NevilleThetaS

Notations

Traditional name

Neville theta function ϑ_s

Traditional notation

$\vartheta_s(z \mid m)$

Mathematica StandardForm notation

`NevilleThetaS[z, m]`

Primary definition

09.12.02.0001.01

$$\vartheta_s(z \mid m) = \frac{\sqrt{2\pi}\,\sqrt[4]{q(m)}}{\sqrt[4]{1-m}\,\sqrt[4]{m}\,\sqrt{K(m)}} \sum_{k=0}^{\infty} (-1)^k\, q(m)^{k(k+1)} \sin\left(\frac{(2k+1)\pi z}{2K(m)}\right)$$

Specific values

Specialized values

For fixed *z*

09.12.03.0001.01

$$\vartheta_s(z \mid 0) = \sin(z)$$

09.12.03.0002.01

$$\vartheta_s\left(z + \frac{\pi}{2} \,\middle|\, 0\right) = \cos(z)$$

For fixed *m*

09.12.03.0003.01

$$\vartheta_s(0 \mid m) = 0$$

General characteristics

Domain and analyticity

$\vartheta_s(z \mid m)$ is an analytical meromorphic function of z and m which is defined over $\mathbb{C}^2$.

09.12.04.0001.01
$(z * m) \longrightarrow \vartheta_s(z \mid m) :: (\mathbb{C} \otimes \mathbb{C}) \longrightarrow \mathbb{C}$

Symmetries and periodicities

Parity

$\vartheta_s(z \mid m)$ is an odd function with respect to z.

09.12.04.0002.01
$$\vartheta_s(-z \mid m) = -\vartheta_s(z \mid m)$$

Mirror symmetry

09.12.04.0003.01
$$\vartheta_s(\bar{z} \mid \overline{m}) = \overline{\vartheta_s(z \mid m)}$$

Periodicity

$\vartheta_s(z \mid m)$ is a periodic function with respect to z with period $4\,K(m)$.

09.12.04.0004.01
$$\vartheta_s(z + 2\,K(m) \mid m) = -\vartheta_s(z \mid m)$$

09.12.04.0005.01
$$\vartheta_s(z + 4\,K(m) \mid m) = \vartheta_s(z \mid m)$$

09.12.04.0006.01
$$\vartheta_s(z + 2\,r\,K(m) \mid m) = (-1)^r\,\vartheta_s(z \mid m) \,/;\, r \in \mathbb{Z}$$

Branch points

Branch points locations: complicated

Branch cuts

Branch cut locations: complicated

Series representations

Generalized power series

09.12.06.0001.01
$$\vartheta_s(z \mid m) = \frac{\sqrt{2\pi}\,\sqrt[4]{q(m)}}{\sqrt[4]{1-m}\,\sqrt[4]{m}\,\sqrt{K(m)}} \sum_{k=0}^{\infty} (-1)^k\, q(m)^{k\,(k+1)} \sin\left(\frac{(2\,k+1)\,\pi\,z}{2\,K(m)}\right)$$

Product representations

09.12.08.0001.01
$$\vartheta_s(z \mid m) = \frac{2^{2/3}\,\sqrt[6]{q(m)}}{\sqrt[6]{1-m}\,\sqrt[6]{m}} \sin\left(\frac{\pi\,z}{2\,K(m)}\right) \prod_{k=1}^{\infty} \left(-2\cos\left(\frac{\pi\,z}{K(m)}\right) q(m)^{2\,k} + q(m)^{4\,k} + 1\right)$$

Differential equations

Partial differential equations

09.12.13.0001.01

$$K(m)\frac{\partial^2 \vartheta_s(z\mid m)}{\partial z^2}+2\,z\,(E(m)+(m-1)\,K(m))\frac{\partial \vartheta_s(z\mid m)}{\partial z}-4\,(m-1)\,m\,K(m)\frac{\partial \vartheta_s(z\mid m)}{\partial m}+(E(m)-m\,K(m))\,\vartheta_s(z\mid m)=0$$

Transformations

Transformations and argument simplifications

Argument involving basic arithmetic operations

09.12.16.0001.01

$$\vartheta_s(K(m)-z\mid m)=\frac{1}{\sqrt[4]{1-m}}\,\vartheta_c(z\mid m)$$

09.12.16.0002.01

$$\vartheta_s(z+(2\,r+1)\,K(m)\mid m)=\frac{(-1)^r}{\sqrt[4]{1-m}}\,\vartheta_c(z\mid m)\;/;\;r\in\mathbb{Z}$$

Differentiation

Low-order differentiation

With respect to *z*

09.12.20.0001.01

$$\frac{\partial \vartheta_s(z\mid m)}{\partial z}=\frac{\pi^{3/2}\sqrt[4]{q(m)}}{\sqrt{2}\,\sqrt[4]{1-m}\,\sqrt[4]{m}\,K(m)^{3/2}}\sum_{k=0}^{\infty}(-1)^k\,(2\,k+1)\,q(m)^{k\,(k+1)}\cos\left(\frac{\pi\,(2\,k\,z+z)}{2\,K(m)}\right)$$

09.12.20.0002.01

$$\frac{\partial^2 \vartheta_s(z\mid m)}{\partial z^2}=-\frac{\pi^{5/2}\sqrt[4]{q(m)}}{2\sqrt{2}\,\sqrt[4]{1-m}\,\sqrt[4]{m}\,K(m)^{5/2}}\sum_{k=0}^{\infty}(-1)^k\,(2\,k+1)^n\,q(m)^{k\,(k+1)}\sin\left(\frac{(2\,k+1)\,\pi\,z}{2\,K(m)}\right)$$

Symbolic differentiation

With respect to *z*

09.12.20.0003.01

$$\frac{\partial^n \vartheta_s(z\mid m)}{\partial z^n}=\frac{2^{\frac{1}{2}-n}\,\pi^{n+\frac{1}{2}}\,K(m)^{-n-\frac{1}{2}}\sqrt[4]{q(m)}}{\sqrt[4]{1-m}\,\sqrt[4]{m}}\sum_{k=0}^{\infty}(-1)^k\,(2\,k+1)^n\,q(m)^{k\,(k+1)}\sin\left(\frac{\pi\,n}{2}+\frac{(2\,k+1)\,\pi\,z}{2\,K(m)}\right)\;/;\;n\in\mathbb{N}^+$$

Fractional integro-differentiation

With respect to *z*

09.12.20.0004.01

$$\frac{\partial^{\alpha}\vartheta_s(z\mid m)}{\partial z^{\alpha}} = \frac{2^{\alpha-\frac{3}{2}}\,\pi^2\,z^{1-\alpha}\,\sqrt[4]{q(m)}}{\sqrt[4]{1-m}\,\sqrt[4]{m}\,K(m)^{3/2}}\sum_{k=0}^{\infty}(-1)^k\,(2k+1)\,q(m)^{k(k+1)}\,{}_1\tilde{F}_2\left(1;\,1-\frac{\alpha}{2},\,\frac{3-\alpha}{2};\,-\frac{(2k+1)^2\,\pi^2\,z^2}{16\,K(m)^2}\right)$$

Integration

Indefinite integration

Involving only one direct function

09.12.21.0001.01

$$\int\vartheta_s(z\mid m)\,dz = \frac{4\sqrt{2}\,\sqrt{K(m)}\,\sqrt[4]{q(m)}}{\sqrt[4]{1-m}\,\sqrt[4]{m}\,\sqrt{\pi}}\sum_{k=0}^{\infty}\frac{(-1)^k\,q(m)^{k(k+1)}}{2k+1}\sin^2\!\left(\frac{(2k+1)\,\pi\,z}{4\,K(m)}\right)$$

Representations through equivalent functions

With related functions

Involving Jacobi and other Neville functions

09.12.27.0001.01

$$\vartheta_s(z\mid m) = \mathrm{sc}(z\mid m)\,\vartheta_c(z\mid m)$$

09.12.27.0002.01

$$\vartheta_s(z\mid m) = \frac{\vartheta_c(z\mid m)}{\mathrm{cs}(z\mid m)}$$

09.12.27.0003.01

$$\vartheta_s(z\mid m) = -\frac{1}{\sqrt[4]{1-m}}\,\vartheta_c(z+K(m)\mid m)$$

09.12.27.0004.01

$$\vartheta_s(z\mid m) = \mathrm{sd}(z\mid m)\,\vartheta_d(z\mid m)$$

09.12.27.0005.01

$$\vartheta_s(z\mid m) = \frac{\vartheta_d(z\mid m)}{\mathrm{ds}(z\mid m)}$$

09.12.27.0006.01

$$\vartheta_s(z\mid m) = \mathrm{sn}(z\mid m)\,\vartheta_n(z\mid m)$$

09.12.27.0007.01

$$\vartheta_s(z\mid m) = \frac{\vartheta_n(z\mid m)}{\mathrm{ns}(z\mid m)}$$

Involving theta functions

09.12.27.0008.01

$$\vartheta_s(z\mid m) = \frac{\sqrt{\pi}}{\sqrt{2}\,\sqrt[4]{1-m}\,\sqrt[4]{m}\,\sqrt{K(m)}}\,\vartheta_1\!\left(\frac{\pi z}{2\,K(m)},\,q(m)\right)$$

History

– K. Weierstrass (1894)
– E. N. Neville (1944)

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