ParabolicCylinderD

Notations

Traditional name

Parabolic cylinder function

Traditional notation

$D_\nu(z)$

Mathematica StandardForm notation

ParabolicCylinderD[ν, z]

Primary definition

07.41.02.0001.01

$$D_\nu(z) = 2^{\nu/2}\sqrt{\pi}\, e^{-\frac{z^2}{4}}\left(\frac{1}{\Gamma\left(\frac{1-\nu}{2}\right)}\, {}_1F_1\left(-\frac{\nu}{2};\frac{1}{2};\frac{z^2}{2}\right)-\frac{\sqrt{2}\,z}{\Gamma\left(-\frac{\nu}{2}\right)}\, {}_1F_1\left(\frac{1-\nu}{2};\frac{3}{2};\frac{z^2}{2}\right)\right)$$

Specific values

Specialized values

For fixed ν

07.41.03.0001.01

$$D_\nu(0) = \frac{2^{\nu/2}\sqrt{\pi}}{\Gamma\left(\frac{1-\nu}{2}\right)}$$

For fixed z

Explicit rational ν

07.41.03.0002.01

$$D_{-\frac{9}{2}}(z) = \frac{2\sqrt{\pi}}{105}\left(\left(2z^4+10z^2+5\right)\sqrt[4]{z^2}\, I_{-\frac{1}{4}}\left(\frac{z^2}{4}\right)-2z\left(z^2+4\right)\left(z^2\right)^{3/4} I_{-\frac{3}{4}}\left(\frac{z^2}{4}\right)+2\left(z^2+4\right)\left(z^2\right)^{5/4} I_{\frac{3}{4}}\left(\frac{z^2}{4}\right)-\frac{1}{z}\left(z^2\right)^{3/4}\left(2z^4+10z^2+5\right) I_{\frac{1}{4}}\left(\frac{z^2}{4}\right)\right)$$

07.41.03.0003.01

$$D_{-4}(z) = -\frac{1}{3}\sqrt{\frac{\pi}{8}}\, z\left(z^2+3\right) e^{\frac{z^2}{4}} + \frac{z^2+2}{6}\, e^{-\frac{z^2}{4}} + \frac{1}{3}\sqrt{\frac{\pi}{8}}\, e^{\frac{z^2}{4}}\, z\left(z^2+3\right)\operatorname{erf}\left(\frac{z}{\sqrt{2}}\right)$$

07.41.03.0004.01

$$D_{-\frac{7}{2}}(z) = \frac{\sqrt{\pi}}{15}\left(-z\left(2z^2+3\right)\sqrt[4]{z^2}\; I_{\frac{3}{4}}\left(\frac{z^2}{4}\right) - z\left(2z^2+5\right)\sqrt[4]{z^2}\; I_{-\frac{1}{4}}\left(\frac{z^2}{4}\right) + \left(2z^2+5\right)\left(z^2\right)^{3/4} I_{\frac{1}{4}}\left(\frac{z^2}{4}\right) + \left(2z^2+3\right)\left(z^2\right)^{3/4} I_{-\frac{3}{4}}\left(\frac{z^2}{4}\right)\right)$$

07.41.03.0005.01

$$D_{-3}(z) = -\frac{z}{2}\, e^{-\frac{z^2}{4}} - \sqrt{\frac{\pi}{8}}\left(z^2+1\right) e^{\frac{z^2}{4}} \operatorname{erf}\left(\frac{z}{\sqrt{2}}\right) + \sqrt{\frac{\pi}{8}}\left(z^2+1\right) e^{\frac{z^2}{4}}$$

07.41.03.0006.01

$$D_{-\frac{5}{2}}(z) = \frac{\sqrt{\pi}}{3}\left(\left(z^2+1\right)\sqrt[4]{z^2}\; I_{-\frac{1}{4}}\left(\frac{z^2}{4}\right) - z\left(z^2\right)^{3/4} I_{-\frac{3}{4}}\left(\frac{z^2}{4}\right) + \left(z^2\right)^{5/4} I_{\frac{3}{4}}\left(\frac{z^2}{4}\right) - \frac{1}{z}\left(z^2\right)^{3/4}\left(z^2+1\right) I_{\frac{1}{4}}\left(\frac{z^2}{4}\right)\right)$$

07.41.03.0007.01

$$D_{-2}(z) = -\sqrt{\frac{\pi}{2}}\, z\, e^{\frac{z^2}{4}} + \sqrt{\frac{\pi}{2}}\, e^{\frac{z^2}{4}}\, z \operatorname{erf}\left(\frac{z}{\sqrt{2}}\right) + e^{-\frac{z^2}{4}}$$

07.41.03.0008.01

$$D_{-\frac{3}{2}}(z) = \frac{\sqrt{\pi}}{2}\left(-z\sqrt[4]{z^2}\; I_{\frac{3}{4}}\left(\frac{z^2}{4}\right) - z\sqrt[4]{z^2}\; I_{-\frac{1}{4}}\left(\frac{z^2}{4}\right) + \left(z^2\right)^{3/4} I_{\frac{1}{4}}\left(\frac{z^2}{4}\right) + \left(z^2\right)^{3/4} I_{-\frac{3}{4}}\left(\frac{z^2}{4}\right)\right)$$

07.41.03.0009.01

$$D_{-1}(z) = \sqrt{\frac{\pi}{2}}\, e^{\frac{z^2}{4}}\left(1-\operatorname{erf}\left(\frac{z}{\sqrt{2}}\right)\right)$$

07.41.03.0010.01

$$D_{-\frac{1}{2}}(z) = \frac{\sqrt{\pi}}{2}\left(\sqrt[4]{z^2}\; I_{-\frac{1}{4}}\left(\frac{z^2}{4}\right) - \frac{z}{\sqrt[4]{z^2}}\, I_{\frac{1}{4}}\left(\frac{z^2}{4}\right)\right)$$

07.41.03.0011.01

$$D_0(z) = e^{-\frac{z^2}{4}}$$

07.41.03.0012.01

$$D_{\frac{1}{2}}(z) = \frac{\sqrt{\pi}}{4}\left(-z\sqrt[4]{z^2}\; I_{\frac{3}{4}}\left(\frac{z^2}{4}\right) + z\sqrt[4]{z^2}\; I_{-\frac{1}{4}}\left(\frac{z^2}{4}\right) - \left(z^2\right)^{3/4} I_{\frac{1}{4}}\left(\frac{z^2}{4}\right) + I_{-\frac{3}{4}}\left(\frac{z^2}{4}\right)\left(z^2\right)^{3/4}\right)$$

07.41.03.0013.01

$$D_1(z) = z\, e^{-\frac{z^2}{4}}$$

07.41.03.0014.01

$$D_{\frac{3}{2}}(z) = \frac{\sqrt{\pi}}{4}\left(\left(z^2-1\right)\sqrt[4]{z^2}\; I_{-\frac{1}{4}}\left(\frac{z^2}{4}\right) + z\left(z^2\right)^{3/4} I_{-\frac{3}{4}}\left(\frac{z^2}{4}\right) - \left(z^2\right)^{5/4} I_{\frac{3}{4}}\left(\frac{z^2}{4}\right) - \frac{\left(z^2\right)^{3/4}\left(z^2-1\right)}{z}\, I_{\frac{1}{4}}\left(\frac{z^2}{4}\right)\right)$$

07.41.03.0015.01

$$D_2(z) = e^{-\frac{z^2}{4}}\left(z^2-1\right)$$

07.41.03.0016.01

$$D_{\frac{5}{2}}(z) = \frac{1}{8}\sqrt{\pi}\left(-z\left(2z^2-3\right)\sqrt[4]{z^2}\, I_{\frac{3}{4}}\left(\frac{z^2}{4}\right) + z\left(2z^2-5\right)\sqrt[4]{z^2}\, I_{-\frac{1}{4}}\left(\frac{z^2}{4}\right) - \left(2z^2-5\right)\left(z^2\right)^{3/4} I_{\frac{1}{4}}\left(\frac{z^2}{4}\right) + \left(2z^2-3\right)\left(z^2\right)^{3/4} I_{-\frac{3}{4}}\left(\frac{z^2}{4}\right)\right)$$

07.41.03.0017.01

$$D_3(z) = z\left(z^2-3\right)e^{-\frac{z^2}{4}}$$

07.41.03.0018.01

$$D_{\frac{7}{2}}(z) = \frac{1}{8}\sqrt{\pi}\left(\left(2z^4-10z^2+5\right)\sqrt[4]{z^2}\, I_{-\frac{1}{4}}\left(\frac{z^2}{4}\right) + 2z\left(z^2-4\right)\left(z^2\right)^{3/4} I_{-\frac{3}{4}}\left(\frac{z^2}{4}\right) - 2\left(z^2-4\right)\left(z^2\right)^{5/4} I_{\frac{3}{4}}\left(\frac{z^2}{4}\right) - \frac{\left(z^2\right)^{3/4}\left(2z^4-10z^2+5\right)}{z} I_{\frac{1}{4}}\left(\frac{z^2}{4}\right)\right)$$

07.41.03.0019.01

$$D_4(z) = \left(z^4-6z^2+3\right)e^{-\frac{z^2}{4}}$$

07.41.03.0020.01

$$D_{\frac{9}{2}}(z) = \frac{1}{16}\sqrt{\pi}\left(-z\left(4z^4-30z^2+21\right)\sqrt[4]{z^2}\, I_{\frac{3}{4}}\left(\frac{z^2}{4}\right) + z\left(4z^4-34z^2+45\right)\sqrt[4]{z^2}\, I_{-\frac{1}{4}}\left(\frac{z^2}{4}\right) - \left(4z^4-34z^2+45\right)\left(z^2\right)^{3/4} I_{\frac{1}{4}}\left(\frac{z^2}{4}\right) + \left(4z^4-30z^2+21\right)\left(z^2\right)^{3/4} I_{-\frac{3}{4}}\left(\frac{z^2}{4}\right)\right)$$

Symbolic rational ν

07.41.03.0021.01

$$D_{2n}(z) = (-1)^n 2^n n!\, e^{-\frac{z^2}{4}} L_n^{-\frac{1}{2}}\left(\frac{z^2}{2}\right) /;\, n \in \mathbb{N}$$

07.41.03.0022.01

$$D_{2n+1}(z) = (-1)^n 2^n n!\, z\, e^{-\frac{z^2}{4}} L_n^{\frac{1}{2}}\left(\frac{z^2}{2}\right) /;\, n \in \mathbb{N}$$

07.41.03.0023.01

$$D_{-n-1}(z) = \frac{2^{-\frac{1}{2}(n+1)}\sqrt{\pi}\, i^n}{n!} e^{\frac{z^2}{4}} H_n\left(\frac{i z}{\sqrt{2}}\right)\operatorname{erfc}\left(\frac{z}{\sqrt{2}}\right) + \frac{2^{\frac{1-n}{2}}\, i^n}{n!} e^{-\frac{z^2}{4}} \sum_{k=1}^{n}\binom{n}{k}(-i)^k H_{n-k}\left(\frac{i z}{\sqrt{2}}\right) H_{k-1}\left(\frac{z}{\sqrt{2}}\right) /;\, n \in \mathbb{N}$$

07.41.03.0024.01

$$D_{2n-\frac{1}{2}}(z) = (-1)^n 2^{n-\frac{5}{4}} e^{-\frac{z^2}{4}}\left(\sqrt{z^2}-z\right)\Gamma\left(n+\frac{1}{4}\right) L_{n-\frac{3}{4}}^{\frac{1}{2}}\left(\frac{z^2}{2}\right) + \frac{(-2)^n \sqrt[4]{z^2}\, n!}{\sqrt{2\pi}} \sum_{k=0}^{n}\frac{\left(-\frac{z^2}{8}\right)^k}{k!} L_{n-k}^{k-\frac{1}{2}}\left(\frac{z^2}{4}\right)\sum_{m=0}^{k}\binom{k}{m} K_{-k+2m+\frac{1}{4}}\left(\frac{z^2}{4}\right) /;\, n \in \mathbb{N}$$

Brychkov Yu.A. (2005)

07.41.03.0025.01

$$D_{2n-\frac{3}{2}}(z) = (-1)^n\, 2^{n-\frac{7}{4}}\, e^{-\frac{z^2}{4}} \left(\sqrt{z^2} - z\right) \Gamma\!\left(n - \frac{1}{4}\right) L_{n-\frac{5}{4}}^{\frac{1}{2}}\!\left(\frac{z^2}{2}\right) + \frac{(-2)^n\, n!\, \left(z^2\right)^{3/4}}{\sqrt{2\pi}} \sum_{k=0}^{n} \frac{\left(-\frac{z^2}{8}\right)^k}{k!}\, L_{n-k}^{k-\frac{1}{2}}\!\left(\frac{z^2}{4}\right) \sum_{m=0}^{k} \binom{k}{m} \left(K_{-k+2m+\frac{3}{4}}\!\left(\frac{z^2}{4}\right) - K_{-k+2m+\frac{1}{4}}\!\left(\frac{z^2}{4}\right)\right) /;\, n \in \mathbb{N}$$

Brychkov Yu.A. (2005)

07.41.03.0026.01

$$D_{-2n-\frac{1}{2}}(z) = (-1)^n\, 2^{-n-\frac{5}{4}}\, e^{-\frac{z^2}{4}} \left(\sqrt{z^2} - z\right) \Gamma\!\left(\frac{1}{4} - n\right) L_{-n-\frac{3}{4}}^{\frac{1}{2}}\!\left(\frac{z^2}{2}\right) + \frac{2^{n-\frac{1}{2}}\, n!\, \sqrt[4]{z^2}}{\sqrt{\pi}\, \left(\frac{1}{2}\right)_{2n}} \sum_{k=0}^{n} \frac{\left(-\frac{z^2}{8}\right)^k}{k!}\, L_{n-k}^{k-\frac{1}{2}}\!\left(-\frac{z^2}{4}\right) \sum_{m=0}^{k} \binom{k}{m} K_{-k+2m+\frac{1}{4}}\!\left(\frac{z^2}{4}\right) /;\, n \in \mathbb{N}$$

Brychkov Yu.A. (2005)

07.41.03.0027.01

$$D_{-2n-\frac{3}{2}}(z) = (-1)^n\, 2^{-n-\frac{7}{4}}\, e^{-\frac{z^2}{4}} \left(\sqrt{z^2} - z\right) \Gamma\!\left(-n - \frac{1}{4}\right) L_{-n-\frac{5}{4}}^{\frac{1}{2}}\!\left(\frac{z^2}{2}\right) + \frac{2^{n-\frac{1}{2}}\, n!\, \left(z^2\right)^{3/4}}{\sqrt{\pi}\, \left(\frac{3}{2}\right)_{2n}} \sum_{k=0}^{n} \frac{\left(-\frac{z^2}{8}\right)^k}{k!}\, L_{n-k}^{k+\frac{1}{2}}\!\left(-\frac{z^2}{4}\right) \sum_{m=0}^{k} \binom{k}{m} \left(K_{-k+2m+\frac{3}{4}}\!\left(\frac{z^2}{4}\right) - K_{-k+2m+\frac{1}{4}}\!\left(\frac{z^2}{4}\right)\right) /;\, n \in \mathbb{N}$$

Brychkov Yu.A. (2005)

Values at fixed points

07.41.03.0028.01

$$D_0(0) = 1$$

Values at infinities

07.41.03.0029.01

$$D_\nu(\infty) = 0$$

07.41.03.0030.01

$$D_\nu\!\left(e^{i\lambda}\, \infty\right) = \begin{cases} 0 & \lambda = 0 \\ \tilde{\infty} & \text{True} \end{cases} /;\, \operatorname{Im}(\lambda) = 0$$

07.41.03.0031.01

$$\lim_{z \to \infty} D_\nu(z) = 0$$

07.41.03.0032.01

$$\lim_{z \to -\infty} D_\nu(z) = \tilde{\infty}$$

07.41.03.0033.01

$$\lim_{z \to i\infty} D_\nu(z) = \tilde{\infty}$$

07.41.03.0034.01

$$\lim_{z\to -i\infty} D_\nu(z) = \tilde{\infty}$$

General characteristics

Domain and analyticity

$D_\nu(z)$ is an analytical function of ν and z, which is defined in $\mathbb{C}^2$.

07.41.04.0001.01

$$(\nu * z) \longrightarrow D_\nu(z) :: (\mathbb{C} \otimes \mathbb{C}) \longrightarrow \mathbb{C}$$

Symmetries and periodicities

Parity

07.41.04.0002.01

$$D_n(-z) = (-1)^n D_n(z) \;/;\; n \in \mathbb{N}$$

Mirror symmetry

07.41.04.0003.01

$$D_{\nu^*}(z^*) = D_\nu(z)^*$$

Periodicity

No periodicity

Poles and essential singularities

With respect to z

For fixed ν, the function $D_\nu(z)$ has an essential singularity at $z = \tilde{\infty}$.

07.41.04.0004.01

$$\mathcal{Sing}_z(D_\nu(z)) = \{\{\tilde{\infty}, \infty\}\}$$

With respect to ν

For fixed z, the function $D_\nu(z)$ has only one singular point at $\nu = \tilde{\infty}$. It is an essential singular point.

07.41.04.0005.01

$$\mathcal{Sing}_\nu(D_\nu(z)) = \{\{\tilde{\infty}, \infty\}\}$$

Branch points

With respect to z

For fixed ν, the function $D_\nu(z)$ does not have branch points.

07.41.04.0006.01

$$\mathcal{BP}_\nu(D_\nu(z)) = \{\}$$

With respect to ν

For fixed z, the function $D_\nu(z)$ does not have branch points.

07.41.04.0007.01

$$\mathcal{BP}_z(D_\nu(z)) = \{\}$$

Branch cuts

With respect to z

For fixed ν, the function $D_\nu(z)$ is an entire function of z and does not have branch cuts.

07.41.04.0008.01

$$\mathcal{BC}_z(D_\nu(z)) = \{\}$$

With respect to ν

For fixed z, the function $D_\nu(z)$ is an entire function of ν and does not have branch cuts.

07.41.04.0009.01

$$\mathcal{BC}_\nu(D_\nu(z)) = \{\}$$

Series representations

Generalized power series

Expansions at generic point $z = z_0$

07.41.06.0001.01

$$D_\nu(z) = D_\nu(z_0) - \left(D_{\nu+1}(z_0) - \frac{1}{2} z_0 D_\nu(z_0)\right)(z - z_0) + \frac{1}{8}\left(4\left(D_{\nu+2}(z_0) - z_0 D_{\nu+1}(z_0)\right) + \left(z_0^2 + 2\right) D_\nu(z_0)\right)(z - z_0)^2 + \ldots /; (z \to z_0)$$

07.41.06.0002.01

$$D_\nu(z) = D_\nu(z_0) - \left(D_{\nu+1}(z_0) - \frac{1}{2} z_0 D_\nu(z_0)\right)(z - z_0) + \frac{1}{8}\left(4\left(D_{\nu+2}(z_0) - z_0 D_{\nu+1}(z_0)\right) + \left(z_0^2 + 2\right) D_\nu(z_0)\right)(z - z_0)^2 + O\left((z - z_0)^3\right)$$

07.41.06.0003.01

$$D_\nu(z) = \sum_{k=0}^{\infty} \frac{1}{k!}\left(-\frac{i}{2}\right)^k \sum_{j=0}^{k} \binom{k}{j} (-2\, i)^j H_{k-j}\left(\frac{i\, z_0}{2}\right) D_{j+\nu}(z_0)\,(z - z_0)^k$$

07.41.06.0004.01

$$D_\nu(z) = \sum_{k=0}^{\infty} \frac{1}{k!}\left(-\frac{1}{2}\right)^k \sum_{j=0}^{k} \binom{k}{j} 2^j (-\nu)_j H_{k-j}\left(\frac{z_0}{2}\right) D_{\nu-j}(z_0)\,(z - z_0)^k$$

07.41.06.0005.01

$$D_\nu(z) \propto D_\nu(z_0)\,(1 + O(z - z_0))$$

Expansions at $z = 0$

07.41.06.0006.01

$$D_\nu(z) \propto \frac{2^{\nu/2}\sqrt{\pi}}{\Gamma\left(\frac{1-\nu}{2}\right)} - \frac{2^{\frac{\nu+1}{2}}\sqrt{\pi}\, z}{\Gamma\left(-\frac{\nu}{2}\right)} - \frac{2^{\frac{\nu}{2}-2}\sqrt{\pi}\,(2\nu + 1)\, z^2}{\Gamma\left(\frac{1-\nu}{2}\right)} + \frac{2^{\frac{\nu-3}{2}}\sqrt{\pi}\,(2\nu + 1)\, z^3}{3\,\Gamma\left(-\frac{\nu}{2}\right)} + \ldots /; (z \to 0)$$

07.41.06.0007.01

$$D_\nu(z) \propto \frac{2^{\nu/2}\sqrt{\pi}}{\Gamma\left(\frac{1-\nu}{2}\right)} - \frac{2^{\frac{\nu+1}{2}}\sqrt{\pi}\,z}{\Gamma\left(-\frac{\nu}{2}\right)} - \frac{2^{\frac{\nu}{2}-2}\sqrt{\pi}\,(2\nu+1)\,z^2}{\Gamma\left(\frac{1-\nu}{2}\right)} + \frac{2^{\frac{\nu-3}{2}}\sqrt{\pi}\,(2\nu+1)\,z^3}{3\,\Gamma\left(-\frac{\nu}{2}\right)} + O\left(z^4\right)$$

07.41.06.0008.01

$$D_\nu(z) = 2^{\nu/2}\sqrt{\pi}\left(\frac{1}{\Gamma\left(\frac{1-\nu}{2}\right)}\sum_{k=0}^{\infty}\sum_{j=0}^{k}\frac{(-1)^{k-j}\,2^{j-2k}\left(-\frac{\nu}{2}\right)_j}{j!\,(k-j)!\left(\frac{1}{2}\right)_j}z^{2k} - \frac{\sqrt{2}\,z}{\Gamma\left(-\frac{\nu}{2}\right)}\sum_{k=0}^{\infty}\sum_{j=0}^{k}\frac{(-1)^{k-j}\,2^{j-2k}\left(\frac{1-\nu}{2}\right)_j}{j!\,(k-j)!\left(\frac{3}{2}\right)_j}z^{2k}\right)$$

07.41.06.0009.01

$$D_\nu(z) = 2^{\nu/2}\sqrt{\pi}\left(\frac{1}{\Gamma\left(\frac{1-\nu}{2}\right)}\sum_{k=0}^{\infty}\frac{(-1)^k\,2^{-2k}\,z^{2k}}{k!}\sum_{k=0}^{\infty}\frac{\left(-\frac{\nu}{2}\right)_k 2^{-k}\,z^{2k}}{\left(\frac{1}{2}\right)_k k!} - \frac{\sqrt{2}\,z}{\Gamma\left(-\frac{\nu}{2}\right)}\sum_{k=0}^{\infty}\frac{(-1)^k\,2^{-2k}\,z^{2k}}{k!}\sum_{k=0}^{\infty}\frac{\left(\frac{1-\nu}{2}\right)_k 2^{-k}\,z^{2k}}{\left(\frac{3}{2}\right)_k k!}\right)$$

07.41.06.0010.01

$$D_\nu(z) = 2^{\nu/2}\sqrt{\pi}\,e^{-\frac{z^2}{4}}\left(\frac{1}{\Gamma\left(\frac{1-\nu}{2}\right)}\,{}_1F_1\left(-\frac{\nu}{2};\frac{1}{2};\frac{z^2}{2}\right) - \frac{\sqrt{2}\,z}{\Gamma\left(-\frac{\nu}{2}\right)}\,{}_1F_1\left(\frac{1-\nu}{2};\frac{3}{2};\frac{z^2}{2}\right)\right)$$

07.41.06.0011.01

$$D_\nu(z) \propto \frac{2^{\nu/2}\sqrt{\pi}}{\Gamma\left(\frac{1-\nu}{2}\right)}\,(1+O(z))$$

Asymptotic series expansions

07.41.06.0012.01

$$D_\nu(z) \propto -\frac{\sin(\pi\nu)}{\sqrt{2\pi}}\left(z^2\right)^{-\frac{\nu}{2}-1}\left(\sqrt{\frac{\pi}{2}}\,e^{-\frac{z^2}{4}}\sqrt{-z^2}\left(-z^4\right)^{\nu/2}\left(\sqrt{-z^2}\csc\left(\frac{\pi\nu}{2}\right)+z\sec\left(\frac{\pi\nu}{2}\right)\right)\left(1-\frac{(\nu-1)\nu}{2z^2}+\frac{(\nu-3)(\nu-2)(\nu-1)\nu}{8z^4}+\ldots\right)+ e^{\frac{z^2}{4}}\left(\sqrt{z^2}-z\right)\Gamma(\nu+1)\left(1+\frac{(\nu+1)(\nu+2)}{2z^2}+\frac{(\nu+1)(\nu+3)(\nu+4)(\nu+2)}{8z^4}+\ldots\right)\right)/;\ (|z|\to\infty)$$

07.41.06.0013.01

$$D_\nu(z) \propto \begin{cases} z^\nu e^{-\frac{z^2}{4}}\left(1-\frac{(\nu-1)\nu}{2z^2}+\frac{(\nu-3)(\nu-2)(\nu-1)\nu}{8z^4}+O\left(\frac{1}{z^6}\right)\right) & -\frac{\pi}{2}<\arg(z)\le\frac{\pi}{2} \\ z^\nu e^{-\frac{z^2}{4}}\left(1-\frac{(\nu-1)\nu}{2z^2}+\frac{(\nu-3)(\nu-2)(\nu-1)\nu}{8z^4}+O\left(\frac{1}{z^6}\right)\right)-\frac{e^{\frac{z^2}{4}-i\pi\nu}\sqrt{2\pi}\,z^{-\nu-1}}{\Gamma(-\nu)}\left(1+\frac{(\nu+1)(\nu+2)}{2z^2}+\frac{(\nu+1)(\nu+3)(\nu+4)(\nu+2)}{8z^4}+O\left(\frac{1}{z^6}\right)\right) & \arg(z)\le-\frac{\pi}{2} \\ z^\nu e^{-\frac{z^2}{4}}\left(1-\frac{(\nu-1)\nu}{2z^2}+\frac{(\nu-3)(\nu-2)(\nu-1)\nu}{8z^4}+O\left(\frac{1}{z^6}\right)\right)-\frac{e^{\frac{z^2}{4}+i\pi\nu}\sqrt{2\pi}\,z^{-\nu-1}}{\Gamma(-\nu)}\left(1+\frac{(\nu+1)(\nu+2)}{2z^2}+\frac{(\nu+1)(\nu+3)(\nu+4)(\nu+2)}{8z^4}+O\left(\frac{1}{z^6}\right)\right) & \text{True} \end{cases}$$

$/;$

$(|z|\to\infty)$

07.41.06.0014.01

$$D_\nu(z) \propto -\frac{\sin(\pi\nu)}{\sqrt{2\pi}} \left(z^2\right)^{-\frac{\nu}{2}-1} \left(\left(\sqrt{\frac{\pi}{2}}\, e^{-\frac{z^2}{4}} \sqrt{-z^2}\, \left(-z^4\right)^{\nu/2} \left(\sqrt{-z^2}\, \csc\!\left(\frac{\pi\nu}{2}\right) + z \sec\!\left(\frac{\pi\nu}{2}\right)\right)\right) \left(\sum_{k=0}^{n} \frac{(-1)^k \left(-\frac{\nu}{2}\right)_k \left(\frac{1-\nu}{2}\right)_k 2^k z^{-2k}}{k!} + O\!\left(z^{-2n-2}\right)\right) + e^{\frac{z^2}{4}} \left(\sqrt{z^2} - z\right) \Gamma(\nu+1) \left(\sum_{k=0}^{n} \frac{\left(\frac{\nu+1}{2}\right)_k \left(\frac{\nu+2}{2}\right)_k 2^k z^{-2k}}{k!} + O\!\left(z^{-2n-2}\right)\right)\right) /; (|z| \to \infty)$$

07.41.06.0015.01

$$D_\nu(z) \propto \begin{cases} z^\nu e^{-\frac{z^2}{4}} \left(\sum_{k=0}^{n} \frac{(-1)^k \left(-\frac{\nu}{2}\right)_k \left(\frac{1-\nu}{2}\right)_k 2^k z^{-2k}}{k!} + O\!\left(z^{-2n-2}\right)\right) & -\frac{\pi}{2} < \arg(z) \le \frac{\pi}{2} \\ z^\nu e^{-\frac{z^2}{4}} \left(\sum_{k=0}^{n} \frac{(-1)^k \left(-\frac{\nu}{2}\right)_k \left(\frac{1-\nu}{2}\right)_k 2^k z^{-2k}}{k!} + O\!\left(z^{-2n-2}\right)\right) - \frac{e^{\frac{z^2}{4} - i\pi\nu} \sqrt{2\pi}\, z^{-\nu-1}}{\Gamma(-\nu)} \left(\sum_{k=0}^{n} \frac{\left(\frac{\nu+1}{2}\right)_k \left(\frac{\nu+2}{2}\right)_k 2^k z^{-2k}}{k!} + O\!\left(z^{-2n-2}\right)\right) & \arg(z) \le -\frac{\pi}{2} \\ z^\nu e^{-\frac{z^2}{4}} \left(\sum_{k=0}^{n} \frac{(-1)^k \left(-\frac{\nu}{2}\right)_k \left(\frac{1-\nu}{2}\right)_k 2^k z^{-2k}}{k!} + O\!\left(z^{-2n-2}\right)\right) - \frac{e^{\frac{z^2}{4} + i\pi\nu} \sqrt{2\pi}\, z^{-\nu-1}}{\Gamma(-\nu)} \left(\sum_{k=0}^{n} \frac{\left(\frac{\nu+1}{2}\right)_k \left(\frac{\nu+2}{2}\right)_k 2^k z^{-2k}}{k!} + O\!\left(z^{-2n-2}\right)\right) & \text{True} \end{cases} /; (|z| \to \infty)$$

07.41.06.0016.01

$$D_\nu(z) \propto -\frac{\left(z^2\right)^{-\frac{\nu}{2}-1} \sin(\pi\nu)}{\sqrt{2\pi}} \left(\sqrt{\frac{\pi}{2}}\, e^{-\frac{z^2}{4}} \sqrt{-z^2}\, \left(-z^4\right)^{\nu/2} \left(\sqrt{-z^2}\, \csc\!\left(\frac{\pi\nu}{2}\right) + z \sec\!\left(\frac{\pi\nu}{2}\right)\right) {}_2F_0\!\left(-\frac{\nu}{2}, \frac{1-\nu}{2};; -\frac{2}{z^2}\right) + e^{\frac{z^2}{4}} \left(\sqrt{z^2} - z\right) \Gamma(\nu+1)\, {}_2F_0\!\left(\frac{\nu+1}{2}, \frac{\nu+2}{2};; \frac{2}{z^2}\right)\right) /; (|z| \to \infty)$$

07.41.06.0017.01

$$D_\nu(z) \propto \begin{cases} z^\nu e^{-\frac{z^2}{4}}\, {}_2F_0\!\left(\frac{1}{2} - \frac{\nu}{2}, -\frac{\nu}{2};; -\frac{2}{z^2}\right) & -\frac{\pi}{2} < \arg(z) \le \frac{\pi}{2} \\ z^\nu e^{-\frac{z^2}{4}}\, {}_2F_0\!\left(\frac{1}{2} - \frac{\nu}{2}, -\frac{\nu}{2};; -\frac{2}{z^2}\right) - \frac{e^{\frac{z^2}{4} - i\pi\nu} \sqrt{2\pi}\, z^{-\nu-1}}{\Gamma(-\nu)}\, {}_2F_0\!\left(\frac{\nu}{2} + \frac{1}{2}, \frac{\nu}{2} + 1;; \frac{2}{z^2}\right) & \arg(z) \le -\frac{\pi}{2} \\ z^\nu e^{-\frac{z^2}{4}}\, {}_2F_0\!\left(\frac{1}{2} - \frac{\nu}{2}, -\frac{\nu}{2};; -\frac{2}{z^2}\right) - \frac{e^{\frac{z^2}{4} + i\pi\nu} \sqrt{2\pi}\, z^{-\nu-1}}{\Gamma(-\nu)}\, {}_2F_0\!\left(\frac{\nu}{2} + \frac{1}{2}, \frac{\nu}{2} + 1;; \frac{2}{z^2}\right) & \text{True} \end{cases} /; (|z| \to \infty)$$

07.41.06.0018.01

$$D_\nu(z) \propto \frac{1}{\Gamma(-\nu)} \sqrt{\frac{\pi}{2}}\, e^{\frac{z^2}{4}} \left(z^2\right)^{-\frac{\nu}{2}-1} \left(\sqrt{z^2} - z\right) \left(1 + O\!\left(\frac{1}{z^2}\right)\right) - \frac{\sin(\pi\nu)}{2} e^{-\frac{z^2}{4}} \sqrt{-z^2}\, \left(z^2\right)^{-\frac{\nu}{2}-1} \left(-z^4\right)^{\nu/2} \left(\sqrt{-z^2}\, \csc\!\left(\frac{\pi\nu}{2}\right) + z \sec\!\left(\frac{\pi\nu}{2}\right)\right) \left(1 + O\!\left(\frac{1}{z^2}\right)\right) /; (|z| \to \infty)$$

07.41.06.0019.01

$$D_\nu(z) \propto \begin{cases} z^\nu e^{-\frac{z^2}{4}} \left(1 + O\left(\frac{1}{z^2}\right)\right) & -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2} \\ z^\nu e^{-\frac{z^2}{4}} \left(1 + O\left(\frac{1}{z^2}\right)\right) - \frac{e^{\frac{z^2}{4} - i\pi\nu} \sqrt{2\pi}\, z^{-\nu-1}}{\Gamma(-\nu)} \left(1 + O\left(\frac{1}{z^2}\right)\right) & \arg(z) \leq -\frac{\pi}{2} \\ z^\nu e^{-\frac{z^2}{4}} \left(1 + O\left(\frac{1}{z^2}\right)\right) - \frac{e^{\frac{z^2}{4} + i\pi\nu} \sqrt{2\pi}\, z^{-\nu-1}}{\Gamma(-\nu)} \left(1 + O\left(\frac{1}{z^2}\right)\right) & \text{True} \end{cases} /; (|z| \to \infty)$$

Integral representations

On the real axis

Of the direct function

07.41.07.0001.01

$$D_\nu(z) = \frac{2^{\frac{\nu+7}{2}}}{\Gamma\left(-\frac{\nu}{2}\right)} \int_{\frac{1}{4}}^{\infty} (4t-1)^{-\frac{\nu}{2}-1} (4t+1)^{\frac{\nu-1}{2}} e^{-z^2 t} \, dt \,/;\, \mathrm{Re}(\nu) < 0 \bigwedge \mathrm{Re}\left(z^2\right) > 0$$

07.41.07.0002.01

$$D_\nu(z) = \frac{2^{\frac{\nu}{2}+1}}{\Gamma(-\nu)\, z^\nu} \int_{\frac{1}{4}}^{\infty} \frac{(4t-1)^{-\nu-1} \left(\sqrt{4t+1} + \sqrt{2}\right)^{\nu+1}}{\sqrt{4t+1}} e^{-z^2 t} \, dt \,/;\, \mathrm{Re}(\nu) < 0 \bigwedge \mathrm{Re}\left(z^2\right) > 0$$

07.41.07.0003.01

$$D_\nu(z) = \frac{2^{\nu/2} z^{2-\nu}}{\Gamma(1-\nu)} \int_{\frac{1}{4}}^{\infty} \frac{\left(\sqrt{4t+1} + \sqrt{2}\right)^\nu e^{-z^2 t}}{(4t-1)^\nu} \, dt \,/;\, \mathrm{Re}(\nu) < 1 \bigwedge \mathrm{Re}\left(z^2\right) > 0$$

07.41.07.0004.01

$$D_\nu(z) = \frac{2^{\frac{\nu}{2}+1} z}{\Gamma\left(\frac{1-\nu}{2}\right)} \int_{\frac{1}{4}}^{\infty} \frac{(4t+1)^{\nu/2} e^{-z^2 t}}{(4t-1)^{\frac{\nu+1}{2}}} \, dt \,/;\, \mathrm{Re}(\nu) < 1 \bigwedge \mathrm{Re}\left(z^2\right) > 0$$

07.41.07.0005.01

$$D_\nu(z) = \frac{e^{-\frac{z^2}{4}}}{\Gamma\left(-\frac{\nu}{2}\right)} \int_0^{\infty} t^{-\frac{\nu}{2}-1} (2t+1)^{\frac{\nu-1}{2}} e^{-z^2 t} \, dt \,/;\, \mathrm{Re}(\nu) < 0 \bigwedge \mathrm{Re}\left(z^2\right) > 0$$

07.41.07.0006.01

$$D_\nu(z) = \frac{2^{-\nu-1} e^{-\frac{z^2}{4}}}{\Gamma(-\nu)\, z^\nu} \int_0^{\infty} \frac{t^{-\nu-1} \left(\sqrt{2t+1} + 1\right)^{\nu+1} e^{-z^2 t}}{\sqrt{2t+1}} \, dt \,/;\, \mathrm{Re}(\nu) < 0 \bigwedge \mathrm{Re}\left(z^2\right) > 0$$

07.41.07.0007.01

$$D_\nu(z) = \frac{z^{2-\nu} e^{-\frac{z^2}{4}}}{2^\nu \Gamma(1-\nu)} \int_0^{\infty} \frac{\left(\sqrt{2t+1} + 1\right)^\nu e^{-z^2 t}}{t^\nu} \, dt \,/;\, \mathrm{Re}(\nu) < 1 \bigwedge \mathrm{Re}\left(z^2\right) > 0$$

07.41.07.0008.01

$$D_\nu(z) = \frac{e^{-\frac{z^2}{4}}}{\Gamma(-\nu)} \int_0^{\infty} t^{-\nu-1} e^{-\frac{t^2}{2} - t z} \, dt \,/;\, \mathrm{Re}(\nu) < 0 \land \mathrm{Re}(z) > 0$$

07.41.07.0009.01

$$D_\nu(z) = -\frac{e^{-\frac{z^2}{4}}}{\Gamma(-\nu)\, z} \int_0^\infty t^{-\nu-2} \left(t^2 + (\nu+1)\right) e^{-\frac{t^2}{2} - t z}\, dt \;/;\; \mathrm{Re}(\nu) < -1 \land \mathrm{Re}(z) > 0$$

07.41.07.0010.01

$$D_\nu(z) = \frac{\sqrt{2}}{\sqrt{\pi}}\, e^{\frac{z^2}{4}} \int_0^\infty t^\nu\, e^{-\frac{t^2}{2}} \cos\left(z\, t - \frac{\pi\, \nu}{2}\right) dt \;/;\; \mathrm{Re}(\nu) > -1$$

07.41.07.0011.01

$$D_\nu(z) = \frac{2^{-\frac{\nu}{2}-1}\, z^\nu}{\Gamma(-\nu)}\, e^{-\frac{z^2}{4}} \int_0^\infty t^{-\frac{\nu}{2}-1}\, e^{-\frac{t}{z^2} - \sqrt{2\, t}}\, dt \;/;\; \mathrm{Re}(\nu) < 0 \bigwedge \mathrm{Re}\left(z^2\right) > 0$$

07.41.07.0012.01

$$D_\nu(z) = \frac{2^{-\frac{\nu}{2}-1}\, z}{\Gamma(-\nu)}\, e^{-\frac{z^2}{4}} \int_0^\infty e^{-z\, t} \left(\Gamma\left(-\frac{\nu}{2}\right) - \Gamma\left(-\frac{\nu}{2}, \frac{t^2}{2}\right)\right) dt \;/;\; \mathrm{Re}(\nu) < 1 \land \mathrm{Re}(z) > 0$$

07.41.07.0013.01

$$D_n(z) = \frac{1}{\sqrt{2\, \pi}}\, e^{\frac{3\, i\, n\, \pi}{2} - \frac{z^2}{4}} \int_{-\infty}^\infty e^{-\frac{1}{2}(t - i\, z)^2}\, t^n\, dt \;/;\; \mathrm{Re}(\nu) > -1$$

Of the products of direct functions

07.41.07.0014.01

$$D_\nu(z)\, D_{-\nu-1}(z) = 2 \int_0^\infty e^{-z\, t} \cos\left(z\, t - \frac{\nu\, \pi}{2}\right) J_{\nu+\frac{1}{2}}\left(t^2\right) dt \;/;\; \mathrm{Re}(\nu) > -1 \land \mathrm{Re}(z) > 0$$

07.41.07.0015.01

$$D_\nu\left(e^{\frac{\pi\, i}{4}}\, z\right) D_\nu\left(e^{-\frac{1}{4}(\pi\, i)}\, z\right) = \frac{\sqrt{\pi}}{\Gamma(-\nu)} \int_0^\infty e^{-z\, t}\, J_{-\nu-\frac{1}{2}}\left(\frac{t^2}{2}\right) dt \;/;\; \mathrm{Re}(\nu) < 0 \land \mathrm{Re}(z) \geq 0$$

07.41.07.0016.01

$$D_\nu\left(e^{\frac{\pi\, i}{4}}\, z\right) D_\nu\left(e^{-\frac{1}{4}(\pi\, i)}\, z\right) = \frac{\sqrt{\frac{8}{\pi}}}{\Gamma(-\nu)} \int_0^\infty e^{-z\, t} \cos\left(z\, t - \frac{\nu\, \pi}{2}\right) K_{\nu+\frac{1}{2}}\left(t^2\right) dt \;/;\; -1 < \mathrm{Re}(\nu) < 0$$

Contour integral representations

07.41.07.0017.01

$$D_\nu(z) = \frac{e^{-\frac{z^2}{4}}}{2^{\frac{\nu+2}{2}}\, \Gamma(-\nu)\, 2\, \pi\, i} \int_{-i\,\infty}^{i\,\infty} \Gamma(-s)\, \Gamma\left(\frac{s-\nu}{2}\right) \left(\sqrt{2}\, z\right)^s ds \;/;\; \arg(z) < \frac{3\, \pi}{4} \bigwedge \neg\, (\nu \in \mathbb{Z} \land \nu > 0)$$

Whittaker,Watson,16, Example12

07.41.07.0018.01

$$D_\nu(z) == -\frac{\Gamma(\nu+1)\, e^{-\frac{z^2}{4}}}{2\, \pi\, i} \int_L (-t)^{-\nu-1}\, e^{-\frac{t^2}{2} - z\, t}\, dt$$

(Hankel's contour integral.) The path of integration L starts at $\infty + i\, 0$ on the real axis, goes to $\epsilon + i\, 0$, circles the origin in the counterclockwise direction with radius ϵ to the point $\epsilon - i\, 0$, and returns to the point $\infty - i\, 0$.

Whittaker,Watson,16.6

07.41.07.0019.01

$$D_\nu(z) = \frac{2^{\frac{\nu+1}{2}}\,\Gamma\left(\frac{\nu}{2}+1\right)}{2\,\pi\,i} \int_L (t+1)^{-\frac{\nu}{2}-1}\,(1-t)^{\frac{\nu-1}{2}}\,e^{\frac{z^2 t}{4}}\,dt\ /;\ \arg(z) < \frac{\pi}{4} \bigwedge \arg(t+1) \leq \pi$$

(Hankel's type contour integral.) The path of integration L starts at $-\infty - i\,0$ on the real axis, goes to $-1-\epsilon-i\,0$, circles the origin in the counterclockwise direction with radius ϵ to the point $-1-\epsilon+i\,0$, and returns to the point $-\infty + i\,0$.

Whittaker,Watson,16,Example 11

Integral representations of negative integer order

Rodrigues-type formula.

07.41.07.0020.01

$$D_n(z) = (-1)^n\, e^{\frac{z^2}{4}}\, \frac{\partial^n e^{-\frac{z^2}{2}}}{\partial z^n}\ /;\ n \in \mathbb{N}$$

Limit representations

07.41.09.0001.01

$$D_\nu(z) = e^{-\frac{z^2}{4}}\,\Gamma(\nu+1)\left(\lim_{\lambda\to\infty} \lambda^{-\frac{\nu}{2}}\, L_\nu^\lambda\left(\lambda - \sqrt{\lambda}\ z\right)\right)$$

07.41.09.0002.01

$$D_\nu(z) = 2^{-\frac{\nu}{2}}\, e^{-\frac{z^2}{4}}\,\Gamma(\nu+1)\left(\lim_{\lambda\to\infty} \lambda^{-\frac{\nu}{2}}\, C_\nu^\lambda\left(\frac{z}{\sqrt{2\,\lambda}}\right)\right)\ /;\ |z| < \sqrt{2}$$

07.41.09.0003.01

$$D_\nu(z) = 2^{\nu/2}\, e^{-\frac{z^2}{4}}\,\Gamma(\nu+1)\left(\lim_{a\to\infty} a^{-\frac{\nu}{2}}\, P_\nu^{(a,a)}\left(\frac{z}{\sqrt{2\,a}}\right)\right)$$

Generating functions

07.41.11.0001.01

$$D_n(z) = 2^{-\frac{n}{2}}\, n!\left([t^n]\, e^{-t^2+\sqrt{2}\, z\, t - \frac{z^2}{4}}\right)\ /;\ n \in \mathbb{N}$$

Differential equations

Ordinary linear differential equations and wronskians

07.41.13.0001.01

$$w''(z) + \left(\nu + \frac{1}{2} - \frac{z^2}{4}\right) w(z) = 0\ /;\ w(z) = c_1\, D_\nu(z) + c_2\, D_{-\nu-1}(i\, z)$$

07.41.13.0002.01

$$W_z(D_\nu(z), D_{-\nu-1}(i z)) = -i e^{-\frac{1}{2} i \pi \nu}$$

07.41.13.0003.01

$$w''(z) - \frac{g''(z)}{g'(z)} w'(z) + \frac{1}{4} \left(-g(z)^2 + 4 \nu + 2\right) g'(z)^2 w(z) = 0 /; w(z) = c_1 D_\nu(g(z)) + c_2 D_{-\nu-1}(i g(z))$$

07.41.13.0004.01

$$W_z(D_\nu(g(z)), D_{-\nu-1}(i g(z))) = -i e^{-\frac{1}{2} i \pi \nu} g'(z)$$

07.41.13.0005.01

$$w''(z) - \left(\frac{2 h'(z)}{h(z)} + \frac{g''(z)}{g'(z)}\right) w'(z) + \left(\frac{1}{4} \left(-g(z)^2 + 4 \nu + 2\right) g'(z)^2 + \frac{2 h'(z)^2}{h(z)^2} + \frac{h'(z) g''(z)}{h(z) g'(z)} - \frac{h''(z)}{h(z)}\right) w(z) = 0 /;$$
$$w(z) = c_1 h(z) D_\nu(g(z)) + c_2 h(z) D_{-\nu-1}(i g(z))$$

07.41.13.0006.01

$$W_z(h(z) D_\nu(g(z)), h(z) D_{-\nu-1}(i g(z))) = -i e^{-\frac{1}{2} i \pi \nu} h(z)^2 g'(z)$$

07.41.13.0007.01

$$w''(z) - \frac{r + 2 s - 1}{z} w'(z) + \left(\frac{1}{4} a^2 r^2 \left(-a^2 z^{2 r} + 4 \nu + 2\right) z^{2 r-2} + \frac{s (r + s)}{z^2}\right) w(z) = 0 /; w(z) = c_1 z^s D_\nu \left(a z^r\right) + c_2 z^s D_{-\nu-1} (i a z^r)$$

07.41.13.0008.01

$$W_z(z^s D_\nu(a z^r), z^s D_{-\nu-1}(i a z^r)) = -i a e^{-\frac{1}{2} i \pi \nu} r z^{r+2 s-1}$$

07.41.13.0009.01

$$w''(z) - (\log(r) + 2 \log(s)) w'(z) + \left(\frac{1}{4} a^2 \left(-a^2 r^{2 z} + 4 \nu + 2\right) \log^2(r) r^{2 z} + \log^2(s) + \log(r) \log(s)\right) w(z) = 0 /;$$
$$w(z) = c_1 s^z D_\nu(a r^z) + c_2 s^z D_{-\nu-1}(i a r^z)$$

07.41.13.0010.01

$$W_z(s^z D_\nu(a r^z), s^z D_{-\nu-1}(i a r^z)) = -i a e^{-\frac{1}{2} i \pi \nu} r^z s^{2 z} \log(r)$$

Transformations

Transformations and argument simplifications

Argument involving basic arithmetic operations

07.41.16.0001.01

$$D_\nu\left(\sqrt{z^2}\right) = D_\nu(z) /; -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

07.41.16.0002.01

$$D_\nu\left(\sqrt{z^2}\right) = D_\nu(z) + 2^{\frac{\nu}{2}-\frac{1}{2}} e^{-\frac{z^2}{4}} \left(\sqrt{z^2} - z\right) \Gamma\left(\frac{\nu+1}{2}\right) \sin\left(\frac{\pi \nu}{2}\right) L_{\frac{\nu-1}{2}}^{\frac{1}{2}}\left(\frac{z^2}{2}\right)$$

07.41.16.0003.01

$$D_\nu\left(\sqrt{z^2}\right) = D_\nu(z) + \frac{2^{\frac{\nu-1}{2}} \left(\sqrt{z^2} - z\right) \nu \Gamma\left(\frac{\nu}{2}\right) \sin\left(\frac{\pi \nu}{2}\right)}{\sqrt{\pi}} e^{-\frac{z^2}{4}} {}_1F_1\left(\frac{1-\nu}{2}; \frac{3}{2}; \frac{z^2}{2}\right)$$

07.41.16.0004.01

$$D_{2n}\left(\sqrt{z^2}\right) = D_{2n}(z) /; n \in \mathbb{N}$$

07.41.16.0005.01

$$D_\nu(-z) = D_\nu(z) - 2^{\frac{\nu+1}{2}} z\, e^{-\frac{z^2}{4}} \Gamma\left(\frac{\nu+1}{2}\right) \sin\left(\frac{\pi\nu}{2}\right) L_{\frac{\nu-1}{2}}^{\frac{1}{2}}\left(\frac{z^2}{2}\right)$$

07.41.16.0006.01

$$D_\nu(-z) = D_\nu(z) - \frac{2^{\frac{\nu+1}{2}} \nu\, \Gamma\left(\frac{\nu}{2}\right) \sin\left(\frac{\pi\nu}{2}\right)}{\sqrt{\pi}} z\, e^{-\frac{z^2}{4}} {}_1F_1\left(\frac{1-\nu}{2}; \frac{3}{2}; \frac{z^2}{2}\right)$$

07.41.16.0007.01

$$D_n(-z) = (-1)^n D_n(z) /; n \in \mathbb{N}$$

Addition formulas

07.41.16.0008.01

$$D_\nu(z_1 - z_2) = e^{\frac{1}{4} z_2 (z_2 - 2 z_1)} \sum_{k=0}^{\infty} \frac{z_2^k}{k!} D_{k+\nu}(z_1)$$

07.41.16.0009.01

$$D_\nu(z_1 - z_2) = e^{\frac{1}{4} z_2 (2 z_1 - z_2)} \sum_{k=0}^{\infty} \frac{(-\nu)_k z_2^k}{k!} D_{\nu-k}(z_1)$$

07.41.16.0010.01

$$D_\nu(\cos(\alpha) z_1 + \sin(\alpha) z_2) = \cos^\nu(\alpha)\, e^{\frac{1}{4} (z_1 \sin(\alpha) - z_2 \cos(\alpha))^2} \sum_{k=0}^{\infty} \frac{(-\tan(\alpha))^k (-\nu)_k}{k!} D_{\nu-k}(z_1) D_k(z_2)$$

Identities

Recurrence identities

Consecutive neighbors

07.41.17.0001.01

$$D_\nu(z) = \frac{1}{\nu+1} (z D_{\nu+1}(z) - D_{\nu+2}(z))$$

07.41.17.0002.01

$$D_\nu(z) = (\nu - 1) D_{\nu-2}(z) - z D_{\nu-1}(z)$$

07.41.17.0003.01

$$D_\nu(z) = \frac{1}{z} (\nu D_{\nu-1}(z) + D_{\nu+1}(z))$$

Distant neighbors

07.41.17.0004.01

$$D_\nu(z) = C_n(\nu, z)\, D_{n+\nu}(z) - \frac{C_{n-1}(\nu, z)}{n+\nu}\, D_{n+\nu+1}(z) \,/;$$
$$C_0(\nu, z) = 1 \bigwedge C_1(\nu, z) = \frac{z}{\nu+1} \bigwedge C_n(\nu, z) = \frac{z}{n+\nu}\, C_{n-1}(\nu, z) - \frac{1}{n+\nu-1}\, C_{n-2}(\nu, z) \bigwedge n \in \mathbb{N}^+$$

07.41.17.0005.01

$$D_\nu(z) = D_{\nu-n}(z)\, C_n(\nu, z) + (n-\nu)\, D_{\nu-n-1}(z)\, C_{n-1}(\nu, z) \,/;$$
$$C_0(\nu, z) = 1 \wedge C_1(\nu, z) = z \wedge C_n(\nu, z) = z\, C_{n-1}(\nu, z) - (\nu - n + 1)\, C_{n-2}(\nu, z) \wedge n \in \mathbb{N}^+$$

Functional identities

Relations between contiguous functions

07.41.17.0006.01

$$D_\nu(z) = \frac{\Gamma(\nu+1)}{\sqrt{2\pi}} \left(e^{-\frac{\pi i \nu}{2}}\, D_{-\nu-1}(-i\, z) + e^{\frac{\nu \pi i}{2}}\, D_{-\nu-1}(i\, z) \right)$$

Relations of special kind

07.41.17.0007.01

$$D_\nu(z) = \frac{i \sqrt{2\pi}\; e^{\frac{\pi i \nu}{2}}}{\Gamma(-\nu)}\, D_{-\nu-1}(-i\, z) + e^{\nu \pi i}\, D_\nu(-z)$$

07.41.17.0008.01

$$D_\nu(z) = e^{-\nu \pi i}\, D_\nu(-z) - \frac{i \sqrt{2\pi}\; e^{-\frac{\pi i \nu}{2}}}{\Gamma(-\nu)}\, D_{-\nu-1}(i\, z)$$

07.41.17.0009.01

$$D_\nu\left(\sqrt{-z}\right) = \frac{2^{\nu+\frac{1}{2}}\, \Gamma\left(\frac{\nu}{2}+1\right)}{\Gamma\left(\frac{1-\nu}{2}\right)}\, D_{-\nu-1}\left(\sqrt{z}\right) - \frac{2^{\frac{\nu+1}{2}} \sqrt{\pi} \left(\sqrt{z}\, \cot\left(\frac{\nu\pi}{2}\right) + \sqrt{-z}\right)}{\Gamma\left(-\frac{\nu}{2}\right) e^{z/4}}\, {}_1F_1\left(\frac{\nu}{2}+1;\, \frac{3}{2};\, \frac{z}{2}\right)$$

07.41.17.0010.01

$$D_\nu\left(\sqrt{-z}\right) = \frac{2^{\nu+\frac{1}{2}} \sqrt{-z}\; \Gamma\left(\frac{\nu+1}{2}\right)}{\sqrt{z}\; \Gamma\left(-\frac{\nu}{2}\right)}\, D_{-\nu-1}\left(\sqrt{z}\right) + \frac{2^{\nu/2} \sqrt{\pi} \left(\sqrt{-z^2} - z \tan\left(\frac{\nu\pi}{2}\right)\right)}{\sqrt{-z^2}\; \Gamma\left(\frac{1-\nu}{2}\right) e^{z/4}}\, {}_1F_1\left(\frac{\nu+1}{2};\, \frac{1}{2};\, \frac{z}{2}\right)$$

Differentiation

Low-order differentiation

With respect to ν

07.41.20.0001.01

$$\frac{\partial D_\nu(z)}{\partial \nu} = \frac{1}{2}\log(2)\,D_\nu(z) + 2^{-\frac{\nu}{2}}\,e^{-\frac{z^2}{4}}\left(\frac{1}{4\,\Gamma(-\nu)}\left(\Gamma\left(-\frac{\nu}{2}\right)\psi\left(\frac{1-\nu}{2}\right){}_1F_1\left(-\frac{\nu}{2};\frac{1}{2};\frac{z^2}{2}\right) - \sqrt{2}\,z\,\Gamma\left(\frac{1-\nu}{2}\right)\psi\left(-\frac{\nu}{2}\right){}_1F_1\left(\frac{1-\nu}{2};\frac{3}{2};\frac{z^2}{2}\right)\right) - \frac{z^2}{12\,\Gamma(-\nu)}\left(3\,\Gamma\left(-\frac{\nu}{2}\right)F_{2\times0\times1}^{1\times1\times2}\left(\begin{matrix}1-\frac{\nu}{2};1;1,-\frac{\nu}{2};\\ 2,\frac{3}{2};;1-\frac{\nu}{2};\end{matrix}\frac{z^2}{2},\frac{z^2}{2}\right) - \sqrt{2}\,z\,\Gamma\left(\frac{1-\nu}{2}\right)F_{2\times0\times1}^{1\times1\times2}\left(\begin{matrix}\frac{3-\nu}{2};1;1,\frac{1-\nu}{2};\\ 2,\frac{5}{2};;\frac{3-\nu}{2};\end{matrix}\frac{z^2}{2},\frac{z^2}{2}\right)\right)\right)$$

07.41.20.0002.01

$$\frac{\partial D_\nu(z)}{\partial \nu} = \frac{2^{\frac{\nu-1}{2}}\sqrt{\pi}\,(\log(2)-2\,\psi(-\nu))\,z}{\Gamma\left(-\frac{\nu}{2}\right)}\,e^{-\frac{z^2}{4}}\,{}_1F_1\left(\frac{1-\nu}{2};\frac{3}{2};\frac{z^2}{2}\right) - \frac{2^{\frac{\nu}{2}-1}\sqrt{\pi}\,(\log(2)-2\,\psi(-\nu))}{\Gamma\left(\frac{1-\nu}{2}\right)}\,e^{-\frac{z^2}{4}}\,{}_1F_1\left(-\frac{\nu}{2};\frac{1}{2};\frac{z^2}{2}\right) - \frac{2^{\frac{\nu}{2}-1}\sqrt{\pi}\,e^{-\frac{z^2}{4}}}{\Gamma\left(\frac{1-\nu}{2}\right)}\sum_{k=0}^{\infty}\frac{\left(-\frac{\nu}{2}\right)_k\psi\left(k-\frac{\nu}{2}\right)z^{2k}}{k!\left(\frac{1}{2}\right)_k 2^k} + \frac{\sqrt{\pi}\,2^{\frac{\nu-1}{2}}\,z\,e^{-\frac{z^2}{4}}}{\Gamma\left(-\frac{\nu}{2}\right)}\sum_{k=0}^{\infty}\frac{\left(\frac{1-\nu}{2}\right)_k\psi\left(k+\frac{1-\nu}{2}\right)z^{2k}}{k!\left(\frac{3}{2}\right)_k 2^k}$$

Brychkov Yu.A. (2006)

07.41.20.0003.01

$$\frac{\partial D_\nu(z)}{\partial \nu} = \frac{1}{2}\log(2)\,D_\nu(z) + 2^{-\frac{\nu}{2}}\,e^{-\frac{z^2}{4}}\left(\frac{1}{4\,\Gamma(-\nu)}\left(\Gamma\left(-\frac{\nu}{2}\right)\psi\left(\frac{1-\nu}{2}\right){}_1F_1\left(-\frac{\nu}{2};\frac{1}{2};\frac{z^2}{2}\right) - \sqrt{2}\,z\,\Gamma\left(\frac{1-\nu}{2}\right)\psi\left(-\frac{\nu}{2}\right){}_1F_1\left(\frac{1-\nu}{2};\frac{3}{2};\frac{z^2}{2}\right)\right) + \frac{2^{\nu-1}\sqrt{\pi}}{\Gamma\left(\frac{1-\nu}{2}\right)}\left(\psi\left(-\frac{\nu}{2}\right){}_1F_1\left(-\frac{\nu}{2};\frac{1}{2};\frac{z^2}{2}\right) - \sum_{k=0}^{\infty}\frac{2^{-k}\left(-\frac{\nu}{2}\right)_k\psi\left(k-\frac{\nu}{2}\right)z^{2k}}{k!\left(\frac{1}{2}\right)_k}\right) - \frac{2^{\nu-\frac{1}{2}}\sqrt{\pi}\,z}{\Gamma\left(-\frac{\nu}{2}\right)}\left(\psi\left(\frac{1-\nu}{2}\right){}_1F_1\left(\frac{1-\nu}{2};\frac{3}{2};\frac{z^2}{2}\right) - \sum_{k=0}^{\infty}\frac{2^{-k}\left(\frac{1-\nu}{2}\right)_k\psi\left(k+\frac{1-\nu}{2}\right)z^{2k}}{k!\left(\frac{3}{2}\right)_k}\right)\right)$$

07.41.20.0004.01

$$D_{2n}^{(1,0)}(z) = n!\left(\frac{1}{2}\right)_n\left(\frac{2^{2n-1}}{(2n)!}\left(-z^2\,{}_2F_2\left(1,1;\frac{3}{2},2;\frac{z^2}{2}\right) + \pi\,\mathrm{erfi}\left(\frac{z}{\sqrt{2}}\right) + \psi\left(\frac{1}{2}-n\right) - \gamma\right)D_{2n}(z) - (-1)^n\,2^{n-1}\,e^{-\frac{z^2}{4}}\sum_{k=0}^{n}\frac{\psi\left(\frac{1}{2}-k\right)(-2)^k z^{2k}}{(n-k)!\,(2k)!} + \frac{(-1)^n\,2^{n-\frac{3}{2}}}{\left(\frac{1}{2}\right)_n}\,e^{\frac{z^2}{4}}\sum_{k=0}^{n-1}\frac{1}{k+1}\left(\sqrt{\pi}\,z\,L_k^{-k-\frac{1}{2}}\left(-\frac{z^2}{2}\right) - \frac{2^{-k-\frac{1}{2}}\,\pi\,z^{2k+2}}{k!}\,L_{-\frac{1}{2}}^{k+\frac{1}{2}}\left(-\frac{z^2}{2}\right)\right)L_{-k+n-1}^{k+\frac{1}{2}}\left(\frac{z^2}{2}\right)\right) - \frac{1}{2}\,D_{2n}(z)\log(2)\;/;\;n\in\mathbb{N}$$

Brychkov Yu.A. (2006)

07.41.20.0005.01

$$D_0^{(1,0)}(z) = \frac{1}{2} e^{-\frac{z^2}{4}} \left(-{}_2F_2\left(1, 1; \frac{3}{2}, 2; \frac{z^2}{2}\right) z^2 + \pi \operatorname{erfi}\left(\frac{z}{\sqrt{2}}\right) - \log(2) - \gamma \right)$$

Brychkov Yu.A. (2006)

07.41.20.0006.01

$$D_2^{(1,0)}(z) = \frac{1}{2} e^{-\frac{z^2}{4}} \left(\left(1 - z^2\right) {}_2F_2\left(1, 1; \frac{3}{2}, 2; \frac{z^2}{2}\right) z^2 - e^{\frac{z^2}{2}} \sqrt{2\pi} \operatorname{erfc}\left(\frac{z}{\sqrt{2}}\right) z + \pi \left(z^2 - 1\right) \operatorname{erfi}\left(\frac{z}{\sqrt{2}}\right) - \left(z^2 - 1\right) (\log(2) + \gamma) - 2 \right)$$

Brychkov Yu.A. (2006)

07.41.20.0007.01

$$\frac{\partial D_\nu(0)}{\partial \nu} = \begin{cases} (-1)^{\frac{\nu+1}{2}} 2^{\frac{\nu}{2}-1} \sqrt{\pi} \; \frac{\nu-1}{2}! & \frac{\nu-1}{2} \in \mathbb{Z} \bigwedge \nu > 0 \\ \frac{2^{\frac{\nu}{2}-1} \sqrt{\pi} \log(2)}{\Gamma\left(\frac{1-\nu}{2}\right)} + \frac{2^{\frac{\nu}{2}-1} \sqrt{\pi} \; \psi\left(\frac{1-\nu}{2}\right)}{\Gamma\left(\frac{1-\nu}{2}\right)} & \text{True} \end{cases}$$

With respect to *z*

07.41.20.0008.01

$$\frac{\partial D_\nu(z)}{\partial z} = \frac{1}{2} z D_\nu(z) - D_{\nu+1}(z)$$

07.41.20.0009.01

$$\frac{\partial D_\nu(z)}{\partial z} = \nu D_{\nu-1}(z) - \frac{1}{2} z D_\nu(z)$$

07.41.20.0010.01

$$\frac{\partial D_\nu(z)}{\partial z} = \frac{1}{2} (\nu D_{\nu-1}(z) - D_{\nu+1}(z))$$

07.41.20.0011.01

$$\frac{\partial^2 D_\nu(z)}{\partial z^2} = \frac{1}{4} \left(4 (\nu - 1) \nu D_{\nu-2}(z) - 4 z \nu D_{\nu-1}(z) + \left(z^2 - 2\right) D_\nu(z)\right)$$

07.41.20.0012.01

$$\frac{\partial^2 D_\nu(z)}{\partial z^2} = \frac{1}{4} \left(z^2 + 2\right) D_\nu(z) - z D_{\nu+1}(z) + D_{\nu+2}(z)$$

Symbolic differentiation

With respect to *ν*

07.41.20.0013.01

$$\frac{\partial^m D_\nu(z)}{\partial \nu^m} = 2^{\frac{\nu}{2}-m} \log^m(2) \sqrt{\pi} \; e^{-\frac{z^2}{4}} \sum_{k=0}^{m} \binom{m}{k} \left(\frac{2}{\log(2)}\right)^k \left(\sum_{j=0}^{\infty} \frac{1}{(2j)!} \frac{\partial^k \frac{\left(-\frac{\nu}{2}\right)_j}{\Gamma\left(\frac{1}{2} - \frac{\nu}{2}\right)}}{\partial \nu^k} \left(2 z^2\right)^j - \sqrt{2} \, z \sum_{j=0}^{\infty} \frac{1}{(2j+1)!} \frac{\partial^k \frac{\left(\frac{1-\nu}{2}\right)_j}{\Gamma\left(-\frac{\nu}{2}\right)}}{\partial \nu^k} \left(2 z^2\right)^j \right) /;$$

$m \in \mathbb{N}$

With respect to *z*

07.41.20.0014.01

$$\frac{\partial^m D_\nu(z)}{\partial \nu^m} = 2^{\frac{\nu}{2}-m} \log^m(2) \sqrt{\pi}\, e^{-\frac{z^2}{4}} \sum_{k=0}^{m} \binom{m}{k} \left(\frac{2}{\log(2)}\right)^k \left(\sum_{j=0}^{\infty} \frac{1}{(2j)!} \frac{\partial^k \frac{\left(-\frac{\nu}{2}\right)_j}{\Gamma\left(\frac{1}{2}-\frac{\nu}{2}\right)}}{\partial \nu^k} \left(2z^2\right)^j - \sqrt{2}\, z \sum_{j=0}^{\infty} \frac{1}{(2j+1)!} \frac{\partial^k \frac{\left(\frac{1-\nu}{2}\right)_j}{\Gamma\left(-\frac{\nu}{2}\right)}}{\partial \nu^k} \left(2z^2\right)^j \right) /;$$

$m \in \mathbb{N}$

07.41.20.0015.01

$$\frac{\partial^m D_\nu(z)}{\partial \nu^m} = 2^{\frac{\nu}{2}-m} \log^m(2) \sqrt{\pi}\, e^{-\frac{z^2}{4}} \sum_{k=0}^{m} \binom{m}{k} \left(\frac{2}{\log(2)}\right)^k \left(\sum_{j=0}^{\infty} \frac{1}{(2j)!} \frac{\partial^k \frac{\left(-\frac{\nu}{2}\right)_j}{\Gamma\left(\frac{1}{2}-\frac{\nu}{2}\right)}}{\partial \nu^k} \left(2z^2\right)^j - \sqrt{2}\, z \sum_{j=0}^{\infty} \frac{1}{(2j+1)!} \frac{\partial^k \frac{\left(\frac{1-\nu}{2}\right)_j}{\Gamma\left(-\frac{\nu}{2}\right)}}{\partial \nu^k} \left(2z^2\right)^j \right) /;$$

$m \in \mathbb{N}$

07.41.20.0016.01

$$\frac{\partial^n D_\nu(z)}{\partial z^n} = 2^{\frac{\nu}{2}-n} e^{-\frac{z^2}{4}} \sqrt{\pi}\, n! \left(\frac{z^{-n}}{\Gamma\left(\frac{1-\nu}{2}\right)} \sum_{k=0}^{n} \frac{(-1)^k k!\, z^{2k}}{(n-k)!\,(2k-n)!} \sum_{j=0}^{k} \frac{(-1)^j 2^j \left(-\frac{\nu}{2}\right)_j}{j!\,(k-j)!\left(\frac{1}{2}\right)_j} \, {}_1F_1\left(j-\frac{\nu}{2}; j+\frac{1}{2}; \frac{z^2}{2}\right) - \right.$$

$$\frac{\sqrt{2}}{\Gamma\left(-\frac{\nu}{2}\right)} \left((n+1)\, z^{1-n} \sum_{k=0}^{n-1} \frac{z^{2k}}{(2k-n+1)!\,(n-k)!} \sum_{j=0}^{k} \frac{(-1)^{k-j} 2^j \binom{k}{j} \left(\frac{1-\nu}{2}\right)_j}{\left(\frac{3}{2}\right)_j} \, {}_1F_1\left(j+\frac{1-\nu}{2}; j+\frac{3}{2}; \frac{z^2}{2}\right) + \right.$$

$$\left.\left. z^{n+1} \sum_{j=0}^{n} \frac{(-1)^{n-j} 2^j \left(\frac{1-\nu}{2}\right)_j}{j!\,(n-j)!\left(\frac{3}{2}\right)_j} \, {}_1F_1\left(j+\frac{1-\nu}{2}; j+\frac{3}{2}; \frac{z^2}{2}\right) \right)\right) /; n \in \mathbb{N}$$

07.41.20.0017.01

$$\frac{\partial^n \left(e^{\frac{z^2}{4}} D_\nu(z) \right)}{\partial z^n} = (-1)^n (-\nu)_n\, e^{\frac{z^2}{4}} D_{\nu-n}(z) /; n \in \mathbb{N}$$

07.41.20.0018.01

$$\frac{\partial^n \left(e^{-\frac{z^2}{4}} D_\nu(z) \right)}{\partial z^n} = (-1)^n e^{-\frac{z^2}{4}} D_{n+\nu}(z) /; n \in \mathbb{N}$$

07.41.20.0019.01

$$\frac{\partial^n \left(z^{n-1} e^{\frac{1}{4z^2}} D_\nu\left(\frac{1}{z}\right) \right)}{\partial z^n} = z^{-n-1} (-\nu)_n\, e^{\frac{1}{4z^2}} D_{\nu-n}\left(\frac{1}{z}\right) /; n \in \mathbb{N}$$

07.41.20.0020.01

$$\frac{\partial^n \left(z^{n-\frac{\nu}{2}-1} e^{z/4} D_\nu\left(\sqrt{z}\right) \right)}{\partial z^n} = \frac{(-\nu)_{2n}\, z^{-\frac{\nu}{2}-1} e^{z/4} D_{\nu-2n}\left(\sqrt{z}\right)}{2^n} /; n \in \mathbb{N}$$

07.41.20.0021.01

$$\frac{\partial^n\left(z^{\frac{\nu-1}{2}+n}\,e^{-\frac{z}{4}}\,D_\nu\left(\sqrt{z}\right)\right)}{\partial z^n}=\frac{z^{\frac{\nu-1}{2}}\,e^{-\frac{z}{4}}\,D_{2n+\nu}\left(\sqrt{z}\right)}{(-2)^n}\;/;\;n\in\mathbb{N}$$

07.41.20.0022.01

$$\frac{\partial^n\left(z^{\nu/2}\,e^{\frac{1}{4z}}\,D_\nu\left(\frac{1}{\sqrt{z}}\right)\right)}{\partial z^n}=\frac{(-\nu)_{2n}\,z^{\frac{\nu}{2}-n}\,e^{\frac{1}{4z}}\,D_{\nu-2n}\left(\frac{1}{\sqrt{z}}\right)}{(-2)^n}\;/;\;n\in\mathbb{N}$$

07.41.20.0023.01

$$\frac{\partial^n\,\frac{e^{-\frac{z}{4}}\,D_{-2n}\left(\sqrt{z}\right)}{\sqrt{z}}}{\partial z^n}=\frac{z^{-n-\frac{1}{2}}\,e^{-\frac{z}{2}}}{(-2)^n}\;/;\;n\in\mathbb{N}$$

07.41.20.0024.01

$$\frac{\partial^n\,\frac{e^{-\frac{z}{4}}\,D_{-2n-1}\left(\sqrt{z}\right)}{z}}{\partial z^n}=(-1)^n\,2^{-n-\frac{1}{2}}\,\sqrt{\pi}\;z^{-n-1}\,\mathrm{erfc}\left(\sqrt{\frac{z}{2}}\right)\;/;\;n\in\mathbb{N}$$

07.41.20.0025.01

$$\frac{\partial^n\left(z^{n-\frac{1}{2}}\,e^{-\frac{1}{4z}}\,D_{-2n}\left(\frac{1}{\sqrt{z}}\right)\right)}{\partial z^n}=\frac{e^{-\frac{1}{2z}}}{2^n\,\sqrt{z}}\;/;\;n\in\mathbb{N}$$

07.41.20.0026.01

$$\frac{\partial^n\left(z^n\,e^{-\frac{1}{4z}}\,D_{-2n-1}\left(\frac{1}{\sqrt{z}}\right)\right)}{\partial z^n}=2^{-n-\frac{1}{2}}\,\sqrt{\pi}\;\mathrm{erfc}\left(\frac{1}{\sqrt{2z}}\right)\;/;\;n\in\mathbb{N}$$

Fractional integro-differentiation

With respect to z

07.41.20.0027.01

$$\frac{\partial^\alpha D_\nu(z)}{\partial z^\alpha}=2^{\nu/2}\,\sqrt{\pi}\left(\frac{1}{\Gamma\left(\frac{1-\nu}{2}\right)}\sum_{k=0}^{\infty}\sum_{j=0}^{k}\frac{(-1)^{k-j}\,2^{j-2k}\left(-\frac{\nu}{2}\right)_j\,(2k)!\,z^{2k-\alpha}}{j!\,(k-j)!\left(\frac{1}{2}\right)_j\,\Gamma(2k-\alpha+1)}-\frac{\sqrt{2}}{\Gamma\left(-\frac{\nu}{2}\right)}\sum_{k=0}^{\infty}\sum_{j=0}^{k}\frac{(-1)^{k-j}\,2^{j-2k}\left(\frac{1-\nu}{2}\right)_j\,(2k+1)!\,z^{2k-\alpha+1}}{j!\,(k-j)!\left(\frac{3}{2}\right)_j\,\Gamma(2k-\alpha+2)}\right)$$

07.41.20.0028.01

$$\frac{\partial^\alpha\left(e^{\frac{z^2}{4}}\,D_\nu(z)\right)}{\partial z^\alpha}=\frac{2^{\alpha+\frac{\nu}{2}}\,\pi\,z^{-\alpha}}{\Gamma\left(\frac{1-\nu}{2}\right)}\,{}_2\tilde{F}_2\left(1,-\frac{\nu}{2};\frac{1-\alpha}{2},1-\frac{\alpha}{2};\frac{z^2}{2}\right)-\frac{2^{\frac{\nu-1}{2}+\alpha}\,\pi\,z^{1-\alpha}}{\Gamma\left(-\frac{\nu}{2}\right)}\,{}_2\tilde{F}_2\left(1,\frac{1-\nu}{2};1-\frac{\alpha}{2},\frac{3-\alpha}{2};\frac{z^2}{2}\right)$$

Integration

Indefinite integration

Involving one direct function and elementary functions

Involving exponential function

07.41.21.0001.01

$$\int e^{\frac{z^2}{4}} D_\nu(z)\, dz = \frac{e^{\frac{z^2}{4}}}{\nu+1} D_{\nu+1}(z)$$

07.41.21.0002.01

$$\int e^{-\frac{z^2}{4}} D_\nu(z)\, dz = -e^{-\frac{z^2}{4}} D_{\nu-1}(z)$$

Involving exponential function and a power function

07.41.21.0003.01

$$\int z^{\alpha-1} e^{\frac{z^2}{4}} D_\nu(z)\, dz = \frac{2^{\nu/2} \sqrt{\pi}\, z^\alpha}{\alpha\, \Gamma\left(\frac{1-\nu}{2}\right)} {}_2F_2\left(\frac{\alpha}{2}, -\frac{\nu}{2}; \frac{1}{2}, \frac{\alpha}{2}+1; \frac{z^2}{2}\right) - \frac{2^{\frac{\nu+1}{2}} \sqrt{\pi}\, z^{\alpha+1}}{(\alpha+1)\, \Gamma\left(-\frac{\nu}{2}\right)} {}_2F_2\left(\frac{\alpha+1}{2}, \frac{1-\nu}{2}; \frac{3}{2}, \frac{\alpha+3}{2}; \frac{z^2}{2}\right)$$

07.41.21.0004.01

$$\int z^{\alpha-1} e^{-\frac{z^2}{4}} D_\nu(t)\, dt = \frac{2^{\nu/2} \sqrt{\pi}\, z^\alpha}{\alpha\, \Gamma\left(\frac{1-\nu}{2}\right)} {}_2F_2\left(\frac{\alpha}{2}, \frac{\nu+1}{2}; \frac{1}{2}, \frac{\alpha}{2}+1; -\frac{z^2}{2}\right) - \frac{2^{\frac{\nu+1}{2}} \sqrt{\pi}\, z^{\alpha+1}}{(\alpha+1)\, \Gamma\left(-\frac{\nu}{2}\right)} {}_2F_2\left(\frac{\alpha+1}{2}, \frac{\nu}{2}+1; \frac{3}{2}, \frac{\alpha+3}{2}; -\frac{z^2}{2}\right)$$

07.41.21.0005.01

$$\int z^n e^{\frac{z^2}{4}} D_\nu(z)\, dz = -\frac{e^{\frac{z^2}{4}}}{n+1} \sum_{k=1}^{n+1} \frac{(-n-1)_k\, z^{n-k+1}}{(\nu+1)_k} D_{k+\nu}(z) \;/;\; n \in \mathbb{N}$$

07.41.21.0006.01

$$\int z\, e^{\frac{z^2}{4}} D_\nu(z)\, dz = \frac{e^{\frac{z^2}{4}}}{\nu+2} (z\, D_{\nu+1}(z) + D_\nu(z))$$

07.41.21.0007.01

$$\int z^2 e^{\frac{z^2}{4}} D_\nu(z)\, dz = \frac{e^{\frac{z^2}{4}}}{\nu+3} \left(\left(z^2 - \frac{2}{\nu+1}\right) D_{\nu+1}(z) + 2\, z\, D_\nu(z)\right)$$

07.41.21.0008.01

$$\int z^{-\nu-3} e^{\frac{z^2}{4}} D_\nu(z)\, dz = \frac{z^{-\nu-2}\, e^{\frac{z^2}{4}}}{(\nu+1)\,(\nu+2)} D_{\nu+2}(z)$$

07.41.21.0009.01

$$\int z^n e^{-\frac{z^2}{4}} D_\nu(z)\, dz = -\frac{e^{-\frac{z^2}{4}}}{n+1} \sum_{k=1}^{n+1} (-1)^k\, (-n-1)_k\, z^{n-k+1}\, D_{\nu-k}(z) \;/;\; n \in \mathbb{N}$$

07.41.21.0010.01

$$\int z\, e^{-\frac{z^2}{4}} D_\nu(z)\, dz = -e^{-\frac{z^2}{4}} (D_{\nu-2}(z) + z\, D_{\nu-1}(z))$$

07.41.21.0011.01

$$\int z^2\, e^{-\frac{z^2}{4}}\, D_\nu(z)\, dz = \frac{e^{-\frac{z^2}{4}}}{\nu - 2}\left(2\, z\, D_\nu(z) - \left(\nu\, z^2 - 2\right) D_{\nu-1}(z)\right)$$

07.41.21.0012.01

$$\int z^{\nu-2}\, e^{-\frac{z^2}{4}}\, D_\nu(z)\, dz = -z^{\nu-1}\, e^{-\frac{z^2}{4}}\, D_{\nu-2}(z)$$

Definite integration

07.41.21.0013.01

$$\int_0^\infty t^{\alpha-1}\, D_\nu(t)\, dt = \frac{2^{\frac{\nu-\alpha}{2}}\sqrt{\pi}\;\Gamma(\alpha)}{\Gamma\left(\frac{\alpha-\nu+1}{2}\right)}\; {}_2F_1\left(\frac{\alpha}{2}, \frac{\alpha+1}{2}; \frac{\alpha-\nu+1}{2}; \frac{1}{2}\right) /; \operatorname{Re}(\alpha) > 0$$

07.41.21.0014.01

$$\int_0^\infty t^{\alpha-1}\, e^{-a t^2}\, D_\nu(t)\, dt = \frac{2^{\frac{\nu-\alpha}{2}}\sqrt{\pi}\;\Gamma(\alpha)}{\Gamma\left(\frac{\alpha-\nu+1}{2}\right)}\; {}_2F_1\left(\frac{\alpha}{2}, \frac{\alpha+1}{2}; \frac{\alpha-\nu+1}{2}; \frac{1}{2} - 2\, a\right) /;$$

$$(\operatorname{Re}(\alpha) > 0 \land \operatorname{Re}(4\, a + 1) > 0) \lor (0 < \operatorname{Re}(\alpha) < -\operatorname{Re}(\nu) \land \operatorname{Re}(4\, a + 1) = 0)$$

07.41.21.0015.01

$$\int_0^\infty t^{\alpha-1}\, e^{\frac{t^2}{4}}\, D_\nu(t)\, dt = \frac{2^{-\frac{\alpha+\nu}{2}-1}\,\Gamma(\alpha)\,\Gamma\left(-\frac{\alpha+\nu}{2}\right)}{\Gamma(-\nu)} /; 0 < \operatorname{Re}(\alpha) < -\operatorname{Re}(\nu)$$

07.41.21.0016.01

$$\int_0^\infty D_\alpha(t)\, D_\nu(t)\, dt = \frac{\Gamma\left(-\frac{\alpha}{2}\right)\Gamma\left(\frac{1-\nu}{2}\right) - \Gamma\left(\frac{1-\alpha}{2}\right)\Gamma\left(-\frac{\nu}{2}\right)}{2^{\frac{1}{2}(\alpha+\nu+3)}\,(\alpha-\nu)\,\Gamma(-\alpha)\,\Gamma(-\nu)}$$

07.41.21.0017.01

$$\int_0^\infty D_\nu(t)^2\, dt = \frac{\sqrt{\pi}}{2^{3/2}\,\Gamma(-\nu)}\left(\psi\left(\frac{1-\nu}{2}\right) - \psi\left(-\frac{\nu}{2}\right)\right)$$

Integral transforms

Laplace transforms

07.41.22.0001.01

$$\mathcal{L}_t\left[t^{\alpha-1}\, D_\nu\left(\sqrt{t}\right)\right](z) = \frac{2^{-\alpha+\frac{\nu}{2}+1}\sqrt{\pi}\;\Gamma(2\,\alpha)}{\Gamma\left(\alpha + \frac{1-\nu}{2}\right)}\; {}_2F_1\left(\alpha, \alpha + \frac{1}{2}; \alpha + \frac{1-\nu}{2}; \frac{1}{2} - 2\, z\right) /;$$

$$(\operatorname{Re}(\alpha) > 0 \land \operatorname{Re}(4\, z + 1) > 0) \bigvee \left(0 < \operatorname{Re}(\alpha) < -\frac{\operatorname{Re}(\nu)}{2} \bigwedge \operatorname{Re}(4\, z + 1) = 0\right)$$

Mellin transforms

07.41.22.0002.01

$$\mathcal{M}_t[D_\nu(t)](z) = \frac{2^{\frac{\nu-z}{2}}\sqrt{\pi}\;\Gamma(z)}{\Gamma\left(\frac{1+z-\nu}{2}\right)}\; {}_2F_1\left(\frac{z}{2}, \frac{z+1}{2}; \frac{1+z-\nu}{2}; \frac{1}{2}\right) /; \operatorname{Re}(z) > 0$$

Summation

Infinite summation

07.41.23.0001.01

$$\sum_{k=0}^{\infty} \frac{w^k D_{k+\nu}(z)}{k!} = e^{\frac{1}{4} w (2 z - w)} D_\nu(z - w)$$

07.41.23.0002.01

$$\sum_{k=0}^{\infty} \frac{(-\nu)_k w^k D_{\nu-k}(z)}{k!} = e^{\frac{1}{4} w (w - 2 z)} D_\nu(z - w)$$

07.41.23.0003.01

$$\sum_{k=0}^{\infty} \frac{w^k D_{2k+\nu}(z)}{k!} = \frac{1}{(2 w + 1)^{\frac{\nu+1}{2}}} e^{\frac{w z^2}{2 (2 w + 1)}} D_\nu\left(\frac{z}{\sqrt{2 w + 1}}\right) /; |w| < \frac{1}{2}$$

Operations

Limit operation

07.41.25.0001.01

$$\lim_{n \to \infty} \frac{1}{n!} \left(-\frac{1}{2}\right)^n e^{\frac{z^2}{8 n}} D_{2n+1}\left(\frac{z}{\sqrt{2} \sqrt{n}}\right) = \frac{\sqrt{2} \sin(z)}{\sqrt{\pi}} /; n \in \mathbb{N}$$

07.41.25.0002.01

$$\lim_{n \to \infty} \frac{\left(-\frac{1}{2}\right)^n e^{\frac{z^2}{8 n}} \sqrt{n} D_{2n}\left(\frac{z}{\sqrt{2} \sqrt{n}}\right)}{n!} = \frac{\cos(z)}{\sqrt{\pi}} /; n \in \mathbb{N}$$

Representations through more general functions

Through hypergeometric functions

Involving $_pF_q$

07.41.26.0001.01

$$D_\nu(z) = 2^{\nu/2} \sqrt{\pi} e^{-\frac{z^2}{4}} \left(\frac{1}{\Gamma\left(\frac{1-\nu}{2}\right)} {}_1F_1\left(-\frac{\nu}{2}; \frac{1}{2}; \frac{z^2}{2}\right) - \frac{\sqrt{2} z}{\Gamma\left(-\frac{\nu}{2}\right)} {}_1F_1\left(\frac{1-\nu}{2}; \frac{3}{2}; \frac{z^2}{2}\right)\right)$$

07.41.26.0002.01

$$D_\nu(z) = 2^{\nu/2} \sqrt{\pi} e^{\frac{z^2}{4}} \left(\frac{1}{\Gamma\left(\frac{1-\nu}{2}\right)} {}_1F_1\left(\frac{1+\nu}{2}; \frac{1}{2}; -\frac{z^2}{2}\right) - \frac{\sqrt{2} z}{\Gamma\left(-\frac{\nu}{2}\right)} {}_1F_1\left(\frac{\nu}{2} + 1; \frac{3}{2}; -\frac{z^2}{2}\right)\right)$$

07.41.26.0003.01

$$D_\nu(i z) D_\nu(z) = \frac{2^\nu \pi}{\Gamma\left(\frac{1-\nu}{2}\right)^2} {}_2F_3\left(\frac{\nu+1}{2}, -\frac{\nu}{2}; \frac{1}{4}, \frac{1}{2}, \frac{3}{4}; \frac{z^4}{16}\right) - \frac{\sqrt[4]{-1} \sqrt{\pi} z}{\Gamma(-\nu)} {}_2F_3\left(\frac{1}{4}(1-2\nu), \frac{1}{4}(2\nu+3); \frac{1}{2}, \frac{3}{4}, \frac{5}{4}; \frac{z^4}{16}\right) + \frac{i 2^{\nu+1} \pi z^2}{\Gamma\left(-\frac{\nu}{2}\right)^2} {}_2F_3\left(\frac{1-\nu}{2}, \frac{\nu}{2}+1; \frac{3}{4}, \frac{5}{4}, \frac{3}{2}; \frac{z^4}{16}\right) + \frac{(-1)^{3/4} \sqrt{\pi} (2\nu+1) z^3}{6 \Gamma(-\nu)} {}_2F_3\left(\frac{1}{4}(3-2\nu), \frac{1}{4}(2\nu+5); \frac{5}{4}, \frac{3}{2}, \frac{7}{4}; \frac{z^4}{16}\right)$$

Involving ${}_1\tilde{F}_1$

07.41.26.0004.01

$$D_\nu(z) = 2^{\nu/2} \pi e^{-\frac{z^2}{4}} \left(\frac{1}{\Gamma\left(\frac{1-\nu}{2}\right)} {}_1\tilde{F}_1\left(-\frac{\nu}{2}; \frac{1}{2}; \frac{z^2}{2}\right) - \frac{z}{\sqrt{2} \Gamma\left(-\frac{\nu}{2}\right)} {}_1\tilde{F}_1\left(\frac{1-\nu}{2}; \frac{3}{2}; \frac{z^2}{2}\right)\right)$$

07.41.26.0005.01

$$D_\nu(z) = 2^{\nu/2} \pi e^{\frac{z^2}{4}} \left(\frac{1}{\Gamma\left(\frac{1-\nu}{2}\right)} {}_1\tilde{F}_1\left(\frac{\nu+1}{2}; \frac{1}{2}; -\frac{z^2}{2}\right) - \frac{z}{\sqrt{2} \Gamma\left(-\frac{\nu}{2}\right)} {}_1\tilde{F}_1\left(\frac{\nu}{2}+1; \frac{3}{2}; -\frac{z^2}{2}\right)\right)$$

07.41.26.0006.01

$$D_\nu(i z) D_\nu(z) = \frac{i 2^{\nu-\frac{3}{2}} \pi^{5/2} z^2}{\Gamma\left(-\frac{\nu}{2}\right)^2} {}_2\tilde{F}_3\left(\frac{1-\nu}{2}, \frac{\nu}{2}+1; \frac{3}{4}, \frac{5}{4}, \frac{3}{2}; \frac{z^4}{16}\right) + \frac{2^{\nu+\frac{1}{2}} \pi^{5/2}}{\Gamma\left(\frac{1-\nu}{2}\right)^2} {}_2\tilde{F}_3\left(\frac{\nu+1}{2}, -\frac{\nu}{2}; \frac{1}{4}, \frac{1}{2}, \frac{3}{4}; \frac{z^4}{16}\right) - \frac{\left(\frac{1}{4}+\frac{i}{4}\right) \pi^2 z}{\Gamma(-\nu)} {}_2\tilde{F}_3\left(\frac{1}{4}(1-2\nu), \frac{1}{4}(2\nu+3); \frac{1}{2}, \frac{3}{4}, \frac{5}{4}; \frac{z^4}{16}\right) - \frac{\left(\frac{1}{64}-\frac{i}{64}\right) \pi^2 z^3 (2\nu+1)}{\Gamma(-\nu)} {}_2\tilde{F}_3\left(\frac{1}{4}(3-2\nu), \frac{1}{4}(2\nu+5); \frac{5}{4}, \frac{3}{2}, \frac{7}{4}; \frac{z^4}{16}\right)$$

Involving hypergeometric U

07.41.26.0007.01

$$D_\nu(z) = 2^{\nu/2} e^{-\frac{z^2}{4}} U\left(-\frac{\nu}{2}, \frac{1}{2}, \frac{z^2}{2}\right) /; -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

07.41.26.0008.01

$$D_\nu(z) = 2^{\frac{\nu-1}{2}} z e^{-\frac{z^2}{4}} U\left(\frac{1-\nu}{2}, \frac{3}{2}, \frac{z^2}{2}\right) /; -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

07.41.26.0009.01

$$D_\nu(z) = \frac{2^{-\frac{\nu}{2}-1} \sin(\pi\nu)}{\sqrt{-z^2} \cos\left(\frac{\pi\nu}{2}\right) + \sqrt{z^2} \sin\left(\frac{\pi\nu}{2}\right)} e^{-\frac{z^2}{4}} \left(2^\nu \left(\sqrt{-z^2} \csc\left(\frac{\pi\nu}{2}\right) + z \sec\left(\frac{\pi\nu}{2}\right)\right) U\left(-\frac{\nu}{2}, \frac{1}{2}, \frac{z^2}{2}\right) - \frac{\sqrt{-z^2} \left(z - \sqrt{z^2}\right) \Gamma(\nu+1)}{\sqrt{2\pi}} e^{\frac{z^2}{2}} U\left(\frac{\nu}{2}+1, \frac{3}{2}, -\frac{z^2}{2}\right)\right)$$

Through Meijer *G*

Classical cases involving exp

07.41.26.0010.01

$$e^{z/4} D_\nu\left(\sqrt{z}\right) = \frac{2^{-\frac{\nu}{2}-1}}{\sqrt{\pi}\,\Gamma(-\nu)} G_{1,2}^{2,1}\left(\frac{z}{2} \,\middle|\, \begin{matrix} \frac{\nu}{2}+1 \\ 0, \frac{1}{2} \end{matrix}\right)$$

07.41.26.0011.01

$$e^{\frac{z^2}{4}} D_\nu(z) = \frac{2^{-\frac{\nu}{2}-1}}{\sqrt{\pi}\,\Gamma(-\nu)} G_{1,2}^{2,1}\left(\frac{z^2}{2} \,\middle|\, \begin{matrix} \frac{\nu}{2}+1 \\ 0, \frac{1}{2} \end{matrix}\right) /; -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

07.41.26.0012.01

$$e^{z/4} D_\nu\left(-\sqrt{z}\right) = \frac{2^{\frac{1-\nu}{2}} \pi^{3/2}}{\Gamma(-\nu)} G_{3,4}^{2,1}\left(\frac{z}{2} \,\middle|\, \begin{matrix} \frac{\nu}{2}+1, \frac{1}{8}, \frac{5}{8} \\ 0, \frac{1}{2}, \frac{1}{8}, \frac{5}{8} \end{matrix}\right)$$

07.41.26.0013.01

$$e^{-\frac{z}{4}} D_\nu\left(\sqrt{z}\right) = 2^{\nu/2} G_{1,2}^{2,0}\left(\frac{z}{2} \,\middle|\, \begin{matrix} \frac{1-\nu}{2} \\ 0, \frac{1}{2} \end{matrix}\right)$$

07.41.26.0014.01

$$e^{-\frac{z^2}{4}} D_\nu(z) = 2^{\nu/2} G_{1,2}^{2,0}\left(\frac{z^2}{2} \,\middle|\, \begin{matrix} \frac{1-\nu}{2} \\ 0, \frac{1}{2} \end{matrix}\right) /; -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

07.41.26.0015.01

$$e^{-\frac{z}{4}} D_\nu\left(-\sqrt{z}\right) = 2^{\nu/2} G_{2,3}^{2,1}\left(\frac{z}{2} \,\middle|\, \begin{matrix} \frac{1-\nu}{2}, \frac{\nu+1}{2} \\ 0, \frac{1}{2}, \frac{\nu+1}{2} \end{matrix}\right)$$

07.41.26.0016.01

$$e^{z/4} \left(D_\nu\left(\sqrt{z}\right) + D_\nu\left(-\sqrt{z}\right)\right) = \frac{2^{\frac{1-\nu}{2}} \pi^{3/2}}{\Gamma(-\nu)} G_{2,3}^{1,1}\left(\frac{z}{2} \,\middle|\, \begin{matrix} \frac{\nu}{2}+1, \frac{1}{4} \\ 0, \frac{1}{2}, \frac{1}{4} \end{matrix}\right)$$

07.41.26.0017.01

$$e^{z/4} \left(D_\nu\left(\sqrt{z}\right) - D_\nu\left(-\sqrt{z}\right)\right) = \frac{2^{\frac{1-\nu}{2}} \pi^{3/2}}{\Gamma(-\nu)} G_{2,3}^{1,1}\left(\frac{z}{2} \,\middle|\, \begin{matrix} \frac{\nu}{2}+1, \frac{1}{4} \\ \frac{1}{2}, 0, \frac{1}{4} \end{matrix}\right)$$

07.41.26.0018.01

$$e^{-\frac{z}{4}} \left(D_\nu\left(\sqrt{z}\right) + D_\nu\left(-\sqrt{z}\right)\right) = 2^{\frac{\nu}{2}+1} \cos\left(\frac{\pi\nu}{2}\right) G_{1,2}^{1,1}\left(\frac{z}{2} \,\middle|\, \begin{matrix} \frac{1-\nu}{2} \\ 0, \frac{1}{2} \end{matrix}\right)$$

07.41.26.0019.01

$$e^{-\frac{z}{4}} \left(D_\nu\left(\sqrt{z}\right) - D_\nu\left(-\sqrt{z}\right)\right) = 2^{\frac{\nu}{2}+1} \sin\left(\frac{\pi\nu}{2}\right) G_{1,2}^{1,1}\left(\frac{z}{2} \,\middle|\, \begin{matrix} \frac{1-\nu}{2} \\ \frac{1}{2}, 0 \end{matrix}\right)$$

Classical cases involving cosh

07.41.26.0020.01

$$\cosh\left(\frac{z^2}{4}\right) D_\nu(z) = 2^{\frac{\nu}{2}-1}\, G_{1,2}^{2,0}\left(\frac{z^2}{2} \,\middle|\, \begin{matrix} \frac{1-\nu}{2} \\ 0, \frac{1}{2} \end{matrix}\right) + \frac{2^{-\frac{\nu}{2}-2}}{\sqrt{\pi}\,\Gamma(-\nu)}\, G_{1,2}^{2,1}\left(\frac{z^2}{2} \,\middle|\, \begin{matrix} \frac{\nu}{2}+1 \\ 0, \frac{1}{2} \end{matrix}\right) /;\ -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

07.41.26.0021.01

$$\cosh\left(\frac{z^2}{4}\right) D_\nu(-z) = 2^{\frac{\nu}{2}-1}\, G_{2,3}^{2,1}\left(\frac{z^2}{2} \,\middle|\, \begin{matrix} \frac{1-\nu}{2}, \frac{\nu+1}{2} \\ 0, \frac{1}{2}, \frac{\nu+1}{2} \end{matrix}\right) + \frac{2^{-\frac{1}{2}(\nu+1)}\,\pi^{3/2}}{\Gamma(-\nu)}\, G_{3,4}^{2,1}\left(\frac{z^2}{2} \,\middle|\, \begin{matrix} \frac{\nu}{2}+1, \frac{1}{8}, \frac{5}{8} \\ 0, \frac{1}{2}, \frac{1}{8}, \frac{5}{8} \end{matrix}\right) /;\ -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

07.41.26.0022.01

$$\cosh\left(\frac{z^2}{4}\right) (D_\nu(-z) + D_\nu(z)) = 2^{\nu/2} \cos\left(\frac{\pi\,\nu}{2}\right) G_{1,2}^{1,1}\left(\frac{z^2}{2} \,\middle|\, \begin{matrix} \frac{1-\nu}{2} \\ 0, \frac{1}{2} \end{matrix}\right) + \frac{2^{-\frac{1}{2}(\nu+1)}\,\pi^{3/2}}{\Gamma(-\nu)}\, G_{2,3}^{1,1}\left(\frac{z^2}{2} \,\middle|\, \begin{matrix} \frac{\nu}{2}+1, \frac{1}{4} \\ 0, \frac{1}{2}, \frac{1}{4} \end{matrix}\right) /;\ -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

07.41.26.0023.01

$$\cosh\left(\frac{z^2}{4}\right) (D_\nu(z) - D_\nu(-z)) = \frac{2^{-\frac{1}{2}(\nu+1)}\,\pi^{3/2}}{\Gamma(-\nu)}\, G_{2,3}^{1,1}\left(\frac{z^2}{2} \,\middle|\, \begin{matrix} \frac{\nu}{2}+1, \frac{1}{4} \\ \frac{1}{2}, 0, \frac{1}{4} \end{matrix}\right) + 2^{\nu/2}\, G_{1,2}^{1,1}\left(\frac{z^2}{2} \,\middle|\, \begin{matrix} \frac{1-\nu}{2} \\ \frac{1}{2}, 0 \end{matrix}\right) \sin\left(\frac{\pi\,\nu}{2}\right) /;\ -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

07.41.26.0024.01

$$\cosh\left(\frac{z}{4}\right) D_\nu\left(\sqrt{z}\right) = 2^{\frac{\nu}{2}-1}\, G_{1,2}^{2,0}\left(\frac{z}{2} \,\middle|\, \begin{matrix} \frac{1-\nu}{2} \\ 0, \frac{1}{2} \end{matrix}\right) + \frac{2^{-\frac{\nu}{2}-2}}{\sqrt{\pi}\,\Gamma(-\nu)}\, G_{1,2}^{2,1}\left(\frac{z}{2} \,\middle|\, \begin{matrix} \frac{\nu}{2}+1 \\ 0, \frac{1}{2} \end{matrix}\right)$$

07.41.26.0025.01

$$\cosh\left(\frac{z}{4}\right) D_\nu\left(-\sqrt{z}\right) = 2^{\frac{\nu}{2}-1}\, G_{2,3}^{2,1}\left(\frac{z}{2} \,\middle|\, \begin{matrix} \frac{1-\nu}{2}, \frac{\nu+1}{2} \\ 0, \frac{1}{2}, \frac{\nu+1}{2} \end{matrix}\right) + \frac{2^{-\frac{1}{2}(\nu+1)}\,\pi^{3/2}}{\Gamma(-\nu)}\, G_{3,4}^{2,1}\left(\frac{z}{2} \,\middle|\, \begin{matrix} \frac{\nu}{2}+1, \frac{1}{8}, \frac{5}{8} \\ 0, \frac{1}{2}, \frac{1}{8}, \frac{5}{8} \end{matrix}\right)$$

07.41.26.0026.01

$$\cosh\left(\frac{z}{4}\right) \left(D_\nu\left(-\sqrt{z}\right) + D_\nu\left(\sqrt{z}\right)\right) = 2^{\nu/2} \cos\left(\frac{\pi\,\nu}{2}\right) G_{1,2}^{1,1}\left(\frac{z}{2} \,\middle|\, \begin{matrix} \frac{1-\nu}{2} \\ 0, \frac{1}{2} \end{matrix}\right) + \frac{2^{-\frac{1}{2}(\nu+1)}\,\pi^{3/2}}{\Gamma(-\nu)}\, G_{2,3}^{1,1}\left(\frac{z}{2} \,\middle|\, \begin{matrix} \frac{\nu}{2}+1, \frac{1}{4} \\ 0, \frac{1}{2}, \frac{1}{4} \end{matrix}\right)$$

07.41.26.0027.01

$$\cosh\left(\frac{z}{4}\right) \left(D_\nu\left(\sqrt{z}\right) - D_\nu\left(-\sqrt{z}\right)\right) = \frac{2^{-\frac{1}{2}(\nu+1)}\,\pi^{3/2}}{\Gamma(-\nu)}\, G_{2,3}^{1,1}\left(\frac{z}{2} \,\middle|\, \begin{matrix} \frac{\nu}{2}+1, \frac{1}{4} \\ \frac{1}{2}, 0, \frac{1}{4} \end{matrix}\right) + 2^{\nu/2} \sin\left(\frac{\pi\,\nu}{2}\right) G_{1,2}^{1,1}\left(\frac{z}{2} \,\middle|\, \begin{matrix} \frac{1-\nu}{2} \\ \frac{1}{2}, 0 \end{matrix}\right)$$

Classical cases involving sinh

07.41.26.0028.01

$$\sinh\left(\frac{z^2}{4}\right) D_\nu(z) = \frac{2^{-\frac{\nu}{2}-2}}{\sqrt{\pi}\,\Gamma(-\nu)}\, G_{1,2}^{2,1}\left(\frac{z^2}{2} \,\middle|\, \begin{matrix} \frac{\nu}{2}+1 \\ 0, \frac{1}{2} \end{matrix}\right) - 2^{\frac{\nu}{2}-1}\, G_{1,2}^{2,0}\left(\frac{z^2}{2} \,\middle|\, \begin{matrix} \frac{1-\nu}{2} \\ 0, \frac{1}{2} \end{matrix}\right) /;\ -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

07.41.26.0029.01

$$\sinh\left(\frac{z^2}{4}\right) D_\nu(-z) = \frac{2^{-\frac{1}{2}(\nu+1)}\,\pi^{3/2}}{\Gamma(-\nu)}\, G_{3,4}^{2,1}\left(\frac{z^2}{2} \,\middle|\, \begin{matrix} \frac{\nu}{2}+1, \frac{1}{8}, \frac{5}{8} \\ 0, \frac{1}{2}, \frac{1}{8}, \frac{5}{8} \end{matrix}\right) - 2^{\frac{\nu}{2}-1}\, G_{2,3}^{2,1}\left(\frac{z^2}{2} \,\middle|\, \begin{matrix} \frac{1-\nu}{2}, \frac{\nu+1}{2} \\ 0, \frac{1}{2}, \frac{\nu+1}{2} \end{matrix}\right) /;\ -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

07.41.26.0030.01

$$\sinh\left(\frac{z^2}{4}\right) (D_\nu(-z) + D_\nu(z)) = \frac{2^{-\frac{1}{2}(\nu+1)}\,\pi^{3/2}}{\Gamma(-\nu)}\, G_{2,3}^{1,1}\left(\frac{z^2}{2} \,\middle|\, \begin{matrix} \frac{\nu}{2}+1, \frac{1}{4} \\ 0, \frac{1}{2}, \frac{1}{4} \end{matrix}\right) - 2^{\nu/2} \cos\left(\frac{\pi\,\nu}{2}\right) G_{1,2}^{1,1}\left(\frac{z^2}{2} \,\middle|\, \begin{matrix} \frac{1-\nu}{2} \\ 0, \frac{1}{2} \end{matrix}\right) /;\ -\frac{\pi}{2} < \arg(z) \leq \frac{\pi}{2}$$

07.41.26.0031.01

$$\sinh\left(\frac{z^2}{4}\right)(D_\nu(z)-D_\nu(-z)) = \frac{2^{-\frac{1}{2}(\nu+1)}\pi^{3/2}}{\Gamma(-\nu)} G_{2,3}^{1,1}\left(\frac{z^2}{2} \,\middle|\, \begin{matrix} \frac{\nu}{2}+1, \frac{1}{4} \\ \frac{1}{2}, 0, \frac{1}{4} \end{matrix}\right) - 2^{\nu/2}\sin\left(\frac{\pi\nu}{2}\right) G_{1,2}^{1,1}\left(\frac{z^2}{2} \,\middle|\, \begin{matrix} \frac{1-\nu}{2} \\ \frac{1}{2}, 0 \end{matrix}\right) /; -\frac{\pi}{2} < \arg(z) \le \frac{\pi}{2}$$

07.41.26.0032.01

$$\sinh\left(\frac{z}{4}\right) D_\nu\left(\sqrt{z}\right) = \frac{2^{-\frac{\nu}{2}-2}}{\sqrt{\pi}\,\Gamma(-\nu)} G_{1,2}^{2,1}\left(\frac{z}{2} \,\middle|\, \begin{matrix} \frac{\nu}{2}+1 \\ 0, \frac{1}{2} \end{matrix}\right) - 2^{\frac{\nu}{2}-1} G_{1,2}^{2,0}\left(\frac{z}{2} \,\middle|\, \begin{matrix} \frac{1-\nu}{2} \\ 0, \frac{1}{2} \end{matrix}\right)$$

07.41.26.0033.01

$$\sinh\left(\frac{z}{4}\right) D_\nu\left(-\sqrt{z}\right) = \frac{2^{-\frac{1}{2}(\nu+1)}\pi^{3/2}}{\Gamma(-\nu)} G_{3,4}^{2,1}\left(\frac{z}{2} \,\middle|\, \begin{matrix} \frac{\nu}{2}+1, \frac{1}{8}, \frac{5}{8} \\ 0, \frac{1}{2}, \frac{1}{8}, \frac{5}{8} \end{matrix}\right) - 2^{\frac{\nu}{2}-1} G_{2,3}^{2,1}\left(\frac{z}{2} \,\middle|\, \begin{matrix} \frac{1-\nu}{2}, \frac{\nu+1}{2} \\ 0, \frac{1}{2}, \frac{\nu+1}{2} \end{matrix}\right)$$

07.41.26.0034.01

$$\sinh\left(\frac{z}{4}\right)\left(D_\nu\left(-\sqrt{z}\right)+D_\nu\left(\sqrt{z}\right)\right) = \frac{2^{-\frac{1}{2}(\nu+1)}\pi^{3/2}}{\Gamma(-\nu)} G_{2,3}^{1,1}\left(\frac{z}{2} \,\middle|\, \begin{matrix} \frac{\nu}{2}+1, \frac{1}{4} \\ 0, \frac{1}{2}, \frac{1}{4} \end{matrix}\right) - 2^{\nu/2}\cos\left(\frac{\pi\nu}{2}\right) G_{1,2}^{1,1}\left(\frac{z}{2} \,\middle|\, \begin{matrix} \frac{1-\nu}{2} \\ 0, \frac{1}{2} \end{matrix}\right)$$

07.41.26.0035.01

$$\sinh\left(\frac{z}{4}\right)\left(D_\nu\left(\sqrt{z}\right)-D_\nu\left(-\sqrt{z}\right)\right) = \frac{2^{-\frac{1}{2}(\nu+1)}\pi^{3/2}}{\Gamma(-\nu)} G_{2,3}^{1,1}\left(\frac{z}{2} \,\middle|\, \begin{matrix} \frac{\nu}{2}+1, \frac{1}{4} \\ \frac{1}{2}, 0, \frac{1}{4} \end{matrix}\right) - 2^{\nu/2}\sin\left(\frac{\pi\nu}{2}\right) G_{1,2}^{1,1}\left(\frac{z}{2} \,\middle|\, \begin{matrix} \frac{1-\nu}{2} \\ \frac{1}{2}, 0 \end{matrix}\right)$$

Classical cases for products of *D*

07.41.26.0036.01

$$D_\nu\left(e^{\frac{\pi i}{4}}\sqrt[4]{z}\right) D_\nu\left(e^{-\frac{\pi i}{4}}\sqrt[4]{z}\right) = \frac{1}{2^{3/2}\,\pi\,\Gamma(-\nu)} G_{2,4}^{4,1}\left(\frac{z}{16} \,\middle|\, \begin{matrix} \frac{\nu}{2}+1, \frac{1-\nu}{2} \\ 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4} \end{matrix}\right)$$

07.41.26.0037.01

$$D_{-\nu-1}\left(\sqrt[4]{z}\right) D_\nu\left(\sqrt[4]{z}\right) = \frac{1}{2\sqrt{\pi}} G_{2,4}^{4,0}\left(\frac{z}{16} \,\middle|\, \begin{matrix} \frac{1-\nu}{2}, \frac{\nu}{2}+1 \\ 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4} \end{matrix}\right)$$

Classical cases involving Exp and Hermite *H*

07.41.26.0038.01

$$e^{\frac{z^2}{4}} H_\nu\left(-\frac{i z}{\sqrt{2}}\right) D_\nu(z) = \frac{2^{\frac{\nu-3}{2}}}{\pi\,\Gamma(-\nu)} G_{2,4}^{4,1}\left(-\frac{z^4}{16} \,\middle|\, \begin{matrix} \frac{\nu}{2}+1, \frac{1-\nu}{2} \\ 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4} \end{matrix}\right) /; 0 < \arg(z) \le \frac{\pi}{2}$$

07.41.26.0039.01

$$e^{-\frac{z^2}{4}} H_{-\nu-1}\left(\frac{z}{\sqrt{2}}\right) D_\nu(z) = \frac{2^{-\frac{1}{2}(\nu+3)}}{\sqrt{\pi}} G_{2,4}^{4,0}\left(\frac{z^4}{16} \,\middle|\, \begin{matrix} \frac{\nu}{2}+1, \frac{1-\nu}{2} \\ 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4} \end{matrix}\right) /; -\frac{\pi}{4} \le \arg(z) \le \frac{\pi}{4}$$

Generalized cases involving exp

07.41.26.0040.01

$$e^{\frac{z^2}{4}} D_\nu(z) = \frac{2^{-\frac{\nu}{2}-1}}{\sqrt{\pi}\,\Gamma(-\nu)} G_{1,2}^{2,1}\left(\frac{z}{\sqrt{2}}, \frac{1}{2} \,\middle|\, \begin{matrix} \frac{\nu}{2}+1 \\ 0, \frac{1}{2} \end{matrix}\right)$$

07.41.26.0041.01

$$e^{-\frac{z^2}{4}} D_\nu(z) = 2^{\nu/2} G_{1,2}^{2,0}\left(\frac{z}{\sqrt{2}}, \frac{1}{2} \middle| \begin{array}{c} \frac{1-\nu}{2} \\ 0, \frac{1}{2} \end{array}\right)$$

07.41.26.0042.01

$$e^{\frac{z^2}{4}} D_\nu(-z) = \frac{2^{\frac{1-\nu}{2}} \pi^{3/2}}{\Gamma(-\nu)} G_{3,4}^{2,1}\left(\frac{z}{\sqrt{2}}, \frac{1}{2} \middle| \begin{array}{c} \frac{\nu}{2}+1, \frac{1}{8}, \frac{5}{8} \\ 0, \frac{1}{2}, \frac{1}{8}, \frac{5}{8} \end{array}\right)$$

07.41.26.0043.01

$$e^{-\frac{z^2}{4}} D_\nu(-z) = 2^{\nu/2} G_{2,3}^{2,1}\left(\frac{z}{\sqrt{2}}, \frac{1}{2} \middle| \begin{array}{c} \frac{1-\nu}{2}, \frac{\nu+1}{2} \\ 0, \frac{1}{2}, \frac{\nu+1}{2} \end{array}\right)$$

07.41.26.0044.01

$$e^{\frac{z^2}{4}} (D_\nu(z) + D_\nu(-z)) = \frac{2^{\frac{1-\nu}{2}} \pi^{3/2}}{\Gamma(-\nu)} G_{2,3}^{1,1}\left(\frac{z}{\sqrt{2}}, \frac{1}{2} \middle| \begin{array}{c} \frac{\nu}{2}+1, \frac{1}{4} \\ 0, \frac{1}{2}, \frac{1}{4} \end{array}\right)$$

07.41.26.0045.01

$$e^{\frac{z^2}{4}} (D_\nu(z) - D_\nu(-z)) = \frac{2^{\frac{1-\nu}{2}} \pi^{3/2}}{\Gamma(-\nu)} G_{2,3}^{1,1}\left(\frac{z}{\sqrt{2}}, \frac{1}{2} \middle| \begin{array}{c} \frac{\nu}{2}+1, \frac{1}{4} \\ \frac{1}{2}, 0, \frac{1}{4} \end{array}\right)$$

07.41.26.0046.01

$$e^{-\frac{z^2}{4}} (D_\nu(z) + D_\nu(-z)) = 2^{\frac{\nu}{2}+1} \cos\left(\frac{\pi \nu}{2}\right) G_{1,2}^{1,1}\left(\frac{z}{\sqrt{2}}, \frac{1}{2} \middle| \begin{array}{c} \frac{1-\nu}{2} \\ 0, \frac{1}{2} \end{array}\right)$$

07.41.26.0047.01

$$e^{-\frac{z^2}{4}} (D_\nu(z) - D_\nu(-z)) = 2^{\frac{\nu}{2}+1} \sin\left(\frac{\pi \nu}{2}\right) G_{1,2}^{1,1}\left(\frac{z}{\sqrt{2}}, \frac{1}{2} \middle| \begin{array}{c} \frac{1-\nu}{2} \\ \frac{1}{2}, 0 \end{array}\right)$$

Generalized cases involving cosh

07.41.26.0048.01

$$\cosh\left(\frac{z^2}{4}\right) D_\nu(z) = 2^{\frac{\nu}{2}-1} G_{1,2}^{2,0}\left(\frac{z}{\sqrt{2}}, \frac{1}{2} \middle| \begin{array}{c} \frac{1-\nu}{2} \\ 0, \frac{1}{2} \end{array}\right) + \frac{2^{-\frac{\nu}{2}-2}}{\sqrt{\pi}\, \Gamma(-\nu)} G_{1,2}^{2,1}\left(\frac{z}{\sqrt{2}}, \frac{1}{2} \middle| \begin{array}{c} \frac{\nu}{2}+1 \\ 0, \frac{1}{2} \end{array}\right)$$

07.41.26.0049.01

$$\cosh\left(\frac{z^2}{4}\right) D_\nu(-z) = 2^{\frac{\nu}{2}-1} G_{2,3}^{2,1}\left(\frac{z}{\sqrt{2}}, \frac{1}{2} \middle| \begin{array}{c} \frac{1-\nu}{2}, \frac{\nu+1}{2} \\ 0, \frac{1}{2}, \frac{\nu+1}{2} \end{array}\right) + \frac{2^{-\frac{1+\nu}{2}} \pi^{3/2}}{\Gamma(-\nu)} G_{3,4}^{2,1}\left(\frac{z}{\sqrt{2}}, \frac{1}{2} \middle| \begin{array}{c} \frac{\nu}{2}+1, \frac{1}{8}, \frac{5}{8} \\ 0, \frac{1}{2}, \frac{1}{8}, \frac{5}{8} \end{array}\right)$$

07.41.26.0050.01

$$\cosh\left(\frac{z^2}{4}\right) (D_\nu(-z) + D_\nu(z)) = 2^{\nu/2} \cos\left(\frac{\pi \nu}{2}\right) G_{1,2}^{1,1}\left(\frac{z}{\sqrt{2}}, \frac{1}{2} \middle| \begin{array}{c} \frac{1-\nu}{2} \\ 0, \frac{1}{2} \end{array}\right) + \frac{2^{-\frac{1+\nu}{2}} \pi^{3/2}}{\Gamma(-\nu)} G_{2,3}^{1,1}\left(\frac{z}{\sqrt{2}}, \frac{1}{2} \middle| \begin{array}{c} \frac{\nu}{2}+1, \frac{1}{4} \\ 0, \frac{1}{2}, \frac{1}{4} \end{array}\right)$$

07.41.26.0051.01

$$\cosh\left(\frac{z^2}{4}\right) (D_\nu(z) - D_\nu(-z)) = \frac{2^{-\frac{1}{2}(\nu+1)} \pi^{3/2}}{\Gamma(-\nu)} G_{2,3}^{1,1}\left(\frac{z}{\sqrt{2}}, \frac{1}{2} \middle| \begin{array}{c} \frac{\nu}{2}+1, \frac{1}{4} \\ \frac{1}{2}, 0, \frac{1}{4} \end{array}\right) + 2^{\nu/2} \sin\left(\frac{\pi \nu}{2}\right) G_{1,2}^{1,1}\left(\frac{z}{\sqrt{2}}, \frac{1}{2} \middle| \begin{array}{c} \frac{1-\nu}{2} \\ \frac{1}{2}, 0 \end{array}\right)$$

Generalized cases involving sinh

07.41.26.0052.01

$$\sinh\left(\frac{z^2}{4}\right) D_\nu(z) = \frac{2^{-\frac{\nu}{2}-2}}{\sqrt{\pi}\ \Gamma(-\nu)} G_{1,2}^{2,1}\left(\frac{z}{\sqrt{2}}, \frac{1}{2} \middle| \begin{array}{c} \frac{\nu}{2}+1 \\ 0, \frac{1}{2} \end{array}\right) - 2^{\frac{\nu}{2}-1} G_{1,2}^{2,0}\left(\frac{z}{\sqrt{2}}, \frac{1}{2} \middle| \begin{array}{c} \frac{1-\nu}{2} \\ 0, \frac{1}{2} \end{array}\right)$$

07.41.26.0053.01

$$\sinh\left(\frac{z^2}{4}\right) D_\nu(-z) = \frac{2^{-\frac{1}{2}(\nu+1)} \pi^{3/2}}{\Gamma(-\nu)} G_{3,4}^{2,1}\left(\frac{z}{\sqrt{2}}, \frac{1}{2} \middle| \begin{array}{c} \frac{\nu}{2}+1, \frac{1}{8}, \frac{5}{8} \\ 0, \frac{1}{2}, \frac{1}{8}, \frac{5}{8} \end{array}\right) - 2^{\frac{\nu}{2}-1} G_{2,3}^{2,1}\left(\frac{z}{\sqrt{2}}, \frac{1}{2} \middle| \begin{array}{c} \frac{1-\nu}{2}, \frac{\nu+1}{2} \\ 0, \frac{1}{2}, \frac{\nu+1}{2} \end{array}\right)$$

07.41.26.0054.01

$$\sinh\left(\frac{z^2}{4}\right) (D_\nu(-z) + D_\nu(z)) = \frac{2^{-\frac{1}{2}(\nu+1)} \pi^{3/2}}{\Gamma(-\nu)} G_{2,3}^{1,1}\left(\frac{z}{\sqrt{2}}, \frac{1}{2} \middle| \begin{array}{c} \frac{\nu}{2}+1, \frac{1}{4} \\ 0, \frac{1}{2}, \frac{1}{4} \end{array}\right) - 2^{\nu/2} \cos\left(\frac{\pi \nu}{2}\right) G_{1,2}^{1,1}\left(\frac{z}{\sqrt{2}}, \frac{1}{2} \middle| \begin{array}{c} \frac{1-\nu}{2} \\ 0, \frac{1}{2} \end{array}\right)$$

07.41.26.0055.01

$$\sinh\left(\frac{z^2}{4}\right) (D_\nu(z) - D_\nu(-z)) = \frac{2^{-\frac{1}{2}(\nu+1)} \pi^{3/2}}{\Gamma(-\nu)} G_{2,3}^{1,1}\left(\frac{z}{\sqrt{2}}, \frac{1}{2} \middle| \begin{array}{c} \frac{\nu}{2}+1, \frac{1}{4} \\ \frac{1}{2}, 0, \frac{1}{4} \end{array}\right) - 2^{\nu/2} \sin\left(\frac{\pi \nu}{2}\right) G_{1,2}^{1,1}\left(\frac{z}{\sqrt{2}}, \frac{1}{2} \middle| \begin{array}{c} \frac{1-\nu}{2} \\ \frac{1}{2}, 0 \end{array}\right)$$

Generalized cases for products of *D*

07.41.26.0056.01

$$D_\nu\left(2 e^{\frac{\pi i}{4}} z\right) D_\nu\left(2 e^{-\frac{\pi i}{4}} z\right) = \frac{1}{2^{3/2} \pi \Gamma(-\nu)} G_{2,4}^{4,1}\left(\frac{z}{2}, \frac{1}{4} \middle| \begin{array}{c} \frac{\nu}{2}+1, \frac{1-\nu}{2} \\ 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4} \end{array}\right)$$

07.41.26.0057.01

$$D_{-\nu-1}(z) D_\nu(z) = \frac{1}{2\sqrt{\pi}} G_{2,4}^{4,0}\left(\frac{z}{2}, \frac{1}{4} \middle| \begin{array}{c} \frac{1-\nu}{2}, \frac{\nu}{2}+1 \\ 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4} \end{array}\right)$$

Generalized cases involving Exp and Hermite *H*

07.41.26.0058.01

$$e^{\frac{z^2}{4}} H_\nu\left(-\frac{i z}{\sqrt{2}}\right) D_\nu(z) = \frac{2^{\frac{\nu-3}{2}}}{\pi \Gamma(-\nu)} G_{2,4}^{4,1}\left(\frac{1}{2} e^{-\frac{1}{4}(i\pi)} z, \frac{1}{4} \middle| \begin{array}{c} \frac{\nu}{2}+1, \frac{1-\nu}{2} \\ 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4} \end{array}\right)$$

07.41.26.0059.01

$$e^{-\frac{z^2}{4}} H_{-\nu-1}\left(\frac{z}{\sqrt{2}}\right) D_\nu(z) = \frac{2^{-\frac{1}{2}(\nu+3)}}{\sqrt{\pi}} G_{2,4}^{4,0}\left(\frac{z}{2}, \frac{1}{4} \middle| \begin{array}{c} \frac{\nu}{2}+1, \frac{1-\nu}{2} \\ 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4} \end{array}\right)$$

Through other functions

Involving some hypergeometric-type functions

07.41.26.0060.01

$$D_\nu(z) = 2^{\frac{\nu}{2}} e^{-\frac{z^2}{4}} \left(\cos\left(\frac{\nu \pi}{2}\right) \Gamma\left(\frac{\nu}{2}+1\right) L_{\frac{\nu}{2}}^{-\frac{1}{2}}\left(\frac{z^2}{2}\right) + \frac{z \Gamma\left(\frac{\nu+1}{2}\right) \sin\left(\frac{\nu\pi}{2}\right)}{\sqrt{2}} L_{\frac{\nu-1}{2}}^{\frac{1}{2}}\left(\frac{z^2}{2}\right)\right)$$

07.41.26.0061.01

$$D_\nu(z) = \lim_{\lambda\to\infty} e^{-\frac{z^2}{4}}\,\Gamma(\nu+1)\,\lambda^{-\frac{\nu}{2}}\,L_\nu^\lambda\left(\lambda - \sqrt{\lambda}\,z\right)$$

07.41.26.0062.01

$$D_\nu(z) = \Gamma(\nu+1)\,2^{-\frac{\nu}{2}}\,e^{-\frac{z^2}{4}}\left(\lim_{\lambda\to\infty}\lambda^{-\frac{\nu}{2}}\,C_\nu^\lambda\left(\frac{z}{\sqrt{2\,\lambda}}\right)\right)/;\ |z| < \sqrt{2}$$

07.41.26.0063.01

$$D_\nu(z) = 2^{\nu/2}\,e^{-\frac{z^2}{4}}\,\Gamma(\nu+1)\left(\lim_{a\to\infty} a^{-\frac{\nu}{2}}\,P_\nu^{(a,a)}\left(\frac{z}{\sqrt{2\,a}}\right)\right)$$

Representations through equivalent functions

With related functions

07.41.27.0001.01

$$D_{2n}(z) = (-1)^n\,2^n\,n!\,e^{-\frac{z^2}{4}}\,L_n^{-\frac{1}{2}}\left(\frac{z^2}{2}\right)/;\ n\in\mathbb{N}$$

07.41.27.0002.01

$$D_{2n+1}(z) = (-1)^n\,2^n\,n!\,z\,e^{-\frac{z^2}{4}}\,L_n^{\frac{1}{2}}\left(\frac{z^2}{2}\right)/;\ n\in\mathbb{N}$$

07.41.27.0003.01

$$D_\nu(z) = 2^{-\frac{\nu}{2}}\,e^{-\frac{z^2}{4}}\,H_\nu\left(\frac{z}{\sqrt{2}}\right)$$

Theorems

History

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