Tanh

Notations

Traditional name

Hyperbolic tangent

Traditional notation

$\tanh(z)$

Mathematica StandardForm notation

Tanh[z]

Primary definition

01.21.02.0001.01

$$\tanh(z) == \frac{\sinh(z)}{\cosh(z)} == \frac{e^z - e^{-z}}{e^z + e^{-z}}$$

Specific values

Specialized values

01.21.03.0001.01

$$\tanh\left(\pi\, i \left(\frac{1}{2} + m\right)\right) == \tilde{\infty} \;/;\; m \in \mathbb{Z}$$

01.21.03.0002.01

$$\tanh(\pi\, i\, m) == 0 \;/;\; m \in \mathbb{Z}$$

Values at fixed points

01.21.03.0003.01

$$\tanh(0) == 0$$

01.21.03.0004.01

$$\tanh\left(\frac{\pi\, i}{12}\right) == i\left(2 - \sqrt{3}\right)$$

01.21.03.0005.01

$$\tanh\left(\frac{\pi\, i}{12}\right) == \left(z;\, z^4 + 14\, z^2 + 1\right)_4^{-1}$$

01.21.03.0006.01

$$\tanh\left(\frac{\pi i}{10}\right) == i\sqrt{1-\frac{2}{\sqrt{5}}}$$

01.21.03.0007.01

$$\tanh\left(\frac{\pi i}{10}\right) == \left(z;\, 5z^4+10z^2+1\right)_4^{-1}$$

01.21.03.0008.01

$$\tanh\left(\frac{\pi i}{9}\right) == -\frac{-\left(-1-i\sqrt{3}\right)^{4/3}+\left(-1+i\sqrt{3}\right)^{4/3}}{\left(-1-i\sqrt{3}\right)^{4/3}+\left(-1+i\sqrt{3}\right)^{4/3}}$$

01.21.03.0009.01

$$\tanh\left(\frac{\pi i}{9}\right) == \left(z;\, z^6+33z^4+27z^2+3\right)_4^{-1}$$

01.21.03.0010.01

$$\tanh\left(\frac{\pi i}{9}\right) == \frac{-1+(-1)^{2/9}}{1+(-1)^{2/9}}$$

01.21.03.0011.01

$$\tanh\left(\frac{\pi i}{8}\right) == i\left(\sqrt{2}-1\right)$$

01.21.03.0012.01

$$\tanh\left(\frac{\pi i}{8}\right) == \left(z;\, z^4+6z^2+1\right)_2^{-1}$$

01.21.03.0013.01

$$\tanh\left(\frac{\pi i}{8}\right) == \frac{-1+\sqrt[4]{-1}}{1+\sqrt[4]{-1}}$$

01.21.03.0014.01

$$\tanh\left(\frac{\pi i}{7}\right) = \left(-2\,i\,2^{2/3}\,7^{5/6}\sqrt[3]{1-3\,i\sqrt{3}} - 2\sqrt{7}\,\left(-i+\sqrt{3}\right)\sqrt[3]{7-\frac{i\sqrt{7}}{2}-\frac{3\sqrt{21}}{2}} + 2\,i\sqrt{7}\,\sqrt[3]{7+\frac{i\sqrt{7}}{2}+\frac{3\sqrt{21}}{2}} + 2\sqrt{21}\,\sqrt[3]{7+\frac{i\sqrt{7}}{2}+\frac{3\sqrt{21}}{2}} + \left(14-i\sqrt{7}-3\sqrt{21}\right)^{2/3}\sqrt[3]{14+i\sqrt{7}+3\sqrt{21}} + i\sqrt{3}\,\left(14-i\sqrt{7}-3\sqrt{21}\right)^{2/3}\sqrt[3]{14+i\sqrt{7}+3\sqrt{21}} - \sqrt[3]{14-i\sqrt{7}-3\sqrt{21}}\,\left(14+i\sqrt{7}+3\sqrt{21}\right)^{2/3} + i\sqrt{3}\,\sqrt[3]{14-i\sqrt{7}-3\sqrt{21}}\,\left(14+i\sqrt{7}+3\sqrt{21}\right)^{2/3}\right) \Big/ \left(2\,2^{2/3}\sqrt[3]{7-21\,i\sqrt{3}} + 2\sqrt{7}\,\left(-i+\sqrt{3}\right)\sqrt[3]{7-\frac{i\sqrt{7}}{2}-\frac{3\sqrt{21}}{2}} + 2\,i\sqrt{7}\,\sqrt[3]{7+\frac{i\sqrt{7}}{2}+\frac{3\sqrt{21}}{2}} + 2\sqrt{21}\,\sqrt[3]{7+\frac{i\sqrt{7}}{2}+\frac{3\sqrt{21}}{2}} + \left(14-i\sqrt{7}-3\sqrt{21}\right)^{2/3}\sqrt[3]{14+i\sqrt{7}+3\sqrt{21}} + i\sqrt{3}\,\left(14-i\sqrt{7}-3\sqrt{21}\right)^{2/3}\sqrt[3]{14+i\sqrt{7}+3\sqrt{21}} + \sqrt[3]{14-i\sqrt{7}-3\sqrt{21}}\,\left(14+i\sqrt{7}+3\sqrt{21}\right)^{2/3} - i\sqrt{3}\,\sqrt[3]{14-i\sqrt{7}-3\sqrt{21}}\,\left(14+i\sqrt{7}+3\sqrt{21}\right)^{2/3}\right)$$

01.21.03.0015.01

$$\tanh\left(\frac{\pi i}{7}\right) = \left(z;\, z^6+21\,z^4+35\,z^2+7\right)_6^{-1}$$

01.21.03.0016.01

$$\tanh\left(\frac{\pi i}{7}\right) = \frac{-1+(-1)^{2/7}}{1+(-1)^{2/7}}$$

01.21.03.0017.01

$$\tanh\left(\frac{\pi i}{6}\right) = \frac{i}{\sqrt{3}}$$

01.21.03.0018.01

$$\tanh\left(\frac{\pi i}{5}\right) = i\sqrt{5-2\sqrt{5}}$$

01.21.03.0019.01

$$\tanh\left(\frac{\pi i}{5}\right) = \left(z;\, z^4+10\,z^2+5\right)_4^{-1}$$

01.21.03.0020.01

$$\tanh\left(\frac{2\pi i}{9}\right) == \frac{-\sqrt[3]{-1-i\sqrt{3}}+\sqrt[3]{-1+i\sqrt{3}}}{\sqrt[3]{-1-i\sqrt{3}}+\sqrt[3]{-1+i\sqrt{3}}}$$

01.21.03.0021.01

$$\tanh\left(\frac{2\pi i}{9}\right) == \left(z;\, z^6+33z^4+27z^2+3\right)_6^{-1}$$

01.21.03.0022.01

$$\tanh\left(\frac{2\pi i}{9}\right) == \frac{-1+(-1)^{4/9}}{1+(-1)^{4/9}}$$

01.21.03.0023.01

$$\tanh\left(\frac{\pi i}{4}\right) == i$$

01.21.03.0024.01

$$\tanh\left(\frac{2\pi i}{7}\right) == \left(\sqrt[3]{\frac{7}{2}\left(1-3i\sqrt{3}\right)}\left(2i\,2^{2/3}\,7^{5/6}\sqrt[3]{1-3i\sqrt{3}}+4i\sqrt{7}\sqrt[3]{7-\frac{i\sqrt{7}}{2}-\frac{3\sqrt{21}}{2}}-2i\sqrt{7}\sqrt[3]{7+\frac{i\sqrt{7}}{2}+\frac{3\sqrt{21}}{2}}+ 2\sqrt{21}\sqrt[3]{7+\frac{i\sqrt{7}}{2}+\frac{3\sqrt{21}}{2}}-\left(14-i\sqrt{7}-3\sqrt{21}\right)^{2/3}\sqrt[3]{14+i\sqrt{7}+3\sqrt{21}}+ i\sqrt{3}\left(14-i\sqrt{7}-3\sqrt{21}\right)^{2/3}\sqrt[3]{14+i\sqrt{7}+3\sqrt{21}}-2\sqrt[3]{14-i\sqrt{7}-3\sqrt{21}}\left(14+i\sqrt{7}+3\sqrt{21}\right)^{2/3}\right)\right) \Big/ \left(2\left(7-21i\sqrt{3}+7\,2^{2/3}\sqrt[3]{7-21i\sqrt{3}}-\sqrt[3]{2}\left(7-21i\sqrt{3}\right)^{2/3}\right)\right)$$

01.21.03.0025.01

$$\tanh\left(\frac{2\pi i}{7}\right) == \left(z;\, z^6+21z^4+35z^2+7\right)_4^{-1}$$

01.21.03.0026.01

$$\tanh\left(\frac{2\pi i}{7}\right) == \frac{-1+(-1)^{4/7}}{1+(-1)^{4/7}}$$

01.21.03.0027.01

$$\tanh\left(\frac{3\pi i}{10}\right) == i\sqrt{1+\frac{2}{\sqrt{5}}}$$

01.21.03.0028.01

$$\tanh\left(\frac{3\pi i}{10}\right) == \left(z;\, 5z^4+10z^2+1\right)_2^{-1}$$

01.21.03.0029.01

$$\tanh\left(\frac{\pi i}{3}\right) = \sqrt{3}\, i$$

01.21.03.0030.01

$$\tanh\left(\frac{3\pi i}{8}\right) = i\left(1+\sqrt{2}\right)$$

01.21.03.0031.01

$$\tanh\left(\frac{3\pi i}{8}\right) = \left(z;\, z^4+6z^2+1\right)_4^{-1}$$

01.21.03.0032.01

$$\tanh\left(\frac{3\pi i}{8}\right) = \frac{-1+(-1)^{3/4}}{1+(-1)^{3/4}}$$

01.21.03.0033.01

$$\tanh\left(\frac{2\pi i}{5}\right) = i\sqrt{5+2\sqrt{5}}$$

01.21.03.0034.01

$$\tanh\left(\frac{2\pi i}{5}\right) = \left(z;\, z^4+10z^2+5\right)_2^{-1}$$

01.21.03.0035.01

$$\tanh\left(\frac{5\pi i}{12}\right) = i\left(2+\sqrt{3}\right)$$

01.21.03.0036.01

$$\tanh\left(\frac{5\pi i}{12}\right) = \left(z;\, z^4+14z^2+1\right)_2^{-1}$$

01.21.03.0037.01

$$\tanh\left(\frac{3\pi i}{7}\right) = \left(-i\, 2^{2/3}\, 7^{5/6} \sqrt[3]{1-3i\sqrt{3}} + \sqrt{7}\left(i+\sqrt{3}\right)\sqrt[3]{7-\frac{i\sqrt{7}}{2}-\frac{3\sqrt{21}}{2}} - \frac{1}{2} i \left(4\sqrt{7}\sqrt[3]{7+\frac{i\sqrt{7}}{2}+\frac{3\sqrt{21}}{2}} + \sqrt[3]{14-42i\sqrt{3}}\left(-2i\sqrt[3]{14-i\sqrt{7}-3\sqrt{21}} + \left(-i+\sqrt{3}\right)\sqrt[3]{14+i\sqrt{7}+3\sqrt{21}}\right)\right)\right) \Big/ \left(-2^{2/3}\sqrt[3]{7-21i\sqrt{3}} + \sqrt{7}\left(i+\sqrt{3}\right)\sqrt[3]{7-\frac{i\sqrt{7}}{2}-\frac{3\sqrt{21}}{2}} + 2i\sqrt{7}\sqrt[3]{7+\frac{i\sqrt{7}}{2}+\frac{3\sqrt{21}}{2}} + \left(14-i\sqrt{7}-3\sqrt{21}\right)^{2/3}\sqrt[3]{14+i\sqrt{7}+3\sqrt{21}} - \frac{1}{2}\sqrt[3]{14-i\sqrt{7}-3\sqrt{21}}\left(14+i\sqrt{7}+3\sqrt{21}\right)^{2/3} - \frac{1}{2} i\sqrt{3}\sqrt[3]{14-i\sqrt{7}-3\sqrt{21}}\left(14+i\sqrt{7}+3\sqrt{21}\right)^{2/3}\right)$$

01.21.03.0038.01

$$\tanh\left(\frac{3\pi i}{7}\right) == \left(z;\, z^6 + 21 z^4 + 35 z^2 + 7\right)_2^{-1}$$

01.21.03.0039.01

$$\tanh\left(\frac{3\pi i}{7}\right) == \frac{-1 + (-1)^{6/7}}{1 + (-1)^{6/7}}$$

01.21.03.0040.01

$$\tanh\left(\frac{4\pi i}{9}\right) == -\frac{\sqrt[3]{-1 + i\sqrt{3}}\,\left(-i + \sqrt{3}\right) + \sqrt[3]{-1 - i\sqrt{3}}\,\left(i + \sqrt{3}\right)}{\sqrt[3]{-1 + i\sqrt{3}}\,\left(-i + \sqrt{3}\right) - \sqrt[3]{-1 - i\sqrt{3}}\,\left(i + \sqrt{3}\right)}$$

01.21.03.0041.01

$$\tanh\left(\frac{4\pi i}{9}\right) == \left(z;\, z^6 + 33 z^4 + 27 z^2 + 3\right)_2^{-1}$$

01.21.03.0042.01

$$\tanh\left(\frac{4\pi i}{9}\right) == \frac{-1 + (-1)^{8/9}}{1 + (-1)^{8/9}}$$

01.21.03.0043.01

$$\tanh\left(\frac{\pi i}{2}\right) == \tilde{\infty}$$

01.21.03.0044.01

$$\tanh\left(\frac{5\pi i}{9}\right) == \frac{\sqrt[3]{-1 + i\sqrt{3}}\,\left(-i + \sqrt{3}\right) + \sqrt[3]{-1 - i\sqrt{3}}\,\left(i + \sqrt{3}\right)}{\sqrt[3]{-1 + i\sqrt{3}}\,\left(-i + \sqrt{3}\right) - \sqrt[3]{-1 - i\sqrt{3}}\,\left(i + \sqrt{3}\right)}$$

01.21.03.0045.01

$$\tanh\left(\frac{5\pi i}{9}\right) == \left(z;\, z^6 + 33 z^4 + 27 z^2 + 3\right)_1^{-1}$$

01.21.03.0046.01

$$\tanh\left(\frac{5\pi i}{9}\right) == \frac{1 + \sqrt[9]{-1}}{-1 + \sqrt[9]{-1}}$$

01.21.03.0047.01

$$\tanh\left(\frac{4\pi i}{7}\right) = \left(2\,2^{2/3}\,7^{5/6}\sqrt[3]{1-3\,i\sqrt{3}} + 2\,i\sqrt{7}\left(i+\sqrt{3}\right)\sqrt[3]{7-\frac{i\sqrt{7}}{2}-\frac{3\sqrt{21}}{2}} + 4\sqrt{7}\sqrt[3]{7+\frac{i\sqrt{7}}{2}+\frac{3\sqrt{21}}{2}} - 2\,i\left(14-i\sqrt{7}-3\sqrt{21}\right)^{2/3}\sqrt[3]{14+i\sqrt{7}+3\sqrt{21}} - i\sqrt[3]{14-i\sqrt{7}-3\sqrt{21}}\left(14+i\sqrt{7}+3\sqrt{21}\right)^{2/3} + \sqrt{3}\sqrt[3]{14-i\sqrt{7}-3\sqrt{21}}\left(14+i\sqrt{7}+3\sqrt{21}\right)^{2/3}\right) \Big/ \left(2\left(7\,i-7\sqrt{3}+i\left(\frac{7}{2}\left(1-3\,i\sqrt{3}\right)\right)^{2/3}+\sqrt{3}\left(\frac{7}{2}-\frac{21\,i\sqrt{3}}{2}\right)^{2/3}+i\,2^{2/3}\sqrt[3]{7-21\,i\sqrt{3}}\right)\right)$$

01.21.03.0048.01

$$\tanh\left(\frac{4\pi i}{7}\right) = \left(z;\, z^6+21\,z^4+35\,z^2+7\right)_1^{-1}$$

01.21.03.0049.01

$$\tanh\left(\frac{4\pi i}{7}\right) = \frac{1+\sqrt[7]{-1}}{-1+\sqrt[7]{-1}}$$

01.21.03.0050.01

$$\tanh\left(\frac{7\pi i}{12}\right) = -i\left(2+\sqrt{3}\right)$$

01.21.03.0051.01

$$\tanh\left(\frac{7\pi i}{12}\right) = \left(z;\, z^4+14\,z^2+1\right)_1^{-1}$$

01.21.03.0052.01

$$\tanh\left(\frac{3\pi i}{5}\right) = -i\sqrt{5+2\sqrt{5}}$$

01.21.03.0053.01

$$\tanh\left(\frac{3\pi i}{5}\right) = \left(z;\, z^4+10\,z^2+5\right)_1^{-1}$$

01.21.03.0054.01

$$\tanh\left(\frac{5\pi i}{8}\right) = -i\left(1+\sqrt{2}\right)$$

01.21.03.0055.01

$$\tanh\left(\frac{5\pi i}{8}\right) = \left(z;\, z^4+6\,z^2+1\right)_3^{-1}$$

01.21.03.0056.01

$$\tanh\left(\frac{5\pi i}{8}\right) = \frac{1+\sqrt[4]{-1}}{-1+\sqrt[4]{-1}}$$

01.21.03.0057.01

$$\tanh\left(\frac{2\pi i}{3}\right) == -\sqrt{3}\, i$$

01.21.03.0058.01

$$\tanh\left(\frac{7\pi i}{10}\right) == -i\sqrt{1+\frac{2}{\sqrt{5}}}$$

01.21.03.0059.01

$$\tanh\left(\frac{7\pi i}{10}\right) == \left(z;\, 5z^4+10z^2+1\right)_1^{-1}$$

01.21.03.0060.01

$$\tanh\left(\frac{5\pi i}{7}\right) == \left(2\, i\, 2^{2/3}\, 7^{5/6}\sqrt[3]{1-3\, i\sqrt{3}} + 4\, i\sqrt{7}\sqrt[3]{7-\frac{i\sqrt{7}}{2}-\frac{3\sqrt{21}}{2}} - 2\, i\sqrt{7}\sqrt[3]{7+\frac{i\sqrt{7}}{2}+\frac{3\sqrt{21}}{2}} + 2\sqrt{21}\sqrt[3]{7+\frac{i\sqrt{7}}{2}+\frac{3\sqrt{21}}{2}} - \left(14-i\sqrt{7}-3\sqrt{21}\right)^{2/3}\sqrt[3]{14+i\sqrt{7}+3\sqrt{21}} + i\sqrt{3}\left(14-i\sqrt{7}-3\sqrt{21}\right)^{2/3}\sqrt[3]{14+i\sqrt{7}+3\sqrt{21}} - 2\sqrt[3]{14-i\sqrt{7}-3\sqrt{21}}\left(14+i\sqrt{7}+3\sqrt{21}\right)^{2/3}\right) \Big/ \left(2\, 2^{2/3}\sqrt[3]{7-21\, i\sqrt{3}} + 4\, i\sqrt{7}\sqrt[3]{7-\frac{i\sqrt{7}}{2}-\frac{3\sqrt{21}}{2}} + 2\, i\sqrt{7}\sqrt[3]{7+\frac{i\sqrt{7}}{2}+\frac{3\sqrt{21}}{2}} - 2\sqrt{21}\sqrt[3]{7+\frac{i\sqrt{7}}{2}+\frac{3\sqrt{21}}{2}} + \left(14-i\sqrt{7}-3\sqrt{21}\right)^{2/3}\sqrt[3]{14+i\sqrt{7}+3\sqrt{21}} - i\sqrt{3}\left(14-i\sqrt{7}-3\sqrt{21}\right)^{2/3}\sqrt[3]{14+i\sqrt{7}+3\sqrt{21}} - 2\sqrt[3]{14-i\sqrt{7}-3\sqrt{21}}\left(14+i\sqrt{7}+3\sqrt{21}\right)^{2/3}\right)$$

01.21.03.0061.01

$$\tanh\left(\frac{5\pi i}{7}\right) == \left(z;\, z^6+21z^4+35z^2+7\right)_3^{-1}$$

01.21.03.0062.01

$$\tanh\left(\frac{5\pi i}{7}\right) == \frac{1+(-1)^{3/7}}{-1+(-1)^{3/7}}$$

01.21.03.0063.01

$$\tanh\left(\frac{3\pi i}{4}\right) == -i$$

01.21.03.0064.01

$$\tanh\left(\frac{7\pi i}{9}\right) = \frac{\sqrt[3]{-1-i\sqrt{3}} - \sqrt[3]{-1+i\sqrt{3}}}{\sqrt[3]{-1-i\sqrt{3}} + \sqrt[3]{-1+i\sqrt{3}}}$$

01.21.03.0065.01

$$\tanh\left(\frac{7\pi i}{9}\right) = \left(z;\, z^6 + 33 z^4 + 27 z^2 + 3\right)_5^{-1}$$

01.21.03.0066.01

$$\tanh\left(\frac{7\pi i}{9}\right) = \frac{1+(-1)^{5/9}}{-1+(-1)^{5/9}}$$

01.21.03.0067.01

$$\tanh\left(\frac{4\pi i}{5}\right) = -i\sqrt{5-2\sqrt{5}}$$

01.21.03.0068.01

$$\tanh\left(\frac{4\pi i}{5}\right) = \left(z;\, z^4 + 10 z^2 + 5\right)_3^{-1}$$

01.21.03.0069.01

$$\tanh\left(\frac{5\pi i}{6}\right) = -\frac{i}{\sqrt{3}}$$

01.21.03.0070.01

$$\tanh\left(\frac{6\pi i}{7}\right) = -\left(\sqrt[3]{\frac{7}{2}\left(1-3i\sqrt{3}\right)}\left(-4i\sqrt{7} + \frac{2\sqrt{7}\left(i+\sqrt{3}\right)}{\sqrt[3]{7-\frac{i\sqrt{7}}{2}-\frac{3\sqrt{21}}{2}}} + \left(2+2i\sqrt{3}\right)\sqrt[3]{7-\frac{i\sqrt{7}}{2}-\frac{3\sqrt{21}}{2}} - \frac{2\sqrt{7}\left(-i+\sqrt{3}\right)}{\sqrt[3]{7+\frac{i\sqrt{7}}{2}+\frac{3\sqrt{21}}{2}}} + 2i\left(i+\sqrt{3}\right)\sqrt[3]{7+\frac{i\sqrt{7}}{2}+\frac{3\sqrt{21}}{2}}\right)\right) \Big/ \left(2\left(7-7i\sqrt{3}+\left(\frac{7}{2}\left(1-3i\sqrt{3}\right)\right)^{2/3} + i\sqrt{3}\left(\frac{7}{2}-\frac{21 i\sqrt{3}}{2}\right)^{2/3} + 2^{2/3}\sqrt[3]{7-21i\sqrt{3}}\right)\right)$$

01.21.03.0071.01

$$\tanh\left(\frac{6\pi i}{7}\right) = \left(z;\, z^6 + 21 z^4 + 35 z^2 + 7\right)_5^{-1}$$

01.21.03.0072.01

$$\tanh\left(\frac{6\pi i}{7}\right) = \frac{1+(-1)^{5/7}}{-1+(-1)^{5/7}}$$

01.21.03.0073.01

$$\tanh\left(\frac{7\pi i}{8}\right) == i\left(1-\sqrt{2}\right)$$

01.21.03.0074.01

$$\tanh\left(\frac{7\pi i}{8}\right) == \left(z;\, z^4+6z^2+1\right)_1^{-1}$$

01.21.03.0075.01

$$\tanh\left(\frac{7\pi i}{8}\right) == \frac{1+(-1)^{3/4}}{-1+(-1)^{3/4}}$$

01.21.03.0076.01

$$\tanh\left(\frac{8\pi i}{9}\right) == \frac{-\left(-1-i\sqrt{3}\right)^{4/3}+\left(-1+i\sqrt{3}\right)^{4/3}}{\left(-1-i\sqrt{3}\right)^{4/3}+\left(-1+i\sqrt{3}\right)^{4/3}}$$

01.21.03.0077.01

$$\tanh\left(\frac{8\pi i}{9}\right) == \left(z;\, z^6+33z^4+27z^2+3\right)_3^{-1}$$

01.21.03.0078.01

$$\tanh\left(\frac{8\pi i}{9}\right) == \frac{1+(-1)^{7/9}}{-1+(-1)^{7/9}}$$

01.21.03.0079.01

$$\tanh\left(\frac{9\pi i}{10}\right) == -i\sqrt{1-\frac{2}{\sqrt{5}}}$$

01.21.03.0080.01

$$\tanh\left(\frac{9\pi i}{10}\right) == \left(z;\, 5z^4+10z^2+1\right)_3^{-1}$$

01.21.03.0081.01

$$\tanh\left(\frac{11\pi i}{12}\right) == -i\left(2-\sqrt{3}\right)$$

01.21.03.0082.01

$$\tanh\left(\frac{11\pi i}{12}\right) == \left(z;\, z^4+14z^2+1\right)_3^{-1}$$

01.21.03.0083.01

$$\tanh(\pi i) == 0$$

01.21.03.0084.01

$$\tanh\left(\frac{\pi i}{17}\right) == i \Big/ \left(\sqrt{\left(\left(\sqrt{\left(2\left(\sqrt{34\left(17-\sqrt{17}\right)}+6\sqrt{17}-8\sqrt{2\left(17+\sqrt{17}\right)}-\sqrt{34-2\sqrt{17}}+34\right)\right)}\right)+\sqrt{17}+\sqrt{34-2\sqrt{17}}+15\right) \Big/ \left(16-2\sqrt{\left(2\left(\sqrt{\left(2\left(-\sqrt{34\left(17-\sqrt{17}\right)}+6\sqrt{17}+8\sqrt{2\left(17+\sqrt{17}\right)}+\sqrt{34-2\sqrt{17}}+34\right)\right)}+\sqrt{17}-\sqrt{34-2\sqrt{17}}+15\right)\right)}\right)}\right)$$

01.21.03.0085.01

$$\tanh\left(\frac{\pi i}{30}\right) == i\sqrt{7-2\sqrt{5}-2\sqrt{15-6\sqrt{5}}}$$

$\tanh\left(\frac{n i \pi}{m}\right)$ can be expressed using only square roots if $n \in \mathbb{Z}$ and m is a product of a power of 2 and distinct Fermat primes $\{3, 5, 17, 257, \ldots\}$.

Values at infinities

01.21.03.0086.01

$$\tanh(\infty) == 1$$

01.21.03.0087.01

$$\tanh(-\infty) == -1$$

01.21.03.0088.01

$$\tanh(\tilde{\infty}) == ¿$$

General characteristics

Domain and analyticity

$\tanh(z)$ is an analytical function of z which is defined over the whole complex z-plane with the exception of countably many points $z = \pi i/2 + k \pi i /; k \in \mathbb{Z}$.

01.21.04.0001.01

$$z \longrightarrow \tanh(z) :: \mathbb{C} \longrightarrow \mathbb{C}$$

Symmetries and periodicities

Parity

$\tanh(z)$ is an odd function.

01.21.04.0002.01

$$\tanh(-z) == -\tanh(z)$$

Mirror symmetry

01.21.04.0003.01

$$\tanh(\bar{z}) == \overline{\tanh(z)}$$

Periodicity

$\tanh(z)$ is a periodic function with period πi.

01.21.04.0009.01

$$\tanh(z + \pi i) == \tanh(z)$$

01.21.04.0004.01

$$\tanh(z + \pi i m) == \tanh(z) \text{ /; } m \in \mathbb{Z}$$

Poles and essential singularities

The function $\tanh(z)$ has an infinite set of singular points:

a) $z == \pi i / 2 + \pi k i$ /; $k \in \mathbb{Z}$ are the simple poles with residues 1;

b) $z == \tilde{\infty}$ is an essential singular point.

01.21.04.0005.01

$$\mathcal{S}ing_z(\tanh(z)) == \left\{\left\{\left\{\frac{\pi i}{2} + \pi k i, 1\right\} \text{ /; } k \in \mathbb{Z}\right\}, \{\tilde{\infty}, \infty\}\right\}$$

01.21.04.0006.01

$$\text{res}_z(\tanh(z))\left(\frac{\pi i}{2} + \pi k i\right) == 1 \text{ /; } k \in \mathbb{Z}$$

Branch points

The function $\tanh(z)$ does not have branch points.

01.21.04.0007.01

$$\mathcal{BP}_z(\tanh(z)) == \{\}$$

Branch cuts

The function $\tanh(z)$ does not have branch cuts.

01.21.04.0008.01

$$\mathcal{BC}_z(\tanh(z)) == \{\}$$

Series representations

Generalized power series

Expansions at $z == z_0$

For the function itself

01.21.06.0019.01

$$\tanh(z) \propto \tanh(z_0) + \text{sech}^2(z_0)(z - z_0) - \frac{1}{2}\sinh(2 z_0)\,\text{sech}^4(z_0)(z - z_0)^2 + \ldots \text{ /; } (z \to z_0)$$

01.21.06.0020.01

$$\tanh(z) \propto \tanh(z_0) + \operatorname{sech}^2(z_0)(z - z_0) - \frac{1}{2}\sinh(2 z_0)\operatorname{sech}^4(z_0)(z - z_0)^2 + O\left((z - z_0)^3\right)$$

01.21.06.0021.01

$$\tanh(z) = \sum_{k=0}^{\infty} \frac{1}{k!} \left(\tanh(z_0)\,\delta_k + \operatorname{sech}^2(z_0)\,\delta_{k-1} - i^k\, k \sum_{m=0}^{k-1} \sum_{j=0}^{m-1} \frac{(-1)^m}{m+1} \binom{k-1}{m} \cosh^{-2m-2}(z_0)\, 2^{k-2m} \binom{2m}{j} (m-j)^{k-1} \sinh\left(\frac{i\pi k}{2} + 2(j-m) z_0\right)\right)(z - z_0)^k$$

01.21.06.0022.01

$$\tanh(z) \propto \tanh(z_0)(1 + O(z - z_0))$$

Expansions at $z = 0$

For the function itself

01.21.06.0001.02

$$\tanh(z) \propto z - \frac{z^3}{3} + \frac{2 z^5}{15} - \ldots \;/;\; (z \to 0)$$

01.21.06.0023.01

$$\tanh(z) \propto z - \frac{z^3}{3} + \frac{2 z^5}{15} - O\left(z^7\right)$$

01.21.06.0002.01

$$\tanh(z) = \sum_{k=1}^{\infty} \frac{2^{2k}\left(2^{2k} - 1\right) B_{2k}\, z^{2k-1}}{(2k)!} \;/;\; |z| < \frac{\pi}{2}$$

01.21.06.0003.02

$$\tanh(z) \propto z + O\left(z^3\right)$$

Expansions at $z = \frac{\pi i}{2}$

For the function itself

01.21.06.0004.02

$$\tanh(z) \propto \frac{1}{z - \frac{\pi i}{2}} + \frac{1}{3}\left(z - \frac{\pi i}{2}\right) - \frac{1}{45}\left(z - \frac{\pi i}{2}\right)^3 + \frac{2}{945}\left(z - \frac{\pi i}{2}\right)^5 - \ldots \;/;\; \left(z \to \frac{\pi i}{2}\right)$$

01.21.06.0024.01

$$\tanh(z) \propto \frac{1}{z - \frac{\pi i}{2}} + \frac{1}{3}\left(z - \frac{\pi i}{2}\right) - \frac{1}{45}\left(z - \frac{\pi i}{2}\right)^3 + \frac{2}{945}\left(z - \frac{\pi i}{2}\right)^5 - O\left(\left(z - \frac{\pi i}{2}\right)^7\right)$$

01.21.06.0005.01

$$\tanh(z) = \frac{1}{z - \frac{\pi i}{2}} + \sum_{k=1}^{\infty} \frac{2^{2k} B_{2k}}{(2k)!}\left(z - \frac{\pi i}{2}\right)^{2k-1} \;/;\; \left|z - \frac{\pi i}{2}\right| < \pi$$

01.21.06.0006.02

$$\tanh(z) \propto \frac{1}{z-\frac{\pi i}{2}} + \frac{1}{3}\left(z-\frac{\pi i}{2}\right) + O\left(\left(z-\frac{\pi i}{2}\right)^3\right)$$

q-series

01.21.06.0007.01

$$\tanh(z) = -1 - 2\sum_{k=1}^{\infty} (-1)^k q^{2k} \;/;\; q = e^z$$

Dirichlet series

01.21.06.0008.01

$$\tanh(z) = 1 - 2\sum_{k=0}^{\infty} (-1)^k e^{-2z(k+1)} \;/;\; \operatorname{Re}(z) > 0$$

01.21.06.0009.01

$$\tanh(z) = 2\sum_{k=0}^{\infty} (-1)^k e^{2z(k+1)} - 1 \;/;\; \operatorname{Re}(z) < 0$$

Asymptotic series expansions

01.21.06.0010.01

$$\tanh(z) \propto 1 - 2e^{-2z}\, {}_1F_0\left(1;\, ;\, -e^{-2z}\right) \;/;\; \operatorname{Re}(z) > 0 \land (|z| \to \infty)$$

01.21.06.0011.01

$$\tanh(z) \propto 1 - 2e^{-2z}\left(1 + O\left(e^{-2z}\right)\right) \;/;\; \operatorname{Re}(z) > 0 \land (|z| \to \infty)$$

01.21.06.0012.01

$$\tanh(z) \propto -1 + 2e^{2z}\, {}_1F_0\left(1;\, ;\, -e^{2z}\right) \;/;\; \operatorname{Re}(z) < 0 \land (|z| \to \infty)$$

01.21.06.0013.01

$$\tanh(z) \propto -1 + 2e^{2z}\left(1 + O\left(e^{2z}\right)\right) \;/;\; \operatorname{Re}(z) < 0 \land (|z| \to \infty)$$

01.21.06.0014.01

$$\tanh(z) \propto \tanh(z) \;/;\; \operatorname{Re}(z) = 0 \land (|z| \to \infty)$$

01.21.06.0015.01

$$\tanh(z) \propto 1 \;/;\; \left(z \to e^{i\phi}\infty\right) \bigwedge -\frac{\pi}{2} < \phi < \frac{\pi}{2}$$

01.21.06.0016.01

$$\tanh(z) \propto -1 \;/;\; \left(z \to e^{i\phi}\infty\right) \bigwedge -\pi < \phi < -\frac{\pi}{2} \bigvee \frac{\pi}{2} < \phi \leq \pi$$

01.21.06.0025.01

$$\tanh(z) \propto \begin{cases} -1 & -\pi < \arg(z) < -\frac{\pi}{2} \bigvee \frac{\pi}{2} < \arg(z) \leq \pi \\ 1 & -\frac{\pi}{2} < \arg(z) < \frac{\pi}{2} \\ \tanh(z) & \text{True} \end{cases} \;/;\; (|z| \to \infty)$$

Other series representations

01.21.06.0017.01

$$\tanh(z) = 8\,z \sum_{k=1}^{\infty} \frac{1}{\pi^2\,(2\,k-1)^2 + 4\,z^2} \;/;\; \frac{i\,z}{\pi} - \frac{1}{2} \notin \mathbb{Z}$$

01.21.06.0018.01

$$\log\left(\frac{\tanh(z)}{z}\right) = -\sum_{k=1}^{\infty} \frac{2^{2\,k}\left(2^{2\,k-1}-1\right) B_{2\,k}\, z^{2\,k}}{k\,(2\,k)!} \;/;\; |z| < \frac{\pi}{2}$$

Integral representations

On the real axis

Of the direct function

01.21.07.0001.01

$$\tanh(z) = \int_0^z \operatorname{sech}^2(t)\,dt$$

01.21.07.0002.01

$$\tanh(z) = -\frac{2\,i}{\pi} \int_0^{\infty} \frac{t^{\frac{2\,i\,z}{\pi}} - 1}{t^2 - 1}\,dt \;/;\; -\frac{\pi}{2} < \operatorname{Im}(z) < 0$$

Limit representations

01.21.09.0001.01

$$\tanh(z) = \lim_{n\to\infty} \sum_{k=-n}^{n} \frac{1}{i\,\pi\left(\frac{1}{2}-k\right) + z} \;/;\; \frac{i\,z}{\pi} - \frac{1}{2} \notin \mathbb{Z}$$

Continued fraction representations

01.21.10.0001.01

$$\tanh(z) = \cfrac{z}{1 + \cfrac{z^2}{3 + \cfrac{z^2}{5 + \cfrac{z^2}{7 + \cfrac{z^2}{9 + \cfrac{z^2}{11 + \ldots}}}}}} \;/;\; \frac{i\,z}{\pi} - \frac{1}{2} \notin \mathbb{Z}$$

01.21.10.0002.01

$$\tanh(z) = \frac{1}{z} \mathrm{K}_k\left(z^2,\, 2\,k-1\right)_1^{\infty} \;/;\; \frac{i\,z}{\pi} - \frac{1}{2} \notin \mathbb{Z}$$

01.21.10.0003.01

$$\tanh(z) = \cfrac{1}{\cfrac{1}{z} + \cfrac{1}{\cfrac{3}{z} + \cfrac{1}{\cfrac{5}{z} + \cfrac{1}{\cfrac{7}{z} + \cfrac{1}{\cfrac{9}{z} + \cfrac{1}{\cfrac{11}{z} + \ldots}}}}}} \;/;\; \frac{i z}{\pi} - \frac{1}{2} \notin \mathbb{Z}$$

01.21.10.0004.01

$$\tanh(z) = \underset{k}{\mathrm{K}}\left(1, \frac{2k-1}{z}\right)_1^{\infty} \;/;\; \frac{i z}{\pi} - \frac{1}{2} \notin \mathbb{Z}$$

Differential equations

Ordinary nonlinear differential equations

01.21.13.0001.01

$$w'(z) + w(z)^2 - 1 = 0 \;/;\; w(z) = \tanh(z) \bigwedge w(0) = 0$$

01.21.13.0002.01

$$w'(z) - a\, w(z)^2 - b\, w(z) - c = 0 \;/;\; w(z) = -\frac{1}{2a}\left(b + \sqrt{b^2 - 4ac}\, \tanh\left(\frac{a\sqrt{b^2-4ac}\, z + \sqrt{b^2-4ac}\, c_1}{2a}\right)\right)$$

01.21.13.0003.01

$$w''(z) + 2\, w(z)\, w'(z) = 0 \;/;\; w(z) = c_1 \tanh\left(\sqrt{c_1}\, z + \sqrt{c_1}\, c_2\right)$$

Transformations

Transformations and argument simplifications

Argument involving basic arithmetic operations

01.21.16.0001.01

$$\tanh(-z) = -\tanh(z)$$

01.21.16.0002.01

$$\tanh(a\,(b\,z^c)^m) = \frac{(b\,z^c)^m \tanh(a\,b^m\,z^{m c})}{b^m\,z^{m c}} \;/;\; 2m \in \mathbb{Z}$$

01.21.16.0003.01

$$\tanh\left(\sqrt{z^2}\right) = \frac{\sqrt{z^2}\,\tanh(z)}{z}$$

Argument involving inverse trigonometric and hyperbolic functions

Involving $\sin^{-1}$

01.21.16.0063.01

$$\tanh\left(\sin^{-1}(z)\right) = \frac{2}{\left(i\,z + \sqrt{1-z^2}\right)^{2\,i} + 1} - 1$$

01.21.16.0004.01

$$\tanh\left(i\,\sin^{-1}(z)\right) == \frac{i\,z}{\sqrt{1-z^2}}$$

01.21.16.0016.01

$$\tanh\left(\frac{i}{2}\sin^{-1}(z)\right) == i\,\frac{1-\sqrt{1-z^2}}{z}$$

01.21.16.0064.01

$$\tanh\left(a\,\sin^{-1}(z)\right) = \frac{2}{\left(i\,z + \sqrt{1-z^2}\right)^{2\,i\,a} + 1} - 1$$

Involving $\cos^{-1}$

01.21.16.0065.01

$$\tanh\left(\cos^{-1}(z)\right) = 1 - \frac{2}{e^{\pi}\left(i\,z + \sqrt{1-z^2}\right)^{2\,i} + 1}$$

01.21.16.0005.01

$$\tanh\left(i\,\cos^{-1}(z)\right) == \frac{i\sqrt{1-z^2}}{z}$$

01.21.16.0017.01

$$\tanh\left(\frac{i}{2}\cos^{-1}(z)\right) == \frac{i\sqrt{1-z}}{\sqrt{1+z}}$$

01.21.16.0066.01

$$\tanh\left(a\,\cos^{-1}(z)\right) = 1 - \frac{2}{e^{a\,\pi}\left(i\,z + \sqrt{1-z^2}\right)^{2\,a\,i} + 1}$$

Involving $\tan^{-1}$

01.21.16.0067.01

$$\tanh\left(\tan^{-1}(z)\right) = \frac{2}{(i\,z+1)^{i}\,(1-i\,z)^{-i} + 1} - 1$$

01.21.16.0068.01

$$\tanh\left(\tan^{-1}(x,\,y)\right) = \frac{2}{\left(\frac{x+i\,y}{\sqrt{x^2+y^2}}\right)^{2\,i} + 1} - 1$$

01.21.16.0006.01

$$\tanh\left(i \tan^{-1}(z)\right) = i z$$

01.21.16.0069.01

$$\tanh\left(i \tan^{-1}(x, y)\right) = \frac{i y}{x}$$

01.21.16.0018.01

$$\tanh\left(\frac{i}{2} \tan^{-1}(z)\right) = \frac{i}{z}\left(\sqrt{1+z^2} - 1\right)$$

01.21.16.0070.01

$$\tanh\left(\frac{1}{2} i \tan^{-1}(x, y)\right) = \frac{i\left(\sqrt{x^2+y^2} - x\right)}{y}$$

01.21.16.0071.01

$$\tanh\left(a \tan^{-1}(z)\right) = \frac{2}{(i z+1)^{i a} (1 - i z)^{-i a} + 1} - 1$$

01.21.16.0072.01

$$\tanh\left(a \tan^{-1}(x, y)\right) = \frac{2}{\left(\frac{x+i y}{\sqrt{x^2+y^2}}\right)^{2 i a} + 1} - 1$$

Involving $\cot^{-1}$

01.21.16.0073.01

$$\tanh\left(\cot^{-1}(z)\right) = \frac{2}{\left(\frac{i+z}{z}\right)^{i} \left(\frac{-i+z}{z}\right)^{-i} + 1} - 1$$

01.21.16.0007.01

$$\tanh\left(i \cot^{-1}(z)\right) = \frac{i}{z}$$

01.21.16.0019.01

$$\tanh\left(\frac{i}{2} \cot^{-1}(z)\right) = i\left(\frac{\sqrt{z}\sqrt{-1-z^2}}{\sqrt{-z}} - z\right)$$

01.21.16.0074.01

$$\tanh\left(a \cot^{-1}(z)\right) = \frac{2}{\left(\frac{i+z}{z}\right)^{i a} \left(\frac{-i+z}{z}\right)^{-i a} + 1} - 1$$

Involving $\csc^{-1}$

01.21.16.0075.01

$$\tanh\left(\csc^{-1}(z)\right) = \frac{2}{\left(\sqrt{1-\frac{1}{z^2}} + \frac{i}{z}\right)^{2 i} + 1} - 1$$

01.21.16.0008.01

$$\tanh\left(i \csc^{-1}(z)\right) = \frac{i\sqrt{z^2}}{z\sqrt{z^2-1}}$$

01.21.16.0020.01

$$\tanh\left(\frac{i}{2}\csc^{-1}(z)\right) = i\,z - \frac{i\sqrt{z^2}\sqrt{-1+z^2}}{z}$$

01.21.16.0076.01

$$\tanh\left(a \csc^{-1}(z)\right) = \frac{2}{\left(\sqrt{1-\frac{1}{z^2}}+\frac{i}{z}\right)^{2\,i\,a}+1} - 1$$

Involving $\sec^{-1}$

01.21.16.0077.01

$$\tanh\left(\sec^{-1}(z)\right) = 1 - \frac{2}{e^{\pi}\left(\sqrt{1-\frac{1}{z^2}}+\frac{i}{z}\right)^{2\,i}+1}$$

01.21.16.0009.01

$$\tanh\left(i \sec^{-1}(z)\right) = i\sqrt{1-\frac{1}{z^2}}\;z$$

01.21.16.0021.01

$$\tanh\left(\frac{i}{2}\sec^{-1}(z)\right) = \frac{i\sqrt{-1+z}\sqrt{-z}}{\sqrt{-1-z}\sqrt{z}}$$

01.21.16.0078.01

$$\tanh\left(a \sec^{-1}(z)\right) = 1 - \frac{2}{e^{a\,\pi}\left(\sqrt{1-\frac{1}{z^2}}+\frac{i}{z}\right)^{2\,a\,i}+1}$$

Involving $\sinh^{-1}$

01.21.16.0010.01

$$\tanh\left(\sinh^{-1}(z)\right) = \frac{z}{\sqrt{1+z^2}}$$

01.21.16.0022.01

$$\tanh\left(\frac{1}{2}\sinh^{-1}(z)\right) = \frac{\sqrt{z^2+1}-1}{z}$$

01.21.16.0079.01

$$\tanh\left(i\sinh^{-1}(z)\right) = 1 - \frac{2}{\left(z+\sqrt{z^2+1}\right)^{2i}+1}$$

01.21.16.0080.01

$$\tanh\left(a\sinh^{-1}(z)\right) = 1 - \frac{2}{\left(z+\sqrt{z^2+1}\right)^{2a}+1}$$

Involving $\cosh^{-1}$

01.21.16.0011.01

$$\tanh\left(\cosh^{-1}(z)\right) == \frac{\sqrt{z-1}\,\sqrt{z+1}}{z}$$

01.21.16.0023.01

$$\tanh\left(\frac{1}{2}\cosh^{-1}(z)\right) == \sqrt{\frac{z-1}{z+1}}$$

01.21.16.0081.01

$$\tanh\left(i\cosh^{-1}(z)\right) = 1 - \frac{2}{\left(z+\sqrt{z-1}\,\sqrt{z+1}\right)^{2i}+1}$$

01.21.16.0082.01

$$\tanh\left(a\cosh^{-1}(z)\right) = 1 - \frac{2}{\left(z+\sqrt{z-1}\,\sqrt{z+1}\right)^{2a}+1}$$

Involving $\tanh^{-1}$

01.21.16.0012.01

$$\tanh\left(\tanh^{-1}(z)\right) == z$$

01.21.16.0024.01

$$\tanh\left(\frac{1}{2}\tanh^{-1}(z)\right) == \frac{1-\sqrt{1-z^2}}{z}$$

01.21.16.0083.01

$$\tanh\left(i\tanh^{-1}(z)\right) = 1 + \frac{1}{-\frac{1}{2}(z+1)^{i}(1-z)^{-i}-\frac{1}{2}}$$

01.21.16.0084.01

$$\tanh\left(a\tanh^{-1}(z)\right) = 1 - \frac{2(1-z)^a}{(1-z)^a+(z+1)^a}$$

01.21.16.0028.01

$$\tanh\left(n\tanh^{-1}(z)\right) == \frac{(z+1)^n-(1-z)^n}{(1-z)^n+(z+1)^n} \;/;\; n\in\mathbb{N}^+$$

Involving $\coth^{-1}$

01.21.16.0013.01

$$\tanh\left(\coth^{-1}(z)\right) = \frac{1}{z}$$

01.21.16.0025.01

$$\tanh\left(\frac{1}{2}\coth^{-1}(z)\right) = z - \frac{\sqrt{z^2}\,\sqrt{z^2-1}}{z}$$

01.21.16.0085.01

$$\tanh\left(i\coth^{-1}(z)\right) = \frac{2}{\left(1-\frac{1}{z}\right)^{i}\left(1+\frac{1}{z}\right)^{-i}+1} - 1$$

01.21.16.0086.01

$$\tanh\left(a\coth^{-1}(z)\right) = \frac{2\left(1+\frac{1}{z}\right)^{a}}{\left(1+\frac{1}{z}\right)^{a}+\left(1-\frac{1}{z}\right)^{a}} - 1$$

Involving csch^{-1}

01.21.16.0014.01

$$\tanh\left(\text{csch}^{-1}(z)\right) = \frac{\sqrt{-z^2}}{z\sqrt{-1-z^2}}$$

01.21.16.0026.01

$$\tanh\left(\frac{1}{2}\text{csch}^{-1}(z)\right) = z\left(\sqrt{1+\frac{1}{z^2}} - 1\right)$$

01.21.16.0087.01

$$\tanh\left(i\,\text{csch}^{-1}(z)\right) = 1 - \frac{2}{\left(\sqrt{1+\frac{1}{z^2}}+\frac{1}{z}\right)^{2i}+1}$$

01.21.16.0088.01

$$\tanh\left(a\,\text{csch}^{-1}(z)\right) = 1 - \frac{2}{\left(\sqrt{1+\frac{1}{z^2}}+\frac{1}{z}\right)^{2a}+1}$$

Involving sech^{-1}

01.21.16.0015.01

$$\tanh\left(\text{sech}^{-1}(z)\right) = \sqrt{\frac{1-z}{1+z}}\,(1+z)$$

01.21.16.0027.01

$$\tanh\left(\frac{1}{2}\operatorname{sech}^{-1}(z)\right) = \sqrt{\frac{1-z}{z+1}}$$

01.21.16.0089.01

$$\tanh\left(i \operatorname{sech}^{-1}(z)\right) = 1 - \frac{2}{\left(\sqrt{\frac{1}{z}-1}\sqrt{1+\frac{1}{z}}+\frac{1}{z}\right)^{2i}+1}$$

01.21.16.0090.01

$$\tanh\left(a \operatorname{sech}^{-1}(z)\right) = 1 - \frac{2}{\left(\sqrt{\frac{1}{z}-1}\sqrt{1+\frac{1}{z}}+\frac{1}{z}\right)^{2a}+1}$$

Addition formulas

01.21.16.0029.01

$$\tanh(a+b) = \frac{\tanh(a)+\tanh(b)}{\tanh(a)\tanh(b)+1}$$

01.21.16.0030.01

$$\tanh\left(a+\frac{\pi i}{4}\right) = \frac{i+\tanh(a)}{i\tanh(a)+1}$$

01.21.16.0031.01

$$\tanh(a-b) = \frac{\tanh(a)-\tanh(b)}{1-\tanh(a)\tanh(b)}$$

01.21.16.0032.01

$$\tanh\left(a-\frac{\pi i}{4}\right) = \frac{\tanh(a)-i}{1-i\tanh(a)}$$

01.21.16.0033.01

$$\tanh(a+b\,i) = \frac{i\sin(2b)+\sinh(2a)}{\cos(2b)+\cosh(2a)}$$

01.21.16.0034.01

$$\tanh(a-i\,b) = \frac{\sinh(2a)-i\sin(2b)}{\cos(2b)+\cosh(2a)}$$

01.21.16.0035.01

$$\tanh(z_1+z_2+z_3) = \frac{\tanh(z_2)\tanh(z_3)\tanh(z_1)+\tanh(z_1)+\tanh(z_2)+\tanh(z_3)}{\tanh(z_1)\tanh(z_2)+\tanh(z_3)\tanh(z_2)+\tanh(z_1)\tanh(z_3)+1}$$

Half-angle formulas

01.21.16.0036.01

$$\tanh\left(\frac{z}{2}\right) = \coth(z)-\operatorname{csch}(z)$$

01.21.16.0037.01

$$\tanh\left(\frac{z}{2}\right) = \frac{\sinh(z)}{\cosh(z)+1}$$

01.21.16.0038.01

$$\tanh\left(\frac{z}{2}\right) = -\frac{\sqrt{-z^2}}{z}\sqrt{\frac{1-\cosh(z)}{1+\cosh(z)}} \;/;\; |\mathrm{Im}(z)| < \pi$$

01.21.16.0039.01

$$\tanh\left(\frac{z}{2}\right) = \frac{\sqrt{z^2}}{z}\sqrt{\frac{\cosh(z)-1}{\cosh(z)+1}}$$

Multiple arguments

Argument involving numeric multiples of variable

01.21.16.0040.01

$$\tanh(2\,z) = \frac{2\tanh(z)}{\tanh^2(z)+1}$$

01.21.16.0041.01

$$\tanh(3\,z) = \frac{\tanh^3(z)+3\tanh(z)}{3\tanh^2(z)+1}$$

Argument involving symbolic multiples of variable

01.21.16.0042.01

$$\tanh(n\,z) = \frac{U_{n-1}(\cosh(z))\sinh(z)}{T_n(\cosh(z))}$$

01.21.16.0052.01

$$\tanh(n\,z) = \frac{\sum_{k=0}^{\left\lfloor\frac{n-1}{2}\right\rfloor}\binom{n}{2\,k+1}\tanh^{2\,k+1}(z)}{\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\binom{n}{2\,k}\tanh^{2\,k}(z)} \;/;\; n \in \mathbb{N}^+$$

Roger Germundsson

Products, sums, and powers of the direct function

Products of the direct function

01.21.16.0043.01

$$\tanh(a)\tanh(b) = \frac{\cosh(a+b)-\cosh(a-b)}{\cosh(a-b)+\cosh(a+b)}$$

Products involving the direct function

01.21.16.0044.01

$$\tanh(a)\coth(b) = \frac{\sinh(a-b)+\sinh(a+b)}{\sinh(a+b)-\sinh(a-b)}$$

Sums of the direct function

01.21.16.0045.01

$\tanh(a) + \tanh(b) = \operatorname{sech}(a)\operatorname{sech}(b)\sinh(a+b)$

01.21.16.0046.01

$\tanh(a) - \tanh(b) = \operatorname{sech}(a)\operatorname{sech}(b)\sinh(a-b)$

Sums involving the direct function

Involving other hyperbolic functions

Involving coth

01.21.16.0053.01

$\tanh(z) + \coth(z) = \cosh(2z)\operatorname{csch}(z)\operatorname{sech}(z)$

01.21.16.0054.01

$\tanh(z) - \coth(z) = -\operatorname{csch}(z)\operatorname{sech}(z)$

01.21.16.0047.01

$\coth(b) + \tanh(a) = \cosh(a+b)\operatorname{sech}(a)\operatorname{csch}(b)$

01.21.16.0048.01

$\tanh(a) - \coth(b) = -\cosh(a-b)\operatorname{sech}(a)\operatorname{csch}(b)$

Involving trigonometric functions

Involving tan

01.21.16.0055.01

$\tanh(z) + i\tan(z) = \sec(z)\operatorname{sech}(z)\sinh\left(\sqrt{2}\, e^{\frac{i\pi}{4}} z\right)$

01.21.16.0056.01

$\tanh(z) - i\tan(z) = \sec(z)\operatorname{sech}(z)\sinh\left(\sqrt{2}\, e^{-\frac{1}{4}(i\pi)} z\right)$

01.21.16.0057.01

$\tanh(a) + i\tan(b) = \sec(b)\operatorname{sech}(a)\sinh(a + b\, i)$

01.21.16.0058.01

$\tanh(a) - i\tan(b) = \sec(b)\operatorname{sech}(a)\sinh(a - b\, i)$

Involving cot

01.21.16.0059.01

$\tanh(z) + i\cot(z) = i\cosh\left(\sqrt{2}\, e^{-\frac{1}{4}(i\pi)} z\right)\csc(z)\operatorname{sech}(z)$

01.21.16.0060.01

$\tanh(z) - i\cot(z) = -i\cosh\left(\sqrt{2}\, e^{\frac{i\pi}{4}} z\right)\csc(z)\operatorname{sech}(z)$

01.21.16.0061.01

$$\tanh(a) + i\cot(b) = i\cosh(a - i\,b)\csc(b)\,\mathrm{sech}(a)$$

01.21.16.0062.01

$$\tanh(a) - i\cot(b) = -i\cosh(a + b\,i)\csc(b)\,\mathrm{sech}(a)$$

Powers of the direct function

01.21.16.0049.01

$$\mathrm{Tanh}[z]^2 = \frac{\mathrm{Cosh}[2\,z] - 1}{\mathrm{Cosh}[2\,z] + 1}$$

01.21.16.0050.01

$$\tanh^3(z) = \frac{\sinh(3\,z) - 3\sinh(z)}{3\cosh(z) + \cosh(3\,z)}$$

Sums of powers involving the direct function

01.21.16.0051.01

$$\tanh^2(a) - \tanh^2(b) = \mathrm{sech}^2(a)\,\mathrm{sech}^2(b)\sinh(a - b)\sinh(a + b)$$

Identities

Functional identities

01.21.17.0001.01

$$\tanh(2\,z)\left(\tanh^2(z) + 1\right) = 2\tanh(z)$$

Complex characteristics

Real part

01.21.19.0001.01

$$\mathrm{Re}(\tanh(x + i\,y)) = \frac{\sinh(2\,x)}{\cos(2\,y) + \cosh(2\,x)}$$

Imaginary part

01.21.19.0002.01

$$\mathrm{Im}(\tanh(x + i\,y)) = \frac{\sin(2\,y)}{\cos(2\,y) + \cosh(2\,x)}$$

Absolute value

01.21.19.0003.01

$$|\tanh(x + i\,y)| = \sqrt{\frac{\sin^2(2\,y) + \sinh^2(2\,x)}{(\cos(2\,y) + \cosh(2\,x))^2}}$$

Argument

01.21.19.0004.01

$$\arg(\tanh(x+i\,y)) = \tan^{-1}\left(\frac{\sinh(2\,x)}{\cos(2\,y)+\cosh(2\,x)}, \frac{\sin(2\,y)}{\cos(2\,y)+\cosh(2\,x)}\right)$$

01.21.19.0005.01

$$\arg(\tanh(x+i\,y)) = \tan^{-1}(\operatorname{csch}(2\,x)\sin(2\,y)) + \frac{\pi}{2}\operatorname{sgn}\left(\frac{\operatorname{sgn}(\sin(2\,y))}{\operatorname{sgn}(\cos(2\,y)+\cosh(2\,x))} + \frac{1}{2}\right)\left(1 - \frac{\operatorname{sgn}(\sinh(2\,x))}{\operatorname{sgn}(\cos(2\,y)+\cosh(2\,x))}\right)$$

Conjugate value

01.21.19.0006.01

$$\overline{\tanh(x+i\,y)} = \frac{\sinh(2\,x) - i\sin(2\,y)}{\cos(2\,y)+\cosh(2\,x)}$$

Differentiation

Low-order differentiation

01.21.20.0001.01

$$\frac{\partial \tanh(z)}{\partial z} = \operatorname{sech}^2(z)$$

01.21.20.0002.01

$$\frac{\partial^2 \tanh(z)}{\partial z^2} = -2\operatorname{sech}^2(z)\tanh(z)$$

Symbolic differentiation

01.21.20.0003.01

$$\frac{\partial^n \tanh(z)}{\partial z^n} = \sum_{k=1}^{\infty} \frac{2^{2k-1}\left(2^{2k}-1\right)}{k\,(2\,k-n-1)!} B_{2k}\, z^{2k-n-1} \;/;\; |z| < \frac{\pi}{2} \bigwedge n \in \mathbb{N}^+$$

01.21.20.0004.01

$$\frac{\partial^n \tanh(z)}{\partial z^n} =$$

$$\tanh(z)\,\delta_n + \operatorname{sech}^2(z)\,\delta_{n-1} - i^n\, n \sum_{k=0}^{n-1}\sum_{j=0}^{k-1} \frac{(-1)^k \cosh^{-2k-2}(z)\, 2^{n-2k}\,(k-j)^{n-1}}{k+1} \binom{n-1}{k}\binom{2\,k}{j} \sinh\left(\frac{i\,n\,\pi}{2} + 2\,(j-k)\,z\right) /;\; n \in \mathbb{N}$$

01.21.20.0006.01

$$\frac{\partial^n \tanh(z)}{\partial z^n} = 2^n\,(\delta_n + \tanh(z) - 1)\sum_{k=0}^{n} \frac{(-1)^k\, k!}{2^k}\, \mathcal{S}_n^{(k)}\, (\tanh(z)+1)^k \;/;\; n \in \mathbb{N}$$

Victor Adamchik (2005)

Fractional integro-differentiation

01.21.20.0005.01

$$\frac{\partial^\alpha \tanh(z)}{\partial z^\alpha} = \sum_{k=1}^{\infty} \frac{2^{2k-1}\left(2^{2k}-1\right) B_{2k}\, z^{2k-\alpha-1}}{\Gamma(2\,k-\alpha)\,k} \;/;\; |z| < \frac{\pi}{2}$$

01.21.20.0007.01

$$\tanh^{(\alpha)}(c\,z) = \frac{(-i)^{1-\alpha}\log(4)}{\Gamma(-\alpha)}(i\,c\,z)^{-\alpha-1} - (-i)^{1-\alpha}\pi^{-\alpha-1}(i\,c\,z)^{-\alpha}\left((-i\,c\,z)^{\alpha}\left(2^{\alpha+1}\psi^{(\alpha)}\left(-\frac{2\,i\,c\,z}{\pi}\right)-\psi^{(\alpha)}\left(-\frac{i\,c\,z}{\pi}\right)\right)+(i\,c\,z)^{\alpha}\left(\psi^{(\alpha)}\left(\frac{i\,c\,z}{\pi}\right)-2^{\alpha+1}\psi^{(\alpha)}\left(\frac{2\,i\,c\,z}{\pi}\right)\right)\right)$$

Integration

Indefinite integration

Involving only one direct function

01.21.21.0014.01

$$\int \tanh(b+a\,z)\,dz = \frac{\log(\cosh(b+a\,z))}{a}$$

01.21.21.0015.01

$$\int \tanh(a\,z)\,dz = \frac{\log(\cosh(a\,z))}{a}$$

01.21.21.0016.01

$$\int \tanh(z)\,dz = \log(\cosh(z))$$

Involving one direct function and elementary functions

Involving power function

Involving power

Involving z^n and linear arguments

01.21.21.0017.01

$$\int z\tanh(b+a\,z)\,dz = \frac{1}{2\,a^2}\Big(-a^2 z^2 - 2\,a\,b\,z + 2\,a\log\left(1+e^{2(b+a\,z)}\right)z + i\,\pi\log\left(1+e^{-2\,a\,z}\right) + 2\,b\log\left(1+e^{2(b+a\,z)}\right) + i\,\pi\log\left(1+e^{2(b+a\,z)}\right) - i\,\pi\log(\cosh(a\,z)) - 2\,b\log(\cosh(b+a\,z)) - i\,\pi\log(\cosh(b+a\,z)) + \mathrm{Li}_2\left(-e^{2(b+a\,z)}\right)\Big)$$

01.21.21.0018.01

$$\int z^n\tanh(a\,z)\,dz = \frac{z^{n+1}}{n+1} - 2\,e^{-2\,a\,z}\,n!\sum_{j=0}^{n}\frac{(-1)^j\,2^{-j-1}\,(-a)^{-j-1}\,z^{n-j}}{(n-j)!}\;{}_{j+2}F_{j+1}\left(1,\,\ldots,\,1;\,2,\,\ldots,\,2;\,-e^{-2\,a\,z}\right)\,/;\,n\in\mathbb{N}$$

01.21.21.0019.01

$$\int z\tanh(a\,z)\,dz = \frac{a\,z\left(a\,z+2\log\left(1+e^{-2\,a\,z}\right)\right)-\mathrm{Li}_2\left(-e^{-2\,a\,z}\right)}{2\,a^2}$$

01.21.21.0020.01

$$\int z^2 \tanh(a z)\, dz = \frac{2 a^2 \left(a z + 3 \log\left(1 + e^{-2 a z}\right)\right) z^2 - 6 a \operatorname{Li}_2\left(-e^{-2 a z}\right) z - 3 \operatorname{Li}_3\left(-e^{-2 a z}\right)}{6 a^3}$$

01.21.21.0021.01

$$\int z^3 \tanh(a z)\, dz = \frac{a^4 z^4 + 4 a^3 \log\left(1 + e^{-2 a z}\right) z^3 - 6 a^2 \operatorname{Li}_2\left(-e^{-2 a z}\right) z^2 - 6 a \operatorname{Li}_3\left(-e^{-2 a z}\right) z - 3 \operatorname{Li}_4\left(-e^{-2 a z}\right)}{4 a^4}$$

01.21.21.0022.01

$$\int z^4 \tanh(a z)\, dz = \frac{1}{a^5} \left(\frac{a^5 z^5}{5} + a^4 \log\left(1 + e^{-2 a z}\right) z^4 - 2 a^3 \operatorname{Li}_2\left(-e^{-2 a z}\right) z^3 - 3 a^2 \operatorname{Li}_3\left(-e^{-2 a z}\right) z^2 - 3 a \operatorname{Li}_4\left(-e^{-2 a z}\right) z - \frac{3}{2} \operatorname{Li}_5\left(-e^{-2 a z}\right) \right)$$

Involving exponential function

Involving exp

Involving $a^{b z}$

01.21.21.0023.01

$$\int a^{b z} \tanh(c z)\, dz = \frac{1}{b \log(a) (2 c + b \log(a))} \left(a^{b z} \left(b\, e^{2 c z} {}_2F_1\left(\frac{b \log(a)}{2 c} + 1, 1; \frac{b \log(a)}{2 c} + 2; -e^{2 c z} \right) \log(a) - {}_2F_1\left(\frac{b \log(a)}{2 c}, 1; \frac{b \log(a)}{2 c} + 1; -e^{2 c z} \right) (2 c + b \log(a)) \right) \right)$$

01.21.21.0024.01

$$\int e^{b z} \tanh(a z)\, dz = \frac{1}{b (2 a + b)} \left(b\, e^{(2 a + b) z} {}_2F_1\left(\frac{b}{2 a} + 1, 1; \frac{b}{2 a} + 2; -e^{2 a z} \right) - (2 a + b)\, e^{b z} {}_2F_1\left(\frac{b}{2 a}, 1; \frac{b}{2 a} + 1; -e^{2 a z} \right) \right)$$

01.21.21.0025.01

$$\int e^{-a z} \tanh(a z)\, dz = \frac{e^{-a z} - 2 \tan^{-1}\left(e^{-a z}\right)}{a}$$

01.21.21.0026.01

$$\int e^{a z} \tanh(a z)\, dz = \frac{e^{a z} - 2 \tan^{-1}\left(e^{a z}\right)}{a}$$

Involving exponential function and a power function

Involving exp and power

Involving $z^n e^{b z}$

01.21.21.0027.01

$$\int z^n e^{bz}\tanh(cz)\,dz = n!\left(e^{(b+2c)z}\sum_{j=0}^{n}\frac{(-1)^j (b+2c)^{-j-1} z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{b+2c}{2c},\ldots,\frac{b+2c}{2c},1;\frac{b+2c}{2c}+1,\ldots,\frac{b+2c}{2c}+1;-e^{2cz}\right)-e^{bz}\sum_{j=0}^{n}\frac{(-1)^j b^{-j-1} z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{b}{2c},\ldots,\frac{b}{2c},1;\frac{b}{2c}+1,\ldots,\frac{b}{2c}+1;-e^{2cz}\right)\right)/;\,n\in\mathbb{N}$$

Arguments involving inverse trigonometric functions

Involving $\sin^{-1}$

01.21.21.0028.01

$$\int\tanh\left(\sin^{-1}(z)\right)dz = \frac{1}{10}e^{-i\sin^{-1}(z)}\left(-5i\,{}_2F_1\left(-\frac{i}{2},1;1-\frac{i}{2};-e^{2\sin^{-1}(z)}\right)+5i\,e^{2i\sin^{-1}(z)}\,{}_2F_1\left(\frac{i}{2},1;1+\frac{i}{2};-e^{2\sin^{-1}(z)}\right)+(2+i)\,e^{2\sin^{-1}(z)}\,{}_2F_1\left(1-\frac{i}{2},1;2-\frac{i}{2};-e^{2\sin^{-1}(z)}\right)+(2-i)\,e^{(2+2i)\sin^{-1}(z)}\,{}_2F_1\left(1+\frac{i}{2},1;2+\frac{i}{2};-e^{2\sin^{-1}(z)}\right)\right)$$

01.21.21.0029.01

$$\int\tanh\left(a\sin^{-1}(z)\right)dz = \frac{1}{2\left(4a^2+1\right)}\left(e^{-i\sin^{-1}(z)}\left((2a-i)\left((1-2ia)\left({}_2F_1\left(-\frac{i}{2a},1;1-\frac{i}{2a};-e^{2a\sin^{-1}(z)}\right)-e^{2i\sin^{-1}(z)}\,{}_2F_1\left(\frac{i}{2a},1;1+\frac{i}{2a};-e^{2a\sin^{-1}(z)}\right)\right)+e^{2(a+i)\sin^{-1}(z)}\,{}_2F_1\left(1+\frac{i}{2a},1;2+\frac{i}{2a};-e^{2a\sin^{-1}(z)}\right)\right)+(2a+i)\,e^{2a\sin^{-1}(z)}\,{}_2F_1\left(1-\frac{i}{2a},1;2-\frac{i}{2a};-e^{2a\sin^{-1}(z)}\right)\right)\right)$$

Involving $\cos^{-1}$

01.21.21.0030.01

$$\int\tanh\left(\cos^{-1}(z)\right)dz = \frac{1}{10}e^{-i\cos^{-1}(z)}\left(-5\,{}_2F_1\left(-\frac{i}{2},1;1-\frac{i}{2};-e^{2\cos^{-1}(z)}\right)-5\,e^{2i\cos^{-1}(z)}\,{}_2F_1\left(\frac{i}{2},1;1+\frac{i}{2};-e^{2\cos^{-1}(z)}\right)+(1-2i)\,e^{2\cos^{-1}(z)}\,{}_2F_1\left(1-\frac{i}{2},1;2-\frac{i}{2};-e^{2\cos^{-1}(z)}\right)+(1+2i)\,e^{(2+2i)\cos^{-1}(z)}\,{}_2F_1\left(1+\frac{i}{2},1;2+\frac{i}{2};-e^{2\cos^{-1}(z)}\right)\right)$$

01.21.21.0031.01

$$\int\tanh\left(a\cos^{-1}(z)\right)dz = \frac{1}{2\left(4a^2+1\right)}\left(e^{-i\cos^{-1}(z)}\left((1+2ia)\left((2a+i)\,i\left(e^{2i\cos^{-1}(z)}\,{}_2F_1\left(\frac{i}{2a},1;1+\frac{i}{2a};-e^{2a\cos^{-1}(z)}\right)+{}_2F_1\left(-\frac{i}{2a},1;1-\frac{i}{2a};-e^{2a\cos^{-1}(z)}\right)\right)+e^{2(a+i)\cos^{-1}(z)}\,{}_2F_1\left(1+\frac{i}{2a},1;2+\frac{i}{2a};-e^{2a\cos^{-1}(z)}\right)\right)+(1-2ia)\,e^{2a\cos^{-1}(z)}\,{}_2F_1\left(1-\frac{i}{2a},1;2-\frac{i}{2a};-e^{2a\cos^{-1}(z)}\right)\right)\right)$$

Arguments involving inverse hyperbolic functions

Involving $\sinh^{-1}$

01.21.21.0032.01

$$\int \tanh\left(\sinh^{-1}(z)\right) dz = \sqrt{z^2+1}$$

01.21.21.0033.01

$$\int \tanh\left(a\sinh^{-1}(z)\right) dz = \frac{1}{8a^2-2}\left(e^{-2\sinh^{-1}(z)}\left((2a-1)\left(e^{(2a+3)\sinh^{-1}(z)}\, {}_2F_1\left(1+\frac{1}{2a}, 1; 2+\frac{1}{2a}; -e^{2a\sinh^{-1}(z)}\right) - (2a+1)\, e^{\sinh^{-1}(z)}\left(e^{2\sinh^{-1}(z)}\, {}_2F_1\left(\frac{1}{2a}, 1; 1+\frac{1}{2a}; -e^{2a\sinh^{-1}(z)}\right) - {}_2F_1\left(-\frac{1}{2a}, 1; 1-\frac{1}{2a}; -e^{2a\sinh^{-1}(z)}\right)\right)\right) + (2a+1)\, e^{2a\sinh^{-1}(z)+\sinh^{-1}(z)}\, {}_2F_1\left(1-\frac{1}{2a}, 1; 2-\frac{1}{2a}; -e^{2a\sinh^{-1}(z)}\right)\right)\right)$$

Involving $\cosh^{-1}$

01.21.21.0034.01

$$\int \tanh\left(\cosh^{-1}(z)\right) dz = \tan^{-1}\left(\frac{1}{\sqrt{z-1}\,\sqrt{z+1}}\right) + \sqrt{z-1}\,\sqrt{z+1}$$

01.21.21.0035.01

$$\int \tanh\left(a\cosh^{-1}(z)\right) dz = -\frac{1}{8a^2-2}\left(e^{-2\cosh^{-1}(z)}\left((2a-1)\left((2a+1)\, e^{\cosh^{-1}(z)}\left(e^{2\cosh^{-1}(z)}\, {}_2F_1\left(\frac{1}{2a}, 1; 1+\frac{1}{2a}; -e^{2a\cosh^{-1}(z)}\right) + {}_2F_1\left(-\frac{1}{2a}, 1; 1-\frac{1}{2a}; -e^{2a\cosh^{-1}(z)}\right)\right) - e^{(2a+3)\cosh^{-1}(z)}\, {}_2F_1\left(1+\frac{1}{2a}, 1; 2+\frac{1}{2a}; -e^{2a\cosh^{-1}(z)}\right)\right) + (2a+1)\, e^{2a\cosh^{-1}(z)+\cosh^{-1}(z)}\, {}_2F_1\left(1-\frac{1}{2a}, 1; 2-\frac{1}{2a}; -e^{2a\cosh^{-1}(z)}\right)\right)\right)$$

Involving $\tanh^{-1}$

01.21.21.0036.01

$$\int \tanh\left(\tanh^{-1}(z)\right) dz = \frac{z^2}{2}$$

Involving $\coth^{-1}$

01.21.21.0037.01

$$\int \tanh\left(\coth^{-1}(z)\right) dz = \log(z)$$

Involving csch^{-1}

01.21.21.0038.01

$$\int \tanh\left(\operatorname{csch}^{-1}(z)\right) dz = \frac{\sqrt{1+\frac{1}{z^2}}\; z \sinh^{-1}(z)}{\sqrt{z^2+1}}$$

Involving sech^{-1}

01.21.21.0039.01

$$\int \tanh\left(\operatorname{sech}^{-1}(z)\right) dz = \frac{1}{2}\sqrt{\frac{1}{z+1}}\;\sqrt{z+1}\left(\sqrt{1-z^2}\; z + \tan^{-1}\left(\frac{z}{\sqrt{1-z^2}}\right)\right)$$

Involving trigonometric functions

Involving sin

Involving $\sin(b\,z)$

01.21.21.0040.01

$$\int \sin(b\,z)\tanh(c\,z)\, dz = \frac{1}{2\left(b^3+4\,c^2\,b\right)}\, e^{-i\,b\,z} \left(-(b+2\,i\,c)\left(b\,e^{2\,(c+i\,b)\,z}\,{}_2F_1\left(1+\frac{i\,b}{2\,c},\,1;\,2+\frac{i\,b}{2\,c};\,-e^{2\,c\,z}\right) - (b-2\,i\,c)\left(e^{2\,i\,b\,z}\,{}_2F_1\left(\frac{i\,b}{2\,c},\,1;\,1+\frac{i\,b}{2\,c};\,-e^{2\,c\,z}\right) + {}_2F_1\left(-\frac{i\,b}{2\,c},\,1;\,1-\frac{i\,b}{2\,c};\,-e^{2\,c\,z}\right)\right)\right) - b\,(b-2\,i\,c)\,e^{2\,c\,z}\,{}_2F_1\left(1-\frac{i\,b}{2\,c},\,1;\,2-\frac{i\,b}{2\,c};\,-e^{2\,c\,z}\right)\right)$$

Involving power of sin

Involving $\sin^m(b\,z)$

01.21.21.0041.01

$$\int \sin^m(b\,z)\tanh(c\,z)\, dz = \frac{2^{-m}\log(\cosh(c\,z))\,(1-m \bmod 2)}{c}\binom{m}{\frac{m}{2}} + i^{-m}\,2^{-m}\sum_{k=0}^{\left\lfloor\frac{m-1}{2}\right\rfloor}(-1)^k\binom{m}{k} \left(\frac{i\,e^{i\,b\,(m-2\,k)\,z}\,{}_2F_1\left(\frac{i\,b\,(m-2\,k)}{2\,c},\,1;\,\frac{2\,c+b\,i\,(m-2\,k)}{2\,c};\,-e^{2\,c\,z}\right)}{b\,(m-2\,k)} + \frac{e^{(2\,c+b\,i\,(m-2\,k))\,z}\,{}_2F_1\left(\frac{2\,c+b\,i\,(m-2\,k)}{2\,c},\,1;\,\frac{4\,c+b\,i\,(m-2\,k)}{2\,c};\,-e^{2\,c\,z}\right)}{2\,c+b\,i\,(m-2\,k)} + \frac{(-1)^m\,e^{(2\,c-i\,b\,(m-2\,k))\,z}\,{}_2F_1\left(\frac{2\,c-i\,b\,(m-2\,k)}{2\,c},\,1;\,\frac{4\,c-i\,b\,(m-2\,k)}{2\,c};\,-e^{2\,c\,z}\right)}{2\,c-i\,b\,(m-2\,k)} - \frac{i\,(-1)^m\,e^{-i\,b\,(m-2\,k)\,z}\,{}_2F_1\left(-\frac{i\,b\,(m-2\,k)}{2\,c},\,1;\,\frac{2\,c-i\,b\,(m-2\,k)}{2\,c};\,-e^{2\,c\,z}\right)}{b\,(m-2\,k)}\right) /;\, m \in \mathbb{N}^+$$

Involving cos

Involving cos(b z)

01.21.21.0042.01

$$\int \cos(b z) \tanh(c z) \, dz = \frac{1}{2 \left(b^3 + 4 c^2 b\right)} i e^{-i b z} \left(b (b - 2 i c) e^{2 c z} {}_2F_1\left(1 - \frac{i b}{2 c}, 1; 2 - \frac{i b}{2 c}; -e^{2 c z}\right) - (b + 2 i c) \left((b - 2 i c) \left({}_2F_1\left(-\frac{i b}{2 c}, 1; 1 - \frac{i b}{2 c}; -e^{2 c z}\right) - e^{2 i b z} {}_2F_1\left(\frac{i b}{2 c}, 1; 1 + \frac{i b}{2 c}; -e^{2 c z}\right)\right) + b e^{2 (c + i b) z} {}_2F_1\left(1 + \frac{i b}{2 c}, 1; 2 + \frac{i b}{2 c}; -e^{2 c z}\right)\right)\right)$$

Involving power of cos

Involving cos^m(b z)

01.21.21.0043.01

$$\int \cos^m(b z) \tanh(c z) \, dz = \frac{2^{-m} \log(\cosh(c z)) (1 - m \bmod 2)}{c} \binom{m}{\frac{m}{2}} + 2^{-m} \sum_{k=0}^{\left\lfloor \frac{m-1}{2} \right\rfloor} \binom{m}{k} \left(\frac{i e^{i b (m - 2 k) z} {}_2F_1\left(\frac{i b (m-2k)}{2 c}, 1; \frac{2 c + b i (m - 2 k)}{2 c}; -e^{2 c z}\right)}{b (m - 2 k)} + \frac{e^{(2 c + b i (m - 2 k)) z} {}_2F_1\left(\frac{2 c + b i (m - 2 k)}{2 c}, 1; \frac{4 c + b i (m - 2 k)}{2 c}; -e^{2 c z}\right)}{2 c + b i (m - 2 k)} + \frac{e^{(2 c - i b (m - 2 k)) z} {}_2F_1\left(\frac{2 c - i b (m - 2 k)}{2 c}, 1; \frac{4 c - i b (m - 2 k)}{2 c}; -e^{2 c z}\right)}{2 c - i b (m - 2 k)} - \frac{i e^{-i b (m - 2 k) z} {}_2F_1\left(-\frac{i b (m - 2 k)}{2 c}, 1; \frac{2 c - i b (m - 2 k)}{2 c}; -e^{2 c z}\right)}{b (m - 2 k)} \right) /; m \in \mathbb{N}^+$$

Involving trigonometric and a power functions

Involving sin and power

Involving z^n sin(a + b z) tanh(c z)

01.21.21.0044.01

$$\int z^n \sin(a+b\,z)\tanh(c\,z)\,dz = \frac{1}{2}\,i\,e^{-i\,a}\,n!\left(e^{(2c-i\,b)z}\sum_{j=0}^{n}\frac{(-1)^j\,(2\,c-i\,b)^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\!\left(\frac{2\,c-i\,b}{2\,c},\ldots,\frac{2\,c-i\,b}{2\,c},1;\frac{2\,c-i\,b}{2\,c}+1,\ldots,\frac{2\,c-i\,b}{2\,c}+1;-e^{2\,c\,z}\right)-e^{-i\,b\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(-i\,b)^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\!\left(-\frac{i\,b}{2\,c},\ldots,-\frac{i\,b}{2\,c},1;1-\frac{i\,b}{2\,c},\ldots,1-\frac{i\,b}{2\,c};-e^{2\,c\,z}\right)\right)-\frac{1}{2}\,i\,e^{i\,a}\,n!\left(e^{(2c+i\,b)z}\sum_{j=0}^{n}\frac{(-1)^j\,(2\,c+i\,b)^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\!\left(\frac{2\,c+i\,b}{2\,c},\ldots,\frac{2\,c+i\,b}{2\,c},1;\frac{2\,c+i\,b}{2\,c}+1,\ldots,\frac{2\,c+i\,b}{2\,c}+1;-e^{2\,c\,z}\right)-e^{i\,b\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(i\,b)^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\!\left(\frac{i\,b}{2\,c},\ldots,\frac{i\,b}{2\,c},1;\frac{i\,b}{2\,c}+1,\ldots,\frac{i\,b}{2\,c}+1;-e^{2\,c\,z}\right)\right)\;/;\;n\in\mathbb{N}$$

01.21.21.0045.01

$$\int z^n \sin(b\,z)\tanh(c\,z)\,dz = \frac{1}{2}\,i\,n!\left(-e^{-i\,b\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(-i\,b)^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\!\left(-\frac{i\,b}{2\,c},\ldots,-\frac{i\,b}{2\,c},1;1-\frac{i\,b}{2\,c},\ldots,1-\frac{i\,b}{2\,c};-e^{2\,c\,z}\right)+e^{i\,b\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(i\,b)^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\!\left(\frac{i\,b}{2\,c},\ldots,\frac{i\,b}{2\,c},1;\frac{i\,b}{2\,c}+1,\ldots,\frac{i\,b}{2\,c}+1;-e^{2\,c\,z}\right)+e^{(2c-i\,b)z}\sum_{j=0}^{n}\frac{(-1)^j\,(2\,c-i\,b)^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\!\left(\frac{2\,c-i\,b}{2\,c},\ldots,\frac{2\,c-i\,b}{2\,c},1;\frac{2\,c-i\,b}{2\,c}+1,\ldots,\frac{2\,c-i\,b}{2\,c}+1;-e^{2\,c\,z}\right)-e^{(2c+i\,b)z}\sum_{j=0}^{n}\frac{(-1)^j\,(2\,c+i\,b)^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\!\left(\frac{i\,b+2\,c}{2\,c},\ldots,\frac{i\,b+2\,c}{2\,c},1;\frac{i\,b+2\,c}{2\,c}+1,\ldots,\frac{i\,b+2\,c}{2\,c}+1;-e^{2\,c\,z}\right)\right)\;/;\;n\in\mathbb{N}$$

Involving powers of sin and power

Involving $z^n \sin^m(b\,z)\tanh(c\,z)$

01.21.21.0046.01

$$\int z^n \sin^m(b z) \tanh(c z)\, dz ==$$

$$2^{-m} \binom{m}{\frac{m}{2}} n!\,(1 - m \bmod 2)\left(2\, e^{2 c z} \sum_{j=0}^{n} \frac{(-1)^j\, 2^{-j-1}\, c^{-j-1}\, z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}\left(1, \ldots, 1; 2, \ldots, 2; -e^{2 c z}\right) - \frac{z^{n+1}}{(n+1)!}\right) +$$

$$2^{-m}\, n!\, i^{-m} \sum_{k=0}^{\left\lfloor \frac{m-1}{2} \right\rfloor} (-1)^k \binom{m}{k} \left(-e^{i b (m-2k) z} \sum_{j=0}^{n} \frac{(-1)^j\, (i b (m-2k))^{-j-1}\, z^{n-j}}{(n-j)!}\right.$$

$${}_{j+2}F_{j+1}\left(\frac{i b (m-2k)}{2c}, \ldots, \frac{i b (m-2k)}{2c}, 1; \frac{i b (m-2k)}{2c} + 1, \ldots, \frac{i b (m-2k)}{2c} + 1; -e^{2 c z}\right) +$$

$$(-1)^m \left(e^{(2c - i b (m-2k)) z} \sum_{j=0}^{n} \frac{(-1)^j\, (2c - i b (m-2k))^{-j-1}\, z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}\left(\frac{2c - i b (m-2k)}{2c}, \ldots,\right.\right.$$

$$\left.\frac{2c - i b (m-2k)}{2c}, 1; \frac{2c - i b (m-2k)}{2c} + 1, \ldots, \frac{2c - i b (m-2k)}{2c} + 1; -e^{2 c z}\right) -$$

$$e^{-i b (m-2k) z} \sum_{j=0}^{n} \frac{(-1)^j\, (-i b (m-2k))^{-j-1}\, z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}\left(-\frac{i b (m-2k)}{2c}, \ldots, -\frac{i b (m-2k)}{2c},\right.$$

$$\left.\left. 1; 1 - \frac{i b (m-2k)}{2c}, \ldots, 1 - \frac{i b (m-2k)}{2c}; -e^{2 c z}\right)\right) +$$

$$e^{(2c + b i (m-2k)) z} \sum_{j=0}^{n} \frac{(-1)^j\, (2c + b i (m-2k))^{-j-1}\, z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}\left(\frac{2c + i b (m-2k)}{2c}, \ldots, \frac{2c + i b (m-2k)}{2c},\right.$$

$$\left.\left. 1; \frac{2c + i b (m-2k)}{2c} + 1, \ldots, \frac{2c + i b (m-2k)}{2c} + 1; -e^{2 c z}\right)\right) /; n \in \mathbb{N} \land m \in \mathbb{N}^+$$

Involving cosh and power

Involving $z^n \cos(a + b z) \tanh(c z)$

01.21.21.0047.01

$$\int z^n \cos(a+b z)\tanh(c z)\,dz = \frac{1}{2} e^{-i a} n!\left(e^{(2c-ib)z}\sum_{j=0}^{n}\frac{(-1)^j (2c-ib)^{-j-1} z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{2c-ib}{2c},\ldots,\frac{2c-ib}{2c},1;\frac{2c-ib}{2c}+1,\ldots,\frac{2c-ib}{2c}+1;-e^{2cz}\right)-e^{-ibz}\sum_{j=0}^{n}\frac{(-1)^j(-ib)^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(-\frac{ib}{2c},\ldots,-\frac{ib}{2c},1;1-\frac{ib}{2c},\ldots,1-\frac{ib}{2c};-e^{2cz}\right)\right)+\frac{1}{2}e^{ia}n!\left(e^{(2c+ib)z}\sum_{j=0}^{n}\frac{(-1)^j(2c+ib)^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{2c+ib}{2c},\ldots,\frac{2c+ib}{2c},1;\frac{2c+ib}{2c}+1,\ldots,\frac{2c+ib}{2c}+1;-e^{2cz}\right)-e^{ibz}\sum_{j=0}^{n}\frac{(-1)^j(ib)^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{ib}{2c},\ldots,\frac{ib}{2c},1;\frac{ib}{2c}+1,\ldots,\frac{ib}{2c}+1;-e^{2cz}\right)\right)\;/;\;n\in\mathbb{N}$$

01.21.21.0048.01

$$\int z^n\cos(bz)\tanh(cz)\,dz = \frac{1}{2}n!\left(-e^{-ibz}\sum_{j=0}^{n}\frac{(-1)^j(-ib)^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(-\frac{ib}{2c},\ldots,-\frac{ib}{2c},1;1-\frac{ib}{2c},\ldots,1-\frac{ib}{2c};-e^{2cz}\right)-e^{ibz}\sum_{j=0}^{n}\frac{(-1)^j(ib)^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{ib}{2c},\ldots,\frac{ib}{2c},1;\frac{ib}{2c}+1,\ldots,\frac{ib}{2c}+1;-e^{2cz}\right)+e^{(2c-ib)z}\sum_{j=0}^{n}\frac{(-1)^j(2c-ib)^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{2c-ib}{2c},\ldots,\frac{2c-ib}{2c},1;\frac{2c-ib}{2c}+1,\ldots,\frac{2c-ib}{2c}+1;-e^{2cz}\right)+e^{(2c+ib)z}\sum_{j=0}^{n}\frac{(-1)^j(2c+ib)^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{ib+2c}{2c},\ldots,\frac{ib+2c}{2c},1;\frac{ib+2c}{2c}+1,\ldots,\frac{ib+2c}{2c}+1;-e^{2cz}\right)\right)\;/;\;n\in\mathbb{N}$$

Involving powers of cos and power

Involving $z^n \cos^m(b z)\tanh(c z)$

01.21.21.0049.01

$$\int z^n \cos^m(b z) \tanh(c z)\, dz = 2^{-m} \binom{m}{\frac{m}{2}} n!\,(1 - m \bmod 2) \left(2 e^{2 c z} \sum_{j=0}^{n} \frac{(-1)^j 2^{-j-1} c^{-j-1} z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}\!\left(1, \ldots, 1; 2, \ldots, 2; -e^{2 c z}\right) - \frac{z^{n+1}}{(n+1)!}\right) + 2^{-m} n! \sum_{k=0}^{\left\lfloor \frac{m-1}{2} \right\rfloor} \binom{m}{k} \left(-e^{-i b (m-2k) z} \sum_{j=0}^{n} \frac{(-1)^j (-i b (m-2k))^{-j-1} z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}\!\left(-\frac{i b (m-2k)}{2c}, \ldots, -\frac{i b (m-2k)}{2c}, 1; 1 - \frac{i b (m-2k)}{2c}, \ldots, 1 - \frac{i b (m-2k)}{2c}; -e^{2 c z}\right) - e^{i b (m-2k) z} \sum_{j=0}^{n} \frac{(-1)^j (i b (m-2k))^{-j-1} z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}\!\left(\frac{i b (m-2k)}{2c}, \ldots, \frac{i b (m-2k)}{2c}, 1; \frac{i b (m-2k)}{2c} + 1, \ldots, \frac{i b (m-2k)}{2c} + 1; -e^{2 c z}\right) + e^{(2c - i b (m-2k)) z} \sum_{j=0}^{n} \frac{(-1)^j (2c - i b (m-2k))^{-j-1} z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}\!\left(\frac{2c - i b (m-2k)}{2c}, \ldots, \frac{2c - i b (m-2k)}{2c}, 1; \frac{2c - i b (m-2k)}{2c} + 1, \ldots, \frac{2c - i b (m-2k)}{2c} + 1; -e^{2 c z}\right) + e^{(2c + b i (m-2k)) z} \sum_{j=0}^{n} \frac{(-1)^j (2c + b i (m-2k))^{-j-1} z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}\!\left(\frac{2c + i b (m-2k)}{2c}, \ldots, \frac{2c + i b (m-2k)}{2c}, 1; \frac{2c + i b (m-2k)}{2c} + 1, \ldots, \frac{2c + i b (m-2k)}{2c} + 1; -e^{2 c z}\right)\right) /;\ n \in \mathbb{N} \land m \in \mathbb{N}^+$$

Involving trigonometric and exponential functions

Involving sin and exp

Involving $e^{p z} \sin(b z)$

01.21.21.0050.01

$$\int e^{p z} \sin(b z) \tanh(c z)\, dz = \frac{1}{2} \left(\frac{e^{(-i b + p) z}\, {}_2F_1\!\left(\frac{-i b + p}{2c}, 1; \frac{2c - i b + p}{2c}; -e^{2 c z}\right)}{b + i p} + \frac{e^{(i b + p) z}\, {}_2F_1\!\left(\frac{i b + p}{2c}, 1; \frac{2c + i b + p}{2c}; -e^{2 c z}\right)}{b - i p} - \frac{e^{(2c - i b + p) z}\, {}_2F_1\!\left(\frac{2c - i b + p}{2c}, 1; \frac{4c - i b + p}{2c}; -e^{2 c z}\right)}{b + i (2c + p)} + \frac{e^{(2c + i b + p) z}\, {}_2F_1\!\left(\frac{2c + i b + p}{2c}, 1; \frac{4c + i b + p}{2c}; -e^{2 c z}\right)}{i (2c + p) - b}\right)$$

01.21.21.0051.01

$$\int e^{i b z} \sin(b z) \tanh(c z)\, dz = \frac{1}{4 b c (c + i b)} \left((c + i b) \left(c\, e^{2 i b z}\, {}_2F_1\!\left(\frac{i b}{c}, 1; 1 + \frac{i b}{c}; -e^{2 c z}\right) + 2 b i \log(\cosh(c z))\right) - i b c\, e^{2 (c + i b) z}\, {}_2F_1\!\left(1 + \frac{i b}{c}, 1; 2 + \frac{i b}{c}; -e^{2 c z}\right)\right)$$

01.21.21.0052.01

$$\int e^{-i b z} \sin(b z) \tanh(c z) \, dz = \frac{1}{4 b c (c - i b)} \left(b c e^{2 (c - i b) z} i \, {}_2F_1\left(1 - \frac{i b}{c}, 1; 2 - \frac{i b}{c}; -e^{2 c z}\right) + (c - i b) \left(c e^{-2 i b z} \, {}_2F_1\left(-\frac{i b}{c}, 1; 1 - \frac{i b}{c}; -e^{2 c z}\right) - 2 i b \log(\cosh(c z)) \right) \right)$$

Involving powers of sin and exp

Involving $e^{p z} \sin^m(b z)$

01.21.21.0053.01

$$\int e^{p z} \sin^m(b z) \tanh(c z) \, dz = 2^{-m} \left(\frac{e^{(2 c + p) z} \, {}_2F_1\left(\frac{2 c + p}{2 c}, 1; \frac{4 c + p}{2 c}; -e^{2 c z}\right)}{2 c + p} - \frac{e^{p z} \, {}_2F_1\left(\frac{p}{2 c}, 1; \frac{2 c + p}{2 c}; -e^{2 c z}\right)}{p} \right) \binom{m}{\frac{m}{2}} (1 - m \bmod 2) + i^{-m} 2^{-m} \sum_{k=0}^{\left\lfloor \frac{m-1}{2} \right\rfloor} (-1)^k \binom{m}{k} \left(\frac{e^{(2 c + b i (m - 2 k) + p) z} \, {}_2F_1\left(\frac{2 c + b i (m - 2 k) + p}{2 c}, 1; \frac{4 c + b i (m - 2 k) + p}{2 c}; -e^{2 c z}\right)}{2 c + b i (m - 2 k) + p} + \frac{(-1)^m e^{(2 c - i b (m - 2 k) + p) z} \, {}_2F_1\left(\frac{2 c - i b (m - 2 k) + p}{2 c}, 1; \frac{4 c - i b (m - 2 k) + p}{2 c}; -e^{2 c z}\right)}{2 c - i b (m - 2 k) + p} - \frac{e^{(b i (m - 2 k) + p) z} \, {}_2F_1\left(\frac{b i (m - 2 k) + p}{2 c}, 1; \frac{2 c + b i (m - 2 k) + p}{2 c}; -e^{2 c z}\right)}{b i (m - 2 k) + p} - \frac{(-1)^m e^{(p - i b (m - 2 k)) z} \, {}_2F_1\left(\frac{p - i b (m - 2 k)}{2 c}, 1; \frac{2 c - i b (m - 2 k) + p}{2 c}; -e^{2 c z}\right)}{p - i b (m - 2 k)} \right) /; m \in \mathbb{N}^+$$

Involving cos and exp

Involving $e^{p z} \cos(b z)$

01.21.21.0054.01

$$\int e^{p z} \cos(b z) \tanh(c z) \, dz = \frac{1}{2} \left(\frac{e^{(-i b + p) z} \, {}_2F_1\left(\frac{-i b + p}{2 c}, 1; \frac{2 c - i b + p}{2 c}; -e^{2 c z}\right)}{i b - p} + \frac{e^{(2 c - i b + p) z} \, {}_2F_1\left(\frac{2 c - i b + p}{2 c}, 1; \frac{4 c - i b + p}{2 c}; -e^{2 c z}\right)}{2 c - i b + p} + \frac{e^{(2 c + i b + p) z} \, {}_2F_1\left(\frac{2 c + i b + p}{2 c}, 1; \frac{4 c + i b + p}{2 c}; -e^{2 c z}\right)}{2 c + i b + p} - \frac{e^{(i b + p) z} \, {}_2F_1\left(\frac{i b + p}{2 c}, 1; \frac{2 c + i b + p}{2 c}; -e^{2 c z}\right)}{i b + p} \right)$$

01.21.21.0055.01

$$\int e^{i b z} \cos(b z) \tanh(c z) \, dz = \frac{1}{4 b c (c + i b)} \left(b c e^{2 (c + i b) z} \, {}_2F_1\left(1 + \frac{i b}{c}, 1; 2 + \frac{i b}{c}; -e^{2 c z}\right) + (c + i b) \left(c e^{2 i b z} i \, {}_2F_1\left(\frac{i b}{c}, 1; 1 + \frac{i b}{c}; -e^{2 c z}\right) + 2 b \log(\cosh(c z)) \right) \right)$$

01.21.21.0056.01

$$\int e^{-i b z}\cos(b z)\tanh(c z)\,dz = \frac{1}{4 b c (c - i b)} e^{-2 i b z} \left(b c\, e^{2 c z}\, {}_2F_1\!\left(1 - \frac{i b}{c}, 1; 2 - \frac{i b}{c}; -e^{2 c z}\right) + (c - i b)\left(2 b\, e^{2 i b z} \log(\cosh(c z)) - i c\, {}_2F_1\!\left(-\frac{i b}{c}, 1; 1 - \frac{i b}{c}; -e^{2 c z}\right)\right)\right)$$

Involving powers of cos and exp

Involving $e^{p z} \cos^m(b z)$

01.21.21.0057.01

$$\int e^{p z}\cos^m(b z)\tanh(c z)\,dz = 2^{-m}\left(\frac{e^{(2 c + p) z}\, {}_2F_1\!\left(\frac{2 c + p}{2 c}, 1; \frac{4 c + p}{2 c}; -e^{2 c z}\right)}{2 c + p} - \frac{e^{p z}\, {}_2F_1\!\left(\frac{p}{2 c}, 1; \frac{2 c + p}{2 c}; -e^{2 c z}\right)}{p}\right)\binom{m}{\frac{m}{2}}(1 - m \bmod 2) + 2^{-m}\sum_{k=0}^{\left\lfloor\frac{m-1}{2}\right\rfloor}\binom{m}{k}\left(\frac{e^{(2 c + b i (m - 2 k) + p) z}\, {}_2F_1\!\left(\frac{2 c + b i (m - 2 k) + p}{2 c}, 1; \frac{4 c + b i (m - 2 k) + p}{2 c}; -e^{2 c z}\right)}{2 c + b i (m - 2 k) + p} + \frac{e^{(2 c - i b (m - 2 k) + p) z}\, {}_2F_1\!\left(\frac{2 c - i b (m - 2 k) + p}{2 c}, 1; \frac{4 c - i b (m - 2 k) + p}{2 c}; -e^{2 c z}\right)}{2 c - i b (m - 2 k) + p} - \frac{e^{(b i (m - 2 k) + p) z}\, {}_2F_1\!\left(\frac{b i (m - 2 k) + p}{2 c}, 1; \frac{2 c + b i (m - 2 k) + p}{2 c}; -e^{2 c z}\right)}{b i (m - 2 k) + p} - \frac{e^{(p - i b (m - 2 k)) z}\, {}_2F_1\!\left(\frac{p - i b (m - 2 k)}{2 c}, 1; \frac{2 c - i b (m - 2 k) + p}{2 c}; -e^{2 c z}\right)}{p - i b (m - 2 k)}\right) /;\ m \in \mathbb{N}^+$$

Involving trigonometric, exponential and a power functions

Involving sin, exp and power

Involving $z^n e^{p z} \sin(a + b z) \tanh(c z)$

01.21.21.0058.01

$$\int z^n e^{p z} \sin(a + b z) \tanh(c z)\, dz = \frac{1}{2} i e^{-i a} n! \left(e^{(2c - i b + p) z} \sum_{j=0}^{n} \frac{(-1)^j (2c - i b + p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{-i b + p + 2c}{2c}, \ldots, \frac{-i b + p + 2c}{2c}, 1; \frac{-i b + p + 2c}{2c} + 1, \ldots, \frac{-i b + p + 2c}{2c} + 1; -e^{2 c z} \right) - e^{(-i b + p) z} \sum_{j=0}^{n} \frac{(-1)^j (-i b + p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{-i b + p}{2c}, \ldots, \frac{-i b + p}{2c}, 1; \frac{-i b + p}{2c} + 1, \ldots, \frac{-i b + p}{2c} + 1; -e^{2 c z} \right) \right) - \frac{1}{2} i e^{i a} n! \left(e^{(2c + i b + p) z} \sum_{j=0}^{n} \frac{(-1)^j (2c + i b + p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{i b + p + 2c}{2c}, \ldots, \frac{i b + p + 2c}{2c}, 1; \frac{i b + p + 2c}{2c} + 1, \ldots, \frac{i b + p + 2c}{2c} + 1; -e^{2 c z} \right) - e^{(i b + p) z} \sum_{j=0}^{n} \frac{(-1)^j (i b + p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{i b + p}{2c}, \ldots, \frac{i b + p}{2c}, 1; \frac{i b + p}{2c} + 1, \ldots, \frac{i b + p}{2c} + 1; -e^{2 c z} \right) \right) /; n \in \mathbb{N}$$

01.21.21.0059.01

$$\int z^n e^{p z} \sin(b z) \tanh(c z)\, dz = \frac{1}{2} i n! \left(-e^{(-i b + p) z} \sum_{j=0}^{n} \frac{(-1)^j (-i b + p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{p - i b}{2c}, \ldots, \frac{p - i b}{2c}, 1; \frac{p - i b}{2c} + 1, \ldots, \frac{p - i b}{2c} + 1; -e^{2 c z} \right) + e^{(i b + p) z} \sum_{j=0}^{n} \frac{(-1)^j (i b + p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{i b + p}{2c}, \ldots, \frac{i b + p}{2c}, 1; \frac{i b + p}{2c} + 1, \ldots, \frac{i b + p}{2c} + 1; -e^{2 c z} \right) + e^{(2c - i b + p) z} \sum_{j=0}^{n} \frac{(-1)^j (2c - i b + p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{-i b + 2c + p}{2c}, \ldots, \frac{-i b + 2c + p}{2c}, 1; \frac{-i b + 2c + p}{2c} + 1, \ldots, \frac{-i b + 2c + p}{2c} + 1; -e^{2 c z} \right) - e^{(2c + i b + p) z} \sum_{j=0}^{n} \frac{(-1)^j (2c + i b + p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{i b + 2c + p}{2c}, \ldots, \frac{i b + 2c + p}{2c}, 1; \frac{i b + 2c + p}{2c} + 1, \ldots, \frac{i b + 2c + p}{2c} + 1; -e^{2 c z} \right) \right) /; n \in \mathbb{N}$$

Involving powers of sin, exp and power

Involving $z^n e^{p z} \sin^m(b z) \tanh(c z)$

01.21.21.0060.01

$$\int z^n e^{p z} \sin^m(b z) \tanh(c z)\, dz = (2 i)^{-m} n! \left(\sum_{k=0}^{\left\lfloor \frac{m-1}{2} \right\rfloor} (-1)^k \binom{m}{k} \left((-1)^m \left(e^{(2c - i b (m-2k)+p) z} \sum_{j=0}^{n} \frac{(-1)^j (2c - i b (m-2k)+p)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{2c - i b (m-2k)+p}{2c}, \ldots, \frac{2c - i b (m-2k)+p}{2c}, 1; \frac{2c - i b (m-2k)+p}{2c} + 1, \ldots, \frac{2c - i b (m-2k)+p}{2c} + 1; -e^{2 c z} \right) - e^{(p - i b (m-2k)) z} \sum_{j=0}^{n} \frac{(-1)^j (p - i b (m-2k))^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{p - i b (m-2k)}{2c}, \ldots, \frac{p - i b (m-2k)}{2c}, 1; \frac{p - i b (m-2k)}{2c} + 1, \ldots, \frac{p - i b (m-2k)}{2c} + 1; -e^{2 c z} \right) \right) - e^{(b i (m-2k)+p) z} \sum_{j=0}^{n} \frac{(-1)^j (b i (m-2k)+p)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{b i (m-2k)+p}{2c}, \ldots, \frac{b i (m-2k)+p}{2c}, 1; \frac{b i (m-2k)+p}{2c} + 1, \ldots, \frac{b i (m-2k)+p}{2c} + 1; -e^{2 c z} \right) + e^{(2c + b i (m-2k)+p) z} \sum_{j=0}^{n} \frac{(-1)^j (2c + b i (m-2k)+p)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{2c + b i (m-2k)+p}{2c}, \ldots, \frac{2c + b i (m-2k)+p}{2c}, 1; \frac{2c + b i (m-2k)+p}{2c} + 1, \ldots, \frac{2c + b i (m-2k)+p}{2c} + 1; -e^{2 c z} \right) \right) \right) + 2^{-m} \binom{m}{\frac{m}{2}} n! \, (1 - m \bmod 2) \left(e^{(2c+p) z} \sum_{j=0}^{n} \frac{(-1)^j (2c+p)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{2c+p}{2c}, \ldots, \frac{2c+p}{2c}, 1; \frac{2c+p}{2c} + 1, \ldots, \frac{2c+p}{2c} + 1; -e^{2 c z} \right) - e^{p z} \sum_{j=0}^{n} \frac{(-1)^j p^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{p}{2c}, \ldots, \frac{p}{2c}, 1; \frac{p}{2c} + 1, \ldots, \frac{p}{2c} + 1; -e^{2 c z} \right) \right) /; n \in \mathbb{N} \land m \in \mathbb{N}^+$$

Involving cos, exp and power

Involving $z^n e^{p z} \cos(a + b z) \tanh(c z)$

01.21.21.0061.01

$$\int z^n e^{p z} \cos(a + b z) \tanh(c z) \, dz = \frac{1}{2} e^{-i a} n! \left(e^{(2c - i b + p) z} \sum_{j=0}^{n} \frac{(-1)^j (2c - i b + p)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{-i b + p + 2c}{2c}, \ldots, \frac{-i b + p + 2c}{2c}, 1; \frac{-i b + p + 2c}{2c} + 1, \ldots, \frac{-i b + p + 2c}{2c} + 1; -e^{2 c z} \right) - e^{(-i b + p) z} \sum_{j=0}^{n} \frac{(-1)^j (-i b + p)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{-i b + p}{2c}, \ldots, \frac{-i b + p}{2c}, 1; \frac{-i b + p}{2c} + 1, \ldots, \frac{-i b + p}{2c} + 1; -e^{2 c z} \right) \right) + \frac{1}{2} e^{i a} n! \left(e^{(2c + i b + p) z} \sum_{j=0}^{n} \frac{(-1)^j (2c + i b + p)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{i b + p + 2c}{2c}, \ldots, \frac{i b + p + 2c}{2c}, 1; \frac{i b + p + 2c}{2c} + 1, \ldots, \frac{i b + p + 2c}{2c} + 1; -e^{2 c z} \right) - e^{(i b + p) z} \sum_{j=0}^{n} \frac{(-1)^j (i b + p)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{i b + p}{2c}, \ldots, \frac{i b + p}{2c}, 1; \frac{i b + p}{2c} + 1, \ldots, \frac{i b + p}{2c} + 1; -e^{2 c z} \right) \right) /; n \in \mathbb{N}$$

01.21.21.0062.01

$$\int z^n \cos(b z) \tanh(c z) \, dz = \frac{1}{2} n! \left(-e^{-i b z} \sum_{j=0}^{n} \frac{(-1)^j (-i b)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(-\frac{i b}{2c}, \ldots, -\frac{i b}{2c}, 1; 1 - \frac{i b}{2c}, \ldots, 1 - \frac{i b}{2c}; -e^{2 c z} \right) - e^{i b z} \sum_{j=0}^{n} \frac{(-1)^j (i b)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{i b}{2c}, \ldots, \frac{i b}{2c}, 1; \frac{i b}{2c} + 1, \ldots, \frac{i b}{2c} + 1; -e^{2 c z} \right) + e^{(2c - i b) z} \sum_{j=0}^{n} \frac{(-1)^j (2c - i b)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{2c - i b}{2c}, \ldots, \frac{2c - i b}{2c}, 1; \frac{2c - i b}{2c} + 1, \ldots, \frac{2c - i b}{2c} + 1; -e^{2 c z} \right) + e^{(2c + i b) z} \sum_{j=0}^{n} \frac{(-1)^j (2c + i b)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{i b + 2c}{2c}, \ldots, \frac{i b + 2c}{2c}, 1; \frac{i b + 2c}{2c} + 1, \ldots, \frac{i b + 2c}{2c} + 1; -e^{2 c z} \right) \right) /; n \in \mathbb{N}$$

Involving powers of cos, exp and power

Involving $z^n e^{p z} \cos^m(b z) \tanh(c z)$

01.21.21.0063.01

$$\int z^n e^{pz}\cos^m(bz)\tanh(cz)\,dz = 2^{-m}\,n!\left(\sum_{k=0}^{\left\lfloor\frac{m-1}{2}\right\rfloor}\binom{m}{k}\left(e^{(2c-ib(m-2k)+p)z}\sum_{j=0}^{n}\frac{(-1)^j\,(2c-ib(m-2k)+p)^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{2c-ib(m-2k)+p}{2c},\ldots,\frac{2c-ib(m-2k)+p}{2c},1;\frac{2c-ib(m-2k)+p}{2c}+1,\ldots,\frac{2c-ib(m-2k)+p}{2c}+1;-e^{2cz}\right)-e^{(p-ib(m-2k))z}\sum_{j=0}^{n}\frac{(-1)^j\,(p-ib(m-2k))^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{p-ib(m-2k)}{2c},\ldots,\frac{p-ib(m-2k)}{2c},1;\frac{p-ib(m-2k)}{2c}+1,\ldots,\frac{p-ib(m-2k)}{2c}+1;-e^{2cz}\right)-e^{(bi(m-2k)+p)z}\sum_{j=0}^{n}\frac{(-1)^j\,(bi(m-2k)+p)^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{bi(m-2k)+p}{2c},\ldots,\frac{bi(m-2k)+p}{2c},1;\frac{bi(m-2k)+p}{2c}+1,\ldots,\frac{bi(m-2k)+p}{2c}+1;-e^{2cz}\right)+e^{(2c+bi(m-2k)+p)z}\sum_{j=0}^{n}\frac{(-1)^j\,(2c+bi(m-2k)+p)^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{2c+bi(m-2k)+p}{2c},\ldots,\frac{2c+bi(m-2k)+p}{2c},1;\frac{2c+bi(m-2k)+p}{2c}+1,\ldots,\frac{2c+bi(m-2k)+p}{2c}+1;-e^{2cz}\right)\right)\right)+2^{-m}\binom{m}{\frac{m}{2}}n!\,(1-m\bmod 2)\left(e^{(2c+p)z}\sum_{j=0}^{n}\frac{(-1)^j\,(2c+p)^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{2c+p}{2c},\ldots,\frac{2c+p}{2c},1;\frac{2c+p}{2c}+1,\ldots,\frac{2c+p}{2c}+1;-e^{2cz}\right)-e^{pz}\sum_{j=0}^{n}\frac{(-1)^j\,p^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{p}{2c},\ldots,\frac{p}{2c},1;\frac{p}{2c}+1,\ldots,\frac{p}{2c}+1;-e^{2cz}\right)\right)\;/;\;n\in\mathbb{N}\land m\in\mathbb{N}^+$$

Involving hyperbolic functions

Involving sinh

Involving sinh($b\,z$)

01.21.21.0064.01

$$\int\sinh(bz)\tanh(cz)\,dz = \frac{1}{2\left(b^3-4bc^2\right)}\,e^{-2bz}\left((b-2c)\,e^{bz}\left(b\,e^{2(b+c)z}\,{}_2F_1\left(\frac{b}{2c}+1,1;\frac{b}{2c}+2;-e^{2cz}\right)-(b+2c)\left(e^{2bz}\,{}_2F_1\left(\frac{b}{2c},1;\frac{b}{2c}+1;-e^{2cz}\right)+{}_2F_1\left(-\frac{b}{2c},1;1-\frac{b}{2c};-e^{2cz}\right)\right)\right)+b(b+2c)\,e^{(b+2c)z}\,{}_2F_1\left(1-\frac{b}{2c},1;2-\frac{b}{2c};-e^{2cz}\right)\right)$$

01.21.21.0065.01

$$\int \sinh(z)\tanh(z)\,dz = \sinh(z) - 2\tan^{-1}\left(\tanh\left(\frac{z}{2}\right)\right)$$

01.21.21.0066.01

$$\int \sinh(z)\tanh(2z)\,dz = \sinh(z) - \frac{\tan^{-1}\left(\sqrt{2}\sinh(z)\right)}{\sqrt{2}}$$

01.21.21.0067.01

$$\int \sinh(z)\tanh(3z)\,dz = -\frac{1}{3}\tan^{-1}(2\sinh(z)) - \frac{2}{3}\tan^{-1}\left(\tanh\left(\frac{z}{2}\right)\right) + \sinh(z)$$

01.21.21.0068.01

$$\int \sinh(z)\tanh(4z)\,dz = \frac{1}{126}e^{-9z}\left(-63e^{10z}\,{}_2F_1\left(\frac{1}{8},1;\frac{9}{8};-e^{8z}\right) - 9e^{16z}\,{}_2F_1\left(\frac{7}{8},1;\frac{15}{8};-e^{8z}\right) + 7e^{18z}\,{}_2F_1\left(\frac{9}{8},1;\frac{17}{8};-e^{8z}\right) - 63e^{8z}\,{}_2F_1\left(-\frac{1}{8},1;\frac{7}{8};-e^{8z}\right)\right)$$

01.21.21.0069.01

$$\int \sinh(2z)\tanh(z)\,dz = \frac{1}{2}\sinh(2z) - z$$

01.21.21.0070.01

$$\int \sinh(3z)\tanh(z)\,dz = 2\tan^{-1}\left(\tanh\left(\frac{z}{2}\right)\right) - 2\sinh(z) + \frac{1}{3}\sinh(3z)$$

01.21.21.0071.01

$$\int \sinh(4z)\tanh(z)\,dz = z - \sinh(2z) + \frac{1}{4}\sinh(4z)$$

Involving power of sinh

Involving $\sinh^{\mu}(b\,z)$

01.21.21.0072.01

$$\int \sinh^m(bz)\tanh(cz)\,dz = \frac{\log(\cosh(cz))\,(1-m \bmod 2)}{c}\left(\frac{i}{2}\right)^m\binom{m}{\frac{m}{2}} + 2^{-m}\sum_{k=0}^{\left\lfloor\frac{m-1}{2}\right\rfloor}(-1)^k\binom{m}{k}\left(-\frac{e^{b(m-2k)z}\,{}_2F_1\left(\frac{b(m-2k)}{2c},1;\frac{2c+b(m-2k)}{2c};-e^{2cz}\right)}{b(m-2k)} + \frac{e^{(2c+b(m-2k))z}\,{}_2F_1\left(\frac{2c+b(m-2k)}{2c},1;\frac{4c+b(m-2k)}{2c};-e^{2cz}\right)}{2c+b(m-2k)} + \frac{(-1)^m e^{(2c-b(m-2k))z}\,{}_2F_1\left(\frac{2c-b(m-2k)}{2c},1;\frac{4c-b(m-2k)}{2c};-e^{2cz}\right)}{2c-b(m-2k)} + \frac{(-1)^m e^{-b(m-2k)z}\,{}_2F_1\left(-\frac{b(m-2k)}{2c},1;\frac{2c-b(m-2k)}{2c};-e^{2cz}\right)}{b(m-2k)}\right)\;/;\; m\in\mathbb{N}^+$$

01.21.21.0073.01

$$\int \sinh^{\mu}(cz)\tanh(cz)\,dz = \frac{\left(1-e^{2cz}\right)^{-\mu}\sinh^{\mu}(cz)}{c\,\mu}F_1\left(-\frac{\mu}{2};-\mu-1,1;\frac{2-\mu}{2};e^{2cz},-e^{2cz}\right)$$

01.21.21.0074.01

$$\int \sinh^{\mu}(c\,z)\tanh(c\,z)\,dz = \frac{\sinh^{\mu}(c\,z)\tanh^2(c\,z)^{-\frac{\mu}{2}}}{c\,\mu}\,{}_2F_1\left(-\frac{\mu}{2}, -\frac{\mu}{2}; 1-\frac{\mu}{2}; \operatorname{sech}^2(c\,z)\right)$$

01.21.21.0075.01

$$\int \sinh^2(z)\tanh(z)\,dz = \frac{1}{4}\left(\cosh(2\,z) - 4\log(\cosh(z))\right)$$

01.21.21.0076.01

$$\int \sinh^3(z)\tanh(z)\,dz = \frac{1}{12}\left(24\tan^{-1}\left(\tanh\left(\frac{z}{2}\right)\right) - 15\sinh(z) + \sinh(3\,z)\right)$$

01.21.21.0077.01

$$\int \sinh^{10}(z)\tanh(z)\,dz = \\ \frac{1}{15\,360}\left(8430\cosh(2\,z) - 1740\cosh(4\,z) + 335\cosh(6\,z) - 45\cosh(8\,z) + 3\cosh(10\,z) - 15\,360\log(\cosh(z))\right)$$

01.21.21.0078.01

$$\int \sinh^2(z)\tanh(2\,z)\,dz = \frac{1}{4}\left(\cosh(2\,z) - \log(\cosh(2\,z))\right)$$

01.21.21.0079.01

$$\int \sinh^3(z)\tanh(3\,z)\,dz = \frac{1}{12}\left(\tan^{-1}(2\sinh(z)) + 8\tan^{-1}\left(\tanh\left(\frac{z}{2}\right)\right) - 9\sinh(z) + \sinh(3\,z)\right)$$

01.21.21.0080.01

$$\int \sinh^{\frac{1}{2}}(c\,z)\tanh(c\,z)\,dz = \frac{2\operatorname{sech}^2(c\,z)\sinh^{\frac{5}{2}}(c\,z)}{c\tanh^2(c\,z)^{5/4}}\,{}_2F_1\left(-\frac{1}{4}, -\frac{1}{4}; \frac{3}{4}; \operatorname{sech}^2(c\,z)\right)$$

01.21.21.0081.01

$$\int \frac{\tanh(c\,z)}{\sinh^{\frac{1}{2}}(c\,z)}\,dz = -\frac{2\sqrt[4]{\tanh^2(c\,z)}}{c\sinh^{\frac{1}{2}}(c\,z)}\,{}_2F_1\left(\frac{1}{4}, \frac{1}{4}; \frac{5}{4}; \operatorname{sech}^2(c\,z)\right)$$

01.21.21.0082.01

$$\int \frac{\tanh(c\,z)}{\sqrt{\sinh^3(2\,c\,z)}}\,dz = \frac{\sinh(2\,c\,z)\left(i\sqrt{i\sinh(2\,c\,z)}\;F\left(\frac{\pi}{4} - i\,c\,z \,\middle|\, 2\right) + \tanh(c\,z)\right)}{3\,c\sqrt{\sinh^3(2\,c\,z)}}$$

Involving rational functions of sinh

Involving $(a + b\sinh(c\,z))^{-n}$

01.21.21.0083.01

$$\int \frac{\tanh(c\,z)}{a + b\sinh(c\,z)}\,dz = \frac{2\,b\tan^{-1}\left(\tanh\left(\frac{c\,z}{2}\right)\right) + a\log(\cosh(c\,z)) - a\log(a + b\sinh(c\,z))}{c\,a^2 + b^2\,c}$$

01.21.21.0084.01

$$\int \frac{A+B\tanh(z)}{a+b\sinh(z)}\,dz = -\frac{1}{\left(-a^2-b^2\right)^{3/2}} \left(2A\left(a^2+b^2\right)\tan^{-1}\left(\frac{b-a\tanh\left(\frac{z}{2}\right)}{\sqrt{-a^2-b^2}}\right)+2b\sqrt{-a^2-b^2}\,B\tan^{-1}\left(\tanh\left(\frac{z}{2}\right)\right)+a\sqrt{-a^2-b^2}\,B\left(\log(\cosh(z))-\log(a+b\sinh(z))\right)\right)$$

01.21.21.0085.01

$$\int \frac{\tanh(z)}{a+b\sinh(2z)}\,dz = \frac{1}{2a}\left(\frac{2b}{\sqrt{-a^2-b^2}}\tan^{-1}\left(\frac{b-a\tanh(z)}{\sqrt{-a^2-b^2}}\right)+2\log(\cosh(z))-\log(a+b\sinh(2z))\right)$$

01.21.21.0086.01

$$\int \frac{(A+B\sinh(z))\tanh(z)}{a+b\sinh(z)}\,dz = \frac{2b(Ab-aB)\tan^{-1}\left(\tanh\left(\frac{z}{2}\right)\right)+b(aA+bB)\log(\cosh(z))+a(aB-Ab)\log(a+b\sinh(z))}{b\left(a^2+b^2\right)}$$

01.21.21.0087.01

$$\int \frac{\tanh(cz)}{(a+b\sinh(cz))^2}\,dz = \left(a\left(-\log(a+b\sinh(cz))\,a^2+a^2+4b\tan^{-1}\left(\tanh\left(\frac{cz}{2}\right)\right)a+b^2+\left(a^2-b^2\right)\log(\cosh(cz))+b^2\log(a+b\sinh(cz))\right)+ b\left(4ab\tan^{-1}\left(\tanh\left(\frac{cz}{2}\right)\right)+\left(a^2-b^2\right)\left(\log(\cosh(cz))-\log(a+b\sinh(cz))\right)\right)\sinh(cz)\right)\Big/\left(\left(a^2+b^2\right)^2 c\,(a+b\sinh(cz))\right)$$

Involving algebraic functions of sinh

Involving $(a+b\sinh(cz))^\beta$

01.21.21.0088.01

$$\int (a+b\sinh(cz))^\beta\tanh(cz)\,dz = -\left(\left((a-ib)\,{}_2F_1\left(\beta+1,1;\beta+2;\frac{a+b\sinh(cz)}{a+ib}\right)+(a+ib)\,{}_2F_1\left(\beta+1,1;\beta+2;\frac{a+b\sinh(cz)}{a-ib}\right)\right)(a+b\sinh(cz))^{\beta+1}\right)\Big/ (2(a-ib)(a+ib)c(\beta+1))$$

01.21.21.0089.01

$$\int \sqrt{a+b\sinh(cz)}\,\tanh(cz)\,dz = -\frac{1}{c}\left(\frac{b\,i\tan^{-1}\left(\frac{\sqrt{a+b\sinh(cz)}}{\sqrt{ib-a}}\right)}{\sqrt{ib-a}}+\frac{a\tanh^{-1}\left(\frac{\sqrt{a+b\sinh(cz)}}{\sqrt{a+ib}}\right)}{\sqrt{a+ib}}+\frac{a\tanh^{-1}\left(\frac{\sqrt{a+b\sinh(cz)}}{\sqrt{a-ib}}\right)}{\sqrt{a-ib}}-2\sqrt{a+b\sinh(cz)}-\frac{i\,b\tan^{-1}\left(\frac{\sqrt{a+b\sinh(cz)}}{\sqrt{-a-ib}}\right)}{\sqrt{-a-ib}}\right)$$

01.21.21.0090.01

$$\int \sqrt{i\sinh(cz)\,a+a}\,\tanh(cz)\,dz = \frac{2\sqrt{i\sinh(cz)\,a+a}}{c\left(\cosh\left(\frac{cz}{2}\right)+i\sinh\left(\frac{cz}{2}\right)\right)}\left(-\sqrt{2}\tanh^{-1}\left(\frac{i\tanh\left(\frac{cz}{4}\right)+1}{\sqrt{2}}\right)+\cosh\left(\frac{cz}{2}\right)+i\sinh\left(\frac{cz}{2}\right)\right)$$

01.21.21.0091.01

$$\int \sqrt{a - i\, a \sinh(c\, z)}\, \tanh(c\, z)\, dz = \frac{2\sqrt{a - i\, a \sinh(c\, z)}}{c} \left(\frac{i\sqrt{2}}{\cosh\left(\frac{c\, z}{2}\right) - i \sinh\left(\frac{c\, z}{2}\right)} \tan^{-1}\left(\frac{i + \tanh\left(\frac{c\, z}{4}\right)}{\sqrt{2}} \right) + 1 \right)$$

01.21.21.0092.01

$$\int \frac{\tanh(c\, z)}{\sqrt{a + b \sinh(c\, z)}}\, dz = -\frac{1}{c} \left(\frac{1}{\sqrt{a + i\, b}} \tanh^{-1}\left(\frac{\sqrt{a + b \sinh(c\, z)}}{\sqrt{a + i\, b}} \right) + \frac{1}{\sqrt{a - i\, b}} \tanh^{-1}\left(\frac{\sqrt{a + b \sinh(c\, z)}}{\sqrt{a - i\, b}} \right) \right)$$

01.21.21.0093.01

$$\int \frac{\tanh(c\, z)}{\sqrt{i \sinh(c\, z)\, a + a}}\, dz = -\frac{\sqrt{2}\, \tanh^{-1}\left(\frac{i \tanh\left(\frac{c\, z}{4}\right) + 1}{\sqrt{2}} \right) \left(\cosh\left(\frac{c\, z}{2}\right) + i \sinh\left(\frac{c\, z}{2}\right) \right) + 1}{c\sqrt{i \sinh(c\, z)\, a + a}}$$

01.21.21.0094.01

$$\int \frac{\tanh(c\, z)}{\sqrt{a - i\, a \sinh(c\, z)}}\, dz = \frac{i\sqrt{2}\, \tan^{-1}\left(\frac{i + \tanh\left(\frac{c\, z}{4}\right)}{\sqrt{2}} \right) \left(\cosh\left(\frac{c\, z}{2}\right) - i \sinh\left(\frac{c\, z}{2}\right) \right) - 1}{c\sqrt{a - i\, a \sinh(c\, z)}}$$

Involving cosh

Involving cosh($b\, z$)

01.21.21.0095.01

$$\int \cosh(b\, z) \tanh(c\, z)\, dz = \frac{1}{2\left(b^3 - 4\, b\, c^2\right)} e^{-2\, b\, z} \left((b - 2\, c)\, e^{b\, z} \left((b + 2\, c) \left({}_2F_1\left(-\frac{b}{2\, c}, 1; 1 - \frac{b}{2\, c}; -e^{2\, c\, z}\right) - e^{2\, b\, z}\, {}_2F_1\left(\frac{b}{2\, c}, 1; \frac{b}{2\, c} + 1; -e^{2\, c\, z}\right) \right) + b\, e^{2\,(b+c)\, z}\, {}_2F_1\left(\frac{b}{2\, c} + 1, 1; \frac{b}{2\, c} + 2; -e^{2\, c\, z}\right) \right) - b\,(b + 2\, c)\, e^{(b+2\, c)\, z}\, {}_2F_1\left(1 - \frac{b}{2\, c}, 1; 2 - \frac{b}{2\, c}; -e^{2\, c\, z}\right) \right)$$

01.21.21.0096.01

$$\int \cosh(z) \tanh(z)\, dz = \cosh(z)$$

01.21.21.0097.01

$$\int \cosh(z) \tanh(2\, z)\, dz = -\frac{i \tan^{-1}\left(-i\sqrt{2} - \tanh\left(\frac{z}{2}\right)\right)}{\sqrt{2}} + \cosh(z) - \frac{\tanh^{-1}\left(i \tanh\left(\frac{z}{2}\right) + \sqrt{2}\right)}{\sqrt{2}}$$

01.21.21.0098.01

$$\int \cosh(z) \tanh(3\, z)\, dz = -\frac{1}{\sqrt{3}} \tanh^{-1}\left(\frac{i \tanh\left(\frac{z}{2}\right) + 2}{\sqrt{3}} \right) + \cosh(z) - \frac{1}{\sqrt{3}} \tanh^{-1}\left(\frac{2 - i \tanh\left(\frac{z}{2}\right)}{\sqrt{3}} \right)$$

01.21.21.0099.01

$$\int \cosh(z) \tanh(4\, z)\, dz = \frac{1}{126} e^{-9\, z} \left(-63\, e^{10\, z}\, {}_2F_1\left(\frac{1}{8}, 1; \frac{9}{8}; -e^{8\, z}\right) + 9\, e^{16\, z}\, {}_2F_1\left(\frac{7}{8}, 1; \frac{15}{8}; -e^{8\, z}\right) + 7\, e^{18\, z}\, {}_2F_1\left(\frac{9}{8}, 1; \frac{17}{8}; -e^{8\, z}\right) + 63\, e^{8\, z}\, {}_2F_1\left(-\frac{1}{8}, 1; \frac{7}{8}; -e^{8\, z}\right) \right)$$

01.21.21.0100.01

$$\int \cosh(2z)\tanh(z)\,dz = \frac{1}{2}(\cosh(2z) - 2\log(\cosh(z)))$$

01.21.21.0101.01

$$\int \cosh(3z)\tanh(z)\,dz = \frac{1}{3}(\cosh(3z) - 6\cosh(z))$$

01.21.21.0102.01

$$\int \cosh(4z)\tanh(z)\,dz = -\cosh(2z) + \frac{1}{4}\cosh(4z) + \log(\cosh(z))$$

Involving power of cosh

Involving $\cosh^{\mu}(b z)$

01.21.21.0103.01

$$\int \cosh^m(bz)\tanh(cz)\,dz = \frac{\log(\cosh(cz))(1 - m \bmod 2)}{c} 2^{-m}\binom{m}{\frac{m}{2}} + 2^{-m}\sum_{k=0}^{\left\lfloor\frac{m-1}{2}\right\rfloor}\binom{m}{k}\left(-\frac{e^{b(m-2k)z}}{b(m-2k)}\,{}_2F_1\left(\frac{b(m-2k)}{2c}, 1; \frac{2c+b(m-2k)}{2c}; -e^{2cz}\right) + \frac{e^{(2c+b(m-2k))z}}{2c+b(m-2k)}\,{}_2F_1\left(\frac{2c+b(m-2k)}{2c}, 1; \frac{4c+b(m-2k)}{2c}; -e^{2cz}\right) + \frac{e^{(2c-b(m-2k))z}}{2c-b(m-2k)}\,{}_2F_1\left(\frac{2c-b(m-2k)}{2c}, 1; \frac{4c-b(m-2k)}{2c}; -e^{2cz}\right) + \frac{e^{-b(m-2k)z}}{b(m-2k)}\,{}_2F_1\left(-\frac{b(m-2k)}{2c}, 1; \frac{2c-b(m-2k)}{2c}; -e^{2cz}\right)\right) /;\ m \in \mathbb{N}^+$$

01.21.21.0104.01

$$\int \cosh^{\mu}(cz)\tanh(cz)\,dz = \frac{\cosh^{\mu}(cz)}{c\,\mu}$$

01.21.21.0105.01

$$\int \cosh^2(cz)\tanh(cz)\,dz = \frac{\cosh^2(cz)}{2c}$$

01.21.21.0106.01

$$\int \cosh^3(cz)\tanh(cz)\,dz = \frac{\cosh^3(cz)}{3c}$$

01.21.21.0107.01

$$\int \sqrt{\cosh^2(z)}\,\tanh(z)\,dz = \sqrt{\cosh^2(z)}$$

01.21.21.0108.01

$$\int \cosh^2(z)\tanh(3z)\,dz = \frac{1}{4}(\cosh(2z) + \log(2\cosh(2z) - 1))$$

01.21.21.0109.01

$$\int \cosh^{\frac{1}{2}}(cz)\tanh(cz)\,dz = \frac{2\cosh^{\frac{1}{2}}(cz)}{c}$$

01.21.21.0110.01

$$\int \frac{\tanh(c\,z)}{\cosh^{\frac{1}{2}}(c\,z)}\,dz = -\frac{2}{c\cosh^{\frac{1}{2}}(c\,z)}$$

01.21.21.0111.01

$$\int \cosh^{\frac{1}{2}}(2\,c\,z)\tanh(c\,z)\,dz = \frac{\cosh^{\frac{1}{2}}(2\,c\,z) - \tan^{-1}\left(\cosh^{\frac{1}{2}}(2\,c\,z)\right)}{c}$$

01.21.21.0112.01

$$\int \frac{\tanh(c\,z)}{\cosh^{\frac{1}{2}}(2\,c\,z)}\,dz = \frac{\tan^{-1}\left(\cosh^{\frac{1}{2}}(2\,c\,z)\right)}{c}$$

Involving rational functions of cosh

Involving $(a + b\cosh(c\,z))^{-n}$

01.21.21.0113.01

$$\int \frac{\tanh(c\,z)}{a + b\cosh(c\,z)}\,dz = \frac{\log(\cosh(c\,z)) - \log(a + b\cosh(c\,z))}{a\,c}$$

01.21.21.0114.01

$$\int \frac{A + B\tanh(z)}{a + b\cosh(z)}\,dz = \frac{B\,(\log(\cosh(z)) - \log(a + b\cosh(z)))}{a} - \frac{2\,A}{\sqrt{b^2 - a^2}}\tan^{-1}\left(\frac{(a-b)\tanh\left(\frac{z}{2}\right)}{\sqrt{b^2 - a^2}}\right)$$

01.21.21.0115.01

$$\int \frac{A + B\tanh(z)}{1 - \cosh(z)}\,dz = A\coth\left(\frac{z}{2}\right) + B\left(\log(\cosh(z)) - 2\log\left(\sinh\left(\frac{z}{2}\right)\right)\right)$$

01.21.21.0116.01

$$\int \frac{A + B\tanh(z)}{\cosh(z) + 1}\,dz = \frac{2\cosh\left(\frac{z}{2}\right)}{\cosh(z) + 1}\left(B\cosh\left(\frac{z}{2}\right)\left(\log(\cosh(z)) - 2\log\left(\cosh\left(\frac{z}{2}\right)\right)\right) + A\sinh\left(\frac{z}{2}\right)\right)$$

01.21.21.0117.01

$$\int \frac{(A + B\cosh(z))\tanh(z)}{a + b\cosh(z)}\,dz = \frac{A\,b\log(\cosh(z)) + (a\,B - A\,b)\log(a + b\cosh(z))}{a\,b}$$

01.21.21.0118.01

$$\int \frac{\tanh(c\,z)}{(a + b\cosh(c\,z))^2}\,dz = \frac{1}{a^2\,c}\left(\frac{a}{a + b\cosh(c\,z)} + \log(\cosh(c\,z)) - \log(a + b\cosh(c\,z))\right)$$

Involving algebraic functions of cosh

Involving $(a + b\cosh(c\,z))^{\beta}$

01.21.21.0119.01

$$\int (a+b\cosh(c z))^{\beta} \tanh(c z)\, dz = \frac{(a+b\cosh(c z))^{\beta}}{c\,\beta}\, {}_2F_1\!\left(-\beta, -\beta; 1-\beta; -\frac{a\operatorname{sech}(c z)}{b}\right)\left(\frac{a\operatorname{sech}(c z)}{b}+1\right)^{-\beta}$$

01.21.21.0120.01

$$\int \sqrt{a+b\cosh(c z)}\, \tanh(c z)\, dz = \frac{2\sqrt{a+b\cosh(c z)}}{c}\left(1-\frac{\sqrt{a}\,\operatorname{sech}^{\frac{1}{2}}(c z)}{\sqrt{b}\,\sqrt{\frac{a\operatorname{sech}(c z)}{b}+1}}\sinh^{-1}\!\left(\frac{\sqrt{a}\,\operatorname{sech}^{\frac{1}{2}}(c z)}{\sqrt{b}}\right)\right)$$

01.21.21.0121.01

$$\int \sqrt{\cosh(c z)\, a + a}\, \tanh(c z)\, dz = \frac{\sqrt{a(\cosh(c z)+1)}\,\operatorname{sech}\left(\frac{c z}{2}\right)}{c}\left(-i\sqrt{2}\tan^{-1}\!\left(-i\sqrt{2}-\tanh\left(\frac{c z}{4}\right)\right)-\sqrt{2}\tanh^{-1}\!\left(i\tanh\left(\frac{c z}{4}\right)+\sqrt{2}\right)+2\cosh\left(\frac{c z}{2}\right)\right)$$

01.21.21.0122.01

$$\int \sqrt{a-a\cosh(c z)}\, \tanh(c z)\, dz = \frac{\sqrt{a-a\cosh(c z)}\,\operatorname{csch}\left(\frac{c z}{2}\right)}{c}\left(2\sinh\left(\frac{c z}{2}\right)-\sqrt{2}\tan^{-1}\!\left(\sqrt{2}\sinh\left(\frac{c z}{2}\right)\right)\right)$$

01.21.21.0123.01

$$\int \frac{\tanh(c z)}{\sqrt{a+b\cosh(c z)}}\, dz = -\frac{2\sqrt{b}}{\sqrt{a}\, c\,\sqrt{a+b\cosh(c z)}\,\operatorname{sech}^{\frac{1}{2}}(c z)}\sqrt{\frac{a\operatorname{sech}(c z)}{b}+1}\,\sinh^{-1}\!\left(\frac{\sqrt{a}\,\operatorname{sech}^{\frac{1}{2}}(c z)}{\sqrt{b}}\right)$$

01.21.21.0124.01

$$\int \frac{\tanh(c z)}{\sqrt{\cosh(c z)\, a + a}}\, dz = -\frac{2\, i\cosh\left(\frac{c z}{2}\right)}{c\sqrt{a\cosh^2\left(\frac{c z}{2}\right)}}\left(\tan^{-1}\!\left(-i\sqrt{2}-\tanh\left(\frac{c z}{4}\right)\right)-i\tanh^{-1}\!\left(i\tanh\left(\frac{c z}{4}\right)+\sqrt{2}\right)\right)$$

01.21.21.0125.01

$$\int \frac{\tanh(c z)}{\sqrt{a-a\cosh(c z)}}\, dz = \frac{2\tan^{-1}\!\left(\sqrt{2}\sinh\left(\frac{c z}{2}\right)\right)\sinh\left(\frac{c z}{2}\right)}{c\sqrt{-a\sinh^2\left(\frac{c z}{2}\right)}}$$

Involving $(a+b\cosh(2 c z))^{\beta}$

01.21.21.0126.01

$$\int (a+b\cosh(2 c z))^{\beta} \tanh(c z)\, dz = \frac{(a+b\cosh(2 c z))^{\beta}}{2 c\,\beta}\, {}_2F_1\!\left(-\beta, -\beta; 1-\beta; \frac{(b-a)\operatorname{sech}^2(c z)}{2 b}\right)\left(\frac{(a-b)\operatorname{sech}^2(c z)}{2 b}+1\right)^{-\beta}$$

01.21.21.0127.01

$$\int \sqrt{a+b\cosh(2 c z)}\, \tanh(c z)\, dz = \frac{1}{c}\left(\sqrt{a+b\cosh(2 c z)}-\sqrt{a-b}\,\tanh^{-1}\!\left(\frac{\sqrt{a+b\cosh(2 c z)}}{\sqrt{a-b}}\right)\right)$$

01.21.21.0128.01

$$\int \sqrt{\cosh(2 c z)\, a + a}\, \tanh(c z)\, dz = \frac{\sqrt{\cosh(2 c z)\, a + a}}{c}$$

01.21.21.0129.01

$$\int \sqrt{a-a\cosh(2cz)}\,\tanh(cz)\,dz = \frac{\sqrt{a-a\cosh(2cz)}\,\operatorname{csch}(cz)\left(\sinh(cz)-2\tan^{-1}\left(\tanh\left(\frac{cz}{2}\right)\right)\right)}{c}$$

01.21.21.0130.01

$$\int \frac{\tanh(cz)}{\sqrt{a+b\cosh(2cz)}}\,dz = -\frac{1}{\sqrt{a-b}\,c}\tanh^{-1}\left(\frac{\sqrt{a+b\cosh(2cz)}}{\sqrt{a-b}}\right)$$

01.21.21.0131.01

$$\int \frac{\tanh(cz)}{\sqrt{\cosh(2cz)\,a+a}}\,dz = -\frac{1}{c\sqrt{\cosh(2cz)\,a+a}}$$

01.21.21.0132.01

$$\int \frac{\tanh(cz)}{\sqrt{a-a\cosh(2cz)}}\,dz = \frac{2\tan^{-1}\left(\tanh\left(\frac{cz}{2}\right)\right)\sinh(cz)}{c\sqrt{a-a\cosh(2cz)}}$$

01.21.21.0133.01

$$\int \frac{\cosh(2cz)\tanh(cz)}{\sqrt{a+b\cosh(2cz)}}\,dz = \cosh^2\left(\frac{cz}{2}\right)\sqrt{\frac{a+b\cosh(2cz)}{(\cosh(cz)+1)^2}}\left(\sqrt{a-b}\,\sqrt{(a+b\cosh(2cz))\operatorname{sech}^4\left(\frac{cz}{2}\right)}\,\cosh(cz)+\right.$$

$$2b\log\left(\frac{2\left((b-a)\tanh^2\left(\frac{cz}{2}\right)+a-b+\sqrt{a-b}\,\sqrt{(a+b\cosh(2cz))\operatorname{sech}^4\left(\frac{cz}{2}\right)}\right)}{\sqrt{a-b}\left(\tanh^2\left(\frac{cz}{2}\right)+1\right)}\right)+$$

$$\left.\sqrt{a-b}\,\sqrt{(a+b\cosh(2cz))\operatorname{sech}^4\left(\frac{cz}{2}\right)}\right)\Big/\left(\sqrt{a-b}\,b\,c\sqrt{a+b\cosh(2cz)}\right)$$

01.21.21.0134.01

$$\int \frac{\cosh(cz)\tanh(cz)}{\sqrt{a+b\cosh(2cz)}}\,dz = \frac{\log\left(\sqrt{2}\,\sqrt{b}\,\cosh(cz)+\sqrt{a+b\cosh(2cz)}\right)}{\sqrt{2}\,\sqrt{b}\,c}$$

01.21.21.0135.01

$$\int \frac{\cosh^2(cz)\tanh(cz)}{\sqrt{a+b\cosh(2cz)}}\,dz = \frac{1}{2bc\sqrt{a+b\cosh(2cz)}}\left(-\sqrt{\frac{b\cosh(2cz)}{a}+1}\,a+a+b\cosh(2cz)\right)$$

Involving $\left((a+b\cosh^m(cz))^n\right)^\beta$

01.21.21.0136.01

$$\int \frac{\tanh(c\,z)}{((a+b\cosh^m(c\,z))^n)^\beta}\,dz = -\frac{((b\cosh^m(c\,z)+a)^n)^{-\beta}}{c\,m\,n\,\beta}\left(\frac{a\cosh^{-m}(c\,z)}{b}+1\right)^{n\,\beta}\,{}_2F_1\!\left(n\,\beta,\,n\,\beta;\,n\,\beta+1;\,-\frac{a\cosh^{-m}(c\,z)}{b}\right)$$

Involving sinh and cosh

01.21.21.0137.01

$$\int \frac{(A+B\cosh(z))\tanh(z)}{a+b\sinh(z)}\,dz = -\frac{1}{b\,(-a^2-b^2)^{3/2}}\Bigg(2\,A\sqrt{-a^2-b^2}\,\tan^{-1}\!\left(\tanh\!\left(\frac{z}{2}\right)\right)b^2 - 2\,a\,(a^2+b^2)\,B\tan^{-1}\!\left(\frac{b-a\tanh\!\left(\frac{z}{2}\right)}{\sqrt{-a^2-b^2}}\right)+\sqrt{-a^2-b^2}\,\left((a^2+b^2)\,B\,z+a\,A\,b\log(\cosh(z))-a\,A\,b\log(a+b\sinh(z))\right)\Bigg)$$

01.21.21.0138.01

$$\int \frac{(A+B\sinh(z))\tanh(z)}{a+b\cosh(z)}\,dz = \frac{1}{a\,b\sqrt{b^2-a^2}}\Bigg(2\,(a^2-b^2)\,B\tan^{-1}\!\left(\frac{(a-b)\tanh\!\left(\frac{z}{2}\right)}{\sqrt{b^2-a^2}}\right) - 2\,b\sqrt{b^2-a^2}\,B\tan^{-1}\!\left(\tanh\!\left(\frac{z}{2}\right)\right)+\sqrt{b^2-a^2}\,(a\,B\,z+A\,b\log(\cosh(z))-A\,b\log(a+b\cosh(z)))\Bigg)$$

01.21.21.0139.01

$$\int \sqrt{\cosh(a\,z)\sinh(a\,z)}\,\tanh(a\,z)\,dz = -\frac{2\,(\cosh(a\,z)\sinh(a\,z))^{3/2}\tanh(a\,z)}{a\,(-\sinh^2(a\,z))^{5/4}}\,{}_2F_1\!\left(\frac{1}{4},\,-\frac{1}{4};\,\frac{5}{4};\,\cosh^2(a\,z)\right)$$

01.21.21.0140.01

$$\int \sinh(c\,z)\sqrt{a+b\cosh(2\,c\,z)}\,\tanh(c\,z)\,dz = \frac{1}{4\sqrt{a-b}\,\sqrt{b}\,c}\Bigg(\sqrt{2}\,(a-3\,b)\sqrt{a-b}\,\tanh^{-1}\!\left(\frac{\sqrt{2}\,\sqrt{b}\,\sinh(c\,z)}{\sqrt{a+b\cosh(2\,c\,z)}}\right)+ 2\sqrt{b}\left(2\,(b-a)\tan^{-1}\!\left(\frac{\sqrt{a-b}\,\sinh(c\,z)}{\sqrt{a+b\cosh(2\,c\,z)}}\right)+\sqrt{a-b}\,\sqrt{a+b\cosh(2\,c\,z)}\,\sinh(c\,z)\right)\Bigg)$$

01.21.21.0141.01

$$\int \frac{\sinh(c\,z)\tanh(c\,z)}{\sqrt{a+b\cosh(2\,c\,z)}}\,dz = \frac{\dfrac{\sqrt{2}\,\tanh^{-1}\!\left(\frac{\sqrt{2}\,\sqrt{b}\,\sinh(c\,z)}{\sqrt{a+b\cosh(2\,c\,z)}}\right)}{\sqrt{b}} - \dfrac{2\tan^{-1}\!\left(\frac{\sqrt{a-b}\,\sinh(c\,z)}{\sqrt{a+b\cosh(2\,c\,z)}}\right)}{\sqrt{a-b}}}{2\,c}$$

01.21.21.0142.01

$$\int \sinh^3(c\,z)\sqrt{a+b\cosh(2\,c\,z)}\,\tanh(c\,z)\,dz = \frac{1}{32\sqrt{a-b}\,b^{3/2}\,c}\Bigg(2\sqrt{b}\left((a-10\,b)\sqrt{a-b}\,\sqrt{a+b\cosh(2\,c\,z)}\,\sinh(c\,z)+b\left(16\,(a-b)\tan^{-1}\!\left(\frac{\sqrt{a-b}\,\sinh(c\,z)}{\sqrt{a+b\cosh(2\,c\,z)}}\right)+ \sqrt{a-b}\,\sqrt{a+b\cosh(2\,c\,z)}\,\sinh(3\,c\,z)\right)\right)-\sqrt{2}\,\sqrt{a-b}\,(a^2+10\,b\,a-23\,b^2)\tanh^{-1}\!\left(\frac{\sqrt{2}\,\sqrt{b}\,\sinh(c\,z)}{\sqrt{a+b\cosh(2\,c\,z)}}\right)\Bigg)$$

01.21.21.0143.01

$$\int \frac{\sinh^3(c z)\tanh(c z)}{\sqrt{a+b\cosh(2 c z)}}\,dz = \frac{1}{8\sqrt{a-b}\,b^{3/2}\,c}\left(2\sqrt{b}\left(4 b \tan^{-1}\left(\frac{\sqrt{a-b}\,\sinh(c z)}{\sqrt{a+b\cosh(2 c z)}}\right)+\sqrt{a-b}\,\sqrt{a+b\cosh(2 c z)}\,\sinh(c z)\right)-\sqrt{2}\,\sqrt{a-b}\,(a+5 b)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sinh(c z)}{\sqrt{a+b\cosh(2 c z)}}\right)\right)$$

Involving hyperbolic and a power functions

Involving sinh and power

Involving $z^n \sinh(a + b z) \tanh(c z)$

01.21.21.0144.01

$$\int z^n \sinh(a+b z)\tanh(c z)\,dz = \frac{1}{2} e^a\, n!\left(e^{(b+2c)z}\sum_{j=0}^{n}\frac{(-1)^j (b+2c)^{-j-1} z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{b+2c}{2c},\ldots,\frac{b+2c}{2c},1;\frac{b+2c}{2c}+1,\ldots,\frac{b+2c}{2c}+1;-e^{2cz}\right)-e^{bz}\sum_{j=0}^{n}\frac{(-1)^j b^{-j-1} z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{b}{2c},\ldots,\frac{b}{2c},1;\frac{b}{2c}+1,\ldots,\frac{b}{2c}+1;-e^{2cz}\right)\right)-\frac{1}{2} e^{-a}\, n!\left(e^{(2c-b)z}\sum_{j=0}^{n}\frac{(-1)^j (2c-b)^{-j-1} z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{2c-b}{2c},\ldots,\frac{2c-b}{2c},1;\frac{2c-b}{2c}+1,\ldots,\frac{2c-b}{2c}+1;-e^{2cz}\right)-e^{-bz}\sum_{j=0}^{n}\frac{(-1)^j (-b)^{-j-1} z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(-\frac{b}{2c},\ldots,-\frac{b}{2c},1;1-\frac{b}{2c},\ldots,1-\frac{b}{2c};-e^{2cz}\right)\right)\;/;\; n\in\mathbb{N}$$

01.21.21.0145.01

$$\int z^n \sinh(b z)\tanh(c z)\,dz = \frac{1}{2} n!\left(e^{(b+2c)z}\sum_{j=0}^{n}\frac{(-1)^j (b+2c)^{-j-1} z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{b+2c}{2c},\ldots,\frac{b+2c}{2c},1;\frac{b+2c}{2c}+1,\ldots,\frac{b+2c}{2c}+1;-e^{2cz}\right)-e^{bz}\sum_{j=0}^{n}\frac{(-1)^j b^{-j-1} z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{b}{2c},\ldots,\frac{b}{2c},1;\frac{b}{2c}+1,\ldots,\frac{b}{2c}+1;-e^{2cz}\right)-e^{(2c-b)z}\sum_{j=0}^{n}\frac{(-1)^j (2c-b)^{-j-1} z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{2c-b}{2c},\ldots,\frac{2c-b}{2c},1;\frac{2c-b}{2c}+1,\ldots,\frac{2c-b}{2c}+1;-e^{2cz}\right)+e^{-bz}\sum_{j=0}^{n}\frac{(-1)^j (-b)^{-j-1} z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(-\frac{b}{2c},\ldots,-\frac{b}{2c},1;1-\frac{b}{2c},\ldots,1-\frac{b}{2c};-e^{2cz}\right)\right)\;/;\; n\in\mathbb{N}$$

Involving powers of sinh and power

Involving $z^n \sinh^u(b z) \tanh(c z)$

01.21.21.0146.01

$$\int z^n \sinh^u(b z) \tanh(c z) \, dz = \binom{u}{\frac{u}{2}} n! \, (1 - u \bmod 2) \left(\frac{i}{2}\right)^u \left(2 e^{2 c z} \sum_{j=0}^{n} \frac{(-1)^j 2^{-j-1} c^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}(1, \ldots, 1, 1; 2, \ldots, 2; -e^{2 c z}) - \frac{z^{n+1}}{(n+1)!}\right) +$$

$$\left(\frac{i}{2}\right)^u n! \left(\sum_{k=0}^{\left\lfloor \frac{u-1}{2} \right\rfloor} (-1)^k \binom{u}{k} \left(e^{\frac{i \pi u}{2}} \left(e^{(2 c - b (u - 2 k)) z} \sum_{j=0}^{n} \frac{(-1)^j (2 c - b (u - 2 k))^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{2 c - b (u - 2 k)}{2 c}, \ldots, \frac{2 c - b (u - 2 k)}{2 c}, 1; \frac{2 c - b (u - 2 k)}{2 c} + 1, \ldots, \frac{2 c - b (u - 2 k)}{2 c} + 1; -e^{2 c z}\right) - e^{-b (u-2 k) z} \sum_{j=0}^{n} \frac{(-1)^j (-b (u - 2 k))^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{-b (u - 2 k)}{2 c}, \ldots, \frac{-b (u - 2 k)}{2 c}, 1; \frac{-b (u - 2 k)}{2 c} + 1, \ldots, \frac{-b (u - 2 k)}{2 c} + 1; -e^{2 c z}\right)\right) + e^{-\frac{1}{2} i \pi u} \left(e^{(2 c + b (u - 2 k)) z} \sum_{j=0}^{n} \frac{(-1)^j (2 c + b (u - 2 k))^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{2 c + b (u - 2 k)}{2 c}, \ldots, \frac{2 c + b (u - 2 k)}{2 c}, 1; \frac{2 c + b (u - 2 k)}{2 c} + 1, \ldots, \frac{2 c + b (u - 2 k)}{2 c} + 1; -e^{2 c z}\right) - e^{b (u-2 k) z} \sum_{j=0}^{n} \frac{(-1)^j (b (u - 2 k))^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{b (u - 2 k)}{2 c}, \ldots, \frac{b (u - 2 k)}{2 c}, 1; \frac{b (u - 2 k)}{2 c} + 1, \ldots, \frac{b (u - 2 k)}{2 c} + 1; -e^{2 c z}\right)\right)\right)\right) /; n \in \mathbb{N} \land u \in \mathbb{N}^+$$

Involving cosh and power

Involving $z^n \cosh(a + b z) \tanh(c z)$

01.21.21.0147.01

$$\int z^n \cosh(a+b\,z)\tanh(c\,z)\,dz = \frac{1}{2}\,e^a\,n!\left(e^{(b+2c)z}\sum_{j=0}^{n}\frac{(-1)^j\,(b+2c)^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{b+2c}{2c},\ldots,\frac{b+2c}{2c},1;\frac{b+2c}{2c}+1,\ldots,\frac{b+2c}{2c}+1;-e^{2cz}\right)-e^{b\,z}\sum_{j=0}^{n}\frac{(-1)^j\,b^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{b}{2c},\ldots,\frac{b}{2c},1;\frac{b}{2c}+1,\ldots,\frac{b}{2c}+1;-e^{2cz}\right)\right)+\frac{1}{2}\,e^{-a}\,n!\left(e^{(2c-b)z}\sum_{j=0}^{n}\frac{(-1)^j\,(2c-b)^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{2c-b}{2c},\ldots,\frac{2c-b}{2c},1;\frac{2c-b}{2c}+1,\ldots,\frac{2c-b}{2c}+1;-e^{2cz}\right)-e^{-b\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(-b)^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(-\frac{b}{2c},\ldots,-\frac{b}{2c},1;1-\frac{b}{2c},\ldots,1-\frac{b}{2c};-e^{2cz}\right)\right)\;/;\;n\in\mathbb{N}$$

01.21.21.0148.01

$$\int z^n \cosh(b\,z)\tanh(c\,z)\,dz = \frac{1}{2}\,n!\left(e^{(b+2c)z}\sum_{j=0}^{n}\frac{(-1)^j\,(b+2c)^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{b+2c}{2c},\ldots,\frac{b+2c}{2c},1;\frac{b+2c}{2c}+1,\ldots,\frac{b+2c}{2c}+1;-e^{2cz}\right)-e^{b\,z}\sum_{j=0}^{n}\frac{(-1)^j\,b^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{b}{2c},\ldots,\frac{b}{2c},1;\frac{b}{2c}+1,\ldots,\frac{b}{2c}+1;-e^{2cz}\right)+e^{(2c-b)z}\sum_{j=0}^{n}\frac{(-1)^j\,(2c-b)^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{2c-b}{2c},\ldots,\frac{2c-b}{2c},1;\frac{2c-b}{2c}+1,\ldots,\frac{2c-b}{2c}+1;-e^{2cz}\right)-e^{-b\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(-b)^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(-\frac{b}{2c},\ldots,-\frac{b}{2c},1;1-\frac{b}{2c},\ldots,1-\frac{b}{2c};-e^{2cz}\right)\right)\;/;\;n\in\mathbb{N}$$

Involving powers of cosh and power

Involving $z^n \cosh^u(b\,z)\tanh(c\,z)$

01.21.21.0149.01

$$\int z^n \cosh^u(b z) \tanh(c z)\, dz = 2^{-u} \binom{u}{\frac{u}{2}} n!\, (1 - u \bmod 2) \left(2\, e^{2 c z} \sum_{j=0}^{n} \frac{(-1)^j\, 2^{-j-1}\, c^{-j-1}\, z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}\!\left(1, \ldots, 1; 2, \ldots, 2; -e^{2 c z}\right) - \frac{z^{n+1}}{(n+1)!}\right) + 2^{-u}\, n! \sum_{k=0}^{\left\lfloor \frac{u-1}{2} \right\rfloor} \binom{u}{k} \left(-e^{-b(u-2k) z} \sum_{j=0}^{n} \frac{(-1)^j\, (-b(u-2k))^{-j-1}\, z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}\!\left(-\frac{b(u-2k)}{2c}, \ldots, -\frac{b(u-2k)}{2c}, 1; 1 - \frac{b(u-2k)}{2c}, \ldots, 1 - \frac{b(u-2k)}{2c}; -e^{2 c z}\right) - e^{b(u-2k) z} \sum_{j=0}^{n} \frac{(-1)^j\, (b(u-2k))^{-j-1}\, z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}\!\left(\frac{b(u-2k)}{2c}, \ldots, \frac{b(u-2k)}{2c}, 1; \frac{b(u-2k)}{2c} + 1, \ldots, \frac{b(u-2k)}{2c} + 1; -e^{2 c z}\right) + e^{(2c - b(u-2k)) z} \sum_{j=0}^{n} \frac{(-1)^j\, (2c - b(u-2k))^{-j-1}\, z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}\!\left(\frac{2c - b(u-2k)}{2c}, \ldots, \frac{2c - b(u-2k)}{2c}, 1; \frac{2c - b(u-2k)}{2c} + 1, \ldots, \frac{2c - b(u-2k)}{2c} + 1; -e^{2 c z}\right) + e^{(2c + b(u-2k)) z} \sum_{j=0}^{n} \frac{(-1)^j\, (2c + b(u-2k))^{-j-1}\, z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}\!\left(\frac{2c + b(u-2k)}{2c}, \ldots, \frac{2c + b(u-2k)}{2c}, 1; \frac{2c + b(u-2k)}{2c} + 1, \ldots, \frac{2c + b(u-2k)}{2c} + 1; -e^{2 c z}\right)\right) /;\ n \in \mathbb{N} \land u \in \mathbb{N}^+$$

Involving hyperbolic and exponential functions

Involving sinh and exp

Involving $e^{p z} \sinh(b z) \operatorname{csch}(c z)$

01.21.21.0150.01

$$\int e^{p z} \sinh(b z) \tanh(c z)\, dz = \frac{1}{2} \left(\frac{e^{(-b+2c+p) z}\, {}_2F_1\!\left(\frac{-b+2c+p}{2c}, 1; \frac{-b+4c+p}{2c}; -e^{2 c z}\right)}{b - 2c - p} + \frac{e^{(b+2c+p) z}\, {}_2F_1\!\left(\frac{b+2c+p}{2c}, 1; \frac{b+4c+p}{2c}; -e^{2 c z}\right)}{b + 2c + p} - \frac{e^{(p-b) z}\, {}_2F_1\!\left(\frac{p-b}{2c}, 1; \frac{p-b}{2c} + 1; -e^{2 c z}\right)}{b - p} - \frac{e^{(b+p) z}\, {}_2F_1\!\left(\frac{b+p}{2c}, 1; \frac{b+p}{2c} + 1; -e^{2 c z}\right)}{b + p}\right)$$

Involving powers of sinh and exp

Involving $e^{p z} \sinh^u(b z) \tanh(c z)$

01.21.21.0151.01

$$\int e^{p z} \sinh^u(b z) \tanh(c z)\, dz = i^u\, 2^{-u} \binom{u}{\frac{u}{2}} \left(\frac{e^{(2c+p)z}\, {}_2F_1\left(\frac{2c+p}{2c}, 1; \frac{4c+p}{2c}; -e^{2cz}\right)}{2c+p} - \frac{e^{pz}\, {}_2F_1\left(\frac{p}{2c}, 1; \frac{2c+p}{2c}; -e^{2cz}\right)}{p} \right) (1 - u \bmod 2) + 2^{-u} \sum_{k=0}^{\left\lfloor \frac{u-1}{2} \right\rfloor} (-1)^k \binom{u}{k} \left(\frac{e^{(2c+p+b(u-2k))z}\, {}_2F_1\left(\frac{2c+p+b(u-2k)}{2c}, 1; \frac{4c+p+b(u-2k)}{2c}; -e^{2cz}\right)}{2c+p+b(u-2k)} - \frac{(-1)^u\, e^{(p-b(u-2k))z}\, {}_2F_1\left(\frac{p-b(u-2k)}{2c}, 1; \frac{2c+p-b(u-2k)}{2c}; -e^{2cz}\right)}{p-b(u-2k)} + \frac{(-1)^u\, e^{(2c+p-b(u-2k))z}\, {}_2F_1\left(\frac{2c+p-b(u-2k)}{2c}, 1; \frac{4c+p-b(u-2k)}{2c}; -e^{2cz}\right)}{2c+p-b(u-2k)} - \frac{e^{(p+b(u-2k))z}\, {}_2F_1\left(\frac{p+b(u-2k)}{2c}, 1; \frac{2c+p+b(u-2k)}{2c}; -e^{2cz}\right)}{p+b(u-2k)} \right) /;\, u \in \mathbb{N}^+$$

01.21.21.0152.01

$$\int e^{p z} \sinh^\mu(c z) \tanh(c z)\, dz = -\frac{e^{pz} \left(1 - e^{2cz}\right)^{-\mu} \sinh^\mu(c z)}{p - c\mu} F_1\left(\frac{p - c\mu}{2c}; -\mu - 1, 1; \frac{1}{2}\left(\frac{p}{c} - \mu + 2\right); e^{2cz}, -e^{2cz}\right)$$

Involving cosh and exp

Involving $e^{p z} \cosh(b z) \tanh(c z)$

01.21.21.0153.01

$$\int e^{p z} \cosh(b z) \tanh(c z)\, dz = \frac{1}{2} \left(\frac{e^{(p-b)z}}{b-p}\, {}_2F_1\left(\frac{p-b}{2c}, 1; \frac{p-b}{2c} + 1; -e^{2cz}\right) + \frac{e^{(b+p+2c)z}}{b+p+2c}\, {}_2F_1\left(\frac{b+p+2c}{2c}, 1; \frac{b+p+4c}{2c}; -e^{2cz}\right) - \frac{e^{(b+p)z}}{b+p}\, {}_2F_1\left(\frac{b+p}{2c}, 1; \frac{b+p}{2c} + 1; -e^{2cz}\right) - \frac{e^{(-b+p+2c)z}}{b-p-2c}\, {}_2F_1\left(\frac{-b+p+2c}{2c}, 1; \frac{-b+p+4c}{2c}; -e^{2cz}\right) \right)$$

01.21.21.0154.01

$$\int e^{b z} \cosh(b z) \tanh(c z)\, dz = \frac{1}{4 b c (b+c)} \left(b \left(c\, e^{2(b+c)z}\, {}_2F_1\left(\frac{b+c}{c}, 1; \frac{b}{c} + 2; -e^{2cz}\right) + 2(b+c) \log(\cosh(c z)) \right) - c (b+c)\, e^{2bz}\, {}_2F_1\left(\frac{b}{c}, 1; \frac{b+c}{c}; -e^{2cz}\right) \right)$$

01.21.21.0155.01

$$\int e^{-b z} \cosh(b z) \tanh(c z)\, dz = \frac{e^{-2bz}}{4 b c (c-b)} \left(b c\, e^{2cz}\, {}_2F_1\left(1 - \frac{b}{c}, 1; 2 - \frac{b}{c}; -e^{2cz}\right) - (b-c) \left(c\, {}_2F_1\left(-\frac{b}{c}, 1; 1 - \frac{b}{c}; -e^{2cz}\right) + 2 b\, e^{2bz} \log(\cosh(c z)) \right) \right)$$

01.21.21.0156.01

$$\int e^{-c z} \cosh(c z) \tanh(c z)\, dz = \frac{2 c z + e^{-2 c z}}{4 c}$$

Involving powers of cosh and exp

Involving $e^{p z} \cosh^u(b z) \tanh(c z)$

01.21.21.0157.01

$$\int e^{p z} \cosh^u(b z) \tanh(c z)\, dz = 2^{-u} \binom{u}{\frac{u}{2}} \left(\frac{e^{(2 c + p) z}\, {}_2F_1\left(\frac{2 c + p}{2 c}, 1; \frac{4 c + p}{2 c}; -e^{2 c z}\right)}{2 c + p} - \frac{e^{p z}\, {}_2F_1\left(\frac{p}{2 c}, 1; \frac{2 c + p}{2 c}; -e^{2 c z}\right)}{p} \right) (1 - u \bmod 2) + 2^{-u} \sum_{k=0}^{\left\lfloor \frac{u-1}{2} \right\rfloor} \binom{u}{k} \left(\frac{e^{(2 c + p + b (u - 2 k)) z}\, {}_2F_1\left(\frac{2 c + p + b (u - 2 k)}{2 c}, 1; \frac{4 c + p + b (u - 2 k)}{2 c}; -e^{2 c z}\right)}{2 c + p + b (u - 2 k)} - \frac{e^{(p - b (u - 2 k)) z}\, {}_2F_1\left(\frac{p - b (u - 2 k)}{2 c}, 1; \frac{2 c + p - b (u - 2 k)}{2 c}; -e^{2 c z}\right)}{p - b (u - 2 k)} + \frac{e^{(2 c + p - b (u - 2 k)) z}\, {}_2F_1\left(\frac{2 c + p - b (u - 2 k)}{2 c}, 1; \frac{4 c + p - b (u - 2 k)}{2 c}; -e^{2 c z}\right)}{2 c + p - b (u - 2 k)} - \frac{e^{(p + b (u - 2 k)) z}\, {}_2F_1\left(\frac{p + b (u - 2 k)}{2 c}, 1; \frac{2 c + p + b (u - 2 k)}{2 c}; -e^{2 c z}\right)}{p + b (u - 2 k)} \right) /; u \in \mathbb{N}^+$$

01.21.21.0158.01

$$\int e^{p z} \cosh^\mu(c z) \tanh(c z)\, dz = \cosh^\mu(c z) \left(1 + e^{2 c z}\right)^{-\mu} \left(\frac{e^{(2 c + p) z}\, {}_2F_1\left(\frac{p - c \mu}{2 c} + 1, 1 - \mu; \frac{p - c \mu}{2 c} + 2; -e^{2 c z}\right)}{-\mu c + 2 c + p} - \frac{e^{p z}\, {}_2F_1\left(\frac{p - c \mu}{2 c}, 1 - \mu; \frac{p - c \mu}{2 c} + 1; -e^{2 c z}\right)}{p - c \mu} \right)$$

Involving hyperbolic and trigonometric functions

Involving sin and sinh

Involving $\sin(a z) \sinh(b z) \tanh(c z)$

01.21.21.0159.01

$$\int \sin(a z)\sinh(b z)\tanh(c z)\,dz = \frac{1}{4} i \left(\frac{e^{(-b-i a) z}\, {}_2F_1\left(\frac{-b-i a}{2c}, 1; \frac{-b-i a}{2c}+1; -e^{2 c z}\right)}{-b-i a} + \frac{e^{(b+i a) z}\, {}_2F_1\left(\frac{b+i a}{2c}, 1; \frac{b+i a}{2c}+1; -e^{2 c z}\right)}{b+i a} - \frac{e^{(-b+2c-i a) z}\, {}_2F_1\left(\frac{-b-i a}{2c}+1, 1; \frac{-b-i a}{2c}+2; -e^{2 c z}\right)}{-b+2c-i a} + \frac{e^{(-b+2c+i a) z}\, {}_2F_1\left(\frac{i a-b}{2c}+1, 1; \frac{i a-b}{2c}+2; -e^{2 c z}\right)}{-b+2c+i a} + \frac{e^{(b+2c-i a) z}\, {}_2F_1\left(\frac{b-i a}{2c}+1, 1; \frac{b-i a}{2c}+2; -e^{2 c z}\right)}{b+2c-i a} - \frac{e^{(i a-b) z}\, {}_2F_1\left(\frac{i a-b}{2c}, 1; \frac{i a-b}{2c}+1; -e^{2 c z}\right)}{i a-b} - \frac{e^{(b-i a) z}\, {}_2F_1\left(\frac{b-i a}{2c}, 1; \frac{b-i a}{2c}+1; -e^{2 c z}\right)}{b-i a} - \frac{e^{(b+2c+i a) z}\, {}_2F_1\left(\frac{b+i a}{2c}+1, 1; \frac{b+i a}{2c}+2; -e^{2 c z}\right)}{b+2c+i a} \right)$$

Involving powers of sin and powers of sinh

Involving $\sin^m(a z)\sinh^u(b z)\tanh(c z)$

01.21.21.0160.01

$$\int \sin^m(a z)\sinh^u(b z)\tanh(c z)\,dz = \frac{i^u\, 2^{-m-u} \log(\cosh(c z))\,(1-m \bmod 2)\,(1-u \bmod 2)}{c}\binom{m}{\frac{m}{2}}\binom{u}{\frac{u}{2}} - i^u\, 2^{-m-u}\binom{u}{\frac{u}{2}}(u \bmod 2-1)\sum_{k=0}^{\left\lfloor\frac{m-1}{2}\right\rfloor}(-1)^k\binom{m}{k}\left(e^{-\frac{1}{2} i m \pi}\left(\frac{i\, e^{i a (m-2k) z}\, {}_2F_1\left(\frac{i a (m-2k)}{2c}, 1; \frac{a i (m-2k)}{2c}+1; -e^{2 c z}\right)}{a (m-2k)} + \frac{e^{(2c+a i (m-2k)) z}\, {}_2F_1\left(\frac{a i (m-2k)}{2c}+1, 1; \frac{a i (m-2k)}{2c}+2; -e^{2 c z}\right)}{2c+a i (m-2k)}\right) + e^{\frac{i m \pi}{2}}\left(\frac{e^{(2c-i a (m-2k)) z}\, {}_2F_1\left(1-\frac{i a (m-2k)}{2c}, 1; 2-\frac{i a (m-2k)}{2c}; -e^{2 c z}\right)}{2c-i a (m-2k)} - \frac{i\, e^{-i a (m-2k) z}\, {}_2F_1\left(-\frac{i a (m-2k)}{2c}, 1; 1-\frac{i a (m-2k)}{2c}; -e^{2 c z}\right)}{a (m-2k)}\right)\right) + 2^{-m-u}\binom{m}{\frac{m}{2}}(1-m \bmod 2)\sum_{k=0}^{\left\lfloor\frac{u-1}{2}\right\rfloor}(-1)^k\binom{u}{k}\left((-1)^u\left(\frac{e^{-b (u-2k) z}\, {}_2F_1\left(-\frac{b (u-2k)}{2c}, 1; 1-\frac{b (u-2k)}{2c}; -e^{2 c z}\right)}{b (u-2k)} + \frac{e^{(2c-b (u-2k)) z}\, {}_2F_1\left(1-\frac{b (u-2k)}{2c}, 1; 2-\frac{b (u-2k)}{2c}; -e^{2 c z}\right)}{2c-b (u-2k)}\right) - \frac{e^{b (u-2k) z}\, {}_2F_1\left(\frac{b (u-2k)}{2c}, 1; \frac{b (u-2k)}{2c}+1; -e^{2 c z}\right)}{b (u-2k)} + \frac{e^{(2c+b (u-2k)) z}\, {}_2F_1\left(\frac{b (u-2k)}{2c}+1, 1; \frac{b (u-2k)}{2c}+2; -e^{2 c z}\right)}{2c+b (u-2k)}\right) + 2^{-m-u}\sum_{s=0}^{\left\lfloor\frac{u-1}{2}\right\rfloor}\sum_{k=0}^{\left\lfloor\frac{m-1}{2}\right\rfloor}(-1)^{k+s}\binom{m}{k}\binom{u}{s}\left(e^{-\frac{1}{2} i m \pi}\left(\left(e^{(2c+a i (m-2k)+b (u-2s)) z}\, {}_2F_1\left(\frac{a i (m-2k)+b (u-2s)}{2c}+1, 1; \frac{a i (m-2k)+b (u-2s)}{2c}+2; -e^{2 c z}\right)\right)\Big/ (2c+a i (m-2k)+b (u-2s)) - \right.\right.$$

$$\left.\frac{e^{(a\,i\,(m-2\,k)+b\,(u-2\,s))\,z}\,{}_2F_1\left(\frac{a\,i\,(m-2\,k)+b\,(u-2\,s)}{2\,c},\,1;\,\frac{a\,i\,(m-2\,k)+b\,(u-2\,s)}{2\,c}+1;\,-e^{2\,c\,z}\right)}{a\,i\,(m-2\,k)+b\,(u-2\,s)}\right)+$$

$$e^{\frac{i\,m\,\pi}{2}}\left(\left(e^{(2\,c-i\,a\,(m-2\,k)+b\,(u-2\,s))\,z}\,{}_2F_1\left(\frac{b\,(u-2\,s)-i\,a\,(m-2\,k)}{2\,c}+1,\,1;\,\frac{b\,(u-2\,s)-i\,a\,(m-2\,k)}{2\,c}+2;\,-e^{2\,c\,z}\right)\right)\Big/\right.$$

$$(2\,c-i\,a\,(m-2\,k)+b\,(u-2\,s))-$$

$$\left.\frac{e^{(b\,(u-2\,s)-i\,a\,(m-2\,k))\,z}\,{}_2F_1\left(\frac{b\,(u-2\,s)-i\,a\,(m-2\,k)}{2\,c},\,1;\,\frac{b\,(u-2\,s)-i\,a\,(m-2\,k)}{2\,c}+1;\,-e^{2\,c\,z}\right)}{b\,(u-2\,s)-i\,a\,(m-2\,k)}\right)+(-1)^u\,e^{-\frac{1}{2}\,i\,m\,\pi}$$

$$\left(\left(e^{(2\,c+a\,i\,(m-2\,k)-b\,(u-2\,s))\,z}\,{}_2F_1\left(\frac{i\,a\,(m-2\,k)-b\,(u-2\,s)}{2\,c}+1,\,1;\,\frac{i\,a\,(m-2\,k)-b\,(u-2\,s)}{2\,c}+2;\,-e^{2\,c\,z}\right)\right)\Big/\right.$$

$$(2\,c+a\,i\,(m-2\,k)-b\,(u-2\,s))-$$

$$\left.\frac{e^{(i\,a\,(m-2\,k)-b\,(u-2\,s))\,z}\,{}_2F_1\left(\frac{i\,a\,(m-2\,k)-b\,(u-2\,s)}{2\,c},\,1;\,\frac{i\,a\,(m-2\,k)-b\,(u-2\,s)}{2\,c}+1;\,-e^{2\,c\,z}\right)}{i\,a\,(m-2\,k)-b\,(u-2\,s)}\right)+(-1)^u\,e^{\frac{i\,m\,\pi}{2}}$$

$$\left(\left(e^{(2\,c-i\,a\,(m-2\,k)-b\,(u-2\,s))\,z}\,{}_2F_1\left(\frac{-i\,a\,(m-2\,k)-b\,(u-2\,s)}{2\,c}+1,\,1;\,\frac{-i\,a\,(m-2\,k)-b\,(u-2\,s)}{2\,c}+2;\,-e^{2\,c\,z}\right)\right)\Big/\right.$$

$$(2\,c-i\,a\,(m-2\,k)-b\,(u-2\,s))-\left(e^{(-i\,a\,(m-2\,k)-b\,(u-2\,s))\,z}\,{}_2F_1\left(\frac{-i\,a\,(m-2\,k)-b\,(u-2\,s)}{2\,c},\,1;\right.\right.$$

$$\left.\left.\left.\frac{-i\,a\,(m-2\,k)-b\,(u-2\,s)}{2\,c}+1;\,-e^{2\,c\,z}\right)\right)\Big/(-i\,a\,(m-2\,k)-b\,(u-2\,s))\right)\right)\,/;\,m\in\mathbb{N}^+\bigwedge u\in\mathbb{N}^+$$

01.21.21.0161.01

$$\int\sin^m(a\,z)\,\sinh^\mu(c\,z)\,\tanh(c\,z)\,dz==$$

$$\frac{2^{-m}\left(1-e^{2\,c\,z}\right)^{-\mu}(1-m\bmod 2)\,\sinh^\mu(c\,z)}{c\,\mu}\binom{m}{\frac{m}{2}}F_1\left(-\frac{\mu}{2};\,-\mu-1,\,1;\,\frac{2-\mu}{2};\,e^{2\,c\,z},\,-e^{2\,c\,z}\right)-2^{-m}\left(1-e^{2\,c\,z}\right)^{-\mu}$$

$$\sinh^\mu(c\,z)\sum_{k=0}^{\left\lfloor\frac{m-1}{2}\right\rfloor}(-1)^k\binom{m}{k}\left(\frac{e^{\frac{i\,m\,\pi}{2}-i\,a\,(m-2\,k)\,z}\,F_1\left(\frac{-i\,a\,(m-2\,k)-c\,\mu}{2\,c};\,-\mu-1,\,1;\,\frac{1}{2}\left(-\frac{i\,a\,(m-2\,k)}{c}-\mu+2\right);\,e^{2\,c\,z},\,-e^{2\,c\,z}\right)}{-i\,a\,(m-2\,k)-c\,\mu}+\right.$$

$$\left.\frac{e^{i\,a\,(m-2\,k)\,z-\frac{i\,m\,\pi}{2}}\,F_1\left(\frac{i\,a\,(m-2\,k)-c\,\mu}{2\,c};\,-\mu-1,\,1;\,\frac{1}{2}\left(\frac{a\,i\,(m-2\,k)}{c}-\mu+2\right);\,e^{2\,c\,z},\,-e^{2\,c\,z}\right)}{i\,a\,(m-2\,k)-c\,\mu}\right)\,/;\,m\in\mathbb{N}^+$$

Involving cos and sinh

Involving cos(*a z*) sinh(*b z*) tanh(*c z*)

01.21.21.0162.01

$$\int \cos(a z)\sinh(b z)\tanh(c z)\,dz = \frac{1}{4}\left(\frac{e^{(-b-i a) z}\,{}_2F_1\left(\frac{-b-i a}{2 c}, 1; \frac{-b-i a}{2 c}+1; -e^{2 c z}\right)}{-b-i a} + \frac{e^{(i a-b) z}\,{}_2F_1\left(\frac{i a-b}{2 c}, 1; \frac{i a-b}{2 c}+1; -e^{2 c z}\right)}{i a-b} - \frac{e^{(-b+2 c-i a) z}\,{}_2F_1\left(\frac{-b-i a}{2 c}+1, 1; \frac{-b-i a}{2 c}+2; -e^{2 c z}\right)}{-b+2 c-i a} + \frac{e^{(b+2 c-i a) z}\,{}_2F_1\left(\frac{b-i a}{2 c}+1, 1; \frac{b-i a}{2 c}+2; -e^{2 c z}\right)}{b+2 c-i a} + \frac{e^{(b+2 c+i a) z}\,{}_2F_1\left(\frac{b+i a}{2 c}+1, 1; \frac{b+i a}{2 c}+2; -e^{2 c z}\right)}{b+2 c+i a} - \frac{e^{(b-i a) z}\,{}_2F_1\left(\frac{b-i a}{2 c}, 1; \frac{b-i a}{2 c}+1; -e^{2 c z}\right)}{b-i a} - \frac{e^{(b+i a) z}\,{}_2F_1\left(\frac{b+i a}{2 c}, 1; \frac{b+i a}{2 c}+1; -e^{2 c z}\right)}{b+i a} - \frac{e^{(-b+2 c+i a) z}\,{}_2F_1\left(\frac{i a-b}{2 c}+1, 1; \frac{i a-b}{2 c}+2; -e^{2 c z}\right)}{-b+2 c+i a}\right)$$

Involving powers of cos and powers of sinh

Involving $\cos^m(a z)\sinh^u(b z)\tanh(c z)$

01.21.21.0163.01

$$\int \cos^m(a z)\sinh^u(b z)\tanh(c z)\,dz = \frac{i^u\,2^{-m-u}\,\log(\cosh(c z))\,(1-m \bmod 2)\,(1-u \bmod 2)}{c}\binom{m}{\frac{m}{2}}\binom{u}{\frac{u}{2}} - i^u\,2^{-m-u}\binom{u}{\frac{u}{2}}(u \bmod 2-1)\sum_{k=0}^{\left\lfloor\frac{m-1}{2}\right\rfloor}\binom{m}{k}\left(\frac{i\,e^{i a (m-2 k) z}\,{}_2F_1\left(\frac{i a (m-2 k)}{2 c}, 1; \frac{a i (m-2 k)}{2 c}+1; -e^{2 c z}\right)}{a (m-2 k)} - \frac{i\,e^{-i a (m-2 k) z}\,{}_2F_1\left(-\frac{i a (m-2 k)}{2 c}, 1; 1-\frac{i a (m-2 k)}{2 c}; -e^{2 c z}\right)}{a (m-2 k)} + \frac{e^{(2 c+a i (m-2 k)) z}\,{}_2F_1\left(\frac{a i (m-2 k)}{2 c}+1, 1; \frac{a i (m-2 k)}{2 c}+2; -e^{2 c z}\right)}{2 c+a i (m-2 k)} + \frac{e^{(2 c-i a (m-2 k)) z}\,{}_2F_1\left(1-\frac{i a (m-2 k)}{2 c}, 1; 2-\frac{i a (m-2 k)}{2 c}; -e^{2 c z}\right)}{2 c-i a (m-2 k)}\right) + 2^{-m-u}\binom{m}{\frac{m}{2}}(1-m \bmod 2)\sum_{k=0}^{\left\lfloor\frac{u-1}{2}\right\rfloor}(-1)^k\binom{u}{k}\left((-1)^u\left(\frac{e^{-b (u-2 k) z}\,{}_2F_1\left(-\frac{b (u-2 k)}{2 c}, 1; 1-\frac{b (u-2 k)}{2 c}; -e^{2 c z}\right)}{b (u-2 k)} + \frac{e^{(2 c-b (u-2 k)) z}\,{}_2F_1\left(1-\frac{b (u-2 k)}{2 c}, 1; 2-\frac{b (u-2 k)}{2 c}; -e^{2 c z}\right)}{2 c-b (u-2 k)}\right) - \frac{e^{b (u-2 k) z}\,{}_2F_1\left(\frac{b (u-2 k)}{2 c}, 1; \frac{b (u-2 k)}{2 c}+1; -e^{2 c z}\right)}{b (u-2 k)} + \frac{e^{(2 c+b (u-2 k)) z}\,{}_2F_1\left(\frac{b (u-2 k)}{2 c}+1, 1; \frac{b (u-2 k)}{2 c}+2; -e^{2 c z}\right)}{2 c+b (u-2 k)}\right) +$$

$$2^{-m-u}\sum_{s=0}^{\left\lfloor\frac{u-1}{2}\right\rfloor}\sum_{k=0}^{\left\lfloor\frac{m-1}{2}\right\rfloor}(-1)^s\binom{m}{k}\binom{u}{s}\left((-1)^u\left(\left(e^{(2 c+a i (m-2 k)-b (u-2 s)) z}\,{}_2F_1\left(\frac{i a (m-2 k)-b (u-2 s)}{2 c}+1, 1; \frac{i a (m-2 k)-b (u-2 s)}{2 c}+2; -e^{2 c z}\right)\right)\middle/(2 c+a i (m-2 k)-b (u-2 s)) -\right.\right.$$

$$\frac{e^{(i\,a\,(m-2\,k)-b\,(u-2\,s))\,z}\,{}_2F_1\!\left(\frac{i\,a\,(m-2\,k)-b\,(u-2\,s)}{2\,c},\,1;\,\frac{i\,a\,(m-2\,k)-b\,(u-2\,s)}{2\,c}+1;\,-e^{2\,c\,z}\right)}{i\,a\,(m-2\,k)-b\,(u-2\,s)}\Bigg)+$$

$$(-1)^u\Bigg(\Bigg(e^{(2\,c-i\,a\,(m-2\,k)-b\,(u-2\,s))\,z}\,{}_2F_1\!\left(\frac{-i\,a\,(m-2\,k)-b\,(u-2\,s)}{2\,c}+1,\,1;\,\frac{-i\,a\,(m-2\,k)-b\,(u-2\,s)}{2\,c}+2;\,-e^{2\,c\,z}\right)\Bigg)\Big/$$

$$(2\,c-i\,a\,(m-2\,k)-b\,(u-2\,s))-\Bigg(e^{(-i\,a\,(m-2\,k)-b\,(u-2\,s))\,z}\,{}_2F_1\!\left(\frac{-i\,a\,(m-2\,k)-b\,(u-2\,s)}{2\,c},\right.$$

$$\left.1;\,\frac{-i\,a\,(m-2\,k)-b\,(u-2\,s)}{2\,c}+1;\,-e^{2\,c\,z}\right)\Bigg)\Big/(-i\,a\,(m-2\,k)-b\,(u-2\,s))\Bigg)+$$

$$\Bigg(e^{(2\,c-i\,a\,(m-2\,k)+b\,(u-2\,s))\,z}\,{}_2F_1\!\left(\frac{b\,(u-2\,s)-i\,a\,(m-2\,k)}{2\,c}+1,\,1;\,\frac{b\,(u-2\,s)-i\,a\,(m-2\,k)}{2\,c}+2;\,-e^{2\,c\,z}\right)\Bigg)\Big/$$

$$(2\,c-i\,a\,(m-2\,k)+b\,(u-2\,s))+$$

$$\Bigg(e^{(2\,c+a\,i\,(m-2\,k)+b\,(u-2\,s))\,z}\,{}_2F_1\!\left(\frac{a\,i\,(m-2\,k)+b\,(u-2\,s)}{2\,c}+1,\,1;\,\frac{a\,i\,(m-2\,k)+b\,(u-2\,s)}{2\,c}+2;\,-e^{2\,c\,z}\right)\Bigg)\Big/$$

$$(2\,c+a\,i\,(m-2\,k)+b\,(u-2\,s))-\frac{e^{(b\,(u-2\,s)-i\,a\,(m-2\,k))\,z}\,{}_2F_1\!\left(\frac{b\,(u-2\,s)-i\,a\,(m-2\,k)}{2\,c},\,1;\,\frac{b\,(u-2\,s)-i\,a\,(m-2\,k)}{2\,c}+1;\,-e^{2\,c\,z}\right)}{b\,(u-2\,s)-i\,a\,(m-2\,k)}-$$

$$\frac{e^{(a\,i\,(m-2\,k)+b\,(u-2\,s))\,z}\,{}_2F_1\!\left(\frac{a\,i\,(m-2\,k)+b\,(u-2\,s)}{2\,c},\,1;\,\frac{a\,i\,(m-2\,k)+b\,(u-2\,s)}{2\,c}+1;\,-e^{2\,c\,z}\right)}{a\,i\,(m-2\,k)+b\,(u-2\,s)}\Bigg)\,/;\,m\in\mathbb{N}^+\bigwedge u\in\mathbb{N}^+$$

01.21.21.0164.01

$$\int\cos^m(a\,z)\sinh^\mu(c\,z)\tanh(c\,z)\,dz = \frac{2^{-m}\left(1-e^{2\,c\,z}\right)^{-\mu}(1-m\bmod 2)\sinh^\mu(c\,z)}{c\,\mu}\binom{m}{\frac{m}{2}}F_1\!\left(-\frac{\mu}{2};\,-\mu-1,\,1;\,\frac{2-\mu}{2};\,e^{2\,c\,z},\,-e^{2\,c\,z}\right)-$$

$$2^{-m}\left(1-e^{2\,c\,z}\right)^{-\mu}\sinh^\mu(c\,z)\sum_{k=0}^{\left\lfloor\frac{m-1}{2}\right\rfloor}\binom{m}{k}\left(\frac{e^{-i\,a\,(m-2\,k)\,z}\,F_1\!\left(\frac{-i\,a\,(m-2\,k)-c\,\mu}{2\,c};\,-\mu-1,\,1;\,\frac{1}{2}\left(-\frac{i\,a\,(m-2\,k)}{c}-\mu+2\right);\,e^{2\,c\,z},\,-e^{2\,c\,z}\right)}{-i\,a\,(m-2\,k)-c\,\mu}+\right.$$

$$\left.\frac{e^{i\,a\,(m-2\,k)\,z}\,F_1\!\left(\frac{i\,a\,(m-2\,k)-c\,\mu}{2\,c};\,-\mu-1,\,1;\,\frac{1}{2}\left(\frac{a\,i\,(m-2\,k)}{c}-\mu+2\right);\,e^{2\,c\,z},\,-e^{2\,c\,z}\right)}{i\,a\,(m-2\,k)-c\,\mu}\right)\,/;\,m\in\mathbb{N}^+$$

Involving sin and cosh

Involving sin(a z) cosh(b z) tanh(c z)

01.21.21.0165.01

$$\int \sin(a z)\cosh(b z)\tanh(c z)\,dz = \frac{1}{4} i \left(-\frac{e^{(-b-i a) z}\, {}_2F_1\left(\frac{-b-i a}{2c}, 1; \frac{-b-i a}{2c}+1; -e^{2cz}\right)}{-b-i a} + \frac{e^{(i a-b) z}\, {}_2F_1\left(\frac{i a-b}{2c}, 1; \frac{i a-b}{2c}+1; -e^{2cz}\right)}{i a-b} + \frac{e^{(b+i a) z}\, {}_2F_1\left(\frac{b+i a}{2c}, 1; \frac{b+i a}{2c}+1; -e^{2cz}\right)}{b+i a} + \frac{e^{(-b+2c-i a) z}\, {}_2F_1\left(\frac{-b-i a}{2c}+1, 1; \frac{-b-i a}{2c}+2; -e^{2cz}\right)}{-b+2c-i a} + \frac{e^{(b+2c-i a) z}\, {}_2F_1\left(\frac{b-i a}{2c}+1, 1; \frac{b-i a}{2c}+2; -e^{2cz}\right)}{b+2c-i a} - \frac{e^{(b-i a) z}\, {}_2F_1\left(\frac{b-i a}{2c}, 1; \frac{b-i a}{2c}+1; -e^{2cz}\right)}{b-i a} - \frac{e^{(-b+2c+i a) z}\, {}_2F_1\left(\frac{i a-b}{2c}+1, 1; \frac{i a-b}{2c}+2; -e^{2cz}\right)}{-b+2c+i a} - \frac{e^{(b+2c+i a) z}\, {}_2F_1\left(\frac{b+i a}{2c}+1, 1; \frac{b+i a}{2c}+2; -e^{2cz}\right)}{b+2c+i a} \right)$$

Involving powers of sin and powers of cosh

Involving $\sin^m(a z) \cosh^u(b z) \tanh(c z)$

01.21.21.0166.01

$$\int \sin^m(a z)\cosh^u(b z)\tanh(c z)\,dz = \frac{2^{-m-u} \log(\cosh(c z))\,(1-m \bmod 2)\,(1-u \bmod 2)}{c} \binom{m}{\frac{m}{2}} \binom{u}{\frac{u}{2}} - 2^{-m-u} \binom{u}{\frac{u}{2}} (u \bmod 2 - 1) \sum_{k=0}^{\left\lfloor \frac{m-1}{2} \right\rfloor} (-1)^k \binom{m}{k} \left(e^{-\frac{1}{2} i m \pi} \left(\frac{i\, e^{i a (m-2k) z}\, {}_2F_1\left(\frac{i a (m-2k)}{2c}, 1; \frac{a i (m-2k)}{2c}+1; -e^{2cz}\right)}{a (m-2k)} + \frac{e^{(2c+a i (m-2k)) z}\, {}_2F_1\left(\frac{a i (m-2k)}{2c}+1, 1; \frac{a i (m-2k)}{2c}+2; -e^{2cz}\right)}{2c + a i (m-2k)} \right) + e^{\frac{i m \pi}{2}} \left(\frac{e^{(2c-i a (m-2k)) z}\, {}_2F_1\left(1-\frac{i a (m-2k)}{2c}, 1; 2-\frac{i a (m-2k)}{2c}; -e^{2cz}\right)}{2c - i a (m-2k)} - \frac{i\, e^{-i a (m-2k) z}\, {}_2F_1\left(-\frac{i a (m-2k)}{2c}, 1; 1-\frac{i a (m-2k)}{2c}; -e^{2cz}\right)}{a (m-2k)} \right) \right) + 2^{-m-u} \binom{m}{\frac{m}{2}} (1-m \bmod 2) \sum_{k=0}^{\left\lfloor \frac{u-1}{2} \right\rfloor} \binom{u}{k} \left(-\frac{e^{b (u-2k) z}\, {}_2F_1\left(\frac{b (u-2k)}{2c}, 1; \frac{b (u-2k)}{2c}+1; -e^{2cz}\right)}{b (u-2k)} + \frac{e^{-b (u-2k) z}\, {}_2F_1\left(-\frac{b (u-2k)}{2c}, 1; 1-\frac{b (u-2k)}{2c}; -e^{2cz}\right)}{b (u-2k)} + \frac{e^{(2c+b (u-2k)) z}\, {}_2F_1\left(\frac{b (u-2k)}{2c}+1, 1; \frac{b (u-2k)}{2c}+2; -e^{2cz}\right)}{2c + b (u-2k)} + \frac{e^{(2c-b (u-2k)) z}\, {}_2F_1\left(1-\frac{b (u-2k)}{2c}, 1; 2-\frac{b (u-2k)}{2c}; -e^{2cz}\right)}{2c - b (u-2k)} \right) + 2^{-m-u} \sum_{s=0}^{\left\lfloor \frac{u-1}{2} \right\rfloor} \sum_{k=0}^{\left\lfloor \frac{m-1}{2} \right\rfloor} (-1)^k \binom{m}{k} \binom{u}{s} \left(e^{-\frac{1}{2} i m \pi} \left(\left(e^{(2c+a i (m-2k)+b (u-2s)) z}\, {}_2F_1\left(\frac{a i (m-2k)+b (u-2s)}{2c}+1, 1; \frac{a i (m-2k)+b (u-2s)}{2c}+2; -e^{2cz}\right)\right) \Big/ (2c + a i (m-2k) + b (u-2s)) - \right.\right.$$

$$\left.\frac{e^{(a\,i\,(m-2\,k)+b\,(u-2\,s))\,z}\;{}_2F_1\!\left(\frac{a\,i\,(m-2\,k)+b\,(u-2\,s)}{2\,c},\,1;\,\frac{a\,i\,(m-2\,k)+b\,(u-2\,s)}{2\,c}+1;\,-e^{2\,c\,z}\right)}{a\,i\,(m-2\,k)+b\,(u-2\,s)}\right)+$$

$$e^{\frac{i\,m\,\pi}{2}}\left(\left(e^{(2\,c-i\,a\,(m-2\,k)+b\,(u-2\,s))\,z}\;{}_2F_1\!\left(\frac{b\,(u-2\,s)-i\,a\,(m-2\,k)}{2\,c}+1,\,1;\,\frac{b\,(u-2\,s)-i\,a\,(m-2\,k)}{2\,c}+2;\,-e^{2\,c\,z}\right)\right)\Big/\right.$$

$$(2\,c-i\,a\,(m-2\,k)+b\,(u-2\,s))-$$

$$\left.\frac{e^{(b\,(u-2\,s)-i\,a\,(m-2\,k))\,z}\;{}_2F_1\!\left(\frac{b\,(u-2\,s)-i\,a\,(m-2\,k)}{2\,c},\,1;\,\frac{b\,(u-2\,s)-i\,a\,(m-2\,k)}{2\,c}+1;\,-e^{2\,c\,z}\right)}{b\,(u-2\,s)-i\,a\,(m-2\,k)}\right)+$$

$$e^{-\frac{1}{2}\,i\,m\,\pi}\left(\left(e^{(2\,c+a\,i\,(m-2\,k)-b\,(u-2\,s))\,z}\;{}_2F_1\!\left(\frac{i\,a\,(m-2\,k)-b\,(u-2\,s)}{2\,c}+1,\,1;\,\frac{i\,a\,(m-2\,k)-b\,(u-2\,s)}{2\,c}+2;\,-e^{2\,c\,z}\right)\right)\Big/\right.$$

$$(2\,c+a\,i\,(m-2\,k)-b\,(u-2\,s))-$$

$$\left.\frac{e^{(i\,a\,(m-2\,k)-b\,(u-2\,s))\,z}\;{}_2F_1\!\left(\frac{i\,a\,(m-2\,k)-b\,(u-2\,s)}{2\,c},\,1;\,\frac{i\,a\,(m-2\,k)-b\,(u-2\,s)}{2\,c}+1;\,-e^{2\,c\,z}\right)}{i\,a\,(m-2\,k)-b\,(u-2\,s)}\right)+$$

$$e^{\frac{i\,m\,\pi}{2}}\left(\left(e^{(2\,c-i\,a\,(m-2\,k)-b\,(u-2\,s))\,z}\;{}_2F_1\!\left(\frac{-i\,a\,(m-2\,k)-b\,(u-2\,s)}{2\,c}+1,\,1;\,\frac{-i\,a\,(m-2\,k)-b\,(u-2\,s)}{2\,c}+2;\,-e^{2\,c\,z}\right)\right)\Big/\right.$$

$$(2\,c-i\,a\,(m-2\,k)-b\,(u-2\,s))-\left(e^{(-i\,a\,(m-2\,k)-b\,(u-2\,s))\,z}\;{}_2F_1\!\left(\frac{-i\,a\,(m-2\,k)-b\,(u-2\,s)}{2\,c},\,1;\right.\right.$$

$$\left.\left.\left.\frac{-i\,a\,(m-2\,k)-b\,(u-2\,s)}{2\,c}+1;\,-e^{2\,c\,z}\right)\right)\Big/(-i\,a\,(m-2\,k)-b\,(u-2\,s))\right)\Bigg)\,/;\,m\in\mathbb{N}^+\bigwedge u\in\mathbb{N}^+$$

01.21.21.0167.01

$$\int\sin^m(a\,z)\cosh^\mu(c\,z)\tanh(c\,z)\,dz==$$

$$2^{-m}\binom{m}{\frac{m}{2}}\cosh^\mu(c\,z)\,(1-m\bmod 2)\left(1+e^{2\,c\,z}\right)^{-\mu}\left(\frac{e^{2\,c\,z}\;{}_2F_1\!\left(\frac{2-\mu}{2},\,1-\mu;\,\frac{4-\mu}{2};\,-e^{2\,c\,z}\right)}{c\,(2-\mu)}+\frac{{}_2F_1\!\left(-\frac{\mu}{2},\,1-\mu;\,\frac{2-\mu}{2};\,-e^{2\,c\,z}\right)}{c\,\mu}\right)+2^{-m}$$

$$\cosh^\mu(c\,z)\left(1+e^{2\,c\,z}\right)^{-\mu}\sum_{k=0}^{\left\lfloor\frac{m-1}{2}\right\rfloor}(-1)^k\binom{m}{k}\left(\frac{e^{\frac{i\,\pi\,m}{2}+(2\,c-i\,a\,(m-2\,k))\,z}\;{}_2F_1\!\left(\frac{c\,(2-\mu)-i\,a\,(m-2\,k)}{2\,c},\,1-\mu;\,\frac{1}{2}\left(-\frac{i\,a\,(m-2\,k)}{c}-\mu+4\right);\,-e^{2\,c\,z}\right)}{c\,(2-\mu)-i\,a\,(m-2\,k)}+\right.$$

$$\frac{e^{(2\,c+a\,i\,(m-2\,k))\,z-\frac{i\,m\,\pi}{2}}\;{}_2F_1\!\left(\frac{a\,i\,(m-2\,k)+c\,(2-\mu)}{2\,c},\,1-\mu;\,\frac{1}{2}\left(\frac{a\,i\,(m-2\,k)}{c}-\mu+4\right);\,-e^{2\,c\,z}\right)}{a\,i\,(m-2\,k)+c\,(2-\mu)}-$$

$$\frac{e^{\frac{i\,m\,\pi}{2}-i\,a\,(m-2\,k)\,z}\;{}_2F_1\!\left(\frac{-i\,a\,(m-2\,k)-c\,\mu}{2\,c},\,1-\mu;\,\frac{1}{2}\left(-\frac{i\,a\,(m-2\,k)}{c}-\mu+2\right);\,-e^{2\,c\,z}\right)}{-i\,a\,(m-2\,k)-c\,\mu}-$$

$$\left.\frac{e^{i\,a\,(m-2\,k)\,z-\frac{i\,m\,\pi}{2}}\;{}_2F_1\!\left(\frac{i\,a\,(m-2\,k)-c\,\mu}{2\,c},\,1-\mu;\,\frac{1}{2}\left(\frac{a\,i\,(m-2\,k)}{c}-\mu+2\right);\,-e^{2\,c\,z}\right)}{i\,a\,(m-2\,k)-c\,\mu}\right)\,/;\,m\in\mathbb{N}^+$$

Involving cos and cosh

Involving cos(*a z*) cosh(*b z*) tanh(*c z*)

01.21.21.0168.01

$$\int \cos(a\,z)\cosh(b\,z)\tanh(c\,z)\,dz = \frac{1}{4}\left(-\frac{e^{(-b-i\,a)\,z}\,{}_2F_1\left(\frac{-b-i\,a}{2\,c}, 1; \frac{-b-i\,a}{2\,c}+1; -e^{2\,c\,z}\right)}{-b-i\,a} + \frac{e^{(-b+2\,c-i\,a)\,z}\,{}_2F_1\left(\frac{-b-i\,a}{2\,c}+1, 1; \frac{-b-i\,a}{2\,c}+2; -e^{2\,c\,z}\right)}{-b+2\,c-i\,a} + \frac{e^{(-b+2\,c+i\,a)\,z}\,{}_2F_1\left(\frac{i\,a-b}{2\,c}+1, 1; \frac{i\,a-b}{2\,c}+2; -e^{2\,c\,z}\right)}{-b+2\,c+i\,a} + \frac{e^{(b+2\,c-i\,a)\,z}\,{}_2F_1\left(\frac{b-i\,a}{2\,c}+1, 1; \frac{b-i\,a}{2\,c}+2; -e^{2\,c\,z}\right)}{b+2\,c-i\,a} + \frac{e^{(b+2\,c+i\,a)\,z}\,{}_2F_1\left(\frac{b+i\,a}{2\,c}+1, 1; \frac{b+i\,a}{2\,c}+2; -e^{2\,c\,z}\right)}{b+2\,c+i\,a} - \frac{e^{(i\,a-b)\,z}\,{}_2F_1\left(\frac{i\,a-b}{2\,c}, 1; \frac{i\,a-b}{2\,c}+1; -e^{2\,c\,z}\right)}{i\,a-b} - \frac{e^{(b-i\,a)\,z}\,{}_2F_1\left(\frac{b-i\,a}{2\,c}, 1; \frac{b-i\,a}{2\,c}+1; -e^{2\,c\,z}\right)}{b-i\,a} - \frac{e^{(b+i\,a)\,z}\,{}_2F_1\left(\frac{b+i\,a}{2\,c}, 1; \frac{b+i\,a}{2\,c}+1; -e^{2\,c\,z}\right)}{b+i\,a}\right)$$

Involving powers of cos and powers of cosh

Involving $\cos^m(a\,z)\cosh^u(b\,z)\tanh(c\,z)$

01.21.21.0169.01

$$\int \cos^m(a z)\cosh^u(b z)\tanh(c z)\,dz = \frac{2^{-m-u}\log(\cosh(c z))(1 - m \bmod 2)(1 - u \bmod 2)}{c}\binom{m}{\frac{m}{2}}\binom{u}{\frac{u}{2}} - 2^{-m-u}\binom{u}{\frac{u}{2}}(u \bmod 2 - 1)$$

$$\sum_{k=0}^{\left\lfloor\frac{m-1}{2}\right\rfloor}\binom{m}{k}\left(\frac{i\, e^{i a (m-2k) z}\,{}_2F_1\left(\frac{i a (m-2k)}{2c}, 1; \frac{a i (m-2k)}{2c} + 1; -e^{2cz}\right)}{a(m-2k)} - \frac{i\, e^{-i a (m-2k) z}\,{}_2F_1\left(-\frac{i a (m-2k)}{2c}, 1; 1 - \frac{i a (m-2k)}{2c}; -e^{2cz}\right)}{a(m-2k)} + \right.$$

$$\frac{e^{(2c + a i (m-2k))z}\,{}_2F_1\left(\frac{a i (m-2k)}{2c} + 1, 1; \frac{a i (m-2k)}{2c} + 2; -e^{2cz}\right)}{2c + a i (m-2k)} +$$

$$\left.\frac{e^{(2c - i a (m-2k))z}\,{}_2F_1\left(1 - \frac{i a (m-2k)}{2c}, 1; 2 - \frac{i a (m-2k)}{2c}; -e^{2cz}\right)}{2c - i a (m-2k)}\right) + 2^{-m-u}\binom{m}{\frac{m}{2}}(1 - m \bmod 2)$$

$$\sum_{k=0}^{\left\lfloor\frac{u-1}{2}\right\rfloor}\binom{u}{k}\left(-\frac{e^{b(u-2k)z}\,{}_2F_1\left(\frac{b(u-2k)}{2c}, 1; \frac{b(u-2k)}{2c} + 1; -e^{2cz}\right)}{b(u-2k)} + \frac{e^{-b(u-2k)z}\,{}_2F_1\left(-\frac{b(u-2k)}{2c}, 1; 1 - \frac{b(u-2k)}{2c}; -e^{2cz}\right)}{b(u-2k)} + \right.$$

$$\left.\frac{e^{(2c + b(u-2k))z}\,{}_2F_1\left(\frac{b(u-2k)}{2c} + 1, 1; \frac{b(u-2k)}{2c} + 2; -e^{2cz}\right)}{2c + b(u-2k)} + \frac{e^{(2c - b(u-2k))z}\,{}_2F_1\left(1 - \frac{b(u-2k)}{2c}, 1; 2 - \frac{b(u-2k)}{2c}; -e^{2cz}\right)}{2c - b(u-2k)}\right) +$$

$$2^{-m-u}\sum_{s=0}^{\left\lfloor\frac{u-1}{2}\right\rfloor}\sum_{k=0}^{\left\lfloor\frac{m-1}{2}\right\rfloor}\binom{m}{k}\binom{u}{s}\left(\frac{e^{(2c - i a (m-2k) + b(u-2s))z}\,{}_2F_1\left(\frac{b(u-2s) - i a (m-2k)}{2c} + 1, 1; \frac{b(u-2s) - i a (m-2k)}{2c} + 2; -e^{2cz}\right)}{2c - i a (m-2k) + b(u-2s)} + \right.$$

$$\left(e^{(2c + a i (m-2k) + b(u-2s))z}\,{}_2F_1\left(\frac{a i (m-2k) + b(u-2s)}{2c} + 1, 1; \frac{a i (m-2k) + b(u-2s)}{2c} + 2; -e^{2cz}\right)\right) \Big/$$

$$(2c + a i (m-2k) + b(u-2s)) + \left(e^{(2c - i a (m-2k) - b(u-2s))z}\,{}_2F_1\left(\frac{-i a (m-2k) - b(u-2s)}{2c} + 1,\right.\right.$$

$$\left.\left. 1; \frac{-i a (m-2k) - b(u-2s)}{2c} + 2; -e^{2cz}\right)\right) \Big/ (2c - i a (m-2k) - b(u-2s)) +$$

$$\left(e^{(2c + a i (m-2k) - b(u-2s))z}\,{}_2F_1\left(\frac{i a (m-2k) - b(u-2s)}{2c} + 1, 1; \frac{i a (m-2k) - b(u-2s)}{2c} + 2; -e^{2cz}\right)\right) \Big/$$

$$(2c + a i (m-2k) - b(u-2s)) - \frac{e^{(b(u-2s) - i a (m-2k))z}\,{}_2F_1\left(\frac{b(u-2s) - i a (m-2k)}{2c}, 1; \frac{b(u-2s) - i a (m-2k)}{2c} + 1; -e^{2cz}\right)}{b(u-2s) - i a (m-2k)} -$$

$$\frac{e^{(a i (m-2k) + b(u-2s))z}\,{}_2F_1\left(\frac{a i (m-2k) + b(u-2s)}{2c}, 1; \frac{a i (m-2k) + b(u-2s)}{2c} + 1; -e^{2cz}\right)}{a i (m-2k) + b(u-2s)} -$$

$$\frac{e^{(-i a (m-2k) - b(u-2s))z}\,{}_2F_1\left(\frac{-i a (m-2k) - b(u-2s)}{2c}, 1; \frac{-i a (m-2k) - b(u-2s)}{2c} + 1; -e^{2cz}\right)}{-i a (m-2k) - b(u-2s)} -$$

$$\left.\frac{e^{(i a (m-2k) - b(u-2s))z}\,{}_2F_1\left(\frac{i a (m-2k) - b(u-2s)}{2c}, 1; \frac{i a (m-2k) - b(u-2s)}{2c} + 1; -e^{2cz}\right)}{i a (m-2k) - b(u-2s)}\right) /;\, m \in \mathbb{N}^+ \land u \in \mathbb{N}^+$$

01.21.21.0170.01

$$\int \cos^m(a z)\cosh^\mu(c z)\tanh(c z)\,dz = 2^{-m}\cosh^\mu(c z)\,(1-m \bmod 2)\left(1+e^{2cz}\right)^{-\mu}\binom{m}{\frac{m}{2}}\left(\frac{e^{2cz}\,{}_2F_1\!\left(\frac{2-\mu}{2},1-\mu;\frac{4-\mu}{2};-e^{2cz}\right)}{c(2-\mu)}+\frac{{}_2F_1\!\left(-\frac{\mu}{2},1-\mu;\frac{2-\mu}{2};-e^{2cz}\right)}{c\mu}\right)+ 2^{-m}\cosh^\mu(c z)\left(1+e^{2cz}\right)^{-\mu}\sum_{k=0}^{\left\lfloor\frac{m-1}{2}\right\rfloor}\binom{m}{k}\left(\frac{e^{(2c-i a(m-2k))z}\,{}_2F_1\!\left(\frac{c(2-\mu)-i a(m-2k)}{2c},1-\mu;\frac{1}{2}\left(-\frac{i a(m-2k)}{c}-\mu+4\right);-e^{2cz}\right)}{c(2-\mu)-i a(m-2k)}+ \frac{e^{(2c+a i(m-2k))z}\,{}_2F_1\!\left(\frac{a i(m-2k)+c(2-\mu)}{2c},1-\mu;\frac{1}{2}\left(\frac{a i(m-2k)}{c}-\mu+4\right);-e^{2cz}\right)}{a i(m-2k)+c(2-\mu)}- \frac{e^{-i a(m-2k)z}\,{}_2F_1\!\left(\frac{-i a(m-2k)-c\mu}{2c},1-\mu;\frac{1}{2}\left(-\frac{i a(m-2k)}{c}-\mu+2\right);-e^{2cz}\right)}{-i a(m-2k)-c\mu}- \frac{e^{i a(m-2k)z}\,{}_2F_1\!\left(\frac{i a(m-2k)-c\mu}{2c},1-\mu;\frac{1}{2}\left(\frac{a i(m-2k)}{c}-\mu+2\right);-e^{2cz}\right)}{i a(m-2k)-c\mu}\right)\;/;\; m\in\mathbb{N}^+$$

Involving hyperbolic, exponential and a power functions

Involving sinh, exp and power

Involving $z^n\, e^{p z} \sinh(a+b z)\tanh(c z)$

01.21.21.0171.01

$$\int z^n e^{p z}\sinh(a+b z)\tanh(c z)\,dz = \frac{1}{2}e^a n!\left(e^{(b+2c+p)z}\sum_{j=0}^{n}\frac{(-1)^j(b+2c+p)^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\!\left(\frac{b+2c+p}{2c},\ldots,\frac{b+2c+p}{2c},1;\frac{b+2c+p}{2c}+1,\ldots,\frac{b+2c+p}{2c}+1;-e^{2cz}\right)- e^{(b+p)z}\sum_{j=0}^{n}\frac{(-1)^j(b+p)^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\!\left(\frac{b+p}{2c},\ldots,\frac{b+p}{2c},1;\frac{b+p}{2c}+1,\ldots,\frac{b+p}{2c}+1;-e^{2cz}\right)\right)- \frac{1}{2}e^{-a}n!\left(e^{(-b+2c+p)z}\sum_{j=0}^{n}\frac{(-1)^j(-b+2c+p)^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\!\left(\frac{-b+2c+p}{2c},\ldots,\frac{-b+2c+p}{2c},1;\frac{-b+2c+p}{2c}+1,\ldots,\frac{-b+2c+p}{2c}+1;-e^{2cz}\right)- e^{(p-b)z}\sum_{j=0}^{n}\frac{(-1)^j(p-b)^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\!\left(\frac{p-b}{2c},\ldots,\frac{p-b}{2c},1;\frac{p-b}{2c}+1,\ldots,\frac{p-b}{2c}+1;-e^{2cz}\right)\right)\;/;\; n\in\mathbb{N}$$

01.21.21.0172.01

$$\int z^n e^{p z} \sinh(b z) \tanh(c z) \, dz = \frac{1}{2} n! \left(e^{(b+2c+p) z} \sum_{j=0}^{n} \frac{(-1)^j (b+2c+p)^{-j-1} z^{n-j}}{(n-j)!} \right.$$
$$ {}_{j+2}F_{j+1}\left(\frac{b+2c+p}{2c}, \ldots, \frac{b+2c+p}{2c}, 1; \frac{b+2c+p}{2c}+1, \ldots, \frac{b+2c+p}{2c}+1; -e^{2cz} \right) -$$
$$ e^{(b+p) z} \sum_{j=0}^{n} \frac{(-1)^j (b+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{b+p}{2c}, \ldots, \frac{b+p}{2c}, 1; \frac{b+p}{2c}+1, \ldots, \frac{b+p}{2c}+1; -e^{2cz} \right) - e^{(-b+2c+p) z}$$
$$ \sum_{j=0}^{n} \frac{(-1)^j (-b+2c+p)^{-j-1} z^{n-j}}{(n-j)!}$$
$$ {}_{j+2}F_{j+1}\left(\frac{-b+2c+p}{2c}, \ldots, \frac{-b+2c+p}{2c}, 1; \frac{-b+2c+p}{2c}+1, \ldots, \frac{-b+2c+p}{2c}+1; -e^{2cz} \right) +$$
$$ \left. e^{(p-b) z} \sum_{j=0}^{n} \frac{(-1)^j (p-b)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{p-b}{2c}, \ldots, \frac{p-b}{2c}, 1; \frac{p-b}{2c}+1, \ldots, \frac{p-b}{2c}+1; -e^{2cz} \right) \right) /; n \in \mathbb{N}$$

01.21.21.0173.01

$$\int z^n e^{b z} \sinh(b z) \tanh(c z) \, dz = \frac{1}{2} n! \left(\frac{z^{n+1}}{(n+1)!} - 2 e^{2cz} \sum_{j=0}^{n} \frac{(-1)^j 2^{-j-1} c^{-j-1} z^{n-j}}{(n-j)!} {}_{j+3}F_{j+2}(1, \ldots, 1, 2; 2, \ldots, 2; -e^{2cz}) - \right.$$
$$ e^{2bz} \sum_{j=0}^{n} \frac{(-1)^j 2^{-j-1} b^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{b}{c}, \ldots, \frac{b}{c}, 1; \frac{b}{c}+1, \ldots, \frac{b}{c}+1; -e^{2cz} \right) +$$
$$ \left. e^{2(b+c) z} \sum_{j=0}^{n} \frac{(-1)^j 2^{-j-1} (b+c)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{b+c}{c}, \ldots, \frac{b+c}{c}, 1; \frac{b+c}{c}+1, \ldots, \frac{b+c}{c}+1; -e^{2cz} \right) \right) /; n \in \mathbb{N}$$

01.21.21.0174.01

$$\int z^n e^{-b z} \sinh(b z) \tanh(c z) \, dz = -\frac{1}{2} n! \left(\frac{z^{n+1}}{(n+1)!} - 2 e^{2cz} \sum_{j=0}^{n} \frac{(-1)^j 2^{-j-1} c^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}(1, \ldots, 1, 1; 2, \ldots, 2; -e^{2cz}) + \right.$$
$$ e^{-2bz} \sum_{j=0}^{n} \frac{2^{-j-1} b^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(-\frac{b}{c}, \ldots, -\frac{b}{c}, 1; 1-\frac{b}{c}, \ldots, 1-\frac{b}{c}; -e^{2cz} \right) + e^{2(-b+c) z}$$
$$ \left. \sum_{j=0}^{n} \frac{(-1)^j 2^{-j-1} (-b+c)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{-b+c}{c}, \ldots, \frac{-b+c}{c}, 1; \frac{-b+c}{c}+1, \ldots, \frac{-b+c}{c}+1; -e^{2cz} \right) \right) /; n \in \mathbb{N}$$

Involving powers of sinh, exp and power

Involving $z^n e^{p z} \sinh^u(b z) \tanh(c z)$

01.21.21.0175.01

$$\int z^n e^{pz}\sinh^u(bz)\tanh(cz)\,dz = \binom{u}{\frac{u}{2}} n!\,(1-u \bmod 2)\left(\frac{i}{2}\right)^u$$
$$\left(e^{(2c+p)z}\sum_{j=0}^{n}\frac{(-1)^j(2c+p)^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{2c+p}{2c},\ldots,\frac{2c+p}{2c},1;\frac{2c+p}{2c}+1,\ldots,\frac{2c+p}{2c}+1;-e^{2cz}\right)-\right.$$
$$\left.e^{pz}\sum_{j=0}^{n}\frac{(-1)^j p^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{p}{2c},\ldots,\frac{p}{2c},1;\frac{p}{2c}+1,\ldots,\frac{p}{2c}+1;-e^{2cz}\right)\right)+$$
$$2^{-u}n!\sum_{k=0}^{\left\lfloor\frac{u-1}{2}\right\rfloor}(-1)^k\binom{u}{k}\left((-1)^u\left(e^{(2c+p-b(u-2k))z}\sum_{j=0}^{n}\frac{(-1)^j(2c+p-b(u-2k))^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{2c+p-b(u-2k)}{2c},\right.\right.\right.$$
$$\left.\ldots,\frac{2c+p-b(u-2k)}{2c},1;\frac{2c+p-b(u-2k)}{2c}+1,\ldots,\frac{2c+p-b(u-2k)}{2c}+1;-e^{2cz}\right)-$$
$$e^{(p-b(u-2k))z}\sum_{j=0}^{n}\frac{(-1)^j(p-b(u-2k))^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{p-b(u-2k)}{2c},\ldots,\frac{p-b(u-2k)}{2c},\right.$$
$$\left.\left.1;\frac{p-b(u-2k)}{2c}+1,\ldots,\frac{p-b(u-2k)}{2c}+1;-e^{2cz}\right)\right)-$$
$$e^{(p+b(u-2k))z}\sum_{j=0}^{n}\frac{(-1)^j(p+b(u-2k))^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{p+b(u-2k)}{2c},\ldots,\frac{p+b(u-2k)}{2c},\right.$$
$$\left.1;\frac{p+b(u-2k)}{2c}+1,\ldots,\frac{p+b(u-2k)}{2c}+1;-e^{2cz}\right)+$$
$$e^{(2c+p+b(u-2k))z}\sum_{j=0}^{n}\frac{(-1)^j(2c+p+b(u-2k))^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{2c+p+b(u-2k)}{2c},\ldots,\frac{2c+p+b(u-2k)}{2c},\right.$$
$$\left.\left.1;\frac{2c+p+b(u-2k)}{2c}+1,\ldots,\frac{2c+p+b(u-2k)}{2c}+1;-e^{2cz}\right)\right)\;/;\;n\in\mathbb{N}\bigwedge u\in\mathbb{N}^+$$

Involving cosh, exp and power

Involving $z^n e^{pz}\cosh(a+bz)\tanh(cz)$

01.21.21.0176.01

$$\int z^n e^{p z} \cosh(a + b z) \tanh(c z) \, dz = \frac{1}{2} e^a n! \left(e^{(b+2c+p)z} \sum_{j=0}^{n} \frac{(-1)^j (b + 2c + p)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{b+2c+p}{2c}, \ldots, \frac{b+2c+p}{2c}, 1; \frac{b+2c+p}{2c} + 1, \ldots, \frac{b+2c+p}{2c} + 1; -e^{2cz} \right) - e^{(b+p)z} \sum_{j=0}^{n} \frac{(-1)^j (b+p)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{b+p}{2c}, \ldots, \frac{b+p}{2c}, 1; \frac{b+p}{2c} + 1, \ldots, \frac{b+p}{2c} + 1; -e^{2cz} \right) \right) + \frac{1}{2} e^{-a} n! \left(e^{(-b+2c+p)z} \sum_{j=0}^{n} \frac{(-1)^j (-b + 2c + p)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{-b+2c+p}{2c}, \ldots, \frac{-b+2c+p}{2c}, 1; \frac{-b+2c+p}{2c} + 1, \ldots, \frac{-b+2c+p}{2c} + 1; -e^{2cz} \right) - e^{(p-b)z} \sum_{j=0}^{n} \frac{(-1)^j (p-b)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{p-b}{2c}, \ldots, \frac{p-b}{2c}, 1; \frac{p-b}{2c} + 1, \ldots, \frac{p-b}{2c} + 1; -e^{2cz} \right) \right) /; n \in \mathbb{N}$$

01.21.21.0177.01

$$\int z^n e^{p z} \cosh(b z) \tanh(c z) \, dz = \frac{1}{2} n! \left(e^{(b+2c+p)z} \sum_{j=0}^{n} \frac{(-1)^j (b + 2c + p)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{b+2c+p}{2c}, \ldots, \frac{b+2c+p}{2c}, 1; \frac{b+2c+p}{2c} + 1, \ldots, \frac{b+2c+p}{2c} + 1; -e^{2cz} \right) - e^{(b+p)z} \sum_{j=0}^{n} \frac{(-1)^j (b+p)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{b+p}{2c}, \ldots, \frac{b+p}{2c}, 1; \frac{b+p}{2c} + 1, \ldots, \frac{b+p}{2c} + 1; -e^{2cz} \right) + e^{(-b+2c+p)z} \sum_{j=0}^{n} \frac{(-1)^j (-b + 2c + p)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{-b+2c+p}{2c}, \ldots, \frac{-b+2c+p}{2c}, 1; \frac{-b+2c+p}{2c} + 1, \ldots, \frac{-b+2c+p}{2c} + 1; -e^{2cz} \right) - e^{(p-b)z} \sum_{j=0}^{n} \frac{(-1)^j (p-b)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{p-b}{2c}, \ldots, \frac{p-b}{2c}, 1; \frac{p-b}{2c} + 1, \ldots, \frac{p-b}{2c} + 1; -e^{2cz} \right) \right) /; n \in \mathbb{N}$$

01.21.21.0178.01

$$\int z^n e^{b z} \cosh(b z) \tanh(c z) \, dz = \frac{1}{2} n! \left(-\frac{z^{n+1}}{(n+1)!} + 2 e^{2cz} \sum_{j=0}^{n} \frac{(-1)^j 2^{-j-1} c^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+3}F_{j+2}\left(1, \ldots, 1, 2; 2, \ldots, 2; -e^{2cz}\right) - e^{2bz} \sum_{j=0}^{n} \frac{(-1)^j 2^{-j-1} b^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{b}{c}, \ldots, \frac{b}{c}, 1; \frac{b}{c} + 1, \ldots, \frac{b}{c} + 1; -e^{2cz} \right) + e^{2(b+c)z} \sum_{j=0}^{n} \frac{(-1)^j 2^{-j-1} (b+c)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{b+c}{c}, \ldots, \frac{b+c}{c}, 1; \frac{b+c}{c} + 1, \ldots, \frac{b+c}{c} + 1; -e^{2cz} \right) \right) /; n \in \mathbb{N}$$

Involving powers of cosh, exp and power

Involving $z^n e^{p z} \cosh^u(b z) \tanh(c z)$

01.21.21.0179.01

$$\int z^n e^{p z} \cosh^u(b z) \tanh(c z)\, dz = \binom{u}{\frac{u}{2}} n!\,(1 - u \bmod 2)\, 2^{-u} \left(e^{(2c+p)z} \sum_{j=0}^{n} \frac{(-1)^j (2c+p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{2c+p}{2c}, \ldots, \frac{2c+p}{2c}, 1; \frac{2c+p}{2c}+1, \ldots, \frac{2c+p}{2c}+1; -e^{2cz}\right) - e^{p z} \sum_{j=0}^{n} \frac{(-1)^j p^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{p}{2c}, \ldots, \frac{p}{2c}, 1; \frac{p}{2c}+1, \ldots, \frac{p}{2c}+1; -e^{2cz}\right)\right) + 2^{-u} n! \sum_{k=0}^{\left\lfloor \frac{u-1}{2} \right\rfloor} \binom{u}{k} \left(e^{(2c+p-b(u-2k))z} \sum_{j=0}^{n} \frac{(-1)^j (2c+p-b(u-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{2c+p-b(u-2k)}{2c}, \ldots, \frac{2c+p-b(u-2k)}{2c}, 1; \frac{2c+p-b(u-2k)}{2c}+1, \ldots, \frac{2c+p-b(u-2k)}{2c}+1; -e^{2cz}\right) - e^{(p-b(u-2k))z} \sum_{j=0}^{n} \frac{(-1)^j (p-b(u-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{p-b(u-2k)}{2c}, \ldots, \frac{p-b(u-2k)}{2c}, 1; \frac{p-b(u-2k)}{2c}+1, \ldots, \frac{p-b(u-2k)}{2c}+1; -e^{2cz}\right) - e^{(p+b(u-2k))z} \sum_{j=0}^{n} \frac{(-1)^j (p+b(u-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{p+b(u-2k)}{2c}, \ldots, \frac{p+b(u-2k)}{2c}, 1; \frac{p+b(u-2k)}{2c}+1, \ldots, \frac{p+b(u-2k)}{2c}+1; -e^{2cz}\right) + e^{(2c+p+b(u-2k))z} \sum_{j=0}^{n} \frac{(-1)^j (2c+p+b(u-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{2c+p+b(u-2k)}{2c}, \ldots, \frac{2c+p+b(u-2k)}{2c}, 1; \frac{2c+p+b(u-2k)}{2c}+1, \ldots, \frac{2c+p+b(u-2k)}{2c}+1; -e^{2cz}\right)\right) /; n \in \mathbb{N} \land u \in \mathbb{N}^+$$

Involving hyperbolic, exponential and trigonometric functions

Involving sin, sinh and exp

Involving $e^{p z} \sin(a z) \sinh(b z) \tanh(c z)$

01.21.21.0180.01

$$\int e^{p z} \sin(a z) \sinh(b z) \tanh(c z)\, dz = \frac{1}{4} i \left(\frac{e^{(-b-i a+p) z} {}_2F_1\left(\frac{-b-i a+p}{2 c}, 1; \frac{-b-i a+p}{2 c}+1; -e^{2 c z}\right)}{-b-i a+p} - \frac{e^{(b-i a+p) z} {}_2F_1\left(\frac{b-i a+p}{2 c}, 1; \frac{b-i a+p}{2 c}+1; -e^{2 c z}\right)}{b-i a+p} + \frac{e^{(b+i a+p) z} {}_2F_1\left(\frac{b+i a+p}{2 c}, 1; \frac{b+i a+p}{2 c}+1; -e^{2 c z}\right)}{b+i a+p} + \frac{e^{(-b+2 c+i a+p) z} {}_2F_1\left(\frac{-b+i a+p}{2 c}+1, 1; \frac{-b+i a+p}{2 c}+2; -e^{2 c z}\right)}{-b+2 c+i a+p} + \frac{e^{(b+2 c-i a+p) z} {}_2F_1\left(\frac{b-i a+p}{2 c}+1, 1; \frac{b-i a+p}{2 c}+2; -e^{2 c z}\right)}{b+2 c-i a+p} - \frac{e^{(b+2 c+i a+p) z} {}_2F_1\left(\frac{b+i a+p}{2 c}+1, 1; \frac{b+i a+p}{2 c}+2; -e^{2 c z}\right)}{b+2 c+i a+p} - \frac{e^{(-b+i a+p) z} {}_2F_1\left(\frac{-b+i a+p}{2 c}, 1; \frac{-b+i a+p}{2 c}+1; -e^{2 c z}\right)}{-b+i a+p} - \frac{e^{(-b+2 c-i a+p) z} {}_2F_1\left(\frac{-b-i a+p}{2 c}+1, 1; \frac{-b-i a+p}{2 c}+2; -e^{2 c z}\right)}{-b+2 c-i a+p} \right)$$

Involving powers of sin, powers of sinh and exp

Involving $e^{p z} \sin^m(a z) \sinh^u(b z) \tanh(c z)$

01.21.21.0181.01

$$\int e^{p z} \sin^m(a z) \sinh^u(b z) \tanh(c z)\, dz = i^u 2^{-m-u} \binom{m}{\frac{m}{2}} \binom{u}{\frac{u}{2}} \left(\frac{e^{(2 c+p) z} {}_2F_1\left(\frac{p}{2 c}+1, 1; \frac{p}{2 c}+2; -e^{2 c z}\right)}{2 c+p} - \frac{e^{p z} {}_2F_1\left(\frac{p}{2 c}, 1; \frac{p}{2 c}+1; -e^{2 c z}\right)}{p} \right) (1 - m \bmod 2)(1 - u \bmod 2) - i^u 2^{-m-u} \binom{u}{\frac{u}{2}} (u \bmod 2 - 1) \sum_{k=0}^{\left\lfloor \frac{m-1}{2} \right\rfloor} (-1)^k \binom{m}{k} \left(e^{-\frac{1}{2} i m \pi} \left(\frac{e^{(2 c+a i (m-2 k)+p) z} {}_2F_1\left(\frac{a i (m-2 k)+p}{2 c}+1, 1; \frac{a i (m-2 k)+p}{2 c}+2; -e^{2 c z}\right)}{2 c+a i (m-2 k)+p} - \frac{e^{(a i (m-2 k)+p) z} {}_2F_1\left(\frac{a i (m-2 k)+p}{2 c}, 1; \frac{a i (m-2 k)+p}{2 c}+1; -e^{2 c z}\right)}{a i (m-2 k)+p} \right) + e^{\frac{i m \pi}{2}} \left(\frac{e^{(2 c-i a (m-2 k)+p) z} {}_2F_1\left(\frac{p-i a (m-2 k)}{2 c}+1, 1; \frac{p-i a (m-2 k)}{2 c}+2; -e^{2 c z}\right)}{2 c-i a (m-2 k)+p} - \frac{e^{(p-i a (m-2 k)) z} {}_2F_1\left(\frac{p-i a (m-2 k)}{2 c}, 1; \frac{p-i a (m-2 k)}{2 c}+1; -e^{2 c z}\right)}{p-i a (m-2 k)} \right) \right) + 2^{-m-u} \binom{m}{\frac{m}{2}} (1 - m \bmod 2) \sum_{k=0}^{\left\lfloor \frac{u-1}{2} \right\rfloor} (-1)^k \binom{u}{k} \left(\left(\frac{e^{(2 c+p+b (u-2 k)) z} {}_2F_1\left(\frac{p+b (u-2 k)}{2 c}+1, 1; \frac{p+b (u-2 k)}{2 c}+2; -e^{2 c z}\right)}{2 c+p+b (u-2 k)} - \right.\right.$$

$$\frac{e^{(p+b(u-2k))z}\,{}_2F_1\left(\frac{p+b(u-2k)}{2c},1;\frac{p+b(u-2k)}{2c}+1;-e^{2cz}\right)}{p+b(u-2k)}\Bigg)+$$

$$(-1)^u\left(\frac{e^{(2c+p-b(u-2k))z}\,{}_2F_1\left(\frac{p-b(u-2k)}{2c}+1,1;\frac{p-b(u-2k)}{2c}+2;-e^{2cz}\right)}{2c+p-b(u-2k)}-\right.$$

$$\left.\frac{e^{(p-b(u-2k))z}\,{}_2F_1\left(\frac{p-b(u-2k)}{2c},1;\frac{p-b(u-2k)}{2c}+1;-e^{2cz}\right)}{p-b(u-2k)}\right)\Bigg)+$$

$$2^{-m-u}\sum_{s=0}^{\left\lfloor\frac{u-1}{2}\right\rfloor}\sum_{k=0}^{\left\lfloor\frac{m-1}{2}\right\rfloor}(-1)^{k+s}\binom{m}{k}\binom{u}{s}\left(e^{-\frac{1}{2}i m\pi}\left(\left(e^{(2c+a i(m-2k)+p+b(u-2s))z}\,{}_2F_1\left(\frac{a i(m-2k)+p+b(u-2s)}{2c}+1,\right.\right.\right.\right.$$

$$\left.\left.1;\frac{a i(m-2k)+p+b(u-2s)}{2c}+2;-e^{2cz}\right)\right)\Big/(2c+a i(m-2k)+p+b(u-2s))-$$

$$\left(e^{(a i(m-2k)+p+b(u-2s))z}\,{}_2F_1\left(\frac{a i(m-2k)+p+b(u-2s)}{2c},1;\frac{a i(m-2k)+p+b(u-2s)}{2c}+1;-e^{2cz}\right)\right)\Big/$$

$$\left.(a i(m-2k)+p+b(u-2s))\right)+e^{\frac{i m\pi}{2}}\left(\left(e^{(2c-i a(m-2k)+p+b(u-2s))z}\,{}_2F_1\right.\right.$$

$$\left.\left(\frac{-i a(m-2k)+p+b(u-2s)}{2c}+1,1;\frac{-i a(m-2k)+p+b(u-2s)}{2c}+2;-e^{2cz}\right)\right)\Big/$$

$$(2c-i a(m-2k)+p+b(u-2s))-\left(e^{(-i a(m-2k)+p+b(u-2s))z}\,{}_2F_1\left(\frac{-i a(m-2k)+p+b(u-2s)}{2c},\right.\right.$$

$$\left.\left.\left.1;\frac{-i a(m-2k)+p+b(u-2s)}{2c}+1;-e^{2cz}\right)\right)\Big/(-i a(m-2k)+p+b(u-2s))\right)+$$

$$(-1)^u\,e^{-\frac{1}{2}i m\pi}\left(\left(e^{(2c+a i(m-2k)+p-b(u-2s))z}\,{}_2F_1\left(\frac{a i(m-2k)+p-b(u-2s)}{2c}+1,1;\right.\right.\right.$$

$$\left.\left.\frac{a i(m-2k)+p-b(u-2s)}{2c}+2;-e^{2cz}\right)\right)\Big/(2c+a i(m-2k)+p-b(u-2s))-$$

$$\left(e^{(a i(m-2k)+p-b(u-2s))z}\,{}_2F_1\left(\frac{a i(m-2k)+p-b(u-2s)}{2c},1;\frac{a i(m-2k)+p-b(u-2s)}{2c}+1;-e^{2cz}\right)\right)\Big/$$

$$\left.(a i(m-2k)+p-b(u-2s))\right)+$$

$$(-1)^u\,e^{\frac{i m\pi}{2}}\left(\left(e^{(2c-i a(m-2k)+p-b(u-2s))z}\,{}_2F_1\left(\frac{-i a(m-2k)+p-b(u-2s)}{2c}+1,1;\right.\right.\right.$$

$$\left.\left.\frac{-i a(m-2k)+p-b(u-2s)}{2c}+2;-e^{2cz}\right)\right)\Big/(2c-i a(m-2k)+p-b(u-2s))-$$

$$\left(e^{(-i a(m-2k)+p-b(u-2s))z}\,{}_2F_1\left(\frac{-i a(m-2k)+p-b(u-2s)}{2c},1;\frac{-i a(m-2k)+p-b(u-2s)}{2c}+1;\right.\right.$$

$$\left.\left.\left.\left.-e^{2cz}\right)\right)\Big/(-i a(m-2k)+p-b(u-2s))\right)\right)\;/;\;m\in\mathbb{N}^+\bigwedge u\in\mathbb{N}^+$$

01.21.21.0182.01

$$\int e^{p z} \sin^m(a z) \sinh^\mu(c z) \tanh(c z)\, dz = -\frac{2^{-m} e^{p z} (1 - m \bmod 2) \sinh^\mu(c z) \left(1 - e^{2 c z}\right)^{-\mu}}{p - c\mu} \binom{m}{\frac{m}{2}} F_1\left(\frac{p - c\mu}{2c}; -\mu - 1, 1; \frac{1}{2}\left(\frac{p}{c} - \mu + 2\right); e^{2 c z}, -e^{2 c z}\right) - 2^{-m} \sinh^\mu(c z) \left(1 - e^{2 c z}\right)^{-\mu} \sum_{k=0}^{\left\lfloor \frac{m-1}{2} \right\rfloor} (-1)^k \binom{m}{k} \left(\left(e^{\frac{i \pi m}{2} + (p - i a (m - 2k)) z} F_1\left(\frac{-i a (m - 2k) + p - c\mu}{2c}; -\mu - 1, 1; \frac{1}{2}\left(\frac{p - i a (m - 2k)}{c} - \mu + 2\right); e^{2 c z}, -e^{2 c z}\right)\right) \Big/ (-i a (m - 2k) + p - c\mu) + \frac{1}{a i (m - 2k) + p - c\mu} \left(e^{(a i (m - 2k) + p) z - \frac{i m \pi}{2}} F_1\left(\frac{a i (m - 2k) + p - c\mu}{2c}; -\mu - 1, 1; \frac{1}{2}\left(\frac{a i (m - 2k) + p}{c} - \mu + 2\right); e^{2 c z}, -e^{2 c z}\right)\right)\right) /; m \in \mathbb{N}^+$$

Involving cos, sinh and exp

Involving $e^{p z} \cos(a z) \sinh(b z) \tanh(c z)$

01.21.21.0183.01

$$\int e^{p z} \cos(a z) \sinh(b z) \tanh(c z)\, dz = \frac{1}{4}\left(\frac{e^{(-b - i a + p) z} {}_2F_1\left(\frac{-b - i a + p}{2c}, 1; \frac{-b - i a + p}{2c} + 1; -e^{2 c z}\right)}{-b - i a + p} + \frac{e^{(-b + i a + p) z} {}_2F_1\left(\frac{-b + i a + p}{2c}, 1; \frac{-b + i a + p}{2c} + 1; -e^{2 c z}\right)}{-b + i a + p} - \frac{e^{(b - i a + p) z} {}_2F_1\left(\frac{b - i a + p}{2c}, 1; \frac{b - i a + p}{2c} + 1; -e^{2 c z}\right)}{b - i a + p} - \frac{e^{(-b + 2c + i a + p) z} {}_2F_1\left(\frac{-b + i a + p}{2c} + 1, 1; \frac{-b + i a + p}{2c} + 2; -e^{2 c z}\right)}{-b + 2c + i a + p} + \frac{e^{(b + 2c - i a + p) z} {}_2F_1\left(\frac{b - i a + p}{2c} + 1, 1; \frac{b - i a + p}{2c} + 2; -e^{2 c z}\right)}{b + 2c - i a + p} + \frac{e^{(b + 2c + i a + p) z} {}_2F_1\left(\frac{b + i a + p}{2c} + 1, 1; \frac{b + i a + p}{2c} + 2; -e^{2 c z}\right)}{b + 2c + i a + p} - \frac{e^{(b + i a + p) z} {}_2F_1\left(\frac{b + i a + p}{2c}, 1; \frac{b + i a + p}{2c} + 1; -e^{2 c z}\right)}{b + i a + p} - \frac{e^{(-b + 2c - i a + p) z} {}_2F_1\left(\frac{-b - i a + p}{2c} + 1, 1; \frac{-b - i a + p}{2c} + 2; -e^{2 c z}\right)}{-b + 2c - i a + p}\right)$$

Involving powers of cos, powers of sinh and exp

Involving $e^{p z} \cos^m(a z) \sinh^u(b z) \tanh(c z)$

01.21.21.0184.01

$$\int e^{p z} \cos^m(a z) \sinh^u(b z) \tanh(c z)\, dz = i^u 2^{-m-u} \binom{m}{\frac{m}{2}} \binom{u}{\frac{u}{2}} \left(\frac{e^{(2c + p) z}}{2c + p} {}_2F_1\left(\frac{p}{2c} + 1, 1; \frac{p}{2c} + 2; -e^{2 c z}\right) - \frac{e^{p z}}{p} {}_2F_1\left(\frac{p}{2c}, 1; \frac{p}{2c} + 1; -e^{2 c z}\right)\right) (1 - m \bmod 2) (1 - u \bmod 2) -$$

$$i^u 2^{-m-u} \binom{u}{\frac{u}{2}} (u \bmod 2 - 1) \sum_{k=0}^{\left\lfloor \frac{m-1}{2} \right\rfloor} \binom{m}{k} \Bigg(\Bigg(\frac{e^{(2c+a i (m-2k)+p) z}}{2c + a i (m-2k) + p} \, {}_2F_1\!\left(\frac{a i (m-2k)+p}{2c} + 1, 1; \frac{a i (m-2k)+p}{2c} + 2; -e^{2cz} \right) -$$

$$\frac{e^{(a i (m-2k)+p) z}}{a i (m-2k) + p} \, {}_2F_1\!\left(\frac{a i (m-2k)+p}{2c}, 1; \frac{a i (m-2k)+p}{2c} + 1; -e^{2cz} \right) \Bigg) +$$

$$\Bigg(\frac{e^{(2c - i a (m-2k)+p) z}}{2c - i a (m-2k) + p} \, {}_2F_1\!\left(\frac{p - i a (m-2k)}{2c} + 1, 1; \frac{p - i a (m-2k)}{2c} + 2; -e^{2cz} \right) -$$

$$\frac{e^{(p - i a (m-2k)) z}}{p - i a (m-2k)} \, {}_2F_1\!\left(\frac{p - i a (m-2k)}{2c}, 1; \frac{p - i a (m-2k)}{2c} + 1; -e^{2cz} \right) \Bigg) \Bigg) +$$

$$2^{-m-u} \binom{m}{\frac{m}{2}} (1 - m \bmod 2) \sum_{k=0}^{\left\lfloor \frac{u-1}{2} \right\rfloor} (-1)^k \binom{u}{k} \Bigg(\Bigg(\frac{e^{(2c+p+b(u-2k)) z}}{2c + p + b(u-2k)} \, {}_2F_1\!\left(\frac{p + b(u-2k)}{2c} + 1, 1; \frac{p + b(u-2k)}{2c} + 2; -e^{2cz} \right) -$$

$$\frac{e^{(p+b(u-2k)) z}}{p + b(u-2k)} \, {}_2F_1\!\left(\frac{p + b(u-2k)}{2c}, 1; \frac{p + b(u-2k)}{2c} + 1; -e^{2cz} \right) \Bigg) +$$

$$(-1)^u \Bigg(\frac{e^{(2c+p-b(u-2k)) z}}{2c + p - b(u-2k)} \, {}_2F_1\!\left(\frac{p - b(u-2k)}{2c} + 1, 1; \frac{p - b(u-2k)}{2c} + 2; -e^{2cz} \right) -$$

$$\frac{e^{(p-b(u-2k)) z}}{p - b(u-2k)} \, {}_2F_1\!\left(\frac{p - b(u-2k)}{2c}, 1; \frac{p - b(u-2k)}{2c} + 1; -e^{2cz} \right) \Bigg) \Bigg) +$$

$$2^{-m-u} \sum_{s=0}^{\left\lfloor \frac{u-1}{2} \right\rfloor} \sum_{k=0}^{\left\lfloor \frac{m-1}{2} \right\rfloor} (-1)^s \binom{m}{k} \binom{u}{s} \Bigg(\frac{e^{(2c + a i (m-2k) + p + b(u-2s)) z}}{2c + a i (m-2k) + p + b(u-2s)} \, {}_2F_1\Bigg(\frac{a i (m-2k) + p + b(u-2s)}{2c} + 1,$$

$$1; \frac{a i (m-2k) + p + b(u-2s)}{2c} + 2; -e^{2cz} \Bigg) - \frac{e^{(a i (m-2k) + p + b(u-2s)) z}}{a i (m-2k) + p + b(u-2s)}$$

$${}_2F_1\!\left(\frac{a i (m-2k) + p + b(u-2s)}{2c}, 1; \frac{a i (m-2k) + p + b(u-2s)}{2c} + 1; -e^{2cz} \right) +$$

$$\frac{e^{(2c - i a (m-2k) + p + b(u-2s)) z}}{2c - i a (m-2k) + p + b(u-2s)} \, {}_2F_1\Bigg(\frac{-i a (m-2k) + p + b(u-2s)}{2c} + 1, 1;$$

$$\frac{-i a (m-2k) + p + b(u-2s)}{2c} + 2; -e^{2cz} \Bigg) - \frac{e^{(-i a (m-2k) + p + b(u-2s)) z}}{-i a (m-2k) + p + b(u-2s)}$$

$${}_2F_1\!\left(\frac{-i a (m-2k) + p + b(u-2s)}{2c}, 1; \frac{-i a (m-2k) + p + b(u-2s)}{2c} + 1; -e^{2cz} \right) +$$

$$(-1)^u \Bigg(\frac{e^{(2c + a i (m-2k) + p - b(u-2s)) z}}{2c + a i (m-2k) + p - b(u-2s)} \, {}_2F_1\Bigg(\frac{a i (m-2k) + p - b(u-2s)}{2c} + 1, 1;$$

$$\frac{a i (m-2k) + p - b(u-2s)}{2c} + 2; -e^{2cz} \Bigg) - \frac{e^{(a i (m-2k) + p - b(u-2s)) z}}{a i (m-2k) + p - b(u-2s)}$$

$${}_2F_1\!\left(\frac{a i (m-2k) + p - b(u-2s)}{2c}, 1; \frac{a i (m-2k) + p - b(u-2s)}{2c} + 1; -e^{2cz} \right) +$$

$$\frac{e^{(2c-i a (m-2k)+p-b(u-2s))z}}{2c-i a (m-2k)+p-b(u-2s)} {}_2F_1\left(\frac{-i a (m-2k)+p-b(u-2s)}{2c}+1, 1; \frac{-i a (m-2k)+p-b(u-2s)}{2c}+2; -e^{2cz}\right) - \frac{e^{(-i a (m-2k)+p-b(u-2s))z}}{-i a (m-2k)+p-b(u-2s)} {}_2F_1\left(\frac{-i a (m-2k)+p-b(u-2s)}{2c}, 1; \frac{-i a (m-2k)+p-b(u-2s)}{2c}+1; -e^{2cz}\right)\Bigg)\Bigg) /; m \in \mathbb{N}^+ \wedge u \in \mathbb{N}^+$$

01.21.21.0185.01

$$\int e^{pz} \cos^m(a z) \sinh^\mu(c z) \tanh(c z)\, dz = -\frac{2^{-m} e^{pz} (1 - m \bmod 2) \sinh^\mu(c z) \left(1-e^{2cz}\right)^{-\mu}}{p - c\mu} \binom{m}{\frac{m}{2}} F_1\left(\frac{p-c\mu}{2c}; -\mu-1, 1; \frac{1}{2}\left(\frac{p}{c}-\mu+2\right); e^{2cz}, -e^{2cz}\right) - 2^{-m} \sinh^\mu(c z)\left(1-e^{2cz}\right)^{-\mu} \sum_{k=0}^{\left\lfloor\frac{m-1}{2}\right\rfloor} \binom{m}{k} \left(\frac{e^{(p-i a (m-2k))z} F_1\left(\frac{-i a (m-2k)+p-c\mu}{2c}; -\mu-1, 1; \frac{1}{2}\left(\frac{p-i a (m-2k)}{c}-\mu+2\right); e^{2cz}, -e^{2cz}\right)}{-i a (m-2k)+p-c\mu} + \frac{e^{(a i (m-2k)+p)z} F_1\left(\frac{a i (m-2k)+p-c\mu}{2c}; -\mu-1, 1; \frac{1}{2}\left(\frac{a i (m-2k)+p}{c}-\mu+2\right); e^{2cz}, -e^{2cz}\right)}{a i (m-2k)+p-c\mu}\right) /; m \in \mathbb{N}^+$$

Involving sin, cosh and exp

Involving $e^{pz} \sin(a z) \cosh(b z) \tanh(c z)$

01.21.21.0186.01

$$\int e^{pz} \sin(a z) \cosh(b z) \tanh(c z)\, dz = \frac{1}{4} i \left(-\frac{e^{(-b-i a+p)z} {}_2F_1\left(\frac{-b-i a+p}{2c}, 1; \frac{-b-i a+p}{2c}+1; -e^{2cz}\right)}{-b-i a+p} + \frac{e^{(-b+i a+p)z} {}_2F_1\left(\frac{-b+i a+p}{2c}, 1; \frac{-b+i a+p}{2c}+1; -e^{2cz}\right)}{-b+i a+p} - \frac{e^{(b-i a+p)z} {}_2F_1\left(\frac{b-i a+p}{2c}, 1; \frac{b-i a+p}{2c}+1; -e^{2cz}\right)}{b-i a+p} + \frac{e^{(b+i a+p)z} {}_2F_1\left(\frac{b+i a+p}{2c}, 1; \frac{b+i a+p}{2c}+1; -e^{2cz}\right)}{b+i a+p} + \frac{e^{(-b+2c-i a+p)z} {}_2F_1\left(\frac{-b-i a+p}{2c}+1, 1; \frac{-b-i a+p}{2c}+2; -e^{2cz}\right)}{-b+2c-i a+p} - \frac{e^{(-b+2c+i a+p)z} {}_2F_1\left(\frac{-b+i a+p}{2c}+1, 1; \frac{-b+i a+p}{2c}+2; -e^{2cz}\right)}{-b+2c+i a+p} + \frac{e^{(b+2c-i a+p)z} {}_2F_1\left(\frac{b-i a+p}{2c}+1, 1; \frac{b-i a+p}{2c}+2; -e^{2cz}\right)}{b+2c-i a+p} - \frac{e^{(b+2c+i a+p)z} {}_2F_1\left(\frac{b+i a+p}{2c}+1, 1; \frac{b+i a+p}{2c}+2; -e^{2cz}\right)}{b+2c+i a+p}\right)$$

Involving powers of sin, powers of cosh and exp

Involving $e^{pz} \sin^m(a z) \cosh^u(b z) \tanh(c z)$

01.21.21.0187.01

$$\int e^{pz}\sin^m(az)\cosh^u(bz)\tanh(cz)\,dz ==$$

$$2^{-m-u}\left(\frac{e^{(2c+p)z}\,{}_2F_1\left(\frac{p}{2c}+1,1;\frac{p}{2c}+2;-e^{2cz}\right)}{2c+p}-\frac{e^{pz}\,{}_2F_1\left(\frac{p}{2c},1;\frac{p}{2c}+1;-e^{2cz}\right)}{p}\right)\binom{m}{\frac{m}{2}}\binom{u}{\frac{u}{2}}(1-m\bmod 2)(1-u\bmod 2)-$$

$$2^{-m-u}\binom{u}{\frac{u}{2}}(u\bmod 2-1)\sum_{k=0}^{\left\lfloor\frac{m-1}{2}\right\rfloor}(-1)^k\binom{m}{k}\left(e^{-\frac{1}{2}im\pi}\left(\frac{e^{(2c+ai(m-2k)+p)z}\,{}_2F_1\left(\frac{ai(m-2k)+p}{2c}+1,1;\frac{ai(m-2k)+p}{2c}+2;-e^{2cz}\right)}{2c+ai(m-2k)+p}-\right.\right.$$

$$\left.\frac{e^{(ai(m-2k)+p)z}\,{}_2F_1\left(\frac{ai(m-2k)+p}{2c},1;\frac{ai(m-2k)+p}{2c}+1;-e^{2cz}\right)}{ai(m-2k)+p}\right)+$$

$$e^{\frac{im\pi}{2}}\left(\frac{e^{(2c-ia(m-2k)+p)z}\,{}_2F_1\left(\frac{p-ia(m-2k)}{2c}+1,1;\frac{p-ia(m-2k)}{2c}+2;-e^{2cz}\right)}{2c-ia(m-2k)+p}-\right.$$

$$\left.\left.\frac{e^{(p-ia(m-2k))z}\,{}_2F_1\left(\frac{p-ia(m-2k)}{2c},1;\frac{p-ia(m-2k)}{2c}+1;-e^{2cz}\right)}{p-ia(m-2k)}\right)\right)+$$

$$2^{-m-u}\binom{m}{\frac{m}{2}}(1-m\bmod 2)\sum_{k=0}^{\left\lfloor\frac{u-1}{2}\right\rfloor}\binom{u}{k}\left(-\frac{e^{(p+b(u-2k))z}\,{}_2F_1\left(\frac{p+b(u-2k)}{2c},1;\frac{p+b(u-2k)}{2c}+1;-e^{2cz}\right)}{p+b(u-2k)}+\right.$$

$$\frac{e^{(2c+p+b(u-2k))z}\,{}_2F_1\left(\frac{p+b(u-2k)}{2c}+1,1;\frac{p+b(u-2k)}{2c}+2;-e^{2cz}\right)}{2c+p+b(u-2k)}+$$

$$\frac{e^{(2c+p-b(u-2k))z}\,{}_2F_1\left(\frac{p-b(u-2k)}{2c}+1,1;\frac{p-b(u-2k)}{2c}+2;-e^{2cz}\right)}{2c+p-b(u-2k)}-$$

$$\left.\frac{e^{(p-b(u-2k))z}\,{}_2F_1\left(\frac{p-b(u-2k)}{2c},1;\frac{p-b(u-2k)}{2c}+1;-e^{2cz}\right)}{p-b(u-2k)}\right)+$$

$$2^{-m-u}\sum_{s=0}^{\left\lfloor\frac{u-1}{2}\right\rfloor}\sum_{k=0}^{\left\lfloor\frac{m-1}{2}\right\rfloor}(-1)^k\binom{m}{k}\binom{u}{s}\left(e^{-\frac{1}{2}im\pi}\left(\left(e^{(2c+ai(m-2k)+p+b(u-2s))z}\,{}_2F_1\left(\frac{ai(m-2k)+p+b(u-2s)}{2c}+1,1;\right.\right.\right.\right.$$

$$\left.\left.\frac{ai(m-2k)+p+b(u-2s)}{2c}+2;-e^{2cz}\right)\right)\Big/(2c+ai(m-2k)+p+b(u-2s))-$$

$$\left(e^{(ai(m-2k)+p+b(u-2s))z}\,{}_2F_1\left(\frac{ai(m-2k)+p+b(u-2s)}{2c},1;\frac{ai(m-2k)+p+b(u-2s)}{2c}+1;-e^{2cz}\right)\right)\Big/$$

$$\left.(ai(m-2k)+p+b(u-2s))\right)+e^{\frac{im\pi}{2}}\left(\left(e^{(2c-ia(m-2k)+p+b(u-2s))z}\,{}_2F_1\right.\right.$$

$$\left.\left.\left(\frac{-ia(m-2k)+p+b(u-2s)}{2c}+1,1;\frac{-ia(m-2k)+p+b(u-2s)}{2c}+2;-e^{2cz}\right)\right)\right)\Big/$$

$$(2c - i a (m-2k) + p + b(u-2s)) - \left(e^{(-i a (m-2k)+p+b(u-2s))z}\, {}_2F_1\left(\frac{-i a (m-2k) + p + b(u-2s)}{2c}, 1; \frac{-i a (m-2k) + p + b(u-2s)}{2c} + 1; -e^{2cz}\right)\right) \Big/ (-i a (m-2k) + p + b(u-2s))\Bigg) +$$

$$e^{-\frac{1}{2} i m \pi} \Bigg(\Bigg(\left(e^{(2c + a i (m-2k) + p - b(u-2s))z}\, {}_2F_1\left(\frac{a i (m-2k) + p - b(u-2s)}{2c} + 1, 1; \frac{a i (m-2k) + p - b(u-2s)}{2c} + 2; -e^{2cz}\right)\right) \Big/ (2c + a i (m-2k) + p - b(u-2s)) -$$

$$\left(e^{(a i (m-2k) + p - b(u-2s))z}\, {}_2F_1\left(\frac{a i (m-2k) + p - b(u-2s)}{2c}, 1; \frac{a i (m-2k) + p - b(u-2s)}{2c} + 1; -e^{2cz}\right)\right) \Big/$$

$$(a i (m-2k) + p - b(u-2s))\Bigg) + e^{\frac{i m \pi}{2}} \Bigg(\left(e^{(2c - i a (m-2k) + p - b(u-2s))z}\, {}_2F_1\left(\frac{-i a (m-2k) + p - b(u-2s)}{2c} + 1, 1; \frac{-i a (m-2k) + p - b(u-2s)}{2c} + 2; -e^{2cz}\right)\right) \Big/ (2c - i a (m-2k) + p - b(u-2s)) -$$

$$\left(e^{(-i a (m-2k) + p - b(u-2s))z}\, {}_2F_1\left(\frac{-i a (m-2k) + p - b(u-2s)}{2c}, 1; \frac{-i a (m-2k) + p - b(u-2s)}{2c} + 1; -e^{2cz}\right)\right) \Big/ (-i a (m-2k) + p - b(u-2s))\Bigg)\Bigg) /;\ m \in \mathbb{N}^+ \bigwedge u \in \mathbb{N}^+$$

01.21.21.0188.01

$$\int e^{pz} \sin^m(az) \cosh^\mu(cz) \tanh(cz)\, dz = 2^{-m} \binom{m}{\frac{m}{2}} \cosh^\mu(cz)\, (1 - m \bmod 2) \left(1 + e^{2cz}\right)^{-\mu}$$

$$\left(\frac{e^{(2c+p)z}\, {}_2F_1\left(\frac{p + c(2-\mu)}{2c}, 1-\mu; \frac{1}{2}\left(\frac{p}{c} - \mu + 4\right); -e^{2cz}\right)}{p + c(2-\mu)} - \frac{e^{pz}\, {}_2F_1\left(\frac{p - c\mu}{2c}, 1-\mu; \frac{1}{2}\left(\frac{p}{c} - \mu + 2\right); -e^{2cz}\right)}{p - c\mu}\right) +$$

$$2^{-m} \cosh^\mu(cz) \left(1 + e^{2cz}\right)^{-\mu} \sum_{k=0}^{\left\lfloor \frac{m-1}{2} \right\rfloor} (-1)^k \binom{m}{k} \Bigg(\left(e^{\frac{i \pi m}{2} + (2c - i a (m-2k) + p)z}\, {}_2F_1\left(\frac{-i a (m-2k) + p + c(2-\mu)}{2c}, 1-\mu; \frac{1}{2}\left(\frac{p - i a (m-2k)}{c} - \mu + 4\right); -e^{2cz}\right)\right) \Big/ (-i a (m-2k) + p + c(2-\mu)) +$$

$$\left(e^{(2c + a i (m-2k) + p)z - \frac{i m \pi}{2}}\, {}_2F_1\left(\frac{a i (m-2k) + p + c(2-\mu)}{2c}, 1-\mu; \frac{1}{2}\left(\frac{a i (m-2k) + p}{c} - \mu + 4\right); -e^{2cz}\right)\right) \Big/$$

$$(a i (m-2k) + p + c(2-\mu)) - \frac{e^{\frac{i \pi m}{2} + (p - i a (m-2k))z}\, {}_2F_1\left(\frac{-i a (m-2k) + p - c\mu}{2c}, 1-\mu; \frac{1}{2}\left(\frac{p - i a (m-2k)}{c} - \mu + 2\right); -e^{2cz}\right)}{-i a (m-2k) + p - c\mu} -$$

$$\frac{e^{(a i (m-2k) + p)z - \frac{i m \pi}{2}}\, {}_2F_1\left(\frac{a i (m-2k) + p - c\mu}{2c}, 1-\mu; \frac{1}{2}\left(\frac{a i (m-2k) + p}{c} - \mu + 2\right); -e^{2cz}\right)}{a i (m-2k) + p - c\mu}\Bigg) /;\ m \in \mathbb{N}^+$$

Involving cos, cosh and exp

Involving $e^{p z} \cos(a z) \cosh(b z) \tanh(c z)$

01.21.21.0189.01

$$\int e^{p z} \cos(a z) \cosh(b z) \tanh(c z)\, dz = \frac{1}{4}\left(-\frac{e^{(-b-i a+p) z}\, {}_2F_1\left(\frac{-b-i a+p}{2 c}, 1; \frac{-b-i a+p}{2 c}+1; -e^{2 c z}\right)}{-b-i a+p} - \frac{e^{(b-i a+p) z}\, {}_2F_1\left(\frac{b-i a+p}{2 c}, 1; \frac{b-i a+p}{2 c}+1; -e^{2 c z}\right)}{b-i a+p} + \frac{e^{(-b+2 c-i a+p) z}\, {}_2F_1\left(\frac{-b-i a+p}{2 c}+1, 1; \frac{-b-i a+p}{2 c}+2; -e^{2 c z}\right)}{-b+2 c-i a+p} + \frac{e^{(-b+2 c+i a+p) z}\, {}_2F_1\left(\frac{-b+i a+p}{2 c}+1, 1; \frac{-b+i a+p}{2 c}+2; -e^{2 c z}\right)}{-b+2 c+i a+p} + \frac{e^{(b+2 c-i a+p) z}\, {}_2F_1\left(\frac{b-i a+p}{2 c}+1, 1; \frac{b-i a+p}{2 c}+2; -e^{2 c z}\right)}{b+2 c-i a+p} + \frac{e^{(b+2 c+i a+p) z}\, {}_2F_1\left(\frac{b+i a+p}{2 c}+1, 1; \frac{b+i a+p}{2 c}+2; -e^{2 c z}\right)}{b+2 c+i a+p} - \frac{e^{(-b+i a+p) z}\, {}_2F_1\left(\frac{-b+i a+p}{2 c}, 1; \frac{-b+i a+p}{2 c}+1; -e^{2 c z}\right)}{-b+i a+p} - \frac{e^{(b+i a+p) z}\, {}_2F_1\left(\frac{b+i a+p}{2 c}, 1; \frac{b+i a+p}{2 c}+1; -e^{2 c z}\right)}{b+i a+p}\right)$$

Involving powers of cos, powers of cosh and exp

Involving $e^{p z} \cos^m(a z) \cosh^u(b z) \tanh(c z)$

01.21.21.0190.01

$$\int e^{p z} \cos^m(a z) \cosh^u(b z) \tanh(c z)\, dz = 2^{-m-u}\left(\frac{e^{(2 c+p) z}\, {}_2F_1\left(\frac{p}{2 c}+1, 1; \frac{p}{2 c}+2; -e^{2 c z}\right)}{2 c+p} - \frac{e^{p z}\, {}_2F_1\left(\frac{p}{2 c}, 1; \frac{p}{2 c}+1; -e^{2 c z}\right)}{p}\right)\binom{m}{\frac{m}{2}}\binom{u}{\frac{u}{2}}(1-m \bmod 2)(1-u \bmod 2) - 2^{-m-u}\binom{u}{\frac{u}{2}}(u \bmod 2-1)\sum_{k=0}^{\left\lfloor\frac{m-1}{2}\right\rfloor}\binom{m}{k}\left(\frac{e^{(2 c-i a (m-2 k)+p) z}\, {}_2F_1\left(\frac{p-i a (m-2 k)}{2 c}+1, 1; \frac{p-i a (m-2 k)}{2 c}+2; -e^{2 c z}\right)}{2 c-i a (m-2 k)+p} + \frac{e^{(2 c+a i (m-2 k)+p) z}\, {}_2F_1\left(\frac{a i (m-2 k)+p}{2 c}+1, 1; \frac{a i (m-2 k)+p}{2 c}+2; -e^{2 c z}\right)}{2 c+a i (m-2 k)+p} - \frac{e^{(p-i a (m-2 k)) z}\, {}_2F_1\left(\frac{p-i a (m-2 k)}{2 c}, 1; \frac{p-i a (m-2 k)}{2 c}+1; -e^{2 c z}\right)}{p-i a (m-2 k)} - \frac{e^{(a i (m-2 k)+p) z}\, {}_2F_1\left(\frac{a i (m-2 k)+p}{2 c}, 1; \frac{a i (m-2 k)+p}{2 c}+1; -e^{2 c z}\right)}{a i (m-2 k)+p}\right) + 2^{-m-u}\binom{m}{\frac{m}{2}}(1-m \bmod 2)\sum_{k=0}^{\left\lfloor\frac{u-1}{2}\right\rfloor}\binom{u}{k}\left(-\frac{e^{(p+b (u-2 k)) z}\, {}_2F_1\left(\frac{p+b (u-2 k)}{2 c}, 1; \frac{p+b (u-2 k)}{2 c}+1; -e^{2 c z}\right)}{p+b (u-2 k)} + \right.$$

$$\frac{e^{(2c+p+b(u-2k))z}\,{}_2F_1\left(\frac{p+b(u-2k)}{2c}+1,\,1;\,\frac{p+b(u-2k)}{2c}+2;\,-e^{2cz}\right)}{2c+p+b(u-2k)}+$$

$$\frac{e^{(2c+p-b(u-2k))z}\,{}_2F_1\left(\frac{p-b(u-2k)}{2c}+1,\,1;\,\frac{p-b(u-2k)}{2c}+2;\,-e^{2cz}\right)}{2c+p-b(u-2k)}-$$

$$\left.\frac{e^{(p-b(u-2k))z}\,{}_2F_1\left(\frac{p-b(u-2k)}{2c},\,1;\,\frac{p-b(u-2k)}{2c}+1;\,-e^{2cz}\right)}{p-b(u-2k)}\right)+$$

$$2^{-m-u}\sum_{s=0}^{\left\lfloor\frac{u-1}{2}\right\rfloor}\sum_{k=0}^{\left\lfloor\frac{m-1}{2}\right\rfloor}\binom{m}{k}\binom{u}{s}\Bigg(\Bigg(e^{(2c-i a(m-2k)+p+b(u-2s))z}\,{}_2F_1\Bigg(\frac{-i a(m-2k)+p+b(u-2s)}{2c}+1,\,1;$$

$$\frac{-i a(m-2k)+p+b(u-2s)}{2c}+2;\,-e^{2cz}\Bigg)\Bigg)\Big/(2c-i a(m-2k)+p+b(u-2s))+$$

$$\left(e^{(2c+a i(m-2k)+p+b(u-2s))z}\,{}_2F_1\left(\frac{a i(m-2k)+p+b(u-2s)}{2c}+1,\,1;\,\frac{a i(m-2k)+p+b(u-2s)}{2c}+2;\,-e^{2cz}\right)\right)\Big/$$

$$(2c+a i(m-2k)+p+b(u-2s))+\Bigg(e^{(2c-i a(m-2k)+p-b(u-2s))z}\,{}_2F_1\Bigg(\frac{-i a(m-2k)+p-b(u-2s)}{2c}+1,$$

$$1;\,\frac{-i a(m-2k)+p-b(u-2s)}{2c}+2;\,-e^{2cz}\Bigg)\Bigg)\Big/(2c-i a(m-2k)+p-b(u-2s))+$$

$$\left(e^{(2c+a i(m-2k)+p-b(u-2s))z}\,{}_2F_1\left(\frac{a i(m-2k)+p-b(u-2s)}{2c}+1,\,1;\,\frac{a i(m-2k)+p-b(u-2s)}{2c}+2;\,-e^{2cz}\right)\right)\Big/$$

$$(2c+a i(m-2k)+p-b(u-2s))-\Bigg(e^{(-i a(m-2k)+p+b(u-2s))z}\,{}_2F_1\Bigg(\frac{-i a(m-2k)+p+b(u-2s)}{2c},$$

$$1;\,\frac{-i a(m-2k)+p+b(u-2s)}{2c}+1;\,-e^{2cz}\Bigg)\Bigg)\Big/(-i a(m-2k)+p+b(u-2s))-$$

$$\left(e^{(a i(m-2k)+p+b(u-2s))z}\,{}_2F_1\left(\frac{a i(m-2k)+p+b(u-2s)}{2c},\,1;\,\frac{a i(m-2k)+p+b(u-2s)}{2c}+1;\,-e^{2cz}\right)\right)\Big/$$

$$(a i(m-2k)+p+b(u-2s))-\Bigg(e^{(-i a(m-2k)+p-b(u-2s))z}\,{}_2F_1\Bigg(\frac{-i a(m-2k)+p-b(u-2s)}{2c},$$

$$1;\,\frac{-i a(m-2k)+p-b(u-2s)}{2c}+1;\,-e^{2cz}\Bigg)\Bigg)\Big/(-i a(m-2k)+p-b(u-2s))-$$

$$\left(e^{(a i(m-2k)+p-b(u-2s))z}\,{}_2F_1\left(\frac{a i(m-2k)+p-b(u-2s)}{2c},\,1;\,\frac{a i(m-2k)+p-b(u-2s)}{2c}+1;\,-e^{2cz}\right)\right)\Big/$$

$$(a i(m-2k)+p-b(u-2s))\Bigg)\,/;\,m\in\mathbb{N}^+\bigwedge u\in\mathbb{N}^+$$

01.21.21.0191.01

$$\int e^{p z} \cos^m(a z) \cosh^{\mu}(c z) \tanh(c z)\, dz = 2^{-m} \binom{m}{\frac{m}{2}} \cosh^{\mu}(c z)\, (1 - m \bmod 2) \left(1 + e^{2 c z}\right)^{-\mu} \left(\frac{e^{(2 c+p) z}\, {}_2F_1\left(\frac{p+c(2-\mu)}{2 c}, 1-\mu; \frac{1}{2}\left(\frac{p}{c} - \mu + 4\right); -e^{2 c z}\right)}{p + c(2-\mu)} - \frac{e^{p z}\, {}_2F_1\left(\frac{p-c\mu}{2 c}, 1-\mu; \frac{1}{2}\left(\frac{p}{c} - \mu + 2\right); -e^{2 c z}\right)}{p - c\mu} \right) + 2^{-m} \cosh^{\mu}(c z) \left(1 + e^{2 c z}\right)^{-\mu} \sum_{k=0}^{\left\lfloor \frac{m-1}{2} \right\rfloor} \binom{m}{k} \left(\left(e^{(2 c - i a (m-2k)+p) z}\, {}_2F_1\left(\frac{-i a (m-2k) + p + c(2-\mu)}{2 c}, 1-\mu; \frac{1}{2}\left(\frac{p - i a (m-2k)}{c} - \mu + 4\right); -e^{2 c z}\right)\right) \Big/ (-i a (m-2k) + p + c(2-\mu)) + \left(e^{(2 c + a i (m-2k)+p) z}\, {}_2F_1\left(\frac{a i (m-2k) + p + c(2-\mu)}{2 c}, 1-\mu; \frac{1}{2}\left(\frac{a i (m-2k) + p}{c} - \mu + 4\right); -e^{2 c z}\right)\right) \Big/ (a i (m-2k) + p + c(2-\mu)) - \frac{e^{(p - i a (m-2k)) z}\, {}_2F_1\left(\frac{-i a (m-2k) + p - c\mu}{2 c}, 1-\mu; \frac{1}{2}\left(\frac{p - i a (m-2k)}{c} - \mu + 2\right); -e^{2 c z}\right)}{-i a (m-2k) + p - c\mu} - \frac{e^{(a i (m-2k)+p) z}\, {}_2F_1\left(\frac{a i (m-2k) + p - c\mu}{2 c}, 1-\mu; \frac{1}{2}\left(\frac{a i (m-2k) + p}{c} - \mu + 2\right); -e^{2 c z}\right)}{a i (m-2k) + p - c\mu} \right) /;\ m \in \mathbb{N}^+$$

Involving hyperbolic, trigonometric and a power functions

Involving sin, sinh and power

Involving $z^n \sin(a z) \sinh(b z) \tanh(c z)$

01.21.21.0192.01

$$\int z^n \sin(a z) \sinh(b z) \tanh(c z)\, dz = \frac{1}{4} i\, n! \left(-e^{(b-i a) z} \sum_{j=0}^{n} \frac{(-1)^j (b - i a)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{b - i a}{2 c}, \ldots, \frac{b - i a}{2 c}, 1; \frac{b - i a}{2 c} + 1, \ldots, \frac{b - i a}{2 c} + 1; -e^{2 c z}\right) + e^{(b+2 c-i a) z} \sum_{j=0}^{n} \frac{(-1)^j (b + 2 c - i a)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{b + 2 c - i a}{2 c}, \ldots, \frac{b + 2 c - i a}{2 c}, 1; \frac{b + 2 c - i a}{2 c} + 1, \ldots, \frac{b + 2 c - i a}{2 c} + 1; -e^{2 c z}\right) - e^{(b+2 c+i a) z} \sum_{j=0}^{n} \frac{(-1)^j (b + 2 c + i a)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{b + 2 c + i a}{2 c}, \ldots, \frac{b + 2 c + i a}{2 c}, 1; \frac{b + 2 c + i a}{2 c} + 1, \ldots, \frac{b + 2 c + i a}{2 c} + 1; -e^{2 c z}\right) + e^{(b+i a) z} \sum_{j=0}^{n} \frac{(-1)^j (b + i a)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{b + i a}{2 c}, \ldots, \frac{b + i a}{2 c}, 1; \frac{b + i a}{2 c} + 1, \ldots, \frac{b + i a}{2 c} + 1; -e^{2 c z}\right) + e^{(-b-i a) z} \sum_{j=0}^{n} \frac{(-1)^j (-b - i a)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{-b - i a}{2 c}, \ldots, \frac{-b - i a}{2 c}, 1; \frac{-b - i a}{2 c} + 1, \ldots, \frac{-b - i a}{2 c} + 1; -e^{2 c z}\right) - e^{(-b+2 c-i a) z} \sum_{j=0}^{n} \frac{(-1)^j (-b + 2 c - i a)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{-b + 2 c - i a}{2 c}, \ldots, \frac{-b + 2 c - i a}{2 c}, 1; \frac{-b + 2 c - i a}{2 c} + 1, \ldots, \frac{-b + 2 c - i a}{2 c} + 1; -e^{2 c z}\right) - e^{(i a-b) z} \sum_{j=0}^{n} \frac{(-1)^j (i a - b)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{i a - b}{2 c}, \ldots, \frac{i a - b}{2 c}, 1; \frac{i a - b}{2 c} + 1, \ldots, \frac{i a - b}{2 c} + 1; -e^{2 c z}\right) + e^{(-b+2 c+i a) z} \sum_{j=0}^{n} \frac{(-1)^j (-b + 2 c + i a)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{-b + 2 c + i a}{2 c}, \ldots, \frac{-b + 2 c + i a}{2 c}, 1; \frac{-b + 2 c + i a}{2 c} + 1, \ldots, \frac{-b + 2 c + i a}{2 c} + 1; -e^{2 c z}\right)\right) /; n \in \mathbb{N}$$

Involving powers of sin, powers of sinh and power

Involving $z^n \sin^m(a z) \sinh^u(b z) \mathrm{Tanh}(c z)$

01.21.21.0193.01

$$\int z^n \sin^m(a z) \sinh^u(b z) \tanh(c z)\, dz = i^u 2^{-m-u} \binom{m}{\frac{m}{2}} \binom{u}{\frac{u}{2}} n! (1 - m \bmod 2)(1 - u \bmod 2) \left(-\frac{z^{n+1}}{(n+1)!} + 2 e^{2 c z} \sum_{j=0}^{n} \frac{(-1)^j 2^{-j-1} c^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}(1, \ldots, 1, 1; 2, \ldots, 2; -e^{2 c z}) \right) + i^u 2^{-m-u} \binom{u}{\frac{u}{2}} n! (1 - u \bmod 2)$$

$$\sum_{k=0}^{\left\lfloor \frac{m-1}{2} \right\rfloor} (-1)^k \binom{m}{k} \left(e^{\frac{i m \pi}{2}} \left(e^{(2c - i a (m-2k)) z} \sum_{j=0}^{n} \frac{(-1)^j (2c - i a (m-2k))^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{2c - i a (m-2k)}{2c}, \ldots, \frac{2c - i a (m-2k)}{2c}, 1; \frac{2c - i a (m-2k)}{2c} + 1, \ldots, \frac{2c - i a (m-2k)}{2c} + 1; -e^{2cz} \right) - e^{-i a (m-2k) z} \sum_{j=0}^{n} \frac{(-1)^j (-i a (m-2k))^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{-i a (m-2k)}{2c}, \ldots, \frac{-i a (m-2k)}{2c}, 1; \frac{-i a (m-2k)}{2c} + 1, \ldots, \frac{-i a (m-2k)}{2c} + 1; -e^{2cz} \right) \right) + e^{-\frac{1}{2} i m \pi} \left(e^{(2c + a i (m-2k)) z} \sum_{j=0}^{n} \frac{(-1)^j (2c + a i (m-2k))^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{2c + a i (m-2k)}{2c}, \ldots, \frac{2c + a i (m-2k)}{2c}, 1; \frac{2c + a i (m-2k)}{2c} + 1, \ldots, \frac{2c + a i (m-2k)}{2c} + 1; -e^{2cz} \right) - e^{i a (m-2k) z} \sum_{j=0}^{n} \frac{(-1)^j (i a (m-2k))^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{i a (m-2k)}{2c}, \ldots, \frac{i a (m-2k)}{2c}, 1; \frac{i a (m-2k)}{2c} + 1, \ldots, \frac{i a (m-2k)}{2c} + 1; -e^{2cz} \right) \right) \right) +$$

$$2^{-m-u} \binom{m}{\frac{m}{2}} n! \, (1 - m \bmod 2) \sum_{k=0}^{\left\lfloor \frac{u-1}{2} \right\rfloor} (-1)^k \binom{u}{k} \left(-e^{b (u-2k) z} \sum_{j=0}^{n} \frac{(-1)^j (b (u-2k))^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{b (u-2k)}{2c}, \ldots, \frac{b (u-2k)}{2c}, 1; \frac{b (u-2k)}{2c} + 1, \ldots, \frac{b (u-2k)}{2c} + 1; -e^{2cz} \right) + e^{(2c + b (u-2k)) z} \sum_{j=0}^{n} \frac{(-1)^j (2c + b (u-2k))^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{2c + b (u-2k)}{2c}, \ldots, \frac{2c + b (u-2k)}{2c}, 1; \frac{2c + b (u-2k)}{2c} + 1, \ldots, \frac{2c + b (u-2k)}{2c} + 1; -e^{2cz} \right) + (-1)^u \left(e^{(2c - b (u-2k)) z} \sum_{j=0}^{n} \frac{(-1)^j (2c - b (u-2k))^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{2c - b (u-2k)}{2c}, \ldots, \frac{2c - b (u-2k)}{2c}, 1; \frac{2c - b (u-2k)}{2c} + 1, \ldots, \frac{2c - b (u-2k)}{2c} + 1; -e^{2cz} \right) - e^{-b (u-2k) z} \sum_{j=0}^{n} \frac{(-1)^j (-b (u-2k))^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{-b (u-2k)}{2c}, \ldots, \frac{-b (u-2k)}{2c}, 1; \frac{-b (u-2k)}{2c} + 1, \ldots, \frac{-b (u-2k)}{2c} + 1; -e^{2cz} \right) \right) \right) +$$

$$2^{-m-u}\, n! \sum_{k=0}^{\left\lfloor \frac{m-1}{2} \right\rfloor} (-1)^k \binom{m}{k} \sum_{i=0}^{\left\lfloor \frac{u-1}{2} \right\rfloor} (-1)^i \binom{u}{i} \left(e^{\frac{i m \pi}{2}} \left(e^{(2c - i a (m-2k) + b (u-2i)) z} \sum_{j=0}^{n} \frac{(-1)^j (2c - i a (m-2k) + b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} \right. \right.$$

$$\left. {}_{j+2}F_{j+1}\left(\frac{2c - i a(m-2k) + b(u-2i)}{2c}, \ldots, \frac{2c - i a(m-2k) + b(u-2i)}{2c}, 1; \frac{2c - i a(m-2k) + b(u-2i)}{2c} + 1, \ldots, \frac{2c - i a(m-2k) + b(u-2i)}{2c} + 1; -e^{2cz} \right) - \right.$$

$$e^{(b(u-2i) - i a(m-2k)) z} \sum_{j=0}^{n} \frac{(-1)^j (b(u-2i) - i a(m-2k))^{-j-1} z^{n-j}}{(n-j)!}$$

$$\left. \left. {}_{j+2}F_{j+1}\left(\frac{b(u-2i) - i a(m-2k)}{2c}, \ldots, \frac{b(u-2i) - i a(m-2k)}{2c}, 1; \frac{b(u-2i) - i a(m-2k)}{2c} + 1, \ldots, \frac{b(u-2i) - i a(m-2k)}{2c} + 1; -e^{2cz} \right) \right) + \right.$$

$$e^{-\frac{1}{2} i m \pi} \left(e^{(2c + a i (m-2k) + b(u-2i)) z} \sum_{j=0}^{n} \frac{(-1)^j (2c + a i (m-2k) + b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} \right.$$

$$\left. {}_{j+2}F_{j+1}\left(\frac{2c + a i (m-2k) + b(u-2i)}{2c}, \ldots, \frac{2c + a i (m-2k) + b(u-2i)}{2c}, 1; \frac{2c + a i (m-2k) + b(u-2i)}{2c} + 1, \ldots, \frac{2c + a i (m-2k) + b(u-2i)}{2c} + 1; -e^{2cz} \right) - \right.$$

$$e^{(a i (m-2k) + b(u-2i)) z} \sum_{j=0}^{n} \frac{(-1)^j (a i (m-2k) + b(u-2i))^{-j-1} z^{n-j}}{(n-j)!}$$

$$\left. \left. {}_{j+2}F_{j+1}\left(\frac{a i (m-2k) + b(u-2i)}{2c}, \ldots, \frac{a i (m-2k) + b(u-2i)}{2c}, 1; \frac{a i (m-2k) + b(u-2i)}{2c} + 1, \ldots, \frac{a i (m-2k) + b(u-2i)}{2c} + 1; -e^{2cz} \right) \right) + \right.$$

$$(-1)^u e^{\frac{i m \pi}{2}} \left(e^{(2c - i a(m-2k) - b(u-2i)) z} \sum_{j=0}^{n} \frac{(-1)^j (2c - i a(m-2k) - b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} \right.$$

$$\left. {}_{j+2}F_{j+1}\left(\frac{2c - i a(m-2k) - b(u-2i)}{2c}, \ldots, \frac{2c - i a(m-2k) - b(u-2i)}{2c}, 1; \frac{2c - i a(m-2k) - b(u-2i)}{2c} + 1, \ldots, \frac{2c - i a(m-2k) - b(u-2i)}{2c} + 1; -e^{2cz} \right) - \right.$$

$$e^{(-i a(m-2k) - b(u-2i)) z} \sum_{j=0}^{n} \frac{(-1)^j (-i a(m-2k) - b(u-2i))^{-j-1} z^{n-j}}{(n-j)!}$$

$$\left. \left. {}_{j+2}F_{j+1}\left(\frac{-i a(m-2k) - b(u-2i)}{2c}, \ldots, \frac{-i a(m-2k) - b(u-2i)}{2c}, 1; \frac{-i a(m-2k) - b(u-2i)}{2c} + 1, \ldots, \frac{-i a(m-2k) - b(u-2i)}{2c} + 1; -e^{2cz} \right) \right) + \right.$$

$$(-1)^u e^{-\frac{1}{2} i m \pi} \left(e^{(2c+a i (m-2k)-b(u-2i))z} \sum_{j=0}^{n} \frac{(-1)^j (2c + a i (m-2k) - b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} \right.$$

$$ {}_{j+2}F_{j+1}\left(\frac{2c + a i (m-2k) - b(u-2i)}{2c}, \ldots, \frac{2c + a i (m-2k) - b(u-2i)}{2c}, 1; \right.$$

$$\left. \frac{2c + a i (m-2k) - b(u-2i)}{2c} + 1, \ldots, \frac{2c + a i (m-2k) - b(u-2i)}{2c} + 1; -e^{2cz} \right) -$$

$$e^{(i a (m-2k)-b(u-2i))z} \sum_{j=0}^{n} \frac{(-1)^j (i a (m-2k) - b(u-2i))^{-j-1} z^{n-j}}{(n-j)!}$$

$$ {}_{j+2}F_{j+1}\left(\frac{i a (m-2k) - b(u-2i)}{2c}, \ldots, \frac{i a (m-2k) - b(u-2i)}{2c}, 1; \frac{i a (m-2k) - b(u-2i)}{2c} + 1, \right.$$

$$\left.\left.\left. \ldots, \frac{i a (m-2k) - b(u-2i)}{2c} + 1; -e^{2cz} \right) \right) \right) /; n \in \mathbb{N} \bigwedge m \in \mathbb{N}^+ \bigwedge u \in \mathbb{N}^+$$

Involving cos, sinh and power

Involving $z^n \cos(a z) \sinh(b z) \tanh(c z)$

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$$\int z^n \cos(a z) \sinh(b z) \tanh(c z)\, dz = \frac{1}{4} n! \left(-e^{(b-i a) z} \sum_{j=0}^{n} \frac{(-1)^j (b - i a)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{b-i a}{2c}, \ldots, \frac{b-i a}{2c}, 1; \frac{b-i a}{2c} + 1, \ldots, \frac{b-i a}{2c} + 1; -e^{2cz}\right) + e^{(b+2c-i a) z} \sum_{j=0}^{n} \frac{(-1)^j (b + 2c - i a)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{b+2c-i a}{2c}, \ldots, \frac{b+2c-i a}{2c}, 1; \frac{b+2c-i a}{2c} + 1, \ldots, \frac{b+2c-i a}{2c} + 1; -e^{2cz}\right) + e^{(b+2c+i a) z} \sum_{j=0}^{n} \frac{(-1)^j (b + 2c + i a)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{b+2c+i a}{2c}, \ldots, \frac{b+2c+i a}{2c}, 1; \frac{b+2c+i a}{2c} + 1, \ldots, \frac{b+2c+i a}{2c} + 1; -e^{2cz}\right) - e^{(b+i a) z} \sum_{j=0}^{n} \frac{(-1)^j (b + i a)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{b+i a}{2c}, \ldots, \frac{b+i a}{2c}, 1; \frac{b+i a}{2c} + 1, \ldots, \frac{b+i a}{2c} + 1; -e^{2cz}\right) + e^{(-b-i a) z} \sum_{j=0}^{n} \frac{(-1)^j (-b - i a)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{-b-i a}{2c}, \ldots, \frac{-b-i a}{2c}, 1; \frac{-b-i a}{2c} + 1, \ldots, \frac{-b-i a}{2c} + 1; -e^{2cz}\right) - e^{(-b+2c-i a) z} \sum_{j=0}^{n} \frac{(-1)^j (-b + 2c - i a)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{-b+2c-i a}{2c}, \ldots, \frac{-b+2c-i a}{2c}, 1; \frac{-b+2c-i a}{2c} + 1, \ldots, \frac{-b+2c-i a}{2c} + 1; -e^{2cz}\right) + e^{(i a-b) z} \sum_{j=0}^{n} \frac{(-1)^j (i a - b)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{i a-b}{2c}, \ldots, \frac{i a-b}{2c}, 1; \frac{i a-b}{2c} + 1, \ldots, \frac{i a-b}{2c} + 1; -e^{2cz}\right) - e^{(-b+2c+i a) z} \sum_{j=0}^{n} \frac{(-1)^j (-b + 2c + i a)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{-b+2c+i a}{2c}, \ldots, \frac{-b+2c+i a}{2c}, 1; \frac{-b+2c+i a}{2c} + 1, \ldots, \frac{-b+2c+i a}{2c} + 1; -e^{2cz}\right)\right) /; n \in \mathbb{N}$$

Involving powers of cos, powers of sinh and power

Involving $z^n \cos^m(a z) \sinh^u(b z) \tanh(c z)$

01.21.21.0195.01

$$\int z^n \cos^m(a z) \sinh^u(b z) \tanh(c z)\, dz = i^u 2^{-m-u} \binom{m}{\frac{m}{2}} \binom{u}{\frac{u}{2}} n! (1 - m \bmod 2) (1 - u \bmod 2) \left(-\frac{z^{n+1}}{(n+1)!} + 2 e^{2cz} \sum_{j=0}^{n} \frac{(-1)^j 2^{-j-1} c^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}(1, \ldots, 1, 1; 2, \ldots, 2; -e^{2cz}) \right) +$$

$$i^u\, 2^{-m-u} \binom{u}{\frac{u}{2}} n!\,(1-u \bmod 2) \sum_{k=0}^{\left\lfloor\frac{m-1}{2}\right\rfloor} \binom{m}{k} \Bigg(e^{(2c-i\,a\,(m-2k))\,z} \sum_{j=0}^{n} \frac{(-1)^j\,(2c-i\,a\,(m-2k))^{-j-1}\,z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}$$

$$\left(\frac{2c-i\,a\,(m-2k)}{2c}, \ldots, \frac{2c-i\,a\,(m-2k)}{2c}, 1; \frac{2c-i\,a\,(m-2k)}{2c}+1, \ldots, \frac{2c-i\,a\,(m-2k)}{2c}+1; -e^{2cz}\right) -$$

$$e^{-i\,a\,(m-2k)\,z} \sum_{j=0}^{n} \frac{(-1)^j\,(-i\,a\,(m-2k))^{-j-1}\,z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}\left(\frac{-i\,a\,(m-2k)}{2c}, \ldots, \frac{-i\,a\,(m-2k)}{2c},\right.$$

$$\left. 1; \frac{-i\,a\,(m-2k)}{2c}+1, \ldots, \frac{-i\,a\,(m-2k)}{2c}+1; -e^{2cz}\right) +$$

$$e^{(2c+a\,i\,(m-2k))\,z} \sum_{j=0}^{n} \frac{(-1)^j\,(2c+a\,i\,(m-2k))^{-j-1}\,z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}\left(\frac{2c+a\,i\,(m-2k)}{2c}, \ldots, \frac{2c+a\,i\,(m-2k)}{2c}, 1;\right.$$

$$\left.\frac{2c+a\,i\,(m-2k)}{2c}+1, \ldots, \frac{2c+a\,i\,(m-2k)}{2c}+1; -e^{2cz}\right) - e^{i\,a\,(m-2k)\,z} \sum_{j=0}^{n} \frac{(-1)^j\,(i\,a\,(m-2k))^{-j-1}\,z^{n-j}}{(n-j)!}$$

$$\left.{}_{j+2}F_{j+1}\left(\frac{i\,a\,(m-2k)}{2c}, \ldots, \frac{i\,a\,(m-2k)}{2c}, 1; \frac{i\,a\,(m-2k)}{2c}+1, \ldots, \frac{i\,a\,(m-2k)}{2c}+1; -e^{2cz}\right)\right) +$$

$$2^{-m-u} \binom{m}{\frac{m}{2}} n!\,(1-m \bmod 2) \sum_{k=0}^{\left\lfloor\frac{u-1}{2}\right\rfloor} (-1)^k \binom{u}{k} \Bigg(-e^{b\,(u-2k)\,z} \sum_{j=0}^{n} \frac{(-1)^j\,(b\,(u-2k))^{-j-1}\,z^{n-j}}{(n-j)!}$$

$${}_{j+2}F_{j+1}\left(\frac{b\,(u-2k)}{2c}, \ldots, \frac{b\,(u-2k)}{2c}, 1; \frac{b\,(u-2k)}{2c}+1, \ldots, \frac{b\,(u-2k)}{2c}+1; -e^{2cz}\right) +$$

$$e^{(2c+b\,(u-2k))\,z} \sum_{j=0}^{n} \frac{(-1)^j\,(2c+b\,(u-2k))^{-j-1}\,z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}\left(\frac{2c+b\,(u-2k)}{2c}, \ldots, \frac{2c+b\,(u-2k)}{2c},\right.$$

$$\left. 1; \frac{2c+b\,(u-2k)}{2c}+1, \ldots, \frac{2c+b\,(u-2k)}{2c}+1; -e^{2cz}\right) +$$

$$(-1)^u \Bigg(e^{(2c-b\,(u-2k))\,z} \sum_{j=0}^{n} \frac{(-1)^j\,(2c-b\,(u-2k))^{-j-1}\,z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}\left(\frac{2c-b\,(u-2k)}{2c}, \ldots,\right.$$

$$\left.\frac{2c-b\,(u-2k)}{2c}, 1; \frac{2c-b\,(u-2k)}{2c}+1, \ldots, \frac{2c-b\,(u-2k)}{2c}+1; -e^{2cz}\right) -$$

$$e^{-b\,(u-2k)\,z} \sum_{j=0}^{n} \frac{(-1)^j\,(-b\,(u-2k))^{-j-1}\,z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}\left(\frac{-b\,(u-2k)}{2c}, \ldots, \frac{-b\,(u-2k)}{2c}, 1;\right.$$

$$\left.\left.\left.\frac{-b\,(u-2k)}{2c}+1, \ldots, \frac{-b\,(u-2k)}{2c}+1; -e^{2cz}\right)\right)\right) +$$

$$2^{-m-u}\, n! \sum_{k=0}^{\left\lfloor\frac{m-1}{2}\right\rfloor} \binom{m}{k} \sum_{i=0}^{\left\lfloor\frac{u-1}{2}\right\rfloor} (-1)^i \binom{u}{i} \Bigg(e^{(2c-i\,a\,(m-2k)+b\,(u-2i))\,z} \sum_{j=0}^{n} \frac{(-1)^j\,(2c-i\,a\,(m-2k)+b\,(u-2i))^{-j-1}\,z^{n-j}}{(n-j)!}$$

$${}_{j+2}F_{j+1}\left(\frac{2c-i\,a\,(m-2k)+b\,(u-2i)}{2c}, \ldots, \frac{2c-i\,a\,(m-2k)+b\,(u-2i)}{2c}, 1;\right.$$

$$\frac{2\,c-\mathbb{i}\,a\,(m-2\,k)+b\,(u-2\,i)}{2\,c}+1,\,\ldots,\,\frac{2\,c-\mathbb{i}\,a\,(m-2\,k)+b\,(u-2\,i)}{2\,c}+1;\,-e^{2\,c\,z}\Bigg)-$$

$$e^{(b\,(u-2\,i)-\mathbb{i}\,a\,(m-2\,k))\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(b\,(u-2\,i)-\mathbb{i}\,a\,(m-2\,k))^{-j-1}\,z^{n-j}}{(n-j)\,!}$$

$${}_{j+2}F_{j+1}\Bigg(\frac{b\,(u-2\,i)-\mathbb{i}\,a\,(m-2\,k)}{2\,c},\,\ldots,\,\frac{b\,(u-2\,i)-\mathbb{i}\,a\,(m-2\,k)}{2\,c},\,1;$$

$$\frac{b\,(u-2\,i)-\mathbb{i}\,a\,(m-2\,k)}{2\,c}+1,\,\ldots,\,\frac{b\,(u-2\,i)-\mathbb{i}\,a\,(m-2\,k)}{2\,c}+1;\,-e^{2\,c\,z}\Bigg)+$$

$$e^{(2\,c+a\,\mathbb{i}\,(m-2\,k)+b\,(u-2\,i))\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(2\,c+a\,\mathbb{i}\,(m-2\,k)+b\,(u-2\,i))^{-j-1}\,z^{n-j}}{(n-j)\,!}$$

$${}_{j+2}F_{j+1}\Bigg(\frac{2\,c+a\,\mathbb{i}\,(m-2\,k)+b\,(u-2\,i)}{2\,c},\,\ldots,\,\frac{2\,c+a\,\mathbb{i}\,(m-2\,k)+b\,(u-2\,i)}{2\,c},\,1;$$

$$\frac{2\,c+a\,\mathbb{i}\,(m-2\,k)+b\,(u-2\,i)}{2\,c}+1,\,\ldots,\,\frac{2\,c+a\,\mathbb{i}\,(m-2\,k)+b\,(u-2\,i)}{2\,c}+1;\,-e^{2\,c\,z}\Bigg)-$$

$$e^{(a\,\mathbb{i}\,(m-2\,k)+b\,(u-2\,i))\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(a\,\mathbb{i}\,(m-2\,k)+b\,(u-2\,i))^{-j-1}\,z^{n-j}}{(n-j)\,!}$$

$${}_{j+2}F_{j+1}\Bigg(\frac{a\,\mathbb{i}\,(m-2\,k)+b\,(u-2\,i)}{2\,c},\,\ldots,\,\frac{a\,\mathbb{i}\,(m-2\,k)+b\,(u-2\,i)}{2\,c},\,1;$$

$$\frac{a\,\mathbb{i}\,(m-2\,k)+b\,(u-2\,i)}{2\,c}+1,\,\ldots,\,\frac{a\,\mathbb{i}\,(m-2\,k)+b\,(u-2\,i)}{2\,c}+1;\,-e^{2\,c\,z}\Bigg)+$$

$$(-1)^u\Bigg(e^{(2\,c-\mathbb{i}\,a\,(m-2\,k)-b\,(u-2\,i))\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(2\,c-\mathbb{i}\,a\,(m-2\,k)-b\,(u-2\,i))^{-j-1}\,z^{n-j}}{(n-j)\,!}$$

$${}_{j+2}F_{j+1}\Bigg(\frac{2\,c-\mathbb{i}\,a\,(m-2\,k)-b\,(u-2\,i)}{2\,c},\,\ldots,\,\frac{2\,c-\mathbb{i}\,a\,(m-2\,k)-b\,(u-2\,i)}{2\,c},\,1;$$

$$\frac{2\,c-\mathbb{i}\,a\,(m-2\,k)-b\,(u-2\,i)}{2\,c}+1,\,\ldots,\,\frac{2\,c-\mathbb{i}\,a\,(m-2\,k)-b\,(u-2\,i)}{2\,c}+1;\,-e^{2\,c\,z}\Bigg)-$$

$$e^{(-\mathbb{i}\,a\,(m-2\,k)-b\,(u-2\,i))\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(-\mathbb{i}\,a\,(m-2\,k)-b\,(u-2\,i))^{-j-1}\,z^{n-j}}{(n-j)\,!}$$

$${}_{j+2}F_{j+1}\Bigg(\frac{-\mathbb{i}\,a\,(m-2\,k)-b\,(u-2\,i)}{2\,c},\,\ldots,\,\frac{-\mathbb{i}\,a\,(m-2\,k)-b\,(u-2\,i)}{2\,c},\,1;$$

$$\frac{-\mathbb{i}\,a\,(m-2\,k)-b\,(u-2\,i)}{2\,c}+1,\,\ldots,\,\frac{-\mathbb{i}\,a\,(m-2\,k)-b\,(u-2\,i)}{2\,c}+1;\,-e^{2\,c\,z}\Bigg)+$$

$$e^{(2\,c+a\,\mathbb{i}\,(m-2\,k)-b\,(u-2\,i))\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(2\,c+a\,\mathbb{i}\,(m-2\,k)-b\,(u-2\,i))^{-j-1}\,z^{n-j}}{(n-j)\,!}$$

$${}_{j+2}F_{j+1}\Bigg(\frac{2\,c+a\,\mathbb{i}\,(m-2\,k)-b\,(u-2\,i)}{2\,c},\,\ldots,\,\frac{2\,c+a\,\mathbb{i}\,(m-2\,k)-b\,(u-2\,i)}{2\,c},\,1;$$

$$\frac{2\,c+a\,\mathbb{i}\,(m-2\,k)-b\,(u-2\,i)}{2\,c}+1,\,\ldots,\,\frac{2\,c+a\,\mathbb{i}\,(m-2\,k)-b\,(u-2\,i)}{2\,c}+1;\,-e^{2\,c\,z}\Bigg)-$$

$$e^{(i a (m-2k)-b(u-2i))z} \sum_{j=0}^{n} \frac{(-1)^j (i a (m-2k)-b(u-2i))^{-j-1} z^{n-j}}{(n-j)!}$$
$$ {}_{j+2}F_{j+1}\left(\frac{i a (m-2k)-b(u-2i)}{2c}, \ldots, \frac{i a (m-2k)-b(u-2i)}{2c}, 1; \frac{i a (m-2k)-b(u-2i)}{2c}+1, \right.$$
$$\left.\left.\left. \ldots, \frac{i a (m-2k)-b(u-2i)}{2c}+1; -e^{2cz}\right)\right)\right) /; n \in \mathbb{N} \bigwedge m \in \mathbb{N}^+ \bigwedge u \in \mathbb{N}^+$$

Involving sin, cosh and power

Involving $z^n \sin(a z) \cosh(b z) \tanh(c z)$

01.21.21.0196.01

$$\int z^n \sin(a z) \cosh(b z) \tanh(c z)\, dz ==$$
$$\frac{1}{4} i n! \left(-e^{(b-i a)z} \sum_{j=0}^{n} \frac{(-1)^j (b-i a)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{b-i a}{2c}, \ldots, \frac{b-i a}{2c}, 1; \frac{b-i a}{2c}+1, \ldots, \frac{b-i a}{2c}+1; -e^{2cz}\right) + \right.$$
$$e^{(b+2c-i a)z} \sum_{j=0}^{n} \frac{(-1)^j (b+2c-i a)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{b+2c-i a}{2c}, \ldots, \frac{b+2c-i a}{2c}, 1; \right.$$
$$\left.\frac{b+2c-i a}{2c}+1, \ldots, \frac{b+2c-i a}{2c}+1; -e^{2cz}\right) - e^{(b+2c+i a)z} \sum_{j=0}^{n} \frac{(-1)^j (b+2c+i a)^{-j-1} z^{n-j}}{(n-j)!}$$
$${}_{j+2}F_{j+1}\left(\frac{b+2c+i a}{2c}, \ldots, \frac{b+2c+i a}{2c}, 1; \frac{b+2c+i a}{2c}+1, \ldots, \frac{b+2c+i a}{2c}+1; -e^{2cz}\right) +$$
$$e^{(b+i a)z} \sum_{j=0}^{n} \frac{(-1)^j (b+i a)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{b+i a}{2c}, \ldots, \frac{b+i a}{2c}, 1; \frac{b+i a}{2c}+1, \ldots, \frac{b+i a}{2c}+1; -e^{2cz}\right) -$$
$$e^{(-b-i a)z} \sum_{j=0}^{n} \frac{(-1)^j (-b-i a)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{-b-i a}{2c}, \ldots, \frac{-b-i a}{2c}, 1; \frac{-b-i a}{2c}+1, \ldots, \frac{-b-i a}{2c}+1; -e^{2cz}\right) +$$
$$e^{(-b+2c-i a)z} \sum_{j=0}^{n} \frac{(-1)^j (-b+2c-i a)^{-j-1} z^{n-j}}{(n-j)!}$$
$${}_{j+2}F_{j+1}\left(\frac{-b+2c-i a}{2c}, \ldots, \frac{-b+2c-i a}{2c}, 1; \frac{-b+2c-i a}{2c}+1, \ldots, \frac{-b+2c-i a}{2c}+1; -e^{2cz}\right) +$$
$$e^{(i a-b)z} \sum_{j=0}^{n} \frac{(-1)^j (i a-b)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{i a-b}{2c}, \ldots, \frac{i a-b}{2c}, 1; \frac{i a-b}{2c}+1, \ldots, \frac{i a-b}{2c}+1; -e^{2cz}\right) -$$
$$e^{(-b+2c+i a)z} \sum_{j=0}^{n} \frac{(-1)^j (-b+2c+i a)^{-j-1} z^{n-j}}{(n-j)!}$$
$$\left.{}_{j+2}F_{j+1}\left(\frac{-b+2c+i a}{2c}, \ldots, \frac{-b+2c+i a}{2c}, 1; \frac{-b+2c+i a}{2c}+1, \ldots, \frac{-b+2c+i a}{2c}+1; -e^{2cz}\right)\right) /; n \in \mathbb{N}$$

Involving powers of sin, powers of cosh and power

Involving $z^n \sin^m(a z) \cosh^u(c z) \tanh(c z)$

01.21.21.0197.01

$$\int z^n \sin^m(a z) \cosh^u(b z) \tanh(c z)\, dz = 2^{-m-u} \binom{m}{\frac{m}{2}} \binom{u}{\frac{u}{2}} n!\, (1 - m \bmod 2)\, (1 - u \bmod 2)$$

$$\left(-\frac{z^{n+1}}{(n+1)!} + 2\, e^{2 c z} \sum_{j=0}^{n} \frac{(-1)^j\, 2^{-j-1}\, c^{-j-1}\, z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}\left(1, \ldots, 1, 1; 2, \ldots, 2; -e^{2 c z}\right)\right) + 2^{-m-u} \binom{u}{\frac{u}{2}} n!\, (1 - u \bmod 2)$$

$$\sum_{k=0}^{\left\lfloor \frac{m-1}{2} \right\rfloor} (-1)^k \binom{m}{k} \left(e^{\frac{i m \pi}{2}} \left(e^{(2 c - i a (m - 2 k)) z} \sum_{j=0}^{n} \frac{(-1)^j\, (2 c - i a (m - 2 k))^{-j-1}\, z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}\left(\frac{2 c - i a (m - 2 k)}{2 c}, \ldots,\right.\right.\right.$$

$$\left.\frac{2 c - i a (m - 2 k)}{2 c}, 1; \frac{2 c - i a (m - 2 k)}{2 c} + 1, \ldots, \frac{2 c - i a (m - 2 k)}{2 c} + 1; -e^{2 c z}\right) -$$

$$e^{-i a (m - 2 k) z} \sum_{j=0}^{n} \frac{(-1)^j\, (-i a (m - 2 k))^{-j-1}\, z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}\left(\frac{-i a (m - 2 k)}{2 c}, \ldots, \frac{-i a (m - 2 k)}{2 c},\right.$$

$$\left.\left. 1; \frac{-i a (m - 2 k)}{2 c} + 1, \ldots, \frac{-i a (m - 2 k)}{2 c} + 1; -e^{2 c z}\right)\right) +$$

$$e^{-\frac{1}{2} i m \pi} \left(e^{(2 c + a i (m - 2 k)) z} \sum_{j=0}^{n} \frac{(-1)^j\, (2 c + a i (m - 2 k))^{-j-1}\, z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}\left(\frac{2 c + a i (m - 2 k)}{2 c}, \ldots,\right.\right.$$

$$\left.\frac{2 c + a i (m - 2 k)}{2 c}, 1; \frac{2 c + a i (m - 2 k)}{2 c} + 1, \ldots, \frac{2 c + a i (m - 2 k)}{2 c} + 1; -e^{2 c z}\right) -$$

$$e^{i a (m - 2 k) z} \sum_{j=0}^{n} \frac{(-1)^j\, (i a (m - 2 k))^{-j-1}\, z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}\left(\frac{i a (m - 2 k)}{2 c}, \ldots, \frac{i a (m - 2 k)}{2 c},\right.$$

$$\left.\left.\left. 1; \frac{i a (m - 2 k)}{2 c} + 1, \ldots, \frac{i a (m - 2 k)}{2 c} + 1; -e^{2 c z}\right)\right)\right) +$$

$$2^{-m-u} \binom{m}{\frac{m}{2}} n!\, (1 - m \bmod 2) \sum_{k=0}^{\left\lfloor \frac{u-1}{2} \right\rfloor} \binom{u}{k} \left(-e^{b (u - 2 k) z} \sum_{j=0}^{n} \frac{(-1)^j\, (b (u - 2 k))^{-j-1}\, z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}\right.$$

$$\left(\frac{b (u - 2 k)}{2 c}, \ldots, \frac{b (u - 2 k)}{2 c}, 1; \frac{b (u - 2 k)}{2 c} + 1, \ldots, \frac{b (u - 2 k)}{2 c} + 1; -e^{2 c z}\right) +$$

$$e^{(2 c + b (u - 2 k)) z} \sum_{j=0}^{n} \frac{(-1)^j\, (2 c + b (u - 2 k))^{-j-1}\, z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}\left(\frac{2 c + b (u - 2 k)}{2 c}, \ldots, \frac{2 c + b (u - 2 k)}{2 c},\right.$$

$$\left. 1; \frac{2 c + b (u - 2 k)}{2 c} + 1, \ldots, \frac{2 c + b (u - 2 k)}{2 c} + 1; -e^{2 c z}\right) +$$

$$e^{(2 c - b (u - 2 k)) z} \sum_{j=0}^{n} \frac{(-1)^j\, (2 c - b (u - 2 k))^{-j-1}\, z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}\left(\frac{2 c - b (u - 2 k)}{2 c}, \ldots, \frac{2 c - b (u - 2 k)}{2 c}, 1;\right.$$

$$\frac{2c-b(u-2k)}{2c}+1,\ \ldots,\ \frac{2c-b(u-2k)}{2c}+1;\ -e^{2cz}\Bigg)-e^{-b(u-2k)z}\sum_{j=0}^{n}\frac{(-1)^j(-b(u-2k))^{-j-1}z^{n-j}}{(n-j)!}$$

$$\left.\left.{}_{j+2}F_{j+1}\left(\frac{-b(u-2k)}{2c},\ \ldots,\ \frac{-b(u-2k)}{2c},\ 1;\ \frac{-b(u-2k)}{2c}+1,\ \ldots,\ \frac{-b(u-2k)}{2c}+1;\ -e^{2cz}\right)\right)\right)+$$

$$2^{-m-u}\,n!\sum_{k=0}^{\left\lfloor\frac{m-1}{2}\right\rfloor}(-1)^k\binom{m}{k}\sum_{i=0}^{\left\lfloor\frac{u-1}{2}\right\rfloor}\binom{u}{i}\Bigg(e^{\frac{i m\pi}{2}}\Bigg(e^{(2c-i a(m-2k)+b(u-2i))z}\sum_{j=0}^{n}\frac{(-1)^j(2c-i a(m-2k)+b(u-2i))^{-j-1}z^{n-j}}{(n-j)!}$$

$${}_{j+2}F_{j+1}\Bigg(\frac{2c-i a(m-2k)+b(u-2i)}{2c},\ \ldots,\ \frac{2c-i a(m-2k)+b(u-2i)}{2c},\ 1;$$

$$\frac{2c-i a(m-2k)+b(u-2i)}{2c}+1,\ \ldots,\ \frac{2c-i a(m-2k)+b(u-2i)}{2c}+1;\ -e^{2cz}\Bigg)-$$

$$e^{(b(u-2i)-i a(m-2k))z}\sum_{j=0}^{n}\frac{(-1)^j(b(u-2i)-i a(m-2k))^{-j-1}z^{n-j}}{(n-j)!}$$

$${}_{j+2}F_{j+1}\Bigg(\frac{b(u-2i)-i a(m-2k)}{2c},\ \ldots,\ \frac{b(u-2i)-i a(m-2k)}{2c},\ 1;$$

$$\left.\left.\frac{b(u-2i)-i a(m-2k)}{2c}+1,\ \ldots,\ \frac{b(u-2i)-i a(m-2k)}{2c}+1;\ -e^{2cz}\right)\right)+$$

$$e^{-\frac{1}{2}i m\pi}\Bigg(e^{(2c+a i(m-2k)+b(u-2i))z}\sum_{j=0}^{n}\frac{(-1)^j(2c+a i(m-2k)+b(u-2i))^{-j-1}z^{n-j}}{(n-j)!}$$

$${}_{j+2}F_{j+1}\Bigg(\frac{2c+a i(m-2k)+b(u-2i)}{2c},\ \ldots,\ \frac{2c+a i(m-2k)+b(u-2i)}{2c},\ 1;$$

$$\frac{2c+a i(m-2k)+b(u-2i)}{2c}+1,\ \ldots,\ \frac{2c+a i(m-2k)+b(u-2i)}{2c}+1;\ -e^{2cz}\Bigg)-$$

$$e^{(a i(m-2k)+b(u-2i))z}\sum_{j=0}^{n}\frac{(-1)^j(a i(m-2k)+b(u-2i))^{-j-1}z^{n-j}}{(n-j)!}$$

$${}_{j+2}F_{j+1}\Bigg(\frac{a i(m-2k)+b(u-2i)}{2c},\ \ldots,\ \frac{a i(m-2k)+b(u-2i)}{2c},\ 1;$$

$$\left.\left.\frac{a i(m-2k)+b(u-2i)}{2c}+1,\ \ldots,\ \frac{a i(m-2k)+b(u-2i)}{2c}+1;\ -e^{2cz}\right)\right)+$$

$$e^{\frac{i m\pi}{2}}\Bigg(e^{(2c-i a(m-2k)-b(u-2i))z}\sum_{j=0}^{n}\frac{(-1)^j(2c-i a(m-2k)-b(u-2i))^{-j-1}z^{n-j}}{(n-j)!}$$

$${}_{j+2}F_{j+1}\Bigg(\frac{2c-i a(m-2k)-b(u-2i)}{2c},\ \ldots,\ \frac{2c-i a(m-2k)-b(u-2i)}{2c},\ 1;$$

$$\frac{2c-i a(m-2k)-b(u-2i)}{2c}+1,\ \ldots,\ \frac{2c-i a(m-2k)-b(u-2i)}{2c}+1;\ -e^{2cz}\Bigg)-$$

$$e^{(-i a (m-2k)-b(u-2i)) z} \sum_{j=0}^{n} \frac{(-1)^j (-i a (m-2k) - b(u-2i))^{-j-1} z^{n-j}}{(n-j)!}$$

$$ {}_{j+2}F_{j+1}\left(\frac{-i a (m-2k) - b(u-2i)}{2c}, \ldots, \frac{-i a (m-2k) - b(u-2i)}{2c}, 1; \right.$$

$$\left.\left. \frac{-i a (m-2k) - b(u-2i)}{2c} + 1, \ldots, \frac{-i a (m-2k) - b(u-2i)}{2c} + 1; -e^{2cz}\right)\right) +$$

$$e^{-\frac{1}{2} i m \pi} \left(e^{(2c + a i (m-2k) - b(u-2i)) z} \sum_{j=0}^{n} \frac{(-1)^j (2c + a i (m-2k) - b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} \right.$$

$$ {}_{j+2}F_{j+1}\left(\frac{2c + a i (m-2k) - b(u-2i)}{2c}, \ldots, \frac{2c + a i (m-2k) - b(u-2i)}{2c}, 1; \right.$$

$$\left. \frac{2c + a i (m-2k) - b(u-2i)}{2c} + 1, \ldots, \frac{2c + a i (m-2k) - b(u-2i)}{2c} + 1; -e^{2cz}\right) -$$

$$e^{(i a (m-2k) - b(u-2i)) z} \sum_{j=0}^{n} \frac{(-1)^j (i a (m-2k) - b(u-2i))^{-j-1} z^{n-j}}{(n-j)!}$$

$$ {}_{j+2}F_{j+1}\left(\frac{i a (m-2k) - b(u-2i)}{2c}, \ldots, \frac{i a (m-2k) - b(u-2i)}{2c}, 1; \frac{i a (m-2k) - b(u-2i)}{2c} + 1, \right.$$

$$\left.\left.\left. \ldots, \frac{i a (m-2k) - b(u-2i)}{2c} + 1; -e^{2cz}\right)\right)\right) /; n \in \mathbb{N} \land m \in \mathbb{N}^+ \land u \in \mathbb{N}^+$$

Involving cos, cosh and power

Involving $z^n \cos(a z) \cosh(b z) \tanh(c z)$

01.21.21.0198.01

$$\int z^n \cos(a z) \cosh(b z) \tanh(c z)\, dz = \frac{1}{4} n! \Bigg(-e^{(b-i a) z} \sum_{j=0}^{n} \frac{(-1)^j (b - i a)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{b-i a}{2 c}, \ldots, \frac{b-i a}{2 c}, 1; \frac{b-i a}{2 c}+1, \ldots, \frac{b-i a}{2 c}+1; -e^{2 c z}\right) + e^{(b+2 c-i a) z} \sum_{j=0}^{n} \frac{(-1)^j (b+2 c-i a)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{b+2 c-i a}{2 c}, \ldots, \frac{b+2 c-i a}{2 c}, 1; \frac{b+2 c-i a}{2 c}+1, \ldots, \frac{b+2 c-i a}{2 c}+1; -e^{2 c z}\right) + e^{(b+2 c+i a) z} \sum_{j=0}^{n} \frac{(-1)^j (b+2 c+i a)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{b+2 c+i a}{2 c}, \ldots, \frac{b+2 c+i a}{2 c}, 1; \frac{b+2 c+i a}{2 c}+1, \ldots, \frac{b+2 c+i a}{2 c}+1; -e^{2 c z}\right) - e^{(b+i a) z} \sum_{j=0}^{n} \frac{(-1)^j (b+i a)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{b+i a}{2 c}, \ldots, \frac{b+i a}{2 c}, 1; \frac{b+i a}{2 c}+1, \ldots, \frac{b+i a}{2 c}+1; -e^{2 c z}\right) - e^{(-b-i a) z} \sum_{j=0}^{n} \frac{(-1)^j (-b-i a)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{-b-i a}{2 c}, \ldots, \frac{-b-i a}{2 c}, 1; \frac{-b-i a}{2 c}+1, \ldots, \frac{-b-i a}{2 c}+1; -e^{2 c z}\right) + e^{(-b+2 c-i a) z} \sum_{j=0}^{n} \frac{(-1)^j (-b+2 c-i a)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{-b+2 c-i a}{2 c}, \ldots, \frac{-b+2 c-i a}{2 c}, 1; \frac{-b+2 c-i a}{2 c}+1, \ldots, \frac{-b+2 c-i a}{2 c}+1; -e^{2 c z}\right) - e^{(i a-b) z} \sum_{j=0}^{n} \frac{(-1)^j (i a-b)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{i a-b}{2 c}, \ldots, \frac{i a-b}{2 c}, 1; \frac{i a-b}{2 c}+1, \ldots, \frac{i a-b}{2 c}+1; -e^{2 c z}\right) + e^{(-b+2 c+i a) z} \sum_{j=0}^{n} \frac{(-1)^j (-b+2 c+i a)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{-b+2 c+i a}{2 c}, \ldots, \frac{-b+2 c+i a}{2 c}, 1; \frac{-b+2 c+i a}{2 c}+1, \ldots, \frac{-b+2 c+i a}{2 c}+1; -e^{2 c z}\right)\Bigg) /; n \in \mathbb{N}$$

Involving powers of cos, powers of cosh and power

Involving $z^n \cos^m(a z) \cosh^u(c z) \tanh^v(c z)$

01.21.21.0199.01

$$\int z^n \cos^m(a z) \cosh^u(b z) \tanh(c z)\, dz = 2^{-m-u} \binom{m}{\frac{m}{2}} \binom{u}{\frac{u}{2}} n! (1 - m \bmod 2) (1 - u \bmod 2) \left(-\frac{z^{n+1}}{(n+1)!} + 2 e^{2 c z} \sum_{j=0}^{n} \frac{(-1)^j 2^{-j-1} c^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(1, \ldots, 1, 1; 2, \ldots, 2; -e^{2 c z}\right)\right) +$$

$$2^{-m-u}\binom{u}{\frac{u}{2}}n!\,(1-u \bmod 2)\sum_{k=0}^{\left\lfloor\frac{m-1}{2}\right\rfloor}\binom{m}{k}\Bigg(e^{(2c-i a(m-2k))z}\sum_{j=0}^{n}\frac{(-1)^j\,(2c-i a(m-2k))^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\Bigg(\frac{2c-i a(m-2k)}{2c},\,\ldots,\,\frac{2c-i a(m-2k)}{2c},\,1;\,\frac{2c-i a(m-2k)}{2c}+1,\,\ldots,\,\frac{2c-i a(m-2k)}{2c}+1;\,-e^{2cz}\Bigg)-$$

$$e^{-i a(m-2k)z}\sum_{j=0}^{n}\frac{(-1)^j\,(-i a(m-2k))^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\Bigg(\frac{-i a(m-2k)}{2c},\,\ldots,\,\frac{-i a(m-2k)}{2c},\,1;\,\frac{-i a(m-2k)}{2c}+1,\,\ldots,\,\frac{-i a(m-2k)}{2c}+1;\,-e^{2cz}\Bigg)+$$

$$e^{(2c+a i(m-2k))z}\sum_{j=0}^{n}\frac{(-1)^j\,(2c+a i(m-2k))^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\Bigg(\frac{2c+a i(m-2k)}{2c},\,\ldots,\,\frac{2c+a i(m-2k)}{2c},\,1;\,\frac{2c+a i(m-2k)}{2c}+1,\,\ldots,\,\frac{2c+a i(m-2k)}{2c}+1;\,-e^{2cz}\Bigg)-e^{i a(m-2k)z}\sum_{j=0}^{n}\frac{(-1)^j\,(i a(m-2k))^{-j-1}\,z^{n-j}}{(n-j)!}$$

$${}_{j+2}F_{j+1}\Bigg(\frac{i a(m-2k)}{2c},\,\ldots,\,\frac{i a(m-2k)}{2c},\,1;\,\frac{i a(m-2k)}{2c}+1,\,\ldots,\,\frac{i a(m-2k)}{2c}+1;\,-e^{2cz}\Bigg)\Bigg)+$$

$$2^{-m-u}\binom{m}{\frac{m}{2}}n!\,(1-m \bmod 2)\sum_{k=0}^{\left\lfloor\frac{u-1}{2}\right\rfloor}\binom{u}{k}\Bigg(-e^{b(u-2k)z}\sum_{j=0}^{n}\frac{(-1)^j\,(b(u-2k))^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\Bigg(\frac{b(u-2k)}{2c},\,\ldots,\,\frac{b(u-2k)}{2c},\,1;\,\frac{b(u-2k)}{2c}+1,\,\ldots,\,\frac{b(u-2k)}{2c}+1;\,-e^{2cz}\Bigg)+e^{(2c+b(u-2k))z}\sum_{j=0}^{n}\frac{(-1)^j\,(2c+b(u-2k))^{-j-1}\,z^{n-j}}{(n-j)!}$$

$${}_{j+2}F_{j+1}\Bigg(\frac{2c+b(u-2k)}{2c},\,\ldots,\,\frac{2c+b(u-2k)}{2c},\,1;\,\frac{2c+b(u-2k)}{2c}+1,\,\ldots,\,\frac{2c+b(u-2k)}{2c}+1;\,-e^{2cz}\Bigg)+$$

$$e^{(2c-b(u-2k))z}\sum_{j=0}^{n}\frac{(-1)^j\,(2c-b(u-2k))^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\Bigg(\frac{2c-b(u-2k)}{2c},\,\ldots,\,\frac{2c-b(u-2k)}{2c},\,1;\,\frac{2c-b(u-2k)}{2c}+1,\,\ldots,\,\frac{2c-b(u-2k)}{2c}+1;\,-e^{2cz}\Bigg)-e^{-b(u-2k)z}\sum_{j=0}^{n}\frac{(-1)^j\,(-b(u-2k))^{-j-1}\,z^{n-j}}{(n-j)!}$$

$${}_{j+2}F_{j+1}\Bigg(\frac{-b(u-2k)}{2c},\,\ldots,\,\frac{-b(u-2k)}{2c},\,1;\,\frac{-b(u-2k)}{2c}+1,\,\ldots,\,\frac{-b(u-2k)}{2c}+1;\,-e^{2cz}\Bigg)\Bigg)+$$

$$2^{-m-u}\,n!\sum_{k=0}^{\left\lfloor\frac{m-1}{2}\right\rfloor}\binom{m}{k}\sum_{i=0}^{\left\lfloor\frac{u-1}{2}\right\rfloor}\binom{u}{i}\Bigg(e^{(2c-i a(m-2k)+b(u-2i))z}\sum_{j=0}^{n}\frac{(-1)^j\,(2c-i a(m-2k)+b(u-2i))^{-j-1}\,z^{n-j}}{(n-j)!}$$

$${}_{j+2}F_{j+1}\Bigg(\frac{2c-i a(m-2k)+b(u-2i)}{2c},\,\ldots,\,\frac{2c-i a(m-2k)+b(u-2i)}{2c},\,1;\,\frac{2c-i a(m-2k)+b(u-2i)}{2c}+1,\,\ldots,\,\frac{2c-i a(m-2k)+b(u-2i)}{2c}+1;\,-e^{2cz}\Bigg)-$$

$$e^{(b(u-2i)-i a(m-2k))z}\sum_{j=0}^{n}\frac{(-1)^j (b(u-2i)-i a(m-2k))^{-j-1} z^{n-j}}{(n-j)!}$$
$${}_{j+2}F_{j+1}\left(\frac{b(u-2i)-i a(m-2k)}{2c}, \ldots, \frac{b(u-2i)-i a(m-2k)}{2c}, 1; \frac{b(u-2i)-i a(m-2k)}{2c}+1, \ldots, \frac{b(u-2i)-i a(m-2k)}{2c}+1; -e^{2cz}\right)+$$
$$e^{(2c+a i(m-2k)+b(u-2i))z}\sum_{j=0}^{n}\frac{(-1)^j (2c+a i(m-2k)+b(u-2i))^{-j-1} z^{n-j}}{(n-j)!}$$
$${}_{j+2}F_{j+1}\left(\frac{2c+a i(m-2k)+b(u-2i)}{2c}, \ldots, \frac{2c+a i(m-2k)+b(u-2i)}{2c}, 1; \frac{2c+a i(m-2k)+b(u-2i)}{2c}+1, \ldots, \frac{2c+a i(m-2k)+b(u-2i)}{2c}+1; -e^{2cz}\right)-$$
$$e^{(a i(m-2k)+b(u-2i))z}\sum_{j=0}^{n}\frac{(-1)^j (a i(m-2k)+b(u-2i))^{-j-1} z^{n-j}}{(n-j)!}$$
$${}_{j+2}F_{j+1}\left(\frac{a i(m-2k)+b(u-2i)}{2c}, \ldots, \frac{a i(m-2k)+b(u-2i)}{2c}, 1; \frac{a i(m-2k)+b(u-2i)}{2c}+1, \ldots, \frac{a i(m-2k)+b(u-2i)}{2c}+1; -e^{2cz}\right)+$$
$$e^{(2c-i a(m-2k)-b(u-2i))z}\sum_{j=0}^{n}\frac{(-1)^j (2c-i a(m-2k)-b(u-2i))^{-j-1} z^{n-j}}{(n-j)!}$$
$${}_{j+2}F_{j+1}\left(\frac{2c-i a(m-2k)-b(u-2i)}{2c}, \ldots, \frac{2c-i a(m-2k)-b(u-2i)}{2c}, 1; \frac{2c-i a(m-2k)-b(u-2i)}{2c}+1, \ldots, \frac{2c-i a(m-2k)-b(u-2i)}{2c}+1; -e^{2cz}\right)-$$
$$e^{(-i a(m-2k)-b(u-2i))z}\sum_{j=0}^{n}\frac{(-1)^j (-i a(m-2k)-b(u-2i))^{-j-1} z^{n-j}}{(n-j)!}$$
$${}_{j+2}F_{j+1}\left(\frac{-i a(m-2k)-b(u-2i)}{2c}, \ldots, \frac{-i a(m-2k)-b(u-2i)}{2c}, 1; \frac{-i a(m-2k)-b(u-2i)}{2c}+1, \ldots, \frac{-i a(m-2k)-b(u-2i)}{2c}+1; -e^{2cz}\right)+$$
$$e^{(2c+a i(m-2k)-b(u-2i))z}\sum_{j=0}^{n}\frac{(-1)^j (2c+a i(m-2k)-b(u-2i))^{-j-1} z^{n-j}}{(n-j)!}$$
$${}_{j+2}F_{j+1}\left(\frac{2c+a i(m-2k)-b(u-2i)}{2c}, \ldots, \frac{2c+a i(m-2k)-b(u-2i)}{2c}, 1; \frac{2c+a i(m-2k)-b(u-2i)}{2c}+1, \ldots, \frac{2c+a i(m-2k)-b(u-2i)}{2c}+1; -e^{2cz}\right)-$$

$$e^{(i a (m-2 k)-b (u-2 i)) z} \sum_{j=0}^{n} \frac{(-1)^j (i a (m-2 k)-b (u-2 i))^{-j-1} z^{n-j}}{(n-j)!}$$

$$ {}_{j+2}F_{j+1}\left(\frac{i a (m-2 k)-b (u-2 i)}{2 c}, \ldots, \frac{i a (m-2 k)-b (u-2 i)}{2 c}, 1; \frac{i a (m-2 k)-b (u-2 i)}{2 c} + 1, \ldots, \frac{i a (m-2 k)-b (u-2 i)}{2 c} + 1; -e^{2 c z}\right)\Bigg) /; n \in \mathbb{N} \bigwedge m \in \mathbb{N}^+ \bigwedge u \in \mathbb{N}^+$$

Involving hyperbolic, exponential, trigonometric and a power functions

Involving sin, sinh, exp and power

Involving $z^n e^{p z} \sin(a z) \sinh(b z) \tanh(c z)$

01.21.21.0200.01

$$\int z^n e^{p z} \sin(a z) \sinh(b z) \tanh(c z) \, dz = \frac{1}{4} i\, n! \left(e^{(-b-i a+p) z} \sum_{j=0}^{n} \frac{(-1)^j (-b - i a + p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{-i a - b + p}{2 c}, \ldots, \frac{-i a - b + p}{2 c}, 1; \frac{-i a - b + p}{2 c} + 1, \ldots, \frac{-i a - b + p}{2 c} + 1; -e^{2 c z} \right) - e^{(-b+i a+p) z} \sum_{j=0}^{n} \frac{(-1)^j (-b + i a + p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{i a - b + p}{2 c}, \ldots, \frac{i a - b + p}{2 c}, 1; \frac{i a - b + p}{2 c} + 1, \ldots, \frac{i a - b + p}{2 c} + 1; -e^{2 c z} \right) - e^{(b-i a+p) z} \sum_{j=0}^{n} \frac{(-1)^j (b - i a + p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{-i a + b + p}{2 c}, \ldots, \frac{-i a + b + p}{2 c}, 1; \frac{-i a + b + p}{2 c} + 1, \ldots, \frac{-i a + b + p}{2 c} + 1; -e^{2 c z} \right) + e^{(b+i a+p) z} \sum_{j=0}^{n} \frac{(-1)^j (b + i a + p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{i a + b + p}{2 c}, \ldots, \frac{i a + b + p}{2 c}, 1; \frac{i a + b + p}{2 c} + 1, \ldots, \frac{i a + b + p}{2 c} + 1; -e^{2 c z} \right) - e^{(-b+2 c-i a+p) z} \sum_{j=0}^{n} \frac{(-1)^j (-b + 2 c - i a + p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{-i a - b + 2 c + p}{2 c}, \ldots, \frac{-i a - b + 2 c + p}{2 c}, 1; \frac{-i a - b + 2 c + p}{2 c} + 1, \ldots, \frac{-i a - b + 2 c + p}{2 c} + 1; -e^{2 c z} \right) + e^{(-b+2 c+i a+p) z} \sum_{j=0}^{n} \frac{(-1)^j (-b + 2 c + i a + p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{i a - b + 2 c + p}{2 c}, \ldots, \frac{i a - b + 2 c + p}{2 c}, 1; \frac{i a - b + 2 c + p}{2 c} + 1, \ldots, \frac{i a - b + 2 c + p}{2 c} + 1; -e^{2 c z} \right) + e^{(b+2 c-i a+p) z} \sum_{j=0}^{n} \frac{(-1)^j (b + 2 c - i a + p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{-i a + b + 2 c + p}{2 c}, \ldots, \frac{-i a + b + 2 c + p}{2 c}, 1; \frac{-i a + b + 2 c + p}{2 c} + 1, \ldots, \frac{-i a + b + 2 c + p}{2 c} + 1; -e^{2 c z} \right) - e^{(b+2 c+i a+p) z} \sum_{j=0}^{n} \frac{(-1)^j (b + 2 c + i a + p)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{i a + b + 2 c + p}{2 c}, \ldots, \frac{i a + b + 2 c + p}{2 c}, 1; \frac{i a + b + 2 c + p}{2 c} + 1, \ldots, \frac{i a + b + 2 c + p}{2 c} + 1; -e^{2 c z} \right) \right) /; n \in \mathbb{N}$$

Involving powers of sin, powers of sinh, exp and power

Involving $z^n e^{p z} \sin^m(a z) \sinh^u(b z) \tanh(c z)$

01.21.21.0201.01

$$\int z^n e^{pz} \sin^m(a z) \sinh^u(b z) \tanh(c z)\, dz = i^u 2^{-m-u} \binom{m}{\frac{m}{2}} \binom{u}{\frac{u}{2}} n!\, (1 - m \bmod 2)\, (1 - u \bmod 2)$$

$$\left(e^{(2c+p)z} \sum_{j=0}^{n} \frac{(-1)^j (2c+p)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{2c+p}{2c}, \ldots, \frac{2c+p}{2c}, 1; \frac{2c+p}{2c}+1, \ldots, \frac{2c+p}{2c}+1; -e^{2cz}\right) - \right.$$

$$\left. e^{pz} \sum_{j=0}^{n} \frac{(-1)^j p^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{p}{2c}, \ldots, \frac{p}{2c}, 1; \frac{p}{2c}+1, \ldots, \frac{p}{2c}+1; -e^{2cz}\right)\right) +$$

$$i^u 2^{-m-u} \binom{u}{\frac{u}{2}} n!\, (1 - u \bmod 2) \sum_{k=0}^{\left\lfloor \frac{m-1}{2} \right\rfloor} (-1)^k \binom{m}{k} \left(e^{\frac{i m \pi}{2}} \left(e^{(2c - i a (m-2k) + p) z} \sum_{j=0}^{n} \frac{(-1)^j (2c - i a (m-2k) + p)^{-j-1} z^{n-j}}{(n-j)!} \right.\right.$$

$${}_{j+2}F_{j+1}\left(\frac{2c - i a (m-2k) + p}{2c}, \ldots, \frac{2c - i a (m-2k) + p}{2c}, 1; \right.$$

$$\left. \frac{2c - i a (m-2k) + p}{2c} + 1, \ldots, \frac{2c - i a (m-2k) + p}{2c} + 1; -e^{2cz}\right) -$$

$$e^{(p - i a (m-2k)) z} \sum_{j=0}^{n} \frac{(-1)^j (p - i a (m-2k))^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{p - i a (m-2k)}{2c}, \ldots, \right.$$

$$\left.\left. \frac{p - i a (m-2k)}{2c}, 1; \frac{p - i a (m-2k)}{2c} + 1, \ldots, \frac{p - i a (m-2k)}{2c} + 1; -e^{2cz}\right)\right) +$$

$$e^{-\frac{1}{2} i m \pi} \left(e^{(2c + a i (m-2k) + p) z} \sum_{j=0}^{n} \frac{(-1)^j (2c + a i (m-2k) + p)^{-j-1} z^{n-j}}{(n-j)!} \right.$$

$${}_{j+2}F_{j+1}\left(\frac{2c + a i (m-2k) + p}{2c}, \ldots, \frac{2c + a i (m-2k) + p}{2c}, 1; \right.$$

$$\left. \frac{2c + a i (m-2k) + p}{2c} + 1, \ldots, \frac{2c + a i (m-2k) + p}{2c} + 1; -e^{2cz}\right) -$$

$$e^{(a i (m-2k) + p) z} \sum_{j=0}^{n} \frac{(-1)^j (a i (m-2k) + p)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{a i (m-2k) + p}{2c}, \ldots, \right.$$

$$\left.\left.\left. \frac{a i (m-2k) + p}{2c}, 1; \frac{a i (m-2k) + p}{2c} + 1, \ldots, \frac{a i (m-2k) + p}{2c} + 1; -e^{2cz}\right)\right)\right) +$$

$$2^{-m-u} \binom{m}{\frac{m}{2}} n!\, (1 - m \bmod 2) \sum_{k=0}^{\left\lfloor \frac{u-1}{2} \right\rfloor} (-1)^k \binom{u}{k} \left(-e^{(p + b (u-2k)) z} \sum_{j=0}^{n} \frac{(-1)^j (p + b (u-2k))^{-j-1} z^{n-j}}{(n-j)!} \right.$$

$${}_{j+2}F_{j+1}\left(\frac{p + b (u-2k)}{2c}, \ldots, \frac{p + b (u-2k)}{2c}, 1; \frac{p + b (u-2k)}{2c} + 1, \ldots, \frac{p + b (u-2k)}{2c} + 1; -e^{2cz}\right) +$$

$$e^{(2c + p + b (u-2k)) z} \sum_{j=0}^{n} \frac{(-1)^j (2c + p + b (u-2k))^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{2c + p + b (u-2k)}{2c}, \ldots, \right.$$

$$\left. \frac{2c + p + b (u-2k)}{2c}, 1; \frac{2c + p + b (u-2k)}{2c} + 1, \ldots, \frac{2c + p + b (u-2k)}{2c} + 1; -e^{2cz}\right) +$$

$$(-1)^u \left(e^{(2c+p-b(u-2k))z} \sum_{j=0}^{n} \frac{(-1)^j (2c+p-b(u-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{2c+p-b(u-2k)}{2c}, \ldots, \frac{2c+p-b(u-2k)}{2c}, 1; \frac{2c+p-b(u-2k)}{2c}+1, \ldots, \frac{2c+p-b(u-2k)}{2c}+1; -e^{2cz} \right) - e^{(p-b(u-2k))z} \sum_{j=0}^{n} \frac{(-1)^j (p-b(u-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{p-b(u-2k)}{2c}, \ldots, \frac{p-b(u-2k)}{2c}, 1; \frac{p-b(u-2k)}{2c}+1, \ldots, \frac{p-b(u-2k)}{2c}+1; -e^{2cz} \right)\right)\Bigg) + 2^{-m-u} n!$$

$$\sum_{k=0}^{\left\lfloor \frac{m-1}{2} \right\rfloor} (-1)^k \binom{m}{k} \sum_{i=0}^{\left\lfloor \frac{u-1}{2} \right\rfloor} (-1)^i \binom{u}{i} \Bigg(e^{\frac{i m \pi}{2}} \Bigg(e^{(2c-i a(m-2k)+p+b(u-2i))z} \sum_{j=0}^{n} \frac{(-1)^j (2c-i a(m-2k)+p+b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{2c-i a(m-2k)+p+b(u-2i)}{2c}, \ldots, \frac{2c-i a(m-2k)+p+b(u-2i)}{2c}, 1; \frac{2c-i a(m-2k)+p+b(u-2i)}{2c}+1, \ldots, \frac{2c-i a(m-2k)+p+b(u-2i)}{2c}+1; -e^{2cz} \right) - e^{(-i a(m-2k)+p+b(u-2i))z} \sum_{j=0}^{n} \frac{(-1)^j (-i a(m-2k)+p+b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{-i a(m-2k)+p+b(u-2i)}{2c}, \ldots, \frac{-i a(m-2k)+p+b(u-2i)}{2c}, 1; \frac{-i a(m-2k)+p+b(u-2i)}{2c}+1, \ldots, \frac{-i a(m-2k)+p+b(u-2i)}{2c}+1; -e^{2cz} \right)\Bigg) +$$

$$e^{-\frac{1}{2} i m \pi} \Bigg(e^{(2c+a i(m-2k)+p+b(u-2i))z} \sum_{j=0}^{n} \frac{(-1)^j (2c+a i(m-2k)+p+b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{2c+a i(m-2k)+p+b(u-2i)}{2c}, \ldots, \frac{2c+a i(m-2k)+p+b(u-2i)}{2c}, 1; \frac{2c+a i(m-2k)+p+b(u-2i)}{2c}+1, \ldots, \frac{2c+a i(m-2k)+p+b(u-2i)}{2c}+1; -e^{2cz} \right) - e^{(a i(m-2k)+p+b(u-2i))z} \sum_{j=0}^{n} \frac{(-1)^j (a i(m-2k)+p+b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{a i(m-2k)+p+b(u-2i)}{2c}, \ldots, \frac{a i(m-2k)+p+b(u-2i)}{2c}, 1; \frac{a i(m-2k)+p+b(u-2i)}{2c}+1, \ldots, \frac{a i(m-2k)+p+b(u-2i)}{2c}+1; -e^{2cz} \right)\Bigg) +$$

$$(-1)^u e^{\frac{i m \pi}{2}} \Bigg(e^{(2c-i a(m-2k)+p-b(u-2i))z} \sum_{j=0}^{n} \frac{(-1)^j (2c-i a(m-2k)+p-b(u-2i))^{-j-1} z^{n-j}}{(n-j)!}$$

$$ {}_{j+2}F_{j+1}\left(\frac{2c - i\,a(m-2k) + p - b(u-2i)}{2c}, \ldots, \frac{2c - i\,a(m-2k) + p - b(u-2i)}{2c}, 1; \frac{2c - i\,a(m-2k) + p - b(u-2i)}{2c} + 1, \ldots, \frac{2c - i\,a(m-2k) + p - b(u-2i)}{2c} + 1; -e^{2cz}\right) - $$

$$ e^{(-i\,a(m-2k)+p-b(u-2i))z} \sum_{j=0}^{n} \frac{(-1)^j (-i\,a(m-2k) + p - b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} $$

$$ {}_{j+2}F_{j+1}\left(\frac{-i\,a(m-2k) + p - b(u-2i)}{2c}, \ldots, \frac{-i\,a(m-2k) + p - b(u-2i)}{2c}, 1; \frac{-i\,a(m-2k) + p - b(u-2i)}{2c} + 1, \ldots, \frac{-i\,a(m-2k) + p - b(u-2i)}{2c} + 1; -e^{2cz}\right)\Bigg) + $$

$$ (-1)^u e^{-\frac{1}{2} i m \pi} \Bigg(e^{(2c+a\,i(m-2k)+p-b(u-2i))z} \sum_{j=0}^{n} \frac{(-1)^j (2c + a\,i(m-2k) + p - b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} $$

$$ {}_{j+2}F_{j+1}\left(\frac{2c + a\,i(m-2k) + p - b(u-2i)}{2c}, \ldots, \frac{2c + a\,i(m-2k) + p - b(u-2i)}{2c}, 1; \frac{2c + a\,i(m-2k) + p - b(u-2i)}{2c} + 1, \ldots, \frac{2c + a\,i(m-2k) + p - b(u-2i)}{2c} + 1; -e^{2cz}\right) - $$

$$ e^{(a\,i(m-2k)+p-b(u-2i))z} \sum_{j=0}^{n} \frac{(-1)^j (a\,i(m-2k) + p - b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} $$

$$ {}_{j+2}F_{j+1}\left(\frac{a\,i(m-2k) + p - b(u-2i)}{2c}, \ldots, \frac{a\,i(m-2k) + p - b(u-2i)}{2c}, 1; \frac{a\,i(m-2k) + p - b(u-2i)}{2c} + 1, \ldots, \frac{a\,i(m-2k) + p - b(u-2i)}{2c} + 1; -e^{2cz}\right)\Bigg)\Bigg) /; n \in \mathbb{N} \land m \in \mathbb{N}^+ \land u \in \mathbb{N}^+ $$

Involving cos, sinh, exp and power

Involving $z^n e^{pz} \cos(a z) \sinh(b z) \tanh(c z)$

01.21.21.0202.01

$$\int z^n e^{p z} \cos(a z) \sinh(b z) \tanh(c z) \, dz = \frac{1}{4} n! \left(e^{(-b-i a+p) z} \sum_{j=0}^{n} \frac{(-1)^j (-b-i a+p)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{-i a-b+p}{2 c}, \ldots, \frac{-i a-b+p}{2 c}, 1; \frac{-i a-b+p}{2 c}+1, \ldots, \frac{-i a-b+p}{2 c}+1; -e^{2 c z} \right) + e^{(-b+i a+p) z} \sum_{j=0}^{n} \frac{(-1)^j (-b+i a+p)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{i a-b+p}{2 c}, \ldots, \frac{i a-b+p}{2 c}, 1; \frac{i a-b+p}{2 c}+1, \ldots, \frac{i a-b+p}{2 c}+1; -e^{2 c z} \right) - e^{(b-i a+p) z} \sum_{j=0}^{n} \frac{(-1)^j (b-i a+p)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{-i a+b+p}{2 c}, \ldots, \frac{-i a+b+p}{2 c}, 1; \frac{-i a+b+p}{2 c}+1, \ldots, \frac{-i a+b+p}{2 c}+1; -e^{2 c z} \right) - e^{(b+i a+p) z} \sum_{j=0}^{n} \frac{(-1)^j (b+i a+p)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{i a+b+p}{2 c}, \ldots, \frac{i a+b+p}{2 c}, 1; \frac{i a+b+p}{2 c}+1, \ldots, \frac{i a+b+p}{2 c}+1; -e^{2 c z} \right) - e^{(-b+2 c-i a+p) z} \sum_{j=0}^{n} \frac{(-1)^j (-b+2 c-i a+p)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{-i a-b+2 c+p}{2 c}, \ldots, \frac{-i a-b+2 c+p}{2 c}, 1; \frac{-i a-b+2 c+p}{2 c}+1, \ldots, \frac{-i a-b+2 c+p}{2 c}+1; -e^{2 c z} \right) - e^{(-b+2 c+i a+p) z} \sum_{j=0}^{n} \frac{(-1)^j (-b+2 c+i a+p)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{i a-b+2 c+p}{2 c}, \ldots, \frac{i a-b+2 c+p}{2 c}, 1; \frac{i a-b+2 c+p}{2 c}+1, \ldots, \frac{i a-b+2 c+p}{2 c}+1; -e^{2 c z} \right) + e^{(b+2 c-i a+p) z} \sum_{j=0}^{n} \frac{(-1)^j (b+2 c-i a+p)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{-i a+b+2 c+p}{2 c}, \ldots, \frac{-i a+b+2 c+p}{2 c}, 1; \frac{-i a+b+2 c+p}{2 c}+1, \ldots, \frac{-i a+b+2 c+p}{2 c}+1; -e^{2 c z} \right) + e^{(b+2 c+i a+p) z} \sum_{j=0}^{n} \frac{(-1)^j (b+2 c+i a+p)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{i a+b+2 c+p}{2 c}, \ldots, \frac{i a+b+2 c+p}{2 c}, 1; \frac{i a+b+2 c+p}{2 c}+1, \ldots, \frac{i a+b+2 c+p}{2 c}+1; -e^{2 c z} \right) \right) /; n \in \mathbb{N}$$

Involving powers of cos, powers of sinh, exp and power

Involving $z^n e^{p z} \cos^m(a z) \sinh^u(b z) \tanh(c z)$

01.21.21.0203.01

$$\int z^n e^{p z} \cos^m(a z) \sinh^u(b z) \tanh(c z)\, dz = i^u\, 2^{-m-u} \binom{m}{\frac{m}{2}} \binom{u}{\frac{u}{2}} n!\, (1 - m \bmod 2)\, (1 - u \bmod 2)$$

$$\Bigg(e^{(2c+p) z} \sum_{j=0}^{n} \frac{(-1)^j (2c+p)^{-j-1} z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}\Big(\frac{2c+p}{2c}, \ldots, \frac{2c+p}{2c}, 1; \frac{2c+p}{2c}+1, \ldots, \frac{2c+p}{2c}+1; -e^{2cz}\Big) -$$

$$e^{p z} \sum_{j=0}^{n} \frac{(-1)^j p^{-j-1} z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}\Big(\frac{p}{2c}, \ldots, \frac{p}{2c}, 1; \frac{p}{2c}+1, \ldots, \frac{p}{2c}+1; -e^{2cz}\Big)\Bigg) +$$

$$i^u\, 2^{-m-u} \binom{u}{\frac{u}{2}} n!\, (1 - u \bmod 2) \sum_{k=0}^{\lfloor \frac{m-1}{2} \rfloor} \binom{m}{k} \Bigg(e^{(2c - i a (m-2k) + p) z} \sum_{j=0}^{n} \frac{(-1)^j (2c - i a (m-2k) + p)^{-j-1} z^{n-j}}{(n-j)!}$$

$${}_{j+2}F_{j+1}\Big(\frac{2c - i a (m-2k) + p}{2c}, \ldots, \frac{2c - i a (m-2k) + p}{2c}, 1; \frac{2c - i a (m-2k) + p}{2c} + 1, \ldots,$$

$$\frac{2c - i a (m-2k) + p}{2c} + 1; -e^{2cz}\Big) - e^{(p - i a (m-2k)) z} \sum_{j=0}^{n} \frac{(-1)^j (p - i a (m-2k))^{-j-1} z^{n-j}}{(n-j)!}$$

$${}_{j+2}F_{j+1}\Big(\frac{p - i a (m-2k)}{2c}, \ldots, \frac{p - i a (m-2k)}{2c}, 1; \frac{p - i a (m-2k)}{2c} + 1, \ldots, \frac{p - i a (m-2k)}{2c} + 1; -e^{2cz}\Big) +$$

$$e^{(2c + a i (m-2k) + p) z} \sum_{j=0}^{n} \frac{(-1)^j (2c + a i (m-2k) + p)^{-j-1} z^{n-j}}{(n-j)!}$$

$${}_{j+2}F_{j+1}\Big(\frac{2c + a i (m-2k) + p}{2c}, \ldots, \frac{2c + a i (m-2k) + p}{2c}, 1; \frac{2c + a i (m-2k) + p}{2c} + 1, \ldots,$$

$$\frac{2c + a i (m-2k) + p}{2c} + 1; -e^{2cz}\Big) - e^{(a i (m-2k) + p) z} \sum_{j=0}^{n} \frac{(-1)^j (a i (m-2k) + p)^{-j-1} z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}$$

$$\Big(\frac{a i (m-2k) + p}{2c}, \ldots, \frac{a i (m-2k) + p}{2c}, 1; \frac{a i (m-2k) + p}{2c} + 1, \ldots, \frac{a i (m-2k) + p}{2c} + 1; -e^{2cz}\Big)\Bigg) +$$

$$2^{-m-u} \binom{m}{\frac{m}{2}} n!\, (1 - m \bmod 2) \sum_{k=0}^{\lfloor \frac{u-1}{2} \rfloor} (-1)^k \binom{u}{k} \Bigg(-e^{(p + b (u-2k)) z} \sum_{j=0}^{n} \frac{(-1)^j (p + b (u-2k))^{-j-1} z^{n-j}}{(n-j)!}$$

$${}_{j+2}F_{j+1}\Big(\frac{p + b (u-2k)}{2c}, \ldots, \frac{p + b (u-2k)}{2c}, 1; \frac{p + b (u-2k)}{2c} + 1, \ldots, \frac{p + b (u-2k)}{2c} + 1; -e^{2cz}\Big) +$$

$$e^{(2c + p + b (u-2k)) z} \sum_{j=0}^{n} \frac{(-1)^j (2c + p + b (u-2k))^{-j-1} z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}\Big(\frac{2c + p + b (u-2k)}{2c}, \ldots,$$

$$\frac{2c + p + b (u-2k)}{2c}, 1; \frac{2c + p + b (u-2k)}{2c} + 1, \ldots, \frac{2c + p + b (u-2k)}{2c} + 1; -e^{2cz}\Big) +$$

$$(-1)^u \Bigg(e^{(2c + p - b (u-2k)) z} \sum_{j=0}^{n} \frac{(-1)^j (2c + p - b (u-2k))^{-j-1} z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}\Big(\frac{2c + p - b (u-2k)}{2c}, \ldots,$$

$$\frac{2c + p - b (u-2k)}{2c}, 1; \frac{2c + p - b (u-2k)}{2c} + 1, \ldots, \frac{2c + p - b (u-2k)}{2c} + 1; -e^{2cz}\Big) -$$

$$e^{(p-b(u-2k))z}\sum_{j=0}^{n}\frac{(-1)^j\,(p-b(u-2k))^{-j-1}\,z^{n-j}}{(n-j)!}\;{}_{j+2}F_{j+1}\left(\frac{p-b(u-2k)}{2c},\,\ldots,\,\frac{p-b(u-2k)}{2c},\right.$$

$$\left.\left.\left.1;\,\frac{p-b(u-2k)}{2c}+1,\,\ldots,\,\frac{p-b(u-2k)}{2c}+1;\,-e^{2cz}\right)\right)\right)+$$

$$2^{-m-u}\,n!\sum_{k=0}^{\left\lfloor\frac{m-1}{2}\right\rfloor}\binom{m}{k}\sum_{i=0}^{\left\lfloor\frac{u-1}{2}\right\rfloor}(-1)^i\binom{u}{i}\left(e^{(2c-i a(m-2k)+p+b(u-2i))z}\sum_{j=0}^{n}\frac{(-1)^j\,(2c-i a(m-2k)+p+b(u-2i))^{-j-1}\,z^{n-j}}{(n-j)!}\right.$$

$${}_{j+2}F_{j+1}\left(\frac{2c-i a(m-2k)+p+b(u-2i)}{2c},\,\ldots,\,\frac{2c-i a(m-2k)+p+b(u-2i)}{2c},\,1;\right.$$

$$\left.\frac{2c-i a(m-2k)+p+b(u-2i)}{2c}+1,\,\ldots,\,\frac{2c-i a(m-2k)+p+b(u-2i)}{2c}+1;\,-e^{2cz}\right)-$$

$$e^{(-i a(m-2k)+p+b(u-2i))z}\sum_{j=0}^{n}\frac{(-1)^j\,(-i a(m-2k)+p+b(u-2i))^{-j-1}\,z^{n-j}}{(n-j)!}$$

$${}_{j+2}F_{j+1}\left(\frac{-i a(m-2k)+p+b(u-2i)}{2c},\,\ldots,\,\frac{-i a(m-2k)+p+b(u-2i)}{2c},\,1;\right.$$

$$\left.\frac{-i a(m-2k)+p+b(u-2i)}{2c}+1,\,\ldots,\,\frac{-i a(m-2k)+p+b(u-2i)}{2c}+1;\,-e^{2cz}\right)+$$

$$e^{(2c+a i(m-2k)+p+b(u-2i))z}\sum_{j=0}^{n}\frac{(-1)^j\,(2c+a i(m-2k)+p+b(u-2i))^{-j-1}\,z^{n-j}}{(n-j)!}$$

$${}_{j+2}F_{j+1}\left(\frac{2c+a i(m-2k)+p+b(u-2i)}{2c},\,\ldots,\,\frac{2c+a i(m-2k)+p+b(u-2i)}{2c},\,1;\right.$$

$$\left.\frac{2c+a i(m-2k)+p+b(u-2i)}{2c}+1,\,\ldots,\,\frac{2c+a i(m-2k)+p+b(u-2i)}{2c}+1;\,-e^{2cz}\right)-$$

$$e^{(a i(m-2k)+p+b(u-2i))z}\sum_{j=0}^{n}\frac{(-1)^j\,(a i(m-2k)+p+b(u-2i))^{-j-1}\,z^{n-j}}{(n-j)!}$$

$${}_{j+2}F_{j+1}\left(\frac{a i(m-2k)+p+b(u-2i)}{2c},\,\ldots,\,\frac{a i(m-2k)+p+b(u-2i)}{2c},\,1;\right.$$

$$\left.\frac{a i(m-2k)+p+b(u-2i)}{2c}+1,\,\ldots,\,\frac{a i(m-2k)+p+b(u-2i)}{2c}+1;\,-e^{2cz}\right)+$$

$$(-1)^u\left(e^{(2c-i a(m-2k)+p-b(u-2i))z}\sum_{j=0}^{n}\frac{(-1)^j\,(2c-i a(m-2k)+p-b(u-2i))^{-j-1}\,z^{n-j}}{(n-j)!}\right.$$

$${}_{j+2}F_{j+1}\left(\frac{2c-i a(m-2k)+p-b(u-2i)}{2c},\,\ldots,\,\frac{2c-i a(m-2k)+p-b(u-2i)}{2c},\,1;\right.$$

$$\left.\frac{2c-i a(m-2k)+p-b(u-2i)}{2c}+1,\,\ldots,\,\frac{2c-i a(m-2k)+p-b(u-2i)}{2c}+1;\,-e^{2cz}\right)-$$

$$e^{(-i a(m-2k)+p-b(u-2i))z}\sum_{j=0}^{n}\frac{(-1)^j\,(-i a(m-2k)+p-b(u-2i))^{-j-1}\,z^{n-j}}{(n-j)!}$$

$$ {}_{j+2}F_{j+1}\left(\frac{-i\,a\,(m-2\,k)+p-b\,(u-2\,i)}{2\,c},\,\ldots,\,\frac{-i\,a\,(m-2\,k)+p-b\,(u-2\,i)}{2\,c},\,1;\right. $$
$$ \left.\frac{-i\,a\,(m-2\,k)+p-b\,(u-2\,i)}{2\,c}+1,\,\ldots,\,\frac{-i\,a\,(m-2\,k)+p-b\,(u-2\,i)}{2\,c}+1;\,-e^{2\,c\,z}\right)+ $$
$$ e^{(2\,c+a\,i\,(m-2\,k)+p-b\,(u-2\,i))\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(2\,c+a\,i\,(m-2\,k)+p-b\,(u-2\,i))^{-j-1}\,z^{n-j}}{(n-j)!} $$
$$ {}_{j+2}F_{j+1}\left(\frac{2\,c+a\,i\,(m-2\,k)+p-b\,(u-2\,i)}{2\,c},\,\ldots,\,\frac{2\,c+a\,i\,(m-2\,k)+p-b\,(u-2\,i)}{2\,c},\,1;\right. $$
$$ \left.\frac{2\,c+a\,i\,(m-2\,k)+p-b\,(u-2\,i)}{2\,c}+1,\,\ldots,\,\frac{2\,c+a\,i\,(m-2\,k)+p-b\,(u-2\,i)}{2\,c}+1;\,-e^{2\,c\,z}\right)- $$
$$ e^{(a\,i\,(m-2\,k)+p-b\,(u-2\,i))\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(a\,i\,(m-2\,k)+p-b\,(u-2\,i))^{-j-1}\,z^{n-j}}{(n-j)!} $$
$$ {}_{j+2}F_{j+1}\left(\frac{a\,i\,(m-2\,k)+p-b\,(u-2\,i)}{2\,c},\,\ldots,\,\frac{a\,i\,(m-2\,k)+p-b\,(u-2\,i)}{2\,c},\right. $$
$$ 1;\,\frac{a\,i\,(m-2\,k)+p-b\,(u-2\,i)}{2\,c}+1,\,\ldots, $$
$$ \left.\left.\left.\frac{a\,i\,(m-2\,k)+p-b\,(u-2\,i)}{2\,c}+1;\,-e^{2\,c\,z}\right)\right)\right)\,/;\,n\in\mathbb{N}\bigwedge m\in\mathbb{N}^+\bigwedge u\in\mathbb{N}^+ $$

Involving sin, cosh, exp and power

Involving $z^n\,e^{p\,z}\,\sin(a\,z)\cosh(b\,z)\tanh(c\,z)$

01.21.21.0204.01

$$\int z^n e^{p z} \sin(a z) \cosh(b z) \tanh(c z)\, dz = \frac{1}{4} i\, n! \Bigg(-e^{(-b-i a+p) z} \sum_{j=0}^{n} \frac{(-1)^j (-b - i a + p)^{-j-1} z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}\left(\frac{-i a - b + p}{2 c}, \ldots, \frac{-i a - b + p}{2 c}, 1; \frac{-i a - b + p}{2 c} + 1, \ldots, \frac{-i a - b + p}{2 c} + 1; -e^{2 c z}\right) + e^{(-b+i a+p) z} \sum_{j=0}^{n} \frac{(-1)^j (-b + i a + p)^{-j-1} z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}\left(\frac{i a - b + p}{2 c}, \ldots, \frac{i a - b + p}{2 c}, 1; \frac{i a - b + p}{2 c} + 1, \ldots, \frac{i a - b + p}{2 c} + 1; -e^{2 c z}\right) - e^{(b-i a+p) z} \sum_{j=0}^{n} \frac{(-1)^j (b - i a + p)^{-j-1} z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}\left(\frac{-i a + b + p}{2 c}, \ldots, \frac{-i a + b + p}{2 c}, 1; \frac{-i a + b + p}{2 c} + 1, \ldots, \frac{-i a + b + p}{2 c} + 1; -e^{2 c z}\right) + e^{(b+i a+p) z} \sum_{j=0}^{n} \frac{(-1)^j (b + i a + p)^{-j-1} z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}\left(\frac{i a + b + p}{2 c}, \ldots, \frac{i a + b + p}{2 c}, 1; \frac{i a + b + p}{2 c} + 1, \ldots, \frac{i a + b + p}{2 c} + 1; -e^{2 c z}\right) + e^{(-b+2 c-i a+p) z} \sum_{j=0}^{n} \frac{(-1)^j (-b + 2 c - i a + p)^{-j-1} z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}\left(\frac{-i a - b + 2 c + p}{2 c}, \ldots, \frac{-i a - b + 2 c + p}{2 c}, 1; \frac{-i a - b + 2 c + p}{2 c} + 1, \ldots, \frac{-i a - b + 2 c + p}{2 c} + 1; -e^{2 c z}\right) - e^{(-b+2 c+i a+p) z} \sum_{j=0}^{n} \frac{(-1)^j (-b + 2 c + i a + p)^{-j-1} z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}\left(\frac{i a - b + 2 c + p}{2 c}, \ldots, \frac{i a - b + 2 c + p}{2 c}, 1; \frac{i a - b + 2 c + p}{2 c} + 1, \ldots, \frac{i a - b + 2 c + p}{2 c} + 1; -e^{2 c z}\right) + e^{(b+2 c-i a+p) z} \sum_{j=0}^{n} \frac{(-1)^j (b + 2 c - i a + p)^{-j-1} z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}\left(\frac{-i a + b + 2 c + p}{2 c}, \ldots, \frac{-i a + b + 2 c + p}{2 c}, 1; \frac{-i a + b + 2 c + p}{2 c} + 1, \ldots, \frac{-i a + b + 2 c + p}{2 c} + 1; -e^{2 c z}\right) - e^{(b+2 c+i a+p) z} \sum_{j=0}^{n} \frac{(-1)^j (b + 2 c + i a + p)^{-j-1} z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}\left(\frac{i a + b + 2 c + p}{2 c}, \ldots, \frac{i a + b + 2 c + p}{2 c}, 1; \frac{i a + b + 2 c + p}{2 c} + 1, \ldots, \frac{i a + b + 2 c + p}{2 c} + 1; -e^{2 c z}\right)\Bigg) /; n \in \mathbb{N}$$

Involving powers of sin, powers of cosh, exp and power

Involving $z^n e^{p z} \sin^m(a z) \cosh^u(b z) \tanh^v(c z)$

01.21.21.0205.01

$$\int z^n e^{pz}\sin^m(az)\cosh^u(bz)\tanh(cz)\,dz = 2^{-m-u}\binom{m}{\frac{m}{2}}\binom{u}{\frac{u}{2}} n!\,(1-m \bmod 2)(1-u \bmod 2)$$

$$\Bigg(e^{(2c+p)z}\sum_{j=0}^{n}\frac{(-1)^j (2c+p)^{-j-1} z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{2c+p}{2c},\ldots,\frac{2c+p}{2c},1;\frac{2c+p}{2c}+1,\ldots,\frac{2c+p}{2c}+1;-e^{2cz}\right)-$$

$$e^{pz}\sum_{j=0}^{n}\frac{(-1)^j p^{-j-1} z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{p}{2c},\ldots,\frac{p}{2c},1;\frac{p}{2c}+1,\ldots,\frac{p}{2c}+1;-e^{2cz}\right)\Bigg)+$$

$$2^{-m-u}\binom{u}{\frac{u}{2}} n!\,(1-u \bmod 2)\sum_{k=0}^{\left\lfloor\frac{m-1}{2}\right\rfloor}(-1)^k\binom{m}{k}\Bigg(e^{\frac{i m\pi}{2}}\Bigg(e^{(2c-i a(m-2k)+p)z}\sum_{j=0}^{n}\frac{(-1)^j (2c-i a(m-2k)+p)^{-j-1} z^{n-j}}{(n-j)!}$$

$${}_{j+2}F_{j+1}\left(\frac{2c-i a(m-2k)+p}{2c},\ldots,\frac{2c-i a(m-2k)+p}{2c},1;\right.$$

$$\left.\frac{2c-i a(m-2k)+p}{2c}+1,\ldots,\frac{2c-i a(m-2k)+p}{2c}+1;-e^{2cz}\right)-$$

$$e^{(p-i a(m-2k))z}\sum_{j=0}^{n}\frac{(-1)^j (p-i a(m-2k))^{-j-1} z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{p-i a(m-2k)}{2c},\ldots,\right.$$

$$\left.\frac{p-i a(m-2k)}{2c},1;\frac{p-i a(m-2k)}{2c}+1,\ldots,\frac{p-i a(m-2k)}{2c}+1;-e^{2cz}\right)\Bigg)+$$

$$e^{-\frac{1}{2} i m\pi}\Bigg(e^{(2c+a i(m-2k)+p)z}\sum_{j=0}^{n}\frac{(-1)^j (2c+a i(m-2k)+p)^{-j-1} z^{n-j}}{(n-j)!}$$

$${}_{j+2}F_{j+1}\left(\frac{2c+a i(m-2k)+p}{2c},\ldots,\frac{2c+a i(m-2k)+p}{2c},1;\right.$$

$$\left.\frac{2c+a i(m-2k)+p}{2c}+1,\ldots,\frac{2c+a i(m-2k)+p}{2c}+1;-e^{2cz}\right)-$$

$$e^{(a i(m-2k)+p)z}\sum_{j=0}^{n}\frac{(-1)^j (a i(m-2k)+p)^{-j-1} z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{a i(m-2k)+p}{2c},\ldots,\right.$$

$$\left.\frac{a i(m-2k)+p}{2c},1;\frac{a i(m-2k)+p}{2c}+1,\ldots,\frac{a i(m-2k)+p}{2c}+1;-e^{2cz}\right)\Bigg)\Bigg)+$$

$$2^{-m-u}\binom{m}{\frac{m}{2}} n!\,(1-m \bmod 2)\sum_{k=0}^{\left\lfloor\frac{u-1}{2}\right\rfloor}\binom{u}{k}\Bigg(-e^{(p+b(u-2k))z}\sum_{j=0}^{n}\frac{(-1)^j (p+b(u-2k))^{-j-1} z^{n-j}}{(n-j)!}$$

$${}_{j+2}F_{j+1}\left(\frac{p+b(u-2k)}{2c},\ldots,\frac{p+b(u-2k)}{2c},1;\frac{p+b(u-2k)}{2c}+1,\ldots,\frac{p+b(u-2k)}{2c}+1;-e^{2cz}\right)+$$

$$e^{(2c+p+b(u-2k))z}\sum_{j=0}^{n}\frac{(-1)^j (2c+p+b(u-2k))^{-j-1} z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{2c+p+b(u-2k)}{2c},\ldots,\right.$$

$$\left.\frac{2c+p+b(u-2k)}{2c},1;\frac{2c+p+b(u-2k)}{2c}+1,\ldots,\frac{2c+p+b(u-2k)}{2c}+1;-e^{2cz}\right)+$$

$$e^{(2c+p-b(u-2k))z}\sum_{j=0}^{n}\frac{(-1)^j (2c+p-b(u-2k))^{-j-1} z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\Bigg(\frac{2c+p-b(u-2k)}{2c}, \ldots, \frac{2c+p-b(u-2k)}{2c}, 1; \frac{2c+p-b(u-2k)}{2c}+1, \ldots, \frac{2c+p-b(u-2k)}{2c}+1; -e^{2cz}\Bigg) -$$

$$e^{(p-b(u-2k))z}\sum_{j=0}^{n}\frac{(-1)^j (p-b(u-2k))^{-j-1} z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\Bigg(\frac{p-b(u-2k)}{2c}, \ldots, \frac{p-b(u-2k)}{2c}, 1; \frac{p-b(u-2k)}{2c}+1, \ldots, \frac{p-b(u-2k)}{2c}+1; -e^{2cz}\Bigg)\Bigg) +$$

$$2^{-m-u}\, n! \sum_{k=0}^{\left\lfloor\frac{m-1}{2}\right\rfloor} (-1)^k \binom{m}{k} \sum_{i=0}^{\left\lfloor\frac{u-1}{2}\right\rfloor} \binom{u}{i} \Bigg(e^{\frac{i m\pi}{2}} \Bigg(e^{(2c-i a(m-2k)+p+b(u-2i))z}\sum_{j=0}^{n}\frac{(-1)^j (2c-i a(m-2k)+p+b(u-2i))^{-j-1} z^{n-j}}{(n-j)!}$$

$${}_{j+2}F_{j+1}\Bigg(\frac{2c-i a(m-2k)+p+b(u-2i)}{2c}, \ldots, \frac{2c-i a(m-2k)+p+b(u-2i)}{2c}, 1; \frac{2c-i a(m-2k)+p+b(u-2i)}{2c}+1, \ldots, \frac{2c-i a(m-2k)+p+b(u-2i)}{2c}+1; -e^{2cz}\Bigg) -$$

$$e^{(-i a(m-2k)+p+b(u-2i))z}\sum_{j=0}^{n}\frac{(-1)^j (-i a(m-2k)+p+b(u-2i))^{-j-1} z^{n-j}}{(n-j)!}$$

$${}_{j+2}F_{j+1}\Bigg(\frac{-i a(m-2k)+p+b(u-2i)}{2c}, \ldots, \frac{-i a(m-2k)+p+b(u-2i)}{2c}, 1; \frac{-i a(m-2k)+p+b(u-2i)}{2c}+1, \ldots, \frac{-i a(m-2k)+p+b(u-2i)}{2c}+1; -e^{2cz}\Bigg)\Bigg) +$$

$$e^{-\frac{1}{2} i m\pi}\Bigg(e^{(2c+a i(m-2k)+p+b(u-2i))z}\sum_{j=0}^{n}\frac{(-1)^j (2c+a i(m-2k)+p+b(u-2i))^{-j-1} z^{n-j}}{(n-j)!}$$

$${}_{j+2}F_{j+1}\Bigg(\frac{2c+a i(m-2k)+p+b(u-2i)}{2c}, \ldots, \frac{2c+a i(m-2k)+p+b(u-2i)}{2c}, 1; \frac{2c+a i(m-2k)+p+b(u-2i)}{2c}+1, \ldots, \frac{2c+a i(m-2k)+p+b(u-2i)}{2c}+1; -e^{2cz}\Bigg) -$$

$$e^{(a i(m-2k)+p+b(u-2i))z}\sum_{j=0}^{n}\frac{(-1)^j (a i(m-2k)+p+b(u-2i))^{-j-1} z^{n-j}}{(n-j)!}$$

$${}_{j+2}F_{j+1}\Bigg(\frac{a i(m-2k)+p+b(u-2i)}{2c}, \ldots, \frac{a i(m-2k)+p+b(u-2i)}{2c}, 1; \frac{a i(m-2k)+p+b(u-2i)}{2c}+1, \ldots, \frac{a i(m-2k)+p+b(u-2i)}{2c}+1; -e^{2cz}\Bigg)\Bigg) +$$

$$e^{\frac{i m\pi}{2}}\Bigg(e^{(2c-i a(m-2k)+p-b(u-2i))z}\sum_{j=0}^{n}\frac{(-1)^j (2c-i a(m-2k)+p-b(u-2i))^{-j-1} z^{n-j}}{(n-j)!}$$

$$ {}_{j+2}F_{j+1}\left(\frac{2c-i a(m-2k)+p-b(u-2i)}{2c},\ldots,\frac{2c-i a(m-2k)+p-b(u-2i)}{2c},1;\right. $$
$$ \left.\frac{2c-i a(m-2k)+p-b(u-2i)}{2c}+1,\ldots,\frac{2c-i a(m-2k)+p-b(u-2i)}{2c}+1;-e^{2cz}\right)- $$
$$ e^{(-i a(m-2k)+p-b(u-2i))z}\sum_{j=0}^{n}\frac{(-1)^j(-i a(m-2k)+p-b(u-2i))^{-j-1}z^{n-j}}{(n-j)!} $$
$$ {}_{j+2}F_{j+1}\left(\frac{-i a(m-2k)+p-b(u-2i)}{2c},\ldots,\frac{-i a(m-2k)+p-b(u-2i)}{2c},1;\right. $$
$$ \left.\left.\frac{-i a(m-2k)+p-b(u-2i)}{2c}+1,\ldots,\frac{-i a(m-2k)+p-b(u-2i)}{2c}+1;-e^{2cz}\right)\right)+ $$
$$ e^{-\frac{1}{2}i m\pi}\left(e^{(2c+a i(m-2k)+p-b(u-2i))z}\sum_{j=0}^{n}\frac{(-1)^j(2c+a i(m-2k)+p-b(u-2i))^{-j-1}z^{n-j}}{(n-j)!}\right. $$
$$ {}_{j+2}F_{j+1}\left(\frac{2c+a i(m-2k)+p-b(u-2i)}{2c},\ldots,\frac{2c+a i(m-2k)+p-b(u-2i)}{2c},1;\right. $$
$$ \left.\frac{2c+a i(m-2k)+p-b(u-2i)}{2c}+1,\ldots,\frac{2c+a i(m-2k)+p-b(u-2i)}{2c}+1;-e^{2cz}\right)- $$
$$ e^{(a i(m-2k)+p-b(u-2i))z}\sum_{j=0}^{n}\frac{(-1)^j(a i(m-2k)+p-b(u-2i))^{-j-1}z^{n-j}}{(n-j)!} $$
$$ {}_{j+2}F_{j+1}\left(\frac{a i(m-2k)+p-b(u-2i)}{2c},\ldots,\frac{a i(m-2k)+p-b(u-2i)}{2c},\right. $$
$$ 1;\frac{a i(m-2k)+p-b(u-2i)}{2c}+1,\ldots, $$
$$ \left.\left.\left.\frac{a i(m-2k)+p-b(u-2i)}{2c}+1;-e^{2cz}\right)\right)\right)/;n\in\mathbb{N}\land m\in\mathbb{N}^+\land u\in\mathbb{N}^+ $$

Involving cos, cosh, exp and power

Involving $z^n e^{pz}\cos(az)\cosh(bz)\tanh(cz)$

01.21.21.0206.01

$$\int z^n e^{p z} \cos(a z) \cosh(b z) \tanh(c z) \, dz = \frac{1}{4} n! \left(-e^{(-b - i a + p) z} \sum_{j=0}^{n} \frac{(-1)^j (-b - i a + p)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{-i a - b + p}{2 c}, \ldots, \frac{-i a - b + p}{2 c}, 1; \frac{-i a - b + p}{2 c} + 1, \ldots, \frac{-i a - b + p}{2 c} + 1; -e^{2 c z} \right) - e^{(-b + i a + p) z} \sum_{j=0}^{n} \frac{(-1)^j (-b + i a + p)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{i a - b + p}{2 c}, \ldots, \frac{i a - b + p}{2 c}, 1; \frac{i a - b + p}{2 c} + 1, \ldots, \frac{i a - b + p}{2 c} + 1; -e^{2 c z} \right) - e^{(b - i a + p) z} \sum_{j=0}^{n} \frac{(-1)^j (b - i a + p)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{-i a + b + p}{2 c}, \ldots, \frac{-i a + b + p}{2 c}, 1; \frac{-i a + b + p}{2 c} + 1, \ldots, \frac{-i a + b + p}{2 c} + 1; -e^{2 c z} \right) - e^{(b + i a + p) z} \sum_{j=0}^{n} \frac{(-1)^j (b + i a + p)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{i a + b + p}{2 c}, \ldots, \frac{i a + b + p}{2 c}, 1; \frac{i a + b + p}{2 c} + 1, \ldots, \frac{i a + b + p}{2 c} + 1; -e^{2 c z} \right) + e^{(-b + 2 c - i a + p) z} \sum_{j=0}^{n} \frac{(-1)^j (-b + 2 c - i a + p)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{-i a - b + 2 c + p}{2 c}, \ldots, \frac{-i a - b + 2 c + p}{2 c}, 1; \frac{-i a - b + 2 c + p}{2 c} + 1, \ldots, \frac{-i a - b + 2 c + p}{2 c} + 1; -e^{2 c z} \right) + e^{(-b + 2 c + i a + p) z} \sum_{j=0}^{n} \frac{(-1)^j (-b + 2 c + i a + p)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{i a - b + 2 c + p}{2 c}, \ldots, \frac{i a - b + 2 c + p}{2 c}, 1; \frac{i a - b + 2 c + p}{2 c} + 1, \ldots, \frac{i a - b + 2 c + p}{2 c} + 1; -e^{2 c z} \right) + e^{(b + 2 c - i a + p) z} \sum_{j=0}^{n} \frac{(-1)^j (b + 2 c - i a + p)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{-i a + b + 2 c + p}{2 c}, \ldots, \frac{-i a + b + 2 c + p}{2 c}, 1; \frac{-i a + b + 2 c + p}{2 c} + 1, \ldots, \frac{-i a + b + 2 c + p}{2 c} + 1; -e^{2 c z} \right) + e^{(b + 2 c + i a + p) z} \sum_{j=0}^{n} \frac{(-1)^j (b + 2 c + i a + p)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{i a + b + 2 c + p}{2 c}, \ldots, \frac{i a + b + 2 c + p}{2 c}, 1; \frac{i a + b + 2 c + p}{2 c} + 1, \ldots, \frac{i a + b + 2 c + p}{2 c} + 1; -e^{2 c z} \right) \right) /; n \in \mathbb{N}$$

Involving powers of cos, powers of cosh, exp and power

Involving $z^n e^{p z} \cos^m(a z) \cosh^u(b z) \tanh^v(c z)$

01.21.21.0207.01

$$\int z^n e^{p z} \cos^m(a z) \cosh^u(b z) \tanh(c z)\, dz = 2^{-m-u} \binom{m}{\frac{m}{2}} \binom{u}{\frac{u}{2}} n!\, (1 - m \bmod 2)\, (1 - u \bmod 2) \left(e^{(2c+p) z} \sum_{j=0}^{n} \frac{(-1)^j (2c+p)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{2c+p}{2c}, \ldots, \frac{2c+p}{2c}, 1; \frac{2c+p}{2c} + 1, \ldots, \frac{2c+p}{2c} + 1; -e^{2 c z} \right) - e^{p z} \sum_{j=0}^{n} \frac{(-1)^j p^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{p}{2c}, \ldots, \frac{p}{2c}, 1; \frac{p}{2c} + 1, \ldots, \frac{p}{2c} + 1; -e^{2 c z} \right) \right) +$$

$$2^{-m-u} \binom{u}{\frac{u}{2}} n!\, (1 - u \bmod 2) \sum_{k=0}^{\left\lfloor \frac{m-1}{2} \right\rfloor} \binom{m}{k} \left(e^{(2c - i a (m-2k) + p) z} \sum_{j=0}^{n} \frac{(-1)^j (2c - i a (m-2k) + p)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{2c - i a (m-2k) + p}{2c}, \ldots, \frac{2c - i a (m-2k) + p}{2c}, 1; \frac{2c - i a (m-2k) + p}{2c} + 1, \ldots, \frac{2c - i a (m-2k) + p}{2c} + 1; -e^{2 c z} \right) - e^{(p - i a (m-2k)) z} \sum_{j=0}^{n} \frac{(-1)^j (p - i a (m-2k))^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{p - i a (m-2k)}{2c}, \ldots, \frac{p - i a (m-2k)}{2c}, 1; \frac{p - i a (m-2k)}{2c} + 1, \ldots, \frac{p - i a (m-2k)}{2c} + 1; -e^{2 c z} \right) + e^{(2c + a i (m-2k) + p) z} \sum_{j=0}^{n} \frac{(-1)^j (2c + a i (m-2k) + p)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{2c + a i (m-2k) + p}{2c}, \ldots, \frac{2c + a i (m-2k) + p}{2c}, 1; \frac{2c + a i (m-2k) + p}{2c} + 1, \ldots, \frac{2c + a i (m-2k) + p}{2c} + 1; -e^{2 c z} \right) - e^{(a i (m-2k) + p) z} \sum_{j=0}^{n} \frac{(-1)^j (a i (m-2k) + p)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{a i (m-2k) + p}{2c}, \ldots, \frac{a i (m-2k) + p}{2c}, 1; \frac{a i (m-2k) + p}{2c} + 1, \ldots, \frac{a i (m-2k) + p}{2c} + 1; -e^{2 c z} \right) \right) +$$

$$2^{-m-u} \binom{m}{\frac{m}{2}} n!\, (1 - m \bmod 2) \sum_{k=0}^{\left\lfloor \frac{u-1}{2} \right\rfloor} \binom{u}{k} \left(-e^{(p + b (u-2k)) z} \sum_{j=0}^{n} \frac{(-1)^j (p + b (u-2k))^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{p + b (u-2k)}{2c}, \ldots, \frac{p + b (u-2k)}{2c}, 1; \frac{p + b (u-2k)}{2c} + 1, \ldots, \frac{p + b (u-2k)}{2c} + 1; -e^{2 c z} \right) + e^{(2c + p + b (u-2k)) z} \sum_{j=0}^{n} \frac{(-1)^j (2c + p + b (u-2k))^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{2c + p + b (u-2k)}{2c}, \ldots, \frac{2c + p + b (u-2k)}{2c}, 1; \frac{2c + p + b (u-2k)}{2c} + 1, \ldots, \frac{2c + p + b (u-2k)}{2c} + 1; -e^{2 c z} \right) + e^{(2c + p - b (u-2k)) z} \sum_{j=0}^{n} \frac{(-1)^j (2c + p - b (u-2k))^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{2c + p - b (u-2k)}{2c}, \ldots, \frac{2c + p - b (u-2k)}{2c}, 1; \frac{2c + p - b (u-2k)}{2c} + 1, \ldots, \frac{2c + p - b (u-2k)}{2c} + 1; -e^{2 c z} \right) - \right.$$

$$e^{(p-b(u-2k))z}\sum_{j=0}^{n}\frac{(-1)^j (p-b(u-2k))^{-j-1} z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{p-b(u-2k)}{2c},\ldots,\frac{p-b(u-2k)}{2c},\right.$$

$$\left.\left.1;\frac{p-b(u-2k)}{2c}+1,\ldots,\frac{p-b(u-2k)}{2c}+1;-e^{2cz}\right)\right)+$$

$$2^{-m-u}n!\sum_{k=0}^{\left\lfloor\frac{m-1}{2}\right\rfloor}\binom{m}{k}\sum_{i=0}^{\left\lfloor\frac{u-1}{2}\right\rfloor}\binom{u}{i}\left(e^{(2c-i a(m-2k)+p+b(u-2i))z}\sum_{j=0}^{n}\frac{(-1)^j (2c-i a(m-2k)+p+b(u-2i))^{-j-1} z^{n-j}}{(n-j)!}\right.$$

$${}_{j+2}F_{j+1}\left(\frac{2c-i a(m-2k)+p+b(u-2i)}{2c},\ldots,\frac{2c-i a(m-2k)+p+b(u-2i)}{2c},1;\right.$$

$$\left.\frac{2c-i a(m-2k)+p+b(u-2i)}{2c}+1,\ldots,\frac{2c-i a(m-2k)+p+b(u-2i)}{2c}+1;-e^{2cz}\right)-$$

$$e^{(-i a(m-2k)+p+b(u-2i))z}\sum_{j=0}^{n}\frac{(-1)^j (-i a(m-2k)+p+b(u-2i))^{-j-1} z^{n-j}}{(n-j)!}$$

$${}_{j+2}F_{j+1}\left(\frac{-i a(m-2k)+p+b(u-2i)}{2c},\ldots,\frac{-i a(m-2k)+p+b(u-2i)}{2c},1;\right.$$

$$\left.\frac{-i a(m-2k)+p+b(u-2i)}{2c}+1,\ldots,\frac{-i a(m-2k)+p+b(u-2i)}{2c}+1;-e^{2cz}\right)+$$

$$e^{(2c+a i(m-2k)+p+b(u-2i))z}\sum_{j=0}^{n}\frac{(-1)^j (2c+a i(m-2k)+p+b(u-2i))^{-j-1} z^{n-j}}{(n-j)!}$$

$${}_{j+2}F_{j+1}\left(\frac{2c+a i(m-2k)+p+b(u-2i)}{2c},\ldots,\frac{2c+a i(m-2k)+p+b(u-2i)}{2c},1;\right.$$

$$\left.\frac{2c+a i(m-2k)+p+b(u-2i)}{2c}+1,\ldots,\frac{2c+a i(m-2k)+p+b(u-2i)}{2c}+1;-e^{2cz}\right)-$$

$$e^{(a i(m-2k)+p+b(u-2i))z}\sum_{j=0}^{n}\frac{(-1)^j (a i(m-2k)+p+b(u-2i))^{-j-1} z^{n-j}}{(n-j)!}$$

$${}_{j+2}F_{j+1}\left(\frac{a i(m-2k)+p+b(u-2i)}{2c},\ldots,\frac{a i(m-2k)+p+b(u-2i)}{2c},1;\right.$$

$$\left.\frac{a i(m-2k)+p+b(u-2i)}{2c}+1,\ldots,\frac{a i(m-2k)+p+b(u-2i)}{2c}+1;-e^{2cz}\right)+$$

$$e^{(2c-i a(m-2k)+p-b(u-2i))z}\sum_{j=0}^{n}\frac{(-1)^j (2c-i a(m-2k)+p-b(u-2i))^{-j-1} z^{n-j}}{(n-j)!}$$

$${}_{j+2}F_{j+1}\left(\frac{2c-i a(m-2k)+p-b(u-2i)}{2c},\ldots,\frac{2c-i a(m-2k)+p-b(u-2i)}{2c},1;\right.$$

$$\left.\frac{2c-i a(m-2k)+p-b(u-2i)}{2c}+1,\ldots,\frac{2c-i a(m-2k)+p-b(u-2i)}{2c}+1;-e^{2cz}\right)-$$

$$e^{(-i a(m-2k)+p-b(u-2i))z}\sum_{j=0}^{n}\frac{(-1)^j (-i a(m-2k)+p-b(u-2i))^{-j-1} z^{n-j}}{(n-j)!}$$

$$ {}_{j+2}F_{j+1}\left(\frac{-i\, a\,(m-2\,k)+p-b\,(u-2\,i)}{2\,c}, \ldots, \frac{-i\, a\,(m-2\,k)+p-b\,(u-2\,i)}{2\,c}, 1; \frac{-i\, a\,(m-2\,k)+p-b\,(u-2\,i)}{2\,c}+1, \ldots, \frac{-i\, a\,(m-2\,k)+p-b\,(u-2\,i)}{2\,c}+1; -e^{2\,c\,z}\right)+ $$

$$ e^{(2\,c+a\,i\,(m-2\,k)+p-b\,(u-2\,i))\,z} \sum_{j=0}^{n} \frac{(-1)^j\,(2\,c+a\,i\,(m-2\,k)+p-b\,(u-2\,i))^{-j-1}\,z^{n-j}}{(n-j)!} $$

$$ {}_{j+2}F_{j+1}\left(\frac{2\,c+a\,i\,(m-2\,k)+p-b\,(u-2\,i)}{2\,c}, \ldots, \frac{2\,c+a\,i\,(m-2\,k)+p-b\,(u-2\,i)}{2\,c}, 1; \frac{2\,c+a\,i\,(m-2\,k)+p-b\,(u-2\,i)}{2\,c}+1, \ldots, \frac{2\,c+a\,i\,(m-2\,k)+p-b\,(u-2\,i)}{2\,c}+1; -e^{2\,c\,z}\right)- $$

$$ e^{(a\,i\,(m-2\,k)+p-b\,(u-2\,i))\,z} \sum_{j=0}^{n} \frac{(-1)^j\,(a\,i\,(m-2\,k)+p-b\,(u-2\,i))^{-j-1}\,z^{n-j}}{(n-j)!} $$

$$ {}_{j+2}F_{j+1}\left(\frac{a\,i\,(m-2\,k)+p-b\,(u-2\,i)}{2\,c}, \ldots, \frac{a\,i\,(m-2\,k)+p-b\,(u-2\,i)}{2\,c}, 1; \frac{a\,i\,(m-2\,k)+p-b\,(u-2\,i)}{2\,c}+1, \ldots, \frac{a\,i\,(m-2\,k)+p-b\,(u-2\,i)}{2\,c}+1; -e^{2\,c\,z}\right)\Bigg) /;\ n \in \mathbb{N} \land m \in \mathbb{N}^+ \land u \in \mathbb{N}^+ $$

Involving functions of the direct function

Involving powers of the direct function

Involving powers of tanh

Linear argument

01.21.21.0208.01

$$ \int \tanh^{\nu}(c\,z)\,dz = \frac{\tanh^{\nu+1}(c\,z)}{\nu\,c+c}\; {}_2F_1\left(\frac{\nu+1}{2}, 1; \frac{\nu+3}{2}; \tanh^2(c\,z)\right) $$

01.21.21.0209.01

$$ \int \tanh^2(c\,z)\,dz = z - \frac{\tanh(c\,z)}{c} $$

01.21.21.0210.01

$$ \int \tanh^3(c\,z)\,dz = \frac{\operatorname{sech}^2(c\,z)+2\log(\cosh(c\,z))}{2\,c} $$

01.21.21.0211.01

$$ \int \tanh^4(c\,z)\,dz = \frac{3\,c\,z+\left(\operatorname{sech}^2(c\,z)-4\right)\tanh(c\,z)}{3\,c} $$

01.21.21.0212.01

$$\int \tanh^5(c z)\, dz = \frac{-\operatorname{sech}^4(c z) + 4 \operatorname{sech}^2(c z) + 4 \log(\cosh(c z))}{4 c}$$

01.21.21.0213.01

$$\int \tanh^6(c z)\, dz = \frac{15 c z + \left(-3 \operatorname{sech}^4(c z) + 11 \operatorname{sech}^2(c z) - 23\right) \tanh(c z)}{15 c}$$

01.21.21.0214.01

$$\int \tanh^7(c z)\, dz = \frac{2 \operatorname{sech}^6(c z) - 9 \operatorname{sech}^4(c z) + 18 \operatorname{sech}^2(c z) + 12 \log(\cosh(c z))}{12 c}$$

01.21.21.0215.01

$$\int \tanh^8(c z)\, dz = \frac{105 c z + \left(15 \operatorname{sech}^6(c z) - 66 \operatorname{sech}^4(c z) + 122 \operatorname{sech}^2(c z) - 176\right) \tanh(c z)}{105 c}$$

01.21.21.0433.01

$$\int \tanh^{2n}(a z)\, dz = z - \frac{\coth(a z)}{a} \sum_{k=1}^{n} \frac{\tanh^{2k}(a z)}{2k - 1} \;/;\; n \in \mathbb{N}$$

01.21.21.0434.01

$$\int \tanh^{2n+1}(a z)\, dz = \frac{\log(\cosh(a z))}{a} - \frac{1}{2a} \sum_{k=1}^{n} \frac{\tanh^{2k}(a z)}{k} - \frac{(-1)^n S_{n+1}^{(2)}}{2 a n!} \;/;\; n \in \mathbb{N}$$

01.21.21.0435.01

$$\int \tanh^{2n}(a z)\, dz = \frac{a z - \tanh^{-1}(\tanh(a z))}{a} + \frac{\tanh^{2n+1}(a z)}{a (2n+1)}\, {}_2F_1\!\left(1, n + \frac{1}{2}; n + \frac{3}{2}; \tanh^2(a z)\right) /;\; n \in \mathbb{N}$$

01.21.21.0436.01

$$\int \tanh^{2n+1}(a z)\, dz = \frac{\tanh^{2n+2}(a z)}{2 a (n+1)}\, {}_2F_1\!\left(n+1, 1; n+2; \tanh^2(a z)\right) + \frac{2 \log(\cosh(a z)) + \log\left(\operatorname{sech}^2(a z)\right)}{2 a} - \frac{(-1)^n S_{n+1}^{(2)}}{2 a n!} \;/;\; n \in \mathbb{N}$$

01.21.21.0216.01

$$\int \tanh^{\frac{1}{2}}(c z)\, dz = -\frac{2 \tan^{-1}\left(\tanh^{\frac{1}{2}}(c z)\right) + \log\left(\tanh^{\frac{1}{2}}(c z) - 1\right) - \log\left(\tanh^{\frac{1}{2}}(c z) + 1\right)}{2 c}$$

01.21.21.0217.01

$$\int \frac{1}{\tanh^{\frac{1}{2}}(c z)}\, dz = \frac{2 \tan^{-1}\left(\tanh^{\frac{1}{2}}(c z)\right) - \log\left(\tanh^{\frac{1}{2}}(c z) - 1\right) + \log\left(\tanh^{\frac{1}{2}}(c z) + 1\right)}{2 c}$$

01.21.21.0218.01

$$\int \frac{1}{\tanh^{\frac{1}{3}}(c z)}\, dz = \frac{1}{4c} \left(2 \sqrt{3} \tan^{-1}\left(\frac{2 \tanh^{\frac{1}{3}}(c z) - 1}{\sqrt{3}}\right) - 2 \sqrt{3} \tan^{-1}\left(\frac{2 \tanh^{\frac{1}{3}}(c z) + 1}{\sqrt{3}}\right) - 2 \log\left(\tanh^{\frac{1}{3}}(c z) - 1\right) - \right.$$
$$\left. 2 \log\left(\tanh^{\frac{1}{3}}(c z) + 1\right) + \log\left(-\tanh^{\frac{1}{3}}(c z) + \tanh^{\frac{2}{3}}(c z) + 1\right) + \log\left(\tanh^{\frac{1}{3}}(c z) + \tanh^{\frac{2}{3}}(c z) + 1\right)\right)$$

Involving products of the direct functions

01.21.21.0219.01

$$\int \tanh(b + a z) \tanh(a z) \, dz = z + \frac{\coth(b) (\log(\cosh(a z)) - \log(\cosh(b + a z)))}{a}$$

01.21.21.0220.01

$$\int \tanh(b - a z) \tanh(a z) \, dz = \frac{\coth(b) (\log(\cosh(a z)) - \log(\cosh(b - a z)))}{a} - z$$

Involving powers of products of the direct function

01.21.21.0221.01

$$\int \sqrt{\tanh(c z) \tanh(2 c z)} \, dz = \frac{1}{c} \left(\tanh^{-1}\left(\frac{\sqrt{2} \cosh(c z)}{\cosh^{\frac{1}{2}}(2 c z)} \right) \cosh^{\frac{1}{2}}(2 c z) \operatorname{csch}(c z) \sqrt{\operatorname{sech}(2 c z) \sinh^2(c z)} \right)$$

Involving rational functions of the direct function

Involving $(a + b \tanh(c z))^{-n}$

01.21.21.0222.01

$$\int \frac{1}{a + b \tanh(c z)} \, dz = \frac{a c z - b \log(a \cosh(c z) + b \sinh(c z))}{a^2 c - b^2 c}$$

01.21.21.0223.01

$$\int \frac{1}{(a + b \tanh(c z))^2} \, dz = \left(\left(\left(a^2 + b^2\right) c z - 2 a b \log(a \cosh(c z) + b \sinh(c z)) \right) a^2 + b \left(c z a^3 - 2 b \log(a \cosh(c z) + b \sinh(c z)) a^2 + b^2 c z a + b^3 - a^2 b \right) \tanh(c z) \right) \Big/ \left(a (a - b)^2 (a + b)^2 c (a + b \tanh(c z)) \right)$$

01.21.21.0224.01

$$\int \frac{A + B \tanh(z)}{(a + b \tanh(z))^2} \, dz = \frac{\operatorname{sech}(z) (a \cosh(z) + b \sinh(z)) (A + B \tanh(z))}{(A \cosh(z) + B \sinh(z)) (a + b \tanh(z))^2} \left(\frac{b (a B - A b) \sinh(z)}{a^3 - a b^2} + \frac{\left(A a^2 - 2 b B a + A b^2\right) z (a \cosh(z) + b \sinh(z))}{(a - b)^2 (a + b)^2} + \frac{\left(B a^2 - 2 A b a + b^2 B\right) \log(a \cosh(z) + b \sinh(z)) (a \cosh(z) + b \sinh(z))}{\left(a^2 - b^2\right)^2} \right)$$

01.21.21.0225.01

$$\int \frac{A + B\tanh(z)}{(a + b\tanh(z))^3}\, dz = \frac{\operatorname{sech}^2(z)\,(a\cosh(z) + b\sinh(z))\,(A + B\tanh(z))}{2\,(A\cosh(z) + B\sinh(z))\,(a + b\tanh(z))^3}\left(\frac{(a\,B - A\,b)\,b^2}{(a-b)^2\,(a+b)^2} + \frac{2\left(2\,B\,a^2 - 3\,A\,b\,a + b^2\,B\right)\sinh(z)\,(a\cosh(z) + b\sinh(z))\,b}{a\,(a-b)^2\,(a+b)^2} + \frac{2\left(A\,a^3 - 3\,b\,B\,a^2 + 3\,A\,b^2\,a - b^3\,B\right)z\,(a\cosh(z) + b\sinh(z))^2}{(a-b)^3\,(a+b)^3} + \frac{2\left(B\,a^3 - 3\,A\,b\,a^2 + 3\,b^2\,B\,a - A\,b^3\right)\log(a\cosh(z) + b\sinh(z))\,(a\cosh(z) + b\sinh(z))^2}{\left(a^2 - b^2\right)^3}\right)$$

01.21.21.0226.01

$$\int \frac{A + B\tanh(z) + C\tanh^2(z)}{(a + b\tanh(z))^3}\, dz = \left(\operatorname{sech}(z)\,(a\cosh(z) + b\sinh(z))\left(\frac{2\left((A+C)\,a^3 - 3\,b\,B\,a^2 + 3\,b^2\,(A+C)\,a - b^3\,B\right)z\,(a\cosh(z) + b\sinh(z))^2}{(a-b)^3\,(a+b)^3} - \frac{2\left(C\,a^3 - 2\,b\,B\,a^2 + b^2\,(3\,A + 2\,C)\,a - b^3\,B\right)\sinh(z)\,(a\cosh(z) + b\sinh(z))}{a\,(a-b)^2\,(a+b)^2} + \frac{1}{\left(a^2 - b^2\right)^3}\left(2\left(B\,a^3 - 3\,b\,(A+C)\,a^2 + 3\,b^2\,B\,a - b^3\,(A+C)\right)\log(a\cosh(z) + b\sinh(z))\,(a\cosh(z) + b\sinh(z))^2\right) - \frac{b\left(A\,b^2 + a\,(a\,C - b\,B)\right)}{(a-b)^2\,(a+b)^2}\right)\left(C\tanh^2(z) + B\tanh(z) + A\right)\right) \Big/ \left((A - C + (A+C)\cosh(2\,z) + B\sinh(2\,z))\,(a + b\tanh(z))^3\right)$$

Involving $\left(a + b\tanh^2(c\,z)\right)^{-n}$

01.21.21.0227.01

$$\int \frac{1}{a + b\tanh^2(c\,z)}\, dz = \frac{1}{a+b}\left(z + \frac{\sqrt{b}}{\sqrt{a}\;c}\tan^{-1}\left(\frac{\sqrt{b}\,\tanh(c\,z)}{\sqrt{a}}\right)\right)$$

01.21.21.0228.01

$$\int \frac{1}{\left(a + b\tanh^2(c\,z)\right)^2}\, dz = \left((a - b + (a+b)\cosh(2\,c\,z))\operatorname{sech}^4(c\,z)\left(2\,z\,(a - b + (a+b)\cosh(2\,c\,z)) + \frac{\sqrt{b}\,(3\,a + b)\tan^{-1}\left(\frac{\sqrt{b}\,\tanh(c\,z)}{\sqrt{a}}\right)(a - b + (a+b)\cosh(2\,c\,z))}{a^{3/2}\,c} + \frac{b\,(a+b)\sinh(2\,c\,z)}{a\,c}\right)\right) \Big/ \left(8\,(a+b)^2\left(b\tanh^2(c\,z) + a\right)^2\right)$$

Involving algebraic functions of the direct function

Involving $(a+b\tanh(c z))^{\beta}$

01.21.21.0229.01

$$\int (a+b\tanh(c z))^{\beta}\, dz = \frac{(a+b\tanh(c z))^{\beta+1}}{2(b-a)(a+b)c(\beta+1)}\left((a+b)\, {}_2F_1\left(\beta+1, 1; \beta+2; \frac{a+b\tanh(c z)}{a-b}\right)+(b-a)\, {}_2F_1\left(\beta+1, 1; \beta+2; \frac{a+b\tanh(c z)}{a+b}\right)\right)$$

01.21.21.0230.01

$$\int \sqrt{a+b\tanh(c z)}\, dz = \frac{(b-a)\sqrt{a+b}\, \tanh^{-1}\left(\frac{\sqrt{a+b\tanh(c z)}}{\sqrt{a-b}}\right)+\sqrt{a-b}\,(a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\tanh(c z)}}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,\sqrt{a+b}\, c}$$

01.21.21.0231.01

$$\int \frac{1}{\sqrt{a+b\tanh(c z)}}\, dz = \frac{1}{c}\left(\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh(c z)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh(c z)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}\right)$$

01.21.21.0232.01

$$\int \tanh(c z)(a+b\tanh(c z))^{\beta}\, dz = \frac{(a+b\tanh(c z))^{\beta+1}}{2(a-b)(a+b)c(\beta+1)}\left((a+b)\, {}_2F_1\left(\beta+1, 1; \beta+2; \frac{a+b\tanh(c z)}{a-b}\right)+(a-b)\, {}_2F_1\left(\beta+1, 1; \beta+2; \frac{a+b\tanh(c z)}{a+b}\right)\right)$$

01.21.21.0233.01

$$\int \tanh(c z)\sqrt{a+b\tanh(c z)}\, dz = \frac{1}{\sqrt{a-b}\,\sqrt{a+b}\, c}\left((a-b)\sqrt{a+b}\, \tanh^{-1}\left(\frac{\sqrt{a+b\tanh(c z)}}{\sqrt{a-b}}\right)+\sqrt{a-b}\left((a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\tanh(c z)}}{\sqrt{a+b}}\right)-2\sqrt{a+b}\,\sqrt{a+b\tanh(c z)}\right)\right)$$

01.21.21.0234.01

$$\int \frac{\tanh(c z)}{\sqrt{a+b\tanh(c z)}}\, dz = \frac{1}{c}\left(\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh(c z)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh(c z)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}\right)$$

Involving $((a+b\tanh(c z))^{n})^{\beta}$

01.21.21.0235.01

$$\int ((a+b\tanh(c z))^{n})^{\beta}\, dz = \frac{1}{2(b-a)(a+b)c(n\beta+1)}\left(\left((a+b)\, {}_2F_1\left(n\beta+1, 1; n\beta+2; \frac{a+b\tanh(c z)}{a-b}\right)+(b-a)\, {}_2F_1\left(n\beta+1, 1; n\beta+2; \frac{a+b\tanh(c z)}{a+b}\right)\right)(a+b\tanh(c z))((a+b\tanh(c z))^{n})^{\beta}\right)$$

01.21.21.0236.01

$$\int \sqrt{(a+b\tanh(c z))^3}\, dz = \left(\cosh(c z)\sqrt{(a+b\tanh(c z))^3}\left(\sqrt{a-b}\left((a+b)^2\tanh^{-1}\left(\frac{\sqrt{a+b\tanh(c z)}}{\sqrt{a+b}}\right)\cosh(c z)\sqrt{a+b\tanh(c z)} - 2b\sqrt{a+b}\,(a\cosh(c z)+b\sinh(c z))\right) - (a-b)^2\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\tanh(c z)}}{\sqrt{a-b}}\right)\cosh(c z)\sqrt{a+b\tanh(c z)}\right)\right) \Big/ \left(\sqrt{a-b}\,\sqrt{a+b}\,c\,(a\cosh(c z)+b\sinh(c z))^2\right)$$

01.21.21.0237.01

$$\int \frac{1}{\sqrt{(a+b\tanh(c z))^3}}\, dz = \left((a+b\tanh(c z))\left(\sqrt{a-b}\left(2\sqrt{a+b}\,b+(a-b)\tanh^{-1}\left(\frac{\sqrt{a+b\tanh(c z)}}{\sqrt{a+b}}\right)\sqrt{a+b\tanh(c z)}\right) - (a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\tanh(c z)}}{\sqrt{a-b}}\right)\sqrt{a+b\tanh(c z)}\right)\right) \Big/ \left((a-b)^{3/2}(a+b)^{3/2}\,c\sqrt{(a+b\tanh(c z))^3}\right)$$

01.21.21.0238.01

$$\int \tanh(c z)\,((a+b\tanh(c z))^n)^\beta\, dz = \frac{1}{2(a-b)(a+b)c(n\beta+1)}\left(\left((a+b)\,{}_2F_1\left(n\beta+1,1;n\beta+2;\frac{a+b\tanh(c z)}{a-b}\right)+(a-b)\,{}_2F_1\left(n\beta+1,1;n\beta+2;\frac{a+b\tanh(c z)}{a+b}\right)\right)(a+b\tanh(c z))\,((a+b\tanh(c z))^n)^\beta\right)$$

01.21.21.0239.01

$$\int \tanh(c z)\sqrt{(a+b\tanh(c z))^3}\, dz = \left(\sqrt{(a+b\tanh(c z))^3}\left(3\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\tanh(c z)}}{\sqrt{a-b}}\right)\sqrt{a+b\tanh(c z)}\,(a-b)^2 + \left(3(a+b)^2\tanh^{-1}\left(\frac{\sqrt{a+b\tanh(c z)}}{\sqrt{a+b}}\right)\sqrt{a+b\tanh(c z)} - 2\sqrt{a+b}\,\left(4a^2+5b\tanh(c z)\,a+b^2\tanh^2(c z)\right)\right)\sqrt{a-b}\right)\right) \Big/ \left(3\sqrt{a-b}\,\sqrt{a+b}\,c\,(a+b\tanh(c z))^2\right)$$

01.21.21.0240.01

$$\int \frac{\tanh(c\,z)}{\sqrt{(a+b\tanh(c\,z))^3}}\,dz = -\left((a+b\tanh(c\,z))\left(\left(1-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh(c\,z)}}{\sqrt{a-b}}\right)\sqrt{a+b\tanh(c\,z)}}{\sqrt{a-b}}\right)(a+b)+\right.\right.$$

$$\left.\left.(a-b)\left(1-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\tanh(c\,z)}}{\sqrt{a+b}}\right)\sqrt{a+b\tanh(c\,z)}}{\sqrt{a+b}}\right)\right)\right)\Big/\left((a-b)\,(a+b)\,c\sqrt{(a+b\tanh(c\,z))^3}\right)$$

Involving $\left(a+b\tanh^2(c\,z)\right)^\beta$

01.21.21.0241.01

$$\int \left(a+b\tanh^2(c\,z)\right)^\beta dz = \frac{\tanh(c\,z)\left(b\tanh^2(c\,z)+a\right)^\beta\left(\frac{b\tanh^2(c\,z)}{a}+1\right)^{-\beta}}{c}\,F_1\left(\frac{1}{2};1,-\beta;\frac{3}{2};\tanh^2(c\,z),-\frac{b\tanh^2(c\,z)}{a}\right)$$

01.21.21.0242.01

$$\int \sqrt{a+b\tanh^2(c\,z)}\,dz = \frac{1}{2\,c\sqrt{a+b}}$$

$$\left(-(a+b)\left(\log\left(\frac{4\left(a-b\tanh(c\,z)+\sqrt{a+b}\sqrt{b\tanh^2(c\,z)+a}\right)}{(a+b)^{3/2}\,(\tanh(c\,z)+1)}\right)-\log\left(-\frac{4\left(a+b\tanh(c\,z)+\sqrt{a+b}\sqrt{b\tanh^2(c\,z)+a}\right)}{(a+b)^{3/2}\,(\tanh(c\,z)-1)}\right)\right)-\right.$$

$$\left.2\sqrt{b}\sqrt{a+b}\,\log\left(2\left(\sqrt{b}\tanh(c\,z)+\sqrt{b\tanh^2(c\,z)+a}\right)\right)\right)$$

01.21.21.0243.01

$$\int \frac{1}{\sqrt{a+b\tanh^2(c\,z)}}\,dz =$$

$$\frac{\sqrt{a-b+(a+b)\cosh(2\,c\,z)}\,\log\left(\sqrt{a-b+(a+b)\cosh(2\,c\,z)}+\sqrt{2}\sqrt{(a+b)\sinh^2(c\,z)}\right)\tanh(c\,z)}{\sqrt{2}\,c\sqrt{(a+b)\sinh^2(c\,z)}\sqrt{b\tanh^2(c\,z)+a}}$$

01.21.21.0244.01

$$\int \tanh(c\,z)\left(a+b\tanh^2(c\,z)\right)^\beta dz = \frac{\left(b\tanh^2(c\,z)+a\right)^{\beta+1}}{2\,(a+b)\,c\,(\beta+1)}\,{}_2F_1\left(\beta+1,1;\beta+2;\frac{b\tanh^2(c\,z)+a}{a+b}\right)$$

01.21.21.0245.01

$$\int \tanh(c\,z)\sqrt{a+b\tanh^2(c\,z)}\,dz = \frac{\sqrt{a+b\tanh^2(c\,z)}}{c}\left(\frac{\sqrt{2}\sqrt{(a+b)\cosh^2(c\,z)}\,\log\left(\sqrt{2}\sqrt{(a+b)\cosh^2(c\,z)}+\sqrt{a-b+(a+b)\cosh(2\,c\,z)}\right)}{\sqrt{a-b+(a+b)\cosh(2\,c\,z)}}-1\right)$$

01.21.21.0246.01

$$\int \frac{\tanh(c\,z)}{\sqrt{a+b\tanh^2(c\,z)}}\,dz = \frac{\sqrt{a-b+(a+b)\cosh(2\,c\,z)}\,\log\left(\sqrt{2}\sqrt{(a+b)\cosh^2(c\,z)}+\sqrt{a-b+(a+b)\cosh(2\,c\,z)}\right)}{\sqrt{2}\,c\sqrt{(a+b)\cosh^2(c\,z)}\sqrt{b\tanh^2(c\,z)+a}}$$

Involving $\left(\left(a+b\tanh^2(c\,z)\right)^n\right)^\beta$

01.21.21.0247.01

$$\int \left(\left(a+b\tanh^2(c\,z)\right)^n\right)^\beta dz = \frac{\tanh(c\,z)\left(\left(b\tanh^2(c\,z)+a\right)^n\right)^\beta\left(\frac{b\tanh^2(c\,z)}{a}+1\right)^{-n\beta}}{c}F_1\left(\frac{1}{2};1,-n\,\beta;\frac{3}{2};\tanh^2(c\,z),-\frac{b\tanh^2(c\,z)}{a}\right)$$

01.21.21.0248.01

$$\int \sqrt{\left(a+b\tanh^2(c\,z)\right)^3}\,dz = -\frac{1}{2\,c\left(b\tanh^2(c\,z)+a\right)^{3/2}}\left(\sqrt{\left(b\tanh^2(c\,z)+a\right)^3}\left(\left(\log\left(\frac{4\left(a-b\tanh(c\,z)+\sqrt{a+b}\sqrt{b\tanh^2(c\,z)+a}\right)}{(a+b)^{5/2}(\tanh(c\,z)+1)}\right)-\log\left(-\frac{4\left(a+b\tanh(c\,z)+\sqrt{a+b}\sqrt{b\tanh^2(c\,z)+a}\right)}{(a+b)^{5/2}(\tanh(c\,z)-1)}\right)\right)(a+b)^{3/2}+\sqrt{b}\,(3\,a+2\,b)\log\left(2\left(\sqrt{b}\tanh(c\,z)+\sqrt{b\tanh^2(c\,z)+a}\right)\right)+b\tanh(c\,z)\sqrt{b\tanh^2(c\,z)+a}\right)\right)$$

01.21.21.0249.01

$$\int \frac{1}{\sqrt{\left(a+b\tanh^2(c\,z)\right)^3}}\,dz = \left(\operatorname{sech}^2(c\,z)\left(\sqrt{2}\operatorname{csch}(2\,c\,z)\log\left(\sqrt{a-b+(a+b)\cosh(2\,c\,z)}+\sqrt{2}\sqrt{(a+b)\sinh^2(c\,z)}\right)\sqrt{(a+b)\sinh^2(c\,z)}\,(a-b+(a+b)\cosh(2\,c\,z))^{3/2}+\frac{b\,(a+b)\tanh(c\,z)\,(a-b+(a+b)\cosh(2\,c\,z))}{a}\right)\right)\bigg/\left(2\,(a+b)^2\,c\sqrt{\left(b\tanh^2(c\,z)+a\right)^3}\right)$$

01.21.21.0250.01

$$\int \tanh(c\,z)\left(\left(a+b\tanh^2(c\,z)\right)^n\right)^\beta\,dz = \frac{\left(b\tanh^2(c\,z)+a\right)\left(\left(b\tanh^2(c\,z)+a\right)^n\right)^\beta}{2\,(a+b)\,c\,(n\,\beta+1)}\,{}_2F_1\!\left(n\,\beta+1,\,1;\,n\,\beta+2;\,\frac{b\tanh^2(c\,z)+a}{a+b}\right)$$

01.21.21.0251.01

$$\int \tanh(c\,z)\sqrt{\left(a+b\tanh^2(c\,z)\right)^5}\,dz =$$

$$\frac{1}{c}\left(\cosh^5(c\,z)\left(\left(4\sqrt{2}\,\left((a+b)\cosh^2(c\,z)\right)^{5/2}\log\left(\sqrt{2}\,\sqrt{(a+b)\cosh^2(c\,z)}+\sqrt{a-b+(a+b)\cosh(2\,c\,z)}\right)\operatorname{sech}^5(c\,z)\right)\Big/\right.\right.$$
$$(a-b+(a+b)\cosh(2\,c\,z))^{5/2}-$$
$$\left.\left.\frac{4\operatorname{sech}(c\,z)\left(3\,b^2\operatorname{sech}^4(c\,z)-11\,b\,(a+b)\operatorname{sech}^2(c\,z)+23\,(a+b)^2\right)}{15\,(a-b+(a+b)\cosh(2\,c\,z))^2}\right)\sqrt{\left(b\tanh^2(c\,z)+a\right)^5}\right)$$

01.21.21.0252.01

$$\int \tanh(c\,z)\sqrt{\left(a+b\tanh^2(c\,z)\right)^3}\,dz =$$

$$\left(\operatorname{csch}^2(c\,z)\operatorname{sech}^2(c\,z)\left(b\sqrt{a-b+(a+b)\cosh(2\,c\,z)}\,\sinh^2(2\,c\,z)-4\,(a+b)\cosh^4(c\,z)\left(4\sqrt{a-b+(a+b)\cosh(2\,c\,z)}-\right.\right.\right.$$
$$\left.\left.3\sqrt{2}\,\sqrt{(a+b)\cosh^2(c\,z)}\,\log\left(\sqrt{2}\,\sqrt{(a+b)\cosh^2(c\,z)}+\sqrt{a-b+(a+b)\cosh(2\,c\,z)}\right)\right)\sinh^2(c\,z)\right)$$
$$\left.\sqrt{\left(b\tanh^2(c\,z)+a\right)^3}\right)\Big/\left(6\,c\,(a-b+(a+b)\cosh(2\,c\,z))^{3/2}\right)$$

01.21.21.0253.01

$$\int \frac{\tanh(c\,z)}{\sqrt{\left(a+b\tanh^2(c\,z)\right)^3}}\,dz =$$

$$-\left((a-b+(a+b)\cosh(2\,c\,z))\left(a+b+(a+b)\cosh(2\,c\,z)-\sqrt{2}\,\sqrt{(a+b)\cosh^2(c\,z)}\,\sqrt{a-b+(a+b)\cosh(2\,c\,z)}\right.\right.$$
$$\left.\left.\log\left(\sqrt{2}\,\sqrt{(a+b)\cosh^2(c\,z)}+\sqrt{a-b+(a+b)\cosh(2\,c\,z)}\right)\right)\operatorname{sech}^4(c\,z)\right)\Big/\left(4\,(a+b)^2\,c\sqrt{\left(b\tanh^2(c\,z)+a\right)^3}\right)$$

01.21.21.0254.01

$$\int \frac{\tanh(c z)}{\sqrt{\left(a+b\tanh^2(c z)\right)^5}}\,dz = -\left((a-b+(a+b)\cosh(2 c z))\left(6 a^2+6 b a-3\sqrt{2}\sqrt{(a+b)\cosh^2(c z)}\sqrt{a-b+(a+b)\cosh(2 c z)}\log\left(\sqrt{2}\sqrt{(a+b)\cosh^2(c z)}+\sqrt{a-b+(a+b)\cosh(2 c z)}\right) a + 2(a+b)^2\cosh(4 c z)+3\sqrt{2}\, b\sqrt{(a+b)\cosh^2(c z)}\sqrt{a-b+(a+b)\cosh(2 c z)}\log\left(\sqrt{2}\sqrt{(a+b)\cosh^2(c z)}+\sqrt{a-b+(a+b)\cosh(2 c z)}\right)+(a+b)\cosh(2 c z)\left(8 a+2 b-3\sqrt{2}\sqrt{(a+b)\cosh^2(c z)}\sqrt{a-b+(a+b)\cosh(2 c z)}\log\left(\sqrt{2}\sqrt{(a+b)\cosh^2(c z)}+\sqrt{a-b+(a+b)\cosh(2 c z)}\right)\right)\right)\operatorname{sech}^6(c z)\right)\Big/\left(24(a+b)^3 c\sqrt{\left(b\tanh^2(c z)+a\right)^5}\right)$$

Involving $\left(a+b\tanh^{\frac{1}{2}}(c z)\right)^\beta$

01.21.21.0255.01

$$\int \left(a+b\tanh^{\frac{1}{2}}(c z)\right)^\beta dz = \frac{1}{2\left(a^4-b^4\right)c(\beta+1)}\left(\left(a+b\tanh^{\frac{1}{2}}(c z)\right)^{\beta+1}\left(\left(a^3+b a^2+b^2 a+b^3\right){}_2F_1\left(\beta+1,1;\beta+2;\frac{a+b\tanh^{\frac{1}{2}}(c z)}{a-b}\right)-(a-b)\left((a-i b)\left((a+b)\,{}_2F_1\left(\beta+1,1;\beta+2;\frac{a+b\tanh^{\frac{1}{2}}(c z)}{a+i b}\right)-(a+i b)\,{}_2F_1\left(\beta+1,1;\beta+2;\frac{a+b\tanh^{\frac{1}{2}}(c z)}{a+b}\right)\right)+\left(a^2+b(1+i)a+b^2 i\right){}_2F_1\left(\beta+1,1;\beta+2;\frac{a+b\tanh^{\frac{1}{2}}(c z)}{a-i b}\right)\right)\right)\right)$$

01.21.21.0256.01

$$\int \sqrt{a+b\tanh^{\frac{1}{2}}(c\,z)}\,dz = \left((a-b)\sqrt{a+b}\,\sqrt{a+i\,b}\,\sqrt{a-i\,b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^{\frac{1}{2}}(c\,z)}}{\sqrt{a-b}}\right)+\right.$$
$$\sqrt{a-b}\left(\sqrt{a-i\,b}\left(\sqrt{a+i\,b}\,(a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^{\frac{1}{2}}(c\,z)}}{\sqrt{a+b}}\right)-(a+i\,b)\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^{\frac{1}{2}}(c\,z)}}{\sqrt{a+i\,b}}\right)\right)-\right.$$
$$\left.\left.(a-i\,b)\sqrt{a+i\,b}\,\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^{\frac{1}{2}}(c\,z)}}{\sqrt{a-i\,b}}\right)\right)\right)\Big/\left(\sqrt{a-b}\,\sqrt{a-i\,b}\,\sqrt{a+i\,b}\,\sqrt{a+b}\,c\right)$$

01.21.21.0257.01

$$\int \frac{1}{\sqrt{a+b\tanh^{\frac{1}{2}}(c\,z)}}\,dz = \frac{1}{\left(a^4-b^4\right)c}\left(\sqrt{a-b}\,\left(a^3+b\,a^2+b^2\,a+b^3\right)\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^{\frac{1}{2}}(c\,z)}}{\sqrt{a-b}}\right)-\right.$$
$$(a-b)\left((a-i\,b)\left(\sqrt{a+i\,b}\,(a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^{\frac{1}{2}}(c\,z)}}{\sqrt{a+i\,b}}\right)-(a+i\,b)\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^{\frac{1}{2}}(c\,z)}}{\sqrt{a+b}}\right)\right)+\right.$$
$$\left.\left.\sqrt{a-i\,b}\,\left(a^2+b\,(1+i)\,a+b^2\,i\right)\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^{\frac{1}{2}}(c\,z)}}{\sqrt{a-i\,b}}\right)\right)\right)$$

01.21.21.0258.01

$$\int \tanh(c\,z)\left(a+b\tanh^{\frac{1}{2}}(c\,z)\right)^{\beta}dz = \frac{1}{2\,c\left(a^4-b^4\right)(\beta+1)}\left(\left(\left(a^3+b\,a^2+b^2\,a+b^3\right){}_2F_1\left(\beta+1,\,1;\,\beta+2;\,\frac{a+b\tanh^{\frac{1}{2}}(c\,z)}{a-b}\right)+\right.\right.$$
$$(a-b)\left((a-i\,b)\left((a+i\,b)\,{}_2F_1\left(\beta+1,\,1;\,\beta+2;\,\frac{a+b\tanh^{\frac{1}{2}}(c\,z)}{a+b}\right)+(a+b)\,{}_2F_1\left(\beta+1,\,1;\,\beta+2;\,\frac{a+b\tanh^{\frac{1}{2}}(c\,z)}{a+i\,b}\right)\right)+\right.$$
$$\left.\left.\left.\left(a^2+b\,(1+i)\,a+b^2\,i\right){}_2F_1\left(\beta+1,\,1;\,\beta+2;\,\frac{a+b\tanh^{\frac{1}{2}}(c\,z)}{a-i\,b}\right)\right)\right)\left(a+b\tanh^{\frac{1}{2}}(c\,z)\right)^{\beta+1}\right)$$

01.21.21.0259.01

$$\int \tanh(c z)\sqrt{a+b\tanh^{\frac{1}{2}}(c z)}\,dz = \left((a-b)\sqrt{a+b}\,\sqrt{a+i b}\,\sqrt{a-i b}\,\sqrt{a+b\tanh^{\frac{1}{2}}(c z)}\,\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^{\frac{1}{2}}(c z)}}{\sqrt{a-b}}\right)+\right.$$
$$\sqrt{a-b}\left(\sqrt{a+b}\,\sqrt{a+i b}\,(a-i b)\sqrt{a+b\tanh^{\frac{1}{2}}(c z)}\,\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^{\frac{1}{2}}(c z)}}{\sqrt{a-i b}}\right)+\right.$$
$$\sqrt{a-i b}\left(\sqrt{a+b}\,(a+i b)\sqrt{a+b\tanh^{\frac{1}{2}}(c z)}\,\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^{\frac{1}{2}}(c z)}}{\sqrt{a+i b}}\right)+\right.$$
$$\left.\left.\left.\left.\sqrt{a+i b}\left((a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^{\frac{1}{2}}(c z)}}{\sqrt{a+b}}\right)\sqrt{a+b\tanh^{\frac{1}{2}}(c z)}\,-4\sqrt{a+b}\left(a+b\tanh^{\frac{1}{2}}(c z)\right)\right)\right)\right)\right)\right)\Big/$$
$$\left(\sqrt{a-b}\,\sqrt{a-i b}\,\sqrt{a+i b}\,\sqrt{a+b}\,c\sqrt{a+b\tanh^{\frac{1}{2}}(c z)}\right)$$

01.21.21.0260.01

$$\int \frac{\tanh(c z)}{\sqrt{a+b\tanh^{\frac{1}{2}}(c z)}}\,dz = \frac{1}{\left(a^4-b^4\right)c}\left(\sqrt{a-b}\left(a^3+b a^2+b^2 a+b^3\right)\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^{\frac{1}{2}}(c z)}}{\sqrt{a-b}}\right)+\right.$$
$$(a-b)\left((a-i b)\left(\sqrt{a+b}\,(a+i b)\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^{\frac{1}{2}}(c z)}}{\sqrt{a+b}}\right)+(a+b)\sqrt{a+i b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^{\frac{1}{2}}(c z)}}{\sqrt{a+i b}}\right)\right)+\right.$$
$$\left.\left.\sqrt{a-i b}\left(a^2+b(1+i)a+b^2 i\right)\tanh^{-1}\left(\frac{\sqrt{a+b\tanh^{\frac{1}{2}}(c z)}}{\sqrt{a-i b}}\right)\right)\right)$$

01.21.21.0261.01

$$\int \frac{\tanh(c\,z)}{\left(a+b\tanh^{\frac{1}{2}}(c\,z)\right)^2}\,dz = -\frac{1}{4\,c}\left(\frac{8\,a^3}{\left(a^4-b^4\right)\left(a+b\tanh^{\frac{1}{2}}(c\,z)\right)}+\frac{2\,i\tan^{-1}\left(\tanh^{\frac{1}{2}}(c\,z)\right)}{(a+i\,b)^2}+\frac{2\log\left(\tanh^{\frac{1}{2}}(c\,z)-1\right)}{(a+b)^2}+\frac{2\log\left(\tanh^{\frac{1}{2}}(c\,z)+1\right)}{(a-b)^2}+\frac{\log(\tanh(c\,z)+1)}{(a+i\,b)^2}+\frac{\log(\tanh(c\,z)+1)}{(a-i\,b)^2}-\frac{2\,i\tan^{-1}\left(\tanh^{\frac{1}{2}}(c\,z)\right)}{(a-i\,b)^2}-\frac{8\left(a^6+3\,b^4\,a^2\right)\log\left(a+b\tanh^{\frac{1}{2}}(c\,z)\right)}{\left(a^4-b^4\right)^2}\right)$$

Involving functions of the direct function and a power function

Involving powers of the direct function and a power function

Involving powers of tanh and power

Involving z^n and linear arguments

01.21.21.0262.01

$$\int z^n\tanh^\nu(c\,z)\,dz = \frac{(-1)^\nu z^{n+1}}{n+1}-(-1)^\nu e^{2\,c\,z}\,\nu\,n!\sum_{j=0}^{n}\frac{(-1)^j\,2^{-j-1}\,c^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+3}F_{j+2}\left(1,\,\ldots,\,1,\,\nu+1;\,2,\,\ldots,\,2;\,-e^{2\,c\,z}\right)+ e^{c\,\nu\,z}\,i^\nu\binom{\nu}{\frac{\nu}{2}}n!\,(1-\nu\bmod 2)\sum_{j=0}^{n}\frac{(-1)^j\,(c\,\nu)^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{\nu}{2},\,\ldots,\,\frac{\nu}{2},\,\nu;\,\frac{\nu}{2}+1,\,\ldots,\,\frac{\nu}{2}+1;\,-e^{2\,c\,z}\right)+ (-1)^\nu\,n!\sum_{s=1}^{\left\lfloor\frac{\nu-1}{2}\right\rfloor}(-1)^s\binom{\nu}{s}e^{2\,s\,c\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(2\,c\,s)^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(s,\,\ldots,\,s,\,\nu;\,s+1,\,\ldots,\,s+1;\,-e^{2\,c\,z}\right)+ n!\sum_{s=0}^{\left\lfloor\frac{\nu-1}{2}\right\rfloor}(-1)^s\binom{\nu}{s}e^{2\,c\,(\nu-s)\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(2\,c\,(\nu-s))^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\nu-s,\,\ldots,\,\nu-s,\,\nu;\,\nu-s+1,\,\ldots,\,\nu-s+1;\,-e^{2\,c\,z}\right)\;/;\;n\in\mathbb{N}\land\nu\in\mathbb{N}^+$$

01.21.21.0263.01

$$\int z\tanh^{\nu}(c\,z)\,dz = \frac{1}{2}(-1)^{\nu} z^2 + e^{2c\nu z}\left(\frac{z\,{}_2F_1\left(\nu,\nu;\nu+1;-e^{2cz}\right)}{2c\nu} - \frac{{}_3F_2\left(\nu,\nu,\nu;\nu+1,\nu+1;-e^{2cz}\right)}{4c^2\nu^2}\right) - (-1)^{\nu} e^{2cz}\,\nu\left(\frac{z\,{}_3F_2\left(1,1,\nu+1;2,2;-e^{2cz}\right)}{2c} - \frac{{}_4F_3\left(1,1,1,\nu+1;2,2,2;-e^{2cz}\right)}{4c^2}\right) + i^{\nu} e^{c\nu z}\binom{\nu}{\frac{\nu}{2}}\left(\frac{z\,{}_2F_1\left(\frac{\nu}{2},\nu;\frac{\nu}{2}+1;-e^{2cz}\right)}{c\nu} - \frac{{}_3F_2\left(\frac{\nu}{2},\frac{\nu}{2},\nu;\frac{\nu}{2}+1,\frac{\nu}{2}+1;-e^{2cz}\right)}{c^2\nu^2}\right)(1-\nu \bmod 2) + \frac{1}{4c^2}\sum_{s=1}^{\left\lfloor\frac{\nu-1}{2}\right\rfloor}(-1)^s\binom{\nu}{s}\left(\frac{(-1)^{\nu}e^{2csz}}{s^2}\left(2csz\,{}_2F_1\left(s,\nu;s+1;-e^{2cz}\right) - {}_3F_2\left(s,s,\nu;s+1,s+1;-e^{2cz}\right)\right) - \frac{1}{(s-\nu)^2}e^{2c(\nu-s)z}\left(2c(s-\nu)z\,{}_2F_1\left(\nu,\nu-s;-s+\nu+1;-e^{2cz}\right) + {}_3F_2\left(\nu,\nu-s,\nu-s;-s+\nu+1,-s+\nu+1;-e^{2cz}\right)\right)\right) /; \nu\in\mathbb{N}^+$$

01.21.21.0264.01

$$\int z\tanh^2(c\,z)\,dz = \frac{z^2}{2} - \frac{\tanh(c\,z)\,z}{c} + \frac{\log(\cosh(c\,z))}{c^2}$$

01.21.21.0265.01

$$\int z\tanh^3(c\,z)\,dz = \frac{c^2z^2 + c\,\mathrm{sech}^2(c\,z)\,z + 2c\log\left(1+e^{-2cz}\right)z - \mathrm{Li}_2\left(-e^{-2cz}\right) - \tanh(c\,z)}{2c^2}$$

01.21.21.0266.01

$$\int z\tanh^4(c\,z)\,dz = \frac{3c^2z^2 - 8c\tanh(c\,z)\,z + 8\log(\cosh(c\,z)) + \mathrm{sech}^2(c\,z)\,(2c\,z\tanh(c\,z)+1)}{6c^2}$$

01.21.21.0267.01

$$\int z\tanh^5(c\,z)\,dz = \frac{1}{12c^2}\left(-3c\,z\,\mathrm{sech}^4(c\,z) + (12c\,z + \tanh(c\,z))\,\mathrm{sech}^2(c\,z) - 6\,\mathrm{Li}_2\left(-e^{-2cz}\right) + 2\left(3c\,z\left(c\,z + 2\log\left(1+e^{-2cz}\right)\right) - 5\tanh(c\,z)\right)\right)$$

01.21.21.0268.01

$$\int z^2\tanh^2(c\,z)\,dz = \frac{c\,z\left(c\,z\,(c\,z+3) + 6\log\left(1+e^{-2cz}\right) - 3c\,z\tanh(c\,z)\right) - 3\,\mathrm{Li}_2\left(-e^{-2cz}\right)}{3c^3}$$

01.21.21.0269.01

$$\int z^3\tanh^3(c\,z)\,dz = \frac{1}{4c^4}\left(c^4z^4 + 2c^3\,\mathrm{sech}^2(c\,z)\,z^3 + 4c^3\log\left(1+e^{-2cz}\right)z^3 + 6c^2z^2 - 6c^2\tanh(c\,z)\,z^2 + 12c\log\left(1+e^{-2cz}\right)z - 6c\,\mathrm{Li}_3\left(-e^{-2cz}\right)z - 6\left(c^2z^2+1\right)\mathrm{Li}_2\left(-e^{-2cz}\right) - 3\,\mathrm{Li}_4\left(-e^{-2cz}\right)\right)$$

Involving functions of the direct function and exponential function

Involving powers of the direct function and exponential function

Involving exp

Involving $e^{b z}$

01.21.21.0270.01

$$\int e^{b z} \tanh^{\nu}(c z)\, dz = \frac{e^{b z} \left(1 - e^{-2 c z}\right)^{-\nu} \left(1 + e^{-2 c z}\right)^{\nu} \tanh^{\nu}(c z)}{b} F_1\left(-\frac{b}{2 c}; \nu, -\nu; 1 - \frac{b}{2 c}; -e^{-2 c z}, e^{-2 c z}\right)$$

01.21.21.0271.01

$$\int e^{2 c z} \tanh^{\nu}(c z)\, dz = \frac{2^{\nu-1} \left(1 - e^{2 c z}\right)^{-\nu} \left(1 + e^{2 c z}\right) \tanh^{\nu}(c z)}{c (1 - \nu)} \, {}_2F_1\left(1 - \nu, -\nu; 2 - \nu; \frac{1}{2} \left(1 + e^{2 c z}\right)\right)$$

01.21.21.0272.01

$$\int e^{c z} \tanh^2(c z)\, dz = \frac{1}{c} \left(e^{c z} \left(1 + \frac{2}{1 + e^{2 c z}}\right) - 2 \tan^{-1}\left(e^{c z}\right)\right)$$

01.21.21.0273.01

$$\int e^{2 c z} \tanh^2(c z)\, dz = \frac{1}{2 c} \left(-4 \log\left(1 + e^{2 c z}\right) + e^{2 c z} - \frac{4}{1 + e^{2 c z}}\right)$$

01.21.21.0274.01

$$\int e^{2 c z} \tanh^4(c z)\, dz = -\frac{24 \log\left(1 + e^{2 c z}\right) \left(1 + e^{2 c z}\right)^3 + 93 e^{2 c z} + 63 e^{4 c z} - 9 e^{6 c z} - 3 e^{8 c z} + 40}{6 c \left(1 + e^{2 c z}\right)^3}$$

01.21.21.0275.01

$$\int e^{-2 c z} \tanh^4(c z)\, dz = \frac{1}{6 c} \left(\frac{8 e^{2 c z} \left(9 + 12 e^{2 c z} + 5 e^{4 c z}\right)}{\left(1 + e^{2 c z}\right)^3} - 3 e^{-2 c z} + 24 \log\left(1 + e^{-2 c z}\right)\right)$$

Involving functions of the direct function, exponential and a power functions

Involving powers of the direct function, exponential and a power functions

Involving exp and power

Involving $z^n e^{b z}$

01.21.21.0276.01

$$\int z^n e^{bz} \tanh^\nu(cz)\,dz = i^\nu e^{(b+c\nu)z} \binom{\nu}{\frac{\nu}{2}} n!\,(1-\nu \bmod 2) \sum_{j=0}^{n} \frac{(-1)^j (b+c\nu)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{b+c\nu}{2c}, \ldots, \frac{b+c\nu}{2c}, \nu; \frac{b+c\nu}{2c}+1, \ldots, \frac{b+c\nu}{2c}+1; -e^{2cz}\right) + n! \sum_{s=0}^{\left\lfloor \frac{\nu-1}{2} \right\rfloor} (-1)^s \binom{\nu}{s} \left((-1)^\nu e^{(b+2cs)z} \sum_{j=0}^{n} \frac{(-1)^j (b+2cs)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{b+2cs}{2c}, \ldots, \frac{b+2cs}{2c}, \nu; \frac{b+2cs}{2c}+1, \ldots, \frac{b+2cs}{2c}+1; -e^{2cz}\right) + e^{(b+2c(\nu-s))z} \sum_{j=0}^{n} \frac{(-1)^j (b+2c(\nu-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{b+2c(\nu-s)}{2c}, \ldots, \frac{b+2c(\nu-s)}{2c}, \nu; \frac{b+2c(\nu-s)}{2c}+1, \ldots, \frac{b+2c(\nu-s)}{2c}+1; -e^{2cz}\right)\right) /; n \in \mathbb{N} \land \nu \in \mathbb{N}^+$$

Involving functions of the direct function and trigonometric functions

Involving powers of the direct function and trigonometric functions

Involving sin

Involving sin($b z$)

01.21.21.0277.01

$$\int \sin(bz) \tanh^\nu(cz)\,dz = -\frac{1}{2b} e^{-ibz} \left(1-e^{-2cz}\right)^{-\nu} \left(1+e^{-2cz}\right)^\nu \tanh^\nu(cz) \left(F_1\left(\frac{ib}{2c}; -\nu, \nu; 1+\frac{ib}{2c}; e^{-2cz}, -e^{-2cz}\right) + e^{2ibz} F_1\left(-\frac{ib}{2c}; -\nu, \nu; 1-\frac{ib}{2c}; e^{-2cz}, -e^{-2cz}\right)\right)$$

Involving powers of sin

Involving $\sin^m(b z)$

01.21.21.0278.01

$$\int \sin^m(bz) \tanh^\nu(cz)\,dz = \frac{2^{-m} \tanh^{\nu+1}(cz)\,(1-m \bmod 2)}{c(\nu+1)} \binom{m}{\frac{m}{2}} {}_2F_1\left(\frac{\nu+1}{2}, 1; \frac{\nu+1}{2}+1; \tanh^2(cz)\right) + \frac{1}{b} 2^{-m} i^{1-m} \left(1-e^{-2cz}\right)^{-\nu} \left(1+e^{-2cz}\right)^\nu \tanh^\nu(cz) \sum_{k=0}^{\left\lfloor \frac{m-1}{2} \right\rfloor} \frac{(-1)^k}{2k-m} e^{-ib(m-2k)z} \binom{m}{k} \left(e^{2ib(m-2k)z} F_1\left(-\frac{ib(m-2k)}{2c}; -\nu, \nu; 1-\frac{ib(m-2k)}{2c}; e^{-2cz}, -e^{-2cz}\right) - (-1)^m F_1\left(\frac{ib(m-2k)}{2c}; -\nu, \nu; \frac{bi(m-2k)}{2c}+1; e^{-2cz}, -e^{-2cz}\right)\right) /; m \in \mathbb{N}^+$$

Involving cos

Involving cos(b z)

01.21.21.0279.01

$$\int \cos(b z) \tanh^{\nu}(c z)\, dz = -\frac{i}{2b} e^{-i b z} \left(1 - e^{-2 c z}\right)^{-\nu} \left(1 + e^{-2 c z}\right)^{\nu} \tanh^{\nu}(c z) \left(e^{2 i b z} F_1\left(-\frac{i b}{2c}; -\nu, \nu; 1 - \frac{i b}{2c}; e^{-2 c z}, -e^{-2 c z}\right) - F_1\left(\frac{i b}{2c}; -\nu, \nu; 1 + \frac{i b}{2c}; e^{-2 c z}, -e^{-2 c z}\right)\right)$$

Involving powers of cos

Involving $\cos^m(b z)$

01.21.21.0280.01

$$\int \cos^m(b z) \tanh^{\nu}(c z)\, dz = \frac{2^{-m} (1 - m \bmod 2) \tanh^{\nu+1}(c z)}{c (\nu + 1)} \binom{m}{\frac{m}{2}} {}_2F_1\left(\frac{\nu+1}{2}, 1; \frac{\nu+1}{2} + 1; \tanh^2(c z)\right) + \frac{1}{b} 2^{-m} i \left(1 - e^{-2 c z}\right)^{-\nu} \left(1 + e^{-2 c z}\right)^{\nu} \tanh^{\nu}(c z) \sum_{k=0}^{\left\lfloor \frac{m-1}{2} \right\rfloor} \frac{1}{m - 2k} \binom{m}{k} \left(e^{-i b (m-2k) z} F_1\left(\frac{i b (m - 2k)}{2c}; -\nu, \nu; \frac{b i (m - 2k)}{2c} + 1; e^{-2 c z}, -e^{-2 c z}\right) - e^{i b (m-2k) z} F_1\left(-\frac{i b (m - 2k)}{2c}; -\nu, \nu; 1 - \frac{i b (m - 2k)}{2c}; e^{-2 c z}, -e^{-2 c z}\right)\right) /; m \in \mathbb{N}^+$$

Involving functions of the direct function, trigonometric and a power functions

Involving powers of the direct function, trigonometric and a power functions

Involving sin and power

Involving $z^n \sin(a + b z) \tanh^{\nu}(c z)$

01.21.21.0281.01

$$\int z^n \sin(a+b\,z)\tanh^\nu(c\,z)\,dz = \frac{i}{2}\,n!\,e^{-i\,a}\Bigg(i^\nu\,e^{(-i\,b+c\,\nu)\,z}\binom{\nu}{\frac{\nu}{2}}(1-\nu \bmod 2)\sum_{j=0}^{n}\frac{(-1)^j\,(-i\,b+c\,\nu)^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\Big(\frac{-i\,b+c\,\nu}{2\,c},\,\ldots,\,\frac{-i\,b+c\,\nu}{2\,c},\,\nu;\,\frac{-i\,b+c\,\nu}{2\,c}+1,\,\ldots,\,\frac{-i\,b+c\,\nu}{2\,c}+1;\,-e^{2\,c\,z}\Big)+\sum_{s=0}^{\left\lfloor\frac{\nu-1}{2}\right\rfloor}(-1)^s\binom{\nu}{s}\Bigg((-1)^\nu\,e^{(-i\,b+2\,c\,s)\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(-i\,b+2\,c\,s)^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\Big(\frac{-i\,b+2\,c\,s}{2\,c},\,\ldots,\,\frac{-i\,b+2\,c\,s}{2\,c},\,\nu;\,\frac{-i\,b+2\,c\,s}{2\,c}+1,\,\ldots,\,\frac{-i\,b+2\,c\,s}{2\,c}+1;\,-e^{2\,c\,z}\Big)+e^{(-i\,b+2\,c\,(\nu-s))\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(-i\,b+2\,c\,(\nu-s))^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\Big(\frac{-i\,b+2\,c\,(\nu-s)}{2\,c},\,\ldots,\,\frac{-i\,b+2\,c\,(\nu-s)}{2\,c},\,\nu;\,\frac{-i\,b+2\,c\,(\nu-s)}{2\,c}+1,\,\ldots,\,\frac{-i\,b+2\,c\,(\nu-s)}{2\,c}+1;\,-e^{2\,c\,z}\Big)\Bigg)\Bigg) - \frac{i}{2}\,n!\,e^{i\,a}\Bigg(i^\nu\,e^{(i\,b+c\,\nu)\,z}\binom{\nu}{\frac{\nu}{2}}(1-\nu \bmod 2)\sum_{j=0}^{n}\frac{(-1)^j\,(i\,b+c\,\nu)^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\Big(\frac{i\,b+c\,\nu}{2\,c},\,\ldots,\,\frac{i\,b+c\,\nu}{2\,c},\,\nu;\,\frac{i\,b+c\,\nu}{2\,c}+1,\,\ldots,\,\frac{i\,b+c\,\nu}{2\,c}+1;\,-e^{2\,c\,z}\Big)+\sum_{s=0}^{\left\lfloor\frac{\nu-1}{2}\right\rfloor}(-1)^s\binom{\nu}{s}\Bigg((-1)^\nu\,e^{(i\,b+2\,c\,s)\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(i\,b+2\,c\,s)^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\Big(\frac{i\,b+2\,c\,s}{2\,c},\,\ldots,\,\frac{i\,b+2\,c\,s}{2\,c},\,\nu;\,\frac{i\,b+2\,c\,s}{2\,c}+1,\,\ldots,\,\frac{i\,b+2\,c\,s}{2\,c}+1;\,-e^{2\,c\,z}\Big)+e^{(i\,b+2\,c\,(\nu-s))\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(i\,b+2\,c\,(\nu-s))^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\Big(\frac{i\,b+2\,c\,(\nu-s)}{2\,c},\,\ldots,\,\frac{i\,b+2\,c\,(\nu-s)}{2\,c},\,\nu;\,\frac{i\,b+2\,c\,(\nu-s)}{2\,c}+1,\,\ldots,\,\frac{i\,b+2\,c\,(\nu-s)}{2\,c}+1;\,-e^{2\,c\,z}\Big)\Bigg)\Bigg)\;/;\;n\in\mathbb{N}\land\nu\in\mathbb{N}^+$$

01.21.21.0282.01

$$\int z^n \sin(b z) \tanh^\nu(c z)\, dz = \frac{i}{2} n! \Bigg(i^\nu e^{(c\nu - i b) z} \binom{\nu}{\frac{\nu}{2}} (1 - \nu \bmod 2) \sum_{j=0}^{n} \frac{(-1)^j (c\nu - i b)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{c\nu - i b}{2c}, \ldots, \frac{c\nu - i b}{2c}, \nu; \frac{c\nu - i b}{2c} + 1, \ldots, \frac{c\nu - i b}{2c} + 1; -e^{2cz}\right) + \sum_{s=0}^{\left\lfloor \frac{\nu-1}{2} \right\rfloor} (-1)^s \binom{\nu}{s} \Bigg((-1)^\nu e^{(-i b + c\nu - c(\nu - 2s)) z} \sum_{j=0}^{n} \frac{(-1)^j (2cs - i b)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{2cs - i b}{2c}, \ldots, \frac{2cs - i b}{2c}, \nu; \frac{2cs - i b}{2c} + 1, \ldots, \frac{2cs - i b}{2c} + 1; -e^{2cz}\right) + e^{(-i b + c\nu + c(\nu - 2s)) z} \sum_{j=0}^{n} \frac{(-1)^j (2c(\nu - s) - i b)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{2c(\nu-s) - i b}{2c}, \ldots, \frac{2c(\nu-s) - i b}{2c}, \nu; \frac{2c(\nu-s) - i b}{2c} + 1, \ldots, \frac{2c(\nu-s) - i b}{2c} + 1; -e^{2cz}\right) \Bigg) - i^\nu e^{(i b + c\nu) z} \binom{\nu}{\frac{\nu}{2}} (1 - \nu \bmod 2) \sum_{j=0}^{n} \frac{(-1)^j (i b + c\nu)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{i b + c\nu}{2c}, \ldots, \frac{i b + c\nu}{2c}, \nu; \frac{i b + c\nu}{2c} + 1, \ldots, \frac{i b + c\nu}{2c} + 1; -e^{2cz}\right) - \sum_{s=0}^{\left\lfloor \frac{\nu-1}{2} \right\rfloor} (-1)^s \binom{\nu}{s} \Bigg(e^{(i b + c\nu + c(\nu - 2s)) z} \sum_{j=0}^{n} \frac{(-1)^j (i b + 2c(\nu - s))^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{i b + 2c(\nu-s)}{2c}, \ldots, \frac{i b + 2c(\nu-s)}{2c}, \nu; \frac{i b + 2c(\nu-s)}{2c} + 1, \ldots, \frac{i b + 2c(\nu-s)}{2c} + 1; -e^{2cz}\right) + (-1)^\nu e^{(i b + c\nu - c(\nu - 2s)) z} \sum_{j=0}^{n} \frac{(-1)^j (i b + 2cs)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{i b + 2cs}{2c}, \ldots, \frac{i b + 2cs}{2c}, \nu; \frac{i b + 2cs}{2c} + 1, \ldots, \frac{i b + 2cs}{2c} + 1; -e^{2cz}\right) \Bigg) \Bigg) \;/;\; n \in \mathbb{N} \land \nu \in \mathbb{N}^+$$

Involving powers of sin and power

Involving $z^n \sin^m(b z) \tanh^\nu(c z)$

01.21.21.0283.01

$$\int z^n \sin^m(b z) \tanh^\nu(c z)\, dz = 2^{-m} \binom{m}{\frac{m}{2}} n! (1 - m \bmod 2) \left(\frac{(-1)^\nu z^{n+1}}{(n+1)!} - (-1)^\nu e^{2cz} \nu \sum_{j=0}^{n} \frac{(-1)^j 2^{-j-1} c^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+3}F_{j+2}\left(1, \ldots, 1, \nu + 1; 2, \ldots, 2; -e^{2cz}\right) + \right.$$

$$e^{c v z} i^v \binom{v}{\frac{v}{2}} (1 - v \bmod 2) \sum_{j=0}^{n} \frac{(-1)^j (c v)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{v}{2}, \ldots, \frac{v}{2}, v; \frac{v}{2}+1, \ldots, \frac{v}{2}+1; -e^{2 c z}\right) +$$

$$(-1)^v \sum_{s=1}^{\left\lfloor \frac{v-1}{2} \right\rfloor} (-1)^s e^{2 c s z} \binom{v}{s} \sum_{j=0}^{n} \frac{(-1)^j 2^{-j-1} (c s)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}(s, \ldots, s, v; s+1, \ldots, s+1; -e^{2 c z}) +$$

$$\sum_{s=0}^{\left\lfloor \frac{v-1}{2} \right\rfloor} (-1)^s e^{2 c (v-s) z} \binom{v}{s} \sum_{j=0}^{n} \frac{(-1)^j 2^{-j-1} (c (v-s))^{-j-1} z^{n-j}}{(n-j)!}$$

$$\left. {}_{j+2}F_{j+1}(v-s, \ldots, v-s, v; -s+v+1, \ldots, -s+v+1; -e^{2 c z}) \right) + 2^{-m} n! \sum_{k=0}^{\left\lfloor \frac{m-1}{2} \right\rfloor} (-1)^k \binom{m}{k}$$

$$\left(e^{\frac{i \pi m}{2}} \left(i^v e^{(c v - i b (m-2k)) z} \binom{v}{\frac{v}{2}} (1 - v \bmod 2) \sum_{j=0}^{n} \frac{(-1)^j (c v - i b (m-2k))^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{c v - i b (m-2k)}{2c}, \right.\right.\right.$$

$$\left. \ldots, \frac{c v - i b (m-2k)}{2c}, v; \frac{c v - i b (m-2k)}{2c} + 1, \ldots, \frac{c v - i b (m-2k)}{2c} + 1; -e^{2 c z} \right) +$$

$$\sum_{s=0}^{\left\lfloor \frac{v-1}{2} \right\rfloor} (-1)^s \binom{v}{s} \left((-1)^v e^{(b i (2k-m) + 2 c s) z} \sum_{j=0}^{n} \frac{(-1)^j (2 c s - i b (m-2k))^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{2 c s - i b (m-2k)}{2c}, \right.\right.$$

$$\left. \ldots, \frac{2 c s - i b (m-2k)}{2c}, v; \frac{2 c s - i b (m-2k)}{2c} + 1, \ldots, \frac{2 c s - i b (m-2k)}{2c} + 1; -e^{2 c z} \right) +$$

$$e^{(b i (2k-m) + 2 c (v-s)) z} \sum_{j=0}^{n} \frac{(-1)^j (2 c (v-s) - i b (m-2k))^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}$$

$$\left(\frac{2 c (v-s) - i b (m-2k)}{2c}, \ldots, \frac{2 c (v-s) - i b (m-2k)}{2c}, v; \right.$$

$$\left.\left.\left. \frac{2 c (v-s) - i b (m-2k)}{2c} + 1, \ldots, \frac{2 c (v-s) - i b (m-2k)}{2c} + 1; -e^{2 c z} \right)\right)\right) +$$

$$e^{-\frac{1}{2} i \pi m} \left(i^v e^{(b i (m-2k) + c v) z} \binom{v}{\frac{v}{2}} (1 - v \bmod 2) \sum_{j=0}^{n} \frac{(-1)^j (b i (m-2k) + c v)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{i b (m-2k) + c v}{2c}, \right.\right.$$

$$\left. \ldots, \frac{i b (m-2k) + c v}{2c}, v; \frac{i b (m-2k) + c v}{2c} + 1, \ldots, \frac{i b (m-2k) + c v}{2c} + 1; -e^{2 c z} \right) +$$

$$\sum_{s=0}^{\left\lfloor \frac{v-1}{2} \right\rfloor} (-1)^s \binom{v}{s} \left((-1)^v e^{(-2 b i k + i b m + 2 c s) z} \sum_{j=0}^{n} \frac{(-1)^j (b i (m-2k) + 2 c s)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{i b (m-2k) + 2 c s}{2c}, \right.\right.$$

$$\left. \ldots, \frac{i b (m-2k) + 2 c s}{2c}, v; \frac{i b (m-2k) + 2 c s}{2c} + 1, \ldots, \frac{i b (m-2k) + 2 c s}{2c} + 1; -e^{2 c z} \right) +$$

$$e^{(-2\,b\,i\,k+i\,b\,m-2\,c\,s+2\,c\,v)\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(b\,i\,(m-2\,k)+2\,c\,(v-s))^{-j-1}\,z^{n-j}}{(n-j)!}\;{}_{j+2}F_{j+1}\left(\frac{i\,b\,(m-2\,k)+2\,c\,(v-s)}{2\,c},\,\ldots,\,\frac{i\,b\,(m-2\,k)+2\,c\,(v-s)}{2\,c},\,v;\,\frac{i\,b\,(m-2\,k)+2\,c\,(v-s)}{2\,c}+1,\,\ldots,\,\frac{i\,b\,(m-2\,k)+2\,c\,(v-s)}{2\,c}+1;\,-e^{2\,c\,z}\right)\Bigg)\Bigg)\Bigg)\,/;\,n\in\mathbb{N}\bigwedge m\in\mathbb{N}^+\bigwedge v\in\mathbb{N}^+$$

Involving cos and power

Involving $z^n \cos(a+b\,z)\tanh^v(c\,z)$

01.21.21.0284.01

$$\int z^n\cos(a+b\,z)\tanh^v(c\,z)\,dz = \frac{1}{2}\,n!\,e^{-i\,a}\Bigg(i^v\,e^{(-i\,b+c\,v)\,z}\binom{v}{\frac{v}{2}}(1-v\bmod 2)\sum_{j=0}^{n}\frac{(-1)^j\,(-i\,b+c\,v)^{-j-1}\,z^{n-j}}{(n-j)!}\;{}_{j+2}F_{j+1}\left(\frac{c\,v-i\,b}{2\,c},\,\ldots,\,\frac{c\,v-i\,b}{2\,c},\,v;\,\frac{c\,v-i\,b}{2\,c}+1,\,\ldots,\,\frac{c\,v-i\,b}{2\,c}+1;\,-e^{2\,c\,z}\right)+$$
$$\sum_{s=0}^{\left\lfloor\frac{v-1}{2}\right\rfloor}(-1)^s\binom{v}{s}\Bigg((-1)^v\,e^{(-i\,b+2\,c\,s)\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(-i\,b+2\,c\,s)^{-j-1}\,z^{n-j}}{(n-j)!}\;{}_{j+2}F_{j+1}\left(\frac{2\,c\,s-i\,b}{2\,c},\,\ldots,\,\frac{2\,c\,s-i\,b}{2\,c},\,v;\,\frac{2\,c\,s-i\,b}{2\,c}+1,\,\ldots,\,\frac{2\,c\,s-i\,b}{2\,c}+1;\,-e^{2\,c\,z}\right)+e^{(-i\,b+2\,c\,(v-s))\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(-i\,b+2\,c\,(v-s))^{-j-1}\,z^{n-j}}{(n-j)!}\;{}_{j+2}F_{j+1}$$
$$\left(\frac{2\,c\,(v-s)-i\,b}{2\,c},\,\ldots,\,\frac{2\,c\,(v-s)-i\,b}{2\,c},\,v;\,\frac{2\,c\,(v-s)-i\,b}{2\,c}+1,\,\ldots,\,\frac{2\,c\,(v-s)-i\,b}{2\,c}+1;\,-e^{2\,c\,z}\right)\Bigg)\Bigg)+$$
$$\frac{1}{2}\,n!\,e^{i\,a}\Bigg(i^v\,e^{(i\,b+c\,v)\,z}\binom{v}{\frac{v}{2}}(1-v\bmod 2)\sum_{j=0}^{n}\frac{(-1)^j\,(i\,b+c\,v)^{-j-1}\,z^{n-j}}{(n-j)!}\;{}_{j+2}F_{j+1}\left(\frac{i\,b+c\,v}{2\,c},\,\ldots,\,\frac{i\,b+c\,v}{2\,c},\,v;\,\frac{i\,b+c\,v}{2\,c}+1,\,\ldots,\,\frac{i\,b+c\,v}{2\,c}+1;\,-e^{2\,c\,z}\right)+\sum_{s=0}^{\left\lfloor\frac{v-1}{2}\right\rfloor}(-1)^s\binom{v}{s}\Bigg((-1)^v\,e^{(i\,b+2\,c\,s)\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(i\,b+2\,c\,s)^{-j-1}\,z^{n-j}}{(n-j)!}$$
$${}_{j+2}F_{j+1}\left(\frac{i\,b+2\,c\,s}{2\,c},\,\ldots,\,\frac{i\,b+2\,c\,s}{2\,c},\,v;\,\frac{i\,b+2\,c\,s}{2\,c}+1,\,\ldots,\,\frac{i\,b+2\,c\,s}{2\,c}+1;\,-e^{2\,c\,z}\right)+$$
$$e^{(i\,b+2\,c\,(v-s))\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(i\,b+2\,c\,(v-s))^{-j-1}\,z^{n-j}}{(n-j)!}\;{}_{j+2}F_{j+1}\left(\frac{i\,b+2\,c\,(v-s)}{2\,c},\,\ldots,\,\frac{i\,b+2\,c\,(v-s)}{2\,c},\,v;\,\frac{i\,b+2\,c\,(v-s)}{2\,c}+1,\,\ldots,\,\frac{i\,b+2\,c\,(v-s)}{2\,c}+1;\,-e^{2\,c\,z}\right)\Bigg)\Bigg)\,/;\,n\in\mathbb{N}\bigwedge v\in\mathbb{N}^+$$

01.21.21.0285.01

$$\int z^n \cos(b z) \tanh^{\nu}(c z)\, dz = \frac{1}{2} n! \left(i^{\nu} e^{(-i b + c \nu) z} \binom{\nu}{\frac{\nu}{2}} (1 - \nu \bmod 2) \sum_{j=0}^{n} \frac{(-1)^j (-i b + c \nu)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{c \nu - i b}{2 c}, \ldots, \frac{c \nu - i b}{2 c}, \nu; \frac{c \nu - i b}{2 c} + 1, \ldots, \frac{c \nu - i b}{2 c} + 1; -e^{2 c z} \right) + \sum_{s=0}^{\left\lfloor \frac{\nu-1}{2} \right\rfloor} (-1)^s \binom{\nu}{s} \left((-1)^{\nu} e^{(-i b + c \nu - c (\nu - 2 s)) z} \sum_{j=0}^{n} \frac{(-1)^j (-i b + 2 c s)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{2 c s - i b}{2 c}, \ldots, \frac{2 c s - i b}{2 c}, \nu; \frac{2 c s - i b}{2 c} + 1, \ldots, \frac{2 c s - i b}{2 c} + 1; -e^{2 c z} \right) + e^{(-i b + c \nu + c (\nu - 2 s)) z} \sum_{j=0}^{n} \frac{(-1)^j (-i b + 2 c (\nu - s))^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{2 c (\nu - s) - i b}{2 c}, \ldots, \frac{2 c (\nu - s) - i b}{2 c}, \nu; \frac{2 c (\nu - s) - i b}{2 c} + 1, \ldots, \frac{2 c (\nu - s) - i b}{2 c} + 1; -e^{2 c z} \right) \right) + i^{\nu} e^{(i b + c \nu) z} \binom{\nu}{\frac{\nu}{2}} (1 - \nu \bmod 2) \sum_{j=0}^{n} \frac{(-1)^j (i b + c \nu)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{i b + c \nu}{2 c}, \ldots, \frac{i b + c \nu}{2 c}, \nu; \frac{i b + c \nu}{2 c} + 1, \ldots, \frac{i b + c \nu}{2 c} + 1; -e^{2 c z} \right) + \sum_{s=0}^{\left\lfloor \frac{\nu-1}{2} \right\rfloor} (-1)^s \binom{\nu}{s} \left((-1)^{\nu} e^{(i b + c \nu - c (\nu - 2 s)) z} \sum_{j=0}^{n} \frac{(-1)^j (i b + 2 c s)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{i b + 2 c s}{2 c}, \ldots, \frac{i b + 2 c s}{2 c}, \nu; \frac{i b + 2 c s}{2 c} + 1, \ldots, \frac{i b + 2 c s}{2 c} + 1; -e^{2 c z} \right) + e^{(i b + c \nu + c (\nu - 2 s)) z} \sum_{j=0}^{n} \frac{(-1)^j (i b + 2 c (\nu - s))^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{i b + 2 c (\nu - s)}{2 c}, \ldots, \frac{i b + 2 c (\nu - s)}{2 c}, \nu; \frac{i b + 2 c (\nu - s)}{2 c} + 1, \ldots, \frac{i b + 2 c (\nu - s)}{2 c} + 1; -e^{2 c z} \right) \right) \right) /; n \in \mathbb{N} \land \nu \in \mathbb{N}^+$$

Involving powers of cos and power

Involving $z^n \cos^m(b z) \tanh^{\nu}(c z)$

01.21.21.0286.01

$$\int z^n \cos^m(b z) \tanh^{\nu}(c z)\, dz = 2^{-m} \binom{m}{\frac{m}{2}} n! (1 - m \bmod 2) \left(\frac{(-1)^{\nu} z^{n+1}}{(n+1)!} - (-1)^{\nu} e^{2 c z} \nu \sum_{j=0}^{n} \frac{(-1)^j 2^{-j-1} c^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+3}F_{j+2}\left(1, \ldots, 1, \nu + 1; 2, \ldots, 2; -e^{2 c z}\right) + \right.$$

$$e^{c v z}\, i^{v} \binom{v}{\frac{v}{2}} (1 - v \bmod 2) \sum_{j=0}^{n} \frac{(-1)^{j}\, (c\, v)^{-j-1}\, z^{n-j}}{(n-j)!}\; {}_{j+2}F_{j+1}\left(\frac{v}{2}, \ldots, \frac{v}{2}, v; \frac{v}{2} + 1, \ldots, \frac{v}{2} + 1; -e^{2 c z}\right) +$$

$$(-1)^{v} \sum_{s=1}^{\left\lfloor \frac{v-1}{2} \right\rfloor} (-1)^{s}\, e^{2 c s z} \binom{v}{s} \sum_{j=0}^{n} \frac{(-1)^{j}\, 2^{-j-1}\, (c\, s)^{-j-1}\, z^{n-j}}{(n-j)!}\; {}_{j+2}F_{j+1}\left(s, \ldots, s, v; s+1, \ldots, s+1; -e^{2 c z}\right) + \sum_{s=0}^{\left\lfloor \frac{v-1}{2} \right\rfloor} (-1)^{s}$$

$$\left. e^{2 c (v-s) z} \binom{v}{s} \sum_{j=0}^{n} \frac{(-1)^{j}\, 2^{-j-1}\, (c\, (v-s))^{-j-1}\, z^{n-j}}{(n-j)!}\; {}_{j+2}F_{j+1}\left(v-s, \ldots, v-s, v; -s+v+1, \ldots, -s+v+1; -e^{2 c z}\right)\right) +$$

$$2^{-m}\, n! \sum_{k=0}^{\left\lfloor \frac{m-1}{2} \right\rfloor} \binom{m}{k} \left(i^{v}\, e^{(c v - i b (m - 2 k)) z} \binom{v}{\frac{v}{2}} (1 - v \bmod 2) \sum_{j=0}^{n} \frac{(-1)^{j}\, (c\, v - i\, b\, (m - 2 k))^{-j-1}\, z^{n-j}}{(n-j)!}\; {}_{j+2}F_{j+1} \right.$$

$$\left(\frac{c\, v - i\, b\, (m - 2 k)}{2 c}, \ldots, \frac{c\, v - i\, b\, (m - 2 k)}{2 c}, v; \frac{c\, v - i\, b\, (m - 2 k)}{2 c} + 1, \ldots, \frac{c\, v - i\, b\, (m - 2 k)}{2 c} + 1; -e^{2 c z}\right) +$$

$$\sum_{s=0}^{\left\lfloor \frac{v-1}{2} \right\rfloor} (-1)^{s} \binom{v}{s} \left((-1)^{v}\, e^{(b\, i\, (2 k - m) + 2 c s) z} \sum_{j=0}^{n} \frac{(-1)^{j}\, (2\, c\, s - i\, b\, (m - 2 k))^{-j-1}\, z^{n-j}}{(n-j)!}\; {}_{j+2}F_{j+1}\left(\frac{2\, c\, s - i\, b\, (m - 2 k)}{2 c}, \right.\right.$$

$$\left. \ldots, \frac{2\, c\, s - i\, b\, (m - 2 k)}{2 c}, v; \frac{2\, c\, s - i\, b\, (m - 2 k)}{2 c} + 1, \ldots, \frac{2\, c\, s - i\, b\, (m - 2 k)}{2 c} + 1; -e^{2 c z}\right) +$$

$$e^{(b\, i\, (2 k - m) + 2 c (v - s)) z} \sum_{j=0}^{n} \frac{(-1)^{j}\, (2\, c\, (v - s) - i\, b\, (m - 2 k))^{-j-1}\, z^{n-j}}{(n-j)!}\; {}_{j+2}F_{j+1}$$

$$\left(\frac{2\, c\, (v - s) - i\, b\, (m - 2 k)}{2 c}, \ldots, \frac{2\, c\, (v - s) - i\, b\, (m - 2 k)}{2 c}, v; \right.$$

$$\left.\left. \frac{2\, c\, (v - s) - i\, b\, (m - 2 k)}{2 c} + 1, \ldots, \frac{2\, c\, (v - s) - i\, b\, (m - 2 k)}{2 c} + 1; -e^{2 c z}\right)\right) +$$

$$i^{v}\, e^{(b\, i\, (m - 2 k) + c v) z} \binom{v}{\frac{v}{2}} (1 - v \bmod 2) \sum_{j=0}^{n} \frac{(-1)^{j}\, (b\, i\, (m - 2 k) + c\, v)^{-j-1}\, z^{n-j}}{(n-j)!}\; {}_{j+2}F_{j+1}\left(\frac{i\, b\, (m - 2 k) + c\, v}{2 c}, \right.$$

$$\left. \ldots, \frac{i\, b\, (m - 2 k) + c\, v}{2 c}, v; \frac{i\, b\, (m - 2 k) + c\, v}{2 c} + 1, \ldots, \frac{i\, b\, (m - 2 k) + c\, v}{2 c} + 1; -e^{2 c z}\right) +$$

$$\sum_{s=0}^{\left\lfloor \frac{v-1}{2} \right\rfloor} (-1)^{s} \binom{v}{s} \left((-1)^{v}\, e^{(-2 b i k + i b m + 2 c s) z} \sum_{j=0}^{n} \frac{(-1)^{j}\, (b\, i\, (m - 2 k) + 2\, c\, s)^{-j-1}\, z^{n-j}}{(n-j)!}\; {}_{j+2}F_{j+1}\left(\frac{i\, b\, (m - 2 k) + 2\, c\, s}{2 c}, \right.\right.$$

$$\left. \ldots, \frac{i\, b\, (m - 2 k) + 2\, c\, s}{2 c}, v; \frac{i\, b\, (m - 2 k) + 2\, c\, s}{2 c} + 1, \ldots, \frac{i\, b\, (m - 2 k) + 2\, c\, s}{2 c} + 1; -e^{2 c z}\right) +$$

$$e^{(-2 b i k + i b m - 2 c s + 2 c v) z} \sum_{j=0}^{n} \frac{(-1)^{j}\, (b\, i\, (m - 2 k) + 2\, c\, (v - s))^{-j-1}\, z^{n-j}}{(n-j)!}\; {}_{j+2}F_{j+1}$$

$$\left(\frac{i\, b\, (m - 2 k) + 2\, c\, (v - s)}{2 c}, \ldots, \frac{i\, b\, (m - 2 k) + 2\, c\, (v - s)}{2 c}, v; \frac{i\, b\, (m - 2 k) + 2\, c\, (v - s)}{2 c} + 1, \right.$$

$$i\, b\, (m - 2 k) + 2\, c\, (v - s) \quad \ldots$$

Involving functions of the direct function, trigonometric and exponential functions

Involving powers of the direct function, trigonometric and exponential functions

Involving sin and exp

Involving $e^{p z} \sin(a z) \tanh^{\nu}(c z)$

01.21.21.0287.01

$$\int e^{p z} \sin(a z) \tanh^{\nu}(c z)\, dz = \frac{1}{2} i \left(1 - e^{-2 c z}\right)^{-\nu} \left(1 + e^{-2 c z}\right)^{\nu} \tanh^{\nu}(c z) \left(\frac{e^{(-i a + p) z} F_1\left(-\frac{-i a + p}{2 c}; -\nu, \nu; 1 - \frac{-i a + p}{2 c}; e^{-2 c z}, -e^{-2 c z}\right)}{-i a + p} - \frac{e^{(i a + p) z} F_1\left(-\frac{i a + p}{2 c}; -\nu, \nu; 1 - \frac{i a + p}{2 c}; e^{-2 c z}, -e^{-2 c z}\right)}{i a + p} \right)$$

01.21.21.0288.01

$$\int e^{i a z} \sin(a z) \tanh^{\nu}(c z)\, dz = \frac{i \tanh^{\nu-1}(c z)}{2 c (1 - \nu)}\, {}_2F_1\left(\frac{1 - \nu}{2}, 1; \frac{3 - \nu}{2}; \coth^2(c z)\right) - \frac{e^{2 i a z} \left(1 - e^{-2 c z}\right)^{-\nu} \left(1 + e^{-2 c z}\right)^{\nu} \tanh^{\nu}(c z)}{4 a} F_1\left(-\frac{i a}{c}; \nu, -\nu; 1 - \frac{i a}{c}; -e^{-2 c z}, e^{-2 c z}\right)$$

01.21.21.0289.01

$$\int e^{-i a z} \sin(a z) \tanh^{\nu}(c z)\, dz = -\frac{i \tanh^{\nu-1}(c z)}{2 c (1 - \nu)}\, {}_2F_1\left(\frac{1 - \nu}{2}, 1; \frac{3 - \nu}{2}; \coth^2(c z)\right) - \frac{e^{-2 i a z} \left(1 - e^{-2 c z}\right)^{-\nu} \left(1 + e^{-2 c z}\right)^{\nu} \tanh^{\nu}(c z)}{4 a} F_1\left(\frac{i a}{c}; \nu, -\nu; 1 + \frac{i a}{c}; -e^{-2 c z}, e^{-2 c z}\right)$$

Involving powers of sin and exp

Involving $e^{p z} \sin^m(a z) \tanh^{\nu}(c z)$

01.21.21.0290.01

$$\int e^{p z} \sin^m(a z) \tanh^{\nu}(c z)\, dz = \frac{2^{-m} e^{p z} \left(1 - e^{-2 c z}\right)^{-\nu} \left(1 + e^{-2 c z}\right)^{\nu} \tanh^{\nu}(c z) (1 - m \bmod 2)}{p} \binom{m}{\frac{m}{2}} F_1\left(-\frac{p}{2 c}; -\nu, \nu; 1 - \frac{p}{2 c}; e^{-2 c z}, -e^{-2 c z}\right) + 2^{-m} i^{-m} \left(1 - e^{-2 c z}\right)^{-\nu} \left(1 + e^{-2 c z}\right)^{\nu} \tanh^{\nu}(c z) \sum_{k=0}^{\left\lfloor \frac{m-1}{2} \right\rfloor} (-1)^k \binom{m}{k} \left(\frac{(-1)^m e^{(p - i a (m - 2 k)) z} F_1\left(-\frac{p - i a (m - 2 k)}{2 c}; -\nu, \nu; 1 - \frac{p - i a (m - 2 k)}{2 c}; e^{-2 c z}, -e^{-2 c z}\right)}{p - i a (m - 2 k)} + \frac{e^{(a i (m - 2 k) + p) z} F_1\left(-\frac{a i (m - 2 k) + p}{2 c}; -\nu, \nu; 1 - \frac{a i (m - 2 k) + p}{2 c}; e^{-2 c z}, -e^{-2 c z}\right)}{a i (m - 2 k) + p} \right) /; m \in \mathbb{N}^+$$

Involving cos and exp

Involving $e^{p z} \cos(a z)$ ianh$^{\nu}(c z)$

01.21.21.0291.01

$$\int e^{p z} \cos(a z) \tanh^{\nu}(c z)\, dz = \frac{1}{2} \left(1 - e^{-2 c z}\right)^{-\nu} \left(1 + e^{-2 c z}\right)^{\nu} \tanh^{\nu}(c z) \left(\frac{e^{(-i a + p) z} F_1\left(-\frac{-i a+p}{2 c}; -\nu, \nu; 1 - \frac{-i a+p}{2 c}; e^{-2 c z}, -e^{-2 c z}\right)}{-i a + p} + \frac{e^{(i a + p) z} F_1\left(-\frac{i a+p}{2 c}; -\nu, \nu; 1 - \frac{i a+p}{2 c}; e^{-2 c z}, -e^{-2 c z}\right)}{i a + p}\right)$$

01.21.21.0292.01

$$\int e^{i a z} \cos(a z) \tanh^{\nu}(c z)\, dz = \frac{\tanh^{\nu-1}(c z)}{2 c (1 - \nu)} \, {}_2F_1\left(\frac{1-\nu}{2}, 1; \frac{3-\nu}{2}; \coth^2(c z)\right) - \frac{i e^{2 i a z} \left(1 - e^{-2 c z}\right)^{-\nu} \left(1 + e^{-2 c z}\right)^{\nu} \tanh^{\nu}(c z)}{4 a} F_1\left(-\frac{i a}{c}; \nu, -\nu; 1 - \frac{i a}{c}; -e^{-2 c z}, e^{-2 c z}\right)$$

01.21.21.0293.01

$$\int e^{-i a z} \cos(a z) \tanh^{\nu}(c z)\, dz = \frac{\tanh^{\nu-1}(c z)}{2 c (1 - \nu)} \, {}_2F_1\left(\frac{1-\nu}{2}, 1; \frac{3-\nu}{2}; \coth^2(c z)\right) + \frac{i e^{-2 i a z} \left(1 - e^{-2 c z}\right)^{-\nu} \left(1 + e^{-2 c z}\right)^{\nu} \tanh^{\nu}(c z)}{4 a} F_1\left(\frac{i a}{c}; \nu, -\nu; 1 + \frac{i a}{c}; -e^{-2 c z}, e^{-2 c z}\right)$$

Involving powers of cos and exp

Involving $e^{p z} \cos^m(a z) \tanh^{\nu}(c z)$

01.21.21.0294.01

$$\int e^{p z} \cos^m(a z) \tanh^{\nu}(c z)\, dz = \frac{2^{-m} e^{p z} \left(1 + e^{-2 c z}\right)^{\nu} (1 - m \bmod 2) \tanh^{\nu}(c z) \left(1 - e^{-2 c z}\right)^{-\nu}}{p} \binom{m}{\frac{m}{2}} F_1\left(-\frac{p}{2 c}; -\nu, \nu; 1 - \frac{p}{2 c}; e^{-2 c z}, -e^{-2 c z}\right) + 2^{-m} \left(1 + e^{-2 c z}\right)^{\nu} \tanh^{\nu}(c z) \left(1 - e^{-2 c z}\right)^{-\nu} \sum_{k=0}^{\left\lfloor \frac{m-1}{2} \right\rfloor} \binom{m}{k} \left(\frac{e^{(p - i a (m - 2 k)) z} F_1\left(-\frac{p - i a (m-2 k)}{2 c}; -\nu, \nu; 1 - \frac{p - i a (m-2 k)}{2 c}; e^{-2 c z}, -e^{-2 c z}\right)}{p - i a (m - 2 k)} + \frac{e^{(a i (m - 2 k) + p) z} F_1\left(-\frac{a i (m-2 k)+p}{2 c}; -\nu, \nu; 1 - \frac{a i (m-2 k)+p}{2 c}; e^{-2 c z}, -e^{-2 c z}\right)}{a i (m - 2 k) + p}\right) /; m \in \mathbb{N}^+$$

Involving functions of the direct function, trigonometric, exponential and a power functions

Involving powers of the direct function, trigonometric, exponential and a power functions

Involving sin, exp and power

Involving $z^n e^{pz} \sin(a+bz)\tanh^{\nu}(cz)$

01.21.21.0295.01

$$\int z^n e^{pz}\sin(a+bz)\tanh^{\nu}(cz)\,dz = \frac{i}{2} n!\, e^{-ia}\Bigg(i^{\nu} e^{(-ib+p+c\nu)z}\binom{\nu}{\frac{\nu}{2}}(1-\nu \bmod 2)\sum_{j=0}^{n}\frac{(-1)^j(-ib+p+c\nu)^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\Bigg(\frac{-ib+p+c\nu}{2c}, \ldots, \frac{-ib+p+c\nu}{2c}, \nu; \frac{-ib+p+c\nu}{2c}+1, \ldots, \frac{-ib+p+c\nu}{2c}+1; -e^{2cz}\Bigg) + \sum_{s=0}^{\left\lfloor\frac{\nu-1}{2}\right\rfloor}(-1)^s\binom{\nu}{s}\Bigg((-1)^{\nu} e^{(-ib+p+2cs)z}\sum_{j=0}^{n}\frac{(-1)^j(-ib+p+2cs)^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\Bigg(\frac{-ib+p+2cs}{2c}, \ldots, \frac{-ib+p+2cs}{2c}, \nu; \frac{-ib+p+2cs}{2c}+1, \ldots, \frac{-ib+p+2cs}{2c}+1; -e^{2cz}\Bigg) + e^{(-ib+p+2c(\nu-s))z}\sum_{j=0}^{n}\frac{(-1)^j(-ib+p+2c(\nu-s))^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\Bigg(\frac{-ib+p+2c(\nu-s)}{2c}, \ldots, \frac{-ib+p+2c(\nu-s)}{2c}, \nu; \frac{-ib+p+2c(\nu-s)}{2c}+1, \ldots, \frac{-ib+p+2c(\nu-s)}{2c}+1; -e^{2cz}\Bigg)\Bigg)\Bigg) - \frac{i}{2} n!\, e^{ia}\Bigg(i^{\nu} e^{(ib+p+c\nu)z}\binom{\nu}{\frac{\nu}{2}}(1-\nu \bmod 2)\sum_{j=0}^{n}\frac{(-1)^j(ib+p+c\nu)^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\Bigg(\frac{ib+p+c\nu}{2c}, \ldots, \frac{ib+p+c\nu}{2c}, \nu; \frac{ib+p+c\nu}{2c}+1, \ldots, \frac{ib+p+c\nu}{2c}+1; -e^{2cz}\Bigg) + \sum_{s=0}^{\left\lfloor\frac{\nu-1}{2}\right\rfloor}(-1)^s\binom{\nu}{s}\Bigg((-1)^{\nu} e^{(ib+p+2cs)z}\sum_{j=0}^{n}\frac{(-1)^j(ib+p+2cs)^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\Bigg(\frac{ib+p+2cs}{2c}, \ldots, \frac{ib+p+2cs}{2c}, \nu; \frac{ib+p+2cs}{2c}+1, \ldots, \frac{ib+p+2cs}{2c}+1; -e^{2cz}\Bigg) + e^{(ib+p+2c(\nu-s))z}\sum_{j=0}^{n}\frac{(-1)^j(ib+p+2c(\nu-s))^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\Bigg(\frac{ib+p+2c(\nu-s)}{2c}, \ldots, \frac{ib+p+2c(\nu-s)}{2c}, \nu; \frac{ib+p+2c(\nu-s)}{2c}+1, \ldots, \frac{ib+p+2c(\nu-s)}{2c}+1; -e^{2cz}\Bigg)\Bigg)\Bigg) /;\ n \in \mathbb{N} \land \nu \in \mathbb{N}^+$$

01.21.21.0296.01

$$\int z^n e^{pz}\sin(bz)\tanh^{\nu}(cz)\,dz = \frac{1}{2}n!\,i^{\nu+1}\binom{\nu}{\frac{\nu}{2}}(1-\nu \bmod 2)\left(e^{(-ib+p+c\nu)z}\sum_{j=0}^{n}\frac{(-1)^j(-ib+p+c\nu)^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{-ib+p+c\nu}{2c},\ldots,\frac{-ib+p+c\nu}{2c},\nu;\frac{-ib+p+c\nu}{2c}+1,\ldots,\frac{-ib+p+c\nu}{2c}+1;-e^{2cz}\right)-e^{(ib+p+c\nu)z}\sum_{j=0}^{n}\frac{(-1)^j(ib+p+c\nu)^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{ib+p+c\nu}{2c},\ldots,\frac{ib+p+c\nu}{2c},\nu;\frac{ib+p+c\nu}{2c}+1,\ldots,\frac{ib+p+c\nu}{2c}+1;-e^{2cz}\right)\right)-$$

$$\frac{1}{2}(i\,n!)\sum_{s=0}^{\left\lfloor\frac{\nu-1}{2}\right\rfloor}(-1)^s\binom{\nu}{s}\left(-(-1)^{\nu}e^{(-ib+p+2cs)z}\sum_{j=0}^{n}\frac{(-1)^j(-ib+p+2cs)^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{-ib+p+2cs}{2c},\ldots,\frac{-ib+p+2cs}{2c},\nu;\frac{-ib+p+2cs}{2c}+1,\ldots,\frac{-ib+p+2cs}{2c}+1;-e^{2cz}\right)+(-1)^{\nu}e^{(ib+p+2cs)z}\sum_{j=0}^{n}\frac{(-1)^j(ib+p+2cs)^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{ib+p+2cs}{2c},\ldots,\frac{ib+p+2cs}{2c},\nu;\frac{ib+p+2cs}{2c}+1,\ldots,\frac{ib+p+2cs}{2c}+1;-e^{2cz}\right)-e^{(-ib+p+2c(\nu-s))z}\sum_{j=0}^{n}\frac{(-1)^j(-ib+p+2c(\nu-s))^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{-ib+p+2c(\nu-s)}{2c},\ldots,\frac{-ib+p+2c(\nu-s)}{2c},\nu;\frac{-ib+p+2c(\nu-s)}{2c}+1,\ldots,\frac{-ib+p+2c(\nu-s)}{2c}+1;-e^{2cz}\right)+e^{(ib+p+2c(\nu-s))z}\sum_{j=0}^{n}\frac{(-1)^j(ib+p+2c(\nu-s))^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{ib+p+2c(\nu-s)}{2c},\ldots,\frac{ib+p+2c(\nu-s)}{2c},\nu;\frac{ib+p+2c(\nu-s)}{2c}+1,\ldots,\frac{ib+p+2c(\nu-s)}{2c}+1;-e^{2cz}\right)\right)\;/;\;n\in\mathbb{N}\land\nu\in\mathbb{N}^+$$

Involving powers of sin, exp and power

Involving $z^n e^{pz}\sin^m(bz)\tanh^{\nu}(cz)$

01.21.21.0297.01

$$\int z^n e^{pz}\sin^m(bz)\tanh^{\nu}(cz)\,dz = 2^{-m}\binom{m}{\frac{m}{2}}(1-m \bmod 2)\,n!\left(i^{\nu}e^{(p+c\nu)z}\binom{\nu}{\frac{\nu}{2}}(1-\nu \bmod 2)\sum_{j=0}^{n}\frac{(-1)^j(p+c\nu)^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{p+c\nu}{2c},\ldots,\frac{p+c\nu}{2c},\nu;\frac{p+c\nu}{2c}+1,\ldots,\frac{p+c\nu}{2c}+1;-e^{2cz}\right)+\right.$$

$$\sum_{s=0}^{\left\lfloor\frac{v-1}{2}\right\rfloor}(-1)^s\binom{v}{s}\left((-1)^v e^{(p+2cs)z}\sum_{j=0}^{n}\frac{(-1)^j(p+2cs)^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{p+2cs}{2c},\ldots,\frac{p+2cs}{2c},v;\frac{p+2cs}{2c}+1,\ldots,\frac{p+2cs}{2c}+1;-e^{2cz}\right)+e^{(p+2c(v-s))z}\sum_{j=0}^{n}\frac{(-1)^j(p+2c(v-s))^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{p+2c(v-s)}{2c},\ldots,\frac{p+2c(v-s)}{2c},v;\frac{p+2c(v-s)}{2c}+1,\ldots,\frac{p+2c(v-s)}{2c}+1;-e^{2cz}\right)\right)\Bigg)+$$

$$i^{-m}2^{-m}n!\sum_{k=0}^{\left\lfloor\frac{m-1}{2}\right\rfloor}(-1)^k\binom{m}{k}\Bigg((-1)^m i^v e^{(-ib(m-2k)+p+cv)z}\binom{v}{\frac{v}{2}}(1-v\bmod 2)\sum_{j=0}^{n}\frac{(-1)^j(-ib(m-2k)+p+cv)^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{-ib(m-2k)+p+cv}{2c},\ldots,\frac{-ib(m-2k)+p+cv}{2c},v;\frac{-ib(m-2k)+p+cv}{2c}+1,\ldots,\frac{-ib(m-2k)+p+cv}{2c}+1;-e^{2cz}\right)+$$

$$(-1)^m\sum_{s=0}^{\left\lfloor\frac{v-1}{2}\right\rfloor}(-1)^s\binom{v}{s}\left((-1)^v e^{(bi(2k-m)+p+2cs)z}\sum_{j=0}^{n}\frac{(-1)^j(-ib(m-2k)+p+2cs)^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{-ib(m-2k)+p+2cs}{2c},\ldots,\frac{-ib(m-2k)+p+2cs}{2c},v;\frac{-ib(m-2k)+p+2cs}{2c}+1,\ldots,\frac{-ib(m-2k)+p+2cs}{2c}+1;-e^{2cz}\right)+e^{(bi(2k-m)+p+2c(v-s))z}\sum_{j=0}^{n}\frac{(-1)^j(-ib(m-2k)+p+2c(v-s))^{-j-1}}{(n-j)!}z^{n-j}\,{}_{j+2}F_{j+1}\left(\frac{-ib(m-2k)+p+2c(v-s)}{2c},\ldots,\frac{-ib(m-2k)+p+2c(v-s)}{2c},v;\frac{-ib(m-2k)+p+2c(v-s)}{2c}+1,\ldots,\frac{-ib(m-2k)+p+2c(v-s)}{2c}+1;-e^{2cz}\right)\right)+$$

$$i^v e^{(bi(m-2k)+p+cv)z}\binom{v}{\frac{v}{2}}(1-v\bmod 2)\sum_{j=0}^{n}\frac{(-1)^j(bi(m-2k)+p+cv)^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{ib(m-2k)+p+cv}{2c},\ldots,\frac{ib(m-2k)+p+cv}{2c},v;\frac{ib(m-2k)+p+cv}{2c}+1,\ldots,\frac{ib(m-2k)+p+cv}{2c}+1;-e^{2cz}\right)+$$

$$\sum_{s=0}^{\left\lfloor\frac{v-1}{2}\right\rfloor}(-1)^s\binom{v}{s}\Bigg((-1)^v e^{(-2bik+ibm+p+2cs)z}\sum_{j=0}^{n}\frac{(-1)^j(bi(m-2k)+p+2cs)^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{ib(m-2k)+p+2cs}{2c},\ldots,\frac{ib(m-2k)+p+2cs}{2c},v;\frac{ib(m-2k)+p+2cs}{2c}+1,\ldots,\frac{ib(m-2k)+p+2cs}{2c}+1;-e^{2cz}\right)+e^{(-2bik+ibm+p-2cs+2cv)z}$$

$$\sum_{j=0}^{n} \frac{(-1)^j \left(b\, i\, (m-2k)+p+2c(v-s)\right)^{-j-1}}{(n-j)!} z^{n-j} \,{}_{j+2}F_{j+1}\left(\frac{i\, b\, (m-2k)+p+2c(v-s)}{2c}, \ldots, \frac{i\, b\, (m-2k)+p+2c(v-s)}{2c}, v; \frac{i\, b\, (m-2k)+p+2c(v-s)}{2c}+1, \ldots, \frac{i\, b\, (m-2k)+p+2c(v-s)}{2c}+1; -e^{2cz}\right)\Bigg)\Bigg) /; n \in \mathbb{N} \bigwedge m \in \mathbb{N}^+ \bigwedge v \in \mathbb{N}^+$$

Involving cos, exp and power

Involving $z^n e^{pz} \cos(a+bz) \tanh^v(cz)$

01.21.21.0298.01

$$\int z^n e^{p z} \cos(a + b z) \tanh^\nu(c z)\, dz = \frac{1}{2} n!\, e^{-i a} \left(i^\nu e^{(-i b + p + c \nu) z} \binom{\nu}{\frac{\nu}{2}} (1 - \nu \bmod 2) \sum_{j=0}^{n} \frac{(-1)^j (-i b + p + c \nu)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{-i b + p + c \nu}{2 c}, \ldots, \frac{-i b + p + c \nu}{2 c}, \nu; \frac{-i b + p + c \nu}{2 c} + 1, \ldots, \frac{-i b + p + c \nu}{2 c} + 1; -e^{2 c z} \right) + \sum_{s=0}^{\left\lfloor \frac{\nu-1}{2} \right\rfloor} (-1)^s \binom{\nu}{s} \left((-1)^\nu e^{(-i b + p + 2 c s) z} \sum_{j=0}^{n} \frac{(-1)^j (-i b + p + 2 c s)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{-i b + p + 2 c s}{2 c}, \ldots, \frac{-i b + p + 2 c s}{2 c}, \nu; \frac{-i b + p + 2 c s}{2 c} + 1, \ldots, \frac{-i b + p + 2 c s}{2 c} + 1; -e^{2 c z} \right) + e^{(-i b + p + 2 c (\nu - s)) z} \sum_{j=0}^{n} \frac{(-1)^j (-i b + p + 2 c (\nu - s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{-i b + p + 2 c (\nu - s)}{2 c}, \ldots, \frac{-i b + p + 2 c (\nu - s)}{2 c}, \nu; \frac{-i b + p + 2 c (\nu - s)}{2 c} + 1, \ldots, \frac{-i b + p + 2 c (\nu - s)}{2 c} + 1; -e^{2 c z} \right) \right) \right) + \frac{1}{2} n!\, e^{i a} \left(i^\nu e^{(i b + p + c \nu) z} \binom{\nu}{\frac{\nu}{2}} (1 - \nu \bmod 2) \sum_{j=0}^{n} \frac{(-1)^j (i b + p + c \nu)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{i b + p + c \nu}{2 c}, \ldots, \frac{i b + p + c \nu}{2 c}, \nu; \frac{i b + p + c \nu}{2 c} + 1, \ldots, \frac{i b + p + c \nu}{2 c} + 1; -e^{2 c z} \right) + \sum_{s=0}^{\left\lfloor \frac{\nu-1}{2} \right\rfloor} (-1)^s \binom{\nu}{s} \left((-1)^\nu e^{(i b + p + 2 c s) z} \sum_{j=0}^{n} \frac{(-1)^j (i b + p + 2 c s)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{i b + p + 2 c s}{2 c}, \ldots, \frac{i b + p + 2 c s}{2 c}, \nu; \frac{i b + p + 2 c s}{2 c} + 1, \ldots, \frac{i b + p + 2 c s}{2 c} + 1; -e^{2 c z} \right) + e^{(i b + p + 2 c (\nu - s)) z} \sum_{j=0}^{n} \frac{(-1)^j (i b + p + 2 c (\nu - s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{i b + p + 2 c (\nu - s)}{2 c}, \ldots, \frac{i b + p + 2 c (\nu - s)}{2 c}, \nu; \frac{i b + p + 2 c (\nu - s)}{2 c} + 1, \ldots, \frac{i b + p + 2 c (\nu - s)}{2 c} + 1; -e^{2 c z} \right) \right) \right) /; n \in \mathbb{N} \land \nu \in \mathbb{N}^+$$

01.21.21.0299.01

$$\int z^n e^{pz}\cos(bz)\tanh^\nu(cz)\,dz = \frac{1}{2}n!\,i^\nu\binom{\nu}{\frac{\nu}{2}}(1-\nu\bmod 2)\Bigg(e^{(-ib+p+c\nu)z}\sum_{j=0}^{n}\frac{(-1)^j(-ib+p+c\nu)^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\Big(\frac{-ib+p+c\nu}{2c},\ldots,\frac{-ib+p+c\nu}{2c},\nu;\frac{-ib+p+c\nu}{2c}+1,\ldots,\frac{-ib+p+c\nu}{2c}+1;-e^{2cz}\Big)+e^{(ib+p+c\nu)z}\sum_{j=0}^{n}\frac{(-1)^j(ib+p+c\nu)^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\Big(\frac{ib+p+c\nu}{2c},\ldots,\frac{ib+p+c\nu}{2c},\nu;\frac{ib+p+c\nu}{2c}+1,\ldots,\frac{ib+p+c\nu}{2c}+1;-e^{2cz}\Big)\Bigg)+$$

$$\frac{1}{2}n!\sum_{s=0}^{\left\lfloor\frac{\nu-1}{2}\right\rfloor}(-1)^s\binom{\nu}{s}\Bigg((-1)^\nu e^{(-ib+p+2cs)z}\sum_{j=0}^{n}\frac{(-1)^j(-ib+p+2cs)^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\Big(\frac{-ib+p+2cs}{2c},\ldots,\frac{-ib+p+2cs}{2c},\nu;\frac{-ib+p+2cs}{2c}+1,\ldots,\frac{-ib+p+2cs}{2c}+1;-e^{2cz}\Big)+(-1)^\nu e^{(ib+p+2cs)z}\sum_{j=0}^{n}\frac{(-1)^j(ib+p+2cs)^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\Big(\frac{ib+p+2cs}{2c},\ldots,\frac{ib+p+2cs}{2c},\nu;\frac{ib+p+2cs}{2c}+1,\ldots,\frac{ib+p+2cs}{2c}+1;-e^{2cz}\Big)+e^{(-ib+p+2c(\nu-s))z}\sum_{j=0}^{n}\frac{(-1)^j(-ib+p+2c(\nu-s))^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\Big(\frac{-ib+p+2c(\nu-s)}{2c},\ldots,\frac{-ib+p+2c(\nu-s)}{2c},\nu;\frac{-ib+p+2c(\nu-s)}{2c}+1,\ldots,\frac{-ib+p+2c(\nu-s)}{2c}+1;-e^{2cz}\Big)+e^{(ib+p+2c(\nu-s))z}\sum_{j=0}^{n}\frac{(-1)^j(ib+p+2c(\nu-s))^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\Big(\frac{ib+p+2c(\nu-s)}{2c},\ldots,\frac{ib+p+2c(\nu-s)}{2c},\nu;\frac{ib+p+2c(\nu-s)}{2c}+1,\ldots,\frac{ib+p+2c(\nu-s)}{2c}+1;-e^{2cz}\Big)\Bigg)\;/;\;n\in\mathbb{N}\land\nu\in\mathbb{N}^+$$

Involving powers of cos, exp and power

Involving $z^n e^{pz}\cos^m(bz)\tanh^\nu(cz)$

01.21.21.0300.01

$$\int z^n e^{pz}\cos^m(bz)\tanh^\nu(cz)\,dz = 2^{-m}\binom{m}{\frac{m}{2}}n!\,(1-m\bmod 2)\Bigg(i^\nu e^{(p+c\nu)z}\binom{\nu}{\frac{\nu}{2}}(1-\nu\bmod 2)\sum_{j=0}^{n}\frac{(-1)^j(p+c\nu)^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\Big(\frac{p+c\nu}{2c},\ldots,\frac{p+c\nu}{2c},\nu;\frac{p+c\nu}{2c}+1,\ldots,\frac{p+c\nu}{2c}+1;-e^{2cz}\Big)+$$

$$\sum_{s=0}^{\left\lfloor \frac{v-1}{2} \right\rfloor} (-1)^s \binom{v}{s} \Bigg((-1)^v e^{(p+2cs)z} \sum_{j=0}^{n} \frac{(-1)^j (p+2cs)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\Bigg(\frac{p+2cs}{2c}, \ldots, \frac{p+2cs}{2c}, v; \frac{p+2cs}{2c} +$$

$$1, \ldots, \frac{p+2cs}{2c} + 1; -e^{2cz}\Bigg) + e^{(p+2c(v-s))z} \sum_{j=0}^{n} \frac{(-1)^j (p+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!}$$

$${}_{j+2}F_{j+1}\Bigg(\frac{p+2c(v-s)}{2c}, \ldots, \frac{p+2c(v-s)}{2c}, v; \frac{p+2c(v-s)}{2c} + 1, \ldots, \frac{p+2c(v-s)}{2c} + 1; -e^{2cz}\Bigg)\Bigg)\Bigg) +$$

$$2^{-m} n! \sum_{k=0}^{\left\lfloor \frac{m-1}{2} \right\rfloor} \binom{m}{k} \Bigg(i^v e^{(-ib(m-2k)+p+cv)z} \binom{v}{\frac{v}{2}} (1 - v \bmod 2) \sum_{j=0}^{n} \frac{(-1)^j (-ib(m-2k)+p+cv)^{-j-1} z^{n-j}}{(n-j)!}$$

$${}_{j+2}F_{j+1}\Bigg(\frac{-ib(m-2k)+p+cv}{2c}, \ldots, \frac{-ib(m-2k)+p+cv}{2c}, v;$$

$$\frac{-ib(m-2k)+p+cv}{2c} + 1, \ldots, \frac{-ib(m-2k)+p+cv}{2c} + 1; -e^{2cz}\Bigg) +$$

$$\sum_{s=0}^{\left\lfloor \frac{v-1}{2} \right\rfloor} (-1)^s \binom{v}{s} \Bigg((-1)^v e^{(bi(2k-m)+p+2cs)z} \sum_{j=0}^{n} \frac{(-1)^j (-ib(m-2k)+p+2cs)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}$$

$$\Bigg(\frac{-ib(m-2k)+p+2cs}{2c}, \ldots, \frac{-ib(m-2k)+p+2cs}{2c}, v;$$

$$\frac{-ib(m-2k)+p+2cs}{2c} + 1, \ldots, \frac{-ib(m-2k)+p+2cs}{2c} + 1; -e^{2cz}\Bigg) +$$

$$e^{(bi(2k-m)+p+2c(v-s))z} \sum_{j=0}^{n} \frac{(-1)^j (-ib(m-2k)+p+2c(v-s))^{-j-1}}{(n-j)!} z^{n-j} \, {}_{j+2}F_{j+1}$$

$$\Bigg(\frac{-ib(m-2k)+p+2c(v-s)}{2c}, \ldots, \frac{-ib(m-2k)+p+2c(v-s)}{2c}, v;$$

$$\frac{-ib(m-2k)+p+2c(v-s)}{2c} + 1, \ldots, \frac{-ib(m-2k)+p+2c(v-s)}{2c} + 1; -e^{2cz}\Bigg)\Bigg) +$$

$$i^v e^{(bi(m-2k)+p+cv)z} \binom{v}{\frac{v}{2}} (1 - v \bmod 2) \sum_{j=0}^{n} \frac{(-1)^j (bi(m-2k)+p+cv)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\Bigg(\frac{ib(m-2k)+p+cv}{2c},$$

$$\ldots, \frac{ib(m-2k)+p+cv}{2c}, v; \frac{ib(m-2k)+p+cv}{2c} + 1, \ldots, \frac{ib(m-2k)+p+cv}{2c} + 1; -e^{2cz}\Bigg) +$$

$$\sum_{s=0}^{\left\lfloor \frac{v-1}{2} \right\rfloor} (-1)^s \binom{v}{s} \Bigg((-1)^v e^{(-2bik+ibm+p+2cs)z} \sum_{j=0}^{n} \frac{(-1)^j (bi(m-2k)+p+2cs)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}$$

$$\Bigg(\frac{ib(m-2k)+p+2cs}{2c}, \ldots, \frac{ib(m-2k)+p+2cs}{2c}, v; \frac{ib(m-2k)+p+2cs}{2c} + 1,$$

$$\ldots, \frac{ib(m-2k)+p+2cs}{2c} + 1; -e^{2cz}\Bigg) + e^{(-2bik+ibm+p-2cs+2cv)z}$$

$$\sum_{j=0}^{n} \frac{(-1)^j \left(b\,i\,(m-2\,k)+p+2\,c\,(v-s)\right)^{-j-1}}{(n-j)!}\, z^{n-j}\, {}_{j+2}F_{j+1}\!\left(\frac{i\,b\,(m-2\,k)+p+2\,c\,(v-s)}{2\,c},\right.$$
$$\ldots,\ \frac{i\,b\,(m-2\,k)+p+2\,c\,(v-s)}{2\,c},\ v;\ \frac{i\,b\,(m-2\,k)+p+2\,c\,(v-s)}{2\,c}+1,\ \ldots,$$
$$\left.\left.\left.\frac{i\,b\,(m-2\,k)+p+2\,c\,(v-s)}{2\,c}+1;\ -e^{2\,c\,z}\right)\right)\right) /;\ n\in\mathbb{N} \land m\in\mathbb{N}^+ \land v\in\mathbb{N}^+$$

Involving functions of the direct function and hyperbolic functions

Involving powers of the direct function and hyperbolic functions

Involving sinh

Involving sinh($b\,z$)

01.21.21.0301.01

$$\int \sinh(b\,z)\tanh^{\nu}(c\,z)\,dz = \frac{1}{2\,b}\, e^{-b\,z}\left(1-e^{-2\,c\,z}\right)^{-\nu}\left(1+e^{-2\,c\,z}\right)^{\nu}\tanh^{\nu}(c\,z)$$
$$\left(F_1\!\left(\frac{b}{2\,c};\ \nu,\ -\nu;\ \frac{b}{2\,c}+1;\ -e^{-2\,c\,z},\ e^{-2\,c\,z}\right)+e^{2\,b\,z}\,F_1\!\left(-\frac{b}{2\,c};\ \nu,\ -\nu;\ 1-\frac{b}{2\,c};\ -e^{-2\,c\,z},\ e^{-2\,c\,z}\right)\right)$$

01.21.21.0302.01

$$\int \sinh(c\,z)\tanh^{\nu}(c\,z)\,dz = \frac{\cosh(c\,z)\left(-\sinh^2(c\,z)\right)^{-\frac{\nu}{2}}\tanh^{\nu}(c\,z)}{c-c\,\nu}\,{}_2F_1\!\left(\frac{1-\nu}{2},\ -\frac{\nu}{2};\ \frac{3-\nu}{2};\ \cosh^2(c\,z)\right)$$

01.21.21.0303.01

$$\int \sinh(c\,z)\tanh^2(c\,z)\,dz = \frac{\cosh(c\,z)+\operatorname{sech}(c\,z)}{c}$$

01.21.21.0304.01

$$\int \sinh(c\,z)\tanh^3(c\,z)\,dz = \frac{-6\tan^{-1}\!\left(\tanh\!\left(\frac{c\,z}{2}\right)\right)+2\sinh(c\,z)+\operatorname{sech}(c\,z)\tanh(c\,z)}{2\,c}$$

01.21.21.0305.01

$$\int \sinh(4\,c\,z)\tanh^4(c\,z)\,dz = \frac{-16\cosh(2\,c\,z)+\cosh(4\,c\,z)+8\left(\operatorname{sech}^2(c\,z)+8\log(\cosh(c\,z))\right)}{4\,c}$$

Involving power of sinh

Involving $\sinh^{\mu}(b\,z)\tanh^{\nu}(c\,z)$

01.21.21.0306.01

$$\int \sinh^u(b\,z)\tanh^\nu(c\,z)\,dz = \frac{\tanh^{\nu+1}(c\,z)\,(1-u \bmod 2)}{c\,(\nu+1)}\left(\frac{i}{2}\right)^u \binom{u}{\frac{u}{2}}\,{}_2F_1\!\left(\frac{\nu+1}{2},\,1;\,\frac{\nu+1}{2}+1;\,\tanh^2(c\,z)\right)+\frac{1}{b}\,2^{-u}\left(1-e^{-2cz}\right)^{-\nu}\left(1+e^{-2cz}\right)^{\nu}$$

$$\tanh^\nu(c\,z)\sum_{k=0}^{\left\lfloor\frac{u-1}{2}\right\rfloor}\frac{(-1)^k}{u-2k}\binom{u}{k}\left(e^{b(u-2k)z}\,F_1\!\left(-\frac{b\,(u-2k)}{2c};\,-\nu,\,\nu;\,1-\frac{b\,(u-2k)}{2c};\,e^{-2cz},\,-e^{-2cz}\right)-(-1)^u\,e^{-b(u-2k)z}\right.$$

$$\left.F_1\!\left(\frac{b\,(u-2k)}{2c};\,-\nu,\,\nu;\,\frac{b\,(u-2k)}{2c}+1;\,e^{-2cz},\,-e^{-2cz}\right)\right)\,/;\,u\in\mathbb{N}^+$$

01.21.21.0307.01

$$\int \sinh^\mu(c\,z)\tanh^\nu(c\,z)\,dz = -\frac{\sinh^\mu(c\,z)\left(1-e^{2cz}\right)^{-\mu-\nu}\tanh^\nu(c\,z)\left(1+e^{2cz}\right)^\nu}{c\,\mu}\,F_1\!\left(-\frac{\mu}{2};\,-\mu-\nu,\,\nu;\,\frac{2-\mu}{2};\,e^{2cz},\,-e^{2cz}\right)$$

01.21.21.0308.01

$$\int \sinh^\mu(c\,z)\tanh^\nu(c\,z)\,dz = \frac{\cosh(c\,z)\,\sinh^{\mu+1}(c\,z)\left(-\sinh^2(c\,z)\right)^{\frac{1}{2}(-\mu-\nu-1)}\tanh^\nu(c\,z)}{c\,(\nu-1)}\,{}_2F_1\!\left(\frac{1-\nu}{2},\,\frac{1}{2}(-\mu-\nu+1);\,\frac{3-\nu}{2};\,\cosh^2(c\,z)\right)$$

01.21.21.0309.01

$$\int \sinh^2(c\,z)\tanh^2(c\,z)\,dz = \frac{-6\,c\,z+\sinh(2\,c\,z)+4\tanh(c\,z)}{4\,c}$$

01.21.21.0310.01

$$\int \frac{\tanh^3(c\,z)}{\sinh^{\frac{1}{2}}(2\,c\,z)}\,dz = \frac{(3\cosh(2\,c\,z)+1)\tanh^2(c\,z)-3\,E\!\left(\frac{\pi}{4}-i\,c\,z\,\middle|\,2\right)\sqrt{i\sinh(2\,c\,z)}}{5\,c\sinh^{\frac{1}{2}}(2\,c\,z)}$$

01.21.21.0311.01

$$\int \sqrt{\sinh^3(2\,c\,z)}\,\tanh^5(c\,z)\,dz = \frac{\sqrt{\sinh^3(2\,c\,z)}}{15\,c}\left(-6\,\mathrm{sech}^4(c\,z)+78\,\mathrm{sech}^2(c\,z)-231\,\mathrm{csch}^2(2\,c\,z)\,E\!\left(\frac{\pi}{4}-i\,c\,z\,\middle|\,2\right)\sqrt{i\sinh(2\,c\,z)}+5\right)$$

Involving powers of products with sinh

Involving $\sqrt{\sinh^m(c\,z)\tanh(c\,z)}$

01.21.21.0312.01

$$\int \sqrt{\sinh(c\,z)\tanh(c\,z)}\,dz = \frac{2\coth(c\,z)\sqrt{\sinh(c\,z)\tanh(c\,z)}}{c}$$

01.21.21.0313.01

$$\int \sqrt{\sinh^4(c\,z)\tanh(c\,z)}\,dz = \frac{1}{4\,c\tanh^{\frac{1}{2}}(c\,z)}\left(\mathrm{csch}^2(c\,z)\left(3\tan^{-1}\!\left(\tanh^{\frac{1}{2}}(c\,z)\right)-3\tanh^{-1}\!\left(\tanh^{\frac{1}{2}}(c\,z)\right)+\sinh(2\,c\,z)\tanh^{\frac{1}{2}}(c\,z)\right)\sqrt{\sinh^4(c\,z)\tanh(c\,z)}\right)$$

Involving algebraic functions of sinh

01.21.21.0314.01

$$\int \sqrt{a+b\sinh(c z)}\;\tanh^2(c z)\,dz = \frac{1}{c\sqrt{a+b\sinh(c z)}}\left(3\,(b+i a)\sqrt{\frac{a+b\sinh(c z)}{a-i b}}\;E\!\left(\frac{1}{4}(\pi-2 i c z)\,\middle|\,-\frac{2 i b}{a-i b}\right)-(a+b\sinh(c z))\tanh(c z)-i a F\!\left(\frac{1}{4}(\pi-2 i c z)\,\middle|\,-\frac{2 i b}{a-i b}\right)\sqrt{\frac{a+b\sinh(c z)}{a-i b}}\right)$$

Involving cosh

Involving cosh$(b z)$

01.21.21.0315.01

$$\int \cosh(b z)\tanh^{\nu}(c z)\,dz = \frac{1}{2b}e^{-b z}\left(1-e^{-2 c z}\right)^{-\nu}\left(1+e^{-2 c z}\right)^{\nu}\tanh^{\nu}(c z)\left(e^{2 b z}F_1\!\left(-\frac{b}{2c};\nu,-\nu;1-\frac{b}{2c};-e^{-2 c z},e^{-2 c z}\right)-F_1\!\left(\frac{b}{2c};\nu,-\nu;\frac{b}{2c}+1;-e^{-2 c z},e^{-2 c z}\right)\right)$$

01.21.21.0316.01

$$\int \cosh(c z)\tanh^{\nu}(c z)\,dz = \frac{\cosh(c z)\left(-\sinh^2(c z)\right)^{\frac{1}{2}-\frac{\nu}{2}}\tanh^{\nu-1}(c z)}{2c-c\nu}\,{}_2F_1\!\left(\frac{2-\nu}{2},\frac{1-\nu}{2};\frac{4-\nu}{2};\cosh^2(c z)\right)$$

01.21.21.0317.01

$$\int \cosh(c z)\tanh^2(c z)\,dz = \frac{\sinh(c z)-2\tan^{-1}\!\left(\tanh\!\left(\frac{c z}{2}\right)\right)}{c}$$

01.21.21.0318.01

$$\int \cosh(c z)\tanh^3(c z)\,dz = \frac{\cosh(c z)+\operatorname{sech}(c z)}{c}$$

01.21.21.0319.01

$$\int \cosh(2 c z)\tanh^3(c z)\,dz = \frac{-\operatorname{sech}^2(c z)+\cosh(2 c z)-6\log(\cosh(c z))}{2c}$$

01.21.21.0320.01

$$\int \cosh(4 c z)\tanh^5(c z)\,dz = \frac{-\operatorname{sech}^4(c z)+20\operatorname{sech}^2(c z)-20\cosh(2 c z)+\cosh(4 c z)+100\log(\cosh(c z))}{4c}$$

01.21.21.0321.01

$$\int \cosh(5 c z)\tanh^5(c z)\,dz = \frac{-25\operatorname{sech}^3(c z)+450\operatorname{sech}(c z)+750\cosh(c z)-50\cosh(3 c z)+3\cosh(5 c z)}{15c}$$

Involving power of cosh

Involving $\cosh^{u}(b z)\tanh^{\nu}(c z)$

01.21.21.0322.01

$$\int \cosh^{u}(b z)\tanh^{\nu}(c z)\,dz = \frac{1}{b}\left(\left(2^{-u}\left(1-e^{-2cz}\right)^{-\nu}\left(1+e^{-2cz}\right)^{\nu}\tanh^{\nu}(c z)\right)\sum_{k=0}^{\left\lfloor\frac{u-1}{2}\right\rfloor}\frac{1}{u-2k}\left(\binom{u}{k}\left(e^{b(u-2k)z}F_1\left(-\frac{b(u-2k)}{2c};-\nu,\nu;1-\frac{b(u-2k)}{2c};\right.\right.\right.\right.$$
$$\left.\left.\left.\left. e^{-2cz},-e^{-2cz}\right)-e^{-b(u-2k)z}F_1\left(\frac{b(u-2k)}{2c};-\nu,\nu;\frac{b(u-2k)}{2c}+1;e^{-2cz},-e^{-2cz}\right)\right)\right)\right)-$$
$$\frac{2^{-u}(u \bmod 2-1)\tanh^{\nu+1}(c z)}{c(\nu+1)}\binom{u}{\frac{u}{2}}\,{}_2F_1\left(\frac{\nu+1}{2},1;\frac{\nu+1}{2}+1;\tanh^2(c z)\right)\;/;\;u\in\mathbb{N}^+$$

01.21.21.0323.01

$$\int \cosh^{\mu}(c z)\tanh^{\nu}(c z)\,dz = -\frac{\cosh^{\mu}(c z)\left(1-e^{2cz}\right)^{-\nu}\tanh^{\nu}(c z)\left(1+e^{2cz}\right)^{\nu-\mu}}{c\mu}F_1\left(-\frac{\mu}{2};-\nu,\nu-\mu;\frac{2-\mu}{2};e^{2cz},-e^{2cz}\right)$$

01.21.21.0324.01

$$\int \cosh^{\mu}(c z)\tanh^{\nu}(c z)\,dz = \frac{\cosh^{\mu+1}(c z)\operatorname{csch}(c z)\left(-\sinh^2(c z)\right)^{\frac{1}{2}-\frac{\nu}{2}}\tanh^{\nu}(c z)}{\mu c-\nu c+c}\,{}_2F_1\left(\frac{1}{2}(\mu-\nu+1),\frac{1-\nu}{2};\frac{1}{2}(\mu-\nu+3);\cosh^2(c z)\right)$$

01.21.21.0325.01

$$\int \cosh(5 c z)\tanh^5(c z)\,dz = \frac{-25\operatorname{sech}^3(c z)+450\operatorname{sech}(c z)+750\cosh(c z)-50\cosh(3 c z)+3\cosh(5 c z)}{15 c}$$

01.21.21.0326.01

$$\int \cosh^2(c z)\tanh^2(c z)\,dz = \frac{\sinh(2 c z)-2 c z}{4 c}$$

01.21.21.0327.01

$$\int \sqrt{\cosh^3(2 c z)}\,\tanh^3(c z)\,dz = \frac{\sqrt{\cosh^3(2 c z)}}{6 c\cosh^{\frac{3}{2}}(2 c z)}\left(24\tan^{-1}\left(\cosh^{\frac{1}{2}}(2 c z)\right)+\cosh^{\frac{1}{2}}(2 c z)\left(2\cosh(2 c z)-3\left(\operatorname{sech}^2(c z)+6\right)\right)\right)$$

Involving powers of products with cosh

Involving $\sqrt{\cosh^{m}(c z)\tanh(c z)}$

01.21.21.0328.01

$$\int \sqrt{\cosh(c z)\tanh(c z)}\,dz = \frac{2\sqrt{i\sinh(c z)}}{c\sinh^{\frac{1}{2}}(c z)}E\left(\frac{1}{4}(\pi-2 i c z)\,\middle|\,2\right)$$

01.21.21.0329.01

$$\int \sqrt{\cosh^m(c\,z)\tanh(c\,z)}\,dz = -\frac{\sqrt{\cosh^{m-1}(c\,z)\sinh(c\,z)}\,\sinh(2\,c\,z)}{c\,(m+1)\left(-\sinh^2(c\,z)\right)^{3/4}}\,{}_2F_1\left(\frac{m+1}{4},\frac{1}{4};\frac{m+5}{4};\cosh^2(c\,z)\right)$$

Involving algebraic functions of cosh

Involving $(a+b\cosh(2\,c\,z))^\beta$

01.21.21.0330.01

$$\int (a+b\cosh(2\,c\,z))^\beta \tanh^\nu(c\,z)\,dz = \frac{1}{\nu\,c+c}\left(F_1\left(\frac{\nu+1}{2};\beta+1,-\beta;\frac{\nu+3}{2};\tanh^2(c\,z),\frac{(a-b)\tanh^2(c\,z)}{a+b}\right)\right.$$
$$\left.(a+b\cosh(2\,c\,z))^\beta\,\operatorname{sech}^2(c\,z)^\beta\,\tanh^{\nu+1}(c\,z)\left(\frac{(b-a)\tanh^2(c\,z)}{a+b}+1\right)^{-\beta}\right)$$

01.21.21.0331.01

$$\int \sqrt{a+b\cosh(2\,c\,z)}\,\tanh^2(c\,z)\,dz =$$
$$\frac{1}{c\sqrt{a+b\cosh(2\,c\,z)}}\left(i\left(\sqrt{\frac{a+b\cosh(2\,c\,z)}{a+b}}\,(a+b)\,F\left(i\,c\,z\,\middle|\,\frac{2\,b}{a+b}\right)+i\,(a+b\cosh(2\,c\,z))\tanh(c\,z)\right)-\right.$$
$$\left.2\,i\,(a+b)\sqrt{\frac{a+b\cosh(2\,c\,z)}{a+b}}\,E\left(i\,c\,z\,\middle|\,\frac{2\,b}{a+b}\right)\right)$$

01.21.21.0332.01

$$\int \sqrt{a-a\cosh(2\,c\,z)}\,\tanh^2(c\,z)\,dz = \frac{(\cosh(2\,c\,z)+3)\sqrt{a-a\cosh(2\,c\,z)}\,\operatorname{csch}(c\,z)\operatorname{sech}(c\,z)}{2\,c}$$

01.21.21.0333.01

$$\int \sqrt{\cosh(2\,c\,z)\,a+a}\,\tanh^2(c\,z)\,dz = \frac{\sqrt{\cosh(2\,c\,z)\,a+a}\,\operatorname{sech}(c\,z)\left(\sinh(c\,z)-2\tan^{-1}\left(\tanh\left(\frac{c\,z}{2}\right)\right)\right)}{c}$$

01.21.21.0334.01

$$\int \sqrt{a+b\cosh(2\,c\,z)}\,\tanh^3(c\,z)\,dz =$$
$$\frac{1}{4\,c}\left(2\sqrt{a+b\cosh(2\,c\,z)}\,\operatorname{sech}^2(c\,z)+\sqrt{a+b\cosh(2\,c\,z)}-\frac{2\,(2\,a-3\,b)\tanh^{-1}\left(\frac{\sqrt{a+b\cosh(2\,c\,z)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}-\right.$$
$$\left.\frac{2\,b\tan^{-1}\left(\frac{\sqrt{a+b\cosh(2\,c\,z)}}{\sqrt{b-a}}\right)}{\sqrt{b-a}}+\frac{6\sqrt{a+b\cosh(2\,c\,z)}}{\sqrt{(a+b\cosh(2\,c\,z))\operatorname{sech}^4\left(\frac{c\,z}{2}\right)}}\sqrt{\frac{a+b\cosh(2\,c\,z)}{(\cosh(c\,z)+1)^2}}\right)$$

01.21.21.0335.01

$$\int \sqrt{a+b\cosh(2cz)}\,\tanh^4(cz)\,dz = \left(-\left(2a^2-2ba-3b^2+4\left(a^2-ba-b^2\right)\cosh(2cz)+b(2a-3b)\cosh(4cz)\right)\tanh(cz)\operatorname{sech}^2(cz) - 2\sqrt{\frac{a+b\cosh(2cz)}{a+b}}\left(7a^2-2ba-9b^2\right)i\,E\!\left(i\,cz\,\middle|\,\frac{2b}{a+b}\right) + 8\sqrt{\frac{a+b\cosh(2cz)}{a+b}}\left(a^2-b^2\right)i\,F\!\left(i\,cz\,\middle|\,\frac{2b}{a+b}\right)\right)\Big/\left(6(a-b)c\sqrt{a+b\cosh(2cz)}\right)$$

01.21.21.0336.01

$$\int \frac{\tanh^2(cz)}{\sqrt{a+b\cosh(2cz)}}\,dz = \frac{-i\sqrt{\frac{a+b\cosh(2cz)}{a+b}}\,(a+b)\,E\!\left(i\,cz\,\middle|\,\frac{2b}{a+b}\right)-(a+b\cosh(2cz))\tanh(cz)}{(a-b)\,c\sqrt{a+b\cosh(2cz)}}$$

01.21.21.0337.01

$$\int \frac{\tanh^3(cz)}{\sqrt{a+b\cosh(2cz)}}\,dz = \frac{\sqrt{a-b}\,\sqrt{a+b\cosh(2cz)}\,\operatorname{sech}^2(cz)-2a\tanh^{-1}\!\left(\frac{\sqrt{a+b\cosh(2cz)}}{\sqrt{a-b}}\right)}{2(a-b)^{3/2}\,c}$$

01.21.21.0338.01

$$\int \frac{\tanh^4(cz)}{\sqrt{a+b\cosh(2cz)}}\,dz = \left(-\left(\left(2a^2+ba+b^2\right)\cosh(2cz)+a(a+2b+b\cosh(4cz))\right)\tanh(cz)\operatorname{sech}^2(cz) - 4a\sqrt{\frac{a+b\cosh(2cz)}{a+b}}\,(a+b)\,i\,E\!\left(i\,cz\,\middle|\,\frac{2b}{a+b}\right) + \sqrt{\frac{a+b\cosh(2cz)}{a+b}}\left(a^2-b^2\right)i\,F\!\left(i\,cz\,\middle|\,\frac{2b}{a+b}\right)\right)\Big/\left(3(a-b)^2\,c\sqrt{a+b\cosh(2cz)}\right)$$

Involving $\cosh(c\,z)\,(a+b\cosh(2\,c\,z))^{\beta}$

01.21.21.0339.01

$$\int \cosh(cz)\,(a+b\cosh(2cz))^{\beta}\tanh^{\nu}(cz)\,dz = \frac{1}{\nu c+c}\left(F_1\!\left(\frac{\nu+1}{2};\beta+\frac{3}{2},-\beta;\frac{\nu+3}{2};\tanh^2(cz),\frac{(a-b)\tanh^2(cz)}{a+b}\right)(a+b\cosh(2cz))^{\beta}\operatorname{sech}(cz)\operatorname{sech}^2(cz)^{\beta-\frac{1}{2}}\tanh^{\nu+1}(cz)\left(\frac{(b-a)\tanh^2(cz)}{a+b}+1\right)^{-\beta}\right)$$

01.21.21.0340.01

$$\int \cosh(c\,z)\,\sqrt{a+b\cosh(2\,c\,z)}\,\tanh^2(c\,z)\,dz = \frac{1}{4\sqrt{a-b}\,\sqrt{b}\,c}\left(\sqrt{2}\,\sqrt{a-b}\,(a-3\,b)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sinh(c\,z)}{\sqrt{a+b\cosh(2\,c\,z)}}\right)+ 2\sqrt{b}\left(2\,(b-a)\tan^{-1}\left(\frac{\sqrt{a-b}\,\sinh(c\,z)}{\sqrt{a+b\cosh(2\,c\,z)}}\right)+\sqrt{a-b}\,\sqrt{a+b\cosh(2\,c\,z)}\,\sinh(c\,z)\right)\right)$$

01.21.21.0341.01

$$\int \frac{\cosh(c\,z)\tanh^2(c\,z)}{\sqrt{a+b\cosh(2\,c\,z)}}\,dz = \frac{1}{2\,c}\left(\frac{\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sinh(c\,z)}{\sqrt{a+b\cosh(2\,c\,z)}}\right)}{\sqrt{b}} - \frac{2\tan^{-1}\left(\frac{\sqrt{a-b}\,\sinh(c\,z)}{\sqrt{a+b\cosh(2\,c\,z)}}\right)}{\sqrt{a-b}}\right)$$

Involving $\cosh(2\,c\,z)\,(a+b\cosh(2\,c\,z))^\beta$

01.21.21.0342.01

$$\int \cosh(2\,c\,z)\,(a+b\cosh(2\,c\,z))^\beta \tanh^\nu(c\,z)\,dz = \frac{1}{c\,(\nu+1)\,(\nu+3)}\left((a+b\cosh(2\,c\,z))^\beta\,\mathrm{sech}^2(c\,z)^\beta\,\tanh^{\nu+1}(c\,z)\left(\frac{(b-a)\tanh^2(c\,z)}{a+b}+1\right)^{-\beta}\left((\nu+1)\,F_1\left(\frac{\nu+3}{2};\beta+2,-\beta;\frac{\nu+5}{2};\tanh^2(c\,z),\frac{(a-b)\tanh^2(c\,z)}{a+b}\right)\tanh^2(c\,z)+ (\nu+3)\,F_1\left(\frac{\nu+1}{2};\beta+2,-\beta;\frac{\nu+3}{2};\tanh^2(c\,z),\frac{(a-b)\tanh^2(c\,z)}{a+b}\right)\right)\right)$$

01.21.21.0343.01

$$\int \cosh(2\,c\,z)\,\sqrt{a+b\cosh(2\,c\,z)}\,\tanh^2(c\,z)\,dz = \frac{1}{3\,b\,c\sqrt{a+b\cosh(2\,c\,z)}}\left(-i\sqrt{\frac{a+b\cosh(2\,c\,z)}{a+b}}\,\left(a^2-8\,b\,a-9\,b^2\right)E\left(i\,c\,z\,\middle|\,\frac{2\,b}{a+b}\right)+ \sqrt{\frac{a+b\cosh(2\,c\,z)}{a+b}}\,\left(a^2-3\,b\,a-4\,b^2\right)i\,F\left(i\,c\,z\,\middle|\,\frac{2\,b}{a+b}\right)+\frac{1}{2}\,b\,(a+b\cosh(2\,c\,z))\,\mathrm{sech}(c\,z)\,(7\sinh(c\,z)+\sinh(3\,c\,z))\right)$$

01.21.21.0344.01

$$\int \cosh(2\,c\,z)\,\sqrt{\cosh(2\,c\,z)\,a+a}\,\tanh^2(c\,z)\,dz = \frac{\sqrt{\cosh(2\,c\,z)\,a+a}\,\mathrm{sech}(c\,z)\left(12\tan^{-1}\left(\tanh\left(\frac{c\,z}{2}\right)\right)-9\sinh(c\,z)+\sinh(3\,c\,z)\right)}{6\,c}$$

01.21.21.0345.01

$$\int \cosh(2\,c\,z)\,\sqrt{a-a\cosh(2\,c\,z)}\,\tanh^2(c\,z)\,dz = \frac{\sqrt{a-a\cosh(2\,c\,z)}\,(-14\cosh(2\,c\,z)+\cosh(4\,c\,z)-27)\,\mathrm{csch}(c\,z)\,\mathrm{sech}(c\,z)}{12\,c}$$

Involving sinh and cosh

01.21.21.0346.01

$$\int \sinh(c z)\,(a+b\cosh(2 c z))^{\beta} \tanh^{\nu}(c z)\,dz = \frac{1}{c(\nu+2)}\left(F_1\left(\frac{\nu+2}{2};\beta+\frac{3}{2},-\beta;\frac{\nu+4}{2};\tanh^2(c z),\frac{(a-b)\tanh^2(c z)}{a+b}\right)(a+b\cosh(2 c z))^{\beta}\operatorname{sech}(c z)\operatorname{sech}^2(c z)^{\beta-\frac{1}{2}}\tanh^{\nu+2}(c z)\left(\frac{(b-a)\tanh^2(c z)}{a+b}+1\right)^{-\beta}\right)$$

01.21.21.0347.01

$$\int \sinh(c z)\sqrt{a+b\cosh(2 c z)}\,\tanh^2(c z)\,dz = \frac{1}{4c}\left(\frac{\sqrt{2}\,(a-5b)\log\left(\sqrt{2}\,\sqrt{b}\,\cosh(c z)+\sqrt{a+b\cosh(2 c z)}\right)}{\sqrt{b}}+(\cosh(2 c z)+5)\sqrt{a+b\cosh(2 c z)}\,\operatorname{sech}(c z)\right)$$

01.21.21.0348.01

$$\int \frac{\sinh(c z)\tanh^2(c z)}{\sqrt{a+b\cosh(2 c z)}}\,dz = \frac{\log\left(\sqrt{2}\,\sqrt{b}\,\cosh(c z)+\sqrt{a+b\cosh(2 c z)}\right)}{\sqrt{2}\,\sqrt{b}\,c}+\frac{\sqrt{a+b\cosh(2 c z)}\,\operatorname{sech}(c z)}{a c-b c}$$

01.21.21.0349.01

$$\int \frac{\sinh(c z)\tanh^3(c z)}{\sqrt{a+b\cosh(2 c z)}}\,dz = \frac{1}{2(a-b)^{3/2}\sqrt{b}\,c}\left(\sqrt{2}\,\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sinh(c z)}{\sqrt{a+b\cosh(2 c z)}}\right)(a-b)^{3/2}+\sqrt{b}\left((b-3a)\tan^{-1}\left(\frac{\sqrt{a-b}\,\sinh(c z)}{\sqrt{a+b\cosh(2 c z)}}\right)+\sqrt{a-b}\,\sqrt{a+b\cosh(2 c z)}\,\operatorname{sech}(c z)\tanh(c z)\right)\right)$$

01.21.21.0350.01

$$\int \frac{\sinh^2(c z)\tanh^2(c z)}{\sqrt{a+b\cosh(2 c z)}}\,dz = \left(-i\left(a^2-2ba-3b^2\right)\sqrt{\frac{a+b\cosh(2 c z)}{a+b}}\,E\left(i c z\,\middle|\,\frac{2b}{a+b}\right)+i\left(a^2-b^2\right)\sqrt{\frac{a+b\cosh(2 c z)}{a+b}}\,F\left(i c z\,\middle|\,\frac{2b}{a+b}\right)+2b(a+b\cosh(2 c z))\tanh(c z)\right)\Big/\left(2(a-b)\,b\,c\sqrt{a+b\cosh(2 c z)}\right)$$

Involving rational functions of the direct function and hyperbolic functions

Involving rational functions of sinh

Involving $(a\sinh(c z)+b\tanh(c z))^{-n}$

01.21.21.0351.01

$$\int \frac{1}{a\sinh(c z)+b\tanh(c z)}\,dz = \frac{-(a+b)\log\left(\cosh\left(\frac{c z}{2}\right)\right)+b\log(b+a\cosh(c z))+(a-b)\log\left(\sinh\left(\frac{c z}{2}\right)\right)}{(a-b)(a+b)c}$$

01.21.21.0352.01

$$\int \frac{1}{(a\sinh(c z)+b\tanh(c z))^2}\,dz = \left(\coth(c z)\left(-a\sinh(c z)\tanh(c z)\,b^2-(b+a\cosh(c z))\left(\left(a^2+b^2\right)\cosh(c z)-2\,a\,b\right)\operatorname{sech}(c z)-\frac{2\,b\left(2\,a^2+b^2\right)(b+a\cosh(c z))\tanh(c z)}{\sqrt{a^2-b^2}}\tan^{-1}\left(\frac{(b-a)\tanh\left(\frac{c z}{2}\right)}{\sqrt{a^2-b^2}}\right)\right)\right)\bigg/\left(\left(a^2-b^2\right)^2 c\,(b+a\cosh(c z))\right)$$

Involving rational functions of cosh

Involving $(a\cosh(c z)+b\tanh(c z))^{-n}$

01.21.21.0353.01

$$\int \frac{1}{a\cosh(c z)+b\tanh(c z)}\,dz = \frac{\log\left(-b-2\,a\sinh(c z)+\sqrt{b^2-4\,a^2}\right)-\log\left(b+2\,a\sinh(c z)+\sqrt{b^2-4\,a^2}\right)}{\sqrt{b^2-4\,a^2}\;c}$$

01.21.21.0354.01

$$\int \frac{1}{(a\cosh(c z)+b\tanh(c z))^2}\,dz = \left(\operatorname{sech}^2(c z)\,(\cosh(2 c z)\,a+a+2 b\sinh(c z))\left(\frac{2\cosh(c z)\,(b+2 a\sinh(c z))}{4 a^2-b^2}+\left(\sqrt{2}\left(b\left(\sqrt{b^2-4a^2}-b\right)-4a^2\right)\tan^{-1}\left(\frac{2a+\left(\sqrt{b^2-4a^2}-b\right)\tanh\left(\frac{c z}{2}\right)}{\sqrt{2}\sqrt{b}\sqrt{\sqrt{b^2-4a^2}-b}}\right)(\cosh(2 c z)\,a+a+2 b\sinh(c z))\right)\Big/\left(\sqrt{b}\left(b^2-4a^2\right)^{3/2}\sqrt{\sqrt{b^2-4a^2}-b}\right)+\left(\sqrt{2}\left(4a^2+b\left(b+\sqrt{b^2-4a^2}\right)\right)\tan^{-1}\left(\frac{2a-\left(b+\sqrt{b^2-4a^2}\right)\tanh\left(\frac{c z}{2}\right)}{\sqrt{2}\sqrt{-b\left(b+\sqrt{b^2-4a^2}\right)}}\right)(\cosh(2 c z)\,a+a+2 b\sinh(c z))\right)\Big/\left(\left(b^2-4a^2\right)^{3/2}\sqrt{-b\left(b+\sqrt{b^2-4a^2}\right)}\right)\right)\right)\Big/\left(4 c\,(a\cosh(c z)+b\tanh(c z))^2\right)$$

Involving algebraic functions of the direct function and hyperbolic functions

Involving sinh

Involving $\sinh(c z)\,(a+b\tanh(c z))^\beta$

01.21.21.0355.01

$$\int \sinh(c z)\sqrt{a+b\tanh(c z)}\,dz = \frac{1}{\sqrt{-a-b}\;c}\left(\cosh(c z)\left(\sqrt{-a-b}\sqrt{a+b\tanh(c z)}-i\,F\!\left(i\sinh^{-1}\left(\frac{\sqrt{-a-b}}{\sqrt{a+b\tanh(c z)}}\right)\middle|\,\frac{a-b}{a+b}\right)\operatorname{sech}(c z)\,(a\cosh(c z)+b\sinh(c z))\sqrt{\frac{b(\tanh(c z)-1)}{a+b\tanh(c z)}}\sqrt{\frac{b(\tanh(c z)+1)}{a+b\tanh(c z)}}\right)\right)$$

01.21.21.0356.01

$$\int \frac{\sinh(c z)}{\sqrt{a+b\tanh(c z)}}\,dz = \frac{1}{(a-b)c}\left(\sqrt{-\frac{1}{a+b}}\cosh(c z)\left(\sqrt{\frac{b(\tanh(c z)+1)}{b-a}}(b-a)\sqrt{\frac{b-b\tanh(c z)}{a+b}}\,i\,E\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\tanh(c z)}\right)\middle|\frac{a+b}{a-b}\right)- i(b-a)F\left(i\sinh^{-1}\left(\sqrt{-\frac{1}{a+b}}\sqrt{a+b\tanh(c z)}\right)\middle|\frac{a+b}{a-b}\right)\sqrt{\frac{b(\tanh(c z)+1)}{b-a}}\sqrt{\frac{b-b\tanh(c z)}{a+b}}+\sqrt{-\frac{1}{a+b}}(b\tanh(c z)-a)\sqrt{a+b\tanh(c z)}\right)\right)$$

Involving $\sinh(c z)\left(a+b\tanh^2(c z)\right)^\beta$

01.21.21.0357.01

$$\int \sinh(c z)\left(a+b\tanh^2(c z)\right)^\beta dz = \frac{\cosh(c z)\left(b\tanh^2(c z)+a\right)^{\beta+1}}{2(a+b)c(\beta+1)}\,{}_2F_1\left(\beta+1,\frac{3}{2};\beta+2;\frac{b\tanh^2(c z)+a}{a+b}\right)\sqrt{\frac{b\,\text{sech}^2(c z)}{a+b}}$$

01.21.21.0358.01

$$\int \sinh(c z)\sqrt{a+b\tanh^2(c z)}\,dz = \left(\sqrt{2}\cosh(c z)\left(\sqrt{4b\tanh^2\left(\frac{c z}{2}\right)+a\left(\tanh^2\left(\frac{c z}{2}\right)+1\right)^2}\cosh^2\left(\frac{c z}{2}\right)+\sqrt{b}\,i\log\left(-\frac{2\left(\sqrt{b}\,i\tanh^2\left(\frac{c z}{2}\right)-i\sqrt{b}+\sqrt{4b\tanh^2\left(\frac{c z}{2}\right)+a\left(\tanh^2\left(\frac{c z}{2}\right)+1\right)^2}\right)}{b\left(\tanh^2\left(\frac{c z}{2}\right)+1\right)}\right)\right)\sqrt{b\tanh^2(c z)+a}\right)\Bigg/\left(c(\cosh(c z)+1)\sqrt{\frac{a-b+(a+b)\cosh(2 c z)}{(\cosh(c z)+1)^2}}\right)$$

01.21.21.0359.01

$$\int \frac{\sinh(c z)}{\sqrt{a+b\tanh^2(c z)}}\,dz = \frac{(a-b)\,\text{sech}(c z)}{2(a+b)c\sqrt{b\tanh^2(c z)+a}}\sqrt{\frac{(a+b)\cosh(2 c z)}{a-b}+1}\left(\sqrt{\frac{(a+b)\cosh(2 c z)}{a-b}+1}-1\right)$$

Involving cosh

Involving $\cosh(c z)\,(a + b\tanh(c z))^{\beta}$

01.21.21.0360.01

$$\int \cosh(c z)\,(a + b\tanh(c z))^{\beta}\,dz = \cosh(c z)\left(\frac{b(\tanh(c z)+1)}{b-a}\right)^{3/2}\sqrt{\frac{b - b\tanh(c z)}{a+b}}\,(a + b\tanh(c z))^{\beta+1}\Big/\left((a+b)\,c\,(\beta+1)(\tanh(c z)+1)\right) F_1\!\left(\beta+1; \frac{3}{2}, \frac{3}{2}; \beta+2; \frac{a+b\tanh(c z)}{a-b}, \frac{a+b\tanh(c z)}{a+b}\right)$$

01.21.21.0361.01

$$\int \cosh(c z)\sqrt{a + b\tanh(c z)}\,dz = \frac{1}{2c}\Bigg(\operatorname{sech}(c z)\Bigg(\frac{1}{\sqrt{-a-b}\,(a-b)\,b}\Bigg(2 i a^2\cosh(c z)\left(E\!\left(i\sinh^{-1}\!\left(\frac{\sqrt{-a-b}}{\sqrt{a+b\tanh(c z)}}\right)\middle|\frac{a-b}{a+b}\right) - F\!\left(i\sinh^{-1}\!\left(\frac{\sqrt{-a-b}}{\sqrt{a+b\tanh(c z)}}\right)\middle|\frac{a-b}{a+b}\right)\right)(a\cosh(c z)+b\sinh(c z))\sqrt{\frac{b(\tanh(c z)+1)}{a+b\tanh(c z)}}\sqrt{1-\frac{a+b}{a+b\tanh(c z)}}\Bigg) + \frac{1}{\sqrt{-a-b}\,b}\Bigg(2 i a\cosh(c z)\,F\!\left(i\sinh^{-1}\!\left(\frac{\sqrt{-a-b}}{\sqrt{a+b\tanh(c z)}}\right)\middle|\frac{a-b}{a+b}\right)(a\cosh(c z)+b\sinh(c z))\sqrt{\frac{b(\tanh(c z)+1)}{a+b\tanh(c z)}}\sqrt{1-\frac{a+b}{a+b\tanh(c z)}}\Bigg) + \sinh(2 c z)\sqrt{a+b\tanh(c z)} - \frac{1}{\sqrt{-a-b}\,(a-b)}\Bigg(2 i b\cosh(c z)\left(E\!\left(i\sinh^{-1}\!\left(\frac{\sqrt{-a-b}}{\sqrt{a+b\tanh(c z)}}\right)\middle|\frac{2a}{a+b}-1\right) - F\!\left(i\sinh^{-1}\!\left(\frac{\sqrt{-a-b}}{\sqrt{a+b\tanh(c z)}}\right)\middle|\frac{2a}{a+b}-1\right)\right)(a\cosh(c z)+b\sinh(c z))\sqrt{\frac{b(\tanh(c z)+1)}{a+b\tanh(c z)}}\sqrt{1-\frac{a+b}{a+b\tanh(c z)}} + \frac{2b}{\sqrt{a+b\tanh(c z)}}\Bigg)\Bigg)\Bigg)$$

01.21.21.0362.01

$$\int \frac{\cosh(c\,z)}{\sqrt{a+b\tanh(c\,z)}}\,dz = \left(\cosh(c\,z)\left(-a\sqrt{\frac{a+b\tanh(c\,z)}{a-b}}\sqrt{\frac{b-b\tanh(c\,z)}{a+b}}\,(a+b)\,E\!\left(\sin^{-1}\!\left(\sqrt{\frac{a+b\tanh(c\,z)}{a-b}}\right)\middle|\frac{a-b}{a+b}\right)\operatorname{sech}(c\,z)\sqrt{1-\tanh(c\,z)}\right.\right.$$
$$(\cosh(c\,z)+\sinh(c\,z))+a\sqrt{\frac{a+b\tanh(c\,z)}{a-b}}\;b\sqrt{\frac{b-b\tanh(c\,z)}{a+b}}\;F\!\left(\sin^{-1}\!\left(\sqrt{\frac{a+b\tanh(c\,z)}{a-b}}\right)\middle|\frac{a-b}{a+b}\right)$$
$$\sqrt{1-\tanh(c\,z)}\,(\tanh(c\,z)+1)+\sqrt{\frac{b\,(\tanh(c\,z)+1)}{b-a}}\left(\sqrt{1-\tanh(c\,z)}\,\left(\left(a^2-b^2\right)\tanh(c\,z)-a\,b\operatorname{sech}^2(c\,z)\right)-\right.$$
$$\left.\left.\left.b^2\,F\!\left(\sin^{-1}\!\left(\frac{\sqrt{1-\tanh(c\,z)}}{\sqrt{2}}\right)\middle|\frac{2\,b}{a+b}\right)(\tanh(c\,z)-1)\sqrt{\tanh(c\,z)+1}\sqrt{\frac{a+b\tanh(c\,z)}{a+b}}\right)\right)\right)\Bigg/$$
$$\left(\left(a^2-b^2\right)c\sqrt{1-\tanh(c\,z)}\sqrt{\frac{b\,(\tanh(c\,z)+1)}{b-a}}\sqrt{a+b\tanh(c\,z)}\right)$$

Involving $\cosh(c\,z)\left(a+b\tanh^2(c\,z)\right)^{\beta}$

01.21.21.0363.01

$$\int \cosh(c\,z)\left(a+b\tanh^2(c\,z)\right)^{\beta}\,dz = \frac{\sqrt{\operatorname{sech}^2(c\,z)}\,\sinh(c\,z)\left(b\tanh^2(c\,z)+a\right)^{\beta}\left(\frac{b\tanh^2(c\,z)}{a}+1\right)^{-\beta}}{c}\,F_1\!\left(\frac{1}{2};\frac{3}{2},-\beta;\frac{3}{2};\tanh^2(c\,z),-\frac{b\tanh^2(c\,z)}{a}\right)$$

01.21.21.0364.01

$$\int \cosh(c\,z)\sqrt{a+b\tanh^2(c\,z)}\,dz = \left(i\sqrt{2}\sqrt{-\frac{1}{a}}\;a\cosh(c\,z)\coth(c\,z)\right.$$
$$\left(E\!\left(i\sinh^{-1}\!\left(\frac{\sqrt{-\frac{1}{a}}\sqrt{a-b+(a+b)\cosh(2\,c\,z)}}{\sqrt{2}}\right)\middle|-\frac{a}{b}\right)-F\!\left(i\sinh^{-1}\!\left(\frac{\sqrt{-\frac{1}{a}}\sqrt{a-b+(a+b)\cosh(2\,c\,z)}}{\sqrt{2}}\right)\middle|-\frac{a}{b}\right)\right)$$
$$\left.\sqrt{-\frac{(a+b)\sinh^2(c\,z)}{a}}\sqrt{b\tanh^2(c\,z)+a}\right)\Bigg/\left(c\sqrt{\frac{(a+b)\cosh^2(c\,z)}{b}}\sqrt{a-b+(a+b)\cosh(2\,c\,z)}\right)$$

01.21.21.0365.01

$$\int \frac{\cosh(c\,z)}{\sqrt{a+b\tanh^2(c\,z)}}\,dz = \left((\cosh(c\,z)+1)\sqrt{\frac{a-b+(a+b)\cosh(2\,c\,z)}{(\cosh(c\,z)+1)^2}}\right.$$
$$\operatorname{sech}(c\,z)\left(\sqrt{-\frac{a+2\left(b+\sqrt{b\,(a+b)}\right)}{a}}\,(a-b+(a+b)\cosh(2\,c\,z))\tanh\left(\frac{c\,z}{2}\right)\operatorname{sech}^2\left(\frac{c\,z}{2}\right)+\right.$$
$$2\left(a+2\left(b+\sqrt{b\,(a+b)}\right)\right)E\left(\sin^{-1}\left(\sqrt{-\frac{a+2\left(b+\sqrt{b\,(a+b)}\right)}{a}}\tanh\left(\frac{c\,z}{2}\right)\right)\middle|\frac{a+2\,b-2\sqrt{b\,(a+b)}}{a+2\left(b+\sqrt{b\,(a+b)}\right)}\right)$$
$$\sqrt{\frac{\left(-b+\left(a+b-\sqrt{b\,(a+b)}\right)\cosh(c\,z)+\sqrt{b\,(a+b)}\right)\operatorname{sech}^2\left(\frac{c\,z}{2}\right)}{a}}$$
$$\sqrt{\frac{\left(-b+\left(a+b+\sqrt{b\,(a+b)}\right)\cosh(c\,z)-\sqrt{b\,(a+b)}\right)\operatorname{sech}^2\left(\frac{c\,z}{2}\right)}{a}}-$$
$$4\sqrt{b\,(a+b)}\,F\left(\sin^{-1}\left(\sqrt{-\frac{a+2\left(b+\sqrt{b\,(a+b)}\right)}{a}}\tanh\left(\frac{c\,z}{2}\right)\right)\middle|\frac{a+2\,b-2\sqrt{b\,(a+b)}}{a+2\left(b+\sqrt{b\,(a+b)}\right)}\right)$$
$$\sqrt{\frac{\left(-b+\left(a+b-\sqrt{b\,(a+b)}\right)\cosh(c\,z)+\sqrt{b\,(a+b)}\right)\operatorname{sech}^2\left(\frac{c\,z}{2}\right)}{a}}$$
$$\left.\left.\sqrt{\frac{\left(-b+\left(a+b+\sqrt{b\,(a+b)}\right)\cosh(c\,z)-\sqrt{b\,(a+b)}\right)\operatorname{sech}^2\left(\frac{c\,z}{2}\right)}{a}}\right)\right)\Big/$$
$$\left(2\,(a+b)\sqrt{-\frac{a+2\left(b+\sqrt{b\,(a+b)}\right)}{a}}\;c\sqrt{(a-b+(a+b)\cosh(2\,c\,z))\operatorname{sech}^4\left(\frac{c\,z}{2}\right)}\sqrt{b\tanh^2(c\,z)+a}\right)$$

Involving sinh and cosh

01.21.21.0366.01

$$\int \frac{\cosh^3(z)\,(\cosh(2z) - 3\tanh(z))}{\left(\sinh^2(z) - \sinh(2z)\right)\sqrt{\sinh^5(2z)}}\,dz = \frac{1}{240\sqrt{\sinh^5(2z)}}\left(\cosh^2(z)\left(-690\sqrt[4]{-1}\,\sqrt{\coth^2\left(\frac{z}{2}\right)+1}\;F\left(i\sinh^{-1}\left(\frac{\sqrt[4]{-1}}{\tanh^{\frac{1}{2}}\left(\frac{z}{2}\right)}\right)\middle|\,-1\right)\tanh^{\frac{1}{2}}\left(\frac{z}{2}\right)\sinh^3(z) + 690\sqrt[4]{-1}\,\sqrt{\coth^2\left(\frac{z}{2}\right)+1}\;\Pi\left(\sqrt[6]{-1};\,i\sinh^{-1}\left(\frac{\sqrt[4]{-1}}{\tanh^{\frac{1}{2}}\left(\frac{z}{2}\right)}\right)\middle|\,-1\right)\tanh^{\frac{1}{2}}\left(\frac{z}{2}\right)\sinh^3(z) + 690\sqrt[4]{-1}\,\sqrt{\coth^2\left(\frac{z}{2}\right)+1}\;\Pi\left((-1)^{5/6};\,i\sinh^{-1}\left(\frac{\sqrt[4]{-1}}{\tanh^{\frac{1}{2}}\left(\frac{z}{2}\right)}\right)\middle|\,-1\right)\tanh^{\frac{1}{2}}\left(\frac{z}{2}\right)\sinh^3(z) - 50\sinh(z) + 51\cosh(z) - 3\cosh(3z) - 50\sinh(3z)\right)\right)$$

Involving functions of the direct function, hyperbolic and a power functions

Involving powers of the direct function, hyperbolic and a power functions

Involving sinh and power

Involving $z^n \sinh(a + b z) \tanh^v(c z)$

01.21.21.0367.01

$$\int z^n \sinh(a+b\,z)\tanh^{\nu}(c\,z)\,dz = -\frac{1}{2}\,n!\,e^{-a}\left(i^{\nu}\,e^{(-b+c\,\nu)\,z}\binom{\nu}{\frac{\nu}{2}}(1-\nu \bmod 2)\sum_{j=0}^{n}\frac{(-1)^j\,(-b+c\,\nu)^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{-b+c\,\nu}{2\,c},\,\ldots,\,\frac{-b+c\,\nu}{2\,c},\,\nu;\,\frac{-b+c\,\nu}{2\,c}+1,\,\ldots,\,\frac{-b+c\,\nu}{2\,c}+1;\,-e^{2\,c\,z}\right)+\sum_{s=0}^{\left\lfloor\frac{\nu-1}{2}\right\rfloor}(-1)^s\binom{\nu}{s}\left((-1)^{\nu}\,e^{(-b+2\,c\,s)\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(-b+2\,c\,s)^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{-b+2\,c\,s}{2\,c},\,\ldots,\,\frac{-b+2\,c\,s}{2\,c},\,\nu;\,\frac{-b+2\,c\,s}{2\,c}+1,\,\ldots,\,\frac{-b+2\,c\,s}{2\,c}+1;\,-e^{2\,c\,z}\right)+e^{(-b+2\,c\,(\nu-s))\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(-b+2\,c\,(\nu-s))^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{-b+2\,c\,(\nu-s)}{2\,c},\,\ldots,\,\frac{-b+2\,c\,(\nu-s)}{2\,c},\,\nu;\,\frac{-b+2\,c\,(\nu-s)}{2\,c}+1,\,\ldots,\,\frac{-b+2\,c\,(\nu-s)}{2\,c}+1;\,-e^{2\,c\,z}\right)\right)\right)+$$

$$\frac{1}{2}\,n!\,e^{a}\left(i^{\nu}\,e^{(b+c\,\nu)\,z}\binom{\nu}{\frac{\nu}{2}}(1-\nu \bmod 2)\sum_{j=0}^{n}\frac{(-1)^j\,(b+c\,\nu)^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{b+c\,\nu}{2\,c},\,\ldots,\,\frac{b+c\,\nu}{2\,c},\,\nu;\,\frac{b+c\,\nu}{2\,c}+1,\,\ldots,\,\frac{b+c\,\nu}{2\,c}+1;\,-e^{2\,c\,z}\right)+\sum_{s=0}^{\left\lfloor\frac{\nu-1}{2}\right\rfloor}(-1)^s\binom{\nu}{s}\left((-1)^{\nu}\,e^{(b+2\,c\,s)\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(b+2\,c\,s)^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{b+2\,c\,s}{2\,c},\,\ldots,\,\frac{b+2\,c\,s}{2\,c},\,\nu;\,\frac{b+2\,c\,s}{2\,c}+1,\,\ldots,\,\frac{b+2\,c\,s}{2\,c}+1;\,-e^{2\,c\,z}\right)+e^{(b+2\,c\,(\nu-s))\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(b+2\,c\,(\nu-s))^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{b+2\,c\,(\nu-s)}{2\,c},\,\ldots,\,\frac{b+2\,c\,(\nu-s)}{2\,c},\,\nu;\,\frac{b+2\,c\,(\nu-s)}{2\,c}+1,\,\ldots,\,\frac{b+2\,c\,(\nu-s)}{2\,c}+1;\,-e^{2\,c\,z}\right)\right)\right)\,/;\,n\in\mathbb{N}\bigwedge\nu\in\mathbb{N}^{+}$$

01.21.21.0368.01

$$\int z^n \sinh(b z) \tanh^\nu(c z)\, dz = -\frac{1}{2} n! \left(i^\nu e^{(c\nu-b) z} \binom{\nu}{\frac{\nu}{2}} (1-\nu \bmod 2) \sum_{j=0}^{n} \frac{(-1)^j (c\nu-b)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{c\nu-b}{2c}, \ldots, \frac{c\nu-b}{2c}, \nu; \frac{c\nu-b}{2c}+1, \ldots, \frac{c\nu-b}{2c}+1; -e^{2cz}\right) + \sum_{s=0}^{\left\lfloor\frac{\nu-1}{2}\right\rfloor} (-1)^s \binom{\nu}{s} \left((-1)^\nu e^{(-b+c\nu-c(\nu-2s))z} \sum_{j=0}^{n} \frac{(-1)^j (2cs-b)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{2cs-b}{2c}, \ldots, \frac{2cs-b}{2c}, \nu; \frac{2cs-b}{2c}+1, \ldots, \frac{2cs-b}{2c}+1; -e^{2cz}\right) + e^{(-b+c\nu+c(\nu-2s))z} \sum_{j=0}^{n} \frac{(-1)^j (2c(\nu-s)-b)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{2c(\nu-s)-b}{2c}, \ldots, \frac{2c(\nu-s)-b}{2c}, \nu; \frac{2c(\nu-s)-b}{2c}+1, \ldots, \frac{2c(\nu-s)-b}{2c}+1; -e^{2cz}\right)\right) - i^\nu e^{(b+c\nu)z} \binom{\nu}{\frac{\nu}{2}} (1-\nu \bmod 2) \sum_{j=0}^{n} \frac{(-1)^j (b+c\nu)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{b+c\nu}{2c}, \ldots, \frac{b+c\nu}{2c}, \nu; \frac{b+c\nu}{2c}+1, \ldots, \frac{b+c\nu}{2c}+1; -e^{2cz}\right) - \sum_{s=0}^{\left\lfloor\frac{\nu-1}{2}\right\rfloor} (-1)^s \binom{\nu}{s} \left(e^{(b+c\nu+c(\nu-2s))z} \sum_{j=0}^{n} \frac{(-1)^j (b+2c(\nu-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{b+2c(\nu-s)}{2c}, \ldots, \frac{b+2c(\nu-s)}{2c}, \nu; \frac{b+2c(\nu-s)}{2c}+1, \ldots, \frac{b+2c(\nu-s)}{2c}+1; -e^{2cz}\right) + (-1)^\nu e^{(b+c\nu-c(\nu-2s))z} \sum_{j=0}^{n} \frac{(-1)^j (b+2cs)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{b+2cs}{2c}, \ldots, \frac{b+2cs}{2c}, \nu; \frac{b+2cs}{2c}+1, \ldots, \frac{b+2cs}{2c}+1; -e^{2cz}\right)\right)\right) /; n \in \mathbb{N} \wedge \nu \in \mathbb{N}^+$$

Involving powers of sinh and power

Involving $z^n \sinh^u(b z) \tanh^\nu(c z)$

01.21.21.0369.01

$$\int z^n \sinh^u(b z) \tanh^\nu(c z)\, dz = \binom{u}{\frac{u}{2}} (1-u \bmod 2) \left(\frac{(-1)^\nu z^{n+1}}{n+1} - (-1)^\nu e^{2cz} \nu\, n! \sum_{j=0}^{n} \frac{(-1)^j 2^{-j-1} c^{-j-1} z^{n-j}}{(n-j)!} {}_{j+3}F_{j+2}(1, \ldots, 1, \nu+1; 2, \ldots, 2; -e^{2cz}) + e^{c\nu z} i^\nu \binom{\nu}{\frac{\nu}{2}} n! (1-\nu \bmod 2) \sum_{j=0}^{n} \frac{(-1)^j (c\nu)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{\nu}{2}, \ldots, \frac{\nu}{2}, \nu; \frac{\nu}{2}+1, \ldots, \frac{\nu}{2}+1; -e^{2cz}\right) + (-1)^\nu n! \sum_{s=1}^{\left\lfloor\frac{\nu-1}{2}\right\rfloor} (-1)^s \binom{\nu}{s} e^{2scz} \sum_{j=0}^{n} \frac{(-1)^j (2cs)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}(s, \ldots, s, \nu; s+1, \ldots, s+1; -e^{2cz}) + n! \sum_{s=0}^{\left\lfloor\frac{\nu-1}{2}\right\rfloor} (-1)^s \binom{\nu}{s}$$

$$e^{2c(v-s)z}\sum_{j=0}^{n}\frac{(-1)^j\,(2c(v-s))^{-j-1}\,z^{n-j}}{(n-j)!}\;{}_{j+2}F_{j+1}\left(v-s,\,\ldots,\,v-s,\,v;\,-s+v+1,\,\ldots,\,-s+v+1;\,-e^{2cz}\right)\Bigg)\left(\frac{i}{2}\right)^u+$$

$$2^{-u}\,n!\sum_{k=0}^{\left\lfloor\frac{u-1}{2}\right\rfloor}(-1)^k\binom{u}{k}\Bigg((-1)^u\,i^v\,e^{(cv-b(u-2k))z}\binom{v}{\frac{v}{2}}(1-v\bmod 2)\sum_{j=0}^{n}\frac{(-1)^j\,(cv-b(u-2k))^{-j-1}\,z^{n-j}}{(n-j)!}$$

$${}_{j+2}F_{j+1}\left(\frac{cv-b(u-2k)}{2c},\,\ldots,\,\frac{cv-b(u-2k)}{2c},\,v;\,\frac{cv-b(u-2k)}{2c}+1,\,\ldots,\,\frac{cv-b(u-2k)}{2c}+1;\,-e^{2cz}\right)+$$

$$(-1)^u\sum_{s=0}^{\left\lfloor\frac{v-1}{2}\right\rfloor}(-1)^s\binom{v}{s}\Bigg((-1)^v\,e^{(2bk+2cs-bu)z}\sum_{j=0}^{n}\frac{(-1)^j\,(2cs-b(u-2k))^{-j-1}\,z^{n-j}}{(n-j)!}\;{}_{j+2}F_{j+1}\Bigg(\frac{2cs-b(u-2k)}{2c},$$

$$\ldots,\,\frac{2cs-b(u-2k)}{2c},\,v;\,\frac{2cs-b(u-2k)}{2c}+1,\,\ldots,\,\frac{2cs-b(u-2k)}{2c}+1;\,-e^{2cz}\Bigg)+$$

$$e^{(2bk-2cs-bu+2cv)z}\sum_{j=0}^{n}\frac{(-1)^j\,(2c(v-s)-b(u-2k))^{-j-1}\,z^{n-j}}{(n-j)!}\;{}_{j+2}F_{j+1}\Bigg(\frac{2c(v-s)-b(u-2k)}{2c},$$

$$\ldots,\,\frac{2c(v-s)-b(u-2k)}{2c},\,v;\,\frac{2c(v-s)-b(u-2k)}{2c}+1,\,\ldots,\,\frac{2c(v-s)-b(u-2k)}{2c}+1;$$

$$-e^{2cz}\Bigg)\Bigg)+i^v\,e^{(b(u-2k)+cv)z}\binom{v}{\frac{v}{2}}(1-v\bmod 2)\sum_{j=0}^{n}\frac{(-1)^j\,(b(u-2k)+cv)^{-j-1}\,z^{n-j}}{(n-j)!}$$

$${}_{j+2}F_{j+1}\left(\frac{b(u-2k)+cv}{2c},\,\ldots,\,\frac{b(u-2k)+cv}{2c},\,v;\,\frac{b(u-2k)+cv}{2c}+1,\,\ldots,\,\frac{b(u-2k)+cv}{2c}+1;\,-e^{2cz}\right)+$$

$$\sum_{s=0}^{\left\lfloor\frac{v-1}{2}\right\rfloor}(-1)^s\binom{v}{s}\Bigg((-1)^v\,e^{(2cs+b(u-2k))z}\sum_{j=0}^{n}\frac{(-1)^j\,(2cs+b(u-2k))^{-j-1}\,z^{n-j}}{(n-j)!}\;{}_{j+2}F_{j+1}\Bigg(\frac{2cs+b(u-2k)}{2c},$$

$$\ldots,\,\frac{2cs+b(u-2k)}{2c},\,v;\,\frac{2cs+b(u-2k)}{2c}+1,\,\ldots,\,\frac{2cs+b(u-2k)}{2c}+1;\,-e^{2cz}\Bigg)+$$

$$e^{(b(u-2k)+2c(v-s))z}\sum_{j=0}^{n}\frac{(-1)^j\,(b(u-2k)+2c(v-s))^{-j-1}\,z^{n-j}}{(n-j)!}\;{}_{j+2}F_{j+1}\Bigg(\frac{b(u-2k)+2c(v-s)}{2c},$$

$$\ldots,\,\frac{b(u-2k)+2c(v-s)}{2c},\,v;\,\frac{b(u-2k)+2c(v-s)}{2c}+1,\,\ldots,$$

$$\frac{b(u-2k)+2c(v-s)}{2c}+1;\,-e^{2cz}\Bigg)\Bigg)\Bigg)\;/;\;n\in\mathbb{N}\land u\in\mathbb{N}^+\land v\in\mathbb{N}^+$$

Involving cosh and power

Involving $z^n\cosh(a+b\,z)\tanh^v(c\,z)$

01.21.21.0370.01

$$\int z^n \cosh(a+b\,z)\tanh^{\nu}(c\,z)\,dz = \frac{1}{2}\,n!\,e^{-a}\Bigg(i^{\nu}\,e^{(-b+c\,\nu)\,z}\binom{\nu}{\frac{\nu}{2}}(1-\nu \bmod 2)\sum_{j=0}^{n}\frac{(-1)^j\,(-b+c\,\nu)^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\Big(\frac{-b+c\,\nu}{2\,c},\,\ldots,\,\frac{-b+c\,\nu}{2\,c},\,\nu;\,\frac{-b+c\,\nu}{2\,c}+1,\,\ldots,\,\frac{-b+c\,\nu}{2\,c}+1;\,-e^{2\,c\,z}\Big)+$$
$$\sum_{s=0}^{\left\lfloor\frac{\nu-1}{2}\right\rfloor}(-1)^s\binom{\nu}{s}\Bigg((-1)^{\nu}\,e^{(-b+2\,c\,s)\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(-b+2\,c\,s)^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\Big(\frac{-b+2\,c\,s}{2\,c},\,\ldots,\,\frac{-b+2\,c\,s}{2\,c},\,\nu;\,\frac{-b+2\,c\,s}{2\,c}+$$
$$1,\,\ldots,\,\frac{-b+2\,c\,s}{2\,c}+1;\,-e^{2\,c\,z}\Big)+e^{(-b+2\,c\,(\nu-s))\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(-b+2\,c\,(\nu-s))^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}$$
$$\Big(\frac{-b+2\,c\,(\nu-s)}{2\,c},\,\ldots,\,\frac{-b+2\,c\,(\nu-s)}{2\,c},\,\nu;\,\frac{-b+2\,c\,(\nu-s)}{2\,c}+1,\,\ldots,\,\frac{-b+2\,c\,(\nu-s)}{2\,c}+1;\,-e^{2\,c\,z}\Big)\Bigg)\Bigg)+$$
$$\frac{1}{2}\,n!\,e^{a}\Bigg(i^{\nu}\,e^{(b+c\,\nu)\,z}\binom{\nu}{\frac{\nu}{2}}(1-\nu \bmod 2)\sum_{j=0}^{n}\frac{(-1)^j\,(b+c\,\nu)^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\Big(\frac{b+c\,\nu}{2\,c},\,\ldots,\,\frac{b+c\,\nu}{2\,c},\,\nu;$$
$$\frac{b+c\,\nu}{2\,c}+1,\,\ldots,\,\frac{b+c\,\nu}{2\,c}+1;\,-e^{2\,c\,z}\Big)+\sum_{s=0}^{\left\lfloor\frac{\nu-1}{2}\right\rfloor}(-1)^s\binom{\nu}{s}\Bigg((-1)^{\nu}\,e^{(b+2\,c\,s)\,z}$$
$$\sum_{j=0}^{n}\frac{(-1)^j\,(b+2\,c\,s)^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\Big(\frac{b+2\,c\,s}{2\,c},\,\ldots,\,\frac{b+2\,c\,s}{2\,c},\,\nu;\,\frac{b+2\,c\,s}{2\,c}+1,\,\ldots,\,\frac{b+2\,c\,s}{2\,c}+1;\,-e^{2\,c\,z}\Big)+$$
$$e^{(b+2\,c\,(\nu-s))\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(b+2\,c\,(\nu-s))^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\Big(\frac{b+2\,c\,(\nu-s)}{2\,c},\,\ldots,\,\frac{b+2\,c\,(\nu-s)}{2\,c},$$
$$\nu;\,\frac{b+2\,c\,(\nu-s)}{2\,c}+1,\,\ldots,\,\frac{b+2\,c\,(\nu-s)}{2\,c}+1;\,-e^{2\,c\,z}\Big)\Bigg)\Bigg)\,/;\,n\in\mathbb{N}\land\nu\in\mathbb{N}^{+}$$

01.21.21.0371.01

$$\int z^n \cosh(b\,z)\tanh^{\nu}(c\,z)\,dz = \frac{1}{2}\,n!\,\Bigg(i^{\nu}\,e^{(c\,\nu-b)\,z}\binom{\nu}{\frac{\nu}{2}}(1-\nu \bmod 2)$$

$$\sum_{j=0}^{n}\frac{(-1)^j\,(c\,\nu-b)^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\!\left(\frac{c\,\nu-b}{2\,c},\,\ldots,\,\frac{c\,\nu-b}{2\,c},\,\nu;\,\frac{c\,\nu-b}{2\,c}+1,\,\ldots,\,\frac{c\,\nu-b}{2\,c}+1;\,-e^{2\,c\,z}\right)+$$

$$\sum_{s=0}^{\left\lfloor\frac{\nu-1}{2}\right\rfloor}(-1)^s\binom{\nu}{s}\Bigg((-1)^{\nu}\,e^{(-b+c\,\nu-c\,(\nu-2\,s))\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(2\,c\,s-b)^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\Bigg(\frac{2\,c\,s-b}{2\,c},\,\ldots,\,\frac{2\,c\,s-b}{2\,c},\,\nu;\,\frac{2\,c\,s-b}{2\,c}+1,$$

$$\ldots,\,\frac{2\,c\,s-b}{2\,c}+1;\,-e^{2\,c\,z}\Bigg)+e^{(-b+c\,\nu+c\,(\nu-2\,s))\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(2\,c\,(\nu-s)-b)^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}$$

$$\left(\frac{2\,c\,(\nu-s)-b}{2\,c},\,\ldots,\,\frac{2\,c\,(\nu-s)-b}{2\,c},\,\nu;\,\frac{2\,c\,(\nu-s)-b}{2\,c}+1,\,\ldots,\,\frac{2\,c\,(\nu-s)-b}{2\,c}+1;\,-e^{2\,c\,z}\right)\Bigg)+i^{\nu}\,e^{(b+c\,\nu)\,z}$$

$$\binom{\nu}{\frac{\nu}{2}}(1-\nu \bmod 2)\sum_{j=0}^{n}\frac{(-1)^j\,(b+c\,\nu)^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\!\left(\frac{b+c\,\nu}{2\,c},\,\ldots,\,\frac{b+c\,\nu}{2\,c},\,\nu;\,\frac{b+c\,\nu}{2\,c}+1,\,\ldots,\,\frac{b+c\,\nu}{2\,c}+1;\,-e^{2\,c\,z}\right)+$$

$$\sum_{s=0}^{\left\lfloor\frac{\nu-1}{2}\right\rfloor}(-1)^s\binom{\nu}{s}\Bigg((-1)^{\nu}\,e^{(b+c\,\nu-c\,(\nu-2\,s))\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(b+2\,c\,s)^{-j-1}\,z^{n-j}}{(n-j)!}$$

$${}_{j+2}F_{j+1}\!\left(\frac{b+2\,c\,s}{2\,c},\,\ldots,\,\frac{b+2\,c\,s}{2\,c},\,\nu;\,\frac{b+2\,c\,s}{2\,c}+1,\,\ldots,\,\frac{b+2\,c\,s}{2\,c}+1;\,-e^{2\,c\,z}\right)+$$

$$e^{(b+c\,\nu+c\,(\nu-2\,s))\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(b+2\,c\,(\nu-s))^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\Bigg(\frac{b+2\,c\,(\nu-s)}{2\,c},\,\ldots,\,\frac{b+2\,c\,(\nu-s)}{2\,c},$$

$$\nu;\,\frac{b+2\,c\,(\nu-s)}{2\,c}+1,\,\ldots,\,\frac{b+2\,c\,(\nu-s)}{2\,c}+1;\,-e^{2\,c\,z}\Bigg)\Bigg)\Bigg)\,/;\,n\in\mathbb{N}\land\nu\in\mathbb{N}^+$$

Involving powers of cosh and power

Involving $z^n \cosh^u(b\,z)\tanh^{\nu}(c\,z)$

01.21.21.0372.01

$$\int z^n \cosh^u(b\,z)\tanh^{\nu}(c\,z)\,dz =$$

$$2^{-u}\binom{u}{\frac{u}{2}}n!\,(1-u \bmod 2)\Bigg(\frac{(-1)^{\nu}\,z^{n+1}}{(n+1)!}-(-1)^{\nu}\,e^{2\,c\,z}\,\nu\sum_{j=0}^{n}\frac{(-1)^j\,2^{-j-1}\,c^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+3}F_{j+2}\!\left(1,\,\ldots,\,1,\,\nu+1;\,2,\,\ldots,\,2;\,-e^{2\,c\,z}\right)+$$

$$e^{c\,\nu\,z}\,i^{\nu}\binom{\nu}{\frac{\nu}{2}}(1-\nu \bmod 2)\sum_{j=0}^{n}\frac{(-1)^j\,(c\,\nu)^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\!\left(\frac{\nu}{2},\,\ldots,\,\frac{\nu}{2},\,\nu;\,\frac{\nu}{2}+1,\,\ldots,\,\frac{\nu}{2}+1;\,-e^{2\,c\,z}\right)+$$

$$(-1)^v \sum_{s=1}^{\left\lfloor \frac{v-1}{2} \right\rfloor} (-1)^s e^{2 c s z} \binom{v}{s} \sum_{j=0}^{n} \frac{(-1)^j \, 2^{-j-1} \, (c\, s)^{-j-1} \, z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\!\left(s, \ldots, s, v; s+1, \ldots, s+1; -e^{2 c z}\right) + \sum_{s=0}^{\left\lfloor \frac{v-1}{2} \right\rfloor} (-1)^s$$

$$e^{2 c (v-s) z} \binom{v}{s} \sum_{j=0}^{n} \frac{(-1)^j \, 2^{-j-1} \, (c\,(v-s))^{-j-1} \, z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\!\left(v-s, \ldots, v-s, v; -s+v+1, \ldots, -s+v+1; -e^{2 c z}\right)\Bigg) +$$

$$2^{-u} \, n! \sum_{k=0}^{\left\lfloor \frac{u-1}{2} \right\rfloor} \binom{u}{k} \Bigg(i^v \, e^{(c v - b (u - 2 k)) z} \binom{v}{\frac{v}{2}} (1 - v \bmod 2) \sum_{j=0}^{n} \frac{(-1)^j \, (c\, v - b\,(u - 2 k))^{-j-1} \, z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\!\Bigg(\frac{c\, v - b\,(u-2k)}{2 c},$$

$$\ldots, \frac{c\, v - b\,(u-2k)}{2 c}, v; \frac{c\, v - b\,(u-2k)}{2 c} + 1, \ldots, \frac{c\, v - b\,(u-2k)}{2 c} + 1; -e^{2 c z}\Bigg) + \sum_{s=0}^{\left\lfloor \frac{v-1}{2} \right\rfloor} (-1)^s \binom{v}{s}$$

$$\Bigg((-1)^v \, e^{(2 b k + 2 c s - b u) z} \sum_{j=0}^{n} \frac{(-1)^j \, (2 c s - b\,(u-2k))^{-j-1} \, z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\!\Bigg(\frac{2 c s - b\,(u-2k)}{2 c}, \ldots, \frac{2 c s - b\,(u-2k)}{2 c},$$

$$v; \frac{2 c s - b\,(u-2k)}{2 c} + 1, \ldots, \frac{2 c s - b\,(u-2k)}{2 c} + 1; -e^{2 c z}\Bigg) + e^{(2 b k - 2 c s - b u + 2 c v) z}$$

$$\sum_{j=0}^{n} \frac{(-1)^j \, (2 c\,(v-s) - b\,(u-2k))^{-j-1} \, z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\!\Bigg(\frac{2 c\,(v-s) - b\,(u-2k)}{2 c}, \ldots, \frac{2 c\,(v-s) - b\,(u-2k)}{2 c},$$

$$v; \frac{2 c\,(v-s) - b\,(u-2k)}{2 c} + 1, \ldots, \frac{2 c\,(v-s) - b\,(u-2k)}{2 c} + 1; -e^{2 c z}\Bigg)\Bigg) +$$

$$i^v \, e^{(b (u - 2 k) + c v) z} \binom{v}{\frac{v}{2}} (1 - v \bmod 2) \sum_{j=0}^{n} \frac{(-1)^j \, (b\,(u-2k) + c\, v)^{-j-1} \, z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\!\Bigg(\frac{b\,(u-2k) + c\, v}{2 c},$$

$$\ldots, \frac{b\,(u-2k) + c\, v}{2 c}, v; \frac{b\,(u-2k) + c\, v}{2 c} + 1, \ldots, \frac{b\,(u-2k) + c\, v}{2 c} + 1; -e^{2 c z}\Bigg) +$$

$$\sum_{s=0}^{\left\lfloor \frac{v-1}{2} \right\rfloor} (-1)^s \binom{v}{s} \Bigg((-1)^v \, e^{(2 c s + b (u - 2 k)) z} \sum_{j=0}^{n} \frac{(-1)^j \, (2 c s + b\,(u-2k))^{-j-1} \, z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\!\Bigg(\frac{2 c s + b\,(u-2k)}{2 c},$$

$$\ldots, \frac{2 c s + b\,(u-2k)}{2 c}, v; \frac{2 c s + b\,(u-2k)}{2 c} + 1, \ldots, \frac{2 c s + b\,(u-2k)}{2 c} + 1; -e^{2 c z}\Bigg) +$$

$$e^{(b (u - 2 k) + 2 c (v - s)) z} \sum_{j=0}^{n} \frac{(-1)^j \, (b\,(u-2k) + 2 c\,(v-s))^{-j-1} \, z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\!\Bigg(\frac{b\,(u-2k) + 2 c\,(v-s)}{2 c},$$

$$\ldots, \frac{b\,(u-2k) + 2 c\,(v-s)}{2 c}, v; \frac{b\,(u-2k) + 2 c\,(v-s)}{2 c} + 1, \ldots,$$

$$\frac{b\,(u-2k) + 2 c\,(v-s)}{2 c} + 1; -e^{2 c z}\Bigg)\Bigg)\Bigg) \,/;\, n \in \mathbb{N} \land u \in \mathbb{N}^+ \land v \in \mathbb{N}^+$$

Involving functions of the direct function, hyperbolic and exponential functions

Involving powers of the direct function, hyperbolic and exponential functions

Involving sinh and exp

Involving $e^{p z} \sinh(b z) \tanh^{\nu}(c z)$

01.21.21.0373.01

$$\int e^{p z} \sinh(b z) \tanh^{\nu}(c z) \, dz = \frac{1}{2} \left(1 - e^{-2 c z}\right)^{-\nu} \left(1 + e^{-2 c z}\right)^{\nu} \tanh^{\nu}(c z) \left(\frac{e^{(b+p) z} F_1\left(-\frac{b+p}{2 c}; -\nu, \nu; 1 - \frac{b+p}{2 c}; e^{-2 c z}, -e^{-2 c z}\right)}{b + p} - \frac{e^{(p-b) z} F_1\left(-\frac{p-b}{2 c}; -\nu, \nu; 1 - \frac{p-b}{2 c}; e^{-2 c z}, -e^{-2 c z}\right)}{p - b} \right)$$

01.21.21.0374.01

$$\int e^{b z} \sinh(b z) \tanh^{\nu}(c z) \, dz = \frac{1}{8 b c (\nu - 1)} \left(e^{-2 c z} \left(1 - e^{-2 c z}\right)^{-\nu} \left(-\sinh^2(c z)\right)^{\frac{1}{2} (-\nu - 1)} \left(2 c e^{2 (b+c) z} \left(1 + e^{-2 c z}\right)^{\nu} (\nu - 1) F_1\left(-\frac{b}{c}; \nu, -\nu; 1 - \frac{b}{c}; -e^{-2 c z}, e^{-2 c z}\right) \left(-\sinh^2(c z)\right)^{\frac{\nu+1}{2}} - b \left(1 - e^{-2 c z}\right)^{\nu} \left(-1 + e^{4 c z}\right) {}_2F_1\left(\frac{1-\nu}{2}, \frac{1-\nu}{2}; \frac{3-\nu}{2}; \cosh^2(c z)\right) \right) \tanh^{\nu}(c z) \right)$$

01.21.21.0375.01

$$\int e^{-b z} \sinh(b z) \tanh^{\nu}(c z) \, dz = \frac{1}{8 b c (\nu - 1)} \left(e^{-2 c z} \left(1 - e^{-2 c z}\right)^{-\nu} \left(-\sinh^2(c z)\right)^{\frac{1}{2} (-\nu - 1)} \left(2 c e^{2 (c-b) z} \left(1 + e^{-2 c z}\right)^{\nu} (\nu - 1) F_1\left(\frac{b}{c}; \nu, -\nu; \frac{b+c}{c}; -e^{-2 c z}, e^{-2 c z}\right) \left(-\sinh^2(c z)\right)^{\frac{\nu+1}{2}} + b \left(1 - e^{-2 c z}\right)^{\nu} \left(-1 + e^{4 c z}\right) {}_2F_1\left(\frac{1-\nu}{2}, \frac{1-\nu}{2}; \frac{3-\nu}{2}; \cosh^2(c z)\right) \right) \tanh^{\nu}(c z) \right)$$

Involving powers of sinh and exp

Involving $e^{p z} \sinh^{u}(b z) \tanh^{\nu}(c z)$

01.21.21.0376.01

$$\int e^{p z} \sinh^{u}(b z) \tanh^{\nu}(c z) \, dz = \frac{1}{p} e^{p z} \left(1 - e^{-2 c z}\right)^{-\nu} \left(1 + e^{-2 c z}\right)^{\nu} F_1\left(-\frac{p}{2 c}; -\nu, \nu; 1 - \frac{p}{2 c}; e^{-2 c z}, -e^{-2 c z}\right) \binom{u}{\frac{u}{2}} (1 - u \bmod 2) \tanh^{\nu}(c z) \left(\frac{i}{2}\right)^{u} + 2^{-u} \left(1 - e^{-2 c z}\right)^{-\nu} \left(1 + e^{-2 c z}\right)^{\nu} \tanh^{\nu}(c z) \sum_{k=0}^{\left\lfloor \frac{u-1}{2} \right\rfloor} (-1)^k \binom{u}{k} \left(\frac{e^{(p+b (u-2 k)) z} F_1\left(-\frac{p+b (u-2 k)}{2 c}; -\nu, \nu; 1 - \frac{p+b (u-2 k)}{2 c}; e^{-2 c z}, -e^{-2 c z}\right)}{p + b (u - 2 k)} + \frac{(-1)^u e^{(p-b (u-2 k)) z} F_1\left(-\frac{p-b (u-2 k)}{2 c}; -\nu, \nu; 1 - \frac{p-b (u-2 k)}{2 c}; e^{-2 c z}, -e^{-2 c z}\right)}{p - b (u - 2 k)} \right) /; u \in \mathbb{N}^+$$

01.21.21.0377.01

$$\int e^{p z} \sinh^{\mu}(c z) \tanh^{\nu}(c z) \, dz = \frac{e^{p z} \sinh^{\mu}(c z) \left(1 - e^{2 c z}\right)^{-\mu-\nu} \tanh^{\nu}(c z) \left(1 + e^{2 c z}\right)^{\nu}}{p - c \mu} F_1\left(\frac{p - c \mu}{2 c}; -\mu - \nu, \nu; \frac{1}{2}\left(\frac{p}{c} - \mu + 2\right); e^{2 c z}, -e^{2 c z}\right)$$

Involving cosh and exp

Involving $e^{p z} \cosh(b z) \tanh^{\nu}(c z)$

01.21.21.0378.01

$$\int e^{p z} \cosh(b z) \tanh^{\nu}(c z) \, dz = \frac{1}{2} \left(1 - e^{-2 c z}\right)^{-\nu} \left(1 + e^{-2 c z}\right)^{\nu} \tanh^{\nu}(c z) \left(\frac{e^{(p-b) z} F_1\left(-\frac{p-b}{2 c}; -\nu, \nu; 1 - \frac{p-b}{2 c}; e^{-2 c z}, -e^{-2 c z}\right)}{p - b} + \frac{e^{(b+p) z} F_1\left(-\frac{b+p}{2 c}; -\nu, \nu; 1 - \frac{b+p}{2 c}; e^{-2 c z}, -e^{-2 c z}\right)}{b + p}\right)$$

01.21.21.0379.01

$$\int e^{b z} \cosh(b z) \tanh^{\nu}(c z) \, dz = \frac{1}{8 b c (\nu - 1)} \left(e^{-2 c z} \left(1 - e^{-2 c z}\right)^{-\nu} \left(-\sinh^2(c z)\right)^{\frac{1}{2}(-\nu-1)} \left(2 c e^{2 (b+c) z} \left(1 + e^{-2 c z}\right)^{\nu} (\nu - 1) F_1\left(-\frac{b}{c}; \nu, -\nu; 1 - \frac{b}{c}; -e^{-2 c z}, e^{-2 c z}\right) \left(-\sinh^2(c z)\right)^{\frac{\nu+1}{2}} + b \left(1 - e^{-2 c z}\right)^{\nu} \left(-1 + e^{4 c z}\right) {}_2F_1\left(\frac{1-\nu}{2}, \frac{1-\nu}{2}; \frac{3-\nu}{2}; \cosh^2(c z)\right)\right) \tanh^{\nu}(c z)\right)$$

01.21.21.0380.01

$$\int e^{-b z} \cosh(b z) \tanh^{\nu}(c z) \, dz = \frac{1}{8 b c (\nu - 1)} \left(e^{-2 c z} \left(1 - e^{-2 c z}\right)^{-\nu} \left(-\sinh^2(c z)\right)^{\frac{1}{2}(-\nu-1)} \left(b \left(1 - e^{-2 c z}\right)^{\nu} \left(-1 + e^{4 c z}\right) {}_2F_1\left(\frac{1-\nu}{2}, \frac{1-\nu}{2}; \frac{3-\nu}{2}; \cosh^2(c z)\right) - 2 c e^{2 (c-b) z} \left(1 + e^{-2 c z}\right)^{\nu} (\nu - 1) F_1\left(\frac{b}{c}; \nu, -\nu; \frac{b+c}{c}; -e^{-2 c z}, e^{-2 c z}\right) \left(-\sinh^2(c z)\right)^{\frac{\nu+1}{2}}\right) \tanh^{\nu}(c z)\right)$$

Involving powers of cosh and exp

Involving $e^{p z} \cosh^{u}(b z) \tanh^{\nu}(c z)$

01.21.21.0381.01

$$\int e^{pz}\cosh^u(bz)\tanh^\nu(cz)\,dz = 2^{-u}\left(1+e^{-2cz}\right)^\nu \tanh^\nu(cz)\left(1-e^{-2cz}\right)^{-\nu}\sum_{k=0}^{\left\lfloor\frac{u-1}{2}\right\rfloor}\binom{u}{k}\left(\frac{e^{(p+b(u-2k))z}F_1\left(-\frac{p+b(u-2k)}{2c};-\nu,\nu;1-\frac{p+b(u-2k)}{2c};e^{-2cz},-e^{-2cz}\right)}{p+b(u-2k)}+\frac{e^{(p-b(u-2k))z}F_1\left(-\frac{p-b(u-2k)}{2c};-\nu,\nu;1-\frac{p-b(u-2k)}{2c};e^{-2cz},-e^{-2cz}\right)}{p-b(u-2k)}\right)+\frac{1}{p}2^{-u}e^{pz}\left(1-e^{-2cz}\right)^{-\nu}\left(1+e^{-2cz}\right)^\nu F_1\left(-\frac{p}{2c};-\nu,\nu;1-\frac{p}{2c};e^{-2cz},-e^{-2cz}\right)\binom{u}{\frac{u}{2}}\tanh^\nu(cz)\,(1-u \bmod 2)\;/;\;u\in\mathbb{N}^+$$

01.21.21.0382.01

$$\int e^{pz}\cosh^\mu(cz)\tanh^\nu(cz)\,dz = \frac{e^{pz}\cosh^\mu(cz)\left(1-e^{2cz}\right)^{-\nu}\tanh^\nu(cz)\left(1+e^{2cz}\right)^{\nu-\mu}}{p-c\mu}F_1\left(\frac{p-c\mu}{2c};-\nu,\nu-\mu;\frac{1}{2}\left(\frac{p}{c}-\mu+2\right);e^{2cz},-e^{2cz}\right)$$

Involving functions of the direct function, hyperbolic and trigonometric functions

Involving powers of the direct function, hyperbolic and trigonometric functions

Involving sin and sinh

Involving $\sin(a z)\sinh(b z)\tanh^\nu(c z)$

01.21.21.0383.01

$$\int \sin(az)\sinh(bz)\tanh^\nu(cz)\,dz = \frac{1}{4}i\left(1-e^{-2cz}\right)^{-\nu}\left(1+e^{-2cz}\right)^\nu\tanh^\nu(cz)\left(-\frac{e^{(-b-ia)z}F_1\left(-\frac{-b-ia}{2c};-\nu,\nu;1-\frac{-b-ia}{2c};e^{-2cz},-e^{-2cz}\right)}{-b-ia}+\frac{e^{(ia-b)z}F_1\left(-\frac{ia-b}{2c};-\nu,\nu;1-\frac{ia-b}{2c};e^{-2cz},-e^{-2cz}\right)}{ia-b}+\frac{e^{(b-ia)z}F_1\left(-\frac{b-ia}{2c};-\nu,\nu;1-\frac{b-ia}{2c};e^{-2cz},-e^{-2cz}\right)}{b-ia}-\frac{e^{(b+ia)z}F_1\left(-\frac{b+ia}{2c};-\nu,\nu;1-\frac{b+ia}{2c};e^{-2cz},-e^{-2cz}\right)}{b+ia}\right)$$

Involving powers of sin and powers of sinh

Involving $\sin^m(a z)\sinh^u(b z)\tanh^\nu(c z)$

01.21.21.0384.01

$$\int \sin^m(a z)\sinh^u(b z)\tanh^\nu(c z)\,dz = \frac{i^u\, 2^{-m-u}\tanh^{\nu+1}(c z)\,(1-m \bmod 2)(1-u \bmod 2)}{c(\nu+1)}\binom{m}{\frac{m}{2}}\binom{u}{\frac{u}{2}}\,{}_2F_1\!\left(\frac{\nu+1}{2},1;\frac{\nu+1}{2}+1;\tanh^2(c z)\right)+$$

$$\frac{1}{a}\, i^{u+1}\, 2^{-m-u}\left(1-e^{-2cz}\right)^{-\nu}\left(1+e^{-2cz}\right)^{\nu}\binom{u}{\frac{u}{2}}\tanh^\nu(c z)(1-u \bmod 2)$$

$$\sum_{s=0}^{\left\lfloor\frac{m-1}{2}\right\rfloor}\frac{(-1)^s}{m-2s}\binom{m}{s}\left(e^{\frac{i m\pi}{2}-i a(m-2s)z}F_1\!\left(\frac{i a(m-2s)}{2c};-\nu,\nu;\frac{a i(m-2s)}{2c}+1;e^{-2cz},-e^{-2cz}\right)-\right.$$

$$\left.e^{i a(m-2s)z-\frac{i m\pi}{2}}F_1\!\left(-\frac{i a(m-2s)}{2c};-\nu,\nu;1-\frac{i a(m-2s)}{2c};e^{-2cz},-e^{-2cz}\right)\right)+\frac{1}{b}\,2^{-m-u}\left(1-e^{-2cz}\right)^{-\nu}\left(1+e^{-2cz}\right)^{\nu}$$

$$\binom{m}{\frac{m}{2}}\tanh^\nu(c z)(1-m \bmod 2)\sum_{k=0}^{\left\lfloor\frac{u-1}{2}\right\rfloor}\frac{(-1)^k}{u-2k}\binom{u}{k}\left(e^{b(u-2k)z}F_1\!\left(-\frac{b(u-2k)}{2c};-\nu,\nu;1-\frac{b(u-2k)}{2c};e^{-2cz},-e^{-2cz}\right)-\right.$$

$$\left.(-1)^u e^{-b(u-2k)z}F_1\!\left(\frac{b(u-2k)}{2c};-\nu,\nu;\frac{b(u-2k)}{2c}+1;e^{-2cz},-e^{-2cz}\right)\right)+$$

$$2^{-m-u}\left(1-e^{-2cz}\right)^{-\nu}\left(1+e^{-2cz}\right)^{\nu}\tanh^\nu(c z)\sum_{k=0}^{\left\lfloor\frac{u-1}{2}\right\rfloor}\sum_{s=0}^{\left\lfloor\frac{m-1}{2}\right\rfloor}(-1)^{k+s}\binom{m}{s}\binom{u}{k}$$

$$\left(\left(e^{\frac{i\pi m}{2}+(b(u-2k)-i a(m-2s))z}F_1\!\left(-\frac{b(u-2k)-i a(m-2s)}{2c};-\nu,\nu;1-\frac{b(u-2k)-i a(m-2s)}{2c};e^{-2cz},-e^{-2cz}\right)\right)\right/$$

$$(b(u-2k)-i a(m-2s))+\left(e^{(a i(m-2s)+b(u-2k))z-\frac{i m\pi}{2}}F_1\!\left(-\frac{a i(m-2s)+b(u-2k)}{2c};-\nu,\right.\right.$$

$$\left.\left.\nu;1-\frac{a i(m-2s)+b(u-2k)}{2c};e^{-2cz},-e^{-2cz}\right)\right)\Big/(a i(m-2s)+b(u-2k))+$$

$$\left((-1)^u e^{\frac{i\pi m}{2}+(-i a(m-2s)-b(u-2k))z}F_1\!\left(-\frac{-i a(m-2s)-b(u-2k)}{2c};-\nu,\nu;1-\frac{-i a(m-2s)-b(u-2k)}{2c};e^{-2cz},\right.\right.$$

$$\left.\left.-e^{-2cz}\right)\right)\Big/(-i a(m-2s)-b(u-2k))+\left((-1)^u e^{(i a(m-2s)-b(u-2k))z-\frac{i m\pi}{2}}F_1\!\left(-\frac{i a(m-2s)-b(u-2k)}{2c};\right.\right.$$

$$\left.\left.\left.-\nu,\nu;1-\frac{i a(m-2s)-b(u-2k)}{2c};e^{-2cz},-e^{-2cz}\right)\right)\Big/(i a(m-2s)-b(u-2k))\right)\Big/;\ m\in\mathbb{N}^+\land u\in\mathbb{N}^+$$

01.21.21.0385.01

$$\int \sin^m(a z) \sinh^\mu(c z) \tanh^\nu(c z)\, dz = 2^{-m} \left(1 - e^{2 c z}\right)^{-\mu-\nu} \left(1 + e^{2 c z}\right)^\nu \sinh^\mu(c z) \tanh^\nu(c z)$$
$$\sum_{k=0}^{\left\lfloor \frac{m-1}{2} \right\rfloor} (-1)^k \binom{m}{k} \left(\frac{e^{\frac{i m \pi}{2} - i a (m-2k) z}}{-i a (m-2k) - c\mu} F_1\left(\frac{-i a (m-2k) - c\mu}{2c}; -\mu-\nu, \nu; \frac{1}{2}\left(-\frac{i a (m-2k)}{c} - \mu + 2\right); e^{2 c z}, -e^{2 c z} \right) + \right.$$
$$\left. \frac{e^{i a (m-2k) z - \frac{i m \pi}{2}}}{i a (m-2k) - c\mu} F_1\left(\frac{i a (m-2k) - c\mu}{2c}; -\mu-\nu, \nu; \frac{1}{2}\left(\frac{a i (m-2k)}{c} - \mu + 2\right); e^{2 c z}, -e^{2 c z} \right) \right) - \frac{1}{c\mu} 2^{-m}$$
$$\left(1 - e^{2 c z}\right)^{-\mu-\nu} \left(1 + e^{2 c z}\right)^\nu \binom{m}{\frac{m}{2}} (1 - m \bmod 2) \sinh^\mu(c z) \tanh^\nu(c z)\, F_1\left(-\frac{\mu}{2}; -\mu-\nu, \nu; \frac{2-\mu}{2}; e^{2 c z}, -e^{2 c z}\right) /; m \in \mathbb{N}^+$$

Involving cos and sinh

Involving $\cos(a z) \sinh(b z) \tanh^\nu(c z)$

01.21.21.0386.01

$$\int \cos(a z) \sinh(b z) \tanh^\nu(c z)\, dz = \frac{1}{4} \left(1 - e^{-2 c z}\right)^{-\nu} \left(1 + e^{-2 c z}\right)^\nu \tanh^\nu(c z)$$
$$\left(-\frac{e^{(-b - i a) z} F_1\left(-\frac{-b - i a}{2c}; -\nu, \nu; 1 - \frac{-b - i a}{2c}; e^{-2 c z}, -e^{-2 c z}\right)}{-b - i a} + \frac{e^{(b - i a) z} F_1\left(-\frac{b - i a}{2c}; -\nu, \nu; 1 - \frac{b - i a}{2c}; e^{-2 c z}, -e^{-2 c z}\right)}{b - i a} + \right.$$
$$\left. \frac{e^{(b + i a) z} F_1\left(-\frac{b + i a}{2c}; -\nu, \nu; 1 - \frac{b + i a}{2c}; e^{-2 c z}, -e^{-2 c z}\right)}{b + i a} - \frac{e^{(i a - b) z} F_1\left(-\frac{i a - b}{2c}; -\nu, \nu; 1 - \frac{i a - b}{2c}; e^{-2 c z}, -e^{-2 c z}\right)}{i a - b} \right)$$

Involving powers of cos and powers of sinh

Involving $\cos^m(a z) \sinh^u(b z) \tanh^\nu(c z)$

01.21.21.0387.01

$$\int \cos^m(a z)\sinh^u(b z)\tanh^\nu(c z)\,dz = \frac{i^u\, 2^{-m-u}\, \tanh^{\nu+1}(c z)\,(1-m \bmod 2)(1-u \bmod 2)}{c(\nu+1)}\binom{m}{\frac{m}{2}}\binom{u}{\frac{u}{2}}\,{}_2F_1\!\left(\frac{\nu+1}{2},1;\frac{\nu+1}{2}+1;\tanh^2(c z)\right)+$$
$$\frac{1}{a}\, i^{u+1}\, 2^{-m-u}\left(1-e^{-2cz}\right)^{-\nu}\left(1+e^{-2cz}\right)^{\nu}\tanh^\nu(c z)(1-u \bmod 2)\binom{u}{\frac{u}{2}}$$
$$\sum_{s=0}^{\left\lfloor\frac{m-1}{2}\right\rfloor}\frac{1}{m-2s}\binom{m}{s}\left(e^{-i a(m-2s)z}F_1\!\left(\frac{i a(m-2s)}{2c};-\nu,\nu;\frac{a i(m-2s)}{2c}+1;e^{-2cz},-e^{-2cz}\right)-\right.$$
$$\left.e^{i a(m-2s)z}F_1\!\left(-\frac{i a(m-2s)}{2c};-\nu,\nu;1-\frac{i a(m-2s)}{2c};e^{-2cz},-e^{-2cz}\right)\right)+\frac{1}{b}\,2^{-m-u}\left(1-e^{-2cz}\right)^{-\nu}\left(1+e^{-2cz}\right)^{\nu}$$
$$\tanh^\nu(c z)(1-m \bmod 2)\binom{m}{\frac{m}{2}}\sum_{k=0}^{\left\lfloor\frac{u-1}{2}\right\rfloor}\frac{(-1)^k}{u-2k}\binom{u}{k}\left(e^{b(u-2k)z}F_1\!\left(-\frac{b(u-2k)}{2c};-\nu,\nu;1-\frac{b(u-2k)}{2c};e^{-2cz},-e^{-2cz}\right)-\right.$$
$$\left.(-1)^u e^{-b(u-2k)z}F_1\!\left(\frac{b(u-2k)}{2c};-\nu,\nu;\frac{b(u-2k)}{2c}+1;e^{-2cz},-e^{-2cz}\right)\right)+$$
$$2^{-m-u}\left(1-e^{-2cz}\right)^{-\nu}\left(1+e^{-2cz}\right)^{\nu}\tanh^\nu(c z)\sum_{k=0}^{\left\lfloor\frac{u-1}{2}\right\rfloor}\sum_{s=0}^{\left\lfloor\frac{m-1}{2}\right\rfloor}(-1)^k\binom{m}{s}\binom{u}{k}$$
$$\left(\left(e^{(b(u-2k)-i a(m-2s))z}F_1\!\left(-\frac{b(u-2k)-i a(m-2s)}{2c};-\nu,\nu;1-\frac{b(u-2k)-i a(m-2s)}{2c};e^{-2cz},-e^{-2cz}\right)\right)\right/$$
$$(b(u-2k)-i a(m-2s))+\left(e^{(a i(m-2s)+b(u-2k))z}F_1\!\left(-\frac{a i(m-2s)+b(u-2k)}{2c};-\nu,\right.\right.$$
$$\left.\left.\nu;1-\frac{a i(m-2s)+b(u-2k)}{2c};e^{-2cz},-e^{-2cz}\right)\right)\Big/(a i(m-2s)+b(u-2k))+$$
$$\left((-1)^u e^{(-i a(m-2s)-b(u-2k))z}F_1\!\left(-\frac{-i a(m-2s)-b(u-2k)}{2c};-\nu,\nu;1-\frac{-i a(m-2s)-b(u-2k)}{2c};e^{-2cz},\right.\right.$$
$$\left.\left.-e^{-2cz}\right)\right)\Big/(-i a(m-2s)-b(u-2k))+\left((-1)^u e^{(i a(m-2s)-b(u-2k))z}F_1\!\left(-\frac{i a(m-2s)-b(u-2k)}{2c};\right.\right.$$
$$\left.\left.\left.-\nu,\nu;1-\frac{i a(m-2s)-b(u-2k)}{2c};e^{-2cz},-e^{-2cz}\right)\right)\Big/(i a(m-2s)-b(u-2k))\right)\text{ /; } m\in\mathbb{N}^+\land u\in\mathbb{N}^+$$

01.21.21.0388.01

$$\int \cos^m(a z)\sinh^\mu(c z)\tanh^\nu(c z)\,dz = 2^{-m}\left(1-e^{2cz}\right)^{-\mu-\nu}\left(1+e^{2cz}\right)^\nu \sinh^\mu(c z) \tanh^\nu(c z)\sum_{k=0}^{\left\lfloor\frac{m-1}{2}\right\rfloor}\binom{m}{k}\left(\frac{e^{-i a (m-2k) z} F_1\left(\frac{-i a(m-2k)-c\mu}{2c}; -\mu-\nu, \nu; \frac{1}{2}\left(-\frac{i a (m-2k)}{c}-\mu+2\right); e^{2cz}, -e^{2cz}\right)}{-i a(m-2k)-c\mu} + \frac{e^{i a (m-2k) z} F_1\left(\frac{i a(m-2k)-c\mu}{2c}; -\mu-\nu, \nu; \frac{1}{2}\left(\frac{a i (m-2k)}{c}-\mu+2\right); e^{2cz}, -e^{2cz}\right)}{i a(m-2k)-c\mu}\right) - \frac{1}{c\mu} 2^{-m}\left(1-e^{2cz}\right)^{-\mu-\nu} \left(1+e^{2cz}\right)^\nu \binom{m}{\frac{m}{2}} (1-m \bmod 2)\sinh^\mu(c z)\tanh^\nu(c z) F_1\left(-\frac{\mu}{2}; -\mu-\nu, \nu; \frac{2-\mu}{2}; e^{2cz}, -e^{2cz}\right) /; m\in\mathbb{N}^+$$

Involving sin and cosh

Involving $\sin(a z)\cosh(b z)\tanh^\nu(c z)$

01.21.21.0389.01

$$\int \sin(a z)\cosh(b z)\tanh^\nu(c z)\,dz = -\frac{1}{4} i \left(1-e^{-2cz}\right)^{-\nu}\left(1+e^{-2cz}\right)^\nu \tanh^\nu(c z)\left(-\frac{e^{(-b-i a) z} F_1\left(-\frac{-b-i a}{2c}; -\nu, \nu; 1-\frac{-b-i a}{2c}; e^{-2cz}, -e^{-2cz}\right)}{-b-i a} + \frac{e^{(i a-b) z} F_1\left(-\frac{i a-b}{2c}; -\nu, \nu; 1-\frac{i a-b}{2c}; e^{-2cz}, -e^{-2cz}\right)}{i a-b} + \frac{e^{(b+i a) z} F_1\left(-\frac{b+i a}{2c}; -\nu, \nu; 1-\frac{b+i a}{2c}; e^{-2cz}, -e^{-2cz}\right)}{b+i a} - \frac{e^{(b-i a) z} F_1\left(-\frac{b-i a}{2c}; -\nu, \nu; 1-\frac{b-i a}{2c}; e^{-2cz}, -e^{-2cz}\right)}{b-i a}\right)$$

Involving powers of sin and powers of cosh

Involving $\sin^m(a z)\cosh^u(b z)\tanh^\nu(c z)$

01.21.21.0390.01

$$\int \sin^m(a z)\cosh^u(b z)\tanh^\nu(c z)\,dz = 2^{-m-u}\left(1+e^{-2cz}\right)^\nu \left(\sum_{k=0}^{\left\lfloor\frac{u-1}{2}\right\rfloor}\sum_{s=0}^{\left\lfloor\frac{m-1}{2}\right\rfloor}(-1)^s\left(\left(e^{\frac{i\pi m}{2}+(b(u-2k)-i a(m-2s))z} F_1\left(-\frac{b(u-2k)-i a(m-2s)}{2c}; -\nu, \nu; 1-\frac{b(u-2k)-i a(m-2s)}{2c}; e^{-2cz}, -e^{-2cz}\right)\right)\Big/(b(u-2k)-i a(m-2s)) + \left(e^{(a i(m-2s)+b(u-2k))z-\frac{i m\pi}{2}} F_1\left(-\frac{a i(m-2s)+b(u-2k)}{2c}; -\nu, \nu; 1-\frac{a i(m-2s)+b(u-2k)}{2c}; e^{-2cz}, -e^{-2cz}\right)\right)\Big/(a i(m-2s)+b(u-2k)) + \left(e^{\frac{i\pi m}{2}+(-i a(m-2s)-b(u-2k))z} F_1\left(-\frac{-i a(m-2s)-b(u-2k)}{2c};\right.\right.\right.\right.$$

$$-\nu,\ \nu;\ 1-\frac{-i\,a\,(m-2\,s)-b\,(u-2\,k)}{2\,c};\ e^{-2\,c\,z},\ -e^{-2\,c\,z}\Big)\Big)\Big/\left(-i\,a\,(m-2\,s)-b\,(u-2\,k)\right)+$$

$$\Bigg(e^{(i\,a\,(m-2\,s)-b\,(u-2\,k))\,z-\frac{i\,m\,\pi}{2}}\,F_1\Bigg(-\frac{i\,a\,(m-2\,s)-b\,(u-2\,k)}{2\,c};\ -\nu,\ \nu;\ 1-\frac{i\,a\,(m-2\,s)-b\,(u-2\,k)}{2\,c};$$

$$e^{-2\,c\,z},\ -e^{-2\,c\,z}\Bigg)\Bigg)\Big/\left(i\,a\,(m-2\,s)-b\,(u-2\,k)\right)\Bigg)\binom{m}{s}\binom{u}{k}\Bigg)\tanh^{\nu}(c\,z)\left(1-e^{-2\,c\,z}\right)^{-\nu}+$$

$$\frac{2^{-m-u}\binom{m}{\frac{m}{2}}\binom{u}{\frac{u}{2}}\,{}_2F_1\!\left(\frac{\nu+1}{2},\ 1;\ \frac{\nu+1}{2}+1;\ \tanh^2(c\,z)\right)(1-m\bmod 2)\,(1-u\bmod 2)\tanh^{\nu+1}(c\,z)}{c\,(\nu+1)}+$$

$$\frac{1}{a}\Bigg(\Bigg(i\,2^{-m-u}\left(1-e^{-2\,c\,z}\right)^{-\nu}\left(1+e^{-2\,c\,z}\right)^{\nu}\binom{u}{\frac{u}{2}}\tanh^{\nu}(c\,z)\,(1-u\bmod 2)\Bigg)$$

$$\sum_{s=0}^{\left\lfloor\frac{m-1}{2}\right\rfloor}\frac{1}{m-2\,s}\Bigg((-1)^s\Bigg(e^{\frac{i\,m\,\pi}{2}-i\,a\,(m-2\,s)\,z}\,F_1\Bigg(\frac{i\,a\,(m-2\,s)}{2\,c};\ -\nu,\ \nu;\ \frac{a\,i\,(m-2\,s)}{2\,c}+1;\ e^{-2\,c\,z},\ -e^{-2\,c\,z}\Bigg)-$$

$$e^{i\,a\,(m-2\,s)\,z-\frac{i\,m\,\pi}{2}}\,F_1\Bigg(-\frac{i\,a\,(m-2\,s)}{2\,c};\ -\nu,\ \nu;\ 1-\frac{i\,a\,(m-2\,s)}{2\,c};\ e^{-2\,c\,z},\ -e^{-2\,c\,z}\Bigg)\Bigg)\binom{m}{s}\Bigg)\Bigg)+$$

$$\frac{1}{b}\Bigg(\Bigg(2^{-m-u}\left(1-e^{-2\,c\,z}\right)^{-\nu}\left(1+e^{-2\,c\,z}\right)^{\nu}\binom{m}{\frac{m}{2}}\tanh^{\nu}(c\,z)\,(1-m\bmod 2)\Bigg)\sum_{k=0}^{\left\lfloor\frac{u-1}{2}\right\rfloor}\frac{1}{u-2\,k}$$

$$\Bigg(\Bigg(e^{b\,(u-2\,k)\,z}\,F_1\Bigg(-\frac{b\,(u-2\,k)}{2\,c};\ -\nu,\ \nu;\ 1-\frac{b\,(u-2\,k)}{2\,c};\ e^{-2\,c\,z},\ -e^{-2\,c\,z}\Bigg)-$$

$$e^{-b\,(u-2\,k)\,z}\,F_1\Bigg(\frac{b\,(u-2\,k)}{2\,c};\ -\nu,\ \nu;\ \frac{b\,(u-2\,k)}{2\,c}+1;\ e^{-2\,c\,z},\ -e^{-2\,c\,z}\Bigg)\Bigg)\binom{u}{k}\Bigg)\Bigg)\ /;\ m\in\mathbb{N}^+\land u\in\mathbb{N}^+$$

01.21.21.0391.01

$$\int \sin^m(a z) \cosh^\mu(c z) \tanh^\nu(c z)\, dz = 2^{-m} \left(1 - e^{2 c z}\right)^{-\nu} \left(1 + e^{2 c z}\right)^{\nu-\mu} \cosh^\mu(c z) \tanh^\nu(c z) \sum_{k=0}^{\left\lfloor \frac{m-1}{2} \right\rfloor} (-1)^k \binom{m}{k} \left(\frac{e^{\frac{i m \pi}{2} - i a (m-2k) z} F_1\left(\frac{-i a (m-2k) - c\mu}{2c}; -\nu, \nu-\mu; \frac{1}{2}\left(-\frac{i a (m-2k)}{c} - \mu + 2\right); e^{2 c z}, -e^{2 c z}\right)}{-i a (m-2k) - c\mu} + \frac{e^{i a (m-2k) z - \frac{i m \pi}{2}} F_1\left(\frac{i a (m-2k) - c\mu}{2c}; -\nu, \nu-\mu; \frac{1}{2}\left(\frac{a i (m-2k)}{c} - \mu + 2\right); e^{2 c z}, -e^{2 c z}\right)}{i a (m-2k) - c\mu} \right) - \frac{1}{c\mu} 2^{-m} \left(1 - e^{2 c z}\right)^{-\nu} \left(1 + e^{2 c z}\right)^{\nu-\mu} \binom{m}{\frac{m}{2}} \cosh^\mu(c z) (1 - m \bmod 2) \tanh^\nu(c z) F_1\left(-\frac{\mu}{2}; -\nu, \nu-\mu; \frac{2-\mu}{2}; e^{2 c z}, -e^{2 c z}\right) /; m \in \mathbb{N}^+$$

Involving cos and cosh

Involving $\cos(a z) \cosh(b z) \tanh^\nu(c z)$

01.21.21.0392.01

$$\int \cos(a z) \cosh(b z) \tanh^\nu(c z)\, dz = \frac{1}{4} \left(1 - e^{-2 c z}\right)^{-\nu} \left(1 + e^{-2 c z}\right)^{\nu} \tanh^\nu(c z) \left(\frac{e^{(-b - i a) z} F_1\left(-\frac{-b - i a}{2c}; -\nu, \nu; 1 - \frac{-b - i a}{2c}; e^{-2 c z}, -e^{-2 c z}\right)}{-b - i a} + \frac{e^{(i a - b) z} F_1\left(-\frac{i a - b}{2c}; -\nu, \nu; 1 - \frac{i a - b}{2c}; e^{-2 c z}, -e^{-2 c z}\right)}{i a - b} + \frac{e^{(b - i a) z} F_1\left(-\frac{b - i a}{2c}; -\nu, \nu; 1 - \frac{b - i a}{2c}; e^{-2 c z}, -e^{-2 c z}\right)}{b - i a} + \frac{e^{(b + i a) z} F_1\left(-\frac{b + i a}{2c}; -\nu, \nu; 1 - \frac{b + i a}{2c}; e^{-2 c z}, -e^{-2 c z}\right)}{b + i a} \right)$$

Involving powers of cos and powers of cosh

Involving $\cos^m(a z) \cosh^u(b z) \tanh^\nu(c z)$

01.21.21.0393.01

$$\int \cos^m(a z)\cosh^u(b z)\tanh^\nu(c z)\,dz = 2^{-m-u}\left(1+e^{-2cz}\right)^\nu \tanh^\nu(c z)\left(1-e^{-2cz}\right)^{-\nu}\sum_{k=0}^{\left\lfloor\frac{m-1}{2}\right\rfloor}\sum_{s=0}^{\left\lfloor\frac{u-1}{2}\right\rfloor}\binom{m}{k}\binom{u}{s}$$
$$\left(\frac{e^{(b(u-2s)-i a(m-2k))z}}{b(u-2s)-i a(m-2k)} F_1\left(-\frac{b(u-2s)-i a(m-2k)}{2c}; -\nu, \nu; 1-\frac{b(u-2s)-i a(m-2k)}{2c}; e^{-2cz}, -e^{-2cz}\right)+\right.$$
$$\frac{e^{(a i(m-2k)+b(u-2s))z}}{a i(m-2k)+b(u-2s)} F_1\left(-\frac{a i(m-2k)+b(u-2s)}{2c}; -\nu, \nu; 1-\frac{a i(m-2k)+b(u-2s)}{2c}; e^{-2cz}, -e^{-2cz}\right)+$$
$$\frac{e^{(-i a(m-2k)-b(u-2s))z}}{-i a(m-2k)-b(u-2s)} F_1\left(-\frac{-i a(m-2k)-b(u-2s)}{2c}; -\nu, \nu;\right.$$
$$\left.1-\frac{-i a(m-2k)-b(u-2s)}{2c}; e^{-2cz}, -e^{-2cz}\right)+\frac{e^{(i a(m-2k)-b(u-2s))z}}{i a(m-2k)-b(u-2s)}$$
$$\left.F_1\left(-\frac{i a(m-2k)-b(u-2s)}{2c}; -\nu, \nu; 1-\frac{i a(m-2k)-b(u-2s)}{2c}; e^{-2cz}, -e^{-2cz}\right)\right)+$$
$$\frac{2^{-m-u}(1-m \bmod 2)(1-u \bmod 2)\tanh^{\nu+1}(c z)}{c(\nu+1)}\binom{m}{\frac{m}{2}}\binom{u}{\frac{u}{2}}\,{}_2F_1\left(\frac{\nu+1}{2}, 1; \frac{\nu+1}{2}+1; \tanh^2(c z)\right)+$$
$$\frac{2^{-m-u}\, i\left(1-e^{-2cz}\right)^{-\nu}\left(1+e^{-2cz}\right)^\nu \tanh^\nu(c z)(1-u \bmod 2)}{a}\binom{u}{\frac{u}{2}}$$
$$\sum_{k=0}^{\left\lfloor\frac{m-1}{2}\right\rfloor}\frac{1}{m-2k}\binom{m}{k}\left(e^{-i a(m-2k)z} F_1\left(\frac{i a(m-2k)}{2c}; -\nu, \nu; \frac{a i(m-2k)}{2c}+1; e^{-2cz}, -e^{-2cz}\right)-\right.$$
$$\left.e^{i a(m-2k)z} F_1\left(-\frac{i a(m-2k)}{2c}; -\nu, \nu; 1-\frac{i a(m-2k)}{2c}; e^{-2cz}, -e^{-2cz}\right)\right)+$$
$$\frac{2^{-m-u}\left(1-e^{-2cz}\right)^{-\nu}\left(1+e^{-2cz}\right)^\nu \tanh^\nu(c z)(1-m \bmod 2)}{b}\binom{m}{\frac{m}{2}}$$
$$\sum_{s=0}^{\left\lfloor\frac{u-1}{2}\right\rfloor}\frac{1}{u-2s}\binom{u}{s}\left(e^{b(u-2s)z} F_1\left(-\frac{b(u-2s)}{2c}; -\nu, \nu; 1-\frac{b(u-2s)}{2c}; e^{-2cz}, -e^{-2cz}\right)-\right.$$
$$\left.e^{-b(u-2s)z} F_1\left(\frac{b(u-2s)}{2c}; -\nu, \nu; \frac{b(u-2s)}{2c}+1; e^{-2cz}, -e^{-2cz}\right)\right)\;/;\; m\in\mathbb{N}^+\land u\in\mathbb{N}^+$$

01.21.21.0394.01

$$\int \cos^m(a z) \cosh^\mu(c z) \tanh^\nu(c z)\, dz = 2^{-m} \left(1 - e^{2 c z}\right)^{-\nu} \left(1 + e^{2 c z}\right)^{\nu-\mu} \cosh^\mu(c z) \tanh^\nu(c z) \sum_{k=0}^{\left\lfloor \frac{m-1}{2} \right\rfloor} \binom{m}{k} \left(\frac{e^{-i a (m-2k) z} F_1\left(\frac{-i a (m-2k) - c\mu}{2c}; -\nu, \nu-\mu; \frac{1}{2}\left(-\frac{i a (m-2k)}{c} - \mu + 2\right); e^{2 c z}, -e^{2 c z}\right)}{-i a (m-2k) - c\mu} + \frac{e^{i a (m-2k) z} F_1\left(\frac{i a (m-2k) - c\mu}{2c}; -\nu, \nu-\mu; \frac{1}{2}\left(\frac{a i (m-2k)}{c} - \mu + 2\right); e^{2 c z}, -e^{2 c z}\right)}{i a (m-2k) - c\mu} \right) - \frac{1}{c\mu} \left(2^{-m} \left(1 - e^{2 c z}\right)^{-\nu} \left(1 + e^{2 c z}\right)^{\nu-\mu} F_1\left(-\frac{\mu}{2}; -\nu, \nu-\mu; \frac{2-\mu}{2}; e^{2 c z}, -e^{2 c z}\right) \binom{m}{\frac{m}{2}} \cosh^\mu(c z) (1 - m \bmod 2) \tanh^\nu(c z) \right) /; m \in \mathbb{N}^+$$

Involving functions of the direct function, hyperbolic, exponential and a power functions

Involving powers of the direct function, hyperbolic, exponential and a power functions

Involving sinh, exp and power

Involving $z^n e^{p z} \sinh(a + b z) \tanh^\nu(c z)$

01.21.21.0395.01

$$\int z^n e^{p z} \sinh(a + b z) \tanh^v(c z) \, dz ==$$

$$-\frac{1}{2} n! \, e^{-a} \left(i^v e^{(-b+p+c v) z} \binom{v}{\frac{v}{2}} (1 - v \bmod 2) \sum_{j=0}^{n} \frac{(-1)^j (-b + p + c v)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{-b + p + c v}{2 c}, \right.\right.$$

$$\left. \ldots, \frac{-b + p + c v}{2 c}, v; \frac{-b + p + c v}{2 c} + 1, \ldots, \frac{-b + p + c v}{2 c} + 1; -e^{2 c z} \right) +$$

$$\sum_{s=0}^{\left\lfloor \frac{v-1}{2} \right\rfloor} (-1)^s \binom{v}{s} \left((-1)^v e^{(-b+p+2 c s) z} \sum_{j=0}^{n} \frac{(-1)^j (-b + p + 2 c s)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{-b + p + 2 c s}{2 c}, \ldots, \right.\right.$$

$$\left. \frac{-b + p + 2 c s}{2 c}, v; \frac{-b + p + 2 c s}{2 c} + 1, \ldots, \frac{-b + p + 2 c s}{2 c} + 1; -e^{2 c z} \right) +$$

$$e^{(-b+p+2 c (v-s)) z} \sum_{j=0}^{n} \frac{(-1)^j (-b + p + 2 c (v - s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{-b + p + 2 c (v - s)}{2 c}, \ldots, \right.$$

$$\left.\left.\left. \frac{-b + p + 2 c (v - s)}{2 c}, v; \frac{-b + p + 2 c (v - s)}{2 c} + 1, \ldots, \frac{-b + p + 2 c (v - s)}{2 c} + 1; -e^{2 c z} \right) \right) \right) +$$

$$\frac{1}{2} n! \, e^{a} \left(i^v e^{(b+p+c v) z} \binom{v}{\frac{v}{2}} (1 - v \bmod 2) \sum_{j=0}^{n} \frac{(-1)^j (b + p + c v)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{b + p + c v}{2 c}, \ldots, \right.\right.$$

$$\left. \frac{b + p + c v}{2 c}, v; \frac{b + p + c v}{2 c} + 1, \ldots, \frac{b + p + c v}{2 c} + 1; -e^{2 c z} \right) +$$

$$\sum_{s=0}^{\left\lfloor \frac{v-1}{2} \right\rfloor} (-1)^s \binom{v}{s} \left((-1)^v e^{(b+p+2 c s) z} \sum_{j=0}^{n} \frac{(-1)^j (b + p + 2 c s)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{b + p + 2 c s}{2 c}, \ldots, \right.\right.$$

$$\left. \frac{b + p + 2 c s}{2 c}, v; \frac{b + p + 2 c s}{2 c} + 1, \ldots, \frac{b + p + 2 c s}{2 c} + 1; -e^{2 c z} \right) +$$

$$e^{(b+p+2 c (v-s)) z} \sum_{j=0}^{n} \frac{(-1)^j (b + p + 2 c (v - s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{b + p + 2 c (v - s)}{2 c}, \ldots, \right.$$

$$\left.\left.\left. \frac{b + p + 2 c (v - s)}{2 c}, v; \frac{b + p + 2 c (v - s)}{2 c} + 1, \ldots, \frac{b + p + 2 c (v - s)}{2 c} + 1; -e^{2 c z} \right) \right) \right) /; n \in \mathbb{N} \land v \in \mathbb{N}^+$$

01.21.21.0396.01

$$\int z^n e^{p z} \sinh(b z) \tanh^v(c z) \, dz = \frac{1}{2} n! \, i^v \binom{v}{\frac{v}{2}} (1 - v \bmod 2) \left(-e^{(-b+p+c v) z} \sum_{j=0}^{n} \frac{(-1)^j (-b+p+c v)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{-b+p+c v}{2 c}, \ldots, \frac{-b+p+c v}{2 c}, v; \frac{-b+p+c v}{2 c} + 1, \ldots, \frac{-b+p+c v}{2 c} + 1; -e^{2 c z} \right) + e^{(b+p+c v) z} \sum_{j=0}^{n} \frac{(-1)^j (b+p+c v)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{b+p+c v}{2 c}, \ldots, \frac{b+p+c v}{2 c}, v; \frac{b+p+c v}{2 c} + 1, \ldots, \frac{b+p+c v}{2 c} + 1; -e^{2 c z} \right) \right) + \frac{1}{2} n! \sum_{s=0}^{\left\lfloor \frac{v-1}{2} \right\rfloor} (-1)^s \binom{v}{s} \left(-(-1)^v e^{(-b+p+2 c s) z} \sum_{j=0}^{n} \frac{(-1)^j (-b+p+2 c s)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{-b+p+2 c s}{2 c}, \ldots, \frac{-b+p+2 c s}{2 c}, v; \frac{-b+p+2 c s}{2 c} + 1, \ldots, \frac{-b+p+2 c s}{2 c} + 1; -e^{2 c z} \right) + (-1)^v e^{(b+p+2 c s) z} \sum_{j=0}^{n} \frac{(-1)^j (b+p+2 c s)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{b+p+2 c s}{2 c}, \ldots, \frac{b+p+2 c s}{2 c}, v; \frac{b+p+2 c s}{2 c} + 1, \ldots, \frac{b+p+2 c s}{2 c} + 1; -e^{2 c z} \right) - e^{(-b+p+2 c (v-s)) z} \sum_{j=0}^{n} \frac{(-1)^j (-b+p+2 c (v-s))^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{-b+p+2 c (v-s)}{2 c}, \ldots, \frac{-b+p+2 c (v-s)}{2 c}, v; \frac{-b+p+2 c (v-s)}{2 c} + 1, \ldots, \frac{-b+p+2 c (v-s)}{2 c} + 1; -e^{2 c z} \right) + e^{(b+p+2 c (v-s)) z} \sum_{j=0}^{n} \frac{(-1)^j (b+p+2 c (v-s))^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{b+p+2 c (v-s)}{2 c}, \ldots, \frac{b+p+2 c (v-s)}{2 c}, v; \frac{b+p+2 c (v-s)}{2 c} + 1, \ldots, \frac{b+p+2 c (v-s)}{2 c} + 1; -e^{2 c z} \right) \right) /; n \in \mathbb{N} \land v \in \mathbb{N}^+$$

01.21.21.0397.01

$$\int z^n e^{bz}\sinh(bz)\tanh^{\nu}(cz)\,dz = \frac{1}{2}\left(-\frac{(-1)^{\nu} z^{n+1}}{n+1} + (-1)^{\nu} e^{2cz}\,\nu\, n!\sum_{j=0}^{n}\frac{(-1)^j 2^{-j-1} c^{-j-1} z^{n-j}}{(n-j)!}\,{}_{j+3}F_{j+2}\left(1,\ldots,1,\nu+1;2,\ldots,2;-e^{2cz}\right) - \right.$$

$$i^{\nu} e^{c\nu z}\binom{\nu}{\frac{\nu}{2}} n!\,(1-\nu \bmod 2)\sum_{j=0}^{n}\frac{(-1)^j (c\nu)^{-j-1} z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{\nu}{2},\ldots,\frac{\nu}{2},\nu;\frac{\nu}{2}+1,\ldots,\frac{\nu}{2}+1;-e^{2cz}\right) +$$

$$e^{(2b+c\nu)z} i^{\nu}\binom{\nu}{\frac{\nu}{2}} n!\,(1-\nu \bmod 2)$$

$$\sum_{j=0}^{n}\frac{(-1)^j (2b+c\nu)^{-j-1} z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{2b+c\nu}{2c},\ldots,\frac{2b+c\nu}{2c},\nu;\frac{2b+c\nu}{2c}+1,\ldots,\frac{2b+c\nu}{2c}+1;-e^{2cz}\right) -$$

$$(-1)^{\nu} n!\sum_{s=1}^{\left\lfloor\frac{\nu-1}{2}\right\rfloor}(-1)^s\binom{\nu}{s} e^{2scz}\sum_{j=0}^{n}\frac{(-1)^j (2cs)^{-j-1} z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(s,\ldots,s,\nu;s+1,\ldots,s+1;-e^{2cz}\right) - n!$$

$$\sum_{s=0}^{\left\lfloor\frac{\nu-1}{2}\right\rfloor}(-1)^s\binom{\nu}{s} e^{2c(\nu-s)z}\sum_{j=0}^{n}\frac{(-1)^j (2c(\nu-s))^{-j-1} z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\nu-s,\ldots,\nu-s,\nu;-s+\nu+1,\ldots,-s+\nu+1;-e^{2cz}\right) +$$

$$n!\sum_{s=0}^{\left\lfloor\frac{\nu-1}{2}\right\rfloor}(-1)^s\binom{\nu}{s}\left((-1)^{\nu} e^{2(b+cs)z}\sum_{j=0}^{n}\frac{(-1)^j (2b+2cs)^{-j-1} z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{b+cs}{c},\ldots,\frac{b+cs}{c},\nu;\frac{b+cs}{c}+\right.\right.$$

$$\left.1,\ldots,\frac{b+cs}{c}+1;-e^{2cz}\right) + e^{2(b+c(\nu-s))z}\sum_{j=0}^{n}\frac{(-1)^j (2b+2c(\nu-s))^{-j-1} z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}$$

$$\left.\left.\left(\frac{b+c(\nu-s)}{c},\ldots,\frac{b+c(\nu-s)}{c},\nu;\frac{b+c(\nu-s)}{c}+1,\ldots,\frac{b+c(\nu-s)}{c}+1;-e^{2cz}\right)\right)\right)\;/;\; n\in\mathbb{N}\land \nu\in\mathbb{N}^+$$

01.21.21.0398.01

$$\int z^n e^{-b z} \sinh(b z) \tanh^{\nu}(c z) \, dz = \frac{1}{2} n! \left(\frac{(-1)^\nu z^{n+1}}{(n+1)!} - (-1)^\nu e^{2 c z} \nu \sum_{j=0}^{n} \frac{(-1)^j 2^{-j-1} c^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+3}F_{j+2}\left(1, \ldots, 1, \nu+1; 2, \ldots, 2; -e^{2 c z}\right) + e^{c \nu z} i^\nu \binom{\nu}{\frac{\nu}{2}} (1 - \nu \bmod 2) \left(\sum_{j=0}^{n} \frac{(-1)^j (c \nu)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{\nu}{2}, \ldots, \frac{\nu}{2}, \nu; \frac{\nu}{2}+1, \ldots, \frac{\nu}{2}+1; -e^{2 c z}\right) - e^{-2 b z} \sum_{j=0}^{n} \frac{(-1)^j (c \nu - 2 b)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{c\nu - 2b}{2c}, \ldots, \frac{c\nu - 2b}{2c}, \nu; \frac{c\nu - 2b}{2c}+1, \ldots, \frac{c\nu - 2b}{2c}+1; -e^{2 c z}\right)\right) + \sum_{s=1}^{\left\lfloor \frac{\nu-1}{2} \right\rfloor} (-1)^s \binom{\nu}{s} (-1)^\nu e^{2 s c z} \sum_{j=0}^{n} \frac{(-1)^j (2 c s)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(s, \ldots, s, \nu; s+1, \ldots, s+1; -e^{2 c z}\right) + \sum_{s=0}^{\left\lfloor \frac{\nu-1}{2} \right\rfloor} (-1)^s \binom{\nu}{s} \left(-(-1)^\nu e^{(2 c s - 2 b) z} \sum_{j=0}^{n} \frac{(-1)^j (2 c s - 2 b)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{c s - b}{c}, \ldots, \frac{c s - b}{c}, \nu; \frac{c s - b}{c}+1, \ldots, \frac{c s - b}{c}+1; -e^{2 c z}\right) + e^{2 c (\nu - s) z} \sum_{j=0}^{n} \frac{(-1)^j (2 c (\nu - s))^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\nu - s, \ldots, \nu - s, \nu; -s+\nu+1, \ldots, -s+\nu+1; -e^{2 c z}\right) - e^{(2 c (\nu - s) - 2 b) z} \sum_{j=0}^{n} \frac{(-1)^j (2 c (\nu - s) - 2 b)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{c(\nu - s) - b}{c}, \ldots, \frac{c(\nu - s) - b}{c}, \nu; \frac{c(\nu - s) - b}{c}+1, \ldots, \frac{c(\nu - s) - b}{c}+1; -e^{2 c z}\right)\right)\right) /; n \in \mathbb{N} \land \nu \in \mathbb{N}^+$$

Involving powers of sinh, exp and power

Involving $z^n e^{p z} \sinh^u(b z) \tanh^\nu(c z)$

01.21.21.0399.01

$$\int z^n e^{p z} \sinh^u(b z) \tanh^{\nu}(c z) \, dz = \left(\frac{i}{2}\right)^u \binom{u}{\frac{u}{2}} n! (1 - u \bmod 2) \left(i^\nu e^{(p + c \nu) z} \binom{\nu}{\frac{\nu}{2}} (1 - \nu \bmod 2) \sum_{j=0}^{n} \frac{(-1)^j (p + c \nu)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{p + c\nu}{2c}, \ldots, \frac{p + c\nu}{2c}, \nu; \frac{p + c\nu}{2c}+1, \ldots, \frac{p + c\nu}{2c}+1; -e^{2 c z}\right) + \sum_{s=0}^{\left\lfloor \frac{\nu-1}{2} \right\rfloor} (-1)^s \binom{\nu}{s} \left((-1)^\nu e^{(p + 2 c s) z} \sum_{j=0}^{n} \frac{(-1)^j (p + 2 c s)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{p + 2cs}{2c}, \ldots, \frac{p + 2cs}{2c}, \nu; \frac{p + 2cs}{2c} + \right.\right.\right.$$

$$1,\ \ldots,\ \frac{p+2\,c\,s}{2\,c}+1;\ -e^{2\,c\,z}\Big)+e^{(p+2\,c\,(v-s))\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(p+2\,c\,(v-s))^{-j-1}\,z^{n-j}}{(n-j)!}$$

$$_{j+2}F_{j+1}\left(\frac{p+2\,c\,(v-s)}{2\,c},\ \ldots,\ \frac{p+2\,c\,(v-s)}{2\,c},\ v;\ \frac{p+2\,c\,(v-s)}{2\,c}+1,\ \ldots,\ \frac{p+2\,c\,(v-s)}{2\,c}+1;\ -e^{2\,c\,z}\right)\Bigg)\Bigg)+$$

$$2^{-u}\,n!\sum_{k=0}^{\left\lfloor\frac{u-1}{2}\right\rfloor}(-1)^k\binom{u}{k}\Bigg((-1)^u\,i^v\,e^{(p-b\,(u-2\,k)+c\,v)\,z}\binom{v}{\frac{v}{2}}(1-v\bmod 2)\sum_{j=0}^{n}\frac{(-1)^j\,(p-b\,(u-2\,k)+c\,v)^{-j-1}\,z^{n-j}}{(n-j)!}$$

$$_{j+2}F_{j+1}\Bigg(\frac{p-b\,(u-2\,k)+c\,v}{2\,c},\ \ldots,\ \frac{p-b\,(u-2\,k)+c\,v}{2\,c},\ v;\ \frac{p-b\,(u-2\,k)+c\,v}{2\,c}+1,$$

$$\ldots,\ \frac{p-b\,(u-2\,k)+c\,v}{2\,c}+1;\ -e^{2\,c\,z}\Bigg)+(-1)^u\sum_{s=0}^{\left\lfloor\frac{v-1}{2}\right\rfloor}(-1)^s\binom{v}{s}$$

$$\Bigg((-1)^v\,e^{(2\,b\,k+p+2\,c\,s-b\,u)\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(p+2\,c\,s-b\,(u-2\,k))^{-j-1}\,z^{n-j}}{(n-j)!}\ _{j+2}F_{j+1}\Bigg(\frac{p+2\,c\,s-b\,(u-2\,k)}{2\,c},\ \ldots,$$

$$\frac{p+2\,c\,s-b\,(u-2\,k)}{2\,c},\ v;\ \frac{p+2\,c\,s-b\,(u-2\,k)}{2\,c}+1,\ \ldots,\ \frac{p+2\,c\,s-b\,(u-2\,k)}{2\,c}+1;\ -e^{2\,c\,z}\Bigg)+$$

$$e^{(2\,b\,k+p-2\,c\,s-b\,u+2\,c\,v)\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(p-b\,(u-2\,k)+2\,c\,(v-s))^{-j-1}\,z^{n-j}}{(n-j)!}\ _{j+2}F_{j+1}$$

$$\Bigg(\frac{p-b\,(u-2\,k)+2\,c\,(v-s)}{2\,c},\ \ldots,\ \frac{p-b\,(u-2\,k)+2\,c\,(v-s)}{2\,c},\ v;$$

$$\frac{p-b\,(u-2\,k)+2\,c\,(v-s)}{2\,c}+1,\ \ldots,\ \frac{p-b\,(u-2\,k)+2\,c\,(v-s)}{2\,c}+1;\ -e^{2\,c\,z}\Bigg)\Bigg)+$$

$$i^v\,e^{(p+b\,(u-2\,k)+c\,v)\,z}\binom{v}{\frac{v}{2}}(1-v\bmod 2)\sum_{j=0}^{n}\frac{(-1)^j\,(p+b\,(u-2\,k)+c\,v)^{-j-1}\,z^{n-j}}{(n-j)!}\ _{j+2}F_{j+1}\Bigg(\frac{p+b\,(u-2\,k)+c\,v}{2\,c},$$

$$\ldots,\ \frac{p+b\,(u-2\,k)+c\,v}{2\,c},\ v;\ \frac{p+b\,(u-2\,k)+c\,v}{2\,c}+1,\ \ldots,\ \frac{p+b\,(u-2\,k)+c\,v}{2\,c}+1;\ -e^{2\,c\,z}\Bigg)+$$

$$\sum_{s=0}^{\left\lfloor\frac{v-1}{2}\right\rfloor}(-1)^s\binom{v}{s}\Bigg((-1)^v\,e^{(p+2\,c\,s+b\,(u-2\,k))\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(p+2\,c\,s+b\,(u-2\,k))^{-j-1}\,z^{n-j}}{(n-j)!}\ _{j+2}F_{j+1}\Bigg(\frac{p+2\,c\,s+b\,(u-2\,k)}{2\,c},$$

$$\ldots,\ \frac{p+2\,c\,s+b\,(u-2\,k)}{2\,c},\ v;\ \frac{p+2\,c\,s+b\,(u-2\,k)}{2\,c}+1,\ \ldots,\ \frac{p+2\,c\,s+b\,(u-2\,k)}{2\,c}+1;\ -e^{2\,c\,z}\Bigg)+$$

$$e^{(p+b\,(u-2\,k)+2\,c\,(v-s))\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(p+b\,(u-2\,k)+2\,c\,(v-s))^{-j-1}\,z^{n-j}}{(n-j)!}\ _{j+2}F_{j+1}$$

$$\Bigg(\frac{p+b\,(u-2\,k)+2\,c\,(v-s)}{2\,c},\ \ldots,\ \frac{p+b\,(u-2\,k)+2\,c\,(v-s)}{2\,c},\ v;\ \frac{p+b\,(u-2\,k)+2\,c\,(v-s)}{2\,c}+1,$$

$$\ldots,\ \frac{p+b\,(u-2\,k)+2\,c\,(v-s)}{2\,c}+1;\ -e^{2\,c\,z}\Bigg)\Bigg)\Bigg)\Bigg)\ /;\ n\in\mathbb{N}\land u\in\mathbb{N}^+\land v\in\mathbb{N}^+$$

Involving cosh, exp and power

Involving $z^n e^{p z} \cosh(a+b z) \tanh^v(c z)$

01.21.21.0400.01

$$\int z^n e^{p z} \cosh(a+b z) \tanh^v(c z)\, dz = \frac{1}{2} n!\, e^{-a} \Bigg(i^v e^{(-b+p+c v) z} \binom{v}{\frac{v}{2}} (1 - v \bmod 2) \sum_{j=0}^{n} \frac{(-1)^j (-b+p+c v)^{-j-1} z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}\Big(\frac{-b+p+c v}{2 c}, \ldots, \frac{-b+p+c v}{2 c}, v; \frac{-b+p+c v}{2 c}+1, \ldots, \frac{-b+p+c v}{2 c}+1; -e^{2 c z}\Big) + \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} (-1)^s \binom{v}{s} \Bigg((-1)^v e^{(-b+p+2 c s) z} \sum_{j=0}^{n} \frac{(-1)^j (-b+p+2 c s)^{-j-1} z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}\Big(\frac{-b+p+2 c s}{2 c}, \ldots, \frac{-b+p+2 c s}{2 c}, v; \frac{-b+p+2 c s}{2 c}+1, \ldots, \frac{-b+p+2 c s}{2 c}+1; -e^{2 c z}\Big) + e^{(-b+p+2 c (v-s)) z} \sum_{j=0}^{n} \frac{(-1)^j (-b+p+2 c (v-s))^{-j-1} z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}\Big(\frac{-b+p+2 c (v-s)}{2 c}, \ldots, \frac{-b+p+2 c (v-s)}{2 c}, v; \frac{-b+p+2 c (v-s)}{2 c}+1, \ldots, \frac{-b+p+2 c (v-s)}{2 c}+1; -e^{2 c z}\Big) \Bigg) \Bigg) + \frac{1}{2} n!\, e^{a} \Bigg(i^v e^{(b+p+c v) z} \binom{v}{\frac{v}{2}} (1 - v \bmod 2) \sum_{j=0}^{n} \frac{(-1)^j (b+p+c v)^{-j-1} z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}\Big(\frac{b+p+c v}{2 c}, \ldots, \frac{b+p+c v}{2 c}, v; \frac{b+p+c v}{2 c}+1, \ldots, \frac{b+p+c v}{2 c}+1; -e^{2 c z}\Big) + \sum_{s=0}^{\lfloor \frac{v-1}{2} \rfloor} (-1)^s \binom{v}{s} \Bigg((-1)^v e^{(b+p+2 c s) z} \sum_{j=0}^{n} \frac{(-1)^j (b+p+2 c s)^{-j-1} z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}\Big(\frac{b+p+2 c s}{2 c}, \ldots, \frac{b+p+2 c s}{2 c}, v; \frac{b+p+2 c s}{2 c}+1, \ldots, \frac{b+p+2 c s}{2 c}+1; -e^{2 c z}\Big) + e^{(b+p+2 c (v-s)) z} \sum_{j=0}^{n} \frac{(-1)^j (b+p+2 c (v-s))^{-j-1} z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}\Big(\frac{b+p+2 c (v-s)}{2 c}, \ldots, \frac{b+p+2 c (v-s)}{2 c}, v; \frac{b+p+2 c (v-s)}{2 c}+1, \ldots, \frac{b+p+2 c (v-s)}{2 c}+1; -e^{2 c z}\Big) \Bigg) \Bigg) /;\ n \in \mathbb{N} \land v \in \mathbb{N}^+$$

01.21.21.0401.01

$$\int z^n e^{p z} \cosh(b z) \tanh^{\nu}(c z)\, dz = \frac{1}{2} n!\, i^{\nu} \binom{\nu}{\frac{\nu}{2}} (1 - \nu \bmod 2) \left(e^{(-b+p+c\nu) z} \sum_{j=0}^{n} \frac{(-1)^j (-b+p+c\nu)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{-b+p+c\nu}{2c}, \ldots, \frac{-b+p+c\nu}{2c}, \nu; \frac{-b+p+c\nu}{2c} + 1, \ldots, \frac{-b+p+c\nu}{2c} + 1; -e^{2cz} \right) + e^{(b+p+c\nu) z} \sum_{j=0}^{n} \frac{(-1)^j (b+p+c\nu)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{b+p+c\nu}{2c}, \ldots, \frac{b+p+c\nu}{2c}, \nu; \frac{b+p+c\nu}{2c} + 1, \ldots, \frac{b+p+c\nu}{2c} + 1; -e^{2cz} \right) \right) +$$

$$\frac{1}{2} n! \sum_{s=0}^{\left\lfloor \frac{\nu-1}{2} \right\rfloor} (-1)^s \binom{\nu}{s} \left((-1)^{\nu} e^{(-b+p+2cs) z} \sum_{j=0}^{n} \frac{(-1)^j (-b+p+2cs)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{-b+p+2cs}{2c}, \ldots, \frac{-b+p+2cs}{2c}, \nu; \frac{-b+p+2cs}{2c} + 1, \ldots, \frac{-b+p+2cs}{2c} + 1; -e^{2cz} \right) + (-1)^{\nu} e^{(b+p+2cs) z} \sum_{j=0}^{n} \frac{(-1)^j (b+p+2cs)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{b+p+2cs}{2c}, \ldots, \frac{b+p+2cs}{2c}, \nu; \frac{b+p+2cs}{2c} + 1, \ldots, \frac{b+p+2cs}{2c} + 1; -e^{2cz} \right) + e^{(-b+p+2c(\nu-s)) z} \sum_{j=0}^{n} \frac{(-1)^j (-b+p+2c(\nu-s))^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{-b+p+2c(\nu-s)}{2c}, \ldots, \frac{-b+p+2c(\nu-s)}{2c}, \nu; \frac{-b+p+2c(\nu-s)}{2c} + 1, \ldots, \frac{-b+p+2c(\nu-s)}{2c} + 1; -e^{2cz} \right) + e^{(b+p+2c(\nu-s)) z} \sum_{j=0}^{n} \frac{(-1)^j (b+p+2c(\nu-s))^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{b+p+2c(\nu-s)}{2c}, \ldots, \frac{b+p+2c(\nu-s)}{2c}, \nu; \frac{b+p+2c(\nu-s)}{2c} + 1, \ldots, \frac{b+p+2c(\nu-s)}{2c} + 1; -e^{2cz} \right) \right) /; n \in \mathbb{N} \land \nu \in \mathbb{N}^+$$

01.21.21.0402.01

$$\int z^n e^{b z} \cosh(b z) \tanh^\nu(c z)\, dz = \frac{1}{2}\left(\frac{(-1)^\nu z^{n+1}}{n+1} - (-1)^\nu e^{2cz} \nu\, n! \sum_{j=0}^{n} \frac{(-1)^j 2^{-j-1} c^{-j-1} z^{n-j}}{(n-j)!}\, {}_{j+3}F_{j+2}\left(1, \ldots, 1, \nu+1; 2, \ldots, 2; -e^{2cz}\right) + e^{c\nu z} i^\nu \binom{\nu}{\frac{\nu}{2}} n!\, (1 - \nu \bmod 2) \sum_{j=0}^{n} \frac{(-1)^j (c\nu)^{-j-1} z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}\left(\frac{\nu}{2}, \ldots, \frac{\nu}{2}, \nu; \frac{\nu}{2}+1, \ldots, \frac{\nu}{2}+1; -e^{2cz}\right) + e^{(2b+c\nu)z} i^\nu \binom{\nu}{\frac{\nu}{2}} n!\, (1 - \nu \bmod 2) \sum_{j=0}^{n} \frac{(-1)^j (2b+c\nu)^{-j-1} z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}\left(\frac{2b+c\nu}{2c}, \ldots, \frac{2b+c\nu}{2c}, \nu; \frac{2b+c\nu}{2c}+1, \ldots, \frac{2b+c\nu}{2c}+1; -e^{2cz}\right) + (-1)^\nu n! \sum_{s=1}^{\left\lfloor \frac{\nu-1}{2} \right\rfloor} (-1)^s \binom{\nu}{s} e^{2scz} \sum_{j=0}^{n} \frac{(-1)^j (2cs)^{-j-1} z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}\left(s, \ldots, s, \nu; s+1, \ldots, s+1; -e^{2cz}\right) + n! \sum_{s=0}^{\left\lfloor \frac{\nu-1}{2} \right\rfloor} (-1)^s \binom{\nu}{s} e^{2c(\nu-s)z} \sum_{j=0}^{n} \frac{(-1)^j (2c(\nu-s))^{-j-1} z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}\left(\nu-s, \ldots, \nu-s, \nu; -s+\nu+1, \ldots, -s+\nu+1; -e^{2cz}\right) + n! \sum_{s=0}^{\left\lfloor \frac{\nu-1}{2} \right\rfloor} (-1)^s \binom{\nu}{s} \left((-1)^\nu e^{2(b+cs)z} \sum_{j=0}^{n} \frac{(-1)^j (2b+2cs)^{-j-1} z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}\left(\frac{b+cs}{c}, \ldots, \frac{b+cs}{c}, \nu; \frac{b+cs}{c}+1, \ldots, \frac{b+cs}{c}+1; -e^{2cz}\right) + e^{2(b+c(\nu-s))z} \sum_{j=0}^{n} \frac{(-1)^j (2b+2c(\nu-s))^{-j-1} z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}\left(\frac{b+c(\nu-s)}{c}, \ldots, \frac{b+c(\nu-s)}{c}, \nu; \frac{b+c(\nu-s)}{c}+1, \ldots, \frac{b+c(\nu-s)}{c}+1; -e^{2cz}\right)\right)\right) /; n \in \mathbb{N} \land \nu \in \mathbb{N}^+$$

01.21.21.0403.01

$$\int z^n e^{-b z} \cosh(b z) \tanh^{\nu}(c z)\, dz = \frac{1}{2} n! \left(\frac{(-1)^{\nu} z^{n+1}}{(n+1)!} - (-1)^{\nu} e^{2 c z} \nu \sum_{j=0}^{n} \frac{(-1)^j 2^{-j-1} c^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+3}F_{j+2}\left(1, \ldots, 1, \nu+1; 2, \ldots, 2; -e^{2 c z}\right) + e^{c \nu z} i^{\nu} \binom{\nu}{\frac{\nu}{2}} (1 - \nu \bmod 2) \left(\sum_{j=0}^{n} \frac{(-1)^j (c \nu)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{\nu}{2}, \ldots, \frac{\nu}{2}, \nu; \frac{\nu}{2}+1, \ldots, \frac{\nu}{2}+1; -e^{2 c z}\right) + e^{-2 b z} \sum_{j=0}^{n} \frac{(-1)^j (c \nu - 2 b)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{c \nu - 2 b}{2 c}, \ldots, \frac{c \nu - 2 b}{2 c}, \nu; \frac{c \nu - 2 b}{2 c}+1, \ldots, \frac{c \nu - 2 b}{2 c}+1; -e^{2 c z}\right)\right) + \sum_{s=1}^{\left\lfloor \frac{\nu-1}{2} \right\rfloor} (-1)^s \binom{\nu}{s} (-1)^{\nu} e^{2 s c z} \sum_{j=0}^{n} \frac{(-1)^j (2 c s)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(s, \ldots, s, \nu; s+1, \ldots, s+1; -e^{2 c z}\right) + \sum_{s=0}^{\left\lfloor \frac{\nu-1}{2} \right\rfloor} (-1)^s \binom{\nu}{s} \left((-1)^{\nu} e^{(2 c s - 2 b) z} \sum_{j=0}^{n} \frac{(-1)^j (2 c s - 2 b)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{c s - b}{c}, \ldots, \frac{c s - b}{c}, \nu; \frac{c s - b}{c}+1, \ldots, \frac{c s - b}{c}+1; -e^{2 c z}\right) + e^{2 c (\nu - s) z} \sum_{j=0}^{n} \frac{(-1)^j (2 c (\nu - s))^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\nu - s, \ldots, \nu - s, \nu; -s+\nu+1, \ldots, -s+\nu+1; -e^{2 c z}\right) + e^{(2 c (\nu - s) - 2 b) z} \sum_{j=0}^{n} \frac{(-1)^j (2 c (\nu - s) - 2 b)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{c (\nu - s) - b}{c}, \ldots, \frac{c (\nu - s) - b}{c}, \nu; \frac{c (\nu - s) - b}{c}+1, \ldots, \frac{c (\nu - s) - b}{c}+1; -e^{2 c z}\right)\right)\right) /; n \in \mathbb{N} \wedge \nu \in \mathbb{N}^+$$

Involving powers of cosh, exp and power

Involving $z^n e^{p z} \cosh^{u}(b z) \tanh^{\nu}(c z)$

01.21.21.0404.01

$$\int z^n e^{p z} \cosh^{u}(b z) \tanh^{\nu}(c z)\, dz = 2^{-u} \binom{u}{\frac{u}{2}} n! (1 - u \bmod 2) \left(i^{\nu} e^{(p + c \nu) z} \binom{\nu}{\frac{\nu}{2}} (1 - \nu \bmod 2) \sum_{j=0}^{n} \frac{(-1)^j (p + c \nu)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{p + c \nu}{2 c}, \ldots, \frac{p + c \nu}{2 c}, \nu; \frac{p + c \nu}{2 c}+1, \ldots, \frac{p + c \nu}{2 c}+1; -e^{2 c z}\right) + \sum_{s=0}^{\left\lfloor \frac{\nu-1}{2} \right\rfloor} (-1)^s \binom{\nu}{s} \left((-1)^{\nu} e^{(p + 2 c s) z} \sum_{j=0}^{n} \frac{(-1)^j (p + 2 c s)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{p + 2 c s}{2 c}, \ldots, \frac{p + 2 c s}{2 c}, \nu; \frac{p + 2 c s}{2 c} + \right.\right.\right.$$

$$1,\ \ldots,\ \frac{p+2\,c\,s}{2\,c}+1;\ -e^{2\,c\,z}\Bigg)+e^{(p+2\,c\,(v-s))\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(p+2\,c\,(v-s))^{-j-1}\,z^{n-j}}{(n-j)!}$$

$$_{j+2}F_{j+1}\Bigg(\frac{p+2\,c\,(v-s)}{2\,c},\ \ldots,\ \frac{p+2\,c\,(v-s)}{2\,c},\ v;\ \frac{p+2\,c\,(v-s)}{2\,c}+1,\ \ldots,\ \frac{p+2\,c\,(v-s)}{2\,c}+1;\ -e^{2\,c\,z}\Bigg)\Bigg)\Bigg)+$$

$$2^{-u}\,n!\sum_{k=0}^{\left\lfloor\frac{u-1}{2}\right\rfloor}\binom{u}{k}\Bigg(i^v\,e^{(p-b\,(u-2\,k)+c\,v)\,z}\binom{v}{\frac{v}{2}}(1-v\bmod 2)\sum_{j=0}^{n}\frac{(-1)^j\,(p-b\,(u-2\,k)+c\,v)^{-j-1}\,z^{n-j}}{(n-j)!}\ {}_{j+2}F_{j+1}\Bigg(\frac{p-b\,(u-2\,k)+c\,v}{2\,c},$$

$$\ldots,\ \frac{p-b\,(u-2\,k)+c\,v}{2\,c},\ v;\ \frac{p-b\,(u-2\,k)+c\,v}{2\,c}+1,\ \ldots,\ \frac{p-b\,(u-2\,k)+c\,v}{2\,c}+1;\ -e^{2\,c\,z}\Bigg)+$$

$$\sum_{s=0}^{\left\lfloor\frac{v-1}{2}\right\rfloor}(-1)^s\binom{v}{s}\Bigg((-1)^v\,e^{(2\,b\,k+p+2\,c\,s-b\,u)\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(p+2\,c\,s-b\,(u-2\,k))^{-j-1}\,z^{n-j}}{(n-j)!}\ {}_{j+2}F_{j+1}\Bigg(\frac{p+2\,c\,s-b\,(u-2\,k)}{2\,c},$$

$$\ldots,\ \frac{p+2\,c\,s-b\,(u-2\,k)}{2\,c},\ v;\ \frac{p+2\,c\,s-b\,(u-2\,k)}{2\,c}+1,\ \ldots,\ \frac{p+2\,c\,s-b\,(u-2\,k)}{2\,c}+1;\ -e^{2\,c\,z}\Bigg)+$$

$$e^{(2\,b\,k+p-2\,c\,s-b\,u+2\,c\,v)\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(p-b\,(u-2\,k)+2\,c\,(v-s))^{-j-1}\,z^{n-j}}{(n-j)!}\ {}_{j+2}F_{j+1}$$

$$\Bigg(\frac{p-b\,(u-2\,k)+2\,c\,(v-s)}{2\,c},\ \ldots,\ \frac{p-b\,(u-2\,k)+2\,c\,(v-s)}{2\,c},\ v;$$

$$\frac{p-b\,(u-2\,k)+2\,c\,(v-s)}{2\,c}+1,\ \ldots,\ \frac{p-b\,(u-2\,k)+2\,c\,(v-s)}{2\,c}+1;\ -e^{2\,c\,z}\Bigg)\Bigg)+$$

$$i^v\,e^{(p+b\,(u-2\,k)+c\,v)\,z}\binom{v}{\frac{v}{2}}(1-v\bmod 2)\sum_{j=0}^{n}\frac{(-1)^j\,(p+b\,(u-2\,k)+c\,v)^{-j-1}\,z^{n-j}}{(n-j)!}\ {}_{j+2}F_{j+1}\Bigg(\frac{p+b\,(u-2\,k)+c\,v}{2\,c},$$

$$\ldots,\ \frac{p+b\,(u-2\,k)+c\,v}{2\,c},\ v;\ \frac{p+b\,(u-2\,k)+c\,v}{2\,c}+1,\ \ldots,\ \frac{p+b\,(u-2\,k)+c\,v}{2\,c}+1;\ -e^{2\,c\,z}\Bigg)+$$

$$\sum_{s=0}^{\left\lfloor\frac{v-1}{2}\right\rfloor}(-1)^s\binom{v}{s}\Bigg((-1)^v\,e^{(p+2\,c\,s+b\,(u-2\,k))\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(p+2\,c\,s+b\,(u-2\,k))^{-j-1}\,z^{n-j}}{(n-j)!}\ {}_{j+2}F_{j+1}\Bigg(\frac{p+2\,c\,s+b\,(u-2\,k)}{2\,c},$$

$$\ldots,\ \frac{p+2\,c\,s+b\,(u-2\,k)}{2\,c},\ v;\ \frac{p+2\,c\,s+b\,(u-2\,k)}{2\,c}+1,\ \ldots,\ \frac{p+2\,c\,s+b\,(u-2\,k)}{2\,c}+1;\ -e^{2\,c\,z}\Bigg)+$$

$$e^{(p+b\,(u-2\,k)+2\,c\,(v-s))\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(p+b\,(u-2\,k)+2\,c\,(v-s))^{-j-1}\,z^{n-j}}{(n-j)!}\ {}_{j+2}F_{j+1}$$

$$\Bigg(\frac{p+b\,(u-2\,k)+2\,c\,(v-s)}{2\,c},\ \ldots,\ \frac{p+b\,(u-2\,k)+2\,c\,(v-s)}{2\,c},\ v;\ \frac{p+b\,(u-2\,k)+2\,c\,(v-s)}{2\,c}+1,$$

$$\ldots,\ \frac{p+b\,(u-2\,k)+2\,c\,(v-s)}{2\,c}+1;\ -e^{2\,c\,z}\Bigg)\Bigg)\Bigg)\Bigg)\ /;\ n\in\mathbb{N}\land u\in\mathbb{N}^+\land v\in\mathbb{N}^+$$

Involving functions of the direct function, hyperbolic, exponential and trigonometric functions

Involving powers of the direct function, hyperbolic, exponential and trigonometric functions

Involving sin, sinh and exp

Involving $e^{p z} \sin(a z) \sinh(b z) \tanh^{\nu}(c z)$

01.21.21.0405.01

$$\int e^{p z} \sin(a z) \sinh(b z) \tanh^{\nu}(c z)\, dz = \frac{1}{4} i \left(1 - e^{-2 c z}\right)^{-\nu} \left(1 + e^{-2 c z}\right)^{\nu} \tanh^{\nu}(c z) \left(-\frac{e^{(-b - i a + p) z} F_1\left(-\frac{-b - i a + p}{2 c}; -\nu, \nu; 1 - \frac{-b - i a + p}{2 c}; e^{-2 c z}, -e^{-2 c z}\right)}{-b - i a + p} + \frac{e^{(-b + i a + p) z} F_1\left(-\frac{-b + i a + p}{2 c}; -\nu, \nu; 1 - \frac{-b + i a + p}{2 c}; e^{-2 c z}, -e^{-2 c z}\right)}{-b + i a + p} + \frac{e^{(b - i a + p) z} F_1\left(-\frac{b - i a + p}{2 c}; -\nu, \nu; 1 - \frac{b - i a + p}{2 c}; e^{-2 c z}, -e^{-2 c z}\right)}{b - i a + p} - \frac{e^{(b + i a + p) z} F_1\left(-\frac{b + i a + p}{2 c}; -\nu, \nu; 1 - \frac{b + i a + p}{2 c}; e^{-2 c z}, -e^{-2 c z}\right)}{b + i a + p}\right)$$

Involving powers of sin, powers of sinh and exp

Involving $e^{p z} \sin^{m}(a z) \sinh^{u}(b z) \tanh^{\nu}(c z)$

01.21.21.0406.01

$$\int e^{p z} \sin^m(a z) \sinh^u(b z) \tanh^\nu(c z)\, dz = \frac{1}{p}\Bigg(i^u 2^{-m-u} e^{p z} \left(1-e^{-2 c z}\right)^{-\nu} \left(1+e^{-2 c z}\right)^{\nu} F_1\left(-\frac{p}{2 c}; -\nu, \nu; 1-\frac{p}{2 c}; e^{-2 c z}, -e^{-2 c z}\right) \binom{m}{\frac{m}{2}} \binom{u}{\frac{u}{2}} \tanh^\nu(c z)\, (1 - m \bmod 2)\, (1 - u \bmod 2)\Bigg) - i^u 2^{-m-u} \binom{u}{\frac{u}{2}} (u \bmod 2 - 1) \left(1-e^{-2 c z}\right)^{-\nu} \left(1+e^{-2 c z}\right)^{\nu} \tanh^\nu(c z) \sum_{s=0}^{\left\lfloor \frac{m-1}{2} \right\rfloor} (-1)^s \binom{m}{s} \Bigg(\frac{1}{p + a i (m-2 s)} \left(e^{(p + a i (m-2 s)) z - \frac{i m \pi}{2}} F_1\left(-\frac{p + a i (m-2 s)}{2 c}; -\nu, \nu; 1-\frac{p + a i (m-2 s)}{2 c}; e^{-2 c z}, -e^{-2 c z}\right)\right) + \frac{1}{p - i a (m-2 s)} \left(e^{\frac{i \pi m}{2} + (p - i a (m-2 s)) z} F_1\left(-\frac{p - i a (m-2 s)}{2 c}; -\nu, \nu; 1-\frac{p - i a (m-2 s)}{2 c}; e^{-2 c z}, -e^{-2 c z}\right)\right)\Bigg) + 2^{-m-u} \binom{m}{\frac{m}{2}} (1 - m \bmod 2) \left(1-e^{-2 c z}\right)^{-\nu} \left(1+e^{-2 c z}\right)^{\nu} \tanh^\nu(c z) \sum_{k=0}^{\left\lfloor \frac{u-1}{2} \right\rfloor} (-1)^k \binom{u}{k} \Bigg(\frac{1}{p + b (u-2 k)} \left(e^{(p + b (u-2 k)) z} F_1\left(-\frac{p + b (u-2 k)}{2 c}; -\nu, \nu; 1-\frac{p + b (u-2 k)}{2 c}; e^{-2 c z}, -e^{-2 c z}\right)\right) + \frac{1}{p - b (u-2 k)} \left((-1)^u e^{(p - b (u-2 k)) z} F_1\left(-\frac{p - b (u-2 k)}{2 c}; -\nu, \nu; 1-\frac{p - b (u-2 k)}{2 c}; e^{-2 c z}, -e^{-2 c z}\right)\right)\Bigg) + 2^{-m-u} \left(1-e^{-2 c z}\right)^{-\nu} \left(1+e^{-2 c z}\right)^{\nu} \tanh^\nu(c z) \sum_{k=0}^{\left\lfloor \frac{u-1}{2} \right\rfloor} \sum_{s=0}^{\left\lfloor \frac{m-1}{2} \right\rfloor} (-1)^{k+s} \binom{m}{s} \binom{u}{k} \Bigg(\left(e^{(p + a i (m-2 s) + b (u-2 k)) z - \frac{i m \pi}{2}} F_1\left(-\frac{p + a i (m-2 s) + b (u-2 k)}{2 c}; -\nu, \nu; 1-\frac{p + a i (m-2 s) + b (u-2 k)}{2 c}; e^{-2 c z}, -e^{-2 c z}\right)\right) \Big/ (p + a i (m-2 s) + b (u-2 k)) + \left(e^{\frac{i \pi m}{2} + (p - i a (m-2 s) + b (u-2 k)) z} F_1\left(-\frac{p - i a (m-2 s) + b (u-2 k)}{2 c}; -\nu, \nu; 1-\frac{p - i a (m-2 s) + b (u-2 k)}{2 c}; e^{-2 c z}, -e^{-2 c z}\right)\right) \Big/ (p - i a (m-2 s) + b (u-2 k)) + \left((-1)^u e^{(p + a i (m-2 s) - b (u-2 k)) z - \frac{i m \pi}{2}} F_1\left(-\frac{p + a i (m-2 s) - b (u-2 k)}{2 c}; -\nu, \nu; 1-\frac{p + a i (m-2 s) - b (u-2 k)}{2 c}; e^{-2 c z}, -e^{-2 c z}\right)\right) \Big/ (p + a i (m-2 s) - b (u-2 k)) + \left((-1)^u e^{\frac{i \pi m}{2} + (p - i a (m-2 s) - b (u-2 k)) z} F_1\left(-\frac{p - i a (m-2 s) - b (u-2 k)}{2 c}; -\nu, \nu; 1-\frac{p - i a (m-2 s) - b (u-2 k)}{2 c}; e^{-2 c z}, -e^{-2 c z}\right)\right) \Big/ (p - i a (m-2 s) - b (u-2 k))\Bigg) /;\ m \in \mathbb{N}^+ \land u \in \mathbb{N}^+$$

01.21.21.0407.01

$$\int e^{pz}\sin^m(az)\sinh^\mu(cz)\tanh^\nu(cz)\,dz = \frac{1}{p-c\mu}2^{-m}e^{pz}\left(1-e^{2cz}\right)^{-\mu-\nu}\left(1+e^{2cz}\right)^\nu\binom{m}{\frac{m}{2}}(1-m \bmod 2)\tanh^\nu(cz)\sinh^\mu(cz)F_1\left(\frac{p-c\mu}{2c};-\mu-\nu,\nu;\frac{1}{2}\left(\frac{p}{c}-\mu+2\right);e^{2cz},-e^{2cz}\right)+2^{-m}\left(1-e^{2cz}\right)^{-\mu-\nu}\left(1+e^{2cz}\right)^\nu\tanh^\nu(cz)\sinh^\mu(cz)\sum_{k=0}^{\left\lfloor\frac{m-1}{2}\right\rfloor}(-1)^k\binom{m}{k}\left(\left(e^{\frac{i\pi m}{2}+(p-ia(m-2k))z}F_1\left(\frac{-ia(m-2k)+p-c\mu}{2c};-\mu-\nu,\nu;\frac{1}{2}\left(\frac{p-ia(m-2k)}{c}-\mu+2\right);e^{2cz},-e^{2cz}\right)\right)\Big/(-ia(m-2k)+p-c\mu)+\frac{1}{ai(m-2k)+p-c\mu}\left(e^{(ai(m-2k)+p)z-\frac{im\pi}{2}}F_1\left(\frac{ai(m-2k)+p-c\mu}{2c};-\mu-\nu,\nu;\frac{1}{2}\left(\frac{ai(m-2k)+p}{c}-\mu+2\right);e^{2cz},-e^{2cz}\right)\right)\right)\ /;\ m\in\mathbb{N}^+$$

Involving cos, sinh and exp

Involving $e^{pz}\cos(az)\sinh(bz)\tanh^\nu(cz)$

01.21.21.0408.01

$$\int e^{pz}\cos(az)\sinh(bz)\tanh^\nu(cz)\,dz = \frac{1}{4}\left(1-e^{-2cz}\right)^{-\nu}\left(1+e^{-2cz}\right)^\nu\tanh^\nu(cz)\left(-\frac{e^{(-b-ia+p)z}F_1\left(-\frac{-b-ia+p}{2c};-\nu,\nu;1-\frac{-b-ia+p}{2c};e^{-2cz},-e^{-2cz}\right)}{-b-ia+p}-\frac{e^{(-b+ia+p)z}F_1\left(-\frac{-b+ia+p}{2c};-\nu,\nu;1-\frac{-b+ia+p}{2c};e^{-2cz},-e^{-2cz}\right)}{-b+ia+p}+\frac{e^{(b-ia+p)z}F_1\left(-\frac{b-ia+p}{2c};-\nu,\nu;1-\frac{b-ia+p}{2c};e^{-2cz},-e^{-2cz}\right)}{b-ia+p}+\frac{e^{(b+ia+p)z}F_1\left(-\frac{b+ia+p}{2c};-\nu,\nu;1-\frac{b+ia+p}{2c};e^{-2cz},-e^{-2cz}\right)}{b+ia+p}\right)$$

Involving powers of cos, powers of sinh and exp

Involving $e^{pz}\cos^m(az)\sinh^\mu(bz)\tanh^\nu(cz)$

01.21.21.0409.01

$$\int e^{p z} \cos^m(a z) \sinh^u(b z) \tanh^\nu(c z)\, dz = \frac{1}{p}\left(i^u 2^{-m-u} e^{p z} \left(1-e^{-2 c z}\right)^{-\nu} \left(1+e^{-2 c z}\right)^\nu F_1\left(-\frac{p}{2c}; -\nu, \nu; 1-\frac{p}{2c}; e^{-2cz}, -e^{-2cz}\right) \binom{m}{\frac{m}{2}} \binom{u}{\frac{u}{2}} \tanh^\nu(c z)\,(1-m \bmod 2)(1-u \bmod 2)\right) - i^u 2^{-m-u} \binom{u}{\frac{u}{2}} (u \bmod 2 - 1) \left(1-e^{-2cz}\right)^{-\nu} \left(1+e^{-2cz}\right)^\nu \tanh^\nu(c z) \sum_{s=0}^{\left\lfloor \frac{m-1}{2} \right\rfloor} \binom{m}{s} \left(\frac{e^{(p+a i (m-2s)) z} F_1\left(-\frac{p+a i (m-2s)}{2c}; -\nu, \nu; 1-\frac{p+a i(m-2s)}{2c}; e^{-2cz}, -e^{-2cz}\right)}{p+a i (m-2s)} + \frac{e^{(p-i a (m-2s)) z} F_1\left(-\frac{p-i a (m-2s)}{2c}; -\nu, \nu; 1-\frac{p-i a (m-2s)}{2c}; e^{-2cz}, -e^{-2cz}\right)}{p-i a (m-2s)} \right) + 2^{-m-u} \binom{m}{\frac{m}{2}} (1-m \bmod 2) \left(1-e^{-2cz}\right)^{-\nu} \left(1+e^{-2cz}\right)^\nu \tanh^\nu(c z) \sum_{k=0}^{\left\lfloor \frac{u-1}{2} \right\rfloor} (-1)^k \binom{u}{k} \left(\frac{e^{(p+b(u-2k)) z} F_1\left(-\frac{p+b(u-2k)}{2c}; -\nu, \nu; 1-\frac{p+b(u-2k)}{2c}; e^{-2cz}, -e^{-2cz}\right)}{p+b(u-2k)} + \frac{(-1)^u e^{(p-b(u-2k)) z} F_1\left(-\frac{p-b(u-2k)}{2c}; -\nu, \nu; 1-\frac{p-b(u-2k)}{2c}; e^{-2cz}, -e^{-2cz}\right)}{p-b(u-2k)} \right) +$$

$$2^{-m-u} \left(1-e^{-2cz}\right)^{-\nu} \left(1+e^{-2cz}\right)^\nu \tanh^\nu(c z) \sum_{k=0}^{\left\lfloor \frac{u-1}{2} \right\rfloor} \sum_{s=0}^{\left\lfloor \frac{m-1}{2} \right\rfloor} (-1)^k \binom{m}{s} \binom{u}{k} \left(\left(e^{(p+a i (m-2s)+b(u-2k)) z} F_1\left(-\frac{p+a i (m-2s)+b(u-2k)}{2c}; -\nu, \nu; 1-\frac{p+a i (m-2s)+b(u-2k)}{2c}; e^{-2cz}, -e^{-2cz}\right)\right) \Big/ (p+a i (m-2s)+b(u-2k)) + \left(e^{(p-i a (m-2s)+b(u-2k)) z} F_1\left(-\frac{p-i a (m-2s)+b(u-2k)}{2c}; -\nu, \nu; 1-\frac{p-i a (m-2s)+b(u-2k)}{2c}; e^{-2cz}, -e^{-2cz}\right)\right) \Big/ (p-i a (m-2s)+b(u-2k)) + \left((-1)^u e^{(p+a i (m-2s)-b(u-2k)) z} F_1\left(-\frac{p+a i (m-2s)-b(u-2k)}{2c}; -\nu, \nu; 1-\frac{p+a i (m-2s)-b(u-2k)}{2c}; e^{-2cz}, -e^{-2cz}\right)\right) \Big/ (p+a i (m-2s)-b(u-2k)) + \left((-1)^u e^{(p-i a (m-2s)-b(u-2k)) z} F_1\left(-\frac{p-i a (m-2s)-b(u-2k)}{2c}; -\nu, \nu; 1-\frac{p-i a (m-2s)-b(u-2k)}{2c}; e^{-2cz}, -e^{-2cz}\right)\right) \Big/ (p-i a (m-2s)-b(u-2k)) \right) /;\ m \in \mathbb{N}^+ \land u \in \mathbb{N}^+$$

01.21.21.0410.01

$$\int e^{p z} \cos^m(a z) \sinh^\mu(c z) \tanh^\nu(c z)\, dz = \frac{1}{p - c\mu} 2^{-m} e^{p z} \left(1 - e^{2 c z}\right)^{-\mu-\nu} \left(1 + e^{2 c z}\right)^\nu \binom{m}{\frac{m}{2}} (1 - m \bmod 2) \tanh^\nu(c z) \sinh^\mu(c z)\, F_1\left(\frac{p - c\mu}{2 c}; -\mu - \nu, \nu; \frac{1}{2}\left(\frac{p}{c} - \mu + 2\right); e^{2 c z}, -e^{2 c z}\right) + 2^{-m} \left(1 - e^{2 c z}\right)^{-\mu-\nu} \left(1 + e^{2 c z}\right)^\nu \tanh^\nu(c z) \sinh^\mu(c z) \sum_{k=0}^{\left\lfloor \frac{m-1}{2} \right\rfloor} \binom{m}{k} \left(\left(e^{(p - i a (m - 2 k)) z} F_1\left(\frac{-i a (m - 2 k) + p - c\mu}{2 c}; -\mu - \nu, \nu; \frac{1}{2}\left(\frac{p - i a (m - 2 k)}{c} - \mu + 2\right); e^{2 c z}, -e^{2 c z}\right)\right) \Big/ (-i a (m - 2 k) + p - c\mu) + \frac{1}{a i (m - 2 k) + p - c\mu} \left(e^{(a i (m - 2 k) + p) z} F_1\left(\frac{a i (m - 2 k) + p - c\mu}{2 c}; -\mu - \nu, \nu; \frac{1}{2}\left(\frac{a i (m - 2 k) + p}{c} - \mu + 2\right); e^{2 c z}, -e^{2 c z}\right)\right)\right) /; m \in \mathbb{N}^+$$

Involving sin, cosh and exp

Involving $e^{p z} \sin(a z) \cosh(b z) \tanh^\nu(c z)$

01.21.21.0411.01

$$\int e^{p z} \sin(a z) \cosh(b z) \tanh^\nu(c z)\, dz = -\frac{1}{4} i \left(1 - e^{-2 c z}\right)^{-\nu} \left(1 + e^{-2 c z}\right)^\nu \tanh^\nu(c z) \left(-\frac{e^{(-b - i a + p) z} F_1\left(-\frac{-b - i a + p}{2 c}; -\nu, \nu; 1 - \frac{-b - i a + p}{2 c}; e^{-2 c z}, -e^{-2 c z}\right)}{-b - i a + p} + \frac{e^{(-b + i a + p) z} F_1\left(-\frac{-b + i a + p}{2 c}; -\nu, \nu; 1 - \frac{-b + i a + p}{2 c}; e^{-2 c z}, -e^{-2 c z}\right)}{-b + i a + p} + \frac{e^{(b + i a + p) z} F_1\left(-\frac{b + i a + p}{2 c}; -\nu, \nu; 1 - \frac{b + i a + p}{2 c}; e^{-2 c z}, -e^{-2 c z}\right)}{b + i a + p} - \frac{e^{(b - i a + p) z} F_1\left(-\frac{b - i a + p}{2 c}; -\nu, \nu; 1 - \frac{b - i a + p}{2 c}; e^{-2 c z}, -e^{-2 c z}\right)}{b - i a + p}\right)$$

Involving powers of sin, powers of cosh and exp

Involving $e^{p z} \sin^m(a z) \cosh^u(b z) \tanh^\nu(c z)$

01.21.21.0412.01

$$\int e^{p z} \sin^m(a z) \cosh^u(b z) \tanh^\nu(c z)\, dz = \frac{1}{p}\left(2^{-m-u} e^{p z} \left(1-e^{-2 c z}\right)^{-\nu} \left(1+e^{-2 c z}\right)^\nu F_1\left(-\frac{p}{2 c}; -\nu, \nu; 1-\frac{p}{2 c}; e^{-2 c z}, -e^{-2 c z}\right) \binom{m}{\frac{m}{2}} \binom{u}{\frac{u}{2}} \tanh^\nu(c z) (1-m \bmod 2)(1-u \bmod 2)\right) + 2^{-m-u} i^{-m} \left(1-e^{-2 c z}\right)^{-\nu} \left(1+e^{-2 c z}\right)^\nu \binom{u}{\frac{u}{2}} \tanh^\nu(c z)(1-u \bmod 2) \sum_{k=0}^{\left\lfloor \frac{m-1}{2} \right\rfloor} (-1)^k \binom{m}{k} \left(\frac{(-1)^m e^{(p-i a (m-2 k)) z} F_1\left(-\frac{p-i a (m-2 k)}{2 c}; -\nu, \nu; 1-\frac{p-i a (m-2 k)}{2 c}; e^{-2 c z}, -e^{-2 c z}\right)}{p-i a (m-2 k)} + \frac{e^{(a i (m-2 k)+p) z} F_1\left(-\frac{a i (m-2 k)+p}{2 c}; -\nu, \nu; 1-\frac{a i (m-2 k)+p}{2 c}; e^{-2 c z}, -e^{-2 c z}\right)}{a i (m-2 k)+p}\right) - 2^{-m-u} \left(1-e^{-2 c z}\right)^{-\nu} \left(1+e^{-2 c z}\right)^\nu \binom{m}{\frac{m}{2}} \tanh^\nu(c z)(m \bmod 2 - 1) \sum_{s=0}^{\left\lfloor \frac{u-1}{2} \right\rfloor} \binom{u}{s} \left(\frac{e^{(p+b (u-2 s)) z} F_1\left(-\frac{p+b (u-2 s)}{2 c}; -\nu, \nu; 1-\frac{p+b (u-2 s)}{2 c}; e^{-2 c z}, -e^{-2 c z}\right)}{p+b (u-2 s)} + \frac{e^{(p-b (u-2 s)) z} F_1\left(-\frac{p-b (u-2 s)}{2 c}; -\nu, \nu; 1-\frac{p-b (u-2 s)}{2 c}; e^{-2 c z}, -e^{-2 c z}\right)}{p-b (u-2 s)}\right) + i^{-m} 2^{-m-u} \left(1-e^{-2 c z}\right)^{-\nu} \left(1+e^{-2 c z}\right)^\nu \tanh^\nu(c z) \sum_{k=0}^{\left\lfloor \frac{m-1}{2} \right\rfloor} \sum_{s=0}^{\left\lfloor \frac{u-1}{2} \right\rfloor} (-1)^k \binom{m}{k} \binom{u}{s} \left(\left((-1)^m e^{(-i a (m-2 k)+p+b (u-2 s)) z} F_1\left(-\frac{-i a (m-2 k)+p+b (u-2 s)}{2 c}; -\nu, \nu; 1-\frac{-i a (m-2 k)+p+b (u-2 s)}{2 c}; e^{-2 c z}, -e^{-2 c z}\right)\right) \Big/ (-i a (m-2 k)+p+b (u-2 s)) + \left(e^{(a i (m-2 k)+p+b (u-2 s)) z} F_1\left(-\frac{a i (m-2 k)+p+b (u-2 s)}{2 c}; -\nu, \nu; 1-\frac{a i (m-2 k)+p+b (u-2 s)}{2 c}; e^{-2 c z}, -e^{-2 c z}\right)\right) \Big/ (a i (m-2 k)+p+b (u-2 s)) + \left((-1)^m e^{(-i a (m-2 k)+p-b (u-2 s)) z} F_1\left(-\frac{-i a (m-2 k)+p-b (u-2 s)}{2 c}; -\nu, \nu; 1-\frac{-i a (m-2 k)+p-b (u-2 s)}{2 c}; e^{-2 c z}, -e^{-2 c z}\right)\right) \Big/ (-i a (m-2 k)+p-b (u-2 s)) + \left(e^{(a i (m-2 k)+p-b (u-2 s)) z} F_1\left(-\frac{a i (m-2 k)+p-b (u-2 s)}{2 c}; -\nu, \nu; 1-\frac{a i (m-2 k)+p-b (u-2 s)}{2 c}; e^{-2 c z}, -e^{-2 c z}\right)\right) \Big/ (a i (m-2 k)+p-b (u-2 s))\right) \;/;\; m \in \mathbb{N}^+ \land u \in \mathbb{N}^+$$

01.21.21.0413.01

$$\int e^{p z} \sin^m(a z) \cosh^\mu(c z) \tanh^\nu(c z)\, dz = \frac{1}{p - c\mu} 2^{-m} e^{p z} \left(1 - e^{2 c z}\right)^{-\nu} \left(1 + e^{2 c z}\right)^{\nu-\mu} \binom{m}{\frac{m}{2}} (1 - m \bmod 2) \tanh^\nu(c z) \cosh^\mu(c z)$$
$$F_1\left(\frac{p - c\mu}{2c}; -\nu, \nu - \mu; \frac{1}{2}\left(\frac{p}{c} - \mu + 2\right); e^{2cz}, -e^{2cz}\right) + 2^{-m} \left(1 - e^{2 c z}\right)^{-\nu} \left(1 + e^{2 c z}\right)^{\nu-\mu} \tanh^\nu(c z) \cosh^\mu(c z) \sum_{k=0}^{\left\lfloor\frac{m-1}{2}\right\rfloor} (-1)^k$$
$$\binom{m}{k} \left(\left(e^{\frac{i\pi m}{2} + (p - i a (m - 2k)) z} F_1\left(\frac{-i a (m - 2k) + p - c\mu}{2c}; -\nu, \nu - \mu; \frac{1}{2}\left(\frac{p - i a (m - 2k)}{c} - \mu + 2\right); e^{2cz}, -e^{2cz}\right)\right) \Big/\right.$$
$$(-i a (m - 2k) + p - c\mu) + \frac{1}{a i (m - 2k) + p - c\mu} \left(e^{(a i (m - 2k) + p) z - \frac{i m \pi}{2}}\right.$$
$$\left.\left. F_1\left(\frac{a i (m - 2k) + p - c\mu}{2c}; -\nu, \nu - \mu; \frac{1}{2}\left(\frac{a i (m - 2k) + p}{c} - \mu + 2\right); e^{2cz}, -e^{2cz}\right)\right)\right) /; m \in \mathbb{N}^+$$

Involving cos, cosh and exp

Involving $e^{p z} \cos(a z) \cosh(b z) \tanh^\nu(c z)$

01.21.21.0414.01

$$\int e^{p z} \cos(a z) \cosh(b z) \tanh^\nu(c z)\, dz =$$
$$\frac{1}{4} \left(1 - e^{-2 c z}\right)^{-\nu} \left(1 + e^{-2 c z}\right)^{\nu} \tanh^\nu(c z) \left(\frac{e^{(-b - i a + p) z} F_1\left(-\frac{-b - i a + p}{2c}; -\nu, \nu; 1 - \frac{-b - i a + p}{2c}; e^{-2cz}, -e^{-2cz}\right)}{-b - i a + p} + \right.$$
$$\frac{e^{(-b + i a + p) z} F_1\left(-\frac{-b + i a + p}{2c}; -\nu, \nu; 1 - \frac{-b + i a + p}{2c}; e^{-2cz}, -e^{-2cz}\right)}{-b + i a + p} +$$
$$\left.\frac{e^{(b - i a + p) z} F_1\left(-\frac{b - i a + p}{2c}; -\nu, \nu; 1 - \frac{b - i a + p}{2c}; e^{-2cz}, -e^{-2cz}\right)}{b - i a + p} + \frac{e^{(b + i a + p) z} F_1\left(-\frac{b + i a + p}{2c}; -\nu, \nu; 1 - \frac{b + i a + p}{2c}; e^{-2cz}, -e^{-2cz}\right)}{b + i a + p}\right)$$

Involving powers of cos, powers of cosh and exp

Involving $e^{p z} \cos^m(a z) \cosh^u(b z) \tanh^\nu(c z)$

01.21.21.0415.01

$$\int e^{pz}\cos^m(az)\cosh^u(bz)\tanh^\nu(cz)\,dz = 2^{-m-u}\left(1+e^{-2cz}\right)^\nu\binom{u}{\frac{u}{2}}(1-u\bmod 2)\tanh^\nu(cz)$$
$$\left(1-e^{-2cz}\right)^{-\nu}\sum_{k=0}^{\left\lfloor\frac{m-1}{2}\right\rfloor}\binom{m}{k}\left(\frac{e^{(p-ia(m-2k))z}F_1\left(-\frac{p-ia(m-2k)}{2c};-\nu,\nu;1-\frac{p-ia(m-2k)}{2c};e^{-2cz},-e^{-2cz}\right)}{p-ia(m-2k)}+\right.$$
$$\left.\frac{e^{(ai(m-2k)+p)z}F_1\left(-\frac{ai(m-2k)+p}{2c};-\nu,\nu;1-\frac{ai(m-2k)+p}{2c};e^{-2cz},-e^{-2cz}\right)}{ai(m-2k)+p}\right)+2^{-m-u}\left(1+e^{-2cz}\right)^\nu\binom{m}{\frac{m}{2}}(1-m\bmod 2)$$
$$\tanh^\nu(cz)\left(1-e^{-2cz}\right)^{-\nu}\sum_{s=0}^{\left\lfloor\frac{u-1}{2}\right\rfloor}\binom{u}{s}\left(\frac{e^{(p+b(u-2s))z}F_1\left(-\frac{p+b(u-2s)}{2c};-\nu,\nu;1-\frac{p+b(u-2s)}{2c};e^{-2cz},-e^{-2cz}\right)}{p+b(u-2s)}+\right.$$
$$\left.\frac{e^{(p-b(u-2s))z}F_1\left(-\frac{p-b(u-2s)}{2c};-\nu,\nu;1-\frac{p-b(u-2s)}{2c};e^{-2cz},-e^{-2cz}\right)}{p-b(u-2s)}\right)+$$
$$2^{-m-u}\left(1+e^{-2cz}\right)^\nu\tanh^\nu(cz)\left(1-e^{-2cz}\right)^{-\nu}\sum_{k=0}^{\left\lfloor\frac{m-1}{2}\right\rfloor}\sum_{s=0}^{\left\lfloor\frac{u-1}{2}\right\rfloor}\binom{m}{k}\binom{u}{s}\left(\left(e^{(-ia(m-2k)+p+b(u-2s))z}F_1\left(-\frac{-ia(m-2k)+p+b(u-2s)}{2c};\right.\right.\right.$$
$$\left.\left.-\nu,\nu;1-\frac{-ia(m-2k)+p+b(u-2s)}{2c};e^{-2cz},-e^{-2cz}\right)\right)\Big/(-ia(m-2k)+p+b(u-2s))+$$
$$\left(e^{(ai(m-2k)+p+b(u-2s))z}F_1\left(-\frac{ai(m-2k)+p+b(u-2s)}{2c};-\nu,\nu;1-\frac{ai(m-2k)+p+b(u-2s)}{2c};\right.\right.$$
$$\left.\left.e^{-2cz},-e^{-2cz}\right)\right)\Big/(ai(m-2k)+p+b(u-2s))+\left(e^{(-ia(m-2k)+p-b(u-2s))z}\right.$$
$$\left.F_1\left(-\frac{-ia(m-2k)+p-b(u-2s)}{2c};-\nu,\nu;1-\frac{-ia(m-2k)+p-b(u-2s)}{2c};e^{-2cz},-e^{-2cz}\right)\right)\Big/$$
$$(-ia(m-2k)+p-b(u-2s))+\left(e^{(ai(m-2k)+p-b(u-2s))z}F_1\left(-\frac{ai(m-2k)+p-b(u-2s)}{2c};\right.\right.$$
$$\left.\left.\left.-\nu,\nu;1-\frac{ai(m-2k)+p-b(u-2s)}{2c};e^{-2cz},-e^{-2cz}\right)\right)\Big/(ai(m-2k)+p-b(u-2s))\right)+$$
$$\frac{1}{p}2^{-m-u}e^{pz}\left(1-e^{-2cz}\right)^{-\nu}\left(1+e^{-2cz}\right)^\nu F_1\left(-\frac{p}{2c};-\nu,\nu;1-\frac{p}{2c};e^{-2cz},-e^{-2cz}\right)$$
$$\binom{m}{\frac{m}{2}}$$
$$\binom{u}{\frac{u}{2}}$$
$$\tanh^\nu(cz)$$
$$(1-m\bmod 2)$$
$$(1-u\bmod 2)\;/;\;m\in\mathbb{N}^+\land u\in\mathbb{N}^+$$

01.21.21.0416.01

$$\int e^{pz}\cos^m(az)\cosh^{\mu}(cz)\tanh^{\nu}(cz)\,dz = \frac{1}{p-c\mu}2^{-m}e^{pz}\left(1-e^{2cz}\right)^{-\nu}\left(1+e^{2cz}\right)^{\nu-\mu}\binom{m}{\frac{m}{2}}(1-m\bmod 2)\tanh^{\nu}(cz)\cosh^{\mu}(cz)\, F_1\!\left(\frac{p-c\mu}{2c};-\nu,\nu-\mu;\frac{1}{2}\left(\frac{p}{c}-\mu+2\right);e^{2cz},-e^{2cz}\right)+2^{-m}\left(1-e^{2cz}\right)^{-\nu}\left(1+e^{2cz}\right)^{\nu-\mu}\tanh^{\nu}(cz)\cosh^{\mu}(cz)\sum_{k=0}^{\left\lfloor\frac{m-1}{2}\right\rfloor}\binom{m}{k}\left(\left(e^{(p-ia(m-2k))z}F_1\!\left(\frac{-ia(m-2k)+p-c\mu}{2c};-\nu,\nu-\mu;\frac{1}{2}\left(\frac{p-ia(m-2k)}{c}-\mu+2\right);e^{2cz},-e^{2cz}\right)\right)\Big/(-ia(m-2k)+p-c\mu)+\frac{1}{ai(m-2k)+p-c\mu}\left(e^{(ai(m-2k)+p)z}F_1\!\left(\frac{ai(m-2k)+p-c\mu}{2c};-\nu,\nu-\mu;\frac{1}{2}\left(\frac{ai(m-2k)+p}{c}-\mu+2\right);e^{2cz},-e^{2cz}\right)\right)\right)\;/;\;m\in\mathbb{N}^+$$

Involving functions of the direct function, hyperbolic, trigonometric and a power functions

Involving powers of the direct function, hyperbolic, trigonometric and a power functions

Involving sin, sinh and power

Involving $z^n\sin(az)\sinh(bz)\tanh^{\nu}(cz)$

01.21.21.0417.01

$$\int z^n\sin(az)\sinh(bz)\tanh^{\nu}(cz)\,dz = \frac{1}{4}i\,n!\left(i^{\nu}e^{(b-ia+c\nu)z}\binom{\nu}{\frac{\nu}{2}}(1-\nu\bmod 2)\sum_{j=0}^{n}\frac{(-1)^j(b-ia+c\nu)^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\!\left(\frac{b-ia+c\nu}{2c},\ldots,\frac{b-ia+c\nu}{2c},\nu;\frac{b-ia+c\nu}{2c}+1,\ldots,\frac{b-ia+c\nu}{2c}+1;-e^{2cz}\right)+\sum_{s=0}^{\left\lfloor\frac{\nu-1}{2}\right\rfloor}(-1)^s\binom{\nu}{s}\left((-1)^{\nu}e^{(-ia+b+2cs)z}\sum_{j=0}^{n}\frac{(-1)^j(b-ia+2cs)^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\!\left(\frac{b-ia+2cs}{2c},\ldots,\frac{b-ia+2cs}{2c},\nu;\frac{b-ia+2cs}{2c}+1,\ldots,\frac{b-ia+2cs}{2c}+1;-e^{2cz}\right)+e^{(b-ia+2c(\nu-s))z}\sum_{j=0}^{n}\frac{(-1)^j(b-ia+2c(\nu-s))^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\!\left(\frac{b-ia+2c(\nu-s)}{2c},\ldots,\frac{b-ia+2c(\nu-s)}{2c},\nu;\frac{b-ia+2c(\nu-s)}{2c}+1,\ldots,\frac{b-ia+2c(\nu-s)}{2c}+1;-e^{2cz}\right)\right)-i^{\nu}e^{(b+ia+c\nu)z}\binom{\nu}{\frac{\nu}{2}}(1-\nu\bmod 2)\sum_{j=0}^{n}\frac{(-1)^j(b+ia+c\nu)^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\!\left(\frac{b+ia+c\nu}{2c},\ldots,\frac{b+ia+c\nu}{2c},\nu;\frac{b+ia+c\nu}{2c}+1,\ldots,\frac{b+ia+c\nu}{2c}+1;-e^{2cz}\right)-$$

$$\sum_{s=0}^{\left\lfloor\frac{\nu-1}{2}\right\rfloor}(-1)^s\binom{\nu}{s}\Bigg(e^{(b+i a+2c(\nu-s))z}\sum_{j=0}^{n}\frac{(-1)^j(b+i a+2c(\nu-s))^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\Bigg(\frac{b+i a+2c(\nu-s)}{2c},\ldots,\frac{b+i a+2c(\nu-s)}{2c},\nu;\frac{b+i a+2c(\nu-s)}{2c}+1,\ldots,\frac{b+i a+2c(\nu-s)}{2c}+1;-e^{2cz}\Bigg)+$$

$$(-1)^\nu e^{(i a+b+2cs)z}\sum_{j=0}^{n}\frac{(-1)^j(b+i a+2cs)^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\Bigg(\frac{b+i a+2cs}{2c},\ldots,\frac{b+i a+2cs}{2c},\nu;\frac{b+i a+2cs}{2c}+1,\ldots,\frac{b+i a+2cs}{2c}+1;-e^{2cz}\Bigg)\Bigg)-$$

$$i^\nu e^{(-b-i a+c\nu)z}\binom{\nu}{\frac{\nu}{2}}(1-\nu\bmod 2)\sum_{j=0}^{n}\frac{(-1)^j(-b-i a+c\nu)^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\Bigg(\frac{-b-i a+c\nu}{2c},\ldots,\frac{-b-i a+c\nu}{2c},\nu;\frac{-b-i a+c\nu}{2c}+1,\ldots,\frac{-b-i a+c\nu}{2c}+1;-e^{2cz}\Bigg)-$$

$$\sum_{s=0}^{\left\lfloor\frac{\nu-1}{2}\right\rfloor}(-1)^s\binom{\nu}{s}\Bigg((-1)^\nu e^{(-i a-b+2cs)z}\sum_{j=0}^{n}\frac{(-1)^j(-b-i a+2cs)^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\Bigg(\frac{-b-i a+2cs}{2c},\ldots,\frac{-b-i a+2cs}{2c},\nu;\frac{-b-i a+2cs}{2c}+1,\ldots,\frac{-b-i a+2cs}{2c}+1;-e^{2cz}\Bigg)+$$

$$e^{(-i a-b-2cs+2c\nu)z}\sum_{j=0}^{n}\frac{(-1)^j(-b-i a+2c(\nu-s))^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\Bigg(\frac{-b-i a+2c(\nu-s)}{2c},\ldots,\frac{-b-i a+2c(\nu-s)}{2c},\nu;\frac{-b-i a+2c(\nu-s)}{2c}+1,\ldots,\frac{-b-i a+2c(\nu-s)}{2c}+1;-e^{2cz}\Bigg)\Bigg)+$$

$$i^\nu e^{(-b+i a+c\nu)z}\binom{\nu}{\frac{\nu}{2}}(1-\nu\bmod 2)\sum_{j=0}^{n}\frac{(-1)^j(-b+i a+c\nu)^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\Bigg(\frac{-b+i a+c\nu}{2c},\ldots,\frac{-b+i a+c\nu}{2c},\nu;\frac{-b+i a+c\nu}{2c}+1,\ldots,\frac{-b+i a+c\nu}{2c}+1;-e^{2cz}\Bigg)+$$

$$\sum_{s=0}^{\left\lfloor\frac{\nu-1}{2}\right\rfloor}(-1)^s\binom{\nu}{s}\Bigg((-1)^\nu e^{(i a-b+2cs)z}\sum_{j=0}^{n}\frac{(-1)^j(-b+i a+2cs)^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\Bigg(\frac{-b+i a+2cs}{2c},\ldots,\frac{-b+i a+2cs}{2c},\nu;\frac{-b+i a+2cs}{2c}+1,\ldots,\frac{-b+i a+2cs}{2c}+1;-e^{2cz}\Bigg)+e^{(i a-b-2cs+2c\nu)z}$$

$$\sum_{j=0}^{n}\frac{(-1)^j(-b+i a+2c(\nu-s))^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\Bigg(\frac{-b+i a+2c(\nu-s)}{2c},\ldots,\frac{-b+i a+2c(\nu-s)}{2c},\nu;\frac{-b+i a+2c(\nu-s)}{2c}+1,\ldots,\frac{-b+i a+2c(\nu-s)}{2c}+1;-e^{2cz}\Bigg)\Bigg)\Bigg)\Bigg/;\ n\in\mathbb{N}\land\nu\in\mathbb{N}^+$$

Involving powers of sin, powers of sinh and power

Involving $z^n \sin^m(a z) \sinh^u(b z) \text{Tanh}^v(c z)$

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$$\int z^n \sin^m(a z) \sinh^u(b z) \tanh^v(c z)\, dz = i^u\, 2^{-m-u} \binom{m}{\frac{m}{2}} \binom{u}{\frac{u}{2}} (1 - m \bmod 2)(1 - u \bmod 2)$$
$$\left(\frac{(-1)^v z^{n+1}}{n+1} - (-1)^v e^{2 c z} v\, n! \sum_{j=0}^{n} \frac{(-1)^j 2^{-j-1} c^{-j-1} z^{n-j}}{(n-j)!}\, {}_{j+3}F_{j+2}\left(1, \ldots, 1, v+1; 2, \ldots, 2; -e^{2 c z}\right) + \right.$$
$$e^{c v z} i^v \binom{v}{\frac{v}{2}} n!\, (1 - v \bmod 2) \sum_{j=0}^{n} \frac{(-1)^j (c v)^{-j-1} z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}\left(\frac{v}{2}, \ldots, \frac{v}{2}, v; \frac{v}{2}+1, \ldots, \frac{v}{2}+1; -e^{2 c z}\right) +$$
$$(-1)^v n! \sum_{s=1}^{\left\lfloor \frac{v-1}{2} \right\rfloor} (-1)^s \binom{v}{s} e^{2 s c z} \sum_{j=0}^{n} \frac{(-1)^j (2 c s)^{-j-1} z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}\left(s, \ldots, s, v; s+1, \ldots, s+1; -e^{2 c z}\right) +$$
$$n! \sum_{s=0}^{\left\lfloor \frac{v-1}{2} \right\rfloor} (-1)^s \binom{v}{s} e^{2 c (v-s) z} \sum_{j=0}^{n} \frac{(-1)^j (2 c (v-s))^{-j-1} z^{n-j}}{(n-j)!}$$
$$\left. {}_{j+2}F_{j+1}\left(v-s, \ldots, v-s, v; -s+v+1, \ldots, -s+v+1; -e^{2 c z}\right)\right) + i^u\, 2^{-m-u} \binom{u}{\frac{u}{2}} n!\, (1 - u \bmod 2) \sum_{k=0}^{\left\lfloor \frac{m-1}{2} \right\rfloor} (-1)^k$$
$$\binom{m}{k} \left(e^{\frac{i m \pi}{2}} \left(i^v e^{(c v - i a (m-2k)) z} \binom{v}{\frac{v}{2}} (1 - v \bmod 2) \sum_{j=0}^{n} \frac{(-1)^j (c v - i a (m-2k))^{-j-1} z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}\left(\frac{c v - i a (m-2k)}{2c},\right.\right.\right.$$
$$\left. \ldots, \frac{c v - i a (m-2k)}{2c}, v; \frac{c v - i a (m-2k)}{2c} + 1, \ldots, \frac{c v - i a (m-2k)}{2c} + 1; -e^{2 c z}\right) +$$
$$\sum_{s=0}^{\left\lfloor \frac{v-1}{2} \right\rfloor} (-1)^s \binom{v}{s} \left((-1)^v e^{(2 c s - i a (m-2k)) z} \sum_{j=0}^{n} \frac{(-1)^j (2 c s - i a (m-2k))^{-j-1} z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}\left(\frac{2 c s - i a (m-2k)}{2c},\right.\right.$$
$$\left. \ldots, \frac{2 c s - i a (m-2k)}{2c}, v; \frac{2 c s - i a (m-2k)}{2c} + 1, \ldots, \frac{2 c s - i a (m-2k)}{2c} + 1; -e^{2 c z}\right) +$$
$$e^{(2 c (v-s) - i a (m-2k)) z} \sum_{j=0}^{n} \frac{(-1)^j (2 c (v-s) - i a (m-2k))^{-j-1} z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}$$
$$\left(\frac{2 c (v-s) - i a (m-2k)}{2c}, \ldots, \frac{2 c (v-s) - i a (m-2k)}{2c}, v;\right.$$
$$\left.\left.\left. \frac{2 c (v-s) - i a (m-2k)}{2c} + 1, \ldots, \frac{2 c (v-s) - i a (m-2k)}{2c} + 1; -e^{2 c z}\right)\right)\right) +$$

$$e^{-\frac{1}{2} i m \pi} \left(i^{v} e^{(a i (m-2k)+c v) z} \binom{v}{\frac{v}{2}} (1 - v \bmod 2) \sum_{j=0}^{n} \frac{(-1)^j (a i (m-2k)+c v)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{i a (m-2k)+c v}{2c}, \right.\right.$$

$$\left. \ldots, \frac{i a (m-2k)+c v}{2c}, v; \frac{i a (m-2k)+c v}{2c}+1, \ldots, \frac{i a (m-2k)+c v}{2c}+1; -e^{2cz} \right) +$$

$$\sum_{s=0}^{\left\lfloor \frac{v-1}{2} \right\rfloor} (-1)^s \binom{v}{s} \left((-1)^v e^{(a i (m-2k)+2cs) z} \sum_{j=0}^{n} \frac{(-1)^j (a i (m-2k)+2cs)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{i a (m-2k)+2cs}{2c}, \right.\right.$$

$$\left. \ldots, \frac{i a (m-2k)+2cs}{2c}, v; \frac{i a (m-2k)+2cs}{2c}+1, \ldots, \frac{i a (m-2k)+2cs}{2c}+1; -e^{2cz} \right) +$$

$$e^{(a i (m-2k)+2c(v-s)) z} \sum_{j=0}^{n} \frac{(-1)^j (a i (m-2k)+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}$$

$$\left(\frac{i a (m-2k)+2c(v-s)}{2c}, \ldots, \frac{i a (m-2k)+2c(v-s)}{2c}, v; \right.$$

$$\left.\left.\left.\left. \frac{i a (m-2k)+2c(v-s)}{2c}+1, \ldots, \frac{i a (m-2k)+2c(v-s)}{2c}+1; -e^{2cz} \right)\right)\right)\right) +$$

$$2^{-m-u} \binom{m}{\frac{m}{2}} n! \, (1 - m \bmod 2) \sum_{k=0}^{\left\lfloor \frac{u-1}{2} \right\rfloor} (-1)^k \binom{u}{k} \left(i^{v} e^{(b(u-2k)+c v) z} \binom{v}{\frac{v}{2}} (1 - v \bmod 2) \sum_{j=0}^{n} \frac{(-1)^j (b(u-2k)+c v)^{-j-1} z^{n-j}}{(n-j)!} \right.$$

$${}_{j+2}F_{j+1}\left(\frac{b(u-2k)+c v}{2c}, \ldots, \frac{b(u-2k)+c v}{2c}, v; \frac{b(u-2k)+c v}{2c}+1, \ldots, \frac{b(u-2k)+c v}{2c}+1; -e^{2cz} \right) +$$

$$(-1)^u i^{v} e^{(c v - b(u-2k)) z} \binom{v}{\frac{v}{2}} (1 - v \bmod 2) \sum_{j=0}^{n} \frac{(-1)^j (c v - b(u-2k))^{-j-1} z^{n-j}}{(n-j)!}$$

$${}_{j+2}F_{j+1}\left(\frac{c v - b(u-2k)}{2c}, \ldots, \frac{c v - b(u-2k)}{2c}, v; \frac{c v - b(u-2k)}{2c}+1, \ldots, \frac{c v - b(u-2k)}{2c}+1; -e^{2cz} \right) +$$

$$\sum_{s=0}^{\left\lfloor \frac{v-1}{2} \right\rfloor} (-1)^s \binom{v}{s} \left((-1)^v e^{(2cs+b(u-2k)) z} \sum_{j=0}^{n} \frac{(-1)^j (2cs+b(u-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{2cs+b(u-2k)}{2c}, \right.\right.$$

$$\left. \ldots, \frac{2cs+b(u-2k)}{2c}, v; \frac{2cs+b(u-2k)}{2c}+1, \ldots, \frac{2cs+b(u-2k)}{2c}+1; -e^{2cz} \right) +$$

$$e^{(b(u-2k)+2c(v-s)) z} \sum_{j=0}^{n} \frac{(-1)^j (b(u-2k)+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}$$

$$\left(\frac{b(u-2k)+2c(v-s)}{2c}, \ldots, \frac{b(u-2k)+2c(v-s)}{2c}, v; \right.$$

$$\left.\left. \frac{b(u-2k)+2c(v-s)}{2c}+1, \ldots, \frac{b(u-2k)+2c(v-s)}{2c}+1; -e^{2cz} \right)\right) +$$

$$(-1)^u \sum_{s=0}^{\left\lfloor \frac{v-1}{2} \right\rfloor} (-1)^s \binom{v}{s} \left((-1)^v e^{(2cs-b(u-2k))z} \sum_{j=0}^{n} \frac{(-1)^j (2cs-b(u-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{2cs-b(u-2k)}{2c}, \right.\right.$$

$$\left. \ldots, \frac{2cs-b(u-2k)}{2c}, v; \frac{2cs-b(u-2k)}{2c}+1, \ldots, \frac{2cs-b(u-2k)}{2c}+1; -e^{2cz} \right) +$$

$$e^{(2c(v-s)-b(u-2k))z} \sum_{j=0}^{n} \frac{(-1)^j (2c(v-s)-b(u-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}$$

$$\left(\frac{2c(v-s)-b(u-2k)}{2c}, \ldots, \frac{2c(v-s)-b(u-2k)}{2c}, v; \right.$$

$$\left.\left.\left. \frac{2c(v-s)-b(u-2k)}{2c}+1, \ldots, \frac{2c(v-s)-b(u-2k)}{2c}+1; -e^{2cz} \right)\right)\right) +$$

$$2^{-m-u} n! \sum_{k=0}^{\left\lfloor \frac{m-1}{2} \right\rfloor} (-1)^k \binom{m}{k} \sum_{i=0}^{\left\lfloor \frac{u-1}{2} \right\rfloor} (-1)^i \binom{u}{i} \left((-1)^u e^{\frac{i m \pi}{2}} \left(i^v e^{(-i a(m-2k)-b(u-2i)+cv)z} \binom{v}{\frac{v}{2}} (1 - v \bmod 2) \right.\right.$$

$$\sum_{j=0}^{n} \frac{(-1)^j (-i a(m-2k)-b(u-2i)+cv)^{-j-1} z^{n-j}}{(n-j)!}$$

$${}_{j+2}F_{j+1}\left(\frac{-i a(m-2k)-b(u-2i)+cv}{2c}, \ldots, \frac{-i a(m-2k)-b(u-2i)+cv}{2c}, v; \right.$$

$$\left. \frac{-i a(m-2k)-b(u-2i)+cv}{2c}+1, \ldots, \frac{-i a(m-2k)-b(u-2i)+cv}{2c}+1; -e^{2cz} \right) +$$

$$\sum_{s=0}^{\left\lfloor \frac{v-1}{2} \right\rfloor} (-1)^s \binom{v}{s} \left((-1)^v e^{(-i a(m-2k)+2cs-b(u-2i))z} \sum_{j=0}^{n} \frac{(-1)^j (-i a(m-2k)+2cs-b(u-2i))^{-j-1} z^{n-j}}{(n-j)!} \right.$$

$${}_{j+2}F_{j+1}\left(\frac{-i a(m-2k)+2cs-b(u-2i)}{2c}, \ldots, \right.$$

$$\frac{-i a(m-2k)+2cs-b(u-2i)}{2c}, v; \frac{-i a(m-2k)+2cs-b(u-2i)}{2c}+1,$$

$$\left. \ldots, \frac{-i a(m-2k)+2cs-b(u-2i)}{2c}+1; -e^{2cz} \right) +$$

$$e^{(-i a(m-2k)-b(u-2i)+2c(v-s))z} \sum_{j=0}^{n} \frac{(-1)^j (-i a(m-2k)-b(u-2i)+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!}$$

$${}_{j+2}F_{j+1}\left(\frac{-i a(m-2k)-b(u-2i)+2c(v-s)}{2c}, \ldots, \right.$$

$$\frac{-i a(m-2k)-b(u-2i)+2c(v-s)}{2c}, v; \frac{-i a(m-2k)-b(u-2i)+2c(v-s)}{2c}+1,$$

$$\left.\left.\left. \ldots, \frac{-i a(m-2k)-b(u-2i)+2c(v-s)}{2c}+1; -e^{2cz} \right)\right)\right) + (-1)^u e^{-\frac{1}{2} i m \pi}$$

$$\Bigg(i^{v} e^{(a i (m-2k)-b(u-2i)+c v) z} \binom{v}{\frac{v}{2}} (1 - v \bmod 2) \sum_{j=0}^{n} \frac{(-1)^j (a i (m-2k) - b(u-2i) + c v)^{-j-1} z^{n-j}}{(n-j)!}$$

$$_{j+2}F_{j+1}\left(\frac{i a (m-2k) - b(u-2i) + c v}{2c}, \ldots, \frac{i a (m-2k) - b(u-2i) + c v}{2c}, v; \frac{i a (m-2k) - b(u-2i) + c v}{2c} + 1, \ldots, \frac{i a (m-2k) - b(u-2i) + c v}{2c} + 1; -e^{2cz}\right) +$$

$$\sum_{s=0}^{\left\lfloor \frac{v-1}{2} \right\rfloor} (-1)^s \binom{v}{s} \Bigg((-1)^v e^{(a i (m-2k)+2cs-b(u-2i)) z} \sum_{j=0}^{n} \frac{(-1)^j (a i (m-2k) + 2cs - b(u-2i))^{-j-1} z^{n-j}}{(n-j)!}$$

$$_{j+2}F_{j+1}\left(\frac{i a (m-2k) + 2cs - b(u-2i)}{2c}, \ldots, \frac{i a (m-2k) + 2cs - b(u-2i)}{2c}, v; \frac{i a (m-2k) + 2cs - b(u-2i)}{2c} + 1, \ldots, \frac{i a (m-2k) + 2cs - b(u-2i)}{2c} + 1; -e^{2cz}\right) +$$

$$e^{(a i (m-2k)-b(u-2i)+2c(v-s)) z} \sum_{j=0}^{n} \frac{(-1)^j (a i (m-2k) - b(u-2i) + 2c(v-s))^{-j-1} z^{n-j}}{(n-j)!}$$

$$_{j+2}F_{j+1}\left(\frac{i a (m-2k) - b(u-2i) + 2c(v-s)}{2c}, \ldots, \frac{i a (m-2k) - b(u-2i) + 2c(v-s)}{2c}, v; \frac{i a (m-2k) - b(u-2i) + 2c(v-s)}{2c} + 1, \ldots, \frac{i a (m-2k) - b(u-2i) + 2c(v-s)}{2c} + 1; -e^{2cz}\right)\Bigg)\Bigg) +$$

$$e^{\frac{i m \pi}{2}} \Bigg(i^{v} e^{(-i a (m-2k)+b(u-2i)+c v) z} \binom{v}{\frac{v}{2}} (1 - v \bmod 2) \sum_{j=0}^{n} \frac{(-1)^j (-i a (m-2k) + b(u-2i) + c v)^{-j-1} z^{n-j}}{(n-j)!}$$

$$_{j+2}F_{j+1}\left(\frac{-i a (m-2k) + b(u-2i) + c v}{2c}, \ldots, \frac{-i a (m-2k) + b(u-2i) + c v}{2c}, v; \frac{-i a (m-2k) + b(u-2i) + c v}{2c} + 1, \ldots, \frac{-i a (m-2k) + b(u-2i) + c v}{2c} + 1; -e^{2cz}\right) +$$

$$\sum_{s=0}^{\left\lfloor \frac{v-1}{2} \right\rfloor} (-1)^s \binom{v}{s} \Bigg((-1)^v e^{(-i a (m-2k)+2cs+b(u-2i)) z} \sum_{j=0}^{n} \frac{(-1)^j (-i a (m-2k) + 2cs + b(u-2i))^{-j-1} z^{n-j}}{(n-j)!}$$

$$_{j+2}F_{j+1}\left(\frac{-i a (m-2k) + 2cs + b(u-2i)}{2c}, \ldots, \frac{-i a (m-2k) + 2cs + b(u-2i)}{2c}, v; \frac{-i a (m-2k) + 2cs + b(u-2i)}{2c} + 1,\right.$$

$$\ldots, \frac{-i\, a\,(m-2\,k)+2\,c\,s+b\,(u-2\,i)}{2\,c}+1;\, -e^{2\,c\,z}\Bigg)+$$

$$e^{(-i\,a\,(m-2\,k)+b\,(u-2\,i)+2\,c\,(v-s))\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(-i\,a\,(m-2\,k)+b\,(u-2\,i)+2\,c\,(v-s))^{-j-1}\,z^{n-j}}{(n-j)!}$$

$${}_{j+2}F_{j+1}\Bigg(\frac{-i\,a\,(m-2\,k)+b\,(u-2\,i)+2\,c\,(v-s)}{2\,c},\, \ldots,$$

$$\frac{-i\,a\,(m-2\,k)+b\,(u-2\,i)+2\,c\,(v-s)}{2\,c},\, v;\, \frac{-i\,a\,(m-2\,k)+b\,(u-2\,i)+2\,c\,(v-s)}{2\,c}+1,$$

$$\ldots, \frac{-i\,a\,(m-2\,k)+b\,(u-2\,i)+2\,c\,(v-s)}{2\,c}+1;\, -e^{2\,c\,z}\Bigg)\Bigg)\Bigg)+$$

$$e^{-\frac{1}{2}\,i\,m\,\pi}\Bigg(i^v\, e^{(a\,i\,(m-2\,k)+b\,(u-2\,i)+c\,v)\,z}\binom{v}{\frac{v}{2}}(1-v \bmod 2)\sum_{j=0}^{n}\frac{(-1)^j\,(a\,i\,(m-2\,k)+b\,(u-2\,i)+c\,v)^{-j-1}\,z^{n-j}}{(n-j)!}$$

$${}_{j+2}F_{j+1}\Bigg(\frac{i\,a\,(m-2\,k)+b\,(u-2\,i)+c\,v}{2\,c},\, \ldots,\, \frac{i\,a\,(m-2\,k)+b\,(u-2\,i)+c\,v}{2\,c},\, v;$$

$$\frac{i\,a\,(m-2\,k)+b\,(u-2\,i)+c\,v}{2\,c}+1,\, \ldots,\, \frac{i\,a\,(m-2\,k)+b\,(u-2\,i)+c\,v}{2\,c}+1;\, -e^{2\,c\,z}\Bigg)+$$

$$\sum_{s=0}^{\left\lfloor\frac{v-1}{2}\right\rfloor}(-1)^s\binom{v}{s}\Bigg((-1)^v\, e^{(a\,i\,(m-2\,k)+2\,c\,s+b\,(u-2\,i))\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(a\,i\,(m-2\,k)+2\,c\,s+b\,(u-2\,i))^{-j-1}\,z^{n-j}}{(n-j)!}$$

$${}_{j+2}F_{j+1}\Bigg(\frac{i\,a\,(m-2\,k)+2\,c\,s+b\,(u-2\,i)}{2\,c},\, \ldots,\, \frac{i\,a\,(m-2\,k)+2\,c\,s+b\,(u-2\,i)}{2\,c},$$

$$v;\, \frac{i\,a\,(m-2\,k)+2\,c\,s+b\,(u-2\,i)}{2\,c}+1,\, \ldots,$$

$$\frac{i\,a\,(m-2\,k)+2\,c\,s+b\,(u-2\,i)}{2\,c}+1;\, -e^{2\,c\,z}\Bigg)+e^{(a\,i\,(m-2\,k)+b\,(u-2\,i)+2\,c\,(v-s))\,z}$$

$$\sum_{j=0}^{n}\frac{(-1)^j\,(a\,i\,(m-2\,k)+b\,(u-2\,i)+2\,c\,(v-s))^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}$$

$$\Bigg(\frac{i\,a\,(m-2\,k)+b\,(u-2\,i)+2\,c\,(v-s)}{2\,c},\, \ldots,\, \frac{i\,a\,(m-2\,k)+b\,(u-2\,i)+2\,c\,(v-s)}{2\,c},\, v;$$

$$\frac{i\,a\,(m-2\,k)+b\,(u-2\,i)+2\,c\,(v-s)}{2\,c}+1,\, \ldots,\, \frac{i\,a\,(m-2\,k)+b\,(u-2\,i)+2\,c\,(v-s)}{2\,c}+$$

$$1;\, -e^{2\,c\,z}\Bigg)\Bigg)\Bigg)\Bigg)\, /;\, n\in\mathbb{N}\land m\in\mathbb{N}^+\land u\in\mathbb{N}^+\land v\in\mathbb{N}^+$$

Involving cos, sinh and power

Involving $z^n \cos(a z) \sinh(b z) \tanh^{\nu}(c z)$

01.21.21.0419.01

$$\int z^n \cos(a z) \sinh(b z) \tanh^{\nu}(c z)\, dz = \frac{1}{4} n! \Bigg(i^{\nu} e^{(b - i a + c \nu) z} \binom{\nu}{\frac{\nu}{2}} (1 - \nu \bmod 2) \sum_{j=0}^{n} \frac{(-1)^j (b - i a + c \nu)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{b - i a + c \nu}{2c}, \ldots, \frac{b - i a + c \nu}{2c}, \nu; \frac{b - i a + c \nu}{2c} + 1, \ldots, \frac{b - i a + c \nu}{2c} + 1; -e^{2 c z}\right) + i^{\nu} e^{(b + i a + c \nu) z} \binom{\nu}{\frac{\nu}{2}} (1 - \nu \bmod 2) \sum_{j=0}^{n} \frac{(-1)^j (b + i a + c \nu)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{b + i a + c \nu}{2c}, \ldots, \frac{b + i a + c \nu}{2c}, \nu; \frac{b + i a + c \nu}{2c} + 1, \ldots, \frac{b + i a + c \nu}{2c} + 1; -e^{2 c z}\right) - i^{\nu} e^{(-b - i a + c \nu) z} \binom{\nu}{\frac{\nu}{2}} (1 - \nu \bmod 2) \sum_{j=0}^{n} \frac{(-1)^j (-b - i a + c \nu)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{-b - i a + c \nu}{2c}, \ldots, \frac{-b - i a + c \nu}{2c}, \nu; \frac{-b - i a + c \nu}{2c} + 1, \ldots, \frac{-b - i a + c \nu}{2c} + 1; -e^{2 c z}\right) - i^{\nu} e^{(-b + i a + c \nu) z} \binom{\nu}{\frac{\nu}{2}} (1 - \nu \bmod 2) \sum_{j=0}^{n} \frac{(-1)^j (-b + i a + c \nu)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{-b + i a + c \nu}{2c}, \ldots, \frac{-b + i a + c \nu}{2c}, \nu; \frac{-b + i a + c \nu}{2c} + 1, \ldots, \frac{-b + i a + c \nu}{2c} + 1; -e^{2 c z}\right) + \sum_{s=0}^{\left\lfloor \frac{\nu-1}{2} \right\rfloor} (-1)^s \binom{\nu}{s} \Bigg((-1)^{\nu} e^{(-i a + b + 2 c s) z} \sum_{j=0}^{n} \frac{(-1)^j (b - i a + 2 c s)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{b - i a + 2 c s}{2c}, \ldots, \frac{b - i a + 2 c s}{2c}, \nu; \frac{b - i a + 2 c s}{2c} + 1, \ldots, \frac{b - i a + 2 c s}{2c} + 1; -e^{2 c z}\right) + e^{(-i a + b + 2 c (-s + \nu)) z} \sum_{j=0}^{n} \frac{(-1)^j (b - i a + 2 c (\nu - s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{b - i a + 2 c (\nu - s)}{2c}, \ldots, \frac{b - i a + 2 c (\nu - s)}{2c}, \nu; \frac{b - i a + 2 c (\nu - s)}{2c} + 1, \ldots, \frac{b - i a + 2 c (\nu - s)}{2c} + 1; -e^{2 c z}\right) \Bigg) + \sum_{s=0}^{\left\lfloor \frac{\nu-1}{2} \right\rfloor} (-1)^s \binom{\nu}{s} \Bigg(e^{(i a + b + 2 c (-s + \nu)) z} \sum_{j=0}^{n} \frac{(-1)^j (b + i a + 2 c (\nu - s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{b + i a + 2 c (\nu - s)}{2c}, \ldots, \frac{b + i a + 2 c (\nu - s)}{2c}, \nu; \frac{b + i a + 2 c (\nu - s)}{2c} + 1, \ldots, \frac{b + i a + 2 c (\nu - s)}{2c} + 1; -e^{2 c z}\right) + (-1)^{\nu} e^{(i a + b + 2 c s) z} \sum_{j=0}^{n} \frac{(-1)^j (b + i a + 2 c s)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{b + i a + 2 c s}{2c}, \ldots, \frac{b + i a + 2 c s}{2c}, \right.$$

$$v;\ \frac{b+i a+2 c s}{2 c}+1,\ \ldots,\ \frac{b+i a+2 c s}{2 c}+1;\ -e^{2 c z}\Bigg)\Bigg)-$$

$$\sum_{s=0}^{\left\lfloor\frac{v-1}{2}\right\rfloor}(-1)^s\binom{v}{s}\Bigg((-1)^v e^{(-i a-b+2 c s) z}\sum_{j=0}^{n}\frac{(-1)^j(-b-i a+2 c s)^{-j-1} z^{n-j}}{(n-j)!}\ {}_{j+2}F_{j+1}\Bigg(\frac{-b-i a+2 c s}{2 c},\ \ldots,$$

$$\frac{-b-i a+2 c s}{2 c},\ v;\ \frac{-b-i a+2 c s}{2 c}+1,\ \ldots,\ \frac{-b-i a+2 c s}{2 c}+1;\ -e^{2 c z}\Bigg)+$$

$$e^{(-i a-b-2 c s+2 c v) z}\sum_{j=0}^{n}\frac{(-1)^j(-b-i a+2 c(v-s))^{-j-1} z^{n-j}}{(n-j)!}\ {}_{j+2}F_{j+1}\Bigg(\frac{-b-i a+2 c(v-s)}{2 c},\ \ldots,$$

$$\frac{-b-i a+2 c(v-s)}{2 c},\ v;\ \frac{-b-i a+2 c(v-s)}{2 c}+1,\ \ldots,\ \frac{-b-i a+2 c(v-s)}{2 c}+1;\ -e^{2 c z}\Bigg)\Bigg)-$$

$$\sum_{s=0}^{\left\lfloor\frac{v-1}{2}\right\rfloor}(-1)^s\binom{v}{s}\Bigg((-1)^v e^{(i a-b+2 c s) z}\sum_{j=0}^{n}\frac{(-1)^j(-b+i a+2 c s)^{-j-1} z^{n-j}}{(n-j)!}\ {}_{j+2}F_{j+1}\Bigg(\frac{-b+i a+2 c s}{2 c},\ \ldots,$$

$$\frac{-b+i a+2 c s}{2 c},\ v;\ \frac{-b+i a+2 c s}{2 c}+1,\ \ldots,\ \frac{-b+i a+2 c s}{2 c}+1;\ -e^{2 c z}\Bigg)+e^{(i a-b-2 c s+2 c v) z}$$

$$\sum_{j=0}^{n}\frac{(-1)^j(-b+i a+2 c(v-s))^{-j-1} z^{n-j}}{(n-j)!}\ {}_{j+2}F_{j+1}\Bigg(\frac{-b+i a+2 c(v-s)}{2 c},\ \ldots,\ \frac{-b+i a+2 c(v-s)}{2 c},$$

$$v;\ \frac{-b+i a+2 c(v-s)}{2 c}+1,\ \ldots,\ \frac{-b+i a+2 c(v-s)}{2 c}+1;\ -e^{2 c z}\Bigg)\Bigg)\Bigg)\ /;\ n\in\mathbb{N}\land v\in\mathbb{N}^+$$

Involving powers of cos, powers of sinh and power

Involving $z^n \cos^m(a z) \sinh^u(b z) \tanh^v(c z)$

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$$\int z^n \cos^m(a z)\sinh^u(b z)\tanh^v(c z)\,dz = i^u\, 2^{-m-u}\binom{m}{\frac{m}{2}}\binom{u}{\frac{u}{2}}(1-m \bmod 2)(1-u \bmod 2)$$

$$\Bigg(\frac{(-1)^v z^{n+1}}{n+1}-(-1)^v e^{2 c z}\, v\, n!\sum_{j=0}^{n}\frac{(-1)^j 2^{-j-1} c^{-j-1} z^{n-j}}{(n-j)!}\ {}_{j+3}F_{j+2}\big(1,\ \ldots,\ 1,\ v+1;\ 2,\ \ldots,\ 2;\ -e^{2 c z}\big)+$$

$$e^{c v z}\, i^v\binom{v}{\frac{v}{2}} n!\,(1-v \bmod 2)\sum_{j=0}^{n}\frac{(-1)^j (c v)^{-j-1} z^{n-j}}{(n-j)!}\ {}_{j+2}F_{j+1}\Big(\frac{v}{2},\ \ldots,\ \frac{v}{2},\ v;\ \frac{v}{2}+1,\ \ldots,\ \frac{v}{2}+1;\ -e^{2 c z}\Big)+$$

$$(-1)^v n!\sum_{s=1}^{\left\lfloor\frac{v-1}{2}\right\rfloor}(-1)^s\binom{v}{s} e^{2 s c z}\sum_{j=0}^{n}\frac{(-1)^j (2 c s)^{-j-1} z^{n-j}}{(n-j)!}\ {}_{j+2}F_{j+1}\big(s,\ \ldots,\ s,\ v;\ s+1,\ \ldots,\ s+1;\ -e^{2 c z}\big)+n!\sum_{s=0}^{\left\lfloor\frac{v-1}{2}\right\rfloor}(-1)^s$$

$$\left.\left.\binom{v}{s} e^{2c(v-s)z} \sum_{j=0}^{n} \frac{(-1)^j (2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(v-s, \ldots, v-s, v; -s+v+1, \ldots, -s+v+1; -e^{2cz}\right)\right)\right) +$$

$$i^u 2^{-m-u} \binom{u}{\frac{u}{2}} n! (1-u \bmod 2) \sum_{k=0}^{\left\lfloor\frac{m-1}{2}\right\rfloor} \binom{m}{k} \left(i^v e^{(cv - i a(m-2k))z} \binom{v}{\frac{v}{2}} (1-v \bmod 2) \sum_{j=0}^{n} \frac{(-1)^j (cv - i a(m-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\right.$$

$$\left(\frac{cv - i a(m-2k)}{2c}, \ldots, \frac{cv - i a(m-2k)}{2c}, v; \frac{cv - i a(m-2k)}{2c} + 1, \ldots, \frac{cv - i a(m-2k)}{2c} + 1; -e^{2cz}\right) +$$

$$\sum_{s=0}^{\left\lfloor\frac{v-1}{2}\right\rfloor} (-1)^s \binom{v}{s} \left((-1)^v e^{(2cs - i a(m-2k))z} \sum_{j=0}^{n} \frac{(-1)^j (2cs - i a(m-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{2cs - i a(m-2k)}{2c},\right.\right.$$

$$\left.\ldots, \frac{2cs - i a(m-2k)}{2c}, v; \frac{2cs - i a(m-2k)}{2c} + 1, \ldots, \frac{2cs - i a(m-2k)}{2c} + 1; -e^{2cz}\right) +$$

$$e^{(2c(v-s) - i a(m-2k))z} \sum_{j=0}^{n} \frac{(-1)^j (2c(v-s) - i a(m-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}$$

$$\left(\frac{2c(v-s) - i a(m-2k)}{2c}, \ldots, \frac{2c(v-s) - i a(m-2k)}{2c}, v;\right.$$

$$\left.\left.\frac{2c(v-s) - i a(m-2k)}{2c} + 1, \ldots, \frac{2c(v-s) - i a(m-2k)}{2c} + 1; -e^{2cz}\right)\right) +$$

$$i^v e^{(a i(m-2k) + cv)z} \binom{v}{\frac{v}{2}} (1-v \bmod 2) \sum_{j=0}^{n} \frac{(-1)^j (a i(m-2k) + cv)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}$$

$$\left(\frac{i a(m-2k) + cv}{2c}, \ldots, \frac{i a(m-2k) + cv}{2c}, v; \frac{i a(m-2k) + cv}{2c} + 1, \ldots, \frac{i a(m-2k) + cv}{2c} + 1; -e^{2cz}\right) +$$

$$\sum_{s=0}^{\left\lfloor\frac{v-1}{2}\right\rfloor} (-1)^s \binom{v}{s} \left((-1)^v e^{(a i(m-2k) + 2cs)z} \sum_{j=0}^{n} \frac{(-1)^j (a i(m-2k) + 2cs)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{i a(m-2k) + 2cs}{2c},\right.\right.$$

$$\left.\ldots, \frac{i a(m-2k) + 2cs}{2c}, v; \frac{i a(m-2k) + 2cs}{2c} + 1, \ldots, \frac{i a(m-2k) + 2cs}{2c} + 1; -e^{2cz}\right) +$$

$$e^{(a i(m-2k) + 2c(v-s))z} \sum_{j=0}^{n} \frac{(-1)^j (a i(m-2k) + 2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}$$

$$\left(\frac{i a(m-2k) + 2c(v-s)}{2c}, \ldots, \frac{i a(m-2k) + 2c(v-s)}{2c}, v;\right.$$

$$\left.\left.\left.\frac{i a(m-2k) + 2c(v-s)}{2c} + 1, \ldots, \frac{i a(m-2k) + 2c(v-s)}{2c} + 1; -e^{2cz}\right)\right)\right) +$$

$$2^{-m-u} \binom{m}{\frac{m}{2}} n! (1-m \bmod 2) \sum_{k=0}^{\left\lfloor\frac{u-1}{2}\right\rfloor} (-1)^k \binom{u}{k} \left(i^v e^{(b(u-2k) + cv)z} \binom{v}{\frac{v}{2}} (1-v \bmod 2) \sum_{j=0}^{n} \frac{(-1)^j (b(u-2k) + cv)^{-j-1} z^{n-j}}{(n-j)!}\right.$$

$$ {}_{j+2}F_{j+1}\left(\frac{b\,(u-2\,k)+c\,v}{2\,c},\,\ldots,\,\frac{b\,(u-2\,k)+c\,v}{2\,c},\,v;\,\frac{b\,(u-2\,k)+c\,v}{2\,c}+1,\,\ldots,\,\frac{b\,(u-2\,k)+c\,v}{2\,c}+1;\,-e^{2\,c\,z}\right)+ $$

$$ (-1)^u\,i^v\,e^{(c\,v-b\,(u-2\,k))\,z}\binom{v}{\frac{v}{2}}(1-v\bmod 2)\sum_{j=0}^{n}\frac{(-1)^j\,(c\,v-b\,(u-2\,k))^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{c\,v-b\,(u-2\,k)}{2\,c},\right. $$

$$ \left.\ldots,\,\frac{c\,v-b\,(u-2\,k)}{2\,c},\,v;\,\frac{c\,v-b\,(u-2\,k)}{2\,c}+1,\,\ldots,\,\frac{c\,v-b\,(u-2\,k)}{2\,c}+1;\,-e^{2\,c\,z}\right)+\sum_{s=0}^{\left\lfloor\frac{v-1}{2}\right\rfloor}(-1)^s\binom{v}{s} $$

$$ \left((-1)^v\,e^{(2\,c\,s+b\,(u-2\,k))\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(2\,c\,s+b\,(u-2\,k))^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{2\,c\,s+b\,(u-2\,k)}{2\,c},\,\ldots,\,\frac{2\,c\,s+b\,(u-2\,k)}{2\,c},\right.\right. $$

$$ \left.v;\,\frac{2\,c\,s+b\,(u-2\,k)}{2\,c}+1,\,\ldots,\,\frac{2\,c\,s+b\,(u-2\,k)}{2\,c}+1;\,-e^{2\,c\,z}\right)+e^{(b\,(u-2\,k)+2\,c\,(v-s))\,z} $$

$$ \sum_{j=0}^{n}\frac{(-1)^j\,(b\,(u-2\,k)+2\,c\,(v-s))^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{b\,(u-2\,k)+2\,c\,(v-s)}{2\,c},\,\ldots,\,\frac{b\,(u-2\,k)+2\,c\,(v-s)}{2\,c},\right. $$

$$ \left.\left.v;\,\frac{b\,(u-2\,k)+2\,c\,(v-s)}{2\,c}+1,\,\ldots,\,\frac{b\,(u-2\,k)+2\,c\,(v-s)}{2\,c}+1;\,-e^{2\,c\,z}\right)\right)+ $$

$$ (-1)^u\sum_{s=0}^{\left\lfloor\frac{v-1}{2}\right\rfloor}(-1)^s\binom{v}{s}\left((-1)^v\,e^{(2\,c\,s-b\,(u-2\,k))\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(2\,c\,s-b\,(u-2\,k))^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{2\,c\,s-b\,(u-2\,k)}{2\,c},\right.\right. $$

$$ \left.\ldots,\,\frac{2\,c\,s-b\,(u-2\,k)}{2\,c},\,v;\,\frac{2\,c\,s-b\,(u-2\,k)}{2\,c}+1,\,\ldots,\,\frac{2\,c\,s-b\,(u-2\,k)}{2\,c}+1;\,-e^{2\,c\,z}\right)+ $$

$$ e^{(2\,c\,(v-s)-b\,(u-2\,k))\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(2\,c\,(v-s)-b\,(u-2\,k))^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{2\,c\,(v-s)-b\,(u-2\,k)}{2\,c},\right. $$

$$ \ldots,\,\frac{2\,c\,(v-s)-b\,(u-2\,k)}{2\,c},\,v;\,\frac{2\,c\,(v-s)-b\,(u-2\,k)}{2\,c}+1,\,\ldots, $$

$$ \left.\left.\left.\frac{2\,c\,(v-s)-b\,(u-2\,k)}{2\,c}+1;\,-e^{2\,c\,z}\right)\right)\right)+2^{-m-u}\,n!\sum_{k=0}^{\left\lfloor\frac{m-1}{2}\right\rfloor}\binom{m}{k} $$

$$ \sum_{i=0}^{\left\lfloor\frac{u-1}{2}\right\rfloor}(-1)^i\binom{u}{i}\left(i^v\,e^{(-i\,a\,(m-2\,k)+b\,(u-2\,i)+c\,v)\,z}\binom{v}{\frac{v}{2}}(1-v\bmod 2)\sum_{j=0}^{n}\frac{(-1)^j\,(-i\,a\,(m-2\,k)+b\,(u-2\,i)+c\,v)^{-j-1}\,z^{n-j}}{(n-j)!}\right. $$

$$ {}_{j+2}F_{j+1}\left(\frac{-i\,a\,(m-2\,k)+b\,(u-2\,i)+c\,v}{2\,c},\,\ldots,\,\frac{-i\,a\,(m-2\,k)+b\,(u-2\,i)+c\,v}{2\,c},\,v;\right. $$

$$ \left.\frac{-i\,a\,(m-2\,k)+b\,(u-2\,i)+c\,v}{2\,c}+1,\,\ldots,\,\frac{-i\,a\,(m-2\,k)+b\,(u-2\,i)+c\,v}{2\,c}+1;\,-e^{2\,c\,z}\right)+ $$

$$ \sum_{s=0}^{\left\lfloor\frac{v-1}{2}\right\rfloor}(-1)^s\binom{v}{s}\left((-1)^v\,e^{(-i\,a\,(m-2\,k)+2\,c\,s+b\,(u-2\,i))\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(-i\,a\,(m-2\,k)+2\,c\,s+b\,(u-2\,i))^{-j-1}\,z^{n-j}}{(n-j)!}\right. $$

$$ {}_{j+2}F_{j+1}\left(\frac{-i\,a\,(m-2\,k)+2\,c\,s+b\,(u-2\,i)}{2\,c},\,\ldots,\,\frac{-i\,a\,(m-2\,k)+2\,c\,s+b\,(u-2\,i)}{2\,c},\,v;\,\frac{-i\,a\,(m-2\,k)+2\,c\,s+b\,(u-2\,i)}{2\,c}+1,\,\ldots,\,\frac{-i\,a\,(m-2\,k)+2\,c\,s+b\,(u-2\,i)}{2\,c}+1;\,-e^{2\,c\,z}\right)+ $$

$$ e^{(-i\,a\,(m-2\,k)+b\,(u-2\,i)+2\,c\,(v-s))\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(-i\,a\,(m-2\,k)+b\,(u-2\,i)+2\,c\,(v-s))^{-j-1}\,z^{n-j}}{(n-j)!} $$

$$ {}_{j+2}F_{j+1}\left(\frac{-i\,a\,(m-2\,k)+b\,(u-2\,i)+2\,c\,(v-s)}{2\,c},\,\ldots,\,\frac{-i\,a\,(m-2\,k)+b\,(u-2\,i)+2\,c\,(v-s)}{2\,c},\,v;\,\frac{-i\,a\,(m-2\,k)+b\,(u-2\,i)+2\,c\,(v-s)}{2\,c}+1,\,\ldots,\,\frac{-i\,a\,(m-2\,k)+b\,(u-2\,i)+2\,c\,(v-s)}{2\,c}+1;\,-e^{2\,c\,z}\right)\Bigg)+ $$

$$ i^v\,e^{(a\,i\,(m-2\,k)+b\,(u-2\,i)+c\,v)\,z}\binom{v}{\frac{v}{2}}(1-v\bmod 2)\sum_{j=0}^{n}\frac{(-1)^j\,(a\,i\,(m-2\,k)+b\,(u-2\,i)+c\,v)^{-j-1}\,z^{n-j}}{(n-j)!} $$

$$ {}_{j+2}F_{j+1}\left(\frac{i\,a\,(m-2\,k)+b\,(u-2\,i)+c\,v}{2\,c},\,\ldots,\,\frac{i\,a\,(m-2\,k)+b\,(u-2\,i)+c\,v}{2\,c},\,v;\,\frac{i\,a\,(m-2\,k)+b\,(u-2\,i)+c\,v}{2\,c}+1,\,\ldots,\,\frac{i\,a\,(m-2\,k)+b\,(u-2\,i)+c\,v}{2\,c}+1;\,-e^{2\,c\,z}\right)+ $$

$$ \sum_{s=0}^{\left\lfloor\frac{v-1}{2}\right\rfloor}(-1)^s\binom{v}{s}\left((-1)^v\,e^{(a\,i\,(m-2\,k)+2\,c\,s+b\,(u-2\,i))\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(a\,i\,(m-2\,k)+2\,c\,s+b\,(u-2\,i))^{-j-1}\,z^{n-j}}{(n-j)!}\right. $$

$$ {}_{j+2}F_{j+1}\left(\frac{i\,a\,(m-2\,k)+2\,c\,s+b\,(u-2\,i)}{2\,c},\,\ldots,\,\frac{i\,a\,(m-2\,k)+2\,c\,s+b\,(u-2\,i)}{2\,c},\,v;\,\frac{i\,a\,(m-2\,k)+2\,c\,s+b\,(u-2\,i)}{2\,c}+1,\,\ldots,\,\frac{i\,a\,(m-2\,k)+2\,c\,s+b\,(u-2\,i)}{2\,c}+1;\,-e^{2\,c\,z}\right)+ $$

$$ e^{(a\,i\,(m-2\,k)+b\,(u-2\,i)+2\,c\,(v-s))\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(a\,i\,(m-2\,k)+b\,(u-2\,i)+2\,c\,(v-s))^{-j-1}\,z^{n-j}}{(n-j)!} $$

$$ {}_{j+2}F_{j+1}\left(\frac{i\,a\,(m-2\,k)+b\,(u-2\,i)+2\,c\,(v-s)}{2\,c},\,\ldots,\,\frac{i\,a\,(m-2\,k)+b\,(u-2\,i)+2\,c\,(v-s)}{2\,c},\,v;\,\frac{i\,a\,(m-2\,k)+b\,(u-2\,i)+2\,c\,(v-s)}{2\,c}+1,\,\ldots,\,\frac{i\,a\,(m-2\,k)+b\,(u-2\,i)+2\,c\,(v-s)}{2\,c}+1;\,-e^{2\,c\,z}\right)\Bigg)+ $$

$$ (-1)^u\left(i^v\,e^{(-i\,a\,(m-2\,k)-b\,(u-2\,i)+c\,v)\,z}\binom{v}{\frac{v}{2}}(1-v\bmod 2)\sum_{j=0}^{n}\frac{(-1)^j\,(-i\,a\,(m-2\,k)-b\,(u-2\,i)+c\,v)^{-j-1}\,z^{n-j}}{(n-j)!}\right. $$

$$ {}_{j+2}F_{j+1}\left(\frac{-i\,a\,(m-2\,k)-b\,(u-2\,i)+c\,v}{2\,c},\,\ldots,\,\frac{-i\,a\,(m-2\,k)-b\,(u-2\,i)+c\,v}{2\,c},\,v;\right. $$

$$\frac{-i\,a\,(m-2\,k)-b\,(u-2\,i)+c\,v}{2\,c}+1,\,\ldots,\,\frac{-i\,a\,(m-2\,k)-b\,(u-2\,i)+c\,v}{2\,c}+1;\,-e^{2\,c\,z}\Bigg)+$$

$$\sum_{s=0}^{\left\lfloor\frac{v-1}{2}\right\rfloor}(-1)^s\binom{v}{s}\Bigg((-1)^v\,e^{(-i\,a\,(m-2\,k)+2\,c\,s-b\,(u-2\,i))\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(-i\,a\,(m-2\,k)+2\,c\,s-b\,(u-2\,i))^{-j-1}\,z^{n-j}}{(n-j)!}$$

$$_{j+2}F_{j+1}\Bigg(\frac{-i\,a\,(m-2\,k)+2\,c\,s-b\,(u-2\,i)}{2\,c},\,\ldots,$$

$$\frac{-i\,a\,(m-2\,k)+2\,c\,s-b\,(u-2\,i)}{2\,c},\,v;\,\frac{-i\,a\,(m-2\,k)+2\,c\,s-b\,(u-2\,i)}{2\,c}+1,$$

$$\ldots,\,\frac{-i\,a\,(m-2\,k)+2\,c\,s-b\,(u-2\,i)}{2\,c}+1;\,-e^{2\,c\,z}\Bigg)+$$

$$e^{(-i\,a\,(m-2\,k)-b\,(u-2\,i)+2\,c\,(v-s))\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(-i\,a\,(m-2\,k)-b\,(u-2\,i)+2\,c\,(v-s))^{-j-1}\,z^{n-j}}{(n-j)!}$$

$$_{j+2}F_{j+1}\Bigg(\frac{-i\,a\,(m-2\,k)-b\,(u-2\,i)+2\,c\,(v-s)}{2\,c},\,\ldots,$$

$$\frac{-i\,a\,(m-2\,k)-b\,(u-2\,i)+2\,c\,(v-s)}{2\,c},\,v;\,\frac{-i\,a\,(m-2\,k)-b\,(u-2\,i)+2\,c\,(v-s)}{2\,c}+1,$$

$$\ldots,\,\frac{-i\,a\,(m-2\,k)-b\,(u-2\,i)+2\,c\,(v-s)}{2\,c}+1;\,-e^{2\,c\,z}\Bigg)\Bigg)+$$

$$i^v\,e^{(a\,i\,(m-2\,k)-b\,(u-2\,i)+c\,v)\,z}\binom{v}{\frac{v}{2}}(1-v\bmod 2)\sum_{j=0}^{n}\frac{(-1)^j\,(a\,i\,(m-2\,k)-b\,(u-2\,i)+c\,v)^{-j-1}\,z^{n-j}}{(n-j)!}$$

$$_{j+2}F_{j+1}\Bigg(\frac{i\,a\,(m-2\,k)-b\,(u-2\,i)+c\,v}{2\,c},\,\ldots,\,\frac{i\,a\,(m-2\,k)-b\,(u-2\,i)+c\,v}{2\,c},\,v;$$

$$\frac{i\,a\,(m-2\,k)-b\,(u-2\,i)+c\,v}{2\,c}+1,\,\ldots,\,\frac{i\,a\,(m-2\,k)-b\,(u-2\,i)+c\,v}{2\,c}+1;\,-e^{2\,c\,z}\Bigg)+$$

$$\sum_{s=0}^{\left\lfloor\frac{v-1}{2}\right\rfloor}(-1)^s\binom{v}{s}\Bigg((-1)^v\,e^{(a\,i\,(m-2\,k)+2\,c\,s-b\,(u-2\,i))\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(a\,i\,(m-2\,k)+2\,c\,s-b\,(u-2\,i))^{-j-1}\,z^{n-j}}{(n-j)!}$$

$$_{j+2}F_{j+1}\Bigg(\frac{i\,a\,(m-2\,k)+2\,c\,s-b\,(u-2\,i)}{2\,c},\,\ldots,\,\frac{i\,a\,(m-2\,k)+2\,c\,s-b\,(u-2\,i)}{2\,c},$$

$$v;\,\frac{i\,a\,(m-2\,k)+2\,c\,s-b\,(u-2\,i)}{2\,c}+1,\,\ldots,$$

$$\frac{i\,a\,(m-2\,k)+2\,c\,s-b\,(u-2\,i)}{2\,c}+1;\,-e^{2\,c\,z}\Bigg)+e^{(a\,i\,(m-2\,k)-b\,(u-2\,i)+2\,c\,(v-s))\,z}$$

$$\sum_{j=0}^{n}\frac{(-1)^j\,(a\,i\,(m-2\,k)-b\,(u-2\,i)+2\,c\,(v-s))^{-j-1}\,z^{n-j}}{(n-j)!}\;{}_{j+2}F_{j+1}$$

$$\left(\frac{i\,a\,(m-2\,k)-b\,(u-2\,i)+2\,c\,(v-s)}{2\,c},\,\ldots,\,\frac{i\,a\,(m-2\,k)-b\,(u-2\,i)+2\,c\,(v-s)}{2\,c},\,v;\; \frac{i\,a\,(m-2\,k)-b\,(u-2\,i)+2\,c\,(v-s)}{2\,c}+1,\,\ldots,\,\frac{i\,a\,(m-2\,k)-b\,(u-2\,i)+2\,c\,(v-s)}{2\,c}+1;\,-e^{2\,c\,z}\right)\Bigg)\Bigg)\Bigg)\Bigg) /;\, n\in\mathbb{N}\land m\in\mathbb{N}^+\land u\in\mathbb{N}^+\land v\in\mathbb{N}^+$$

Involving sin, cosh and power

Involving $z^n \sin(a\,z)\cosh(b\,z)\tanh^v(c\,z)$

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$$\int z^n \sin(a\,z)\cosh(b\,z)\tanh^v(c\,z)\,dz = \frac{1}{4}\,i\,n!\,\Bigg(i^v\,e^{(b-i\,a+c\,v)\,z}\binom{v}{\frac{v}{2}}(1-v \bmod 2)\sum_{j=0}^{n}\frac{(-1)^j\,(b-i\,a+c\,v)^{-j-1}\,z^{n-j}}{(n-j)!}\; {}_{j+2}F_{j+1}\!\left(\frac{b-i\,a+c\,v}{2\,c},\,\ldots,\,\frac{b-i\,a+c\,v}{2\,c},\,v;\,\frac{b-i\,a+c\,v}{2\,c}+1,\,\ldots,\,\frac{b-i\,a+c\,v}{2\,c}+1;\,-e^{2\,c\,z}\right)+$$

$$\sum_{s=0}^{\left\lfloor\frac{v-1}{2}\right\rfloor}(-1)^s\binom{v}{s}\Bigg((-1)^v\,e^{(-i\,a+b+2\,c\,s)\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(b-i\,a+2\,c\,s)^{-j-1}\,z^{n-j}}{(n-j)!}\; {}_{j+2}F_{j+1}\!\left(\frac{b-i\,a+2\,c\,s}{2\,c},\,\ldots,\,\frac{b-i\,a+2\,c\,s}{2\,c},\,v;\,\frac{b-i\,a+2\,c\,s}{2\,c}+1,\,\ldots,\,\frac{b-i\,a+2\,c\,s}{2\,c}+1;\,-e^{2\,c\,z}\right)+$$

$$e^{(-i\,a+b+2\,c\,(-s+v))\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(b-i\,a+2\,c\,(v-s))^{-j-1}\,z^{n-j}}{(n-j)!}\; {}_{j+2}F_{j+1}\!\left(\frac{b-i\,a+2\,c\,(v-s)}{2\,c},\,\ldots,\,\frac{b-i\,a+2\,c\,(v-s)}{2\,c},\,v;\,\frac{b-i\,a+2\,c\,(v-s)}{2\,c}+1,\,\ldots,\,\frac{b-i\,a+2\,c\,(v-s)}{2\,c}+1;\,-e^{2\,c\,z}\right)\Bigg)-$$

$$i^v\,e^{(b+i\,a+c\,v)\,z}\binom{v}{\frac{v}{2}}(1-v \bmod 2)\sum_{j=0}^{n}\frac{(-1)^j\,(b+i\,a+c\,v)^{-j-1}\,z^{n-j}}{(n-j)!}\; {}_{j+2}F_{j+1}\!\left(\frac{b+i\,a+c\,v}{2\,c},\,\ldots,\,\frac{b+i\,a+c\,v}{2\,c},\,v;\,\frac{b+i\,a+c\,v}{2\,c}+1,\,\ldots,\,\frac{b+i\,a+c\,v}{2\,c}+1;\,-e^{2\,c\,z}\right)-$$

$$\sum_{s=0}^{\left\lfloor\frac{v-1}{2}\right\rfloor}(-1)^s\binom{v}{s}\Bigg(e^{(i\,a+b+2\,c\,(-s+v))\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(b+i\,a+2\,c\,(v-s))^{-j-1}\,z^{n-j}}{(n-j)!}\; {}_{j+2}F_{j+1}\!\left(\frac{b+i\,a+2\,c\,(v-s)}{2\,c},\,\ldots,\,\frac{b+i\,a+2\,c\,(v-s)}{2\,c},\,v;\,\frac{b+i\,a+2\,c\,(v-s)}{2\,c}+1,\,\ldots,\,\frac{b+i\,a+2\,c\,(v-s)}{2\,c}+1;\,-e^{2\,c\,z}\right)+$$

$$(-1)^v\,e^{(i\,a+b+2\,c\,s)\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(b+i\,a+2\,c\,s)^{-j-1}\,z^{n-j}}{(n-j)!}\; {}_{j+2}F_{j+1}\!\left(\frac{b+i\,a+2\,c\,s}{2\,c},\,\ldots,\,\frac{b+i\,a+2\,c\,s}{2\,c},\right.$$

$$v;\ \frac{b+i a+2 c s}{2 c}+1,\ \ldots,\ \frac{b+i a+2 c s}{2 c}+1;\ -e^{2 c z}\Bigg)\Bigg)+$$

$$i^v e^{(-b-i a+c v) z}\binom{v}{\frac{v}{2}}(1-v \bmod 2)\sum_{j=0}^n \frac{(-1)^j (-b-i a+c v)^{-j-1} z^{n-j}}{(n-j)!}\ {}_{j+2}F_{j+1}\Bigg(\frac{-b-i a+c v}{2 c},\ \ldots,$$

$$\frac{-b-i a+c v}{2 c},\ v;\ \frac{-b-i a+c v}{2 c}+1,\ \ldots,\ \frac{-b-i a+c v}{2 c}+1;\ -e^{2 c z}\Bigg)+$$

$$\sum_{s=0}^{\left\lfloor\frac{v-1}{2}\right\rfloor}(-1)^s\binom{v}{s}\Bigg((-1)^v e^{(-i a-b+2 c s) z}\sum_{j=0}^n \frac{(-1)^j (-b-i a+2 c s)^{-j-1} z^{n-j}}{(n-j)!}\ {}_{j+2}F_{j+1}\Bigg(\frac{-b-i a+2 c s}{2 c},\ \ldots,$$

$$\frac{-b-i a+2 c s}{2 c},\ v;\ \frac{-b-i a+2 c s}{2 c}+1,\ \ldots,\ \frac{-b-i a+2 c s}{2 c}+1;\ -e^{2 c z}\Bigg)+$$

$$e^{(-i a-b-2 c s+2 c v) z}\sum_{j=0}^n \frac{(-1)^j (-b-i a+2 c (v-s))^{-j-1} z^{n-j}}{(n-j)!}\ {}_{j+2}F_{j+1}\Bigg(\frac{-b-i a+2 c (v-s)}{2 c},\ \ldots,$$

$$\frac{-b-i a+2 c (v-s)}{2 c},\ v;\ \frac{-b-i a+2 c (v-s)}{2 c}+1,\ \ldots,\ \frac{-b-i a+2 c (v-s)}{2 c}+1;\ -e^{2 c z}\Bigg)\Bigg)-$$

$$i^v e^{(-b+i a+c v) z}\binom{v}{\frac{v}{2}}(1-v \bmod 2)\sum_{j=0}^n \frac{(-1)^j (-b+i a+c v)^{-j-1} z^{n-j}}{(n-j)!}\ {}_{j+2}F_{j+1}\Bigg(\frac{-b+i a+c v}{2 c},\ \ldots,$$

$$\frac{-b+i a+c v}{2 c},\ v;\ \frac{-b+i a+c v}{2 c}+1,\ \ldots,\ \frac{-b+i a+c v}{2 c}+1;\ -e^{2 c z}\Bigg)-$$

$$\sum_{s=0}^{\left\lfloor\frac{v-1}{2}\right\rfloor}(-1)^s\binom{v}{s}\Bigg((-1)^v e^{(i a-b+2 c s) z}\sum_{j=0}^n \frac{(-1)^j (-b+i a+2 c s)^{-j-1} z^{n-j}}{(n-j)!}\ {}_{j+2}F_{j+1}\Bigg(\frac{-b+i a+2 c s}{2 c},\ \ldots,$$

$$\frac{-b+i a+2 c s}{2 c},\ v;\ \frac{-b+i a+2 c s}{2 c}+1,\ \ldots,\ \frac{-b+i a+2 c s}{2 c}+1;\ -e^{2 c z}\Bigg)+e^{(i a-b-2 c s+2 c v) z}$$

$$\sum_{j=0}^n \frac{(-1)^j (-b+i a+2 c (v-s))^{-j-1} z^{n-j}}{(n-j)!}\ {}_{j+2}F_{j+1}\Bigg(\frac{-b+i a+2 c (v-s)}{2 c},\ \ldots,\ \frac{-b+i a+2 c (v-s)}{2 c},$$

$$v;\ \frac{-b+i a+2 c (v-s)}{2 c}+1,\ \ldots,\ \frac{-b+i a+2 c (v-s)}{2 c}+1;\ -e^{2 c z}\Bigg)\Bigg)\Bigg)\ /;\ n\in\mathbb{N}\bigwedge v\in\mathbb{N}^+$$

Involving powers of sin, powers of cosh and power

Involving $z^n \sin^m(a z) \cosh^u(c z) \tanh^v(c z)$

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$$\int z^n \sin^m(a z) \cosh^u(b z) \tanh^v(c z)\, dz = 2^{-m-u} \binom{m}{\frac{m}{2}} \binom{u}{\frac{u}{2}} (1 - m \bmod 2)\, (1 - u \bmod 2)$$

$$\Bigg(\frac{(-1)^v z^{n+1}}{n+1} - (-1)^v e^{2 c z}\, v\, n! \sum_{j=0}^{n} \frac{(-1)^j\, 2^{-j-1}\, c^{-j-1}\, z^{n-j}}{(n-j)!}\, {}_{j+3}F_{j+2}\big(1, \ldots, 1, v+1; 2, \ldots, 2; -e^{2 c z}\big) +$$

$$e^{c v z}\, i^v \binom{v}{\frac{v}{2}} n!\, (1 - v \bmod 2) \sum_{j=0}^{n} \frac{(-1)^j\, (c v)^{-j-1}\, z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}\Big(\frac{v}{2}, \ldots, \frac{v}{2}, v; \frac{v}{2}+1, \ldots, \frac{v}{2}+1; -e^{2 c z}\Big) +$$

$$(-1)^v\, n! \sum_{s=1}^{\left\lfloor \frac{v-1}{2} \right\rfloor} (-1)^s \binom{v}{s} e^{2 s c z} \sum_{j=0}^{n} \frac{(-1)^j\, (2 c s)^{-j-1}\, z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}\big(s, \ldots, s, v; s+1, \ldots, s+1; -e^{2 c z}\big) +$$

$$n! \sum_{s=0}^{\left\lfloor \frac{v-1}{2} \right\rfloor} (-1)^s \binom{v}{s} e^{2 c (v-s) z} \sum_{j=0}^{n} \frac{(-1)^j\, (2 c (v-s))^{-j-1}\, z^{n-j}}{(n-j)!}$$

$${}_{j+2}F_{j+1}\big(v-s, \ldots, v-s, v; -s+v+1, \ldots, -s+v+1; -e^{2 c z}\big)\Bigg) + 2^{-m-u} \binom{u}{\frac{u}{2}} n!\, (1 - u \bmod 2) \sum_{k=0}^{\left\lfloor \frac{m-1}{2} \right\rfloor} (-1)^k$$

$$\binom{m}{k} \Bigg(e^{\frac{i m \pi}{2}} \Bigg(i^v e^{(c v - i a (m-2k)) z} \binom{v}{\frac{v}{2}} (1 - v \bmod 2) \sum_{j=0}^{n} \frac{(-1)^j\, (c v - i a (m-2k))^{-j-1}\, z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}\Big(\frac{c v - i a (m-2k)}{2 c},$$

$$\ldots, \frac{c v - i a (m-2k)}{2 c}, v; \frac{c v - i a (m-2k)}{2 c} + 1, \ldots, \frac{c v - i a (m-2k)}{2 c} + 1; -e^{2 c z}\Big) +$$

$$\sum_{s=0}^{\left\lfloor \frac{v-1}{2} \right\rfloor} (-1)^s \binom{v}{s} \Bigg((-1)^v e^{(2 c s - i a (m-2k)) z} \sum_{j=0}^{n} \frac{(-1)^j\, (2 c s - i a (m-2k))^{-j-1}\, z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}\Big(\frac{2 c s - i a (m-2k)}{2 c},$$

$$\ldots, \frac{2 c s - i a (m-2k)}{2 c}, v; \frac{2 c s - i a (m-2k)}{2 c} + 1, \ldots, \frac{2 c s - i a (m-2k)}{2 c} + 1; -e^{2 c z}\Big) +$$

$$e^{(2 c (v-s) - i a (m-2k)) z} \sum_{j=0}^{n} \frac{(-1)^j\, (2 c (v-s) - i a (m-2k))^{-j-1}\, z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}$$

$$\Big(\frac{2 c (v-s) - i a (m-2k)}{2 c}, \ldots, \frac{2 c (v-s) - i a (m-2k)}{2 c}, v;$$

$$\frac{2 c (v-s) - i a (m-2k)}{2 c} + 1, \ldots, \frac{2 c (v-s) - i a (m-2k)}{2 c} + 1; -e^{2 c z}\Big)\Bigg)\Bigg) +$$

$$e^{-\frac{1}{2} i m \pi} \Bigg(i^v e^{(a i (m-2k) + c v) z} \binom{v}{\frac{v}{2}} (1 - v \bmod 2) \sum_{j=0}^{n} \frac{(-1)^j\, (a i (m-2k) + c v)^{-j-1}\, z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}\Big(\frac{i a (m-2k) + c v}{2 c},$$

$$\ldots, \frac{i a (m-2k) + c v}{2 c}, v; \frac{i a (m-2k) + c v}{2 c} + 1, \ldots, \frac{i a (m-2k) + c v}{2 c} + 1; -e^{2 c z}\Big) +$$

$$\sum_{s=0}^{\left\lfloor\frac{v-1}{2}\right\rfloor}(-1)^s\binom{v}{s}\Bigg((-1)^v e^{(a\,i\,(m-2k)+2cs)z}\sum_{j=0}^{n}\frac{(-1)^j\,(a\,i\,(m-2k)+2cs)^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\Bigg(\frac{i\,a\,(m-2k)+2cs}{2c},$$

$$\ldots,\frac{i\,a\,(m-2k)+2cs}{2c},v;\frac{i\,a\,(m-2k)+2cs}{2c}+1,\ldots,\frac{i\,a\,(m-2k)+2cs}{2c}+1;-e^{2cz}\Bigg)+$$

$$e^{(a\,i\,(m-2k)+2c(v-s))z}\sum_{j=0}^{n}\frac{(-1)^j\,(a\,i\,(m-2k)+2c(v-s))^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}$$

$$\Bigg(\frac{i\,a\,(m-2k)+2c(v-s)}{2c},\ldots,\frac{i\,a\,(m-2k)+2c(v-s)}{2c},v;$$

$$\frac{i\,a\,(m-2k)+2c(v-s)}{2c}+1,\ldots,\frac{i\,a\,(m-2k)+2c(v-s)}{2c}+1;-e^{2cz}\Bigg)\Bigg)\Bigg)\Bigg)+$$

$$2^{-m-u}\binom{m}{\frac{m}{2}}n!\,(1-m\bmod 2)\sum_{k=0}^{\left\lfloor\frac{u-1}{2}\right\rfloor}\binom{u}{k}\Bigg(i^v e^{(b(u-2k)+cv)z}\binom{v}{\frac{v}{2}}(1-v\bmod 2)\sum_{j=0}^{n}\frac{(-1)^j\,(b(u-2k)+cv)^{-j-1}\,z^{n-j}}{(n-j)!}$$

$${}_{j+2}F_{j+1}\Bigg(\frac{b(u-2k)+cv}{2c},\ldots,\frac{b(u-2k)+cv}{2c},v;\frac{b(u-2k)+cv}{2c}+1,\ldots,\frac{b(u-2k)+cv}{2c}+1;-e^{2cz}\Bigg)+$$

$$i^v e^{(cv-b(u-2k))z}\binom{v}{\frac{v}{2}}(1-v\bmod 2)\sum_{j=0}^{n}\frac{(-1)^j\,(cv-b(u-2k))^{-j-1}\,z^{n-j}}{(n-j)!}$$

$${}_{j+2}F_{j+1}\Bigg(\frac{cv-b(u-2k)}{2c},\ldots,\frac{cv-b(u-2k)}{2c},v;\frac{cv-b(u-2k)}{2c}+1,\ldots,\frac{cv-b(u-2k)}{2c}+1;-e^{2cz}\Bigg)+$$

$$\sum_{s=0}^{\left\lfloor\frac{v-1}{2}\right\rfloor}(-1)^s\binom{v}{s}\Bigg((-1)^v e^{(2cs+b(u-2k))z}\sum_{j=0}^{n}\frac{(-1)^j\,(2cs+b(u-2k))^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\Bigg(\frac{2cs+b(u-2k)}{2c},$$

$$\ldots,\frac{2cs+b(u-2k)}{2c},v;\frac{2cs+b(u-2k)}{2c}+1,\ldots,\frac{2cs+b(u-2k)}{2c}+1;-e^{2cz}\Bigg)+$$

$$e^{(b(u-2k)+2c(v-s))z}\sum_{j=0}^{n}\frac{(-1)^j\,(b(u-2k)+2c(v-s))^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}$$

$$\Bigg(\frac{b(u-2k)+2c(v-s)}{2c},\ldots,\frac{b(u-2k)+2c(v-s)}{2c},v;$$

$$\frac{b(u-2k)+2c(v-s)}{2c}+1,\ldots,\frac{b(u-2k)+2c(v-s)}{2c}+1;-e^{2cz}\Bigg)\Bigg)+$$

$$\sum_{s=0}^{\left\lfloor\frac{v-1}{2}\right\rfloor}(-1)^s\binom{v}{s}\Bigg((-1)^v e^{(2cs-b(u-2k))z}\sum_{j=0}^{n}\frac{(-1)^j\,(2cs-b(u-2k))^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\Bigg(\frac{2cs-b(u-2k)}{2c},$$

$$\ldots,\frac{2cs-b(u-2k)}{2c},v;\frac{2cs-b(u-2k)}{2c}+1,\ldots,\frac{2cs-b(u-2k)}{2c}+1;-e^{2cz}\Bigg)+$$

$$e^{(2c(v-s)-b(u-2k))z}\sum_{j=0}^{n}\frac{(-1)^j(2c(v-s)-b(u-2k))^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\Bigg(\frac{2c(v-s)-b(u-2k)}{2c},$$

$$\ldots,\frac{2c(v-s)-b(u-2k)}{2c},v;\frac{2c(v-s)-b(u-2k)}{2c}+1,\ldots,$$

$$\frac{2c(v-s)-b(u-2k)}{2c}+1;-e^{2cz}\Bigg)\Bigg)\Bigg)+2^{-m-u}n!\sum_{k=0}^{\left\lfloor\frac{m-1}{2}\right\rfloor}(-1)^k\binom{m}{k}$$

$$\sum_{i=0}^{\left\lfloor\frac{u-1}{2}\right\rfloor}\binom{u}{i}\Bigg(e^{\frac{im\pi}{2}}\Bigg(i^v e^{(-ia(m-2k)-b(u-2i)+cv)z}\binom{v}{\frac{v}{2}}(1-v\bmod 2)\sum_{j=0}^{n}\frac{(-1)^j(-ia(m-2k)-b(u-2i)+cv)^{-j-1}z^{n-j}}{(n-j)!}$$

$${}_{j+2}F_{j+1}\Bigg(\frac{-ia(m-2k)-b(u-2i)+cv}{2c},\ldots,\frac{-ia(m-2k)-b(u-2i)+cv}{2c},v;$$

$$\frac{-ia(m-2k)-b(u-2i)+cv}{2c}+1,\ldots,\frac{-ia(m-2k)-b(u-2i)+cv}{2c}+1;-e^{2cz}\Bigg)+$$

$$\sum_{s=0}^{\left\lfloor\frac{v-1}{2}\right\rfloor}(-1)^s\binom{v}{s}\Bigg((-1)^v e^{(-ia(m-2k)+2cs-b(u-2i))z}\sum_{j=0}^{n}\frac{(-1)^j(-ia(m-2k)+2cs-b(u-2i))^{-j-1}z^{n-j}}{(n-j)!}$$

$${}_{j+2}F_{j+1}\Bigg(\frac{-ia(m-2k)+2cs-b(u-2i)}{2c},\ldots,$$

$$\frac{-ia(m-2k)+2cs-b(u-2i)}{2c},v;\frac{-ia(m-2k)+2cs-b(u-2i)}{2c}+1,$$

$$\ldots,\frac{-ia(m-2k)+2cs-b(u-2i)}{2c}+1;-e^{2cz}\Bigg)+$$

$$e^{(-ia(m-2k)-b(u-2i)+2c(v-s))z}\sum_{j=0}^{n}\frac{(-1)^j(-ia(m-2k)-b(u-2i)+2c(v-s))^{-j-1}z^{n-j}}{(n-j)!}$$

$${}_{j+2}F_{j+1}\Bigg(\frac{-ia(m-2k)-b(u-2i)+2c(v-s)}{2c},\ldots,$$

$$\frac{-ia(m-2k)-b(u-2i)+2c(v-s)}{2c},v;\frac{-ia(m-2k)-b(u-2i)+2c(v-s)}{2c}+1,$$

$$\ldots,\frac{-ia(m-2k)-b(u-2i)+2c(v-s)}{2c}+1;-e^{2cz}\Bigg)\Bigg)\Bigg)+$$

$$e^{-\frac{1}{2}im\pi}\Bigg(i^v e^{(ai(m-2k)-b(u-2i)+cv)z}\binom{v}{\frac{v}{2}}(1-v\bmod 2)\sum_{j=0}^{n}\frac{(-1)^j(ai(m-2k)-b(u-2i)+cv)^{-j-1}z^{n-j}}{(n-j)!}$$

$${}_{j+2}F_{j+1}\Bigg(\frac{ia(m-2k)-b(u-2i)+cv}{2c},\ldots,\frac{ia(m-2k)-b(u-2i)+cv}{2c},v;$$

$$\frac{ia(m-2k)-b(u-2i)+cv}{2c}+1,\ldots,\frac{ia(m-2k)-b(u-2i)+cv}{2c}+1;-e^{2cz}\Bigg)+$$

$$\sum_{s=0}^{\left\lfloor \frac{v-1}{2} \right\rfloor} (-1)^s \binom{v}{s} \Bigg((-1)^v e^{(a i (m-2k)+2cs-b(u-2i))z} \sum_{j=0}^{n} \frac{(-1)^j (a i (m-2k)+2cs-b(u-2i))^{-j-1} z^{n-j}}{(n-j)!}$$

$$_{j+2}F_{j+1}\Bigg(\frac{i a (m-2k)+2cs-b(u-2i)}{2c}, \ldots, \frac{i a (m-2k)+2cs-b(u-2i)}{2c}, v; \frac{i a (m-2k)+2cs-b(u-2i)}{2c}+1, \ldots, \frac{i a (m-2k)+2cs-b(u-2i)}{2c}+1; -e^{2cz} \Bigg) +$$

$$e^{(a i (m-2k)-b(u-2i)+2c(v-s))z} \sum_{j=0}^{n} \frac{(-1)^j (a i (m-2k)-b(u-2i)+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!}$$

$$_{j+2}F_{j+1}\Bigg(\frac{i a (m-2k)-b(u-2i)+2c(v-s)}{2c}, \ldots, \frac{i a (m-2k)-b(u-2i)+2c(v-s)}{2c}, v; \frac{i a (m-2k)-b(u-2i)+2c(v-s)}{2c}+1, \ldots, \frac{i a (m-2k)-b(u-2i)+2c(v-s)}{2c}+1; -e^{2cz} \Bigg)\Bigg)\Bigg) +$$

$$e^{\frac{i m \pi}{2}} \Bigg(i^v e^{(-i a (m-2k)+b(u-2i)+cv)z} \binom{v}{\frac{v}{2}} (1 - v \bmod 2) \sum_{j=0}^{n} \frac{(-1)^j (-i a (m-2k)+b(u-2i)+cv)^{-j-1} z^{n-j}}{(n-j)!}$$

$$_{j+2}F_{j+1}\Bigg(\frac{-i a (m-2k)+b(u-2i)+cv}{2c}, \ldots, \frac{-i a (m-2k)+b(u-2i)+cv}{2c}, v; \frac{-i a (m-2k)+b(u-2i)+cv}{2c}+1, \ldots, \frac{-i a (m-2k)+b(u-2i)+cv}{2c}+1; -e^{2cz} \Bigg) +$$

$$\sum_{s=0}^{\left\lfloor \frac{v-1}{2} \right\rfloor} (-1)^s \binom{v}{s} \Bigg((-1)^v e^{(-i a (m-2k)+2cs+b(u-2i))z} \sum_{j=0}^{n} \frac{(-1)^j (-i a (m-2k)+2cs+b(u-2i))^{-j-1} z^{n-j}}{(n-j)!}$$

$$_{j+2}F_{j+1}\Bigg(\frac{-i a (m-2k)+2cs+b(u-2i)}{2c}, \ldots, \frac{-i a (m-2k)+2cs+b(u-2i)}{2c}, v; \frac{-i a (m-2k)+2cs+b(u-2i)}{2c}+1, \ldots, \frac{-i a (m-2k)+2cs+b(u-2i)}{2c}+1; -e^{2cz} \Bigg) +$$

$$e^{(-i a (m-2k)+b(u-2i)+2c(v-s))z} \sum_{j=0}^{n} \frac{(-1)^j (-i a (m-2k)+b(u-2i)+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!}$$

$$_{j+2}F_{j+1}\Bigg(\frac{-i a (m-2k)+b(u-2i)+2c(v-s)}{2c}, \ldots, \frac{-i a (m-2k)+b(u-2i)+2c(v-s)}{2c}, v; \frac{-i a (m-2k)+b(u-2i)+2c(v-s)}{2c}+1,$$

$$\ldots, \frac{-i\, a\,(m-2k)+b\,(u-2i)+2\,c\,(v-s)}{2\,c}+1; -e^{2cz}\Bigg)\Bigg)\Bigg) +$$

$$e^{-\frac{1}{2} i m \pi}\Bigg(i^v\, e^{(a\,i\,(m-2k)+b\,(u-2i)+c\,v)\,z}\binom{v}{\frac{v}{2}}(1-v \bmod 2)\sum_{j=0}^{n}\frac{(-1)^j\,(a\,i\,(m-2k)+b\,(u-2i)+c\,v)^{-j-1}\,z^{n-j}}{(n-j)!}$$

$$_{j+2}F_{j+1}\Bigg(\frac{i\,a\,(m-2k)+b\,(u-2i)+c\,v}{2\,c}, \ldots, \frac{i\,a\,(m-2k)+b\,(u-2i)+c\,v}{2\,c}, v;$$

$$\frac{i\,a\,(m-2k)+b\,(u-2i)+c\,v}{2\,c}+1, \ldots, \frac{i\,a\,(m-2k)+b\,(u-2i)+c\,v}{2\,c}+1; -e^{2cz}\Bigg) +$$

$$\sum_{s=0}^{\left\lfloor\frac{v-1}{2}\right\rfloor}(-1)^s\binom{v}{s}\Bigg((-1)^v\, e^{(a\,i\,(m-2k)+2\,c\,s+b\,(u-2i))\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(a\,i\,(m-2k)+2\,c\,s+b\,(u-2i))^{-j-1}\,z^{n-j}}{(n-j)!}$$

$$_{j+2}F_{j+1}\Bigg(\frac{i\,a\,(m-2k)+2\,c\,s+b\,(u-2i)}{2\,c}, \ldots, \frac{i\,a\,(m-2k)+2\,c\,s+b\,(u-2i)}{2\,c},$$

$$v; \frac{i\,a\,(m-2k)+2\,c\,s+b\,(u-2i)}{2\,c}+1, \ldots,$$

$$\frac{i\,a\,(m-2k)+2\,c\,s+b\,(u-2i)}{2\,c}+1; -e^{2cz}\Bigg) + e^{(a\,i\,(m-2k)+b\,(u-2i)+2\,c\,(v-s))\,z}$$

$$\sum_{j=0}^{n}\frac{(-1)^j\,(a\,i\,(m-2k)+b\,(u-2i)+2\,c\,(v-s))^{-j-1}\,z^{n-j}}{(n-j)!}\; {}_{j+2}F_{j+1}$$

$$\Bigg(\frac{i\,a\,(m-2k)+b\,(u-2i)+2\,c\,(v-s)}{2\,c}, \ldots, \frac{i\,a\,(m-2k)+b\,(u-2i)+2\,c\,(v-s)}{2\,c}, v;$$

$$\frac{i\,a\,(m-2k)+b\,(u-2i)+2\,c\,(v-s)}{2\,c}+1, \ldots, \frac{i\,a\,(m-2k)+b\,(u-2i)+2\,c\,(v-s)}{2\,c}+$$

$$1; -e^{2cz}\Bigg)\Bigg)\Bigg)\Bigg) /; n\in\mathbb{N}\land m\in\mathbb{N}^+\land u\in\mathbb{N}^+\land v\in\mathbb{N}^+$$

Involving cos, cosh and power

Involving $z^n \cos(a\,z) \cosh(b\,z) \tanh^v(c\,z)$

01.21.21.0423.01

$$\int z^n \cos(a\,z)\cosh(b\,z)\tanh^v(c\,z)\,dz = \frac{1}{4}\,n!\Bigg(i^v\, e^{(b-i\,a+c\,v)\,z}\binom{v}{\frac{v}{2}}(1-v \bmod 2)\sum_{j=0}^{n}\frac{(-1)^j\,(b-i\,a+c\,v)^{-j-1}\,z^{n-j}}{(n-j)!}$$

$$_{j+2}F_{j+1}\Bigg(\frac{b-i\,a+c\,v}{2\,c}, \ldots, \frac{b-i\,a+c\,v}{2\,c}, v; \frac{b-i\,a+c\,v}{2\,c}+1, \ldots, \frac{b-i\,a+c\,v}{2\,c}+1; -e^{2cz}\Bigg) +$$

$$\sum_{s=0}^{\left\lfloor\frac{v-1}{2}\right\rfloor}(-1)^s\binom{v}{s}\Bigg((-1)^v e^{(-i a+b+2 c s) z}\sum_{j=0}^n\frac{(-1)^j (b-i a+2 c s)^{-j-1} z^{n-j}}{(n-j)!}$$

$$_{j+2}F_{j+1}\left(\frac{b-i a+2 c s}{2 c},\ldots,\frac{b-i a+2 c s}{2 c},v;\frac{b-i a+2 c s}{2 c}+1,\ldots,\frac{b-i a+2 c s}{2 c}+1;-e^{2 c z}\right)+$$

$$e^{(-i a+b+2 c(-s+v)) z}\sum_{j=0}^n\frac{(-1)^j (b-i a+2 c(v-s))^{-j-1} z^{n-j}}{(n-j)!}\,_{j+2}F_{j+1}\Bigg(\frac{b-i a+2 c(v-s)}{2 c},\ldots,$$

$$\frac{b-i a+2 c(v-s)}{2 c},v;\frac{b-i a+2 c(v-s)}{2 c}+1,\ldots,\frac{b-i a+2 c(v-s)}{2 c}+1;-e^{2 c z}\Bigg)\Bigg)+$$

$$i^v e^{(b+i a+c v) z}\binom{v}{\frac{v}{2}}(1-v \bmod 2)\sum_{j=0}^n\frac{(-1)^j (b+i a+c v)^{-j-1} z^{n-j}}{(n-j)!}\,_{j+2}F_{j+1}\Bigg(\frac{b+i a+c v}{2 c},\ldots,$$

$$\frac{b+i a+c v}{2 c},v;\frac{b+i a+c v}{2 c}+1,\ldots,\frac{b+i a+c v}{2 c}+1;-e^{2 c z}\Bigg)+$$

$$\sum_{s=0}^{\left\lfloor\frac{v-1}{2}\right\rfloor}(-1)^s\binom{v}{s}\Bigg(e^{(i a+b+2 c(-s+v)) z}\sum_{j=0}^n\frac{(-1)^j (b+i a+2 c(v-s))^{-j-1} z^{n-j}}{(n-j)!}\,_{j+2}F_{j+1}\Bigg(\frac{b+i a+2 c(v-s)}{2 c},$$

$$\ldots,\frac{b+i a+2 c(v-s)}{2 c},v;\frac{b+i a+2 c(v-s)}{2 c}+1,\ldots,\frac{b+i a+2 c(v-s)}{2 c}+1;-e^{2 c z}\Bigg)+$$

$$(-1)^v e^{(i a+b+2 c s) z}\sum_{j=0}^n\frac{(-1)^j (b+i a+2 c s)^{-j-1} z^{n-j}}{(n-j)!}\,_{j+2}F_{j+1}\Bigg(\frac{b+i a+2 c s}{2 c},\ldots,\frac{b+i a+2 c s}{2 c},$$

$$v;\frac{b+i a+2 c s}{2 c}+1,\ldots,\frac{b+i a+2 c s}{2 c}+1;-e^{2 c z}\Bigg)\Bigg)+$$

$$i^v e^{(-b-i a+c v) z}\binom{v}{\frac{v}{2}}(1-v \bmod 2)\sum_{j=0}^n\frac{(-1)^j (-b-i a+c v)^{-j-1} z^{n-j}}{(n-j)!}\,_{j+2}F_{j+1}\Bigg(\frac{-b-i a+c v}{2 c},\ldots,$$

$$\frac{-b-i a+c v}{2 c},v;\frac{-b-i a+c v}{2 c}+1,\ldots,\frac{-b-i a+c v}{2 c}+1;-e^{2 c z}\Bigg)+$$

$$\sum_{s=0}^{\left\lfloor\frac{v-1}{2}\right\rfloor}(-1)^s\binom{v}{s}\Bigg((-1)^v e^{(-i a-b+2 c s) z}\sum_{j=0}^n\frac{(-1)^j (-b-i a+2 c s)^{-j-1} z^{n-j}}{(n-j)!}\,_{j+2}F_{j+1}\Bigg(\frac{-b-i a+2 c s}{2 c},\ldots,$$

$$\frac{-b-i a+2 c s}{2 c},v;\frac{-b-i a+2 c s}{2 c}+1,\ldots,\frac{-b-i a+2 c s}{2 c}+1;-e^{2 c z}\Bigg)+$$

$$e^{(-i a-b-2 c s+2 c v) z}\sum_{j=0}^n\frac{(-1)^j (-b-i a+2 c(v-s))^{-j-1} z^{n-j}}{(n-j)!}\,_{j+2}F_{j+1}\Bigg(\frac{-b-i a+2 c(v-s)}{2 c},\ldots,$$

$$\frac{-b-i a+2 c(v-s)}{2 c},v;\frac{-b-i a+2 c(v-s)}{2 c}+1,\ldots,\frac{-b-i a+2 c(v-s)}{2 c}+1;-e^{2 c z}\Bigg)\Bigg)+$$

$$i^{v} e^{(-b+i a+c v) z}\binom{v}{\frac{v}{2}}(1-v \bmod 2) \sum_{j=0}^{n} \frac{(-1)^{j}(-b+i a+c v)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{-b+i a+c v}{2 c}, \ldots, \frac{-b+i a+c v}{2 c}, v ; \frac{-b+i a+c v}{2 c}+1, \ldots, \frac{-b+i a+c v}{2 c}+1 ;-e^{2 c z}\right)+$$

$$\sum_{s=0}^{\left\lfloor\frac{v-1}{2}\right\rfloor}(-1)^{s}\binom{v}{s}\left((-1)^{v} e^{(i a-b+2 c s) z} \sum_{j=0}^{n} \frac{(-1)^{j}(-b+i a+2 c s)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{-b+i a+2 c s}{2 c}, \ldots, \frac{-b+i a+2 c s}{2 c}, v ; \frac{-b+i a+2 c s}{2 c}+1, \ldots, \frac{-b+i a+2 c s}{2 c}+1 ;-e^{2 c z}\right)+e^{(i a-b-2 c s+2 c v) z} \sum_{j=0}^{n} \frac{(-1)^{j}(-b+i a+2 c(v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{-b+i a+2 c(v-s)}{2 c}, \ldots, \frac{-b+i a+2 c(v-s)}{2 c}, v ; \frac{-b+i a+2 c(v-s)}{2 c}+1, \ldots, \frac{-b+i a+2 c(v-s)}{2 c}+1 ;-e^{2 c z}\right)\right)\Bigg) /; n \in \mathbb{N} \wedge v \in \mathbb{N}^{+}$$

Involving powers of cos, powers of cosh and power

Involving $z^n \cos^m(a z) \cosh^u(c z) \tanh^v(c z)$

01.21.21.0424.01

$$\int z^{n} \cos^{m}(a z) \cosh^{u}(b z) \tanh^{v}(c z)\, dz = 2^{-m-u}\binom{m}{\frac{m}{2}}\binom{u}{\frac{u}{2}}(1-m \bmod 2)(1-u \bmod 2)$$

$$\left(\frac{(-1)^{v} z^{n+1}}{n+1}-(-1)^{v} e^{2 c z} v\, n! \sum_{j=0}^{n} \frac{(-1)^{j} 2^{-j-1} c^{-j-1} z^{n-j}}{(n-j)!} {}_{j+3}F_{j+2}\left(1, \ldots, 1, v+1 ; 2, \ldots, 2 ;-e^{2 c z}\right)+\right.$$

$$e^{c v z} i^{v}\binom{v}{\frac{v}{2}} n!(1-v \bmod 2) \sum_{j=0}^{n} \frac{(-1)^{j}(c v)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{v}{2}, \ldots, \frac{v}{2}, v ; \frac{v}{2}+1, \ldots, \frac{v}{2}+1 ;-e^{2 c z}\right)+$$

$$(-1)^{v} n! \sum_{s=1}^{\left\lfloor\frac{v-1}{2}\right\rfloor}(-1)^{s}\binom{v}{s} e^{2 s c z} \sum_{j=0}^{n} \frac{(-1)^{j}(2 c s)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(s, \ldots, s, v ; s+1, \ldots, s+1 ;-e^{2 c z}\right)+n! \sum_{s=0}^{\left\lfloor\frac{v-1}{2}\right\rfloor}(-1)^{s}$$

$$\left.\binom{v}{s} e^{2 c(v-s) z} \sum_{j=0}^{n} \frac{(-1)^{j}(2 c(v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(v-s, \ldots, v-s, v ;-s+v+1, \ldots,-s+v+1 ;-e^{2 c z}\right)\right)+$$

$$2^{-m-u}\binom{u}{\frac{u}{2}} n!(1-u \bmod 2) \sum_{k=0}^{\left\lfloor\frac{m-1}{2}\right\rfloor}\binom{m}{k}\left(i^{v} e^{(c v-i a(m-2 k)) z}\binom{v}{\frac{v}{2}}(1-v \bmod 2) \sum_{j=0}^{n} \frac{(-1)^{j}(c v-i a(m-2 k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\right.$$

$$\left(\frac{c v-i a(m-2 k)}{2 c}, \ldots, \frac{c v-i a(m-2 k)}{2 c}, v ; \frac{c v-i a(m-2 k)}{2 c}+1, \ldots, \frac{c v-i a(m-2 k)}{2 c}+1 ;-e^{2 c z}\right)+$$

$$\sum_{s=0}^{\left\lfloor\frac{v-1}{2}\right\rfloor}(-1)^s\binom{v}{s}\Bigg((-1)^v e^{(2cs-ia(m-2k))z}\sum_{j=0}^{n}\frac{(-1)^j(2cs-ia(m-2k))^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\Bigg(\frac{2cs-ia(m-2k)}{2c},$$

$$\ldots,\frac{2cs-ia(m-2k)}{2c},v;\frac{2cs-ia(m-2k)}{2c}+1,\ldots,\frac{2cs-ia(m-2k)}{2c}+1;-e^{2cz}\Bigg)+$$

$$e^{(2c(v-s)-ia(m-2k))z}\sum_{j=0}^{n}\frac{(-1)^j(2c(v-s)-ia(m-2k))^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}$$

$$\Bigg(\frac{2c(v-s)-ia(m-2k)}{2c},\ldots,\frac{2c(v-s)-ia(m-2k)}{2c},v;$$

$$\frac{2c(v-s)-ia(m-2k)}{2c}+1,\ldots,\frac{2c(v-s)-ia(m-2k)}{2c}+1;-e^{2cz}\Bigg)\Bigg)+$$

$$i^v e^{(ai(m-2k)+cv)z}\binom{v}{\frac{v}{2}}(1-v\bmod 2)\sum_{j=0}^{n}\frac{(-1)^j(ai(m-2k)+cv)^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}$$

$$\left(\frac{ia(m-2k)+cv}{2c},\ldots,\frac{ia(m-2k)+cv}{2c},v;\frac{ia(m-2k)+cv}{2c}+1,\ldots,\frac{ia(m-2k)+cv}{2c}+1;-e^{2cz}\right)+$$

$$\sum_{s=0}^{\left\lfloor\frac{v-1}{2}\right\rfloor}(-1)^s\binom{v}{s}\Bigg((-1)^v e^{(ai(m-2k)+2cs)z}\sum_{j=0}^{n}\frac{(-1)^j(ai(m-2k)+2cs)^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\Bigg(\frac{ia(m-2k)+2cs}{2c},$$

$$\ldots,\frac{ia(m-2k)+2cs}{2c},v;\frac{ia(m-2k)+2cs}{2c}+1,\ldots,\frac{ia(m-2k)+2cs}{2c}+1;-e^{2cz}\Bigg)+$$

$$e^{(ai(m-2k)+2c(v-s))z}\sum_{j=0}^{n}\frac{(-1)^j(ai(m-2k)+2c(v-s))^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}$$

$$\Bigg(\frac{ia(m-2k)+2c(v-s)}{2c},\ldots,\frac{ia(m-2k)+2c(v-s)}{2c},v;$$

$$\frac{ia(m-2k)+2c(v-s)}{2c}+1,\ldots,\frac{ia(m-2k)+2c(v-s)}{2c}+1;-e^{2cz}\Bigg)\Bigg)\Bigg)+$$

$$2^{-m-u}\binom{m}{\frac{m}{2}}n!\,(1-m\bmod 2)\sum_{k=0}^{\left\lfloor\frac{u-1}{2}\right\rfloor}\binom{u}{k}\Bigg(i^v e^{(b(u-2k)+cv)z}\binom{v}{\frac{v}{2}}(1-v\bmod 2)\sum_{j=0}^{n}\frac{(-1)^j(b(u-2k)+cv)^{-j-1}z^{n-j}}{(n-j)!}$$

$${}_{j+2}F_{j+1}\left(\frac{b(u-2k)+cv}{2c},\ldots,\frac{b(u-2k)+cv}{2c},v;\frac{b(u-2k)+cv}{2c}+1,\ldots,\frac{b(u-2k)+cv}{2c}+1;-e^{2cz}\right)+$$

$$i^v e^{(cv-b(u-2k))z}\binom{v}{\frac{v}{2}}(1-v\bmod 2)\sum_{j=0}^{n}\frac{(-1)^j(cv-b(u-2k))^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\Bigg(\frac{cv-b(u-2k)}{2c},$$

$$\ldots,\frac{cv-b(u-2k)}{2c},v;\frac{cv-b(u-2k)}{2c}+1,\ldots,\frac{cv-b(u-2k)}{2c}+1;-e^{2cz}\Bigg)+\sum_{s=0}^{\left\lfloor\frac{v-1}{2}\right\rfloor}(-1)^s\binom{v}{s}$$

$$\left((-1)^v e^{(2cs+b(u-2k))z} \sum_{j=0}^{n} \frac{(-1)^j (2cs+b(u-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{2cs+b(u-2k)}{2c}, \ldots, \frac{2cs+b(u-2k)}{2c},\right.\right.$$

$$\left. v; \frac{2cs+b(u-2k)}{2c}+1, \ldots, \frac{2cs+b(u-2k)}{2c}+1; -e^{2cz}\right) + e^{(b(u-2k)+2c(v-s))z}$$

$$\sum_{j=0}^{n} \frac{(-1)^j (b(u-2k)+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{b(u-2k)+2c(v-s)}{2c}, \ldots, \frac{b(u-2k)+2c(v-s)}{2c},\right.$$

$$\left.\left. v; \frac{b(u-2k)+2c(v-s)}{2c}+1, \ldots, \frac{b(u-2k)+2c(v-s)}{2c}+1; -e^{2cz}\right)\right) + \sum_{s=0}^{\left\lfloor\frac{v-1}{2}\right\rfloor} (-1)^s \binom{v}{s}$$

$$\left((-1)^v e^{(2cs-b(u-2k))z} \sum_{j=0}^{n} \frac{(-1)^j (2cs-b(u-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{2cs-b(u-2k)}{2c}, \ldots, \frac{2cs-b(u-2k)}{2c},\right.\right.$$

$$\left. v; \frac{2cs-b(u-2k)}{2c}+1, \ldots, \frac{2cs-b(u-2k)}{2c}+1; -e^{2cz}\right) + e^{(2c(v-s)-b(u-2k))z}$$

$$\sum_{j=0}^{n} \frac{(-1)^j (2c(v-s)-b(u-2k))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{2c(v-s)-b(u-2k)}{2c}, \ldots, \frac{2c(v-s)-b(u-2k)}{2c},\right.$$

$$\left.\left.\left. v; \frac{2c(v-s)-b(u-2k)}{2c}+1, \ldots, \frac{2c(v-s)-b(u-2k)}{2c}+1; -e^{2cz}\right)\right)\right) + 2^{-m-u} n!$$

$$\sum_{k=0}^{\left\lfloor\frac{m-1}{2}\right\rfloor} \binom{m}{k} \sum_{i=0}^{\left\lfloor\frac{u-1}{2}\right\rfloor} \binom{u}{i} \left(i^v e^{(-i a(m-2k)+b(u-2i)+cv)z} \binom{v}{\frac{v}{2}} (1 - v \bmod 2) \sum_{j=0}^{n} \frac{(-1)^j (-i a(m-2k)+b(u-2i)+cv)^{-j-1} z^{n-j}}{(n-j)!}\right.$$

$${}_{j+2}F_{j+1}\left(\frac{-i a(m-2k)+b(u-2i)+cv}{2c}, \ldots, \frac{-i a(m-2k)+b(u-2i)+cv}{2c}, v;\right.$$

$$\left.\frac{-i a(m-2k)+b(u-2i)+cv}{2c}+1, \ldots, \frac{-i a(m-2k)+b(u-2i)+cv}{2c}+1; -e^{2cz}\right) +$$

$$\sum_{s=0}^{\left\lfloor\frac{v-1}{2}\right\rfloor} (-1)^s \binom{v}{s} \left((-1)^v e^{(-i a(m-2k)+2cs+b(u-2i))z} \sum_{j=0}^{n} \frac{(-1)^j (-i a(m-2k)+2cs+b(u-2i))^{-j-1} z^{n-j}}{(n-j)!}\right.$$

$${}_{j+2}F_{j+1}\left(\frac{-i a(m-2k)+2cs+b(u-2i)}{2c}, \ldots, \frac{-i a(m-2k)+2cs+b(u-2i)}{2c}, v;\right.$$

$$\left.\frac{-i a(m-2k)+2cs+b(u-2i)}{2c}+1, \ldots, \frac{-i a(m-2k)+2cs+b(u-2i)}{2c}+1; -e^{2cz}\right) +$$

$$e^{(-i a(m-2k)+b(u-2i)+2c(v-s))z} \sum_{j=0}^{n} \frac{(-1)^j (-i a(m-2k)+b(u-2i)+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!}$$

$${}_{j+2}F_{j+1}\left(\frac{-i a(m-2k)+b(u-2i)+2c(v-s)}{2c}, \ldots,\right.$$

$$\frac{-i a(m-2k)+b(u-2i)+2c(v-s)}{2c}, v; \frac{-i a(m-2k)+b(u-2i)+2c(v-s)}{2c}+1,$$

$$\ldots,\ \frac{-i\,a\,(m-2\,k)+b\,(u-2\,i)+2\,c\,(v-s)}{2\,c}+1;\ -e^{2\,c\,z}\Bigg)\Bigg)+$$

$$i^{v}\,e^{(a\,i\,(m-2\,k)+b\,(u-2\,i)+c\,v)\,z}\binom{v}{\frac{v}{2}}(1-v\bmod 2)\sum_{j=0}^{n}\frac{(-1)^{j}\,(a\,i\,(m-2\,k)+b\,(u-2\,i)+c\,v)^{-j-1}\,z^{n-j}}{(n-j)!}$$

$$_{j+2}F_{j+1}\Bigg(\frac{i\,a\,(m-2\,k)+b\,(u-2\,i)+c\,v}{2\,c},\ \ldots,\ \frac{i\,a\,(m-2\,k)+b\,(u-2\,i)+c\,v}{2\,c},\ v;$$

$$\frac{i\,a\,(m-2\,k)+b\,(u-2\,i)+c\,v}{2\,c}+1,\ \ldots,\ \frac{i\,a\,(m-2\,k)+b\,(u-2\,i)+c\,v}{2\,c}+1;\ -e^{2\,c\,z}\Bigg)+$$

$$\sum_{s=0}^{\left\lfloor\frac{v-1}{2}\right\rfloor}(-1)^{s}\binom{v}{s}\Bigg((-1)^{v}\,e^{(a\,i\,(m-2\,k)+2\,c\,s+b\,(u-2\,i))\,z}\sum_{j=0}^{n}\frac{(-1)^{j}\,(a\,i\,(m-2\,k)+2\,c\,s+b\,(u-2\,i))^{-j-1}\,z^{n-j}}{(n-j)!}$$

$$_{j+2}F_{j+1}\Bigg(\frac{i\,a\,(m-2\,k)+2\,c\,s+b\,(u-2\,i)}{2\,c},\ \ldots,\ \frac{i\,a\,(m-2\,k)+2\,c\,s+b\,(u-2\,i)}{2\,c},\ v;$$

$$\frac{i\,a\,(m-2\,k)+2\,c\,s+b\,(u-2\,i)}{2\,c}+1,\ \ldots,\ \frac{i\,a\,(m-2\,k)+2\,c\,s+b\,(u-2\,i)}{2\,c}+1;\ -e^{2\,c\,z}\Bigg)+$$

$$e^{(a\,i\,(m-2\,k)+b\,(u-2\,i)+2\,c\,(v-s))\,z}\sum_{j=0}^{n}\frac{(-1)^{j}\,(a\,i\,(m-2\,k)+b\,(u-2\,i)+2\,c\,(v-s))^{-j-1}\,z^{n-j}}{(n-j)!}$$

$$_{j+2}F_{j+1}\Bigg(\frac{i\,a\,(m-2\,k)+b\,(u-2\,i)+2\,c\,(v-s)}{2\,c},\ \ldots,$$

$$\frac{i\,a\,(m-2\,k)+b\,(u-2\,i)+2\,c\,(v-s)}{2\,c},\ v;\ \frac{i\,a\,(m-2\,k)+b\,(u-2\,i)+2\,c\,(v-s)}{2\,c}+1,$$

$$\ldots,\ \frac{i\,a\,(m-2\,k)+b\,(u-2\,i)+2\,c\,(v-s)}{2\,c}+1;\ -e^{2\,c\,z}\Bigg)\Bigg)+$$

$$i^{v}\,e^{(-i\,a\,(m-2\,k)-b\,(u-2\,i)+c\,v)\,z}\binom{v}{\frac{v}{2}}(1-v\bmod 2)\sum_{j=0}^{n}\frac{(-1)^{j}\,(-i\,a\,(m-2\,k)-b\,(u-2\,i)+c\,v)^{-j-1}\,z^{n-j}}{(n-j)!}$$

$$_{j+2}F_{j+1}\Bigg(\frac{-i\,a\,(m-2\,k)-b\,(u-2\,i)+c\,v}{2\,c},\ \ldots,\ \frac{-i\,a\,(m-2\,k)-b\,(u-2\,i)+c\,v}{2\,c},\ v;$$

$$\frac{-i\,a\,(m-2\,k)-b\,(u-2\,i)+c\,v}{2\,c}+1,\ \ldots,\ \frac{-i\,a\,(m-2\,k)-b\,(u-2\,i)+c\,v}{2\,c}+1;\ -e^{2\,c\,z}\Bigg)+$$

$$\sum_{s=0}^{\left\lfloor\frac{v-1}{2}\right\rfloor}(-1)^{s}\binom{v}{s}\Bigg((-1)^{v}\,e^{(-i\,a\,(m-2\,k)+2\,c\,s-b\,(u-2\,i))\,z}\sum_{j=0}^{n}\frac{(-1)^{j}\,(-i\,a\,(m-2\,k)+2\,c\,s-b\,(u-2\,i))^{-j-1}\,z^{n-j}}{(n-j)!}$$

$$_{j+2}F_{j+1}\Bigg(\frac{-i\,a\,(m-2\,k)+2\,c\,s-b\,(u-2\,i)}{2\,c},\ \ldots,\ \frac{-i\,a\,(m-2\,k)+2\,c\,s-b\,(u-2\,i)}{2\,c},\ v;$$

$$\frac{-i\,a\,(m-2\,k)+2\,c\,s-b\,(u-2\,i)}{2\,c}+1,\ \ldots,\ \frac{-i\,a\,(m-2\,k)+2\,c\,s-b\,(u-2\,i)}{2\,c}+1;\ -e^{2\,c\,z}\Bigg)+$$

$$e^{(-i\,a\,(m-2\,k)-b\,(u-2\,i)+2\,c\,(v-s))\,z}\sum_{j=0}^{n}\frac{(-1)^{j}\,(-i\,a\,(m-2\,k)-b\,(u-2\,i)+2\,c\,(v-s))^{-j-1}\,z^{n-j}}{(n-j)!}$$

$$ {}_{j+2}F_{j+1}\left(\frac{-i\,a\,(m-2\,k)-b\,(u-2\,i)+2\,c\,(v-s)}{2\,c},\,\ldots,\,\frac{-i\,a\,(m-2\,k)-b\,(u-2\,i)+2\,c\,(v-s)}{2\,c},\,v;\,\frac{-i\,a\,(m-2\,k)-b\,(u-2\,i)+2\,c\,(v-s)}{2\,c}+1,\,\ldots,\,\frac{-i\,a\,(m-2\,k)-b\,(u-2\,i)+2\,c\,(v-s)}{2\,c}+1;\,-e^{2\,c\,z}\right)\Bigg)+ $$

$$ i^v\,e^{(a\,i\,(m-2\,k)-b\,(u-2\,i)+c\,v)\,z}\binom{v}{\frac{v}{2}}(1-v \bmod 2)\sum_{j=0}^{n}\frac{(-1)^j\,(a\,i\,(m-2\,k)-b\,(u-2\,i)+c\,v)^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{i\,a\,(m-2\,k)-b\,(u-2\,i)+c\,v}{2\,c},\,\ldots,\,\frac{i\,a\,(m-2\,k)-b\,(u-2\,i)+c\,v}{2\,c},\,v;\,\frac{i\,a\,(m-2\,k)-b\,(u-2\,i)+c\,v}{2\,c}+1,\,\ldots,\,\frac{i\,a\,(m-2\,k)-b\,(u-2\,i)+c\,v}{2\,c}+1;\,-e^{2\,c\,z}\right)+ $$

$$ \sum_{s=0}^{\left\lfloor\frac{v-1}{2}\right\rfloor}(-1)^s\binom{v}{s}\Bigg((-1)^v\,e^{(a\,i\,(m-2\,k)+2\,c\,s-b\,(u-2\,i))\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(a\,i\,(m-2\,k)+2\,c\,s-b\,(u-2\,i))^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{i\,a\,(m-2\,k)+2\,c\,s-b\,(u-2\,i)}{2\,c},\,\ldots,\,\frac{i\,a\,(m-2\,k)+2\,c\,s-b\,(u-2\,i)}{2\,c},\,v;\,\frac{i\,a\,(m-2\,k)+2\,c\,s-b\,(u-2\,i)}{2\,c}+1,\,\ldots,\,\frac{i\,a\,(m-2\,k)+2\,c\,s-b\,(u-2\,i)}{2\,c}+1;\,-e^{2\,c\,z}\right)+ $$

$$ e^{(a\,i\,(m-2\,k)-b\,(u-2\,i)+2\,c\,(v-s))\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(a\,i\,(m-2\,k)-b\,(u-2\,i)+2\,c\,(v-s))^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{i\,a\,(m-2\,k)-b\,(u-2\,i)+2\,c\,(v-s)}{2\,c},\,\ldots,\,\frac{i\,a\,(m-2\,k)-b\,(u-2\,i)+2\,c\,(v-s)}{2\,c},\,v;\,\frac{i\,a\,(m-2\,k)-b\,(u-2\,i)+2\,c\,(v-s)}{2\,c}+1,\,\ldots,\,\frac{i\,a\,(m-2\,k)-b\,(u-2\,i)+2\,c\,(v-s)}{2\,c}+1;\,-e^{2\,c\,z}\right)\Bigg)\Bigg)\Bigg)\;/;\;n\in\mathbb{N}\land m\in\mathbb{N}^+\land u\in\mathbb{N}^+\land v\in\mathbb{N}^+ $$

Involving functions of the direct function, hyperbolic, exponential, trigonometric and a power functions

Involving powers of the direct function, hyperbolic, exponential, trigonometric and a power functions

Involving sin, sinh, exp and power

Involving $z^n\,e^{p\,z}\,\sin(a\,z)\,\sinh(b\,z)\,\tanh^v(c\,z)$

01.21.21.0425.01

$$\int z^n e^{p z} \sin(a z) \sinh(b z) \tanh^{\nu}(c z) \, dz = \frac{1}{4} i \, n! \left(i^{\nu} \binom{\nu}{\frac{\nu}{2}} (1 - \nu \bmod 2) \left(e^{(b - i a + p + c \nu) z} \sum_{j=0}^{n} \frac{(-1)^j (b - i a + p + c \nu)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{b - i a + p + c \nu}{2 c}, \ldots, \frac{b - i a + p + c \nu}{2 c}, \nu; \frac{b - i a + p + c \nu}{2 c} + 1, \ldots, \frac{b - i a + p + c \nu}{2 c} + 1; -e^{2 c z} \right) - e^{(b + i a + p + c \nu) z} \sum_{j=0}^{n} \frac{(-1)^j (b + i a + p + c \nu)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{b + i a + p + c \nu}{2 c}, \ldots, \frac{b + i a + p + c \nu}{2 c}, \nu; \frac{b + i a + p + c \nu}{2 c} + 1, \ldots, \frac{b + i a + p + c \nu}{2 c} + 1; -e^{2 c z} \right) - e^{(-b - i a + p + c \nu) z} \sum_{j=0}^{n} \frac{(-1)^j (-b - i a + p + c \nu)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{-b - i a + p + c \nu}{2 c}, \ldots, \frac{-b - i a + p + c \nu}{2 c}, \nu; \frac{-b - i a + p + c \nu}{2 c} + 1, \ldots, \frac{-b - i a + p + c \nu}{2 c} + 1; -e^{2 c z} \right) + e^{(-b + i a + p + c \nu) z} \sum_{j=0}^{n} \frac{(-1)^j (-b + i a + p + c \nu)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{-b + i a + p + c \nu}{2 c}, \ldots, \frac{-b + i a + p + c \nu}{2 c}, \nu; \frac{-b + i a + p + c \nu}{2 c} + 1, \ldots, \frac{-b + i a + p + c \nu}{2 c} + 1; -e^{2 c z} \right) \right) + \sum_{s=0}^{\left\lfloor \frac{\nu-1}{2} \right\rfloor} (-1)^s \binom{\nu}{s} \left((-1)^{\nu} e^{(b - i a + p + 2 c s) z} \sum_{j=0}^{n} \frac{(-1)^j (b - i a + p + 2 c s)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{b - i a + p + 2 c s}{2 c}, \ldots, \frac{b - i a + p + 2 c s}{2 c}, \nu; \frac{b - i a + p + 2 c s}{2 c} + 1, \ldots, \frac{b - i a + p + 2 c s}{2 c} + 1; -e^{2 c z} \right) + e^{(b - i a + p + 2 c (\nu - s)) z} \sum_{j=0}^{n} \frac{(-1)^j (b - i a + p + 2 c (\nu - s))^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{b - i a + p + 2 c (\nu - s)}{2 c}, \ldots, \frac{b - i a + p + 2 c (\nu - s)}{2 c}, \nu; \frac{b - i a + p + 2 c (\nu - s)}{2 c} + 1, \ldots, \frac{b - i a + p + 2 c (\nu - s)}{2 c} + 1; -e^{2 c z} \right) - e^{(b + i a + p + 2 c (\nu - s)) z} \sum_{j=0}^{n} \frac{(-1)^j (b + i a + p + 2 c (\nu - s))^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{b + i a + p + 2 c (\nu - s)}{2 c}, \ldots, \frac{b + i a + p + 2 c (\nu - s)}{2 c}, \nu; \frac{b + i a + p + 2 c (\nu - s)}{2 c} + 1, \ldots, \frac{b + i a + p + 2 c (\nu - s)}{2 c} + 1; -e^{2 c z} \right) - (-1)^{\nu} e^{(b + i a + p + 2 c s) z} \sum_{j=0}^{n} \frac{(-1)^j (b + i a + p + 2 c s)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{b + i a + p + 2 c s}{2 c}, \ldots, \frac{b + i a + p + 2 c s}{2 c}, \nu; \frac{b + i a + p + 2 c s}{2 c} + 1, \ldots, \frac{b + i a + p + 2 c s}{2 c} + 1; -e^{2 c z} \right) -$$

$$(-1)^v e^{(-b-i a+p+2 c s) z} \sum_{j=0}^{n} \frac{(-1)^j (-b-i a+p+2 c s)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{-b-i a+p+2 c s}{2 c}, \ldots, \frac{-b-i a+p+2 c s}{2 c}, v; \frac{-b-i a+p+2 c s}{2 c}+1, \ldots, \frac{-b-i a+p+2 c s}{2 c}+1; -e^{2 c z}\right) -$$

$$e^{(-b-i a+p+2 c (v-s)) z} \sum_{j=0}^{n} \frac{(-1)^j (-b-i a+p+2 c (v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{-b-i a+p+2 c (v-s)}{2 c}, \ldots, \frac{-b-i a+p+2 c (v-s)}{2 c}, v; \frac{-b-i a+p+2 c (v-s)}{2 c}+1, \ldots, \frac{-b-i a+p+2 c (v-s)}{2 c}+1; -e^{2 c z}\right) +$$

$$(-1)^v e^{(-b+i a+p+2 c s) z} \sum_{j=0}^{n} \frac{(-1)^j (-b+i a+p+2 c s)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{-b+i a+p+2 c s}{2 c}, \ldots, \frac{-b+i a+p+2 c s}{2 c}, v; \frac{-b+i a+p+2 c s}{2 c}+1, \ldots, \frac{-b+i a+p+2 c s}{2 c}+1; -e^{2 c z}\right) +$$

$$e^{(-b+i a+p+2 c (v-s)) z} \sum_{j=0}^{n} \frac{(-1)^j (-b+i a+p+2 c (v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{-b+i a+p+2 c (v-s)}{2 c}, \ldots, \frac{-b+i a+p+2 c (v-s)}{2 c}, v; \frac{-b+i a+p+2 c (v-s)}{2 c}+1, \ldots, \frac{-b+i a+p+2 c (v-s)}{2 c}+1; -e^{2 c z}\right)\Bigg)\Bigg)\Bigg) /; n \in \mathbb{N} \land v \in \mathbb{N}^+$$

Involving powers of sin, powers of sinh, exp and power

Involving $z^n e^{p z} \sin^m(a z) \sinh^u(b z) \tanh^v(c z)$

01.21.21.0426.01

$$\int z^n e^{p z} \sin^m(a z) \sinh^u(b z) \tanh^v(c z)\, dz = i^{u+v} 2^{-m-u} e^{(p+c v) z} \binom{m}{\frac{m}{2}} \binom{u}{\frac{u}{2}} \binom{v}{\frac{v}{2}} n!\, (1-m \bmod 2)(1-u \bmod 2)(1-v \bmod 2)$$

$$\sum_{j=0}^{n} \frac{(-1)^j (p+c v)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{p+c v}{2 c}, \ldots, \frac{p+c v}{2 c}, v; \frac{p+c v}{2 c}+1, \ldots, \frac{p+c v}{2 c}+1; -e^{2 c z}\right) +$$

$$i^u 2^{-m-u} \binom{m}{\frac{m}{2}} \binom{u}{\frac{u}{2}} n!\, (1-m \bmod 2)(1-u \bmod 2)$$

$$\sum_{s=0}^{\left\lfloor \frac{v-1}{2} \right\rfloor} (-1)^s \binom{v}{s} \Bigg((-1)^v e^{(p+2 c s) z} \sum_{j=0}^{n} \frac{(-1)^j (p+2 c s)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{p+2 c s}{2 c}, \ldots, \frac{p+2 c s}{2 c}, v; \frac{p+2 c s}{2 c}+1, \ldots, \frac{p+2 c s}{2 c}+1; -e^{2 c z}\right) + e^{(p+2 c (v-s)) z} \sum_{j=0}^{n} \frac{(-1)^j (p+2 c (v-s))^{-j-1} z^{n-j}}{(n-j)!}$$

$$ {}_{j+2}F_{j+1}\left(\frac{p+2c(v-s)}{2c}, \ldots, \frac{p+2c(v-s)}{2c}, v; \frac{p+2c(v-s)}{2c}+1, \ldots, \frac{p+2c(v-s)}{2c}+1; -e^{2cz}\right)\Bigg) + $$

$$ i^u 2^{-m-u}\binom{u}{\frac{u}{2}} n!\,(1-u \bmod 2) \sum_{k=0}^{\left\lfloor\frac{m-1}{2}\right\rfloor} (-1)^k \binom{m}{k} \Bigg(e^{\frac{i m\pi}{2}}\Bigg(i^v e^{(-i a(m-2k)+p+c v)z}\binom{v}{\frac{v}{2}}(1-v \bmod 2) $$

$$ \sum_{j=0}^{n} \frac{(-1)^j(-i a(m-2k)+p+c v)^{-j-1} z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}\left(\frac{-i a(m-2k)+p+c v}{2c}, \ldots, \frac{-i a(m-2k)+p+c v}{2c},\right. $$

$$ \left. v; \frac{-i a(m-2k)+p+c v}{2c}+1, \ldots, \frac{-i a(m-2k)+p+c v}{2c}+1; -e^{2cz}\right) + $$

$$ \sum_{s=0}^{\left\lfloor\frac{v-1}{2}\right\rfloor} (-1)^s \binom{v}{s} \Bigg((-1)^v e^{(-i a(m-2k)+p+2cs)z} \sum_{j=0}^{n} \frac{(-1)^j(-i a(m-2k)+p+2cs)^{-j-1} z^{n-j}}{(n-j)!} $$

$$ {}_{j+2}F_{j+1}\left(\frac{-i a(m-2k)+p+2cs}{2c}, \ldots, \frac{-i a(m-2k)+p+2cs}{2c}, v;\right. $$

$$ \left. \frac{-i a(m-2k)+p+2cs}{2c}+1, \ldots, \frac{-i a(m-2k)+p+2cs}{2c}+1; -e^{2cz}\right) + $$

$$ e^{(-i a(m-2k)+p+2c(v-s))z} \sum_{j=0}^{n} \frac{(-1)^j(-i a(m-2k)+p+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!} $$

$$ {}_{j+2}F_{j+1}\left(\frac{-i a(m-2k)+p+2c(v-s)}{2c}, \ldots, \frac{-i a(m-2k)+p+2c(v-s)}{2c}, v;\right. $$

$$ \left. \frac{-i a(m-2k)+p+2c(v-s)}{2c}+1, \ldots, \frac{-i a(m-2k)+p+2c(v-s)}{2c}+1; -e^{2cz}\right)\Bigg)\Bigg) + $$

$$ e^{-\frac{1}{2} i m\pi}\Bigg(i^v e^{(a i(m-2k)+p+c v)z}\binom{v}{\frac{v}{2}}(1-v \bmod 2) \sum_{j=0}^{n} \frac{(-1)^j(a i(m-2k)+p+c v)^{-j-1} z^{n-j}}{(n-j)!} $$

$$ {}_{j+2}F_{j+1}\left(\frac{i a(m-2k)+p+c v}{2c}, \ldots, \frac{i a(m-2k)+p+c v}{2c}, v;\right. $$

$$ \left. \frac{i a(m-2k)+p+c v}{2c}+1, \ldots, \frac{i a(m-2k)+p+c v}{2c}+1; -e^{2cz}\right) + \sum_{s=0}^{\left\lfloor\frac{v-1}{2}\right\rfloor} (-1)^s \binom{v}{s} $$

$$ \Bigg((-1)^v e^{(a i(m-2k)+p+2cs)z} \sum_{j=0}^{n} \frac{(-1)^j(a i(m-2k)+p+2cs)^{-j-1} z^{n-j}}{(n-j)!}\, {}_{j+2}F_{j+1}\left(\frac{i a(m-2k)+p+2cs}{2c},\right. $$

$$ \ldots, \frac{i a(m-2k)+p+2cs}{2c}, v; \frac{i a(m-2k)+p+2cs}{2c}+1, \ldots, \frac{i a(m-2k)+p+2cs}{2c}+ $$

$$ \left. 1; -e^{2cz}\right) + e^{(a i(m-2k)+p+2c(v-s))z} \sum_{j=0}^{n} \frac{(-1)^j(a i(m-2k)+p+2c(v-s))^{-j-1}}{(n-j)!} $$

$$z^{n-j}\,{}_{j+2}F_{j+1}\left(\frac{i\,a\,(m-2\,k)+p+2\,c\,(v-s)}{2\,c},\,\ldots,\,\frac{i\,a\,(m-2\,k)+p+2\,c\,(v-s)}{2\,c},\,v;\right.$$

$$\left.\left.\left.\left.\frac{i\,a\,(m-2\,k)+p+2\,c\,(v-s)}{2\,c}+1,\,\ldots,\,\frac{i\,a\,(m-2\,k)+p+2\,c\,(v-s)}{2\,c}+1;\,-e^{2\,c\,z}\right)\right)\right)\right)+$$

$$2^{-m-u}\binom{m}{\frac{m}{2}}n!\,(1-m\bmod 2)\sum_{k=0}^{\left\lfloor\frac{u-1}{2}\right\rfloor}(-1)^k\binom{u}{k}\left(\left(i^v\,e^{(p+b\,(u-2\,k)+c\,v)\,z}\binom{v}{\frac{v}{2}}(1-v\bmod 2)\right.\right.$$

$$\sum_{j=0}^{n}\frac{(-1)^j\,(p+b\,(u-2\,k)+c\,v)^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{p+b\,(u-2\,k)+c\,v}{2\,c},\,\ldots,\right.$$

$$\left.\frac{p+b\,(u-2\,k)+c\,v}{2\,c},\,v;\,\frac{p+b\,(u-2\,k)+c\,v}{2\,c}+1,\,\ldots,\,\frac{p+b\,(u-2\,k)+c\,v}{2\,c}+1;\,-e^{2\,c\,z}\right)+$$

$$(-1)^u\,i^v\,e^{(p-b\,(u-2\,k)+c\,v)\,z}\binom{v}{\frac{v}{2}}(1-v\bmod 2)\sum_{j=0}^{n}\frac{(-1)^j\,(p-b\,(u-2\,k)+c\,v)^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{p-b\,(u-2\,k)+c\,v}{2\,c},\right.$$

$$\left.\ldots,\,\frac{p-b\,(u-2\,k)+c\,v}{2\,c},\,v;\,\frac{p-b\,(u-2\,k)+c\,v}{2\,c}+1,\,\ldots,\,\frac{p-b\,(u-2\,k)+c\,v}{2\,c}+1;\,-e^{2\,c\,z}\right)+$$

$$\sum_{s=0}^{\left\lfloor\frac{v-1}{2}\right\rfloor}(-1)^s\binom{v}{s}\left((-1)^v\,e^{(p+2\,c\,s+b\,(u-2\,k))\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(p+2\,c\,s+b\,(u-2\,k))^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{p+2\,c\,s+b\,(u-2\,k)}{2\,c},\right.\right.$$

$$\left.\ldots,\,\frac{p+2\,c\,s+b\,(u-2\,k)}{2\,c},\,v;\,\frac{p+2\,c\,s+b\,(u-2\,k)}{2\,c}+1,\,\ldots,\,\frac{p+2\,c\,s+b\,(u-2\,k)}{2\,c}+1;\,-e^{2\,c\,z}\right)+$$

$$e^{(p+b\,(u-2\,k)+2\,c\,(v-s))\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(p+b\,(u-2\,k)+2\,c\,(v-s))^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}$$

$$\left(\frac{p+b\,(u-2\,k)+2\,c\,(v-s)}{2\,c},\,\ldots,\,\frac{p+b\,(u-2\,k)+2\,c\,(v-s)}{2\,c},\,v;\,\frac{p+b\,(u-2\,k)+2\,c\,(v-s)}{2\,c}+1,\right.$$

$$\left.\left.\ldots,\,\frac{p+b\,(u-2\,k)+2\,c\,(v-s)}{2\,c}+1;\,-e^{2\,c\,z}\right)\right)+(-1)^u\sum_{s=0}^{\left\lfloor\frac{v-1}{2}\right\rfloor}(-1)^s\binom{v}{s}$$

$$\left((-1)^v\,e^{(p+2\,c\,s-b\,(u-2\,k))\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(p+2\,c\,s-b\,(u-2\,k))^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{p+2\,c\,s-b\,(u-2\,k)}{2\,c},\,\ldots,\right.\right.$$

$$\left.\frac{p+2\,c\,s-b\,(u-2\,k)}{2\,c},\,v;\,\frac{p+2\,c\,s-b\,(u-2\,k)}{2\,c}+1,\,\ldots,\,\frac{p+2\,c\,s-b\,(u-2\,k)}{2\,c}+1;\,-e^{2\,c\,z}\right)+$$

$$e^{(p-b\,(u-2\,k)+2\,c\,(v-s))\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(p-b\,(u-2\,k)+2\,c\,(v-s))^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}$$

$$\left(\frac{p-b\,(u-2\,k)+2\,c\,(v-s)}{2\,c},\,\ldots,\,\frac{p-b\,(u-2\,k)+2\,c\,(v-s)}{2\,c},\,v;\right.$$

$$\left.\left.\left.\frac{p-b(u-2k)+2c(v-s)}{2c}+1,\ \ldots,\ \frac{p-b(u-2k)+2c(v-s)}{2c}+1;\ -e^{2cz}\right)\right)\right)+$$

$$2^{-m-u}\,n!\sum_{k=0}^{\left\lfloor\frac{m-1}{2}\right\rfloor}(-1)^k\binom{m}{k}\sum_{i=0}^{\left\lfloor\frac{u-1}{2}\right\rfloor}(-1)^i\binom{u}{i}\left((-1)^u e^{\frac{i m\pi}{2}}\left(i^v e^{(-i a(m-2k)+p-b(u-2i)+cv)z}\binom{v}{\frac{v}{2}}(1-v \bmod 2)\right.\right.$$

$$\sum_{j=0}^{n}\frac{(-1)^j(-i a(m-2k)+p-b(u-2i)+cv)^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{-i a(m-2k)+p-b(u-2i)+cv}{2c},\right.$$

$$\ldots,\ \frac{-i a(m-2k)+p-b(u-2i)+cv}{2c},\ v;\ \frac{-i a(m-2k)+p-b(u-2i)+cv}{2c}+1,$$

$$\left.\ldots,\ \frac{-i a(m-2k)+p-b(u-2i)+cv}{2c}+1;\ -e^{2cz}\right)+\sum_{s=0}^{\left\lfloor\frac{v-1}{2}\right\rfloor}(-1)^s\binom{v}{s}$$

$$\left((-1)^v e^{(-i a(m-2k)+p+2cs-b(u-2i))z}\sum_{j=0}^{n}\frac{(-1)^j(-i a(m-2k)+p+2cs-b(u-2i))^{-j-1}z^{n-j}}{(n-j)!}\right.$$

$${}_{j+2}F_{j+1}\left(\frac{-i a(m-2k)+p+2cs-b(u-2i)}{2c},\ \ldots,\right.$$

$$\frac{-i a(m-2k)+p+2cs-b(u-2i)}{2c},\ v;\ \frac{-i a(m-2k)+p+2cs-b(u-2i)}{2c}+1,$$

$$\left.\ldots,\ \frac{-i a(m-2k)+p+2cs-b(u-2i)}{2c}+1;\ -e^{2cz}\right)+$$

$$e^{(-i a(m-2k)+p-b(u-2i)+2c(v-s))z}\sum_{j=0}^{n}\frac{(-1)^j(-i a(m-2k)+p-b(u-2i)+2c(v-s))^{-j-1}z^{n-j}}{(n-j)!}$$

$${}_{j+2}F_{j+1}\left(\frac{-i a(m-2k)+p-b(u-2i)+2c(v-s)}{2c},\right.$$

$$\ldots,\ \frac{-i a(m-2k)+p-b(u-2i)+2c(v-s)}{2c},\ v;$$

$$\frac{-i a(m-2k)+p-b(u-2i)+2c(v-s)}{2c}+1,\ \ldots,$$

$$\left.\left.\left.\frac{-i a(m-2k)+p-b(u-2i)+2c(v-s)}{2c}+1;\ -e^{2cz}\right)\right)\right)+(-1)^u e^{-\frac{1}{2}i m\pi}$$

$$\left(i^v e^{(a i(m-2k)+p-b(u-2i)+cv)z}\binom{v}{\frac{v}{2}}(1-v \bmod 2)\sum_{j=0}^{n}\frac{(-1)^j(a i(m-2k)+p-b(u-2i)+cv)^{-j-1}z^{n-j}}{(n-j)!}\right.$$

$${}_{j+2}F_{j+1}\left(\frac{i a(m-2k)+p-b(u-2i)+cv}{2c},\ \ldots,\ \frac{i a(m-2k)+p-b(u-2i)+cv}{2c},\ v;\right.$$

$$\left.\frac{i a(m-2k)+p-b(u-2i)+cv}{2c}+1,\ \ldots,\ \frac{i a(m-2k)+p-b(u-2i)+cv}{2c}+1;\ -e^{2cz}\right)+$$

$$\sum_{s=0}^{\left\lfloor \frac{v-1}{2} \right\rfloor} (-1)^s \binom{v}{s} \Bigg((-1)^v e^{(a i (m-2k)+p+2cs-b(u-2i))z} \sum_{j=0}^{n} \frac{(-1)^j (a i (m-2k)+p+2cs-b(u-2i))^{-j-1} z^{n-j}}{(n-j)!}$$

$$_{j+2}F_{j+1}\Bigg(\frac{i a (m-2k)+p+2cs-b(u-2i)}{2c}, \ldots, \frac{i a (m-2k)+p+2cs-b(u-2i)}{2c}, v; \frac{i a (m-2k)+p+2cs-b(u-2i)}{2c}+1, \ldots, \frac{i a (m-2k)+p+2cs-b(u-2i)}{2c}+1; -e^{2cz} \Bigg) +$$

$$e^{(a i (m-2k)+p-b(u-2i)+2c(v-s))z} \sum_{j=0}^{n} \frac{(-1)^j (a i (m-2k)+p-b(u-2i)+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!}$$

$$_{j+2}F_{j+1}\Bigg(\frac{i a (m-2k)+p-b(u-2i)+2c(v-s)}{2c}, \ldots, \frac{i a (m-2k)+p-b(u-2i)+2c(v-s)}{2c}, v; \frac{i a (m-2k)+p-b(u-2i)+2c(v-s)}{2c}+1, \ldots, \frac{i a (m-2k)+p-b(u-2i)+2c(v-s)}{2c}+1; -e^{2cz} \Bigg) \Bigg) \Bigg) +$$

$$e^{\frac{i m \pi}{2}} \Bigg(i^v e^{(-i a (m-2k)+p+b(u-2i)+cv)z} \binom{v}{\frac{v}{2}} (1 - v \bmod 2) \sum_{j=0}^{n} \frac{(-1)^j (-i a (m-2k)+p+b(u-2i)+cv)^{-j-1} z^{n-j}}{(n-j)!}$$

$$_{j+2}F_{j+1}\Bigg(\frac{-i a (m-2k)+p+b(u-2i)+cv}{2c}, \ldots, \frac{-i a (m-2k)+p+b(u-2i)+cv}{2c}, v; \frac{-i a (m-2k)+p+b(u-2i)+cv}{2c}+1, \ldots, \frac{-i a (m-2k)+p+b(u-2i)+cv}{2c}+1; -e^{2cz} \Bigg) + \sum_{s=0}^{\left\lfloor \frac{v-1}{2} \right\rfloor} (-1)^s \binom{v}{s}$$

$$\Bigg((-1)^v e^{(-i a (m-2k)+p+2cs+b(u-2i))z} \sum_{j=0}^{n} \frac{(-1)^j (-i a (m-2k)+p+2cs+b(u-2i))^{-j-1} z^{n-j}}{(n-j)!}$$

$$_{j+2}F_{j+1}\Bigg(\frac{-i a (m-2k)+p+2cs+b(u-2i)}{2c}, \ldots, \frac{-i a (m-2k)+p+2cs+b(u-2i)}{2c}, v; \frac{-i a (m-2k)+p+2cs+b(u-2i)}{2c}+1, \ldots, \frac{-i a (m-2k)+p+2cs+b(u-2i)}{2c}+1; -e^{2cz} \Bigg) +$$

$$e^{(-i a (m-2k)+p+b(u-2i)+2c(v-s))z} \sum_{j=0}^{n} \frac{(-1)^j (-i a (m-2k)+p+b(u-2i)+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!}$$

$$_{j+2}F_{j+1}\left(\frac{-i a (m-2k)+p+b(u-2i)+2c(v-s)}{2c}, \ldots, \frac{-i a (m-2k)+p+b(u-2i)+2c(v-s)}{2c}, v; \frac{-i a (m-2k)+p+b(u-2i)+2c(v-s)}{2c}+1, \ldots, \frac{-i a (m-2k)+p+b(u-2i)+2c(v-s)}{2c}+1; -e^{2cz}\right)\Bigg)\Bigg) + e^{-\frac{1}{2} i m \pi}$$

$$\Bigg(i^v e^{(a i (m-2k)+p+b(u-2i)+c v)z} \binom{v}{\frac{v}{2}} (1 - v \bmod 2) \sum_{j=0}^{n} \frac{(-1)^j (a i (m-2k)+p+b(u-2i)+c v)^{-j-1} z^{n-j}}{(n-j)!}$$

$$_{j+2}F_{j+1}\left(\frac{i a (m-2k)+p+b(u-2i)+c v}{2c}, \ldots, \frac{i a (m-2k)+p+b(u-2i)+c v}{2c}, v; \frac{i a (m-2k)+p+b(u-2i)+c v}{2c}+1, \ldots, \frac{i a (m-2k)+p+b(u-2i)+c v}{2c}+1; -e^{2cz}\right)+$$

$$\sum_{s=0}^{\left\lfloor\frac{v-1}{2}\right\rfloor} (-1)^s \binom{v}{s} \Bigg((-1)^v e^{(a i (m-2k)+p+2cs+b(u-2i))z} \sum_{j=0}^{n} \frac{(-1)^j (a i (m-2k)+p+2cs+b(u-2i))^{-j-1} z^{n-j}}{(n-j)!}$$

$$_{j+2}F_{j+1}\left(\frac{i a (m-2k)+p+2cs+b(u-2i)}{2c}, \ldots, \frac{i a (m-2k)+p+2cs+b(u-2i)}{2c}, v; \frac{i a (m-2k)+p+2cs+b(u-2i)}{2c}+1, \ldots, \frac{i a (m-2k)+p+2cs+b(u-2i)}{2c}+1; -e^{2cz}\right)+$$

$$e^{(a i (m-2k)+p+b(u-2i)+2c(v-s))z} \sum_{j=0}^{n} \frac{(-1)^j (a i (m-2k)+p+b(u-2i)+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!}$$

$$_{j+2}F_{j+1}\left(\frac{i a (m-2k)+p+b(u-2i)+2c(v-s)}{2c}, \ldots, \frac{i a (m-2k)+p+b(u-2i)+2c(v-s)}{2c}, v; \frac{i a (m-2k)+p+b(u-2i)+2c(v-s)}{2c}+1, \ldots, \frac{i a (m-2k)+p+b(u-2i)+2c(v-s)}{2c}+1; -e^{2cz}\right)\Bigg)\Bigg)\Bigg) /; n \in \mathbb{N} \land m \in \mathbb{N}^+ \land u \in \mathbb{N}^+ \land v \in \mathbb{N}^+$$

Involving cos, sinh, exp and power

Involving $z^n e^{p z} \cos(a z) \sinh(b z) \tanh^{\nu}(c z)$

01.21.21.0427.01

$$\int z^n e^{p z} \cos(a z) \sinh(b z) \tanh^{\nu}(c z) \, dz = \frac{1}{4} n! \Bigg(i^{\nu} \binom{\nu}{\frac{\nu}{2}} (1 - \nu \bmod 2) \Bigg(e^{(b - i a + p + c \nu) z} \sum_{j=0}^{n} \frac{(-1)^j (b - i a + p + c \nu)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\Bigg(\frac{b - i a + p + c \nu}{2 c}, \ldots, \frac{b - i a + p + c \nu}{2 c}, \nu; \frac{b - i a + p + c \nu}{2 c} + 1, \ldots, \frac{b - i a + p + c \nu}{2 c} + 1; -e^{2 c z} \Bigg) + e^{(b + i a + p + c \nu) z} \sum_{j=0}^{n} \frac{(-1)^j (b + i a + p + c \nu)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\Bigg(\frac{b + i a + p + c \nu}{2 c}, \ldots, \frac{b + i a + p + c \nu}{2 c}, \nu; \frac{b + i a + p + c \nu}{2 c} + 1, \ldots, \frac{b + i a + p + c \nu}{2 c} + 1; -e^{2 c z} \Bigg) - e^{(-b - i a + p + c \nu) z} \sum_{j=0}^{n} \frac{(-1)^j (-b - i a + p + c \nu)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\Bigg(\frac{-b - i a + p + c \nu}{2 c}, \ldots, \frac{-b - i a + p + c \nu}{2 c}, \nu; \frac{-b - i a + p + c \nu}{2 c} + 1, \ldots, \frac{-b - i a + p + c \nu}{2 c} + 1; -e^{2 c z} \Bigg) - e^{(-b + i a + p + c \nu) z} \sum_{j=0}^{n} \frac{(-1)^j (-b + i a + p + c \nu)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\Bigg(\frac{-b + i a + p + c \nu}{2 c}, \ldots, \frac{-b + i a + p + c \nu}{2 c}, \nu; \frac{-b + i a + p + c \nu}{2 c} + 1, \ldots, \frac{-b + i a + p + c \nu}{2 c} + 1; -e^{2 c z} \Bigg) \Bigg) + \sum_{s=0}^{\left\lfloor \frac{\nu-1}{2} \right\rfloor} (-1)^s \binom{\nu}{s} \Bigg((-1)^{\nu} e^{(b - i a + p + 2 c s) z} \sum_{j=0}^{n} \frac{(-1)^j (b - i a + p + 2 c s)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\Bigg(\frac{b - i a + p + 2 c s}{2 c}, \ldots, \frac{b - i a + p + 2 c s}{2 c}, \nu; \frac{b - i a + p + 2 c s}{2 c} + 1, \ldots, \frac{b - i a + p + 2 c s}{2 c} + 1; -e^{2 c z} \Bigg) + e^{(b - i a + p + 2 c (\nu - s)) z} \sum_{j=0}^{n} \frac{(-1)^j (b - i a + p + 2 c (\nu - s))^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\Bigg(\frac{b - i a + p + 2 c (\nu - s)}{2 c}, \ldots, \frac{b - i a + p + 2 c (\nu - s)}{2 c}, \nu; \frac{b - i a + p + 2 c (\nu - s)}{2 c} + 1, \ldots, \frac{b - i a + p + 2 c (\nu - s)}{2 c} + 1; -e^{2 c z} \Bigg) + e^{(b + i a + p + 2 c (\nu - s)) z} \sum_{j=0}^{n} \frac{(-1)^j (b + i a + p + 2 c (\nu - s))^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\Bigg(\frac{b + i a + p + 2 c (\nu - s)}{2 c}, \ldots, \frac{b + i a + p + 2 c (\nu - s)}{2 c}, \nu; \frac{b + i a + p + 2 c (\nu - s)}{2 c} + 1, \ldots, \frac{b + i a + p + 2 c (\nu - s)}{2 c} + 1; -e^{2 c z} \Bigg) +$$

$$(-1)^v e^{(b+i a+p+2 c s) z} \sum_{j=0}^{n} \frac{(-1)^j (b+i a+p+2 c s)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{b+i a+p+2 c s}{2 c}, \ldots, \frac{b+i a+p+2 c s}{2 c}, v; \frac{b+i a+p+2 c s}{2 c}+1, \ldots, \frac{b+i a+p+2 c s}{2 c}+1; -e^{2 c z}\right) -$$

$$(-1)^v e^{(-b-i a+p+2 c s) z} \sum_{j=0}^{n} \frac{(-1)^j (-b-i a+p+2 c s)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{-b-i a+p+2 c s}{2 c}, \ldots, \frac{-b-i a+p+2 c s}{2 c}, v; \frac{-b-i a+p+2 c s}{2 c}+1, \ldots, \frac{-b-i a+p+2 c s}{2 c}+1; -e^{2 c z}\right) -$$

$$e^{(-b-i a+p+2 c (v-s)) z} \sum_{j=0}^{n} \frac{(-1)^j (-b-i a+p+2 c (v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{-b-i a+p+2 c (v-s)}{2 c}, \ldots, \frac{-b-i a+p+2 c (v-s)}{2 c}, v; \frac{-b-i a+p+2 c (v-s)}{2 c}+1, \ldots, \frac{-b-i a+p+2 c (v-s)}{2 c}+1; -e^{2 c z}\right) -$$

$$(-1)^v e^{(-b+i a+p+2 c s) z} \sum_{j=0}^{n} \frac{(-1)^j (-b+i a+p+2 c s)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{-b+i a+p+2 c s}{2 c}, \ldots, \frac{-b+i a+p+2 c s}{2 c}, v; \frac{-b+i a+p+2 c s}{2 c}+1, \ldots, \frac{-b+i a+p+2 c s}{2 c}+1; -e^{2 c z}\right) -$$

$$e^{(-b+i a+p+2 c (v-s)) z} \sum_{j=0}^{n} \frac{(-1)^j (-b+i a+p+2 c (v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{-b+i a+p+2 c (v-s)}{2 c}, \ldots, \frac{-b+i a+p+2 c (v-s)}{2 c}, v; \frac{-b+i a+p+2 c (v-s)}{2 c}+1, \ldots, \frac{-b+i a+p+2 c (v-s)}{2 c}+1; -e^{2 c z}\right)\Bigg)\Bigg) /; n \in \mathbb{N} \land v \in \mathbb{N}^+$$

Involving powers of cos, powers of sinh, exp and power

Involving $z^n e^{p z} \cos^m(a z) \sinh^u(b z) \tanh^v(c z)$

01.21.21.0428.01

$$\int z^n e^{p z} \cos^m(a z) \sinh^u(b z) \tanh^v(c z)\, dz = i^{u+v} 2^{-m-u} e^{(p+c v) z} \binom{m}{\frac{m}{2}} \binom{u}{\frac{u}{2}} \binom{v}{\frac{v}{2}} n! (1-m \bmod 2)(1-u \bmod 2)(1-v \bmod 2) \sum_{j=0}^{n} \frac{(-1)^j (p+c v)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{p+c v}{2 c}, \ldots, \frac{p+c v}{2 c}, v; \frac{p+c v}{2 c}+1, \ldots, \frac{p+c v}{2 c}+1; -e^{2 c z}\right) +$$

$$i^u 2^{-m-u} \binom{m}{\frac{m}{2}} \binom{u}{\frac{u}{2}} n! (1-m \bmod 2)(1-u \bmod 2)$$

$$\sum_{s=0}^{\left\lfloor \frac{v-1}{2} \right\rfloor} (-1)^s \binom{v}{s} \Bigg((-1)^v e^{(p+2 c s) z} \sum_{j=0}^{n} \frac{(-1)^j (p+2 c s)^{-j-1} z^{n-j}}{(n-j)!} \; {}_{j+2}F_{j+1}\Big(\frac{p+2 c s}{2 c}, \ldots, \frac{p+2 c s}{2 c}, v;$$

$$\frac{p+2 c s}{2 c}+1, \ldots, \frac{p+2 c s}{2 c}+1; -e^{2 c z} \Big) + e^{(p+2 c (v-s)) z} \sum_{j=0}^{n} \frac{(-1)^j (p+2 c (v-s))^{-j-1} z^{n-j}}{(n-j)!}$$

$${}_{j+2}F_{j+1}\Big(\frac{p+2 c (v-s)}{2 c}, \ldots, \frac{p+2 c (v-s)}{2 c}, v; \frac{p+2 c (v-s)}{2 c}+1, \ldots, \frac{p+2 c (v-s)}{2 c}+1; -e^{2 c z} \Big) \Bigg) \Bigg) +$$

$$i^u \, 2^{-m-u} \binom{u}{\frac{u}{2}} n! \, (1-u \bmod 2) \sum_{k=0}^{\left\lfloor \frac{m-1}{2} \right\rfloor} \binom{m}{k} \Bigg(i^v \, e^{(-i a (m-2 k)+p+c v) z} \binom{v}{\frac{v}{2}} (1-v \bmod 2)$$

$$\sum_{j=0}^{n} \frac{(-1)^j (-i a (m-2 k)+p+c v)^{-j-1} z^{n-j}}{(n-j)!} \; {}_{j+2}F_{j+1}\Big(\frac{-i a (m-2 k)+p+c v}{2 c}, \ldots, \frac{-i a (m-2 k)+p+c v}{2 c},$$

$$v; \frac{-i a (m-2 k)+p+c v}{2 c}+1, \ldots, \frac{-i a (m-2 k)+p+c v}{2 c}+1; -e^{2 c z} \Big) + \sum_{s=0}^{\left\lfloor \frac{v-1}{2} \right\rfloor} (-1)^s \binom{v}{s}$$

$$\Bigg((-1)^v e^{(-i a (m-2 k)+p+2 c s) z} \sum_{j=0}^{n} \frac{(-1)^j (-i a (m-2 k)+p+2 c s)^{-j-1} z^{n-j}}{(n-j)!} \; {}_{j+2}F_{j+1}\Big(\frac{-i a (m-2 k)+p+2 c s}{2 c}, \ldots,$$

$$\frac{-i a (m-2 k)+p+2 c s}{2 c}, v; \frac{-i a (m-2 k)+p+2 c s}{2 c}+1, \ldots, \frac{-i a (m-2 k)+p+2 c s}{2 c}+1;$$

$$-e^{2 c z} \Big) + e^{(-i a (m-2 k)+p+2 c (v-s)) z} \sum_{j=0}^{n} \frac{(-1)^j (-i a (m-2 k)+p+2 c (v-s))^{-j-1}}{(n-j)!}$$

$$z^{n-j} \; {}_{j+2}F_{j+1}\Big(\frac{-i a (m-2 k)+p+2 c (v-s)}{2 c}, \ldots, \frac{-i a (m-2 k)+p+2 c (v-s)}{2 c}, v;$$

$$\frac{-i a (m-2 k)+p+2 c (v-s)}{2 c}+1, \ldots, \frac{-i a (m-2 k)+p+2 c (v-s)}{2 c}+1; -e^{2 c z} \Big) \Bigg) +$$

$$i^v \, e^{(a i (m-2 k)+p+c v) z} \binom{v}{\frac{v}{2}} (1-v \bmod 2) \sum_{j=0}^{n} \frac{(-1)^j (a i (m-2 k)+p+c v)^{-j-1} z^{n-j}}{(n-j)!} \; {}_{j+2}F_{j+1}\Big(\frac{i a (m-2 k)+p+c v}{2 c},$$

$$\ldots, \frac{i a (m-2 k)+p+c v}{2 c}, v; \frac{i a (m-2 k)+p+c v}{2 c}+1, \ldots, \frac{i a (m-2 k)+p+c v}{2 c}+1; -e^{2 c z} \Big) +$$

$$\sum_{s=0}^{\left\lfloor \frac{v-1}{2} \right\rfloor} (-1)^s \binom{v}{s} \Bigg((-1)^v e^{(a i (m-2 k)+p+2 c s) z} \sum_{j=0}^{n} \frac{(-1)^j (a i (m-2 k)+p+2 c s)^{-j-1} z^{n-j}}{(n-j)!} \; {}_{j+2}F_{j+1}$$

$$\Big(\frac{i a (m-2 k)+p+2 c s}{2 c}, \ldots, \frac{i a (m-2 k)+p+2 c s}{2 c}, v;$$

$$\frac{i a (m-2 k)+p+2 c s}{2 c}+1, \ldots, \frac{i a (m-2 k)+p+2 c s}{2 c}+1; -e^{2 c z} \Big) +$$

$$e^{(a\,i\,(m-2\,k)+p+2\,c\,(v-s))\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(a\,i\,(m-2\,k)+p+2\,c\,(v-s))^{-j-1}\,z^{n-j}}{(n-j)!}\;{}_{j+2}F_{j+1}\Bigg(\frac{i\,a\,(m-2\,k)+p+2\,c\,(v-s)}{2\,c},\,\ldots,\,\frac{i\,a\,(m-2\,k)+p+2\,c\,(v-s)}{2\,c},\,v;\;\frac{i\,a\,(m-2\,k)+p+2\,c\,(v-s)}{2\,c}+1,\,\ldots,\,\frac{i\,a\,(m-2\,k)+p+2\,c\,(v-s)}{2\,c}+1;\,-e^{2\,c\,z}\Bigg)\Bigg)\Bigg)+$$

$$2^{-m-u}\binom{m}{\frac{m}{2}}n!\,(1-m\bmod 2)\sum_{k=0}^{\left\lfloor\frac{u-1}{2}\right\rfloor}(-1)^k\binom{u}{k}\Bigg(i^v\,e^{(p+b\,(u-2\,k)+c\,v)\,z}\binom{v}{\frac{v}{2}}(1-v\bmod 2)$$

$$\sum_{j=0}^{n}\frac{(-1)^j\,(p+b\,(u-2\,k)+c\,v)^{-j-1}\,z^{n-j}}{(n-j)!}\;{}_{j+2}F_{j+1}\Bigg(\frac{p+b\,(u-2\,k)+c\,v}{2\,c},\,\ldots,\,\frac{p+b\,(u-2\,k)+c\,v}{2\,c},\,v;\;\frac{p+b\,(u-2\,k)+c\,v}{2\,c}+1,\,\ldots,\,\frac{p+b\,(u-2\,k)+c\,v}{2\,c}+1;\,-e^{2\,c\,z}\Bigg)+$$

$$(-1)^u\,i^v\,e^{(p-b\,(u-2\,k)+c\,v)\,z}\binom{v}{\frac{v}{2}}(1-v\bmod 2)\sum_{j=0}^{n}\frac{(-1)^j\,(p-b\,(u-2\,k)+c\,v)^{-j-1}\,z^{n-j}}{(n-j)!}\;{}_{j+2}F_{j+1}\Bigg(\frac{p-b\,(u-2\,k)+c\,v}{2\,c},\,\ldots,\,\frac{p-b\,(u-2\,k)+c\,v}{2\,c},\,v;\;\frac{p-b\,(u-2\,k)+c\,v}{2\,c}+1,\,\ldots,\,\frac{p-b\,(u-2\,k)+c\,v}{2\,c}+1;\,-e^{2\,c\,z}\Bigg)+$$

$$\sum_{s=0}^{\left\lfloor\frac{v-1}{2}\right\rfloor}(-1)^s\binom{v}{s}\Bigg((-1)^v\,e^{(p+2\,c\,s+b\,(u-2\,k))\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(p+2\,c\,s+b\,(u-2\,k))^{-j-1}\,z^{n-j}}{(n-j)!}\;{}_{j+2}F_{j+1}\Bigg(\frac{p+2\,c\,s+b\,(u-2\,k)}{2\,c},\,\ldots,\,\frac{p+2\,c\,s+b\,(u-2\,k)}{2\,c},\,v;\;\frac{p+2\,c\,s+b\,(u-2\,k)}{2\,c}+1,\,\ldots,\,\frac{p+2\,c\,s+b\,(u-2\,k)}{2\,c}+1;\,-e^{2\,c\,z}\Bigg)+$$

$$e^{(p+b\,(u-2\,k)+2\,c\,(v-s))\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(p+b\,(u-2\,k)+2\,c\,(v-s))^{-j-1}\,z^{n-j}}{(n-j)!}\;{}_{j+2}F_{j+1}\Bigg(\frac{p+b\,(u-2\,k)+2\,c\,(v-s)}{2\,c},\,\ldots,\,\frac{p+b\,(u-2\,k)+2\,c\,(v-s)}{2\,c},\,v;\;\frac{p+b\,(u-2\,k)+2\,c\,(v-s)}{2\,c}+1,\,\ldots,\,\frac{p+b\,(u-2\,k)+2\,c\,(v-s)}{2\,c}+1;\,-e^{2\,c\,z}\Bigg)\Bigg)+(-1)^u\sum_{s=0}^{\left\lfloor\frac{v-1}{2}\right\rfloor}(-1)^s\binom{v}{s}$$

$$\Bigg((-1)^v\,e^{(p+2\,c\,s-b\,(u-2\,k))\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(p+2\,c\,s-b\,(u-2\,k))^{-j-1}\,z^{n-j}}{(n-j)!}\;{}_{j+2}F_{j+1}\Bigg(\frac{p+2\,c\,s-b\,(u-2\,k)}{2\,c},\,\ldots,\,\frac{p+2\,c\,s-b\,(u-2\,k)}{2\,c},\,v;\;\frac{p+2\,c\,s-b\,(u-2\,k)}{2\,c}+1,\,\ldots,\,\frac{p+2\,c\,s-b\,(u-2\,k)}{2\,c}+1;\,-e^{2\,c\,z}\Bigg)+$$

$$e^{(p-b\,(u-2\,k)+2\,c\,(v-s))\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(p-b\,(u-2\,k)+2\,c\,(v-s))^{-j-1}\,z^{n-j}}{(n-j)!}\;{}_{j+2}F_{j+1}$$

$$\left(\frac{p-b(u-2k)+2c(v-s)}{2c}, \ldots, \frac{p-b(u-2k)+2c(v-s)}{2c}, v; \frac{p-b(u-2k)+2c(v-s)}{2c}+1, \ldots, \frac{p-b(u-2k)+2c(v-s)}{2c}+1; -e^{2cz}\right)\Bigg)\Bigg)+$$

$$2^{-m-u}\, n! \sum_{k=0}^{\left\lfloor\frac{m-1}{2}\right\rfloor} \binom{m}{k} \sum_{i=0}^{\left\lfloor\frac{u-1}{2}\right\rfloor} (-1)^i \binom{u}{i} \Bigg(i^v\, e^{(-i a(m-2k)+p+b(u-2i)+cv)z} \binom{v}{\frac{v}{2}} (1-v \bmod 2)$$

$$\sum_{j=0}^{n} \frac{(-1)^j (-i a(m-2k)+p+b(u-2i)+cv)^{-j-1} z^{n-j}}{(n-j)!}$$

$$_{j+2}F_{j+1}\left(\frac{-i a(m-2k)+p+b(u-2i)+cv}{2c}, \ldots, \frac{-i a(m-2k)+p+b(u-2i)+cv}{2c}, v; \frac{-i a(m-2k)+p+b(u-2i)+cv}{2c}+1, \ldots, \frac{-i a(m-2k)+p+b(u-2i)+cv}{2c}+1; -e^{2cz}\right)+$$

$$\sum_{s=0}^{\left\lfloor\frac{v-1}{2}\right\rfloor} (-1)^s \binom{v}{s} \Bigg((-1)^v e^{(-i a(m-2k)+p+2cs+b(u-2i))z} \sum_{j=0}^{n} \frac{(-1)^j (-i a(m-2k)+p+2cs+b(u-2i))^{-j-1} z^{n-j}}{(n-j)!}$$

$$_{j+2}F_{j+1}\left(\frac{-i a(m-2k)+p+2cs+b(u-2i)}{2c}, \ldots, \frac{-i a(m-2k)+p+2cs+b(u-2i)}{2c}, v; \frac{-i a(m-2k)+p+2cs+b(u-2i)}{2c}+1, \ldots, \frac{-i a(m-2k)+p+2cs+b(u-2i)}{2c}+1; -e^{2cz}\right)+$$

$$e^{(-i a(m-2k)+p+b(u-2i)+2c(v-s))z} \sum_{j=0}^{n} \frac{(-1)^j (-i a(m-2k)+p+b(u-2i)+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!}$$

$$_{j+2}F_{j+1}\left(\frac{-i a(m-2k)+p+b(u-2i)+2c(v-s)}{2c}, \ldots, \frac{-i a(m-2k)+p+b(u-2i)+2c(v-s)}{2c}, v; \frac{-i a(m-2k)+p+b(u-2i)+2c(v-s)}{2c}+1, \ldots, \frac{-i a(m-2k)+p+b(u-2i)+2c(v-s)}{2c}+1; -e^{2cz}\right)\Bigg)+$$

$$i^v\, e^{(a i(m-2k)+p+b(u-2i)+cv)z} \binom{v}{\frac{v}{2}} (1-v \bmod 2) \sum_{j=0}^{n} \frac{(-1)^j (a i(m-2k)+p+b(u-2i)+cv)^{-j-1} z^{n-j}}{(n-j)!}$$

$$_{j+2}F_{j+1}\left(\frac{i a(m-2k)+p+b(u-2i)+cv}{2c}, \ldots, \frac{i a(m-2k)+p+b(u-2i)+cv}{2c}, v; \frac{i a(m-2k)+p+b(u-2i)+cv}{2c}+1, \ldots, \frac{i a(m-2k)+p+b(u-2i)+cv}{2c}+1; -e^{2cz}\right)+$$

$$\sum_{s=0}^{\left\lfloor \frac{v-1}{2} \right\rfloor} (-1)^s \binom{v}{s} \Bigg((-1)^v e^{(a i (m-2k)+p+2cs+b(u-2i))z} \sum_{j=0}^{n} \frac{(-1)^j (a i (m-2k)+p+2cs+b(u-2i))^{-j-1} z^{n-j}}{(n-j)!}$$

$$_{j+2}F_{j+1}\Bigg(\frac{i a (m-2k)+p+2cs+b(u-2i)}{2c}, \ldots, \frac{i a (m-2k)+p+2cs+b(u-2i)}{2c}, v; \frac{i a (m-2k)+p+2cs+b(u-2i)}{2c}+1, \ldots, \frac{i a (m-2k)+p+2cs+b(u-2i)}{2c}+1; -e^{2cz}\Bigg) +$$

$$e^{(a i (m-2k)+p+b(u-2i)+2c(v-s))z} \sum_{j=0}^{n} \frac{(-1)^j (a i (m-2k)+p+b(u-2i)+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!}$$

$$_{j+2}F_{j+1}\Bigg(\frac{i a (m-2k)+p+b(u-2i)+2c(v-s)}{2c}, \ldots, \frac{i a (m-2k)+p+b(u-2i)+2c(v-s)}{2c}, v; \frac{i a (m-2k)+p+b(u-2i)+2c(v-s)}{2c}+1, \ldots, \frac{i a (m-2k)+p+b(u-2i)+2c(v-s)}{2c}+1; -e^{2cz}\Bigg)\Bigg) +$$

$$(-1)^u \Bigg(i^v e^{(-i a (m-2k)+p-b(u-2i)+cv)z} \binom{v}{\frac{v}{2}} (1 - v \bmod 2) \sum_{j=0}^{n} \frac{(-1)^j (-i a (m-2k)+p-b(u-2i)+cv)^{-j-1} z^{n-j}}{(n-j)!}$$

$$_{j+2}F_{j+1}\Bigg(\frac{-i a (m-2k)+p-b(u-2i)+cv}{2c}, \ldots, \frac{-i a (m-2k)+p-b(u-2i)+cv}{2c}, v; \frac{-i a (m-2k)+p-b(u-2i)+cv}{2c}+1, \ldots, \frac{-i a (m-2k)+p-b(u-2i)+cv}{2c}+1; -e^{2cz}\Bigg) + \sum_{s=0}^{\left\lfloor \frac{v-1}{2} \right\rfloor} (-1)^s \binom{v}{s}$$

$$\Bigg((-1)^v e^{(-i a (m-2k)+p+2cs-b(u-2i))z} \sum_{j=0}^{n} \frac{(-1)^j (-i a (m-2k)+p+2cs-b(u-2i))^{-j-1} z^{n-j}}{(n-j)!}$$

$$_{j+2}F_{j+1}\Bigg(\frac{-i a (m-2k)+p+2cs-b(u-2i)}{2c}, \ldots, \frac{-i a (m-2k)+p+2cs-b(u-2i)}{2c}, v; \frac{-i a (m-2k)+p+2cs-b(u-2i)}{2c}+1, \ldots, \frac{-i a (m-2k)+p+2cs-b(u-2i)}{2c}+1; -e^{2cz}\Bigg) +$$

$$e^{(-i a (m-2k)+p-b(u-2i)+2c(v-s))z} \sum_{j=0}^{n} \frac{(-1)^j (-i a (m-2k)+p-b(u-2i)+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!}$$

$$_{j+2}F_{j+1}\Bigg(\frac{-i a (m-2k)+p-b(u-2i)+2c(v-s)}{2c},$$

$$\ldots, \frac{-i\,a\,(m-2\,k)+p-b\,(u-2\,i)+2\,c\,(v-s)}{2\,c}, v; \frac{-i\,a\,(m-2\,k)+p-b\,(u-2\,i)+2\,c\,(v-s)}{2\,c}+1, \ldots, \frac{-i\,a\,(m-2\,k)+p-b\,(u-2\,i)+2\,c\,(v-s)}{2\,c}+1; -e^{2\,c\,z}\Bigg)\Bigg)+$$

$$i^{v}\,e^{(a\,i\,(m-2\,k)+p-b\,(u-2\,i)+c\,v)\,z}\binom{v}{\frac{v}{2}}(1-v \bmod 2)\sum_{j=0}^{n}\frac{(-1)^{j}\,(a\,i\,(m-2\,k)+p-b\,(u-2\,i)+c\,v)^{-j-1}\,z^{n-j}}{(n-j)!}$$

$$_{j+2}F_{j+1}\Bigg(\frac{i\,a\,(m-2\,k)+p-b\,(u-2\,i)+c\,v}{2\,c}, \ldots, \frac{i\,a\,(m-2\,k)+p-b\,(u-2\,i)+c\,v}{2\,c}, v; \frac{i\,a\,(m-2\,k)+p-b\,(u-2\,i)+c\,v}{2\,c}+1, \ldots, \frac{i\,a\,(m-2\,k)+p-b\,(u-2\,i)+c\,v}{2\,c}+1; -e^{2\,c\,z}\Bigg)+$$

$$\sum_{s=0}^{\left\lfloor\frac{v-1}{2}\right\rfloor}(-1)^{s}\binom{v}{s}\Bigg((-1)^{v}\,e^{(a\,i\,(m-2\,k)+p+2\,c\,s-b\,(u-2\,i))\,z}\sum_{j=0}^{n}\frac{(-1)^{j}\,(a\,i\,(m-2\,k)+p+2\,c\,s-b\,(u-2\,i))^{-j-1}\,z^{n-j}}{(n-j)!}$$

$$_{j+2}F_{j+1}\Bigg(\frac{i\,a\,(m-2\,k)+p+2\,c\,s-b\,(u-2\,i)}{2\,c}, \ldots, \frac{i\,a\,(m-2\,k)+p+2\,c\,s-b\,(u-2\,i)}{2\,c}, v; \frac{i\,a\,(m-2\,k)+p+2\,c\,s-b\,(u-2\,i)}{2\,c}+1, \ldots, \frac{i\,a\,(m-2\,k)+p+2\,c\,s-b\,(u-2\,i)}{2\,c}+1; -e^{2\,c\,z}\Bigg)+$$

$$e^{(a\,i\,(m-2\,k)+p-b\,(u-2\,i)+2\,c\,(v-s))\,z}\sum_{j=0}^{n}\frac{(-1)^{j}\,(a\,i\,(m-2\,k)+p-b\,(u-2\,i)+2\,c\,(v-s))^{-j-1}\,z^{n-j}}{(n-j)!}$$

$$_{j+2}F_{j+1}\Bigg(\frac{i\,a\,(m-2\,k)+p-b\,(u-2\,i)+2\,c\,(v-s)}{2\,c}, \ldots, \frac{i\,a\,(m-2\,k)+p-b\,(u-2\,i)+2\,c\,(v-s)}{2\,c}, v; \frac{i\,a\,(m-2\,k)+p-b\,(u-2\,i)+2\,c\,(v-s)}{2\,c}+1, \ldots, \frac{i\,a\,(m-2\,k)+p-b\,(u-2\,i)+2\,c\,(v-s)}{2\,c}+1;$$

$$-e^{2\,c\,z}\Bigg)\Bigg)\Bigg)\Bigg) /; n \in \mathbb{N} \land m \in \mathbb{N}^{+} \land u \in \mathbb{N}^{+} \land v \in \mathbb{N}^{+}$$

Involving sin, cosh, exp and power

Involving $z^{n}\,e^{p\,z}\sin(a\,z)\cosh(b\,z)\tanh^{v}(c\,z)$

01.21.21.0429.01

$$\int z^n e^{pz} \sin(a z) \cosh(b z) \tanh^v(c z) \, dz = \frac{1}{4} i\, n! \Bigg(i^v \binom{v}{\frac{v}{2}} (1 - v \bmod 2) \Bigg(e^{(b - i a + p + c v) z} \sum_{j=0}^{n} \frac{(-1)^j (b - i a + p + c v)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\Bigg(\frac{b - i a + p + c v}{2c}, \ldots, \frac{b - i a + p + c v}{2c}, v; \frac{b - i a + p + c v}{2c} + 1, \ldots, \frac{b - i a + p + c v}{2c} + 1; -e^{2cz}\Bigg) -$$

$$e^{(b + i a + p + c v) z} \sum_{j=0}^{n} \frac{(-1)^j (b + i a + p + c v)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\Bigg(\frac{b + i a + p + c v}{2c}, \ldots, \frac{b + i a + p + c v}{2c}, v; \frac{b + i a + p + c v}{2c} + 1, \ldots, \frac{b + i a + p + c v}{2c} + 1; -e^{2cz}\Bigg) +$$

$$e^{(-b - i a + p + c v) z} \sum_{j=0}^{n} \frac{(-1)^j (-b - i a + p + c v)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\Bigg(\frac{-b - i a + p + c v}{2c}, \ldots, \frac{-b - i a + p + c v}{2c}, v; \frac{-b - i a + p + c v}{2c} + 1, \ldots, \frac{-b - i a + p + c v}{2c} + 1; -e^{2cz}\Bigg) -$$

$$e^{(-b + i a + p + c v) z} \sum_{j=0}^{n} \frac{(-1)^j (-b + i a + p + c v)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\Bigg(\frac{-b + i a + p + c v}{2c}, \ldots, \frac{-b + i a + p + c v}{2c}, v; \frac{-b + i a + p + c v}{2c} + 1, \ldots, \frac{-b + i a + p + c v}{2c} + 1; -e^{2cz}\Bigg)\Bigg) +$$

$$\sum_{s=0}^{\left\lfloor \frac{v-1}{2} \right\rfloor} (-1)^s \binom{v}{s} \Bigg((-1)^v e^{(b - i a + p + 2 c s) z} \sum_{j=0}^{n} \frac{(-1)^j (b - i a + p + 2 c s)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\Bigg(\frac{b - i a + p + 2 c s}{2c}, \ldots, \frac{b - i a + p + 2 c s}{2c}, v; \frac{b - i a + p + 2 c s}{2c} + 1, \ldots, \frac{b - i a + p + 2 c s}{2c} + 1; -e^{2cz}\Bigg) +$$

$$e^{(b - i a + p + 2 c (v - s)) z} \sum_{j=0}^{n} \frac{(-1)^j (b - i a + p + 2 c (v - s))^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\Bigg(\frac{b - i a + p + 2 c (v - s)}{2c}, \ldots, \frac{b - i a + p + 2 c (v - s)}{2c}, v; \frac{b - i a + p + 2 c (v - s)}{2c} + 1, \ldots, \frac{b - i a + p + 2 c (v - s)}{2c} + 1; -e^{2cz}\Bigg) -$$

$$e^{(b + i a + p + 2 c (v - s)) z} \sum_{j=0}^{n} \frac{(-1)^j (b + i a + p + 2 c (v - s))^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\Bigg(\frac{b + i a + p + 2 c (v - s)}{2c}, \ldots, \frac{b + i a + p + 2 c (v - s)}{2c}, v; \frac{b + i a + p + 2 c (v - s)}{2c} + 1, \ldots, \frac{b + i a + p + 2 c (v - s)}{2c} + 1; -e^{2cz}\Bigg) -$$

$$(-1)^v e^{(b + i a + p + 2 c s) z} \sum_{j=0}^{n} \frac{(-1)^j (b + i a + p + 2 c s)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\Bigg(\frac{b + i a + p + 2 c s}{2c}, \ldots, \frac{b + i a + p + 2 c s}{2c}, v; \frac{b + i a + p + 2 c s}{2c} + 1, \ldots, \frac{b + i a + p + 2 c s}{2c} + 1; -e^{2cz}\Bigg) +$$

$$(-1)^v e^{(-b-i a+p+2 c s) z} \sum_{j=0}^{n} \frac{(-1)^j (-b-i a+p+2 c s)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{-b-i a+p+2 c s}{2 c}, \ldots, \frac{-b-i a+p+2 c s}{2 c}, v; \frac{-b-i a+p+2 c s}{2 c}+1, \ldots, \frac{-b-i a+p+2 c s}{2 c}+1; -e^{2 c z}\right)+$$

$$e^{(-b-i a+p+2 c (v-s)) z} \sum_{j=0}^{n} \frac{(-1)^j (-b-i a+p+2 c (v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{-b-i a+p+2 c (v-s)}{2 c}, \ldots, \frac{-b-i a+p+2 c (v-s)}{2 c}, v; \frac{-b-i a+p+2 c (v-s)}{2 c}+1, \ldots, \frac{-b-i a+p+2 c (v-s)}{2 c}+1; -e^{2 c z}\right)-$$

$$(-1)^v e^{(-b+i a+p+2 c s) z} \sum_{j=0}^{n} \frac{(-1)^j (-b+i a+p+2 c s)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{-b+i a+p+2 c s}{2 c}, \ldots, \frac{-b+i a+p+2 c s}{2 c}, v; \frac{-b+i a+p+2 c s}{2 c}+1, \ldots, \frac{-b+i a+p+2 c s}{2 c}+1; -e^{2 c z}\right)-$$

$$e^{(-b+i a+p+2 c (v-s)) z} \sum_{j=0}^{n} \frac{(-1)^j (-b+i a+p+2 c (v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{-b+i a+p+2 c (v-s)}{2 c}, \ldots, \frac{-b+i a+p+2 c (v-s)}{2 c}, v; \frac{-b+i a+p+2 c (v-s)}{2 c}+1, \ldots, \frac{-b+i a+p+2 c (v-s)}{2 c}+1; -e^{2 c z}\right)\Bigg)\Bigg) /; n \in \mathbb{N} \land v \in \mathbb{N}^+$$

Involving powers of sin, powers of cosh, exp and power

Involving $z^n e^{p z} \sin^m(a z) \cosh^u(b z) \tanh^v(c z)$

01.21.21.0430.01

$$\int z^n e^{p z} \sin^m(a z) \cosh^u(b z) \tanh^v(c z)\, dz = i^v 2^{-m-u} e^{(p+c v) z} \binom{m}{\frac{m}{2}} \binom{u}{\frac{u}{2}} \binom{v}{\frac{v}{2}} n! (1-m \bmod 2)(1-u \bmod 2)(1-v \bmod 2)$$

$$\sum_{j=0}^{n} \frac{(-1)^j (p+c v)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{p+c v}{2 c}, \ldots, \frac{p+c v}{2 c}, v; \frac{p+c v}{2 c}+1, \ldots, \frac{p+c v}{2 c}+1; -e^{2 c z}\right)+$$

$$2^{-m-u} \binom{m}{\frac{m}{2}} \binom{u}{\frac{u}{2}} n! (1-m \bmod 2)(1-u \bmod 2)$$

$$\sum_{s=0}^{\left\lfloor \frac{v-1}{2} \right\rfloor} (-1)^s \binom{v}{s} \Bigg((-1)^v e^{(p+2 c s) z} \sum_{j=0}^{n} \frac{(-1)^j (p+2 c s)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{p+2 c s}{2 c}, \ldots, \frac{p+2 c s}{2 c}, v; \frac{p+2 c s}{2 c}+1, \ldots, \frac{p+2 c s}{2 c}+1; -e^{2 c z}\right)+e^{(p+2 c (v-s)) z} \sum_{j=0}^{n} \frac{(-1)^j (p+2 c (v-s))^{-j-1} z^{n-j}}{(n-j)!}$$

$$ {}_{j+2}F_{j+1}\left(\frac{p+2c(v-s)}{2c},\ldots,\frac{p+2c(v-s)}{2c},v;\frac{p+2c(v-s)}{2c}+1,\ldots,\frac{p+2c(v-s)}{2c}+1;-e^{2cz}\right)\Bigg)+ $$

$$ 2^{-m-u}\binom{u}{\frac{u}{2}}n!\,(1-u\bmod 2)\sum_{k=0}^{\left\lfloor\frac{m-1}{2}\right\rfloor}(-1)^k\binom{m}{k}\Bigg(e^{\frac{i m\pi}{2}}\Bigg(i^v e^{(-i a(m-2k)+p+cv)z}\binom{v}{\frac{v}{2}}(1-v\bmod 2) $$

$$ \sum_{j=0}^{n}\frac{(-1)^j(-i a(m-2k)+p+cv)^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{-i a(m-2k)+p+cv}{2c},\ldots,\frac{-i a(m-2k)+p+cv}{2c},\right. $$

$$ \left.v;\frac{-i a(m-2k)+p+cv}{2c}+1,\ldots,\frac{-i a(m-2k)+p+cv}{2c}+1;-e^{2cz}\right)+ $$

$$ \sum_{s=0}^{\left\lfloor\frac{v-1}{2}\right\rfloor}(-1)^s\binom{v}{s}\Bigg((-1)^v e^{(-i a(m-2k)+p+2cs)z}\sum_{j=0}^{n}\frac{(-1)^j(-i a(m-2k)+p+2cs)^{-j-1}z^{n-j}}{(n-j)!} $$

$$ {}_{j+2}F_{j+1}\left(\frac{-i a(m-2k)+p+2cs}{2c},\ldots,\frac{-i a(m-2k)+p+2cs}{2c},v;\right. $$

$$ \left.\frac{-i a(m-2k)+p+2cs}{2c}+1,\ldots,\frac{-i a(m-2k)+p+2cs}{2c}+1;-e^{2cz}\right)+ $$

$$ e^{(-i a(m-2k)+p+2c(v-s))z}\sum_{j=0}^{n}\frac{(-1)^j(-i a(m-2k)+p+2c(v-s))^{-j-1}z^{n-j}}{(n-j)!} $$

$$ {}_{j+2}F_{j+1}\left(\frac{-i a(m-2k)+p+2c(v-s)}{2c},\ldots,\frac{-i a(m-2k)+p+2c(v-s)}{2c},v;\right. $$

$$ \left.\frac{-i a(m-2k)+p+2c(v-s)}{2c}+1,\ldots,\frac{-i a(m-2k)+p+2c(v-s)}{2c}+1;-e^{2cz}\right)\Bigg)\Bigg)+ $$

$$ e^{-\frac{1}{2}i m\pi}\Bigg(i^v e^{(a i(m-2k)+p+cv)z}\binom{v}{\frac{v}{2}}(1-v\bmod 2)\sum_{j=0}^{n}\frac{(-1)^j(a i(m-2k)+p+cv)^{-j-1}z^{n-j}}{(n-j)!} $$

$$ {}_{j+2}F_{j+1}\left(\frac{i a(m-2k)+p+cv}{2c},\ldots,\frac{i a(m-2k)+p+cv}{2c},v;\right. $$

$$ \left.\frac{i a(m-2k)+p+cv}{2c}+1,\ldots,\frac{i a(m-2k)+p+cv}{2c}+1;-e^{2cz}\right)+\sum_{s=0}^{\left\lfloor\frac{v-1}{2}\right\rfloor}(-1)^s\binom{v}{s} $$

$$ \Bigg((-1)^v e^{(a i(m-2k)+p+2cs)z}\sum_{j=0}^{n}\frac{(-1)^j(a i(m-2k)+p+2cs)^{-j-1}z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{i a(m-2k)+p+2cs}{2c},\right. $$

$$ \ldots,\frac{i a(m-2k)+p+2cs}{2c},v;\frac{i a(m-2k)+p+2cs}{2c}+1,\ldots,\frac{i a(m-2k)+p+2cs}{2c}+ $$

$$ \left.1;-e^{2cz}\right)+e^{(a i(m-2k)+p+2c(v-s))z}\sum_{j=0}^{n}\frac{(-1)^j(a i(m-2k)+p+2c(v-s))^{-j-1}z^{n-j}}{(n-j)!} $$

$$ {}_{j+2}F_{j+1}\left(\frac{i\,a\,(m-2\,k)+p+2\,c\,(v-s)}{2\,c},\,\ldots,\,\frac{i\,a\,(m-2\,k)+p+2\,c\,(v-s)}{2\,c},\,v;\right. $$

$$ \left.\left.\left.\left.\frac{i\,a\,(m-2\,k)+p+2\,c\,(v-s)}{2\,c}+1,\,\ldots,\,\frac{i\,a\,(m-2\,k)+p+2\,c\,(v-s)}{2\,c}+1;\,-e^{2\,c\,z}\right)\right)\right)\right)+ $$

$$ 2^{-m-u}\binom{m}{\frac{m}{2}}n!\,(1-m\bmod 2)\sum_{k=0}^{\left\lfloor\frac{u-1}{2}\right\rfloor}\binom{u}{k}\left(i^v\,e^{(p+b\,(u-2\,k)+c\,v)\,z}\binom{v}{\frac{v}{2}}(1-v\bmod 2)\sum_{j=0}^{n}\frac{(-1)^j\,(p+b\,(u-2\,k)+c\,v)^{-j-1}\,z^{n-j}}{(n-j)!}\right. $$

$$ {}_{j+2}F_{j+1}\left(\frac{p+b\,(u-2\,k)+c\,v}{2\,c},\,\ldots,\,\frac{p+b\,(u-2\,k)+c\,v}{2\,c},\,v;\right. $$

$$ \left.\frac{p+b\,(u-2\,k)+c\,v}{2\,c}+1,\,\ldots,\,\frac{p+b\,(u-2\,k)+c\,v}{2\,c}+1;\,-e^{2\,c\,z}\right)+ $$

$$ i^v\,e^{(p-b\,(u-2\,k)+c\,v)\,z}\binom{v}{\frac{v}{2}}(1-v\bmod 2)\sum_{j=0}^{n}\frac{(-1)^j\,(p-b\,(u-2\,k)+c\,v)^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{p-b\,(u-2\,k)+c\,v}{2\,c},\right. $$

$$ \left.\ldots,\,\frac{p-b\,(u-2\,k)+c\,v}{2\,c},\,v;\,\frac{p-b\,(u-2\,k)+c\,v}{2\,c}+1,\,\ldots,\,\frac{p-b\,(u-2\,k)+c\,v}{2\,c}+1;\,-e^{2\,c\,z}\right)+ $$

$$ \sum_{s=0}^{\left\lfloor\frac{v-1}{2}\right\rfloor}(-1)^s\binom{v}{s}\left((-1)^v\,e^{(p+2\,c\,s+b\,(u-2\,k))\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(p+2\,c\,s+b\,(u-2\,k))^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{p+2\,c\,s+b\,(u-2\,k)}{2\,c},\right.\right. $$

$$ \left.\ldots,\,\frac{p+2\,c\,s+b\,(u-2\,k)}{2\,c},\,v;\,\frac{p+2\,c\,s+b\,(u-2\,k)}{2\,c}+1,\,\ldots,\,\frac{p+2\,c\,s+b\,(u-2\,k)}{2\,c}+1;\,-e^{2\,c\,z}\right)+ $$

$$ e^{(p+b\,(u-2\,k)+2\,c\,(v-s))\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(p+b\,(u-2\,k)+2\,c\,(v-s))^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1} $$

$$ \left(\frac{p+b\,(u-2\,k)+2\,c\,(v-s)}{2\,c},\,\ldots,\,\frac{p+b\,(u-2\,k)+2\,c\,(v-s)}{2\,c},\,v;\right. $$

$$ \left.\left.\frac{p+b\,(u-2\,k)+2\,c\,(v-s)}{2\,c}+1,\,\ldots,\,\frac{p+b\,(u-2\,k)+2\,c\,(v-s)}{2\,c}+1;\,-e^{2\,c\,z}\right)\right)+ $$

$$ \sum_{s=0}^{\left\lfloor\frac{v-1}{2}\right\rfloor}(-1)^s\binom{v}{s}\left((-1)^v\,e^{(p+2\,c\,s-b\,(u-2\,k))\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(p+2\,c\,s-b\,(u-2\,k))^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{p+2\,c\,s-b\,(u-2\,k)}{2\,c},\right.\right. $$

$$ \left.\ldots,\,\frac{p+2\,c\,s-b\,(u-2\,k)}{2\,c},\,v;\,\frac{p+2\,c\,s-b\,(u-2\,k)}{2\,c}+1,\,\ldots,\,\frac{p+2\,c\,s-b\,(u-2\,k)}{2\,c}+1;\,-e^{2\,c\,z}\right)+ $$

$$ e^{(p-b\,(u-2\,k)+2\,c\,(v-s))\,z}\sum_{j=0}^{n}\frac{(-1)^j\,(p-b\,(u-2\,k)+2\,c\,(v-s))^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1} $$

$$ \left(\frac{p-b\,(u-2\,k)+2\,c\,(v-s)}{2\,c},\,\ldots,\,\frac{p-b\,(u-2\,k)+2\,c\,(v-s)}{2\,c},\,v;\right. $$

$$\frac{p-b(u-2k)+2c(v-s)}{2c}+1,\ \ldots,\ \frac{p-b(u-2k)+2c(v-s)}{2c}+1;\ -e^{2cz}\Bigg)\Bigg)\Bigg)\Bigg)+$$

$$2^{-m-u}\,n!\sum_{k=0}^{\left\lfloor\frac{m-1}{2}\right\rfloor}(-1)^k\binom{m}{k}\sum_{i=0}^{\left\lfloor\frac{u-1}{2}\right\rfloor}\binom{u}{i}\Bigg(e^{\frac{i m\pi}{2}}\Bigg(i^v\,e^{(-i a(m-2k)+p-b(u-2i)+cv)z}\binom{v}{\frac{v}{2}}(1-v\bmod 2)$$

$$\sum_{j=0}^{n}\frac{(-1)^j\,(-i a(m-2k)+p-b(u-2i)+cv)^{-j-1}\,z^{n-j}}{(n-j)!}\;{}_{j+2}F_{j+1}\Bigg(\frac{-i a(m-2k)+p-b(u-2i)+cv}{2c},$$

$$\ldots,\ \frac{-i a(m-2k)+p-b(u-2i)+cv}{2c},\ v;\ \frac{-i a(m-2k)+p-b(u-2i)+cv}{2c}+1,$$

$$\ldots,\ \frac{-i a(m-2k)+p-b(u-2i)+cv}{2c}+1;\ -e^{2cz}\Bigg)+\sum_{s=0}^{\left\lfloor\frac{v-1}{2}\right\rfloor}(-1)^s\binom{v}{s}$$

$$\Bigg((-1)^v\,e^{(-i a(m-2k)+p+2cs-b(u-2i))z}\sum_{j=0}^{n}\frac{(-1)^j\,(-i a(m-2k)+p+2cs-b(u-2i))^{-j-1}\,z^{n-j}}{(n-j)!}$$

$${}_{j+2}F_{j+1}\Bigg(\frac{-i a(m-2k)+p+2cs-b(u-2i)}{2c},\ \ldots,$$

$$\frac{-i a(m-2k)+p+2cs-b(u-2i)}{2c},\ v;\ \frac{-i a(m-2k)+p+2cs-b(u-2i)}{2c}+1,$$

$$\ldots,\ \frac{-i a(m-2k)+p+2cs-b(u-2i)}{2c}+1;\ -e^{2cz}\Bigg)+$$

$$e^{(-i a(m-2k)+p-b(u-2i)+2c(v-s))z}\sum_{j=0}^{n}\frac{(-1)^j\,(-i a(m-2k)+p-b(u-2i)+2c(v-s))^{-j-1}\,z^{n-j}}{(n-j)!}$$

$${}_{j+2}F_{j+1}\Bigg(\frac{-i a(m-2k)+p-b(u-2i)+2c(v-s)}{2c},$$

$$\ldots,\ \frac{-i a(m-2k)+p-b(u-2i)+2c(v-s)}{2c},\ v;$$

$$\frac{-i a(m-2k)+p-b(u-2i)+2c(v-s)}{2c}+1,\ \ldots,$$

$$\frac{-i a(m-2k)+p-b(u-2i)+2c(v-s)}{2c}+1;\ -e^{2cz}\Bigg)\Bigg)\Bigg)+e^{-\frac{1}{2}i m\pi}$$

$$\Bigg(i^v\,e^{(a i(m-2k)+p-b(u-2i)+cv)z}\binom{v}{\frac{v}{2}}(1-v\bmod 2)\sum_{j=0}^{n}\frac{(-1)^j\,(a i(m-2k)+p-b(u-2i)+cv)^{-j-1}\,z^{n-j}}{(n-j)!}$$

$${}_{j+2}F_{j+1}\Bigg(\frac{i a(m-2k)+p-b(u-2i)+cv}{2c},\ \ldots,\ \frac{i a(m-2k)+p-b(u-2i)+cv}{2c},\ v;$$

$$\frac{i a(m-2k)+p-b(u-2i)+cv}{2c}+1,\ \ldots,\ \frac{i a(m-2k)+p-b(u-2i)+cv}{2c}+1;\ -e^{2cz}\Bigg)+$$

$$\sum_{s=0}^{\left\lfloor \frac{v-1}{2} \right\rfloor} (-1)^s \binom{v}{s} \Bigg((-1)^v e^{(a i (m-2k)+p+2cs-b(u-2i))z} \sum_{j=0}^{n} \frac{(-1)^j (a i (m-2k)+p+2cs-b(u-2i))^{-j-1} z^{n-j}}{(n-j)!}$$

$$_{j+2}F_{j+1}\Bigg(\frac{i a (m-2k)+p+2cs-b(u-2i)}{2c}, \ldots, \frac{i a (m-2k)+p+2cs-b(u-2i)}{2c}, v; \frac{i a (m-2k)+p+2cs-b(u-2i)}{2c}+1, \ldots, \frac{i a (m-2k)+p+2cs-b(u-2i)}{2c}+1; -e^{2cz}\Bigg) +$$

$$e^{(a i (m-2k)+p-b(u-2i)+2c(v-s))z} \sum_{j=0}^{n} \frac{(-1)^j (a i (m-2k)+p-b(u-2i)+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!}$$

$$_{j+2}F_{j+1}\Bigg(\frac{i a (m-2k)+p-b(u-2i)+2c(v-s)}{2c}, \ldots, \frac{i a (m-2k)+p-b(u-2i)+2c(v-s)}{2c}, v; \frac{i a (m-2k)+p-b(u-2i)+2c(v-s)}{2c}+1, \ldots, \frac{i a (m-2k)+p-b(u-2i)+2c(v-s)}{2c}+1; -e^{2cz}\Bigg)\Bigg)\Bigg) +$$

$$e^{\frac{i m \pi}{2}} \Bigg(i^v e^{(-i a (m-2k)+p+b(u-2i)+cv)z} \binom{v}{\frac{v}{2}} (1 - v \bmod 2) \sum_{j=0}^{n} \frac{(-1)^j (-i a (m-2k)+p+b(u-2i)+cv)^{-j-1} z^{n-j}}{(n-j)!}$$

$$_{j+2}F_{j+1}\Bigg(\frac{-i a (m-2k)+p+b(u-2i)+cv}{2c}, \ldots, \frac{-i a (m-2k)+p+b(u-2i)+cv}{2c}, v; \frac{-i a (m-2k)+p+b(u-2i)+cv}{2c}+1, \ldots, \frac{-i a (m-2k)+p+b(u-2i)+cv}{2c}+1; -e^{2cz}\Bigg) + \sum_{s=0}^{\left\lfloor \frac{v-1}{2} \right\rfloor} (-1)^s \binom{v}{s}$$

$$\Bigg((-1)^v e^{(-i a (m-2k)+p+2cs+b(u-2i))z} \sum_{j=0}^{n} \frac{(-1)^j (-i a (m-2k)+p+2cs+b(u-2i))^{-j-1} z^{n-j}}{(n-j)!}$$

$$_{j+2}F_{j+1}\Bigg(\frac{-i a (m-2k)+p+2cs+b(u-2i)}{2c}, \ldots, \frac{-i a (m-2k)+p+2cs+b(u-2i)}{2c}, v; \frac{-i a (m-2k)+p+2cs+b(u-2i)}{2c}+1, \ldots, \frac{-i a (m-2k)+p+2cs+b(u-2i)}{2c}+1; -e^{2cz}\Bigg) +$$

$$e^{(-i a (m-2k)+p+b(u-2i)+2c(v-s))z} \sum_{j=0}^{n} \frac{(-1)^j (-i a (m-2k)+p+b(u-2i)+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!}$$
$$_{j+2}F_{j+1}\Bigg(\frac{-i a (m-2k)+p+b(u-2i)+2c(v-s)}{2c}, \ldots, \frac{-i a (m-2k)+p+b(u-2i)+2c(v-s)}{2c}, v; \frac{-i a (m-2k)+p+b(u-2i)+2c(v-s)}{2c}+1, \ldots, \frac{-i a (m-2k)+p+b(u-2i)+2c(v-s)}{2c}+1; -e^{2cz}\Bigg)\Bigg)\Bigg) + e^{-\frac{1}{2} i m \pi}$$

$$\Bigg(i^v e^{(a i (m-2k)+p+b(u-2i)+cv)z} \binom{v}{\frac{v}{2}} (1 - v \bmod 2) \sum_{j=0}^{n} \frac{(-1)^j (a i (m-2k)+p+b(u-2i)+cv)^{-j-1} z^{n-j}}{(n-j)!}$$
$$_{j+2}F_{j+1}\Bigg(\frac{i a (m-2k)+p+b(u-2i)+cv}{2c}, \ldots, \frac{i a (m-2k)+p+b(u-2i)+cv}{2c}, v; \frac{i a (m-2k)+p+b(u-2i)+cv}{2c}+1, \ldots, \frac{i a (m-2k)+p+b(u-2i)+cv}{2c}+1; -e^{2cz}\Bigg) +$$
$$\sum_{s=0}^{\left\lfloor\frac{v-1}{2}\right\rfloor} (-1)^s \binom{v}{s} \Bigg((-1)^v e^{(a i (m-2k)+p+2cs+b(u-2i))z} \sum_{j=0}^{n} \frac{(-1)^j (a i (m-2k)+p+2cs+b(u-2i))^{-j-1} z^{n-j}}{(n-j)!}$$
$$_{j+2}F_{j+1}\Bigg(\frac{i a (m-2k)+p+2cs+b(u-2i)}{2c}, \ldots, \frac{i a (m-2k)+p+2cs+b(u-2i)}{2c}, v; \frac{i a (m-2k)+p+2cs+b(u-2i)}{2c}+1, \ldots, \frac{i a (m-2k)+p+2cs+b(u-2i)}{2c}+1; -e^{2cz}\Bigg) +$$
$$e^{(a i (m-2k)+p+b(u-2i)+2c(v-s))z} \sum_{j=0}^{n} \frac{(-1)^j (a i (m-2k)+p+b(u-2i)+2c(v-s))^{-j-1} z^{n-j}}{(n-j)!}$$
$$_{j+2}F_{j+1}\Bigg(\frac{i a (m-2k)+p+b(u-2i)+2c(v-s)}{2c}, \ldots, \frac{i a (m-2k)+p+b(u-2i)+2c(v-s)}{2c}, v; \frac{i a (m-2k)+p+b(u-2i)+2c(v-s)}{2c}+1, \ldots, \frac{i a (m-2k)+p+b(u-2i)+2c(v-s)}{2c}+1;$$
$$-e^{2cz}\Bigg)\Bigg)\Bigg)\Bigg) /; n \in \mathbb{N} \land m \in \mathbb{N}^+ \land u \in \mathbb{N}^+ \land v \in \mathbb{N}^+$$

Involving cos, cosh, exp and power

Involving $z^n e^{p z} \cos(a z) \cosh(b z) \tanh^{\nu}(c z)$

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$$\int z^n e^{p z} \cos(a z) \cosh(b z) \tanh^{\nu}(c z)\, dz =$$

$$\frac{1}{4} n! \left(i^{\nu} \binom{\nu}{\frac{\nu}{2}} (1 - \nu \bmod 2) \left(e^{(-b - i a + p + c\nu) z} \sum_{j=0}^{n} \frac{(-1)^j (-b - i a + p + c \nu)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{-i a - b + p + c\nu}{2c}, \right. \right. \right.$$

$$\left. \ldots, \frac{-i a - b + p + c\nu}{2c}, \nu; \frac{-i a - b + p + c\nu}{2c} + 1, \ldots, \frac{-i a - b + p + c\nu}{2c} + 1; -e^{2 c z} \right) +$$

$$e^{(-b + i a + p + c\nu) z} \sum_{j=0}^{n} \frac{(-1)^j (-b + i a + p + c \nu)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{i a - b + p + c\nu}{2c}, \ldots, \right.$$

$$\left. \frac{i a - b + p + c\nu}{2c}, \nu; \frac{i a - b + p + c\nu}{2c} + 1, \ldots, \frac{i a - b + p + c\nu}{2c} + 1; -e^{2 c z} \right) +$$

$$e^{(b - i a + p + c\nu) z} \sum_{j=0}^{n} \frac{(-1)^j (b - i a + p + c \nu)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{-i a + b + p + c\nu}{2c}, \ldots, \frac{-i a + b + p + c\nu}{2c}, \right.$$

$$\left. \nu; \frac{-i a + b + p + c\nu}{2c} + 1, \ldots, \frac{-i a + b + p + c\nu}{2c} + 1; -e^{2 c z} \right) +$$

$$e^{(b + i a + p + c\nu) z} \sum_{j=0}^{n} \frac{(-1)^j (b + i a + p + c \nu)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{i a + b + p + c\nu}{2c}, \ldots, \frac{i a + b + p + c\nu}{2c}, \right.$$

$$\left. \left. \left. \nu; \frac{i a + b + p + c\nu}{2c} + 1, \ldots, \frac{i a + b + p + c\nu}{2c} + 1; -e^{2 c z} \right) \right) \right) +$$

$$\sum_{s=0}^{\left\lfloor \frac{\nu-1}{2} \right\rfloor} (-1)^s \binom{\nu}{s} \left((-1)^{\nu} e^{(-b - i a + p + 2 c s) z} \sum_{j=0}^{n} \frac{(-1)^j (-b - i a + p + 2 c s)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{-i a - b + p + 2 c s}{2c}, \right. \right.$$

$$\left. \ldots, \frac{-i a - b + p + 2 c s}{2c}, \nu; \frac{-i a - b + p + 2 c s}{2c} + 1, \ldots, \frac{-i a - b + p + 2 c s}{2c} + 1; -e^{2 c z} \right) +$$

$$(-1)^{\nu} e^{(-b + i a + p + 2 c s) z} \sum_{j=0}^{n} \frac{(-1)^j (-b + i a + p + 2 c s)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{i a - b + p + 2 c s}{2c}, \ldots, \right.$$

$$\left. \frac{i a - b + p + 2 c s}{2c}, \nu; \frac{i a - b + p + 2 c s}{2c} + 1, \ldots, \frac{i a - b + p + 2 c s}{2c} + 1; -e^{2 c z} \right) +$$

$$(-1)^{\nu} e^{(b - i a + p + 2 c s) z} \sum_{j=0}^{n} \frac{(-1)^j (b - i a + p + 2 c s)^{-j-1} z^{n-j}}{(n-j)!} \, {}_{j+2}F_{j+1}\left(\frac{-i a + b + p + 2 c s}{2c}, \ldots, \right.$$

$$\left. \frac{-i a + b + p + 2 c s}{2c}, \nu; \frac{-i a + b + p + 2 c s}{2c} + 1, \ldots, \frac{-i a + b + p + 2 c s}{2c} + 1; -e^{2 c z} \right) +$$

$$(-1)^v e^{(b+i a+p+2 c s) z} \sum_{j=0}^{n} \frac{(-1)^j (b+i a+p+2 c s)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{i a+b+p+2 c s}{2 c}, \ldots, \frac{i a+b+p+2 c s}{2 c}, v; \frac{i a+b+p+2 c s}{2 c}+1, \ldots, \frac{i a+b+p+2 c s}{2 c}+1; -e^{2 c z}\right)+$$

$$e^{(-b-i a+p+2 c (v-s)) z} \sum_{j=0}^{n} \frac{(-1)^j (-b-i a+p+2 c (v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{-i a-b+p+2 c (v-s)}{2 c}, \ldots, \frac{-i a-b+p+2 c (v-s)}{2 c}, v; \frac{-i a-b+p+2 c (v-s)}{2 c}+1, \ldots, \frac{-i a-b+p+2 c (v-s)}{2 c}+1; -e^{2 c z}\right)+ e^{(-b+i a+p+2 c (v-s)) z}$$

$$\sum_{j=0}^{n} \frac{(-1)^j (-b+i a+p+2 c (v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{i a-b+p+2 c (v-s)}{2 c}, \ldots, \frac{i a-b+p+2 c (v-s)}{2 c}, v; \frac{i a-b+p+2 c (v-s)}{2 c}+1, \ldots, \frac{i a-b+p+2 c (v-s)}{2 c}+1; -e^{2 c z}\right)+ e^{(b-i a+p+2 c (v-s)) z}$$

$$\sum_{j=0}^{n} \frac{(-1)^j (b-i a+p+2 c (v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{-i a+b+p+2 c (v-s)}{2 c}, \ldots, \frac{-i a+b+p+2 c (v-s)}{2 c}, v; \frac{-i a+b+p+2 c (v-s)}{2 c}+1, \ldots, \frac{-i a+b+p+2 c (v-s)}{2 c}+1; -e^{2 c z}\right)+ e^{(b+i a+p+2 c (v-s)) z}$$

$$\sum_{j=0}^{n} \frac{(-1)^j (b+i a+p+2 c (v-s))^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{i a+b+p+2 c (v-s)}{2 c}, \ldots, \frac{i a+b+p+2 c (v-s)}{2 c}, v; \frac{i a+b+p+2 c (v-s)}{2 c}+1, \ldots, \frac{i a+b+p+2 c (v-s)}{2 c}+1; -e^{2 c z}\right)\Bigg)\Bigg) /; n \in \mathbb{N} \land v \in \mathbb{N}^+$$

Involving powers of cos, powers of cosh, exp and power

Involving $z^n e^{p z} \cos^m(a z) \cosh^u(b z) \tanh^v(c z)$

01.21.21.0432.01

$$\int z^n e^{p z} \cos^m(a z) \cosh^u(b z) \tanh^v(c z)\, dz = i^v 2^{-m-u} e^{(p+c v) z} \binom{m}{\frac{m}{2}} \binom{u}{\frac{u}{2}} \binom{v}{\frac{v}{2}} n! (1-m \bmod 2)(1-u \bmod 2)(1-v \bmod 2)$$

$$\sum_{j=0}^{n} \frac{(-1)^j (p+c v)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{p+c v}{2 c}, \ldots, \frac{p+c v}{2 c}, v; \frac{p+c v}{2 c}+1, \ldots, \frac{p+c v}{2 c}+1; -e^{2 c z}\right)+$$

$$2^{-m-u} \binom{m}{\frac{m}{2}} \binom{u}{\frac{u}{2}} n! (1-m \bmod 2)(1-u \bmod 2)$$

$$\sum_{s=0}^{\left\lfloor \frac{v-1}{2} \right\rfloor} (-1)^s \binom{v}{s} \Bigg((-1)^v e^{(p+2 c s) z} \sum_{j=0}^{n} \frac{(-1)^j (p+2 c s)^{-j-1} z^{n-j}}{(n-j)!} {}_{j+2}F_{j+1}\left(\frac{p+2 c s}{2 c}, \ldots, \frac{p+2 c s}{2 c}, v;\right.$$

$$\frac{p+2cs}{2c}+1,\ \ldots,\ \frac{p+2cs}{2c}+1;\ -e^{2cz}\Big)+e^{(p+2c(v-s))z}\sum_{j=0}^{n}\frac{(-1)^j\,(p+2c(v-s))^{-j-1}\,z^{n-j}}{(n-j)!}$$

$$_{j+2}F_{j+1}\Big(\frac{p+2c(v-s)}{2c},\ \ldots,\ \frac{p+2c(v-s)}{2c},\ v;\ \frac{p+2c(v-s)}{2c}+1,\ \ldots,\ \frac{p+2c(v-s)}{2c}+1;\ -e^{2cz}\Big)\Big)+$$

$$2^{-m-u}\binom{u}{\frac{u}{2}}n!\,(1-u\bmod 2)\sum_{k=0}^{\left\lfloor\frac{m-1}{2}\right\rfloor}\binom{m}{k}\Bigg(i^v\,e^{(-ia(m-2k)+p+cv)z}\binom{v}{\frac{v}{2}}(1-v\bmod 2)$$

$$\sum_{j=0}^{n}\frac{(-1)^j\,(-ia(m-2k)+p+cv)^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\Big(\frac{-ia(m-2k)+p+cv}{2c},\ \ldots,\ \frac{-ia(m-2k)+p+cv}{2c},$$

$$v;\ \frac{-ia(m-2k)+p+cv}{2c}+1,\ \ldots,\ \frac{-ia(m-2k)+p+cv}{2c}+1;\ -e^{2cz}\Big)+\sum_{s=0}^{\left\lfloor\frac{v-1}{2}\right\rfloor}(-1)^s\binom{v}{s}$$

$$\Big((-1)^v\,e^{(-ia(m-2k)+p+2cs)z}\sum_{j=0}^{n}\frac{(-1)^j\,(-ia(m-2k)+p+2cs)^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\Big(\frac{-ia(m-2k)+p+2cs}{2c},\ \ldots,$$

$$\frac{-ia(m-2k)+p+2cs}{2c},\ v;\ \frac{-ia(m-2k)+p+2cs}{2c}+1,\ \ldots,\ \frac{-ia(m-2k)+p+2cs}{2c}+1;$$

$$-e^{2cz}\Big)+e^{(-ia(m-2k)+p+2c(v-s))z}\sum_{j=0}^{n}\frac{(-1)^j\,(-ia(m-2k)+p+2c(v-s))^{-j-1}\,z^{n-j}}{(n-j)!}$$

$$_{j+2}F_{j+1}\Big(\frac{-ia(m-2k)+p+2c(v-s)}{2c},\ \ldots,\ \frac{-ia(m-2k)+p+2c(v-s)}{2c},\ v;$$

$$\frac{-ia(m-2k)+p+2c(v-s)}{2c}+1,\ \ldots,\ \frac{-ia(m-2k)+p+2c(v-s)}{2c}+1;\ -e^{2cz}\Big)\Big)+$$

$$i^v\,e^{(a\,i(m-2k)+p+cv)z}\binom{v}{\frac{v}{2}}(1-v\bmod 2)\sum_{j=0}^{n}\frac{(-1)^j\,(a\,i(m-2k)+p+cv)^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\Big(\frac{i\,a(m-2k)+p+cv}{2c},$$

$$\ldots,\ \frac{i\,a(m-2k)+p+cv}{2c},\ v;\ \frac{i\,a(m-2k)+p+cv}{2c}+1,\ \ldots,\ \frac{i\,a(m-2k)+p+cv}{2c}+1;\ -e^{2cz}\Big)+$$

$$\sum_{s=0}^{\left\lfloor\frac{v-1}{2}\right\rfloor}(-1)^s\binom{v}{s}\Big((-1)^v\,e^{(a\,i(m-2k)+p+2cs)z}\sum_{j=0}^{n}\frac{(-1)^j\,(a\,i(m-2k)+p+2cs)^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}$$

$$\Big(\frac{i\,a(m-2k)+p+2cs}{2c},\ \ldots,\ \frac{i\,a(m-2k)+p+2cs}{2c},\ v;$$

$$\frac{i\,a(m-2k)+p+2cs}{2c}+1,\ \ldots,\ \frac{i\,a(m-2k)+p+2cs}{2c}+1;\ -e^{2cz}\Big)+$$

$$e^{(a\,i(m-2k)+p+2c(v-s))z}\sum_{j=0}^{n}\frac{(-1)^j\,(a\,i(m-2k)+p+2c(v-s))^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}$$

$$\Big(\frac{i\,a(m-2k)+p+2c(v-s)}{2c},\ \ldots,\ \frac{i\,a(m-2k)+p+2c(v-s)}{2c},\ v;$$

$$\left.\left.\left.\frac{i\,a\,(m-2\,k)+p+2\,c\,(v-s)}{2\,c}+1,\,\ldots,\,\frac{i\,a\,(m-2\,k)+p+2\,c\,(v-s)}{2\,c}+1;\,-e^{2\,c\,z}\right)\right)\right)+$$

$$2^{-m-u}\binom{m}{\frac{m}{2}}n!\,(1-m\bmod 2)\sum_{k=0}^{\left\lfloor\frac{u-1}{2}\right\rfloor}\binom{u}{k}\left(i^{v}\,e^{(p+b\,(u-2\,k)+c\,v)\,z}\binom{v}{\frac{v}{2}}(1-v\bmod 2)\sum_{j=0}^{n}\frac{(-1)^{j}\,(p+b\,(u-2\,k)+c\,v)^{-j-1}\,z^{n-j}}{(n-j)!}\right.$$

$${}_{j+2}F_{j+1}\left(\frac{p+b\,(u-2\,k)+c\,v}{2\,c},\,\ldots,\,\frac{p+b\,(u-2\,k)+c\,v}{2\,c},\,v;\right.$$

$$\left.\frac{p+b\,(u-2\,k)+c\,v}{2\,c}+1,\,\ldots,\,\frac{p+b\,(u-2\,k)+c\,v}{2\,c}+1;\,-e^{2\,c\,z}\right)+$$

$$i^{v}\,e^{(p-b\,(u-2\,k)+c\,v)\,z}\binom{v}{\frac{v}{2}}(1-v\bmod 2)\sum_{j=0}^{n}\frac{(-1)^{j}\,(p-b\,(u-2\,k)+c\,v)^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{p-b\,(u-2\,k)+c\,v}{2\,c},\right.$$

$$\left.\ldots,\,\frac{p-b\,(u-2\,k)+c\,v}{2\,c},\,v;\,\frac{p-b\,(u-2\,k)+c\,v}{2\,c}+1,\,\ldots,\,\frac{p-b\,(u-2\,k)+c\,v}{2\,c}+1;\,-e^{2\,c\,z}\right)+$$

$$\sum_{s=0}^{\left\lfloor\frac{v-1}{2}\right\rfloor}(-1)^{s}\binom{v}{s}\left((-1)^{v}\,e^{(p+2\,c\,s+b\,(u-2\,k))\,z}\sum_{j=0}^{n}\frac{(-1)^{j}\,(p+2\,c\,s+b\,(u-2\,k))^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{p+2\,c\,s+b\,(u-2\,k)}{2\,c},\right.\right.$$

$$\left.\ldots,\,\frac{p+2\,c\,s+b\,(u-2\,k)}{2\,c},\,v;\,\frac{p+2\,c\,s+b\,(u-2\,k)}{2\,c}+1,\,\ldots,\,\frac{p+2\,c\,s+b\,(u-2\,k)}{2\,c}+1;\,-e^{2\,c\,z}\right)+$$

$$e^{(p+b\,(u-2\,k)+2\,c\,(v-s))\,z}\sum_{j=0}^{n}\frac{(-1)^{j}\,(p+b\,(u-2\,k)+2\,c\,(v-s))^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}$$

$$\left(\frac{p+b\,(u-2\,k)+2\,c\,(v-s)}{2\,c},\,\ldots,\,\frac{p+b\,(u-2\,k)+2\,c\,(v-s)}{2\,c},\,v;\right.$$

$$\left.\left.\frac{p+b\,(u-2\,k)+2\,c\,(v-s)}{2\,c}+1,\,\ldots,\,\frac{p+b\,(u-2\,k)+2\,c\,(v-s)}{2\,c}+1;\,-e^{2\,c\,z}\right)\right)+$$

$$\sum_{s=0}^{\left\lfloor\frac{v-1}{2}\right\rfloor}(-1)^{s}\binom{v}{s}\left((-1)^{v}\,e^{(p+2\,c\,s-b\,(u-2\,k))\,z}\sum_{j=0}^{n}\frac{(-1)^{j}\,(p+2\,c\,s-b\,(u-2\,k))^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}\left(\frac{p+2\,c\,s-b\,(u-2\,k)}{2\,c},\right.\right.$$

$$\left.\ldots,\,\frac{p+2\,c\,s-b\,(u-2\,k)}{2\,c},\,v;\,\frac{p+2\,c\,s-b\,(u-2\,k)}{2\,c}+1,\,\ldots,\,\frac{p+2\,c\,s-b\,(u-2\,k)}{2\,c}+1;\,-e^{2\,c\,z}\right)+$$

$$e^{(p-b\,(u-2\,k)+2\,c\,(v-s))\,z}\sum_{j=0}^{n}\frac{(-1)^{j}\,(p-b\,(u-2\,k)+2\,c\,(v-s))^{-j-1}\,z^{n-j}}{(n-j)!}\,{}_{j+2}F_{j+1}$$

$$\left(\frac{p-b\,(u-2\,k)+2\,c\,(v-s)}{2\,c},\,\ldots,\,\frac{p-b\,(u-2\,k)+2\,c\,(v-s)}{2\,c},\,v;\,\frac{p-b\,(u-2\,k)+2\,c\,(v-s)}{2\,c}+1,\right.$$

$$\left.\left.\left.\ldots,\,\frac{p-b\,(u-2\,k)+2\,c\,(v-s)}{2\,c}+1;\,-e^{2\,c\,z}\right)\right)\right)+2^{-m-u}\,n!\sum_{k=0}^{\left\lfloor\frac{m-1}{2}\right\rfloor}\binom{m}{k}$$

$$\sum_{i=0}^{\left\lfloor\frac{u-1}{2}\right\rfloor}\binom{u}{i}\left(i^v e^{(-i a(m-2k)+p+b(u-2i)+c v)z}\binom{v}{\frac{v}{2}}(1-v \bmod 2)\sum_{j=0}^{n}\frac{(-1)^j(-i a(m-2k)+p+b(u-2i)+c v)^{-j-1}z^{n-j}}{(n-j)!}\right.$$

$$_{j+2}F_{j+1}\left(\frac{-i a(m-2k)+p+b(u-2i)+c v}{2c},\ldots,\frac{-i a(m-2k)+p+b(u-2i)+c v}{2c},v;\right.$$

$$\left.\frac{-i a(m-2k)+p+b(u-2i)+c v}{2c}+1,\ldots,\frac{-i a(m-2k)+p+b(u-2i)+c v}{2c}+1;-e^{2cz}\right)+$$

$$\sum_{s=0}^{\left\lfloor\frac{v-1}{2}\right\rfloor}(-1)^s\binom{v}{s}\left((-1)^v e^{(-i a(m-2k)+p+2cs+b(u-2i))z}\sum_{j=0}^{n}\frac{1}{(n-j)!}\left((-1)^j(-i a(m-2k)+p+2cs+b(u-2i))^{-j-1}\right.\right.$$

$$\left.z^{n-j}\right)_{j+2}F_{j+1}\left(\frac{-i a(m-2k)+p+2cs+b(u-2i)}{2c},\ldots,\right.$$

$$\frac{-i a(m-2k)+p+2cs+b(u-2i)}{2c},v;\frac{-i a(m-2k)+p+2cs+b(u-2i)}{2c}+1,$$

$$\left.\ldots,\frac{-i a(m-2k)+p+2cs+b(u-2i)}{2c}+1;-e^{2cz}\right)+e^{(-i a(m-2k)+p+b(u-2i)+2c(v-s))z}$$

$$\sum_{j=0}^{n}\frac{1}{(n-j)!}\left((-1)^j(-i a(m-2k)+p+b(u-2i)+2c(v-s))^{-j-1}z^{n-j}\right)_{j+2}F_{j+1}$$

$$\left(\frac{-i a(m-2k)+p+b(u-2i)+2c(v-s)}{2c},\ldots,\frac{-i a(m-2k)+p+b(u-2i)+2c(v-s)}{2c},\right.$$

$$v;\frac{-i a(m-2k)+p+b(u-2i)+2c(v-s)}{2c}+1,\ldots,$$

$$\left.\left.\frac{-i a(m-2k)+p+b(u-2i)+2c(v-s)}{2c}+1;-e^{2cz}\right)\right)+$$

$$i^v e^{(a i(m-2k)+p+b(u-2i)+c v)z}\binom{v}{\frac{v}{2}}(1-v \bmod 2)\sum_{j=0}^{n}\frac{(-1)^j(a i(m-2k)+p+b(u-2i)+c v)^{-j-1}z^{n-j}}{(n-j)!}$$

$$_{j+2}F_{j+1}\left(\frac{i a(m-2k)+p+b(u-2i)+c v}{2c},\ldots,\frac{i a(m-2k)+p+b(u-2i)+c v}{2c},v;\right.$$

$$\left.\frac{i a(m-2k)+p+b(u-2i)+c v}{2c}+1,\ldots,\frac{i a(m-2k)+p+b(u-2i)+c v}{2c}+1;-e^{2cz}\right)+$$

$$\sum_{s=0}^{\left\lfloor\frac{v-1}{2}\right\rfloor}(-1)^s\binom{v}{s}\left((-1)^v e^{(a i(m-2k)+p+2cs+b(u-2i))z}\sum_{j=0}^{n}\frac{1}{(n-j)!}\left((-1)^j(a i(m-2k)+p+2cs+b(u-2i))^{-j-1}\right.\right.$$

$$\left.z^{n-j}\right)_{j+2}F_{j+1}\left(\frac{i a(m-2k)+p+2cs+b(u-2i)}{2c},\ldots,\frac{i a(m-2k)+p+2cs+b(u-2i)}{2c},\right.$$

$$v;\frac{i a(m-2k)+p+2cs+b(u-2i)}{2c}+1,\ldots,\frac{i a(m-2k)+p+2cs+b(u-2i)}{2c}+1;$$

$$\left.-e^{2cz}\right)+e^{(a i(m-2k)+p+b(u-2i)+2c(v-s))z}\sum_{j=0}^{n}\frac{1}{(n-j)!}\left((-1)^j(a i(m-2k)+p+b(u-2i)+\right.$$

$$2\,c\,(v-s))^{-j-1}\,z^{n-j}\Big)\,{}_{j+2}F_{j+1}\Bigg(\frac{i\,a\,(m-2\,k)+p+b\,(u-2\,i)+2\,c\,(v-s)}{2\,c},\,\ldots,$$

$$\frac{i\,a\,(m-2\,k)+p+b\,(u-2\,i)+2\,c\,(v-s)}{2\,c},\,v;\,\frac{i\,a\,(m-2\,k)+p+b\,(u-2\,i)+2\,c\,(v-s)}{2\,c}+1,$$

$$\ldots,\,\frac{i\,a\,(m-2\,k)+p+b\,(u-2\,i)+2\,c\,(v-s)}{2\,c}+1;\,-e^{2\,c\,z}\Bigg)\Bigg)+$$

$$i^{v}\,e^{(-i\,a\,(m-2\,k)+p-b\,(u-2\,i)+c\,v)\,z}\binom{v}{\frac{v}{2}}(1-v\bmod 2)\sum_{j=0}^{n}\frac{(-1)^{j}\,(-i\,a\,(m-2\,k)+p-b\,(u-2\,i)+c\,v)^{-j-1}\,z^{n-j}}{(n-j)!}$$

$${}_{j+2}F_{j+1}\Bigg(\frac{-i\,a\,(m-2\,k)+p-b\,(u-2\,i)+c\,v}{2\,c},\,\ldots,\,\frac{-i\,a\,(m-2\,k)+p-b\,(u-2\,i)+c\,v}{2\,c},\,v;$$

$$\frac{-i\,a\,(m-2\,k)+p-b\,(u-2\,i)+c\,v}{2\,c}+1,\,\ldots,\,\frac{-i\,a\,(m-2\,k)+p-b\,(u-2\,i)+c\,v}{2\,c}+1;\,-e^{2\,c\,z}\Bigg)+$$

$$\sum_{s=0}^{\left\lfloor\frac{v-1}{2}\right\rfloor}(-1)^{s}\binom{v}{s}\Bigg((-1)^{v}\,e^{(-i\,a\,(m-2\,k)+p+2\,c\,s-b\,(u-2\,i))\,z}\sum_{j=0}^{n}\frac{1}{(n-j)!}\Big((-1)^{j}\,(-i\,a\,(m-2\,k)+p+2\,c\,s-b\,(u-2\,i))^{-j-1}$$

$$z^{n-j}\Big)\,{}_{j+2}F_{j+1}\Bigg(\frac{-i\,a\,(m-2\,k)+p+2\,c\,s-b\,(u-2\,i)}{2\,c},\,\ldots,$$

$$\frac{-i\,a\,(m-2\,k)+p+2\,c\,s-b\,(u-2\,i)}{2\,c},\,v;\,\frac{-i\,a\,(m-2\,k)+p+2\,c\,s-b\,(u-2\,i)}{2\,c}+1,$$

$$\ldots,\,\frac{-i\,a\,(m-2\,k)+p+2\,c\,s-b\,(u-2\,i)}{2\,c}+1;\,-e^{2\,c\,z}\Bigg)+e^{(-i\,a\,(m-2\,k)+p-b\,(u-2\,i)+2\,c\,(v-s))\,z}$$

$$\sum_{j=0}^{n}\frac{1}{(n-j)!}\Big((-1)^{j}\,(-i\,a\,(m-2\,k)+p-b\,(u-2\,i)+2\,c\,(v-s))^{-j-1}\,z^{n-j}\Big)\,{}_{j+2}F_{j+1}$$

$$\Bigg(\frac{-i\,a\,(m-2\,k)+p-b\,(u-2\,i)+2\,c\,(v-s)}{2\,c},\,\ldots,\,\frac{-i\,a\,(m-2\,k)+p-b\,(u-2\,i)+2\,c\,(v-s)}{2\,c},$$

$$v;\,\frac{-i\,a\,(m-2\,k)+p-b\,(u-2\,i)+2\,c\,(v-s)}{2\,c}+1,\,\ldots,$$

$$\frac{-i\,a\,(m-2\,k)+p-b\,(u-2\,i)+2\,c\,(v-s)}{2\,c}+1;\,-e^{2\,c\,z}\Bigg)\Bigg)+$$

$$i^{v}\,e^{(a\,i\,(m-2\,k)+p-b\,(u-2\,i)+c\,v)\,z}\binom{v}{\frac{v}{2}}(1-v\bmod 2)\sum_{j=0}^{n}\frac{(-1)^{j}\,(a\,i\,(m-2\,k)+p-b\,(u-2\,i)+c\,v)^{-j-1}\,z^{n-j}}{(n-j)!}$$

$${}_{j+2}F_{j+1}\Bigg(\frac{i\,a\,(m-2\,k)+p-b\,(u-2\,i)+c\,v}{2\,c},\,\ldots,\,\frac{i\,a\,(m-2\,k)+p-b\,(u-2\,i)+c\,v}{2\,c},\,v;$$

$$\frac{i\,a\,(m-2\,k)+p-b\,(u-2\,i)+c\,v}{2\,c}+1,\,\ldots,\,\frac{i\,a\,(m-2\,k)+p-b\,(u-2\,i)+c\,v}{2\,c}+1;\,-e^{2\,c\,z}\Bigg)+$$

$$\sum_{s=0}^{\left\lfloor\frac{v-1}{2}\right\rfloor}(-1)^{s}\binom{v}{s}\Bigg((-1)^{v}\,e^{(a\,i\,(m-2\,k)+p+2\,c\,s-b\,(u-2\,i))\,z}\sum_{j=0}^{n}\frac{1}{(n-j)!}\Big((-1)^{j}\,(a\,i\,(m-2\,k)+p+2\,c\,s-b\,(u-2\,i))^{-j-1}$$

$$z^{n-j}\Big)\,{}_{j+2}F_{j+1}\Bigg(\frac{i\,a\,(m-2\,k)+p+2\,c\,s-b\,(u-2\,i)}{2\,c},\,\ldots,\,\frac{i\,a\,(m-2\,k)+p+2\,c\,s-b\,(u-2\,i)}{2\,c},$$

$$v;\,\frac{i\,a\,(m-2\,k)+p+2\,c\,s-b\,(u-2\,i)}{2\,c}+1,\,\ldots,\,\frac{i\,a\,(m-2\,k)+p+2\,c\,s-b\,(u-2\,i)}{2\,c}+1;$$

$$-e^{2\,c\,z}\Bigg)+e^{(a\,i\,(m-2\,k)+p-b\,(u-2\,i)+2\,c\,(v-s))\,z}\sum_{j=0}^{n}\frac{1}{(n-j)!}\Big((-1)^j\,(a\,i\,(m-2\,k)+p-b\,(u-2\,i)+$$

$$2\,c\,(v-s))^{-j-1}\,z^{n-j}\Big)\,{}_{j+2}F_{j+1}\Bigg(\frac{i\,a\,(m-2\,k)+p-b\,(u-2\,i)+2\,c\,(v-s)}{2\,c},\,\ldots,$$

$$\frac{i\,a\,(m-2\,k)+p-b\,(u-2\,i)+2\,c\,(v-s)}{2\,c},\,v;\,\frac{i\,a\,(m-2\,k)+p-b\,(u-2\,i)+2\,c\,(v-s)}{2\,c}+1,$$

$$\ldots,\,\frac{i\,a\,(m-2\,k)+p-b\,(u-2\,i)+2\,c\,(v-s)}{2\,c}+1;$$

$$-e^{2\,c\,z}\Bigg)\Bigg)\Bigg)\,/;\,n\in\mathbb{N}\bigwedge m\in\mathbb{N}^+\bigwedge u\in\mathbb{N}^+\bigwedge v\in\mathbb{N}^+$$

Definite integration

For the direct function itself

01.21.21.0009.01

$$\int_a^b \tanh(t)\,dt = \log(\cosh(b))-\log(\cosh(a))$$

Involving related functions

01.21.21.0010.01

$$\int_0^\infty e^{-t}\tanh(t)\,dt = \frac{\pi}{2}-1$$

01.21.21.0011.01

$$\int_0^\infty t\,e^{-t}\tanh(t)\,dt = 2\,C-1$$

01.21.21.0012.01

$$\int_0^\infty t^2\,e^{-t}\tanh(t)\,dt = \frac{\pi^3}{8}-2$$

01.21.21.0013.01

$$\int_0^\infty \frac{e^{-t}\tanh(t)}{t}\,dt = -\log(4)+2\log\left(\Gamma\left(\frac{1}{4}\right)\right)-2\log\left(\Gamma\left(\frac{3}{4}\right)\right)$$

Summation

Finite summation

01.21.23.0001.01

$$\sum_{k=0}^{n}\frac{1}{2^k}\tanh\left(\frac{a}{2^k}\right) = 2\coth(2\,a)-\frac{1}{2^n}\coth\left(\frac{a}{2^n}\right)$$

01.21.23.0002.01

$$\sum_{k=0}^{n-1} \tanh^2\left(\frac{i\,\pi\,k}{n} + z\right) == \coth^2\left(z\,n + \frac{i\,\pi\,n}{2}\right) n^2 - n^2 + n \;/;\; n \in \mathbb{N}^+$$

Infinite summation

01.21.23.0003.01

$$\sum_{k=1}^{\infty} \frac{\tanh\left(\frac{z}{2^k}\right)}{2^k} == \coth(z) - \frac{1}{z}$$

01.21.23.0004.01

$$\sum_{k=1}^{\infty} \frac{\tanh^2\left(\frac{z}{2^k}\right)}{2^{2\,k}} == \operatorname{csch}^2(z) + \frac{1}{3} - \frac{1}{z^2}$$

Representations through more general functions

Through hypergeometric functions

01.21.26.0007.01

$$\tanh(z) = \frac{8\,z}{4\,z^2 + \pi^2}\, {}_3F_2\left(1, \frac{1}{2} - \frac{i\,z}{\pi}, \frac{i\,z}{\pi} + \frac{1}{2}; \frac{3}{2} - \frac{i\,z}{\pi}, \frac{i\,z}{\pi} + \frac{3}{2}; 1\right)$$

Brychkov Yu.A. (2005)

Through other functions

Involving Jacobi functions

01.21.26.0001.01

$$\tanh(z) == -i\operatorname{cs}\left(\frac{\pi}{2} - i\,z \,\middle|\, 0\right)$$

01.21.26.0002.01

$$\tanh(z) == -\operatorname{ns}\left(\frac{\pi\,i}{2} - z \,\middle|\, 1\right)$$

01.21.26.0003.01

$$\tanh(z) == -i\operatorname{sc}(i\,z \mid 0)$$

01.21.26.0004.01

$$\tanh(z) == \operatorname{sn}(z \mid 1)$$

Involving Mathieu functions

01.21.26.0005.01

$$\tanh\left(\sqrt{a}\; z\right) == -\frac{i\,\text{MathieuS}(a, 0, i\,z)}{\text{MathieuC}(a, 0, i\,z)}$$

01.21.26.0006.01

$$\tanh\left(\sqrt{a}\; z\right) == \frac{i\,\text{MathieuCPrime}(a, 0, i\,z)}{\text{MathieuSPrime}(a, 0, i\,z)}$$

Representations through equivalent functions

With inverse function

01.21.27.0001.01

$$\tanh\left(\tanh^{-1}(z)\right) = z$$

01.21.27.0002.01

$$\tanh\left(n \tanh^{-1}(z)\right) = \frac{(1+z)^n - (1-z)^n}{(1+z)^n + (1-z)^n} /; n \in \mathbb{N}^+$$

01.21.27.0003.02

$$\tanh^{-1}(\tanh(z)) = z /; -\frac{\pi}{2} < \mathrm{Im}(z) < \frac{\pi}{2} \bigvee \left(\mathrm{Im}(z) = -\frac{\pi}{2} \bigwedge \mathrm{Re}(z) > 0\right) \bigvee \left(\mathrm{Im}(z) = \frac{\pi}{2} \bigwedge \mathrm{Re}(z) < 0\right)$$

01.21.27.0081.01

$$\tanh^{-1}(\tanh(z)) = z + \pi i /; -\frac{3\pi}{2} < \mathrm{Im}(z) < -\frac{\pi}{2} \bigvee \mathrm{Im}(z) = -\frac{3\pi}{2} \bigwedge \mathrm{Re}(z) > 0 \bigvee \mathrm{Im}(z) = -\frac{\pi}{2} \bigwedge \mathrm{Re}(z) < 0$$

01.21.27.0082.01

$$\tanh^{-1}(\tanh(z)) = z - \pi i /; \frac{\pi}{2} < \mathrm{Im}(z) < \frac{3\pi}{2} \bigvee \mathrm{Im}(z) = \frac{\pi}{2} \bigwedge \mathrm{Re}(z) > 0 \bigvee \mathrm{Im}(z) = \frac{3\pi}{2} \bigwedge \mathrm{Re}(z) < 0$$

01.21.27.0083.01

$$\tanh^{-1}(\tanh(z)) = z - \pi i k /; \left(k\pi - \frac{\pi}{2} < \mathrm{Im}(z) < \pi k + \frac{\pi}{2} \bigvee \mathrm{Im}(z) = k\pi - \frac{\pi}{2} \bigwedge \mathrm{Re}(z) > 0 \bigvee \mathrm{Im}(z) = \pi k + \frac{\pi}{2} \bigwedge \mathrm{Re}(z) < 0\right) \bigwedge k \in \mathbb{Z}$$

01.21.27.0004.01

$$\tanh^{-1}(\tanh(z)) = z + i\pi \left\lfloor \frac{1}{2} - \frac{\mathrm{Im}(z)}{\pi} \right\rfloor - \frac{\pi i}{2} \left(1 + (-1)^{\left\lfloor \frac{\mathrm{Im}(z)}{\pi} + \frac{1}{2} \right\rfloor + \left\lfloor -\frac{\mathrm{Im}(z)}{\pi} - \frac{1}{2} \right\rfloor}\right) \theta(\mathrm{Re}(z)) /; \frac{i z}{\pi} - \frac{1}{2} \notin \mathbb{Z}$$

01.21.27.0084.01

$$\tanh^{-1}(\tanh(z)) = \begin{cases} ¿ & \frac{\pi - 2 i z}{2\pi} \in \mathbb{Z} \\ z - \pi i \left\lfloor \frac{2\,\mathrm{Im}(z) - \pi}{2\pi} \right\rfloor & \frac{2\,\mathrm{Im}(z) + \pi}{2\pi} \in \mathbb{Z} \bigwedge \mathrm{Re}(z) < 0 \\ z - \pi i \left\lfloor \frac{2\,\mathrm{Im}(z) + \pi}{2\pi} \right\rfloor & \mathrm{True} \end{cases}$$

With related functions

Involving exp

01.21.27.0005.01

$$\tanh(z) = \frac{e^z - e^{-z}}{e^z + e^{-z}}$$

01.21.27.0006.01

$$\tanh(z) = \frac{2}{e^{-2z} + 1} - 1$$

Involving sin

01.21.27.0007.01

$$\tanh(z) = -\frac{i\sin(i z)}{\sin\left(\frac{\pi}{2} - i z\right)}$$

01.21.27.0008.01

$$\tanh(z) = -\frac{i\sin(i z)}{\sin\left(\frac{\pi}{2} + i z\right)}$$

01.21.27.0009.01

$$\tanh(z) = -\frac{i\sin(i z)}{\sqrt{1-\sin^2(i z)}} \;/;\; |\text{Im}(z)| < \frac{\pi}{2}$$

01.21.27.0010.01

$$\tanh(z) = -\frac{i\sin(i z)}{\sqrt{1-\sin^2(i z)}} (-1)^{\left\lfloor \frac{\text{Im}(z)}{\pi} + \frac{1}{2} \right\rfloor} \left(1 - \left(1 + (-1)^{\left\lfloor \frac{\text{Im}(z)}{\pi} + \frac{1}{2} \right\rfloor + \left\lfloor -\frac{\text{Im}(z)}{\pi} - \frac{1}{2} \right\rfloor}\right) \theta(\text{Re}(z))\right)$$

01.21.27.0011.01

$$\tanh^2(z) = \frac{\sin^2(i z)}{\sin^2(i z) - 1}$$

Involving cos

01.21.27.0012.01

$$\tanh(z) = -\frac{i\cos\left(\frac{\pi}{2} - i z\right)}{\cos(i z)}$$

01.21.27.0013.01

$$\tanh(z) = \frac{i\cos\left(\frac{\pi}{2} + i z\right)}{\cos(i z)}$$

01.21.27.0014.01

$$\tanh(z) = \frac{\sqrt{z^2}}{z} \frac{\sqrt{\cos^2(i z) - 1}}{\cos(i z)} \;/;\; |\text{Im}(z)| < \frac{\pi}{2}$$

01.21.27.0015.01

$$\tanh(z) = -\frac{\sqrt{-z^2}}{z} \frac{\sqrt{1 - \cos^2(i z)}}{\cos(i z)} \;/;\; |\text{Im}(z)| < \pi$$

01.21.27.0016.01

$$\tanh(z) = \frac{i\sqrt{1-\cos^2(i z)}}{\cos(i z)} \;/;\; 0 < \text{Im}(z) < \pi$$

01.21.27.0017.01

$$\tanh(z) = -\frac{i\sqrt{1-\cos^2(i z)}}{\cos(i z)} (-1)^{\left\lfloor -\frac{\text{Im}(z)}{\pi} \right\rfloor} \left(1 - \left(1 + (-1)^{\left\lfloor \frac{\text{Im}(z)}{\pi} \right\rfloor + \left\lfloor -\frac{\text{Im}(z)}{\pi} \right\rfloor}\right) \theta(-\text{Re}(z))\right)$$

01.21.27.0018.01

$$\tanh^2(z) = \frac{\cos^2(i z) - 1}{\cos^2(i z)}$$

Involving tan

01.21.27.0019.01

$$\tanh(z) = -i\tan(i\,z)$$

01.21.27.0020.01

$$\tanh(i\,z) = i\tan(z)$$

Involving cot

01.21.27.0021.01

$$\tanh(z) = -i\cot\left(\frac{\pi}{2} - i\,z\right)$$

01.21.27.0022.01

$$\tanh(z) = i\cot\left(\frac{\pi}{2} + i\,z\right)$$

01.21.27.0023.01

$$\tanh(z) = -\frac{i}{\cot(i\,z)}$$

Involving csc

01.21.27.0024.01

$$\tanh(z) = -\frac{i\csc\left(\frac{\pi}{2} - i\,z\right)}{\csc(i\,z)}$$

01.21.27.0025.01

$$\tanh(z) = -\frac{i\csc\left(\frac{\pi}{2} + i\,z\right)}{\csc(i\,z)}$$

01.21.27.0026.01

$$\tanh(z) = e^z\csc\left(\frac{\pi}{2} - i\,z\right) - 1$$

01.21.27.0027.01

$$\tanh(z) = 1 - e^{-z}\csc\left(\frac{\pi}{2} - i\,z\right)$$

01.21.27.0028.01

$$\tanh(z) = \frac{\sqrt{z^2}}{z}\,\frac{1}{\sqrt{1-\csc^2(i\,z)}}\ /;\ \mathrm{Re}(z) \neq 0$$

01.21.27.0029.01

$$\tanh(z) = \frac{i}{\sqrt{\csc^2(i\,z) - 1}}\ /;\ 0 < \mathrm{Im}(z) < \frac{\pi}{2}$$

01.21.27.0030.01

$$\tanh(z) = z\sqrt{-\frac{1}{z^2}}\,\frac{i}{\sqrt{\csc^2(i\,z) - 1}}\ /;\ |\mathrm{Im}(z)| < \frac{\pi}{2}$$

01.21.27.0031.01

$$\tanh(z) = -\frac{i}{\sqrt{\csc^2(i z) - 1}} (-1)^{\left\lfloor -\frac{2 \operatorname{Im}(z)}{\pi} \right\rfloor} \left(1 - \left(1 + (-1)^{\left\lfloor \frac{\operatorname{Im}(z)}{\pi} - \frac{1}{2} \right\rfloor + \left\lfloor \frac{1}{2} - \frac{\operatorname{Im}(z)}{\pi} \right\rfloor}\right) \theta(-\operatorname{Re}(z))\right) \left(1 - \left(1 + (-1)^{\left\lfloor \frac{\operatorname{Im}(z)}{\pi} \right\rfloor + \left\lfloor -\frac{\operatorname{Im}(z)}{\pi} \right\rfloor}\right) \theta(\operatorname{Re}(z))\right)$$

01.21.27.0032.01

$$\tanh^2(z) = \frac{1}{1 - \csc^2(i z)}$$

01.21.27.0033.01

$$\tanh^2(z) = 1 - \csc^2\left(\frac{\pi}{2} - i z\right)$$

Involving sec

01.21.27.0034.01

$$\tanh(z) = -\frac{i \sec(i z)}{\sec\left(\frac{\pi}{2} - i z\right)}$$

01.21.27.0035.01

$$\tanh(z) = \frac{i \sec(i z)}{\sec\left(\frac{\pi}{2} + i z\right)}$$

01.21.27.0036.01

$$\tanh(z) = 1 - e^{-z} \sec(i z)$$

01.21.27.0037.01

$$\tanh(z) = e^{z} \sec(i z) - 1$$

01.21.27.0038.01

$$\tanh(z) = \frac{\sqrt{z^2}}{z} \sqrt{1 - \sec^2(i z)} \;/;\; \operatorname{Im}(z) \neq 0$$

01.21.27.0039.01

$$\tanh(z) = i \sqrt{\sec^2(i z) - 1} \;/;\; 0 < \operatorname{Im}(z) < \frac{\pi}{2}$$

01.21.27.0040.01

$$\tanh(z) = -\frac{\sqrt{-z^2}}{z} \sqrt{\sec^2(i z) - 1} \;/;\; |\operatorname{Im}(z)| < \frac{\pi}{2}$$

01.21.27.0041.01

$$\tanh(z) = -i \sqrt{\sec^2(i z) - 1} \, (-1)^{\left\lfloor -\frac{2 \operatorname{Im}(z)}{\pi} \right\rfloor} \left(1 - \left(1 + (-1)^{\left\lfloor \frac{\operatorname{Im}(z)}{\pi} \right\rfloor + \left\lfloor -\frac{\operatorname{Im}(z)}{\pi} \right\rfloor}\right) \theta(-\operatorname{Re}(z))\right) \left(1 - \left(1 + (-1)^{\left\lfloor \frac{\operatorname{Im}(z)}{\pi} - \frac{1}{2} \right\rfloor + \left\lfloor \frac{1}{2} - \frac{\operatorname{Im}(z)}{\pi} \right\rfloor}\right) \theta(\operatorname{Re}(z))\right)$$

01.21.27.0042.01

$$\tanh^2(z) = 1 - \sec^2(i z)$$

Involving sinh

01.21.27.0043.01

$$\tanh(z) = \frac{i \sinh(z)}{\sinh\left(\frac{\pi i}{2} - z\right)}$$

01.21.27.0044.01

$$\tanh(z) = \frac{i \sinh(z)}{\sinh\left(\frac{\pi i}{2} + z\right)}$$

01.21.27.0045.01

$$\tanh(z) = \frac{\sinh(z)}{\sqrt{\sinh^2(z) + 1}} \;/;\; |\mathrm{Im}(z)| < \frac{\pi}{2}$$

01.21.27.0046.01

$$\tanh(z) = \frac{\sinh(z)}{\sqrt{1 + \sinh^2(z)}} (-1)^{\left\lfloor \frac{\mathrm{Im}(z)}{\pi} + \frac{1}{2} \right\rfloor} \left(1 - \left(1 + (-1)^{\left\lfloor \frac{\mathrm{Im}(z)}{\pi} + \frac{1}{2} \right\rfloor + \left\lfloor -\frac{\mathrm{Im}(z)}{\pi} - \frac{1}{2} \right\rfloor}\right) \theta(\mathrm{Re}(z))\right)$$

01.21.27.0047.01

$$\tanh^2(z) = \frac{\sinh^2(z)}{\sinh^2(z) + 1}$$

Involving cosh

01.21.27.0048.01

$$\tanh(z) = \frac{i \cosh\left(\frac{\pi i}{2} - z\right)}{\cosh(z)}$$

01.21.27.0049.01

$$\tanh(z) = -\frac{i \cosh\left(\frac{\pi i}{2} + z\right)}{\cosh(z)}$$

01.21.27.0050.01

$$\tanh(z) = -\frac{\sqrt{-z^2} \sqrt{1 - \cosh^2(z)}}{z \cosh(z)} \;/;\; |\mathrm{Im}(z)| < \pi$$

01.21.27.0051.01

$$\tanh(z) = \frac{i \sqrt{1 - \cosh^2(z)}}{\cosh(z)} \;/;\; 0 < \mathrm{Im}(z) < \pi$$

01.21.27.0052.01

$$\tanh(z) = \frac{\sqrt{z^2}}{z} \frac{\sqrt{\cosh^2(z) - 1}}{\cosh(z)} \;/;\; |\mathrm{Im}(z)| < \frac{\pi}{2}$$

01.21.27.0053.01

$$\tanh(z) = \frac{\sqrt{z^2}}{z} \frac{\sqrt{\cosh^2(z) - 1}}{\cosh(z)} (-1)^{\left\lfloor \frac{\mathrm{Im}(z)}{\pi} + \frac{1}{2} \right\rfloor} \left(1 - \left((-1)^{\left\lfloor \frac{\mathrm{Im}(z)}{\pi} + \frac{1}{2} \right\rfloor + \left\lfloor -\frac{\mathrm{Im}(z)}{\pi} - \frac{1}{2} \right\rfloor} + 1\right) \theta(\mathrm{Re}(z))\right)$$

01.21.27.0054.01

$$\tanh^2(z) = \frac{\cosh^2(z) - 1}{\cosh^2(z)}$$

Involving coth

01.21.27.0055.01

$$\tanh(z) = -\coth\left(\frac{\pi i}{2} - z\right)$$

01.21.27.0056.01

$$\tanh(z) = \coth\left(\frac{\pi i}{2} + z\right)$$

01.21.27.0057.01

$$\tanh(z) = \coth\left(z - \frac{\pi i}{2}\right)$$

01.21.27.0058.01

$$\tanh(z) = \frac{1}{\coth(z)}$$

01.21.27.0059.01

$$\tanh(z) = \frac{2\coth\left(\frac{z}{2}\right)}{\coth^2\left(\frac{z}{2}\right) + 1}$$

01.21.27.0060.01

$$\tanh\left(\frac{\pi i}{2} + z\right) = \coth(z)$$

01.21.27.0061.01

$$\tanh\left(\frac{\pi i}{2} - z\right) = -\coth(z)$$

01.21.27.0062.01

$$\tanh(z) = 2\coth(2z) - \coth(z)$$

Involving csch

01.21.27.0063.01

$$\tanh(z) = \frac{i\,\mathrm{csch}\left(\frac{\pi i}{2} - z\right)}{\mathrm{csch}(z)}$$

01.21.27.0064.01

$$\tanh(z) = \frac{i\,\mathrm{csch}\left(\frac{\pi i}{2} + z\right)}{\mathrm{csch}(z)}$$

01.21.27.0065.01

$$\tanh(z) = 1 - i\,e^{-z}\,\mathrm{csch}\left(\frac{\pi i}{2} - z\right)$$

01.21.27.0066.01

$$\tanh(z) = i\,e^{z}\,\mathrm{csch}\left(\frac{\pi i}{2} - z\right) - 1$$

01.21.27.0067.01

$$\tanh(z) = \frac{\sqrt{z^2}}{z}\,\frac{1}{\sqrt{1 + \mathrm{csch}^2(z)}} \;/;\; \mathrm{Re}(z) \neq 0$$

01.21.27.0068.01

$$\tanh^2(z) == \frac{1}{\operatorname{csch}^2(z) + 1}$$

01.21.27.0069.01

$$\tanh^2(z) == \operatorname{csch}^2\left(\frac{\pi i}{2} - z\right) + 1$$

Involving sech

01.21.27.0070.01

$$\tanh(z) == \frac{i \operatorname{sech}(z)}{\operatorname{sech}\left(\frac{\pi i}{2} - z\right)}$$

01.21.27.0071.01

$$\tanh(z) == -\frac{i \operatorname{sech}(z)}{\operatorname{sech}\left(\frac{\pi i}{2} + z\right)}$$

01.21.27.0072.01

$$\tanh(z) == 1 - e^{-z} \operatorname{sech}(z)$$

01.21.27.0073.01

$$\tanh(z) == e^z \operatorname{sech}(z) - 1$$

01.21.27.0074.01

$$\tanh(z) == \frac{\sqrt{z^2}}{z} \sqrt{1 - \operatorname{sech}^2(z)} \;/;\; \operatorname{Re}(z) \neq 0$$

01.21.27.0075.01

$$\tanh(z) == i \sqrt{\operatorname{sech}^2(z) - 1} \;/;\; 0 < \operatorname{Im}(z) < \frac{\pi}{2}$$

01.21.27.0076.01

$$\tanh(z) == -\frac{\sqrt{-z^2}}{z} \sqrt{\operatorname{sech}^2(z) - 1} \;/;\; |\operatorname{Im}(z)| < \frac{\pi}{2}$$

01.21.27.0077.01

$$\tanh(z) == -i \sqrt{\operatorname{sech}^2(z) - 1}\, (-1)^{\left\lfloor -\frac{2 \operatorname{Im}(z)}{\pi} \right\rfloor} \left(1 - \left(1 + (-1)^{\left\lfloor \frac{\operatorname{Im}(z)}{\pi} \right\rfloor + \left\lfloor -\frac{\operatorname{Im}(z)}{\pi} \right\rfloor}\right) \theta(-\operatorname{Re}(z))\right) \left(1 - \left(1 + (-1)^{\left\lfloor \frac{\operatorname{Im}(z)}{\pi} - \frac{1}{2} \right\rfloor + \left\lfloor \frac{1}{2} - \frac{\operatorname{Im}(z)}{\pi} \right\rfloor}\right) \theta(\operatorname{Re}(z))\right)$$

01.21.27.0078.01

$$\tanh^2(z) == 1 - \operatorname{sech}^2(z)$$

Involving trigonometric and hyperbolic functions

01.21.27.0079.01

$$\tanh(z) == \frac{\sinh(z)}{\cosh(z)}$$

01.21.27.0080.01

$$\tanh(z) - \coth(z) == -2 \operatorname{csch}(2 z)$$

Inequalities

01.21.29.0001.01

$\tanh(x) < x \;/;\; x > 0 \land x \in \mathbb{R}$

01.21.29.0002.01

$|\tanh(x)| < 1 \;/;\; x \in \mathbb{R}$

Theorems

Solution of the associated Camassa-Holm equation

The function $p(x, t) == \left(h + \frac{\lambda}{h}\right) \tanh^2\left(\sqrt{\frac{1}{\lambda} + \frac{1}{h^2}}\, (x + h\, t\, \lambda)\right) - \frac{\lambda}{h}$ is a solution of the associated Camassa–Holm equation

$$\frac{\partial p(x, t)}{\partial t} == (p\,(x, t))^2\, \frac{\partial}{\partial x}\left(\frac{1}{4}\, p(x, t)\, \frac{\partial^2 \log\,(p\,(x, t))}{\partial x\, \partial t} - \frac{1}{2}\, (p\,(x, t))^2\right).$$

History

– L'Abbe Sauri (1774)

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