

NevilleThetaS

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Notations

Traditional name

Neville theta function ϑ_s

Traditional notation

 $\vartheta_s(z | m)$

Mathematica StandardForm notation

NevilleThetaS[z , m]

Primary definition

09.12.02.0001.01

$$\vartheta_s(z | m) = \frac{\sqrt{2\pi} \sqrt[4]{q(m)}}{\sqrt[4]{1-m} \sqrt[4]{m} \sqrt{K(m)}} \sum_{k=0}^{\infty} (-1)^k q(m)^{k(k+1)} \sin\left(\frac{(2k+1)\pi z}{2K(m)}\right)$$

Specific values

Specialized values

For fixed z

09.12.03.0001.01

$$\vartheta_s(z | 0) = \sin(z)$$

09.12.03.0002.01

$$\vartheta_s\left(z + \frac{\pi}{2} \middle| 0\right) = \cos(z)$$

For fixed m

09.12.03.0003.01

$$\vartheta_s(0 | m) = 0$$

General characteristics

Domain and analyticity

$\vartheta_s(z | m)$ is an analytical meromorphic function of z and m which is defined over \mathbb{C}^2 .

09.12.04.0001.01

$$(z * m) \rightarrow \vartheta_s(z | m) :: (\mathbb{C} \otimes \mathbb{C}) \rightarrow \mathbb{C}$$

Symmetries and periodicities

Parity

$\vartheta_s(z | m)$ is an odd function with respect to z .

09.12.04.0002.01

$$\vartheta_s(-z | m) = -\vartheta_s(z | m)$$

Mirror symmetry

09.12.04.0003.01

$$\vartheta_s(\bar{z} | \bar{m}) = \overline{\vartheta_s(z | m)}$$

Periodicity

$\vartheta_s(z | m)$ is a periodic function with respect to z with period $4K(m)$.

09.12.04.0004.01

$$\vartheta_s(z + 2K(m) | m) = -\vartheta_s(z | m)$$

09.12.04.0005.01

$$\vartheta_s(z + 4K(m) | m) = \vartheta_s(z | m)$$

09.12.04.0006.01

$$\vartheta_s(z + 2rK(m) | m) = (-1)^r \vartheta_s(z | m) ; r \in \mathbb{Z}$$

Branch points

Branch points locations: complicated

Branch cuts

Branch cut locations: complicated

Series representations

Generalized power series

09.12.06.0001.01

$$\vartheta_s(z | m) = \frac{\sqrt{2\pi} \sqrt[4]{q(m)}}{\sqrt[4]{1-m} \sqrt[4]{m} \sqrt{K(m)}} \sum_{k=0}^{\infty} (-1)^k q(m)^{k(k+1)} \sin\left(\frac{(2k+1)\pi z}{2K(m)}\right)$$

Product representations

09.12.08.0001.01

$$\vartheta_s(z | m) = \frac{2^{2/3} \sqrt[6]{q(m)}}{\sqrt[6]{1-m} \sqrt[6]{m}} \sin\left(\frac{\pi z}{2K(m)}\right) \prod_{k=1}^{\infty} \left(-2 \cos\left(\frac{\pi z}{K(m)}\right) q(m)^{2k} + q(m)^{4k} + 1\right)$$

Differential equations

Partial differential equations

09.12.13.0001.01

$$K(m) \frac{\partial^2 \vartheta_s(z | m)}{\partial z^2} + 2z(E(m) + (m-1)K(m)) \frac{\partial \vartheta_s(z | m)}{\partial z} - 4(m-1)mK(m) \frac{\partial \vartheta_s(z | m)}{\partial m} + (E(m) - mK(m)) \vartheta_s(z | m) = 0$$

Transformations

Transformations and argument simplifications

Argument involving basic arithmetic operations

09.12.16.0001.01

$$\vartheta_s(K(m) - z | m) = \frac{1}{\sqrt[4]{1-m}} \vartheta_c(z | m)$$

09.12.16.0002.01

$$\vartheta_s(z + (2r+1)K(m) | m) = \frac{(-1)^r}{\sqrt[4]{1-m}} \vartheta_c(z | m) ; r \in \mathbb{Z}$$

Differentiation

Low-order differentiation

With respect to z

09.12.20.0001.01

$$\frac{\partial \vartheta_s(z | m)}{\partial z} = \frac{\pi^{3/2} \sqrt[4]{q(m)}}{\sqrt{2} \sqrt[4]{1-m} \sqrt[4]{m} K(m)^{3/2}} \sum_{k=0}^{\infty} (-1)^k (2k+1) q(m)^{k(k+1)} \cos\left(\frac{\pi(2kz+z)}{2K(m)}\right)$$

09.12.20.0002.01

$$\frac{\partial^2 \vartheta_s(z | m)}{\partial z^2} = -\frac{\pi^{5/2} \sqrt[4]{q(m)}}{2\sqrt{2} \sqrt[4]{1-m} \sqrt[4]{m} K(m)^{5/2}} \sum_{k=0}^{\infty} (-1)^k (2k+1)^n q(m)^{k(k+1)} \sin\left(\frac{(2k+1)\pi z}{2K(m)}\right)$$

Symbolic differentiation

With respect to z

09.12.20.0003.01

$$\frac{\partial^n \vartheta_s(z | m)}{\partial z^n} = \frac{2^{\frac{1}{2}-n} \pi^{n+\frac{1}{2}} K(m)^{-n-\frac{1}{2}} \sqrt[4]{q(m)}}{\sqrt[4]{1-m} \sqrt[4]{m}} \sum_{k=0}^{\infty} (-1)^k (2k+1)^n q(m)^{k(k+1)} \sin\left(\frac{\pi n}{2} + \frac{(2k+1)\pi z}{2K(m)}\right) ; n \in \mathbb{N}^+$$

Fractional integro-differentiation

With respect to z

09.12.20.0004.01

$$\frac{\partial^\alpha \vartheta_s(z | m)}{\partial z^\alpha} = \frac{2^{\alpha-\frac{3}{2}} \pi^2 z^{1-\alpha} \sqrt[4]{q(m)}}{\sqrt[4]{1-m} \sqrt[4]{m} K(m)^{3/2}} \sum_{k=0}^{\infty} (-1)^k (2k+1) q(m)^{k(k+1)} {}_1\tilde{F}_2 \left(1; 1 - \frac{\alpha}{2}, \frac{3-\alpha}{2}; -\frac{(2k+1)^2 \pi^2 z^2}{16 K(m)^2} \right)$$

Integration

Indefinite integration

Involving only one direct function

09.12.21.0001.01

$$\int \vartheta_s(z | m) dz = \frac{4 \sqrt{2} \sqrt{K(m)} \sqrt[4]{q(m)}}{\sqrt[4]{1-m} \sqrt[4]{m} \sqrt{\pi}} \sum_{k=0}^{\infty} \frac{(-1)^k q(m)^{k(k+1)}}{2k+1} \sin^2 \left(\frac{(2k+1)\pi z}{4 K(m)} \right)$$

Representations through equivalent functions

With related functions

Involving Jacobi and other Neville functions

09.12.27.0001.01

$$\vartheta_s(z | m) = \operatorname{sc}(z | m) \vartheta_c(z | m)$$

09.12.27.0002.01

$$\vartheta_s(z | m) = \frac{\vartheta_c(z | m)}{\operatorname{cs}(z | m)}$$

09.12.27.0003.01

$$\vartheta_s(z | m) = -\frac{1}{\sqrt[4]{1-m}} \vartheta_c(z + K(m) | m)$$

09.12.27.0004.01

$$\vartheta_s(z | m) = \operatorname{sd}(z | m) \vartheta_d(z | m)$$

09.12.27.0005.01

$$\vartheta_s(z | m) = \frac{\vartheta_d(z | m)}{\operatorname{ds}(z | m)}$$

09.12.27.0006.01

$$\vartheta_s(z | m) = \operatorname{sn}(z | m) \vartheta_n(z | m)$$

09.12.27.0007.01

$$\vartheta_s(z | m) = \frac{\vartheta_n(z | m)}{\operatorname{ns}(z | m)}$$

Involving theta functions

09.12.27.0008.01

$$\vartheta_s(z | m) = \frac{\sqrt{\pi}}{\sqrt{2} \sqrt[4]{1-m} \sqrt[4]{m} \sqrt{K(m)}} \vartheta_1 \left(\frac{\pi z}{2 K(m)}, q(m) \right)$$

History

- K. Weierstrass (1894)
- E. N. Neville (1944)

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