SphericalBesselJ

Notations

Traditional name

Spherical Bessel function of the first kind

Traditional notation

$j_\nu(z)$

Mathematica StandardForm notation

`SphericalBesselJ[ν, z]`

Primary definition

03.21.02.0001.01

$$j_\nu(z) = \sqrt{\frac{\pi}{2}} \frac{1}{\sqrt{z}} J_{\nu+\frac{1}{2}}(z)$$

Specific values

Specialized values

For fixed ν

03.21.03.0001.01

$$j_\nu(0) = 0 \;/;\; \mathrm{Re}(\nu) > 1$$

03.21.03.0002.01

$$j_\nu(0) = \tilde{\infty} \;/;\; \mathrm{Re}(\nu) < 0$$

03.21.03.0003.01

$$j_\nu(0) = ¿ \;/;\; \mathrm{Re}(\nu) = 0$$

For fixed z

Explicit rational ν

03.21.03.0004.01

$$j_{-\frac{31}{6}}(z) = \frac{\sqrt{\pi}}{162\,\sqrt[6]{2}\;3^{5/6}\,z^{31/6}}\left(-288\sqrt{3}\,\left(9z^2-110\right)\mathrm{Ai}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)z^{4/3} + 288\left(9z^2-110\right)\mathrm{Bi}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)z^{4/3} - 3\sqrt[3]{2}\;\sqrt[6]{3}\,\left(81z^4-4320z^2+14\,080\right)\mathrm{Ai}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) + \sqrt[3]{2}\;3^{2/3}\left(81z^4-4320z^2+14\,080\right)\mathrm{Bi}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right)$$

03.21.03.0005.01

$$j_{-5}(z) = \frac{\left(z^4-45z^2+105\right)\cos(z)+5z\left(21-2z^2\right)\sin(z)}{z^5}$$

03.21.03.0006.01

$$j_{-\frac{29}{6}}(z) = \frac{1}{54\;6^{5/6}\,z^{29/6}}\sqrt{\pi}\left(-168\sqrt[6]{3}\,\left(9z^2-80\right)\mathrm{Ai}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)z^{2/3} + 56\;3^{2/3}\left(80-9z^2\right)\mathrm{Bi}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)z^{2/3} + 2^{2/3}\sqrt{3}\,\left(81z^4-3024z^2+4480\right)\mathrm{Ai}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) + 2^{2/3}\left(81z^4-3024z^2+4480\right)\mathrm{Bi}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right)$$

03.21.03.0007.01

$$j_{-\frac{25}{6}}(z) = \frac{1}{54\,\sqrt[6]{2}\;3^{5/6}\,z^{25/6}}\sqrt{\pi}\left(9\sqrt{3}\,\left(9z^2-160\right)\mathrm{Ai}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)z^{4/3} + 9\left(160-9z^2\right)\mathrm{Bi}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)z^{4/3} - 60\sqrt[3]{2}\;\sqrt[6]{3}\,\left(9z^2-32\right)\mathrm{Ai}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) + 20\sqrt[3]{2}\;3^{2/3}\left(9z^2-32\right)\mathrm{Bi}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right)$$

03.21.03.0008.01

$$j_{-4}(z) = \frac{3\left(2z^2-5\right)\cos(z)+z\left(z^2-15\right)\sin(z)}{z^4}$$

03.21.03.0009.01

$$j_{-\frac{23}{6}}(z) = \frac{1}{9\;6^{5/6}\,z^{23/6}}\sqrt{\pi}\left(3\sqrt[6]{3}\,\left(9z^2-112\right)\mathrm{Ai}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)z^{2/3} + 3^{2/3}\left(9z^2-112\right)\mathrm{Bi}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)z^{2/3} + 8\;2^{2/3}\sqrt{3}\,\left(9z^2-14\right)\mathrm{Ai}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) + 8\;2^{2/3}\left(9z^2-14\right)\mathrm{Bi}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right)$$

03.21.03.0010.01

$$j_{-\frac{19}{6}}(z) = -\frac{1}{18\,\sqrt[6]{2}\;3^{5/6}\,z^{19/6}}\sqrt{\pi}\left(-90\sqrt{3}\;\mathrm{Ai}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)z^{4/3} + 90\,\mathrm{Bi}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)z^{4/3} - 3\sqrt[3]{2}\;\sqrt[6]{3}\,\left(9z^2-40\right)\mathrm{Ai}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) + \sqrt[3]{2}\;3^{2/3}\left(9z^2-40\right)\mathrm{Bi}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right)$$

03.21.03.0011.01

$$j_{-3}(z) = \frac{3z\sin(z)-\left(z^2-3\right)\cos(z)}{z^3}$$

03.21.03.0012.01

$$j_{-\frac{17}{6}}(z) = -\frac{\sqrt{\pi}}{6\,6^{5/6}\,z^{17/6}}\left(-48\sqrt[6]{3}\,\mathrm{Ai}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) z^{2/3} - 16\,3^{2/3}\,\mathrm{Bi}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) z^{2/3} + 2^{2/3}\sqrt{3}\,(9z^2-16)\,\mathrm{Ai}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) + 2^{2/3}\,(9z^2-16)\,\mathrm{Bi}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right)$$

03.21.03.0013.01

$$j_{-\frac{13}{6}}(z) = \frac{\sqrt{\pi}}{6\sqrt[6]{2}\,3^{5/6}\,z^{13/6}}\left(-9\sqrt{3}\,\mathrm{Ai}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) z^{4/3} + 9\,\mathrm{Bi}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) z^{4/3} + 12\sqrt[3]{2}\sqrt[6]{3}\,\mathrm{Ai}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) - 4\sqrt[3]{2}\,3^{2/3}\,\mathrm{Bi}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right)$$

03.21.03.0014.01

$$j_{-2}(z) = -\frac{\cos(z) + z\sin(z)}{z^2}$$

03.21.03.0015.01

$$j_{-\frac{11}{6}}(z) = -\frac{\sqrt{\pi}}{6^{5/6}\,z^{11/6}}\left(3\sqrt[6]{3}\,\mathrm{Ai}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) z^{2/3} + 3^{2/3}\,\mathrm{Bi}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) z^{2/3} + 2^{2/3}\sqrt{3}\,\mathrm{Ai}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) + 2^{2/3}\,\mathrm{Bi}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right)$$

03.21.03.0016.01

$$j_{-\frac{7}{6}}(z) = \frac{\sqrt{\pi}}{2^{5/6}\sqrt[6]{3}\,z^{7/6}}\left(\mathrm{Bi}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) - \sqrt{3}\,\mathrm{Ai}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right)$$

03.21.03.0017.01

$$j_{-1}(z) = \frac{\cos(z)}{z}$$

03.21.03.0018.01

$$j_{-\frac{5}{6}}(z) = \frac{\sqrt{\pi}}{2\sqrt[6]{2}\sqrt[3]{3}\,z^{5/6}}\left(3\,\mathrm{Ai}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) + \sqrt{3}\,\mathrm{Bi}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right)$$

03.21.03.0019.01

$$j_{-\frac{1}{6}}(z) = -\frac{\sqrt{\pi}}{2\sqrt[6]{2}\sqrt[3]{3}\,z^{5/6}}\left(\sqrt{3}\,\mathrm{Bi}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) - 3\,\mathrm{Ai}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right)$$

03.21.03.0020.01

$$j_0(z) = \mathrm{sinc}(z)$$

03.21.03.0021.01

$$j_{\frac{1}{6}}(z) = \frac{\sqrt{\pi}}{2^{5/6}\sqrt[6]{3}\,z^{7/6}}\left(\sqrt{3}\,\mathrm{Ai}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) + \mathrm{Bi}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right)$$

03.21.03.0022.01

$$j_{\frac{5}{6}}(z) = -\frac{\sqrt{\pi}}{6^{5/6} z^{11/6}} \left(-3 \sqrt[6]{3} \operatorname{Ai}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) z^{2/3} + 3^{2/3} \operatorname{Bi}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) z^{2/3} - 2^{2/3} \sqrt{3} \operatorname{Ai}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) + 2^{2/3} \operatorname{Bi}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right)$$

03.21.03.0023.01

$$j_1(z) = \frac{\sin(z) - z\cos(z)}{z^2}$$

03.21.03.0024.01

$$j_{\frac{7}{6}}(z) = -\frac{\sqrt{\pi}}{6 \sqrt[6]{2}\, 3^{5/6} z^{13/6}} \left(9 \sqrt{3} \operatorname{Ai}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) z^{4/3} + 9 \operatorname{Bi}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) z^{4/3} - 12 \sqrt[3]{2} \sqrt[6]{3} \operatorname{Ai}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) - 4 \sqrt[3]{2}\, 3^{2/3} \operatorname{Bi}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right)$$

03.21.03.0025.01

$$j_{\frac{11}{6}}(z) = -\frac{\sqrt{\pi}}{6\, 6^{5/6} z^{17/6}} \left(-48 \sqrt[6]{3} \operatorname{Ai}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) z^{2/3} + 16\, 3^{2/3} \operatorname{Bi}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) z^{2/3} + 2^{2/3} \sqrt{3} \left(9 z^2 - 16\right) \operatorname{Ai}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) + 2^{2/3} \left(16 - 9 z^2\right) \operatorname{Bi}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right)$$

03.21.03.0026.01

$$j_2(z) = -\frac{3 z \cos(z) + \left(z^2 - 3\right) \sin(z)}{z^3}$$

03.21.03.0027.01

$$j_{\frac{13}{6}}(z) = -\frac{\sqrt{\pi}}{18 \sqrt[6]{2}\, 3^{5/6} z^{19/6}} \left(90 \sqrt{3} \operatorname{Ai}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) z^{4/3} + 90 \operatorname{Bi}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) z^{4/3} + 3 \sqrt[3]{2} \sqrt[6]{3} \left(9 z^2 - 40\right) \operatorname{Ai}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) + \sqrt[3]{2}\, 3^{2/3} \left(9 z^2 - 40\right) \operatorname{Bi}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right)$$

03.21.03.0028.01

$$j_{\frac{17}{6}}(z) = -\frac{1}{9\, 6^{5/6} z^{23/6}} \sqrt{\pi} \left(3 \sqrt[6]{3} \left(9 z^2 - 112\right) \operatorname{Ai}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) z^{2/3} + 3^{2/3} \left(112 - 9 z^2\right) \operatorname{Bi}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) z^{2/3} + 8\, 2^{2/3} \sqrt{3} \left(9 z^2 - 14\right) \operatorname{Ai}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) + 8\, 2^{2/3} \left(14 - 9 z^2\right) \operatorname{Bi}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right)$$

03.21.03.0029.01

$$j_3(z) = \frac{z \left(z^2 - 15\right) \cos(z) + 3 \left(5 - 2 z^2\right) \sin(z)}{z^4}$$

03.21.03.0030.01

$$j_{\frac{19}{6}}(z) = \frac{\sqrt{\pi}}{54\sqrt[6]{2}\,3^{5/6}\,z^{25/6}}\left(9\sqrt{3}\,(9z^2-160)\,\mathrm{Ai}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) z^{4/3} + 9\,(9z^2-160)\,\mathrm{Bi}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) z^{4/3} - 60\sqrt[3]{2}\sqrt[6]{3}\,(9z^2-32)\,\mathrm{Ai}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) + 20\sqrt[3]{2}\,3^{2/3}\,(32-9z^2)\,\mathrm{Bi}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right)$$

03.21.03.0031.01

$$j_{\frac{23}{6}}(z) = \frac{\sqrt{\pi}}{54\,6^{5/6}\,z^{29/6}}\left(-168\sqrt[6]{3}\,(9z^2-80)\,\mathrm{Ai}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) z^{2/3} - 56\,3^{2/3}\,(80-9z^2)\,\mathrm{Bi}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) z^{2/3} - 2^{2/3}\sqrt{3}\,(-81z^4+3024z^2-4480)\,\mathrm{Ai}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) - 2^{2/3}\,(81z^4-3024z^2+4480)\,\mathrm{Bi}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right)$$

03.21.03.0032.01

$$j_4(z) = \frac{5z\,(2z^2-21)\cos(z) + (z^4-45z^2+105)\sin(z)}{z^5}$$

03.21.03.0033.01

$$j_{\frac{25}{6}}(z) = \frac{1}{162\sqrt[6]{2}\,3^{5/6}\,z^{31/6}}\sqrt{\pi}\left(288\sqrt{3}\,(9z^2-110)\,\mathrm{Ai}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) z^{4/3} + 288\,(9z^2-110)\,\mathrm{Bi}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) z^{4/3} + 3\sqrt[3]{2}\sqrt[6]{3}\,(81z^4-4320z^2+14\,080)\,\mathrm{Ai}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) + \sqrt[3]{2}\,3^{2/3}\,(81z^4-4320z^2+14\,080)\,\mathrm{Bi}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right)$$

03.21.03.0034.01

$$j_{\frac{29}{6}}(z) = -\frac{1}{81\,3^{5/6}\,z^{35/6}}\,29\,120\sqrt[6]{2}\sqrt{\pi}\left(\sqrt[6]{3}\left(\frac{27\,(3z^2-280)\,z^2}{58\,240}+1\right)\left(\sqrt{3}\,\mathrm{Bi}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) - 3\,\mathrm{Ai}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right) z^{2/3} + 2^{2/3}\left(\frac{243z^4}{8320}-\frac{9z^2}{13}+1\right)\left(\mathrm{Bi}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) - \sqrt{3}\,\mathrm{Ai}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right)\right)$$

03.21.03.0035.01

$$j_{\frac{31}{6}}(z) = -\frac{1}{243\,z^{37/6}}\left(24\,640\left(\frac{2}{3}\right)^{5/6}\sqrt{\pi}\right)\left(9z^{4/3}\left(\frac{81z^4}{98\,560}-\frac{27z^2}{280}+1\right)\left(\sqrt{3}\,\mathrm{Ai}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) + \mathrm{Bi}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right) - 4\sqrt[3]{2}\sqrt[6]{3}\left(\frac{243z^4}{24\,640}-\frac{9z^2}{28}+1\right)\left(3\,\mathrm{Ai}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) + \sqrt{3}\,\mathrm{Bi}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right)\right)\right)$$

Symbolic rational ν

03.21.03.0036.01

$$j_\nu(z) = \sin\left(z-\frac{\pi\nu}{2}\right)\sum_{j=0}^{\left\lfloor\frac{1}{4}\left(2\left|\nu+\frac{1}{2}\right|-1\right)\right\rfloor}\frac{(-1)^j\,2^{-2j}\,z^{-2j-1}\left(2j+\left|\nu+\frac{1}{2}\right|-\frac{1}{2}\right)!}{(2j)!\left(-2j+\left|\nu+\frac{1}{2}\right|-\frac{1}{2}\right)!} + \cos\left(z-\frac{\pi\nu}{2}\right)\sum_{j=0}^{\left\lfloor\frac{1}{4}\left(2\left|\nu+\frac{1}{2}\right|-3\right)\right\rfloor}\frac{(-1)^j\,2^{-2j-1}\,z^{-2j-2}\left(2j+\left|\nu+\frac{1}{2}\right|+\frac{1}{2}\right)!}{(2j+1)!\left(-2j+\left|\nu+\frac{1}{2}\right|-\frac{3}{2}\right)!} \;/;\; \nu\in\mathbb{Z}$$

03.21.03.0037.01

$$j_\nu(z) = \frac{i^{\left(\left|\nu+\frac{1}{2}\right|-\frac{1}{3}\right)\left(\operatorname{sgn}\left(\nu+\frac{1}{2}\right)+1\right)} 2^{\left|\nu+\frac{1}{2}\right|-\frac{13}{6}} \sqrt{\pi}\, z^{-\left|\nu+\frac{1}{2}\right|-\frac{1}{2}}\, \Gamma\left(-\frac{1}{3}\right) \operatorname{sgn}\left(\nu+\frac{1}{2}\right)}{3^{5/6}\, \Gamma\left(1-\left|\nu+\frac{1}{2}\right|\right)}$$

$$\left(\sqrt[6]{3}\, z^{2/3}\left(\sqrt{3}\, \operatorname{Bi}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) - 3\, \operatorname{Ai}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) \operatorname{sgn}\left(\nu+\frac{1}{2}\right)\right)\left(\sum_{k=0}^{\left|\nu+\frac{1}{2}\right|-\frac{4}{3}} \frac{4^{-k} \left(z^2\right)^k \left(-k+\left|\nu+\frac{1}{2}\right|-\frac{4}{3}\right)!}{k!\left(-2k+\left|\nu+\frac{1}{2}\right|-\frac{4}{3}\right)! \left(\frac{4}{3}\right)_k \left(1-\left|\nu+\frac{1}{2}\right|\right)_k}\right) + \right.$$

$$2^{2/3}\left(\operatorname{Bi}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) - \sqrt{3}\, \operatorname{Ai}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) \operatorname{sgn}\left(\nu+\frac{1}{2}\right)\right)$$

$$\left.\sum_{k=0}^{\left|\nu+\frac{1}{2}\right|-\frac{1}{3}} \frac{4^{-k} \left(z^2\right)^k \left(-k+\left|\nu+\frac{1}{2}\right|-\frac{1}{3}\right)!}{k!\left(-2k+\left|\nu+\frac{1}{2}\right|-\frac{1}{3}\right)! \left(\frac{1}{3}\right)_k \left(1-\left|\nu+\frac{1}{2}\right|\right)_k}\right) /;\ \left|\nu+\frac{1}{2}\right|-\frac{1}{3} \in \mathbb{Z}$$

03.21.03.0038.01

$$j_\nu(z) = \frac{i^{\left(\left|\nu+\frac{1}{2}\right|-\frac{2}{3}\right)\left(\operatorname{sgn}\left(\nu+\frac{1}{2}\right)+1\right)} 2^{\left|\nu+\frac{1}{2}\right|-\frac{17}{6}} \sqrt{\pi}\, z^{-\left|\nu+\frac{1}{2}\right|-\frac{1}{2}}\, \Gamma\left(-\frac{2}{3}\right) \operatorname{sgn}\left(\nu+\frac{1}{2}\right)}{3\, 3^{5/6}\, \Gamma\left(1-\left|\nu+\frac{1}{2}\right|\right)}$$

$$\left(9\, z^{4/3}\left(\sqrt{3}\, \operatorname{Ai}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) + \operatorname{Bi}\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) \operatorname{sgn}\left(\nu+\frac{1}{2}\right)\right) \sum_{k=0}^{\left|\nu+\frac{1}{2}\right|-\frac{5}{3}} \frac{4^{-k} \left(z^2\right)^k \left(-k+\left|\nu+\frac{1}{2}\right|-\frac{5}{3}\right)!}{k!\left(-2k+\left|\nu+\frac{1}{2}\right|-\frac{5}{3}\right)! \left(\frac{5}{3}\right)_k \left(1-\left|\nu+\frac{1}{2}\right|\right)_k} - \right.$$

$$4\sqrt[3]{2}\, \sqrt[6]{3}\left(3\, \operatorname{Ai}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) + \sqrt{3}\, \operatorname{Bi}'\left(-\left(\frac{3}{2}\right)^{2/3} z^{2/3}\right) \operatorname{sgn}\left(\nu+\frac{1}{2}\right)\right)$$

$$\left.\sum_{k=0}^{\left|\nu+\frac{1}{2}\right|-\frac{2}{3}} \frac{4^{-k} \left(z^2\right)^k \left(-k+\left|\nu+\frac{1}{2}\right|-\frac{2}{3}\right)!}{k!\left(-2k+\left|\nu+\frac{1}{2}\right|-\frac{2}{3}\right)! \left(\frac{2}{3}\right)_k \left(1-\left|\nu+\frac{1}{2}\right|\right)_k}\right) /;\ \left|\nu+\frac{1}{2}\right|-\frac{2}{3} \in \mathbb{Z}$$

Values at fixed points

03.21.03.0039.01

$$j_0(0) = 1$$

Values at infinities

03.21.03.0040.01

$$\lim_{x\to\infty} j_\nu(x) = 0$$

03.21.03.0041.01

$$\lim_{x\to-\infty} j_\nu(x) = 0$$

03.21.03.0042.01

$$j_\nu\left(e^{i\lambda}\,\infty\right) = \begin{cases} 0 & \lambda = 0 \lor \lambda = \pi \\ \sim & \text{True} \end{cases} /;\ \operatorname{Im}(\lambda) = 0$$

03.21.03.0043.01

$$j_\nu(\infty) = 0$$

03.21.03.0044.01

$j_\nu(-\infty) = 0$

General characteristics

Domain and analyticity

$j_\nu(z)$ is an analytical function of ν and z which is defined over $\mathbb{C}^2$.

03.21.04.0001.01

$(\nu * z) \longrightarrow j_\nu(z) :: (\mathbb{C} \otimes \mathbb{C}) \longrightarrow \mathbb{C}$

Symmetries and periodicities

Parity

03.21.04.0002.01

$j_\nu(-z) = (-z)^\nu z^{-\nu} j_\nu(z)$

03.21.04.0003.01

$j_{-n-\frac{1}{2}}(z) = (-1)^n j_{n-\frac{1}{2}}(z) /; n + \frac{1}{2} \in \mathbb{Z}$

Mirror symmetry

03.21.04.0004.01

$j_{\bar{\nu}}(\bar{z}) == \overline{j_\nu(z)} /; z \notin (-\infty, 0)$

Periodicity

No periodicity

Poles and essential singularities

With respect to z

For fixed ν, the function $j_\nu(z)$ has an essential singularity at $z == \tilde{\infty}$. At the same time, the point $z == \tilde{\infty}$ is a branch point for generic ν.

03.21.04.0005.01

$\mathcal{Sing}_z(j_\nu(z)) == \{\{\tilde{\infty}, \infty\}\}$

With respect to ν

For fixed z, the function $j_\nu(z)$ has only one singular point at $\nu = \tilde{\infty}$. It is an essential singular point.

03.21.04.0006.01

$\mathcal{Sing}_\nu(j_\nu(z)) == \{\{\tilde{\infty}, \infty\}\}$

Branch points

With respect to z

For fixed noninteger $\nu + \frac{1}{2}$, the function $j_\nu(z)$ has two branch points: $z == 0$, $z == \tilde{\infty}$. At the same time, the point $z == \tilde{\infty}$ is an essential singularity.

03.21.04.0007.01

$$\mathcal{BP}_z(j_\nu(z)) == \{0, \tilde{\infty}\} /; \nu + \frac{1}{2} \notin \mathbb{Z}$$

03.21.04.0008.01

$$\mathcal{BP}_z(j_\nu(z)) == \{\} /; \nu + \frac{1}{2} \in \mathbb{Z}$$

03.21.04.0009.01

$$\mathcal{R}_z(j_\nu(z), 0) == \log /; \nu \notin \mathbb{Q}$$

03.21.04.0010.01

$$\mathcal{R}_z\left(j_{\frac{p}{q}}(z), 0\right) == q /; p \in \mathbb{Z} \bigwedge q - 1 \in \mathbb{N}^+ \bigwedge \gcd(p, q) == 1$$

03.21.04.0011.01

$$\mathcal{R}_z(j_\nu(z), \tilde{\infty}) == \log /; \nu \notin \mathbb{Q}$$

03.21.04.0012.01

$$\mathcal{R}_z\left(j_{\frac{p}{q}}(z), \tilde{\infty}\right) == q /; p \in \mathbb{Z} \bigwedge q - 1 \in \mathbb{N}^+ \bigwedge \gcd(p, q) == 1$$

With respect to ν

For fixed z, the function $j_\nu(z)$ does not have branch points.

03.21.04.0013.01

$$\mathcal{BP}_\nu(j_\nu(z)) == \{\}$$

Branch cuts

With respect to z

When $\nu + \frac{1}{2}$ is an integer, $j_\nu(z)$ is an entire function of z. For fixed noninteger $\nu + \frac{1}{2}$, it has one infinitely long branch cut. For fixed noninteger $\nu + \frac{1}{2}$, the function $j_\nu(z)$ is a single-valued function on the z-plane cut along the interval $(-\infty, 0)$, where it is continuous from above.

03.21.04.0014.01

$$\mathcal{BC}_z(j_\nu(z)) == \{\{(-\infty, 0), -i\}\} /; \nu + \frac{1}{2} \notin \mathbb{Z}$$

03.21.04.0015.01

$$\mathcal{BC}_z(j_\nu(z)) == \{\} /; \nu + \frac{1}{2} \in \mathbb{Z}$$

03.21.04.0016.01

$$\lim_{\epsilon \to +0} j_\nu(x + i\,\epsilon) = j_\nu(x) /; x \in \mathbb{R} \land x < 0$$

03.21.04.0017.01

$$\lim_{\epsilon \to +0} j_\nu(x - i\,\epsilon) = e^{-2 i \pi \nu}\, j_\nu(x) /; x \in \mathbb{R} \land x < 0$$

With respect to ν

For fixed z, the function $j_\nu(z)$ is an entire function of ν and does not have branch cuts.

03.21.04.0018.01

$$\mathcal{BC}_\nu(j_\nu(z)) = \{\}$$

Series representations

Generalized power series

Expansions at $\nu = \pm n$

03.21.06.0001.01

$$j_\nu(z) \propto j_n(z) + \frac{2\,(2\,z)^{-n-1}}{n!} \left(2\,z \sum_{k=0}^{\left\lfloor \frac{n-1}{2} \right\rfloor} (-1)^k\, 2^{2k} \binom{n}{2\,k+1} (-2\,k+2\,n-1)!\left(-\cos(z)\,\text{Ci}(2\,z) + \cos(z)\left(\psi\left(k+\frac{3}{2}\right) - \psi\left(k-n+\frac{1}{2}\right)\right) - \sin(z)\,\text{Si}(2\,z)\right) z^{2k} - \sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} (-1)^k\, 2^{2k} \binom{n}{2\,k} (2\,n-2\,k)!\left(-\text{Ci}(2\,z)\sin(z) + \left(\psi\left(k+\frac{1}{2}\right) - \psi\left(k-n+\frac{1}{2}\right)\right)\sin(z) + \cos(z)\,\text{Si}(2\,z)\right) z^{2k}\right)(\nu - n) + \ldots \;/;\; (\nu \to n) \land n \in \mathbb{N}$$

03.21.06.0002.01

$$j_\nu(z) \propto j_{-n}(z) - \frac{(-1)^n\, 2^{1-n}\, z^{-n}}{(n-1)!} \left(\sum_{k=0}^{\left\lfloor \frac{n-1}{2} \right\rfloor} (-4)^k\, z^{2k} \binom{n-1}{2\,k} (-2\,k+2\,n-2)!\left(\cos(z)\,\text{Ci}(2\,z) + \cos(z)\left(\psi\left(k+\frac{1}{2}\right) - \psi\left(k-n+\frac{3}{2}\right)\right) + \sin(z)\,\text{Si}(2\,z)\right) + 2\,z \sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor - 1} (-4)^k\, z^{2k} \binom{n-1}{2\,k+1} (-2\,k+2\,n-3)!\left(\text{Ci}(2\,z)\sin(z) + \left(\psi\left(k+\frac{3}{2}\right) - \psi\left(k-n+\frac{3}{2}\right)\right)\sin(z) - \cos(z)\,\text{Si}(2\,z)\right)\right)(n + \nu) + \ldots \;/;\; (\nu \to -n) \bigwedge n \in \mathbb{N}^+$$

Expansions at generic point $z = z_0$

For the function itself

03.21.06.0003.01

$$j_\nu(z) \propto \left(\frac{1}{z_0}\right)^{\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} z_0^{\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} \left(j_\nu(z_0) + \left(j_{\nu-1}(z_0) - \frac{\nu+1}{z_0}\, j_\nu(z_0)\right)(z - z_0) + \frac{\left(\nu^2 + 3\,\nu - z_0^2 + 2\right) j_\nu(z_0) - 2\,z_0\, j_{\nu-1}(z_0)}{2\,z_0^2}\,(z - z_0)^2 + \ldots\right) /;\; (z \to z_0)$$

03.21.06.0004.01

$$j_\nu(z) \propto \left(\frac{1}{z_0}\right)^{\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} z_0^{\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} \left(j_\nu(z_0) + \left(j_{\nu-1}(z_0) - \frac{\nu+1}{z_0} j_\nu(z_0)\right)(z-z_0) + \frac{\left(\nu^2 + 3\nu - z_0^2 + 2\right) j_\nu(z_0) - 2 z_0 j_{\nu-1}(z_0)}{2 z_0^2} (z-z_0)^2 + O\left((z-z_0)^3\right)\right)$$

03.21.06.0005.01

$$j_\nu(z) = \left(\frac{1}{z_0}\right)^{\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} z_0^{\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} \sum_{k=0}^{\infty} \frac{j_\nu^{(0,k)}(z_0)}{k!} (z-z_0)^k$$

03.21.06.0006.01

$$j_\nu(z) = \frac{\pi}{2} \Gamma(\nu+1) \left(\frac{z_0}{4}\right)^\nu \left(\frac{1}{z_0}\right)^{\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} z_0^{\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} \sum_{k=0}^{\infty} \frac{2^k z_0^{-k}}{k!} {}_2\tilde{F}_3\left(\frac{\nu+1}{2}, \frac{\nu+2}{2}; \frac{\nu-k+1}{2}, \frac{\nu-k+2}{2}, \nu+\frac{3}{2}; -\frac{z_0^2}{4}\right)(z-z_0)^k$$

03.21.06.0007.01

$$j_\nu(z) = \left(\frac{1}{z_0}\right)^{\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} z_0^{\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} \sqrt{2}\,\pi \sum_{k=0}^{\infty} 2^{-k} \sum_{j=0}^{2k} \frac{2^{2j} z_0^{-j}}{j!} {}_2\tilde{F}_3\left(-\frac{j}{2}, \frac{1-j}{2}; -j+k+1, \frac{1}{4}(1-2j), \frac{1}{4}(3-2j); -\frac{z_0^2}{4}\right) j_{j-k+\nu}(z_0)(z-z_0)^k$$

03.21.06.0008.01

$$j_\nu(z) = \left(\frac{1}{z_0}\right)^{\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} z_0^{\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} \sum_{k=0}^{\infty} \frac{1}{k!} z_0^{-k} \sum_{i=0}^{k} \binom{k}{i} \left(i-k+\frac{1}{2}\right)_{k-i} \sum_{m=0}^{i} (-1)^{i+m} \binom{i}{m} \left(-\nu-\frac{1}{2}\right)_{i-m} \sum_{u=0}^{m} \frac{(-1)^{u-1} 2^{2u-m} (-m)_{2(m-u)} \left(\nu+\frac{1}{2}\right)_u}{(m-u)!} \left(\frac{1}{2} z_0 \sum_{j=0}^{u-1} \frac{(-j+u-1)!\, 4^{-j} z_0^{2j}}{j!\,(-2j+u-1)!\left(-u-\nu+\frac{1}{2}\right)_j \left(\nu+\frac{1}{2}\right)_{j+1}} j_{\nu-1}(z_0) - \sum_{j=0}^{u} \frac{(u-j)!\, 4^{-j} z_0^{2j}}{j!\,(u-2j)!\left(-u-\nu+\frac{1}{2}\right)_j \left(\nu+\frac{1}{2}\right)_j} j_\nu(z_0)\right)(z-z_0)^k$$

03.21.06.0009.01

$$j_\nu(z) \propto \left(\frac{1}{z_0}\right)^{\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} z_0^{\nu \left\lfloor \frac{\arg(z-z_0)}{2\pi} \right\rfloor} j_\nu(z_0)(1 + O(z-z_0))$$

Expansions on branch cuts

For the function itself

03.21.06.0010.01

$$j_\nu(z) \propto e^{2\nu\pi i \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor} \left(j_\nu(x) + \left(j_{\nu-1}(x) - \frac{\nu+1}{x} j_\nu(x)\right)(z-x) + \frac{\left(-x^2 + \nu^2 + 3\nu + 2\right) j_\nu(x) - 2x\, j_{\nu-1}(x)}{2x^2} (z-x)^2 + \ldots\right) /; (z \to x) \land x \in \mathbb{R} \land x < 0$$

03.21.06.0011.01

$$j_\nu(z) \propto e^{2\nu\pi i \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor} \left(j_\nu(x) + \left(j_{\nu-1}(x) - \frac{\nu+1}{x} j_\nu(x) \right)(z-x) + \frac{\left(-x^2+\nu^2+3\nu+2\right) j_\nu(x) - 2x\, j_{\nu-1}(x)}{2x^2} (z-x)^2 + O\left((z-x)^3\right) \right) /;$$
$$x \in \mathbb{R} \land x < 0$$

03.21.06.0012.01

$$j_\nu(z) = e^{2\nu\pi i \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor} \sum_{k=0}^{\infty} \frac{j_\nu^{(0,k)}(z_0)}{k!} (z-x)^k$$

03.21.06.0013.01

$$j_\nu(z) = \frac{\pi}{2} \Gamma(\nu+1) \left(\frac{x}{4}\right)^\nu e^{2\nu\pi i \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor} \sum_{k=0}^{\infty} \frac{2^k x^{-k}}{k!} \, {}_2\tilde{F}_3\left(\frac{\nu+1}{2}, \frac{\nu+2}{2}; \frac{1}{2}(-k+\nu+1), \frac{1}{2}(-k+\nu+2), \nu+\frac{3}{2}; -\frac{x^2}{4} \right) (z-x)^k /;$$
$$x \in \mathbb{R} \land x < 0$$

03.21.06.0014.01

$$j_\nu(z) = e^{2\nu\pi i \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor} \sqrt{2}\, \pi \sum_{k=0}^{\infty} 2^{-k} \sum_{j=0}^{2k} \frac{2^{2j} x^{-j}}{j!} \, {}_2\tilde{F}_3\left(-\frac{j}{2}, \frac{1-j}{2}; -j+k+1, \frac{1}{4}(1-2j), \frac{1}{4}(3-2j); -\frac{x^2}{4} \right) j_{j-k+\nu}(x) (z-x)^k /;$$
$$x \in \mathbb{R} \land x < 0$$

03.21.06.0015.01

$$j_\nu(z) = e^{2\nu\pi i \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor} \sum_{k=0}^{\infty} \frac{x^{-k}}{k!} \sum_{i=0}^{k} \binom{k}{i} \left(i-k+\frac{1}{2}\right)_{k-i} \sum_{m=0}^{i} (-1)^{i+m} \binom{i}{m} \left(-\nu-\frac{1}{2}\right)_{i-m}$$
$$\sum_{u=0}^{m} \frac{(-1)^{u-1} 2^{2u-m} (-m)_{2(m-u)} \left(\nu+\frac{1}{2}\right)_u}{(m-u)!} \left(\frac{1}{2} x \sum_{j=0}^{u-1} \frac{\left((-j+u-1)!\, 4^{-j} x^{2j}\right) j_{\nu-1}(x)}{j!\,(-2j+u-1)! \left(-u-\nu+\frac{1}{2}\right)_j \left(\nu+\frac{1}{2}\right)_{j+1}} - \right.$$
$$\left. \sum_{j=0}^{u} \frac{\left((u-j)!\, 4^{-j} x^{2j}\right) j_\nu(x)}{j!\,(u-2j)! \left(-u-\nu+\frac{1}{2}\right)_j \left(\nu+\frac{1}{2}\right)_j} \right) (z-x)^k /; x \in \mathbb{R} \land x < 0$$

03.21.06.0016.01

$$j_\nu(z) \propto e^{2\nu\pi i \left\lfloor \frac{\arg(z-x)}{2\pi} \right\rfloor} j_\nu(x) (1 + O(z-x)) /; x \in \mathbb{R} \land x < 0$$

Expansions at $z == 0$

For the function itself

General case

03.21.06.0017.01

$$j_\nu(z) \propto \frac{\sqrt{\pi}}{2\,\Gamma\left(\nu+\frac{3}{2}\right)} \left(\frac{z}{2}\right)^\nu \left(1 - \frac{z^2}{2(3+2\nu)} + \frac{z^4}{8(3+2\nu)(5+2\nu)} - \ldots \right) /; (z \to 0) \bigwedge -\nu - \frac{1}{2} \notin \mathbb{N}^+$$

03.21.06.0018.01

$$j_\nu(z) \propto \frac{\sqrt{\pi}}{2\,\Gamma\left(\nu+\frac{3}{2}\right)} \left(\frac{z}{2}\right)^\nu \left(1 - \frac{z^2}{2(3+2\nu)} + \frac{z^4}{8(3+2\nu)(5+2\nu)} - O\left(z^6\right) \right) /; -\nu - \frac{1}{2} \notin \mathbb{N}^+$$

03.21.06.0019.01

$$j_\nu(z) = \frac{\sqrt{\pi}}{2} \left(\frac{z}{2}\right)^\nu \sum_{k=0}^{\infty} \frac{(-1)^k z^{2k}}{4^k \, \Gamma\left(k+\nu+\frac{3}{2}\right) k!}$$

03.21.06.0020.01

$$j_\nu(z) = \frac{\sqrt{\pi}}{2\,\Gamma\left(\nu+\frac{3}{2}\right)} \left(\frac{z}{2}\right)^\nu \sum_{k=0}^{\infty} \frac{(-1)^k z^{2k}}{4^k \left(\nu+\frac{3}{2}\right)_k k!} \;/;\; -\nu-\frac{1}{2} \notin \mathbb{N}^+$$

03.21.06.0021.01

$$j_\nu(z) = \frac{2^{-\nu-1} \sqrt{\pi}\, z^\nu}{\Gamma\left(\nu+\frac{3}{2}\right)} \, {}_0F_1\left(;\nu+\frac{3}{2}; -\frac{z^2}{4}\right) /;\; -\nu-\frac{1}{2} \notin \mathbb{N}^+$$

03.21.06.0022.01

$$j_\nu(z) = 2^{-\nu-1} \sqrt{\pi}\, z^\nu \, {}_0\tilde{F}_1\left(;\nu+\frac{3}{2}; -\frac{z^2}{4}\right)$$

03.21.06.0023.01

$$j_\nu(z) \propto \frac{\sqrt{\pi}}{2\,\Gamma\left(\nu+\frac{3}{2}\right)} \left(\frac{z}{2}\right)^\nu + O\left(z^{\nu+2}\right) /;\; -\nu-\frac{1}{2} \notin \mathbb{N}^+$$

03.21.06.0024.01

$$j_\nu(z) = F_\infty(z, \nu) \;/;$$
$$\left(\left(F_n(z,\nu) = \frac{\sqrt{\pi}}{2} \left(\frac{z}{2}\right)^\nu \sum_{k=0}^{n} \frac{(-1)^k z^{2k}}{4^k\, \Gamma\left(k+\nu+\frac{3}{2}\right) k!} = j_\nu(z) + (-1)^n\, 2^{-2n-\nu-3} \sqrt{\pi}\, z^{2n+\nu+2}\, {}_1\tilde{F}_2\left(1; n+2, n+\nu+\frac{5}{2}; -\frac{z^2}{4}\right)\right) \bigwedge n \in \mathbb{N}\right)$$

Summed form of the truncated series expansion.

Special cases

03.21.06.0025.01

$$j_\nu(z) \propto \frac{i\,(-2)^\nu \sqrt{\pi}\, z^{-\nu-1}}{\Gamma\left(\frac{1}{2}-\nu\right)} \left(1 + \frac{z^2}{2\,(2\nu-1)} + \frac{z^4}{8\,(2\nu-1)\,(2\nu-3)} + \ldots\right) /;\; (z \to 0) \bigwedge -\nu-\frac{1}{2} \in \mathbb{N}^+$$

03.21.06.0026.01

$$j_\nu(z) \propto \frac{i\,(-2)^\nu \sqrt{\pi}\, z^{-\nu-1}}{\Gamma\left(\frac{1}{2}-\nu\right)} \left(1 + \frac{z^2}{2\,(2\nu-1)} + \frac{z^4}{8\,(2\nu-1)\,(2\nu-3)} + O\left(z^6\right)\right) /;\; -\nu-\frac{1}{2} \in \mathbb{N}^+$$

03.21.06.0027.01

$$j_\nu(z) = \frac{(-1)^{\nu+\frac{1}{2}} \sqrt{\pi}}{z} \sum_{k=0}^{\infty} \frac{(-1)^k}{\Gamma\left(k-\nu+\frac{1}{2}\right) k!} \left(\frac{z}{2}\right)^{2k-\nu} /;\; -\nu-\frac{1}{2} \in \mathbb{N}^+$$

03.21.06.0028.01

$$j_\nu(z) = \frac{(-1)^{\nu+\frac{1}{2}} \sqrt{\pi}}{z\,\Gamma\left(\frac{1}{2}-\nu\right)} \sum_{k=0}^{\infty} \frac{(-1)^k}{\left(\frac{1}{2}-\nu\right)_k k!} \left(\frac{z}{2}\right)^{2k-\nu} /;\; -\nu-\frac{1}{2} \in \mathbb{N}^+$$

03.21.06.0029.01

$$j_\nu(z) = \frac{(-1)^{\nu+\frac{1}{2}}\, 2^\nu \sqrt{\pi}\; z^{-\nu-1}}{\Gamma\left(\frac{1}{2}-\nu\right)}\, {}_0F_1\left(;\frac{1}{2}-\nu;-\frac{z^2}{4}\right) /; -\nu-\frac{1}{2} \in \mathbb{N}^+$$

03.21.06.0030.01

$$j_\nu(z) = (-1)^{\nu+\frac{1}{2}}\, 2^\nu \sqrt{\pi}\; z^{-\nu-1}\, {}_0\tilde{F}_1\left(;\frac{1}{2}-\nu;-\frac{z^2}{4}\right) /; -\nu-\frac{1}{2} \in \mathbb{N}^+$$

03.21.06.0031.01

$$j_\nu(z) \propto \frac{i\,(-2)^\nu \sqrt{\pi}\; z^{-\nu-1}}{\Gamma\left(\frac{1}{2}-\nu\right)} + O\left(z^{1-\nu}\right) /; -\nu-\frac{1}{2} \in \mathbb{N}^+$$

For small integer powers of the function

For the second power

03.21.06.0032.01

$$j_\nu(z)^2 \propto \frac{2^{-2\nu-2}\,\pi\, z^{2\nu}}{\Gamma\left(\nu+\frac{3}{2}\right)^2}\left(1-\frac{z^2}{3+2\nu}+\frac{z^4\,(2+\nu)}{(3+2\nu)^2\,(5+2\nu)}+\ldots\right) /; (z\to 0)$$

03.21.06.0033.01

$$j_\nu(z)^2 \propto \frac{2^{-2\nu-2}\,\pi\, z^{2\nu}}{\Gamma\left(\nu+\frac{3}{2}\right)^2}\left(1-\frac{z^2}{3+2\nu}+\frac{z^4\,(2+\nu)}{(3+2\nu)^2\,(5+2\nu)}+O\left(z^6\right)\right)$$

03.21.06.0034.01

$$j_\nu(z)^2 = \frac{2^{-2\nu-2}\,\pi\, z^{2\nu}}{\Gamma\left(\nu+\frac{3}{2}\right)^2}\sum_{k=0}^{\infty}\frac{(-1)^k\,(\nu+1)_k\, z^{2k}}{\left(\nu+\frac{3}{2}\right)_k\,(2\nu+2)_k\, k!}$$

03.21.06.0035.01

$$j_\nu(z)^2 = \frac{2^{-2\nu-2}\,\pi\, z^{2\nu}}{\Gamma\left(\nu+\frac{3}{2}\right)^2}\, {}_1F_2\left(\nu+1;\nu+\frac{3}{2},2\nu+2;-z^2\right)$$

03.21.06.0036.01

$$j_\nu(z)^2 = \frac{\sqrt{\pi}}{2}\,\Gamma(\nu+1)\; z^{2\nu}\; {}_1\tilde{F}_2\left(\nu+1;\nu+\frac{3}{2},2\nu+2;-z^2\right)$$

03.21.06.0037.01

$$j_\nu(z)^2 \propto \frac{2^{-2\nu-2}\,\pi\, z^{2\nu}}{\Gamma\left(\nu+\frac{3}{2}\right)^2}\left(1+O\left(z^2\right)\right)$$

03.21.06.0038.01

$$j_\nu(z)^2 = F_\infty(z.\nu) \;/;\; \left(\left(F_n(z,\nu) = \frac{2^{-2\nu-2}\,\pi\, z^{2\nu}}{\Gamma\left(\nu+\frac{3}{2}\right)^2} \sum_{k=0}^{n} \frac{(-1)^k\,(\nu+1)_k\, z^{2k}}{\left(\nu+\frac{3}{2}\right)_k (2\nu+2)_k\, k!} = j_\nu(z)^2 + \frac{1}{2}(-1)^n \sqrt{\pi}\, z^{2n+2\nu+2}\, \Gamma(n+\nu+2)\, {}_2\tilde{F}_3\left(1, n+\nu+2; n+2, n+\nu+\frac{5}{2}, n+2\nu+3; -z^2\right)\right) \bigwedge n \in \mathbb{N}\right)$$

Summed form of the truncated series expansion.

Asymptotic series expansions

Expansions inside Stokes sectors

Expansions containing $z \to \infty$

In exponential form ||| In exponential form

03.21.06.0039.01

$$j_\nu(z) \propto \frac{1}{2z}\left(e^{\frac{1}{2} i(-2z+\pi\nu+\pi)}\left(1 - \frac{i\,\nu(\nu+1)}{2z} + \frac{(1-\nu)\,\nu\,(\nu+2)(\nu+1)}{8z^2} + \ldots\right) + e^{\frac{1}{2}(-i)(-2z+\pi\nu+\pi)}\left(1 + \frac{i\,\nu(\nu+1)}{2z} - \frac{(\nu-1)\,\nu\,(\nu+2)(\nu+1)}{8z^2} + \ldots\right)\right) \;/;\; |\arg(z)| < \pi \bigwedge (|z| \to \infty)$$

03.21.06.0040.01

$$j_\nu(z) \propto \frac{1}{2z}\left(e^{\frac{1}{2} i(-2z+\pi\nu+\pi)}\left(\sum_{k=0}^{n} \frac{(-\nu)_k\,(\nu+1)_k}{k!}\left(\frac{i}{2z}\right)^k + O\left(\frac{1}{z^{n+1}}\right)\right) + e^{-\frac{i}{2}(-2z+\pi\nu+\pi)}\left(\sum_{k=0}^{n} \frac{(-\nu)_k\,(\nu+1)_k}{k!}\left(-\frac{i}{2z}\right)^k + O\left(\frac{1}{z^{n+1}}\right)\right)\right) \;/;\; |\arg(z)| < \pi \bigwedge (|z| \to \infty)$$

03.21.06.0041.01

$$j_\nu(z) \propto \frac{1}{2z}\left(e^{\frac{1}{2} i(-2z+\pi\nu+\pi)}\, {}_2F_0\left(-\nu, \nu+1;; \frac{i}{2z}\right) + e^{\frac{1}{2}(-i)(-2z+\pi\nu+\pi)}\, {}_2F_0\left(-\nu, \nu+1;; -\frac{i}{2z}\right)\right) \;/;\; |\arg(z)| < \pi \bigwedge (|z| \to \infty)$$

03.21.06.0042.01

$$j_\nu(z) \propto \frac{1}{2z}\left(e^{-\frac{1}{2} i(-2z+\pi\nu+\pi)}\left(1 + O\left(\frac{1}{z}\right)\right) + e^{\frac{1}{2} i(-2z+\pi\nu+\pi)}\left(1 + O\left(\frac{1}{z}\right)\right)\right) \;/;\; |\arg(z)| < \pi \bigwedge (|z| \to \infty)$$

In trigonometric form ||| In trigonometric form

03.21.06.0043.01

$$j_\nu(z) \propto \frac{1}{z}\sin\left(z - \frac{\pi\nu}{2}\right)\left(1 - \frac{(-1+\nu)\,\nu\,(1+\nu)(2+\nu)}{8z^2} + \frac{(-3+\nu)(-2+\nu)(-1+\nu)\,\nu\,(1+\nu)(2+\nu)(3+\nu)(4+\nu)}{384z^4} + \ldots\right) + \frac{\nu(\nu+1)}{2z^2}\cos\left(z - \frac{\pi\nu}{2}\right)\left(1 - \frac{(-2+\nu)(-1+\nu)(2+\nu)(3+\nu)}{24z^2} + \frac{(-4+\nu)(-3+\nu)(-2+\nu)(-1+\nu)(2+\nu)(3+\nu)(4+\nu)(5+\nu)}{1920z^4} + \ldots\right) \;/;\; |\arg(z)| < \pi \bigwedge (|z| \to \infty)$$

03.21.06.0044.01

$$j_\nu(z) \propto \frac{1}{z}\sin\left(z-\frac{\pi\nu}{2}\right)\left(\sum_{k=0}^{n}\frac{\left(\frac{1}{2}-\frac{\nu}{2}\right)_k\left(\frac{\nu}{2}+1\right)_k\left(-\frac{\nu}{2}\right)_k\left(\frac{\nu+1}{2}\right)_k}{k!\left(\frac{1}{2}\right)_k}\left(-\frac{1}{z^2}\right)^k+O\left(\frac{1}{z^{2(n+1)}}\right)\right)+$$
$$\frac{\nu(\nu+1)}{2z^2}\cos\left(z-\frac{\pi\nu}{2}\right)\left(\sum_{k=0}^{n}\frac{\left(\frac{1}{2}-\frac{\nu}{2}\right)_k\left(1-\frac{\nu}{2}\right)_k\left(\frac{\nu}{2}+1\right)_k\left(\frac{\nu+3}{2}\right)_k}{k!\left(\frac{3}{2}\right)_k}\left(-\frac{1}{z^2}\right)^k+O\left(\frac{1}{z^{2(n+1)}}\right)\right)\text{/; }|\arg(z)|<\pi\bigwedge(|z|\to\infty)$$

03.21.06.0045.01

$$j_\nu(z) \propto \frac{i\,e^{i\pi\nu}\sqrt{-z}}{z^{3/2}}\left(\left(\sin\left(z+\frac{\pi\nu}{2}\right)\sum_{k=0}^{n}\frac{\left(\frac{1}{2}-\frac{\nu}{2}\right)_k\left(\frac{\nu}{2}+1\right)_k\left(-\frac{\nu}{2}\right)_k\left(\frac{\nu+1}{2}\right)_k}{k!\left(\frac{1}{2}\right)_k}\left(-\frac{1}{z^2}\right)^k+O\left(\frac{1}{z^{2(n+1)}}\right)\right)+\right.$$
$$\left.\frac{\nu(\nu+1)}{2z}\cos\left(z+\frac{\pi\nu}{2}\right)\left(\sum_{k=0}^{n}\frac{\left(\frac{1}{2}-\frac{\nu}{2}\right)_k\left(1-\frac{\nu}{2}\right)_k\left(\frac{\nu}{2}+1\right)_k\left(\frac{\nu+3}{2}\right)_k}{k!\left(\frac{3}{2}\right)_k}\left(-\frac{1}{z^2}\right)^k+O\left(\frac{1}{z^{2(n+1)}}\right)\right)\right)\text{/; }0<\arg(z)\le\pi\bigwedge(|z|\to\infty)$$

03.21.06.0046.01

$$j_\nu(z) \propto \frac{1}{z}\sin\left(z-\frac{\pi\nu}{2}\right){}_4F_1\left(\frac{1}{2}-\frac{\nu}{2},\frac{\nu}{2}+\frac{1}{2},\frac{\nu}{2}+1,-\frac{\nu}{2};\frac{1}{2};-\frac{1}{z^2}\right)+$$
$$\frac{\nu(\nu+1)}{2z^2}\cos\left(z-\frac{\pi\nu}{2}\right){}_4F_1\left(\frac{1}{2}-\frac{\nu}{2},1-\frac{\nu}{2},\frac{\nu}{2}+1,\frac{\nu}{2}+\frac{3}{2};\frac{3}{2};-\frac{1}{z^2}\right)\text{/; }|\arg(z)|<\pi\bigwedge(|z|\to\infty)$$

03.21.06.0047.01

$$j_\nu(z) \propto \frac{1}{z}\sin\left(z-\frac{\pi\nu}{2}\right)\left(1+O\left(\frac{1}{z^2}\right)\right)+\frac{\nu(\nu+1)}{2z^2}\cos\left(z-\frac{\pi\nu}{2}\right)\left(1+O\left(\frac{1}{z^2}\right)\right)\text{/; }|\arg(z)|<\pi\bigwedge(|z|\to\infty)$$

Expansions containing $z \to -\infty$

In exponential form ||| In exponential form

03.21.06.0048.01

$$j_\nu(z) \propto \frac{(-1)^\nu}{2\sqrt{-z^2}}\left(e^{-\frac{1}{2}i(2z+\pi\nu)}\left(1-\frac{i\nu(1+\nu)}{2z}+\frac{(1-\nu)\nu(1+\nu)(2+\nu)}{8z^2}+\ldots\right)-\right.$$
$$\left.e^{\frac{1}{2}i(2z+\pi\nu)}\left(1+\frac{i\nu(1+\nu)}{2z}+\frac{(1-\nu)\nu(1+\nu)(2+\nu)}{8z^2}+\ldots\right)\right)\text{/; }0<\arg(z)\le\pi\bigwedge(|z|\to\infty)$$

03.21.06.0049.01

$$j_\nu(z) \propto \frac{(-1)^\nu}{2\sqrt{-z^2}}\left(e^{-\frac{1}{2}i(2z+\pi\nu)}\left(\sum_{k=0}^{n}\frac{(-\nu)_k(\nu+1)_k}{k!}\left(\frac{i}{2z}\right)^k+O\left(\frac{1}{z^{n+1}}\right)\right)-e^{\frac{1}{2}i(2z+\pi\nu)}\left(\sum_{k=0}^{n}\frac{(-\nu)_k(\nu+1)_k}{k!}\left(-\frac{i}{2z}\right)^k+O\left(\frac{1}{z^{n+1}}\right)\right)\right)\text{/;}$$
$$0<\arg(z)\le\pi\bigwedge(|z|\to\infty)$$

03.21.06.0050.01

$$j_\nu(z) \propto \frac{(-1)^\nu}{2\sqrt{-z^2}}\left(e^{-\frac{1}{2}i(2z+\pi\nu)}\,{}_2F_0\left(-\nu,\nu+1;;\frac{i}{2z}\right)-e^{\frac{1}{2}i(2z+\pi\nu)}\,{}_2F_0\left(-\nu,\nu+1;;-\frac{i}{2z}\right)\right)\text{/; }0<\arg(z)\le\pi\bigwedge(|z|\to\infty)$$

03.21.06.0051.01

$$j_\nu(z) \propto \frac{(-1)^\nu}{2\sqrt{-z^2}} \left(e^{-\frac{1}{2} i (2z+\pi\nu)} \left(1 + O\left(\frac{1}{z}\right)\right) - e^{\frac{1}{2} i (2z+\pi\nu)} \left(1 + O\left(\frac{1}{z}\right)\right)\right) /; 0 < \arg(z) \leq \pi \bigwedge (|z| \to \infty)$$

In trigonometric form ||| In trigonometric form

03.21.06.0052.01

$$j_\nu(z) \propto \frac{i (-1)^\nu \sqrt{-z}}{z^{3/2}} \left(\sin\left(z + \frac{\pi\nu}{2}\right)\left(1 - \frac{(-1+\nu)\nu(1+\nu)(2+\nu)}{8 z^2} + \frac{(-3+\nu)(-2+\nu)(-1+\nu)\nu(1+\nu)(2+\nu)(3+\nu)(4+\nu)}{384 z^4} + \ldots\right) + \frac{\nu(\nu+1)}{2z} \cos\left(z + \frac{\pi\nu}{2}\right)\left(1 - \frac{(-2+\nu)(-1+\nu)(2+\nu)(3+\nu)}{24 z^2} + \frac{(-4+\nu)(-3+\nu)(-2+\nu)(-1+\nu)(2+\nu)(3+\nu)(4+\nu)(5+\nu)}{1920 z^4} + \ldots\right)\right) /; 0 < \arg(z) \leq \pi \bigwedge (|z| \to \infty)$$

03.21.06.0053.01

$$j_\nu(z) \propto \frac{i (-1)^\nu \sqrt{-z}}{z^{3/2}} \left(\sin\left(z + \frac{\pi\nu}{2}\right)\left(\sum_{k=0}^{n} \frac{\left(\frac{1}{2} - \frac{\nu}{2}\right)_k \left(\frac{\nu}{2} + 1\right)_k \left(-\frac{\nu}{2}\right)_k \left(\frac{\nu+1}{2}\right)_k}{k! \left(\frac{1}{2}\right)_k} \left(-\frac{1}{z^2}\right)^k + O\left(\frac{1}{z^{2(n+1)}}\right)\right) + \frac{\nu(\nu+1)}{2z} \cos\left(z + \frac{\pi\nu}{2}\right)\left(\sum_{k=0}^{n} \frac{\left(\frac{1}{2} - \frac{\nu}{2}\right)_k \left(1 - \frac{\nu}{2}\right)_k \left(\frac{\nu}{2} + 1\right)_k \left(\frac{\nu+3}{2}\right)_k}{k! \left(\frac{3}{2}\right)_k} \left(-\frac{1}{z^2}\right)^k + O\left(\frac{1}{z^{2(n+1)}}\right)\right)\right) /; 0 < \arg(z) \leq \pi \bigwedge (|z| \to \infty)$$

03.21.06.0054.01

$$j_\nu(z) \propto \frac{i (-1)^\nu \sqrt{-z}}{z^{3/2}} \left(\frac{\nu(\nu+1)}{2z} \cos\left(z + \frac{\pi\nu}{2}\right) {}_4F_1\left(\frac{1}{2} - \frac{\nu}{2}, 1 - \frac{\nu}{2}, \frac{\nu}{2} + 1, \frac{\nu}{2} + \frac{3}{2}; \frac{3}{2}; -\frac{1}{z^2}\right) + \sin\left(z + \frac{\pi\nu}{2}\right) {}_4F_1\left(\frac{1}{2} - \frac{\nu}{2}, \frac{\nu}{2} + \frac{1}{2}, \frac{\nu}{2} + 1, -\frac{\nu}{2}; \frac{1}{2}; -\frac{1}{z^2}\right)\right) /; 0 < \arg(z) \leq \pi \bigwedge (|z| \to \infty)$$

03.21.06.0055.01

$$j_\nu(z) \propto \frac{i (-1)^\nu \sqrt{-z}}{z^{3/2}} \left(\frac{\nu(\nu+1)}{2z} \cos\left(z + \frac{\pi\nu}{2}\right)\left(1 + O\left(\frac{1}{z^2}\right)\right) + \sin\left(z + \frac{\pi\nu}{2}\right)\left(1 + O\left(\frac{1}{z^2}\right)\right)\right) /; 0 < \arg(z) \leq \pi \bigwedge (|z| \to \infty)$$

Expansions for any z in exponential form

Using exponential function with branch cut-containing arguments

03.21.06.0056.01

$$j_\nu(z) \propto \frac{i}{2} z^\nu \left(z^2\right)^{\frac{1}{2}(-\nu-1)} \left(e^{-i\sqrt{z^2} + \frac{i\pi\nu}{2}} \left(1 - \frac{i\nu(1+\nu)}{2\sqrt{z^2}} - \frac{(-1+\nu)\nu(1+\nu)(2+\nu)}{8 z^2} + \ldots\right) - e^{i\sqrt{z^2} - \frac{i\pi\nu}{2}} \left(1 + \frac{i\nu(1+\nu)}{2\sqrt{z^2}} - \frac{(-1+\nu)\nu(1+\nu)(2+\nu)}{8 z^2} + \ldots\right)\right) /; (|z| \to \infty)$$

03.21.06.0057.01

$$j_\nu(z) \propto \frac{i}{2} z^\nu \left(z^2\right)^{\frac{1}{2}(-\nu-1)} \left(e^{-i\sqrt{z^2}+\frac{i\pi\nu}{2}} \left(\sum_{k=0}^{n} \frac{(-\nu)_k (\nu+1)_k}{k!} \left(\frac{i}{2\sqrt{z^2}} \right)^k + O\left(\frac{1}{z^{n+1}}\right) \right) - e^{i\sqrt{z^2}-\frac{i\pi\nu}{2}} \left(\sum_{k=0}^{n} \frac{(-\nu)_k (\nu+1)_k}{k!} \left(-\frac{i}{2\sqrt{z^2}} \right)^k + O\left(\frac{1}{z^{n+1}}\right) \right) \right) /; (|z| \to \infty)$$

03.21.06.0058.01

$$j_\nu(z) \propto \frac{i}{2} z^\nu \left(z^2\right)^{\frac{1}{2}(-\nu-1)} \left(e^{-i\sqrt{z^2}+\frac{i\pi\nu}{2}} {}_2F_0\left(-\nu, \nu+1; ; \frac{i}{2\sqrt{z^2}}\right) - e^{i\sqrt{z^2}-\frac{i\pi\nu}{2}} {}_2F_0\left(-\nu, \nu+1; ; -\frac{i}{2\sqrt{z^2}}\right) \right) /; (|z| \to \infty)$$

03.21.06.0059.01

$$j_\nu(z) \propto \frac{i}{2} z^\nu \left(z^2\right)^{\frac{1}{2}(-\nu-1)} \left(e^{-i\sqrt{z^2}+\frac{i\pi\nu}{2}} \left(1 + O\left(\frac{1}{z}\right)\right) - e^{i\sqrt{z^2}-\frac{i\pi\nu}{2}} \left(1 + O\left(\frac{1}{z}\right)\right) \right) /; (|z| \to \infty)$$

Using exponential function with branch cut-free arguments

03.21.06.0060.01

$$j_\nu(z) \propto \frac{i\, i^\nu}{2} (-z)^{-\nu-1} z^\nu \left(e^{-iz} \left(\frac{\sqrt{z}\, \sin(\pi\nu)}{\sqrt{-z}} - \cos(\pi\nu) \right) \left(1 - \frac{i\,\nu(1+\nu)}{2z} - \frac{(-1+\nu)\,\nu\,(1+\nu)(2+\nu)}{8z^2} + \ldots \right) + e^{iz} \left(1 + \frac{i\,\nu(1+\nu)}{2z} - \frac{(-1+\nu)\,\nu\,(1+\nu)(2+\nu)}{8z^2} + \ldots \right) \right) /; (|z| \to \infty)$$

03.21.06.0061.01

$$j_\nu(z) \propto \frac{i\, i^\nu}{2} (-z)^{-\nu-1} z^\nu \left(e^{-iz} \left(\frac{\sqrt{z}\, \sin(\pi\nu)}{\sqrt{-z}} - \cos(\pi\nu) \right) \left(\sum_{k=0}^{n} \frac{(-\nu)_k (\nu+1)_k}{k!} \left(\frac{i}{2z}\right)^k + O\left(\frac{1}{z^{n+1}}\right) \right) + e^{iz} \left(\sum_{k=0}^{n} \frac{(-\nu)_k (\nu+1)_k}{k!} \left(-\frac{i}{2z}\right)^k + O\left(\frac{1}{z^{n+1}}\right) \right) \right) /; (|z| \to \infty)$$

03.21.06.0062.01

$$j_\nu(z) \propto \frac{i\, i^\nu}{2} (-z)^{-\nu-1} z^\nu \left(e^{iz} {}_2F_0\left(-\nu, \nu+1; ; -\frac{i}{2z}\right) + e^{-iz} \left(\frac{\sqrt{z}\, \sin(\pi\nu)}{\sqrt{-z}} - \cos(\pi\nu) \right) {}_2F_0\left(-\nu, \nu+1; ; \frac{i}{2z}\right) \right) /; (|z| \to \infty)$$

03.21.06.0063.01

$$j_\nu(z) \propto \frac{i\, i^\nu}{2} (-z)^{-\nu-1} z^\nu \left(e^{iz} \left(1 + O\left(\frac{1}{z}\right)\right) + e^{-iz} \left(\frac{\sqrt{z}\, \sin(\pi\nu)}{\sqrt{-z}} - \cos(\pi\nu) \right) \left(1 + O\left(\frac{1}{z}\right)\right) \right) /; (|z| \to \infty)$$

Expansions for any z in trigonometric form

Using trigonometric functions with branch cut-containing arguments

03.21.06.0064.01

$$j_\nu(z) \propto z^\nu \left(z^2\right)^{-\frac{\nu}{2}-\frac{1}{2}} \left(\sin\left(\sqrt{z^2} - \frac{\pi\nu}{2}\right)\left(1 - \frac{(-1+\nu)\,\nu\,(1+\nu)\,(2+\nu)}{8\,z^2} + \frac{(-3+\nu)\,(-2+\nu)\,(-1+\nu)\,\nu\,(1+\nu)\,(2+\nu)\,(3+\nu)\,(4+\nu)}{384\,z^4} + \ldots\right) + \frac{\nu\,(\nu+1)}{2\sqrt{z^2}} \cos\left(\sqrt{z^2} - \frac{\pi\nu}{2}\right)\left(1 - \frac{(-2+\nu)\,(-1+\nu)\,(2+\nu)\,(3+\nu)}{24\,z^2} + \frac{(-4+\nu)\,(-3+\nu)\,(-2+\nu)\,(-1+\nu)\,(2+\nu)\,(3+\nu)\,(4+\nu)\,(5+\nu)}{1920\,z^4} + \ldots\right)\right) /;\ (|z| \to \infty)$$

03.21.06.0065.01

$$j_\nu(z) \propto z^\nu \left(z^2\right)^{-\frac{\nu}{2}-\frac{1}{2}} \left(\sin\left(\sqrt{z^2} - \frac{\pi\nu}{2}\right)\left(\sum_{k=0}^{n} \frac{\left(\frac{1}{2}-\frac{\nu}{2}\right)_k \left(\frac{\nu}{2}+1\right)_k \left(-\frac{\nu}{2}\right)_k \left(\frac{\nu+1}{2}\right)_k}{k!\left(\frac{1}{2}\right)_k}\left(-\frac{1}{z^2}\right)^k + O\left(\frac{1}{z^{2n+2}}\right)\right) + \frac{\nu\,(\nu+1)}{2\sqrt{z^2}} \cos\left(\sqrt{z^2} - \frac{\pi\nu}{2}\right)\left(\sum_{k=0}^{n} \frac{\left(\frac{1}{2}-\frac{\nu}{2}\right)_k \left(1-\frac{\nu}{2}\right)_k \left(\frac{\nu}{2}+1\right)_k \left(\frac{\nu+3}{2}\right)_k}{k!\left(\frac{3}{2}\right)_k}\left(-\frac{1}{z^2}\right)^k + O\left(\frac{1}{z^{2n+2}}\right)\right)\right) /;\ (|z| \to \infty)$$

03.21.06.0066.01

$$j_\nu(z) \propto z^\nu \left(z^2\right)^{-\frac{\nu}{2}-\frac{1}{2}} \left(\frac{\nu\,(\nu+1)}{2\sqrt{z^2}} \cos\left(\sqrt{z^2} - \frac{\pi\nu}{2}\right) {}_4F_1\left(\frac{1}{2}-\frac{\nu}{2}, 1-\frac{\nu}{2}, \frac{\nu}{2}+1, \frac{\nu}{2}+\frac{3}{2}; \frac{3}{2}; -\frac{1}{z^2}\right) + \sin\left(\sqrt{z^2} - \frac{\pi\nu}{2}\right) {}_4F_1\left(\frac{1}{2}-\frac{\nu}{2}, \frac{\nu}{2}+\frac{1}{2}, \frac{\nu}{2}+1, -\frac{\nu}{2}; \frac{1}{2}; -\frac{1}{z^2}\right)\right) /;\ (|z| \to \infty)$$

03.21.06.0067.01

$$j_\nu(z) \propto z^\nu \left(z^2\right)^{-\frac{\nu}{2}-\frac{1}{2}} \left(\frac{\nu\,(\nu+1)}{2\sqrt{z^2}} \cos\left(\sqrt{z^2} - \frac{\pi\nu}{2}\right)\left(1 + O\left(\frac{1}{z^2}\right)\right) + \sin\left(\sqrt{z^2} - \frac{\pi\nu}{2}\right)\left(1 + O\left(\frac{1}{z^2}\right)\right)\right) /;\ (|z| \to \infty)$$

Using trigonometric functions with branch cut-free arguments

03.21.06.0068.01

$$j_\nu(z) \propto \frac{1}{2z}\left(e^{i\pi\nu}\left(1 + \frac{i\sqrt{-z}}{\sqrt{z}}\right)\sin\left(z + \frac{\pi\nu}{2}\right) + \left(1 - \frac{i\sqrt{-z}}{\sqrt{z}}\right)\sin\left(z - \frac{\pi\nu}{2}\right)\right) \left(1 - \frac{(-1+\nu)\,\nu\,(1+\nu)\,(2+\nu)}{8\,z^2} + \frac{(-3+\nu)\,(-2+\nu)\,(-1+\nu)\,\nu\,(1+\nu)\,(2+\nu)\,(3+\nu)\,(4+\nu)}{384\,z^4} + \ldots\right) + \frac{\nu\,(\nu+1)}{4\,z^2}\left(e^{i\pi\nu}\left(1 + \frac{i\sqrt{-z}}{\sqrt{z}}\right)\cos\left(z + \frac{\pi\nu}{2}\right) + \left(1 - \frac{i\sqrt{-z}}{\sqrt{z}}\right)\cos\left(z - \frac{\pi\nu}{2}\right)\right) \left(1 - \frac{(-2+\nu)\,(-1+\nu)\,(2+\nu)\,(3+\nu)}{24\,z^2} + \frac{(-4+\nu)\,(-3+\nu)\,(-2+\nu)\,(-1+\nu)\,(2+\nu)\,(3+\nu)\,(4+\nu)\,(5+\nu)}{1920\,z^4} + \ldots\right) /;\ (|z| \to \infty)$$

03.21.06.0069.01

$$j_\nu(z) \propto \frac{1}{2z}\left(e^{i\pi\nu}\left(1+\frac{i\sqrt{-z}}{\sqrt{z}}\right)\sin\left(z+\frac{\pi\nu}{2}\right)+\left(1-\frac{i\sqrt{-z}}{\sqrt{z}}\right)\sin\left(z-\frac{\pi\nu}{2}\right)\right)\left(\sum_{k=0}^{n}\frac{\left(\frac{1}{2}-\frac{\nu}{2}\right)_k\left(\frac{\nu}{2}+1\right)_k\left(-\frac{\nu}{2}\right)_k\left(\frac{\nu+1}{2}\right)_k}{k!\left(\frac{1}{2}\right)_k}\left(-\frac{1}{z^2}\right)^k+O\left(\frac{1}{z^{2n+2}}\right)\right)+\frac{\nu(\nu+1)}{4z^2}\left(e^{i\pi\nu}\left(1+\frac{i\sqrt{-z}}{\sqrt{z}}\right)\cos\left(z+\frac{\pi\nu}{2}\right)+\left(1-\frac{i\sqrt{-z}}{\sqrt{z}}\right)\cos\left(z-\frac{\pi\nu}{2}\right)\right)\left(\sum_{k=0}^{n}\frac{\left(\frac{1}{2}-\frac{\nu}{2}\right)_k\left(1-\frac{\nu}{2}\right)_k\left(\frac{\nu}{2}+1\right)_k\left(\frac{\nu+3}{2}\right)_k}{k!\left(\frac{3}{2}\right)_k}\left(-\frac{1}{z^2}\right)^k+O\left(\frac{1}{z^{2n+2}}\right)\right)\,/;\,(|z|\to\infty)$$

03.21.06.0070.01

$$j_\nu(z) \propto \frac{1}{2z}\left(e^{i\pi\nu}\left(1+\frac{i\sqrt{-z}}{\sqrt{z}}\right)\sin\left(z+\frac{\pi\nu}{2}\right)+\left(1-\frac{i\sqrt{-z}}{\sqrt{z}}\right)\sin\left(z-\frac{\pi\nu}{2}\right)\right){}_4F_1\left(\frac{1}{2}-\frac{\nu}{2},\frac{\nu}{2}+\frac{1}{2},\frac{\nu}{2}+1,-\frac{\nu}{2};\frac{1}{2};-\frac{1}{z^2}\right)+\frac{\nu(\nu+1)}{4z^2}\left(e^{i\pi\nu}\left(1+\frac{i\sqrt{-z}}{\sqrt{z}}\right)\cos\left(z+\frac{\pi\nu}{2}\right)+\left(1-\frac{i\sqrt{-z}}{\sqrt{z}}\right)\cos\left(z-\frac{\pi\nu}{2}\right)\right){}_4F_1\left(\frac{1}{2}-\frac{\nu}{2},1-\frac{\nu}{2},\frac{\nu}{2}+1,\frac{\nu}{2}+\frac{3}{2};\frac{3}{2};-\frac{1}{z^2}\right)\,/;\,(|z|\to\infty)$$

03.21.06.0071.01

$$j_\nu(z) \propto \frac{1}{2z}\left(e^{i\pi\nu}\left(1+\frac{i\sqrt{-z}}{\sqrt{z}}\right)\sin\left(z+\frac{\pi\nu}{2}\right)+\left(1-\frac{i\sqrt{-z}}{\sqrt{z}}\right)\sin\left(z-\frac{\pi\nu}{2}\right)\right)\left(1+O\left(\frac{1}{z^2}\right)\right)+\frac{\nu(\nu+1)}{4z^2}\left(e^{i\pi\nu}\left(1+\frac{i\sqrt{-z}}{\sqrt{z}}\right)\cos\left(z+\frac{\pi\nu}{2}\right)+\left(1-\frac{i\sqrt{-z}}{\sqrt{z}}\right)\cos\left(z-\frac{\pi\nu}{2}\right)\right)\left(1+O\left(\frac{1}{z^2}\right)\right)\,/;\,(|z|\to\infty)$$

Residue representations

03.21.06.0072.01

$$j_\nu(z) = 2^{-\nu-1}\pi^{3/2}z^\nu\sum_{j=0}^{\infty}\operatorname{res}_s\left(\frac{\left(-\frac{z^2}{4}\right)^{-s}\Gamma(s)}{\Gamma\left(s+\frac{1}{2}\right)\Gamma\left(\frac{1}{2}-s\right)\Gamma\left(-s+\nu+\frac{3}{2}\right)}\right)(-j)$$

03.21.06.0073.01

$$j_\nu(z) = \sqrt{\frac{\pi}{2}}\,z^\nu\left(z^2\right)^{-\frac{\nu}{2}-\frac{1}{4}}\sum_{j=0}^{\infty}\operatorname{res}_s\left(\frac{\left(\frac{z^2}{4}\right)^{-s}\Gamma\left(s+\frac{\nu}{2}+\frac{1}{4}\right)}{\Gamma\left(\frac{1}{4}(-4s+2\nu+5)\right)}\right)\left(-j-\frac{\nu}{2}-\frac{1}{4}\right)$$

03.21.06.0074.01

$$j_\nu(z) = \frac{\pi^{3/2}z^\nu\left(-z^2\right)^{-\frac{\nu}{2}-\frac{1}{4}}}{\sqrt{2}}\sum_{j=0}^{\infty}\operatorname{res}_s\left(\frac{\left(-\frac{z^2}{4}\right)^{-s}}{\Gamma\left(s+\frac{\nu}{2}+\frac{3}{4}\right)\Gamma\left(-s-\frac{\nu}{2}+\frac{1}{4}\right)\Gamma\left(-s+\frac{\nu}{2}+\frac{5}{4}\right)}\Gamma\left(s+\frac{\nu}{2}+\frac{1}{4}\right)\right)\left(-j-\frac{\nu}{2}-\frac{1}{4}\right)$$

03.21.06.0075.01

$$j_\nu(z) = \frac{\sqrt{\pi}}{2}\sum_{j=0}^{\infty}\operatorname{res}_s\left(\frac{\left(\frac{z}{2}\right)^{-2s}}{\Gamma\left(\frac{1}{2}(-2s+\nu+3)\right)}\Gamma\left(s+\frac{\nu}{2}\right)\right)\left(-\frac{1}{2}(2j+\nu)\right)$$

03.21.06.0076.01

$$j_\nu(z) = \frac{\pi^{3/2} (i z)^{-\nu-\frac{1}{2}} z^\nu}{\sqrt{2}} \sum_{j=0}^{\infty} \operatorname{res}_s\left(\frac{\left(\frac{i z}{2}\right)^{-2 s}}{\Gamma\left(-s-\frac{\nu}{2}+\frac{1}{4}\right)\Gamma\left(-s+\frac{\nu}{2}+\frac{5}{4}\right)\Gamma\left(s+\frac{\nu}{2}+\frac{3}{4}\right)} \Gamma\left(s+\frac{\nu}{2}+\frac{1}{4}\right)\right)\left(-j-\frac{\nu}{2}-\frac{1}{4}\right)$$

Integral representations

On the real axis

Of the direct function

03.21.07.0001.01

$$j_\nu(z) = \frac{2^{-\nu} z^\nu}{\Gamma(\nu+1)} \int_0^1 \left(1-t^2\right)^\nu \cos(t z)\, dt \,/;\, \operatorname{Re}(\nu) > -1$$

03.21.07.0002.01

$$j_\nu(z) = \frac{2^{-\nu-1} z^\nu}{\Gamma(\nu+1)} \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^{2\left(\nu+\frac{1}{2}\right)}(t) \cos(z \sin(t))\, dt \,/;\, \operatorname{Re}(\nu) > -1$$

03.21.07.0003.01

$$j_\nu(z) = \frac{2^{-\nu-1} z^\nu}{\Gamma(\nu+1)} \int_0^\pi \cos(z \cos(t)) \sin^{2\nu+1}(t)\, dt \,/;\, \operatorname{Re}(\nu) > -1$$

03.21.07.0004.01

$$j_\nu(z) = \frac{1}{\sqrt{2\pi}\,\sqrt{z}} \int_0^\pi \cos\left(t\left(\nu+\frac{1}{2}\right) - z \sin(t)\right) dt - \frac{\cos(\pi \nu)}{\sqrt{2\pi}\,\sqrt{z}} \int_0^\infty e^{-t\left(\nu+\frac{1}{2}\right) - z \sinh(t)}\, dt \,/;\, \arg(z) < \frac{\pi}{2}$$

03.21.07.0005.01

$$j_\nu(z) = \frac{i^{-\nu-\frac{1}{2}}}{\sqrt{2\pi}\,\sqrt{z}} \int_0^\pi e^{i z \cos(t)} \cos\left(\left(\nu+\frac{1}{2}\right) t\right) dt \,/;\, \nu+\frac{1}{2} \in \mathbb{N}^+$$

03.21.07.0006.01

$$j_\nu(z) = \frac{1}{\sqrt{2\pi}\,\sqrt{z}} \int_0^\pi \cos\left(\left(\nu+\frac{1}{2}\right) t - z \sin(t)\right) dt \,/;\, \nu+\frac{1}{2} \in \mathbb{Z}$$

Contour integral representations

03.21.07.0007.01

$$j_\nu(z) = -\frac{i\, 2^{-\nu-2} z^\nu}{\sqrt{\pi}} \int_{(\gamma-i)\infty}^{(\gamma+i)\infty} e^{t-\frac{z^2}{4t}}\, t^{-\nu-\frac{3}{2}}\, dt \,/;\, \gamma > 0 \bigwedge \operatorname{Re}(\nu) > -\frac{1}{2}$$

03.21.07.0008.01

$$j_\nu(x) = \frac{1}{4\pi i} \sqrt{\frac{\pi}{2}} \int_{(\gamma-i)\infty}^{\gamma+i\infty} \frac{\Gamma(s)}{\Gamma\left(\frac{3}{2}-s+\nu\right)} \left(\frac{x}{2}\right)^{\nu-2s} ds \,/;\, x > 0 \bigwedge 0 < \gamma < \frac{\operatorname{Re}(\nu)}{2} + 1$$

03.21.07.0009.01

$$j_\nu(z) = \frac{1}{2\pi i} \sqrt{\frac{\pi}{2}}\, z^\nu \left(z^2\right)^{-\frac{\nu}{2}-\frac{1}{4}} \int_{\mathcal{L}} \frac{\Gamma\left(s+\frac{\nu}{2}+\frac{1}{4}\right)}{\Gamma\left(-s+\frac{\nu}{2}+\frac{5}{4}\right)} \left(\frac{z^2}{4}\right)^{-s} ds$$

03.21.07.0010.01

$$j_\nu(z) = \frac{1}{2\pi i} \frac{\pi^{3/2} z^\nu \left(-z^2\right)^{-\frac{\nu}{2}-\frac{1}{4}}}{\sqrt{2}} \int_{\mathcal{L}} \frac{\Gamma\left(s+\frac{\nu}{2}+\frac{1}{4}\right)\left(-\frac{z^2}{4}\right)^{-s}}{\Gamma\left(-s-\frac{\nu}{2}+\frac{1}{4}\right)\Gamma\left(-s+\frac{\nu}{2}+\frac{5}{4}\right)\Gamma\left(s+\frac{\nu}{2}+\frac{3}{4}\right)} \, ds$$

03.21.07.0011.01

$$j_\nu(z) = -\frac{1}{2\pi i} \frac{i}{2\sqrt{2\pi}\sqrt{z}} \int_{\mathcal{L}} \frac{\Gamma\left(s+\frac{\nu}{2}+\frac{1}{4}\right)}{\Gamma\left(-s+\frac{\nu}{2}+\frac{5}{4}\right)} \left(\frac{z}{2}\right)^{-2s} ds$$

03.21.07.0012.01

$$j_\nu(z) = \frac{1}{2\pi i} \frac{\pi^{3/2} (i z)^{-\nu-\frac{1}{2}} z^\nu}{\sqrt{2}} \int_{\mathcal{L}} \frac{\Gamma\left(s+\frac{\nu}{2}+\frac{1}{4}\right)}{\Gamma\left(s+\frac{\nu}{2}+\frac{3}{4}\right)\Gamma\left(-s-\frac{\nu}{2}+\frac{1}{4}\right)\Gamma\left(-s+\frac{\nu}{2}+\frac{5}{4}\right)} \left(\frac{i z}{2}\right)^{-2s} ds$$

Integral representations of negative integer order BAD

03.21.07.0013.01

$$j_n(z) = \sum_{k=0}^{\left\lfloor \frac{l}{2} \right\rfloor} c_{k,n} \frac{\partial^{n-2k} j_0(z)}{\partial x^{n-2k}} /; n \in \mathbb{N}^+ \bigwedge c_{0,0} = 0 \bigwedge c_{1,2} = \frac{1}{2} \bigwedge$$

$$c_{0,n} = \frac{(-1)^n (2n-1)!!}{n!} \bigwedge \left(c_{k,n} = \frac{n-1}{n} c_{k-1,n-2} - \frac{2n-1}{n} c_{k,n-1} /; n > 0 \land k \le n\right) \bigwedge (c_{k,n} = 0 /; k > n)$$

Limit representations

03.21.09.0001.01

$$j_\nu(z) = \sqrt{\frac{\pi}{2}} \frac{1}{\sqrt{z}} \lim_{\lambda\to\infty} \lambda^{\nu+\frac{1}{2}} P_\lambda^{-\nu-\frac{1}{2}}\left(\cos\left(\frac{z}{\lambda}\right)\right)$$

03.21.09.0002.01

$$j_\nu(z) = 2^{-\nu-1} \sqrt{\pi} z^\nu \lim_{n\to\infty} n^{-\nu-\frac{1}{2}} P_n^{\left(\nu+\frac{1}{2},b\right)}\left(\cos\left(\frac{z}{n}\right)\right)$$

03.21.09.0003.01

$$j_\nu(z) = 2^{-\nu-1} \sqrt{\pi} z^\nu \lim_{n\to\infty} n^{-\nu-\frac{1}{2}} L_n^{\nu+\frac{1}{2}}\left(\frac{z^2}{4n}\right)$$

03.21.09.0004.01

$$j_\nu(z) = \frac{2^{-\nu-1}\sqrt{\pi} z^\nu}{\Gamma\left(\nu+\frac{3}{2}\right)} \lim_{a\to\infty} {}_1F_1\left(a; \nu+\frac{3}{2}; -\frac{z^2}{4a}\right)$$

Generating functions

03.21.11.0001.01

$$\sum_{k=-\infty}^{\infty} j_{k-\frac{1}{2}}(z) = \sqrt{\frac{\pi}{2}} \frac{e^{\frac{1}{2}\left(t-\frac{1}{t}\right)z}}{\sqrt{z}}$$

03.21.11.0002.01

$$\sum_{k=-\infty}^{\infty} e^{i k q} j_{k-\frac{1}{2}}(z) = \sqrt{\frac{\pi}{2}} \frac{e^{i z \sin(q)}}{\sqrt{z}}$$

P. Abbott

Differential equations

Ordinary linear differential equations and wronskians

For the direct function itself

03.21.13.0001.01

$$z^2 w''(z) + 2 z\, w'(z) + \left(z^2 - \nu(\nu+1)\right) w(z) = 0 /; w(z) = c_1 j_\nu(z) + c_2 y_\nu(z)$$

03.21.13.0002.01

$$W_z(j_\nu(z), y_\nu(z)) = \frac{1}{z^2}$$

03.21.13.0003.01

$$w''(z) z^2 + 2 w'(z) z + \left(z^2 - \nu(\nu+1)\right) w(z) = 0 /; w(z) = c_1 j_\nu(z) + c_2 j_{-\nu-1}(z) \bigwedge \nu + \frac{1}{2} \notin \mathbb{Z}$$

03.21.13.0004.01

$$W_z(j_\nu(z), j_{-\nu-1}(z)) = -\frac{\cos(\pi \nu)}{z^2}$$

03.21.13.0005.01

$$4 z^2 w''(z) + 4 z(-2 p + q + 1) w'(z) + \left(4 p^2 - 4 q p + q^2 \left(4 m^2 z^{2 q} - 4 \nu^2 + 1\right)\right) w(z) = 0 /; w(z) = c_1 z^p j_\nu(m z^q) + c_2 z^p y_\nu(m z^q)$$

03.21.13.0006.01

$$W_z(z^p j_\nu(m z^q), z^p y_\nu(m z^q)) = \frac{q z^{2 p - q - 1}}{m}$$

03.21.13.0007.01

$$4 z^2 w''(z) + 4 z (-2 p + q + 1) w'(z) + \left(4 p^2 - 4 q p + q^2 \left(4 m^2 z^{2 q} - 4 \nu^2 + 1\right)\right) w(z) = 0 /;$$
$$w(z) = c_1 z^p j_\nu(m z^q) + c_2 z^p j_{-\nu-1}(m z^q) \bigwedge \nu + \frac{1}{2} \notin \mathbb{Z}$$

03.21.13.0008.01

$$W_z(z^p j_\nu(m z^q), z^p j_{-\nu-1}(m z^q)) = -\frac{q z^{2 p - q - 1} \cos(\pi \nu)}{m}$$

03.21.13.0009.01

$$w''(z) - \left(\frac{g''(z)}{g'(z)} - \frac{2 g'(z)}{g(z)}\right) w'(z) - \left(\frac{\nu^2 + \nu}{g(z)^2} - 1\right) g'(z)^2 w(z) = 0 /; w(z) = c_1 j_\nu(g(z)) + c_2 y_\nu(g(z))$$

03.21.13.0010.01

$$W_z(j_\nu(g(z)), y_\nu(g(z))) = \frac{g'(z)}{g(z)^2}$$

03.21.13.0011.01

$$w''(z) - \left(-\frac{2\,g'(z)}{g(z)} + \frac{2\,h'(z)}{h(z)} + \frac{g''(z)}{g'(z)}\right) w'(z) - \left(\left(\frac{\nu^2+\nu}{g(z)^2} - 1\right) g'(z)^2 + \frac{2\,h'(z)\,g'(z)}{g(z)\,h(z)} + \frac{h(z)\,h''(z) - 2\,h'(z)^2}{h(z)^2} - \frac{h'(z)\,g''(z)}{h(z)\,g'(z)}\right) w(z) = 0 \;/;\; w(z) = c_1\,h(z)\,j_\nu(g(z)) + c_2\,h(z)\,y_\nu(g(z))$$

03.21.13.0012.01

$$W_z(h(z)\,j_\nu(g(z)),\, h(z)\,y_\nu(g(z))) = \frac{h(z)^2\,g'(z)}{g(z)^2}$$

03.21.13.0013.01

$$z^2\,w''(z) + z\,(r - 2\,s + 1)\,w'(z) + \left(\left(a^2\,z^{2\,r} - \nu\,(\nu+1)\right) r^2 + s^2 - r\,s\right) w(z) = 0 \;/;\; w(z) = c_1\,z^s\,j_\nu(a\,z^r) + c_2\,z^s\,y_\nu(a\,z^r)$$

03.21.13.0014.01

$$W_z(z^s\,j_\nu(a\,z^r),\, z^s\,y_\nu(a\,z^r)) = \frac{r\,z^{-r+2\,s-1}}{a}$$

03.21.13.0015.01

$$w''(z) + (\log(r) - 2\log(s))\,w'(z) + \left(\left(a^2\,r^{2\,z} - \nu\,(\nu+1)\right)\log^2(r) - \log(s)\log(r) + \log^2(s)\right) w(z) = 0 \;/;\; w(z) = c_1\,s^z\,j_\nu(a\,r^z) + c_2\,s^z\,y_\nu(a\,r^z)$$

03.21.13.0016.01

$$W_z(s^z\,j_\nu(a\,r^z),\, s^z\,y_\nu(a\,r^z)) = \frac{r^{-z}\,s^{2\,z}\log(r)}{a}$$

Transformations

Transformations and argument simplifications

Argument involving basic arithmetic operations

03.21.16.0001.01

$$j_\nu(-z) = (-z)^\nu\,z^{-\nu}\,j_\nu(z)$$

03.21.16.0002.01

$$j_\nu(i\,z) = \sqrt{\frac{\pi}{2}}\,(i\,z)^\nu\,z^{-\nu-\frac{1}{2}}\,I_{\nu+\frac{1}{2}}(z)$$

03.21.16.0003.01

$$j_\nu(-i\,z) = \sqrt{\frac{\pi}{2}}\,(-i\,z)^\nu\,z^{-\nu-\frac{1}{2}}\,I_{\nu+\frac{1}{2}}(z)$$

03.21.16.0004.01

$$j_\nu\left(\sqrt{z^2}\right) = z^{-\nu}\left(z^2\right)^{\nu/2} j_\nu(z)$$

Addition formulas

03.21.16.0005.01

$$j_\nu(z_1 - z_2) = \sqrt{\frac{2}{\pi}}\,\frac{\sqrt{z_1}\,\sqrt{z_2}}{\sqrt{z_1 - z_2}}\sum_{k=-\infty}^{\infty} j_{k+\nu}(z_1)\,j_{k-\frac{1}{2}}(z_2) \;/;\; \left|\frac{z_2}{z_1}\right| < 1 \bigvee \nu + \frac{1}{2} \in \mathbb{Z}$$

03.21.16.0006.01

$$j_\nu(z_1+z_2)=\sqrt{\frac{2}{\pi}}\,\frac{\sqrt{z_1}\,\sqrt{z_2}}{\sqrt{z_1+z_2}}\sum_{k=-\infty}^{\infty} j_{k-\frac{1}{2}}(z_2)\, j_{\nu-k}(z_1)\ /;\ \left|\frac{z_2}{z_1}\right|<1 \bigvee \nu+\frac{1}{2}\in\mathbb{Z}$$

Multiple arguments

03.21.16.0007.01

$$j_\nu(z_1\, z_2)=\frac{z_1^{\nu+\frac{1}{2}}\sqrt{z_2}}{\sqrt{z_1\, z_2}}\sum_{k=0}^{\infty}\frac{(-1)^k\left(z_1^2-1\right)^k}{k!}\, j_{k+\nu}(z_2)\left(\frac{z_2}{2}\right)^k$$

03.21.16.0008.01

$$j_\nu(z_1\, z_2)=\frac{\sqrt{\pi}\, z_1^{-\nu-\frac{1}{2}}\sqrt{z_2}}{\sqrt{z_1\, z_2}}\sum_{k=0}^{\infty}\frac{\left(z_1^2-1\right)^k}{\sqrt{\pi}\, k!}\, j_{\nu-k}(z_2)\left(\frac{z_2}{2}\right)^k$$

Identities

Recurrence identities

Consecutive neighbors

03.21.17.0001.01

$$j_\nu(z)=\frac{2\,\nu+3}{z}\, j_{\nu+1}(z)-j_{\nu+2}(z)$$

03.21.17.0002.01

$$j_\nu(z)=\frac{2\,\nu-1}{z}\, j_{\nu-1}(z)-j_{\nu-2}(z)$$

Distant neighbors

Increasing

03.21.17.0003.01

$$j_\nu(z)=2^{n-1}\, z^{-n}\left(\nu+\frac{3}{2}\right)_{n-1}\left((2\,n+2\,\nu+1)\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{2^{-2\,k}\,(n-k)!\, j_{n+\nu}(z)\, z^{2\,k}}{k!\,(n-2\,k)!\left(-n-\nu-\frac{1}{2}\right)_k\left(\nu+\frac{3}{2}\right)_k}-z\sum_{k=0}^{\left\lfloor\frac{n-1}{2}\right\rfloor}\frac{2^{-2\,k}\,(n-k-1)!\, j_{n+\nu+1}(z)\, z^{2\,k}}{k!\,(n-2\,k-1)!\left(\frac{1}{2}-n-\nu\right)_k\left(\nu+\frac{3}{2}\right)_k}\right)$$

03.21.17.0004.01

$$j_\nu(z)=2^{n-1}\, z^{-n}\left(\nu+\frac{3}{2}\right)_{n-1}\left((2\,n+2\,\nu+1)\,{}_3F_4\left(1,\frac{1-n}{2},-\frac{n}{2};1,-n,-n-\nu-\frac{1}{2},\nu+\frac{3}{2};-z^2\right) j_{n+\nu}(z)-\right.$$
$$\left. z\,{}_3F_4\left(1,\frac{1-n}{2},1-\frac{n}{2};1,1-n,-n-\nu+\frac{1}{2},\nu+\frac{3}{2};-z^2\right) j_{n+\nu+1}(z)\right)\ /;\ n\in\mathbb{N}$$

03.21.17.0005.01

$$j_\nu(z)=\frac{1}{z^2}\left(\left((2\,\nu+3)\,(2\,\nu+5)-z^2\right) j_{\nu+2}(z)-z\,(2\,\nu+3)\, j_{\nu+3}(z)\right)$$

03.21.17.0006.01

$$j_\nu(z)=\frac{1}{z^3}\left((2\,\nu+5)\left(-2\,z^2+4\,\nu^2+20\,\nu+21\right) j_{\nu+3}(z)+z\left(z^2-(3+2\,\nu)\,(5+2\,\nu)\right) j_{\nu+4}(z)\right)$$

03.21.17.0007.01

$$j_\nu(z) = \frac{1}{z^4}\left(\left(z^4 - 3z^2(2\nu+5)(2\nu+7)\left(16\nu^2+96\nu+109\right)\right)j_{\nu+4}(z) + z(2\nu+5)\left(2z^2-(2\nu+3)(2\nu+7)\right)j_{\nu+5}(z)\right)$$

03.21.17.0008.01

$$j_\nu(z) = \frac{1}{z^5}\left((2\nu+7)\left(3z^4 - 4(2\nu+5)(2\nu+9)z^2 + (2\nu+3)(2\nu+5)(2\nu+9)(2\nu+11)\right)j_{\nu+5}(z) - z\left(z^4 - 3(2\nu+5)(2\nu+7)z^2 + (2\nu+3)(2\nu+5)(2\nu+7)(2\nu+9)\right)j_{\nu+6}(z)\right)$$

03.21.17.0009.01

$$j_\nu(z) = C_n(\nu, z)\, j_{n+\nu}(z) - C_{n-1}(\nu, z)\, j_{n+\nu+1}(z) \;/;\; C_0(\nu, z) = 1 \bigwedge C_1(\nu, z) = \frac{2\nu+3}{z} \bigwedge C_n(\nu, z) = \frac{(2n+2\nu+1)}{z} C_{n-1}(\nu, z) - C_{n-2}(\nu, z) \bigwedge n \in \mathbb{N}^+$$

03.21.17.0010.01

$$j_\nu(z) = C_n(\nu, z)\, j_{n+\nu}(z) - C_{n-1}(\nu, z)\, j_{n+\nu+1}(z) \;/;\; C_n(\nu, z) = 2^n z^{-n}\left(\nu+\frac{3}{2}\right)_n {}_2F_3\!\left(\frac{1-n}{2}, -\frac{n}{2}; \nu+\frac{3}{2}, -n, -n-\nu-\frac{1}{2}; -z^2\right) \bigwedge n \in \mathbb{N}^+$$

Decreasing

03.21.17.0011.01

$$j_\nu(z) = 2^{n-1}(-z)^{-n}\left(\frac{1}{2}-\nu\right)_{n-1} \left((2n-2\nu-1)\sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} \frac{2^{-2k}(n-k)!\, j_{\nu-n}(z)\, z^{2k}}{k!\,(n-2k)!\left(\frac{1}{2}-\nu\right)_k \left(\nu-n+\frac{1}{2}\right)_k} + z\sum_{k=0}^{\left\lfloor \frac{n-1}{2} \right\rfloor} \frac{2^{-2k}(n-k-1)!\, j_{\nu-n-1}(z)\, z^{2k}}{k!\,(n-2k-1)!\left(\frac{1}{2}-\nu\right)_k \left(\nu-n+\frac{3}{2}\right)_k}\right) /;\; n \in \mathbb{N}$$

03.21.17.0012.01

$$j_\nu(z) = 2^{n-1}(-z)^{-n}\left(\frac{1}{2}-\nu\right)_{n-1}\left(z\, {}_3F_4\!\left(1, \frac{1-n}{2}, 1-\frac{n}{2}; 1, 1-n, \frac{1}{2}-\nu, \nu-n+\frac{3}{2}; -z^2\right) j_{\nu-n-1}(z) + (2n-2\nu-1)\, {}_3F_4\!\left(1, \frac{1-n}{2}, -\frac{n}{2}; 1, -n, \frac{1}{2}-\nu, \nu-n+\frac{1}{2}; -z^2\right) j_{\nu-n}(z)\right) /;\; n \in \mathbb{N}$$

03.21.17.0013.01

$$j_\nu(z) = -\frac{1}{z^2}\left(z(2\nu-1)\, j_{\nu-3}(z) + \left(z^2-(2\nu-3)(2\nu-1)\right) j_{\nu-2}(z)\right)$$

03.21.17.0014.01

$$j_\nu(z) = \frac{1}{z^3}\left(z\left(z^2-(2\nu-3)(2\nu-1)\right) j_{\nu-4}(z) - (2\nu-3)\left(2z^2-(2\nu-5)(2\nu-1)\right) j_{\nu-3}(z)\right)$$

03.21.17.0015.01

$$j_\nu(z) = \frac{1}{z^4}\left(z(2\nu-3)\left(2z^2-(2\nu-5)(2\nu-1)\right) j_{\nu-5}(z) + \left(z^4 - 3(2\nu-5)(2\nu-3)z^2 + (2\nu-7)(2\nu-5)(2\nu-3)(2\nu-1)\right) j_{\nu-4}(z)\right)$$

03.21.17.0016.01

$$j_\nu(z) = -\frac{1}{z^5}\left(z\left(z^4 - 3(2\nu-5)(2\nu-3)z^2 + (2\nu-7)(2\nu-5)(2\nu-3)(2\nu-1)\right) j_{\nu-6}(z) - (2\nu-5)\left(3z^4 - 4(2\nu-7)(2\nu-3)z^2 + (2\nu-9)(2\nu-7)(2\nu-3)(2\nu-1)\right) j_{\nu-5}(z)\right)$$

03.21.17.0017.01

$$j_\nu(z) = C_n(\nu, z)\, j_{\nu-n}(z) - C_{n-1}(\nu, z)\, j_{\nu-n-1}(z) \,/;\; C_0(\nu, z) = 1 \bigwedge C_1(\nu, z) = \frac{2\nu - 1}{z} \bigwedge C_n(\nu, z) = \frac{2\nu - 2n + 1}{z} C_{n-1}(\nu, z) - C_{n-2}(\nu, z) \bigwedge n \in \mathbb{N}^+$$

03.21.17.0018.01

$$j_\nu(z) = C_n(\nu, z)\, j_{\nu-n}(z) - C_{n-1}(\nu, z)\, j_{\nu-n-1}(z) \,/;\; C_n(\nu, z) = (-2)^n z^{-n} \left(\frac{1}{2}(1 - 2\nu)\right)_n {}_2F_3\left(\frac{1-n}{2}, -\frac{n}{2}; \frac{1}{2}(1 - 2\nu), -n, \frac{1}{2} - n + \nu; -z^2\right) \bigwedge n \in \mathbb{N}^+$$

Functional identities

Relations between contiguous functions

03.21.17.0019.01

$$j_\nu(z) = \frac{z}{2\nu + 1}(j_{\nu-1}(z) + j_{\nu+1}(z))$$

Relations of special kind

03.21.17.0020.01

$$j_{-\nu-1}(z)\, j_{\nu-1}(z) + j_{-\nu}(z)\, j_\nu(z) = \frac{\cos(\pi \nu)}{z^2}$$

Differentiation

Low-order differentiation

With respect to ν

03.21.20.0001.01

$$j_\nu^{(1,0)}(z) = j_\nu(z) \log\left(\frac{z}{2}\right) - \frac{\sqrt{\pi}}{2} \sum_{k=0}^{\infty} \frac{(-1)^k \psi\left(k + \nu + \frac{3}{2}\right)}{k!\, \Gamma\left(k + \nu + \frac{3}{2}\right)} \left(\frac{z}{2}\right)^{2k+\nu}$$

03.21.20.0002.01

$$j_\nu^{(1,0)}(z) = \frac{2^{-\nu-2} \sqrt{\pi}\, z^{\nu+2}}{(2\nu + 3)\, \Gamma\left(\nu + \frac{5}{2}\right)} F_{2\times 0\times 1}^{0\times 1\times 2}\left(\begin{matrix} ;\, 1;\, 1,\, \nu + \frac{3}{2}; \\ 2,\, \nu + \frac{5}{2};;\, \nu + \frac{5}{2}; \end{matrix}\; -\frac{z^2}{4}, -\frac{z^2}{4}\right) + \left(-\log(2) + \log(z) - \psi\left(\nu + \frac{3}{2}\right)\right) j_\nu(z)$$

03.21.20.0003.01

$$j_n^{(1,0)}(z) = \frac{2(2z)^{-n}}{n!} \sum_{k=0}^{\left\lfloor \frac{n-1}{2} \right\rfloor} (-1)^k 2^{2k} \binom{n}{2k+1} (2n - 2k - 1)! \left(-\cos(z)\, \text{Ci}(2z) + \cos(z)\left(\psi\left(k + \frac{3}{2}\right) - \psi\left(k - n + \frac{1}{2}\right)\right) - \sin(z)\, \text{Si}(2z)\right) z^{2k} - \frac{2(2z)^{-n-1}}{n!} \sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} (-1)^k 2^{2k} \binom{n}{2k} (2n - 2k)! \left(-\text{Ci}(2z)\sin(z) + \left(\psi\left(k + \frac{1}{2}\right) - \psi\left(k - n + \frac{1}{2}\right)\right)\sin(z) + \cos(z)\, \text{Si}(2z)\right) z^{2k} \,/;\; n \in \mathbb{N}$$

Brychkov Yu.A. (2005)

03.21.20.0004.01

$$j_{-n}^{(1,0)}(z) == -\frac{(-1)^n 2^{1-n} z^{-n}}{(n-1)!} \left(\sum_{k=0}^{\left\lfloor \frac{n-1}{2} \right\rfloor} (-4)^k z^{2k} \binom{n-1}{2k} (2n-2k-2)! \left(\cos(z)\,\text{Ci}(2z) + \cos(z) \left(\psi\left(k+\frac{1}{2}\right) - \psi\left(k-n+\frac{3}{2}\right) \right) + \sin(z)\,\text{Si}(2z) \right) + 2z \sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor - 1} (-4)^k z^{2k} \binom{n-1}{2k+1} (2n-2k-3)! \left(\text{Ci}(2z) \sin(z) + \left(\psi\left(k+\frac{3}{2}\right) - \psi\left(k-n+\frac{3}{2}\right) \right) \sin(z) - \cos(z)\,\text{Si}(2z) \right) \right) /; n \in \mathbb{N}^+$$

Brychkov Yu.A. (2005)

03.21.20.0005.01

$$j_{n-\frac{1}{2}}^{(1,0)}(z) = \frac{1}{2} \pi\, y_{n-\frac{1}{2}}(z) + \frac{1}{2} n! \left(\frac{z}{2}\right)^{-n} \sum_{k=0}^{n-1} \frac{j_{k-\frac{1}{2}}(z)}{(n-k)\, k!} \left(\frac{z}{2}\right)^k /; n \in \mathbb{N}$$

03.21.20.0006.01

$$j_{-n-\frac{1}{2}}^{(1,0)}(z) == \frac{n!}{2} \left(-\frac{z}{2}\right)^{-n} \sum_{k=0}^{n-1} \frac{1}{(n-k)\, k!} j_{k-\frac{1}{2}}(z) \left(\frac{z}{2}\right)^k + \left(-\frac{z}{2}\right)^n \sqrt{\frac{\pi}{2}} \frac{1}{\sqrt{z}\, n!} \sum_{j=1}^{n} \frac{1}{j}\, {}_1F_2\left(j; j+1, n+1; -\frac{z^2}{4}\right) + \frac{(-1)^n \pi}{2} y_{n-\frac{1}{2}}(z) + \sqrt{\frac{\pi}{2}} \frac{(-1)^{n-1}}{\sqrt{z}} \sum_{k=0}^{n-1} \frac{(-k+n-1)! \left(\frac{z}{2}\right)^{2k-n}}{k!} /; n \in \mathbb{N}$$

With respect to z

03.21.20.0007.01

$$\frac{\partial j_\nu(z)}{\partial z} = j_{\nu-1}(z) - \frac{\nu+1}{z} j_\nu(z)$$

03.21.20.0008.01

$$\frac{\partial j_\nu(z)}{\partial z} = \frac{\nu}{z} j_\nu(z) - j_{\nu+1}(z)$$

03.21.20.0009.01

$$\frac{\partial j_\nu(z)}{\partial z} = \frac{j_{\nu-1}(z) - j_{\nu+1}(z)}{2} - \frac{j_\nu(z)}{2z}$$

03.21.20.0010.01

$$\frac{\partial j_0(z)}{\partial z} = -j_1(z)$$

03.21.20.0011.01

$$\frac{\partial j_{-1}(z)}{\partial z} = j_{-2}(z)$$

03.21.20.0012.01

$$\frac{\partial \left(z^{\nu+1} j_\nu(z)\right)}{\partial z} = z^{\nu+1} j_{\nu-1}(z)$$

03.21.20.0013.01

$$\frac{\partial(z^{-\nu}\, j_\nu(z))}{\partial z} = -z^{-\nu}\, j_{\nu+1}(z)$$

03.21.20.0014.01

$$\frac{\partial^2 j_\nu(z)}{\partial z^2} = \frac{1}{4}\, j_{\nu-2}(z) + \frac{3-2z^2}{4z^2}\, j_\nu(z) + \frac{1}{2z}\, j_{\nu+1}(z) + \frac{1}{4}\, j_{\nu+2}(z) - \frac{1}{2z}\, j_{\nu-1}(z)$$

Symbolic differentiation

With respect to ν

03.21.20.0015.01

$$j_\nu^{(m,0)}(z) = \frac{\sqrt{\pi}}{2} \sum_{k=0}^{\infty} \frac{(-1)^k}{k!} \left(\frac{z}{2}\right)^{2k} \frac{\partial^m \frac{\left(\frac{z}{2}\right)^\nu}{\Gamma\left(k+\nu+\frac{3}{2}\right)}}{\partial \nu^m} \;/;\; m \in \mathbb{N}$$

With respect to z

03.21.20.0016.01

$$j_\nu^{(0,n)}(0) = 0 \;/;\; n \in \mathbb{N}^+ \bigwedge \nu \in \mathbb{N} \bigwedge \left(\nu > n \bigvee \frac{n-\nu-1}{2} \in \mathbb{N}\right)$$

03.21.20.0017.01

$$j_\nu^{(0,n)}(0) = \frac{i^{n-\nu}\, 2^{-n-1} \sqrt{\pi}\, \Gamma(n+1)}{\Gamma\left(\frac{1}{2}(n-\nu+2)\right) \Gamma\left(\frac{1}{2}(n+\nu+3)\right)} \;/;\; n \in \mathbb{N}^+ \bigwedge \nu \in \mathbb{N} \bigwedge \frac{n-\nu}{2} \in \mathbb{N}$$

03.21.20.0018.01

$$\frac{\partial^n j_\nu(z)}{\partial z^n} = z^{-n} \sum_{i=0}^{n} \binom{n}{i} \left(i-n+\frac{1}{2}\right)_{n-i} \sum_{m=0}^{i} (-1)^{i+m} \binom{i}{m} \left(-\nu-\frac{1}{2}\right)_{i-m} \sum_{k=0}^{m} \frac{(-1)^{k-1}\, 2^{2k-m}\, (-m)_{2(m-k)} \left(\nu+\frac{1}{2}\right)_k}{(m-k)!} \left(\frac{1}{2} z \sum_{j=0}^{k-1} \frac{(-j+k-1)!\, 4^{-j} z^{2j}}{j!\, (-2j+k-1)! \left(-k-\nu+\frac{1}{2}\right)_j \left(\nu+\frac{1}{2}\right)_{j+1}}\, j_{\nu-1}(z) - \sum_{j=0}^{k} \frac{(k-j)!\, 4^{-j} z^{2j}}{j!\, (k-2j)! \left(-k-\nu+\frac{1}{2}\right)_j \left(\nu+\frac{1}{2}\right)_j}\, j_\nu(z)\right) \;/;\; n \in \mathbb{N}$$

03.21.20.0019.01

$$\frac{\partial^n j_\nu(z)}{\partial z^n} = 2^{n-2\nu-1} \pi\, z^{\nu-n}\, \Gamma(\nu+1)\, {}_2\tilde{F}_3\left(\frac{\nu+1}{2}, \frac{\nu+2}{2}; \frac{1}{2}(-n+\nu+1), \frac{1}{2}(-n+\nu+2), \nu+\frac{3}{2}; -\frac{z^2}{4}\right) \;/;\; n \in \mathbb{N}$$

03.21.20.0020.01

$$\frac{\partial^n j_\nu(z)}{\partial z^n} = 2^{\frac{1}{2}-n} \pi\, n! \sum_{k=0}^{2n} \frac{2^{2k} z^{-k}}{k!}\, {}_2\tilde{F}_3\left(-\frac{k}{2}, \frac{1-k}{2}; -k+n+1, \frac{1-2k}{4}, \frac{3-2k}{4}; -\frac{z^2}{4}\right) j_{k-n+\nu}(z) \;/;\; n \in \mathbb{N}$$

Fractional integro-differentiation

With respect to z

03.21.20.0021.01

$$\frac{\partial^\alpha j_\nu(z)}{\partial z^\alpha} = 2^{\alpha-2\nu-1}\,\pi\, z^{\nu-\alpha}\,\Gamma(\nu+1)\,{}_2\tilde{F}_3\left(\frac{\nu+1}{2}, \frac{\nu+2}{2}; \frac{1}{2}(-\alpha+\nu+1), \frac{1}{2}(-\alpha+\nu+2), \nu+\frac{3}{2}; -\frac{z^2}{4}\right) /; -\nu-\frac{1}{2} \notin \mathbb{N}^+$$

03.21.20.0022.01

$$\frac{\partial^\alpha j_\nu(z)}{\partial z^\alpha} = (-1)^{\nu+\frac{1}{2}}\, 2^{\alpha+2\nu+1}\,\pi\, z^{-\alpha-\nu-1}\,\Gamma(-\nu)\,{}_2\tilde{F}_3\left(\frac{1-\nu}{2}, -\frac{\nu}{2}; \frac{1}{2}-\nu, \frac{1}{2}(-\alpha-\nu), \frac{1}{2}(-\alpha-\nu+1); -\frac{z^2}{4}\right) /; -\nu-\frac{1}{2} \in \mathbb{N}^+$$

Integration

Indefinite integration

Involving only one direct function

03.21.21.0001.01

$$\int j_\nu(a\,z)\,dz = 2^{-\nu-2}\sqrt{\pi}\; z\,(a\,z)^\nu\,\Gamma\left(\frac{\nu+1}{2}\right)\,{}_1\tilde{F}_2\left(\frac{\nu+1}{2}; \nu+\frac{3}{2}, \frac{\nu+3}{2}; -\frac{1}{4}a^2 z^2\right)$$

03.21.21.0002.01

$$\int j_\nu(z)\,dz = 2^{-\nu-2}\sqrt{\pi}\; z^{\nu+1}\,\Gamma\left(\frac{\nu+1}{2}\right)\,{}_1\tilde{F}_2\left(\frac{\nu+1}{2}; \nu+\frac{3}{2}, \frac{\nu+3}{2}; -\frac{z^2}{4}\right)$$

03.21.21.0003.01

$$\int j_0(z)\,dz = \mathrm{Si}(z)$$

03.21.21.0004.01

$$\int j_1(z)\,dz = -\frac{\sin(z)}{z}$$

Involving one direct function and elementary functions

Involving power function

Involving power

Linear arguments

03.21.21.0005.01

$$\int z^{\alpha-1}\, j_\nu(a\,z)\,dz = 2^{-\nu-2}\sqrt{\pi}\; z^\alpha\,(a\,z)^\nu\,\Gamma\left(\frac{\alpha+\nu}{2}\right)\,{}_1\tilde{F}_2\left(\frac{\alpha+\nu}{2}; \nu+\frac{3}{2}, \frac{1}{2}(\alpha+\nu+2); -\frac{1}{4}a^2 z^2\right)$$

03.21.21.0006.01

$$\int z^{\alpha-1}\, j_\nu(z)\,dz = 2^{-\nu-2}\sqrt{\pi}\; z^{\alpha+\nu}\,\Gamma\left(\frac{\alpha+\nu}{2}\right)\,{}_1\tilde{F}_2\left(\frac{\alpha+\nu}{2}; \nu+\frac{3}{2}, \frac{1}{2}(\alpha+\nu+2); -\frac{z^2}{4}\right)$$

03.21.21.0007.01

$$\int z^{\alpha-1}\, j_0(z)\,dz = \frac{1}{2} z^\alpha \left(z^2\right)^{-\alpha} \left(\Gamma(\alpha-1, i\,z)\,(-i\,z)^\alpha + (i\,z)^\alpha\,\Gamma(\alpha-1, -i\,z)\right)$$

03.21.21.0008.01

$$\int z^{1-\nu}\, j_\nu(z)\, dz = \frac{2^{-\nu-\frac{1}{2}}\, z^{-\nu}}{\Gamma(\nu+2)} \left(\sqrt{\pi}\, z^\nu\, (\nu+1) - 2^{\nu+\frac{1}{2}}\, \sqrt{z}\, \Gamma(\nu+2)\, j_{\nu-\frac{1}{2}}(z)\right)$$

03.21.21.0009.01

$$\int z\, j_0(z)\, dz = -\cos(z)$$

03.21.21.0010.01

$$\int \frac{j_0(z)}{z}\, dz = \text{Ci}(z) - \frac{\sin(z)}{z}$$

Power arguments

03.21.21.0011.01

$$\int z^{\alpha-1}\, j_\nu(a\, z^r)\, dz = \frac{2^{-\nu-2}\, \sqrt{\pi}\, z^\alpha\, (a\, z^r)^\nu}{r}\, \Gamma\!\left(\frac{\alpha+r\,\nu}{2\,r}\right) {}_1\tilde{F}_2\!\left(\frac{\alpha+r\,\nu}{2\,r};\, \nu+\frac{3}{2},\, \frac{\alpha+r\,(\nu+2)}{2\,r};\, -\frac{1}{4}\, a^2\, z^{2\,r}\right)$$

Involving exponential function

Involving exp

Linear arguments

03.21.21.0012.01

$$\int e^{-i\,a\,z}\, j_\nu(a\,z)\, dz = \frac{2^{-\nu-1}\, \sqrt{\pi}\, z\, (a\,z)^\nu}{(\nu+1)\, \Gamma\!\left(\nu+\frac{3}{2}\right)}\, {}_2F_2(\nu+1,\, \nu+1;\, \nu+2,\, 2\,\nu+2;\, -2\,i\,a\,z)$$

03.21.21.0013.01

$$\int e^{i\,a\,z}\, j_\nu(a\,z)\, dz = \frac{2^{-\nu-1}\, \sqrt{\pi}\, z\, (a\,z)^\nu}{(\nu+1)\, \Gamma\!\left(\nu+\frac{3}{2}\right)}\, {}_2F_2(\nu+1,\, \nu+1;\, \nu+2,\, 2\,\nu+2;\, 2\,i\,a\,z)$$

Power arguments

03.21.21.0014.01

$$\int e^{-i\,a\,z^r}\, j_\nu(a\,z^r)\, dz = \frac{2^{-\nu-1}\, \sqrt{\pi}\, z\, (a\,z^r)^\nu}{(r\,\nu+1)\, \Gamma\!\left(\nu+\frac{3}{2}\right)}\, {}_2F_2\!\left(\nu+1,\, \nu+\frac{1}{r};\, \nu+\frac{1}{r}+1,\, 2\,\nu+2;\, -2\,i\,a\,z^r\right)$$

03.21.21.0015.01

$$\int e^{i\,a\,z^r}\, j_\nu(a\,z^r)\, dz = \frac{2^{-\nu-1}\, \sqrt{\pi}\, z\, (a\,z^r)^\nu}{(r\,\nu+1)\, \Gamma\!\left(\nu+\frac{3}{2}\right)}\, {}_2F_2\!\left(\nu+1,\, \nu+\frac{1}{r};\, \nu+\frac{1}{r}+1,\, 2\,\nu+2;\, 2\,i\,a\,z^r\right)$$

Involving exponential function and a power function

Involving exp and power

Linear arguments

03.21.21.0016.01

$$\int z^{\alpha-1} e^{-i a z} j_\nu(a z)\, dz = \frac{2^{-\nu-1} \sqrt{\pi}\, z^\alpha (a z)^\nu}{(\alpha+\nu)\, \Gamma\left(\nu+\frac{3}{2}\right)} \, {}_2F_2(\nu+1, \alpha+\nu; \alpha+\nu+1, 2\nu+2; -2 i a z)$$

03.21.21.0017.01

$$\int z^{-\nu} e^{-i a z} j_\nu(a z)\, dz = \frac{i\, 2^{-\nu-\frac{1}{2}} e^{-i a z} z^{-\nu-\frac{1}{2}}}{a (2\nu+1)\, \Gamma(\nu+1)} \left(-e^{i a z} \sqrt{\pi}\, (a z)^{\nu+\frac{1}{2}} + 2^{\nu+\frac{1}{2}} a\, i\, z\, \Gamma(\nu+1)\, j_{\nu+\frac{1}{2}}(a z) + 2^{\nu+\frac{1}{2}} a\, z\, \Gamma(\nu+1)\, j_{\nu-\frac{1}{2}}(a z)\right)$$

03.21.21.0018.01

$$\int z^{\alpha-1} e^{i a z} j_\nu(a z)\, dz = \frac{2^{-\nu-1} \sqrt{\pi}\, z^\alpha (a z)^\nu}{(\alpha+\nu)\, \Gamma\left(\nu+\frac{3}{2}\right)} \, {}_2F_2(\nu+1, \alpha+\nu; \alpha+\nu+1, 2\nu+2; 2 i a z)$$

03.21.21.0019.01

$$\int z^{-\nu} e^{i a z} j_\nu(a z)\, dz = \frac{i\, 2^{-\nu-\frac{1}{2}} \sqrt{\pi}\, z^{\frac{1}{2}-\nu}}{\sqrt{a z}\, (2\nu+1)\, \Gamma(\nu+1)} \left((a z)^\nu - 2^\nu e^{i a z} J_\nu(a z)\, \Gamma(\nu+1) + 2^\nu e^{i a z}\, i\, J_{\nu+1}(a z)\, \Gamma(\nu+1)\right)$$

Power arguments

03.21.21.0020.01

$$\int z^{\alpha-1} e^{-i a z^r} j_\nu(a z^r)\, dz = \frac{2^{-\nu-1} \sqrt{\pi}\, z^\alpha (a z^r)^\nu}{(\alpha+r\nu)\, \Gamma\left(\nu+\frac{3}{2}\right)} \, {}_2F_2\left(\nu+1, \frac{\alpha}{r}+\nu; \frac{\alpha}{r}+\nu+1, 2\nu+2; -2 i a z^r\right)$$

03.21.21.0021.01

$$\int z^{\alpha-1} e^{i a z^r} j_\nu(a z^r)\, dz = \frac{2^{-\nu-1} \sqrt{\pi}\, z^\alpha (a z^r)^\nu}{(\alpha+r\nu)\, \Gamma\left(\nu+\frac{3}{2}\right)} \, {}_2F_2\left(\nu+1, \frac{\alpha}{r}+\nu; \frac{\alpha}{r}+\nu+1, 2\nu+2; 2 i a z^r\right)$$

Involving trigonometric functions

Involving sin

Linear arguments

03.21.21.0022.01

$$\int \sin(a z)\, j_\nu(a z)\, dz = \frac{2^{-\nu-1} \sqrt{\pi}\, z (a z)^{\nu+1}}{(\nu+2)\, \Gamma\left(\nu+\frac{3}{2}\right)} \, {}_3F_4\left(\frac{\nu}{2}+1, \frac{\nu}{2}+1, \frac{\nu}{2}+\frac{3}{2}; \frac{3}{2}, \frac{\nu}{2}+2, \nu+\frac{3}{2}, \nu+2; -a^2 z^2\right)$$

03.21.21.0023.01

$$\int \sin(b+a z)\, j_\nu(a z)\, dz = \frac{2^{-\nu-1} \sqrt{\pi}\, z (a z)^\nu}{\left(\nu^2+3\nu+2\right) \Gamma\left(\nu+\frac{3}{2}\right)} \left(a z (\nu+1) \cos(b)\, {}_3F_4\left(\frac{\nu}{2}+1, \frac{\nu}{2}+1, \frac{\nu}{2}+\frac{3}{2}; \frac{3}{2}, \frac{\nu}{2}+2, \nu+\frac{3}{2}, \nu+2; -a^2 z^2\right) + \right.$$
$$\left. (\nu+2) \sin(b)\, {}_3F_4\left(\frac{\nu}{2}+\frac{1}{2}, \frac{\nu}{2}+\frac{1}{2}, \frac{\nu}{2}+1; \frac{1}{2}, \frac{\nu}{2}+\frac{3}{2}, \nu+1, \nu+\frac{3}{2}; -a^2 z^2\right)\right)$$

Power arguments

03.21.21.0024.01

$$\int \sin(a z^r)\, j_\nu(a z^r)\, dz = \frac{2^{-\nu-1}\sqrt{\pi}\, z\,(a z^r)^{\nu+1}}{(\nu r + r + 1)\,\Gamma\left(\nu + \frac{3}{2}\right)}\, {}_3F_4\left(\frac{\nu}{2}+1, \frac{\nu}{2}+\frac{3}{2}, \frac{\nu}{2}+\frac{1}{2}+\frac{1}{2r}; \frac{3}{2}, \frac{\nu}{2}+\frac{3}{2}+\frac{1}{2r}, \nu+\frac{3}{2}, \nu+2; -a^2 z^{2r}\right)$$

03.21.21.0025.01

$$\int \sin(a z^r + b)\, j_\nu(a z^r)\, dz = \frac{2^{-\nu-1}\sqrt{\pi}\, z\,(a z^r)^{\nu}}{\left(\nu(\nu+1) r^2 + 2\nu r + r + 1\right)\Gamma\left(\nu+\frac{3}{2}\right)}$$
$$\left(a(r\nu+1)\cos(b)\, {}_3F_4\left(\frac{\nu}{2}+1, \frac{\nu}{2}+\frac{3}{2}, \frac{\nu}{2}+\frac{1}{2}+\frac{1}{2r}; \frac{3}{2}, \frac{\nu}{2}+\frac{3}{2}+\frac{1}{2r}, \nu+\frac{3}{2}, \nu+2; -a^2 z^{2r}\right) z^r + \right.$$
$$\left. (\nu r + r + 1)\sin(b)\, {}_3F_4\left(\frac{\nu}{2}+\frac{1}{2}, \frac{\nu}{2}+1, \frac{\nu}{2}+\frac{1}{2r}; \frac{1}{2}, \frac{\nu}{2}+\frac{1}{2r}+1, \nu+1, \nu+\frac{3}{2}; -a^2 z^{2r}\right)\right)$$

Involving cos

Linear arguments

03.21.21.0026.01

$$\int \cos(a z)\, j_\nu(a z)\, dz = \frac{2^{-\nu-1}\sqrt{\pi}\, z\,(a z)^{\nu}}{(\nu+1)\,\Gamma\left(\nu+\frac{3}{2}\right)}\, {}_3F_4\left(\frac{\nu}{2}+\frac{1}{2}, \frac{\nu}{2}+\frac{1}{2}, \frac{\nu}{2}+1; \frac{1}{2}, \frac{\nu}{2}+\frac{3}{2}, \nu+1, \nu+\frac{3}{2}; -a^2 z^2\right)$$

03.21.21.0027.01

$$\int \cos(b + a z)\, j_\nu(a z)\, dz =$$
$$-\frac{2^{-\nu-1}\sqrt{\pi}\, z\,(a z)^{\nu}}{\left(\nu^2+3\nu+2\right)\Gamma\left(\nu+\frac{3}{2}\right)}\left(a z(\nu+1)\sin(b)\, {}_3F_4\left(\frac{\nu}{2}+1, \frac{\nu}{2}+1, \frac{\nu}{2}+\frac{3}{2}; \frac{3}{2}, \frac{\nu}{2}+2, \nu+\frac{3}{2}, \nu+2; -a^2 z^2\right) - \right.$$
$$\left. (\nu+2)\cos(b)\, {}_3F_4\left(\frac{\nu}{2}+\frac{1}{2}, \frac{\nu}{2}+\frac{1}{2}, \frac{\nu}{2}+1; \frac{1}{2}, \frac{\nu}{2}+\frac{3}{2}, \nu+1, \nu+\frac{3}{2}; -a^2 z^2\right)\right)$$

Power arguments

03.21.21.0028.01

$$\int \cos(a z^r)\, j_\nu(a z^r)\, dz = \frac{2^{-\nu-1}\sqrt{\pi}\, z\,(a z^r)^{\nu}}{(r\nu+1)\,\Gamma\left(\nu+\frac{3}{2}\right)}\, {}_3F_4\left(\frac{\nu}{2}+\frac{1}{2}, \frac{\nu}{2}+1, \frac{\nu}{2}+\frac{1}{2r}; \frac{1}{2}, \frac{\nu}{2}+\frac{1}{2r}+1, \nu+1, \nu+\frac{3}{2}; -a^2 z^{2r}\right)$$

03.21.21.0029.01

$$\int \cos(a z^r + b)\, j_\nu(a z^r)\, dz =$$
$$\frac{2^{-\nu-1}\sqrt{\pi}\, z\,(a z^r)^{\nu}}{\left(\nu(\nu+1) r^2 + 2\nu r + r + 1\right)\Gamma\left(\nu+\frac{3}{2}\right)}\left((\nu r + r + 1)\cos(b)\, {}_3F_4\left(\frac{\nu}{2}+\frac{1}{2}, \frac{\nu}{2}+1, \frac{\nu}{2}+\frac{1}{2r}; \frac{1}{2}, \frac{\nu}{2}+\frac{1}{2r}+1, \nu+1, \nu+\frac{3}{2}; -a^2 z^{2r}\right) - \right.$$
$$\left. a z^r (r\nu+1)\sin(b)\, {}_3F_4\left(\frac{\nu}{2}+1, \frac{\nu}{2}+\frac{3}{2}, \frac{\nu}{2}+\frac{1}{2}+\frac{1}{2r}; \frac{3}{2}, \frac{\nu}{2}+\frac{3}{2}+\frac{1}{2r}, \nu+\frac{3}{2}, \nu+2; -a^2 z^{2r}\right)\right)$$

Involving trigonometric functions and a power function

Involving sin and power

Linear arguments

03.21.21.0030.01

$$\int z^{\alpha-1}\sin(a z)\, j_\nu(a z)\, dz = \frac{2^{-\nu-1}\sqrt{\pi}\, z^\alpha\, (a z)^{\nu+1}}{(\alpha+\nu+1)\,\Gamma\left(\nu+\frac{3}{2}\right)}\, {}_3F_4\left(\frac{\nu}{2}+1, \frac{\nu}{2}+\frac{3}{2}, \frac{\alpha}{2}+\frac{\nu}{2}+\frac{1}{2}; \frac{3}{2}, \frac{\alpha}{2}+\frac{\nu}{2}+\frac{3}{2}, \nu+\frac{3}{2}, \nu+2; -a^2 z^2\right)$$

03.21.21.0031.01

$$\int z^{\alpha-1}\sin(b+a z)\, j_\nu(a z)\, dz = \frac{2^{-\nu-1}\sqrt{\pi}\, z^\alpha\, (a z)^\nu}{\left(\alpha^2+2\nu\alpha+\alpha+\nu^2+\nu\right)\Gamma\left(\nu+\frac{3}{2}\right)}\left(a z(\alpha+\nu)\cos(b)\, {}_3F_4\left(\frac{\nu}{2}+1, \frac{\nu}{2}+\frac{3}{2}, \frac{\alpha}{2}+\frac{\nu}{2}+\frac{1}{2}; \frac{3}{2}, \frac{\alpha}{2}+\frac{\nu}{2}+\frac{3}{2}, \nu+\frac{3}{2}, \nu+2; -a^2 z^2\right) + (\alpha+\nu+1)\sin(b)\, {}_3F_4\left(\frac{\nu}{2}+\frac{1}{2}, \frac{\nu}{2}+1, \frac{\alpha}{2}+\frac{\nu}{2}; \frac{1}{2}, \frac{\alpha}{2}+\frac{\nu}{2}+1, \nu+1, \nu+\frac{3}{2}; -a^2 z^2\right)\right)$$

Power arguments

03.21.21.0032.01

$$\int z^{\alpha-1}\sin(a z^r)\, j_\nu(a z^r)\, dz = \frac{2^{-\nu-1}\sqrt{\pi}\, z^\alpha\, (a z^r)^{\nu+1}}{(\nu r+r+\alpha)\,\Gamma\left(\nu+\frac{3}{2}\right)}\, {}_3F_4\left(\frac{\nu}{2}+1, \frac{\nu}{2}+\frac{3}{2}, \frac{\alpha}{2r}+\frac{\nu}{2}+\frac{1}{2}; \frac{3}{2}, \frac{\alpha}{2r}+\frac{\nu}{2}+\frac{3}{2}, \nu+\frac{3}{2}, \nu+2; -a^2 z^{2r}\right)$$

03.21.21.0033.01

$$\int z^{\alpha-1}\sin(a z^r+b)\, j_\nu(a z^r)\, dz = \frac{2^{-\nu-1}\sqrt{\pi}\, z^\alpha\, (a z^r)^\nu}{\left(\nu(\nu+1) r^2+(2\nu\alpha+\alpha) r+\alpha^2\right)\Gamma\left(\nu+\frac{3}{2}\right)} \left(a(\alpha+r\nu)\cos(b)\, {}_3F_4\left(\frac{\nu}{2}+1, \frac{\nu}{2}+\frac{3}{2}, \frac{\alpha}{2r}+\frac{\nu}{2}+\frac{1}{2}; \frac{3}{2}, \frac{\alpha}{2r}+\frac{\nu}{2}+\frac{3}{2}, \nu+\frac{3}{2}, \nu+2; -a^2 z^{2r}\right) z^r + (\nu r+r+\alpha)\sin(b)\, {}_3F_4\left(\frac{\nu}{2}+\frac{1}{2}, \frac{\nu}{2}+1, \frac{\alpha}{2r}+\frac{\nu}{2}; \frac{1}{2}, \frac{\alpha}{2r}+\frac{\nu}{2}+1, \nu+1, \nu+\frac{3}{2}; -a^2 z^{2r}\right)\right)$$

Involving cos and power

Linear arguments

03.21.21.0034.01

$$\int z^{\alpha-1}\cos(a z)\, j_\nu(a z)\, dz = \frac{2^{-\nu-1}\sqrt{\pi}\, z^\alpha\, (a z)^\nu}{(\alpha+\nu)\,\Gamma\left(\nu+\frac{3}{2}\right)}\, {}_3F_4\left(\frac{\nu}{2}+\frac{1}{2}, \frac{\nu}{2}+1, \frac{\alpha}{2}+\frac{\nu}{2}; \frac{1}{2}, \frac{\alpha}{2}+\frac{\nu}{2}+1, \nu+1, \nu+\frac{3}{2}; -a^2 z^2\right)$$

03.21.21.0035.01

$$\int z^{\alpha-1}\cos(b+a\,z)\,j_\nu(a\,z)\,dz = \frac{2^{-\nu-1}\sqrt{\pi}\,z^\alpha\,(a\,z)^\nu}{(\alpha^2+2\,\nu\,\alpha+\alpha+\nu^2+\nu)\,\Gamma\left(\nu+\frac{3}{2}\right)}\left((\alpha+\nu+1)\cos(b)\,{}_3F_4\left(\frac{\nu}{2}+\frac{1}{2},\frac{\nu}{2}+1,\frac{\alpha}{2}+\frac{\nu}{2};\frac{1}{2},\frac{\alpha}{2}+\frac{\nu}{2}+1,\nu+1,\nu+\frac{3}{2};-a^2z^2\right)-\\ a\,z\,(\alpha+\nu)\sin(b)\,{}_3F_4\left(\frac{\nu}{2}+1,\frac{\nu}{2}+\frac{3}{2},\frac{\alpha}{2}+\frac{\nu}{2}+\frac{1}{2};\frac{3}{2},\frac{\alpha}{2}+\frac{\nu}{2}+\frac{3}{2},\nu+\frac{3}{2},\nu+2;-a^2z^2\right)\right)$$

Power arguments

03.21.21.0036.01

$$\int z^{\alpha-1}\cos(a\,z^r)\,j_\nu(a\,z^r)\,dz = \frac{2^{-\nu-1}\sqrt{\pi}\,z^\alpha\,(a\,z^r)^\nu}{(\alpha+r\,\nu)\,\Gamma\left(\nu+\frac{3}{2}\right)}\,{}_3F_4\left(\frac{\nu}{2}+\frac{1}{2},\frac{\nu}{2}+1,\frac{\alpha}{2\,r}+\frac{\nu}{2};\frac{1}{2},\frac{\alpha}{2\,r}+\frac{\nu}{2}+1,\nu+1,\nu+\frac{3}{2};-a^2z^{2\,r}\right)$$

03.21.21.0037.01

$$\int z^{\alpha-1}\cos(a\,z^r+b)\,j_\nu(a\,z^r)\,dz = \frac{2^{-\nu-1}\sqrt{\pi}\,z^\alpha\,(a\,z^r)^\nu}{(\nu\,(\nu+1)\,r^2+(2\,\nu\,\alpha+\alpha)\,r+\alpha^2)\,\Gamma\left(\nu+\frac{3}{2}\right)}\\ \left((\nu\,r+r+\alpha)\cos(b)\,{}_3F_4\left(\frac{\nu}{2}+\frac{1}{2},\frac{\nu}{2}+1,\frac{\alpha}{2\,r}+\frac{\nu}{2};\frac{1}{2},\frac{\alpha}{2\,r}+\frac{\nu}{2}+1,\nu+1,\nu+\frac{3}{2};-a^2z^{2\,r}\right)-\\ a\,z^r\,(\alpha+r\,\nu)\sin(b)\,{}_3F_4\left(\frac{\nu}{2}+1,\frac{\nu}{2}+\frac{3}{2},\frac{\alpha}{2\,r}+\frac{\nu}{2}+\frac{1}{2};\frac{3}{2},\frac{\alpha}{2\,r}+\frac{\nu}{2}+\frac{3}{2},\nu+\frac{3}{2},\nu+2;-a^2z^{2\,r}\right)\right)$$

Involving functions of the direct function

Involving elementary functions of the direct function

Involving powers of the direct function

Linear arguments

03.21.21.0038.01

$$\int j_\nu(a\,z)^2\,dz = \frac{4^{-\nu}\,\pi\,z\,(a\,z)^{2\,\nu}}{(2\,\nu+1)^3\,\Gamma\left(\nu+\frac{1}{2}\right)^2}\,{}_2F_3\left(\nu+\frac{1}{2},\nu+1;\nu+\frac{3}{2},\nu+\frac{3}{2},2\,\nu+2;-a^2z^2\right)$$

03.21.21.0039.01

$$\int j_\nu(z)^2\,dz = \frac{4^{-\nu}\,\pi\,z^{2\,\nu+1}}{(2\,\nu+1)^3\,\Gamma\left(\nu+\frac{1}{2}\right)^2}\,{}_2F_3\left(\nu+\frac{1}{2},\nu+1;\nu+\frac{3}{2},\nu+\frac{3}{2},2\,\nu+2;-z^2\right)$$

03.21.21.0040.01

$$\int \frac{1}{z^2\,j_{-\nu-1}(z)\,j_\nu(z)}\,dz = -\csc\left(\pi\left(\nu+\frac{1}{2}\right)\right)\log\left(\frac{j_{-\nu-1}(z)}{j_\nu(z)}\right)$$

Power arguments

03.21.21.0041.01

$$\int j_\nu(a z^r)^2 \, dz = \frac{2^{-2\nu-2} \pi z (a z^r)^{2\nu}}{(2 r \nu + 1) \Gamma\left(\nu + \frac{3}{2}\right)^2} \, {}_2F_3\left(\nu + 1, \nu + \frac{1}{2r}; \nu + \frac{3}{2}, \nu + \frac{1}{2r} + 1, 2\nu + 2; -a^2 z^{2r}\right)$$

Involving products of the direct function

Linear arguments

03.21.21.0042.01

$$\int j_\mu(a z) \, j_\nu(a z) \, dz = -\frac{2z}{-4\mu^2 - 4\mu + 4\nu^2 + 8\nu + 3} \left(2 a z \, j_{\mu-1}(a z) \, j_{\nu+\frac{1}{2}}(a z) + j_\mu(a z) \left((-2\mu + 2\nu + 1) \, j_{\nu+\frac{1}{2}}(a z) - 2 a z \, j_{\nu-\frac{1}{2}}(a z)\right)\right)$$

03.21.21.0043.01

$$\int j_\nu(a z) \, j_{\nu+1}(a z) \, dz = \frac{z}{2\nu + 3} \left(a z \, j_{\nu+\frac{1}{2}}(a z)^2 - j_{\nu+\frac{3}{2}}(a z) \, j_{\nu+\frac{1}{2}}(a z) - a z \, j_{\nu-\frac{1}{2}}(a z) \, j_{\nu+\frac{3}{2}}(a z)\right)$$

03.21.21.0044.01

$$\int j_0(a z) \, j_1(a z) \, dz = -\frac{\sin^2(a z)}{2 a^3 z^2}$$

Power arguments

03.21.21.0045.01

$$\int j_\mu(a z^r) \, j_\nu(a z^r) \, dz = \frac{2^{-\mu-\nu-2} \pi z (a z^r)^{\mu+\nu}}{(r(\mu + \nu) + 1) \Gamma\left(\mu + \frac{3}{2}\right) \Gamma\left(\nu + \frac{3}{2}\right)}$$

$${}_3F_4\left(\frac{\mu}{2} + \frac{\nu}{2} + 1, \frac{\mu}{2} + \frac{\nu}{2} + \frac{3}{2}, \frac{\mu}{2} + \frac{\nu}{2} + \frac{1}{2r}; \mu + \frac{3}{2}, \frac{\mu}{2} + \frac{\nu}{2} + \frac{1}{2r} + 1, \nu + \frac{3}{2}, \mu + \nu + 2; -a^2 z^{2r}\right)$$

Involving functions of the direct function and elementary functions

Involving elementary functions of the direct function and elementary functions

Involving powers of the direct function and a power function

Linear arguments

03.21.21.0046.01

$$\int z^{\alpha-1} \, j_\nu(a z)^2 \, dz = \frac{2^{-2\nu-2} \pi z^\alpha (a z)^{2\nu}}{(\alpha + 2\nu) \Gamma\left(\nu + \frac{3}{2}\right)^2} \, {}_2F_3\left(\nu + 1, \frac{\alpha}{2} + \nu; \nu + \frac{3}{2}, \frac{\alpha}{2} + \nu + 1, 2\nu + 2; -a^2 z^2\right)$$

03.21.21.0047.01

$$\int z^{1-2\nu} \, j_\nu(a z)^2 \, dz = -\frac{2^{-2(\nu+1)} z^{-2\nu}}{(2 \nu a + a) \Gamma(\nu + 1)^2} \left(-\pi (a z)^{2\nu} + 2^{2\nu+1} a z \Gamma(\nu + 1)^2 \, j_{\nu+\frac{1}{2}}(a z)^2 + 2^{2\nu+1} a z \Gamma(\nu + 1)^2 \, j_{\nu-\frac{1}{2}}(a z)^2\right)$$

03.21.21.0048.01

$$\int z\, j_0(a\, z)^2\, dz = \frac{\log(z) - \text{Ci}(2\, a\, z)}{2\, a^2}$$

03.21.21.0049.01

$$\int \frac{1}{z^2\, j_\nu(z)^2}\, dz = \frac{y_\nu(z)}{j_\nu(z)}$$

03.21.21.0050.01

$$\int \frac{j_\nu(a\, z)^2}{z}\, dz = \frac{\left(2\, a^2\, z^2 - 2\, \nu - 1\right) j_{\nu+\frac{1}{2}}(a\, z)^2 - 2\, a\, z\, (2\, \nu + 1)\, j_{\nu-\frac{1}{2}}(a\, z)\, j_{\nu+\frac{1}{2}}(a\, z) + 2\, a^2\, z^2\, j_{\nu-\frac{1}{2}}(a\, z)^2}{4\, \nu^2 + 8\, \nu + 3}$$

Power arguments

03.21.21.0051.01

$$\int z^{\alpha-1}\, j_\nu(a\, z^r)^2\, dz = \frac{2^{-2(\nu+1)}\, \pi\, z^\alpha\, (a\, z^r)^{2\nu}}{(\alpha + 2\, r\, \nu)\, \Gamma\left(\nu + \frac{3}{2}\right)^2}\, {}_2F_3\left(\nu + 1, \frac{\alpha}{2\, r} + \nu; \nu + \frac{3}{2}, \frac{\alpha}{2\, r} + \nu + 1, 2\, \nu + 2; -a^2\, z^{2r}\right)$$

Involving products of the direct function and a power function

Linear arguments

03.21.21.0052.01

$$\int z^{\alpha-1}\, j_\mu(a\, z)\, j_\nu(a\, z)\, dz =$$
$$\frac{2^{-\mu-\nu-2}\, \pi\, z^\alpha\, (a\, z)^{\mu+\nu}}{(\alpha + \mu + \nu)\, \Gamma\left(\mu + \frac{3}{2}\right) \Gamma\left(\nu + \frac{3}{2}\right)}\, {}_3F_4\left(\frac{\mu}{2} + \frac{\nu}{2} + 1, \frac{\mu}{2} + \frac{\nu}{2} + \frac{3}{2}, \frac{\alpha}{2} + \frac{\mu}{2} + \frac{\nu}{2}; \mu + \frac{3}{2}, \frac{\alpha}{2} + \frac{\mu}{2} + \frac{\nu}{2} + 1, \nu + \frac{3}{2}, \mu + \nu + 2; -a^2\, z^2\right)$$

03.21.21.0053.01

$$\int z^2\, j_\nu(a\, z)\, j_\nu(b\, z)\, dz = \frac{z^2\, (b\, j_{\nu-1}(b\, z)\, j_\nu(a\, z) - a\, j_{\nu-1}(a\, z)\, j_\nu(b\, z))}{a^2 - b^2}$$

03.21.21.0054.01

$$\int z^2\, j_{-\nu-1}(a\, z)\, j_\nu(b\, z)\, dz = -\frac{z^2}{a^2 - b^2}\, (a\, j_{-\nu-2}(a\, z)\, j_\nu(b\, z) + b\, j_{-\nu-1}(a\, z)\, j_{\nu+1}(b\, z))$$

03.21.21.0055.01

$$\int \left(\left(a^2 - b^2\right) z^2 + \mu^2 - \left(\nu + \frac{1}{2}\right)^2\right) j_{\mu-\frac{1}{2}}(b\, z)\, j_\nu(a\, z)\, dz = z\left(b\, z\, j_{\mu-\frac{3}{2}}(b\, z)\, j_\nu(a\, z) + j_{\mu-\frac{1}{2}}(b\, z) \left(\left(-\mu + \nu + \frac{1}{2}\right) j_\nu(a\, z) - a\, z\, j_{\nu-1}(a\, z)\right)\right)$$

03.21.21.0056.01

$$\int \left(\left(a^2 - b^2\right) z^2 + \mu^2 - \left(\nu + \frac{1}{2}\right)^2\right) j_{\mu-\frac{1}{2}}(b\, z)\, j_\nu(a\, z)\, dz = z\left(b\, z\, j_{\mu-\frac{3}{2}}(b\, z)\, j_\nu(a\, z) + j_{\mu-\frac{1}{2}}(b\, z) \left(\left(-\mu + \nu + \frac{1}{2}\right) j_\nu(a\, z) - a\, z\, j_{\nu-1}(a\, z)\right)\right)$$

Power arguments

03.21.21.0057.01

$$\int z^{\alpha-1}\, j_\mu(a\,z^r)\, j_\nu(a\,z^r)\, dz = \frac{2^{-\mu-\nu-2}\,\pi\, z^\alpha\,(a\,z^r)^{\mu+\nu}}{(\alpha+r\,(\mu+\nu))\,\Gamma\left(\mu+\frac{3}{2}\right)\Gamma\left(\nu+\frac{3}{2}\right)}$$
$$\,_3F_4\left(\frac{\mu}{2}+\frac{\nu}{2}+1,\, \frac{\mu}{2}+\frac{\nu}{2}+\frac{3}{2},\, \frac{\alpha}{2\,r}+\frac{\mu}{2}+\frac{\nu}{2};\, \mu+\frac{3}{2},\, \frac{\alpha}{2\,r}+\frac{\mu}{2}+\frac{\nu}{2}+1,\, \nu+\frac{3}{2},\, \mu+\nu+2;\, -a^2\,z^{2\,r}\right)$$

03.21.21.0058.01

$$\int \sqrt{z}\; j_\nu\left(a\sqrt{z}\right) j_\nu\left(b\sqrt{z}\right) dz = \frac{2\,z}{a^2-b^2}\left(b\, j_{\nu-1}\left(b\sqrt{z}\right) j_\nu\left(a\sqrt{z}\right) - a\, j_{\nu-1}\left(a\sqrt{z}\right) j_\nu\left(b\sqrt{z}\right)\right)$$

03.21.21.0059.01

$$\int \sqrt{z}\; j_{-\nu-1}\left(a\sqrt{z}\right) j_\nu\left(b\sqrt{z}\right) dz = -\frac{2\,z}{a^2-b^2}\left(a\, j_{-\nu-2}\left(a\sqrt{z}\right) j_\nu\left(b\sqrt{z}\right) + b\, j_{-\nu-1}\left(a\sqrt{z}\right) j_{\nu+1}\left(b\sqrt{z}\right)\right)$$

Definite integration

For the direct function itself

03.21.21.0060.01

$$\int_0^\infty j_\nu(t)\, dt = \frac{\sqrt{\pi}\;\Gamma\left(\frac{\nu+1}{2}\right)}{2\,\Gamma\left(\frac{\nu}{2}+1\right)} \;/;\; \mathrm{Re}(\nu) > -1$$

03.21.21.0061.01

$$\int_0^\infty j_\nu(t)\, dt = \frac{2^{\alpha-2}\sqrt{\pi}\;\Gamma\left(\frac{\alpha+\nu}{2}\right)}{\Gamma\left(\frac{1}{2}\,(-\alpha+\nu+3)\right)} \;/;\; \mathrm{Re}(\alpha+\nu) > 0 \land \mathrm{Re}(\alpha) < 2$$

Involving the direct function

03.21.21.0062.01

$$\int_0^\infty j_\nu(a\,t)\, j_\nu(b\,t)\, dt = \frac{\pi\, a^{-\nu-1}\, b^\nu}{4\,\nu+2} \;/;\; \mathrm{Re}(\nu) > -\frac{1}{2} \bigwedge 0 < b < a$$

03.21.21.0063.01

$$\int_0^\infty j_\nu(t)^2\, dt = \frac{\pi}{4\,\nu+2} \;/;\; \mathrm{Re}(\nu) > -\frac{1}{2} \;/;\; \mathrm{Re}(\nu) > -\frac{1}{2}$$

03.21.21.0064.01

$$\int_0^\infty t^{\alpha-1}\, j_\nu(t)^2\, dt = -\frac{\sqrt{\pi}\;\Gamma\left(1-\frac{\alpha}{2}\right)\Gamma\left(\frac{\alpha}{2}+\nu\right)}{2\,(\alpha-1)\,\Gamma\left(\frac{1-\alpha}{2}\right)\Gamma\left(-\frac{\alpha}{2}+\nu+2\right)} \;/;\; \mathrm{Re}(\alpha+2\,\nu) > 0 \land \mathrm{Re}(\alpha) < 2$$

03.21.21.0065.01

$$\int_0^\infty t^2\, j_\nu(a\,t)\, j_\nu(b\,t)\, dt = \frac{\pi\,\delta(a-b)}{2\,a^{3/2}\sqrt{b}} \;/;\; a \in \mathbb{R} \bigwedge b \in \mathbb{R} \bigwedge \nu \in \mathbb{R}$$

03.21.21.0066.01

$$\int_0^\infty t^{\alpha-1}\, j_\lambda(a\,t)\, j_\mu(b\,t)\, j_\nu(c\,t)\, dt = \frac{2^{\alpha-4}\, a^\lambda\, b^\mu\, c^{-\alpha-\lambda-\mu}\,\pi^{3/2}\,\Gamma\left(\frac{\alpha+\lambda+\mu+\nu}{2}\right)}{\Gamma\left(\lambda+\frac{3}{2}\right)\Gamma\left(\mu+\frac{3}{2}\right)\Gamma\left(\frac{3-\alpha-\lambda-\mu+\nu}{2}\right)}\, F_{0,1,1}^{2,0,0}\left(\begin{matrix}\frac{\alpha+\lambda+\mu+\nu}{2},\, \frac{\alpha+\lambda+\mu-\nu-1}{2};;; \\ ;\lambda+\frac{3}{2};\, \mu+\frac{3}{2};\end{matrix}\; \frac{a^2}{c^2},\, \frac{b^2}{c^2}\right) /;$$

$$a \in \mathbb{R} \bigwedge b \in \mathbb{R} \bigwedge c \in \mathbb{R} \bigwedge a > 0 \bigwedge b > 0 \bigwedge \mathrm{Re}(\alpha+\lambda+\mu+\nu) > 0 \bigwedge a+b < c \bigwedge \mathrm{Re}(\alpha) < 4$$

Integral transforms

Fourier cos transforms

03.21.22.0001.01

$$\mathcal{F}c_t[j_\nu(t)](z) = \frac{\theta(1-z)\sqrt{2}\,\Gamma\left(\frac{\nu+1}{2}\right)}{\nu\,\Gamma\left(\frac{\nu}{2}\right)}\,{}_2F_1\left(-\frac{\nu}{2}, \frac{\nu+1}{2}; \frac{1}{2}; z^2\right) - \frac{\theta(z-1)\,2^{-\nu-\frac{1}{2}}\,z^{-\nu-1}\,\Gamma(\nu+1)\sin\left(\frac{\pi\nu}{2}\right)}{\Gamma\left(\nu+\frac{3}{2}\right)}\,{}_2F_1\left(\frac{\nu+1}{2}, \frac{\nu+2}{2}; \nu+\frac{3}{2}; \frac{1}{z^2}\right) /; z>0 \land z \neq 1 \land \mathrm{Re}(\nu) > -1$$

Fourier sin transforms

03.21.22.0002.01

$$\mathcal{F}s_t[j_\nu(t)](z) = \frac{\theta(z-1)\,2^{-\nu-\frac{1}{2}}\,z^{-\nu-1}\cos\left(\frac{\pi\nu}{2}\right)\Gamma(\nu+1)}{\Gamma\left(\nu+\frac{3}{2}\right)}\,{}_2F_1\left(\frac{\nu+1}{2}, \frac{\nu+2}{2}; \nu+\frac{3}{2}; \frac{1}{z^2}\right) - \frac{\theta(1-z)}{2\sqrt{2}\,z\,\Gamma\left(\frac{\nu+3}{2}\right)}\,\Gamma\left(\frac{\nu}{2}\right) \left(\left(1-3z^2\right){}_2F_1\left(\frac{1-\nu}{2}, \frac{\nu+2}{2}; \frac{1}{2}; z^2\right) + \left(z^2-1\right){}_2F_1\left(\frac{1-\nu}{2}, \frac{\nu+2}{2}; -\frac{1}{2}; z^2\right)\right) /; z>0 \land z\neq 1 \land \mathrm{Re}(\nu) > -2$$

Laplace transforms

03.21.22.0003.01

$$\mathcal{L}_t[j_\nu(t)](z) = 2^{-\nu-1}\,z^{-\nu-1}\sqrt{\pi}\;\Gamma(\nu+1)\,{}_2\tilde{F}_1\left(\frac{\nu+1}{2}, \frac{\nu+2}{2}; \nu+\frac{3}{2}; -\frac{1}{z^2}\right) /; \mathrm{Re}(z) > 0 \land \mathrm{Re}(\nu) > -1$$

03.21.22.0004.01

$$\mathcal{L}_t[j_n(t)](z) = i^{n+1}\,Q_n^0(i\,z) /; n \in \mathbb{N} \land \mathrm{Re}(z) > 0$$

03.21.22.0005.01

$$\mathcal{L}_t\left[t^{\alpha-1}\,j_\nu(t)\right](z) = 2^{-\nu-1}\,z^{-\alpha-\nu}\sqrt{\pi}\;\Gamma(\alpha+\nu)\,{}_2\tilde{F}_1\left(\frac{\alpha+\nu}{2}, \frac{1}{2}(\alpha+\nu+1); \nu+\frac{3}{2}; -\frac{1}{z^2}\right) /; \mathrm{Re}(z) > 0 \land \mathrm{Re}(\alpha) > 0 \land \mathrm{Re}(\alpha+\nu) > 0$$

03.21.22.0006.01

$$\mathcal{L}_t[j_n(t)](z) = z^n\left(\tan^{-1}\left(\frac{1}{z}\right)\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor} c_{k,l}\,z^{-2k} - \sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor} c_{k,l}\sum_{m=0}^{\left\lfloor\frac{n-1}{2}\right\rfloor-k}\frac{(-1)^m\,z^{-2k-2m-1}}{2m+1}\right) /; n\in\mathbb{N}^+ \bigwedge c_{0,0}=0 \bigwedge c_{1,2}=\frac{1}{2} \bigwedge c_{0,n} = \frac{(-1)^n\,(2n-1)!!}{n!} \bigwedge \left(c_{k,n} = \frac{(n-1)\,c_{k-1,n-2}}{n} - \frac{(2n-1)\,c_{k,n-1}}{n} /; n\geq 1 \land k \leq n\right) \bigwedge (c_{k,n} = 0 /; k > n)$$

Mellin transforms

03.21.22.0007.01

$$\mathcal{M}_t[j_\nu(t)](z) = \frac{2^{z-2}\sqrt{\pi}\;\Gamma\left(\frac{z+\nu}{2}\right)}{\Gamma\left(\frac{1}{2}(-z+\nu+3)\right)} /; \mathrm{Re}(z) < 2 \land \mathrm{Re}(z+\nu) > 0$$

Hankel transforms

03.21.22.0008.01

$$\mathcal{H}_{t;\mu}[j_\nu(t)](z) = \frac{\theta(1-z)}{\Gamma\left(\frac{1}{4}(-2\mu+2\nu+3)\right)} (-1)^{\mu/4} \sqrt{\frac{\pi}{2}} \sqrt{z} \left(-(-1)^{3/4} z\right)^{\mu} \Gamma\left(\frac{1}{4}(2\mu+2\nu+3)\right)$$
$$ {}_2\tilde{F}_1\left(\frac{1}{4}(2\mu-2\nu+1), \frac{1}{4}(2\mu+2\nu+3); \mu+1; z^2\right) + \frac{\theta(z-1)}{\Gamma\left(\frac{1}{4}(2\mu-2\nu+1)\right)} \sqrt{\frac{\pi}{2}} z^{-\nu-\frac{3}{2}} \Gamma\left(\frac{1}{4}(2\mu+2\nu+3)\right)$$
$$ {}_2\tilde{F}_1\left(\frac{1}{4}(-2\mu+2\nu+3), \frac{1}{4}(2\mu+2\nu+3); \nu+\frac{3}{2}; \frac{1}{z^2}\right) /; z>0 \bigwedge z\neq 1 \bigwedge \mathrm{Re}(\mu+\nu) > -\frac{3}{2}$$

Summation

Infinite summation

03.21.23.0001.01

$$\sum_{k=0}^{\infty} \frac{j_{k+\nu}(x)\, x^k}{k!} = \sqrt{\frac{\pi}{2}} \frac{1}{\sqrt{x}} I_{\nu+\frac{1}{2}}(x)$$

03.21.23.0002.01

$$\sum_{k=0}^{\infty} \frac{\left(2k+\nu+\frac{1}{2}\right) \Gamma\left(k+\nu+\frac{1}{2}\right) j_{2k+\nu}(x)}{k!} = 2^{-\nu-\frac{1}{2}} \sqrt{\pi}\, x^{\nu-\frac{1}{2}}$$

03.21.23.0003.01

$$\sum_{k=1}^{\infty} \frac{(-1)^k}{k} j_{2k-\frac{1}{2}}(x) = \frac{1}{4}\left(2\left(\log\left(\frac{x}{2}\right)+\gamma\right) j_{-\frac{1}{2}}(x) - \pi\, y_{-\frac{1}{2}}(x)\right)$$

03.21.23.0004.01

$$\sum_{k=1}^{\infty} \frac{(-1)^k \left(2k+\nu+\frac{1}{2}\right) j_{2k+\nu}(x)}{k\left(k+\nu+\frac{1}{2}\right)} =$$
$$-2^{\nu-\frac{1}{2}} \left(\nu+\frac{1}{2}\right)!\, x^{-\nu-\frac{1}{2}} \sum_{k=0}^{\nu-\frac{1}{2}} \frac{j_{k-\frac{1}{2}}(x)\, 2^{-k} x^k}{\left(-k+\nu+\frac{1}{2}\right) k!} + \left(\log\left(\frac{x}{2}\right) - \psi^{(0)}\left(\nu+\frac{3}{2}\right)\right) j_\nu(x) - \frac{\pi}{2} y_\nu(x) /; \nu+\frac{1}{2} \in \mathbb{N}$$

03.21.23.0005.01

$$\sum_{k=-\infty}^{\infty} j_{k-\frac{1}{2}}(x)\, t^k = \sqrt{\frac{\pi}{2}} \frac{1}{\sqrt{x}} e^{\frac{1}{2}\left(t-\frac{1}{t}\right)x}$$

03.21.23.0006.01

$$\sum_{k=1}^{\infty} \cos(2kt)\, j_{2k-\frac{1}{2}}(x) = \sqrt{\frac{\pi}{2}} \frac{\cos(x\sin(t))}{2\sqrt{x}} - \frac{1}{2} j_{-\frac{1}{2}}(x)$$

03.21.23.0007.01

$$\sum_{k=1}^{\infty} j_{k-\frac{1}{2}}(x)^2 = \frac{\pi}{4x} - \frac{1}{2} j_{-\frac{1}{2}}(x)^2$$

03.21.23.0008.01

$$\sum_{k=0}^{\infty} \sin((2k+1)t)\, j_{2k+\frac{1}{2}}(x) = \sqrt{\frac{\pi}{2}}\, \frac{\sin(x\sin(t))}{2\sqrt{x}}$$

03.21.23.0009.01

$$\sum_{k=1}^{\infty} (-1)^k \cos(2kt)\, j_{2k-\frac{1}{2}}(x) = \sqrt{\frac{\pi}{2}}\, \frac{\cos(x\cos(t))}{2\sqrt{x}} - \frac{1}{2}\, j_{-\frac{1}{2}}(x)$$

03.21.23.0010.01

$$\sum_{k=0}^{\infty} (-1)^k \cos((2k+1)t)\, j_{2k+\frac{1}{2}}(x) = \sqrt{\frac{\pi}{2}}\, \frac{\sin(x\cos(t))}{2\sqrt{x}}$$

03.21.23.0011.01

$$\sum_{k=1}^{\infty} j_{2k-\frac{1}{2}}(x) = \sqrt{\frac{\pi}{2}}\, \frac{1}{2\sqrt{x}} - \frac{1}{2}\, j_{-\frac{1}{2}}(x)$$

03.21.23.0012.01

$$\sum_{k=1}^{\infty} (-1)^k\, j_{2k-\frac{1}{2}}(x) = \sqrt{\frac{\pi}{2}}\, \frac{\cos(x)}{2\sqrt{x}} - \frac{1}{2}\, j_{-\frac{1}{2}}(x)$$

03.21.23.0013.01

$$\sum_{k=0}^{\infty} (-1)^k\, j_{2k+\frac{1}{2}}(x) = \sqrt{\frac{\pi}{2}}\, \frac{\sin(x)}{2\sqrt{x}}$$

03.21.23.0014.01

$$\sum_{k=0}^{\infty} i^{kn}\, j_{kn-\frac{1}{2}}(z) = \frac{1}{2}\, j_{-\frac{1}{2}}(z) + \sqrt{\frac{\pi}{2}}\, \frac{1}{2n\sqrt{z}} \sum_{k=0}^{n-1} e^{iz\cos\left(\frac{2k\pi}{n}\right)} \text{ /; } n \in \mathbb{N}$$

03.21.23.0015.01

$$\sum_{k=0}^{\infty} (-i)^{kn}\, j_{kn-\frac{1}{2}}(z) = \frac{1}{2}\, j_{-\frac{1}{2}}(z) + \sqrt{\frac{\pi}{2}}\, \frac{1}{2n\sqrt{z}} \sum_{k=0}^{n-1} e^{-iz\cos\left(\frac{2k\pi}{n}\right)} \text{ /; } n \in \mathbb{N}$$

03.21.23.0016.01

$$\sum_{k=0}^{\infty} (4k+2\nu+1)\, j_{2k+\nu}(w)\, j_{2k+\nu}(z) = \frac{(wz)\,(z\, j_{\nu-1}(w)\, j_\nu(z) - w\, j_{\nu-1}(z)\, j_\nu(w))}{z^2-w^2} \text{ /; } \operatorname{Re}(\nu) > -\frac{1}{2}$$

Representations through more general functions

Through hypergeometric functions

Involving ${}_0\tilde{F}_1$

03.21.26.0001.01

$$j_\nu(z) = 2^{-\nu-1}\sqrt{\pi}\, z^\nu\, {}_0\tilde{F}_1\left(;\nu+\frac{3}{2}; -\frac{z^2}{4}\right)$$

Involving ${}_0F_1$

03.21.26.0002.01

$$j_\nu(z) = \frac{2^{-\nu-1}\sqrt{\pi}\, z^\nu}{\Gamma\left(\nu+\frac{3}{2}\right)}\, {}_0F_1\left(;\nu+\frac{3}{2};-\frac{z^2}{4}\right) /; -\nu-\frac{1}{2} \notin \mathbb{N}^+$$

Involving ${}_1F_1$

03.21.26.0003.01

$$j_\nu(z) = \frac{2^{-\nu-1}\, e^{-i z}\sqrt{\pi}\, z^\nu}{\Gamma\left(\nu+\frac{3}{2}\right)}\, {}_1F_1(\nu+1;2\nu+2;2 i z)$$

03.21.26.0004.01

$$j_\nu(z) = \frac{2^{-\nu-1}\sqrt{\pi}\, z^\nu}{\Gamma\left(\nu+\frac{3}{2}\right)} \lim_{a\to\infty} {}_1F_1\left(a;\nu+\frac{3}{2};-\frac{z^2}{4a}\right)$$

Through Meijer G

Classical cases for the direct function itself

03.21.26.0005.01

$$j_\nu(z) = \frac{1}{2}\sqrt{\pi}\, z^\nu \left(z^2\right)^{-\frac{\nu}{2}} G_{0,2}^{1,0}\left(\frac{z^2}{4} \,\middle|\, \frac{\nu}{2}, -\frac{1}{2}(\nu+1)\right)$$

03.21.26.0006.01

$$j_\nu(z) = \frac{1}{2}\pi^{3/2} z^\nu \left(-z^2\right)^{-\frac{\nu}{2}} G_{1,3}^{1,0}\left(-\frac{z^2}{4} \,\middle|\, \begin{matrix} \frac{\nu+1}{2} \\ \frac{\nu}{2}, -\frac{1}{2}(\nu+1), \frac{\nu+1}{2} \end{matrix}\right)$$

03.21.26.0007.01

$$j_\nu\left(\sqrt{z}\right) = \frac{1}{2}\sqrt{\pi}\, G_{0,2}^{1,0}\left(\frac{z}{4} \,\middle|\, \frac{\nu}{2}, -\frac{\nu+1}{2}\right)$$

03.21.26.0008.01

$$j_\nu\left(\sqrt{z}\right) + j_{-\nu-1}\left(\sqrt{z}\right) = \sqrt{\pi}\, \cos\left(\frac{1}{4}(2\pi\nu+\pi)\right) G_{1,3}^{2,0}\left(\frac{z}{4} \,\middle|\, \begin{matrix} -\frac{1}{4} \\ -\frac{1}{2}(\nu+1), \frac{\nu}{2}, -\frac{1}{4} \end{matrix}\right)$$

03.21.26.0009.01

$$j_\nu\left(\sqrt{z}\right) - j_{-\nu-1}\left(\sqrt{z}\right) = -\sqrt{\pi}\, \sin\left(\frac{1}{4}\pi(2\nu+1)\right) G_{1,3}^{2,0}\left(\frac{z}{4} \,\middle|\, \begin{matrix} \frac{1}{4} \\ -\frac{\nu+1}{2}, \frac{\nu}{2}, \frac{1}{4} \end{matrix}\right)$$

Classical cases involving cos

03.21.26.0010.01

$$\cos\left(\sqrt{z}\right) j_\nu\left(\sqrt{z}\right) = \frac{1}{2}\sqrt{\pi}\, G_{2,4}^{1,2}\left(z \,\middle|\, \begin{matrix} 0, \frac{1}{2} \\ \frac{\nu}{2}, -\frac{1+\nu}{2}, \frac{\nu+1}{2}, -\frac{\nu}{2} \end{matrix}\right)$$

03.21.26.0011.01

$$\cos\left(a+\sqrt{z}\right) j_\nu\left(\sqrt{z}\right) = \frac{1}{2}\sqrt{\pi}\, G_{3,5}^{2,2}\left(z \,\middle|\, \begin{matrix} 0, \frac{1}{2}, \frac{a}{\pi}+\frac{\nu+1}{2} \\ \frac{\nu}{2}, \frac{\nu+1}{2}, -\frac{1}{2}(\nu+1), -\frac{\nu}{2}, \frac{a}{\pi}+\frac{\nu+1}{2} \end{matrix}\right)$$

Classical cases involving sin

03.21.26.0012.01

$$\sin\left(\sqrt{z}\right) j_\nu\left(\sqrt{z}\right) = \frac{1}{2}\sqrt{\pi}\, G_{2,4}^{1,2}\left(z \,\middle|\, \begin{array}{c} 0, \frac{1}{2} \\ \frac{\nu+1}{2}, -\frac{1}{2}(\nu+1), -\frac{\nu}{2}, \frac{\nu}{2} \end{array}\right)$$

03.21.26.0013.01

$$\sin\left(a+\sqrt{z}\right) j_\nu\left(\sqrt{z}\right) = \frac{1}{2}\sqrt{\pi}\, G_{3,5}^{2,2}\left(z \,\middle|\, \begin{array}{c} 0, \frac{1}{2}, \frac{a}{\pi}+\frac{\nu}{2} \\ \frac{\nu}{2}, \frac{\nu+1}{2}, -\frac{1+\nu}{2}, -\frac{\nu}{2}, \frac{a}{\pi}+\frac{\nu}{2} \end{array}\right)$$

Classical cases involving cos, sin

03.21.26.0014.01

$$\cos\left(\sqrt{z}\right) j_{-\nu-1}\left(\sqrt{z}\right) + \sin\left(\sqrt{z}\right) j_\nu\left(\sqrt{z}\right) = -\sqrt{\pi}\, \sin\left(\frac{\pi\nu}{2}\right) G_{2,4}^{2,1}\left(z \,\middle|\, \begin{array}{c} \frac{1}{2}, 0 \\ -\frac{1}{2}(\nu+1), \frac{\nu+1}{2}, -\frac{\nu}{2}, \frac{\nu}{2} \end{array}\right)$$

03.21.26.0015.01

$$\cos\left(\sqrt{z}\right) j_{-\nu-1}\left(\sqrt{z}\right) - \sin\left(\sqrt{z}\right) j_\nu\left(\sqrt{z}\right) = \sqrt{\pi}\, \cos\left(\frac{\pi\nu}{2}\right) G_{2,4}^{2,1}\left(z \,\middle|\, \begin{array}{c} 0, \frac{1}{2} \\ -\frac{1}{2}(\nu+1), \frac{\nu+1}{2}, -\frac{\nu}{2}, \frac{\nu}{2} \end{array}\right)$$

Classical cases for powers of spherical Bessel *j*

03.21.26.0016.01

$$j_\nu\left(\sqrt{z}\right)^2 = \frac{1}{2}\sqrt{\pi}\, G_{1,3}^{1,1}\left(z \,\middle|\, \begin{array}{c} 0 \\ \nu, -\frac{1}{2}, -\nu-1 \end{array}\right)$$

03.21.26.0017.01

$$j_{-\nu-1}\left(\sqrt{z}\right)^2 + j_\nu\left(\sqrt{z}\right)^2 = -\sqrt{\pi}\, \sin(\pi\nu)\, G_{2,4}^{2,1}\left(z \,\middle|\, \begin{array}{c} 0, -\frac{1}{2} \\ -\nu-1, \nu, -\frac{1}{2}, -\frac{1}{2} \end{array}\right)$$

03.21.26.0018.01

$$j_{-\nu-1}\left(\sqrt{z}\right)^2 - j_\nu\left(\sqrt{z}\right)^2 = \sqrt{\pi}\, \cos(\pi\nu)\, G_{1,3}^{2,0}\left(z \,\middle|\, \begin{array}{c} 0 \\ -\nu-1, \nu, -\frac{1}{2} \end{array}\right)$$

Classical cases for products of spherical Bessel *j*

03.21.26.0019.01

$$j_{-\nu-1}\left(\sqrt{z}\right) j_\nu\left(\sqrt{z}\right) = \frac{1}{2}\sqrt{\pi}\, G_{1,3}^{1,1}\left(z \,\middle|\, \begin{array}{c} 0 \\ -\frac{1}{2}, \nu, -\nu-1 \end{array}\right)$$

03.21.26.0020.01

$$j_{\nu-1}\left(\sqrt{z}\right) j_\nu\left(\sqrt{z}\right) = \frac{1}{2}\sqrt{\pi}\, G_{1,3}^{1,1}\left(z \,\middle|\, \begin{array}{c} -\frac{1}{2} \\ \nu-\frac{1}{2}, -1, -\nu-\frac{1}{2} \end{array}\right)$$

03.21.26.0021.01

$$j_\mu\left(\sqrt{z}\right) j_\nu\left(\sqrt{z}\right) = \frac{1}{2}\sqrt{\pi}\, G_{2,4}^{1,2}\left(z \,\middle|\, \begin{array}{c} 0, -\frac{1}{2} \\ \frac{\mu+\nu}{2}, \frac{1}{2}(-\mu-\nu-2), \frac{1}{2}(\mu-\nu-1), \frac{1}{2}(-\mu+\nu-1) \end{array}\right)$$

03.21.26.0022.01

$$j_\mu\left(\sqrt{z}\right) j_\nu\left(\sqrt{z}\right) + j_{-\mu-1}\left(\sqrt{z}\right) j_{-\nu-1}\left(\sqrt{z}\right) = -\sqrt{\pi}\, \sin\left(\frac{1}{2}\pi(\mu+\nu)\right) G_{2,4}^{2,1}\left(z \,\middle|\, \begin{array}{c} 0, -\frac{1}{2} \\ \frac{1}{2}(-\mu-\nu-2), \frac{\mu+\nu}{2}, \frac{1}{2}(\mu-\nu-1), \frac{1}{2}(-\mu+\nu-1) \end{array}\right)$$

03.21.26.0023.01

$$j_\mu\left(\sqrt{z}\right) j_\nu\left(\sqrt{z}\right) - j_{-\mu-1}\left(\sqrt{z}\right) j_{-\nu-1}\left(\sqrt{z}\right) = -\sqrt{\pi}\, \cos\left(\frac{1}{2}\pi(\mu+\nu)\right) G_{2,4}^{2,1}\left(z \,\middle|\, \begin{array}{c} -\frac{1}{2}, 0 \\ \frac{1}{2}(-\mu-\nu-2), \frac{\mu+\nu}{2}, \frac{1}{2}(\mu-\nu-1), \frac{1}{2}(-\mu+\nu-1) \end{array}\right)$$

Classical cases involving Bessel *J*

03.21.26.0024.01

$$J_{-\nu-\frac{1}{2}}\left(\sqrt{z}\right) j_\nu\left(\sqrt{z}\right) = \frac{1}{\sqrt{2}} G_{1,3}^{1,1}\left(z \,\middle|\, \begin{array}{c} \frac{1}{4} \\ -\frac{1}{4}, -\nu-\frac{3}{4}, \nu+\frac{1}{4} \end{array}\right)$$

03.21.26.0025.01

$$J_{\nu-\frac{1}{2}}\left(\sqrt{z}\right) j_\nu\left(\sqrt{z}\right) = \frac{1}{\sqrt{2}} G_{1,3}^{1,1}\left(z \,\middle|\, \begin{array}{c} -\frac{1}{4} \\ \nu-\frac{1}{4}, -\frac{3}{4}, -\nu-\frac{1}{4} \end{array}\right)$$

03.21.26.0026.01

$$J_\mu\left(\sqrt{z}\right) j_\nu\left(\sqrt{z}\right) = \frac{1}{\sqrt{2}} G_{2,4}^{1,2}\left(z \,\middle|\, \begin{array}{c} \frac{1}{4}, -\frac{1}{4} \\ \frac{\mu+\nu}{2}, -\frac{1+\mu+\nu}{2}, \frac{\mu-\nu-1}{2}, \frac{\nu-\mu}{2} \end{array}\right)$$

03.21.26.0027.01

$$J_\mu\left(\sqrt{z}\right) j_\nu\left(\sqrt{z}\right) + J_{-\mu}\left(\sqrt{z}\right) j_{-\nu-1}\left(\sqrt{z}\right) = -\sqrt{2}\, \sin\left(\frac{1}{2}\pi\left(\mu+\nu-\frac{1}{2}\right)\right) G_{2,4}^{2,1}\left(z \,\middle|\, \begin{array}{c} \frac{1}{4}, -\frac{1}{4} \\ -\frac{1+\mu+\nu}{2}, \frac{\mu+\nu}{2}, \frac{1}{2}(\mu-\nu-1), \frac{\nu-\mu}{2} \end{array}\right)$$

03.21.26.0028.01

$$J_\mu\left(\sqrt{z}\right) j_\nu\left(\sqrt{z}\right) - J_{-\mu}\left(\sqrt{z}\right) j_{-\nu-1}\left(\sqrt{z}\right) = -\sqrt{2}\, \cos\left(\frac{1}{2}\pi\left(\mu+\nu-\frac{1}{2}\right)\right) G_{2,4}^{2,1}\left(z \,\middle|\, \begin{array}{c} -\frac{1}{4}, \frac{1}{4} \\ -\frac{1+\mu+\nu}{2}, \frac{\mu+\nu}{2}, \frac{1}{2}(\mu-\nu-1), \frac{\nu-\mu}{2} \end{array}\right)$$

Classical cases involving Bessel *Y*

03.21.26.0029.01

$$Y_{\nu+\frac{1}{2}}\left(\sqrt{z}\right) j_\nu\left(\sqrt{z}\right) = -\frac{1}{\sqrt{2}} G_{1,3}^{2,0}\left(z \,\middle|\, \begin{array}{c} \frac{1}{4} \\ -\frac{1}{4}, \nu+\frac{1}{4}, -\nu-\frac{3}{4} \end{array}\right)$$

03.21.26.0030.01

$$Y_{-\nu-\frac{1}{2}}\left(\sqrt{z}\right) j_\nu\left(\sqrt{z}\right) = \frac{1}{\sqrt{2}} G_{2,4}^{2,1}\left(z \,\middle|\, \begin{array}{c} \frac{1}{4}, \nu-\frac{1}{4} \\ -\frac{1}{4}, \nu+\frac{1}{4}, \nu-\frac{1}{4}, -\nu-\frac{3}{4} \end{array}\right)$$

03.21.26.0031.01

$$Y_{\nu-\frac{1}{2}}\left(\sqrt{z}\right) j_\nu\left(\sqrt{z}\right) = \frac{1}{\sqrt{2}} G_{2,4}^{2,1}\left(z \,\middle|\, \begin{array}{c} \frac{1}{4}, -\frac{1}{4} \\ \frac{1}{4}, \nu-\frac{1}{4}, -\frac{3}{4}, -\nu-\frac{1}{4} \end{array}\right)$$

03.21.26.0032.01

$$Y_{\nu-\frac{3}{2}}\left(\sqrt{z}\right) j_\nu\left(\sqrt{z}\right) = \frac{1}{\sqrt{2}} G_{2,4}^{2,1}\left(z \,\middle|\, \begin{array}{c} -\frac{1}{4}, \frac{1}{4} \\ \frac{3}{4}, \nu-\frac{3}{4}, -\frac{5}{4}, \frac{1}{4}-\nu \end{array}\right)$$

03.21.26.0033.01

$$Y_\mu(\sqrt{z})\, j_\nu(\sqrt{z}) = -\frac{1}{\sqrt{2}} G_{3,5}^{2,2}\left(z \,\middle|\, \begin{matrix} -\frac{1}{4}, \frac{1}{4}, \frac{1}{2}(-\mu+\nu+1) \\ \frac{\nu-\mu}{2}, \frac{\mu+\nu}{2}, -\frac{1+\mu+\nu}{2}, \frac{1}{2}(\mu-\nu-1), \frac{1}{2}(-\mu+\nu+1) \end{matrix}\right)$$

Classical cases involving Bessel *J*, *Y*, *y*

03.21.26.0034.01

$$Y_{-\nu-\frac{1}{2}}(\sqrt{z})\, j_\nu(\sqrt{z}) + J_{-\nu-\frac{1}{2}}(\sqrt{z})\, y_\nu(\sqrt{z}) = -\sqrt{2}\, G_{1,3}^{2,0}\left(z \,\middle|\, \begin{matrix} \frac{1}{4} \\ -\nu-\frac{3}{4}, \nu+\frac{1}{4}, -\frac{1}{4} \end{matrix}\right)$$

03.21.26.0035.01

$$Y_{-\nu-\frac{1}{2}}(\sqrt{z})\, j_\nu(\sqrt{z}) - J_{-\nu-\frac{1}{2}}(\sqrt{z})\, y_\nu(\sqrt{z}) = -\frac{\sin(2\pi\nu)}{\sqrt{2}\,\pi^2} G_{1,3}^{3,1}\left(z \,\middle|\, \begin{matrix} \frac{1}{4} \\ -\frac{1}{4}, -\nu-\frac{3}{4}, \nu+\frac{1}{4} \end{matrix}\right)$$

03.21.26.0036.01

$$Y_\mu(\sqrt{z})\, j_\nu(\sqrt{z}) + J_\mu(\sqrt{z})\, y_\nu(\sqrt{z}) = -\sqrt{2}\, G_{2,4}^{3,0}\left(z \,\middle|\, \begin{matrix} -\frac{1}{4}, \frac{1}{4} \\ \frac{1}{2}(\mu-\nu-1), \frac{\nu-\mu}{2}, \frac{\mu+\nu}{2}, -\frac{\mu+\nu+1}{2} \end{matrix}\right)$$

03.21.26.0037.01

$$Y_\mu(\sqrt{z})\, j_\nu(\sqrt{z}) - J_\mu(\sqrt{z})\, y_\nu(\sqrt{z}) = \frac{\cos(\pi(\mu-\nu))}{\sqrt{2}\,\pi^2} G_{2,4}^{3,2}\left(z \,\middle|\, \begin{matrix} -\frac{1}{4}, \frac{1}{4} \\ \frac{1}{2}(\mu-\nu-1), \frac{\nu-\mu}{2}, \frac{\mu+\nu}{2}, -\frac{\mu+\nu+1}{2} \end{matrix}\right)$$

03.21.26.0038.01

$$J_{-\nu-\frac{1}{2}}(\sqrt{z})\, j_\nu(\sqrt{z}) - Y_{-\nu-\frac{1}{2}}(\sqrt{z})\, y_\nu(\sqrt{z}) = -\sqrt{2}\, G_{2,4}^{3,0}\left(z \,\middle|\, \begin{matrix} \frac{1}{4}, \frac{1}{4} \\ -\frac{1}{4}, -\nu-\frac{3}{4}, \nu+\frac{1}{4}, \frac{1}{4} \end{matrix}\right)$$

03.21.26.0039.01

$$J_\mu(\sqrt{z})\, j_\nu(\sqrt{z}) - Y_\mu(\sqrt{z})\, y_\nu(\sqrt{z}) = -\sqrt{2}\, G_{3,5}^{4,0}\left(z \,\middle|\, \begin{matrix} -\frac{1}{4}, \frac{1}{4}, -\frac{\mu+\nu}{2} \\ -\frac{\mu+\nu+1}{2}, \frac{1}{2}(\mu-\nu-1), \frac{\nu-\mu}{2}, \frac{\mu+\nu}{2}, -\frac{\mu+\nu}{2} \end{matrix}\right)$$

03.21.26.0040.01

$$J_\mu(\sqrt{z})\, j_\nu(\sqrt{z}) + Y_\mu(\sqrt{z})\, y_\nu(\sqrt{z}) = \frac{1}{\sqrt{2}\,\pi^2}\left(\cos(\pi\mu)\, G_{2,4}^{3,2}\left(z \,\middle|\, \begin{matrix} -\frac{1}{4}, \frac{1}{4} \\ -\frac{\mu+\nu+1}{2}, \frac{1}{2}(\mu-\nu-1), \frac{\mu+\nu}{2}, \frac{\nu-\mu}{2} \end{matrix}\right) - \sin(\pi\nu)\, G_{2,4}^{3,2}\left(z \,\middle|\, \begin{matrix} -\frac{1}{4}, \frac{1}{4} \\ -\frac{\mu+\nu+1}{2}, \frac{\nu-\mu}{2}, \frac{\mu+\nu}{2}, \frac{1}{2}(\mu-\nu-1) \end{matrix}\right)\right)$$

Classical cases involving Bessel *I*

03.21.26.0041.01

$$I_{\nu+\frac{1}{2}}(\sqrt[4]{z})\, j_\nu(\sqrt[4]{z}) = \frac{\pi}{2\sqrt[4]{2}} G_{0,4}^{1,0}\left(\frac{z}{64} \,\middle|\, \frac{\nu}{2}+\frac{1}{8}, -\frac{1}{8}, \frac{3}{8}, -\frac{\nu}{2}-\frac{3}{8}\right)$$

03.21.26.0042.01

$$I_{-\nu-\frac{1}{2}}(\sqrt[4]{z})\, j_\nu(\sqrt[4]{z}) = \frac{\pi}{2\sqrt[4]{2}} G_{1,5}^{2,0}\left(\frac{z}{64} \,\middle|\, \begin{matrix} \frac{1}{8}(1-4\nu) \\ -\frac{1}{8}, \frac{3}{8}, \frac{1}{8}(-4\nu-3), \frac{1}{8}(1-4\nu), \frac{1}{8}(4\nu+1) \end{matrix}\right)$$

Classical cases involving Bessel *K*

03.21.26.0043.01

$$I_{-\nu-\frac{1}{2}}\left(\sqrt[4]{z}\right) j_\nu\left(\sqrt[4]{z}\right) = \frac{1}{8\sqrt[4]{2}} G_{0,4}^{3,0}\left(\frac{z}{64} \,\middle|\, -\tfrac{1}{8}, \tfrac{3}{8}, \tfrac{1}{8}(4\nu+1), -\tfrac{1}{8}(4\nu+3)\right)$$

03.21.26.0044.01

$$K_{\nu+\frac{1}{2}}\left(\sqrt[4]{z}\right)\left(j_{-\nu-1}\left(\sqrt[4]{z}\right) + j_\nu\left(\sqrt[4]{z}\right)\right) = \frac{\cos\left(\frac{\pi}{4}(1+2\nu)\right)}{2\sqrt{2}\sqrt[8]{64}} G_{0,4}^{3,0}\left(\frac{z}{64} \,\middle|\, \tfrac{3}{8}, -\tfrac{1}{8}(4\nu+3), \tfrac{1}{8}(4\nu+1), -\tfrac{1}{8}\right)$$

03.21.26.0045.01

$$K_{\nu+\frac{1}{2}}\left(\sqrt[4]{z}\right)\left(j_{-\nu-1}\left(\sqrt[4]{z}\right) - j_\nu\left(\sqrt[4]{z}\right)\right) = \frac{\sin\left(\frac{1}{2}\pi\left(\nu+\frac{1}{2}\right)\right)}{4\sqrt[4]{2}} G_{0,4}^{3,0}\left(\frac{z}{64} \,\middle|\, -\tfrac{1}{8}, -\tfrac{1}{8}(4\nu+3), \tfrac{1}{8}(4\nu+1), \tfrac{3}{8}\right)$$

Classical cases involving spherical Bessel *y*

03.21.26.0046.01

$$\cos(a\pi)\, j_\nu\left(\sqrt{z}\right) + \sin(a\pi)\, y_\nu\left(\sqrt{z}\right) = \frac{1}{2}\sqrt{\pi}\, G_{1,3}^{2,0}\left(\frac{z}{4} \,\middle|\, \begin{matrix} -\frac{1}{2}(2a+\nu+1) \\ -\frac{1}{2}(\nu+1), \frac{\nu}{2}, -\frac{1}{2}(2a+\nu+1) \end{matrix}\right)$$

03.21.26.0047.01

$$j_\nu\left(\sqrt{z}\right) y_\nu\left(\sqrt{z}\right) = -\frac{\sqrt{\pi}}{2} G_{1,3}^{2,0}\left(z \,\middle|\, \begin{matrix} 0 \\ -\frac{1}{2}, \nu, -\nu-1 \end{matrix}\right)$$

03.21.26.0048.01

$$j_{-\nu-1}\left(\sqrt{z}\right) y_\nu\left(\sqrt{z}\right) = \frac{1}{2}\sqrt{\pi}\, G_{2,4}^{2,1}\left(z \,\middle|\, \begin{matrix} 0, -\nu-\frac{3}{2} \\ -\frac{1}{2}, -\nu-1, -\nu-\frac{3}{2}, \nu \end{matrix}\right)$$

03.21.26.0049.01

$$j_{\nu+1}\left(\sqrt{z}\right) y_\nu\left(\sqrt{z}\right) = \frac{1}{2}\sqrt{\pi}\, G_{2,4}^{2,1}\left(z \,\middle|\, \begin{matrix} 0, -\frac{1}{2} \\ 0, \nu+\frac{1}{2}, -1, -\nu-\frac{3}{2} \end{matrix}\right)$$

03.21.26.0050.01

$$j_{\nu+2}\left(\sqrt{z}\right) y_\nu\left(\sqrt{z}\right) = \frac{1}{2}\sqrt{\pi}\, G_{2,4}^{2,1}\left(z \,\middle|\, \begin{matrix} -\frac{1}{2}, 0 \\ \frac{1}{2}, \nu+1, -\frac{3}{2}, -\nu-2 \end{matrix}\right)$$

03.21.26.0051.01

$$j_\mu\left(\sqrt{z}\right) y_\nu\left(\sqrt{z}\right) = -\frac{\sqrt{\pi}}{2} G_{3,5}^{2,2}\left(z \,\middle|\, \begin{matrix} 0, -\frac{1}{2}, \frac{\mu-\nu}{2} \\ \frac{\mu-\nu-1}{2}, \frac{\mu+\nu}{2}, -\frac{\mu+\nu+2}{2}, \frac{\mu-\nu}{2}, \frac{\nu-\mu-1}{2} \end{matrix}\right)$$

03.21.26.0052.01

$$j_\nu\left(\sqrt{z}\right) y_{-\nu-1}\left(\sqrt{z}\right) + j_{-\nu-1}\left(\sqrt{z}\right) y_\nu\left(\sqrt{z}\right) = -\sqrt{\pi}\, G_{1,3}^{2,0}\left(z \,\middle|\, \begin{matrix} 0 \\ -\nu-1, \nu, -\frac{1}{2} \end{matrix}\right)$$

03.21.26.0053.01

$$j_\nu\left(\sqrt{z}\right) y_{-\nu-1}\left(\sqrt{z}\right) - j_{-\nu-1}\left(\sqrt{z}\right) y_\nu\left(\sqrt{z}\right) = -\frac{\sin(2\pi\nu)}{2\pi^{3/2}} G_{1,3}^{3,1}\left(z \,\middle|\, \begin{matrix} 0 \\ -\frac{1}{2}, -\nu-1, \nu \end{matrix}\right)$$

03.21.26.0054.01

$$j_\nu(\sqrt{z})\,y_\mu(\sqrt{z})+j_\mu(\sqrt{z})\,y_\nu(\sqrt{z})=-\sqrt{\pi}\,G_{2,4}^{3,0}\left(z\,\middle|\,\begin{array}{c}0,-\frac{1}{2}\\ \frac{1}{2}(\mu-\nu-1),\frac{1}{2}(-\mu+\nu-1),\frac{\mu+\nu}{2},\frac{1}{2}(-\mu-\nu-2)\end{array}\right)$$

03.21.26.0055.01

$$j_\nu(\sqrt{z})\,y_\mu(\sqrt{z})-j_\mu(\sqrt{z})\,y_\nu(\sqrt{z})=\frac{\sin(\pi(\nu-\mu))}{2\pi^{3/2}}\,G_{2,4}^{3,2}\left(z\,\middle|\,\begin{array}{c}0,-\frac{1}{2}\\ \frac{1}{2}(\mu-\nu-1),\frac{1}{2}(-\mu+\nu-1),\frac{\mu+\nu}{2},\frac{1}{2}(-\mu-\nu-2)\end{array}\right)$$

03.21.26.0056.01

$$j_\nu(\sqrt{z})^2+y_\nu(\sqrt{z})^2=-\frac{\sin(\pi\nu)}{\pi^{3/2}}\,G_{1,3}^{3,1}\left(z\,\middle|\,\begin{array}{c}0\\ -\frac{1}{2},-\nu-1,\nu\end{array}\right)$$

03.21.26.0057.01

$$j_\nu(\sqrt{z})^2-y_\nu(\sqrt{z})^2=-\sqrt{\pi}\,G_{2,4}^{3,0}\left(z\,\middle|\,\begin{array}{c}0,-\nu-\frac{1}{2}\\ -\frac{1}{2},-\nu-1,\nu,-\nu-\frac{1}{2}\end{array}\right)$$

03.21.26.0058.01

$$j_{-\nu-1}(\sqrt{z})\,j_\nu(\sqrt{z})-y_{-\nu-1}(\sqrt{z})\,y_\nu(\sqrt{z})=-\sqrt{\pi}\,G_{2,4}^{3,0}\left(z\,\middle|\,\begin{array}{c}0,0\\ -\frac{1}{2},-\nu-1,\nu,0\end{array}\right)$$

03.21.26.0059.01

$$j_\mu(\sqrt{z})\,j_\nu(\sqrt{z})-y_\mu(\sqrt{z})\,y_\nu(\sqrt{z})=-\sqrt{\pi}\,G_{3,5}^{4,0}\left(z\,\middle|\,\begin{array}{c}0,-\frac{1}{2},\frac{1}{2}(-\mu-\nu-1)\\ \frac{1}{2}(-\mu-\nu-2),\frac{1}{2}(\mu-\nu-1),\frac{1}{2}(-\mu+\nu-1),\frac{\mu+\nu}{2},\frac{1}{2}(-\mu-\nu-1)\end{array}\right)$$

03.21.26.0060.01

$$j_\mu(\sqrt{z})\,j_\nu(\sqrt{z})+y_\mu(\sqrt{z})\,y_\nu(\sqrt{z})=-\frac{\sin(\pi\mu)}{2\pi^{3/2}}\,G_{2,4}^{3,2}\left(z\,\middle|\,\begin{array}{c}0,-\frac{1}{2}\\ \frac{1}{2}(-\mu-\nu-2),\frac{1}{2}(\mu-\nu-1),\frac{\mu+\nu}{2},\frac{1}{2}(-\mu+\nu-1)\end{array}\right)-$$
$$\frac{\sin(\pi\nu)}{2\pi^{3/2}}\,G_{2,4}^{3,2}\left(z\,\middle|\,\begin{array}{c}0,-\frac{1}{2}\\ \frac{1}{2}(-\mu-\nu-2),\frac{1}{2}(-\mu+\nu-1),\frac{\mu+\nu}{2},\frac{1}{2}(\mu-\nu-1)\end{array}\right)$$

Classical cases involving cos, sin, y

03.21.26.0061.01

$$\sin(\sqrt{z})\,j_\nu(\sqrt{z})+\cos(\sqrt{z})\,y_\nu(\sqrt{z})=-\sqrt{\pi}\,G_{2,4}^{3,0}\left(z\,\middle|\,\begin{array}{c}0,\frac{1}{2}\\ -\frac{1+\nu}{2},\frac{\nu}{2},\frac{\nu+1}{2},-\frac{\nu}{2}\end{array}\right)$$

03.21.26.0062.01

$$\cos(\sqrt{z})\,j_\nu(\sqrt{z})-\sin(\sqrt{z})\,y_\nu(\sqrt{z})=\sqrt{\pi}\,G_{2,4}^{3,0}\left(z\,\middle|\,\begin{array}{c}0,\frac{1}{2}\\ -\frac{\nu}{2},\frac{\nu}{2},\frac{\nu+1}{2},-\frac{\nu+1}{2}\end{array}\right)$$

03.21.26.0063.01

$$\sin(\sqrt{z})\,j_\nu(\sqrt{z})-\cos(\sqrt{z})\,y_\nu(\sqrt{z})=-\frac{\sin(\pi\nu)}{2\pi^{3/2}}\,G_{2,4}^{3,2}\left(z\,\middle|\,\begin{array}{c}0,\frac{1}{2}\\ -\frac{\nu+1}{2},\frac{\nu}{2},\frac{\nu+1}{2},-\frac{\nu}{2}\end{array}\right)$$

03.21.26.0064.01

$$\cos\left(\sqrt{z}\right) j_\nu\left(\sqrt{z}\right) + \sin\left(\sqrt{z}\right) y_\nu\left(\sqrt{z}\right) = -\frac{\sin(\pi \nu)}{2 \pi^{3/2}} G_{2,4}^{3,2}\left(z \,\middle|\, \begin{matrix} 0, \frac{1}{2} \\ -\frac{\nu}{2}, \frac{\nu}{2}, \frac{\nu+1}{2}, -\frac{\nu+1}{2} \end{matrix}\right)$$

03.21.26.0065.01

$$\sin\left(a + \sqrt{z}\right) j_\nu\left(\sqrt{z}\right) - \cos\left(a + \sqrt{z}\right) y_\nu\left(\sqrt{z}\right) = -\frac{\sin(\pi \nu)}{2 \pi^{3/2}} G_{3,5}^{4,2}\left(z \,\middle|\, \begin{matrix} 0, \frac{1}{2}, -\frac{a}{\pi} - \frac{\nu}{2} \\ -\frac{\nu+1}{2}, -\frac{\nu}{2}, \frac{\nu}{2}, \frac{\nu+1}{2}, -\frac{a}{\pi} - \frac{\nu}{2} \end{matrix}\right)$$

Classical cases involving ${}_0F_1$

03.21.26.0066.01

$${}_0F_1\left(; b; -\frac{z^2}{4}\right) j_\nu(z) = 2^{b-\frac{3}{2}} \Gamma(b) G_{2,4}^{1,2}\left(z^2 \,\middle|\, \begin{matrix} \frac{1}{4} - \frac{b}{2}, \frac{3}{4} - \frac{b}{2} \\ \frac{\nu}{2}, -\frac{\nu}{2} - \frac{1}{2}, -b + \frac{\nu}{2} + 1, -b - \frac{\nu}{2} + \frac{1}{2} \end{matrix}\right) /; \operatorname{Re}(z) > 0$$

03.21.26.0067.01

$${}_0F_1\left(; \nu + \frac{3}{2}; \frac{z^2}{4}\right) j_\nu(z) = 2^{-\frac{\nu}{2}-\frac{3}{2}} \pi z^\nu \left(z^4\right)^{-\frac{\nu}{4}} \Gamma\left(\nu + \frac{3}{2}\right) G_{0,4}^{1,0}\left(\frac{z^4}{64} \,\middle|\, \frac{\nu}{4}, -\frac{\nu+1}{4}, \frac{1-\nu}{4}, -\frac{3\nu+2}{4}\right)$$

03.21.26.0068.01

$${}_0F_1\left(; \frac{1}{2} - \nu; \frac{z^2}{4}\right) j_\nu(z) = 2^{\frac{\nu}{2}-1} \pi \Gamma\left(\frac{1}{2} - \nu\right) G_{1,5}^{2,0}\left(\frac{z^4}{64} \,\middle|\, \begin{matrix} \frac{1-\nu}{4} \\ \frac{\nu}{4}, \frac{\nu+2}{4}, -\frac{\nu+1}{4}, \frac{1-\nu}{4}, \frac{3\nu+1}{4} \end{matrix}\right) /; \operatorname{Re}(z) > 0$$

Classical cases involving ${}_0\tilde{F}_1$

03.21.26.0069.01

$${}_0\tilde{F}_1\left(; b; -\frac{z^2}{4}\right) j_\nu(z) = 2^{b-\frac{3}{2}} G_{2,4}^{1,2}\left(z^2 \,\middle|\, \begin{matrix} \frac{1}{4} - \frac{b}{2}, \frac{3}{4} - \frac{b}{2} \\ \frac{\nu}{2}, -\frac{\nu}{2} - \frac{1}{2}, -b + \frac{\nu}{2} + 1, -b - \frac{\nu}{2} + \frac{1}{2} \end{matrix}\right) /; \operatorname{Re}(z) > 0$$

03.21.26.0070.01

$${}_0\tilde{F}_1\left(; \nu + \frac{3}{2}; \frac{z^2}{4}\right) j_\nu(z) = 2^{-\frac{\nu}{2}-\frac{3}{2}} \pi z^\nu \left(z^4\right)^{-\frac{\nu}{4}} G_{0,4}^{1,0}\left(\frac{z^4}{64} \,\middle|\, \frac{\nu}{4}, -\frac{\nu+1}{4}, \frac{1-\nu}{4}, -\frac{3\nu+2}{4}\right)$$

03.21.26.0071.01

$${}_0\tilde{F}_1\left(; \frac{1}{2} - \nu; \frac{z^2}{4}\right) j_\nu(z) = 2^{\frac{\nu}{2}-1} \pi G_{1,5}^{2,0}\left(\frac{z^4}{64} \,\middle|\, \begin{matrix} \frac{1-\nu}{4} \\ \frac{\nu}{4}, \frac{\nu+2}{4}, -\frac{\nu+1}{4}, \frac{1-\nu}{4}, \frac{3\nu+1}{4} \end{matrix}\right) /; \operatorname{Re}(z) > 0$$

Generalized cases for the direct function itself

03.21.26.0072.01

$$j_\nu(z) = \frac{\sqrt{\pi}}{2} G_{0,2}^{1,0}\left(\frac{z}{2}, \frac{1}{2} \,\middle|\, \frac{\nu}{2}, -\frac{\nu+1}{2}\right)$$

03.21.26.0073.01

$$j_\nu(z) = \frac{1}{2} \pi^{3/2} (i z)^{-\nu} z^\nu G_{1,3}^{1,0}\left(\frac{i z}{2}, \frac{1}{2} \,\middle|\, \begin{matrix} \frac{\nu+1}{2} \\ \frac{\nu}{2}, -\frac{\nu+1}{2}, \frac{\nu+1}{2} \end{matrix}\right)$$

Generalized cases involving cos

03.21.26.0074.01

$$\cos(z)\, j_\nu(z) = \frac{1}{2}\sqrt{\pi}\, G_{2,4}^{1,2}\left(z, \frac{1}{2} \,\middle|\, \begin{array}{c} 0, \frac{1}{2} \\ \frac{\nu}{2}, -\frac{\nu+1}{2}, \frac{\nu+1}{2}, -\frac{\nu}{2} \end{array}\right)$$

03.21.26.0075.01

$$\cos(a+z)\, j_\nu(z) = \frac{1}{2}\sqrt{\pi}\, G_{3,5}^{2,2}\left(z, \frac{1}{2} \,\middle|\, \begin{array}{c} 0, \frac{1}{2}, \frac{\nu+1}{2} + \frac{a}{\pi} \\ \frac{\nu}{2}, \frac{\nu+1}{2}, -\frac{\nu+1}{2}, -\frac{\nu}{2}, \frac{a}{\pi} + \frac{\nu+1}{2} \end{array}\right)$$

Generalized cases involving sin

03.21.26.0076.01

$$\sin(z)\, j_\nu(z) = \frac{1}{2}\sqrt{\pi}\, G_{2,4}^{1,2}\left(z, \frac{1}{2} \,\middle|\, \begin{array}{c} 0, \frac{1}{2} \\ \frac{\nu+1}{2}, -\frac{1}{2}(\nu+1), -\frac{\nu}{2}, \frac{\nu}{2} \end{array}\right)$$

03.21.26.0077.01

$$\sin(a+z)\, j_\nu(z) = \frac{1}{2}\sqrt{\pi}\, G_{3,5}^{2,2}\left(z, \frac{1}{2} \,\middle|\, \begin{array}{c} 0, \frac{1}{2}, \frac{a}{\pi} + \frac{\nu}{2} \\ \frac{\nu}{2}, \frac{\nu+1}{2}, -\frac{\nu+1}{2}, -\frac{\nu}{2}, \frac{a}{\pi} + \frac{\nu}{2} \end{array}\right)$$

Generalized cases involving cos, sin

03.21.26.0078.01

$$\cos(z)\, j_{-\nu-1}(z) + \sin(z)\, j_\nu(z) = -\sqrt{\pi}\, \sin\left(\frac{\pi\nu}{2}\right) G_{2,4}^{2,1}\left(z, \frac{1}{2} \,\middle|\, \begin{array}{c} \frac{1}{2}, 0 \\ -\frac{\nu+1}{2}, \frac{\nu+1}{2}, -\frac{\nu}{2}, \frac{\nu}{2} \end{array}\right)$$

03.21.26.0079.01

$$\cos(z)\, j_{-\nu-1}(z) - \sin(z)\, j_\nu(z) = \sqrt{\pi}\, \cos\left(\frac{\pi\nu}{2}\right) G_{2,4}^{2,1}\left(z, \frac{1}{2} \,\middle|\, \begin{array}{c} 0, \frac{1}{2} \\ -\frac{1}{2}(\nu+1), \frac{\nu+1}{2}, -\frac{\nu}{2}, \frac{\nu}{2} \end{array}\right)$$

Generalized cases for powers of spherical Bessel j

03.21.26.0080.01

$$j_\nu(z)^2 = \frac{1}{2}\sqrt{\pi}\, G_{1,3}^{1,1}\left(z, \frac{1}{2} \,\middle|\, \begin{array}{c} 0 \\ \nu, -\frac{1}{2}, -\nu-1 \end{array}\right)$$

03.21.26.0081.01

$$j_{-\nu-1}(z)^2 + j_\nu(z)^2 = -\sqrt{\pi}\, \sin(\pi\nu)\, G_{2,4}^{2,1}\left(z, \frac{1}{2} \,\middle|\, \begin{array}{c} 0, -\frac{1}{2} \\ -\nu-1, \nu, -\frac{1}{2}, -\frac{1}{2} \end{array}\right)$$

03.21.26.0082.01

$$j_{-\nu-1}(z)^2 - j_\nu(z)^2 = \sqrt{\pi}\, \cos(\pi\nu)\, G_{1,3}^{2,0}\left(z, \frac{1}{2} \,\middle|\, \begin{array}{c} 0 \\ -\nu-1, \nu, -\frac{1}{2} \end{array}\right)$$

Generalized cases for products of spherical Bessel j

03.21.26.0083.01

$$j_{-\nu-1}(z)\, j_\nu(z) = \frac{1}{2}\sqrt{\pi}\, G_{1,3}^{1,1}\left(z, \frac{1}{2} \,\middle|\, \begin{array}{c} 0 \\ -\frac{1}{2}, \nu, -\nu-1 \end{array}\right)$$

03.21.26.0084.01

$$j_{\nu-1}(z)\,j_\nu(z)=\frac{1}{2}\sqrt{\pi}\;G_{1,3}^{1,1}\!\left(z,\,\frac{1}{2}\,\middle|\,\begin{array}{c}-\frac{1}{2}\\ \nu-\frac{1}{2},\,-1,\,-\nu-\frac{1}{2}\end{array}\right)$$

03.21.26.0085.01

$$j_\mu(z)\,j_\nu(z)=\frac{1}{2}\sqrt{\pi}\;G_{2,4}^{1,2}\!\left(z,\,\frac{1}{2}\,\middle|\,\begin{array}{c}0,\,-\frac{1}{2}\\ \frac{\mu+\nu}{2},\,\frac{1}{2}(-\mu-\nu-2),\,\frac{1}{2}(\mu-\nu-1),\,\frac{1}{2}(-\mu+\nu-1)\end{array}\right)$$

03.21.26.0086.01

$$j_{-\mu-1}(z)\,j_{-\nu-1}(z)+j_\mu(z)\,j_\nu(z)=-\sqrt{\pi}\,\sin\!\left(\frac{1}{2}\pi(\mu+\nu)\right)G_{2,4}^{2,1}\!\left(z,\,\frac{1}{2}\,\middle|\,\begin{array}{c}0,\,-\frac{1}{2}\\ \frac{1}{2}(-\mu-\nu-2),\,\frac{\mu+\nu}{2},\,\frac{1}{2}(\mu-\nu-1),\,\frac{1}{2}(-\mu+\nu-1)\end{array}\right)$$

03.21.26.0087.01

$$j_\mu(z)\,j_\nu(z)-j_{-\mu-1}(z)\,j_{-\nu-1}(z)=-\sqrt{\pi}\,\cos\!\left(\frac{1}{2}\pi(\mu+\nu)\right)G_{2,4}^{2,1}\!\left(z,\,\frac{1}{2}\,\middle|\,\begin{array}{c}-\frac{1}{2},\,0\\ \frac{1}{2}(-\mu-\nu-2),\,\frac{\mu+\nu}{2},\,\frac{1}{2}(\mu-\nu-1),\,\frac{1}{2}(-\mu+\nu-1)\end{array}\right)$$

Generalized cases involving Bessel *J*

03.21.26.0088.01

$$J_{-\nu-\frac{1}{2}}(z)\,j_\nu(z)=\frac{1}{\sqrt{2}}\,G_{1,3}^{1,1}\!\left(z,\,\frac{1}{2}\,\middle|\,\begin{array}{c}\frac{1}{4}\\ -\frac{1}{4},\,-\nu-\frac{3}{4},\,\nu+\frac{1}{4}\end{array}\right)$$

03.21.26.0089.01

$$J_{\nu-\frac{1}{2}}(z)\,j_\nu(z)=\frac{1}{\sqrt{2}}\,G_{1,3}^{1,1}\!\left(z,\,\frac{1}{2}\,\middle|\,\begin{array}{c}-\frac{1}{4}\\ \nu-\frac{1}{4},\,-\frac{3}{4},\,-\nu-\frac{1}{4}\end{array}\right)$$

03.21.26.0090.01

$$J_\mu(z)\,j_\nu(z)=\frac{1}{\sqrt{2}}\,G_{2,4}^{1,2}\!\left(z,\,\frac{1}{2}\,\middle|\,\begin{array}{c}\frac{1}{4},\,-\frac{1}{4}\\ \frac{\mu+\nu}{2},\,-\frac{1}{2}(\mu+\nu+1),\,\frac{1}{2}(\mu-\nu-1),\,\frac{\nu-\mu}{2}\end{array}\right)$$

03.21.26.0091.01

$$J_{-\mu}(z)\,j_{-\nu-1}(z)+J_\mu(z)\,j_\nu(z)=-\sqrt{2}\,\sin\!\left(\frac{1}{2}\pi\left(\mu+\nu-\frac{1}{2}\right)\right)G_{2,4}^{2,1}\!\left(z,\,\frac{1}{2}\,\middle|\,\begin{array}{c}\frac{1}{4},\,-\frac{1}{4}\\ -\frac{1}{2}(\mu+\nu+1),\,\frac{\mu+\nu}{2},\,\frac{1}{2}(\mu-\nu-1),\,\frac{\nu-\mu}{2}\end{array}\right)$$

03.21.26.0092.01

$$J_\mu(z)\,j_\nu(z)-J_{-\mu}(z)\,j_{-\nu-1}(z)=-\sqrt{2}\,\cos\!\left(\frac{1}{2}\pi\left(\mu+\nu-\frac{1}{2}\right)\right)G_{2,4}^{2,1}\!\left(z,\,\frac{1}{2}\,\middle|\,\begin{array}{c}-\frac{1}{4},\,\frac{1}{4}\\ -\frac{1}{2}(\mu+\nu+1),\,\frac{\mu+\nu}{2},\,\frac{1}{2}(\mu-\nu-1),\,\frac{\nu-\mu}{2}\end{array}\right)$$

Generalized cases involving Bessel *Y*

03.21.26.0093.01

$$Y_{\nu+\frac{1}{2}}(z)\,j_\nu(z)=-\frac{1}{\sqrt{2}}\,G_{1,3}^{2,0}\!\left(z,\,\frac{1}{2}\,\middle|\,\begin{array}{c}\frac{1}{4}\\ -\frac{1}{4},\,\nu+\frac{1}{4},\,-\nu-\frac{3}{4}\end{array}\right)$$

03.21.26.0094.01

$$Y_{-\nu-\frac{1}{2}}(z)\,j_\nu(z)=\frac{1}{\sqrt{2}}\,G_{2,4}^{2,1}\!\left(z,\,\frac{1}{2}\,\middle|\,\begin{array}{c}\frac{1}{4},\,\nu-\frac{1}{4}\\ -\frac{1}{4},\,\nu+\frac{1}{4},\,\nu-\frac{1}{4},\,-\nu-\frac{3}{4}\end{array}\right)$$

03.21.26.0095.01

$$Y_{\nu-\frac{1}{2}}(z)\, j_\nu(z) = \frac{1}{\sqrt{2}}\, G_{2,4}^{2,1}\!\left(z \,\middle|\, \begin{matrix} \frac{1}{4}, -\frac{1}{4} \\ \frac{1}{4}, \nu-\frac{1}{4}, -\frac{3}{4}, -\nu-\frac{1}{4} \end{matrix}\right)$$

03.21.26.0096.01

$$Y_{\nu-\frac{3}{2}}(z)\, j_\nu(z) = \frac{1}{\sqrt{2}}\, G_{2,4}^{2,1}\!\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} -\frac{1}{4}, \frac{1}{4} \\ \frac{3}{4}, \nu-\frac{3}{4}, -\frac{5}{4}, \frac{1}{4}-\nu \end{matrix}\right)$$

03.21.26.0097.01

$$Y_\mu(z)\, j_\nu(z) = -\frac{1}{\sqrt{2}}\, G_{3,5}^{2,2}\!\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} -\frac{1}{4}, \frac{1}{4}, \frac{1}{2}(-\mu+\nu+1) \\ \frac{\nu-\mu}{2}, \frac{\mu+\nu}{2}, -\frac{1}{2}(\mu+\nu+1), \frac{1}{2}(\mu-\nu-1), \frac{1}{2}(-\mu+\nu+1) \end{matrix}\right)$$

Generalized cases involving Bessel *J*, *Y*, *y*

03.21.26.0098.01

$$Y_{-\nu-\frac{1}{2}}(z)\, j_\nu(z) + J_{-\nu-\frac{1}{2}}(z)\, y_\nu(z) = -\sqrt{2}\, G_{1,3}^{2,0}\!\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} \frac{1}{4} \\ -\nu-\frac{3}{4}, \nu+\frac{1}{4}, -\frac{1}{4} \end{matrix}\right)$$

03.21.26.0099.01

$$Y_{-\nu-\frac{1}{2}}(z)\, j_\nu(z) - J_{-\nu-\frac{1}{2}}(z)\, y_\nu(z) = -\frac{\sin(2\pi\nu)}{\sqrt{2}\,\pi^2}\, G_{1,3}^{3,1}\!\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} \frac{1}{4} \\ -\frac{1}{4}, -\nu-\frac{3}{4}, \nu+\frac{1}{4} \end{matrix}\right)$$

03.21.26.0100.01

$$Y_\mu(z)\, j_\nu(z) + J_\mu(z)\, y_\nu(z) = -\sqrt{2}\, G_{2,4}^{3,0}\!\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} -\frac{1}{4}, \frac{1}{4} \\ \frac{1}{2}(\mu-\nu-1), \frac{\nu-\mu}{2}, \frac{\mu+\nu}{2}, -\frac{1}{2}(\mu+\nu+1) \end{matrix}\right)$$

03.21.26.0101.01

$$Y_\mu(z)\, j_\nu(z) - J_\mu(z)\, y_\nu(z) = \frac{\cos(\pi(\mu-\nu))}{\sqrt{2}\,\pi^2}\, G_{2,4}^{3,2}\!\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} -\frac{1}{4}, \frac{1}{4} \\ \frac{1}{2}(\mu-\nu-1), \frac{\nu-\mu}{2}, \frac{\mu+\nu}{2}, -\frac{1}{2}(\mu+\nu+1) \end{matrix}\right)$$

03.21.26.0102.01

$$J_{-\nu-\frac{1}{2}}(z)\, j_\nu(z) - Y_{-\nu-\frac{1}{2}}(z)\, y_\nu(z) = -\sqrt{2}\, G_{2,4}^{3,0}\!\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} \frac{1}{4}, \frac{1}{4} \\ -\frac{1}{4}, -\nu-\frac{3}{4}, \nu+\frac{1}{4}, \frac{1}{4} \end{matrix}\right)$$

03.21.26.0103.01

$$J_\mu(z)\, j_\nu(z) - Y_\mu(z)\, y_\nu(z) = -\sqrt{2}\, G_{3,5}^{4,0}\!\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} -\frac{1}{4}, \frac{1}{4}, -\frac{1}{2}(\mu+\nu) \\ -\frac{1}{2}(\mu+\nu+1), \frac{1}{2}(\mu-\nu-1), \frac{\nu-\mu}{2}, \frac{\mu+\nu}{2}, -\frac{1}{2}(\mu+\nu) \end{matrix}\right)$$

03.21.26.0104.01

$$J_\mu(z)\, j_\nu(z) + Y_\mu(z)\, y_\nu(z) = \frac{1}{\sqrt{2}\,\pi^2}\left(\cos(\pi\mu)\, G_{2,4}^{3,2}\!\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} -\frac{1}{4}, \frac{1}{4} \\ -\frac{1}{2}(\mu+\nu+1), \frac{1}{2}(\mu-\nu-1), \frac{\mu+\nu}{2}, \frac{\nu-\mu}{2} \end{matrix}\right) - \right.$$
$$\left. \sin(\pi\nu)\, G_{2,4}^{3,2}\!\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} -\frac{1}{4}, \frac{1}{4} \\ -\frac{1}{2}(\mu+\nu+1), \frac{\nu-\mu}{2}, \frac{\mu+\nu}{2}, \frac{1}{2}(\mu-\nu-1) \end{matrix}\right)\right)$$

Generalized cases involving Bessel *I*

03.21.26.0105.01

$$I_{\nu+\frac{1}{2}}(z)\, j_\nu(z) = \frac{\pi}{2\sqrt[4]{2}}\, G_{0,4}^{1,0}\left(\frac{z}{2\sqrt{2}}, \frac{1}{4} \,\middle|\, \frac{4\nu+1}{8}, -\frac{1}{8}, \frac{3}{8}, -\frac{4\nu+3}{8}\right)$$

03.21.26.0106.01

$$I_{-\nu-\frac{1}{2}}(z)\, j_\nu(z) = \frac{\pi}{2\sqrt[4]{2}}\, G_{1,5}^{2,0}\left(\frac{z}{2\sqrt{2}}, \frac{1}{4} \,\middle|\, \begin{array}{c} \frac{1-4\nu}{8} \\ -\frac{1}{8}, \frac{3}{8}, -\frac{4\nu+3}{8}, \frac{1-4\nu}{8}, \frac{4\nu+1}{8} \end{array}\right)$$

03.21.26.0107.01

$$I_{\nu+\frac{1}{2}}(z)\, (j_{-\nu-1}(z) + j_\nu(z)) = \frac{\pi \cos\left(\frac{1}{4}\pi(2\nu+1)\right)}{\sqrt[4]{2}}\, G_{2,6}^{3,0}\left(\frac{z}{2\sqrt{2}}, \frac{1}{4} \,\middle|\, \begin{array}{c} \frac{\nu}{4}, \frac{\nu+2}{4} \\ -\frac{1}{8}, \frac{3}{8}, \frac{4\nu+1}{8}, -\frac{4\nu+3}{8}, \frac{\nu}{4}, \frac{\nu+2}{4} \end{array}\right)$$

03.21.26.0108.01

$$I_{\nu+\frac{1}{2}}(z)\, (j_{-\nu-1}(z) - j_\nu(z)) = \frac{\pi \sin\left(\frac{1}{4}\pi(2\nu+1)\right)}{\sqrt[4]{2}}\, G_{2,6}^{3,0}\left(\frac{z}{2\sqrt{2}}, \frac{1}{4} \,\middle|\, \begin{array}{c} \frac{\nu+1}{4}, \frac{\nu+3}{4} \\ -\frac{1}{8}, \frac{3}{8}, \frac{4\nu+1}{8}, -\frac{4\nu+3}{8}, \frac{\nu+1}{4}, \frac{\nu+3}{4} \end{array}\right)$$

Generalized cases involving Bessel *K*

03.21.26.0109.01

$$K_{\nu+\frac{1}{2}}(z)\, j_\nu(z) = \frac{1}{8\sqrt[4]{2}}\, G_{0,4}^{3,0}\left(\frac{z}{2\sqrt{2}}, \frac{1}{4} \,\middle|\, -\frac{1}{8}, \frac{3}{8}, \frac{4\nu+1}{8}, -\frac{4\nu+3}{8}\right)$$

03.21.26.0110.01

$$K_{\nu+\frac{1}{2}}(z)\, (j_{-\nu-1}(z) + j_\nu(z)) = \frac{\cos\left(\frac{1}{4}\pi(2\nu+1)\right)}{4\sqrt[4]{2}}\, G_{0,4}^{3,0}\left(\frac{z}{2\sqrt{2}}, \frac{1}{4} \,\middle|\, \frac{3}{8}, -\frac{4\nu+3}{8}, \frac{4\nu+1}{8}, -\frac{1}{8}\right)$$

03.21.26.0111.01

$$K_{\nu+\frac{1}{2}}(z)\, (j_{-\nu-1}(z) - j_\nu(z)) = \frac{\sin\left(\frac{1}{4}\pi(2\nu+1)\right)}{4\sqrt[4]{2}}\, G_{0,4}^{3,0}\left(\frac{z}{2\sqrt{2}}, \frac{1}{4} \,\middle|\, -\frac{1}{8}, -\frac{1}{8}(4\nu+3), \frac{1}{8}(4\nu+1), \frac{3}{8}\right)$$

Generalized cases involving spherical Bessel *y*

03.21.26.0112.01

$$\cos(a\pi)\, j_\nu(z) + \sin(a\pi)\, y_\nu(z) = \frac{1}{2}\sqrt{\pi}\, G_{1,3}^{2,0}\left(\frac{z}{2}, \frac{1}{2} \,\middle|\, \begin{array}{c} -\frac{1}{2}(2a+\nu+1) \\ -\frac{1}{2}(\nu+1), \frac{\nu}{2}, -\frac{1}{2}(2a+\nu+1) \end{array}\right)$$

03.21.26.0113.01

$$j_\nu(z)\, y_\nu(z) = -\frac{\sqrt{\pi}}{2}\, G_{1,3}^{2,0}\left(z, \frac{1}{2} \,\middle|\, \begin{array}{c} 0 \\ -\frac{1}{2}, \nu, -\nu-1 \end{array}\right)$$

03.21.26.0114.01

$$j_{-\nu-1}(z)\, y_\nu(z) = \frac{\sqrt{\pi}}{2}\, G_{2,4}^{2,1}\left(z, \frac{1}{2} \,\middle|\, \begin{array}{c} 0, -\nu-\frac{3}{2} \\ -\frac{1}{2}, -\nu-1, -\nu-\frac{3}{2}, \nu \end{array}\right)$$

03.21.26.0115.01

$$j_{\nu+1}(z)\, y_\nu(z) = \frac{1}{2}\sqrt{\pi}\, G_{2,4}^{2,1}\left(z, \frac{1}{2} \,\middle|\, \begin{array}{c} 0, -\frac{1}{2} \\ 0, \nu+\frac{1}{2}, -1, -\nu-\frac{3}{2} \end{array}\right)$$

03.21.26.0116.01

$$j_{\nu+2}(z)\,y_\nu(z) = \frac{1}{2}\sqrt{\pi}\;G_{2,4}^{2,1}\!\left(z,\;\frac{1}{2}\;\middle|\;\begin{array}{c} -\frac{1}{2},\,0 \\ \frac{1}{2},\,\nu+1,\,-\frac{3}{2},\,-\nu-2 \end{array}\right)$$

03.21.26.0117.01

$$j_\mu(z)\,y_\nu(z) = \frac{1}{2}\left(-\sqrt{\pi}\right)G_{3,5}^{2,2}\!\left(z,\;\frac{1}{2}\;\middle|\;\begin{array}{c} 0,\,-\frac{1}{2},\,\frac{\mu-\nu}{2} \\ \frac{\mu-\nu-1}{2},\,\frac{\mu+\nu}{2},\,-\frac{\mu+\nu+2}{2},\,\frac{\mu-\nu}{2},\,\frac{\nu-\mu-1}{2} \end{array}\right)$$

03.21.26.0118.01

$$j_\nu(z)\,y_{-\nu-1}(z) + j_{-\nu-1}(z)\,y_\nu(z) = -\sqrt{\pi}\;G_{1,3}^{2,0}\!\left(z,\;\frac{1}{2}\;\middle|\;\begin{array}{c} 0 \\ -\nu-1,\,\nu,\,-\frac{1}{2} \end{array}\right)$$

03.21.26.0119.01

$$j_\nu(z)\,y_{-\nu-1}(z) - j_{-\nu-1}(z)\,y_\nu(z) = -\frac{\sin(2\pi\nu)}{2\pi^{3/2}}\,G_{1,3}^{3,1}\!\left(z,\;\frac{1}{2}\;\middle|\;\begin{array}{c} 0 \\ -\frac{1}{2},\,-\nu-1,\,\nu \end{array}\right)$$

03.21.26.0120.01

$$j_\nu(z)\,y_\mu(z) + j_\mu(z)\,y_\nu(z) = -\sqrt{\pi}\;G_{2,4}^{3,0}\!\left(z,\;\frac{1}{2}\;\middle|\;\begin{array}{c} 0,\,-\frac{1}{2} \\ \frac{\mu-\nu-1}{2},\,\frac{\nu-\mu-1}{2},\,\frac{\mu+\nu}{2},\,-\frac{\mu+\nu+2}{2} \end{array}\right)$$

03.21.26.0121.01

$$j_\nu(z)\,y_\mu(z) - j_\mu(z)\,y_\nu(z) = \frac{\sin(\pi(\nu-\mu))}{2\pi^{3/2}}\,G_{2,4}^{3,2}\!\left(z,\;\frac{1}{2}\;\middle|\;\begin{array}{c} 0,\,-\frac{1}{2} \\ \frac{\mu-\nu-1}{2},\,\frac{\nu-\mu-1}{2},\,\frac{\mu+\nu}{2},\,-\frac{\mu+\nu+2}{2} \end{array}\right)$$

03.21.26.0122.01

$$j_\nu(z)^2 + y_\nu(z)^2 = -\frac{\sin(\pi\nu)}{\pi^{3/2}}\,G_{1,3}^{3,1}\!\left(z,\;\frac{1}{2}\;\middle|\;\begin{array}{c} 0 \\ -\frac{1}{2},\,-\nu-1,\,\nu \end{array}\right)$$

03.21.26.0123.01

$$j_\nu(z)^2 - y_\nu(z)^2 = -\sqrt{\pi}\;G_{2,4}^{3,0}\!\left(z,\;\frac{1}{2}\;\middle|\;\begin{array}{c} 0,\,-\nu-\frac{1}{2} \\ -\frac{1}{2},\,-\nu-1,\,\nu,\,-\nu-\frac{1}{2} \end{array}\right)$$

03.21.26.0124.01

$$j_{-\nu-1}(z)\,j_\nu(z) + y_{-\nu-1}(z)\,y_\nu(z) = \frac{\sin^2(\pi\nu)}{\pi^{3/2}}\,G_{1,3}^{3,1}\!\left(z,\;\frac{1}{2}\;\middle|\;\begin{array}{c} 0 \\ -\frac{1}{2},\,-\nu-1,\,\nu \end{array}\right)$$

03.21.26.0125.01

$$j_{-\nu-1}(z)\,j_\nu(z) - y_{-\nu-1}(z)\,y_\nu(z) = -\sqrt{\pi}\;G_{2,4}^{3,0}\!\left(z,\;\frac{1}{2}\;\middle|\;\begin{array}{c} 0,\,0 \\ -\frac{1}{2},\,-\nu-1,\,\nu,\,0 \end{array}\right)$$

03.21.26.0126.01

$$j_\mu(z)\,j_\nu(z) - y_\mu(z)\,y_\nu(z) = -\sqrt{\pi}\;G_{3,5}^{4,0}\!\left(z,\;\frac{1}{2}\;\middle|\;\begin{array}{c} -\frac{1}{2},\,0,\,-\frac{\mu+\nu+1}{2} \\ -\frac{\mu+\nu+2}{2},\,\frac{\mu-\nu-1}{2},\,\frac{\nu-\mu-1}{2},\,\frac{\mu+\nu}{2},\,-\frac{\mu+\nu+1}{2} \end{array}\right)$$

03.21.26.0127.01

$$j_\mu(z)\, j_\nu(z) + y_\mu(z)\, y_\nu(z) = -\frac{\sin(\pi\mu)}{2\pi^{3/2}}\, G_{2,4}^{3,2}\!\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} 0, -\frac{1}{2} \\ \frac{1}{2}(-\mu-\nu-2), \frac{1}{2}(\mu-\nu-1), \frac{\mu+\nu}{2}, \frac{1}{2}(-\mu+\nu-1) \end{matrix}\right) - \frac{\sin(\pi\nu)}{2\pi^{3/2}}\, G_{2,4}^{3,2}\!\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} 0, -\frac{1}{2} \\ \frac{1}{2}(-\mu-\nu-2), \frac{1}{2}(-\mu+\nu-1), \frac{\mu+\nu}{2}, \frac{1}{2}(\mu-\nu-1) \end{matrix}\right)$$

Generalized cases involving cos, sin, y

03.21.26.0128.01

$$\sin(z)\, j_\nu(z) + \cos(z)\, y_\nu(z) = -\sqrt{\pi}\, G_{2,4}^{3,0}\!\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} 0, \frac{1}{2} \\ -\frac{1}{2}(\nu+1), \frac{\nu}{2}, \frac{\nu+1}{2}, -\frac{\nu}{2} \end{matrix}\right)$$

03.21.26.0129.01

$$\cos(z)\, j_\nu(z) - \sin(z)\, y_\nu(z) = \sqrt{\pi}\, G_{2,4}^{3,0}\!\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} 0, \frac{1}{2} \\ -\frac{\nu}{2}, \frac{\nu}{2}, \frac{\nu+1}{2}, -\frac{1+\nu}{2} \end{matrix}\right)$$

03.21.26.0130.01

$$\sin(z)\, j_\nu(z) - \cos(z)\, y_\nu(z) = -\frac{\sin(\pi\nu)}{2\pi^{3/2}}\, G_{2,4}^{3,2}\!\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} 0, \frac{1}{2} \\ -\frac{1}{2}(\nu+1), \frac{\nu}{2}, \frac{\nu+1}{2}, -\frac{\nu}{2} \end{matrix}\right)$$

03.21.26.0131.01

$$\cos(z)\, j_\nu(z) + \sin(z)\, y_\nu(z) = -\frac{\sin(\pi\nu)}{2\pi^{3/2}}\, G_{2,4}^{3,2}\!\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} 0, \frac{1}{2} \\ -\frac{\nu}{2}, \frac{\nu}{2}, \frac{\nu+1}{2}, -\frac{\nu+1}{2} \end{matrix}\right)$$

03.21.26.0132.01

$$\sin(a+z)\, j_\nu(z) - \cos(a+z)\, y_\nu(z) = -\frac{\sin(\pi\nu)}{2\pi^{3/2}}\, G_{3,5}^{4,2}\!\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} 0, \frac{1}{2}, -\frac{a}{\pi} - \frac{\nu}{2} \\ -\frac{1}{2}(\nu+1), -\frac{\nu}{2}, \frac{\nu}{2}, \frac{\nu+1}{2}, -\frac{a}{\pi} - \frac{\nu}{2} \end{matrix}\right)$$

Generalized cases involving ${}_0F_1$

03.21.26.0133.01

$${}_0F_1\!\left(; b; -\frac{z^2}{4}\right) j_\nu(z) = 2^{b-\frac{3}{2}}\, \Gamma(b)\, G_{2,4}^{1,2}\!\left(z, \frac{1}{2} \,\middle|\, \begin{matrix} \frac{1}{4}(1-2b), \frac{1}{4}(3-2b) \\ \frac{\nu}{2}, -\frac{1}{2}(\nu+1), \frac{1}{2}(-2b+\nu+2), \frac{1}{2}(-2b-\nu+1) \end{matrix}\right)$$

03.21.26.0134.01

$${}_0F_1\!\left(; \nu + \frac{3}{2}; \frac{z^2}{4}\right) j_\nu(z) = 2^{-\frac{\nu}{2}-\frac{3}{2}}\, \pi\, \Gamma\!\left(\nu + \frac{3}{2}\right) G_{0,4}^{1,0}\!\left(\frac{z}{2\sqrt{2}}, \frac{1}{4} \,\middle|\, \frac{\nu}{4}, -\frac{1}{4}(\nu+1), \frac{1-\nu}{4}, -\frac{1}{4}(3\nu+2)\right)$$

03.21.26.0135.01

$${}_0F_1\!\left(; \frac{1}{2} - \nu; \frac{z^2}{4}\right) j_\nu(z) = 2^{\frac{\nu}{2}-1}\, \pi\, \Gamma\!\left(\frac{1}{2} - \nu\right) G_{1,5}^{2,0}\!\left(\frac{z}{2\sqrt{2}}, \frac{1}{4} \,\middle|\, \begin{matrix} \frac{1-\nu}{4} \\ \frac{\nu}{4}, \frac{\nu+2}{4}, -\frac{1}{4}(\nu+1), \frac{1-\nu}{4}, \frac{1}{4}(3\nu+1) \end{matrix}\right)$$

Generalized cases involving ${}_0\tilde{F}_1$

03.21.26.0136.01

$${}_0\tilde{F}_1\left(;b;-\frac{z^2}{4}\right) j_\nu(z) = 2^{b-\frac{3}{2}} G_{2,4}^{1,2}\left(z, \frac{1}{2} \middle| \begin{array}{c} \frac{1}{4}(1-2b), \frac{1}{4}(3-2b) \\ \frac{\nu}{2}, -\frac{1}{2}(\nu+1), \frac{1}{2}(-2b+\nu+2), \frac{1}{2}(-2b-\nu+1) \end{array}\right)$$

03.21.26.0137.01

$${}_0\tilde{F}_1\left(;\nu+\frac{3}{2};\frac{z^2}{4}\right) j_\nu(z) = 2^{-\frac{\nu}{2}-\frac{3}{2}} \pi\, G_{0,4}^{1,0}\left(\frac{z}{2\sqrt{2}}, \frac{1}{4} \middle| \frac{\nu}{4}, -\frac{1}{4}(\nu+1), \frac{1-\nu}{4}, -\frac{1}{4}(3\nu+2)\right)$$

03.21.26.0138.01

$${}_0\tilde{F}_1\left(;\frac{1}{2}-\nu;\frac{z^2}{4}\right) j_\nu(z) = 2^{\frac{\nu}{2}-1} \pi\, G_{1,5}^{2,0}\left(\frac{z}{2\sqrt{2}}, \frac{1}{4} \middle| \begin{array}{c} \frac{1-\nu}{4} \\ \frac{\nu}{4}, \frac{\nu+2}{4}, -\frac{1}{4}(\nu+1), \frac{1-\nu}{4}, \frac{1}{4}(3\nu+1) \end{array}\right)$$

Through other functions

03.21.26.0139.01

$$j_\nu(z) = \sqrt{\frac{\pi}{2}} \frac{1}{\sqrt{z}} J_{\nu+\frac{1}{2}}(z)$$

03.21.26.0140.01

$$j_\nu(z) = \frac{(-1)^\nu}{\sqrt{z}} \sqrt{\frac{\pi}{2}} \boldsymbol{H}_{-\nu-\frac{1}{2}}(z) \;/;\; \nu \in \mathbb{N}$$

Representations through equivalent functions

With related functions

03.21.27.0001.01

$$j_\nu(z) = \sqrt{\frac{\pi}{2}} (i\,z)^{-\nu-\frac{1}{2}} z^\nu I_{\nu+\frac{1}{2}}(i\,z)$$

03.21.27.0002.01

$$j_\nu(i\,z) = \sqrt{\frac{\pi}{2}} (i\,z)^\nu z^{-\nu-\frac{1}{2}} I_{\nu+\frac{1}{2}}(z)$$

03.21.27.0003.01

$$j_\nu(z) = \sec(\pi\,\nu)\, y_{-\nu-1}(z) + y_\nu(z) \tan(\pi\,\nu)$$

03.21.27.0004.01

$$j_\nu(z)\, y_{\nu+1}(z) - j_{\nu+1}(z)\, y_\nu(z) = -\frac{1}{z^2}$$

03.21.27.0005.01

$$j_\nu(z) = 2^{-\nu-1} \sqrt{\pi}\, z^\nu\, {}_0\tilde{F}_1\left(;\nu+\frac{3}{2};-\frac{z^2}{4}\right)$$

03.21.27.0006.01

$$j_\nu(z) = \frac{2^{-\nu-1}\sqrt{\pi}\, z^\nu}{\Gamma\left(\nu+\frac{3}{2}\right)} {}_0F_1\left(;\nu+\frac{3}{2};-\frac{z^2}{4}\right) \;/;\; -\nu-\frac{1}{2} \notin \mathbb{N}^+$$

03.21.27.0007.01

$$j_\nu(z) = \frac{2^{-\nu-1} \sqrt{\pi}\, z^\nu}{\Gamma\left(\nu + \frac{3}{2}\right)} \, {}_0F_1\left(; \nu + \frac{3}{2}; -\frac{z^2}{4}\right) /; -\nu - \frac{1}{2} \notin \mathbb{N}^+$$

03.21.27.0008.01

$$j_\nu(z) = e^{\frac{3}{8} i \pi (2\nu+1)} \sqrt{\frac{\pi}{2}} \, z^\nu \left(-(-1)^{3/4} z\right)^{-\nu-\frac{1}{2}} \left(\mathrm{ber}_{\nu+\frac{1}{2}}\left(-(-1)^{3/4} z\right) - i \, \mathrm{bei}_{\nu+\frac{1}{2}}\left(-(-1)^{3/4} z\right)\right)$$

03.21.27.0009.01

$$j_\nu\left(\sqrt[4]{-1}\, z\right) = e^{\frac{3}{8} i \pi (2\nu+1)} \sqrt{\frac{\pi}{2}} \, z^{-\nu-\frac{1}{2}} \left(\sqrt[4]{-1}\, z\right)^\nu \left(\mathrm{ber}_{\nu+\frac{1}{2}}(z) - i \, \mathrm{bei}_{\nu+\frac{1}{2}}(z)\right)$$

Copyright