RamanujanTauZ

Notations

Traditional name

Ramanujan tau Z

Traditional notation

$\tau Z(z)$

Mathematica StandardForm notation

`RamanujanTauZ[z]`

Primary definition

10.11.02.0001.01

$$\tau Z(z) = 2^{-i z}\, \pi^{-i z - \frac{1}{2}}\, \tau L(6 + i\, z)\, \Gamma(6 + i\, z)\, \sqrt{\dfrac{\sinh(\pi\, z)}{z\,(z^2 + 1)\,(z^2 + 4)\,(z^2 + 9)\,(z^2 + 16)\,(z^2 + 25)}}$$

Specific values

Specialized values

10.11.03.0001.01

$$\tau Z(n) = 0 \ /; \ n \in \mathbb{Z} \bigwedge |n| \geq 5$$

Values at fixed points

10.11.03.0002.01

$$\tau Z(6) = \tau L(6)$$

General characteristics

Domain and analyticity

$\tau Z(z)$ is an analytical function of z which is defined over the complex z-plane.

10.11.04.0001.01

$$z \longrightarrow \tau Z(z) :: \mathbb{C} \longrightarrow \mathbb{C}$$

Symmetries and periodicities

Parity

$\tau Z(z)$ is an even function.

10.11.04.0002.01

$$\tau Z(-z) = \tau Z(z)$$

Mirror symmetry

10.11.04.0003.01

$$\tau Z(\bar{z}) == \overline{\tau Z(z)}$$

Periodicity

No periodicity

Poles and essential singularities

The function $\tau Z(z)$ does not have poles and essential singularities.

10.11.04.0004.01

$$Sing_z(\tau Z(z)) == \{\}$$

Branch points

The function RamanujanTauZ(z) does not have branch points.

10.11.04.0005.01

$$\mathcal{BP}_z(\tau Z(z)) == \{\}$$

Branch cuts

The function RamanujanTauZ(z) does not have branch cuts.

10.11.04.0006.01

$$\mathcal{BC}_z(\tau Z(z)) == \{\}$$

Differentiation

Low-order differentiation

10.11.20.0001.01

$$\frac{\partial \tau Z(z)}{\partial z} =$$

$$\left(\left(-11\left(z^8 + 45 z^6 + 651 z^4 + 3475 z^2 + 5748\right) z^2 + \left(z^2+1\right)\left(z^2+4\right)\left(z^2+9\right)\left(z^2+16\right)\left(z^2+25\right)\left(\pi \coth(\pi z) - i \log\left(4\pi^2\right)\right) z + \right.$$
$$\left. 2 i \left(z^2+1\right)\left(z^2+4\right)\left(z^2+9\right)\left(z^2+16\right)\left(z^2+25\right)\psi(i z + 6) z - 14\,400\right) \text{RamanujanTauZ}(z)\right) \Big/$$

$$\left(2 z \left(z^2+1\right)\left(z^2+4\right)\left(z^2+9\right)\left(z^2+16\right)\left(z^2+25\right)\right) + 2^{-i z} i \pi^{-i z - \frac{1}{2}} \sqrt{\frac{\sinh(\pi z)}{z^{11} + 55 z^9 + 1023 z^7 + 7645 z^5 + 21\,076 z^3 + 14\,400 z}}$$

$$\Gamma(i z + 6)\,\tau L'(i z + 6)$$

10.11.20.0002.01

$$\frac{\partial^2 \tau Z(z)}{\partial z^2} =$$

$$\frac{1}{4}\Bigg(\Big(-11\left(z^8+45z^6+651z^4+3475z^2+5748\right)z^2+\left(z^2+1\right)\left(z^2+4\right)\left(z^2+9\right)\left(z^2+16\right)\left(z^2+25\right)\left(\pi\coth(\pi z)-i\log\!\left(4\pi^2\right)\right)z+$$

$$2i\left(z^2+1\right)\left(z^2+4\right)\left(z^2+9\right)\left(z^2+16\right)\left(z^2+25\right)\psi(iz+6)z-14400\Big)^2\Big/$$

$$\left(z^2\left(z^2+1\right)^2\left(z^2+4\right)^2\left(z^2+9\right)^2\left(z^2+16\right)^2\left(z^2+25\right)^2\right)-\Big(2\left(\pi^2z\left(z^2+1\right)\left(z^2+4\right)\left(z^2+9\right)\left(z^2+16\right)\left(z^2+25\right)\operatorname{csch}^2(\pi z)+$$

$$22z\left(5z^8+180z^6+1953z^4+6950z^2+5748\right)-\pi\left(11\left(z^8+45z^6+651z^4+3475z^2+5748\right)z^2+14400\right)$$

$$\coth(\pi z)+i\left(11\left(z^8+45z^6+651z^4+3475z^2+5748\right)z^2+14400\right)\log\!\left(4\pi^2\right)-$$

$$2i\left(11z^{10}+495z^8+7161z^6+38225z^4+63228z^2+14400\right)\psi(iz+6)+$$

$$2z\left(z^2+1\right)\left(z^2+4\right)\left(z^2+9\right)\left(z^2+16\right)\left(z^2+25\right)\psi^{(1)}(iz+6)\big)\Big)\Big/\left(z\left(z^2+1\right)\left(z^2+4\right)\left(z^2+9\right)\left(z^2+16\right)\left(z^2+25\right)\right)-$$

$$\Big(2\left(-11\left(z^8+45z^6+651z^4+3475z^2+5748\right)z^2+\left(z^2+1\right)\left(z^2+4\right)\left(z^2+9\right)\left(z^2+16\right)\left(z^2+25\right)\right.$$

$$\left(\pi\coth(\pi z)-i\log\!\left(4\pi^2\right)\right)z+2i\left(z^2+1\right)\left(z^2+4\right)\left(z^2+9\right)\left(z^2+16\right)\left(z^2+25\right)\psi(iz+6)z-14400\big)\Big)\Big/$$

$$\left(z^2\left(z^2+1\right)\left(z^2+4\right)\left(z^2+9\right)\left(z^2+16\right)\left(z^2+25\right)\right)-\Big(4\big(-11\left(z^8+45z^6+651z^4+3475z^2+5748\right)z^2+$$

$$\left(z^2+1\right)\left(z^2+4\right)\left(z^2+9\right)\left(z^2+16\right)\left(z^2+25\right)\left(\pi\coth(\pi z)-i\log\!\left(4\pi^2\right)\right)z+$$

$$2i\left(z^2+1\right)\left(z^2+4\right)\left(z^2+9\right)\left(z^2+16\right)\left(z^2+25\right)\psi^{(0)}(iz+6)z-14400\big)\Big)\Big/$$

$$\left(\left(z^2+1\right)^2\left(z^2+4\right)\left(z^2+9\right)\left(z^2+16\right)\left(z^2+25\right)\right)-\Big(4\big(-11\left(z^8+45z^6+651z^4+3475z^2+5748\right)z^2+$$

$$\left(z^2+1\right)\left(z^2+4\right)\left(z^2+9\right)\left(z^2+16\right)\left(z^2+25\right)\left(\pi\coth(\pi z)-i\log\!\left(4\pi^2\right)\right)z+2i\left(z^2+1\right)\left(z^2+4\right)\left(z^2+9\right)$$

$$\left(z^2+16\right)\left(z^2+25\right)\psi^{(0)}(iz+6)z-14400\big)\Big)\Big/\left(\left(z^2+1\right)\left(z^2+4\right)^2\left(z^2+9\right)\left(z^2+16\right)\left(z^2+25\right)\right)-$$

$$\Big(4\big(-11\left(z^8+45z^6+651z^4+3475z^2+5748\right)z^2+\left(z^2+1\right)\left(z^2+4\right)\left(z^2+9\right)\left(z^2+16\right)\left(z^2+25\right)$$

$$\left(\pi\coth(\pi z)-i\log\!\left(4\pi^2\right)\right)z+2i\left(z^2+1\right)\left(z^2+4\right)\left(z^2+9\right)\left(z^2+16\right)\left(z^2+25\right)\psi^{(0)}(iz+6)z-14400\big)\Big)\Big/$$

$$\left(\left(z^2+1\right)\left(z^2+4\right)\left(z^2+9\right)^2\left(z^2+16\right)\left(z^2+25\right)\right)-\Big(4\big(-11\left(z^8+45z^6+651z^4+3475z^2+5748\right)z^2+$$

$$\left(z^2+1\right)\left(z^2+4\right)\left(z^2+9\right)\left(z^2+16\right)\left(z^2+25\right)\left(\pi\coth(\pi z)-i\log\!\left(4\pi^2\right)\right)z+2i\left(z^2+1\right)\left(z^2+4\right)\left(z^2+9\right)$$

$$\left(z^2+16\right)\left(z^2+25\right)\psi^{(0)}(iz+6)z-14400\big)\Big)\Big/\left(\left(z^2+1\right)\left(z^2+4\right)\left(z^2+9\right)\left(z^2+16\right)^2\left(z^2+25\right)\right)-$$

$$\Big(4\big(-11\left(z^8+45z^6+651z^4+3475z^2+5748\right)z^2+\left(z^2+1\right)\left(z^2+4\right)\left(z^2+9\right)\left(z^2+16\right)\left(z^2+25\right)$$

$$\left(\pi\coth(\pi z)-i\log\!\left(4\pi^2\right)\right)z+2i\left(z^2+1\right)\left(z^2+4\right)\left(z^2+9\right)\left(z^2+16\right)\left(z^2+25\right)\psi^{(0)}(iz+6)z-14400\big)\Big)\Big/$$

$$\left(\left(z^2+1\right)\left(z^2+4\right)\left(z^2+9\right)\left(z^2+16\right)\left(z^2+25\right)^2\right)\Bigg)\,\mathrm{RamanujanTauZ}(z)-$$

$$2^{-iz}\,\pi^{-iz-\frac{1}{2}}\,\Gamma(iz+6)\sqrt{\frac{\sinh(\pi z)}{z\left(z^2+1\right)\left(z^2+4\right)\left(z^2+9\right)\left(z^2+16\right)\left(z^2+25\right)}}$$

$$\tau\mathrm{L}''(iz+6)+$$

$$\Bigg(\Big(2^{-iz-1}\,\pi^{-iz-\frac{1}{2}}\operatorname{csch}(\pi z)\,\Gamma(iz+6)\big(2i\pi z\left(z^2+1\right)\left(z^2+4\right)\left(z^2+9\right)\left(z^2+16\right)\left(z^2+25\right)\cosh(\pi z)+\left(z\left(z^2+1\right)\left(z^2+4\right)\left(z^2+9\right)\right.$$

$$\left(z^2+16\right)\left(z^2+25\right)\log\!\left(16\pi^4\right)-2i\left(11\left(z^8+45z^6+651z^4+3475z^2+5748\right)z^2+14400\right)\big)\sinh(\pi z)-$$

$$4z\left(z^2+1\right)\left(z^2+4\right)\left(z^2+9\right)\left(z^2+16\right)\left(z^2+25\right)\psi^{(0)}(iz+6)\sinh(\pi z)\big)\tau\mathrm{L}'(iz+6)\Big)$$

$$\sqrt{\frac{\sinh(\pi z)}{z\left(z^2+1\right)\left(z^2+4\right)\left(z^2+9\right)\left(z^2+16\right)\left(z^2+25\right)}}\Bigg)\Big/\left(z\left(z^2+1\right)\left(z^2+4\right)\left(z^2+9\right)\left(z^2+16\right)\left(z^2+25\right)\right)$$

Representations through equivalent functions

With related functions

10.11.27.0001.01

$$\tau Z(z) = e^{i \, \text{RamanujanTauTheta}(z)} \, \tau L(6 + i \, z)$$

10.11.27.0002.01

$$\tau Z(z) = \frac{e^{\frac{1}{2} (\log \Gamma(i \, z + 6) - \log \Gamma(6 - i \, z))} \, (2 \, \pi)^{-i \, z} \, \sqrt{\Gamma(6 - i \, z)}}{\sqrt{i \, z \left(z^2 + 1\right) \left(z^2 + 4\right) \left(z^2 + 9\right) \left(z^2 + 16\right) \left(z^2 + 25\right) \Gamma(-i \, z - 5)}} \, \tau L(i \, z + 6)$$

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