \propto L_{\nu_0}\,(1 + O(\nu - \nu_0))$$

Expansions at $\nu == 0$

04.22.06.0005.01

$$L_\nu \propto 2 + \left(\log^2(\phi) - \frac{\pi^2}{2}\right)\nu^2 + \frac{\pi^2 \log(\phi)}{2}\,\nu^3 + \ldots /; (\nu \to 0)$$

04.22.06.0006.01

$$L_\nu \propto 2 + \left(\log^2(\phi) - \frac{\pi^2}{2}\right)\nu^2 + \frac{\pi^2 \log(\phi)}{2}\,\nu^3 + O\left(\nu^4\right)$$

04.22.06.0007.01

$$L_\nu = \sum_{k=0}^{\infty} \frac{\operatorname{csch}^{-1}(2)^k + \frac{1}{2}\left(\left(\pi\, i - \operatorname{csch}^{-1}(2)\right)^k + (-1)^k \left(\pi\, i + \operatorname{csch}^{-1}(2)\right)^k\right)}{k!}\,\nu^k$$

04.22.06.0008.01

$$L_\nu \propto 2 + O\left(\nu^2\right)$$

Asymptotic series expansions

04.22.06.0009.01

$$L_\nu \propto \phi^\nu + \cos(\nu\,\pi)\,\phi^{-\nu} /; (|\nu| \to \infty)$$

04.22.06.0010.01

$$L_\nu \propto \begin{cases} \phi^\nu & \operatorname{Im}(\nu) < 0 \land \operatorname{Re}(\nu) - \pi\,|\operatorname{Im}(\nu)| > 0 \\ \frac{1}{2}\,e^{i\,\nu\,\pi - \nu\operatorname{csch}^{-1}(2)} & \operatorname{Im}(\nu) < 0 \land \operatorname{Re}(\nu) + \pi\operatorname{Im}(\nu) < 0 \\ \frac{1}{2}\,e^{-i\,\pi\,\nu - \operatorname{csch}^{-1}(2)\,\nu} & \operatorname{Im}(\nu) > 0 \land \operatorname{Re}(\nu) - \pi\operatorname{Im}(\nu) < 0 \\ \phi^{-\nu}\cos(\nu\,\pi) + \phi^\nu & \text{True} \end{cases} /; (|\nu| \to \infty)$$

04.22.06.0011.01

$$L_\nu \propto \phi^\nu /; (\nu \to \infty)$$

Other series representations

04.22.06.0012.01

$$L_n = 2^{1-n} \sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} \binom{n}{2\,k} 5^k /; n \in \mathbb{N}$$

Integral representations

On the real axis

Of the direct function

04.22.07.0001.01

$$L_{2\,n+1} = \frac{1}{4}\left(\frac{3}{2}\right)^{n-1} \int_0^\pi \left(\frac{1}{3}\sqrt{5}\,\cos(t) + 1\right)^{n-1} \left(5\,n + \sqrt{5}\,(n+1)\cos(t) + 3\right)\sin(t)\,dt /; n \in \mathbb{Z}$$

Generating functions

04.22.11.0001.01

$$L_n == \left([t^n]\ \frac{2-t}{1-t-t^2}\right) /; n \in \mathbb{N}$$

Differential equations

Ordinary linear differential equations and wronskians

04.22.13.0001.01

$$w^{(3)}(\nu) + \log(\phi)\, w''(\nu) + \left(\pi^2 - \log^2(\phi)\right) w'(\nu) - \log(\phi) \left(\log^2(\phi) + \pi^2\right) w(\nu) == 0 /; w(\nu) = c_1 L_\nu + c_2 F_\nu\ + c_3 \phi^{-\nu} \sin(\pi \nu)$$

Transformations

Addition formulas

04.22.16.0001.01

$$L_{m+n} = \frac{1}{2} (5 F_m F_n + L_m L_n) /; n \in \mathbb{Z} \land m \in \mathbb{Z}$$

04.22.16.0002.01

$$L_{m-n} = \frac{1}{2} (-1)^{n-1} (5 F_n F_m - L_n L_m) /; n \in \mathbb{Z} \land m \in \mathbb{Z}$$

04.22.16.0003.01

$$L_{\nu+1} = \frac{1}{2} (5 F_\nu + L_\nu)$$

Multiple arguments

04.22.16.0004.01

$$L_{2\nu} = \frac{1}{5} \left(3 L_\nu^2 - 2 L_{\nu+1} L_\nu + 2 L_{\nu+1}^2 - 5 \phi^{-2\nu} \sin^2(\pi \nu)\right)$$

04.22.16.0005.01

$$L_{2\nu+1} = \frac{1}{5} \left(-L_\nu^2 + 4 L_{\nu+1} L_\nu + L_{\nu+1}^2 + 5 \phi^{-2\nu-1} \sin^2(\pi \nu)\right)$$

04.22.16.0006.01

$$L_{2n} = \frac{1}{2} \left(5 F_n^2 + L_n^2\right) /; n \in \mathbb{Z}$$

04.22.16.0007.01

$$L_{2\nu} = 3 L_{2(\nu-1)} - L_{2(\nu-2)}$$

04.22.16.0008.01

$$L_{m\nu} = L_m L_{m(\nu-1)} - (-1)^m L_{m(\nu-2)} /; m \in \mathbb{Z}$$

04.22.16.0009.01

$$L_{m n} = 2^{1-m} \sum_{k=0}^{\left\lfloor \frac{m}{2} \right\rfloor} \binom{m}{2k} F_n^{2k} L_n^{m-2k} 5^k /; n \in \mathbb{N}^+ \bigwedge m \in \mathbb{N}$$

04.22.16.0010.01

$$L_{m n} = \sum_{k=0}^{\left\lfloor \frac{m}{2} \right\rfloor} \frac{m}{m-k} \binom{m-k}{k} (-1)^{k (n+1)} L_n^{m-2 k} \,/;\, n \in \mathbb{Z} \bigwedge m \in \mathbb{N}^+$$

04.22.16.0011.01

$$L_{m n} = L_n^{m-2 \left\lfloor \frac{m}{2} \right\rfloor} \sum_{k=0}^{\left\lfloor \frac{m}{2} \right\rfloor} \left(\frac{m}{m-k} \right)^{2 \left\lfloor \frac{m}{2} \right\rfloor - m+1} \binom{2 \left\lfloor \frac{m}{2} \right\rfloor - k}{k} (-1)^{k n} 5^{\left\lfloor \frac{m}{2} \right\rfloor - k} F_n^{2 \left\lfloor \frac{m}{2} \right\rfloor - 2 k} \,/;\, n \in \mathbb{Z} \bigwedge n \neq 0 \bigwedge m \in \mathbb{N}^+$$

04.22.16.0012.01

$$L_{m n} = \sum_{k=0}^{m} \binom{m}{k} L_k F_n^k F_{n-1}^{m-k} \,/;\, n \in \mathbb{Z} \bigwedge n \neq 0 \bigwedge n \neq 1 \bigwedge m \in \mathbb{N}^+$$

Products, sums, and powers of the direct function

Products of the direct function

04.22.16.0013.01

$$L_{\nu+1} L_{\nu-1} = L_\nu^2 - 5 \cos(\pi \nu)$$

04.22.16.0014.01

$$L_m L_n = (-1)^n L_{m-n} + L_{m+n} \,/;\, m \in \mathbb{Z} \land n \in \mathbb{Z}$$

Powers of the direct function

04.22.16.0015.01

$$L_\nu^2 = L_{\nu-1} L_{\nu+1} + 5 \cos(\pi \nu)$$

04.22.16.0016.01

$$L_n^2 = L_{2 n} + 2 (-1)^n \,/;\, n \in \mathbb{Z}$$

04.22.16.0017.01

$$L_n^3 = L_{3 n} + 3 (-1)^n L_n \,/;\, n \in \mathbb{Z}$$

04.22.16.0018.01

$$L_n^4 = L_{4 n} + 4 (-1)^n L_{2 n} + 6 \,/;\, n \in \mathbb{Z}$$

04.22.16.0019.01

$$L_n^5 = L_{5 n} + 5 (-1)^n L_{3 n} + 10 L_n \,/;\, n \in \mathbb{Z}$$

04.22.16.0020.01

$$L_n^m = \frac{1}{2} \sum_{k=0}^{m} (-1)^{k n} \binom{m}{k} L_{n (m-2 k)} \,/;\, n \in \mathbb{Z} \bigwedge m \in \mathbb{N}^+$$

Identities

Recurrence identities

Consecutive neighbors

04.22.17.0001.01

$$L_\nu = L_{\nu+2} - L_{\nu+1}$$

04.22.17.0002.01

$$L_\nu = L_{\nu-2} + L_{\nu-1}$$

04.22.17.0003.01

$$L_\nu = \frac{L_{\nu+1} - \sqrt{5}\ \phi^\nu}{1-\phi}$$

04.22.17.0004.01

$$L_\nu = (1-\phi)\, L_{\nu-1} + \sqrt{5}\ \phi^{\nu-1}$$

Distant neighbors

04.22.17.0005.01

$$L_\nu = i^{m+1}\, U_{\frac{m-1}{2}}\left(-\frac{3}{2}\right) L_{m+\nu} + i^m\, U_{\frac{m}{2}-1}\left(-\frac{3}{2}\right) L_{m+\nu+1} \;/;\; m \in \mathbb{N}^+$$

04.22.17.0006.01

$$L_\nu = i^{1-m}\, U_{\frac{m-1}{2}}\left(-\frac{3}{2}\right) L_{\nu-m} - (-i)^m\, U_{\frac{m}{2}-1}\left(-\frac{3}{2}\right) L_{\nu-m-1} \;/;\; m \in \mathbb{N}^+$$

Functional identities

Functional equations

04.22.17.0007.01

$$w(z) = w(z-2) + w(z-1) \;/;\; w(z) = c_1\, F_z + c_2\, L_z$$

Relations of special kind

04.22.17.0008.01

$$L_\nu^2 - L_{\nu-1}\, L_{\nu+1} = 5\cos(\pi\,\nu)$$

04.22.17.0009.01

$$\sum_{k=1}^{n} \frac{(-1)^k}{L_k\, L_{k+m}} = \frac{F_n}{F_m} \sum_{k=1}^{m} \frac{(-1)^k}{L_k\, L_{k+n}} \;/;\; n \in \mathbb{Z} \land m \in \mathbb{Z} \land n\, m > 0$$

Complex characteristics

Real part

04.22.19.0001.01

$$\operatorname{Re}(L_{x+i\,y}) = \phi^x \cos(y \log(\phi)) + \phi^{-x} \cos(\pi\,x) \cosh(\pi\,y) \cos(y \log(\phi)) - \phi^{-x} \sin(\pi\,x) \sin(y \log(\phi)) \sinh(\pi\,y)$$

04.22.19.0002.01

$$\operatorname{Re}(L_{x+i\,y}) = \phi^x \cos\left(y \operatorname{csch}^{-1}(2)\right) + \phi^{-x} \cos(\pi\,x) \cosh(\pi\,y) \cos\left(y \operatorname{csch}^{-1}(2)\right) - \phi^{-x} \sin(\pi\,x) \sin\left(y \operatorname{csch}^{-1}(2)\right) \sinh(\pi\,y)$$

Imaginary part

04.22.19.0003.01

$$\operatorname{Im}(L_{x+i\,y}) = \phi^x \sin(y \log(\phi)) - \phi^{-x} \cos(\pi\,x) \cosh(\pi\,y) \sin(y \log(\phi)) - \phi^{-x} \cos(y \log(\phi)) \sin(\pi\,x) \sinh(\pi\,y)$$

04.22.19.0004.01

$$\operatorname{Im}(L_{x+i\,y}) = \phi^x \sin\left(y \operatorname{csch}^{-1}(2)\right) - \phi^{-x} \cos(\pi\,x) \cosh(\pi\,y) \sin\left(y \operatorname{csch}^{-1}(2)\right) - \phi^{-x} \cos\left(y \operatorname{csch}^{-1}(2)\right) \sin(\pi\,x) \sinh(\pi\,y)$$

Absolute value

04.22.19.0005.01

$$\left|L_{x+i y}\right| = \frac{1}{\sqrt{2}} \sqrt{\left(\left(1+\sqrt{5}\right)^{-2x} \left(4^x \cos(2\pi x) + 2\left(\left(3+\sqrt{5}\right)^{2x} + 2^{-2 i y} \left(1+\sqrt{5}\right)^{2(x-i y)} \left(\cos(\pi(x-i y)) \left(1+\sqrt{5}\right)^{4 i y} + 2^{4 i y} \cos(\pi(x+i y))\right)\right) + 4^x \cosh(2\pi y)\right)\right)}$$

Argument

04.22.19.0006.01

$$\arg\left(L_{x+i y}\right) = \tan^{-1}(\phi^x \cos(y \log(\phi)) + \phi^{-x} \cos(\pi x) \cosh(\pi y) \cos(y \log(\phi)) - \phi^{-x} \sin(\pi x) \sin(y \log(\phi)) \sinh(\pi y), \phi^x \sin(y \log(\phi)) - \phi^{-x} \cos(\pi x) \cosh(\pi y) \sin(y \log(\phi)) - \phi^{-x} \cos(y \log(\phi)) \sin(\pi x) \sinh(\pi y))$$

Conjugate value

04.22.19.0007.01

$$\overline{L_{x+i y}} = \phi^x \cos(y \log(\phi)) + \phi^{-x} \cos(\pi x) \cosh(\pi y) \cos(y \log(\phi)) - \phi^{-x} \sin(\pi x) \sin(y \log(\phi)) \sinh(\pi y) - i\,(\phi^x \sin(y \log(\phi)) - \phi^{-x} \cos(\pi x) \cosh(\pi y) \sin(y \log(\phi)) - \phi^{-x} \cos(y \log(\phi)) \sin(\pi x) \sinh(\pi y))$$

Signum value

04.22.19.0008.01

$$\operatorname{sgn}\left(L_{x+i y}\right) = \phi^x \cos(y \log(\phi)) + \left(2^{\frac{1}{2}-x} \left(1+\sqrt{5}\right)^{-x} \left(i \sin\left(y \operatorname{csch}^{-1}(2)\right) \left(1+\sqrt{5}\right)^{2x} + 4^x \cos(\pi(x+i y)) \left(\cos\left(y \operatorname{csch}^{-1}(2)\right) - i \sin\left(y \operatorname{csch}^{-1}(2)\right)\right)\right)\right) \Big/ \left(\sqrt{\left(\left(1+\sqrt{5}\right)^{-2x} \left(4^x \cos(2\pi x) + 2\left(\left(3+\sqrt{5}\right)^{2x} + 2^{-2 i y} \left(1+\sqrt{5}\right)^{2(x-i y)} \left(\cos(\pi(x-i y)) \left(1+\sqrt{5}\right)^{4 i y} + 2^{4 i y} \cos(\pi(x+i y))\right)\right) + 4^x \cosh(2\pi y)\right)\right)}\right)$$

Differentiation

Low-order differentiation

04.22.20.0001.01

$$\frac{\partial L_\nu}{\partial \nu} = \phi^\nu \log(\phi) - \phi^{-\nu} \cos(\pi \nu) \log(\phi) - \phi^{-\nu} \pi \sin(\pi \nu)$$

04.22.20.0002.01

$$\frac{\partial^2 L_\nu}{\partial \nu^2} = \phi^\nu \log^2(\phi) + \phi^{-\nu} \cos(\pi \nu) \log^2(\phi) + 2\,\phi^{-\nu} \pi \sin(\pi \nu) \log(\phi) - \phi^{-\nu} \pi^2 \cos(\pi \nu)$$

Symbolic differentiation

04.22.20.0003.01

$$\frac{\partial^n L_\nu}{\partial \nu^n} = \phi^\nu \log^n(\phi) + \frac{1}{2} \phi^{-\nu} \left(e^{\pi i \nu} (\pi i - \log(\phi))^n + (-1)^n e^{-\pi i \nu} (\pi i + \log(\phi))^n\right) /; n \in \mathbb{N}$$

04.22.20.0004.01

$$\frac{\partial^n L_\nu}{\partial \nu^n} = \phi^\nu \log^n(\phi) + \phi^{-\nu} \sum_{k=0}^{n} (-1)^{n-k} \pi^k \binom{n}{k} \cos\left(\frac{\pi (k+2\nu)}{2}\right) \log^{n-k}(\phi) \;/;\; n \in \mathbb{N}$$

Fractional integro-differentiation

04.22.20.0005.01

$$\frac{\partial^\alpha L_\nu}{\partial \nu^\alpha} = -\frac{\nu^{-\alpha}}{2} \Big(\big(Q\big(-\alpha, \big(i\pi - \text{csch}^{-1}(2)\big)\nu\big) - 1\big) e^{\left(i\pi - \text{csch}^{-1}(2)\right)\nu} \big(\nu \big(\pi i - \text{csch}^{-1}(2)\big)\big)^\alpha + \big(Q\big(-\alpha, -\big(i\pi + \text{csch}^{-1}(2)\big)\nu\big) - 1\big) e^{-\left(i\pi + \text{csch}^{-1}(2)\right)\nu} \big(\nu \big(-i\pi - \text{csch}^{-1}(2)\big)\big)^\alpha + 2\big(Q\big(-\alpha, \nu\, \text{csch}^{-1}(2)\big) - 1\big) e^{\nu\, \text{csch}^{-1}(2)} \nu^\alpha\, \text{csch}^{-1}(2)^\alpha \Big)$$

Integration

Indefinite integration

Involving only one direct function

04.22.21.0001.01

$$\int L_{a\nu}\, d\nu = \frac{\phi^{-a\nu} (\pi \sin(\pi a \nu) - \cos(\pi a \nu) \log(\phi))}{a \left(\log^2(\phi) + \pi^2\right)} + \frac{\phi^{a\nu}}{a \log(\phi)}$$

04.22.21.0002.01

$$\int L_\nu\, d\nu = \frac{\phi^{-\nu} (\pi \sin(\pi \nu) - \cos(\pi \nu) \log(\phi))}{\log^2(\phi) + \pi^2} + \frac{\phi^\nu}{\log(\phi)}$$

Involving one direct function and elementary functions

Involving power function

04.22.21.0003.01

$$\int \nu^{\alpha-1} L_{a\nu}\, d\nu = -\frac{1}{2} \nu^\alpha \left(2 E_{1-\alpha}\big(-a \nu\, \text{csch}^{-1}(2)\big) + E_{1-\alpha}\big(a \nu \big(-i\pi + \text{csch}^{-1}(2)\big)\big) + E_{1-\alpha}\big(a \nu \big(i\pi + \text{csch}^{-1}(2)\big)\big)\right)$$

04.22.21.0004.01

$$\int \nu^{\alpha-1} L_\nu\, d\nu = -\frac{1}{2} \nu^\alpha \left(2 E_{1-\alpha}\big(-\nu\, \text{csch}^{-1}(2)\big) + E_{1-\alpha}\big(\nu \big(-i\pi + \text{csch}^{-1}(2)\big)\big) + E_{1-\alpha}\big(\nu \big(i\pi + \text{csch}^{-1}(2)\big)\big)\right)$$

Integral transforms

Laplace transforms

04.22.22.0001.01

$$\mathcal{L}_t[L_t](z) = \frac{z + \text{csch}^{-1}(2)}{\left(z + \text{csch}^{-1}(2)\right)^2 + \pi^2} + \frac{1}{z - \text{csch}^{-1}(2)} \;/;\; \text{Re}(z) > \log(\phi)$$

Summation

Finite summation

04.22.23.0001.01

$$\sum_{k=0}^{n} L_k = L_{n+2} - 1$$

04.22.23.0002.01

$$\sum_{k=0}^{n} \binom{n}{k} L_k = L_{2n} \;/;\; n \in \mathbb{N}$$

04.22.23.0003.01

$$\sum_{k=0}^{n} \binom{n}{k} 2^k L_k = L_{3n} \;/;\; n \in \mathbb{N}$$

04.22.23.0004.01

$$\sum_{k=0}^{n} L_k z^k = \frac{(z L_n + L_{n+1}) z^{n+1} + z - 2}{z^2 + z - 1} \;/;\; n \in \mathbb{N}$$

04.22.23.0005.01

$$\sum_{k=0}^{n} L_{k p+q} z^k = \frac{L_q - z^{n+1} L_{(n+1) p+q} + (-1)^p z^{n+2} L_{n p+q} - (-1)^p z L_{q-p}}{(-1)^p z^2 - z L_p + 1} \;/;\; p \in \mathbb{Z} \bigwedge q \in \mathbb{Z} \bigwedge n \in \mathbb{N}$$

04.22.23.0006.01

$$\sum_{k=0}^{n} L_k L_{n-k} = F_n + (n+2) L_n \;/;\; n \in \mathbb{N}$$

04.22.23.0007.01

$$\sum_{k=1}^{n} L_k^2 = L_n L_{n+1} - 2 \;/;\; n \in \mathbb{N}$$

Infinite summation

04.22.23.0008.01

$$\sum_{k=0}^{\infty} L_k z^k = \frac{z-2}{z^2 + z - 1}$$

Operations

Limit operation

04.22.25.0001.01

$$\lim_{\nu \to \infty} \frac{L_\nu}{F_\nu} = \sqrt{5}$$

04.22.25.0002.01

$$\lim_{\nu \to \infty} \frac{L_{\alpha+\nu}}{L_\nu} = \phi^\alpha$$

04.22.25.0003.01

$$\lim_{\nu\to\infty}\frac{\sum_{k=0}^{m-1}L_{k+\nu}}{L_{m+\nu}-L_\nu}=\phi\;/;\;m\in\mathbb{N}^+$$

Representations through more general functions

Through hypergeometric functions

Involving ${}_2F_1$

04.22.26.0001.01

$$L_\nu=2\cos^2\left(\frac{\pi\nu}{2}\right){}_2F_1\left(-\frac{\nu}{2},\frac{\nu}{2};\frac{1}{2};-\frac{1}{4}\right)+\nu\sin^2\left(\frac{\pi\nu}{2}\right){}_2F_1\left(\frac{1-\nu}{2},\frac{\nu+1}{2};\frac{3}{2};-\frac{1}{4}\right)$$

04.22.26.0002.01

$$L_\nu=\cos(\pi\nu)\,{}_2F_1\left(\frac{\nu}{2},\frac{\nu+1}{2};\nu+1;-4\right)+{}_2F_1\left(-\frac{\nu}{2},\frac{1}{2}-\frac{\nu}{2};1-\nu;-4\right)/;\;\nu\notin\mathbb{Z}$$

04.22.26.0003.01

$$L_\nu=2\left(\cos^3\left(\frac{\pi\nu}{2}\right)-i\sin^3\left(\frac{\pi\nu}{2}\right)\right){}_2F_1\left(-\frac{\nu}{2},\frac{\nu}{2};\frac{1}{2};\frac{5}{4}\right)+\frac{\sqrt{5}}{2}\,e^{\frac{i\pi\nu}{2}}\,\nu\sin(\pi\nu)\,{}_2F_1\left(\frac{1-\nu}{2},\frac{\nu+1}{2};\frac{3}{2};\frac{5}{4}\right)$$

04.22.26.0004.01

$$L_n=\frac{1}{2^{n-1}}\,{}_2F_1\left(\frac{1-n}{2},-\frac{n}{2};\frac{1}{2};5\right)/;\;n\in\mathbb{Z}$$

04.22.26.0005.01

$$L_n={}_2F_1\left(\frac{1-n}{2},-\frac{n}{2};1-n;-4\right)/;\;n\in\mathbb{N}^+$$

Through Meijer *G*

Classical cases for the direct function itself

04.22.26.0006.01

$$L_\nu=-\frac{\nu\sin(\pi\nu)}{2\sqrt{\pi}}\,G_{3,3}^{2,2}\left(\frac{1}{4}\,\middle|\,\begin{matrix}\frac{\nu}{2}+1,\,1-\frac{\nu}{2},\,\frac{\nu+1}{2}\\0,\,\frac{1}{2},\,\frac{\nu+1}{2}\end{matrix}\right)/;\;\nu\notin\mathbb{Z}$$

04.22.26.0007.01

$$L_\nu=-\frac{\nu\sin(\pi\nu)}{2\sqrt{\pi}}\,G_{3,3}^{2,2}\left(4\,\middle|\,\begin{matrix}\frac{1}{2},\,1,\,\frac{1-\nu}{2}\\\frac{\nu}{2},\,-\frac{\nu}{2},\,\frac{1-\nu}{2}\end{matrix}\right)/;\;\nu\notin\mathbb{Z}$$

Generalized cases for the direct function itself

04.22.26.0008.01

$$L_\nu=-\frac{\nu\sin(\pi\nu)}{2\sqrt{\pi}}\,G_{3,3}^{2,2}\left(\frac{1}{2},\frac{1}{2}\,\middle|\,\begin{matrix}\frac{\nu}{2}+1,\,1-\frac{\nu}{2},\,\frac{\nu+1}{2}\\0,\,\frac{1}{2},\,\frac{\nu+1}{2}\end{matrix}\right)/;\;\nu\notin\mathbb{Z}$$

Representations through equivalent functions

With elementary functions

04.22.27.0001.01

$$L_\nu = (\cos(\pi\nu) + 1)\cosh\left(\nu\,\operatorname{csch}^{-1}(2)\right) - (\cos(\pi\nu) - 1)\sinh\left(\nu\,\operatorname{csch}^{-1}(2)\right)$$

04.22.27.0002.01

$$L_\nu = 2^{-\nu}\left(\cos(\pi\nu)\left(-1 + \sqrt{5}\right)^\nu + \left(1 + \sqrt{5}\right)^\nu\right)$$

04.22.27.0003.01

$$L_\nu = 2\cos\left(\nu\csc^{-1}\left(\frac{2}{\sqrt{5}}\right)\right)\left(\cos^3\left(\frac{\pi\nu}{2}\right) - i\sin^3\left(\frac{\pi\nu}{2}\right)\right) + e^{\frac{i\pi\nu}{2}}\sin(\pi\nu)\sin\left(\nu\csc^{-1}\left(\frac{2}{\sqrt{5}}\right)\right)$$

04.22.27.0004.01

$$L_\nu = (1-\phi)^\nu + (\cos(\pi\nu) - (-1)^\nu)\,\phi^{-\nu} + \phi^\nu$$

With Fibonacci numbers

04.22.27.0005.01

$$L_\nu = F_{\nu-1} + F_{\nu+1}$$

04.22.27.0006.01

$$L_n = \frac{(-1)^n F_{m-n} + F_{m+n}}{F_m} \;/;\; m \in \mathbb{Z} \land m \neq 0 \land n \in \mathbb{Z}$$

04.22.27.0007.01

$$L_\nu = \frac{F_{2\nu}}{F_\nu} - \frac{\phi^{-2\nu}\sin^2(\pi\nu)}{\sqrt{5}\,F_\nu}$$

History

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