RiemannSiegelZ

Notations

Traditional name

Riemann-Siegel function Z

Traditional notation

$Z(z)$

Mathematica StandardForm notation

`RiemannSiegelZ[z]`

Primary definition

10.04.02.0001.01

$$Z(z) == e^{i\,\vartheta(z)}\,\zeta\left(i\,z+\frac{1}{2}\right)$$

Specific values

Specialized values

10.04.03.0001.01

$$Z(x) == \pi^{-\frac{i\,x}{2}}\exp\left(i\,\mathrm{Im}\left(\log\Gamma\left(\frac{i\,x}{2}+\frac{1}{4}\right)\right)\right)\zeta\left(i\,x+\frac{1}{2}\right)\;/;\;x\in\mathbb{R}$$

10.04.03.0002.01

$$Z\left(\frac{i}{2}+2\,i\,k\right) == 0\;/;\;k\in\mathbb{N}^+$$

10.04.03.0003.01

$$Z\left(-\frac{i}{2}-2\,i\,k\right) == 0\;/;\;k\in\mathbb{N}^+$$

10.04.03.0004.01

$$Z\left(\frac{3\,i}{2}+2\,i\,k\right) == \frac{i\,(-i)^k\,2^{k-\frac{1}{2}}\,\pi^{k+1}}{(k+1)\sqrt{(2\,k+1)!}}\,B_{2\,k+2}\;/;\;k\in\mathbb{N}$$

10.04.03.0005.01

$$Z\left(-\frac{3\,i}{2}-2\,i\,k\right) == -\frac{i^{k+1}\,2^{k-\frac{1}{2}}\,\pi^{k+1}}{(k+1)\sqrt{(2\,k+1)!}}\,B_{2\,k+2}\;/;\;k\in\mathbb{N}$$

Values at fixed points

10.04.03.0006.01

$$Z(0) == \zeta\left(\frac{1}{2}\right)$$

10.04.03.0007.01

$$Z\left(\frac{i}{2}\right) == \tilde{\infty}$$

10.04.03.0008.01

$$Z\left(\frac{3\,i}{2}\right) == \frac{i\,\pi}{6\sqrt{2}}$$

10.04.03.0009.01

$$Z\left(-\frac{i}{2}\right) == \tilde{\infty}$$

10.04.03.0010.01

$$Z\left(-\frac{3\,i}{2}\right) == -\frac{i\,\pi}{6\sqrt{2}}$$

General characteristics

Domain and analyticity

$Z(z)$ is an analytical function of z which is defined over the whole complex z-plane.

10.04.04.0001.01

$$z \longrightarrow Z(z) :: \mathbb{C} \longrightarrow \mathbb{C}$$

Symmetries and periodicities

Parity

$Z(z)$ is an even function.

10.04.04.0002.01

$$Z(-z) == Z(z)$$

Mirror symmetry

10.04.04.0003.01

$$Z(\bar{z}) == \overline{Z(z)} \;/;\; i\,z \notin \left(-\infty, -\frac{1}{2}\right) \bigwedge i\,z \notin \left(\frac{1}{2}, \infty\right)$$

Periodicity

No periodicity

Poles and essential singularities

The function $Z(z)$ does not have poles and essential singularities.

10.04.04.0004.01

$$\mathcal{S}ing_z(Z(z)) = \{\}$$

Branch points

The function $Z(z)$ has infinitely many branch points: $z = \pm i\left(\frac{1}{2} + 2k\right) /; k \in \mathbb{N}$ and $z = \tilde{\infty}$. All these are square root-type branch points.

10.04.04.0005.01

$$\mathcal{BP}_z(Z(z)) = \left\{\left\{\frac{i}{2} + 2\,k\,i /; k \in \mathbb{N}\right\}, \left\{-\frac{i}{2} - 2\,k\,i /; k \in \mathbb{N}\right\}, \tilde{\infty}\right\}$$

10.04.04.0006.01

$$\mathcal{R}_z\left(Z(z), \frac{i}{2} + 2\,k\,i\right) = 2 /; k \in \mathbb{N}$$

10.04.04.0007.01

$$\mathcal{R}_z\left(Z(z), -\frac{i}{2} - 2\,k\,i\right) = 2 /; k \in \mathbb{N}$$

10.04.04.0008.01

$$\mathcal{R}_z(Z(z), \tilde{\infty}) = 2$$

Branch cuts

The function $Z(z)$ is a single-valued function on the z-plane cut along the intervals $\left\{-i\,\infty, -\frac{i}{2}\right\}$ and $\left\{\frac{i}{2}, i\,\infty\right\}$. At $i\,z \in \left\{-i\,\infty, -\frac{i}{2}\right\} \bigvee i\,z \in \left\{\frac{i}{2}, i\,\infty\right\}$ potentially multiple branch cuts are situated over each other (at $i\,z$ there are $\left\lfloor \frac{i\,z}{2} + \frac{1}{4} \right\rfloor$, respectively $\left\lfloor \frac{1}{4} - \frac{i\,z}{2} \right\rfloor$ branch cuts overlapping).

The function $Z(z)$ is continuous from the left on the interval $\left\{-i\,\infty, -\frac{i}{2}\right\}$ and from the right on the interval $\left\{\frac{i}{2}, i\,\infty\right\}$.

10.04.04.0009.01

$$\mathcal{BC}_z(Z(z)) = \left\{\left\{\left\{-i\,\infty, -\frac{i}{2}\right\}, 1\right\}, \left\{\left\{\frac{i}{2}, i\,\infty\right\}, -1\right\}\right\}$$

10.04.04.0010.01

$$\lim_{\epsilon \to +0} Z(x - \epsilon) = Z(x) /; i\,x > \frac{1}{2}$$

10.04.04.0011.01

$$\lim_{\epsilon \to +0} Z(x + \epsilon) = Z(x) \exp\left(i\,\pi \left\lfloor \frac{1}{4} - \frac{i\,x}{2} \right\rfloor\right) /; i\,x > \frac{1}{2}$$

10.04.04.0012.01

$$\lim_{\epsilon \to +0} Z(x + \epsilon) = Z(x) /; i\,x < -\frac{1}{2}$$

10.04.04.0013.01

$$\lim_{\epsilon \to +0} Z(x - \epsilon) = Z(x) \exp\left(-i\,\pi \left\lfloor \frac{i\,x}{2} + \frac{1}{4} \right\rfloor\right) /; i\,x < -\frac{1}{2}$$

Series representations

Generalized power series

Expansions at $z == \frac{i}{2}$

10.04.06.0013.01

$$Z(z) \propto -\frac{1}{\sqrt{2\,i\left(z-\frac{i}{2}\right)}}\left(1-\frac{i}{2}(\gamma-\log(2\pi))\left(z-\frac{i}{2}\right)+\frac{1}{48}\left(-6\,(\log(2\pi)+\gamma)\,(\gamma-3\log(2\pi))+\pi^2+48\,\zeta''(0)\right)\left(z-\frac{i}{2}\right)^2-\frac{i}{96}\left(-18\,\gamma^2\log(2\pi)+\left(2\log^2(2\pi)+\pi^2+48\,\gamma_1\right)\log(2\pi)-10\,\gamma^3-\gamma\left(6\left(\log^2(2\pi)-8\,\gamma_1\right)+\pi^2\right)-96\,\gamma_2+16\,\zeta(3)\right)\left(z-\frac{i}{2}\right)^3+\ldots\right)/;\left(z\to\frac{i}{2}\right)$$

10.04.06.0001.02

$$Z(z) \propto -\frac{1}{\sqrt{2\,i\left(z-\frac{i}{2}\right)}}\left(1-\frac{i}{2}(\gamma-\log(2\pi))\left(z-\frac{i}{2}\right)+\frac{1}{48}\left(-6\,(\log(2\pi)+\gamma)\,(\gamma-3\log(2\pi))+\pi^2+48\,\zeta''(0)\right)\left(z-\frac{i}{2}\right)^2-\frac{i}{96}\left(-18\,\gamma^2\log(2\pi)+\left(2\log^2(2\pi)+\pi^2+48\,\gamma_1\right)\log(2\pi)-10\,\gamma^3-\gamma\left(6\left(\log^2(2\pi)-8\,\gamma_1\right)+\pi^2\right)-96\,\gamma_2+16\,\zeta(3)\right)\left(z-\frac{i}{2}\right)^3+O\left(\left(z-\frac{i}{2}\right)^4\right)\right)$$

10.04.06.0002.02

$$Z(z) \propto -\frac{1}{\sqrt{2\,i\left(z-\frac{i}{2}\right)}}\left(1+O\left(z-\frac{i}{2}\right)\right)$$

Expansions at $z == -\frac{i}{2}$

10.04.06.0014.01

$$Z(z) \propto -\frac{1}{\sqrt{-2\,i\left(z+\frac{i}{2}\right)}}\left(1+\frac{i}{2}(\gamma-\log(2\pi))\left(z+\frac{i}{2}\right)+\frac{1}{48}\left(-6\,(\log(2\pi)+\gamma)\,(\gamma-3\log(2\pi))+\pi^2+48\,\zeta''(0)\right)\left(z+\frac{i}{2}\right)^2+\frac{i}{96}\left(-18\,\gamma^2\log(2\pi)+\left(2\log^2(2\pi)+\pi^2+48\,\gamma_1\right)\log(2\pi)-10\,\gamma^3-\gamma\left(6\left(\log^2(2\pi)-8\,\gamma_1\right)+\pi^2\right)-96\,\gamma_2+16\,\zeta(3)\right)\left(z+\frac{i}{2}\right)^3+\ldots\right)/;\left(z\to-\frac{i}{2}\right)$$

10.04.06.0003.02

$$Z(z) \propto -\frac{1}{\sqrt{-2\,i\left(z+\frac{i}{2}\right)}}\left(1+\frac{i}{2}(\gamma-\log(2\pi))\left(z+\frac{i}{2}\right)+\frac{1}{48}\left(-6(\log(2\pi)+\gamma)(\gamma-3\log(2\pi))+\pi^2+48\,\zeta''(0)\right)\left(z+\frac{i}{2}\right)^2+\frac{i}{96}\left(-18\gamma^2\log(2\pi)+\left(2\log^2(2\pi)+\pi^2+48\gamma_1\right)\log(2\pi)-10\gamma^3-\gamma\left(6\left(\log^2(2\pi)-8\gamma_1\right)+\pi^2\right)-96\gamma_2+16\,\zeta(3)\right)\left(z+\frac{i}{2}\right)^3+O\left(\left(z+\frac{i}{2}\right)^4\right)\right)$$

10.04.06.0004.02

$$Z(z) \propto -\frac{1}{\sqrt{-2\,i\left(z+\frac{i}{2}\right)}}\left(1+O\left(z+\frac{i}{2}\right)\right)$$

Expansions at $z = z_0$ /; $z_0 \neq -n$

10.04.06.0005.02

$$Z(z) \propto Z(z_0)\left(1-\frac{i}{4}\left(2\log(\pi)-\psi\left(\frac{i z_0}{2}+\frac{1}{4}\right)-\psi\left(\frac{1}{4}-\frac{i z_0}{2}\right)-\frac{4\,\zeta'\left(i z_0+\frac{1}{2}\right)}{\zeta\left(i z_0+\frac{1}{2}\right)}\right)(z-z_0)+\frac{1}{4\,\zeta\left(i z_0+\frac{1}{2}\right)}\left(\frac{1}{8}\left(2\left(\zeta\left(2,\frac{1}{4}-\frac{i z_0}{2}\right)-\zeta\left(2,\frac{i z_0}{2}+\frac{1}{4}\right)\right)-\left(\psi\left(\frac{1}{4}+\frac{i z_0}{2}\right)+\psi\left(\frac{1}{4}-\frac{i z_0}{2}\right)-2\log(\pi)\right)^2\right)\zeta\left(i z_0+\frac{1}{2}\right)-\left(\psi\left(\frac{1}{4}+\frac{i z_0}{2}\right)+\psi\left(\frac{1}{4}-\frac{i z_0}{2}\right)-2\log(\pi)\right)\zeta'\left(i z_0+\frac{1}{2}\right)-2\,\zeta''\left(i z_0+\frac{1}{2}\right)\right)(z-z_0)^2+O\left((z-z_0)^3\right)\right) /; (z\to z_0)\land n\notin \mathbb{N}^+$$

Expansions at $z = \frac{i}{2} + 2\,i\,k$

10.04.06.0006.01

$$Z(z) \propto (-1)^k\, 2^{-2k-\frac{1}{2}}\, e^{-\frac{1}{2}\log\Gamma\left(k+\frac{1}{2}\right)}\, \pi^{\frac{1}{4}-k}\, (2k)!\, \zeta(2k+1) \left(\prod_{j=0}^{k-1} \frac{2}{\sqrt{1+4j+2iz}}\right) \sqrt{i\left(z-\frac{i}{2}-2ik\right)}$$
$$\left(1-\frac{i}{4\zeta(2k+1)}\left(\left(\left(4\psi(2k+1)+\gamma-2\log(4\pi)-\psi\left(k+\frac{1}{2}\right)\right)\zeta(2k+1)+4\zeta'(2k+1)\right)\left(z-\frac{i}{2}-2ik\right)-\right.\right.$$
$$\frac{1}{32\zeta(2k+1)}\left(\zeta(2k+1)\left(4\log^2(\pi)-16\log(2\pi)\log(\pi)+4\gamma\log(\pi)+\gamma^2-\pi^2+16\log^2(2\pi)+\right.\right.$$
$$\psi\left(k+\frac{1}{2}\right)^2+16\psi(2k+1)^2-8\gamma\log(2\pi)+8(-\log(16)-2\log(\pi)+\gamma)\psi(2k+1)+$$
$$\left.\psi\left(k+\frac{1}{2}\right)(\log(256)+4\log(\pi)-8\psi(2k+1)-2\gamma)+16\psi^{(1)}(2k+1)-2\zeta\left(2,k+\frac{1}{2}\right)\right)+$$
$$\left.8\left(-\log(16)-2\log(\pi)-\psi\left(k+\frac{1}{2}\right)+4\psi(2k+1)+\gamma\right)\zeta'(2k+1)+16\zeta''(2k+1)\right)$$
$$\left.\left(z-\frac{i}{2}-2ik\right)^2+O\left(\left(z-\frac{i}{2}-2ik\right)^3\right)\right)\ /;\ \left(z\to\frac{i}{2}+2ik\right)\bigwedge k\in\mathbb{N}^+$$

10.04.06.0007.01

$$Z(z) \propto (-1)^k\, 2^{-2k-1}\, e^{-\frac{1}{2}\log\Gamma\left(k+\frac{1}{2}\right)}\, \pi^{\frac{1}{4}-k}\, (2k)!\, \zeta(2k+1) \left(\prod_{j=0}^{k-1} \frac{2}{\sqrt{1+4j+2iz}}\right) \sqrt{2i\left(z-\frac{i}{2}-2ik\right)}\, \left(1+O\left(z-\frac{i}{2}-2ik\right)\right)/;$$
$$\left(z\to\frac{i}{2}+2ik\right)\bigwedge k\in\mathbb{N}^+$$

Expansions at $z == -\frac{i}{2} - 2ik$

10.04.06.0008.01

$$Z(z) \propto (-1)^k\, 2^{-2k-1}\, e^{-\frac{1}{2}\log\Gamma\left(k+\frac{1}{2}\right)}\, \pi^{\frac{1}{4}-k}\, (2k)!\, \zeta(2k+1) \left(\prod_{j=0}^{k-1} \frac{2}{\sqrt{1+4j-2iz}}\right) \sqrt{-2i\left(z+\frac{i}{2}+2ik\right)}$$
$$\left(1+\frac{i}{4\zeta(2k+1)}\left(\left(\left(4\psi(2k+1)+\gamma-2\log(4\pi)-\psi\left(k+\frac{1}{2}\right)\right)\zeta(2k+1)+4\zeta'(2k+1)\right)\left(z+\frac{i}{2}+2ik\right)-\right.\right.$$
$$\frac{1}{32\zeta(2k+1)}\left(\zeta(2k+1)\left(4\log^2(\pi)-16\log(2\pi)\log(\pi)+4\gamma\log(\pi)+\gamma^2-\pi^2+16\log^2(2\pi)+\right.\right.$$
$$\psi\left(k+\frac{1}{2}\right)^2+16\psi(2k+1)^2-8\gamma\log(2\pi)+8(-\log(16)-2\log(\pi)+\gamma)\psi(2k+1)+$$
$$\left.\psi\left(k+\frac{1}{2}\right)(\log(256)+4\log(\pi)-8\psi(2k+1)-2\gamma)+16\psi^{(1)}(2k+1)-2\zeta\left(2,k+\frac{1}{2}\right)\right)+$$
$$\left.8\left(-\log(16)-2\log(\pi)-\psi\left(k+\frac{1}{2}\right)+4\psi(2k+1)+\gamma\right)\zeta'(2k+1)+16\zeta''(2k+1)\right)$$
$$\left.\left(z+\frac{i}{2}+2ik\right)^2+O\left(\left(z+\frac{i}{2}+2ik\right)^3\right)\right)\ /;\ \left(z\to-\frac{i}{2}-2ik\right)\bigwedge k\in\mathbb{N}^+$$

10.04.06.0009.01

$$Z(z) \propto (-1)^k\, 2^{-2k-1}\, e^{-\frac{1}{2}\log\Gamma\left(k+\frac{1}{2}\right)}\, \pi^{\frac{1}{4}-k}\, (2k)!\, \zeta(2k+1) \left(\prod_{j=0}^{k-1} \frac{2}{\sqrt{1+4j-2iz}}\right) \sqrt{-2i\left(z+\frac{i}{2}+2ik\right)} \left(1+O\left(z+\frac{i}{2}+2ik\right)\right) /; \left(z \to -\frac{i}{2}-2ik\right) \bigwedge k \in \mathbb{N}^+$$

Asymptotic series expansions

10.04.06.0010.01

$$Z(x) \propto 2\sum_{k=1}^{\nu} \frac{\cos(\vartheta(x) - x\log(k))}{\sqrt{k}} + (-1)^{\nu-1} \sqrt[4]{2\pi} \left(\frac{\Omega(p)}{\sqrt[4]{x}} - \frac{\Omega^{(3)}(p)}{48\sqrt{2}\,\pi^{3/2}} x^{-3/4} + 2\pi\left(\frac{\Omega''(p)}{64\pi^2} + \frac{\Omega^{(6)}(p)}{18432\pi^4}\right) x^{-5/4} - (2\pi)^{3/2}\left(\frac{\Omega'(p)}{64\pi^2} + \frac{\Omega^{(5)}(p)}{3840\pi^4} + \frac{\Omega^{(9)}(p)}{5308416\pi^6}\right) x^{-7/4}\right) /; \nu = \left\lfloor \sqrt{\frac{x}{2\pi}} \right\rfloor \bigwedge p = \sqrt{\frac{x}{2\pi}} - \nu \bigwedge \Omega(p) = \frac{1}{\cos(2\pi p)} \cos\left(2\pi\left(p^2 - p - \frac{1}{16}\right)\right) \bigwedge x \in \mathbb{R} \bigwedge (x \to \infty)$$

10.04.06.0011.01

$$Z(z) \propto 4^{-\frac{iz}{4}} \exp\left(-\frac{i\left(4z^2 + \pi\sqrt{z^2}\right)}{8z}\right) \pi^{-\frac{iz}{2}} \left(z^2\right)^{\frac{iz}{4}} \left(1 + \frac{3i}{16z} - \frac{9}{512z^2} + \frac{183i}{8192z^3} - \frac{2277}{524288z^4} + \frac{212829i}{8388608z^5} - \frac{1364445}{268435456z^6} + \frac{326341455i}{4294967296z^7} - \frac{8198081325}{549755813888z^8} + \frac{3781776345585i}{8796093022208z^9} - \frac{23339010744567}{281474976710656z^{10}} + \frac{17654423117199729i}{4503599627370496z^{11}} - \frac{215619469740469809}{288230376151711744z^{12}} + \frac{241858525676475612513i}{4611686018427387904z^{13}} - \frac{1468114834103562061701}{147573952589676412928z^{14}} + \frac{2284179415871077852696767i}{2361183241434822606848z^{15}} + O\left(\frac{1}{z^{16}}\right)\right) \zeta\left(iz + \frac{1}{2}\right) /; \left|\arg\left(z^2\right)\right| < \pi \bigwedge (|z| \to \infty)$$

10.04.06.0012.01

$$Z(z) \propto 4^{-\frac{iz}{4}} \exp\left(-\frac{i\left(4z^2 + \pi\sqrt{z^2}\right)}{8z}\right) \pi^{-\frac{iz}{2}} \left(z^2\right)^{\frac{iz}{4}} \zeta\left(iz + \frac{1}{2}\right) \left(1 + O\left(\frac{1}{z}\right)\right) /; \left|\arg\left(z^2\right)\right| < \pi \bigwedge (|z| \to \infty)$$

Identities

Recurrence identities

10.04.17.0001.01

$$Z(z + 2i) = \frac{4\pi\,\zeta\left(iz - \frac{3}{2}\right)}{\sqrt{1-2iz}\,\sqrt{2iz-3}\,\zeta\left(iz + \frac{1}{2}\right)} Z(z)$$

10.04.17.0002.01

$$Z(z+2\,i\,n) = \frac{(4\,\pi)^n\,\zeta\left(i\,(2\,i\,n+z)+\frac{1}{2}\right)}{\left(\prod_{k=1}^{n}\sqrt{(4\,k-2\,i\,z-3)\,(1-4\,k+2\,i\,z)}\right)\zeta\left(i\,z+\frac{1}{2}\right)}\,Z(z)\ /;\ n\in\mathbb{N}$$

10.04.17.0003.01

$$Z(z-2\,i) = \frac{\sqrt{-2\,i\,z-3}\ \sqrt{2\,i\,z+1}\ \zeta\left(i\,z+\frac{5}{2}\right)}{4\,\pi\,\zeta\left(i\,z+\frac{1}{2}\right)}\,Z(z)$$

10.04.17.0004.01

$$Z(z-2\,i\,n) = \frac{(4\,\pi)^{-n}\,\zeta\left(2\,n+i\,z+\frac{1}{2}\right)}{\zeta\left(i\,z+\frac{1}{2}\right)}\left(\prod_{k=1}^{n}\sqrt{(-13+4\,k-2\,i\,z)\,(15-4\,k+2\,i\,z)}\right)Z\,(z)\ /;\ n\in\mathbb{N}$$

Differentiation

Low-order differentiation

10.04.20.0001.01

$$\frac{\partial Z(z)}{\partial z} = Z(z)\left(\frac{i}{4}\left(-2\log(\pi)+\psi\left(\frac{i\,z}{2}+\frac{1}{4}\right)+\psi\left(\frac{1}{4}-\frac{i\,z}{2}\right)\right)+\frac{\zeta'\left(i\,z+\frac{1}{2}\right)}{\zeta\left(i\,z+\frac{1}{2}\right)}\right)$$

10.04.20.0002.01

$$\frac{\partial^2 Z(z)}{\partial z^2} = \frac{1}{16}\,Z(z)\left(2\left(\psi^{(1)}\left(\frac{1}{4}-\frac{i\,z}{2}\right)-\psi^{(1)}\left(\frac{1}{4}+\frac{i\,z}{2}\right)\right)-\left(\psi\left(\frac{1}{4}+\frac{i\,z}{2}\right)+\psi\left(\frac{1}{4}-\frac{i\,z}{2}\right)-2\log(\pi)\right)^2-\right.$$
$$\left.\frac{8}{\zeta\left(i\,z+\frac{1}{2}\right)}\,\zeta'\left(i\,z+\frac{1}{2}\right)\left(\psi\left(\frac{1}{4}+\frac{i\,z}{2}\right)+\psi\left(\frac{1}{4}-\frac{i\,z}{2}\right)-2\log(\pi)\right)-\frac{16}{\zeta\left(i\,z+\frac{1}{2}\right)}\,\zeta''\left(i\,z+\frac{1}{2}\right)\right)$$

Symbolic differentiation

10.04.20.0003.02

$$\frac{\partial^n Z(z)}{\partial z^n} = Z(z)\sum_{k=0}^{n}\sum_{m=0}^{k}\sum_{j=0}^{m}\frac{(-1)^j\,i^{m+n-k}\,\vartheta(z)^j}{m!\,\zeta\left(i\,z+\frac{1}{2}\right)}\binom{n}{k}\binom{m}{j}\frac{\partial^k\,\vartheta(z)^{m-j}}{\partial z^k}\,\zeta^{(n-k)}\left(i\,z+\frac{1}{2}\right)\ /;\ n\in\mathbb{N}$$

Integration

Definite integration

10.04.21.0001.01

$$\int_0^\infty \frac{\left(3-\sqrt{8}\,\cos(\log(2)\,t)\right)Z(t)^2}{t^2+\frac{1}{4}}\,dt = \pi\log(2)$$

Representations through equivalent functions

With related functions

10.04.27.0001.01

$$Z(z) == e^{i \vartheta(z)} \zeta\left(i z + \frac{1}{2}\right)$$

History

– B. Riemann (1859)

– C. L. Siegel (1932)

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