Tan

Notations

Traditional name

Tangent

Traditional notation

$\tan(z)$

Mathematica StandardForm notation

Tan[z]

Primary definition

01.08.02.0001.01

$$\tan(z) = \frac{\sin(z)}{\cos(z)} = -\frac{i\left(e^{i z} - e^{-i z}\right)}{e^{i z} + e^{-i z}}$$

Specific values

Specialized values

01.08.03.0001.01

$$\tan\left(\pi\left(\frac{1}{2} + m\right)\right) = \tilde{\infty} /; m \in \mathbb{Z}$$

01.08.03.0002.01

$$\tan(\pi m) = 0 /; m \in \mathbb{Z}$$

Values at fixed points

01.08.03.0003.01

$$\tan(0) = 0$$

01.08.03.0004.01

$$\tan\left(\frac{\pi}{12}\right) = 2 - \sqrt{3}$$

01.08.03.0005.01

$$\tan\left(\frac{\pi}{10}\right) = \sqrt{1 - \frac{2}{\sqrt{5}}}$$

01.08.03.0006.01

$$\tan\left(\frac{\pi}{10}\right) == \left(z; 5z^4 - 10z^2 + 1\right)_3^{-1}$$

01.08.03.0007.01

$$\tan\left(\frac{\pi}{9}\right) == \frac{\left(-1 - i\sqrt{3}\right)^{4/3} - \left(-1 + i\sqrt{3}\right)^{4/3}}{\sqrt[3]{-1 - i\sqrt{3}}\left(-i + \sqrt{3}\right) - \sqrt[3]{-1 + i\sqrt{3}}\left(i + \sqrt{3}\right)}$$

01.08.03.0008.01

$$\tan\left(\frac{\pi}{9}\right) == \left(z; z^6 - 33z^4 + 27z^2 - 3\right)_4^{-1}$$

01.08.03.0009.01

$$\tan\left(\frac{\pi}{9}\right) == -\frac{i\left(-1 + (-1)^{2/9}\right)}{1 + (-1)^{2/9}}$$

01.08.03.0010.01

$$\tan\left(\frac{\pi}{8}\right) == \sqrt{2} - 1$$

01.08.03.0011.01

$$\tan\left(\frac{\pi}{7}\right) == \left(2\sqrt{7}\sqrt[3]{7 + \frac{i\sqrt{7}}{2} + \frac{3\sqrt{21}}{2}} - 2i\sqrt{21}\sqrt[3]{7 + \frac{i\sqrt{7}}{2} + \frac{3\sqrt{21}}{2}} - \right.$$
$$2\,2^{2/3}\,7^{5/6}\sqrt[3]{1 - 3i\sqrt{3}} - i\left(14 - i\sqrt{7} - 3\sqrt{21}\right)^{2/3}\sqrt[3]{14 + i\sqrt{7} + 3\sqrt{21}} +$$
$$\sqrt{3}\left(14 + i\sqrt{7} + 3\sqrt{21}\right)^{2/3}\sqrt[3]{14 - i\sqrt{7} - 3\sqrt{21}} + \sqrt{3}\sqrt[3]{14 + i\sqrt{7} + 3\sqrt{21}}\left(14 - i\sqrt{7} - 3\sqrt{21}\right)^{2/3} +$$
$$\left. 2\sqrt[3]{7 - \frac{i\sqrt{7}}{2} - \frac{3\sqrt{21}}{2}}\left(\sqrt{7} + i\sqrt{21}\right) + \left(14 + i\sqrt{7} + 3\sqrt{21}\right)^{2/3}\sqrt[3]{14 - i\sqrt{7} - 3\sqrt{21}}\, i\right) \Big/$$
$$\left(2\sqrt{7}\, i\sqrt[3]{7 + \frac{i\sqrt{7}}{2} + \frac{3\sqrt{21}}{2}} + 2\sqrt{21}\sqrt[3]{7 + \frac{i\sqrt{7}}{2} + \frac{3\sqrt{21}}{2}} + 2\,2^{2/3}\sqrt[3]{7 - 21i\sqrt{3}} + \right.$$
$$\left(14 + i\sqrt{7} + 3\sqrt{21}\right)^{2/3}\sqrt[3]{14 - i\sqrt{7} - 3\sqrt{21}} - i\sqrt{3}\sqrt[3]{14 - i\sqrt{7} - 3\sqrt{21}}\left(14 + i\sqrt{7} + 3\sqrt{21}\right)^{2/3} +$$
$$\sqrt[3]{14 + i\sqrt{7} + 3\sqrt{21}}\left(14 - i\sqrt{7} - 3\sqrt{21}\right)^{2/3} + 2\sqrt{7}\sqrt[3]{7 - \frac{i\sqrt{7}}{2} - \frac{3\sqrt{21}}{2}}\left(-i + \sqrt{3}\right) +$$
$$\left.\sqrt{3}\sqrt[3]{14 + i\sqrt{7} + 3\sqrt{21}}\left(14 - i\sqrt{7} - 3\sqrt{21}\right)^{2/3} i\right)$$

01.08.03.0012.01

$$\tan\left(\frac{\pi}{7}\right) == \left(z; z^6 - 21z^4 + 35z^2 - 7\right)_4^{-1}$$

01.08.03.0013.01

$$\tan\left(\frac{\pi}{7}\right) == -\frac{i\left(-1+(-1)^{2/7}\right)}{1+(-1)^{2/7}}$$

01.08.03.0014.01

$$\tan\left(\frac{\pi}{6}\right) == \frac{1}{\sqrt{3}}$$

01.08.03.0015.01

$$\tan\left(\frac{\pi}{5}\right) == \sqrt{5-2\sqrt{5}}$$

01.08.03.0016.01

$$\tan\left(\frac{\pi}{5}\right) == \left(z;\, z^4-10\,z^2+5\right)_3^{-1}$$

01.08.03.0017.01

$$\tan\left(\frac{2\pi}{9}\right) == \frac{i\left(\sqrt[3]{-\frac{1}{2}\,i\left(-i+\sqrt{3}\right)}-\sqrt[3]{\frac{1}{2}\,i\left(i+\sqrt{3}\right)}\right)}{\sqrt[3]{-\frac{1}{2}\,i\left(-i+\sqrt{3}\right)}+\sqrt[3]{\frac{1}{2}\,i\left(i+\sqrt{3}\right)}}$$

01.08.03.0018.01

$$\tan\left(\frac{2\pi}{9}\right) == \left(z;\, z^6-33\,z^4+27\,z^2-3\right)_5^{-1}$$

01.08.03.0019.01

$$\tan\left(\frac{2\pi}{9}\right) == -\frac{i\left(-1+(-1)^{4/9}\right)}{1+(-1)^{4/9}}$$

01.08.03.0020.01

$$\tan\left(\frac{\pi}{4}\right) == 1$$

01.08.03.0021.01

$$\tan\left(\frac{2\pi}{7}\right) == \left(\sqrt[3]{1-3\,i\sqrt{3}}\left(4\,7^{5/6}\sqrt[3]{2-6\,i\sqrt{3}}+\sqrt[3]{2}\left(2\left(\sqrt[3]{28-84\,i\sqrt{3}}+\sqrt{7}\,i-\sqrt{21}\right)i\sqrt[3]{14+i\sqrt{7}+3\sqrt{21}}+4\sqrt{7}\sqrt[3]{14-i\sqrt{7}-3\sqrt{21}}+\left(i+\sqrt{3}\right)\sqrt[3]{28+2\,i\sqrt{7}+6\sqrt{21}}\left(14-i\sqrt{7}-3\sqrt{21}\right)^{2/3}\right)\right)\right)\Big/\left(4\left(7\,2^{2/3}\sqrt[3]{1-3\,i\sqrt{3}}-\sqrt[3]{14}\left(1-3\,i\sqrt{3}\right)^{2/3}+7^{2/3}\left(1-3\,i\sqrt{3}\right)\right)\right)$$

01.08.03.0022.01

$$\tan\left(\frac{2\pi}{7}\right) == \left(z;\, z^6-21\,z^4+35\,z^2-7\right)_5^{-1}$$

01.08.03.0023.01

$$\tan\left(\frac{2\pi}{7}\right) == -\frac{i\left(-1+(-1)^{4/7}\right)}{1+(-1)^{4/7}}$$

01.08.03.0024.01

$$\tan\left(\frac{3\pi}{10}\right) == \sqrt{1+\frac{2}{\sqrt{5}}}$$

01.08.03.0025.01

$$\tan\left(\frac{3\pi}{10}\right) == \left(z;\, 5\,z^4 - 10\,z^2 + 1\right)_4^{-1}$$

01.08.03.0026.01

$$\tan\left(\frac{\pi}{3}\right) == \sqrt{3}$$

01.08.03.0027.01

$$\tan\left(\frac{3\pi}{8}\right) == 1+\sqrt{2}$$

01.08.03.0028.01

$$\tan\left(\frac{2\pi}{5}\right) == \sqrt{5+2\sqrt{5}}$$

01.08.03.0029.01

$$\tan\left(\frac{2\pi}{5}\right) == \left(z;\, z^4 - 10\,z^2 + 5\right)_4^{-1}$$

01.08.03.0030.01

$$\tan\left(\frac{5\pi}{12}\right) == 2+\sqrt{3}$$

01.08.03.0031.01

$$\tan\left(\frac{3\pi}{7}\right) == \left(\sqrt[3]{2}\left(-4\sqrt{7}\,\sqrt[3]{14+i\sqrt{7}+3\sqrt{21}} - \left(-i+\sqrt{3}\right)\sqrt[3]{28-2\,i\sqrt{7}-6\sqrt{21}}\left(14+i\sqrt{7}+3\sqrt{21}\right)^{2/3} + 2\sqrt[3]{14-i\sqrt{7}-3\sqrt{21}}\left(\sqrt{7}-i\sqrt{21}\right) + 2\sqrt[3]{28+2\,i\sqrt{7}+6\sqrt{21}}\left(14-i\sqrt{7}-3\sqrt{21}\right)^{2/3} i\right) - 4\,7^{5/6}\sqrt[3]{2-6\,i\sqrt{3}}\right) \Big/ \left(\sqrt[3]{2}\left(4\sqrt{7}\,i\sqrt[3]{14+i\sqrt{7}+3\sqrt{21}} + \left(-1-i\sqrt{3}\right)\sqrt[3]{28-2\,i\sqrt{7}-6\sqrt{21}}\left(14+i\sqrt{7}+3\sqrt{21}\right)^{2/3} + 2\sqrt{7}\left(i+\sqrt{3}\right)\sqrt[3]{14-i\sqrt{7}-3\sqrt{21}} + 2\sqrt[3]{28+2\,i\sqrt{7}+6\sqrt{21}}\left(14-i\sqrt{7}-3\sqrt{21}\right)^{2/3}\right) - 4\sqrt[3]{14-42\,i\sqrt{3}}\right)$$

01.08.03.0032.01

$$\tan\left(\frac{3\pi}{7}\right) == \left(z;\, z^6 - 21\,z^4 + 35\,z^2 - 7\right)_6^{-1}$$

01.08.03.0033.01

$$\tan\left(\frac{3\pi}{7}\right) == -\frac{i\left(-1+(-1)^{6/7}\right)}{1+(-1)^{6/7}}$$

01.08.03.0034.01

$$\tan\left(\frac{4\pi}{9}\right) = \frac{\left(1 - i\sqrt{3}\right)\sqrt[3]{-\frac{1}{2}\,i\left(-i+\sqrt{3}\right)} + \sqrt[3]{\frac{1}{2}\,i\left(i+\sqrt{3}\right)}\left(-1 - i\sqrt{3}\right)}{\sqrt[3]{-\frac{1}{2}\,i\left(-i+\sqrt{3}\right)}\left(i+\sqrt{3}\right) - \left(-i+\sqrt{3}\right)\sqrt[3]{\frac{1}{2}\,i\left(i+\sqrt{3}\right)}}$$

01.08.03.0035.01

$$\tan\left(\frac{4\pi}{9}\right) = \left(z;\, z^6 - 33\,z^4 + 27\,z^2 - 3\right)_6^{-1}$$

01.08.03.0036.01

$$\tan\left(\frac{4\pi}{9}\right) = -\frac{i\left(-1 + (-1)^{8/9}\right)}{1 + (-1)^{8/9}}$$

01.08.03.0037.01

$$\tan\left(\frac{\pi}{2}\right) = \tilde{\infty}$$

01.08.03.0038.01

$$\tan\left(\frac{5\pi}{9}\right) = \frac{\left(i+\sqrt{3}\right) i\,\sqrt[3]{-\frac{1}{2}\,i\left(-i+\sqrt{3}\right)} + \sqrt[3]{\frac{1}{2}\,i\left(i+\sqrt{3}\right)}\left(1 + i\sqrt{3}\right)}{\sqrt[3]{-\frac{1}{2}\,i\left(-i+\sqrt{3}\right)}\left(i+\sqrt{3}\right) - \left(-i+\sqrt{3}\right)\sqrt[3]{\frac{1}{2}\,i\left(i+\sqrt{3}\right)}}$$

01.08.03.0039.01

$$\tan\left(\frac{5\pi}{9}\right) = \left(z;\, z^6 - 33\,z^4 + 27\,z^2 - 3\right)_1^{-1}$$

01.08.03.0040.01

$$\tan\left(\frac{5\pi}{9}\right) = -\frac{i\left(1 + \sqrt[9]{-1}\right)}{-1 + \sqrt[9]{-1}}$$

01.08.03.0041.01

$$\tan\left(\frac{4\pi}{7}\right) = \left(-4\,7^{5/6}\,i\,\sqrt[3]{1 - 3\,i\sqrt{3}} - 2\left(14 - i\sqrt{7} - 3\sqrt{21}\right)^{2/3}\sqrt[3]{28 + 2\,i\sqrt{7} + 6\sqrt{21}} + 2\sqrt{7}\left(i+\sqrt{3}\right)\sqrt[3]{14 - i\sqrt{7} - 3\sqrt{21}} - i\,\sqrt[3]{14 + i\sqrt{7} + 3\sqrt{21}}\left(2^{2/3}\left(-i+\sqrt{3}\right)\sqrt[3]{7 - 21\,i\sqrt{3}} + 4\sqrt{7}\right)\right) \Big/ \left(2^{2/3}\left(2\,i\,\sqrt[3]{14 - 42\,i\sqrt{3}} + \left(i+\sqrt{3}\right)\left(7 - 21\,i\sqrt{3}\right)^{2/3} - 7\,2^{2/3}\left(-i+\sqrt{3}\right)\right)\right)$$

01.08.03.0042.01

$$\tan\left(\frac{4\pi}{7}\right) = \left(z;\, z^6 - 21\,z^4 + 35\,z^2 - 7\right)_1^{-1}$$

01.08.03.0043.01

$$\tan\left(\frac{4\pi}{7}\right) = -\frac{i\left(1 + \sqrt[7]{-1}\right)}{-1 + \sqrt[7]{-1}}$$

01.08.03.0044.01

$$\tan\left(\frac{7\pi}{12}\right) = -2 - \sqrt{3}$$

01.08.03.0045.01

$$\tan\left(\frac{3\pi}{5}\right) = -\sqrt{5+2\sqrt{5}}$$

01.08.03.0046.01

$$\tan\left(\frac{3\pi}{5}\right) = \left(z;\, z^4-10z^2+5\right)_1^{-1}$$

01.08.03.0047.01

$$\tan\left(\frac{5\pi}{8}\right) = -1-\sqrt{2}$$

01.08.03.0048.01

$$\tan\left(\frac{2\pi}{3}\right) = -\sqrt{3}$$

01.08.03.0049.01

$$\tan\left(\frac{7\pi}{10}\right) = -\sqrt{1+\frac{2}{\sqrt{5}}}$$

01.08.03.0050.01

$$\tan\left(\frac{7\pi}{10}\right) = \left(z;\, 5z^4-10z^2+1\right)_1^{-1}$$

01.08.03.0051.01

$$\tan\left(\frac{5\pi}{7}\right) = \left(\sqrt[3]{2}\left(2\left(\sqrt[3]{28-84i\sqrt{3}}+\sqrt{7}\,i-\sqrt{21}\right)\sqrt[3]{14+i\sqrt{7}+3\sqrt{21}}+\left(1-i\sqrt{3}\right)\sqrt[3]{28+2i\sqrt{7}+6\sqrt{21}}\right.\right.$$
$$\left.\left.\left(14-i\sqrt{7}-3\sqrt{21}\right)^{2/3}-4\sqrt{7}\,\sqrt[3]{14-i\sqrt{7}-3\sqrt{21}}\;i\right)-4i\,7^{5/6}\sqrt[3]{2-6i\sqrt{3}}\right)\Big/$$
$$\left(\sqrt[3]{2}\left(2\left(i\sqrt[3]{28-84i\sqrt{3}}+\sqrt{21}\,i+\sqrt{7}\right)\sqrt[3]{14+i\sqrt{7}+3\sqrt{21}}-\left(i+\sqrt{3}\right)\left(14-i\sqrt{7}-3\sqrt{21}\right)^{2/3}\right.\right.$$
$$\left.\left.\sqrt[3]{28+2i\sqrt{7}+6\sqrt{21}}+4\sqrt{7}\,\sqrt[3]{14-i\sqrt{7}-3\sqrt{21}}\right)-4i\sqrt[3]{14-42i\sqrt{3}}\right)$$

01.08.03.0052.01

$$\tan\left(\frac{5\pi}{7}\right) = \left(z;\, z^6-21z^4+35z^2-7\right)_2^{-1}$$

01.08.03.0053.01

$$\tan\left(\frac{5\pi}{7}\right) = -\frac{i\left(1+(-1)^{3/7}\right)}{-1+(-1)^{3/7}}$$

01.08.03.0054.01

$$\tan\left(\frac{3\pi}{4}\right) = -1$$

01.08.03.0055.01

$$\tan\left(\frac{7\pi}{9}\right) = -\frac{i\left(\sqrt[3]{-\frac{1}{2}i\left(-i+\sqrt{3}\right)}-\sqrt[3]{\frac{1}{2}i\left(i+\sqrt{3}\right)}\right)}{\sqrt[3]{-\frac{1}{2}i\left(-i+\sqrt{3}\right)}+\sqrt[3]{\frac{1}{2}i\left(i+\sqrt{3}\right)}}$$

01.08.03.0056.01

$$\tan\left(\frac{7\pi}{9}\right) = \left(z;\, z^6 - 33 z^4 + 27 z^2 - 3\right)_2^{-1}$$

01.08.03.0057.01

$$\tan\left(\frac{7\pi}{9}\right) = -\frac{i\left(1 + (-1)^{5/9}\right)}{-1 + (-1)^{5/9}}$$

01.08.03.0058.01

$$\tan\left(\frac{4\pi}{5}\right) = -\sqrt{5 - 2\sqrt{5}}$$

01.08.03.0059.01

$$\tan\left(\frac{4\pi}{5}\right) = \left(z;\, z^4 - 10 z^2 + 5\right)_2^{-1}$$

01.08.03.0060.01

$$\tan\left(\frac{5\pi}{6}\right) = -\frac{1}{\sqrt{3}}$$

01.08.03.0061.01

$$\tan\left(\frac{6\pi}{7}\right) = \left(-4\, 7^{5/6}\, i \sqrt[3]{1 - 3 i \sqrt{3}} - 2\sqrt{7}\left(-i + \sqrt{3}\right)\sqrt[3]{14 - i\sqrt{7} - 3\sqrt{21}} + \left(1 + i\sqrt{3}\right)\sqrt[3]{28 + 2 i \sqrt{7} + 6\sqrt{21}}\left(14 - i\sqrt{7} - 3\sqrt{21}\right)^{2/3} + 2\sqrt[3]{7}\left(i + \sqrt{3}\right)\left(\sqrt[6]{7}\sqrt[3]{14 + i\sqrt{7} + 3\sqrt{21}} + \sqrt[3]{7 - 21 i \sqrt{3} - 13 i \sqrt{7} + 3\sqrt{21}\, i}\right)\right) \Big/ \left(-4 i \sqrt[3]{7 - 21 i \sqrt{3}} + \left(-i + \sqrt{3}\right)\left(14 - 42 i \sqrt{3}\right)^{2/3} - 14\sqrt[3]{2}\left(i + \sqrt{3}\right)\right)$$

01.08.03.0062.01

$$\tan\left(\frac{6\pi}{7}\right) = \left(z;\, z^6 - 21 z^4 + 35 z^2 - 7\right)_3^{-1}$$

01.08.03.0063.01

$$\tan\left(\frac{6\pi}{7}\right) = -\frac{i\left(1 + (-1)^{5/7}\right)}{-1 + (-1)^{5/7}}$$

01.08.03.0064.01

$$\tan\left(\frac{7\pi}{8}\right) = 1 - \sqrt{2}$$

01.08.03.0065.01

$$\tan\left(\frac{8\pi}{9}\right) = \frac{i\sqrt[3]{-1 + i\sqrt{3}}\left(i + \sqrt{3}\right) - \left(-1 - i\sqrt{3}\right)^{4/3}}{\sqrt[3]{-1 - i\sqrt{3}}\left(-i + \sqrt{3}\right) - \sqrt[3]{-1 + i\sqrt{3}}\left(i + \sqrt{3}\right)}$$

01.08.03.0066.01

$$\tan\left(\frac{8\pi}{9}\right) = \left(z;\, z^6 - 33 z^4 + 27 z^2 - 3\right)_3^{-1}$$

01.08.03.0067.01

$$\tan\left(\frac{8\pi}{9}\right) == -\frac{i\left(1+(-1)^{7/9}\right)}{-1+(-1)^{7/9}}$$

01.08.03.0068.01

$$\tan\left(\frac{9\pi}{10}\right) == -\sqrt{1-\frac{2}{\sqrt{5}}}$$

01.08.03.0069.01

$$\tan\left(\frac{9\pi}{10}\right) == \left(z;\, 5z^4-10z^2+1\right)_2^{-1}$$

01.08.03.0070.01

$$\tan\left(\frac{11\pi}{12}\right) == -2+\sqrt{3}$$

01.08.03.0071.01

$$\tan(\pi) == 0$$

01.08.03.0072.01

$$\tan\left(\frac{\pi}{17}\right) == 1 \Bigg/ \left(\sqrt{\left(\left(\sqrt{\left(2\left(\sqrt{34\left(17-\sqrt{17}\right)}+6\sqrt{17}-8\sqrt{2\left(17+\sqrt{17}\right)}-\sqrt{34-2\sqrt{17}}+34\right)\right)}+\sqrt{17}+\sqrt{34-2\sqrt{17}}+15\right) \Bigg/ \left(16-2\sqrt{\left(2\left(\sqrt{\left(2\left(-\sqrt{34\left(17-\sqrt{17}\right)}+6\sqrt{17}+8\sqrt{2\left(17+\sqrt{17}\right)}+\sqrt{34-2\sqrt{17}}+34\right)\right)}+\sqrt{17}-\sqrt{34-2\sqrt{17}}+15\right)\right)}\right)\right)}\right)$$

01.08.03.0073.01

$$\tan\left(\frac{\pi}{30}\right) == \sqrt{7-2\sqrt{5}-2\sqrt{15-6\sqrt{5}}}$$

$\tan\left(\frac{n\pi}{m}\right)$ can be expressed using only square roots if $n \in \mathbb{Z}$ and m is a product of a power of 2 and distinct Fermat primes $\{3, 5, 17, 257, \ldots\}$.

Values at infinities

01.08.03.0074.01

$$\tan(i\,\infty) == i$$

01.08.03.0075.01

$$\tan(-i\,\infty) == -i$$

01.08.03.0076.01

$$\tan(\tilde{\infty}) == ¿$$

General characteristics

Domain and analyticity

$\tan(z)$ is an analytical function of z which is defined over the whole complex z-plane with the exception of countably many points $z = \pi/2 + k\pi /; k \in \mathbb{Z}$.

01.08.04.0001.01

$$z \longrightarrow \tan(z) :: \mathbb{C} \longrightarrow \mathbb{C}$$

Symmetries and periodicities

Parity

$\tan(z)$ is an odd function.

01.08.04.0002.01

$$\tan(-z) = -\tan(z)$$

Mirror symmetry

01.08.04.0003.01

$$\tan(\bar{z}) = \overline{\tan(z)}$$

Periodicity

$\tan(z)$ is a periodic function with period π.

01.08.04.0009.01

$$\tan(z + \pi) = \tan(z)$$

01.08.04.0004.01

$$\tan(z + \pi\, m) = \tan(z) /; m \in \mathbb{Z}$$

Poles and essential singularities

The function $\tan(z)$ has an infinite set of singular points:

a) $z = \pi/2 + \pi\, k /; k \in \mathbb{Z}$ are the simple poles with residues -1;

b) $z = \tilde{\infty}$ is an essential singular point.

01.08.04.0005.01

$$\mathcal{S}ing_z(\tan(z)) = \left\{\left\{\left\{\frac{\pi}{2} + \pi\, k,\ 1\right\} /; k \in \mathbb{Z}\right\},\ \{\tilde{\infty},\ \infty\}\right\}$$

01.08.04.0006.01

$$\operatorname{res}_z(\tan(z))\left(\frac{\pi}{2} + \pi\, k\right) = -1 /; k \in \mathbb{Z}$$

Branch points

The function $\tan(z)$ does not have branch points.

01.08.04.0007.01

$$\mathcal{BP}_z(\tan(z)) = \{\}$$

Branch cuts

The function $\tan(z)$ does not have branch cuts.

01.08.04.0008.01

$$\mathcal{BC}_z(\tan(z)) == \{\}$$

Series representations

Generalized power series

Expansions at $z == z_0$

For the function itself

01.08.06.0019.01

$$\tan(z) \propto \tan(z_0) + \sec^2(z_0)(z-z_0) + \frac{1}{2}\sin(2z_0)\sec^4(z_0)(z-z_0)^2 + \ldots \;/;\; (z \to z_0)$$

01.08.06.0020.01

$$\tan(z) \propto \tan(z_0) + \sec^2(z_0)(z-z_0) + \frac{1}{2}\sin(2z_0)\sec^4(z_0)(z-z_0)^2 + O\left((z-z_0)^3\right)$$

01.08.06.0021.01

$$\tan(z) == \sum_{k=0}^{\infty} \frac{1}{k!} \left(\delta_k \tan(z_0) + \delta_{k-1}\sec^2(z_0) + k\sum_{m=0}^{k-1}\sum_{j=0}^{m-1} \frac{(-1)^m}{m+1}\binom{k-1}{m}\cos^{-2m-2}(z_0)\, 2^{k-2m}\binom{2m}{j}(m-j)^{k-1}\sin\left(\frac{\pi k}{2} + 2(m-j)z_0\right)\right)(z-z_0)^k$$

01.08.06.0022.01

$$\tan(z) \propto \tan(z_0)(1 + O(z-z_0))$$

Expansions at $z == 0$

For the function itself

01.08.06.0001.02

$$\tan(z) \propto z + \frac{z^3}{3} + \frac{2z^5}{15} + \ldots \;/;\; (z \to 0)$$

01.08.06.0023.01

$$\tan(z) \propto z + \frac{z^3}{3} + \frac{2z^5}{15} - O\left(z^7\right)$$

01.08.06.0002.01

$$\tan(z) == \sum_{k=1}^{\infty} \frac{(-1)^{k-1}\, 2^{2k}\left(2^{2k}-1\right)B_{2k}}{(2k)!} z^{2k-1} \;/;\; |z| < \frac{\pi}{2}$$

01.08.06.0003.02

$$\tan(z) \propto z + O\left(z^3\right)$$

Expansions at $z == \frac{\pi}{2}$

For the function itself

01.08.06.0004.02

$$\tan(z) \propto -\frac{1}{z-\frac{\pi}{2}} + \frac{1}{3}\left(z-\frac{\pi}{2}\right) + \frac{1}{45}\left(z-\frac{\pi}{2}\right)^3 + \frac{2}{945}\left(z-\frac{\pi}{2}\right)^5 + \ldots \;/;\; \left(z \to \frac{\pi}{2}\right)$$

01.08.06.0024.01

$$\tan(z) \propto -\frac{1}{z-\frac{\pi}{2}} + \frac{1}{3}\left(z-\frac{\pi}{2}\right) + \frac{1}{45}\left(z-\frac{\pi}{2}\right)^3 + \frac{2}{945}\left(z-\frac{\pi}{2}\right)^5 + O\left(\left(z-\frac{\pi}{2}\right)^7\right)$$

01.08.06.0005.01

$$\tan(z) = -\frac{1}{z-\frac{\pi}{2}} - \sum_{k=1}^{\infty} \frac{(-1)^k\, 2^{2k}\, B_{2k}}{(2k)!} \left(z-\frac{\pi}{2}\right)^{2k-1} \;/;\; \left|z-\frac{\pi}{2}\right| < \pi$$

01.08.06.0006.02

$$\tan(z) \propto -\frac{1}{z-\frac{\pi}{2}} + \frac{1}{3}\left(z-\frac{\pi}{2}\right) + O\left(\left(z-\frac{\pi}{2}\right)^3\right)$$

q-series

01.08.06.0007.01

$$\tan(z) = i\left(1 + 2\sum_{k=1}^{\infty}(-1)^k q^{2k}\right) /;\; q = e^{iz}$$

Dirichlet series

01.08.06.0008.01

$$\tan(z) = i - 2i\sum_{k=0}^{\infty}(-1)^k e^{2iz(k+1)} \;/;\; \mathrm{Im}(z) > 0$$

01.08.06.0009.01

$$\tan(z) = -i + 2i\sum_{k=0}^{\infty}(-1)^k e^{-2iz(k+1)} \;/;\; \mathrm{Im}(z) < 0$$

Asymptotic series expansions

01.08.06.0010.01

$$\tan(z) \propto i - 2i\, e^{2iz}\, {}_1F_0\left(1;\, ;\, -e^{2iz}\right) /;\; \mathrm{Im}(z) > 0 \land (|z| \to \infty)$$

01.08.06.0011.01

$$\tan(z) \propto i - 2i\, e^{2iz}\left(1 + O\left(e^{2iz}\right)\right) /;\; \mathrm{Im}(z) > 0 \land (|z| \to \infty)$$

01.08.06.0012.01

$$\tan(z) \propto -i + 2i\, e^{-2iz}\, {}_1F_0\left(1;\, ;\, -e^{-2iz}\right) /;\; \mathrm{Im}(z) < 0 \land (|z| \to \infty)$$

01.08.06.0013.01

$$\tan(z) \propto -i + 2i\, e^{-2iz}\left(1 + O\left(e^{-2iz}\right)\right) /;\; \mathrm{Im}(z) < 0 \land (|z| \to \infty)$$

01.08.06.0014.01

$$\tan(z) \propto \tan(z) \;/;\; \mathrm{Im}(z) = 0 \land (|z| \to \infty)$$

01.08.06.0015.01

$$\tan(z) \propto i \,/;\, \left(z \to e^{i\phi}\,\infty\right) \bigwedge 0 < \phi < \pi$$

01.08.06.0016.01

$$\tan(z) \propto -i \,/;\, \left(z \to e^{i\phi}\,\infty\right) \bigwedge -\pi < \phi < 0$$

01.08.06.0025.01

$$\tan(z) \propto \begin{cases} -i & -\pi < \arg(z) < 0 \\ i & 0 < \arg(z) < \pi \\ \tan(z) & \text{True} \end{cases} \,/;\, (|z| \to \infty)$$

Other series representations

01.08.06.0017.01

$$\tan(z) = 8\,z \sum_{k=1}^{\infty} \frac{1}{(2\,k-1)^2\,\pi^2 - 4\,z^2} \,/;\, \frac{z}{\pi} - \frac{1}{2} \notin \mathbb{Z}$$

01.08.06.0018.01

$$\log\left(\frac{\tan(z)}{z}\right) = \sum_{k=1}^{\infty} \frac{1}{k\,(2\,k)!} (-1)^{k-1}\, 2^{2\,k} \left(2^{2\,k-1} - 1\right) B_{2\,k}\, z^{2\,k} \,/;\, |z| < \frac{\pi}{2}$$

Integral representations

On the real axis

Of the direct function

01.08.07.0001.01

$$\tan(z) = \int_0^z \sec^2(t)\,dt$$

01.08.07.0002.01

$$\tan(z) = \frac{2}{\pi} \int_0^{\infty} \frac{t^{\frac{2z}{\pi}} - 1}{t^2 - 1}\,dt \,/;\, 0 < \text{Re}(z) < \frac{\pi}{2}$$

Limit representations

01.08.09.0001.01

$$\tan(z) = \lim_{n\to\infty} \sum_{k=-n}^{n} \frac{1}{\pi\left(\frac{1}{2} - k\right) - z} \,/;\, \frac{z}{\pi} - \frac{1}{2} \notin \mathbb{Z}$$

Continued fraction representations

01.08.10.0001.01

$$\tan(z) = \cfrac{z}{1 - \cfrac{z^2}{3 - \cfrac{z^2}{5 - \cfrac{z^2}{7 - \cfrac{z^2}{9 - \cfrac{z^2}{11 - \ldots}}}}}} \;/;\; \frac{z}{\pi} - \frac{1}{2} \notin \mathbb{Z}$$

01.08.10.0002.01

$$\tan(z) = -\frac{1}{z} \mathrm{K}_k\left(-z^2, 2k-1\right)_1^{\infty} \;/;\; \frac{z}{\pi} - \frac{1}{2} \notin \mathbb{Z}$$

01.08.10.0003.01

$$\tan(z) = \cfrac{1}{\cfrac{1}{z} - \cfrac{1}{\cfrac{3}{z} - \cfrac{1}{\cfrac{5}{z} - \cfrac{1}{\cfrac{7}{z} - \cfrac{1}{\cfrac{9}{z} - \cfrac{1}{\cfrac{11}{z} - \ldots}}}}}} \;/;\; \frac{z}{\pi} - \frac{1}{2} \notin \mathbb{Z}$$

01.08.10.0004.01

$$\tan(z) = -\mathrm{K}_k\left(-1, \frac{2k-1}{z}\right)_1^{\infty} \;/;\; \frac{z}{\pi} - \frac{1}{2} \notin \mathbb{Z}$$

Differential equations

Ordinary nonlinear differential equations

01.08.13.0001.01

$$w'(z) - w(z)^2 - 1 = 0 \;/;\; w(z) = \tan(z) \land w(0) = 0$$

01.08.13.0002.01

$$w'(z) - a\, w(z)^2 - b\, w(z) - c = 0 \;/;\; w(z) = \frac{1}{2a}\left(\sqrt{4ac - b^2}\, \tan\left(\frac{a\sqrt{4ac-b^2}\, z + \sqrt{4ac-b^2}\, c_1}{2a}\right) - b\right)$$

Transformations

Transformations and argument simplifications

Argument involving basic arithmetic operations

01.08.16.0001.01

$$\tan(-z) = -\tan(z)$$

01.08.16.0002.01

$$\tan(a\,(b\,z^c)^m) = \frac{(b\,z^c)^m}{b^m\,z^{m\,c}}\tan(a\,b^m\,z^{m\,c})\;/;\;2\,m\in\mathbb{Z}$$

01.08.16.0003.01

$$\tan\left(\sqrt{z^2}\right) = \frac{\sqrt{z^2}\,\tan(z)}{z}$$

Argument involving inverse trigonometric and hyperbolic functions

Involving $\sin^{-1}$

01.08.16.0004.01

$$\tan\left(\sin^{-1}(z)\right) = \frac{z}{\sqrt{1-z^2}}$$

01.08.16.0016.01

$$\tan\left(\frac{1}{2}\sin^{-1}(z)\right) = \frac{1-\sqrt{1-z^2}}{z}$$

01.08.16.0066.01

$$\tan\left(i\sin^{-1}(z)\right) = -\frac{i\left(\left(i\,z+\sqrt{1-z^2}\right)^{2\,i}-1\right)}{\left(i\,z+\sqrt{1-z^2}\right)^{2\,i}+1}$$

01.08.16.0067.01

$$\tan\left(a\sin^{-1}(z)\right) = -\frac{i\left(\left(i\,z+\sqrt{1-z^2}\right)^{2\,a}-1\right)}{\left(i\,z+\sqrt{1-z^2}\right)^{2\,a}+1}$$

Involving $\cos^{-1}$

01.08.16.0005.01

$$\tan\left(\cos^{-1}(z)\right) = \frac{\sqrt{1-z^2}}{z}$$

01.08.16.0017.01

$$\tan\left(\frac{1}{2}\cos^{-1}(z)\right) = \frac{\sqrt{1-z}}{\sqrt{1+z}}$$

01.08.16.0068.01

$$\tan\left(i\cos^{-1}(z)\right) = i-\frac{2\,i}{e^{\pi}\left(i\,z+\sqrt{1-z^2}\right)^{2\,i}+1}$$

01.08.16.0069.01

$$\tan\left(a\cos^{-1}(z)\right) = i - \frac{2\,i}{e^{-i\,a\,\pi}\left(i\,z + \sqrt{1-z^2}\right)^{2\,a} + 1}$$

Involving $\tan^{-1}$

01.08.16.0006.01

$$\tan\left(\tan^{-1}(z)\right) = z$$

01.08.16.0070.01

$$\tan\left(\tan^{-1}(x, y)\right) = \frac{y}{x}$$

01.08.16.0018.01

$$\tan\left(\frac{1}{2}\tan^{-1}(z)\right) = \frac{\sqrt{z^2+1} - 1}{z}$$

01.08.16.0071.01

$$\tan\left(\frac{1}{2}\tan^{-1}(x, y)\right) = \frac{\sqrt{x^2+y^2} - x}{y}$$

01.08.16.0072.01

$$\tan\left(i\tan^{-1}(z)\right) = -i + \frac{2\,i}{(i\,z+1)^{i}\,(1-i\,z)^{-i} + 1}$$

01.08.16.0073.01

$$\tan\left(i\tan^{-1}(x, y)\right) = -\frac{i\left(\left(\frac{x+i\,y}{\sqrt{x^2+y^2}}\right)^{2\,i} - 1\right)}{\left(\frac{x+i\,y}{\sqrt{x^2+y^2}}\right)^{2\,i} + 1}$$

01.08.16.0074.01

$$\tan\left(a\tan^{-1}(z)\right) = \frac{2\,i\,(1-i\,z)^{a}}{(i\,z+1)^{a} + (1-i\,z)^{a}} - i$$

01.08.16.0075.01

$$\tan\left(a\tan^{-1}(x, y)\right) = -\frac{i\left(\left(\frac{x+i\,y}{\sqrt{x^2+y^2}}\right)^{2\,a} - 1\right)}{\left(\frac{x+i\,y}{\sqrt{x^2+y^2}}\right)^{2\,a} + 1}$$

01.08.16.0028.01

$$\tan\left(n\tan^{-1}(z)\right) = -\frac{i\,((i\,z+1)^{n} - (1-i\,z)^{n})}{(i\,z+1)^{n} + (1-i\,z)^{n}} \;/;\; n \in \mathbb{N}^{+}$$

01.08.16.0076.01

$$\tan\left(i \tan^{-1}(x, y)\right) = -\frac{i\left(\left(\frac{x+i y}{\sqrt{x^2+y^2}}\right)^{2 i} - 1\right)}{\left(\frac{x+i y}{\sqrt{x^2+y^2}}\right)^{2 i} + 1}$$

01.08.16.0077.01

$$\tan\left(a \tan^{-1}(x, y)\right) = -\frac{i\left(\left(\frac{x+i y}{\sqrt{x^2+y^2}}\right)^{2 a} - 1\right)}{\left(\frac{x+i y}{\sqrt{x^2+y^2}}\right)^{2 a} + 1}$$

Involving $\cot^{-1}$

01.08.16.0007.01

$$\tan\left(\cot^{-1}(z)\right) = \frac{1}{z}$$

01.08.16.0019.01

$$\tan\left(\frac{1}{2} \cot^{-1}(z)\right) = z\left(\sqrt{1 + \frac{1}{z^2}} - 1\right)$$

01.08.16.0078.01

$$\tan\left(i \cot^{-1}(z)\right) = -i + \frac{2 i}{1 + \left(\frac{-i+z}{z}\right)^{-i} \left(\frac{i+z}{z}\right)^{i}}$$

01.08.16.0079.01

$$\tan\left(a \cot^{-1}(z)\right) = -i + \frac{2 i}{\left(\frac{i+z}{z}\right)^{a} \left(\frac{-i+z}{z}\right)^{-a} + 1}$$

Involving $\csc^{-1}$

01.08.16.0008.01

$$\tan\left(\csc^{-1}(z)\right) = \frac{\sqrt{z^2}}{z \sqrt{z^2 - 1}}$$

01.08.16.0020.01

$$\tan\left(\frac{1}{2} \csc^{-1}(z)\right) = z\left(1 - \sqrt{1 - \frac{1}{z^2}}\right)$$

01.08.16.0080.01

$$\tan(i\csc^{-1}(z)) = -\frac{i\left(\left(\sqrt{1-\frac{1}{z^2}}+\frac{i}{z}\right)^{2i}-1\right)}{\left(\sqrt{1-\frac{1}{z^2}}+\frac{i}{z}\right)^{2i}+1}$$

01.08.16.0081.01

$$\tan(a\csc^{-1}(z)) = -\frac{i\left(\left(\sqrt{1-\frac{1}{z^2}}+\frac{i}{z}\right)^{2a}-1\right)}{\left(\sqrt{1-\frac{1}{z^2}}+\frac{i}{z}\right)^{2a}+1}$$

Involving $\sec^{-1}$

01.08.16.0009.01

$$\tan(\sec^{-1}(z)) = z\sqrt{1-\frac{1}{z^2}}$$

01.08.16.0021.01

$$\tan\left(\frac{1}{2}\sec^{-1}(z)\right) = \frac{\sqrt{-1+z}\sqrt{-z}}{\sqrt{-1-z}\sqrt{z}}$$

01.08.16.0082.01

$$\tan(i\sec^{-1}(z)) = i - \frac{2i}{e^{\pi}\left(\sqrt{1-\frac{1}{z^2}}+\frac{i}{z}\right)^{2i}+1}$$

01.08.16.0083.01

$$\tan(a\sec^{-1}(z)) = i - \frac{2i}{e^{-ia\pi}\left(\sqrt{1-\frac{1}{z^2}}+\frac{i}{z}\right)^{2a}+1}$$

Involving $\sinh^{-1}$

01.08.16.0084.01

$$\tan(\sinh^{-1}(z)) = -\frac{i\left(\left(z+\sqrt{z^2+1}\right)^{2i}-1\right)}{\left(z+\sqrt{z^2+1}\right)^{2i}+1}$$

01.08.16.0010.01

$$\tan(i\sinh^{-1}(z)) = \frac{iz}{\sqrt{1+z^2}}$$

01.08.16.0022.01

$$\tan\left(\frac{i}{2}\sinh^{-1}(z)\right) == \frac{i}{z}\left(\sqrt{1+z^2}-1\right)$$

01.08.16.0085.01

$$\tan\left(a\sinh^{-1}(z)\right) = -\frac{i\left(\left(z+\sqrt{z^2+1}\right)^{2ia}-1\right)}{\left(z+\sqrt{z^2+1}\right)^{2ia}+1}$$

Involving $\cosh^{-1}$

01.08.16.0086.01

$$\tan\left(\cosh^{-1}(z)\right) = -\frac{i\left(\left(z+\sqrt{z-1}\sqrt{z+1}\right)^{2i}-1\right)}{\left(z+\sqrt{z-1}\sqrt{z+1}\right)^{2i}+1}$$

01.08.16.0011.01

$$\tan\left(i\cosh^{-1}(z)\right) == \frac{i}{z}\sqrt{z-1}\sqrt{z+1}$$

01.08.16.0023.01

$$\tan\left(\frac{i}{2}\cosh^{-1}(z)\right) == i\sqrt{\frac{z-1}{z+1}}$$

01.08.16.0087.01

$$\tan\left(a\cosh^{-1}(z)\right) = -\frac{i\left(\left(z+\sqrt{z-1}\sqrt{z+1}\right)^{2ia}-1\right)}{\left(z+\sqrt{z-1}\sqrt{z+1}\right)^{2ia}+1}$$

Involving $\tanh^{-1}$

01.08.16.0088.01

$$\tan\left(\tanh^{-1}(z)\right) = \frac{2i(1-z)^i}{(1-z)^i+(z+1)^i}-i$$

01.08.16.0012.01

$$\tan\left(i\tanh^{-1}(z)\right) == i z$$

01.08.16.0024.01

$$\tan\left(\frac{i}{2}\tanh^{-1}(z)\right) == \frac{i}{z}\left(1-\sqrt{1-z^2}\right)$$

01.08.16.0089.01

$$\tan\left(a\tanh^{-1}(z)\right) = -i+\frac{2i}{(z+1)^{ia}(1-z)^{-ia}+1}$$

Involving $\coth^{-1}$

01.08.16.0090.01

$$\tan\left(\coth^{-1}(z)\right) = i - \frac{2\,i}{\left(\frac{z-1}{z}\right)^{i}\left(1+\frac{1}{z}\right)^{-i}+1}$$

01.08.16.0013.01

$$\tan\left(i\coth^{-1}(z)\right) = \frac{i}{z}$$

01.08.16.0025.01

$$\tan\left(\frac{i}{2}\coth^{-1}(z)\right) = \frac{i\,z}{\sqrt{z^2}}\left(\sqrt{z^2}-\sqrt{z^2-1}\right)$$

01.08.16.0091.01

$$\tan\left(a\coth^{-1}(z)\right) = i - \frac{2\,i}{\left(\frac{z-1}{z}\right)^{i\,a}\left(1+\frac{1}{z}\right)^{-i\,a}+1}$$

Involving csch^{-1}

01.08.16.0092.01

$$\tan\left(\operatorname{csch}^{-1}(z)\right) = -\frac{i\left(\left(\sqrt{1+\frac{1}{z^2}}+\frac{1}{z}\right)^{2\,i}-1\right)}{\left(\sqrt{1+\frac{1}{z^2}}+\frac{1}{z}\right)^{2\,i}+1}$$

01.08.16.0014.01

$$\tan\left(i\operatorname{csch}^{-1}(z)\right) = \frac{i\sqrt{-z^2}}{z\sqrt{-1-z^2}}$$

01.08.16.0026.01

$$\tan\left(\frac{i}{2}\operatorname{csch}^{-1}(z)\right) = i\,z\left(\sqrt{1+\frac{1}{z^2}}-1\right)$$

01.08.16.0093.01

$$\tan\left(a\operatorname{csch}^{-1}(z)\right) = -\frac{i\left(\left(\sqrt{1+\frac{1}{z^2}}+\frac{1}{z}\right)^{2\,i\,a}-1\right)}{\left(\sqrt{1+\frac{1}{z^2}}+\frac{1}{z}\right)^{2\,i\,a}+1}$$

Involving sech^{-1}

01.08.16.0094.01

$$\tan(\operatorname{sech}^{-1}(z)) = -\frac{i\left(\left(\sqrt{\frac{1}{z}-1}\,\sqrt{1+\frac{1}{z}}+\frac{1}{z}\right)^{2i}-1\right)}{\left(\sqrt{\frac{1}{z}-1}\,\sqrt{1+\frac{1}{z}}+\frac{1}{z}\right)^{2i}+1}$$

01.08.16.0015.01

$$\tan(i\operatorname{sech}^{-1}(z)) = i\sqrt{\frac{1-z}{1+z}}\,(1+z)$$

01.08.16.0027.01

$$\tan\left(\frac{i}{2}\operatorname{sech}^{-1}(z)\right) = i\sqrt{\frac{1-z}{1+z}}$$

01.08.16.0095.01

$$\tan(a\operatorname{sech}^{-1}(z)) = -\frac{i\left(\left(\sqrt{\frac{1}{z}-1}\,\sqrt{1+\frac{1}{z}}+\frac{1}{z}\right)^{2ia}-1\right)}{\left(\sqrt{\frac{1}{z}-1}\,\sqrt{1+\frac{1}{z}}+\frac{1}{z}\right)^{2ia}+1}$$

Addition formulas

01.08.16.0029.01

$$\tan(a+b) = \frac{\tan(a)+\tan(b)}{1-\tan(a)\tan(b)}$$

01.08.16.0030.01

$$\tan\left(a+\frac{\pi}{4}\right) = \frac{1+\tan(a)}{1-\tan(a)}$$

01.08.16.0031.01

$$\tan(a-b) = \frac{\tan(a)-\tan(b)}{\tan(a)\tan(b)+1}$$

01.08.16.0032.01

$$\tan\left(a-\frac{\pi}{4}\right) = \frac{\tan(a)-1}{\tan(a)+1}$$

01.08.16.0033.01

$$\tan(a+b\,i) = \frac{\sin(2a)+i\sinh(2b)}{\cos(2a)+\cosh(2b)}$$

01.08.16.0034.01

$$\tan(a-i\,b) = \frac{\sin(2a)-i\sinh(2b)}{\cos(2a)+\cosh(2b)}$$

01.08.16.0035.01

$$\tan(z_1+z_2+z_3) = \frac{-\tan(z_2)\tan(z_3)\tan(z_1)+\tan(z_1)+\tan(z_2)+\tan(z_3)}{-\tan(z_1)\tan(z_2)-\tan(z_3)\tan(z_2)-\tan(z_1)\tan(z_3)+1}$$

Half-angle formulas

01.08.16.0036.01

$$\tan\left(\frac{z}{2}\right) = \csc(z) - \cot(z)$$

01.08.16.0037.01

$$\tan\left(\frac{z}{2}\right) = \frac{\sin(z)}{\cos(z)+1}$$

01.08.16.0096.01

$$\tan\left(\frac{z}{2}\right) = -\frac{\sqrt{-z^2}}{z}\sqrt{\frac{\cos(z)-1}{\cos(z)+1}}$$

01.08.16.0038.02

$$\tan\left(\frac{z}{2}\right) = \frac{\sqrt{z^2}}{z}\sqrt{\frac{1-\cos(z)}{\cos(z)+1}} \;/;\; |\mathrm{Re}(z)| < \pi \lor \mathrm{Re}(z) = -\pi \land \mathrm{Im}(z) < 0 \lor \mathrm{Re}(z) = \pi \land \mathrm{Im}(z) > 0$$

01.08.16.0039.02

$$\tan\left(\frac{z}{2}\right) = \sqrt{\frac{1-\cos(z)}{1+\cos(z)}} \;/;\; 0 < \mathrm{Re}(z) < \pi \lor \mathrm{Re}(z) = 0 \land \mathrm{Im}(z) \geq 0 \lor \mathrm{Re}(z) = \pi \land \mathrm{Im}(z) > 0$$

01.08.16.0040.01

$$\tan\left(\frac{z}{2}\right) = (-1)^{\mathrm{Round}\left(\frac{\mathrm{Re}(z)}{\pi}-\frac{1}{2}\right)}\sqrt{\frac{1-\cos(z)}{1+\cos(z)}}\left(1-\left(1+(-1)^{\left\lfloor\frac{\mathrm{Re}(z)}{\pi}\right\rfloor+\left\lfloor-\frac{\mathrm{Re}(z)}{\pi}\right\rfloor}\right)\theta(-\mathrm{Im}(z))\right)$$

Multiple arguments

Argument involving numeric multiples of variable

01.08.16.0041.01

$$\tan(2z) = \frac{2\tan(z)}{1-\tan^2(z)}$$

01.08.16.0042.01

$$\tan(3z) = \frac{3\tan(z)-\tan^3(z)}{1-3\tan^2(z)}$$

Argument involving symbolic multiples of variable

01.08.16.0043.01

$$\tan(nz) = \frac{1}{\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}(-1)^k\binom{n}{2k}\tan^{2k}(z)}\sum_{k=0}^{\left\lfloor\frac{n-1}{2}\right\rfloor}(-1)^k\binom{n}{2k+1}\tan^{2k+1}(z) \;/;\; n \in \mathbb{N}^+$$

01.08.16.0044.01

$$\tan(nz) = \frac{U_{n-1}(\cos(z))\sin(z)}{T_n(\cos(z))}$$

Products, sums, and powers of the direct function

Products of the direct function

01.08.16.0045.01

$$\tan(a)\tan(b) == \frac{\cos(a-b)-\cos(a+b)}{\cos(a-b)+\cos(a+b)}$$

Products involving the direct function

01.08.16.0046.01

$$\tan(a)\cot(b) == \frac{\sin(a-b)+\sin(a+b)}{\sin(a+b)-\sin(a-b)}$$

Sums of the direct function

01.08.16.0047.01

$$\tan(a)+\tan(b) == \sec(a)\sec(b)\sin(a+b)$$

01.08.16.0048.01

$$\tan(a)-\tan(b) == \sec(a)\sec(b)\sin(a-b)$$

Sums involving the direct function

Involving other trigonometric functions

Involving cot

01.08.16.0055.01

$$\tan(z)+\cot(z) == \sec(z)\csc(z)$$

01.08.16.0056.01

$$\tan(z)-\cot(z) == -\cos(2z)\sec(z)\csc(z)$$

01.08.16.0049.01

$$\cot(b)+\tan(a) == \cos(a-b)\sec(a)\csc(b)$$

01.08.16.0050.01

$$\tan(a)-\cot(b) == -\cos(a+b)\sec(a)\csc(b)$$

01.08.16.0057.01

$$a\tan(z)+b\cot(z) == (b-a)\cot(2z)+(a+b)\csc(2z)$$

Involving hyperbolic functions

Involving tanh

01.08.16.0058.01

$$\tan(z)+i\tanh(z) == \sec(z)\operatorname{sech}(z)\sin\left(z\sqrt{2}\,e^{\frac{i\pi}{4}}\right)$$

01.08.16.0059.01

$$\tan(z)-i\tanh(z) == \sec(z)\operatorname{sech}(z)\sin\left(z\sqrt{2}\,e^{-\frac{1}{4}(i\pi)}\right)$$

01.08.16.0060.01

$$\tan(a) + i\tanh(b) = \sec(a)\,\mathrm{sech}(b)\sin(a + b\,i)$$

01.08.16.0061.01

$$\tan(a) - i\tanh(b) = \sec(a)\,\mathrm{sech}(b)\sin(a - b\,i)$$

Involving coth

01.08.16.0062.01

$$\tan(z) + i\coth(z) = i\cos\left(z\sqrt{2}\,e^{\frac{i\pi}{4}}\right)\sec(z)\,\mathrm{csch}(z)$$

01.08.16.0063.01

$$\tan(z) - i\coth(z) = -i\cos\left(z\sqrt{2}\,e^{-\frac{1}{4}(i\pi)}\right)\sec(z)\,\mathrm{csch}(z)$$

01.08.16.0064.01

$$\tan(a) + i\coth(b) = i\cos(a + b\,i)\sec(a)\,\mathrm{csch}(b)$$

01.08.16.0065.01

$$\tan(a) - i\coth(b) = -i\cos(a - i\,b)\sec(a)\,\mathrm{csch}(b)$$

Powers of the direct function

01.08.16.0051.01

$$\tan^2(z) = \frac{1 - \cos(2z)}{\cos(2z) + 1}$$

01.08.16.0052.01

$$\tan^3(z) = \frac{3\sin(z) - \sin(3z)}{3\cos(z) + \cos(3z)}$$

Sums of powers involving the direct function

01.08.16.0053.01

$$\tan^2(a) - \tan^2(b) = \sec^2(a)\sec^2(b)\sin(a - b)\sin(a + b)$$

Related transformations

01.08.16.0054.01

$$\tan\left(z - \frac{\pi}{3}\right)\tan(z) + \tan\left(z + \frac{\pi}{3}\right)\tan(z) + \tan\left(z - \frac{\pi}{3}\right)\tan\left(z + \frac{\pi}{3}\right) = -3$$

Identities

Functional identities

01.08.17.0001.01

$$\tan(2z)\left(1 - \tan^2(z)\right) = 2\tan(z)$$

01.08.17.0002.01

$$w((2n+1)z) = w(z)\prod_{k=1}^{n} w\left(\frac{\pi k}{2n+1} + z\right) w\left(\frac{k\pi}{2n+1} - z\right) /;\ w(z) = \tan(z) \bigwedge n \in \mathbb{N}^+$$

Complex characteristics

Real part

01.08.19.0001.01

$$\operatorname{Re}(\tan(x + i\, y)) == \frac{\sin(2\, x)}{\cos(2\, x) + \cosh(2\, y)}$$

Imaginary part

01.08.19.0002.01

$$\operatorname{Im}(\tan(x + i\, y)) == \frac{\sinh(2\, y)}{\cos(2\, x) + \cosh(2\, y)}$$

Absolute value

01.08.19.0003.01

$$|\tan(x + i\, y)| == \sqrt{\frac{\sin^2(2\, x) + \sinh^2(2\, y)}{(\cos(2\, x) + \cosh(2\, y))^2}}$$

Argument

01.08.19.0004.01

$$\arg(\tan(x + i\, y)) == \tan^{-1}\left(\frac{\sin(2\, x)}{\cos(2\, x) + \cosh(2\, y)}, \frac{\sinh(2\, y)}{\cos(2\, x) + \cosh(2\, y)}\right)$$

01.08.19.0005.01

$$\arg(\tan(x + i\, y)) == \tan^{-1}(\csc(2\, x) \sinh(2\, y)) + \frac{\pi}{2} \operatorname{sgn}\left(\frac{\operatorname{sgn}(\sinh(2\, y))}{\operatorname{sgn}(\cos(2\, x) + \cosh(2\, y))} + \frac{1}{2}\right)\left(1 - \frac{\operatorname{sgn}(\sin(2\, x))}{\operatorname{sgn}(\cos(2\, x) + \cosh(2\, y))}\right)$$

Conjugate value

01.08.19.0006.01

$$\overline{\tan(x + i\, y)} == \frac{\sin(2\, x) - i \sinh(2\, y)}{\cos(2\, x) + \cosh(2\, y)}$$

Differentiation

Low-order differentiation

01.08.20.0001.01

$$\frac{\partial \tan(z)}{\partial z} == \sec^2(z)$$

01.08.20.0002.01

$$\frac{\partial^2 \tan(z)}{\partial z^2} == 2 \sec^2(z) \tan(z)$$

Symbolic differentiation

01.08.20.0003.01

$$\frac{\partial^n \tan(z)}{\partial z^n} == \sum_{k=1}^{\infty} \frac{(-1)^{k-1} 2^{2k-1} \left(2^{2k}-1\right)}{k\,(2k-n-1)!} B_{2k}\, z^{2k-n-1} \;/;\; |z| < \frac{\pi}{2} \bigwedge n \in \mathbb{N}^+$$

01.08.20.0004.01

$$\frac{\partial^n \tan(z)}{\partial z^n} == \delta_n \tan(z) + \delta_{n-1} \sec^2(z) + n \sum_{k=0}^{n-1} \sum_{j=0}^{k-1} \frac{(-1)^k\, (k-j)^{n-1}\, 2^{n-2k} \cos^{-2k-2}(z)}{k+1} \binom{n-1}{k} \binom{2k}{j} \sin\!\left(\frac{\pi n}{2} + 2(k-j)z\right) /;\; n \in \mathbb{N}$$

01.08.20.0006.01

$$\frac{\partial^n \tan(z)}{\partial z^n} = -i^{n+1}\, 2^n \left(\delta_n + i \tan(z) - 1\right) \sum_{k=0}^{n} \frac{(-1)^k\, k!}{2^k}\, \mathcal{S}_n^{(k)}\, (i \tan(z) + 1)^k \;/;\; n \in \mathbb{N}$$

Victor Adamchik (2005)

Fractional integro-differentiation

01.08.20.0005.01

$$\frac{\partial^\alpha \tan(z)}{\partial z^\alpha} == \sum_{k=1}^{\infty} \frac{(-1)^{k-1} 2^{2k-1} \left(2^{2k}-1\right) B_{2k}\, z^{2k-\alpha-1}}{\Gamma(2k-\alpha)\, k} \;/;\; |z| < \frac{\pi}{2}$$

01.08.20.0007.01

$$\tan^{(\alpha)}(c\,z) = \frac{\log(4)\,(c\,z)^{-\alpha-1}}{\Gamma(-\alpha)} - \pi^{-\alpha-1}\,(c\,z)^{-\alpha} \left((-c\,z)^{\alpha} \left(2^{\alpha+1}\, \psi^{(\alpha)}\!\left(-\frac{2\,c\,z}{\pi}\right) - \psi^{(\alpha)}\!\left(-\frac{c\,z}{\pi}\right)\right) + (c\,z)^{\alpha} \left(\psi^{(\alpha)}\!\left(\frac{c\,z}{\pi}\right) - 2^{\alpha+1}\, \psi^{(\alpha)}\!\left(\frac{2\,c\,z}{\pi}\right)\right)\right)$$

Integration

Indefinite integration

Involving only one direct function

01.08.21.0013.01

$$\int \tan(b + a\,z)\, dz == -\frac{\log(\cos(b + a\,z))}{a}$$

01.08.21.0014.01

$$\int \tan(a\,z)\, dz == -\frac{\log(\cos(a\,z))}{a}$$

01.08.21.0015.01

$$\int \tan(z)\, dz == -\log(\cos(z))$$

Involving one direct function and elementary functions

Involving power function

Involving power

Involving z^n and linear arguments

01.08.21.0016.01

$$\int z\tan(a\,z+b)\,dz = -\frac{1}{2\,a^2}\left(a^2\,i\,z^2+2\,i\,a\,b\,z+2\,a\log\left(1+e^{-2\,i\,(b+a\,z)}\right)z-\pi\log\left(1+e^{2\,i\,a\,z}\right)+2\,b\log\left(1+e^{-2\,i\,(b+a\,z)}\right)-\pi\log\left(1+e^{-2\,i\,(b+a\,z)}\right)+\pi\log(\cos(a\,z))-2\,b\log(\cos(b+a\,z))+\pi\log(\cos(b+a\,z))+i\,\mathrm{Li}_2\left(-e^{-2\,i\,(b+a\,z)}\right)\right)$$

01.08.21.0017.01

$$\int z^n\tan(a\,z)\,dz = i\,\frac{z^{1+n}}{1+n}-2\,i\,e^{2\,i\,a\,z}\,n!\sum_{j=0}^{n}\frac{(-1)^j\,2^{-j-1}\,(i\,a)^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(1,\,\ldots,\,1,\,1;\,2,\,\ldots,\,2;\,-e^{2\,i\,a\,z}\right)\,/;\,n\in\mathbb{N}$$

01.08.21.0018.01

$$\int z\tan(a\,z)\,dz = \frac{i\left(a\,z\left(a\,z+2\,i\log\left(1+e^{2\,i\,a\,z}\right)\right)+\mathrm{Li}_2\left(-e^{2\,i\,a\,z}\right)\right)}{2\,a^2}$$

01.08.21.0019.01

$$\int z^2\tan(a\,z)\,dz = \frac{2\,a^2\,i\left(a\,z+3\,i\log\left(1+e^{2\,i\,a\,z}\right)\right)z^2+6\,a\,i\,\mathrm{Li}_2\left(-e^{2\,i\,a\,z}\right)z-3\,\mathrm{Li}_3\left(-e^{2\,i\,a\,z}\right)}{6\,a^3}$$

01.08.21.0020.01

$$\int z^3\tan(a\,z)\,dz = \frac{1}{4\,a^4}\left(i\left(a^4\,z^4+4\,a^3\,i\log\left(1+e^{2\,i\,a\,z}\right)z^3+6\,a^2\,\mathrm{Li}_2\left(-e^{2\,i\,a\,z}\right)z^2+6\,a\,i\,\mathrm{Li}_3\left(-e^{2\,i\,a\,z}\right)z-3\,\mathrm{Li}_4\left(-e^{2\,i\,a\,z}\right)\right)\right)$$

01.08.21.0021.01

$$\int z^4\tan(a\,z)\,dz = \frac{1}{a^5}\left(\frac{1}{5}\,a^5\,i\,z^5-a^4\log\left(1+e^{2\,i\,a\,z}\right)z^4+2\,a^3\,i\,\mathrm{Li}_2\left(-e^{2\,i\,a\,z}\right)z^3-3\,a^2\,\mathrm{Li}_3\left(-e^{2\,i\,a\,z}\right)z^2-3\,i\,a\,\mathrm{Li}_4\left(-e^{2\,i\,a\,z}\right)z+\frac{3}{2}\,\mathrm{Li}_5\left(-e^{2\,i\,a\,z}\right)\right)$$

Involving exponential function

Involving exp

Involving $a^{b\,z}$

01.08.21.0022.01

$$\int a^{b\,z}\tan(c\,z)\,dz = \frac{1}{b\log(a)\,(i\,b\log(a)-2\,c)}\left(a^{b\,z}\left(b\,e^{2\,i\,c\,z}\,{}_2F_1\left(1-\frac{i\,b\log(a)}{2\,c},\,1;\,2-\frac{i\,b\log(a)}{2\,c};\,-e^{2\,i\,c\,z}\right)\log(a)+{}_2F_1\left(-\frac{i\,b\log(a)}{2\,c},\,1;\,1-\frac{i\,b\log(a)}{2\,c};\,-e^{2\,i\,c\,z}\right)(-2\,i\,c-b\log(a))\right)\right)$$

01.08.21.0023.01

$$\int e^{b\,z}\tan(a\,z)\,dz = -\frac{1}{b\,(b+2\,i\,a)}\left((2\,a-i\,b)\,e^{b\,z}\,{}_2F_1\left(-\frac{i\,b}{2\,a},\,1;\,1-\frac{i\,b}{2\,a};\,-e^{2\,i\,a\,z}\right)+b\,e^{(b+2\,i\,a)\,z}\,i\,{}_2F_1\left(1-\frac{i\,b}{2\,a},\,1;\,2-\frac{i\,b}{2\,a};\,-e^{2\,i\,a\,z}\right)\right)$$

01.08.21.0024.01

$$\int e^{-i a z} \tan(a z)\, dz = -\frac{e^{-i a z} - 2\tan^{-1}\left(e^{-i a z}\right)}{a}$$

01.08.21.0025.01

$$\int e^{i a z} \tan(a z)\, dz = -\frac{e^{i a z} - 2\tan^{-1}\left(e^{i a z}\right)}{a}$$

Involving exponential function and a power function

Involving exp and power

Involving $z^n e^{b z}$

01.08.21.0026.01

$$\int z^n e^{b z} \tan(c z)\, dz = -i\, n! \left(e^{(b+2 i c) z} \sum_{j=0}^{n} \frac{(-1)^j (b+2 i c)^{-j-1} z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}\left(\frac{-i b+2 c}{2 c}, \ldots, \frac{-i b+2 c}{2 c}, 1; \frac{-i b+2 c}{2 c}+1, \ldots, \frac{-i b+2 c}{2 c}+1; -e^{2 i c z}\right) - e^{b z} \sum_{j=0}^{n} \frac{(-1)^j b^{-j-1} z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}\left(-\frac{i b}{2 c}, \ldots, -\frac{i b}{2 c}, 1; 1-\frac{i b}{2 c}, \ldots, 1-\frac{i b}{2 c}; -e^{2 i c z}\right)\right) /; n \in \mathbb{N}$$

Arguments involving inverse trigonometric functions

Involving $\sin^{-1}$

01.08.21.0027.01

$$\int \tan\left(\sin^{-1}(z)\right) dz = -\sqrt{1-z^2}$$

01.08.21.0028.01

$$\int \tan\left(a \sin^{-1}(z)\right) dz = -\frac{1}{8 a^2 - 2}\left(e^{-2 i \sin^{-1}(z)}\left((2 a - 1)\left(e^{i (2 a+3) \sin^{-1}(z)}\, {}_2F_1\left(1+\frac{1}{2 a}, 1; 2+\frac{1}{2 a}; -e^{2 i a \sin^{-1}(z)}\right) - (2 a + 1)\, e^{i \sin^{-1}(z)}\left(e^{2 i \sin^{-1}(z)}\, {}_2F_1\left(\frac{1}{2 a}, 1; 1+\frac{1}{2 a}; -e^{2 i a \sin^{-1}(z)}\right) - {}_2F_1\left(-\frac{1}{2 a}, 1; 1-\frac{1}{2 a}; -e^{2 i a \sin^{-1}(z)}\right)\right)\right) + (2 a + 1)\, e^{i (2 a+1) \sin^{-1}(z)}\, {}_2F_1\left(1-\frac{1}{2 a}, 1; 2-\frac{1}{2 a}; -e^{2 i a \sin^{-1}(z)}\right)\right)\right)$$

Involving $\cos^{-1}$

01.08.21.0029.01

$$\int \tan\left(\cos^{-1}(z)\right) dz = \log(z) - \log\left(\sqrt{1-z^2} + 1\right) + \sqrt{1-z^2}$$

01.08.21.0030.01

$$\int \tan\left(a\cos^{-1}(z)\right) dz = \frac{1}{8a^2-2}\left(i\, e^{-2i\cos^{-1}(z)} \left((2a-1)\left((2a+1)\, e^{i\cos^{-1}(z)}\left(e^{2i\cos^{-1}(z)}\, {}_2F_1\left(\frac{1}{2a}, 1; 1+\frac{1}{2a}; -e^{2ia\cos^{-1}(z)}\right) + {}_2F_1\left(-\frac{1}{2a}, 1; 1-\frac{1}{2a}; -e^{2ia\cos^{-1}(z)}\right)\right) - e^{i(2a+3)\cos^{-1}(z)}\, {}_2F_1\left(1+\frac{1}{2a}, 1; 2+\frac{1}{2a}; -e^{2ia\cos^{-1}(z)}\right)\right) + (2a+1)\, e^{i(2a+1)\cos^{-1}(z)}\, {}_2F_1\left(1-\frac{1}{2a}, 1; 2-\frac{1}{2a}; -e^{2ia\cos^{-1}(z)}\right)\right)\right)$$

Involving $\tan^{-1}$

01.08.21.0031.01

$$\int \tan\left(\tan^{-1}(z)\right) dz = \frac{z^2}{2}$$

Involving $\cot^{-1}$

01.08.21.0032.01

$$\int \tan\left(\cot^{-1}(z)\right) dz = \log(z)$$

Involving $\csc^{-1}$

01.08.21.0033.01

$$\int \tan\left(\csc^{-1}(z)\right) dz = \frac{\sqrt{1-\frac{1}{z^2}}\; z \log\left(z+\sqrt{z^2-1}\right)}{\sqrt{z^2-1}}$$

Involving $\sec^{-1}$

01.08.21.0034.01

$$\int \tan\left(\sec^{-1}(z)\right) dz = \frac{\sqrt{1-\frac{1}{z^2}}\; z\left(z\sqrt{z^2-1} - \log\left(z+\sqrt{z^2-1}\right)\right)}{2\sqrt{z^2-1}}$$

Arguments involving inverse hyperbolic functions

Involving $\sinh^{-1}$

01.08.21.0035.01

$$\int \tan\left(\sinh^{-1}(z)\right) dz = \frac{1}{10}\left(-5\, i\, e^{-\sinh^{-1}(z)}\, {}_2F_1\left(\frac{i}{2}, 1; 1+\frac{i}{2}; -e^{2i\sinh^{-1}(z)}\right) + 5\, e^{\sinh^{-1}(z)}\, i\, {}_2F_1\left(-\frac{i}{2}, 1; 1-\frac{i}{2}; -e^{2i\sinh^{-1}(z)}\right) - (2-i)\, e^{(-1+2i)\sinh^{-1}(z)}\, {}_2F_1\left(1+\frac{i}{2}, 1; 2+\frac{i}{2}; -e^{2i\sinh^{-1}(z)}\right) - (2+i)\, e^{(1+2i)\sinh^{-1}(z)}\, {}_2F_1\left(1-\frac{i}{2}, 1; 2-\frac{i}{2}; -e^{2i\sinh^{-1}(z)}\right)\right)$$

01.08.21.0036.01

$$\int \tan\left(a\sinh^{-1}(z)\right) dz = \frac{1}{2\left(4a^2+1\right)}\left(i\, e^{-\sinh^{-1}(z)}\left((2a-i)\left((2a+i)\left(e^{2\sinh^{-1}(z)}\, {}_2F_1\left(-\frac{i}{2a}, 1; 1-\frac{i}{2a}; -e^{2ia\sinh^{-1}(z)}\right) - {}_2F_1\left(\frac{i}{2a}, 1; 1+\frac{i}{2a}; -e^{2ia\sinh^{-1}(z)}\right)\right) + e^{2ia\sinh^{-1}(z)}\, i\, {}_2F_1\left(1+\frac{i}{2a}, 1; 2+\frac{i}{2a}; -e^{2ia\sinh^{-1}(z)}\right)\right) + (2a+i)\, e^{2(1+ia)\sinh^{-1}(z)}\, i\, {}_2F_1\left(1-\frac{i}{2a}, 1; 2-\frac{i}{2a}; -e^{2ia\sinh^{-1}(z)}\right)\right)\right)$$

Involving $\cosh^{-1}$

01.08.21.0037.01

$$\int \tan\left(\cosh^{-1}(z)\right) dz = \frac{1}{10}\left(5\, e^{-\cosh^{-1}(z)}\, i\, {}_2F_1\left(\frac{i}{2}, 1; 1+\frac{i}{2}; -e^{2i\cosh^{-1}(z)}\right) + 5\, e^{\cosh^{-1}(z)}\, i\, {}_2F_1\left(-\frac{i}{2}, 1; 1-\frac{i}{2}; -e^{2i\cosh^{-1}(z)}\right) + e^{(-1+2i)\cosh^{-1}(z)}\,(2-i)\, {}_2F_1\left(1+\frac{i}{2}, 1; 2+\frac{i}{2}; -e^{2i\cosh^{-1}(z)}\right) - (2+i)\, e^{(1+2i)\cosh^{-1}(z)}\, {}_2F_1\left(1-\frac{i}{2}, 1; 2-\frac{i}{2}; -e^{2i\cosh^{-1}(z)}\right)\right)$$

01.08.21.0038.01

$$\int \tan\left(a\cosh^{-1}(z)\right) dz = \frac{1}{2\left(4a^2+1\right)}\left(e^{-\cosh^{-1}(z)}\left((2a-i)\left((2a+i)\, i\left({}_2F_1\left(\frac{i}{2a}, 1; 1+\frac{i}{2a}; -e^{2ia\cosh^{-1}(z)}\right) + e^{2\cosh^{-1}(z)}\, {}_2F_1\left(-\frac{i}{2a}, 1; 1-\frac{i}{2a}; -e^{2ia\cosh^{-1}(z)}\right)\right) + e^{2ia\cosh^{-1}(z)}\, {}_2F_1\left(1+\frac{i}{2a}, 1; 2+\frac{i}{2a}; -e^{2ia\cosh^{-1}(z)}\right)\right) - (2a+i)\, e^{2(1+ia)\cosh^{-1}(z)}\, {}_2F_1\left(1-\frac{i}{2a}, 1; 2-\frac{i}{2a}; -e^{2ia\cosh^{-1}(z)}\right)\right)\right)$$

Involving trigonometric functions

Involving sin

Involving $\sin(b\,z)$

01.08.21.0039.01

$$\int \sin(b\,z)\tan(c\,z)\, dz = \frac{1}{2\left(b^3-4bc^2\right)}\left(i\, e^{-2ibz}\left((b-2c)\left(b\, e^{i(3b+2c)z}\, {}_2F_1\left(\frac{b}{2c}+1, 1; \frac{b}{2c}+2; -e^{2icz}\right) - (b+2c)\, e^{ibz}\left(e^{2ibz}\, {}_2F_1\left(\frac{b}{2c}, 1; \frac{b}{2c}+1; -e^{2icz}\right) + {}_2F_1\left(-\frac{b}{2c}, 1; 1-\frac{b}{2c}; -e^{2icz}\right)\right)\right) + b(b+2c)\, e^{i(b+2c)z}\, {}_2F_1\left(1-\frac{b}{2c}, 1; 2-\frac{b}{2c}; -e^{2icz}\right)\right)\right)$$

Involving powers of sin

Involving $\sin^m(b\,z)$

01.08.21.0040.01

$$\int \sin^m(b z)\tan(c z)\,dz = -\frac{2^{-m}\log(\cos(c z))\,(1-m \bmod 2)}{c}\binom{m}{\frac{m}{2}} + i^{-m}\,2^{-m}\sum_{k=0}^{\left\lfloor\frac{m-1}{2}\right\rfloor}(-1)^k\binom{m}{k}\left(\frac{e^{i b(m-2k)z}\,{}_2F_1\left(\frac{b(m-2k)}{2c},1;\frac{2c+b(m-2k)}{2c};-e^{2icz}\right)}{b(m-2k)} - \frac{(-1)^m e^{-i b(m-2k)z}\,{}_2F_1\left(-\frac{b(m-2k)}{2c},1;\frac{2c-b(m-2k)}{2c};-e^{2icz}\right)}{b(m-2k)} - \frac{e^{i(2c+b(m-2k))z}\,{}_2F_1\left(\frac{2c+b(m-2k)}{2c},1;\frac{4c+b(m-2k)}{2c};-e^{2icz}\right)}{2c+b(m-2k)} - \frac{(-1)^m e^{i(2c-b(m-2k))z}\,{}_2F_1\left(\frac{2c-b(m-2k)}{2c},1;\frac{4c-b(m-2k)}{2c};-e^{2icz}\right)}{2c-b(m-2k)}\right) /;\ m\in\mathbb{N}^+$$

01.08.21.0041.01

$$\int \sin^\mu(c z)\tan(c z)\,dz = -\frac{\sin^\mu(c z)\left(-\tan^2(c z)\right)^{-\frac{\mu}{2}}}{c\,\mu}\,{}_2F_1\left(-\frac{\mu}{2},-\frac{\mu}{2};1-\frac{\mu}{2};\sec^2(c z)\right)$$

Involving algebraic functions of sin

01.08.21.0042.01

$$\int (a+b\sin(c z))^\beta\tan(c z)\,dz = \frac{(a+b\sin(c z))^{\beta+1}}{2(a-b)(a+b)c(\beta+1)}\left((a+b)\,{}_2F_1\left(\beta+1,1;\beta+2;\frac{a+b\sin(c z)}{a-b}\right)+(a-b)\,{}_2F_1\left(\beta+1,1;\beta+2;\frac{a+b\sin(c z)}{a+b}\right)\right)$$

01.08.21.0043.01

$$\int \sqrt{a+b\sin(c z)}\,\tan(c z)\,dz = \frac{\sqrt{a-b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c z)}}{\sqrt{a-b}}\right)+\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c z)}}{\sqrt{a+b}}\right)-2\sqrt{a+b\sin(c z)}}{c}$$

01.08.21.0044.01

$$\int \frac{\tan(c z)}{\sqrt{a+b\sin(c z)}}\,dz = \frac{1}{c}\left(\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c z)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin(c z)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}\right)$$

Involving cos

Involving cos($b z$)

01.08.21.0045.01

$$\int \cos(b\,z)\tan(c\,z)\,dz = \frac{1}{2\left(b^3-4\,b\,c^2\right)} \left(e^{-2\,i\,b\,z}\left(b\,(b+2\,c)\,e^{i\,(b+2\,c)\,z}\,{}_2F_1\!\left(1-\frac{b}{2\,c},1;2-\frac{b}{2\,c};-e^{2\,i\,c\,z}\right)-(b-2\,c)\left(b\,e^{i\,(3\,b+2\,c)\,z}\,{}_2F_1\!\left(\frac{b}{2\,c}+1,1;\frac{b}{2\,c}+2;-e^{2\,i\,c\,z}\right)-(b+2\,c)\,e^{i\,b\,z}\left(e^{2\,i\,b\,z}\,{}_2F_1\!\left(\frac{b}{2\,c},1;\frac{b}{2\,c}+1;-e^{2\,i\,c\,z}\right)-{}_2F_1\!\left(-\frac{b}{2\,c},1;1-\frac{b}{2\,c};-e^{2\,i\,c\,z}\right)\right)\right)\right)\right)$$

Involving powers of cos

Involving $\cos^m(b\,z)$

01.08.21.0046.01

$$\int \cos^m(b\,z)\tan(c\,z)\,dz = -\frac{2^{-m}\log(\cos(c\,z))\,(1-m \bmod 2)}{c}\binom{m}{\frac{m}{2}} + 2^{-m}\sum_{k=0}^{\left\lfloor\frac{m-1}{2}\right\rfloor}\binom{m}{k}\left(\frac{e^{i\,b\,(m-2\,k)\,z}\,{}_2F_1\!\left(\frac{b\,(m-2\,k)}{2\,c},1;\frac{2\,c+b\,(m-2\,k)}{2\,c};-e^{2\,i\,c\,z}\right)}{b\,(m-2\,k)} - \frac{e^{-i\,b\,(m-2\,k)\,z}\,{}_2F_1\!\left(-\frac{b\,(m-2\,k)}{2\,c},1;\frac{2\,c-b\,(m-2\,k)}{2\,c};-e^{2\,i\,c\,z}\right)}{b\,(m-2\,k)} - \frac{e^{i\,(2\,c+b\,(m-2\,k))\,z}\,{}_2F_1\!\left(\frac{2\,c+b\,(m-2\,k)}{2\,c},1;\frac{4\,c+b\,(m-2\,k)}{2\,c};-e^{2\,i\,c\,z}\right)}{2\,c+b\,(m-2\,k)} - \frac{e^{i\,(2\,c-b\,(m-2\,k))\,z}\,{}_2F_1\!\left(\frac{2\,c-b\,(m-2\,k)}{2\,c},1;\frac{4\,c-b\,(m-2\,k)}{2\,c};-e^{2\,i\,c\,z}\right)}{2\,c-b\,(m-2\,k)}\right) /;\ m\in\mathbb{N}^+$$

01.08.21.0047.01

$$\int \cos^\mu(c\,z)\tan(c\,z)\,dz = -\frac{\cos^\mu(c\,z)}{c\,\mu}$$

Involving algebraic functions of cos

Involving $(a+b\cos(c\,z))^\beta$

01.08.21.0048.01

$$\int (a+b\cos(c\,z))^\beta\tan(c\,z)\,dz = -\frac{(a+b\cos(c\,z))^\beta}{c\,\beta}\left(\frac{a\sec(c\,z)}{b}+1\right)^{-\beta}{}_2F_1\!\left(-\beta,-\beta;1-\beta;-\frac{a\sec(c\,z)}{b}\right)$$

01.08.21.0049.01

$$\int \sqrt{a+b\cos(c\,z)}\,\tan(c\,z)\,dz = \frac{2\sqrt{a+b\cos(c\,z)}}{c}\left(\frac{\sqrt{a}\,\sec^{\frac{1}{2}}(c\,z)}{\sqrt{b}\sqrt{\frac{a\sec(c\,z)}{b}+1}}\sinh^{-1}\!\left(\frac{\sqrt{a}\,\sec^{\frac{1}{2}}(c\,z)}{\sqrt{b}}\right)-1\right)$$

01.08.21.0050.01

$$\int \frac{\tan(c\,z)}{\sqrt{a+b\cos(c\,z)}}\,dz = \frac{2\sqrt{b}}{\sqrt{a}\;c\sqrt{a+b\cos(c\,z)}\;\sec^{\frac{1}{2}}(c\,z)}\sinh^{-1}\left(\frac{\sqrt{a}\;\sec^{\frac{1}{2}}(c\,z)}{\sqrt{b}}\right)\sqrt{\frac{a\sec(c\,z)}{b}+1}$$

Involving $(a+b\cos(2\,c\,z))^{\beta}$

01.08.21.0051.01

$$\int (a+b\cos(2\,c\,z))^{\beta}\tan(c\,z)\,dz = -\frac{(a+b\cos(2\,c\,z))^{\beta}}{2\,c\,\beta}\left(\frac{(a-b)\sec^2(c\,z)}{2\,b}+1\right)^{-\beta}{}_2F_1\!\left(-\beta,-\beta;1-\beta;-\frac{(a-b)\sec^2(c\,z)}{2\,b}\right)$$

01.08.21.0052.01

$$\int \sqrt{a+b\cos(2\,c\,z)}\;\tan(c\,z)\,dz = \frac{1}{c}\left(\sqrt{a-b}\;\tanh^{-1}\left(\frac{\sqrt{a+b\cos(2\,c\,z)}}{\sqrt{a-b}}\right)-\sqrt{a+b\cos(2\,c\,z)}\right)$$

01.08.21.0053.01

$$\int \frac{\tan(c\,z)}{\sqrt{a+b\cos(2\,c\,z)}}\,dz = \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cos(2\,c\,z)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}\;c}$$

Involving $\cos(2\,c\,z)\,(a+b\cos(2\,c\,z))^{\beta}$

01.08.21.0054.01

$$\int \cos(2\,c\,z)\,(a+b\cos(2\,c\,z))^{\beta}\tan(c\,z)\,dz = \frac{1}{2\,b\,c\,\beta\,(\beta+1)}\Bigg((a+b\cos(2\,c\,z))^{\beta}\left(\frac{a+b\cos(2\,c\,z)}{a-b}\right)^{-\beta}$$
$$\left(\frac{(a-b)\sec^2(c\,z)}{2\,b}+1\right)^{-\beta}\left(b\,(\beta+1)\left(\frac{a+b\cos(2\,c\,z)}{a-b}\right)^{\beta}{}_2F_1\!\left(-\beta,-\beta;1-\beta;-\frac{(a-b)\sec^2(c\,z)}{2\,b}\right)-\right.$$
$$\left.\beta\left(b\cos(2\,c\,z)\left(\frac{a+b\cos(2\,c\,z)}{a-b}\right)^{\beta}+a\left(\left(\frac{a+b\cos(2\,c\,z)}{a-b}\right)^{\beta}-1\right)+b\right)\left(\frac{(a-b)\sec^2(c\,z)}{2\,b}+1\right)^{\beta}\right)\Bigg)$$

01.08.21.0055.01

$$\int \cos(2\,c\,z)\sqrt{a+b\cos(2\,c\,z)}\,\tan(c\,z)\,dz = -\left((a-3\,b)\sqrt{\frac{a+b\cos(2\,c\,z)}{(\cos(c\,z)+1)^2}}\left(\sqrt{a-b}\sqrt{(a+b\cos(2\,c\,z))\sec^4\left(\frac{c\,z}{2}\right)}\cos(c\,z)+2\,b\log\left(\frac{2\left((a-b)\tan^2\left(\frac{c\,z}{2}\right)+a-b+\sqrt{a-b}\sqrt{(a+b\cos(2\,c\,z))\sec^4\left(\frac{c\,z}{2}\right)}\right)}{\sqrt{a-b}\left(\tan^2\left(\frac{c\,z}{2}\right)-1\right)}\right)+\sqrt{a-b}\sqrt{(a+b\cos(2\,c\,z))\sec^4\left(\frac{c\,z}{2}\right)}\right)\cos^2\left(\frac{c\,z}{2}\right)+\sqrt{a-b}\,b^2\cos^2(2\,c\,z)+a\sqrt{a-b}\,b\cos(2\,c\,z)+2\,a\,b\tanh^{-1}\left(\frac{\sqrt{a+b\cos(2\,c\,z)}}{\sqrt{a-b}}\right)\sqrt{a+b\cos(2\,c\,z)}\right)\Big/\left(3\sqrt{a-b}\,b\,c\sqrt{a+b\cos(2\,c\,z)}\right)$$

01.08.21.0056.01

$$\int \frac{\cos(2\,c\,z)\tan(c\,z)}{\sqrt{a+b\cos(2\,c\,z)}}\,dz = -\left(\cos^2\left(\frac{c\,z}{2}\right)\sqrt{\frac{a+b\cos(2\,c\,z)}{(\cos(c\,z)+1)^2}}\left(\sqrt{a-b}\sqrt{(a+b\cos(2\,c\,z))\sec^4\left(\frac{c\,z}{2}\right)}\cos(c\,z)+2\,b\log\left(\frac{2\left((a-b)\tan^2\left(\frac{c\,z}{2}\right)+a-b+\sqrt{a-b}\sqrt{(a+b\cos(2\,c\,z))\sec^4\left(\frac{c\,z}{2}\right)}\right)}{\sqrt{a-b}\left(\tan^2\left(\frac{c\,z}{2}\right)-1\right)}\right)+\sqrt{a-b}\sqrt{(a+b\cos(2\,c\,z))\sec^4\left(\frac{c\,z}{2}\right)}\right)\right)\Big/\left(\sqrt{a-b}\,b\,c\sqrt{a+b\cos(2\,c\,z)}\right)$$

Involving sin and cos

Involving $\sin(c\,z)\,(a+b\cos(2\,c\,z))^\beta$

01.08.21.0057.01

$$\int \sin(c\,z)\,(a+b\cos(2\,c\,z))^\beta\tan(c\,z)\,dz = \frac{1}{3\,c}\left(F_1\left(\frac{3}{2};\beta+\frac{3}{2},-\beta;\frac{5}{2};-\tan^2(c\,z),-\frac{(a-b)\tan^2(c\,z)}{a+b}\right)(a+b\cos(2\,c\,z))^\beta\sec(c\,z)\sec^2(c\,z)^{\beta-\frac{1}{2}}\left(\frac{(a+b\cos(2\,c\,z))\sec^2(c\,z)}{a+b}\right)^{-\beta}\tan^3(c\,z)\right)$$

01.08.21.0058.01

$$\int \sin(c\,z)\sqrt{a+b\cos(2\,c\,z)}\,\tan(c\,z)\,dz = \frac{1}{4\sqrt{a-b}\sqrt{-b}\,c}\left(-4\sqrt{-b}\,(b-a)\tanh^{-1}\left(\frac{\sqrt{a-b}\sin(c\,z)}{\sqrt{a+b\cos(2\,c\,z)}}\right)-\sqrt{a-b}\left(\sqrt{2}\,(a-3\,b)\log\left(\sqrt{2}\sqrt{-b}\sin(c\,z)+\sqrt{a+b\cos(2\,c\,z)}\right)+2\sqrt{-b}\sqrt{a+b\cos(2\,c\,z)}\sin(c\,z)\right)\right)$$

01.08.21.0059.01

$$\int \frac{\sin(c\,z)\tan(c\,z)}{\sqrt{a+b\cos(2\,c\,z)}}\,dz = \frac{\tanh^{-1}\left(\frac{\sqrt{a-b}\,\sin(c\,z)}{\sqrt{a+b\cos(2\,c\,z)}}\right)}{\sqrt{a-b}\,c} - \frac{\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sin(c\,z)}{\sqrt{a+b\cos(2\,c\,z)}}\right)}{\sqrt{2}\,\sqrt{b}\,c}$$

Involving trigonometric and a power functions

Involving sin and power

Involving $z^n \sin(a+b\,z)\tan(c\,z)$

01.08.21.0060.01

$$\int z^n \sin(a+b\,z)\tan(c\,z)\,dz = \frac{1}{2} e^{-i\,a}\,n!\left(e^{(2\,i\,c-i\,b)\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(2\,i\,c-i\,b)^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{2\,c-b}{2\,c},\ldots,\frac{2\,c-b}{2\,c},1;\frac{2\,c-b}{2\,c}+1,\ldots,\frac{2\,c-b}{2\,c}+1;-e^{2\,i\,c\,z}\right) - e^{-i\,b\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(-i\,b)^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(-\frac{b}{2\,c},\ldots,-\frac{b}{2\,c},1;1-\frac{b}{2\,c},\ldots,1-\frac{b}{2\,c};-e^{2\,i\,c\,z}\right)\right) - \frac{1}{2} e^{i\,a}\,n!\left(e^{(2\,i\,c+i\,b)\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(2\,i\,c+i\,b)^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{b+2\,c}{2\,c},\ldots,\frac{b+2\,c}{2\,c},1;\frac{b+2\,c}{2\,c}+1,\ldots,\frac{b+2\,c}{2\,c}+1;-e^{2\,i\,c\,z}\right) - e^{i\,b\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(i\,b)^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{b}{2\,c},\ldots,\frac{b}{2\,c},1;\frac{b}{2\,c}+1,\ldots,\frac{b}{2\,c}+1;-e^{2\,i\,c\,z}\right)\right) /;\, n\in\mathbb{N}$$

01.08.21.0061.01

$$\int z^n \sin(b\,z)\tan(c\,z)\,dz = \frac{1}{2}\,n!\left(e^{(2\,i\,c-i\,b)\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(2\,i\,c-i\,b)^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{2\,c-b}{2\,c},\ldots,\frac{2\,c-b}{2\,c},1;\frac{2\,c-b}{2\,c}+1,\ldots,\frac{2\,c-b}{2\,c}+1;-e^{2\,i\,c\,z}\right) - e^{-i\,b\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(-i\,b)^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(-\frac{b}{2\,c},\ldots,-\frac{b}{2\,c},1;1-\frac{b}{2\,c},\ldots,1-\frac{b}{2\,c};-e^{2\,i\,c\,z}\right) - e^{(2\,i\,c+i\,b)\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(2\,i\,c+i\,b)^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{b+2\,c}{2\,c},\ldots,\frac{b+2\,c}{2\,c},1;\frac{b+2\,c}{2\,c}+1,\ldots,\frac{b+2\,c}{2\,c}+1;-e^{2\,i\,c\,z}\right) + e^{i\,b\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(i\,b)^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{b}{2\,c},\ldots,\frac{b}{2\,c},1;\frac{b}{2\,c}+1,\ldots,\frac{b}{2\,c}+1;-e^{2\,i\,c\,z}\right)\right) /;\, n\in\mathbb{N}$$

Involving powers of sin and power

Involving $z^n \sin^m(b\,z)\tan(c\,z)$

01.08.21.0062.01

$$\int z^n \sin^m(b z) \tan(c z) \, dz = 2^{-m} \binom{m}{\frac{m}{2}} n! \, (1 - m \bmod 2) \left(\frac{i z^{n+1}}{(n+1)!} - 2 i e^{2 i c z} \sum_{j=0}^{n} \frac{(-1)^j 2^{-j-1} (i c)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(1, \ldots, 1; 2, \ldots, 2; -e^{2 i c z}\right) \right) -$$
$$2^{-m} i^{1-m} n! \sum_{k=0}^{\left\lfloor \frac{m-1}{2} \right\rfloor} (-1)^k \binom{m}{k} \left(-e^{i b (m-2k) z} \sum_{j=0}^{n} \frac{(-1)^j (i b (m-2k))^{-j-1} z^{n-j}}{(n-j)!} \right.$$
$${}_{j+2}F_{j+1}\left(\frac{b(m-2k)}{2c}, \ldots, \frac{b(m-2k)}{2c}, 1; \frac{b(m-2k)}{2c} + 1, \ldots, \frac{b(m-2k)}{2c} + 1; -e^{2 i c z} \right) +$$
$$(-1)^m \left(e^{(2 i c - i b (m-2k)) z} \sum_{j=0}^{n} \frac{(-1)^j (2 i c - i b (m-2k))^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{2c - b(m-2k)}{2c}, \ldots, \right. \right.$$
$$\left. \frac{2c - b(m-2k)}{2c}, 1; \frac{2c - b(m-2k)}{2c} + 1, \ldots, \frac{2c - b(m-2k)}{2c} + 1; -e^{2 i c z} \right) -$$
$$e^{-i b (m-2k) z} \sum_{j=0}^{n} \frac{(-1)^j (-i b (m-2k))^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(-\frac{b(m-2k)}{2c}, \ldots, -\frac{b(m-2k)}{2c}, \right.$$
$$\left. \left. 1; 1 - \frac{b(m-2k)}{2c}, \ldots, 1 - \frac{b(m-2k)}{2c}; -e^{2 i c z} \right) \right) +$$
$$e^{(2 i c + i b (m-2k)) z} \sum_{j=0}^{n} \frac{(-1)^j (2 i c + i b (m-2k))^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{2c + b(m-2k)}{2c}, \ldots, \frac{2c + b(m-2k)}{2c}, \right.$$
$$\left. \left. 1; \frac{2c + b(m-2k)}{2c} + 1, \ldots, \frac{2c + b(m-2k)}{2c} + 1; -e^{2 i c z} \right) \right) /; n \in \mathbb{N} \land m \in \mathbb{N}^+$$

Involving cosh and power

Involving $z^n \cos(a + b z) \tan(c z)$

01.08.21.0063.01

$$\int z^n \cos(a+b z)\tan(c z)\,dz = -\frac{i}{2} e^{-i a} n! \left(e^{(2 i c - i b) z} \sum_{j=0}^{n} \frac{(-1)^j (2 i c - i b)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{2c-b}{2c}, \ldots, \frac{2c-b}{2c}, 1; \frac{2c-b}{2c}+1, \ldots, \frac{2c-b}{2c}+1; -e^{2 i c z}\right) - e^{-i b z} \sum_{j=0}^{n} \frac{(-1)^j (-i b)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(-\frac{b}{2c}, \ldots, -\frac{b}{2c}, 1; 1-\frac{b}{2c}, \ldots, 1-\frac{b}{2c}; -e^{2 i c z}\right)\right) - \frac{i}{2} e^{i a} n! \left(e^{(2 i c + i b) z} \sum_{j=0}^{n} \frac{(-1)^j (2 i c + i b)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{2c+b}{2c}, \ldots, \frac{2c+b}{2c}, 1; \frac{2c+b}{2c}+1, \ldots, \frac{2c+b}{2c}+1; -e^{2 i c z}\right) - e^{i b z} \sum_{j=0}^{n} \frac{(-1)^j (i b)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{b}{2c}, \ldots, \frac{b}{2c}, 1; \frac{b}{2c}+1, \ldots, \frac{b}{2c}+1; -e^{2 i c z}\right)\right) /; n \in \mathbb{N}$$

01.08.21.0064.01

$$\int z^n \cos(b z)\tan(c z)\,dz = \frac{i}{2} n! \left(-e^{(2 i c - i b) z} \sum_{j=0}^{n} \frac{(-1)^j (2 i c - i b)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{2c-b}{2c}, \ldots, \frac{2c-b}{2c}, 1; \frac{2c-b}{2c}+1, \ldots, \frac{2c-b}{2c}+1; -e^{2 i c z}\right) + e^{-i b z} \sum_{j=0}^{n} \frac{(-1)^j (-i b)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(-\frac{b}{2c}, \ldots, -\frac{b}{2c}, 1; 1-\frac{b}{2c}, \ldots, 1-\frac{b}{2c}; -e^{2 i c z}\right) - e^{(2 i c + i b) z} \sum_{j=0}^{n} \frac{(-1)^j (2 i c + i b)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{2c+b}{2c}, \ldots, \frac{2c+b}{2c}, 1; \frac{2c+b}{2c}+1, \ldots, \frac{2c+b}{2c}+1; -e^{2 i c z}\right) + e^{i b z} \sum_{j=0}^{n} \frac{(-1)^j (i b)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{b}{2c}, \ldots, \frac{b}{2c}, 1; \frac{b}{2c}+1, \ldots, \frac{b}{2c}+1; -e^{2 i c z}\right)\right) /; n \in \mathbb{N}$$

Involving powers of cos and power

Involving $z^n \cos^m(b z) \tan(c z)$

01.08.21.0065.01

$$\int z^n \cos^m(b z) \tan(c z)\, dz = -i\, 2^{-m} \binom{m}{\frac{m}{2}} n!\,(1 - m \bmod 2) \left(2 e^{2 i c z} \sum_{j=0}^{n} \frac{(-1)^j\, 2^{-j-1} (i c)^{-j-1} z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}(1, \ldots, 1; 2, \ldots, 2; -e^{2 i c z}) - \frac{z^{n+1}}{(n+1)!}\right) - i\, 2^{-m} n! \sum_{k=0}^{\left\lfloor \frac{m-1}{2} \right\rfloor} \binom{m}{k} \left(-e^{-i b (m-2k) z} \sum_{j=0}^{n} \frac{(-1)^j (-i b (m-2k))^{-j-1} z^{n-j}}{(n-j)!}\right.$$
$$ {}_{j+2}F_{j+1}\left(-\frac{b(m-2k)}{2c}, \ldots, -\frac{b(m-2k)}{2c}, 1; 1 - \frac{b(m-2k)}{2c}, \ldots, 1 - \frac{b(m-2k)}{2c}; -e^{2 i c z}\right) - e^{i b (m-2k) z} \sum_{j=0}^{n} \frac{(-1)^j (i b (m-2k))^{-j-1} z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}\left(\frac{b(m-2k)}{2c}, \ldots, \frac{b(m-2k)}{2c}, 1; \frac{b(m-2k)}{2c} + 1, \right.$$
$$\left. \ldots, \frac{b(m-2k)}{2c} + 1; -e^{2 i c z}\right) + e^{(2 i c - i b (m-2k)) z} \sum_{j=0}^{n} \frac{(-1)^j (2 i c - i b (m-2k))^{-j-1} z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}$$
$$\left(\frac{2c - b(m-2k)}{2c}, \ldots, \frac{2c - b(m-2k)}{2c}, 1; \frac{2c - b(m-2k)}{2c} + 1, \ldots, \frac{2c - b(m-2k)}{2c} + 1; -e^{2 i c z}\right) +$$
$$e^{(2 i c + i b (m-2k)) z} \sum_{j=0}^{n} \frac{(-1)^j (2 i c + i b (m-2k))^{-j-1} z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}\left(\frac{2c + b(m-2k)}{2c}, \ldots, \frac{2c + b(m-2k)}{2c},\right.$$
$$\left.\left. 1; \frac{2c + b(m-2k)}{2c} + 1, \ldots, \frac{2c + b(m-2k)}{2c} + 1; -e^{2 i c z}\right)\right) /;\ n \in \mathbb{N} \land m \in \mathbb{N}^+$$

Involving trigonometric and exponential functions

Involving sin and exp

Involving $e^{p z} \sin(b z)$

01.08.21.0066.01

$$\int e^{p z} \sin(b z) \tan(c z)\, dz = \frac{1}{2} i \left(-\frac{e^{(i b + p) z}\, {}_2F_1\left(\frac{b - i p}{2c}, 1; \frac{b + 2c - i p}{2c}; -e^{2 i c z}\right)}{b - i p} + \frac{e^{(i b + 2 i c + p) z}\, {}_2F_1\left(\frac{b + 2c - i p}{2c}, 1; \frac{b + 4c - i p}{2c}; -e^{2 i c z}\right)}{b + 2c - i p} - \frac{e^{(-i b + p) z}\, {}_2F_1\left(-\frac{b + i p}{2c}, 1; -\frac{b - 2c + i p}{2c}; -e^{2 i c z}\right)}{b + i p} + \frac{e^{(-i b + 2 i c + p) z}\, {}_2F_1\left(-\frac{b - 2c + i p}{2c}, 1; -\frac{b - 4c + i p}{2c}; -e^{2 i c z}\right)}{b - 2c + i p}\right)$$

01.08.21.0067.01

$$\int e^{i b z} \sin(b z) \tan(c z)\, dz = -\frac{1}{4 b c (b + c)} \left(i \left(c (b + c) e^{2 i b z}\, {}_2F_1\left(\frac{b}{c}, 1; \frac{b + c}{c}; -e^{2 i c z}\right) + b \left(2 (b + c) \log(\cos(c z)) - c\, e^{2 i (b + c) z}\, {}_2F_1\left(\frac{b + c}{c}, 1; \frac{b}{c} + 2; -e^{2 i c z}\right)\right)\right)\right)$$

01.08.21.0068.01

$$\int e^{-i b z} \sin(b z) \tan(c z) \, dz = \frac{1}{4 b (b-c) c} \left(i e^{-2 i b z} \left(b c e^{2 i c z} {}_2F_1\left(1 - \frac{b}{c}, 1; 2 - \frac{b}{c}; -e^{2 i c z}\right) + (b-c)\left(2 b e^{2 i b z} \log(\cos(c z)) - c\, {}_2F_1\left(-\frac{b}{c}, 1; 1 - \frac{b}{c}; -e^{2 i c z}\right)\right)\right)\right)$$

Involving powers of sin and exp

Involving $e^{p z} \sin^m(b z)$

01.08.21.0069.01

$$\int e^{p z} \sin^m(b z) \tan(c z) \, dz = -i\, 2^{-m} \left(\frac{1}{p (2 i c + p)} \binom{m}{\frac{m}{2}} \left(e^{p z} (2 i c + p)\, {}_2F_1\left(-\frac{i p}{2 c}, 1; 1 - \frac{i p}{2 c}; -e^{2 i c z}\right) - e^{(2 i c + p) z} p\, {}_2F_1\left(1 - \frac{i p}{2 c}, 1; 2 - \frac{i p}{2 c}; -e^{2 i c z}\right)\right) (m \bmod 2 - 1) + i^{-m} \sum_{k=0}^{\left\lfloor \frac{m-1}{2} \right\rfloor} (-1)^k \binom{m}{k} \left(\frac{e^{(b i (m - 2 k) + p) z}\, {}_2F_1\left(\frac{-2 b k + b m - i p}{2 c}, 1; \frac{2 c - 2 b k + b m - i p}{2 c}; -e^{2 i c z}\right)}{2 i b k - i b m - p} - \left((-1)^m e^{(p - i b (m - 2 k)) z}\, {}_2F_1\left(-\frac{-2 b k + b m + i p}{2 c}, 1; \frac{2 c + 2 b k - b m - i p}{2 c}; -e^{2 i c z}\right)\right) \Big/ (2 i b k - i b m + p) - \left(i e^{(2 i c + i b (m - 2 k) + p) z}\, {}_2F_1\left(\frac{2 c - 2 b k + b m - i p}{2 c}, 1; \frac{4 c - 2 b k + b m - i p}{2 c}; -e^{2 i c z}\right)\right) \Big/ (2 c - 2 b k + b m - i p) - \left(i (-1)^m e^{(2 i c - i b (m - 2 k) + p) z}\, {}_2F_1\left(\frac{2 c + 2 b k - b m - i p}{2 c}, 1; \frac{4 c + 2 b k - b m - i p}{2 c}; -e^{2 i c z}\right)\right) \Big/ (2 c + 2 b k - b m - i p)\right)\right) /; m \in \mathbb{N}^+$$

Involving cos and exp

Involving $e^{p z} \cos(b z)$

01.08.21.0070.01

$$\int e^{p z} \cos(b z) \tan(c z) \, dz = -\frac{1}{2} i \left(-\frac{e^{(i b + p) z}\, {}_2F_1\left(\frac{b - i p}{2 c}, 1; \frac{b + 2 c - i p}{2 c}; -e^{2 i c z}\right)}{i b + p} - \frac{i e^{(i b + 2 i c + p) z}\, {}_2F_1\left(\frac{b + 2 c - i p}{2 c}, 1; \frac{b + 4 c - i p}{2 c}; -e^{2 i c z}\right)}{b + 2 c - i p} + \frac{e^{(-i b + p) z}\, {}_2F_1\left(-\frac{b + i p}{2 c}, 1; -\frac{b - 2 c + i p}{2 c}; -e^{2 i c z}\right)}{i b - p} + \frac{i e^{(-i b + 2 i c + p) z}\, {}_2F_1\left(-\frac{b - 2 c + i p}{2 c}, 1; -\frac{b - 4 c + i p}{2 c}; -e^{2 i c z}\right)}{b - 2 c + i p}\right)$$

01.08.21.0071.01

$$\int e^{i b z} \cos(b z) \tan(c z)\, dz = \frac{1}{4 b c (b+c)} \left(c (b+c) e^{2 i b z} {}_2F_1\left(\frac{b}{c}, 1; \frac{b+c}{c}; -e^{2 i c z}\right) - b \left(c\, e^{2 i (b+c) z} {}_2F_1\left(\frac{b+c}{c}, 1; \frac{b}{c}+2; -e^{2 i c z}\right) + 2 (b+c) \log(\cos(c z)) \right) \right)$$

01.08.21.0072.01

$$\int e^{-i b z} \cos(b z) \tan(c z)\, dz = \frac{1}{4 b (b-c) c} \left(e^{-2 i b z} \left(b c\, e^{2 i c z} {}_2F_1\left(1-\frac{b}{c}, 1; 2-\frac{b}{c}; -e^{2 i c z}\right) - (b-c) \left(c\, {}_2F_1\left(-\frac{b}{c}, 1; 1-\frac{b}{c}; -e^{2 i c z}\right) + 2 b\, e^{2 i b z} \log(\cos(c z)) \right) \right) \right)$$

Involving powers of cos and exp

Involving $e^{p z} \cos^m(b z)$

01.08.21.0073.01

$$\int e^{p z} \cos^m(b z) \tan(c z)\, dz = -i\, 2^{-m} \left(\frac{1}{p (2 i c + p)} \binom{m}{\frac{m}{2}} \left(e^{p z} (2 i c + p)\, {}_2F_1\left(-\frac{i p}{2 c}, 1; 1-\frac{i p}{2 c}; -e^{2 i c z}\right) - e^{(2 i c + p) z} p\, {}_2F_1\left(1-\frac{i p}{2 c}, 1; 2-\frac{i p}{2 c}; -e^{2 i c z}\right) \right) (m \bmod 2 - 1) + \sum_{k=0}^{\left\lfloor \frac{m-1}{2} \right\rfloor} \binom{m}{k} \left(\frac{e^{(b i (m-2k) + p) z} {}_2F_1\left(\frac{-2 b k + b m - i p}{2 c}, 1; \frac{2 c - 2 b k + b m - i p}{2 c}; -e^{2 i c z}\right)}{2 i b k - i b m - p} - \left(i\, e^{(2 i c - i b (m-2k) + p) z} {}_2F_1\left(\frac{2 c + 2 b k - b m - i p}{2 c}, 1; \frac{4 c + 2 b k - b m - i p}{2 c}; -e^{2 i c z}\right) \right) \Big/ (2 c + 2 b k - b m - i p) - \frac{e^{(p - i b (m-2k)) z} {}_2F_1\left(-\frac{-2 b k + b m + i p}{2 c}, 1; \frac{2 c + 2 b k - b m - i p}{2 c}; -e^{2 i c z}\right)}{2 i b k - i b m + p} - \left(i\, e^{(2 i c + i b (m-2k) + p) z} {}_2F_1\left(\frac{2 c - 2 b k + b m - i p}{2 c}, 1; \frac{4 c - 2 b k + b m - i p}{2 c}; -e^{2 i c z}\right) \right) \Big/ (2 c - 2 b k + b m - i p) \right) \right) /; m \in \mathbb{N}^+$$

Involving trigonometric, exponential and a power functions

Involving sin, exp and power

Involving $z^n e^{p z} \sin(a + b z) \tan(c z)$

01.08.21.0074.01

$$\int z^n e^{pz}\sin(a+bz)\tan(cz)\,dz = \frac{1}{2}e^{-ia}n!\left(e^{(2ic-ib+p)z}\sum_{j=0}^{n}\frac{(-1)^j(2ic-ib+p)^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{-b+2c-ip}{2c},\ldots,\frac{-b+2c-ip}{2c},1;\frac{-b+2c-ip}{2c}+1,\ldots,\frac{-b+2c-ip}{2c}+1;-e^{2icz}\right)-e^{(-ib+p)z}\sum_{j=0}^{n}\frac{(-1)^j(-ib+p)^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{-b-ip}{2c},\ldots,\frac{-b-ip}{2c},1;\frac{-b-ip}{2c}+1,\ldots,\frac{-b-ip}{2c}+1;-e^{2icz}\right)\right)-\frac{1}{2}e^{ia}n!\left(e^{(2ic+ib+p)z}\sum_{j=0}^{n}\frac{(-1)^j(2ic+ib+p)^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{b+2c-ip}{2c},\ldots,\frac{b+2c-ip}{2c},1;\frac{b+2c-ip}{2c}+1,\ldots,\frac{b+2c-ip}{2c}+1;-e^{2icz}\right)-e^{(ib+p)z}\sum_{j=0}^{n}\frac{(-1)^j(ib+p)^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{b-ip}{2c},\ldots,\frac{b-ip}{2c},1;\frac{b-ip}{2c}+1,\ldots,\frac{b-ip}{2c}+1;-e^{2icz}\right)\right)\;/;\;n\in\mathbb{N}$$

01.08.21.0075.01

$$\int z^n e^{pz}\sin(bz)\tan(cz)\,dz = \frac{1}{2}n!\left(e^{(2ic-ib+p)z}\sum_{j=0}^{n}\frac{(-1)^j(2ic-ib+p)^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{-b+2c-ip}{2c},\ldots,\frac{-b+2c-ip}{2c},1;\frac{-b+2c-ip}{2c}+1,\ldots,\frac{-b+2c-ip}{2c}+1;-e^{2icz}\right)-e^{(-ib+p)z}\sum_{j=0}^{n}\frac{(-1)^j(-ib+p)^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{-b-ip}{2c},\ldots,\frac{-b-ip}{2c},1;\frac{-b-ip}{2c}+1,\ldots,\frac{-b-ip}{2c}+1;-e^{2icz}\right)-e^{(2ic+ib+p)z}\sum_{j=0}^{n}\frac{(-1)^j(2ic+ib+p)^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{b+2c-ip}{2c},\ldots,\frac{b+2c-ip}{2c},1;\frac{b+2c-ip}{2c}+1,\ldots,\frac{b+2c-ip}{2c}+1;-e^{2icz}\right)+e^{(ib+p)z}\sum_{j=0}^{n}\frac{(-1)^j(ib+p)^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{b-ip}{2c},\ldots,\frac{b-ip}{2c},1;\frac{b-ip}{2c}+1,\ldots,\frac{b-ip}{2c}+1;-e^{2icz}\right)\right)\;/;\;n\in\mathbb{N}$$

Involving powers of sin, exp and power

Involving $z^n e^{pz}\sin^m(bz)\tan(cz)$

01.08.21.0076.01

$$\int z^n e^{p z} \sin^m(b z) \tan(c z)\, dz = -2^{-m} i^{1-m} n! \left(\sum_{k=0}^{\left\lfloor \frac{m-1}{2} \right\rfloor} (-1)^k \binom{m}{k} \left((-1)^m \left(e^{(2 i c - i b (m-2k) + p) z} \sum_{j=0}^{n} \frac{(-1)^j (2 i c - i b (m-2k) + p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{2c - b(m-2k) - i p}{2c}, \ldots, \frac{2c - b(m-2k) - i p}{2c}, 1; \frac{2c - b(m-2k) - i p}{2c} + 1, \ldots, \frac{2c - b(m-2k) - i p}{2c} + 1; -e^{2 i c z}\right) - e^{(p - i b (m-2k)) z} \sum_{j=0}^{n} \frac{(-1)^j (p - i b (m-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{-i p - b(m-2k)}{2c}, \ldots, \frac{-i p - b(m-2k)}{2c}, 1; \frac{-i p - b(m-2k)}{2c} + 1, \ldots, \frac{-i p - b(m-2k)}{2c} + 1; -e^{2 i c z}\right)\right) - e^{(b i (m-2k) + p) z} \sum_{j=0}^{n} \frac{(-1)^j (b i (m-2k) + p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{b(m-2k) - i p}{2c}, \ldots, \frac{b(m-2k) - i p}{2c}, 1; \frac{b(m-2k) - i p}{2c} + 1, \ldots, \frac{b(m-2k) - i p}{2c} + 1; -e^{2 i c z}\right) + e^{(2 i c + b i (m-2k) + p) z} \sum_{j=0}^{n} \frac{(-1)^j (2 i c + b i (m-2k) + p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{2c + b(m-2k) - i p}{2c}, \ldots, \frac{2c + b(m-2k) - i p}{2c}, 1; \frac{2c + b(m-2k) - i p}{2c} + 1, \ldots, \frac{2c + b(m-2k) - i p}{2c} + 1; -e^{2 i c z}\right)\right)\right) - i\, 2^{-m} \binom{m}{\frac{m}{2}} n! (1 - m \bmod 2) \left(e^{(2 i c + p) z} \sum_{j=0}^{n} \frac{(-1)^j (2 i c + p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{2c - i p}{2c}, \ldots, \frac{2c - i p}{2c}, 1; \frac{2c - i p}{2c} + 1, \ldots, \frac{2c - i p}{2c} + 1; -e^{2 i c z}\right) - e^{p z} \sum_{j=0}^{n} \frac{(-1)^j p^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(-\frac{i p}{2c}, \ldots, -\frac{i p}{2c}, 1; 1 - \frac{i p}{2c}, \ldots, 1 - \frac{i p}{2c}; -e^{2 i c z}\right)\right) /; n \in \mathbb{N} \wedge m \in \mathbb{N}^+$$

Involving cos, exp and power

Involving $z^n e^{p z} \cos(a + b z) \tan(c z)$

01.08.21.0077.01

$$\int z^n e^{p z} \cos(a + b z) \tan(c z) \, dz = -\frac{i}{2} e^{-i a} n! \left(e^{(2 i c - i b + p) z} \sum_{j=0}^{n} \frac{(-1)^j (2 i c - i b + p)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{-b + 2 c - i p}{2 c}, \ldots, \frac{-b + 2 c - i p}{2 c}, 1; \frac{-b + 2 c - i p}{2 c} + 1, \ldots, \frac{-b + 2 c - i p}{2 c} + 1; -e^{2 i c z} \right) - e^{(-i b + p) z} \sum_{j=0}^{n} \frac{(-1)^j (-i b + p)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{-b - i p}{2 c}, \ldots, \frac{-b - i p}{2 c}, 1; \frac{-b - i p}{2 c} + 1, \ldots, \frac{-b - i p}{2 c} + 1; -e^{2 i c z} \right) \right) - \frac{i}{2} e^{i a} n! \left(e^{(2 i c + i b + p) z} \sum_{j=0}^{n} \frac{(-1)^j (2 i c + i b + p)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{b + 2 c - i p}{2 c}, \ldots, \frac{b + 2 c - i p}{2 c}, 1; \frac{b + 2 c - i p}{2 c} + 1, \ldots, \frac{b + 2 c - i p}{2 c} + 1; -e^{2 i c z} \right) - e^{(i b + p) z} \sum_{j=0}^{n} \frac{(-1)^j (i b + p)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{b - i p}{2 c}, \ldots, \frac{b - i p}{2 c}, 1; \frac{b - i p}{2 c} + 1, \ldots, \frac{b - i p}{2 c} + 1; -e^{2 i c z} \right) \right) /; n \in \mathbb{N}$$

01.08.21.0078.01

$$\int z^n e^{p z} \cos(b z) \tan(c z) \, dz = \frac{i}{2} n! \left(-e^{(2 i c - i b + p) z} \sum_{j=0}^{n} \frac{(-1)^j (2 i c - i b + p)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{-b + 2 c - i p}{2 c}, \ldots, \frac{-b + 2 c - i p}{2 c}, 1; \frac{-b + 2 c - i p}{2 c} + 1, \ldots, \frac{-b + 2 c - i p}{2 c} + 1; -e^{2 i c z} \right) + e^{(-i b + p) z} \sum_{j=0}^{n} \frac{(-1)^j (-i b + p)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{-b - i p}{2 c}, \ldots, \frac{-b - i p}{2 c}, 1; \frac{-b - i p}{2 c} + 1, \ldots, \frac{-b - i p}{2 c} + 1; -e^{2 i c z} \right) - e^{(2 i c + i b + p) z} \sum_{j=0}^{n} \frac{(-1)^j (2 i c + i b + p)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{b + 2 c - i p}{2 c}, \ldots, \frac{b + 2 c - i p}{2 c}, 1; \frac{b + 2 c - i p}{2 c} + 1, \ldots, \frac{b + 2 c - i p}{2 c} + 1; -e^{2 i c z} \right) + e^{(i b + p) z} \sum_{j=0}^{n} \frac{(-1)^j (i b + p)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{b - i p}{2 c}, \ldots, \frac{b - i p}{2 c}, 1; \frac{b - i p}{2 c} + 1, \ldots, \frac{b - i p}{2 c} + 1; -e^{2 i c z} \right) \right) /; n \in \mathbb{N}$$

Involving powers of cos, exp and power

Involving $z^n e^{p z} \cos^m(b z) \tan(c z)$

01.08.21.0079.01

$$\int z^n e^{pz}\cos^m(bz)\tan(cz)\,dz = -2^{-m}\,i\,n!\left(\sum_{k=0}^{\left\lfloor\frac{m-1}{2}\right\rfloor}\binom{m}{k}\left(e^{(2ic-ib(m-2k)+p)z}\sum_{j=0}^{n}\frac{(-1)^j(2ic-ib(m-2k)+p)^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{2c-b(m-2k)-ip}{2c},\ldots,\frac{2c-b(m-2k)-ip}{2c},1;\frac{2c-b(m-2k)-ip}{2c}+1,\ldots,\frac{2c-b(m-2k)-ip}{2c}+1;-e^{2icz}\right)-e^{(p-ib(m-2k))z}\sum_{j=0}^{n}\frac{(-1)^j(p-ib(m-2k))^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{-ip-b(m-2k)}{2c},\ldots,\frac{-ip-b(m-2k)}{2c},1;\frac{-ip-b(m-2k)}{2c}+1,\ldots,\frac{-ip-b(m-2k)}{2c}+1;-e^{2icz}\right)-e^{(bi(m-2k)+p)z}\sum_{j=0}^{n}\frac{(-1)^j(bi(m-2k)+p)^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{b(m-2k)-ip}{2c},\ldots,\frac{b(m-2k)-ip}{2c},1;\frac{b(m-2k)-ip}{2c}+1,\ldots,\frac{b(m-2k)-ip}{2c}+1;-e^{2icz}\right)+e^{(2ic+bi(m-2k)+p)z}\sum_{j=0}^{n}\frac{(-1)^j(2ic+bi(m-2k)+p)^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{2c+b(m-2k)-ip}{2c},\ldots,\frac{2c+b(m-2k)-ip}{2c},1;\frac{2c+b(m-2k)-ip}{2c}+1,\ldots,\frac{2c+b(m-2k)-ip}{2c}+1;-e^{2icz}\right)\right)\right)-i\,2^{-m}\binom{m}{\frac{m}{2}}n!\,(1-m\bmod 2)\left(e^{(2ic+p)z}\sum_{j=0}^{n}\frac{(-1)^j(2ic+p)^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{2c-ip}{2c},\ldots,\frac{2c-ip}{2c},1;\frac{2c-ip}{2c}+1,\ldots,\frac{2c-ip}{2c}+1;-e^{2icz}\right)-e^{pz}\sum_{j=0}^{n}\frac{(-1)^j p^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(-\frac{ip}{2c},\ldots,-\frac{ip}{2c},1;1-\frac{ip}{2c},\ldots,1-\frac{ip}{2c};-e^{2icz}\right)\right)\;/;\;n\in\mathbb{N}\land m\in\mathbb{N}^+$$

Involving functions of the direct function

Involving powers of the direct function

Involving powers of tanh

Linear argument

01.08.21.0080.01

$$\int\tan^\nu(cz)\,dz = \frac{\tan^{\nu+1}(cz)}{\nu c+c}\,{}_2F_1\left(\frac{\nu+1}{2},1;\frac{\nu+3}{2};-\tan^2(cz)\right)$$

01.08.21.0081.01

$$\int\tan^2(cz)\,dz = \frac{\tan(cz)}{c}-z$$

01.08.21.0082.01

$$\int \tan^3(c\,z)\,dz = \frac{\sec^2(c\,z)}{2\,c} + \frac{\log(\cos(c\,z))}{c}$$

01.08.21.0083.01

$$\int \tan^4(c\,z)\,dz = \frac{\tan(c\,z)\sec^2(c\,z)}{3\,c} + z - \frac{4\tan(c\,z)}{3\,c}$$

01.08.21.0084.01

$$\int \tan^5(c\,z)\,dz = \frac{\sec^4(c\,z)}{4\,c} - \frac{\sec^2(c\,z)}{c} - \frac{\log(\cos(c\,z))}{c}$$

01.08.21.0085.01

$$\int \tan^6(c\,z)\,dz = \frac{\tan(c\,z)\sec^4(c\,z)}{5\,c} - \frac{11\tan(c\,z)\sec^2(c\,z)}{15\,c} - z + \frac{23\tan(c\,z)}{15\,c}$$

01.08.21.0086.01

$$\int \tan^7(c\,z)\,dz = \frac{\sec^6(c\,z)}{6\,c} - \frac{3\sec^4(c\,z)}{4\,c} + \frac{3\sec^2(c\,z)}{2\,c} + \frac{\log(\cos(c\,z))}{c}$$

01.08.21.0087.01

$$\int \tan^8(c\,z)\,dz = \frac{\tan(c\,z)\sec^6(c\,z)}{7\,c} - \frac{22\tan(c\,z)\sec^4(c\,z)}{35\,c} + \frac{122\tan(c\,z)\sec^2(c\,z)}{105\,c} + z - \frac{176\tan(c\,z)}{105\,c}$$

01.08.21.0234.01

$$\int \tan^{2n}(a\,z)\,dz = (-1)^n\,z + \frac{(-1)^n\cot(a\,z)}{a}\sum_{k=1}^{n}\frac{(-1)^k\tan^{2k}(a\,z)}{2\,k-1} \;/;\; n \in \mathbb{N}$$

01.08.21.0235.01

$$\int \tan^{2n+1}(a\,z)\,dz = -\frac{(-1)^n\log(\cos(a\,z))}{a} + \frac{S_{n+1}^{(2)}}{2\,a\,n!} + \frac{(-1)^n}{2\,a}\sum_{k=1}^{n}\frac{(-1)^k\tan^{2k}(a\,z)}{k} \;/;\; n \in \mathbb{N}$$

01.08.21.0236.01

$$\int \tan^{2n}(a\,z)\,dz = \frac{(-1)^n\left(a\,z - \tan^{-1}(\tan(a\,z))\right)}{a} + \frac{\tan^{2n+1}(a\,z)}{a\,(2\,n+1)}\,{}_2F_1\!\left(1,\, n+\frac{1}{2};\, n+\frac{3}{2};\, -\tan^2(a\,z)\right) /;\; n \in \mathbb{N}$$

01.08.21.0237.01

$$\int \tan^{2n+1}(a\,z)\,dz = \frac{\tan^{2n+2}(a\,z)}{2\,a\,(n+1)}\,{}_2F_1\!\left(n+1,\, 1;\, n+2;\, -\tan^2(a\,z)\right) + \frac{S_{n+1}^{(2)}}{2\,a\,n!} - \frac{(-1)^n\left(2\log(\cos(a\,z)) + \log\left(\sec^2(a\,z)\right)\right)}{2\,a} \;/;\; n \in \mathbb{N}$$

01.08.21.0088.01

$$\int \tan^{\frac{1}{2}}(c\,z)\,dz = \frac{1}{2\sqrt{2}\,c}\left(2\tan^{-1}\left(\sqrt{2}\,\tan^{\frac{1}{2}}(c\,z)+1\right) - 2\tan^{-1}\left(1-\sqrt{2}\,\tan^{\frac{1}{2}}(c\,z)\right) + \log\left(-\tan(c\,z)+\sqrt{2}\,\tan^{\frac{1}{2}}(c\,z)-1\right) - \log\left(\tan(c\,z)+\sqrt{2}\,\tan^{\frac{1}{2}}(c\,z)+1\right)\right)$$

01.08.21.0089.01

$$\int \frac{1}{\tan^{\frac{1}{2}}(c\,z)}\,dz = \frac{1}{2\sqrt{2}\,c}\left(2\tan^{-1}\left(\sqrt{2}\,\tan^{\frac{1}{2}}(c\,z)+1\right) - 2\tan^{-1}\left(1-\sqrt{2}\,\tan^{\frac{1}{2}}(c\,z)\right) - \log\left(-\tan(c\,z)+\sqrt{2}\,\tan^{\frac{1}{2}}(c\,z)-1\right) + \log\left(\tan(c\,z)+\sqrt{2}\,\tan^{\frac{1}{2}}(c\,z)+1\right)\right)$$

01.08.21.0090.01

$$\int \frac{1}{\tan^{\frac{1}{3}}(c z)} \, dz = -\frac{1}{4 c} \left(2\sqrt{3} \tan^{-1}\left(2 \tan^{\frac{1}{3}}(c z) + \sqrt{3} \right) + 2\sqrt{3} \tan^{-1}\left(\sqrt{3} - 2 \tan^{\frac{1}{3}}(c z) \right) + \log\left(\sqrt{3} \tan^{\frac{1}{3}}(c z) - \tan^{\frac{2}{3}}(c z) - 1 \right) - 2 \log\left(\tan^{\frac{2}{3}}(c z) + 1 \right) + \log\left(\sqrt{3} \tan^{\frac{1}{3}}(c z) + \tan^{\frac{2}{3}}(c z) + 1 \right) \right)$$

Involving products of the direct functions

01.08.21.0091.01

$$\int \tan(b + a z) \tan(a z) \, dz = \frac{\cot(b) (\log(\cos(a z)) - \log(\cos(b + a z))) - a z}{a}$$

01.08.21.0092.01

$$\int \tan(b - a z) \tan(a z) \, dz = \frac{a z + \cot(b) (\log(\cos(a z)) - \log(\cos(b - a z)))}{a}$$

Involving powers of products of the direct function

01.08.21.0093.01

$$\int \sqrt{\tan(c z) \tan(2 c z)} \, dz = -\frac{\cos^{\frac{1}{2}}(2 c z) \csc(c z) \sqrt{\sec(2 c z) \sin^2(c z)}}{c} \tanh^{-1}\left(\frac{\sqrt{2} \cos(c z)}{\cos^{\frac{1}{2}}(2 c z)} \right)$$

Involving rational functions of the direct function

Involving $(a + b \tan(c z))^{-n}$

01.08.21.0094.01

$$\int \frac{1}{a + b \tan(c z)} \, dz = \frac{a c z + b \log(a \cos(c z) + b \sin(c z))}{c a^2 + b^2 c}$$

01.08.21.0095.01

$$\int \frac{1}{(a + b \tan(c z))^2} \, dz = \left(\left(\left(a^2 - b^2 \right) c z + 2 a b \log(a \cos(c z) + b \sin(c z)) \right) a^2 + b \left(c z a^3 + b a^2 + 2 b \log(a \cos(c z) + b \sin(c z)) a^2 - b^2 c z a + b^3 \right) \tan(c z) \right) \Big/ \left(a \left(a^2 + b^2 \right)^2 c (a + b \tan(c z)) \right)$$

01.08.21.0096.01

$$\int \frac{A + B \tan(z)}{(a + b \tan(z))^2} \, dz = \left(\sec(z) (a \cos(z) + b \sin(z)) \left(\frac{b \left(a^2 + b^2 \right) (A b - a B) \sin(z)}{a} + \left(A a^2 + 2 b B a - A b^2 \right) z (a \cos(z) + b \sin(z)) + \left(-B a^2 + 2 A b a + b^2 B \right) \log(a \cos(z) + b \sin(z)) (a \cos(z) + b \sin(z)) \right) (A + B \tan(z)) \right) \Big/ \left(\left(a^2 + b^2 \right)^2 (A \cos(z) + B \sin(z)) (a + b \tan(z))^2 \right)$$

01.08.21.0097.01

$$\int \frac{A + B\tan(z)}{(a + b\tan(z))^3}\,dz = \left(\sec^2(z)\,(a\cos(z) + b\sin(z))\left(\left(a^2 + b^2\right)(a B - A b)\,b^2 + 2\left(A a^3 + 3 b B a^2 - 3 A b^2 a - b^3 B\right) z\,(a\cos(z) + b\sin(z))^2 - 2\left(B a^3 - 3 A b a^2 - 3 b^2 B a + A b^3\right)\log(a\cos(z) + b\sin(z))\,(a\cos(z) + b\sin(z))^2 + \frac{2 b\left(a^2 + b^2\right)\left(-2 B a^2 + 3 A b a + b^2 B\right)\sin(z)\,(a\cos(z) + b\sin(z))}{a}\right)(A + B\tan(z))\right) \Big/ \left(2\left(a^2 + b^2\right)^3 (A\cos(z) + B\sin(z))\,(a + b\tan(z))^3\right)$$

01.08.21.0098.01

$$\int \frac{A + B\tan(z) + C\tan^2(z)}{(a + b\tan(z))^3}\,dz = \left(\sec(z)\,(a\cos(z) + b\sin(z))\left(2\left((A - C)\,a^3 + 3 b B a^2 + 3 b^2 (C - A)\,a - b^3 B\right) z\,(a\cos(z) + b\sin(z))^2 - 2\left(B a^3 + 3 b (C - A)\,a^2 - 3 b^2 B a + b^3 (A - C)\right)\log(a\cos(z) + b\sin(z))\,(a\cos(z) + b\sin(z))^2 - b\left(a^2 + b^2\right)\left(A b^2 + a\,(a C - b B)\right) + \frac{1}{a}\left(2\left(a^2 + b^2\right)\left(C a^3 - 2 b B a^2 + b^2 (3 A - 2 C)\,a + b^3 B\right)\sin(z)\,(a\cos(z) + b\sin(z))\right)\right)\left(C\tan^2(z) + B\tan(z) + A\right)\right) \Big/ \left(\left(a^2 + b^2\right)^3 (A + C + (A - C)\cos(2 z) + B\sin(2 z))\,(a + b\tan(z))^3\right)$$

Involving $\left(a + b\tan^2(c z)\right)^{-n}$

01.08.21.0099.01

$$\int \frac{1}{a + b\tan^2(c z)}\,dz = \frac{1}{a c - b c}\left(c z - \frac{\sqrt{b}}{\sqrt{a}}\tan^{-1}\left(\frac{\sqrt{b}\,\tan(c z)}{\sqrt{a}}\right)\right)$$

01.08.21.0100.01

$$\int \frac{1}{\left(a + b\tan^2(c z)\right)^2}\,dz = \left((a + b + (a - b)\cos(2 c z))\sec^4(c z)\left(\sqrt{a}\,(2 a (a + b)\,c z + 2 a (a - b)\,c\cos(2 c z)\,z - (a - b)\,b\sin(2 c z)) - \sqrt{b}\,(b - 3 a)\tan^{-1}\left(\frac{\sqrt{b}\,\tan(c z)}{\sqrt{a}}\right)(-a - b + (b - a)\cos(2 c z))\right)\right) \Big/ \left(8 a^{3/2} (a - b)^2 c\left(b\tan^2(c z) + a\right)^2\right)$$

Involving algebraic functions of the direct function

Involving $(a + b\tan(c z))^{\beta}$

01.08.21.0101.01

$$\int (a+b\tan(c z))^{\beta}\, dz = -\frac{i\,(a+b\tan(c z))^{\beta+1}}{2\,(-a-i\,b)\,(a-i\,b)\,c\,(\beta+1)}\left((a-i\,b)\,{}_2F_1\!\left(\beta+1,1;\beta+2;\frac{a+b\tan(c z)}{a+i\,b}\right)+(-a-i\,b)\,{}_2F_1\!\left(\beta+1,1;\beta+2;\frac{a+b\tan(c z)}{a-i\,b}\right)\right)$$

01.08.21.0102.01

$$\int \sqrt{a+b\tan(c z)}\, dz = -\frac{i}{c}\left(\sqrt{a-i\,b}\,\tanh^{-1}\!\left(\frac{\sqrt{a+b\tan(c z)}}{\sqrt{a-i\,b}}\right)-\sqrt{a+i\,b}\,\tanh^{-1}\!\left(\frac{\sqrt{a+b\tan(c z)}}{\sqrt{a+i\,b}}\right)\right)$$

01.08.21.0103.01

$$\int \frac{1}{\sqrt{a+b\tan(c z)}}\, dz = -\frac{i}{c}\left(\frac{\tanh^{-1}\!\left(\frac{\sqrt{a+b\tan(c z)}}{\sqrt{a-i\,b}}\right)}{\sqrt{a-i\,b}}-\frac{\tanh^{-1}\!\left(\frac{\sqrt{a+b\tan(c z)}}{\sqrt{a+i\,b}}\right)}{\sqrt{a+i\,b}}\right)$$

01.08.21.0104.01

$$\int \tan(c z)\,(a+b\tan(c z))^{\beta}\, dz = -\frac{(a+b\tan(c z))^{\beta+1}}{2\,(a-i\,b)\,(a+i\,b)\,c\,(\beta+1)}\left((a-i\,b)\,{}_2F_1\!\left(\beta+1,1;\beta+2;\frac{a+b\tan(c z)}{a+i\,b}\right)+(a+i\,b)\,{}_2F_1\!\left(\beta+1,1;\beta+2;\frac{a+b\tan(c z)}{a-i\,b}\right)\right)$$

01.08.21.0105.01

$$\int \tan(c z)\sqrt{a+b\tan(c z)}\, dz = -\frac{\sqrt{a+i\,b}\,\tanh^{-1}\!\left(\frac{\sqrt{a+b\tan(c z)}}{\sqrt{a+i\,b}}\right)+\sqrt{a-i\,b}\,\tanh^{-1}\!\left(\frac{\sqrt{a+b\tan(c z)}}{\sqrt{a-i\,b}}\right)-2\sqrt{a+b\tan(c z)}}{c}$$

01.08.21.0106.01

$$\int \frac{\tan(c z)}{\sqrt{a+b\tan(c z)}}\, dz = -\frac{1}{c}\left(\frac{\tanh^{-1}\!\left(\frac{\sqrt{a+b\tan(c z)}}{\sqrt{a+i\,b}}\right)}{\sqrt{a+i\,b}}+\frac{\tanh^{-1}\!\left(\frac{\sqrt{a+b\tan(c z)}}{\sqrt{a-i\,b}}\right)}{\sqrt{a-i\,b}}\right)$$

Involving $((a+b\tan(c z))^n)^{\beta}$

01.08.21.0107.01

$$\int ((a+b\tan(c z))^n)^{\beta}\, dz = \left(\left((a+i\,b)\,{}_2F_1\!\left(n\,\beta+1,1;n\,\beta+2;\frac{a+b\tan(c z)}{a-i\,b}\right)-(a-i\,b)\,{}_2F_1\!\left(n\,\beta+1,1;n\,\beta+2;\frac{a+b\tan(c z)}{a+i\,b}\right)\right)(a+b\tan(c z))\,((a+b\tan(c z))^n)^{\beta}\right)\Big/\left(2\,(a+i\,b)\,(b+i\,a)\,c\,(n\,\beta+1)\right)$$

01.08.21.0108.01

$$\int \sqrt{(a+b\tan(c z))^3}\, dz = -\Bigg(i\cos(c z)\sqrt{(a+b\tan(c z))^3}\Bigg(\sqrt{a+i b}\left(\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c z)}}{\sqrt{a-i b}}\right)\cos(c z)\sqrt{a+b\tan(c z)}\,(a-i b)^2 + 2 b i (a\cos(c z)+b\sin(c z))\sqrt{a-i b}\right) - \sqrt{a-i b}\,(a+i b)^2\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c z)}}{\sqrt{a+i b}}\right)\cos(c z)\sqrt{a+b\tan(c z)}\Bigg)\Bigg) \Big/ \left(\sqrt{a-i b}\sqrt{a+i b}\, c\,(a\cos(c z)+b\sin(c z))^2\right)$$

01.08.21.0109.01

$$\int \frac{1}{\sqrt{(a+b\tan(c z))^3}}\, dz = -\Bigg(i(a+b\tan(c z))\Bigg(\sqrt{a+i b}\left(-2 i\sqrt{a-i b}\, b + (a+i b)\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c z)}}{\sqrt{a-i b}}\right)\sqrt{a+b\tan(c z)}\right) - (a-i b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c z)}}{\sqrt{a+i b}}\right)\sqrt{a+b\tan(c z)}\Bigg)\Bigg) \Big/ \left((a-i b)^{3/2}(a+i b)^{3/2}\, c\sqrt{(a+b\tan(c z))^3}\right)$$

01.08.21.0110.01

$$\int \tan(c z)\left((a+b\tan(c z))^n\right)^\beta\, dz = -\left(\left((a-i b)\, {}_2F_1\left(n\beta+1, 1; n\beta+2; \frac{a+b\tan(c z)}{a+i b}\right) + (a+i b)\, {}_2F_1\left(n\beta+1, 1; n\beta+2; \frac{a+b\tan(c z)}{a-i b}\right)\right)(a+b\tan(c z))\left((a+b\tan(c z))^n\right)^\beta\right) \Big/ (2(a-i b)(a+i b)\, c\,(n\beta+1))$$

01.08.21.0111.01

$$\int \tan(c z)\sqrt{(a+b\tan(c z))^3}\, dz = -\Bigg(\sqrt{(a+b\tan(c z))^3}\Bigg(3\sqrt{a-i b}\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c z)}}{\sqrt{a+i b}}\right)\sqrt{a+b\tan(c z)}\,(a+i b)^2 + \left(3(a-i b)^2\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c z)}}{\sqrt{a-i b}}\right)\sqrt{a+b\tan(c z)} - 2\sqrt{a-i b}\left(4a^2+5b\tan(c z)\, a + b^2\tan^2(c z)\right)\right)\sqrt{a+i b}\Bigg)\Bigg) \Big/ \left(3\sqrt{a-i b}\sqrt{a+i b}\, c\,(a+b\tan(c z))^2\right)$$

01.08.21.0112.01

$$\int \frac{\tan(c z)}{\sqrt{(a+b\tan(c z))^3}}\, dz = \left((a+b\tan(c z))\left((a-i b)\left(1-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c z)}}{\sqrt{a+i b}}\right)\sqrt{a+b\tan(c z)}}{\sqrt{a+i b}}\right)+(a+i b)\left(1-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tan(c z)}}{\sqrt{a-i b}}\right)\sqrt{a+b\tan(c z)}}{\sqrt{a-i b}}\right)\right)\right) \Big/ \left((a-i b)(a+i b)\, c \sqrt{(a+b\tan(c z))^3}\right)$$

Involving $\left(a+b\tan^2(c z)\right)^\beta$

01.08.21.0113.01

$$\int \left(a+b\tan^2(c z)\right)^\beta dz = \frac{\tan(c z)\left(b\tan^2(c z)+a\right)^\beta \left(\frac{b\tan^2(c z)}{a}+1\right)^{-\beta}}{c} F_1\left(\frac{1}{2}; 1, -\beta; \frac{3}{2}; -\tan^2(c z), -\frac{b\tan^2(c z)}{a}\right)$$

01.08.21.0114.01

$$\int \sqrt{a+b\tan^2(c z)}\, dz = -\frac{1}{2\sqrt{a-b}\, c} i \left(-(a-b)\left(\log\left(\frac{4\left(a+b\, i\tan(c z)+\sqrt{a-b}\sqrt{b\tan^2(c z)+a}\right)}{(a-b)^{3/2}(i\tan(c z)+1)}\right)-\log\left(-\frac{4\left(a-i b\tan(c z)+\sqrt{a-b}\sqrt{b\tan^2(c z)+a}\right)}{(a-b)^{3/2}(i\tan(c z)-1)}\right)\right)-2\sqrt{a-b}\sqrt{-b}\log\left(2\left(\sqrt{-b}\, i\tan(c z)+\sqrt{b\tan^2(c z)+a}\right)\right)\right)$$

01.08.21.0115.01

$$\int \frac{1}{\sqrt{a+b\tan^2(c z)}}\, dz = \left(\sqrt{a+b+(a-b)\cos(2 c z)}\log\left(\sqrt{a+b+(a-b)\cos(2 c z)}+\sqrt{2}\sqrt{-(a-b)\sin^2(c z)}\right)\tan(c z)\right) \Big/ \left(\sqrt{2}\, c\sqrt{-(a-b)\sin^2(c z)}\sqrt{b\tan^2(c z)+a}\right)$$

01.08.21.0116.01

$$\int \tan(c z)\left(a+b\tan^2(c z)\right)^\beta dz = -\frac{\left(b\tan^2(c z)+a\right)^{\beta+1}}{2(a-b)\, c\,(\beta+1)}\, {}_2F_1\left(\beta+1, 1; \beta+2; \frac{b\tan^2(c z)+a}{a-b}\right)$$

01.08.21.0117.01

$$\int \tan(c z)\sqrt{a+b\tan^2(c z)}\, dz = -\frac{\sqrt{b\tan^2(c z)+a}}{c}\left(\frac{\sqrt{2}\sqrt{(a-b)\cos^2(c z)}\log\left(\sqrt{2}\sqrt{(a-b)\cos^2(c z)}+\sqrt{a+b+(a-b)\cos(2 c z)}\right)}{\sqrt{a+b+(a-b)\cos(2 c z)}}-1\right)$$

01.08.21.0118.01

$$\int \frac{\tan(c z)}{\sqrt{a + b \tan^2(c z)}} \, dz = -\frac{\sqrt{a + b + (a - b)\cos(2 c z)} \log\left(\sqrt{2} \sqrt{(a - b)\cos^2(c z)} + \sqrt{a + b + (a - b)\cos(2 c z)}\right)}{\sqrt{2} \, c \sqrt{(a - b)\cos^2(c z)} \sqrt{b \tan^2(c z) + a}}$$

Involving $\left(\left(a + b \tan^2(c z)\right)^n\right)^\beta$

01.08.21.0119.01

$$\int \left(\left(a + b \tan^2(c z)\right)^n\right)^\beta dz = \frac{\tan(c z) \left(\left(b \tan^2(c z) + a\right)^n\right)^\beta \left(\frac{b \tan^2(c z)}{a} + 1\right)^{-n \beta}}{c} F_1\left(\frac{1}{2}; 1, -n \beta; \frac{3}{2}; -\tan^2(c z), -\frac{b \tan^2(c z)}{a}\right)$$

01.08.21.0120.01

$$\int \sqrt{\left(a + b \tan^2(c z)\right)^3} \, dz = \frac{1}{2 c \left(b \tan^2(c z) + a\right)^{3/2}} \left(i \sqrt{\left(b \tan^2(c z) + a\right)^3} \left(\left(\log\left(\frac{4\left(a + b i \tan(c z) + \sqrt{a - b} \sqrt{b \tan^2(c z) + a}\right)}{(a - b)^{5/2} (i \tan(c z) + 1)}\right) - \log\left(-\frac{4\left(a - i b \tan(c z) + \sqrt{a - b} \sqrt{b \tan^2(c z) + a}\right)}{(a - b)^{5/2} (i \tan(c z) - 1)}\right)\right) (a - b)^{3/2} + \sqrt{-b} \, (3 a - 2 b) \log\left(2\left(\sqrt{-b} \, i \tan(c z) + \sqrt{b \tan^2(c z) + a}\right)\right) - i b \tan(c z) \sqrt{b \tan^2(c z) + a}\right)\right)$$

01.08.21.0121.01

$$\int \frac{1}{\sqrt{\left(a + b \tan^2(c z)\right)^3}} \, dz = -\left(i \sec^2(c z) \left(-i \sqrt{2} \csc(2 c z) \log\left(\sqrt{a + b + (a - b)\cos(2 c z)} + \sqrt{2} \sqrt{-(a - b)\sin^2(c z)}\right) \sqrt{-(a - b)\sin^2(c z)} \, (a + b + (a - b)\cos(2 c z))^{3/2} - \frac{i (a - b) b \tan(c z) (a + b + (a - b)\cos(2 c z))}{a}\right)\right) \Big/ \left(2 (a - b)^2 c \sqrt{\left(b \tan^2(c z) + a\right)^3}\right)$$

01.08.21.0122.01

$$\int \tan(c z) \left(\left(a + b \tan^2(c z)\right)^n\right)^\beta dz = -\frac{\left(b \tan^2(c z) + a\right) \left(\left(b \tan^2(c z) + a\right)^n\right)^\beta}{2 (a - b) c (n \beta + 1)} \, {}_2F_1\left(n \beta + 1, 1; n \beta + 2; \frac{b \tan^2(c z) + a}{a - b}\right)$$

01.08.21.0123.01

$$\int \tan(c\,z)\sqrt{\left(a+b\tan^2(c\,z)\right)^5}\,dz = \frac{1}{15\,c}\left(2\cos^4(c\,z)\left(\frac{6\,b^2\sec^4(c\,z)+22\,(a-b)\,b\sec^2(c\,z)+46\,(a-b)^2}{(a+b+(a-b)\cos(2\,c\,z))^2}-\left(15\sqrt{2}\,\csc^2(2\,c\,z)\left(\log\left(-2\,(a-b)\sin^2(c\,z)\right)-\log\left(-a\sqrt{a+b+(a-b)\cos(2\,c\,z)}+b\sqrt{a+b+(a-b)\cos(2\,c\,z)}+(a-b)\cos(2\,c\,z)\sqrt{a+b+(a-b)\cos(2\,c\,z)}+\sqrt{2}\sqrt{(a-b)\sin^2(c\,z)}\sqrt{(a-b)^2\sin^2(2\,c\,z)}\right)\right)\left((a-b)^2\sin^2(2\,c\,z)\right)^{3/2}\right)\Big/\left((a+b+(a-b)\cos(2\,c\,z))^{5/2}\sqrt{(a-b)\sin^2(c\,z)}\right)\right)\sqrt{\left(b\tan^2(c\,z)+a\right)^5}\right)$$

01.08.21.0124.01

$$\int \tan(c\,z)\sqrt{\left(a+b\tan^2(c\,z)\right)^3}\,dz = \left(\csc^2(c\,z)\sec^2(c\,z)\left(4\,(a-b)\left(4\sqrt{a+b+(a-b)\cos(2\,c\,z)}-3\sqrt{2}\sqrt{(a-b)\cos^2(c\,z)}\log\left(\sqrt{2}\sqrt{(a-b)\cos^2(c\,z)}+\sqrt{a+b+(a-b)\cos(2\,c\,z)}\right)\right)\sin^2(c\,z)\cos^4(c\,z)+b\sqrt{a+b+(a-b)\cos(2\,c\,z)}\sin^2(2\,c\,z)\right)\sqrt{\left(b\tan^2(c\,z)+a\right)^3}\right)\Big/\left(6\,c\,(a+b+(a-b)\cos(2\,c\,z))^{3/2}\right)$$

01.08.21.0125.01

$$\int \frac{\tan(c\,z)}{\sqrt{\left(a+b\tan^2(c\,z)\right)^3}}\,dz = \left((a+b+(a-b)\cos(2\,c\,z))\left(a-b+(a-b)\cos(2\,c\,z)-\sqrt{2}\sqrt{(a-b)\cos^2(c\,z)}\sqrt{a+b+(a-b)\cos(2\,c\,z)}\log\left(\sqrt{2}\sqrt{(a-b)\cos^2(c\,z)}+\sqrt{a+b+(a-b)\cos(2\,c\,z)}\right)\right)\sec^4(c\,z)\right)\Big/\left(4\,(a-b)^2\,c\sqrt{\left(b\tan^2(c\,z)+a\right)^3}\right)$$

01.08.21.0126.01

$$\int \frac{\tan(c z)}{\sqrt{\left(a+b\tan^2(c z)\right)^5}}\, dz = \left((a+b+(a-b)\cos(2 c z))\left(6 a^2-6 b a-3\sqrt{2}\sqrt{(a-b)\cos^2(c z)}\sqrt{a+b+(a-b)\cos(2 c z)}\log\left(\sqrt{2}\sqrt{(a-b)\cos^2(c z)}+\sqrt{a+b+(a-b)\cos(2 c z)}\right) a+2(a-b)^2\cos(4 c z)-3\sqrt{2}\, b\sqrt{(a-b)\cos^2(c z)}\sqrt{a+b+(a-b)\cos(2 c z)}\log\left(\sqrt{2}\sqrt{(a-b)\cos^2(c z)}+\sqrt{a+b+(a-b)\cos(2 c z)}\right)+(a-b)\cos(2 c z)\left(8 a-2 b-3\sqrt{2}\sqrt{(a-b)\cos^2(c z)}\sqrt{a+b+(a-b)\cos(2 c z)}\log\left(\sqrt{2}\sqrt{(a-b)\cos^2(c z)}+\sqrt{a+b+(a-b)\cos(2 c z)}\right)\right)\right)\sec^6(c z)\right)\Big/\left(24 (a-b)^3 c\sqrt{\left(b\tan^2(c z)+a\right)^5}\right)$$

Involving $\left(a+b\tan^{\frac{1}{2}}(c z)\right)^{\beta}$

01.08.21.0127.01

$$\int \left(a+b\tan^{\frac{1}{2}}(c z)\right)^{\beta} dz = \frac{1}{2\left(a^4+b^4\right) c\,(\beta+1)}\left(i\left(-{}_2F_1\left(\beta+1,1;\beta+2;\frac{a+b\tan^{\frac{1}{2}}(c z)}{a+(-1)^{3/4} b}\right) a^3-{}_2F_1\left(\beta+1,1;\beta+2;\frac{a+b\tan^{\frac{1}{2}}(c z)}{a-(-1)^{3/4} b}\right) a^3+(-1)^{3/4} b\,{}_2F_1\left(\beta+1,1;\beta+2;\frac{a+b\tan^{\frac{1}{2}}(c z)}{a+(-1)^{3/4} b}\right) a^2-(-1)^{3/4} b\,{}_2F_1\left(\beta+1,1;\beta+2;\frac{a+b\tan^{\frac{1}{2}}(c z)}{a-(-1)^{3/4} b}\right) a^2+b^2 i\,{}_2F_1\left(\beta+1,1;\beta+2;\frac{a+b\tan^{\frac{1}{2}}(c z)}{a+(-1)^{3/4} b}\right) a+b^2 i\,{}_2F_1\left(\beta+1,1;\beta+2;\frac{a+b\tan^{\frac{1}{2}}(c z)}{a-(-1)^{3/4} b}\right) a+\left(a^3-\sqrt[4]{-1}\, b a^2+b^2 i a-(-1)^{3/4} b^3\right){}_2F_1\left(\beta+1,1;\beta+2;\frac{a+b\tan^{\frac{1}{2}}(c z)}{a+\sqrt[4]{-1}\, b}\right)+\sqrt[4]{-1}\, b^3\,{}_2F_1\left(\beta+1,1;\beta+2;\frac{a+b\tan^{\frac{1}{2}}(c z)}{a+(-1)^{3/4} b}\right)+\left(a^3+\sqrt[4]{-1}\, b a^2+b^2 i a+(-1)^{3/4} b^3\right){}_2F_1\left(\beta+1,1;\beta+2;\frac{a+b\tan^{\frac{1}{2}}(c z)}{a-\sqrt[4]{-1}\, b}\right)-\sqrt[4]{-1}\, b^3\,{}_2F_1\left(\beta+1,1;\beta+2;\frac{a+b\tan^{\frac{1}{2}}(c z)}{a-(-1)^{3/4} b}\right)\right)\left(a+b\tan^{\frac{1}{2}}(c z)\right)^{\beta+1}\right)$$

01.08.21.0128.01

$$\int \sqrt{a+b\tan^{\frac{1}{2}}(c\,z)}\,dz = \left(\sqrt{a-\sqrt[4]{-1}\,b}\left(\sqrt{a+(-1)^{3/4}\,b}\,\left((-1)^{3/4}\,b+i\,a\right)\sqrt{a-(-1)^{3/4}\,b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\tan^{\frac{1}{2}}(c\,z)}}{\sqrt{a+\sqrt[4]{-1}\,b}}\right)+\sqrt{a+\sqrt[4]{-1}\,b}\left(\left(\sqrt[4]{-1}\,b-i\,a\right)\sqrt{a-(-1)^{3/4}\,b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\tan^{\frac{1}{2}}(c\,z)}}{\sqrt{a+(-1)^{3/4}\,b}}\right)-i\left(a-(-1)^{3/4}\,b\right)\sqrt{a+(-1)^{3/4}\,b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\tan^{\frac{1}{2}}(c\,z)}}{\sqrt{a-(-1)^{3/4}\,b}}\right)\right)\right)+\sqrt{a+\sqrt[4]{-1}\,b}\,\sqrt{a+(-1)^{3/4}\,b}\,\left(a-\sqrt[4]{-1}\,b\right)\sqrt{a-(-1)^{3/4}\,b}\,i\tanh^{-1}\left(\frac{\sqrt{a+b\tan^{\frac{1}{2}}(c\,z)}}{\sqrt{a-\sqrt[4]{-1}\,b}}\right)\right)\Big/\left(\sqrt{a-\sqrt[4]{-1}\,b}\,\sqrt{a+\sqrt[4]{-1}\,b}\,\sqrt{a-(-1)^{3/4}\,b}\,\sqrt{a+(-1)^{3/4}\,b}\,c\right)$$

01.08.21.0129.01

$$\int \frac{1}{\sqrt{a+b\tan^{\frac{1}{2}}(c z)}}\, dz = \frac{1}{\left(a^4+b^4\right) c}\left(i\left(-\sqrt{a+(-1)^{3/4} b}\, \tanh^{-1}\left(\frac{\sqrt{a+b\tan^{\frac{1}{2}}(c z)}}{\sqrt{a+(-1)^{3/4} b}}\right) a^3-\sqrt{a-(-1)^{3/4} b}\, \tanh^{-1}\left(\frac{\sqrt{a+b\tan^{\frac{1}{2}}(c z)}}{\sqrt{a-(-1)^{3/4} b}}\right) a^3+\right.\right.$$

$$(-1)^{3/4} b\sqrt{a+(-1)^{3/4} b}\, \tanh^{-1}\left(\frac{\sqrt{a+b\tan^{\frac{1}{2}}(c z)}}{\sqrt{a+(-1)^{3/4} b}}\right) a^2-(-1)^{3/4} b\sqrt{a-(-1)^{3/4} b}\, \tanh^{-1}\left(\frac{\sqrt{a+b\tan^{\frac{1}{2}}(c z)}}{\sqrt{a-(-1)^{3/4} b}}\right) a^2+$$

$$b^2\sqrt{a+(-1)^{3/4} b}\, i\tanh^{-1}\left(\frac{\sqrt{a+b\tan^{\frac{1}{2}}(c z)}}{\sqrt{a+(-1)^{3/4} b}}\right) a+b^2\sqrt{a-(-1)^{3/4} b}\, i\tanh^{-1}\left(\frac{\sqrt{a+b\tan^{\frac{1}{2}}(c z)}}{\sqrt{a-(-1)^{3/4} b}}\right) a+$$

$$\sqrt{a+\sqrt[4]{-1}\, b}\,\left(a^3-\sqrt[4]{-1}\, b a^2+b^2 i a-(-1)^{3/4} b^3\right)\tanh^{-1}\left(\frac{\sqrt{a+b\tan^{\frac{1}{2}}(c z)}}{\sqrt{a+\sqrt[4]{-1}\, b}}\right)+$$

$$\sqrt[4]{-1}\, b^3\sqrt{a+(-1)^{3/4} b}\, \tanh^{-1}\left(\frac{\sqrt{a+b\tan^{\frac{1}{2}}(c z)}}{\sqrt{a+(-1)^{3/4} b}}\right)+\sqrt{a-\sqrt[4]{-1}\, b}\,\left(a^3+\sqrt[4]{-1}\, b a^2+b^2 i a+(-1)^{3/4} b^3\right)$$

$$\left.\left.\tanh^{-1}\left(\frac{\sqrt{a+b\tan^{\frac{1}{2}}(c z)}}{\sqrt{a-\sqrt[4]{-1}\, b}}\right)-\sqrt[4]{-1}\, b^3\sqrt{a-(-1)^{3/4} b}\, \tanh^{-1}\left(\frac{\sqrt{a+b\tan^{\frac{1}{2}}(c z)}}{\sqrt{a-(-1)^{3/4} b}}\right)\right)\right)$$

01.08.21.0130.01

$$\int \tan(c\,z)\left(a+b\tan^{\frac{1}{2}}(c\,z)\right)^{\beta} dz = -\frac{1}{2\left(a^4+b^4\right)c\,(\beta+1)}\left(\left({}_2F_1\left(\beta+1,\,1;\,\beta+2;\,\frac{a+b\tan^{\frac{1}{2}}(c\,z)}{a+(-1)^{3/4}\,b}\right)a^3+{}_2F_1\left(\beta+1,\,1;\,\beta+2;\,\frac{a+b\tan^{\frac{1}{2}}(c\,z)}{a-(-1)^{3/4}\,b}\right)a^3-\right.\right.$$
$$(-1)^{3/4}\,b\,{}_2F_1\left(\beta+1,\,1;\,\beta+2;\,\frac{a+b\tan^{\frac{1}{2}}(c\,z)}{a+(-1)^{3/4}\,b}\right)a^2+(-1)^{3/4}\,b\,{}_2F_1\left(\beta+1,\,1;\,\beta+2;\,\frac{a+b\tan^{\frac{1}{2}}(c\,z)}{a-(-1)^{3/4}\,b}\right)a^2-$$
$$i\,b^2\,{}_2F_1\left(\beta+1,\,1;\,\beta+2;\,\frac{a+b\tan^{\frac{1}{2}}(c\,z)}{a+(-1)^{3/4}\,b}\right)a-i\,b^2\,{}_2F_1\left(\beta+1,\,1;\,\beta+2;\,\frac{a+b\tan^{\frac{1}{2}}(c\,z)}{a-(-1)^{3/4}\,b}\right)a+$$
$$\left(a^3-\sqrt[4]{-1}\,b\,a^2+b^2\,i\,a-(-1)^{3/4}\,b^3\right){}_2F_1\left(\beta+1,\,1;\,\beta+2;\,\frac{a+b\tan^{\frac{1}{2}}(c\,z)}{a+\sqrt[4]{-1}\,b}\right)-$$
$$\sqrt[4]{-1}\,b^3\,{}_2F_1\left(\beta+1,\,1;\,\beta+2;\,\frac{a+b\tan^{\frac{1}{2}}(c\,z)}{a+(-1)^{3/4}\,b}\right)+\left(a^3+\sqrt[4]{-1}\,b\,a^2+b^2\,i\,a+(-1)^{3/4}\,b^3\right)$$
$$\left.\left.{}_2F_1\left(\beta+1,\,1;\,\beta+2;\,\frac{a+b\tan^{\frac{1}{2}}(c\,z)}{a-\sqrt[4]{-1}\,b}\right)+\sqrt[4]{-1}\,b^3\,{}_2F_1\left(\beta+1,\,1;\,\beta+2;\,\frac{a+b\tan^{\frac{1}{2}}(c\,z)}{a-(-1)^{3/4}\,b}\right)\right)\left(a+b\tan^{\frac{1}{2}}(c\,z)\right)^{\beta+1}\right)$$

01.08.21.0131.01

$$\int \tan(c\,z)\sqrt{a+b\tan^{\frac{1}{2}}(c\,z)}\,dz = \frac{1}{c}\Bigg(\Bigg(\sqrt{a+\sqrt[4]{-1}\,b}\,\Bigg(\sqrt{a-(-1)^{3/4}\,b}\,\Bigg(4\sqrt{a+(-1)^{3/4}\,b}\,\left(a+b\tan^{\frac{1}{2}}(c\,z)\right)-\left(a+(-1)^{3/4}\,b\right)\tanh^{-1}\!\left(\frac{\sqrt{a+b\tan^{\frac{1}{2}}(c\,z)}}{\sqrt{a+(-1)^{3/4}\,b}}\right)\sqrt{a+b\tan^{\frac{1}{2}}(c\,z)}\Bigg)-\left(a-(-1)^{3/4}\,b\right)\sqrt{a+(-1)^{3/4}\,b}\,\tanh^{-1}\!\left(\frac{\sqrt{a+b\tan^{\frac{1}{2}}(c\,z)}}{\sqrt{a-(-1)^{3/4}\,b}}\right)\sqrt{a+b\tan^{\frac{1}{2}}(c\,z)}\Bigg)-\left(a+\sqrt[4]{-1}\,b\right)\sqrt{a-(-1)^{3/4}\,b}\,\sqrt{a+(-1)^{3/4}\,b}\,\tanh^{-1}\!\left(\frac{\sqrt{a+b\tan^{\frac{1}{2}}(c\,z)}}{\sqrt{a+\sqrt[4]{-1}\,b}}\right)\sqrt{a+b\tan^{\frac{1}{2}}(c\,z)}\Bigg)\Bigg/\left(\sqrt{a+\sqrt[4]{-1}\,b}\,\sqrt{a-(-1)^{3/4}\,b}\,\sqrt{a+(-1)^{3/4}\,b}\,\sqrt{a+b\tan^{\frac{1}{2}}(c\,z)}\right)-\sqrt{a-\sqrt[4]{-1}\,b}\,\tanh^{-1}\!\left(\frac{\sqrt{a+b\tan^{\frac{1}{2}}(c\,z)}}{\sqrt{a-\sqrt[4]{-1}\,b}}\right)\Bigg)$$

01.08.21.0132.01

$$\int \frac{\tan(c z)}{\sqrt{a + b \tan^{\frac{1}{2}}(c z)}} \, dz = = -\frac{1}{(a^4 + b^4) c} \left(\sqrt{a + (-1)^{3/4} b} \tanh^{-1}\left(\frac{\sqrt{a + b \tan^{\frac{1}{2}}(c z)}}{\sqrt{a + (-1)^{3/4} b}} \right) a^3 + \sqrt{a - (-1)^{3/4} b} \tanh^{-1}\left(\frac{\sqrt{a + b \tan^{\frac{1}{2}}(c z)}}{\sqrt{a - (-1)^{3/4} b}} \right) a^3 - \right.$$

$$(-1)^{3/4} b \sqrt{a + (-1)^{3/4} b} \tanh^{-1}\left(\frac{\sqrt{a + b \tan^{\frac{1}{2}}(c z)}}{\sqrt{a + (-1)^{3/4} b}} \right) a^2 + (-1)^{3/4} b \sqrt{a - (-1)^{3/4} b} \tanh^{-1}\left(\frac{\sqrt{a + b \tan^{\frac{1}{2}}(c z)}}{\sqrt{a - (-1)^{3/4} b}} \right) a^2 -$$

$$i b^2 \sqrt{a + (-1)^{3/4} b} \tanh^{-1}\left(\frac{\sqrt{a + b \tan^{\frac{1}{2}}(c z)}}{\sqrt{a + (-1)^{3/4} b}} \right) a - i b^2 \sqrt{a - (-1)^{3/4} b} \tanh^{-1}\left(\frac{\sqrt{a + b \tan^{\frac{1}{2}}(c z)}}{\sqrt{a - (-1)^{3/4} b}} \right) a +$$

$$\sqrt{a + \sqrt[4]{-1} b} \left(a^3 - \sqrt[4]{-1} b a^2 + b^2 i a - (-1)^{3/4} b^3 \right) \tanh^{-1}\left(\frac{\sqrt{a + b \tan^{\frac{1}{2}}(c z)}}{\sqrt{a + \sqrt[4]{-1} b}} \right) -$$

$$\sqrt[4]{-1} b^3 \sqrt{a + (-1)^{3/4} b} \tanh^{-1}\left(\frac{\sqrt{a + b \tan^{\frac{1}{2}}(c z)}}{\sqrt{a + (-1)^{3/4} b}} \right) + \sqrt{a - \sqrt[4]{-1} b} \left(a^3 + \sqrt[4]{-1} b a^2 + b^2 i a + (-1)^{3/4} b^3 \right)$$

$$\left. \tanh^{-1}\left(\frac{\sqrt{a + b \tan^{\frac{1}{2}}(c z)}}{\sqrt{a - \sqrt[4]{-1} b}} \right) + \sqrt[4]{-1} b^3 \sqrt{a - (-1)^{3/4} b} \tanh^{-1}\left(\frac{\sqrt{a + b \tan^{\frac{1}{2}}(c z)}}{\sqrt{a - (-1)^{3/4} b}} \right) \right)$$

01.08.21.0133.01

$$\int \frac{\tan(c\,z)}{\left(a + b\tan^{\frac{1}{2}}(c\,z)\right)^2}\,dz = \frac{1}{2\left(a^4+b^4\right)^2 c}\left(\frac{4\left(a^4+b^4\right)a^3}{a + b\tan^{\frac{1}{2}}(c\,z)} - \sqrt{2}\,b\left(a^4 - 2b^2a^2 - b^4\right)\log\left(-\tan(c\,z) + \sqrt{2}\,\tan^{\frac{1}{2}}(c\,z) - 1\right)a + \sqrt{2}\,b\left(a^4 - 2b^2a^2 - b^4\right)\log\left(\tan(c\,z) + \sqrt{2}\,\tan^{\frac{1}{2}}(c\,z) + 1\right)a - 2b\left(-\sqrt{2}\,a^5 - 3ba^4 - 2\sqrt{2}\,b^2a^3 + \sqrt{2}\,b^4a + b^5\right)\tan^{-1}\left(\sqrt{2}\,\tan^{\frac{1}{2}}(c\,z) + 1\right) - 2b\left(\sqrt{2}\,a^5 - 3ba^4 + 2\sqrt{2}\,b^2a^3 - \sqrt{2}\,b^4a + b^5\right)\tan^{-1}\left(1 - \sqrt{2}\,\tan^{\frac{1}{2}}(c\,z)\right) - 4\left(a^6 - 3a^2b^4\right)\log\left(a + b\tan^{\frac{1}{2}}(c\,z)\right) + \left(a^6 - 3a^2b^4\right)\log\left(\tan^2(c\,z)+1\right)\right)$$

Involving functions of the direct function and a power function

Involving powers of the direct function and a power function

Involving powers of tanh and power

Involving z^n and linear arguments

01.08.21.0134.01

$$\int z^n \tan^\nu(c\,z)\,dz = \frac{i^\nu z^{n+1}}{n+1} - i^\nu e^{2icz}\,\nu\,n!\sum_{j=0}^{n}\frac{(-1)^j\,2^{-j-1}\,(i\,c)^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+3}F_{j+2}\left(1,\ldots,1,\nu+1;2,\ldots,2;-e^{2icz}\right) + e^{ic\nu z}\binom{\nu}{\frac{\nu}{2}}n!\,(1-\nu \bmod 2)\sum_{j=0}^{n}\frac{(-1)^j\,(i\,c\,\nu)^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{\nu}{2},\ldots,\frac{\nu}{2},\nu;\frac{\nu}{2}+1,\ldots,\frac{\nu}{2}+1;-e^{2icz}\right) + i^\nu n!\sum_{s=1}^{\left\lfloor\frac{\nu-1}{2}\right\rfloor}(-1)^s\binom{\nu}{s}e^{2iscz}\sum_{j=0}^{n}\frac{(-1)^j\,(2\,i\,c\,s)^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(s,\ldots,s,\nu;s+1,\ldots,s+1;-e^{2icz}\right) + i^{-\nu}n!\sum_{s=0}^{\left\lfloor\frac{\nu-1}{2}\right\rfloor}(-1)^s\binom{\nu}{s}e^{2ic(\nu-s)z}\sum_{j=0}^{n}\frac{(-1)^j\,(2\,i\,c\,(\nu-s))^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\nu-s,\ldots,\nu-s,\nu;-s+\nu+1,\ldots,-s+\nu+1;-e^{2icz}\right)\,/;\,n\in\mathbb{N}\land\nu\in\mathbb{N}^+$$

01.08.21.0135.01

$$\int z\tan^{\nu}(c\,z)\,dz = \frac{i^{\nu} z^2}{2} + \frac{i^{-\nu} e^{2icvz}}{2\,c\,\nu}\left(\frac{{}_3F_2\left(\nu,\nu,\nu;\nu+1,\nu+1;-e^{2icz}\right)}{2\,c\,\nu} - i\,z\,{}_2F_1\left(\nu,\nu;\nu+1;-e^{2icz}\right)\right) - \frac{i^{\nu} e^{2icz}\,\nu}{2\,c}\left(\frac{{}_4F_3\left(1,1,1,\nu+1;2,2,2;-e^{2icz}\right)}{2\,c} - i\,z\,{}_3F_2\left(1,1,\nu+1;2,2;-e^{2icz}\right)\right) + \frac{e^{icvz}\,(1-\nu \bmod 2)}{c\,\nu}\binom{\nu}{\frac{\nu}{2}}\left(\frac{{}_3F_2\left(\frac{\nu}{2},\frac{\nu}{2},\nu;\frac{\nu}{2}+1,\frac{\nu}{2}+1;-e^{2icz}\right)}{c\,\nu} - i\,z\,{}_2F_1\left(\frac{\nu}{2},\nu;\frac{\nu}{2}+1;-e^{2icz}\right)\right) - \frac{1}{4c^2}\,i^{-\nu}\sum_{s=1}^{\left\lfloor\frac{\nu-1}{2}\right\rfloor}(-1)^s\binom{\nu}{s}\left(\frac{1}{s^2}\left((-1)^{\nu} e^{2icsz}\left(2\,i\,c\,s\,z\,{}_2F_1\left(s,\nu;s+1;-e^{2icz}\right) - {}_3F_2\left(s,s,\nu;s+1,s+1;-e^{2icz}\right)\right)\right) - \frac{1}{(s-\nu)^2}\left(e^{2ic(\nu-s)z}\left(2\,c\,i\,(s-\nu)\,z\,{}_2F_1\left(\nu,\nu-s;-s+\nu+1;-e^{2icz}\right) + {}_3F_2\left(\nu,\nu-s,\nu-s;-s+\nu+1,-s+\nu+1;-e^{2icz}\right)\right)\right)\right)\ /;\ \nu\in\mathbb{N}^+$$

01.08.21.0136.01

$$\int z\tan^2(c\,z)\,dz = -\frac{z^2}{2} + \frac{\tan(c\,z)\,z}{c} + \frac{\log(\cos(c\,z))}{c^2}$$

01.08.21.0137.01

$$\int z\tan^3(c\,z)\,dz = -\frac{c^2\,i\,z^2 - c\sec^2(c\,z)\,z - 2\,c\log\left(1+e^{2icz}\right)z + i\,\mathrm{Li}_2\left(-e^{2icz}\right) + \tan(c\,z)}{2\,c^2}$$

01.08.21.0138.01

$$\int z\tan^4(c\,z)\,dz = \frac{3\,c^2 z^2 - 8\,c\tan(c\,z)\,z - 8\log(\cos(c\,z)) + \sec^2(c\,z)\,(2\,c\,z\tan(c\,z) - 1)}{6\,c^2}$$

01.08.21.0139.01

$$\int z\tan^5(c\,z)\,dz = \frac{1}{12\,c^2}\left(3\,c\,z\sec^4(c\,z) - (12\,c\,z + \tan(c\,z))\sec^2(c\,z) + 6\,i\,\mathrm{Li}_2\left(-e^{2icz}\right) + 2\left(3\,c\,i\,z\left(c\,z + 2\,i\log\left(1+e^{2icz}\right)\right) + 5\tan(c\,z)\right)\right)$$

01.08.21.0140.01

$$\int z^2\tan^2(c\,z)\,dz = \frac{c\,z\left(-c\,z\,(3\,i + c\,z) + 6\log\left(1+e^{2icz}\right) + 3\,c\,z\tan(c\,z)\right) - 3\,i\,\mathrm{Li}_2\left(-e^{2icz}\right)}{3\,c^3}$$

01.08.21.0141.01

$$\int z^3\tan^3(c\,z)\,dz = \frac{1}{4\,c^4}\left(-i\,c^4 z^4 + 2\,c^3\sec^2(c\,z)\,z^3 + 4\,c^3\log\left(1+e^{2icz}\right)z^3 + 6\,c^2\,i\,z^2 - 6\,c^2\tan(c\,z)\,z^2 - 12\,c\log\left(1+e^{2icz}\right)z + 6\,c\,\mathrm{Li}_3\left(-e^{2icz}\right)z - 6\,i\left(c^2 z^2 - 1\right)\mathrm{Li}_2\left(-e^{2icz}\right) + 3\,i\,\mathrm{Li}_4\left(-e^{2icz}\right)\right)$$

Involving functions of the direct function and exponential function

Involving powers of the direct function and exponential function

Involving exp

Involving $e^{b z}$

01.08.21.0142.01

$$\int e^{b z} \tan^{\nu}(c z)\, dz = \frac{e^{b z} \left(1 - e^{2 i c z}\right)^{-\nu} \left(1 + e^{2 i c z}\right)^{\nu} \tan^{\nu}(c z)}{b} F_1\left(-\frac{i b}{2 c}; \nu, -\nu; 1 - \frac{i b}{2 c}; -e^{2 i c z}, e^{2 i c z}\right)$$

01.08.21.0143.01

$$\int e^{2 i c z} \tan^{\nu}(c z)\, dz = -\frac{i\, 2^{\nu-1} \left(1 - e^{2 i c z}\right)^{-\nu} \left(1 + e^{2 i c z}\right) \tan^{\nu}(c z)}{c (1 - \nu)} {}_2F_1\left(1 - \nu, -\nu; 2 - \nu; \frac{1}{2} \left(1 + e^{2 i c z}\right)\right)$$

01.08.21.0144.01

$$\int e^{i c z} \tan^2(c z)\, dz = \frac{i \left(e^{i c z} \left(3 + e^{2 i c z}\right) - 2 \left(1 + e^{2 i c z}\right) \tan^{-1}\left(e^{i c z}\right)\right)}{c \left(1 + e^{2 i c z}\right)}$$

01.08.21.0145.01

$$\int e^{2 i c z} \tan^2(c z)\, dz = \frac{i \left(-4 \log\left(1 + e^{2 i c z}\right) + e^{2 i c z} - \frac{4}{1 + e^{2 i c z}}\right)}{2 c}$$

01.08.21.0146.01

$$\int e^{2 i c z} \tan^4(c z)\, dz = -\frac{i \left(-24 \log\left(1 + e^{2 i c z}\right) \left(1 + e^{2 i c z}\right)^3 - 93 e^{2 i c z} - 63 e^{4 i c z} + 9 e^{6 i c z} + 3 e^{8 i c z} - 40\right)}{6 c \left(1 + e^{2 i c z}\right)^3}$$

01.08.21.0147.01

$$\int e^{-2 i c z} \tan^4(c z)\, dz = -\frac{i}{6 c} \left(\frac{8 e^{2 i c z} \left(9 + 12 e^{2 i c z} + 5 e^{4 i c z}\right)}{\left(1 + e^{2 i c z}\right)^3} - 3 e^{-2 i c z} + 24 \log\left(1 + e^{-2 i c z}\right)\right)$$

Involving functions of the direct function, exponential and a power functions

Involving powers of the direct function, exponential and a power functions

Involving exp and power

Involving $z^n e^{b z}$

01.08.21.0148.01

$$\int z^n e^{bz}\tan^\nu(cz)\,dz = e^{(b+icv)z}\binom{\nu}{\frac{\nu}{2}} n!\,(1-\nu \bmod 2)\sum_{j=0}^{n}\frac{(-1)^j (b+icv)^{-j-1} z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{cv-ib}{2c},\ldots,\frac{cv-ib}{2c},\nu;\frac{cv-ib}{2c}+1,\ldots,\frac{cv-ib}{2c}+1;-e^{2icz}\right)+$$

$$i^{-\nu} n!\sum_{s=0}^{\left\lfloor\frac{\nu-1}{2}\right\rfloor}(-1)^s\binom{\nu}{s}\left((-1)^\nu e^{(b+2ics)z}\sum_{j=0}^{n}\frac{(-1)^j (b+2ics)^{-j-1} z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{-ib+2cs}{2c},\ldots,\frac{-ib+2cs}{2c},\nu;\frac{-ib+2cs}{2c}+1,\ldots,\frac{-ib+2cs}{2c}+1;-e^{2icz}\right)+\right.$$

$$\left.e^{(b+2ic(\nu-s))z}\sum_{j=0}^{n}\frac{(-1)^j (b+2ic(\nu-s))^{-j-1} z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{-ib+2c(-s+\nu)}{2c},\ldots,\frac{-ib+2c(-s+\nu)}{2c},\nu;\frac{-ib+2c(-s+\nu)}{2c}+1,\ldots,\frac{-ib+2c(-s+\nu)}{2c}+1;-e^{2icz}\right)\right)\;/;\;n\in\mathbb{N}\land\nu\in\mathbb{N}^+$$

Involving functions of the direct function and trigonometric functions

Involving powers of the direct function and trigonometric functions

Involving sin

Involving $\sin(b z)$

01.08.21.0149.01

$$\int \sin(bz)\tan^\nu(cz)\,dz = -\frac{1}{2b}e^{-ibz}\left(1-e^{-2icz}\right)^{-\nu}\left(1+e^{-2icz}\right)^{\nu}\tan^\nu(cz)\left(F_1\left(\frac{b}{2c};-\nu,\nu;\frac{b}{2c}+1;e^{-2icz},-e^{-2icz}\right)+e^{2ibz}F_1\left(-\frac{b}{2c};-\nu,\nu;1-\frac{b}{2c};e^{-2icz},-e^{-2icz}\right)\right)$$

01.08.21.0150.01

$$\int \sin(cz)\tan^\nu(cz)\,dz = \frac{\cos(cz)\sin^2(cz)^{-\frac{\nu}{2}}\tan^\nu(cz)}{c\nu-c}\,{}_2F_1\left(\frac{1-\nu}{2},-\frac{\nu}{2};\frac{3-\nu}{2};\cos^2(cz)\right)$$

01.08.21.0151.01

$$\int \sin(cz)\tan^2(cz)\,dz = \frac{\cos(cz)+\sec(cz)}{c}$$

01.08.21.0152.01

$$\int \sin(cz)\tan^3(cz)\,dz = \frac{3\log\left(\cos\left(\frac{cz}{2}\right)-\sin\left(\frac{cz}{2}\right)\right)-3\log\left(\cos\left(\frac{cz}{2}\right)+\sin\left(\frac{cz}{2}\right)\right)+2\sin(cz)+\sec(cz)\tan(cz)}{2c}$$

01.08.21.0153.01

$$\int \sin(4cz)\tan^4(cz)\,dz = -\frac{-16\cos(2cz)+\cos(4cz)+8\left(\sec^2(cz)+8\log(\cos(cz))\right)}{4c}$$

Involving powers of sin

Involving $\sin^m(b z)$

01.08.21.0154.01

$$\int \sin^m(b z) \tan^\nu(c z)\, dz = \frac{1}{b} i^{1-m} 2^{-m} \left(1 - e^{-2 i c z}\right)^{-\nu} \left(1 + e^{-2 i c z}\right)^\nu \tan^\nu(c z) \sum_{k=0}^{\left\lfloor \frac{m-1}{2} \right\rfloor} \frac{(-1)^k}{2k - m} \binom{m}{k} e^{-i b (m-2k) z} \left(e^{2 i b (m-2k) z} F_1\left(-\frac{b(m-2k)}{2c}; -\nu, \nu; 1 - \frac{b(m-2k)}{2c}; e^{-2 i c z}, -e^{-2 i c z}\right) - (-1)^m F_1\left(\frac{b(m-2k)}{2c}; -\nu, \nu; \frac{b(m-2k)}{2c} + 1; e^{-2 i c z}, -e^{-2 i c z}\right)\right) + \frac{2^{-m} (1 - m \bmod 2) \tan^{\nu+1}(c z)}{c(\nu+1)} \binom{m}{\frac{m}{2}} {}_2F_1\left(\frac{\nu+1}{2}, 1; \frac{\nu+1}{2} + 1; -\tan^2(c z)\right) /; m \in \mathbb{N}^+$$

01.08.21.0155.01

$$\int \sin^\mu(c z) \tan^\nu(c z)\, dz = \frac{i \left(1 - e^{2 i c z}\right)^{-\mu-\nu} \left(1 + e^{2 i c z}\right)^\nu \sin^\mu(c z) \tan^\nu(c z)}{c \mu} F_1\left(-\frac{\mu}{2}; -\mu - \nu, \nu; \frac{2-\mu}{2}; e^{2 i c z}, -e^{2 i c z}\right)$$

01.08.21.0156.01

$$\int \sin^\mu(c z) \tan^\nu(c z)\, dz = -\frac{\cos(c z) \sin^{\mu+1}(c z) \sin^2(c z)^{\frac{1}{2}(-\mu-\nu-1)} \tan^\nu(c z)}{c(1-\nu)} {}_2F_1\left(\frac{1-\nu}{2}, \frac{1}{2}(-\mu-\nu+1); \frac{3-\nu}{2}; \cos^2(c z)\right)$$

01.08.21.0157.01

$$\int \sin^2(c z) \tan^2(c z)\, dz = \frac{-6 c z + \sin(2 c z) + 4 \tan(c z)}{4 c}$$

01.08.21.0158.01

$$\int \frac{\tan^3(c z)}{\sin^{\frac{1}{2}}(2 c z)}\, dz = \frac{\left(\sec^2(c z) - 3\right) \sin^{\frac{1}{2}}(2 c z) \tan(c z) - 3 E\left(\frac{\pi}{4} - c z \,\middle|\, 2\right)}{5 c}$$

01.08.21.0159.01

$$\int \sqrt{\sin^3(2 c z)} \tan^5(c z)\, dz = -\frac{1}{15 c \sin^{\frac{3}{2}}(2 c z)} \left(\sqrt{\sin^3(2 c z)} \left(231 E\left(\frac{\pi}{4} - c z \,\middle|\, 2\right) + \sin^{\frac{1}{2}}(2 c z) \left(5 \sin(2 c z) - 12 \left(\sec^2(c z) - 13\right) \tan(c z)\right)\right)\right)$$

Involving powers of products with sin

Involving $\sqrt{\sin^m(c z) \tan(c z)}$

01.08.21.0160.01

$$\int \sqrt{\sin(c z) \tan(c z)}\, dz = -\frac{2 \cot(c z) \sqrt{\sin(c z) \tan(c z)}}{c}$$

01.08.21.0161.01

$$\int \sqrt{\sin^4(c z) \tan(c z)}\, dz = -\frac{1}{8 c} \left(\csc^3(c z) \left(3 \sin^{-1}(\cos(c z) - \sin(c z)) + 3 \log\left(\cos(c z) + \sin(c z) + \sin^{\frac{1}{2}}(2 c z)\right) + 2 \sin(c z) \sin^{\frac{1}{2}}(2 c z)\right) \sin^{\frac{1}{2}}(2 c z) \sqrt{\sin^4(c z) \tan(c z)}\right)$$

Involving algebraic functions of sin

01.08.21.0162.01

$$\int \sqrt{a + b \sin(c z)}\, \tan^2(c z)\, dz = \frac{1}{c \sqrt{a + b \sin(c z)}} \left(3 \sqrt{\frac{a + b \sin(c z)}{a + b}}\, (a + b) E\left(\frac{1}{4} (\pi - 2 c z) \,\middle|\, \frac{2 b}{a + b}\right) + (a + b \sin(c z)) \tan(c z) - a F\left(\frac{1}{4} (\pi - 2 c z) \,\middle|\, \frac{2 b}{a + b}\right) \sqrt{\frac{a + b \sin(c z)}{a + b}}\right)$$

Involving cos

Involving cos(*b z*)

01.08.21.0163.01

$$\int \cos(b z) \tan^\nu(c z)\, dz = -\frac{1}{2 b}\, i\, e^{-i b z} \left(1 - e^{-2 i c z}\right)^{-\nu} \left(1 + e^{-2 i c z}\right)^{\nu} \tan^\nu(c z) \left(e^{2 i b z} F_1\left(-\frac{b}{2 c}; -\nu, \nu; 1 - \frac{b}{2 c}; e^{-2 i c z}, -e^{-2 i c z}\right) - F_1\left(\frac{b}{2 c}; -\nu, \nu; \frac{b}{2 c} + 1; e^{-2 i c z}, -e^{-2 i c z}\right)\right)$$

01.08.21.0164.01

$$\int \cos(c z) \tan^\nu(c z)\, dz = \frac{\cos^2(c z) \sin(c z) \sin^2(c z)^{-\frac{\nu}{2} - \frac{1}{2}} \tan^\nu(c z)}{c \nu - 2 c}\, {}_2F_1\left(1 - \frac{\nu}{2}, \frac{1}{2} - \frac{\nu}{2}; 2 - \frac{\nu}{2}; \cos^2(c z)\right)$$

01.08.21.0165.01

$$\int \cos(c z) \tan^2(c z)\, dz = \frac{1}{c} \left(2 \tanh^{-1}\left(\tan\left(\frac{c z}{2}\right)\right) - \sin(c z)\right)$$

01.08.21.0166.01

$$\int \cos(c z) \tan^3(c z)\, dz = \frac{\cos(c z) + \sec(c z)}{c}$$

01.08.21.0167.01

$$\int \cos(2 c z) \tan^3(c z)\, dz = \frac{-\sec^2(c z) + \cos(2 c z) - 6 \log(\cos(c z))}{2 c}$$

01.08.21.0168.01

$$\int \cos(4 c z) \tan^5(c z)\, dz = \frac{\sec^4(c z) - 20 \sec^2(c z) + 20 \cos(2 c z) - \cos(4 c z) - 100 \log(\cos(c z))}{4 c}$$

01.08.21.0169.01

$$\int \cos(5 c z) \tan^5(c z)\, dz = \frac{25 \sec^3(c z) - 450 \sec(c z) - 750 \cos(c z) + 50 \cos(3 c z) - 3 \cos(5 c z)}{15 c}$$

Involving powers of cos

Involving $\cos^m(b z)$

01.08.21.0170.01

$$\int \cos^m(b z) \tan^\nu(c z) \, dz = \frac{i}{b} 2^{-m} \left(1 + e^{-2 i c z}\right)^\nu \left(1 - e^{-2 i c z}\right)^{-\nu} \tan^\nu(c z) \sum_{k=0}^{\left\lfloor \frac{m-1}{2} \right\rfloor} \frac{1}{2 k - m} \binom{m}{k} e^{-i b (m-2 k) z} \left(e^{2 i b (m-2 k) z} F_1\left(-\frac{b (m-2 k)}{2 c}; -\nu, \nu; 1 - \frac{b (m-2 k)}{2 c}; e^{-2 i c z}, -e^{-2 i c z}\right) - F_1\left(\frac{b (m-2 k)}{2 c}; -\nu, \nu; \frac{b (m-2 k)}{2 c} + 1; e^{-2 i c z}, -e^{-2 i c z}\right)\right) + \frac{2^{-m} (1 - m \bmod 2) \tan^{\nu+1}(c z)}{c (\nu + 1)} \binom{m}{\frac{m}{2}} {}_2F_1\left(\frac{\nu+1}{2}, 1; \frac{\nu+1}{2} + 1; -\tan^2(c z)\right) /; m \in \mathbb{N}^+$$

01.08.21.0171.01

$$\int \cos^\mu(c z) \tan^\nu(c z) \, dz = \frac{i \cos^\mu(c z) \left(1 - e^{2 i c z}\right)^{-\nu} \tan^\nu(c z) \left(1 + e^{2 i c z}\right)^{\nu-\mu}}{c \mu} F_1\left(-\frac{\mu}{2}; -\nu, \nu - \mu; \frac{2-\mu}{2}; e^{2 i c z}, -e^{2 i c z}\right)$$

01.08.21.0172.01

$$\int \cos^\mu(c z) \tan^\nu(c z) \, dz = -\frac{\cos^{\mu+1}(c z) \sin(c z) \sin^2(c z)^{-\frac{\nu}{2}-\frac{1}{2}} \tan^\nu(c z)}{\mu c - \nu c + c} {}_2F_1\left(\frac{1}{2} (\mu - \nu + 1), \frac{1-\nu}{2}; \frac{1}{2} (\mu - \nu + 3); \cos^2(c z)\right)$$

01.08.21.0173.01

$$\int \cos(5 c z) \tan^5(c z) \, dz = \frac{25 \sec^3(c z) - 450 \sec(c z) - 750 \cos(c z) + 50 \cos(3 c z) - 3 \cos(5 c z)}{15 c}$$

01.08.21.0174.01

$$\int \cos^2(c z) \tan^2(c z) \, dz = \frac{z}{2} - \frac{\sin(2 c z)}{4 c}$$

01.08.21.0175.01

$$\int \sqrt{\cos^3(2 c z)} \tan^3(c z) \, dz = \frac{\sqrt{\cos^3(2 c z)}}{6 c \cos^{\frac{3}{2}}(2 c z)} \left(24 \tan^{-1}\left(\cos^{\frac{1}{2}}(2 c z)\right) + \cos^{\frac{1}{2}}(2 c z) \left(2 \cos(2 c z) - 3 \left(\sec^2(c z) + 6\right)\right)\right)$$

Involving powers of products with cos

Involving $\sqrt{\cos^m(c z) \tan(c z)}$

01.08.21.0176.01

$$\int \sqrt{\cos(c z) \tan(c z)} \, dz = -\frac{2}{c} E\left(\frac{1}{4} (\pi - 2 c z) \middle| 2\right)$$

01.08.21.0177.01

$$\int \sqrt{\cos^m(c\,z)\tan(c\,z)}\,dz = -\frac{\sqrt{\cos^{m-1}(c\,z)\sin(c\,z)}\,\sin(2\,c\,z)}{c\,(m+1)\sin^2(c\,z)^{3/4}}\,{}_2F_1\!\left(\frac{m+1}{4},\frac{1}{4};\frac{m+5}{4};\cos^2(c\,z)\right)$$

Involving algebraic functions of cos

Involving $(a+b\cos(2\,c\,z))^{\beta}$

01.08.21.0178.01

$$\int (a+b\cos(2\,c\,z))^{\beta}\tan^{\nu}(c\,z)\,dz = \frac{1}{\nu\,c+c}(a+b\cos(2\,c\,z))^{\beta}\sec^2(c\,z)^{\beta}\tan^{\nu+1}(c\,z)\left(1-\frac{(b-a)\tan^2(c\,z)}{a+b}\right)^{-\beta} F_1\!\left(\frac{\nu+1}{2};\beta+1,-\beta;\frac{\nu+3}{2};-\tan^2(c\,z),-\frac{(a-b)\tan^2(c\,z)}{a+b}\right)$$

01.08.21.0179.01

$$\int \sqrt{a+b\cos(2\,c\,z)}\,\tan^2(c\,z)\,dz = \frac{1}{c\sqrt{a+b\cos(2\,c\,z)}}\left(-2\sqrt{\frac{a+b\cos(2\,c\,z)}{a+b}}\,(a+b)\,E\!\left(c\,z\,\middle|\,\frac{2\,b}{a+b}\right)+\sqrt{\frac{a+b\cos(2\,c\,z)}{a+b}}\,(a+b)\,F\!\left(c\,z\,\middle|\,\frac{2\,b}{a+b}\right)+(a+b\cos(2\,c\,z))\tan(c\,z)\right)$$

01.08.21.0180.01

$$\int \sqrt{a-a\cos(2\,c\,z)}\,\tan^2(c\,z)\,dz = \frac{a\,(\cos(2\,c\,z)+3)\tan(c\,z)}{\sqrt{2}\,c\sqrt{a\sin^2(c\,z)}}$$

01.08.21.0181.01

$$\int \sqrt{\cos(2\,c\,z)\,a+a}\,\tan^2(c\,z)\,dz = -\frac{\sqrt{\cos(2\,c\,z)\,a+a}\,\sec(c\,z)}{c}\left(\log\!\left(\cos\!\left(\frac{c\,z}{2}\right)-\sin\!\left(\frac{c\,z}{2}\right)\right)-\log\!\left(\cos\!\left(\frac{c\,z}{2}\right)+\sin\!\left(\frac{c\,z}{2}\right)\right)+\sin(c\,z)\right)$$

01.08.21.0182.01

$$\int \sqrt{a+b\cos(2\,c\,z)}\,\tan^3(c\,z)\,dz = \left(2\sqrt{\frac{a+b\cos(2\,c\,z)}{(\cos(c\,z)+1)^2}}\left(\sqrt{a-b}\,\sqrt{(a+b\cos(2\,c\,z))\sec^4\!\left(\frac{c\,z}{2}\right)}\,\cos(c\,z)+2\,b\log\!\left(\frac{2\left((a-b)\tan^2\!\left(\frac{c\,z}{2}\right)+a-b+\sqrt{a-b}\,\sqrt{(a+b\cos(2\,c\,z))\sec^4\!\left(\frac{c\,z}{2}\right)}\right)}{\sqrt{a-b}\,\left(\tan^2\!\left(\frac{c\,z}{2}\right)-1\right)}\right)+\sqrt{a-b}\,\sqrt{(a+b\cos(2\,c\,z))\sec^4\!\left(\frac{c\,z}{2}\right)}\right)\cos^2\!\left(\frac{c\,z}{2}\right)+\sqrt{a-b}\,(a+b\cos(2\,c\,z))\sec^2(c\,z)-2\,(a-b)\tanh^{-1}\!\left(\frac{\sqrt{a+b\cos(2\,c\,z)}}{\sqrt{a-b}}\right)\sqrt{a+b\cos(2\,c\,z)}\right)\Big/\left(2\sqrt{a-b}\,c\sqrt{a+b\cos(2\,c\,z)}\right)$$

01.08.21.0183.01

$$\int \sqrt{a+b\cos(2cz)}\,\tan^4(cz)\,dz = \left(\left(-2a^2+2ba+3b^2+4\left(-a^2+ba+b^2\right)\cos(2cz)-(2a-3b)\,b\cos(4cz)\right)\tan(cz)\sec^2(cz)+2\sqrt{\frac{a+b\cos(2cz)}{a+b}}\left(7a^2-2ba-9b^2\right)E\!\left(cz\left|\frac{2b}{a+b}\right.\right)-8\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(2cz)}{a+b}}\,F\!\left(cz\left|\frac{2b}{a+b}\right.\right)\right)\Big/\left(6(a-b)\,c\sqrt{a+b\cos(2cz)}\right)$$

01.08.21.0184.01

$$\int \frac{\tan^2(cz)}{\sqrt{a+b\cos(2cz)}}\,dz = \frac{1}{(a-b)\,c\sqrt{a+b\cos(2cz)}}\left((a+b\cos(2cz))\tan(cz)-(a+b)\sqrt{\frac{a+b\cos(2cz)}{a+b}}\,E\!\left(cz\left|\frac{2b}{a+b}\right.\right)\right)$$

01.08.21.0185.01

$$\int \frac{\tan^3(cz)}{\sqrt{a+b\cos(2cz)}}\,dz = \frac{\sqrt{a-b}\,\sqrt{a+b\cos(2cz)}\,\sec^2(cz)-2a\tanh^{-1}\!\left(\frac{\sqrt{a+b\cos(2cz)}}{\sqrt{a-b}}\right)}{2(a-b)^{3/2}\,c}$$

01.08.21.0186.01

$$\int \frac{\tan^4(cz)}{\sqrt{a+b\cos(2cz)}}\,dz = \left(-\left(\left(2a^2+ba+b^2\right)\cos(2cz)+a\,(a+2b+b\cos(4cz))\right)\tan(cz)\sec^2(cz)+4a\sqrt{\frac{a+b\cos(2cz)}{a+b}}\,(a+b)\,E\!\left(cz\left|\frac{2b}{a+b}\right.\right)-\left(a^2-b^2\right)\sqrt{\frac{a+b\cos(2cz)}{a+b}}\,F\!\left(cz\left|\frac{2b}{a+b}\right.\right)\right)\Big/\left(3(a-b)^2\,c\sqrt{a+b\cos(2cz)}\right)$$

Involving $\cos(cz)\,(a+b\cos(2cz))^{\beta}$

01.08.21.0187.01

$$\int \cos(cz)\,(a+b\cos(2cz))^{\beta}\tan^{\nu}(cz)\,dz = \frac{1}{\nu c+c}\,(a+b\cos(2cz))^{\beta}\sec(cz)\sec^2(cz)^{\beta-\frac{1}{2}}\tan^{\nu+1}(cz)\left(1-\frac{(b-a)\tan^2(cz)}{a+b}\right)^{-\beta}F_1\!\left(\frac{\nu+1}{2};\beta+\frac{3}{2},-\beta;\frac{\nu+3}{2};-\tan^2(cz),-\frac{(a-b)\tan^2(cz)}{a+b}\right)$$

01.08.21.0188.01

$$\int \cos(cz)\sqrt{a+b\cos(2cz)}\,\tan^2(cz)\,dz = \frac{1}{4\sqrt{a-b}\,\sqrt{-b}\,c}\left(-4\sqrt{-b}\,(b-a)\tanh^{-1}\!\left(\frac{\sqrt{a-b}\,\sin(cz)}{\sqrt{a+b\cos(2cz)}}\right)-\sqrt{a-b}\left(\sqrt{2}\,(a-3b)\log\!\left(\sqrt{2}\,\sqrt{-b}\,\sin(cz)+\sqrt{a+b\cos(2cz)}\right)+2\sqrt{-b}\,\sqrt{a+b\cos(2cz)}\,\sin(cz)\right)\right)$$

01.08.21.0189.01

$$\int \frac{\cos(c\,z)\tan^2(c\,z)}{\sqrt{a+b\cos(2\,c\,z)}}\,dz = \frac{\tanh^{-1}\left(\frac{\sqrt{a-b}\,\sin(c\,z)}{\sqrt{a+b\cos(2\,c\,z)}}\right)}{\sqrt{a-b}\,c} - \frac{\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sin(c\,z)}{\sqrt{a+b\cos(2\,c\,z)}}\right)}{\sqrt{2}\,\sqrt{b}\,c}$$

Involving $\cos(2\,c\,z)\,(a+b\cos(2\,c\,z))^\beta$

01.08.21.0190.01

$$\int \cos(2\,c\,z)\,(a+b\cos(2\,c\,z))^\beta \tan^\nu(c\,z)\,dz = \frac{1}{c\,(\nu+1)\,(\nu+3)}\left((a+b\cos(2\,c\,z))^\beta \sec^2(c\,z)^\beta \tan^{\nu+1}(c\,z)\left(1-\frac{(b-a)\tan^2(c\,z)}{a+b}\right)^{-\beta}\left((\nu+3)\,F_1\left(\frac{\nu+1}{2};\beta+2,-\beta;\frac{\nu+3}{2};-\tan^2(c\,z),-\frac{(a-b)\tan^2(c\,z)}{a+b}\right)-(\nu+1)\,F_1\left(\frac{\nu+3}{2};\beta+2,-\beta;\frac{\nu+5}{2};-\tan^2(c\,z),-\frac{(a-b)\tan^2(c\,z)}{a+b}\right)\tan^2(c\,z)\right)\right)$$

01.08.21.0191.01

$$\int \cos(2\,c\,z)\sqrt{a+b\cos(2\,c\,z)}\,\tan^2(c\,z)\,dz = \frac{1}{6\,b\,c\sqrt{a+b\cos(2\,c\,z)}}\left(-2\sqrt{\frac{a+b\cos(2\,c\,z)}{a+b}}\left(a^2-8\,b\,a-9\,b^2\right)E\left(c\,z\,\middle|\,\frac{2\,b}{a+b}\right)+2\sqrt{\frac{a+b\cos(2\,c\,z)}{a+b}}\left(a^2-3\,b\,a-4\,b^2\right)F\left(c\,z\,\middle|\,\frac{2\,b}{a+b}\right)-b\,(a+b\cos(2\,c\,z))\sec(c\,z)\,(7\sin(c\,z)+\sin(3\,c\,z))\right)$$

01.08.21.0192.01

$$\int \cos(2\,c\,z)\sqrt{\cos(2\,c\,z)\,a+a}\,\tan^2(c\,z)\,dz = -\frac{\sqrt{\cos(2\,c\,z)\,a+a}\,\sec(c\,z)}{6\,c}\left(-6\log\left(\cos\left(\frac{c\,z}{2}\right)-\sin\left(\frac{c\,z}{2}\right)\right)+6\log\left(\cos\left(\frac{c\,z}{2}\right)+\sin\left(\frac{c\,z}{2}\right)\right)-9\sin(c\,z)+\sin(3\,c\,z)\right)$$

01.08.21.0193.01

$$\int \cos(2\,c\,z)\sqrt{a-a\cos(2\,c\,z)}\,\tan^2(c\,z)\,dz = \frac{a\,(-14\cos(2\,c\,z)+\cos(4\,c\,z)-27)\tan(c\,z)}{6\sqrt{2}\,c\sqrt{a\sin^2(c\,z)}}$$

Involving sin and cos

01.08.21.0194.01

$$\int \sin(c\,z)\,(a+b\cos(2\,c\,z))^\beta \tan^\nu(c\,z)\,dz = \frac{1}{c\,(\nu+2)}\,(a+b\cos(2\,c\,z))^\beta \sec(c\,z)\sec^2(c\,z)^{\beta-\frac{1}{2}} \tan^{\nu+2}(c\,z)\left(1-\frac{(b-a)\tan^2(c\,z)}{a+b}\right)^{-\beta} F_1\left(\frac{\nu+2}{2};\beta+\frac{3}{2},-\beta;\frac{\nu+4}{2};-\tan^2(c\,z),-\frac{(a-b)\tan^2(c\,z)}{a+b}\right)$$

01.08.21.0195.01

$$\int \sin(c z) \sqrt{a + b \cos(2 c z)} \tan^2(c z) \, dz = \frac{1}{4 c} \left(\frac{\sqrt{2} (a - 5 b) \log\left(\sqrt{2} \sqrt{b} \cos(c z) + \sqrt{a + b \cos(2 c z)}\right)}{\sqrt{b}} + (\cos(2 c z) + 5) \sqrt{a + b \cos(2 c z)} \sec(c z) \right)$$

01.08.21.0196.01

$$\int \frac{\sin(c z) \tan^2(c z)}{\sqrt{a + b \cos(2 c z)}} \, dz = \frac{\log\left(\sqrt{2} \sqrt{b} \cos(c z) + \sqrt{a + b \cos(2 c z)}\right)}{\sqrt{2} \sqrt{b} \, c} + \frac{\sqrt{a + b \cos(2 c z)} \sec(c z)}{a c - b c}$$

01.08.21.0197.01

$$\int \frac{\sin(c z) \tan^3(c z)}{\sqrt{a + b \cos(2 c z)}} \, dz = \frac{1}{2 (a - b)^{3/2} \sqrt{-b} \, c} \left(\sqrt{-b} (b - 3 a) \tanh^{-1}\left(\frac{\sqrt{a - b} \sin(c z)}{\sqrt{a + b \cos(2 c z)}} \right) + \sqrt{a - b} \left(\sqrt{2} (a - b) \log\left(\sqrt{2} \sqrt{-b} \sin(c z) + \sqrt{a + b \cos(2 c z)}\right) + \sqrt{-b} \sqrt{a + b \cos(2 c z)} \sec(c z) \tan(c z) \right) \right)$$

01.08.21.0198.01

$$\int \frac{\sin^2(c z) \tan^2(c z)}{\sqrt{a + b \cos(2 c z)}} \, dz = \left(\sqrt{\frac{a + b \cos(2 c z)}{a + b}} \left(a^2 - 2 b a - 3 b^2\right) E\left(c z \,\middle|\, \frac{2 b}{a + b}\right) - \left(a^2 - b^2\right) \sqrt{\frac{a + b \cos(2 c z)}{a + b}} F\left(c z \,\middle|\, \frac{2 b}{a + b}\right) + 2 b (a + b \cos(2 c z)) \tan(c z) \right) \Big/ \left(2 (a - b) b c \sqrt{a + b \cos(2 c z)} \right)$$

Involving rational functions of the direct function and trigonometric functions

Involving rational functions of sin

Involving $(a \sin(c z) + b \tan(c z))^{-n}$

01.08.21.0199.01

$$\int \frac{1}{a \sin(c z) + b \tan(c z)} \, dz = \frac{-(a + b) \log\left(\cos\left(\frac{c z}{2}\right)\right) + b \log(b + a \cos(c z)) + (a - b) \log\left(\sin\left(\frac{c z}{2}\right)\right)}{(a - b) (a + b) c}$$

01.08.21.0200.01

$$\int \frac{1}{(a \sin(c z) + b \tan(c z))^2} \, dz = \frac{1}{\left(a^2 - b^2\right)^2 c (b + a \cos(c z))} \left(\cot(c z) \left(a \sin(c z) \tan(c z) b^2 - (b + a \cos(c z)) \left(\left(a^2 + b^2\right) \cos(c z) - 2 a b \right) \sec(c z) + \frac{2 b \left(2 a^2 + b^2\right) (b + a \cos(c z)) \tan(c z)}{\sqrt{a^2 - b^2}} \tanh^{-1}\left(\frac{(b - a) \tan\left(\frac{c z}{2}\right)}{\sqrt{a^2 - b^2}} \right) \right) \right)$$

Involving rational functions of cosh

Involving $(a\cos(c z)+b\tan(c z))^{-n}$

01.08.21.0201.01

$$\int \frac{1}{a\cos(c z)+b\tan(c z)}\,dz = \frac{2\tan^{-1}\left(\frac{\sqrt{-4a^2-b^2}}{2a\sin(c z)-b}\right)}{\sqrt{-4a^2-b^2}\;c}$$

01.08.21.0202.01

$$\int \frac{1}{(a\cos(c z)+b\tan(c z))^2}\,dz = \frac{1}{4c\,(a\cos(c z)+b\tan(c z))^2}\left(\sec^2(c z)\,(\cos(2 c z)\,a+a+2b\sin(c z))\left(-\frac{2\cos(c z)\,(b-2a\sin(c z))}{4a^2+b^2}-\left(\sqrt{2}\left(4 i a^2+b\left(-i b+\sqrt{-4a^2-b^2}\right)\right)\tan^{-1}\left(\frac{2a+\left(-b-i\sqrt{-4a^2-b^2}\right)\tan\left(\frac{c z}{2}\right)}{\sqrt{2}\sqrt{b}\sqrt{b+i\sqrt{-4a^2-b^2}}}\right)(\cos(2 c z)\,a+a+2b\sin(c z))\right)\Big/\left(\sqrt{b}\left(-4a^2-b^2\right)^{3/2}\sqrt{b+i\sqrt{-4a^2-b^2}}\right)-\left(\sqrt{2}\left(b\left(i b+\sqrt{-4a^2-b^2}\right)-4 i a^2\right)\tan^{-1}\left(\frac{2a+\left(i b+\sqrt{-4a^2-b^2}\right) i\tan\left(\frac{c z}{2}\right)}{\sqrt{2}\sqrt{b}\sqrt{b-i\sqrt{-4a^2-b^2}}}\right)(\cos(2 c z)\,a+a+2b\sin(c z))\right)\Big/\left(\sqrt{b}\left(-4a^2-b^2\right)^{3/2}\sqrt{b-i\sqrt{-4a^2-b^2}}\right)\right)\right)$$

Involving algebraic functions of the direct function and trigonometric functions

Involving sin

Involving $\sin(c z)\,(a+b\tan(c z))^{\beta}$

01.08.21.0203.01

$$\int \sin(c z)\sqrt{a+b\tan(c z)}\,dz = -\frac{1}{\sqrt{i b-a}\,c}\left(\cos(c z)\left(\sqrt{i b-a}\sqrt{a+b\tan(c z)} - i F\left(i\sinh^{-1}\left(\frac{\sqrt{i b-a}}{\sqrt{a+b\tan(c z)}}\right)\middle|\frac{a+i b}{a-i b}\right)\sec(c z)\right.\right.$$
$$\left.\left.(a\cos(c z)+b\sin(c z))\sqrt{-\frac{i b(i\tan(c z)-1)}{a+b\tan(c z)}}\sqrt{-\frac{i b(i\tan(c z)+1)}{a+b\tan(c z)}}\right)\right)$$

01.08.21.0204.01

$$\int \frac{\sin(c z)}{\sqrt{a+b\tan(c z)}}\,dz = -\frac{1}{(a+i b)c}\left(\sqrt{-\frac{1}{a-i b}}\cos(c z)\right.$$
$$\left(\sqrt{-\frac{i b(i\tan(c z)+1)}{-a-i b}}(-a-i b)\sqrt{\frac{-i b-b\tan(c z)}{a-i b}}\, i E\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a-i b}}\sqrt{a+b\tan(c z)}\right)\middle|\frac{a-i b}{a+i b}\right)-\right.$$
$$i(-a-i b)F\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a-i b}}\sqrt{a+b\tan(c z)}\right)\middle|\frac{a-i b}{a+i b}\right)\sqrt{-\frac{i b(i\tan(c z)+1)}{-a-i b}}\sqrt{\frac{-i b-b\tan(c z)}{a-i b}}+$$
$$\left.\left.\sqrt{-\frac{1}{a-i b}}(b\tan(c z)-a)\sqrt{a+b\tan(c z)}\right)\right)$$

Involving $\sin(c z)\left(a+b\tan^2(c z)\right)^\beta$

01.08.21.0205.01

$$\int \sin(c z)\left(a+b\tan^2(c z)\right)^\beta dz = -\frac{\cos(c z)\left(b\tan^2(c z)+a\right)^{\beta+1}}{2(a-b)c(\beta+1)}\,{}_2F_1\left(\beta+1,\frac{3}{2};\beta+2;\frac{b\tan^2(c z)+a}{a-b}\right)\sqrt{-\frac{b\sec^2(c z)}{a-b}}$$

01.08.21.0206.01

$$\int \sin(c z)\sqrt{a+b\tan^2(c z)}\,dz =$$
$$\left(\sqrt{2}\cos(c z)\left(\sqrt{b}\log\left(\frac{2\left(\sqrt{b}\tan^2\left(\frac{c z}{2}\right)+\sqrt{b}+\sqrt{4 b\tan^2\left(\frac{c z}{2}\right)+a\left(\tan^2\left(\frac{c z}{2}\right)-1\right)^2}\right)}{b\left(\tan^2\left(\frac{c z}{2}\right)-1\right)}\right)-\cos^2\left(\frac{c z}{2}\right)\right.\right.$$
$$\left.\left.\sqrt{4 b\tan^2\left(\frac{c z}{2}\right)+a\left(\tan^2\left(\frac{c z}{2}\right)-1\right)^2}\right)\sqrt{b\tan^2(c z)+a}\right)\Big/\left(c(\cos(c z)+1)\sqrt{\frac{a+b+(a-b)\cos(2 c z)}{(\cos(c z)+1)^2}}\right)$$

01.08.21.0207.01

$$\int \frac{\sin(c z)}{\sqrt{a + b \tan^2(c z)}} \, dz = -\frac{((a+b)\sec(c z))}{2 (a-b) c \sqrt{a + b \tan^2(c z)}} \sqrt{\frac{(a-b)\cos(2 c z)}{a+b} + 1} \left(\sqrt{\frac{(a-b)\cos(2 c z)}{a+b} + 1} - 1 \right)$$

Involving cos

Involving $\cos(c z)(a + b \tan(c z))^{\beta}$

01.08.21.0208.01

$$\int \cos(c z)(a + b\tan(c z))^{\beta} \, dz = \frac{b \cos(c z)}{(a - i b)(a + i b) c (\beta+1)} \sqrt{\frac{b (i \tan(c z) + 1)}{b - i a}} \sqrt{-\frac{b (i + \tan(c z))}{a - i b}} (a + b \tan(c z))^{\beta+1} F_1\left(\beta+1; \frac{3}{2}, \frac{3}{2}; \beta+2; \frac{a + b\tan(c z)}{a + i b}, \frac{a + b \tan(c z)}{a - i b}\right)$$

01.08.21.0209.01

$$\int \cos(c z) \sqrt{a + b \tan(c z)} \, dz = \left(\cos(c z) \left(b \left(F\left(i \sinh^{-1}\left(\frac{\sqrt{-a - i b}}{\sqrt{a + b \tan(c z)}} \right) \middle| \frac{a - i b}{a + i b} \right) \sqrt{\frac{b (-i + \tan(c z))}{a + b \tan(c z)}} \sqrt{\frac{b (i + \tan(c z))}{a + b \tan(c z)}} (a + b \tan(c z))^2 + \sqrt{-a - i b} (a \tan(c z) - b) \sqrt{a + b \tan(c z)} \right) - (b - i a) E\left(i \sinh^{-1}\left(\frac{\sqrt{-a - i b}}{\sqrt{a + b \tan(c z)}} \right) \middle| \frac{a - i b}{a + i b} \right) \sqrt{\frac{b (-i + \tan(c z))}{a + b \tan(c z)}} \sqrt{\frac{b (i + \tan(c z))}{a + b \tan(c z)}} (a + b \tan(c z))^2 \right) \right) \Big/ \left(\sqrt{-a - i b} \, b c (a + b \tan(c z)) \right)$$

01.08.21.0210.01

$$\int \frac{\cos(c\,z)}{\sqrt{a+b\tan(c\,z)}}\,dz = \left(\cos(c\,z)\left(\frac{1}{\sqrt{\frac{b\,(1-i\tan(c\,z))}{b+i\,a}}}\left(\sec(c\,z)\,(i\cos(c\,z)+\sin(c\,z))\sqrt{\frac{a+b\tan(c\,z)}{a-i\,b}}\right.\right.\right.$$
$$\left(b\left(2\,a\sqrt{\frac{b\,(i\tan(c\,z)+1)}{b-i\,a}}\;i\,F\!\left(\sin^{-1}\!\left(\sqrt{\frac{a+b\tan(c\,z)}{a-i\,b}}\right)\middle|\,\frac{a-i\,b}{a+i\,b}\right)+b\,F\!\left(\sin^{-1}\!\left(\sqrt{-\frac{b\,(i+\tan(c\,z))}{a-i\,b}}\right)\middle|\,\frac{b+i\,a}{2\,b}\right)\right.\right.$$
$$\left.\sqrt{2\,i\tan(c\,z)+2}\right)-2\,a\,(a+i\,b)\,E\!\left(\sin^{-1}\!\left(\sqrt{\frac{a+b\tan(c\,z)}{a-i\,b}}\right)\middle|\,\frac{a-i\,b}{a+i\,b}\right)\sqrt{-\frac{b\,(-i+\tan(c\,z))}{a+i\,b}}\Bigg)\Bigg)+$$
$$\left.\left.2\,(b+a\tan(c\,z))\,(a+b\tan(c\,z))\right)\right)\Big/\left(2\left(a^2+b^2\right)c\sqrt{a+b\tan(c\,z)}\right)$$

Involving $\cos(c\,z)\left(a+b\tan^2(c\,z)\right)^{\beta}$

01.08.21.0211.01

$$\int \cos(c\,z)\left(a+b\tan^2(c\,z)\right)^{\beta}dz =$$
$$\frac{\sqrt{\sec^2(c\,z)}\,\sin(c\,z)\left(b\tan^2(c\,z)+a\right)^{\beta}}{c}\left(\frac{b\tan^2(c\,z)}{a}+1\right)^{-\beta}F_1\!\left(\frac{1}{2};\frac{3}{2},-\beta;\frac{3}{2};-\tan^2(c\,z),-\frac{b\tan^2(c\,z)}{a}\right)$$

01.08.21.0212.01

$$\int \cos(c\,z)\sqrt{a+b\tan^2(c\,z)}\,dz = \left(i\sqrt{2}\;a\sqrt{\frac{(b-a)\cos^2(c\,z)}{b}}\;\csc(c\,z)\right.$$
$$\left(E\!\left(i\sinh^{-1}\!\left(\frac{\sqrt{-\frac{1}{b}}\sqrt{a+b+(a-b)\cos(2\,c\,z)}}{\sqrt{2}}\right)\middle|\,\frac{b}{a}\right)-F\!\left(i\sinh^{-1}\!\left(\frac{\sqrt{-\frac{1}{b}}\sqrt{a+b+(a-b)\cos(2\,c\,z)}}{\sqrt{2}}\right)\middle|\,\frac{b}{a}\right)\right)$$
$$\left.\sqrt{\frac{(a-b)\sin^2(c\,z)}{a}}\sqrt{b\tan^2(c\,z)+a}\right)\Big/\left(\sqrt{-\frac{1}{b}}\,(b-a)\,c\sqrt{a+b+(a-b)\cos(2\,c\,z)}\right)$$

01.08.21.0213.01

$$\int \frac{\cos(c z)}{\sqrt{a + b \tan^2(c z)}} \, dz = \Bigg((\cos(c z) + 1) \sqrt{\frac{a + b + (a - b)\cos(2 c z)}{(\cos(c z) + 1)^2}}$$

$$\sec(c z) \left(\sqrt{-\frac{a - 2 b + 2\sqrt{b(b - a)}}{a}} \, (a + b + (a - b)\cos(2 c z)) \tan\left(\frac{c z}{2}\right) \sec^2\left(\frac{c z}{2}\right) - \right.$$

$$2\left(a - 2 b + 2\sqrt{b(b - a)}\right) i \, E\left(i \sinh^{-1}\left(\sqrt{-\frac{a}{a - 2\left(b + \sqrt{b(b - a)}\right)}} \tan\left(\frac{c z}{2}\right)\right) \middle| \frac{a - 2\left(b + \sqrt{b(b - a)}\right)}{a - 2 b + 2\sqrt{b(b - a)}} \right)$$

$$\sqrt{\frac{\left(b + \left(a - b + \sqrt{b(b - a)}\right)\cos(c z) - \sqrt{b(b - a)}\right)\sec^2\left(\frac{c z}{2}\right)}{a}}$$

$$\sqrt{\frac{\left(b - \left(-a + b + \sqrt{b(b - a)}\right)\cos(c z) + \sqrt{b(b - a)}\right)\sec^2\left(\frac{c z}{2}\right)}{a}} +$$

$$4\sqrt{b(b - a)} \; i \, F\left(i \sinh^{-1}\left(\sqrt{-\frac{a}{a - 2\left(b + \sqrt{b(b - a)}\right)}} \tan\left(\frac{c z}{2}\right)\right) \middle| \frac{a - 2\left(b + \sqrt{b(b - a)}\right)}{a - 2 b + 2\sqrt{b(b - a)}} \right)$$

$$\sqrt{\frac{\left(b + \left(a - b + \sqrt{b(b - a)}\right)\cos(c z) - \sqrt{b(b - a)}\right)\sec^2\left(\frac{c z}{2}\right)}{a}}$$

$$\left. \sqrt{\frac{\left(b - \left(-a + b + \sqrt{b(b - a)}\right)\cos(c z) + \sqrt{b(b - a)}\right)\sec^2\left(\frac{c z}{2}\right)}{a}} \right) \Bigg) \Bigg/$$

$$\left(2(a - b) \sqrt{-\frac{a}{a - 2\left(b + \sqrt{b(b - a)}\right)}} \; c \sqrt{(a + b + (a - b)\cos(2 c z))\sec^4\left(\frac{c z}{2}\right)} \sqrt{b \tan^2(c z) + a} \right)$$

Involving functions of the direct function, trigonometric and a power functions

Involving powers of the direct function, trigonometric and a power functions

Involving sin and power

Involving $z^n \sin(a + b z) \tan^v(c z)$

01.08.21.0214.01

$$\int z^n \sin(a+b\,z)\tan^\nu(c\,z)\,dz = \frac{i}{2}\,n!\,e^{-i\,a}\left(e^{(-i\,b+i\,c\,\nu)z}\binom{\nu}{\frac{\nu}{2}}(1-\nu \bmod 2)\sum_{j=0}^{n}\frac{(-1)^j\,(-i\,b+i\,c\,\nu)^{-j-1}\,z^{n-j}}{(n-j)!}\;{}_{j+2}F_{j+1}\left(\frac{c\,\nu-b}{2\,c},\,\ldots,\,\frac{c\,\nu-b}{2\,c},\,\nu;\,\frac{c\,\nu-b}{2\,c}+1,\,\ldots,\,\frac{c\,\nu-b}{2\,c}+1;\,-e^{2\,i\,c\,z}\right)+\sum_{s=0}^{\left\lfloor\frac{\nu-1}{2}\right\rfloor}(-1)^s\binom{\nu}{s}\left(i^\nu\,e^{(-i\,b+2\,i\,c\,s)z}\sum_{j=0}^{n}\frac{(-1)^j\,(-i\,b+2\,i\,c\,s)^{-j-1}\,z^{n-j}}{(n-j)!}\;{}_{j+2}F_{j+1}\left(\frac{-b+2\,c\,s}{2\,c},\,\ldots,\,\frac{-b+2\,c\,s}{2\,c},\,\nu;\,\frac{-b+2\,c\,s}{2\,c}+1,\,\ldots,\,\frac{-b+2\,c\,s}{2\,c}+1;\,-e^{2\,i\,c\,z}\right)+i^{-\nu}\,e^{(-i\,b+2\,i\,c\,(\nu-s))z}\sum_{j=0}^{n}\frac{(-1)^j\,(-i\,b+2\,i\,c\,(\nu-s))^{-j-1}\,z^{n-j}}{(n-j)!}\;{}_{j+2}F_{j+1}\left(\frac{-b+2\,c\,(-s+\nu)}{2\,c},\,\ldots,\,\frac{-b+2\,c\,(-s+\nu)}{2\,c},\,\nu;\,\frac{-b+2\,c\,(-s+\nu)}{2\,c}+1,\,\ldots,\,\frac{-b+2\,c\,(-s+\nu)}{2\,c}+1;\,-e^{2\,i\,c\,z}\right)\right)\right)-$$

$$\frac{i}{2}\,n!\,e^{i\,a}\left(e^{(i\,b+i\,c\,\nu)z}\binom{\nu}{\frac{\nu}{2}}(1-\nu \bmod 2)\sum_{j=0}^{n}\frac{(-1)^j\,(i\,b+i\,c\,\nu)^{-j-1}\,z^{n-j}}{(n-j)!}\;{}_{j+2}F_{j+1}\left(\frac{c\,\nu+b}{2\,c},\,\ldots,\,\frac{c\,\nu+b}{2\,c},\,\nu;\,\frac{c\,\nu+b}{2\,c}+1,\,\ldots,\,\frac{c\,\nu+b}{2\,c}+1;\,-e^{2\,i\,c\,z}\right)+\sum_{s=0}^{\left\lfloor\frac{\nu-1}{2}\right\rfloor}(-1)^s\binom{\nu}{s}\left(i^\nu\,e^{(i\,b+2\,i\,c\,s)z}\sum_{j=0}^{n}\frac{(-1)^j\,(i\,b+2\,i\,c\,s)^{-j-1}\,z^{n-j}}{(n-j)!}\;{}_{j+2}F_{j+1}\left(\frac{b+2\,c\,s}{2\,c},\,\ldots,\,\frac{b+2\,c\,s}{2\,c},\,\nu;\,\frac{b+2\,c\,s}{2\,c}+1,\,\ldots,\,\frac{b+2\,c\,s}{2\,c}+1;\,-e^{2\,i\,c\,z}\right)+i^{-\nu}\,e^{(i\,b+2\,i\,c\,(\nu-s))z}\sum_{j=0}^{n}\frac{(-1)^j\,(i\,b+2\,i\,c\,(\nu-s))^{-j-1}\,z^{n-j}}{(n-j)!}\;{}_{j+2}F_{j+1}\left(\frac{b+2\,c\,(\nu-s)}{2\,c},\,\ldots,\,\frac{b+2\,c\,(\nu-s)}{2\,c},\,\nu;\,\frac{b+2\,c\,(\nu-s)}{2\,c}+1,\,\ldots,\,\frac{b+2\,c\,(\nu-s)}{2\,c}+1;\,-e^{2\,i\,c\,z}\right)\right)\right)\,/;\,n\in\mathbb{N}\land\nu\in\mathbb{N}^+$$

01.08.21.0215.01

$$\int z^n \sin(b\,z)\tan^\nu(c\,z)\,dz = \frac{i\,n!}{2}\left(e^{(-i\,b+i\,c\,\nu)\,z}\binom{\nu}{\frac{\nu}{2}}(1-\nu \bmod 2)\right.$$

$$\sum_{j=0}^{n}\frac{(-1)^j\,(-i\,b+i\,c\,\nu)^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{c\,\nu-b}{2\,c},\,\ldots,\,\frac{c\,\nu-b}{2\,c},\,\nu;\,\frac{c\,\nu-b}{2\,c}+1,\,\ldots,\,\frac{c\,\nu-b}{2\,c}+1;\,-e^{2\,i\,c\,z}\right)+$$

$$\sum_{s=0}^{\left\lfloor\frac{\nu-1}{2}\right\rfloor}(-1)^s\binom{\nu}{s}\left(i^\nu\;e^{(-i\,b+2\,i\,c\,s)\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(-i\,b+2\,i\,c\,s)^{-j-1}\,z^{n-j}}{(n-j)!}\right.$$

$${}_{j+2}F_{j+1}\left(\frac{-b+2\,c\,s}{2\,c},\,\ldots,\,\frac{-b+2\,c\,s}{2\,c},\,\nu;\,\frac{-b+2\,c\,s}{2\,c}+1,\,\ldots,\,\frac{-b+2\,c\,s}{2\,c}+1;\,-e^{2\,i\,c\,z}\right)+$$

$$i^{-\nu}\,e^{(-i\,b+2\,i\,c\,(\nu-s))\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(-i\,b+2\,i\,c\,(\nu-s))^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{-b+2\,c\,(-s+\nu)}{2\,c},\,\ldots,\,\frac{-b+2\,c\,(-s+\nu)}{2\,c},\right.$$

$$\left.\left.\nu;\,\frac{-b+2\,c\,(-s+\nu)}{2\,c}+1,\,\ldots,\,\frac{-b+2\,c\,(-s+\nu)}{2\,c}+1;\,-e^{2\,i\,c\,z}\right)\right)-e^{(i\,b+i\,c\,\nu)\,z}\binom{\nu}{\frac{\nu}{2}}(1-\nu \bmod 2)$$

$$\sum_{j=0}^{n}\frac{(-1)^j\,(i\,b+i\,c\,\nu)^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{c\,\nu+b}{2\,c},\,\ldots,\,\frac{c\,\nu+b}{2\,c},\,\nu;\,\frac{c\,\nu+b}{2\,c}+1,\,\ldots,\,\frac{c\,\nu+b}{2\,c}+1;\,-e^{2\,i\,c\,z}\right)-$$

$$\sum_{s=0}^{\left\lfloor\frac{\nu-1}{2}\right\rfloor}(-1)^s\binom{\nu}{s}\left(i^\nu\;e^{(i\,b+2\,i\,c\,s)\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(i\,b+2\,i\,c\,s)^{-j-1}\,z^{n-j}}{(n-j)!}\right.$$

$${}_{j+2}F_{j+1}\left(\frac{b+2\,c\,s}{2\,c},\,\ldots,\,\frac{b+2\,c\,s}{2\,c},\,\nu;\,\frac{b+2\,c\,s}{2\,c}+1,\,\ldots,\,\frac{b+2\,c\,s}{2\,c}+1;\,-e^{2\,i\,c\,z}\right)+$$

$$i^{-\nu}\,e^{(i\,b+2\,i\,c\,(\nu-s))\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(i\,b+2\,i\,c\,(\nu-s))^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{b+2\,c\,(\nu-s)}{2\,c},\,\ldots,\,\frac{b+2\,c\,(\nu-s)}{2\,c},\right.$$

$$\left.\left.\left.\nu;\,\frac{b+2\,c\,(\nu-s)}{2\,c}+1,\,\ldots,\,\frac{b+2\,c\,(\nu-s)}{2\,c}+1;\,-e^{2\,i\,c\,z}\right)\right)\right)\;/;\;n\in\mathbb{N}\land\nu\in\mathbb{N}^+$$

Involving powers of sin and power

Involving $z^n \sin^m(b\,z)\tan^\nu(c\,z)$

01.08.21.0216.01

$$\int z^n \sin^m(b\,z)\tan^\nu(c\,z)\,dz =$$

$$2^{-m}\binom{m}{\frac{m}{2}}(1-m \bmod 2)\,n!\left(\frac{i^\nu\,z^{n+1}}{(n+1)!}-i^\nu\,e^{2\,i\,c\,z}\,\nu\sum_{j=0}^{n}\frac{(-1)^j\,2^{-j-1}\,(i\,c)^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+3}F_{j+2}\left(1,\,\ldots,\,1,\,\nu+1;\,2,\,\ldots,\,2;\,-e^{2\,i\,c\,z}\right)+\right.$$

$$e^{i c v z}\binom{v}{\frac{v}{2}}(1-v \bmod 2)\sum_{j=0}^{n}\frac{(-1)^j (i c v)^{-j-1} z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}\left(\frac{v}{2}, \ldots, \frac{v}{2}, v; \frac{v}{2}+1, \ldots, \frac{v}{2}+1; -e^{2 i c z}\right)+$$

$$i^{v}\sum_{s=1}^{\left\lfloor\frac{v-1}{2}\right\rfloor}(-1)^s\binom{v}{s} e^{2 i s c z}\sum_{j=0}^{n}\frac{(-1)^j (2 i c s)^{-j-1} z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}(s, \ldots, s, v; s+1, \ldots, s+1; -e^{2 i c z})+i^{-v}\sum_{s=0}^{\left\lfloor\frac{v-1}{2}\right\rfloor}(-1)^s\binom{v}{s}$$

$$\left. e^{2 i c (v-s) z}\sum_{j=0}^{n}\frac{(-1)^j (2 i c (v-s))^{-j-1} z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}(v-s, \ldots, v-s, v; -s+v+1, \ldots, -s+v+1; -e^{2 i c z})\right)+$$

$$i^{-m} 2^{-m} n!\sum_{k=0}^{\left\lfloor\frac{m-1}{2}\right\rfloor}(-1)^k\binom{m}{k}\Bigg((-1)^m e^{(i c v - i b (m-2k)) z}\binom{v}{\frac{v}{2}}(1-v \bmod 2)\sum_{j=0}^{n}\frac{(-1)^j (i c v - i b (m-2k))^{-j-1} z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}$$

$$\left(\frac{c v - b (m-2k)}{2 c}, \ldots, \frac{c v - b (m-2k)}{2 c}, v; \frac{c v - b (m-2k)}{2 c}+1, \ldots, \frac{c v - b (m-2k)}{2 c}+1; -e^{2 i c z}\right)+$$

$$e^{(b i (m-2k) + i c v) z}\binom{v}{\frac{v}{2}}(1-v \bmod 2)\sum_{j=0}^{n}\frac{(-1)^j (b i (m-2k) + i c v)^{-j-1} z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}$$

$$\left(\frac{b (m-2k) + c v}{2 c}, \ldots, \frac{b (m-2k) + c v}{2 c}, v; \frac{b (m-2k) + c v}{2 c}+1, \ldots, \frac{b (m-2k) + c v}{2 c}+1; -e^{2 i c z}\right)+$$

$$(-1)^m\sum_{s=0}^{\left\lfloor\frac{v-1}{2}\right\rfloor}(-1)^s\binom{v}{s}\Bigg(i^{v} e^{(b i (2k-m) + 2 i c s) z}\sum_{j=0}^{n}\frac{(-1)^j (2 i c s - i b (m-2k))^{-j-1} z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}\Bigg(\frac{2 c s - b (m-2k)}{2 c},$$

$$\left.\ldots, \frac{2 c s - b (m-2k)}{2 c}, v; \frac{2 c s - b (m-2k)}{2 c}+1, \ldots, \frac{2 c s - b (m-2k)}{2 c}+1; -e^{2 i c z}\right)+$$

$$i^{-v} e^{(b i (2k-m) + 2 c i (v-s)) z}\sum_{j=0}^{n}\frac{(-1)^j (2 i c (v-s) - i b (m-2k))^{-j-1} z^{n-j}}{(n-j)!}$$

$${}_{j+2}F_{j+1}\Bigg(\frac{2 c (v-s) - b (m-2k)}{2 c}, \ldots, \frac{2 c (v-s) - b (m-2k)}{2 c}, v;$$

$$\left.\left.\frac{2 c (v-s) - b (m-2k)}{2 c}+1, \ldots, \frac{2 c (v-s) - b (m-2k)}{2 c}+1; -e^{2 i c z}\right)\right)+$$

$$\sum_{s=0}^{\left\lfloor\frac{v-1}{2}\right\rfloor}(-1)^s\binom{v}{s}\Bigg(i^{v} e^{(-2 b i k + i b m + 2 i c s) z}\sum_{j=0}^{n}\frac{(-1)^j (b i (m-2k) + 2 i c s)^{-j-1} z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}\Bigg(\frac{b (m-2k) + 2 c s}{2 c},$$

$$\left.\ldots, \frac{b (m-2k) + 2 c s}{2 c}, v; \frac{b (m-2k) + 2 c s}{2 c}+1, \ldots, \frac{b (m-2k) + 2 c s}{2 c}+1; -e^{2 i c z}\right)+$$

$$i^{-v} e^{(-2 b i k + i b m - 2 i c s + 2 i c v) z}\sum_{j=0}^{n}\frac{(-1)^j (b i (m-2k) + 2 c i (v-s))^{-j-1} z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}$$

$$\Bigg(\frac{b (m-2k) + 2 c (v-s)}{2 c}, \ldots, \frac{b (m-2k) + 2 c (v-s)}{2 c}, v; \frac{b (m-2k) + 2 c (v-s)}{2 c}+1,$$

$$b (m-2k) + 2 c (v-s) \ldots$$

Involving cos and power

Involving $z^n \cos(a + b z) \tan^{\nu}(c z)$

01.08.21.0217.01

$$\int z^n \cos(a + b z) \tan^{\nu}(c z) \, dz = \frac{1}{2} n! \, e^{-i a} \left(e^{(-i b + i c \nu) z} \binom{\nu}{\frac{\nu}{2}} (1 - \nu \bmod 2) \sum_{j=0}^{n} \frac{(-1)^j (-i b + i c \nu)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{c \nu - b}{2 c}, \ldots, \frac{c \nu - b}{2 c}, \nu; \frac{c \nu - b}{2 c} + 1, \ldots, \frac{c \nu - b}{2 c} + 1; -e^{2 i c z} \right) + \sum_{s=0}^{\left\lfloor \frac{\nu - 1}{2} \right\rfloor} (-1)^s \binom{\nu}{s} \left(i^{\nu} e^{(-i b + 2 i c s) z} \sum_{j=0}^{n} \frac{(-1)^j (-i b + 2 i c s)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{-b + 2 c s}{2 c}, \ldots, \frac{-b + 2 c s}{2 c}, \nu; \frac{-b + 2 c s}{2 c} + 1, \ldots, \frac{-b + 2 c s}{2 c} + 1; -e^{2 i c z} \right) + i^{-\nu} e^{(-i b + 2 i c (\nu - s)) z} \sum_{j=0}^{n} \frac{(-1)^j (-i b + 2 i c (\nu - s))^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{-b + 2 c (-s + \nu)}{2 c}, \ldots, \frac{-b + 2 c (-s + \nu)}{2 c}, \nu; \frac{-b + 2 c (-s + \nu)}{2 c} + 1, \ldots, \frac{-b + 2 c (-s + \nu)}{2 c} + 1; -e^{2 i c z} \right) \right) \right) + \frac{1}{2} n! \, e^{i a} \left(e^{(i b + i c \nu) z} \binom{\nu}{\frac{\nu}{2}} (1 - \nu \bmod 2) \sum_{j=0}^{n} \frac{(-1)^j (i b + i c \nu)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{c \nu + b}{2 c}, \ldots, \frac{c \nu + b}{2 c}, \nu; \frac{c \nu + b}{2 c} + 1, \ldots, \frac{c \nu + b}{2 c} + 1; -e^{2 i c z} \right) + \sum_{s=0}^{\left\lfloor \frac{\nu - 1}{2} \right\rfloor} (-1)^s \binom{\nu}{s} \left(i^{\nu} e^{(i b + 2 i c s) z} \sum_{j=0}^{n} \frac{(-1)^j (i b + 2 i c s)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{b + 2 c s}{2 c}, \ldots, \frac{b + 2 c s}{2 c}, \nu; \frac{b + 2 c s}{2 c} + 1, \ldots, \frac{b + 2 c s}{2 c} + 1; -e^{2 i c z} \right) + i^{-\nu} e^{(i b + 2 i c (\nu - s)) z} \sum_{j=0}^{n} \frac{(-1)^j (i b + 2 i c (\nu - s))^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{b + 2 c (\nu - s)}{2 c}, \ldots, \frac{b + 2 c (\nu - s)}{2 c}, \nu; \frac{b + 2 c (\nu - s)}{2 c} + 1, \ldots, \frac{b + 2 c (\nu - s)}{2 c} + 1; -e^{2 i c z} \right) \right) \right) /; n \in \mathbb{N} \land \nu \in \mathbb{N}^+$$

01.08.21.0218.01

$$\int z^n \cos(b z) \tan^\nu(c z)\, dz = \frac{n!}{2} \left(e^{(-i b + i c \nu) z} \binom{\nu}{\frac{\nu}{2}} (1 - \nu \bmod 2) \right.$$

$$\sum_{j=0}^{n} \frac{(-1)^j (-i b + i c \nu)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{c \nu - b}{2 c}, \ldots, \frac{c \nu - b}{2 c}, \nu; \frac{c \nu - b}{2 c} + 1, \ldots, \frac{c \nu - b}{2 c} + 1; -e^{2 i c z} \right) +$$

$$\sum_{s=0}^{\left\lfloor \frac{\nu-1}{2} \right\rfloor} (-1)^s \binom{\nu}{s} \left(i^\nu \, e^{(-i b + 2 i c s) z} \sum_{j=0}^{n} \frac{(-1)^j (-i b + 2 i c s)^{-j-1} z^{n-j}}{(n-j)!} \right.$$

$${}_{j+2}F_{j+1}\left(\frac{-b + 2 c s}{2 c}, \ldots, \frac{-b + 2 c s}{2 c}, \nu; \frac{-b + 2 c s}{2 c} + 1, \ldots, \frac{-b + 2 c s}{2 c} + 1; -e^{2 i c z} \right) +$$

$$i^{-\nu} \, e^{(-i b + 2 i c (\nu - s)) z} \sum_{j=0}^{n} \frac{(-1)^j (-i b + 2 i c (\nu - s))^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{-b + 2 c (-s + \nu)}{2 c}, \ldots, \frac{-b + 2 c (-s + \nu)}{2 c}, \right.$$

$$\left.\left. \nu; \frac{-b + 2 c (-s + \nu)}{2 c} + 1, \ldots, \frac{-b + 2 c (-s + \nu)}{2 c} + 1; -e^{2 i c z} \right) \right) + e^{(i b + i c \nu) z} \binom{\nu}{\frac{\nu}{2}} (1 - \nu \bmod 2)$$

$$\sum_{j=0}^{n} \frac{(-1)^j (i b + i c \nu)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{c \nu + b}{2 c}, \ldots, \frac{c \nu + b}{2 c}, \nu; \frac{c \nu + b}{2 c} + 1, \ldots, \frac{c \nu + b}{2 c} + 1; -e^{2 i c z} \right) +$$

$$\sum_{s=0}^{\left\lfloor \frac{\nu-1}{2} \right\rfloor} (-1)^s \binom{\nu}{s} \left(i^\nu \, e^{(i b + 2 i c s) z} \sum_{j=0}^{n} \frac{(-1)^j (i b + 2 i c s)^{-j-1} z^{n-j}}{(n-j)!} \right.$$

$${}_{j+2}F_{j+1}\left(\frac{b + 2 c s}{2 c}, \ldots, \frac{b + 2 c s}{2 c}, \nu; \frac{b + 2 c s}{2 c} + 1, \ldots, \frac{b + 2 c s}{2 c} + 1; -e^{2 i c z} \right) +$$

$$i^{-\nu} \, e^{(i b + 2 i c (\nu - s)) z} \sum_{j=0}^{n} \frac{(-1)^j (i b + 2 i c (\nu - s))^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{b + 2 c (\nu - s)}{2 c}, \ldots, \frac{b + 2 c (\nu - s)}{2 c}, \right.$$

$$\left.\left.\left. \nu; \frac{b + 2 c (\nu - s)}{2 c} + 1, \ldots, \frac{b + 2 c (\nu - s)}{2 c} + 1; -e^{2 i c z} \right) \right) \right) /; n \in \mathbb{N} \land \nu \in \mathbb{N}^+$$

Involving powers of cos and power

Involving $z^n \cos^m(b z) \tan^\nu(c z)$

01.08.21.0219.01

$$\int z^n \cos^m(b z) \tan^\nu(c z)\, dz =$$

$$2^{-m} \binom{m}{\frac{m}{2}} (1 - m \bmod 2)\, n! \left(\frac{i^\nu z^{n+1}}{(n+1)!} - i^\nu e^{2 i c z} \nu \sum_{j=0}^{n} \frac{(-1)^j 2^{-j-1} (i c)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+3}F_{j+2}\left(1, \ldots, 1, \nu + 1; 2, \ldots, 2; -e^{2 i c z}\right) + \right.$$

$$e^{i c v z} \binom{v}{\frac{v}{2}} (1 - v \bmod 2) \sum_{j=0}^{n} \frac{(-1)^j (i c v)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{v}{2}, \ldots, \frac{v}{2}, v; \frac{v}{2}+1, \ldots, \frac{v}{2}+1; -e^{2 i c z}\right) +$$

$$i^{v} \sum_{s=1}^{\left\lfloor \frac{v-1}{2} \right\rfloor} (-1)^s \binom{v}{s} e^{2 i s c z} \sum_{j=0}^{n} \frac{(-1)^j (2 i c s)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}(s, \ldots, s, v; s+1, \ldots, s+1; -e^{2 i c z}) + i^{-v} \sum_{s=0}^{\left\lfloor \frac{v-1}{2} \right\rfloor} (-1)^s \binom{v}{s}$$

$$\left. e^{2 i c (v-s) z} \sum_{j=0}^{n} \frac{(-1)^j (2 i c (v-s))^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}(v-s, \ldots, v-s, v; -s+v+1, \ldots, -s+v+1; -e^{2 i c z}) \right) +$$

$$2^{-m} n! \, i^{-v} \sum_{k=0}^{\left\lfloor \frac{m-1}{2} \right\rfloor} \binom{m}{k} \left(i^{v} e^{(i c v - i b (m-2k)) z} \binom{v}{\frac{v}{2}} (1 - v \bmod 2) \sum_{j=0}^{n} \frac{(-1)^j (i c v - i b (m-2k))^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1} \right.$$

$$\left(\frac{c v - b (m-2k)}{2 c}, \ldots, \frac{c v - b (m-2k)}{2 c}, v; \frac{c v - b (m-2k)}{2 c} + 1, \ldots, \frac{c v - b (m-2k)}{2 c} + 1; -e^{2 i c z}\right) +$$

$$\sum_{s=0}^{\left\lfloor \frac{v-1}{2} \right\rfloor} (-1)^s \binom{v}{s} \left((-1)^{v} e^{(2 i b k - i b m + 2 i c s) z} \sum_{j=0}^{n} \frac{(-1)^j (2 i c s - i b (m-2k))^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{2 c s - b (m-2k)}{2 c}, \right. \right.$$

$$\left. \ldots, \frac{2 c s - b (m-2k)}{2 c}, v; \frac{2 c s - b (m-2k)}{2 c} + 1, \ldots, \frac{2 c s - b (m-2k)}{2 c} + 1; -e^{2 i c z}\right) +$$

$$e^{(2 i b k - i b m - 2 i c s + 2 i c v) z} \sum_{j=0}^{n} \frac{(-1)^j (2 i c (v-s) - i b (m-2k))^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}$$

$$\left(\frac{2 c (v-s) - b (m-2k)}{2 c}, \ldots, \frac{2 c (v-s) - b (m-2k)}{2 c}, v; \right.$$

$$\left. \left. \frac{2 c (v-s) - b (m-2k)}{2 c} + 1, \ldots, \frac{2 c (v-s) - b (m-2k)}{2 c} + 1; -e^{2 i c z}\right)\right) +$$

$$i^{v} e^{(b i (m-2k) + i c v) z} \binom{v}{\frac{v}{2}} (1 - v \bmod 2) \sum_{j=0}^{n} \frac{(-1)^j (b i (m-2k) + i c v)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}$$

$$\left(\frac{b (m-2k) + c v}{2 c}, \ldots, \frac{b (m-2k) + c v}{2 c}, v; \frac{b (m-2k) + c v}{2 c} + 1, \ldots, \frac{b (m-2k) + c v}{2 c} + 1; -e^{2 i c z}\right) +$$

$$\sum_{s=0}^{\left\lfloor \frac{v-1}{2} \right\rfloor} (-1)^s \binom{v}{s} \left((-1)^{v} e^{(b i (m-2k) + 2 i c s) z} \sum_{j=0}^{n} \frac{(-1)^j (b i (m-2k) + 2 i c s)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{b (m-2k) + 2 c s}{2 c}, \right. \right.$$

$$\left. \ldots, \frac{b (m-2k) + 2 c s}{2 c}, v; \frac{b (m-2k) + 2 c s}{2 c} + 1, \ldots, \frac{b (m-2k) + 2 c s}{2 c} + 1; -e^{2 i c z}\right) +$$

$$e^{(b i (m-2k) + 2 c i (v-s)) z} \sum_{j=0}^{n} \frac{(-1)^j (b i (m-2k) + 2 c i (v-s))^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}$$

$$\left(\frac{b (m-2k) + 2 c (v-s)}{2 c}, \ldots, \frac{b (m-2k) + 2 c (v-s)}{2 c}, v; \frac{b (m-2k) + 2 c (v-s)}{2 c} + 1, \right.$$

$$b (m-2k) + 2 c (v-s) \quad \ldots))$$

Involving functions of the direct function, trigonometric and exponential functions

Involving powers of the direct function, trigonometric and exponential functions

Involving sin and exp

Involving $e^{p z} \sin(a z) \tan^{\nu}(c z)$

01.08.21.0220.01

$$\int e^{p z} \sin(a z) \tan^{\nu}(c z)\, dz = \frac{1}{2} i \left(1 - e^{-2 i c z}\right)^{-\nu} \left(1 + e^{-2 i c z}\right)^{\nu} \tan^{\nu}(c z) \left(\frac{i\, e^{(-i a + p) z}}{a + i p} F_1\!\left(\frac{a + i p}{2 c}; -\nu, \nu; \frac{a + 2 c + i p}{2 c}; e^{-2 i c z}, -e^{-2 i c z}\right) - \frac{e^{(i a + p) z}}{i a + p} F_1\!\left(-\frac{a - i p}{2 c}; -\nu, \nu; 1 - \frac{a - i p}{2 c}; e^{-2 i c z}, -e^{-2 i c z}\right) \right)$$

01.08.21.0221.01

$$\int e^{i a z} \sin(a z) \tan^{\nu}(c z)\, dz = \frac{1}{4} \tan^{\nu}(c z) \left(\frac{2 i \cot(c z)}{c \nu - c}\, {}_2F_1\!\left(\frac{1 - \nu}{2}, 1; \frac{3 - \nu}{2}; -\cot^2(c z)\right) - \frac{e^{2 i a z} \left(1 - e^{-2 i c z}\right)^{-\nu} \left(1 + e^{-2 i c z}\right)^{\nu}}{a} F_1\!\left(-\frac{a}{c}; \nu, -\nu; 1 - \frac{a}{c}; -e^{-2 i c z}, e^{-2 i c z}\right) \right)$$

01.08.21.0222.01

$$\int e^{-i a z} \sin(a z) \tan^{\nu}(c z)\, dz = \frac{1}{4} \tan^{\nu}(c z) \left(\frac{2 i \cot(c z)}{c - c \nu}\, {}_2F_1\!\left(\frac{1 - \nu}{2}, 1; \frac{3 - \nu}{2}; -\cot^2(c z)\right) - \frac{e^{-2 i a z} \left(1 - e^{-2 i c z}\right)^{-\nu} \left(1 + e^{-2 i c z}\right)^{\nu}}{a} F_1\!\left(\frac{a}{c}; \nu, -\nu; \frac{a + c}{c}; -e^{-2 i c z}, e^{-2 i c z}\right) \right)$$

Involving powers of sin and exp

Involving $e^{p z} \sin^{m}(a z) \tan^{\nu}(c z)$

01.08.21.0223.01

$$\int e^{p z} \sin^{m}(a z) \tan^{\nu}(c z)\, dz = \frac{1}{p} 2^{-m} i^{-m} \left(1 - e^{-2 i c z}\right)^{-\nu} \left(1 + e^{-2 i c z}\right)^{\nu} \tan^{\nu}(c z) \left(p \sum_{k=0}^{\left\lfloor \frac{m-1}{2} \right\rfloor} (-1)^k \binom{m}{k} \left(\frac{(-1)^m e^{(p - i a (m - 2 k)) z}}{p - i a (m - 2 k)} F_1\!\left(\frac{-2 a k + a m + i p}{2 c}; -\nu, \nu; \frac{2 c - 2 a k + a m + i p}{2 c}; e^{-2 i c z}, -e^{-2 i c z}\right) + \frac{e^{(a i (m - 2 k) + p) z}}{a i (m - 2 k) + p} F_1\!\left(\frac{2 a k - a m + i p}{2 c}; -\nu, \nu; \frac{2 a k - a m + i p}{2 c} + 1; e^{-2 i c z}, -e^{-2 i c z}\right) \right) - i^m e^{p z} \binom{m}{\frac{m}{2}} (m \bmod 2 - 1) F_1\!\left(\frac{i p}{2 c}; -\nu, \nu; \frac{i p}{2 c} + 1; e^{-2 i c z}, -e^{-2 i c z}\right) \right) /; m \in \mathbb{N}^+$$

Involving cos and exp

Involving $e^{p z} \cos(a z) \tan^{\nu}(c z)$

01.08.21.0224.01

$$\int e^{p z} \cos(a z) \tan^{\nu}(c z)\, dz = \frac{1}{2} \left(1 - e^{-2 i c z}\right)^{-\nu} \left(1 + e^{-2 i c z}\right)^{\nu} \tan^{\nu}(c z) \left(\frac{i\, e^{(-i a + p) z}}{a + i p} F_1\left(\frac{a + i p}{2 c}; -\nu, \nu; \frac{a + 2 c + i p}{2 c}; e^{-2 i c z}, -e^{-2 i c z}\right) + \frac{e^{(i a + p) z}}{i a + p} F_1\left(-\frac{a - i p}{2 c}; -\nu, \nu; 1 - \frac{a - i p}{2 c}; e^{-2 i c z}, -e^{-2 i c z}\right)\right)$$

01.08.21.0225.01

$$\int e^{i a z} \cos(a z) \tan^{\nu}(c z)\, dz = \frac{1}{4} \tan^{\nu}(c z) \left(\frac{2 \cot(c z)}{c \nu - c} {}_2F_1\left(\frac{1 - \nu}{2}, 1; \frac{3 - \nu}{2}; -\cot^2(c z)\right) - \frac{i\, e^{2 i a z} \left(1 - e^{-2 i c z}\right)^{-\nu} \left(1 + e^{-2 i c z}\right)^{\nu}}{a} F_1\left(-\frac{a}{c}; \nu, -\nu; 1 - \frac{a}{c}; -e^{-2 i c z}, e^{-2 i c z}\right)\right)$$

01.08.21.0226.01

$$\int e^{-i a z} \cos(a z) \tan^{\nu}(c z)\, dz = \frac{1}{4} \tan^{\nu}(c z) \left(\frac{i\, e^{-2 i a z} \left(1 - e^{-2 i c z}\right)^{-\nu} \left(1 + e^{-2 i c z}\right)^{\nu}}{a} F_1\left(\frac{a}{c}; \nu, -\nu; \frac{a + c}{c}; -e^{-2 i c z}, e^{-2 i c z}\right) + \frac{2 \cot(c z)}{c \nu - c} {}_2F_1\left(\frac{1 - \nu}{2}, 1; \frac{3 - \nu}{2}; -\cot^2(c z)\right)\right)$$

Involving powers of cos and exp

Involving $e^{p z} \cos^m(a z) \tan^{\nu}(c z)$

01.08.21.0227.01

$$\int e^{p z} \cos^m(a z) \tan^{\nu}(c z)\, dz = 2^{-m} \left(1 - e^{-2 i c z}\right)^{-\nu} \left(1 + e^{-2 i c z}\right)^{\nu} \tan^{\nu}(c z) \left(\sum_{k=0}^{\left\lfloor \frac{m-1}{2} \right\rfloor} \binom{m}{k} \left(\frac{e^{(p - i a (m - 2 k)) z}}{p - i a (m - 2 k)} F_1\left(\frac{-2 a k + a m + i p}{2 c}; -\nu, \nu; \frac{2 c - 2 a k + a m + i p}{2 c}; e^{-2 i c z}, -e^{-2 i c z}\right) + \frac{e^{(a i (m - 2 k) + p) z}}{a i (m - 2 k) + p} F_1\left(\frac{2 a k - a m + i p}{2 c}; -\nu, \nu; \frac{2 a k - a m + i p}{2 c} + 1; e^{-2 i c z}, -e^{-2 i c z}\right)\right) - \frac{1}{p} e^{p z} F_1\left(\frac{i p}{2 c}; -\nu, \nu; \frac{i p}{2 c} + 1; e^{-2 i c z}, -e^{-2 i c z}\right) \binom{m}{\frac{m}{2}} (m \bmod 2 - 1) \right) /; m \in \mathbb{N}^+$$

Involving functions of the direct function, trigonometric, exponential and a power functions

Involving powers of the direct function, trigonometric, exponential and a power functions

Involving sin, exp and power

Involving $z^n e^{p z} \sin(a + b z) \tan^\nu(c z)$

01.08.21.0228.01

$$\int z^n e^{p z} \sin(a + b z) \tan^\nu(c z)\, dz = \frac{i}{2} n!\, e^{-i a} \Bigg(e^{(-i b+p+i c \nu) z} \binom{\nu}{\frac{\nu}{2}} (1 - \nu \bmod 2) \sum_{j=0}^{n} \frac{(-1)^j (-i b + p + i c \nu)^{-j-1} z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}\bigg(\frac{c \nu - b - i p}{2 c}, \ldots, \frac{c \nu - b - i p}{2 c}, \nu; \frac{c \nu - b - i p}{2 c} + 1, \ldots, \frac{c \nu - b - i p}{2 c} + 1; -e^{2 i c z}\bigg) + \sum_{s=0}^{\left\lfloor \frac{\nu-1}{2} \right\rfloor} (-1)^s \binom{\nu}{s} \bigg(i^\nu e^{(-i b+p+2 i c s) z} \sum_{j=0}^{n} \frac{(-1)^j (-i b + p + 2 i c s)^{-j-1} z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}\bigg(\frac{-b - i p + 2 c s}{2 c}, \ldots, \frac{-b - i p + 2 c s}{2 c}, \nu; \frac{-b - i p + 2 c s}{2 c} + 1, \ldots, \frac{-b - i p + 2 c s}{2 c} + 1; -e^{2 i c z}\bigg) + i^{-\nu} e^{(-i b+p+2 i c (\nu-s)) z} \sum_{j=0}^{n} \frac{(-1)^j (-i b + p + 2 i c (\nu - s))^{-j-1} z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}\bigg(\frac{-b - i p + 2 c (-s + \nu)}{2 c}, \ldots, \frac{-b - i p + 2 c (-s + \nu)}{2 c}, \nu; \frac{-b - i p + 2 c (-s + \nu)}{2 c} + 1, \ldots, \frac{-b - i p + 2 c (-s + \nu)}{2 c} + 1; -e^{2 i c z}\bigg)\bigg)\Bigg) - \frac{i}{2} n!\, e^{i a} \Bigg(e^{(i b+p+i c \nu) z} \binom{\nu}{\frac{\nu}{2}} (1 - \nu \bmod 2) \sum_{j=0}^{n} \frac{(-1)^j (i b + p + i c \nu)^{-j-1} z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}\bigg(\frac{c \nu + b - i p}{2 c}, \ldots, \frac{c \nu + b - i p}{2 c}, \nu; \frac{c \nu + b - i p}{2 c} + 1, \ldots, \frac{c \nu + b - i p}{2 c} + 1; -e^{2 i c z}\bigg) + \sum_{s=0}^{\left\lfloor \frac{\nu-1}{2} \right\rfloor} (-1)^s \binom{\nu}{s} \bigg(i^\nu e^{(i b+p+2 i c s) z} \sum_{j=0}^{n} \frac{(-1)^j (i b + p + 2 i c s)^{-j-1} z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}\bigg(\frac{b - i p + 2 c s}{2 c}, \ldots, \frac{b - i p + 2 c s}{2 c}, \nu; \frac{b - i p + 2 c s}{2 c} + 1, \ldots, \frac{b - i p + 2 c s}{2 c} + 1; -e^{2 i c z}\bigg) + i^{-\nu} e^{(i b+p+2 i c (\nu-s)) z} \sum_{j=0}^{n} \frac{(-1)^j (i b + p + 2 i c (\nu - s))^{-j-1} z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}\bigg(\frac{b - i p + 2 c (-s + \nu)}{2 c}, \ldots, \frac{b - i p + 2 c (-s + \nu)}{2 c}, \nu; \frac{b - i p + 2 c (-s + \nu)}{2 c} + 1, \ldots, \frac{b - i p + 2 c (-s + \nu)}{2 c} + 1; -e^{2 i c z}\bigg)\bigg)\Bigg) \;/;\; n \in \mathbb{N} \land \nu \in \mathbb{N}^+$$

01.08.21.0229.01

$$\int z^n e^{p z} \sin(b z) \tan^{\nu}(c z)\, dz = \frac{i}{2}\, n! \left(e^{(-i b + p + i c \nu) z} \binom{\nu}{\frac{\nu}{2}} (1 - \nu \bmod 2) \sum_{j=0}^{n} \frac{(-1)^j (-i b + p + i c \nu)^{-j-1} z^{n-j}}{(n-j)!} \right.$$
$$\,_{j+2}F_{j+1}\left(\frac{c \nu - b - i p}{2 c}, \ldots, \frac{c \nu - b - i p}{2 c}, \nu; \frac{c \nu - b - i p}{2 c} + 1, \ldots, \frac{c \nu - b - i p}{2 c} + 1; -e^{2 i c z}\right) +$$
$$\sum_{s=0}^{\left\lfloor \frac{\nu-1}{2} \right\rfloor} (-1)^s \binom{\nu}{s} \left(i^{\nu} e^{(-i b + p + 2 i c s) z} \sum_{j=0}^{n} \frac{(-1)^j (-i b + p + 2 i c s)^{-j-1} z^{n-j}}{(n-j)!} \right.$$
$$\,_{j+2}F_{j+1}\left(\frac{-b - i p + 2 c s}{2 c}, \ldots, \frac{-b - i p + 2 c s}{2 c}, \nu; \frac{-b - i p + 2 c s}{2 c} + 1, \ldots, \frac{-b - i p + 2 c s}{2 c} + 1; -e^{2 i c z}\right) +$$
$$i^{-\nu} e^{(-i b + p + 2 i c (\nu - s)) z} \sum_{j=0}^{n} \frac{(-1)^j (-i b + p + 2 i c (\nu - s))^{-j-1} z^{n-j}}{(n-j)!} \,_{j+2}F_{j+1}\left(\frac{-b - i p + 2 c (-s + \nu)}{2 c}, \ldots,\right.$$
$$\left.\left.\frac{-b - i p + 2 c (-s + \nu)}{2 c}, \nu; \frac{-b - i p + 2 c (-s + \nu)}{2 c} + 1, \ldots, \frac{-b - i p + 2 c (-s + \nu)}{2 c} + 1; -e^{2 i c z}\right)\right) -$$
$$e^{(i b + p + i c \nu) z} \binom{\nu}{\frac{\nu}{2}} (1 - \nu \bmod 2) \sum_{j=0}^{n} \frac{(-1)^j (i b + p + i c \nu)^{-j-1} z^{n-j}}{(n-j)!} \,_{j+2}F_{j+1}\left(\frac{c \nu + b - i p}{2 c}, \ldots,\right.$$
$$\left.\frac{c \nu + b - i p}{2 c}, \nu; \frac{c \nu + b - i p}{2 c} + 1, \ldots, \frac{c \nu + b - i p}{2 c} + 1; -e^{2 i c z}\right) -$$
$$\sum_{s=0}^{\left\lfloor \frac{\nu-1}{2} \right\rfloor} (-1)^s \binom{\nu}{s} \left(i^{\nu} e^{(i b + p + 2 i c s) z} \sum_{j=0}^{n} \frac{(-1)^j (i b + p + 2 i c s)^{-j-1} z^{n-j}}{(n-j)!} \,_{j+2}F_{j+1}\left(\frac{b - i p + 2 c s}{2 c}, \ldots,\right.\right.$$
$$\left.\frac{b - i p + 2 c s}{2 c}, \nu; \frac{b - i p + 2 c s}{2 c} + 1, \ldots, \frac{b - i p + 2 c s}{2 c} + 1; -e^{2 i c z}\right) + i^{-\nu} e^{(i b + p + 2 i c (\nu - s)) z}$$
$$\sum_{j=0}^{n} \frac{(-1)^j (i b + p + 2 i c (\nu - s))^{-j-1} z^{n-j}}{(n-j)!} \,_{j+2}F_{j+1}\left(\frac{b - i p + 2 c (-s + \nu)}{2 c}, \ldots, \frac{b - i p + 2 c (-s + \nu)}{2 c},\right.$$
$$\left.\left.\left.\nu; \frac{b - i p + 2 c (-s + \nu)}{2 c} + 1, \ldots, \frac{b - i p + 2 c (-s + \nu)}{2 c} + 1; -e^{2 i c z}\right)\right)\right) /; n \in \mathbb{N} \bigwedge \nu \in \mathbb{N}^+$$

Involving powers of sin, exp and power

Involving $z^n e^{p z} \sin^m(b z) \tan^{\nu}(c z)$

01.08.21.0230.01

$$\int z^n e^{p z} \sin^m(b z) \tan^{\nu}(c z)\, dz =$$
$$2^{-m} \binom{m}{\frac{m}{2}} n! (1 - m \bmod 2) \left(e^{(p + i c \nu) z} \binom{\nu}{\frac{\nu}{2}} (1 - \nu \bmod 2) \sum_{j=0}^{n} \frac{(-1)^j (p + i c \nu)^{-j-1} z^{n-j}}{(n-j)!} \,_{j+2}F_{j+1}\left(\frac{c \nu - i p}{2 c}, \ldots, \frac{c \nu - i p}{2 c}, \nu;\right.\right.$$

$$\frac{c\,v - i\,p}{2\,c} + 1, \ldots, \frac{c\,v - i\,p}{2\,c} + 1; -e^{2\,i\,c\,z}\Bigg) + \sum_{s=0}^{\left\lfloor \frac{v-1}{2} \right\rfloor} (-1)^s \binom{v}{s} \Bigg(i^v\, e^{(p+2\,i\,c\,s)\,z} \sum_{j=0}^{n} \frac{(-1)^j\,(p + 2\,i\,c\,s)^{-j-1}\,z^{n-j}}{(n-j)!}$$

$$_{j+2}F_{j+1}\Bigg(\frac{-i\,p + 2\,c\,s}{2\,c}, \ldots, \frac{-i\,p + 2\,c\,s}{2\,c}, v; \frac{-i\,p + 2\,c\,s}{2\,c} + 1, \ldots, \frac{-i\,p + 2\,c\,s}{2\,c} + 1; -e^{2\,i\,c\,z}\Bigg) +$$

$$i^{-v}\, e^{(p+2\,i\,c\,(v-s))\,z} \sum_{j=0}^{n} \frac{(-1)^j\,(p + 2\,i\,c\,(v-s))^{-j-1}\,z^{n-j}}{(n-j)!}\; {}_{j+2}F_{j+1}\Bigg(\frac{-i\,p + 2\,c\,(v-s)}{2\,c}, \ldots,$$

$$\frac{-i\,p + 2\,c\,(v-s)}{2\,c}, v; \frac{-i\,p + 2\,c\,(v-s)}{2\,c} + 1, \ldots, \frac{-i\,p + 2\,c\,(v-s)}{2\,c} + 1; -e^{2\,i\,c\,z}\Bigg)\Bigg)\Bigg) +$$

$$i^{-m}\, 2^{-m}\, n! \sum_{k=0}^{\left\lfloor \frac{m-1}{2} \right\rfloor} (-1)^k \binom{m}{k} \Bigg((-1)^m\, e^{(p-i\,b\,(m-2\,k)+i\,c\,v)\,z} \binom{v}{\frac{v}{2}} (1 - v \bmod 2) \sum_{j=0}^{n} \frac{(-1)^j\,(p - i\,b\,(m-2\,k) + i\,c\,v)^{-j-1}\,z^{n-j}}{(n-j)!}$$

$$_{j+2}F_{j+1}\Bigg(\frac{-i\,p - b\,(m-2\,k) + c\,v}{2\,c}, \ldots, \frac{-i\,p - b\,(m-2\,k) + c\,v}{2\,c}, v;$$

$$\frac{-i\,p - b\,(m-2\,k) + c\,v}{2\,c} + 1, \ldots, \frac{-i\,p - b\,(m-2\,k) + c\,v}{2\,c} + 1; -e^{2\,i\,c\,z}\Bigg) +$$

$$(-1)^m \sum_{s=0}^{\left\lfloor \frac{v-1}{2} \right\rfloor} (-1)^s \binom{v}{s} \Bigg(i^v\, e^{(i\,b\,(2\,k-m)+p+2\,i\,c\,s)\,z} \sum_{j=0}^{n} \frac{(-1)^j\,(p + 2\,i\,c\,s - i\,b\,(m-2\,k))^{-j-1}\,z^{n-j}}{(n-j)!}$$

$$_{j+2}F_{j+1}\Bigg(\frac{-i\,p + 2\,c\,s - b\,(m-2\,k)}{2\,c}, \ldots, \frac{-i\,p + 2\,c\,s - b\,(m-2\,k)}{2\,c}, v;$$

$$\frac{-i\,p + 2\,c\,s - b\,(m-2\,k)}{2\,c} + 1, \ldots, \frac{-i\,p + 2\,c\,s - b\,(m-2\,k)}{2\,c} + 1; -e^{2\,i\,c\,z}\Bigg) +$$

$$i^{-v}\, e^{(i\,b\,(2\,k-m)+p+2\,i\,c\,(-s+v))\,z} \sum_{j=0}^{n} \frac{(-1)^j\,(p - i\,b\,(m-2\,k) + 2\,i\,c\,(v-s))^{-j-1}\,z^{n-j}}{(n-j)!}$$

$$_{j+2}F_{j+1}\Bigg(\frac{-i\,p - b\,(m-2\,k) + 2\,c\,(v-s)}{2\,c}, \ldots, \frac{-i\,p - b\,(m-2\,k) + 2\,c\,(v-s)}{2\,c}, v;$$

$$\frac{-i\,p - b\,(m-2\,k) + 2\,c\,(v-s)}{2\,c} + 1, \ldots, \frac{-i\,p - b\,(m-2\,k) + 2\,c\,(v-s)}{2\,c} + 1; -e^{2\,i\,c\,z}\Bigg)\Bigg) +$$

$$e^{(p+i\,b\,(m-2\,k)+i\,c\,v)\,z} \binom{v}{\frac{v}{2}} (1 - v \bmod 2) \sum_{j=0}^{n} \frac{(-1)^j\,(p + i\,b\,(m-2\,k) + i\,c\,v)^{-j-1}\,z^{n-j}}{(n-j)!}\; {}_{j+2}F_{j+1}\Bigg(\frac{-i\,p + b\,(m-2\,k) + c\,v}{2\,c},$$

$$\ldots, \frac{-i\,p + b\,(m-2\,k) + c\,v}{2\,c}, v; \frac{-i\,p + b\,(m-2\,k) + c\,v}{2\,c} + 1, \ldots, \frac{-i\,p + b\,(m-2\,k) + c\,v}{2\,c} + 1; -e^{2\,i\,c\,z}\Bigg) +$$

$$\sum_{s=0}^{\left\lfloor \frac{v-1}{2} \right\rfloor} (-1)^s \binom{v}{s} \Bigg(i^v\, e^{(-2\,i\,b\,k+i\,b\,m+p+2\,i\,c\,s)\,z} \sum_{j=0}^{n} \frac{(-1)^j\,(p + 2\,i\,c\,s + i\,b\,(m-2\,k))^{-j-1}\,z^{n-j}}{(n-j)!}\; {}_{j+2}F_{j+1}$$

$$\Bigg(\frac{-i\,p + 2\,c\,s + b\,(m-2\,k)}{2\,c}, \ldots, \frac{-i\,p + 2\,c\,s + b\,(m-2\,k)}{2\,c}, v; \frac{-i\,p + 2\,c\,s + b\,(m-2\,k)}{2\,c} + 1,$$

$$\ldots, \frac{-i\, p + 2\, c\, s + b\,(m - 2\, k)}{2\, c} + 1;\, -e^{2\, i\, c\, z}\Bigg) + i^{-\nu}\, e^{(-2\, i\, b\, k + i\, b\, m + p - 2\, i\, c\, s + 2\, i\, c\, \nu)\, z}$$

$$\sum_{j=0}^{n} \frac{(-1)^j\, (p + i\, b\,(m - 2\, k) + 2\, i\, c\,(\nu - s))^{-j-1}\, z^{n-j}}{(n - j)!}\; {}_{j+2}F_{j+1}\Bigg(\frac{-i\, p + b\,(m - 2\, k) + 2\, c\,(\nu - s)}{2\, c},$$

$$\ldots, \frac{-i\, p + b\,(m - 2\, k) + 2\, c\,(\nu - s)}{2\, c}, \nu;\, \frac{-i\, p + b\,(m - 2\, k) + 2\, c\,(\nu - s)}{2\, c} + 1, \ldots,$$

$$\frac{-i\, p + b\,(m - 2\, k) + 2\, c\,(\nu - s)}{2\, c} + 1;\, -e^{2\, i\, c\, z}\Bigg)\Bigg)\Bigg) /;\, n \in \mathbb{N} \bigwedge m \in \mathbb{N}^+ \bigwedge \nu \in \mathbb{N}^+$$

Involving cos, exp and power

Involving $z^n\, e^{p\, z} \cos(a + b\, z) \tan^{\nu}(c\, z)$

01.08.21.0231.01

$$\int z^n e^{pz}\cos(a+bz)\tan^{\nu}(cz)\,dz == \frac{1}{2}\,n!\,e^{-ia}\left(e^{(-ib+p+icv)z}\binom{v}{\frac{v}{2}}(1-v\bmod 2)\sum_{j=0}^{n}\frac{(-1)^j(-ib+p+icv)^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{-b-ip+cv}{2c},\ldots,\frac{-b-ip+cv}{2c},v;\frac{-b-ip+cv}{2c}+1,\ldots,\frac{-b-ip+cv}{2c}+1;-e^{2icz}\right)+\right.$$

$$i^{-v}\sum_{s=0}^{\left\lfloor\frac{v-1}{2}\right\rfloor}(-1)^s\binom{v}{s}\left((-1)^v e^{(-ib+p+2ics)z}\sum_{j=0}^{n}\frac{(-1)^j(-ib+p+2ics)^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{-b-ip+2cs}{2c},\ldots,\frac{-b-ip+2cs}{2c},v;\frac{-b-ip+2cs}{2c}+1,\ldots,\frac{-b-ip+2cs}{2c}+1;-e^{2icz}\right)+\right.$$

$$\left.\left.e^{(-ib+p+2ic(v-s))z}\sum_{j=0}^{n}\frac{(-1)^j(-ib+p+2ic(v-s))^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{-b-ip+2c(v-s)}{2c},\ldots,\frac{-b-ip+2c(v-s)}{2c},v;\frac{-b-ip+2c(v-s)}{2c}+1,\ldots,\frac{-b-ip+2c(v-s)}{2c}+1;-e^{2icz}\right)\right)\right)+$$

$$\frac{1}{2}\,n!\,e^{ia}\left(e^{(ib+p+icv)z}\binom{v}{\frac{v}{2}}(1-v\bmod 2)\sum_{j=0}^{n}\frac{(-1)^j(ib+p+icv)^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{b-ip+cv}{2c},\ldots,\frac{b-ip+cv}{2c},v;\frac{b-ip+cv}{2c}+1,\ldots,\frac{b-ip+cv}{2c}+1;-e^{2icz}\right)+\right.$$

$$i^{-v}\sum_{s=0}^{\left\lfloor\frac{v-1}{2}\right\rfloor}(-1)^s\binom{v}{s}\left((-1)^v e^{(ib+p+2ics)z}\sum_{j=0}^{n}\frac{(-1)^j(ib+p+2ics)^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{b-ip+2cs}{2c},\ldots,\frac{b-ip+2cs}{2c},v;\frac{b-ip+2cs}{2c}+1,\ldots,\frac{b-ip+2cs}{2c}+1;-e^{2icz}\right)+e^{(ib+p+2ic(v-s))z}\right.$$

$$\left.\left.\sum_{j=0}^{n}\frac{(-1)^j(ib+p+2ic(v-s))^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{b-ip+2c(v-s)}{2c},\ldots,\frac{b-ip+2c(v-s)}{2c},v;\frac{b-ip+2c(v-s)}{2c}+1,\ldots,\frac{b-ip+2c(v-s)}{2c}+1;-e^{2icz}\right)\right)\right)\,/;\,n\in\mathbb{N}\land v\in\mathbb{N}^+$$

01.08.21.0232.01

$$\int z^n e^{pz}\cos(bz)\tan^\nu(cz)\,dz = \frac{1}{2}n!\left(e^{(-ib+p+icv)z}\binom{\nu}{\frac{\nu}{2}}(1-\nu \bmod 2)\sum_{j=0}^{n}\frac{(-1)^j(-ib+p+icv)^{-j-1}z^{n-j}}{(n-j)!}\right.$$
$${}_{j+2}F_{j+1}\left(\frac{cv-b-ip}{2c},\ldots,\frac{cv-b-ip}{2c},v;\frac{cv-b-ip}{2c}+1,\ldots,\frac{cv-b-ip}{2c}+1;-e^{2icz}\right)+$$
$$\sum_{s=0}^{\left\lfloor\frac{\nu-1}{2}\right\rfloor}(-1)^s\binom{\nu}{s}\left(i^\nu e^{(-ib+p+2ics)z}\sum_{j=0}^{n}\frac{(-1)^j(-ib+p+2ics)^{-j-1}z^{n-j}}{(n-j)!}\right.$$
$${}_{j+2}F_{j+1}\left(\frac{-b-ip+2cs}{2c},\ldots,\frac{-b-ip+2cs}{2c},v;\frac{-b-ip+2cs}{2c}+1,\ldots,\frac{-b-ip+2cs}{2c}+1;-e^{2icz}\right)+$$
$$i^{-\nu}e^{(-ib+p+2ic(\nu-s))z}\sum_{j=0}^{n}\frac{(-1)^j(-ib+p+2ic(\nu-s))^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{-b-ip+2c(-s+\nu)}{2c},\ldots,\right.$$
$$\left.\left.\frac{-b-ip+2c(-s+\nu)}{2c},v;\frac{-b-ip+2c(-s+\nu)}{2c}+1,\ldots,\frac{-b-ip+2c(-s+\nu)}{2c}+1;-e^{2icz}\right)\right)+$$
$$e^{(ib+p+icv)z}\binom{\nu}{\frac{\nu}{2}}(1-\nu \bmod 2)\sum_{j=0}^{n}\frac{(-1)^j(ib+p+icv)^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{cv+b-ip}{2c},\ldots,\right.$$
$$\left.\frac{cv+b-ip}{2c},v;\frac{cv+b-ip}{2c}+1,\ldots,\frac{cv+b-ip}{2c}+1;-e^{2icz}\right)+$$
$$\sum_{s=0}^{\left\lfloor\frac{\nu-1}{2}\right\rfloor}(-1)^s\binom{\nu}{s}\left(i^\nu e^{(ib+p+2ics)z}\sum_{j=0}^{n}\frac{(-1)^j(ib+p+2ics)^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{b-ip+2cs}{2c},\ldots,\right.\right.$$
$$\left.\frac{b-ip+2cs}{2c},v;\frac{b-ip+2cs}{2c}+1,\ldots,\frac{b-ip+2cs}{2c}+1;-e^{2icz}\right)+i^{-\nu}e^{(ib+p+2ic(\nu-s))z}$$
$$\sum_{j=0}^{n}\frac{(-1)^j(ib+p+2ic(\nu-s))^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{b-ip+2c(-s+\nu)}{2c},\ldots,\frac{b-ip+2c(-s+\nu)}{2c},\right.$$
$$\left.\left.\left.v;\frac{b-ip+2c(-s+\nu)}{2c}+1,\ldots,\frac{b-ip+2c(-s+\nu)}{2c}+1;-e^{2icz}\right)\right)\right)\;/;\;n\in\mathbb{N}\land\nu\in\mathbb{N}^+$$

Involving powers of cos, exp and power

Involving $z^n e^{pz}\cos^m(bz)\tan^\nu(cz)$

01.08.21.0233.01

$$\int z^n e^{pz}\cos^m(bz)\tan^\nu(cz)\,dz = 2^{-m}\binom{m}{\frac{m}{2}}(1-m\bmod 2)\,n!\left(e^{(p+icv)z}\binom{v}{\frac{v}{2}}(1-v\bmod 2)\right.$$

$$\sum_{j=0}^{n}\frac{(-1)^j(p+icv)^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{cv-ip}{2c},\ldots,\frac{cv-ip}{2c},v;\frac{cv-ip}{2c}+1,\ldots,\frac{cv-ip}{2c}+1;-e^{2icz}\right)+$$

$$i^{-v}\sum_{s=0}^{\left\lfloor\frac{v-1}{2}\right\rfloor}(-1)^s\binom{v}{s}\left((-1)^v e^{(p+2ics)z}\sum_{j=0}^{n}\frac{(-1)^j(p+2ics)^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{2cs-ip}{2c},\ldots,\frac{2cs-ip}{2c},v;\frac{2cs-ip}{2c}+\right.\right.$$

$$\left.1,\ldots,\frac{2cs-ip}{2c}+1;-e^{2icz}\right)+e^{(p+2ci(v-s))z}\sum_{j=0}^{n}\frac{(-1)^j(p+2ci(v-s))^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}$$

$$\left.\left.\left(\frac{2c(v-s)-ip}{2c},\ldots,\frac{2c(v-s)-ip}{2c},v;\frac{2c(v-s)-ip}{2c}+1,\ldots,\frac{2c(v-s)-ip}{2c}+1;-e^{2icz}\right)\right)\right)+$$

$$2^{-m}\,n!\,i^{-v}\sum_{k=0}^{\left\lfloor\frac{m-1}{2}\right\rfloor}\binom{m}{k}\left(i^v e^{(-ib(m-2k)+p+icv)z}\binom{v}{\frac{v}{2}}(1-v\bmod 2)\sum_{j=0}^{n}\frac{(-1)^j(-ib(m-2k)+p+icv)^{-j-1}z^{n-j}}{(n-j)!}\right.$$

$${}_{j+2}F_{j+1}\left(\frac{-b(m-2k)-ip+cv}{2c},\ldots,\frac{-b(m-2k)-ip+cv}{2c},v;\right.$$

$$\left.\frac{-b(m-2k)-ip+cv}{2c}+1,\ldots,\frac{-b(m-2k)-ip+cv}{2c}+1;-e^{2icz}\right)+$$

$$\sum_{s=0}^{\left\lfloor\frac{v-1}{2}\right\rfloor}(-1)^s\binom{v}{s}\left((-1)^v e^{(2ibk-ibm+p+2ics)z}\sum_{j=0}^{n}\frac{(-1)^j(-ib(m-2k)+p+2ics)^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\right.$$

$$\left(\frac{-b(m-2k)-ip+2cs}{2c},\ldots,\frac{-b(m-2k)-ip+2cs}{2c},v;\right.$$

$$\left.\frac{-b(m-2k)-ip+2cs}{2c}+1,\ldots,\frac{-b(m-2k)-ip+2cs}{2c}+1;-e^{2icz}\right)+$$

$$e^{(2ibk-ibm+p-2ics+2icv)z}\sum_{j=0}^{n}\frac{(-1)^j(-ib(m-2k)+p+2ci(v-s))^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}$$

$$\left(\frac{-b(m-2k)-ip+2c(v-s)}{2c},\ldots,\frac{-b(m-2k)-ip+2c(v-s)}{2c},v;\right.$$

$$\left.\left.\frac{-b(m-2k)-ip+2c(v-s)}{2c}+1,\ldots,\frac{-b(m-2k)-ip+2c(v-s)}{2c}+1;-e^{2icz}\right)\right)+$$

$$i^v e^{(bi(m-2k)+p+icv)z}\binom{v}{\frac{v}{2}}(1-v\bmod 2)\sum_{j=0}^{n}\frac{(-1)^j(bi(m-2k)+p+icv)^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{b(m-2k)-ip+cv}{2c},\right.$$

$$\left.\ldots,\frac{b(m-2k)-ip+cv}{2c},v;\frac{b(m-2k)-ip+cv}{2c}+1,\ldots,\frac{b(m-2k)-ip+cv}{2c}+1;-e^{2icz}\right)+$$

$$\sum_{s=0}^{\left\lfloor \frac{v-1}{2} \right\rfloor} (-1)^s \binom{v}{s} \left((-1)^v e^{(b\, i\, (m-2\,k)+p+2\, i\, c\, s)\, z} \sum_{j=0}^{n} \frac{(-1)^j \,(b\, i\, (m-2\,k)+p+2\, i\, c\, s)^{-j-1}\, z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{b\,(m-2\,k)-i\,p+2\,c\,s}{2\,c}, \ldots, \frac{b\,(m-2\,k)-i\,p+2\,c\,s}{2\,c}, v; \frac{b\,(m-2\,k)-i\,p+2\,c\,s}{2\,c}+1, \ldots, \frac{b\,(m-2\,k)-i\,p+2\,c\,s}{2\,c}+1; -e^{2\,i\,c\,z} \right) + e^{(b\, i\, (m-2\,k)+p+2\,c\, i\, (v-s))\, z} \sum_{j=0}^{n} \frac{(-1)^j\, (b\, i\, (m-2\,k)+p+2\,c\, i\, (v-s))^{-j-1}\, z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{b\,(m-2\,k)-i\,p+2\,c\,(v-s)}{2\,c}, \ldots, \frac{b\,(m-2\,k)-i\,p+2\,c\,(v-s)}{2\,c}, v; \frac{b\,(m-2\,k)-i\,p+2\,c\,(v-s)}{2\,c}+1, \ldots, \frac{b\,(m-2\,k)-i\,p+2\,c\,(v-s)}{2\,c}+1; -e^{2\,i\,c\,z} \right) \right) /;\; n \in \mathbb{N} \land u \in \mathbb{N}^+ \land v \in \mathbb{N}^+$$

Definite integration

For the direct function itself

01.08.21.0009.01

$$\int_0^{\frac{\pi}{4}} t \tan(t)\, dt = \frac{1}{16} \left(8\, C + i\, \pi^2 - 4\, \pi \log(1+i) \right)$$

01.08.21.0010.01

$$\int_0^{\frac{\pi}{4}} t^2 \tan(t)\, dt = \frac{1}{64} \left(-\pi^2 \log(4) - 21\, \zeta(3) + 16\, C\, \pi \right)$$

Involving the direct function

01.08.21.0238.01

$$\int_0^{z} \tan^{2\,n}(t)\, dt = (-1)^n z + (-1)^n \cot(z) \sum_{j=1}^{n} \frac{(-1)^j \tan^{2\,j}(z)}{2\,j-1} /;\; n \in \mathbb{N}$$

01.08.21.0011.01

$$\int_0^{\frac{\pi}{2}} \tan^n(t)\, dt = \frac{1}{2} \pi \sec\left(\frac{n\, \pi}{2} \right) /;\; |\mathrm{Re}(n)| < 1$$

Involving related functions

01.08.21.0012.01

$$\int_0^{\frac{\pi}{4}} \log(\tan(t))\, dt = -C$$

01.08.21.0239.01

$$\int_0^{z} \cos(t) \tan^{2\,n}(t)\, dt = \frac{\sin^{2\,n+1}(z)}{2\,n+1} \, {}_2F_1\left(n+\frac{1}{2}, n; n+\frac{3}{2}; \sin^2(z) \right) /;\; n \in \mathbb{N}$$

Summation

Finite summation

01.08.23.0001.02

$$\sum_{k=0}^{n} \frac{1}{2^k} \tan\left(\frac{z}{2^k}\right) = \frac{1}{2^n} \cot\left(\frac{z}{2^n}\right) - 2\cot(2z) /; n \in \mathbb{N}$$

01.08.23.0002.01

$$\sum_{k=1}^{n-1} \tan^2\left(\frac{\pi k}{n}\right) = n(n-1) /; \frac{n-1}{2} \in \mathbb{N}^+$$

01.08.23.0003.01

$$n \sum_{k=1}^{m-1} \frac{\tan\left(\frac{n\pi k}{m}\right)}{\tan\left(\frac{2\pi k}{m}\right)} + m \sum_{k=1}^{n-1} \frac{\tan\left(\frac{m\pi k}{n}\right)}{\tan\left(\frac{2\pi k}{n}\right)} = -\frac{1}{2}(m-n)^2 /; m \in \mathbb{N}^+ \bigwedge n \in \mathbb{N}^+ \bigwedge \gcd(n, m) = 1$$

01.08.23.0004.01

$$\sum_{k=0}^{n-1} (-1)^k \tan\left(\frac{(2k+1)\pi}{4n}\right) = (-1)^{n-1} n /; n \in \mathbb{N}$$

01.08.23.0005.01

$$\sum_{k=0}^{n-1} \tan^2\left(\frac{\pi k}{n} + z\right) = \cot^2\left(z n + \frac{\pi n}{2}\right) n^2 + n^2 - n /; n \in \mathbb{N}^+$$

01.08.23.0006.01

$$\sum_{k=1}^{n} \tan^4\left(\frac{k\pi}{2n+1}\right) = \frac{1}{3} n(2n+1)\left(4n^2 + 6n - 1\right) /; n \in \mathbb{N}$$

01.08.23.0007.01

$$\sum_{k=1}^{\left\lfloor \frac{n-1}{2} \right\rfloor} \tan^2\left(\frac{k\pi}{n}\right) = \frac{1}{6}(n-1)\left(-(-1)^n (n+1) + 2n - 1\right) /; n \in \mathbb{N}^+$$

Infinite summation

01.08.23.0008.01

$$\sum_{k=1}^{\infty} \frac{\tan\left(\frac{z}{2^k}\right)}{2^k} = \frac{1}{z} - \cot(z)$$

01.08.23.0009.01

$$\sum_{k=1}^{\infty} \frac{\tan^2\left(\frac{z}{2^k}\right)}{2^{2k}} = \csc^2(z) - \frac{1}{3} - \frac{1}{z^2}$$

Products

Finite products

01.08.24.0001.01

$$\prod_{k=1}^{n-1} \tan\left(\frac{k\pi}{n}\right) = (-1)^{(n-1)/2} n /; 2n+1 \in \mathbb{N}^+$$

Infinite Products

01.08.24.0002.01

$$\left|\prod_{k=0}^{\infty} \sqrt[2^k]{\tan(2^k x)}\right| = 4 \sin^2(x) \;/;\; x \in \mathbb{R}$$

Representations through more general functions

Through hypergeometric functions

01.08.26.0007.01

$$\tan(z) = \frac{8 z}{\pi^2 - 4 z^2} \, {}_3F_2\left(1, \frac{1}{2} - \frac{z}{\pi}, \frac{z}{\pi} + \frac{1}{2}; \frac{3}{2} - \frac{z}{\pi}, \frac{z}{\pi} + \frac{3}{2}; 1\right)$$

Brychkov Yu.A. (2005)

Through other functions

Involving Jacobi functions

01.08.26.0001.01

$$\tan(z) = \mathrm{cs}\left(\frac{\pi}{2} - z \,\middle|\, 0\right)$$

01.08.26.0002.01

$$\tan(z) = i \,\mathrm{ns}\left(\frac{\pi i}{2} - i z \,\middle|\, 1\right)$$

01.08.26.0003.01

$$\tan(z) = \mathrm{sc}(z \mid 0)$$

01.08.26.0004.01

$$\tan(z) = -i \,\mathrm{sn}(i z \mid 1)$$

Involving Mathieu functions

01.08.26.0005.01

$$\tan\left(\sqrt{a}\, z\right) = \frac{\mathrm{Se}(a, 0, z)}{\mathrm{Ce}(a, 0, z)}$$

01.08.26.0006.01

$$\tan\left(\sqrt{a}\, z\right) = -\frac{\mathrm{Ce}_z(a, 0, z)}{\mathrm{Se}_z(a, 0, z)}$$

Representations through equivalent functions

With inverse function

01.08.27.0001.01

$$\tan\left(\tan^{-1}(z)\right) = z$$

01.08.27.0002.01

$$\tan\left(n\tan^{-1}(z)\right) = -\frac{i\left((1+i z)^n - (1-i z)^n\right)}{(1+i z)^n + (1-i z)^n} \;/;\; n \in \mathbb{N}^+$$

01.08.27.0003.01

$$\tan^{-1}(\tan(z)) = z \;/;\; |\mathrm{Re}(z)| < \frac{\pi}{2} \bigvee \mathrm{Re}(z) = -\frac{\pi}{2} \bigwedge \mathrm{Im}(z) < 0 \bigvee \mathrm{Re}(z) = \frac{\pi}{2} \bigwedge \mathrm{Im}(z) > 0$$

01.08.27.0085.01

$$\tan^{-1}(\tan(z)) = z + \pi \;/;\; -\frac{3\pi}{2} < \mathrm{Re}(z) < -\frac{\pi}{2} \bigvee \mathrm{Re}(z) = -\frac{3\pi}{2} \bigwedge \mathrm{Im}(z) < 0 \bigvee \mathrm{Re}(z) = -\frac{\pi}{2} \bigwedge \mathrm{Im}(z) > 0$$

01.08.27.0086.01

$$\tan^{-1}(\tan(z)) = z - \pi \;/;\; \frac{\pi}{2} < \mathrm{Re}(z) < \frac{3\pi}{2} \bigvee \mathrm{Re}(z) = \frac{\pi}{2} \bigwedge \mathrm{Im}(z) < 0 \bigvee \mathrm{Re}(z) = \frac{3\pi}{2} \bigwedge \mathrm{Im}(z) > 0$$

01.08.27.0087.01

$$\tan^{-1}(\tan(z)) = z - \pi k \;/;\; \left(k\pi - \frac{\pi}{2} < \mathrm{Re}(z) < \pi k + \frac{\pi}{2} \bigvee \mathrm{Re}(z) = k\pi - \frac{\pi}{2} \bigwedge \mathrm{Im}(z) < 0 \bigvee \mathrm{Re}(z) = \pi k + \frac{\pi}{2} \bigwedge \mathrm{Im}(z) > 0\right) \bigwedge k \in \mathbb{Z}$$

01.08.27.0004.01

$$\tan^{-1}(\tan(z)) = z - \pi\left\lfloor \frac{\mathrm{Re}(z)}{\pi} + \frac{1}{2} \right\rfloor + \frac{1}{2}\left(1 + (-1)^{\left\lfloor \frac{\mathrm{Re}(z)}{\pi} - \frac{1}{2} \right\rfloor + \left\lfloor \frac{1}{2} - \frac{\mathrm{Re}(z)}{\pi} \right\rfloor}\right)\pi\,\theta(\mathrm{Im}(z)) \;/;\; \frac{z}{\pi} - \frac{1}{2} \notin \mathbb{Z}$$

01.08.27.0088.01

$$\tan^{-1}(\tan(z)) = \begin{cases} ¿ & \frac{2z+\pi}{2\pi} \in \mathbb{Z} \\ z - \pi\left\lfloor \frac{2\,\mathrm{Re}(z)-\pi}{2\pi} \right\rfloor & \frac{2\,\mathrm{Re}(z)+\pi}{2\pi} \in \mathbb{Z} \bigwedge \mathrm{Im}(z) > 0 \\ z - \pi\left\lfloor \frac{2\,\mathrm{Re}(z)+\pi}{2\pi} \right\rfloor & \text{True} \end{cases}$$

With related functions

Involving exp

01.08.27.0005.01

$$\tan(z) = \frac{i\left(e^{-iz} - e^{iz}\right)}{e^{-iz} + e^{iz}}$$

01.08.27.0006.01

$$\tan(z) = \frac{2i}{e^{2iz} + 1} - i$$

Involving sin

01.08.27.0007.01

$$\tan(z) = \frac{\sin(z)}{\sin\left(\frac{\pi}{2} - z\right)}$$

01.08.27.0008.01

$$\tan(z) = \frac{\sin(z)}{\sin\left(\frac{\pi}{2} + z\right)}$$

01.08.27.0009.01

$$\tan(z) = -\frac{\sqrt{-z^2}}{z} \frac{\sin(z)}{\sqrt{\sin^2(z) - 1}} \,/;\, 0 < \operatorname{Re}(z) < \pi$$

01.08.27.0010.01

$$\tan(z) = \frac{\sin(z)}{\sqrt{1 - \sin^2(z)}} \,/;\, |\operatorname{Re}(z)| < \frac{\pi}{2}$$

01.08.27.0011.01

$$\tan(z) = \frac{\sin(z)\,(-1)^{\left\lfloor \frac{1}{2} - \frac{\operatorname{Re}(z)}{\pi} \right\rfloor}}{\sqrt{1 - \sin^2(z)}} \left(1 - \left(1 + (-1)^{\left\lfloor \frac{\operatorname{Re}(z)}{\pi} - \frac{1}{2} \right\rfloor + \left\lfloor \frac{1}{2} - \frac{\operatorname{Re}(z)}{\pi} \right\rfloor}\right) \theta(\operatorname{Im}(z))\right)$$

01.08.27.0012.01

$$\tan^2(z) = \frac{\sin^2(z)}{1 - \sin^2(z)}$$

Involving cos

01.08.27.0013.01

$$\tan(z) = \frac{\cos\left(\frac{\pi}{2} - z\right)}{\cos(z)}$$

01.08.27.0014.01

$$\tan(z) = -\frac{\cos\left(\frac{\pi}{2} + z\right)}{\cos(z)}$$

01.08.27.0015.01

$$\tan(z) = -\frac{\sqrt{-z^2}}{z} \frac{\sqrt{\cos^2(z) - 1}}{\cos(z)} \,/;\, |\operatorname{Re}(z)| < \frac{\pi}{2}$$

01.08.27.0016.01

$$\tan(z) = \frac{\sqrt{z^2}}{z} \frac{\sqrt{1 - \cos^2(z)}}{\cos(z)} \,/;\, |\operatorname{Re}(z)| < \pi$$

01.08.27.0017.01

$$\tan(z) = \frac{\sqrt{1 - \cos^2(z)}}{\cos(z)} \,/;\, 0 < \operatorname{Re}(z) < \pi$$

01.08.27.0018.01

$$\tan(z) = \frac{\sqrt{1 - \cos^2(z)}}{\cos(z)} (-1)^{\left\lfloor \frac{\operatorname{Re}(z)}{\pi} \right\rfloor} \left(1 - \left((-1)^{\left\lfloor \frac{\operatorname{Re}(z)}{\pi} \right\rfloor + \left\lfloor -\frac{\operatorname{Re}(z)}{\pi} \right\rfloor} + 1\right) \theta(-\operatorname{Im}(z))\right)$$

01.08.27.0019.01

$$\tan^2(z) = \frac{1 - \cos^2(z)}{\cos^2(z)}$$

Involving cot

01.08.27.0020.01

$$\tan(z) = \cot\left(\frac{\pi}{2} - z\right)$$

01.08.27.0021.01

$$\tan(z) = -\cot\left(\frac{\pi}{2} + z\right)$$

01.08.27.0022.01

$$\tan(z) = -\cot\left(z - \frac{\pi}{2}\right)$$

01.08.27.0023.01

$$\tan(z) = \frac{1}{\cot(z)}$$

01.08.27.0024.01

$$\tan(z) = \frac{2\cot\left(\frac{z}{2}\right)}{\cot^2\left(\frac{z}{2}\right) - 1}$$

01.08.27.0025.01

$$\tan\left(\frac{\pi}{2} + z\right) = -\cot(z)$$

01.08.27.0026.01

$$\tan\left(\frac{\pi}{2} - z\right) = \cot(z)$$

01.08.27.0027.01

$$\tan(z) = \cot(z) - 2\cot(2z)$$

Involving csc

01.08.27.0028.01

$$\tan(z) = \frac{\csc\left(\frac{\pi}{2} - z\right)}{\csc(z)}$$

01.08.27.0029.01

$$\tan(z) = \frac{\csc\left(\frac{\pi}{2} + z\right)}{\csc(z)}$$

01.08.27.0030.01

$$\tan(z) = i\, e^{-i z} \csc\left(\frac{\pi}{2} - z\right) - i$$

01.08.27.0031.01

$$\tan(z) = i - i\, e^{i z} \csc\left(\frac{\pi}{2} - z\right)$$

01.08.27.0032.01

$$\tan(z) = -\frac{\sqrt{-z^2}}{z} \frac{1}{\sqrt{1 - \csc^2(z)}} \;/;\; \operatorname{Im}(z) \neq 0$$

01.08.27.0033.01

$$\tan(z) = \frac{1}{\sqrt{\csc^2(z) - 1}} \;/;\; 0 < \text{Re}(z) < \frac{\pi}{2}$$

01.08.27.0034.01

$$\tan(z) = z \sqrt{\frac{1}{z^2}} \, \frac{1}{\sqrt{\csc^2(z) - 1}} \;/;\; |\text{Re}(z)| < \frac{\pi}{2}$$

01.08.27.0035.01

$$\tan(z) = \frac{1}{\sqrt{\csc^2(z) - 1}} (-1)^{\left\lfloor \frac{2\,\text{Re}(z)}{\pi} \right\rfloor} \left(1 - \left(1 + (-1)^{\left\lfloor \frac{\text{Re}(z)}{\pi} \right\rfloor + \left\lfloor -\frac{\text{Re}(z)}{\pi} \right\rfloor}\right) \theta(\text{Im}(z))\right) \left(1 - \left(1 + (-1)^{\left\lfloor \frac{\text{Re}(z)}{\pi} + \frac{1}{2} \right\rfloor + \left\lfloor -\frac{\text{Re}(z)}{\pi} - \frac{1}{2} \right\rfloor}\right) \theta(-\text{Im}(z))\right)$$

01.08.27.0036.01

$$\tan^2(z) = \frac{1}{\csc^2(z) - 1}$$

01.08.27.0037.01

$$\tan^2(z) = \csc^2\left(\frac{\pi}{2} - z\right) - 1$$

Involving sec

01.08.27.0038.01

$$\tan(z) = \frac{\sec(z)}{\sec\left(\frac{\pi}{2} - z\right)}$$

01.08.27.0039.01

$$\tan(z) = -\frac{\sec(z)}{\sec\left(\frac{\pi}{2} + z\right)}$$

01.08.27.0040.01

$$\tan(z) = i - i\, e^{i z} \sec(z)$$

01.08.27.0041.01

$$\tan(z) = -i + i\, e^{-i z} \sec(z)$$

01.08.27.0042.01

$$\tan(z) = -\frac{\sqrt{-z^2}}{z} \sqrt{1 - \sec^2(z)} \;/;\; \text{Im}(z) \neq 0$$

01.08.27.0043.01

$$\tan(z) = \sqrt{\sec^2(z) - 1} \;/;\; 0 < \text{Re}(z) < \frac{\pi}{2}$$

01.08.27.0044.01

$$\tan(z) = \frac{\sqrt{z^2}}{z} \sqrt{\sec^2(z) - 1} \;/;\; |\text{Re}(z)| < \frac{\pi}{2}$$

01.08.27.0045.01

$$\tan(z) = \sqrt{\sec^2(z) - 1}\, (-1)^{\left\lfloor \frac{2\,\text{Re}(z)}{\pi} \right\rfloor} \left(1 - \left(1 + (-1)^{\left\lfloor \frac{\text{Re}(z)}{\pi} \right\rfloor + \left\lfloor -\frac{\text{Re}(z)}{\pi} \right\rfloor}\right) \theta(-\text{Im}(z))\right) \left(1 - \left(1 + (-1)^{\left\lfloor \frac{\text{Re}(z)}{\pi} + \frac{1}{2} \right\rfloor + \left\lfloor -\frac{\text{Re}(z)}{\pi} - \frac{1}{2} \right\rfloor}\right) \theta(\text{Im}(z))\right)$$

01.08.27.0046.01

$$\tan^2(z) = \sec^2(z) - 1$$

Involving sinh

01.08.27.0047.01

$$\tan(z) = \frac{\sinh(i\,z)}{\sinh\left(\frac{i\,\pi}{2} - i\,z\right)}$$

01.08.27.0048.01

$$\tan(z) = \frac{\sinh(i\,z)}{\sinh\left(\frac{i\,\pi}{2} + i\,z\right)}$$

01.08.27.0049.01

$$\tan(z) = -\frac{i\sinh(i\,z)}{\sqrt{1+\sinh^2(i\,z)}} \;/;\; |\mathrm{Re}(z)| < \frac{\pi}{2}$$

01.08.27.0050.01

$$\tan(z) = -\frac{i\sinh(i\,z)}{\sqrt{1+\sinh^2(i\,z)}} (-1)^{\left\lfloor \frac{1}{2} - \frac{\mathrm{Re}(z)}{\pi} \right\rfloor} \left(1 - \left(1 + (-1)^{\left\lfloor \frac{\mathrm{Re}(z)}{\pi} - \frac{1}{2} \right\rfloor + \left\lfloor \frac{1}{2} - \frac{\mathrm{Re}(z)}{\pi} \right\rfloor}\right) \theta(\mathrm{Im}(z))\right)$$

01.08.27.0051.01

$$\tan^2(z) = -\frac{\sinh^2(i\,z)}{\sinh^2(i\,z) + 1}$$

Involving cosh

01.08.27.0052.01

$$\tan(z) = \frac{\cosh\left(\frac{i\,\pi}{2} - i\,z\right)}{\cosh(i\,z)}$$

01.08.27.0053.01

$$\tan(z) = -\frac{\cosh\left(\frac{i\,\pi}{2} + i\,z\right)}{\cosh(i\,z)}$$

01.08.27.0054.01

$$\tan(z) = -\frac{\sqrt{-z^2}}{z} \frac{\sqrt{\cosh^2(i\,z) - 1}}{\cosh(i\,z)} \;/;\; |\mathrm{Re}(z)| < \frac{\pi}{2}$$

01.08.27.0055.01

$$\tan(z) = \frac{\sqrt{z^2}}{z} \frac{\sqrt{1 - \cosh^2(i\,z)}}{\cosh(i\,z)} \;/;\; |\mathrm{Re}(z)| < \pi$$

01.08.27.0056.01

$$\tan(z) = \frac{\sqrt{1 - \cosh^2(i\,z)}}{\cosh(i\,z)} \;/;\; 0 < \mathrm{Re}(z) < \pi$$

01.08.27.0057.01

$$\tan(z) = -\frac{\sqrt{1-\cosh^2(i z)}}{\cosh(i z)} (-1)^{\left\lfloor -\frac{\mathrm{Re}(z)}{\pi} \right\rfloor} \left(1 - \left(1 + (-1)^{\left\lfloor \frac{\mathrm{Re}(z)}{\pi} \right\rfloor + \left\lfloor -\frac{\mathrm{Re}(z)}{\pi} \right\rfloor}\right) \theta(\mathrm{Im}(z))\right)$$

01.08.27.0058.01

$$\tan^2(z) = \frac{1-\cosh^2(i z)}{\cosh^2(i z)}$$

Involving tanh

01.08.27.0059.01

$$\tan(z) = -i \tanh(i z)$$

01.08.27.0060.01

$$\tan(i z) = i \tanh(z)$$

Involving coth

01.08.27.0061.01

$$\tan(z) = i \coth\left(\frac{\pi i}{2} - i z\right)$$

01.08.27.0062.01

$$\tan(z) = -i \coth\left(\frac{\pi i}{2} + i z\right)$$

01.08.27.0063.01

$$\tan(z) = -\frac{i}{\coth(i z)}$$

Involving csch

01.08.27.0064.01

$$\tan(z) = \frac{\operatorname{csch}\left(\frac{\pi i}{2} - i z\right)}{\operatorname{csch}(i z)}$$

01.08.27.0065.01

$$\tan(z) = \frac{\operatorname{csch}\left(\frac{\pi i}{2} + i z\right)}{\operatorname{csch}(i z)}$$

01.08.27.0066.01

$$\tan(z) = e^{i z} \operatorname{csch}\left(\frac{\pi i}{2} - i z\right) + i$$

01.08.27.0067.01

$$\tan(z) = -e^{-i z} \operatorname{csch}\left(\frac{\pi i}{2} - i z\right) - i$$

01.08.27.0068.01

$$\tan(z) = -\frac{\sqrt{-z^2}}{z} \frac{1}{\sqrt{\operatorname{csch}^2(i z) + 1}} /; \mathrm{Im}(z) \neq 0$$

01.08.27.0069.01

$$\tan(z) = \frac{1}{\sqrt{-\operatorname{csch}^2(i z) - 1}} \;/;\; 0 < \operatorname{Re}(z) < \frac{\pi}{2}$$

01.08.27.0070.01

$$\tan(z) = z \sqrt{\frac{1}{z^2}} \frac{1}{\sqrt{-\operatorname{csch}^2(i z) - 1}} \;/;\; |\operatorname{Re}(z)| < \frac{\pi}{2}$$

01.08.27.0071.01

$$\tan(z) = \frac{1}{\sqrt{-\operatorname{csch}^2(i z) - 1}} (-1)^{\left\lfloor \frac{2 \operatorname{Re}(z)}{\pi} \right\rfloor} \left(1 - \left(1 + (-1)^{\left\lfloor \frac{\operatorname{Re}(z)}{\pi} \right\rfloor + \left\lfloor -\frac{\operatorname{Re}(z)}{\pi} \right\rfloor}\right) \theta(\operatorname{Im}(z))\right) \left(1 - \left(1 + (-1)^{\left\lfloor \frac{\operatorname{Re}(z)}{\pi} + \frac{1}{2} \right\rfloor + \left\lfloor -\frac{\operatorname{Re}(z)}{\pi} - \frac{1}{2} \right\rfloor}\right) \theta(-\operatorname{Im}(z))\right)$$

01.08.27.0072.01

$$\tan^2(z) = -\frac{1}{\operatorname{csch}^2(i z) + 1}$$

01.08.27.0073.01

$$\tan^2(z) = -\operatorname{csch}^2\left(\frac{\pi i}{2} - i z\right) - 1$$

Involving sech

01.08.27.0074.01

$$\tan(z) = \frac{\operatorname{sech}(i z)}{\operatorname{sech}\left(\frac{\pi i}{2} - i z\right)}$$

01.08.27.0075.01

$$\tan(z) = -\frac{\operatorname{sech}(i z)}{\operatorname{sech}\left(\frac{\pi i}{2} + i z\right)}$$

01.08.27.0076.01

$$\tan(z) = i - i e^{i z} \operatorname{sech}(i z)$$

01.08.27.0077.01

$$\tan(z) = i e^{-i z} \operatorname{sech}(i z) - i$$

01.08.27.0078.01

$$\tan(z) = -\frac{\sqrt{-z^2}}{z} \sqrt{1 - \operatorname{sech}^2(i z)} \;/;\; \operatorname{Re}(z) \neq 0$$

01.08.27.0079.01

$$\tan(z) = \sqrt{\operatorname{sech}^2(i z) - 1} \;/;\; 0 < \operatorname{Re}(z) < \frac{\pi}{2}$$

01.08.27.0080.01

$$\tan(z) = \frac{\sqrt{z^2}}{z} \sqrt{\operatorname{sech}^2(i z) - 1} \;/;\; |\operatorname{Re}(z)| < \frac{\pi}{2}$$

01.08.27.0081.01

$$\tan(z) = \sqrt{\operatorname{sech}^2(i z) - 1} \, (-1)^{\left\lfloor \frac{2 \operatorname{Re}(z)}{\pi} \right\rfloor} \left(1 - \left(1 + (-1)^{\left\lfloor \frac{\operatorname{Re}(z)}{\pi} \right\rfloor + \left\lfloor -\frac{\operatorname{Re}(z)}{\pi} \right\rfloor}\right) \theta(-\operatorname{Im}(z))\right) \left(1 - \left(1 + (-1)^{\left\lfloor \frac{\operatorname{Re}(z)}{\pi} + \frac{1}{2} \right\rfloor + \left\lfloor -\frac{\operatorname{Re}(z)}{\pi} - \frac{1}{2} \right\rfloor}\right) \theta(\operatorname{Im}(z))\right)$$

01.08.27.0082.01

$\tan^2(z) == \operatorname{sech}^2(i z) - 1$

Involving trigonometric and hyperbolic functions

01.08.27.0083.01

$$\tan(z) == \frac{\sin(z)}{\cos(z)}$$

01.08.27.0084.01

$\tan(z) + \cot(z) == 2\csc(2 z)$

Inequalities

01.08.29.0001.01

$$\tan(x) \geq x\ /;\ 0 \leq x < \frac{\pi}{2} \bigwedge x \in \mathbb{R}$$

Theorems

The law of tangents

For a triangle in the Euclidean plane with edges a, b, c and opposite angles α, β,γ, the following holds:

$$\frac{a+b}{a-b} == \frac{\tan((\alpha+\beta)/2)}{\tan((\alpha-\beta)/2)}, \frac{a+c}{a-c} == \frac{\tan((\alpha+\gamma)/2)}{\tan((\alpha-\gamma)/2)}, \frac{b+c}{b-c} == \frac{\tan((\beta+\gamma)/2)}{\tan((\beta-\gamma)/2)}.$$

Conformal mapping of the parabola

The function $w(z) = \tan^2\left(\pi \sqrt{z} \,/\, 4\right)$ maps the parabola with focus $z = 0$ and vertex $z = 1$ conformally to the unit disk.

Other information

Value properties

01.08.33.0001.01

$(x \in \mathbb{Q} \bigwedge \tan(x^\circ) \in \mathbb{Q}) \Rightarrow \tan(x) == 0 \vee \tan(x) == -1 \vee \tan(x) == 1$

History

– T. Finck (1583) used the word "tangent"
– E. Gunter (1624) used the notation "tan"
– E. Warina (1762)
– J. A. Seaner (1767)
– J. H. Lambert (1770) found a continued fraction representation of tan

The function tan is encountered often in mathematics and the natural sciences.

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