MathieuCharacteristicExponent

Notations

Traditional name

Characteristic exponent of a Mathieu function

Traditional notation

$r(a, q)$

Mathematica StandardForm notation

`MathieuCharacteristicExponent[a, q]`

Primary definition

11.07.02.0001.01

$r(a, q)$

$r(a, q)$ is the characteristic exponent for Mathieu functions $e^{i\, r(a,q)\, z} f(z)$ with parameter q, such that there exists a solution of the corresponding Mathieu differential equation $w''(z) + (a - 2\, q \cos(2\, z))\, w(z) = 0$ that is of the form $w(z) = e^{i\, r(a,q)\, z} f(z)$, where $f(z)$ has period 2π, with characteristic value a and parameter q.

Specific values

Specialized values

11.07.03.0001.01

$r(a, 0) = \sqrt{a}$

11.07.03.0002.01

$r(a_r(q), q) = r \;/;\; r > 0 \land q \in \mathbb{R}$

11.07.03.0003.01

$r(b_r(q), q) = r \;/;\; r > 0 \land q \in \mathbb{R}$

Values at fixed points

11.07.03.0004.01

$r(0, 0) = 0$

General characteristics

Domain and analyticity

$r(a, q)$ is an analytical function of a and q which is defined over $\mathbb{C}^2$.

11.07.04.0001.01

$$(a * q) \longrightarrow r(a, q) :: (\mathbb{C} \otimes \mathbb{C}) \longrightarrow \mathbb{C}$$

Symmetries and periodicities

Parity

11.07.04.0002.01

$$r(a, -q) == \overline{r(a, q)}$$

Mirror symmetry

11.07.04.0003.01

$$r(\overline{a}, \overline{q}) == \overline{r(a, q)}$$

Periodicity

No periodicity

Branch points

Branch points locations: complicated

Branch cuts

Branch cut locations: complicated

History

– E. L. Mathieu (1868, 1873)
– H. Weber (1869)
– G.W. Hill (1877)
– E. Heine (1878)
– G. Floquet (1883)
– R. C. Maclaurin (1898)
– J. Dougall (1916, 1926)

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