SpheroidalEigenvalue

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Notations

Traditional name

Eigenvalue of the spheroidal wave functions

Traditional notation

$\lambda_{\nu,\mu}(\gamma)$

Mathematica StandardForm notation

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SpheroidalEigenvalue[ν, μ, γ]
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Primary definition

11.16.02.0001.01

$\lambda_{\nu,\mu}(\gamma)$

$\lambda == \lambda_{\nu,\mu}(\gamma)$ is the spheroidal eigenvalue of degree ν and order μ of the corresponding Sturm-Liouville wave differential equation $\left(1-z^2\right) w''(z) - 2\,z\,w'(z) + \left(\lambda + \gamma^2\left(1-z^2\right) - \mu^2 \left/ \left(1-z^2\right)\right.\right) w(z) == 0$. The parameter ν enumerates the spheroidal eigenvalues in such a manner, that in the spherical limit ($\gamma \to 0$), the eigenvalues are $\lambda_{\nu,\mu}(0) == \nu(\nu+1)$. $\lambda_{\nu,\mu}(\gamma)$ is an analytical function of the variables ν, μ and γ.

Specific values

Specialized values

For fixed ν, μ

11.16.03.0001.01

$$\lambda_{\nu,\mu}(0) = \nu\,(\nu+1)$$

For fixed ν, λ

11.16.03.0002.01

$$\lambda_{\nu,-\frac{1}{2}}(\gamma) = -\frac{\gamma^2}{2} - \frac{1}{4} + a_{\nu+\frac{1}{2}}\left(\frac{\gamma^2}{4}\right)$$

11.16.03.0003.01

$$\lambda_{\nu,\frac{1}{2}}(\gamma) = -\frac{\gamma^2}{2} - \frac{1}{4} + a_{\nu+\frac{1}{2}}\left(\frac{\gamma^2}{4}\right)$$

For fixed λ

11.16.03.0004.01
$$\lambda_{0,1}(\gamma) = 0$$

11.16.03.0005.01
$$\lambda_{0,2}(\gamma) = 1 - \sqrt{1 - 4\,\gamma^2}$$

11.16.03.0006.01
$$\lambda_{1,2}(\gamma) = 1 + \sqrt{1 - 4\,\gamma^2}$$

General characteristics

Domain and analyticity

$\lambda_{\nu,\mu}(\gamma)$ is an analytical function of ν, μ and γ which is defined in $\mathbb{C}^3$.

11.16.04.0001.01
$$(\nu * \mu * \gamma) \longrightarrow \lambda_{\nu,\mu}(\gamma) :: (\mathbb{C} \otimes \mathbb{C} \otimes \mathbb{C}) \longrightarrow \mathbb{C}$$

Symmetries and periodicities

Parity

11.16.04.0002.01
$$\lambda_{-\nu,\mu}(\gamma) = \lambda_{\nu-1,\mu}(\gamma)$$

11.16.04.0003.01
$$\lambda_{\nu,-\mu}(\gamma) = \lambda_{\nu,\mu}(\gamma)$$

Mirror symmetry

11.16.04.0004.01
$$\lambda_{\overline{\nu},\overline{\mu}}(\overline{\gamma}) == \overline{\lambda_{\nu,\mu}(\gamma)}$$

Periodicity

No periodicity

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