PartitionsQ

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Notations

Traditional name

Number of partitions of an integer into distinct parts

Traditional notation

$q(n)$

Mathematica StandardForm notation

`PartitionsQ[n]`

Primary definition

04.17.02.0001.01

$$q(n) == \left([t^n] \prod_{k=1}^{\infty} (t^k + 1)\right) /; n \in \mathbb{N}$$

$q(n)$ is the number of restricted partitions of the positive integer n into a sum of distinct positive numbers which add up to n when order does not matter and repetitions are not allowed.

For example, $q(5) == 3$. There are 3 possible ways to express 5 as a sum of positive integers without repetitions: $5 == 1 + 4 == 2 + 3$.

04.17.02.0002.01

$$q(n) == 0 /; n \in \mathbb{Z} \land n < 0$$

Specific values

Values at fixed points

04.17.03.0001.01

$$q(0) == 1$$

04.17.03.0002.01

$$q(1) == 1$$

04.17.03.0003.01

$$q(2) == 1$$

04.17.03.0004.01

$$q(3) == 2$$

04.17.03.0005.01
$q(4) = 2$

04.17.03.0006.01
$q(5) = 3$

04.17.03.0007.01
$q(6) = 4$

04.17.03.0008.01
$q(7) = 5$

04.17.03.0009.01
$q(8) = 6$

04.17.03.0010.01
$q(9) = 8$

04.17.03.0011.01
$q(10) = 10$

04.17.03.0013.01
$q(11) = 12$

04.17.03.0014.01
$q(12) = 15$

04.17.03.0015.01
$q(13) = 18$

04.17.03.0016.01
$q(14) = 22$

04.17.03.0017.01
$q(15) = 27$

04.17.03.0018.01
$q(16) = 32$

04.17.03.0019.01
$q(17) = 38$

04.17.03.0020.01
$q(18) = 46$

04.17.03.0021.01
$q(19) = 54$

04.17.03.0022.01
$q(20) = 64$

04.17.03.0023.01
$q(21) = 76$

04.17.03.0024.01
$q(22) = 89$

04.17.03.0025.01
$q(23) = 104$

04.17.03.0026.01
$q(24) == 122$

04.17.03.0027.01
$q(25) == 142$

04.17.03.0028.01
$q(26) == 165$

04.17.03.0029.01
$q(27) == 192$

04.17.03.0030.01
$q(28) == 222$

04.17.03.0031.01
$q(29) == 256$

04.17.03.0032.01
$q(30) == 296$

04.17.03.0033.01
$q(31) == 340$

04.17.03.0034.01
$q(32) == 390$

04.17.03.0035.01
$q(33) == 448$

04.17.03.0036.01
$q(34) == 512$

04.17.03.0037.01
$q(35) == 585$

04.17.03.0038.01
$q(36) == 668$

04.17.03.0039.01
$q(37) == 760$

04.17.03.0040.01
$q(38) == 864$

04.17.03.0041.01
$q(39) == 982$

04.17.03.0042.01
$q(40) == 1113$

04.17.03.0043.01
$q(41) == 1260$

04.17.03.0044.01
$q(42) == 1426$

04.17.03.0045.01
$q(43) == 1610$

04.17.03.0046.01

$q(44) = 1816$

04.17.03.0047.01

$q(45) = 2048$

04.17.03.0048.01

$q(46) = 2304$

04.17.03.0049.01

$q(47) = 2590$

04.17.03.0050.01

$q(48) = 2910$

04.17.03.0051.01

$q(49) = 3264$

04.17.03.0052.01

$q(50) = 3658$

Values at infinities

04.17.03.0012.01

$q(\infty) = \infty$

General characteristics

Domain and analyticity

The partitions $q(n)$ is a nonanalytical function which is defined only for integers.

04.17.04.0001.01

$n \longrightarrow q(n) :: \mathbb{N} \longrightarrow \mathbb{N}^+$

Symmetries and periodicities

Symmetry

No symmetry

Periodicity

No periodicity

Series representations

Generalized power series

04.17.06.0001.01

$$q(n) = \frac{1}{\sqrt{2}} \sum_{k=1}^{\infty} A(2k-1, n) \frac{\partial J_0\left(\frac{\pi i}{2k-1} \sqrt{\frac{1}{3}} \sqrt{n+\frac{1}{24}}\right)}{\partial n} /; A(k, n) = \sum_{h=1}^{k} \delta_{\gcd(h,k),1} \exp\left(\pi i \sum_{j=1}^{k-1} \frac{1}{k} j \left(\frac{h j}{k} - \left\lfloor \frac{h j}{k} \right\rfloor - \frac{1}{2}\right) - \frac{2 \pi i h n}{k}\right)$$

04.17.06.0002.01

$$q(n) = \frac{\pi^2\sqrt{2}}{24}\sum_{k=1}^{\infty}\frac{A(2k-1,n)}{(1-2k)^2}\,{}_0F_1\!\left(;2;\frac{\pi^2\left(n+\frac{1}{24}\right)}{12(1-2k)^2}\right)/;\ A(k,n) = \sum_{h=1}^{k}\delta_{\gcd(h,k),1}\,\exp\!\left(\pi i\sum_{j=1}^{k-1}\frac{1}{k}\,j\left(\frac{hj}{k}-\left\lfloor\frac{hj}{k}\right\rfloor-\frac{1}{2}\right)-\frac{2\pi i h n}{k}\right)$$

Asymptotic series expansions

04.17.06.0003.01

$$q(n) \propto \frac{1}{4\sqrt[4]{3}\,n^{3/4}}\exp\!\left(\pi\sqrt{\frac{n}{3}}\right)\left(1+O\!\left(\frac{1}{n}\right)\right)/;\ (n\to\infty)$$

Generating functions

04.17.11.0001.01

$$q(n) = \left([t^n]\prod_{k=1}^{\infty}\left(t^k+1\right)\right)/;\ n\in\mathbb{N}$$

04.17.11.0002.01

$$q(n) = \left([t^n]\prod_{k=1}^{\infty}\frac{1}{1-t^{2k-1}}\right)/;\ n\in\mathbb{N}$$

Identities

Functional identities

04.17.17.0001.01

$$q(n) = \frac{1}{n}\sum_{k=1}^{n}\sigma_1(k)\,q(n-k)-\frac{2}{n}\sum_{k=1}^{\left\lfloor\frac{n}{2}\right\rfloor}\sigma_1(k)\,q(n-2k)/;\ n\in\mathbb{N}^+$$

Complex characteristics

Real part

04.17.19.0001.01

$$\mathrm{Re}(q(n)) = q(n)$$

Imaginary part

04.17.19.0002.01

$$\mathrm{Im}(q(n)) = 0$$

Absolute value

04.17.19.0003.01

$$|q(n)| = q(n)$$

Argument

04.17.19.0004.01

$$\arg(q(n)) == 0$$

Conjugate value

04.17.19.0005.01

$$\overline{q(n)} == q(n)$$

Representations through equivalent functions

With related functions

04.17.27.0001.01

$$p(n) == \sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} q(n-2k)\, p(k)$$

Inequalities

04.17.29.0001.01

$$q(n) \leq \frac{1}{2}\,(q(n-1)+q(n+1)) \;/;\; n-3 \in \mathbb{N}^+$$

History

– G.W. Leibniz (1669) investigated the number of ways a given positive integer can be decomposed into smaller ones
– L. Euler (1740)
– S. Ramanujan (1917)
– G. H. Hardy (1920) introduced the notation $q(n)$

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