Notations

Numbers, variables, and functions

Domains and domain membership

$\mathbb{N}$

The set of natural numbers n: $\{n \in \mathbb{N}) \Leftrightarrow (n \in \{0, 1, 2, \ldots\})$.

$\mathbb{N}^+$

The set of positive natural numbers n: $\{n \in \mathbb{N}^+) \Leftrightarrow (n \in \{1, 2, \ldots\})$.

$\mathbb{Z}$

The set of integer numbers n: $\{n \in \mathbb{Z}) \Leftrightarrow (n \in \{0, \pm 1, \pm 2, \ldots\})$.

$\mathbb{Q}$

The set of rational numbers r: $\{r \in \mathbb{Q}) \Leftrightarrow \left(r == \frac{p}{q} /;\ p \in \mathbb{Z}, q \in \mathbb{N}^+\right)$.

$\mathbb{R}$

The set of real numbers x: $\{x \in \mathbb{R}) \Leftrightarrow (x /; \operatorname{Im}(x) == 0)$.

$\mathbb{C}$

The set of complex numbers z: $\{z \in \mathbb{C}) \Leftrightarrow (z == a + i\, b /;\ a, b \in \mathbb{R})$.

$\mathbb{P}$

The set of prime numbers p: $\{p \in \mathbb{P}) \Leftrightarrow (p \in \{2, 3, 5, 7, 11, \ldots\})$.

$\{\}$

The empty set.

$\{a_m, a_{m+1}, \ldots, a_n\}$

The finite set of elements $a_m, a_{m+1}, \ldots, a_n /; m \leq n$.

{*listElement* /; *domainSpecification*}

A sequence of elements *listElement*. Inside a list {…} the construction {*listElement* /; *domainSpecification*} is understood to splice all occurences of *listElement* into the list.

$\mathbb{A}^p$

The Cartesian product of p copies of sets $\mathbb{A}$. (Tensor product of p sets $\mathbb{A}$.)

$\mathbb{A} \otimes \mathbb{B} \otimes \ldots$

The Cartesian product of the sets $\mathbb{A}$, $\mathbb{B}$, ….

$\{\mathbb{A} \otimes \mathbb{B} \otimes \ldots\}$

The ordered set of sets $\mathbb{A}$, $\mathbb{B}$,….

Types of variables

As a rule, the following notation style is supported for all variables, numbers, and indices.

$z, z_1, z_2, \ldots, w, w_1, w_2, \ldots$

Generic complex variables.

$x, x_1, x_2, \ldots, y, y_1, y_2, \ldots, a, a_1, \ldots$

Generic real variables. (Relations of the form $x > y$, $x < y$, $x \geq y$, and $x \leq y$ imply that x and y are real.)

$m, m_1, m_2, \ldots, n, n_1, n_2, \ldots, p, p_1, p_2, \ldots, q, q_1, q_2, \ldots$

Integer variables.

$k, k_1, k_2, \ldots, j, j_1, j_2, \ldots$

Dummy variables used in sums and products.

t, τ, s, υ

Integration dummy variables in definite integrals or integral transforms.

Set membership

$a \in \mathbb{A}$

The element a does belong to the set $\mathbb{A}$.

$a \notin \mathbb{A}$

The element a does not belong to the set $\mathbb{A}$.

$x \in (a, b)$

The number x lies within the specified interval (a, b) (excluding a and b). It is True if the number x lies within the specified interval (a, b) (including its ends), and False otherwise.

$z \in [a, b)$

The number x lies within the specified interval (a, b) (including a and excluding b).

$z \notin (a, b)$

The number z does not belong to the specified interval (a, b).

Types of functions

$f^{(-1)}(z)$

The inverse of the function f. The value of u for which the function $f(u) == z$: $f\left(f^{(-1)}(z)\right) == z$.

$f(z) \in C^n(\mathbb{A})$

The function $f(z)$ defined on the set $\mathbb{A}$ is continuous and has all derivatives of orders $k \leq n$.

$\chi_{\mathbb{A}}(a)$

The characteristic function of a set $\mathbb{A}$ has the value 1 when its argument a is an element of the specified set $\mathbb{A}$, and the value 0 otherwise.

boole(*cond*)

Gives 1 if *cond* is true, and 0 if it is false.

boole(*cond*, *expr*)

Gives *expr* if *cond* is true, and 0 otherwise.

Logical operators and conditions

Logical operators

$a \bigwedge b$

Logical "*a* and *b*".

$a \bigvee b$

Logical "*a* or *b*".

$\neg a$

The logical negation of a.

$\forall$

The universal quantor "for all".

$\exists$

The existential quantor "exists".

Conditionals, equality and ordering operators

a /; b

Relation a holds under the condition b. (Returns a if condition b is satisfied.)

$a == b$

The expression a is mathematically identical to b. (Returns True if a and b are identical.)

$a \neq b$

The expression a is mathematically different from b. (Returns True if a and b are different.)

$x > y$

The real number x is greater than the real number y. (Yields True if x is determined to be greater to y.)

$x \geq y$

The real number x is greater than or equal to y. (Yields True if x is determined to be greater than or equal to y.)

$x < y$

The real number x is less than the real number y. (Yields True if x is determined to be less than y.)

$x \leq y$

$\mathcal{NT}(\{a_1, \ldots, a_p\})$

The sequence of values $\{a_1, \ldots, a_p\}$ leads to a nonterminating hypergeometric series.

Operations

Domain and range

$z \longrightarrow f(z) :: \mathbb{A} \longrightarrow \mathbb{B}$

The function $f(z)$ is defined on domain $\mathbb{A}$: $z \in \mathbb{A}$, and it acts from this domain to domain $\mathbb{B}$: $f(z) \in \mathbb{B}$.

Branch cuts and points, singularities, and discontinuities

$\mathcal{AB}_z(f(z)) == boundary$

The natural boundary of analyticity of the function $f(z)$ with respect to z is the set *boundary*.

$\mathcal{BC}_z(f(z), z) == branchCuts$

$\mathcal{BC}_z(f(z), z)$ represents the branch cuts *branchCuts* of the function $f(z)$ with respect to z. Each branch cut is of the form {*interval*, *direction*} indicating a branch cut along *interval* and continuity of $f(z)$ from the direction *direction*.

$\mathcal{BP}_z(f(z)) == \{z_1, z_2, \ldots, z_n\}$

Gives a list of lists of the branch points $z_1, z_2, \ldots, z_n$ (if present, including infinity) of the function f over the complex z-plane.

$\mathcal{R}_z(f(z), z_0)$

The ramification index for function $f(z)$ in the branch point $z == z_0$.

$\mathcal{S}ing_z(f(z))$

The set of poles (with their orders) and essential singularities of $f(z)$ with respect to z. (The order of essential singularity is ∞.)

$\mathcal{DS}_z(f(z))$

The list of the (parametrized) intervals where the function $f(z)$ is discontinuous over the complex z-plane.

Asymptotics and series

$f(z) \propto g(z) /; (|z| \to \infty)$

$g(z)$ is the main term of asymptotic expansion of $f(z)$ at $\tilde{\infty}$ that reflects the property: $\lim_{|z|\to\infty} \frac{f(z)}{g(z)} == 1$.

$f(z) \propto g(z) + O\left(\frac{1}{z^n}\right) /; (|z| \to \infty)$

Asymptotic relation that reflects the boundedness of $z^n(f(z) - g(z))$ near point $\tilde{\infty}$.

$f(z) \propto g(z) + O((z-a)^n) /; (z \to a)$

Asymptotic relation that reflects the boundedness of $\frac{f(z)-g(z)}{(z-a)^n}$ near point $z == a$.

$\mathcal{P}_{z_0}^{[L,M]}(f(z), z)$

The $[L, M]$ Padé approximant of $f(z)$ at $z = z_0$.

$([z^n]\, f(z))$

Coefficient of the z^n term in the series expansion around $z = 0$ of the function $f(z)$: $f(z) == \sum_{n=0}^{\infty} ([z^n]\, f(z))\, z^n$.

$([(z-a)^n]\, f(z))$

Coefficient of the $(z-a)^n$ term in the series expansion around $z = a$ of the function $f(z)$: $f(z) == \sum_{n=0}^{\infty} ([(z-a)^n]\, f(z))\, (z-a)^n$.

$\left([z_1^{n_1}, z_2^{n_2}, \ldots, z_m^{n_m}]\, (f(z_1, z_2, \ldots, z_m))\right)$

Coefficient of the $z_1^{n_1} z_2^{n_2} \ldots z_m^{n_m}$ term in the series expansion around $(z_1, z_2, \ldots, z_m) = (0, 0, \ldots, 0)$ of the function $f(z_1, z_2, \ldots, z_m)$: $f(z_1, z_2, \ldots, z_m) == \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \ldots \sum_{k_m=0}^{\infty} \left([z_1^{n_1}, z_2^{n_2}, \ldots, z_m^{n_m}]\, (f(z_1, z_2, \ldots, z_m))\right) z_1^{n_1} z_2^{n_2} \ldots z_m^{n_m}$.

$\operatorname{res}_z(f(z))\,(a)$

The residue of $f(z)$ at the point $z == a$ that is equal to the coefficient of the $(z-a)^{-1}$ term in the series expansion around $z = a$ of the function $f(z)$: $\operatorname{res}_z(f(z))\,(a) == \left([(z-a)^{-1}]\, f(z)\right)$.

$\operatorname{res}_{z_1, z_2, \ldots, z_m}(f(z_1, z_2, \ldots, z_m))\,(a_1, a_2, \ldots, a_m)$

The residue of $f(z_1, z_2, \ldots, z_m)$ at the point $z_1 == a_1, z_2 == a_2, \ldots, z_m == a_m$ that is equal to the coefficient of the $\prod_{j=1}^{m} (z_j - a_j)^{-1}$ term in the series expansion around $z_1 == a_1, z_2 == a_2, \ldots, z_m == a_m$ of the function $f(z_1, z_2, \ldots, z_m)$: $\operatorname{res}_{z_1, z_2, \ldots, z_m}(f(z_1, z_2, \ldots, z_m))\,(a_1, a_2, \ldots, a_m) == \left([(z_1-a_1)^{-1}, (z_2-a_2)^{-1}, \ldots, (z_m-a_m)^{-1}]\, f(z_1, z_2, \ldots, z_m)\right)$.

$\Gamma\text{Res}\begin{pmatrix} a_1, \ldots, a_{\mathcal{A}}; & b_1, \ldots, b_{\mathcal{B}}; & \\ c_1, \ldots, c_{\mathcal{C}}; & d_1, \ldots, d_{\mathcal{D}}; & a_n, n, m; z \end{pmatrix}$

The residue of the function $f(s) == \frac{\left(\prod_{k=1}^{\mathcal{A}} \Gamma(a_k+s)\right)\left(\prod_{k=1}^{\mathcal{B}} \Gamma(b_k-s)\right)}{\left(\prod_{k=1}^{\mathcal{C}} \Gamma(c_k+s)\right)\left(\prod_{k=1}^{\mathcal{D}} \Gamma(d_k-s)\right)} z^{-s}$ at the point $s == -a_n - m\, /;\, m \in \mathbb{N}$, where this function has the pole of order n because $a_j - a_{j-1} \in \mathbb{N} \bigwedge 2 \le j \le n$:

$$\Gamma\text{Res}\begin{pmatrix} a_1, \ldots, a_{\mathcal{A}}; & b_1, \ldots, b_{\mathcal{B}}; & \\ c_1, \ldots, c_{\mathcal{C}}; & d_1, \ldots, d_{\mathcal{D}}; & a_n, n, m; z \end{pmatrix} == \operatorname{res}_s\left(\frac{\left(\prod_{k=1}^{\mathcal{A}} \Gamma(a_k+s)\right)\left(\prod_{k=1}^{\mathcal{B}} \Gamma(b_k-s)\right)}{\left(\prod_{k=1}^{\mathcal{C}} \Gamma(c_k+s)\right)\left(\prod_{k=1}^{\mathcal{D}} \Gamma(d_k-s)\right)} z^{-s}\right)(-a_n - m)\, /;$$
$$n \in \mathbb{N} \bigwedge m \in \mathbb{N} \bigwedge a_j - a_{j-1} \in \mathbb{N} \bigwedge 2 \le j \le n \bigwedge a_j - a_1 \notin \mathbb{Z} \bigwedge n+1 \le j \le \mathcal{A} \bigwedge$$
$$-b_j - a_n \notin \mathbb{N} \bigwedge 1 \le j \le \mathcal{B} \bigwedge -c_j + a_n + m \notin \mathbb{N} \bigwedge 1 \le j \le \mathcal{C} \bigwedge -d_j - a_n - m \notin \mathbb{N} \bigwedge 1 \le j \le \mathcal{D}.$$

$\mathcal{E}_k^{(q)}(\{a_1, \ldots, a_{q+1}\}, \{b_1, \ldots, b_q\})$

The main factor in the coefficient of the series representation of the function ${}_{q+1}\tilde{F}_q(a_1, \ldots, a_{q+1}; b_1, \ldots, b_q; z)$ through Gauss functions ${}_2\tilde{F}_1(a_1, a_2; a_1 + a_2 + \psi_q + k; z)$:

$${}_{q+1}\tilde{F}_q(a_1, \ldots, a_{q+1}; b_1, \ldots, b_q; z) == \frac{1}{\prod_{j=3}^{q+1} \Gamma(a_j)} \sum_{k=0}^{\infty} \mathcal{E}_k^{(q)}(\{a_1, \ldots, a_{q+1}\}, \{b_1, \ldots, b_q\})\, {}_2\tilde{F}_1(a_1, a_2; a_1 + a_2 + \psi_q + k; z)\, /;$$
$$\psi_q == \sum_{j=1}^{q} b_j - \sum_{j=1}^{q+1} a_j.$$

$\mathcal{A}_{\tilde{F}}\begin{pmatrix} a_1, \ldots, a_{q+1}; & \\ b_1, \ldots, b_q; & \{z, 1, h\} \end{pmatrix}$

The part of the series representation of the function ${}_p\tilde{F}_q(a_1, \ldots, a_{q+1}; b_1, \ldots, b_q; z)$ at the point $z == 1$ that includes h terms of the series expansions of the regular and singular components:

$$\mathcal{A}_{\tilde{F}}\begin{pmatrix} a_1, \ldots, a_{q+1}; \\ b_1, \ldots, b_q; \end{pmatrix} \{z, 1, h\} == \frac{\Gamma(-\psi_q)}{\prod_{k=1}^{q+1}\Gamma(a_k)} (1-z)^{\psi_q} \sum_{k=0}^{h} \frac{c_{k,q}}{(\psi_q+1)_k} (1-z)^k + \frac{1}{\prod_{k=1}^{q+1}\Gamma(a_k)} \sum_{k=0}^{h} g_k(0) (1-z)^k /;$$

$$|z-1| < 1 \bigwedge \psi_q == \sum_{j=1}^{q} b_j - \sum_{j=1}^{q+1} a_j \bigwedge Q(t) == \prod_{k=1}^{q} (t + b_k - 1) \bigwedge$$

$$R(t) == \prod_{k=1}^{q+1} (t + a_k) \bigwedge \Delta^n f(x) == \sum_{k=0}^{n} (-1)^{n-k} \binom{n}{k} f(k+x) \bigwedge (c_{k,q} == 0 /; k < 0) \bigwedge$$

$$c_{0,q} == 1 \bigwedge c_{k,1} == \frac{(b_1-a_1)_k (b_1-a_2)_k}{k!} \bigwedge c_{1,q} == -\left(\frac{\Delta^{q-2} Q(\psi_q)}{(q-2)!} - \frac{\Delta^{q-1} R(\psi_q-1)}{(q-1)!}\right) c_{0,q} \bigwedge$$

$$c_{k,q} == -\frac{1}{k}\left((-1)^q R(k-q+\psi_q) c_{k-q,q} + (-1)^q \sum_{j=1}^{q-1} \left(\frac{\Delta^{j-1} Q(k-q+\psi_q+1)}{(j-1)!} - \frac{\Delta^j R(k-q+\psi_q)}{j!}\right) c_{j+k-q,q}\right) \bigwedge$$

$$\psi_q \notin \mathbb{Z} \bigwedge g_k(0) == \frac{(-1)^k \Gamma(k+a_1) \Gamma(k+a_2) \Gamma(\psi_q-k)}{k!} \sum_{j=0}^{\infty} \frac{(\psi_q-k)_j \mathcal{E}_j^{(q)}(\{a_1, \ldots, a_{q+1}\}, \{b_1, \ldots, b_q\})}{\Gamma(j+a_1+\psi_q) \Gamma(j+a_2+\psi_q)} \bigwedge \text{Re}(\psi_q) > h \bigwedge h \in \mathbb{N}.$$

$$\mathcal{A}_{\tilde{F}}\begin{pmatrix} a_1, \ldots, a_p; \\ b_1, \ldots, b_q; \end{pmatrix} \{z, \tilde{\infty}, h\}$$

The asymptotic expansion of the function ${}_p\tilde{F}_q(a_1, \ldots, a_p; b_1, \ldots, b_q; z)$ at the point $z == \tilde{\infty}$ that includes h terms of the asymptotic expansions of the regular and exponential type components:

$$\mathcal{A}_{\tilde{F}}\begin{pmatrix} a_1, \ldots, a_p; \\ b_1, \ldots, b_q; \end{pmatrix} \{z, \tilde{\infty}, h\} == \mathcal{A}_{\tilde{F}}^{(\text{power})}\begin{pmatrix} a_1, \ldots, a_p; \\ b_1, \ldots, b_q; \end{pmatrix} \{z, \tilde{\infty}, h\} + .$$

$$\delta_{q,p+1} \mathcal{A}_{\tilde{F}}^{(\text{trig})}\begin{pmatrix} a_1, \ldots, a_p; \\ b_1, \ldots, b_{p+1}; \end{pmatrix} \{z, \tilde{\infty}, h\} + (\theta(q-p) - \delta_{q,p+1}) \mathcal{A}_{\tilde{F}}^{(\text{exp})}\begin{pmatrix} a_1, \ldots, a_p; \\ b_1, \ldots, b_q; \end{pmatrix} \{z, \tilde{\infty}, h\}$$

$$\mathcal{A}_{\tilde{F}}\begin{pmatrix} a_1, \ldots, a_p; \\ b_1, \ldots, b_q; \end{pmatrix} \{z, a, \infty\}$$

Infinite series or asymptotic representation of the function ${}_p\tilde{F}_q(a_1, \ldots, a_p; b_1, \ldots, b_q; z)$ at the point $z == a /; a \in \{1, \tilde{\infty}\}$: ${}^{(\mathcal{B})}\lim_{h\to\infty} \mathcal{A}_{\tilde{F}}\begin{pmatrix} a_1, \ldots, a_p; \\ b_1, \ldots, b_q; \end{pmatrix} \{z, a, h\}$ where ${}^{(\mathcal{B})}\lim_{h\to\infty}$ means the limit of a convergent series or a Borel-regularized infinite sum.

$$\mathcal{A}_{\tilde{F}}^{(\text{power})}\begin{pmatrix} a_1, \ldots, a_p; \\ b_1, \ldots, b_q; \end{pmatrix} \{z, \tilde{\infty}, h\}$$

The nonexponential part of the asymptotic expansion (or series representation for $p == q+1$) of the function ${}_p\tilde{F}_q(a_1, \ldots, a_p; b_1, \ldots, b_q; z)$ at the point $z == \tilde{\infty}$ that includes h terms of each series expansion:

$$\mathcal{A}_{\tilde{F}}^{(\text{power})}\begin{pmatrix} a_1, \ldots, a_p; \\ b_1, \ldots, b_q; \end{pmatrix} \{z, \tilde{\infty}, h\} == -\frac{1}{\prod_{k=1}^{p}\Gamma(a_k)} \sum_{j=1}^{p} \sum_{k=0}^{h} \Gamma\text{Res}\begin{pmatrix} 0; & 1-a_1, \ldots, 1-a_p; \\ ; & 1-b_1, \ldots, 1-b_q; \end{pmatrix} 1-a_j, 1, k; -z \Big) /;.$$

$$\forall_{\{j,k\},\{j,k\}\in\mathbb{Z}\wedge j\neq k\wedge 1\leq j\leq p\wedge 1\leq k\leq p} (a_j - a_k \notin \mathbb{Z}) \wedge h \in \mathbb{N}.$$

In the cases where two or more a_j differ by integer values, the function $\mathcal{A}_{\tilde{F}}^{(\text{power})}\left(\begin{matrix} a_1, \ldots, a_p; \\ b_1, \ldots, b_q; \end{matrix} \{z, \tilde{\infty}, h\}\right)$ is defined by continuity. After evaluation of the corresponding limit, the general formula includes powers of $\log(z)$ and the psi function $\psi^{(k)}(w)$, and in such logarithmic cases the representations are very complicated.

$\mathcal{A}_{\tilde{F}}^{(\exp)}\left(\begin{matrix} a_1, \ldots, a_p; \\ b_1, \ldots, b_q; \end{matrix} \{z, \tilde{\infty}, h\}\right)$

The exponential part of the asymptotic expansion of the function ${}_p\tilde{F}_q(a_1, \ldots, a_p; b_1, \ldots, b_q; z)$ (for $q == p$ or $q > p+1$) at the point $z == \tilde{\infty}$ that includes h terms of the series expansion:

$$\mathcal{A}_{\tilde{F}}^{(\exp)}\left(\begin{matrix} a_1, \ldots, a_p; \\ b_1, \ldots, b_q; \end{matrix} \{z, \tilde{\infty}, h\}\right) == \frac{(2\pi)^{\frac{1-\beta}{2}}}{\sqrt{\beta}\prod_{k=1}^p \Gamma(a_k)} z^\chi \exp\left(\beta z^{1/\beta}\right) \sum_{k=0}^h \beta^{-k} c_k z^{-\frac{k}{\beta}} /; .$$
$$\beta == q-p+1 \bigwedge A_p == \sum_{k=1}^p a_k \bigwedge B_q == \sum_{k=1}^q b_k \bigwedge \mathfrak{A} == \sum_{s=2}^p \sum_{j=1}^{s-1} a_s a_j \bigwedge$$
$$\mathfrak{B} == \sum_{s=2}^q \sum_{j=1}^{s-1} b_s b_j \bigwedge \chi == \frac{1}{\beta}\left(\frac{\beta-1}{2} + A_p - B_q\right) \bigwedge (c_k == 0 /; k < 0) \bigwedge c_0 == 1 \bigwedge$$
$$c_1 == 2\left(\frac{(\beta-11)(\beta-1)}{24\beta} - \mathfrak{A} + \mathfrak{B} + \frac{1}{2\beta}\left(A_p - B_q + \beta\left(A_p + B_q\right) - 2\right)\left(A_p - B_q\right)\right) \bigwedge$$
$$c_k == \frac{1}{k\beta}\left(\sum_{s=1}^q T_{q-s}(s-k) c_{k-s} - \sum_{s=1}^{p-1} U_{p-s-1}(s-k) c_{k-s}\right) \bigwedge T_s(k) == \sum_{r=0}^s \frac{(-1)^{s-r} T(k+r)}{r!(s-r)!} \bigwedge$$
$$T(t) == (t + \beta\chi) \prod_{j=1}^q \left(t + \left(\chi + b_j - 1\right)\beta\right) \bigwedge U_s(k) == \sum_{r=0}^s \frac{(-1)^{s-r} U(k+r)}{r!(s-r)!} \bigwedge U(t) == \prod_{j=1}^p \left(t + \beta\left(\chi + a_j\right)\right) \bigwedge h \in \mathbb{N}.$$

$\mathcal{A}_{\tilde{F}}^{(\text{trig})}\left(\begin{matrix} a_1, \ldots, a_p; \\ b_1, \ldots, b_{p+1}; \end{matrix} \{z, \tilde{\infty}, h\}\right)$

The trigonometric type part of the asymptotic expansion of the function ${}_p\tilde{F}_q(a_1, \ldots, a_p; b_1, \ldots, b_q; z)$ (for $q == p+1$) at the point $z == \tilde{\infty}$ that includes h terms of series expansion:

$$\mathcal{A}_{\tilde{F}}^{(\text{trig})}\left(\begin{matrix} a_1, \ldots, a_p; \\ b_1, \ldots, b_{p+1}; \end{matrix} \{z, \tilde{\infty}, h\}\right) == .$$
$$\frac{1}{2\sqrt{\pi}\prod_{k=1}^p \Gamma(a_k)} (-z)^\chi \left(e^{i\left(\pi\chi + 2\sqrt{-z}\right)} \sum_{k=0}^h (-i)^k 2^{-k} c_k (-z)^{-\frac{k}{2}} + e^{-i\left(\pi\chi + 2\sqrt{-z}\right)} \sum_{k=0}^h i^k 2^{-k} c_k (-z)^{-\frac{k}{2}}\right) /;$$
$$A_p == \sum_{k=1}^p a_k \bigwedge B_{p+1} == \sum_{k=1}^{p+1} b_k \bigwedge \mathfrak{A} == \sum_{s=2}^p \sum_{j=1}^{s-1} a_s a_j \bigwedge \mathfrak{B} == \sum_{s=2}^{p+1} \sum_{j=1}^{s-1} b_s b_j \bigwedge \chi == \frac{1}{2}\left(A_p - B_{p+1} + \frac{1}{2}\right) \bigwedge$$
$$(c_k == 0 /; k < 0) \bigwedge c_0 == 1 \bigwedge c_1 == 2\left(\mathfrak{B} - \mathfrak{A} + \frac{1}{4}\left(3A_p + B_{p+1} - 2\right)\left(A_p - B_{p+1}\right) - \frac{3}{16}\right) \bigwedge$$
$$c_k == \frac{1}{2k}\left(\sum_{s=1}^{p+1} T_{p+1-s}(s-k) c_{k-s} - \sum_{s=1}^{p-1} U_{p-s-1}(s-k) c_{k-s}\right) \bigwedge T_s(k) == \sum_{r=0}^s \frac{(-1)^{s-r} T(k+r)}{r!(s-r)!} \bigwedge$$
$$T(t) == (t + 2\chi) \prod_{j=1}^{p+1} \left(2\left(\chi + b_j - 1\right) + t\right) \bigwedge U_s(k) == \sum_{r=0}^s \frac{(-1)^{s-r} U(k+r)}{r!(s-r)!} \bigwedge U(t) == \prod_{j=1}^p \left(t + 2\left(\chi + a_j\right)\right) \bigwedge h \in \mathbb{N}.$$

$\mathcal{A}_{\tilde{F}}^{(t)}\left(\begin{matrix} a_1, \ldots, a_p; \\ b_1, \ldots, b_q; \end{matrix} \{z, \tilde{\infty}, \infty\}\right)$

Infinite series or the asymptotic representation of the function ${}_p\tilde{F}_q(a_1, \ldots, a_p; b_1, \ldots, b_q; z)$ at the point $z == \tilde{\infty}$: ${}^{(\mathcal{B})}\lim_{h\to\infty} \mathcal{A}_{\tilde{F}}^{(t)}\left(\begin{matrix} a_1, \ldots, a_p; \\ b_1, \ldots, b_q; \end{matrix} \{z, a, h\}\right)$ where ${}^{(\mathcal{B})}\lim_{h\to\infty}$ means the limit of a convergent series or a Borel-regularized infinite sum and $t \in \{\text{power}, \exp, \text{trig}\}$.

$$\mathcal{A}_G^{(\text{power})}\left(\begin{matrix} a_1, \ldots, a_n; & a_{n+1}, \ldots, a_p; \\ b_1, \ldots, b_m; & b_{m+1}, \ldots, b_q; \end{matrix} \{z, 0, h\}\right)$$

The nonexponential part of the asymptotic expansion (or series representation for $p == q$) of the function $G_{p,q}^{m,n}\left(z \,\middle|\, \begin{matrix} a_1, \ldots, a_n, a_{n+1}, \ldots, a_p \\ b_1, \ldots, b_m, b_{m+1}, \ldots, b_q \end{matrix}\right)$ at the point $z == 0$ that includes h terms of each series expansion. In particular,

$$\mathcal{A}_G^{(\text{power})}\left(\begin{matrix} a_1, \ldots, a_n; & a_{n+1}, \ldots, a_p; \\ b_1, \ldots, b_m; & b_{m+1}, \ldots, b_q; \end{matrix} \{z, 0, h\}\right) == \sum_{j=1}^m \sum_{k=0}^h \Gamma\text{Res}\left(\begin{matrix} b_1, \ldots, b_m; & 1-a_1, \ldots, 1-a_n; \\ a_{n+1}, \ldots, a_p; & 1-b_{m+1}, \ldots, 1-b_q; \end{matrix} b_j, 1, k; z\right) /;$$
$$\forall_{\{j,k\},\{j,k\}\in\mathbb{Z}\land j\neq k\land 1\leq j\leq n\land 1\leq k\leq n} \left(a_j - a_k \notin \mathbb{Z}\right) \bigwedge h \in \mathbb{N}.$$

In cases where two or more $b_j /; 1 \leq j \leq m$ differ by integer values, the function $\mathcal{A}_G^{(\text{power})}\left(\begin{matrix} a_1, \ldots, a_n; & a_{n+1}, \ldots, a_p; \\ b_1, \ldots, b_m; & b_{m+1}, \ldots, b_q; \end{matrix} \{z, 0, h\}\right)$ is defined by continuity. After evaluation of the corresponding limit, the general formula includes powers of $\log(z)$ and the psi function $\psi^{(k)}(w)$. It is too complicated for presentation here. The following formulas include the most important ones for application cases where one, two, three, or four b_j all differ by an integer.

$$\mathcal{A}_G^{(\exp)}\left(\begin{matrix} a_1, \ldots, a_n; & a_{n+1}, \ldots, a_{q+1}; \\ b_1, \ldots, b_m; & b_{m+1}, \ldots, b_q; \end{matrix} \{z, 0, h\}\right)$$

The exponential part of the asymptotic expansion of the function $G_{p,p+1}^{m,n}\left(z \,\middle|\, \begin{matrix} a_1, \ldots, a_n, a_{n+1}, \ldots, a_{q+1} \\ b_1, \ldots, b_m, b_{m+1}, \ldots, b_q \end{matrix}\right)$ for $p == q+1$ at the point $z == 0$ that includes h terms of series expansion:

$$\mathcal{A}_G^{(\exp)}\left(\begin{matrix} a_1, \ldots, a_n; & a_{n+1}, \ldots, a_p; \\ b_1, \ldots, b_m; & b_{m+1}, \ldots, b_q; \end{matrix} \{z, 0, h\}\right) ==$$
$$\pi^{m+n-q-1} \exp\left(\frac{(-1)^{q-m-n}}{z}\right) \sum_{r=1}^n \frac{\prod_{j=m+1}^q \sin(\pi(a_r - b_j))}{\prod_{\substack{j=1 \\ j\neq r}}^n \sin(\pi(a_r - a_j))} z^{a_r - 1} \left(\frac{(-1)^{q-m-n}}{z}\right)^{\chi + a_r - 1} \sum_{k=0}^h c_k (-1)^{(q-m-n)k} z^k /;$$
$$p - q == 1 \bigwedge (z \to 0) \bigwedge \chi == \sum_{j=1}^q b_j - \sum_{j=1}^p a_j + 1 \bigwedge (c_k == 0 /; k < 0) \bigwedge c_0 == 1 \bigwedge$$
$$c_1 == \frac{1}{2}\left(\sum_{j=1}^p a_j - \sum_{j=1}^q b_j\right)^2 + \sum_{s=2}^p \sum_{j=1}^{s-1} a_s a_j - \sum_{s=2}^q \sum_{j=1}^{s-1} b_s b_j + \frac{1}{2}\left(\left(\sum_{j=1}^q b_j\right)^2 - \left(\sum_{j=1}^p a_j\right)^2\right) \bigwedge$$
$$c_k == \frac{1}{k}\left(\sum_{s=1}^{p-1} T_{p-s-1}(s-k)\, c_{k-s} - \sum_{s=1}^{q-1} U_{q-s-1}(s-k)\, c_{k-s}\right) \bigwedge T_s(k) == \sum_{r=0}^s \frac{(-1)^{s-r}\, T(k+r)}{r!\,(s-r)!} \bigwedge$$
$$T(t) == \prod_{j=1}^p \left(t + \chi + a_j - 1\right) \bigwedge U_s(k) == \sum_{r=0}^s \frac{(-1)^{s-r}\, U(k+r)}{r!\,(s-r)!} \bigwedge U(t) == \prod_{j=1}^q \left(t + \chi + b_j\right) \bigwedge h \in \mathbb{N}$$

$$\mathcal{A}_G^{(\text{trig})}\left(\begin{matrix} a_1, \ldots, a_n; & a_{n+1}, \ldots, a_{q+2}; \\ b_1, \ldots, b_m; & b_{m+1}, \ldots, b_q; \end{matrix} \{z, 0, h\}\right)$$

The trigonometric part of the asymptotic expansion of function $G_{q+2,q}^{m,n}\left(z \,\middle|\, \begin{matrix} a_1, \ldots, a_n, a_{n+1}, \ldots, a_{q+2} \\ b_1, \ldots, b_m, b_{m+1}, \ldots, b_q \end{matrix}\right)$ for $p = q+2$ at the point $z = 0$ that includes h terms of series expansion:

$$\mathcal{A}_G^{(\text{trig})}\left(\begin{matrix} a_1, \ldots, a_n; & a_{n+1}, \ldots, a_p; \\ b_1, \ldots, b_m; & b_{m+1}, \ldots, b_q; \end{matrix} \{z, 0, h\}\right) = \frac{\pi^{m+n-q-\frac{3}{2}}}{2} \sum_{r=1}^{n} \frac{\prod_{j=m+1}^{q} \sin(\pi(a_r - b_j))}{\prod_{\substack{j=1 \\ j \neq r}}^{n} \sin(\pi(a_r - a_j))} z^{a_r - 1}$$

$$\left(\frac{(-1)^{q-m-n-1}}{z}\right)^{\chi + a_r - 1} \left(\exp\left(i\left(\pi(\chi + a_r - 1) + 2\sqrt{\frac{(-1)^{q-m-n-1}}{z}}\right)\right) \sum_{k=0}^{h} (-i)^k 2^{-k} c_k \left(\frac{(-1)^{q-m-n-1}}{z}\right)^{-\frac{k}{2}} + \right.$$

$$\left. \exp\left(-i\left(\pi(\chi + a_r - 1) + 2\sqrt{\frac{(-1)^{q-m-n-1}}{z}}\right)\right) \sum_{k=0}^{h} i^k 2^{-k} c_k \left(\frac{(-1)^{q-m-n-1}}{z}\right)^{-\frac{k}{2}}\right) /;$$

$$p - q = 2 \bigwedge (z \to 0) \bigwedge \chi = \frac{1}{2}\left(\sum_{j=1}^{q} b_j - \sum_{j=1}^{p} a_j + \frac{3}{2}\right) \bigwedge (c_k = 0 /; k < 0) \bigwedge c_0 = 1 \bigwedge$$

$$c_1 = \frac{1}{4}\left(\sum_{j=1}^{p} a_j - \sum_{j=1}^{q} b_j\right)^2 + \frac{1}{2}\left(\left(\sum_{j=1}^{q} b_j\right)^2 - \left(\sum_{j=1}^{p} a_j\right)^2\right) + \sum_{s=2}^{p} \sum_{j=1}^{s-1} a_s a_j - \sum_{s=2}^{q} \sum_{j=1}^{s-1} b_s b_j + \frac{1}{16} \bigwedge$$

$$c_k = \frac{1}{2k}\left(\sum_{s=1}^{p-1} T_{p-s-1}(s-k) c_{k-s} - \sum_{s=1}^{q-1} U_{q-s-1}(s-k) c_{k-s}\right) \bigwedge$$

$$T_s(k) = \sum_{r=0}^{s} \frac{(-1)^{s-r} T(k+r)}{r!(s-r)!} \bigwedge T(t) = \prod_{j=1}^{p} \left(t + 2(\chi + a_j - 1)\right) \bigwedge$$

$$U_s(k) = \sum_{r=0}^{s} \frac{(-1)^{s-r} U(k+r)}{r!(s-r)!} \bigwedge U(t) = \prod_{j=1}^{q} \left(t + 2(\chi + b_j)\right) \bigwedge h \in \mathbb{N}$$

$$\mathcal{A}_G^{(\text{hyp})}\left(\begin{matrix} a_1, \ldots, a_n; & a_{n+1}, \ldots, a_p; \\ b_1, \ldots, b_m; & b_{m+1}, \ldots, b_q; \end{matrix} \{z, 0, h\}\right)$$

The hyperbolic part of the asymptotic expansion of the function $G_{p,q}^{m,n}\left(z \,\middle|\, \begin{matrix} a_1, \ldots, a_n, a_{n+1}, \ldots, a_p \\ b_1, \ldots, b_m, b_{m+1}, \ldots, b_q \end{matrix}\right)$ for $p \geq q+3$ at the point $z = 0$ that includes h terms of series expansions:

$$\mathcal{A}_G^{(\text{hyp})}\left(\begin{matrix} a_1, \ldots, a_n; & a_{n+1}, \ldots, a_p; \\ b_1, \ldots, b_m; & b_{m+1}, \ldots, b_q; \end{matrix} \{z, 0, h\}\right) = G(i) + G(-i) /; p - q \geq 3 \bigwedge (z \to 0) \bigwedge \beta = p - q \bigwedge G(w) =$$

$$\frac{(2\pi)^{\frac{1-\beta}{2}} \pi^{m+n-q-1}}{\sqrt{\beta}} \exp\left(\beta e^{\frac{\pi w (q-m-n)}{\beta}} \left(\frac{1}{z}\right)^{1/\beta}\right) z^{-\chi} \sum_{r=1}^{n} \frac{\prod_{j=m+1}^{q} \sin(\pi(a_r - b_j))}{\prod_{\substack{j=1 \\ j \neq r}}^{n} \sin(\pi(a_r - a_j))} e^{\pi w (q-m-n)(\chi + a_r - 1)} \sum_{k=0}^{h} \beta^{-k} c_k e^{-\frac{\pi w (q-m-n) k}{\beta}} z^{\frac{k}{\beta}} \bigwedge$$

$$\chi = \frac{1}{\beta}\left(\frac{1+\beta}{2} - \sum_{j=1}^{p} a_j + \sum_{j=1}^{q} b_j\right) \bigwedge (c_k = 0 /; k < 0) \bigwedge c_0 = 1 \bigwedge$$

$$c_1 = \frac{1}{2\beta}\left(\sum_{j=1}^{p} a_j - \sum_{j=1}^{q} b_j\right)^2 + \sum_{s=2}^{p} \sum_{j=1}^{s-1} a_s a_j - \sum_{s=2}^{q} \sum_{j=1}^{s-1} b_s b_j + \frac{1}{2}\left(\left(\sum_{j=1}^{q} b_j\right)^2 - \left(\sum_{j=1}^{p} a_j\right)^2\right) + \frac{\beta^2 - 1}{24\beta} \bigwedge$$

$$c_k = \frac{1}{k\beta}\left(\sum_{s=1}^{p-1} T_{p-s-1}(s-k) c_{k-s} - \sum_{s=1}^{q-1} U_{q-s-1}(s-k) c_{k-s}\right) \bigwedge T_s(k) = \sum_{r=0}^{s} \frac{(-1)^{s-r} T(k+r)}{r!(s-r)!} \bigwedge$$

$$T(t) = \prod_{j=1}^{p} \left(t + \beta(\chi + a_j - 1)\right) \bigwedge U_s(k) = \sum_{r=0}^{s} \frac{(-1)^{s-r} U(k+r)}{r!(s-r)!} \bigwedge U(t) = \prod_{j=1}^{q} \left(t + \beta(\chi + b_j)\right) \bigwedge h \in \mathbb{N}$$

$$\mathcal{A}_G^{(\text{power})}\left(\begin{matrix} a_1, \ldots, a_n; & a_{n+1}, \ldots, a_p; \\ b_1, \ldots, b_m; & b_{m+1}, \ldots, b_q; \end{matrix} \{z, \tilde{\infty}, h\}\right)$$

The nonexponential part of the asymptotic expansion (or series representation for $p == q$) of the function $G_{p,q}^{m,n}\left(z \,\middle|\, \begin{matrix} a_1, \ldots, a_n, a_{n+1}, \ldots, a_p \\ b_1, \ldots, b_m, b_{m+1}, \ldots, b_q \end{matrix}\right)$ at the point $z == \tilde{\infty}$ which includes h terms of each series expansion. In particular,

$$\mathcal{A}_G^{(\text{power})}\left(\begin{matrix} a_1, \ldots, a_n; & a_{n+1}, \ldots, a_p; \\ b_1, \ldots, b_m; & b_{m+1}, \ldots, b_q; \end{matrix} \{z, \tilde{\infty}, h\}\right) == .$$

$$-\sum_{j=1}^{n} \sum_{k=0}^{h} \Gamma\text{Res}\left(\begin{matrix} b_1, \ldots, b_m; & 1-a_1, \ldots, 1-a_n; \\ a_{n+1}, \ldots, a_p; & 1-b_{m+1}, \ldots, 1-b_q; \end{matrix} 1-a_j, 1, k; z\right) /;$$

$$\forall_{\{j,k\},\{j,k\}\in\mathbb{Z}\wedge j\neq k\wedge 1\leq j\leq n\wedge 1\leq k\leq n} \left(a_j - a_k \notin \mathbb{Z}\right) \wedge h \in \mathbb{N}$$

In the cases where two or more $a_j /; 1 \leq j \leq n$ differ by integer values, the function $\mathcal{A}_G^{(\text{power})}\left(\begin{matrix} a_1, \ldots, a_n; & a_{n+1}, \ldots, a_p; \\ b_1, \ldots, b_m; & b_{m+1}, \ldots, b_q; \end{matrix} \{z, \tilde{\infty}, h\}\right)$ is defined by continuity. After evaluation of the corresponding limit, the general formula includes powers of $\log(z)$ and the psi function $\psi^{(k)}(w)$. It is too complicated for presentation here. The following formulas include the most important one for applications of cases when only two, three, or four a_j differ by integers.

$$\mathcal{A}_G^{(\exp)}\left(\begin{matrix} a_1, \ldots, a_n; & a_{n+1}, \ldots, a_p; \\ b_1, \ldots, b_m; & b_{m+1}, \ldots, b_{p+1}; \end{matrix} \{z, \tilde{\infty}, h\}\right)$$

The exponential part of the asymptotic expansion of the function $G_{p,p+1}^{m,n}\left(z \,\middle|\, \begin{matrix} a_1, \ldots, a_n, a_{n+1}, \ldots, a_p \\ b_1, \ldots, b_m, b_{m+1}, \ldots, b_{p+1} \end{matrix}\right)$ for $q == p+1$ at the point $z == \tilde{\infty}$ that includes h terms of series expansion:

$$\mathcal{A}_G^{(\exp)}\left(\begin{matrix} a_1, \ldots, a_n; & a_{n+1}, \ldots, a_p; \\ b_1, \ldots, b_m; & b_{m+1}, \ldots, b_q; \end{matrix} \{z, \tilde{\infty}, h\}\right) == .$$

$$\pi^{m+n-p-1} \exp\left((-1)^{p-m-n} z\right) \sum_{r=1}^{m} \frac{\prod_{j=n+1}^{p} \sin\left(\pi\left(a_j - b_r\right)\right)}{\prod_{\substack{j=1 \\ j\neq r}}^{m} \sin\left(\pi\left(b_j - b_r\right)\right)} z^{b_r} \left((-1)^{p-m-n} z\right)^{\chi - b_r} \sum_{k=0}^{h} c_k\, e^{-\pi i (p-m-n) k} z^{-k} /;$$

$$q - p == 1 \bigwedge (|z| \to \infty) \bigwedge \chi == \sum_{j=1}^{q} b_j - \sum_{j=1}^{p} a_j \bigwedge (c_k == 0 /; k < 0) \bigwedge c_0 == 1 \bigwedge$$
$$c_1 == \frac{1}{2}\left(\sum_{j=1}^{p} a_j - \sum_{j=1}^{q} b_j\right)^2 - \sum_{s=2}^{p} \sum_{j=1}^{s-1} a_s a_j + \sum_{s=2}^{q} \sum_{j=1}^{s-1} b_s b_j + \frac{1}{2}\left(\left(\sum_{j=1}^{p} a_j\right)^2 - \left(\sum_{j=1}^{q} b_j\right)^2\right) \bigwedge$$
$$c_k == \frac{1}{k}\left(\sum_{s=1}^{q-1} T_{q-s-1}(s-k)\, c_{k-s} - \sum_{s=1}^{p-1} U_{p-s-1}(s-k)\, c_{k-s}\right) \bigwedge T_s(k) == \sum_{r=0}^{s} \frac{(-1)^{s-r}\, T(k+r)}{r!\,(s-r)!} \bigwedge$$
$$T(t) == \prod_{j=1}^{q}\left(t + \chi - b_j\right) \bigwedge U_s(k) == \sum_{r=0}^{s} \frac{(-1)^{s-r}\, U(k+r)}{r!\,(s-r)!} \bigwedge U(t) == \prod_{j=1}^{p}\left(t + \chi - a_j + 1\right) \bigwedge h \in \mathbb{N}$$

$$\mathcal{A}_G^{(\text{trig})}\left(\begin{matrix} a_1, \ldots, a_n; & a_{n+1}, \ldots, a_p; \\ b_1, \ldots, b_m; & b_{m+1}, \ldots, b_{p+2}; \end{matrix} \{z, \tilde{\infty}, h\}\right)$$

The trigonometric type part of the asymptotic expansion of the function $G_{p,p+2}^{m,n}\left(z \,\middle|\, \begin{matrix} a_1, \ldots, a_n, a_{n+1}, \ldots, a_p \\ b_1, \ldots, b_m, b_{m+1}, \ldots, b_{p+2} \end{matrix}\right)$ for $q == p+2$ at the point $z == \tilde{\infty}$ that includes h terms of series expansion:

$$\mathcal{A}_G^{(\text{trig})}\left(\begin{matrix} a_1, \ldots, a_n; & a_{n+1}, \ldots, a_p; \\ b_1, \ldots, b_m; & b_{m+1}, \ldots, b_q; \end{matrix} \{z, \tilde{\infty}, h\}\right) = \frac{\pi^{m+n-p-\frac{3}{2}}}{2} \sum_{r=1}^{m} \frac{\prod_{j=n+1}^{p} \sin(\pi(a_j - b_r))}{\prod_{\substack{j=1 \\ j\neq r}}^{m} \sin(\pi(b_j - b_r))} z^{b_r} \left((-1)^{p-m-n-1} z\right)^{\chi - b_r} .$$

$$\left(\exp\left(i\left(\pi(\chi - b_r) + 2\sqrt{(-1)^{-m-n+p-1} z}\right)\right) \sum_{k=0}^{h} (-i)^k 2^{-k} c_k \left((-1)^{-m-n+p-1} z\right)^{-\frac{k}{2}} + \exp\left(-i\left(\pi(\chi - b_r) + 2\sqrt{(-1)^{-m-n+p-1} z}\right)\right) \sum_{k=0}^{h} i^k 2^{-k} c_k \left((-1)^{-m-n+p-1} z\right)^{-\frac{k}{2}}\right) /;$$

$$q - p = 2 \bigwedge (|z| \to \infty) \bigwedge \chi = \frac{1}{2}\left(\sum_{j=1}^{q} b_j - \sum_{j=1}^{p} a_j - \frac{1}{2}\right) \bigwedge (c_k = 0 /; k < 0) \bigwedge c_0 = 1 \bigwedge$$

$$c_1 = \frac{1}{4}\left(\sum_{j=1}^{p} a_j - \sum_{j=1}^{q} b_j\right)^2 + \frac{1}{2}\left(\left(\sum_{j=1}^{p} a_j\right)^2 - \left(\sum_{j=1}^{q} b_j\right)^2\right) - \sum_{s=2}^{p} \sum_{j=1}^{s-1} a_s a_j + \sum_{s=2}^{q} \sum_{j=1}^{s-1} b_s b_j + \frac{1}{16} \bigwedge$$

$$c_k = \frac{1}{2k}\left(\sum_{s=1}^{q-1} T_{q-s-1}(s-k) c_{k-s} - \sum_{s=1}^{p-1} U_{p-s-1}(s-k) c_{k-s}\right) \bigwedge$$

$$T_s(k) = \sum_{r=0}^{s} \frac{(-1)^{s-r} T(k+r)}{r!(s-r)!} \bigwedge T(t) = \prod_{j=1}^{q} \left(t + 2(\chi - b_j)\right) \bigwedge$$

$$U_s(k) = \sum_{r=0}^{s} \frac{(-1)^{s-r} U(k+r)}{r!(s-r)!} \bigwedge U(t) = \prod_{j=1}^{p} \left(t + 2(\chi - a_j + 1)\right) \bigwedge h \in \mathbb{N}$$

$$\mathcal{A}_G^{(\text{hyp})}\left(\begin{matrix} a_1, \ldots, a_n; & a_{n+1}, \ldots, a_p; \\ b_1, \ldots, b_m; & b_{m+1}, \ldots, b_q; \end{matrix} \{z, \tilde{\infty}, h\}\right)$$

The hyperbolic type part of the asymptotic expansion of the function $G_{p,q}^{m,n}\left(z \,\middle|\, \begin{matrix} a_1, \ldots, a_n, a_{n+1}, \ldots, a_p \\ b_1, \ldots, b_m, b_{m+1}, \ldots, b_q \end{matrix}\right)$ for $q \geq p + 3$ at the point $z = \tilde{\infty}$ that includes h terms of series expansions:

$$\mathcal{A}_G^{(\text{hyp})}\left(\begin{matrix} a_1, \ldots, a_n; & a_{n+1}, \ldots, a_p; \\ b_1, \ldots, b_m; & b_{m+1}, \ldots, b_q; \end{matrix} \{z, \tilde{\infty}, h\}\right) = G(i) + G(-i) /; q - p \geq 3 \bigwedge (|z| \to \infty) \bigwedge \beta = q - p \bigwedge G(w) =$$

$$\frac{(2\pi)^{\frac{1-\beta}{2}} \pi^{m+n-p-1}}{\sqrt{\beta}} \exp\left(\beta\, e^{\frac{\pi w (p-m-n)}{\beta}} z^{1/\beta}\right) z^{\chi} \sum_{r=1}^{m} \frac{\prod_{j=n+1}^{p} \sin(\pi(a_j - b_r))}{\prod_{\substack{j=1 \\ j\neq r}}^{m} \sin(\pi(b_j - b_r))} e^{\pi w (p-m-n)(\chi - b_r)} \sum_{k=0}^{h} \beta^{-k} c_k\, e^{-\frac{\pi w (p-m-n) k}{\beta}} z^{-\frac{k}{\beta}} \bigwedge$$

$$\chi = \frac{1}{\beta}\left(\frac{1-\beta}{2} - \sum_{j=1}^{p} a_j + \sum_{j=1}^{q} b_j\right) \bigwedge (c_k = 0 /; k < 0) \bigwedge c_0 = 1 \bigwedge$$

$$c_1 = \frac{1}{2\beta}\left(\sum_{j=1}^{p} a_j - \sum_{j=1}^{q} b_j\right)^2 - \sum_{s=2}^{p} \sum_{j=1}^{s-1} a_s a_j + \sum_{s=2}^{q} \sum_{j=1}^{s-1} b_s b_j + \frac{1}{2}\left(\left(\sum_{j=1}^{p} a_j\right)^2 - \left(\sum_{j=1}^{q} b_j\right)^2\right) + \frac{\beta^2 - 1}{24\beta} \bigwedge$$

$$c_k = \frac{1}{k\beta}\left(\sum_{s=1}^{q-1} T_{q-s-1}(s-k) c_{k-s} - \sum_{s=1}^{p-1} U_{p-s-1}(s-k) c_{k-s}\right) \bigwedge T_s(k) = \sum_{r=0}^{s} \frac{(-1)^{s-r} T(k+r)}{r!(s-r)!} \bigwedge$$

$$T(t) = \prod_{j=1}^{q} \left(t + \beta(\chi - b_j)\right) \bigwedge U_s(k) = \sum_{r=0}^{s} \frac{(-1)^{s-r} U(k+r)}{r!(s-r)!} \bigwedge U(t) = \prod_{j=1}^{p} \left(t + \beta(\chi - a_j + 1)\right) \bigwedge h \in \mathbb{N}$$

$$\mathcal{A}_G^{(t)}\left(\begin{matrix} a_1, \ldots, a_n; & a_{n+1}, \ldots, a_p; \\ b_1, \ldots, b_m; & b_{m+1}, \ldots, b_q; \end{matrix} \{z, a, \infty\}\right)$$

Infinite series or the asymptotic representation of the function $G_{p,q}^{m,n}\left(z \,\middle|\, \begin{matrix} a_1, \ldots, a_n, a_{n+1}, \ldots, a_p \\ b_1, \ldots, b_m, b_{m+1}, \ldots, b_q \end{matrix}\right)$ at the point $z = a /; a \in \{0, \tilde{\infty}\}$: $^{(\mathcal{B})}\lim_{h\to\infty} \mathcal{A}_G^{(t)}\left(\begin{matrix} a_1, \ldots, a_n; & a_{n+1}, \ldots, a_p; \\ b_1, \ldots, b_m; & b_{m+1}, \ldots, b_q; \end{matrix} \{z, a, h\}\right)$, where $^{(\mathcal{B})}\lim_{h\to\infty}$ means the limit of a convergent series or Borel-regularized infinite sums and $t \in \{\text{power}, \exp, \text{trig}, \text{hyp}\}$.

$$\mathcal{A}_G\left(\begin{matrix} a_1, \ldots, a_n; & a_{n+1}, \ldots, a_q; \\ b_1, \ldots, b_m; & b_{m+1}, \ldots, b_q; \end{matrix} \left\{z, (-1)^{m+n-q}, h\right\}\right)$$

The part of the series representation of the function $G_{p,q}^{m,n}\left(z \left| \begin{matrix} a_1, \ldots, a_n, a_{n+1}, \ldots, a_p \\ b_1, \ldots, b_m, b_{m+1}, \ldots, b_q \end{matrix}\right.\right)$ at the point $z == (-1)^{m+n-q}$ that includes h terms of the series expansions of the regular and singular components, and reflects asymptotic behavior at least in the circle $|z| < 1$:

$$\mathcal{A}_G\left(\begin{matrix} a_1, \ldots, a_n; & a_{n+1}, \ldots, a_q; \\ b_1, \ldots, b_m; & b_{m+1}, \ldots, b_q; \end{matrix} \left\{z, (-1)^{m+n-q}, h\right\}\right) == -\frac{\pi^{m+n-q}}{\sin(\psi_q \pi)} \cdot$$

$$\sum_{h=1}^{m} \frac{\prod_{k=n+1}^{q} \sin((a_k - b_h)\pi)}{\prod_{\substack{k=1 \\ k \neq h}}^{m} \sin((b_k - b_h)\pi)} z^{b_h} \left(\sum_{j=0}^{h} b_{j,q,h} \left((-1)^{q-m-n} z - 1\right)^j + \left(1 - (-1)^{q-m-n} z\right)^{\psi_q} \sum_{j=0}^{h} \frac{c_{j,q,h} \left(1 - (-1)^{q-m-n} z\right)^j}{\Gamma(\psi_q + j + 1)}\right) /;$$

$$\left(\psi_q == -\mu == \sum_{j=1}^{q} \left(a_j - b_j\right) - 1 /; p == q\right) \bigwedge Q(t) == \prod_{k=1}^{q} (t - b_k) \bigwedge R(t) == \prod_{k=1}^{q} (t - a_k + 1) \bigwedge$$

$$\Delta^n f(x) == \sum_{k=0}^{n} (-1)^{n-k} \binom{n}{k} f(k + x) \bigwedge c_{0,q,h} == 1 \bigwedge c_{j,1,1} == 0 /;$$

$$(j \in \mathbb{N}^+) \bigwedge c_{1,2,h} == R(b_h + \psi_2) \bigwedge 2\, c_{2,2,h} == R(b_h + \psi_2 + 1)\, c_{1,2,h} \bigwedge \ldots \bigwedge$$

$$j\, c_{j,2,h} == R(j + b_h + \psi_2 - 1)\, c_{j-1,2,h} \bigwedge c_{1,3,h} == \Delta \mathrm{R}(b_h + \psi_3 - 1) - Q(b_h + \psi_3) \bigwedge$$

$$2\, c_{2,3,h} == (\Delta \mathrm{R}(b_h + \psi_3) - Q(b_h + \psi_3 + 1))\, c_{1,3,h} - R(b_h + \psi_3) \bigwedge \ldots \bigwedge$$

$$j\, c_{j,3,h} == (\Delta \mathrm{R}(j + b_h + \psi_3 - 2) - Q(j + b_h + \psi_3 - 1))\, c_{j-1,3,h} - R(j + b_h + \psi_3 - 2)\, c_{j-2,3,h} \bigwedge$$

$$c_{1,q,h} == \frac{\Delta^{q-2} R(b_h + \psi_q - q + 2)}{(q-2)!} - \frac{\Delta^{q-3} Q(b_h + \psi_q - q + 3)}{(q-3)!} \bigwedge$$

$$2\, c_{2,q,h} == \left(\frac{\Delta^{q-2} R(b_h + \psi_q - q + 3)}{(q-2)!} - \frac{\Delta^{q-3} Q(b_h + \psi_q - q + 4)}{(q-3)!}\right) c_{1,q,h} - \left(\frac{\Delta^{q-3} R(b_h + \psi_q - q + 3)}{(q-3)!} - \frac{\Delta^{q-4} Q(b_h + \psi_q - q + 4)}{(q-4)!}\right) \bigwedge \ldots \bigwedge j\, c_{j,q,h} ==$$

$$(-1)^q R\left(j - q + b_h + \psi_q + 1\right) c_{j-q+1,q,h} + \sum_{k=1}^{q-2} (-1)^{q-k} \left(\frac{\Delta^k Q(j - q + b_h + \psi_q + 1)}{k!} - \frac{\Delta^{k-1} R(j - q + b_h + \psi_q + 2)}{(k-1)!}\right) c_{j+k-q+1,q,h} \bigwedge$$

$$\left(b_{j,1,1} == 0 /; j \geq 0\right) \bigwedge b_{j,q,h} == \frac{1}{2\pi i} \int_{\gamma - i\infty}^{\gamma + i\infty} \frac{t^{-b_h}}{(t-1)^{j+1}}\, \Xi\left(\left\{a_1, \ldots, a_q\right\}, \left\{b_1, \ldots, b_q\right\}, h, t\right) dt /;$$

$$0 < \gamma < 1 \bigwedge \psi_q \notin \mathbb{Z} \bigwedge |z| < 1 \bigwedge \left|1 - (-1)^{q-m-n} z\right| < 1 \bigwedge h \in \mathbb{N}$$

More detailed descriptions of $\mathcal{A}_G\left(\begin{matrix} a_1, \ldots, a_n; & a_{n+1}, \ldots, a_q; \\ b_1, \ldots, b_m; & b_{m+1}, \ldots, b_q; \end{matrix} \left\{z, (-1)^{m+n-q}, h\right\}\right)$

$$\mathcal{A}_G\left(\begin{matrix} a_1, \ldots, a_n; & a_{n+1}, \ldots, a_p; \\ b_1, \ldots, b_m; & b_{m+1}, \ldots, b_q; \end{matrix} \{z, a, h\}\right)$$

The asymptotic expansion of the function $G_{p,q}^{m,n}\left(z \left| \begin{matrix} a_1, \ldots, a_n, a_{n+1}, \ldots, a_p \\ b_1, \ldots, b_m, b_{m+1}, \ldots, b_q \end{matrix}\right.\right)$ at the point $z == a /; a \in \{0, \tilde{\infty}\}$ that includes h terms of the asymptotic expansions of the regular and exponential components:

$$\mathcal{A}_G\left(\begin{matrix} a_1, \ldots, a_n; & a_{n+1}, \ldots, a_p; \\ b_1, \ldots, b_m; & b_{m+1}, \ldots, b_q; \end{matrix} \{z, 0, h\}\right) = \mathcal{A}_G^{(\text{power})}\left(\begin{matrix} a_1, \ldots, a_n; & a_{n+1}, \ldots, a_p; \\ b_1, \ldots, b_m; & b_{m+1}, \ldots, b_q; \end{matrix} \{z, 0, h\}\right) + \delta_{p,q+1}\, \mathcal{A}_G^{(\text{exp})}\left(\begin{matrix} a_1, \ldots, a_n; & a_{n+1}, \ldots, a_{q+1}; \\ b_1, \ldots, b_m; & b_{m+1}, \ldots, b_q; \end{matrix} \{z, 0, h\}\right) + \delta_{p,q+2}\, \mathcal{A}_G^{(\text{trig})}\left(\begin{matrix} a_1, \ldots, a_n; & a_{n+1}, \ldots, a_{q+2}; \\ b_1, \ldots, b_m; & b_{m+1}, \ldots, b_q; \end{matrix} \{z, 0, h\}\right) + (1 - \delta_{p,q+1})(-\delta_{p,q+1} - \delta_{p,q+2} + \theta(p - q - 2))\, \mathcal{A}_G^{(\text{hyp})}\left(\begin{matrix} a_1, \ldots, a_n; & a_{n+1}, \ldots, a_p; \\ b_1, \ldots, b_m; & b_{m+1}, \ldots, b_q; \end{matrix} \{z, 0, h\}\right).$$

$$\mathcal{A}_G\left(\begin{matrix} a_1, \ldots, a_n; & a_{n+1}, \ldots, a_p; \\ b_1, \ldots, b_m; & b_{m+1}, \ldots, b_q; \end{matrix} \{z, \tilde{\infty}, h\}\right) = \mathcal{A}_G^{(\text{power})}\left(\begin{matrix} a_1, \ldots, a_n; & a_{n+1}, \ldots, a_p; \\ b_1, \ldots, b_m; & b_{m+1}, \ldots, b_q; \end{matrix} \{z, \tilde{\infty}, h\}\right) + \delta_{q,p+1}\, \mathcal{A}_G^{(\text{exp})}\left(\begin{matrix} a_1, \ldots, a_n; & a_{n+1}, \ldots, a_p; \\ b_1, \ldots, b_m; & b_{m+1}, \ldots, b_{p+1}; \end{matrix} \{z, \tilde{\infty}, h\}\right) + \delta_{q,p+2}\, \mathcal{A}_G^{(\text{trig})}\left(\begin{matrix} a_1, \ldots, a_n; & a_{n+1}, \ldots, a_p; \\ b_1, \ldots, b_m; & b_{m+1}, \ldots, b_{p+2}; \end{matrix} \{z, \tilde{\infty}, h\}\right) + (1 - \delta_{q,p+1})(-\delta_{q,p+1} - \delta_{q,p+2} + \theta(q - p - 2))\, \mathcal{A}_G^{(\text{hyp})}\left(\begin{matrix} a_1, \ldots, a_n; & a_{n+1}, \ldots, a_p; \\ b_1, \ldots, b_m; & b_{m+1}, \ldots, b_q; \end{matrix} \{z, \tilde{\infty}, h\}\right).$$

$$\mathcal{A}_G\left(\begin{matrix} a_1, \ldots, a_n; & a_{n+1}, \ldots, a_p; \\ b_1, \ldots, b_m; & b_{m+1}, \ldots, b_q; \end{matrix} \{z, a, \infty\}\right)$$

Infinite series or asymptotic representation of the function $G_{p,q}^{m,n}\left(z \,\middle|\, \begin{matrix} a_1, \ldots, a_n, a_{n+1}, \ldots, a_p \\ b_1, \ldots, b_m, b_{m+1}, \ldots, b_q \end{matrix}\right)$ at the point $z = a /; a \in \{0, (-1)^{m+n-q}, \tilde{\infty}\}$: ${}^{(\mathcal{B})}\lim_{h \to \infty} \mathcal{A}_G\left(\begin{matrix} a_1, \ldots, a_n; & a_{n+1}, \ldots, a_p; \\ b_1, \ldots, b_m; & b_{m+1}, \ldots, b_q; \end{matrix} \{z, a, h\}\right)$ where ${}^{(\mathcal{B})}\lim_{h \to \infty}$ means the limit of a convergent series or a Borel-regularized infinite sum.

Summations

$\sum_{k=m}^{n} f(k)$

Sum of terms $f(m),\ f(m+1),\ \ldots,\ f(n)$: $\sum_{k=m}^{n} f(k) = f(m) + f(m+1) + \ldots + f(n) /; n \geq m;\ \sum_{k=m}^{n} f(k) = 0 /; n < m$.

$\sum_{\substack{k=m \\ k \neq l}}^{n} f(k)$

Sum of terms $f(m),\ f(m+1),\ \ldots,\ f(n)$ excluding the term $f(l)$.

$\sum_{k=m}^{\infty} f(k)$

Limit of the finite sum (infinite sum): $\lim_{n \to \infty} \sum_{k=m}^{n} f(k)$.

$\sum_{\substack{k=m, \\ \Delta k=2}}^{\infty} f(k)$

Limit of the finite sum (infinite sum): $\lim_{n \to \infty} \sum_{k=0}^{n} f(m + 2k)$.

$$\sum_{k=0}^{\infty} \begin{cases} 0 & \frac{k}{2} \in \mathbb{Z} \\ \frac{1}{k!} & \text{True} \end{cases} = \sqrt{\frac{\pi}{2}}\, I_{\frac{1}{2}}(1)$$

$\sum_{\rho_k} f(\rho_k) /; g(\rho_k) == 0$

Sum over all solutions of the equation $g(\rho_k) == 0$.

$\sum_{k \in \mathbb{K}} f(k)$

Sum over the set $\mathbb{K}$.

$\sum_{d|n} f(d)$

Sum of $f(d)$ over all divisors of n.

$\sum_{k_1=m_1}^{n_1} \sum_{k_2=m_2}^{n_2} \ldots \sum_{k_l=m_l}^{n_l} f(k_1, k_2, \ldots, k_l)$

Multiple sum of function $f(k_1, k_2, \ldots, k_l)$ over the sets $(m_1, n_1) \times (m_2, n_2) \times \ldots \times (m_l, n_l)$.

$\sum_{\substack{m, n=-\infty \\ \{m,n\} \neq \{0,0\}}}^{\infty} f(m, n)$

Double sum of $f(m, n)$ over all integers m, n except $m == n == 0$.

Products

$\prod_{k=m}^{n} f(k)$

Product of terms $f(m)$, $f(m+1)$, $\ldots$, $f(n)$: $\prod_{k=m}^{n} f(k) == f(m) f(m+1) \ldots, f(n) /; n \geq m$, $\prod_{k=m}^{n} f(k) == 1 /; n < m$.

$\prod_{\substack{k=m \\ k \neq l}}^{n} f(k)$

Product of $f(m)$, $f(m+1)$, $\ldots$, $f(n)$ excluding $f(l)$.

$\prod_{k=m}^{\infty} f(k)$

Limit of finite product $\lim_{n \to \infty} \prod_{k=m}^{n} f(k)$.

$\prod_{k \in \mathbb{K}} f(k)$

Product over set $\mathbb{K}$.

$\prod_{d|n} f(d)$

Product of $f(d)$ over all divisors of n.

$\prod_{\substack{m, n=-\infty \\ \{m,n\} \neq \{0,0\}}}^{\infty} f(m, n)$

Double product of function $f(m, n)$ by all integers m, n excluding the term $f(0, 0)$.

Differentiations

$f'(z)$

Derivative of a function f of argument z: $f'(z) == \lim_{\epsilon \to 0} \frac{f(z+\epsilon) - f(z)}{\epsilon}$.

$f''(z)$

Second derivative of a function f of argument z: $f''(z) == \frac{\partial}{\partial z} f'(z) == \lim_{\epsilon \to 0} \frac{f(z) - 2 f(z+\epsilon) + f(z+2\epsilon)}{\epsilon^2}$.

$f^{(n)}(z)$

The n^{th} derivative of a function f of argument z: $f^{(n)}(z) == \frac{\partial}{\partial z} f^{(n-1)}(z) == \lim_{\epsilon \to 0} \frac{1}{\epsilon^n} \sum_{k=0}^{n} (-1)^{n-k} \binom{n}{k} f(z + k\,\epsilon) /; n \in \mathbb{N}^+$.

$f^{(0)}(z) == f(z)$

The 0^{th} derivative of a function f coincides with function f: $f^{(0)}(z) == f(z)$.

$\frac{\partial f(z)}{\partial z}$

Partial derivative of f with respect to z: $\frac{\partial f(z)}{\partial z} == \lim_{\epsilon \to 0} \frac{f(z+\epsilon) - f(z)}{\epsilon}$.

$\frac{\partial^n f(z)}{\partial z^n}$

The n^{th} partial derivative of f with respect to z: $\frac{\partial^n f(z)}{\partial z^n} == \frac{\partial}{\partial z} \frac{\partial^{n-1} f(z)}{\partial z^{n-1}} == \lim_{\epsilon \to 0} \frac{1}{\epsilon^n} \sum_{k=0}^{n} (-1)^{n-k} \binom{n}{k} f(z + k\,\epsilon) /; n \in \mathbb{N}^+$.

$f^{(n_1, n_2, \ldots, n_m)}(z_1, z_2, \ldots, z_m)$

The general form, representing a function obtained from f by differentiating n_1 times with respect to the first argument, n_2 times with respect to the second argument, and so on.

$W_z(f(z), g(z))$

The Wronskian determinant including two functions and its derivatives: $W_z(f(z), g(z)) == \begin{pmatrix} f(z) & g(z) \\ f'(z) & g'(z) \end{pmatrix} == f(z)\, g'(z) - f'(z)\, g(z)$.

The Wronskian determinant for second order linear differential equation $w''(z) + a_1(z)\, w'(z) + a_2(z)\, w(z) == F(z)$ can be evaluated by the Liouville formula $W(z) == W_z(f(z), g(z)) == W(z_0) \exp\left(-\int_{z_0}^{z} a_1(t)\, dt\right)$. The system $f(z), g(z)$ forms a fundamental (linearly independent) set of solutions for this differential equation in a neighborhood z_0 provided W does not vanish at that point.

$W_z(f_1(z), f_2(z), \ldots, f_n(z))$

The Wronskian determinant including n functions and its derivatives:

$$W_z(f_1(z), f_2(z), \ldots, f_n(z)) = \left| \begin{pmatrix} f_1(z) & f_2(z) & \ldots & f_n(z) \\ f_1'(z) & f_2'(z) & \ldots & f_n'(z) \\ \ldots & \ldots & \ldots & \ldots \\ f_1^{(n-1)}(z) & f_2^{(n-1)}(z) & \ldots & f_n^{(n-1)}(z) \end{pmatrix} \right|.$$

The Wronskian determinant for linear differential equations of the form $f^{(n)}(z) + a_1(z) f^{(n-1)}(z) + a_2(z) f^{(n-2)}(z) + \ldots + a_{n-1}(z) f'(z) + a_n(z) f(z) = F(z)$ can be evaluated by the Liouville formula $W(z) = W_z(f_1(z), f_2(z), \ldots, f_n(z)) = W(z_0) \exp\left(-\int_{z_0}^{z} a_1(t) \, dt\right)$. The system $f_1(z), f_2(z), \ldots, f_n(z)$ forms a fundamental (linearly independent) set of solutions for this differential equation in a neighborhood z_0 provided W does not vanish at that point.

Fractional integro-differentiations

$\mathcal{FC}_{\exp}^{(\alpha)}(z, a)$

Fractional differentiation power constant:

$$\mathcal{FC}_{\exp}^{(\alpha)}(z, a) = z^{\alpha-a}\left(\frac{\partial^\alpha}{\partial z^\alpha} z^a\right) = \begin{cases} (-1)^\alpha (-a)_\alpha & -a \in \mathbb{N}^+ \wedge \alpha \in \mathbb{Z} \wedge a < \alpha \\ \frac{(-1)^{a-1} (\log(z) + \psi(-a) - \psi(a-\alpha+1))}{(-a-1)! \, \Gamma(a-\alpha+1)} & -a \in \mathbb{N}^+ \\ \frac{\Gamma(a+1)}{\Gamma(a-\alpha+1)} & \text{True} \end{cases}.$$

$$\mathcal{FC}_{\exp}^{(\alpha)}(z, a+n) = \frac{(a+1)_n}{(a-\alpha+1)_n} \mathcal{FC}_{\exp}^{(\alpha)}(z, a) \,/;\, n \in \mathbb{Z} \wedge \neg (a \in \mathbb{Z} \wedge a < 0) \vee n \in \mathbb{Z} \wedge \alpha \in \mathbb{Z} \wedge a \in \mathbb{Z} \wedge a < \min(0, \alpha) \wedge n < \alpha - a$$

$\mathcal{FC}_{\log}^{(\alpha)}(z) = \mathcal{FC}_{\exp}^{(\alpha-1)}(z, -1)$

gives the logarithmic fractional differentiation constant of order α with respect to z:

$$\mathcal{FC}_{\log}^{(\alpha)}(z) = \mathcal{FC}_{\log}^{(\alpha)}(z, 0) = \begin{cases} (-1)^{\alpha-1} (\alpha-1)! & \alpha \in \mathbb{N}^+ \\ \frac{\log(z) - \psi(1-\alpha) - \gamma}{\Gamma(1-\alpha)} & \text{True} \end{cases}.$$

$\mathcal{FC}_{\log}^{(\alpha)}(z, a)$

gives the logarithmic fractional differentiation constant of order α with respect to z:

$$\mathcal{FC}_{\log}^{(\alpha)}(z, a) = \begin{cases} (-1)^{\alpha-a-1} \Gamma(a+1) (\alpha-a-1)! & \alpha - a \in \mathbb{N}^+ \\ z^{\alpha-a} \frac{\partial \frac{\Gamma(a+1) z^{a-\alpha}}{\Gamma(a-\alpha+1)}}{\partial a} = \frac{\Gamma(a+1)(\log(z) + \psi(a+1) - \psi(a-\alpha+1))}{\Gamma(a-\alpha+1)} & \text{True} \end{cases}.$$

$\mathcal{FC}_{\log}^{(\alpha)}(z, a, n)$

gives the logarithmic fractional differentiation constant of order α with respect to z:

$$\mathcal{FC}_{\log}^{(\alpha)}(z, a, n) = z^{\alpha-a} \frac{\partial^n}{\partial a^n} \left(z^{a-\alpha} \mathcal{F} C_{\exp}^{(\alpha)}(z, a)\right) = z^{\alpha-a} \frac{\partial^n}{\partial a^n} \left(\frac{\partial^\alpha}{\partial z^\alpha} z^a\right) /; n \in \mathbb{N}.$$

$\frac{\partial^\alpha f(z)}{\partial z^\alpha} = f^{(\alpha)}(z)$

The α^{th} fractional integro-derivative of f with respect to z (which provides the Riemann-Liouville-Hadamard fractional left-sided integro-differentiation beginning at point 0):

$$\frac{\partial^\alpha}{\partial z^\alpha}\left(\log^n(z)\sum_{k=-\infty}^{\infty} c_k\, z^{k+a}\right) = \sum_{k=-\infty}^{\infty} c_k\, \mathcal{FC}_{\log}^{(\alpha)}(z,\, k+a,\, n)\, z^{k+a-\alpha} \;/;\; n \in \mathbb{N};$$

$$\frac{\partial^\alpha}{\partial z^\alpha}\sum_{k=0}^{\infty} c_k\, z^k = \sum_{k=0}^{\infty} \frac{k!\, c_k}{\Gamma(k-\alpha+1)}\, z^{k-\alpha};$$

$$\frac{\partial^\alpha f(z)}{\partial z^\alpha} = \int_0^z \frac{f(t)\,(z-t)^{-\alpha-1}}{\Gamma(-\alpha)}\, dt \;/;\; \mathrm{Re}(-\alpha) > 0;$$

$$\frac{\partial^\alpha f(z)}{\partial z^\alpha} = \frac{\partial^n}{\partial z^n}\left(\int_0^z \frac{f(t)\,(z-t)^{n+\alpha-1}}{\Gamma(n+\alpha)}\, dt\right) /;\; n = \lfloor \mathrm{Re}(-\alpha) \rfloor + 1 \bigwedge \mathrm{Re}(-\alpha) \le 0$$

The value $\frac{\partial^\alpha f(z)}{\partial z^\alpha} = f^{(\alpha)}(z)$ is defined for analytical functions in the following way.

Suppose that the function $f(z)$ near point $z = 0$ can be represented through the Laurent type series

$$f(z) = \log^n(z)\sum_{k=-\infty}^{\infty} c_k\, z^{k+a} \;/;\; n \in \mathbb{N}$$

In particular for $n = 0$, $a = 0$, $c_k = 0$ /; $k < 0$, this function is analytical near the point $z = 0$. In this case the value $f^{(\alpha)}(z) = \frac{\partial^\alpha f(z)}{\partial z^\alpha}$ can be defined for arbitrary complex order α by the formula

$$\frac{\partial^\alpha f(z)}{\partial z^\alpha} = f^{(\alpha)}(z) = \sum_{k=-\infty}^{\infty} c_k\, \frac{\partial^\alpha}{\partial z^\alpha}\left(\log^n(z)\, z^{k+a}\right) =$$

$$\sum_{k=-\infty}^{\infty} c_k\, \frac{\partial^n}{\partial a^n}\left(\frac{\partial^\alpha}{\partial z^\alpha}\, z^{k+a}\right) = \sum_{k=-\infty}^{\infty} c_k\, \mathcal{FC}_{\log}^{(\alpha)}(z,\, k+a,\, n)\, z^{k+a-\alpha}.$$

In particular, for $n = 0$

$$\mathcal{FC}_{\log}^{(\alpha)}(z,\, b,\, 0) = z^{\alpha-b}\left(\frac{\partial^\alpha}{\partial z^\alpha}\, z^b\right) = \mathcal{FC}_{\exp}^{(\alpha)}(z,\, b).$$

Such an approach allows the integro-derivative of fractional (generically complex) order α to be defined for all functions of the hypergeometric type, including the Meijer G function, because all such functions can be represented as finite sums of the above Laurent type series.

Integrations

$\int_L f(t)\, dt$

Contour integral of function $f(t)$ by contour L.

For the bounded open contour $L = L(a, b)$ with t ranging from a to b and arbitrary points $t_k \in L$ placed in order between a and b ($t_0 == a,\ t_1,\ \ldots,\ t_n == b$). Thus the contour L is divided into subcontours $L(t_{k-1}, t_k)$. Then $\int_L f(t)\, dt == \lim_{\Delta\to 0} \left(\sum_{k=1}^{n} f(\tau_k) \left| t_k - t_{k-1} \right|\right) /;$

$\tau_k \in L(t_{k-1}, t_k) \bigwedge \Delta == \max(|t_1 - t_0|,\ |t_2 - t_1|,\ \ldots,\ |t_n - t_{n-1}|) \bigwedge f(\tau) \in C^0(L(a, b))$.

For a closed contour L (such as the circle $|t| == 1$), the above procedure can be applied where b on L is "near" a: $\int_L f(t)\, dt == \lim_{b\to a} \int_{L(a,b)} f(t)\, dt$.

For an unbounded contour L with one finite end a, the above procedure can be applied where b on L is "near" $\tilde{\infty}$: $\int_L f(t)\, dt == \int_{L(a,\tilde{\infty})} f(t)\, dt == \lim_{b\to\tilde{\infty}} \int_{L(a,b)} f(t)\, dt$.

For an unbounded contour L with both ends infinite (such as the special contour $\mathcal{L}$ used in the definition of the Meijer G function) define $\int_L f(t)\, dt == \int_{L_1} f(t)\, dt - \int_{L_2} f(t)\, dt$ such as L is divided into semi-unbounded contours $L_1$and L_2 by some arbitrary finite point $a \in L$, such that the directions of L and L_1 coincide.

$\int f(z)\, dz$

Indefinite integral (antiderivative) of function $f(z)$. Inverse operation to differentiation: $\frac{\partial}{\partial z} \int f(z)\, dz == f(z)$.

$\int_a^b f(t)\, dt$

Definite integral of the function $f(t)$ over interval (a, b):

$\int_a^b f(t)\, dt == \lim_{\Delta\to 0} \left(\sum_{k=1}^{n} f(\tau_k)\, (t_k - t_{k-1})\right) /;$

$a == t_0 < t_1 < \ldots < t_n == b \bigwedge t_{k-1} < \tau_k < t_k \bigwedge \Delta == \max(t_1 - t_0,\ t_2 - t_1,\ \ldots,\ t_n - t_{n-1}) \bigwedge f(\tau) \in C^0((a, b))$.

$\int_{a_1}^{b_1} \int_{a_2}^{b_2} \ldots \int_{a_n}^{b_n} f(t_1, t_2, \ldots, t_n)\, dt_n\, dt_{n-1} \ldots dt_1$

Multiple definite integral of the function $f(t_1, t_2, \ldots, t_n)$ by the intervals $(a_1, b_1) \times (a_2, b_2) \times \ldots \times (a_n, b_n)$.

$$\underbrace{\int_a^x \int_a^t \ldots \int_a^t f(t)\, dt\, dt \ldots dt}_{n-\text{times}} == \int_a^x f(t)\, \frac{(x-t)^{n-1}}{(n-1)!}\, dt$$

The repeated (n-times) integral of function $f(t)$ by interval (a, x).

$\mathcal{P} \int_a^b \frac{f(t)}{t-x}\, dt$

Cauchy principal value of a singular integral: $\mathcal{P} \int_a^b \frac{f(t)}{t-x}\, dt == \lim_{\epsilon\to 0} \left(\int_a^{x-\epsilon} \frac{f(t)}{t-x}\, dt + \int_{x+\epsilon}^b \frac{f(t)}{t-x}\, dt\right) /; a < x < b$

$\mathcal{L}$

The special contour $\mathcal{L}$, which is used in the definition of the Meijer G function and its numerous particular cases.

There are three possibilities for the contour $\mathcal{L}$:

(i) $\mathcal{L}$ runs from γ-$i\,\infty$ to γ+$i\,\infty$ (where $\mathrm{Im}(\gamma)=0$) so that all poles of $\Gamma(b_i+s)$, $i=1,\ldots,m$ are to the left of $\mathcal{L}$, and all poles of $\Gamma(1-a_i-s)$, $i=1,\ldots,n$, are to the right.
This contour can be a straight line $(\gamma-i\,\infty,\ \gamma+i\,\infty)$ if $\mathrm{Re}(b_i-a_k)>-1$ (then $-\mathrm{Re}(b_i)<\gamma<1-\mathrm{Re}(a_k)$). (In this case, the integral converges if $p+q<2\,(m+n)$, $|\mathrm{Arg}(z)|<\left(m+n-\frac{p+q}{2}\right)\pi$. If $m+n-\frac{p+q}{2}=0$, then z must be real and positive, and the additional condition $(q-p)\,\gamma+\mathrm{Re}(\mu)<0$, $\mu == \sum_{l=1}^{q} b_l-\sum_{k=1}^{p} a_k+\frac{p-q}{2}+1$ should be added.

(ii) $\mathcal{L}$ is a loop on the left side of the complex plane, starting and ending at -∞ and encircling all poles of $\Gamma(b_i+s)$, $i=1,\ldots,m$, once in the clockwise direction, but none of the poles of $\Gamma(1-a_i-s)$, $i=1,\ldots,n$.
(In this case, the integral converges if $q\geq 1$ and either $q>p$, or $q=p$ and $|z|<1$, or $q=p$ and $|z|==1$ and both $m+n-\frac{p+q}{2}\geq 0$ and $\mathrm{Re}(\mu)<0$.)

(iii) $\mathcal{L}$ is a loop on the right side of the complex plane, starting and ending at +∞ and encircling all poles of $\Gamma(1-a_i-s)$, $i=1,\ldots,n$, once in the counterclockwise direction, but none of the poles of $\Gamma(b_i+s)$, $i=1,\ldots,m$.
(In this case, the integral converges if $p\geq 1$ and either $p>q$, or $p=q$ and $|z|>1$, or $q=p$ and $|z|==1$ and both $m+n-\frac{p+q}{2}\geq 0$ and $\mathrm{Re}(\mu)<0$.)

Integral transforms

$\mathcal{F}_t[f(t)](z)$

Exponential Fourier integral transform of the function f with respect to the variable t: $\mathcal{F}_t[f(t)](z) == \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(t)\,e^{i\,t\,z}\,dt$. (If this integral does not converge, the value of $\mathcal{F}_t[f(t)](z)$ is defined in the sense of generalized functions.)

$\mathcal{F}_t^{-1}[f(t)](z)$

Inverse exponential Fourier integral transform of the function f with respect to the variable t: $\mathcal{F}_t^{-1}[f(t)](z) == \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(t)\,e^{-i\,t\,z}\,dt == \mathcal{F}_t[f(t)](-z)$. (If this integral does not converge, the value of $\mathcal{F}_t^{-1}[f(t)](z)$ is defined in the sense of generalized functions.)

$\mathcal{F}_{\{t_1,t_2\}}[f(t_1,t_2)](z_1,z_2)$

Fourier double integral transform of the function f with respect to the variables t_1, t_2: $\mathcal{F}_{\{t_1,t_2\}}[f(t_1,t_2)](z_1,z_2) == \frac{1}{2\pi}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(t_1,t_2)\,e^{i\,(t_1 z_1+t_2 z_2)}\,dt_1\,dt_2$. (If this integral does not converge, the value of $\mathcal{F}_{\{t_1,t_2\}}[f(t_1,t_2)](z_1,z_2)$ is defined in the sense of generalized functions.)

$\mathcal{F}c_t[f(t)](z)$

Fourier cosine integral transform of the function f with respect to the variable t: $\mathcal{F}c_t[f(t)](z) == \sqrt{2/\pi}\int_0^\infty f(t)\cos(t\,z)\,dt$. (If this integral does not converge, the value of $\mathcal{F}c_t[f(t)](z)$ is defined in the sense of generalized functions.)

$\mathcal{F}s_t[f(t)](z)$

Fourier sine integral transform of the function f with respect to the variable t: $\mathcal{F}s_t[f(t)](z) == \sqrt{2/\pi}\int_0^\infty f(t)\sin(t\,z)\,dt$. (If this integral does not converge, the value of $\mathcal{F}s_t[f(t)](z)$ is defined in the sense of generalized functions.)

$\mathcal{H}_t[f(t)](x)$

Hilbert transform of the function f with respect to the variable t: $\mathcal{H}_t[f(t)](x) == \frac{1}{\pi}\mathcal{P}\int_{-\infty}^\infty \frac{f(t)}{t-x}\,dt\ /;\ x \in \mathbb{R}$.

$\mathcal{H}_{\nu;t}[f(t)](z)$

Hankel integral transform of the function f with respect to the variable t: $\mathcal{H}_{\nu;t}[f(t)](z) == \int_0^\infty f(t)\sqrt{t\,z}\,J_\nu(t\,z)\,dt$. (If this integral does not converge, the value $\mathcal{H}_{\nu;t}[f(t)](z)$ is defined in the sense of generalized functions.)

$\mathcal{L}_t[f(t)](z)$

Laplace integral transform of the function f with respect to the variable t: $\mathcal{L}_t[f(t)](z) == \int_0^\infty f(t)\,e^{-t\,z}\,dt$.

$\mathcal{L}_t^{-1}[f(t)](p)$

Inverse Laplace integral transform of the function f with respect to the variable t: $\mathcal{L}_{\gamma;t}^{-1}[f(t)](p) == \frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty} f(t)\,e^{t\,p}\,dt$. (If this integral does not converge, the value of $\mathcal{L}_t^{-1}[f(t)](p)$ is defined in the sense of generalized functions.)

$\mathcal{L}_{\gamma;t}^{-1}[f(t)](p)$

Inverse Laplace integral transform of the function f with respect to the variable t: $\mathcal{L}_{\gamma;t}^{-1}[f(t)](p) == \frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty} f(t)\,e^{t\,p}\,dt$. (If this integral does not converge, the value of $\mathcal{L}_{\gamma;t}^{-1}[f(t)](p)$ is defined in the sense of generalized functions.)

$\mathcal{L}_t^{-1}[f(t)](p)$

Inverse Laplace integral transform of the function f with respect to the variable t: $\mathcal{L}_t^{-1}[f(t)](p) == \mathcal{L}_{\gamma;t}^{-1}[f(t)](p)$ for appropriately chosen γ.

$\mathcal{L}_{\{t_1,t_2\}}[f(t_1, t_2)](z_1, z_2)$

Laplace double integral transform of the function f with respect to the variables t_1, t_2: $\mathcal{L}_{\{t_1,t_2\}}[f(t_1, t_2)](z_1, z_2) == \int_0^\infty\int_0^\infty f(t_1, t_2)\,e^{-t_1 z_1 - t_2 z_2}\,dt_1\,dt_2$.

$\mathcal{M}_t[f(t)](z)$

Mellin integral transform of the function f with respect to the variable t: $\mathcal{M}_t[f(t)](z) = \int_0^\infty f(t)\, t^{z-1}\, dt$. (If this integral does not converge, the value of $\mathcal{M}_t[f(t)](z)$ is defined in the sense of generalized functions.)

$\mathcal{M}_{\gamma;s}^{-1}[f(s)](t)$

Inverse Mellin integral transform of the function f with respect to the variable s: $\mathcal{M}_{\gamma;s}^{-1}[f(s)](t) = \frac{1}{2\pi i} \int_{\gamma-i\infty}^{\gamma+i\infty} f(s)\, t^{-s}\, ds$. (If this integral does not converge, the value of $\mathcal{M}_{\gamma;s}^{-1}[f(s)](t) = \frac{1}{2\pi i} \int_{\gamma-i\infty}^{\gamma+i\infty} f(s)\, t^{-s}\, ds$ is defined in the sense of generalized functions.) The condition on γ is typically indicated in the result.

$\mathcal{W}_y[\psi_k(y)](x, p)$

Wigner integral transform: $\mathcal{W}_y[\psi_k(y)](x, p) = \int_{-\infty}^{\infty} e^{-i y p}\, \overline{\psi}_k\left(x - \frac{y}{2}\right) \psi_k\left(x + \frac{y}{2}\right) dy$. (If this integral does not converge, the value of $\mathcal{W}_y[\psi_k(y)](x, p)$ is defined in the sense of generalized functions.)

Limits

$\lim_{z \to a} f(z)$

The limiting value of $f(z)$ when z approaches a in any direction: $\left(\lim_{z\to a} f(z) = F\right) = \forall_{\epsilon,\epsilon>0} \left(\exists_{\delta,\delta>0} \left(\forall_{z,|z-a|<\delta} |f(z) - F| < \epsilon\right)\right)$.

$\lim_{z\to a+0} f(z) = \lim_{z\to a^+} f(z)$

The limiting value of $f(z)$ when z approaches a in direction -1: $\left(\lim_{z\to a+0} f(z) = \lim_{z\to a^+} f(z) = F\right) = \forall_{\epsilon,\epsilon>0} \left(\exists_{\delta,\delta>0} \left(\forall_{z,z-a<\delta \land z>a} |f(z) - F| < \epsilon\right)\right)$.

$\lim_{z\to a-0} f(z) = \lim_{z\to a^-} f(z)$

The limiting value of $f(z)$ when z approaches a in direction 1: $\left(\lim_{z\to a-0} f(z) = \lim_{z\to a^-} f(z) = F\right) = \forall_{\epsilon,\epsilon>0} \left(\exists_{\delta,\delta>0} \left(\forall_{z,a-z<\delta \land z<a} |f(z) - F| < \epsilon\right)\right)$.

$\lim_{z\to a+i\,0} f(z)$

The limiting value of $f(z)$ when z approaches a in direction -i: $\left(\lim_{z\to a+i\,0} f(z) = F\right) = \forall_{\epsilon,\epsilon>0} \left(\exists_{\delta,\delta>0} \left(\forall_{z,|z-a|<\delta \land -i\,(z-a)>0} |f(z) - F| < \epsilon\right)\right)$.

$\lim_{z\to a-i\,0} f(z)$

The limiting value of $f(z)$ when z approaches a in direction i: $\left(\lim_{z\to a-i\,0} f(z) = F\right) = \forall_{\epsilon,\epsilon>0} \left(\exists_{\delta,\delta>0} \left(\forall_{z,|z-a|<\delta \land i(z-a)>0} |f(z) - F| < \epsilon\right)\right)$.

$\lim_{z\to\infty} f(z)$

The limiting value of $f(z)$ when z approaches ∞: $(\lim_{z\to\infty} f(z) == F) == \forall_{\epsilon,\epsilon>0} (\exists_{\Delta,\Delta>0} (\forall_{z,z>\Delta} |f(z) - F| < \epsilon))$.

$\lim_{z\to-\infty} f(z)$

The limiting value of $f(z)$ when z approaches $-\infty$: $(\lim_{z\to-\infty} f(z) == F) == \forall_{\epsilon,\epsilon>0} (\exists_{\Delta,\Delta>0} (\forall_{z,z<-\Delta} |f(z) - F| < \epsilon))$.

$\lim_{z\to\tilde{\infty}} f(z)$

The limiting value of $f(z)$ when z approaches $\tilde{\infty}$: $(\lim_{z\to\tilde{\infty}} f(z) == F) == \forall_{\epsilon,\epsilon>0} (\exists_{\Delta,\Delta>0} (\forall_{z,|z|>\Delta} |f(z) - F| < \epsilon))$.

Continued fractions

$\underset{k=m}{\overset{n}{\mathrm{K}}} \frac{a_k}{b_k}$

A finite continued fraction $\cfrac{a_m}{b_m + \cfrac{a_{m+1}}{b_{m+1} + \cfrac{a_{m+2}}{b_{m+2} + \cfrac{a_{m+3}}{\ddots\, b_{n-1} + a_n}}}}$.

$\underset{k=1}{\overset{\infty}{\mathrm{K}}} \frac{a_k}{b_k}$

Limit of the finite continued fraction $\lim_{n\to\infty} \underset{k=1}{\overset{n}{\mathrm{K}}} \frac{a_k}{b_k} = \cfrac{a_1}{b_1 + \cfrac{a_2}{b_2 + \cfrac{a_3}{b_3 + \cfrac{a_4}{\ddots}}}}$.

Matrices and determinants

$(a_{j,k})_{\substack{0\le j\le n \\ 0\le k\le n}}$

The $n \times n$ matrix with elements $a_{j,k}$.

$\left|(a_{j,k})_{\substack{0\le j\le n \\ 0\le k\le n}}\right|$

The determinant of the $n \times n$ matrix with elements $a_{j,k}$.

Symbols used for functions

$\sqrt{z} == z^{1/2}$

Sqrt root: $\left(\sqrt{z}\right)^2 == z$.

$|z|$

The absolute value of the real or complex number z:
$|z| == \begin{cases} z & z \geq 0 \\ -z & z < 0 \end{cases}$ for real z and $|z| == \sqrt{\operatorname{Re}(z)^2 + \operatorname{Im}(z)^2}$ for complex z.

$\bar{z} == z^*$

Complex conjugate of the number z : $\bar{z} == \operatorname{Re}(z) - i \operatorname{Im}(z)$; $\overline{x} == x$ /; $\operatorname{Im}(x) == 0$.

$n!$

Factorial of n: $n! == \Gamma(n+1)$; $n! == 1 \times 2 \times 3 \ldots (n-1)\, n$ /; $n \in \mathbb{N}^+$.

$n!!$

Double factorial of n: $n!! == 2^{\frac{n}{2} - \frac{1}{4} \cos(n \pi) + \frac{1}{4}} \pi^{\frac{1}{4} \cos(n \pi) - \frac{1}{4}} \Gamma\left(\frac{n}{2} + 1\right)$; $(2k)!! == 2 \times 4 \ldots (2k-2)\, 2k$ /; $k \in \mathbb{N}^+$;
$(2k+1)!! == 1 \times 3 \ldots (2k-1)(2k+1)$ /; $k \in \mathbb{N}^+$.

$\binom{n}{k}$

Binomial coefficient: $\binom{n}{k} == \binom{n}{n-k} == \frac{n!}{k!\,(n-k)!} == \frac{\Gamma(n+1)}{\Gamma(k+1)\,\Gamma(n-k+1)}$.

$(n; n_1, n_2, \ldots, n_m)$

Multinomial coefficient: $(n; n_1, n_2, \ldots, n_m) == \frac{n!}{\prod_{k=1}^{m} n_k!}$ /; $n == \sum_{k=1}^{m} n_k$.

$(a)_n$

Pochhammer symbol representing the product:
$(a)_n == \frac{\Gamma(a+n)}{\Gamma(a)}$; $(a)_n == \prod_{k=0}^{n-1} (a+k) == a(a+1)(a+2) \ldots (a+n-1)$ /; $n \in \mathbb{N}^+$.

z^a

Power function: $z^a == \sum_{k=0}^{\infty} \frac{\log^k(z)\, a^k}{k!}$; $z^k == \underbrace{z \times z \times \ldots \times z}_{\text{k- times}} == z\, z^{k-1}$ /; $k \in \mathbb{N}^+$.

$\left(z; \sum_{j=0}^{n} a_j z^j\right)_k^{-1}$

The k^{th} root z_k of algebraic equation $\sum_{j=0}^{n} a_j z^j == 0$: $\sum_{j=0}^{n} a_j z^j == 0$ /; $z == z_k == \left(z; \sum_{j=0}^{n} a_j z^j\right)_k^{-1}$.

∞

Positive infinity symbol.

$\tilde{\infty}$

Symbolic value of a complex number when its absolute value tends to infinity.

$z\,\infty$

Symbolic value of a complex number when its absolute value tends to infinity and its argument coincides with Arg(z): Arg($z\,\infty$) == Arg(z).

¿

Indeterminate value symbol.

$\circ$

One degree °: $1\,° == \frac{\pi}{180} \approx 0.01745329\,\ldots$

$\lfloor z \rfloor$

The greatest integer less than or equal to z: $\lfloor x \rfloor == n$ /; $x \in \mathbb{R} \bigwedge n \in \mathbb{Z} \bigwedge n \leq x < n+1$.

$\lceil z \rceil$

The smallest integer greater than or equal to z: $\lceil x \rceil == n$ /; $x \in \mathbb{R} \bigwedge n \in \mathbb{Z} \bigwedge n-1 < x \leq n$.

$\lfloor z \rceil$

The integer closest to z: $\lfloor x \rceil == n$ /; $x \in \mathbb{R} \bigwedge n \in \mathbb{Z} \bigwedge |x-n| < \frac{1}{2}$; $\lfloor n + \frac{1}{2} \rceil == n$ /; $\frac{n}{2} \in \mathbb{Z}$; $\lfloor n + \frac{1}{2} \rceil == n+1$ /; $\frac{n+1}{2} \in \mathbb{Z}$.

$m \bmod n$

The congruence (mod) (the remainder on division of m by n): $m \bmod n == m - n \left\lfloor \frac{m}{n} \right\rfloor$.

$\langle j_1\, j_2\, m_1\, m_2 \mid j_1\, j_2\, j\, m \rangle$

The Clebsch-Gordan coefficient for the decomposition of $|j\, m\rangle$ in terms of $|j_1\, m_1\rangle \otimes |j_2\, m_2\rangle$.

$\begin{pmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & m_3 \end{pmatrix}$

The value of the Wigner $3j$-symbol.

$\begin{Bmatrix} j_1 & j_2 & j_3 \\ j_4 & j_5 & j_6 \end{Bmatrix}$

The value of the Racah $6\,j$-symbol.

$\left\{s_b^{(1)}(n),\, s_b^{(2)}(n),\, \ldots,\, s_b^{(b-1)}(n),\, s_b^{(0)}(n)\right\}$

The list of the numbers of $1,\ 2,\ \ldots,\ b-1,\ 0$ digits in the base b representation of n. $s_b^{(k)}(n)$ is the number of times the digit k appears in the base b representation of the integer n; $s_2^{(1)}(n) == n - \sum_{k=1}^{\infty} \left\lfloor \frac{n}{2^k} \right\rfloor$.
$s_b^{(k)}(n)$ is the number of times the digit k appears in the base b representation of the integer n.

$\left(\frac{n}{m}\right)$

Jacobi symbol, an integer function of n and m: $\left(\frac{n}{m}\right) = \prod_{k=1}^{j} \left(\frac{n}{p_k}\right) /;$

$\frac{m-1}{2} \in \mathbb{N} \bigwedge m = \prod_{k=1}^{j} p_k \bigwedge p_k \in \mathbb{P} \bigwedge \left(\frac{n}{p}\right) = \left(1 - \delta_{\frac{n}{p} - \left\lfloor \frac{n}{p} \right\rfloor, 0}\right) \left(2 \operatorname{sgn}\left(\sum_{j=1}^{p} \delta_{j^2 \bmod p,\, n \bmod p}\right) - 1\right) /;\ p \in \mathbb{P}.$

Jacobi symbol is identical to Kronecker symbol.

$\left(\frac{n}{m}\right)$

Kronecker symbol, an integer function of n and m: $\left(\frac{n}{m}\right) = \prod_{k=1}^{j} \left(\frac{n}{p_k}\right) /;$

$\frac{m-1}{2} \in \mathbb{N} \bigwedge m = \prod_{k=1}^{j} p_k \bigwedge p_k \in \mathbb{P} \bigwedge \left(\frac{n}{p}\right) = \left(1 - \delta_{\frac{n}{p} - \left\lfloor \frac{n}{p} \right\rfloor, 0}\right) \left(2 \operatorname{sgn}\left(\sum_{j=1}^{p} \delta_{j^2 \bmod p,\, n \bmod p}\right) - 1\right) /;\ p \in \mathbb{P}.$

Kronecker symbol is identical to Jacobi symbol.

Functions in alphabetical order

A

$a_r(q)$

The characteristic value a for even Mathieu functions $w(z) = \mathrm{Ce}(a, q, z)$ with characteristic exponent r and parameter q, such that there exists a solution of the corresponding Mathieu differential equation $w''(z) + (a - 2\, q \cos(2\, z))\, w(z) = 0$ that is of the form $w(z) = e^{i\, r\, z}\, f(z)$, where $f(z)$ is an even function of z with period $2\,\pi$.

A

The Glaisher constant A: $A \approx 1.2824271\ldots$

$\operatorname{agm}(a, b)$

The arithmetic-geometric mean of a and b: $\operatorname{agm}(a, b) = \pi\,(a + b) \Big/ \left(4\, K\!\left(\left(\frac{a-b}{a+b}\right)^2\right)\right)$.

ai_k

The k^{th} root of the equation $\mathrm{Ai}(z) = 0$: $(\mathrm{Ai}(z) /; z = \mathrm{ai}_k) = 0 /; k \in \mathbb{N}^+$.

$\mathrm{Ai}(z)$

The Airy function Ai: $\mathrm{Ai}(z) = \frac{1}{3^{2/3}\,\Gamma\left(\frac{2}{3}\right)}\, {}_0F_1\left(;\, \frac{2}{3};\, \frac{z^3}{9}\right) - \frac{z}{\sqrt[3]{3}\,\Gamma\left(\frac{1}{3}\right)}\, {}_0F_1\left(;\, \frac{4}{3};\, \frac{z^3}{9}\right)$.

$\mathrm{Ai}'(z)$

The first derivative of the Airy function Ai: $\text{Ai}'(z) == \frac{z^2}{2\,3^{2/3}\,\Gamma\left(\frac{2}{3}\right)}\;{}_0F_1\left(;\frac{5}{3};\frac{z^3}{9}\right) - \frac{1}{\sqrt[3]{3}\;\Gamma\left(\frac{1}{3}\right)}\;{}_0F_1\left(;\frac{1}{3};\frac{z^3}{9}\right)$.

$\text{am}(z \mid m)$

Jacobi amplitude function with module m. The value of u for which the elliptic integral of the first kind $F(u \mid m)$ has the value z: $\text{am}(z \mid m) == 2\sum_{k=-\infty}^{\infty} \tan^{-1}\left(\tanh\left(\frac{\pi}{2}\,\frac{K(m)}{K(1-m)}\left(k + \frac{z}{2\,K(m)}\right)\right)\right)$.

$\arg(z) == \text{Arg}(z)$

The argument of the complex number z (where $z == |z|\,e^{i\arg(z)}$): $\arg(z) == -i\log\left(\frac{z}{|z|}\right)$.

B

$b_r(q)$

The characteristic value b for odd Mathieu functions $w(z) == \text{Se}(a, q, z)$ with characteristic exponent r and parameter q, such that there exists a solution of the corresponding Mathieu differential equation $w''(z) + (a - 2\,q\cos(2\,z))\,w(z) == 0$ that is of the form $w(z) == e^{i\,r\,z}\,f(z)$, where $f(z)$ is an odd function of z with period $2\,\pi$.

B_n

The n^{th} Bell number: $B_n = n!\,\left([t^n]\,e^{e^t-1}\right) /; n \in \mathbb{N};\; B_n = B_n(1) /; n \in \mathbb{N}$.

$B_n(z)$

B_n

The n^{th} Bernoulli number: $B_n == n!\left([t^n]\,\frac{t}{e^t-1}\right) /; n \in \mathbb{N};\; B_n == B_n(0) /; n \in \mathbb{N}$.

$B_n(z)$

$B_n^{(z)}$

$B_n^{(\alpha)}(z)$

$\text{bei}(z)$

The Kelvin function of the first kind bei: $\text{bei}(z) == \frac{z^2}{4}\sum_{k=0}^{\infty} \frac{(-1)^k}{((2\,k+1)!)^2}\left(\frac{z}{2}\right)^{4\,k}$; $\text{bei}(z) == \text{bei}_0(z)$.

$\text{bei}_\nu(z)$

The Kelvin function of the first kind bei: $\text{bei}_\nu(z) = \frac{\cos\left(\frac{3\pi\nu}{4}\right)}{\Gamma(\nu+2)}\left(\frac{z}{2}\right)^{\nu+2}\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{z}{4}\right)^{4\,k}}{\left(\frac{\nu}{2}+1\right)_k\left(\frac{\nu+3}{2}\right)_k\left(\frac{3}{2}\right)_k\,k!} + \frac{\sin\left(\frac{3\pi\nu}{4}\right)}{\Gamma(\nu+1)}\left(\frac{z}{2}\right)^{\nu}\sum_{k=0}^{\infty}\frac{(-1)^k\left(\frac{z}{4}\right)^{4\,k}}{\left(\frac{\nu+1}{2}\right)_k\left(\frac{\nu}{2}+1\right)_k\left(\frac{1}{2}\right)_k\,k!} /; -\nu \notin \mathbb{N}^+$.

ber(z)

The Kelvin function of the first kind ber: $\text{ber}(z) = \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{z}{2}\right)^{4k}}{((2k)!)^2}$; $\text{ber}(z) == \text{ber}_0(z)$.

$\text{ber}_\nu(z)$

The Kelvin function of the first kind ber: $\text{ber}_\nu(z) = \frac{\cos\left(\frac{3\pi\nu}{4}\right)}{\Gamma(\nu+1)} \left(\frac{z}{2}\right)^\nu \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{z}{4}\right)^{4k}}{\left(\frac{\nu+1}{2}\right)_k \left(\frac{\nu}{2}+1\right)_k \left(\frac{1}{2}\right)_k k!} - \frac{\sin\left(\frac{3\pi\nu}{4}\right)}{\Gamma(\nu+2)} \left(\frac{z}{2}\right)^{\nu+2} \sum_{k=0}^{\infty} \frac{(-1)^k \left(\frac{z}{4}\right)^{4k}}{\left(\frac{\nu}{2}+1\right)_k \left(\frac{\nu+3}{2}\right)_k \left(\frac{3}{2}\right)_k k!} \;/;\; -\nu \notin \mathbb{N}^+$.

bi_k

The k^{th} root of the equation $\text{Bi}(z) == 0$: $(\text{Bi}(z) \;/;\; z = \text{bi}_k) == 0 \;/;\; k \in \mathbb{N}^+$.

Bi(z)

The Airy function Bi: $\text{Bi}(z) == \frac{1}{\sqrt[6]{3}\,\Gamma\left(\frac{2}{3}\right)}\, {}_0F_1\left(;\frac{2}{3};\frac{z^3}{9}\right) + \frac{\sqrt[6]{3}\,z}{\Gamma\left(\frac{1}{3}\right)}\, {}_0F_1\left(;\frac{4}{3};\frac{z^3}{9}\right)$.

Bi′(z)

The first derivative of the Airy function Bi: $\text{Bi}'(z) == \frac{z^2}{2\sqrt[6]{3}\,\Gamma\left(\frac{2}{3}\right)}\, {}_0F_1\left(;\frac{5}{3};\frac{z^3}{9}\right) + \frac{\sqrt[6]{3}}{\Gamma\left(\frac{1}{3}\right)}\, {}_0F_1\left(;\frac{1}{3};\frac{z^3}{9}\right)$.

C

C

The Catalan constant C: $C \approx 0.9159655\ldots$

$C(z)$

C_z

The Catalan numbers: $C_z = \frac{2^{2z}\,\Gamma\left(z+\frac{1}{2}\right)}{\sqrt{\pi}\,\Gamma(z+2)}$.

$C_n(z)$

The cyclotomic polynomial of order n in z: $C_n(z) == \prod_{\substack{k=1 \\ \gcd(k,n)=1}}^{n} \left(z - e^{2\pi i k/n}\right)$.

$C_\nu^{(0)}(z) == C_\nu(z)$

The renormalized form of the ν^{th} Gegenbauer function in z: $C_\nu^{(0)}(z) == \frac{2}{\nu} T_\nu(z) == \frac{2}{\nu} \cos\left(\nu \cos^{-1}(z)\right)$. For the nonnegative integer ν, the function $C_\nu^{(0)}(z)$ is a polynomial in z.

$C_\nu^\lambda(z)$

The ν^{th} Gegenbauer function in z for parameter λ: $C_\nu^\lambda(z) == \frac{2^{1-2\lambda}\sqrt{\pi}\,\Gamma(\nu+2\lambda)}{\nu!\,\Gamma(\lambda)}\, {}_2\tilde{F}_1\left(-\nu, \nu+2\lambda; \lambda+\frac{1}{2}; \frac{1-z}{2}\right)$. For the nonnegative integer ν, the function $C_\nu^\lambda(z)$ is a polynomial in z.

$\text{cd}(z \mid m)$

The Jacobi elliptic function cd: $\text{cd}(z \mid m) == \frac{\text{cn}(z|m)}{\text{dn}(z|m)} == \frac{1}{\text{dc}(z|m)}$.

$\text{cd}^{-1}(z \mid m)$

The inverse of the Jacobi elliptic function cd is the value of u for which the Jacobi elliptic function cd, such that $\text{cd}(u \mid m) == z:\ \text{cd}^{-1}(z \mid m) == \int_z^1 \frac{1}{\sqrt{1-t^2}\,\sqrt{1-m\,t^2}}\,dt\ /;\ -1 < z < 1 \land m < 1$.

$\text{Ce}(a, q, z)$

The even Mathieu function with characteristic value a and parameter q.

$\text{Ce}_z(a, q, z) == \text{Ce}'(a, q, z)$

The derivative with respect to z of the even Mathieu function with characteristic value a and parameter q: $\text{Ce}_z(a, q, z) == \frac{\partial \text{Ce}(a,q,z)}{\partial z}$.

$\text{Chi}(z)$

The hyperbolic cosine integral function: $\text{Chi}(z) == \int_0^z \frac{\cosh(t)-1}{t}\,dt + \log(z) + \gamma == \log(z) + \gamma + \frac{1}{2}\sum_{k=1}^\infty \frac{z^{2k}}{k\,(2k)!}$.

$\text{Ci}(z)$

The cosine integral function: $\text{Ci}(z) == \int_0^z \frac{\cos(t)-1}{t}\,dt + \log(z) + \gamma == -\int_z^\infty \frac{\cos(t)}{t}\,dt == \log(z) + \gamma + \frac{1}{2}\sum_{k=1}^\infty \frac{(-1)^k z^{2k}}{k\,(2k)!}\ /;\ |\text{Arg}(z)| < \pi$.

$\text{cn}(z \mid m)$

The Jacobi elliptic function cn: $\text{cn}(z \mid m) == \cos(\text{am}(z \mid m))$.

$\text{cn}^{-1}(z \mid m)$

The inverse of the Jacobi elliptic function cn. The value of u such that $\text{cn}(u \mid m) == z:\ \text{cn}^{-1}(z \mid m) == \int_z^1 \frac{1}{\sqrt{1-t^2}\,\sqrt{m\,t^2-m+1}}\,dt\ /;\ -1 < z < 1 \land m\left(z^2-1\right) > -1$.

$\cos(z)$

The cosine function: $\cos(z) == \frac{e^{iz}+e^{-iz}}{2} == \sum_{k=0}^\infty \frac{(-1)^k z^{2k}}{(2k)!}$.

$\cos^{-1}(z)$

The inverse cosine function: $\cos^{-1}(z) = \frac{\pi}{2} - \sin^{-1}(z) = \frac{\pi}{2} + i \log\left(i z + \sqrt{1 - z^2}\right)$.

$\cosh(z)$

The hyperbolic cosine function: $\cosh(z) = \frac{e^z + e^{-z}}{2} = \cos(i z) = \sum_{k=0}^{\infty} \frac{z^{2k}}{(2k)!}$.

$\cosh^{-1}(z)$

The inverse hyperbolic cosine function: $\cosh^{-1}(z) = \log\left(z + \sqrt{z-1} \sqrt{z+1}\right) = \frac{\sqrt{z-1}}{\sqrt{1-z}} \cos^{-1}(z)$.

$\cot(z)$

The cotangent function: $\cot(z) = \frac{\cos(z)}{\sin(z)} = \frac{1}{\tan(z)}$.

$\cot^{-1}(z)$

The inverse cotangent function: $\cot^{-1}(z) = \tan^{-1}\left(\frac{1}{z}\right) = \frac{i}{2}\left(\log\left(1 - \frac{i}{z}\right) - \log\left(1 + \frac{i}{z}\right)\right) /; z \neq 0$.

$\coth(z)$

The hyperbolic cotangent function: $\coth(z) = \frac{\cosh(z)}{\sinh(z)} = \frac{1}{\tanh(z)} = i \cot(i z)$.

$\coth^{-1}(z)$

$\mathrm{cs}(z \mid m)$

The Jacobi elliptic function cs: $\mathrm{cs}(z \mid m) = \frac{\mathrm{cn}(z \mid m)}{\mathrm{sn}(z \mid m)} = \frac{1}{\mathrm{sc}(z \mid m)}$.

$\mathrm{cs}^{-1}(z \mid m)$

The inverse of the Jacobi elliptic function cs. The value of u such that $\mathrm{cs}(u \mid m) = z$: $\mathrm{cs}^{-1}(z \mid m) = \int_z^{\infty} \frac{1}{\sqrt{t^2+1} \sqrt{t^2-m+1}} \, dt /; z \in \mathbb{R} \wedge z^2 - m > -1$.

$\csc(z)$

The cosecant function: $\csc(z) = \frac{1}{\sin(z)}$.

$\csc^{-1}(z)$

The inverse cosecant function: $\csc^{-1}(z) = \sin^{-1}\left(\frac{1}{z}\right) = -i \log\left(\frac{i}{z} + \sqrt{1 - \frac{1}{z^2}}\right)$.

$\mathrm{csch}(z)$

The hyperbolic cosecant function: $\operatorname{csch}(z) == \frac{1}{\sinh(z)} == i \csc(i\, z)$.

$\operatorname{csch}^{-1}(z)$

The inverse hyperbolic cosecant function: $\operatorname{csch}^{-1}(z) == \sinh^{-1}\left(\frac{1}{z}\right) == i \csc^{-1}(i\, z) == \log\left(\sqrt{1 + \frac{1}{z^2}} + \frac{1}{z}\right)$.

D

$d^J_{M,M'}(\beta)$

The Wigner d-function:
$d^J_{M,M'}(\beta) == (-1)^{J-M'} \sqrt{(J+M)!\,(J-M)!\,(J+M')!\,(J-M')!} \sum_{k=\max(0,-M-M')}^{\min(J-M,J-M')} (-1)^k \frac{\cos^{M+M'+2k}\left(\frac{\beta}{2}\right) \sin^{2J-M-M'-2k}\left(\frac{\beta}{2}\right)}{k!\,(J-M-k)!\,(J-M'-k)!\,(M+M'+k)!}$ /;
$\{2J, 2M, 2M', J-M, J-M'\} \in \mathbb{Z} \wedge |M| \leq J \wedge |M'| \leq J$

$D_\nu(z)$

The parabolic cylinder function D: $D_\nu(z) == 2^{\nu/2} \sqrt{\pi}\, e^{-\frac{z^2}{4}} \left(\frac{1}{\Gamma\left(\frac{1-\nu}{2}\right)}\, {}_1F_1\left(-\frac{\nu}{2}; \frac{1}{2}; \frac{z^2}{2}\right) - \frac{\sqrt{2}\, z}{\Gamma\left(-\frac{\nu}{2}\right)}\, {}_1F_1\left(\frac{1-\nu}{2}; \frac{3}{2}; \frac{z^2}{2}\right)\right)$.

$D^J_{M,M'}(\alpha, \beta, \gamma)$

$\operatorname{dc}(z \mid m)$

The Jacobi elliptic function dc: $\operatorname{dc}(z \mid m) == \frac{\operatorname{dn}(z|m)}{\operatorname{cn}(z|m)} == \frac{1}{\operatorname{cd}(z|m)}$.

$\operatorname{dc}^{-1}(z \mid m)$

The inverse of the Jacobi elliptic function dc. The value of u such that $\operatorname{dc}(u \mid m) == z$: $\operatorname{dc}^{-1}(z \mid m) == \int_1^z \frac{1}{\sqrt{t^2-1}\,\sqrt{t^2-m}}\, dt$ /; $z \in \mathbb{R} \wedge z^2 > 1 \wedge z^2 - m > 0 \wedge m < 1$.

$\operatorname{den}(r)$

The denominator of r.

$\operatorname{divisors}(n)$

$\operatorname{dn}(z \mid m)$

The Jacobi elliptic function dn: $\operatorname{dn}(z \mid m) == \sqrt{1 - m \sin^2(\operatorname{am}(z \mid m))}$ /; $m < 1$.

$\operatorname{dn}^{-1}(z \mid m)$

The inverse of the Jacobi elliptic function dn. The value of u such that $\operatorname{dn}(u \mid m) == z$: $\operatorname{dn}^{-1}(z \mid m) == \int_z^1 \frac{1}{\sqrt{1-t^2}\,\sqrt{t^2+m-1}}\, dt$ /; $-1 < z < 1 \wedge z^2 + m > 1$.

$\mathrm{ds}(z \mid m)$

The Jacobi elliptic function ds: $\mathrm{ds}(z \mid m) = \frac{\mathrm{dn}(z|m)}{\mathrm{sn}(z|m)} = \frac{1}{\mathrm{sd}(z|m)}$.

$\mathrm{ds}^{-1}(z \mid m)$

The inverse of the Jacobi elliptic function ds. The value of u such that $\mathrm{ds}(u \mid m) = z : \mathrm{ds}^{-1}(z \mid m) = \int_z^\infty \frac{1}{\sqrt{t^2+m}\,\sqrt{t^2+m-1}}\,dt \,/;\, z \in \mathbb{R} \land z^2 + m > 1$.

E

e

The Euler exponential constant e: $e \approx 2.7182818\ldots$

$e^z = \exp(z)$

Exponential function: $e^z = \sum_{k=0}^{\infty} \frac{z^k}{k!}$.

$\{e_1, e_2, e_3\} = \{e_1(g_2, g_3), e_2(g_2, g_3), e_3(g_2, g_3)\}$

The values of the Weierstrass $\wp$ function at the half-periods $\{\omega_1, \omega_2, \omega_3\}$: $\{e_1, e_2, e_3\} = \{e_1(g_2, g_3), e_2(g_2, g_3), e_3(g_2, g_3)\} = \{\wp(\omega_1; g_2, g_3), \wp(\omega_2; g_2, g_3), \wp(\omega_3; g_2, g_3)\} \,/;\, \omega_2 = -\omega_1 - \omega_3$.

$\{e_1', e_2', e_3'\} = \{e_1'(g_2, g_3), e_2'(g_2, g_3), e_3'(g_2, g_3)\}$

The values of the Weierstrass $\wp'$ function at the half-periods $\{\omega_1, \omega_2, \omega_3\}$: $\{e_1', e_2', e_3'\} = \{e_1'(g_2, g_3), e_2'(g_2, g_3), e_3'(g_2, g_3)\} = \{\wp'(\omega_1; g_2, g_3), \wp'(\omega_2; g_2, g_3), \wp'(\omega_3; g_2, g_3)\} \,/;\, \omega_2 = -\omega_1 - \omega_3$.

$E(z)$

The complete elliptic integral of the second kind: $E(z) = E\left(\frac{\pi}{2} \middle| z\right) = \int_0^{\frac{\pi}{2}} \sqrt{1 - z\sin^2(t)}\,dt \,/;\, |\mathrm{Arg}(1-z)| < \pi$.

$E(z \mid m)$

The elliptic integral of the second kind: $E(z \mid m) = \int_0^z \sqrt{1 - m\sin^2(t)}\,dt$.

E_n

The n^{th} Euler number: $E_n = 2^{n+1}\, n!\left([t^n]\,\frac{e^{t/2}}{e^t+1}\right) /;\, n \in \mathbb{N}$

$E_n(z)$

The Euler polynomial of order n in z: $E_n(z) = 2\, n!\left([t^n]\,\frac{e^{z\,t}}{e^t+1}\right) /;\, n \in \mathbb{N}$.

$E_\nu(z)$

The exponential integral E: $E_\nu(z) == \int_1^\infty \frac{e^{-z t}}{t^\nu} \, dt \;/;\; \mathrm{Re}(z) > 0$.

eexp$(z; a, b)$

The elliptic exponential function eexp$(z; a, b) == \{x, y\}$. The values $\{x, y\}$ such that $z == \mathrm{elog}(x, y; a, b) \;/;\; y^2 - x\left(x^2 + a x + b\right) == 0$.

$\mathrm{eexp}'_z(z; a, b)$

The first derivative of the elliptic exponential function with respect to z: $\mathrm{eexp}'_z(z; a, b) == \frac{\partial \mathrm{eexp}(z;a,b)}{\partial z}$.

egcd(m, n)

The extended greatest common divisor of the integers m and n:
$\mathrm{egcd}(m, n) == \{g, \{r, s\}\} \;/;\; g == \gcd(m, n) \bigwedge g == m r + n s \bigwedge \mathrm{Re}(m), \mathrm{Im}(m), \mathrm{Re}(n), \mathrm{Im}(n) \in \mathbb{Z}$.

elog$(z_1, z_2; a, b)$

The generalized elliptic logarithm associated with the elliptic curve $z_1^3 + a z_1^2 + b z_1 - z_2^2 == 0$:
$\mathrm{elog}(z_1, z_2; a, b) == \frac{\sqrt{z_2^2}}{2 z_2} \int_\infty^{z_1} \frac{1}{\sqrt{t^3 + a t^2 + b t}} \, dt \;/;\; z_1^3 + a z_1^2 + b z_1 - z_2^2 == 0 \bigwedge a \in \mathbb{R} \bigwedge b \in \mathbb{R}$.

erf(z)

The error function: $\mathrm{erf}(z) == \frac{2}{\sqrt{\pi}} \int_0^z e^{-t^2} \, dt == \frac{2}{\sqrt{\pi}} \sum_{k=0}^\infty \frac{(-1)^k z^{2k+1}}{k! \, (2k+1)}$.

$\mathrm{erf}^{-1}(z)$

The inverse of the error function. The value of u such that $\mathrm{erf}(u) == z$.

$\mathrm{erf}(z_1, z_2)$

The generalized error function: $\mathrm{erf}(z_1, z_2) == \frac{2}{\sqrt{\pi}} \int_{z_1}^{z_2} e^{-t^2} \, dt == \mathrm{erf}(z_2) - \mathrm{erf}(z_1)$.

$\mathrm{erf}^{-1}(z_1, z_2)$

The inverse of the generalized error function. The value of u such that $\mathrm{erf}(z_1, u) == z_2$.

erfc(z)

The complementary error function: $\mathrm{erfc}(z) == \frac{2}{\sqrt{\pi}} \int_z^\infty e^{-t^2} \, dt == 1 - \mathrm{erf}(z) == 1 - \frac{2}{\sqrt{\pi}} \sum_{k=0}^\infty \frac{(-1)^k z^{2k+1}}{k! \, (2k+1)}$.

$\mathrm{erfc}^{-1}(z)$

The inverse of the complementary error function. The value of u such that $\mathrm{erfc}(u) == z$.

erfi(z)

The imaginary error function: $\text{erfi}(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{t^2} \, dt = -i \, \text{erf}(i\, z) = \frac{2}{\sqrt{\pi}} \sum_{k=0}^{\infty} \frac{z^{2k+1}}{k! \, (2k+1)}$.

$\text{Ei}(z)$

F

F_ν

$F_\nu(z)$

The Fibonacci polynomial of order n in z: $F_\nu(z) = \frac{1}{\sqrt{z^2+4}} \left(2^{-\nu} \left(z + \sqrt{z^2+4} \right)^{\nu} - \cos(\nu\pi) \, 2^{\nu} \left(z + \sqrt{z^2+4} \right)^{-\nu} \right)$.

$F(z \mid m)$

The elliptic integral of the first kind: $F(z \mid m) = \int_0^z \frac{1}{\sqrt{1 - m \sin^2(t)}} \, dt$.

$F_1(a; b_1, b_2; c; z_1, z_2)$

The Appell hypergeometric function of two variables $F_1 : F_1(a; b_1, b_2; c; z_1, z_2) = \sum_{k=0}^{\infty} \sum_{l=0}^{\infty} \frac{(a)_{k+l} \, (b_1)_k \, (b_2)_l \, z_1^k \, z_2^l}{(c)_{k+l} \, k! \, l!} \, /; \, |z_1| < 1 \wedge |z_2| < 1$.

${}_0F_0(;\,;z)$

The generalized hypergeometric function ${}_0F_0 : {}_0F_0(;\,;z) = e^z$.

${}_1F_0(a;\,;z)$

The generalized hypergeometric function ${}_1F_0 : {}_1F_0(a;\,;z) = (1-z)^{-a}$.

${}_0F_1(;b;z)$

The generalized hypergeometric function ${}_0F_1 : {}_0F_1(;b;z) = \sum_{k=0}^{\infty} \frac{z^k}{(b)_k \, k!}$.

${}_0\tilde{F}_1(;b;z)$

The regularized generalized hypergeometric function ${}_0\tilde{F}_1 : {}_0\tilde{F}_1(;b;z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(b+k) \, k!}$.

${}_1F_1(a;b;z)$

The Kummer confluent hypergeometric function ${}_1F_1 : {}_1F_1(a;b;z) = \sum_{k=0}^{\infty} \frac{(a)_k \, z^k}{(b)_k \, k!}$.

${}_1\tilde{F}_1(a;b;z)$

The regularized confluent hypergeometric function ${}_1\tilde{F}_1 : {}_1\tilde{F}_1(a;b;z) = \sum_{k=0}^{\infty} \frac{(a)_k \, z^k}{\Gamma(b+k) \, k!}$.

${}_2F_1(a, b; c; z)$

The Gauss hypergeometric function ${}_2F_1$: ${}_2F_1(a, b; c; z) == \sum_{k=0}^{\infty} \frac{(a)_k (b)_k z^k}{(c)_k k!}$ /; $|z| < 1$.

${}_2\tilde{F}_1(a, b; c; z)$

The regularized Gauss hypergeometric function ${}_2\tilde{F}_1$: ${}_2\tilde{F}_1(a, b; c; z) == \sum_{k=0}^{\infty} \frac{(a)_k (b)_k z^k}{\Gamma(c+k) k!}$ /; $|z| < 1$.

${}_1F_2(a_1; b_1, b_2; z)$

The generalized hypergeometric function ${}_1F_2$: ${}_1F_2(a_1; b_1, b_2; z) == \sum_{k=0}^{\infty} \frac{(a_1)_k z^k}{(b_1)_k (b_2)_k k!}$.

${}_2F_2(a_1, a_2; b_1, b_2; z)$

The generalized hypergeometric function ${}_2F_2$: ${}_2F_2(a_1, a_2; b_1, b_2; z) == \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k z^k}{(b_1)_k (b_2)_k k!}$.

${}_2F_3(a_1, a_2; b_1, b_2, b_3; z)$

The generalized hypergeometric function ${}_2F_3$: ${}_2F_3(a_1, a_2; b_1, b_2, b_3; z) == \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k z^k}{(b_1)_k (b_2)_k (b_3)_k k!}$.

${}_3F_2(a_1, a_2, a_3; b_1, b_2; z)$

The generalized hypergeometric function ${}_3F_2$: ${}_3F_2(a_1, a_2, a_3; b_1, b_2; z) == \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k (a_3)_k z^k}{(b_1)_k (b_2)_k k!}$ /; $|z| < 1$.

${}_4F_3(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z)$

The generalized hypergeometric function ${}_4F_3$: ${}_4F_3(a_1, a_2, a_3, a_4; b_1, b_2, b_3; z) == \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k (a_3)_k (a_4)_k z^k}{(b_1)_k (b_2)_k (b_3)_k k!}$ /; $|z| < 1$.

${}_pF_q(a_1, a_2, \ldots, a_p; b_1, b_2, \ldots, b_q; z)$

The generalized hypergeometric function ${}_pF_q$: ${}_pF_q(a_1, a_2, \ldots, a_p; b_1, b_2, \ldots, b_q; z) == \sum_{k=0}^{\infty} \frac{\prod_{j=1}^{p} (a_j)_k z^k}{\prod_{j=1}^{q} (b_j)_k k!}$ /; $q == p - 1 \bigwedge |z| < 1 \bigvee q \geq p$.

${}_p\tilde{F}_q(a_1, a_2, \ldots, a_p; b_1, b_2, \ldots, b_q; z)$

The regularized generalized hypergeometric function ${}_p\tilde{F}_q$: ${}_p\tilde{F}_q(a_1, a_2, \ldots, a_p; b_1, b_2, \ldots, b_q; z) == \sum_{k=0}^{\infty} \frac{\prod_{j=1}^{p} (a_j)_k z^k}{\prod_{j=1}^{q} \Gamma(k+b_j) k!}$ /; $q == p - 1 \bigwedge |z| < 1 \bigvee q \geq p$.

$$F_{P,Q,S}^{A,B,C}\left(\begin{matrix} a_1, \ldots, a_A; b_1, \ldots, b_B; c_1, \ldots, c_C; \\ p_1, \ldots, p_P; q_1, \ldots, q_Q; s_1, \ldots, s_S; \end{matrix} z, w\right)$$

The generalized hypergeometric function of two variables (Kampe de Feriet function): $F_{P,Q,S}^{A,B,C}\left(\begin{matrix} a_1, \ldots, a_A; b_1, \ldots, b_B; c_1, \ldots, c_C; \\ p_1, \ldots, p_P; q_1, \ldots, q_Q; s_1, \ldots, s_S; \end{matrix} z, w\right) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{\prod_{j=1}^{A} (a_j)_{m+n} \prod_{j=1}^{B} (b_j)_m \prod_{j=1}^{C} (c_j)_n z^m w^n}{m!\, n! \prod_{j=1}^{P} (p_j)_{m+n} \prod_{j=1}^{Q} (q_j)_m \prod_{j=1}^{S} (s_j)_n}$.

$$F_{P,Q,S}^{A,B,C}\left(\begin{matrix} a_1, \ldots, a_A; b_1, \ldots, b_B; c_1, \ldots, c_C; \\ p_1, \ldots, p_P; q_1, \ldots, q_Q; s_1, \ldots, s_S; \end{matrix} z, w\right) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{\prod_{j=1}^{A} (a_j)_{m+n} \prod_{j=1}^{B} (b_j)_m \prod_{j=1}^{C} (c_j)_n z^m w^n}{m!\, n! \prod_{j=1}^{P} (p_j)_{m+n} \prod_{j=1}^{Q} (q_j)_m \prod_{j=1}^{S} (s_j)_n} =$$

$$\sum_{m=0}^{\infty} \frac{\prod_{j=1}^{A} (a_j)_m \prod_{j=1}^{B} (b_j)_m z^m}{m! \prod_{j=1}^{P} (p_j)_m \prod_{j=1}^{Q} (q_j)_m} \, {}_{A+C}F_{P+S}(a_1 + m, a_2 + m, \ldots, a_A + m, c_1, \ldots, c_C;$$

$$p_1 + m, p_2 + m, \ldots, p_P + m, s_1, \ldots, s_S; w) = \sum_{m=0}^{\infty} \frac{\prod_{j=1}^{A} (a_j)_m \prod_{j=1}^{B} (b_j)_m z^m}{m! \prod_{j=1}^{P} (p_j)_m \prod_{j=1}^{Q} (q_j)_m}$$

$$\left(\frac{\prod_{k=1}^{q} \Gamma(b_k)}{\prod_{k=1}^{q+1} \Gamma(a_k)} \sum_{k=1}^{A} \frac{\Gamma(a_k + m) \prod_{\substack{j=1 \\ j \neq k}}^{A} \Gamma(a_j - a_k) \prod_{j=1}^{C} \Gamma(c_j - a_k - m)}{\prod_{j=1}^{q} \Gamma(b_j - a_k)} (-w)^{-a_k - m} \, {}_{P+S+1}F_{A+C-1} \right.$$

$$\Big(a_k + m, 1 + a_k - p_1, \ldots, 1 + a_k - p_P, 1 + a_k + m - s_1, \ldots, 1 + a_k + m - s_S;$$

$$1 + a_k - a_1, \ldots, 1 + a_k - a_{k-1}, 1 + a_k - a_{k+1}, \ldots, 1 + a_k - a_A,$$

$$1 + a_k + m - c_1, \ldots, 1 + a_k + m - c_C; \frac{1}{w}\Big) +$$

$$\frac{\prod_{k=1}^{q} \Gamma(b_k)}{\prod_{k=1}^{q+1} \Gamma(a_k)} \sum_{k=1}^{C} \frac{\Gamma(a_k) \prod_{\substack{j=1 \\ j \neq k}}^{q+1} \Gamma(a_j - a_k)}{\prod_{j=1}^{q} \Gamma(b_j - a_k)} (-z)^{-a_k} \, {}_{q+1}F_q \Big(a_k, 1 + a_k - b_1, \ldots, 1 + a_k - b_q;$$

$$\left. 1 + a_k - a_1, \ldots, 1 + a_k - a_{k-1}, 1 + a_k - a_{k+1}, \ldots, 1 + a_k - a_{q+1}; \frac{1}{w}\Big) \right)$$

$\tilde{F}_{P,Q,S}^{A,B,C}\left(\begin{matrix} a_1, \ldots, a_A; b_1, \ldots, b_B; c_1, \ldots, c_C; \\ p_1, \ldots, p_P; q_1, \ldots, q_Q; s_1, \ldots, s_S; \end{matrix} z, w\right)$

The regularized generalized hypergeometric function of two variables (regularized Kampe de Feriet function): $\tilde{F}_{P,Q,S}^{A,B,C}\left(\begin{matrix} a_1, \ldots, a_A; b_1, \ldots, b_B; c_1, \ldots, c_C; \\ p_1, \ldots, p_P; q_1, \ldots, q_Q; s_1, \ldots, s_S; \end{matrix} z, w\right) = \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{\prod_{j=1}^{A} (a_j)_{m+n} \prod_{j=1}^{B} (b_j)_m \prod_{j=1}^{C} (c_j)_n z^m w^n}{m!\, n! \prod_{j=1}^{P} \Gamma(m+n+p_j) \prod_{j=1}^{Q} \Gamma(m+q_j) \prod_{j=1}^{S} \Gamma(n+s_j)}$.

$F_A^{(n)}(a, b_1, \ldots, b_n; c_1, \ldots, c_n; z_1, \ldots, z_n)$

The Lauricella function A of n variables: $F_A^{(n)}(a, b_1, \ldots, b_n; c_1, \ldots, c_n; z_1, \ldots, z_n) = \sum_{k_1=0}^{\infty} \cdots \sum_{k_n=0}^{\infty} \frac{(a)_{k_1+\ldots+k_n} (b_1)_{k_1} \cdots (b_n)_{k_n}}{(c_1)_{k_1} \cdots (c_n)_{k_n}} \frac{z_1^{k_1} \ldots z_n^{k_n}}{k_1! \ldots k_n!} /; |z_1| + \ldots + |z_n| < 1.$

$F_B^{(n)}(a_1, \ldots, a_n, b_1, \ldots, b_n; c; z_1, \ldots, z_n)$

The Lauricella function B of n variables: $F_B^{(n)}(a_1, \ldots, a_n, b_1, \ldots, b_n; c; z_1, \ldots, z_n) = \sum_{k_1=0}^{\infty} \cdots \sum_{k_n=0}^{\infty} \frac{(a_1)_{k_1} \cdots (a_n)_{k_n} (b_1)_{k_1} \cdots (b_n)_{k_n}}{(c)_{k_1+\ldots+k_n}} \frac{z_1^{k_1} \ldots z_n^{k_n}}{k_1! \ldots k_n!} /; \max(|z_1|, \ldots, |z_n|) < 1.$

$F_C^{(n)}(a, b; c_1, \ldots, c_n; z_1, \ldots, z_n)$

The Lauricella function C of n variables: $F_C^{(n)}(a, b; c_1, \ldots, c_n; z_1, \ldots, z_n) = \sum_{k_1=0}^{\infty} \cdots \sum_{k_n=0}^{\infty} \frac{(a)_{k_1+\ldots+k_n} (b)_{k_1+\ldots+k_n}}{(c_1)_{k_1} \cdots (c_n)_{k_n}} \frac{z_1^{k_1} \ldots z_n^{k_n}}{k_1! \ldots k_n!} /; \sqrt{|z_1|} + \ldots + \sqrt{|z_n|} < 1.$

$F_D^{(n)}(a, b_1, \ldots, b_n; c; z_1, \ldots, z_n)$

The Lauricella function D of n variables: $F_D^{(n)}(a, b_1, \ldots, b_n; c; z_1, \ldots, z_n) = \sum_{k_1=0}^{\infty} \cdots \sum_{k_n=0}^{\infty} \frac{(a)_{k_1+\ldots+k_n} (b_1)_{k_1} \cdots (b_n)_{k_n}}{(c)_{k_1+\ldots+k_n}} \frac{z_1^{k_1} \ldots z_n^{k_n}}{k_1! \ldots k_n!} /; \max(|z_1|, \ldots, |z_n|) < 1.$

factors(n)

The prime factors of the integer n, together with their exponents.

frac (z)

The fractional part of number z: $\mathrm{frac}(x) = x - n /; x \in \mathbb{R} \wedge n \in \mathbb{Z} \wedge 0 \leq \mathrm{sgn}(x)(x-n) < 1 \wedge x \neq 0.$

G

$$G_{p,q}^{m,n}\left(z \,\middle|\, \begin{matrix} a_1, \ldots, a_n, a_{n+1}, \ldots, a_p \\ b_1, \ldots, b_m, b_{m+1}, \ldots, b_q \end{matrix}\right)$$

The Meijer G function: $G_{p,q}^{m,n}\left(z \,\middle|\, \begin{matrix} a_1, \ldots, a_n, a_{n+1}, \ldots, a_p \\ b_1, \ldots, b_m, b_{m+1}, \ldots, b_q \end{matrix}\right) = \frac{1}{2\pi i} \int_{\mathcal{L}} \frac{\prod_{k=1}^{m} \Gamma(b_k+s) \prod_{k=1}^{n} \Gamma(1-a_k-s)}{\prod_{k=n+1}^{p} \Gamma(a_k+s) \prod_{k=m+1}^{q} \Gamma(1-b_k-s)} z^{-s} \, ds /; 0 \leq m \leq q, \; 0 \leq n \leq p.$

The infinite contour of integration $\mathcal{L}$ separates the poles of $\Gamma(1-a_k-s)$ at $s = 1-a_k+j$, $j \in \mathbb{N}$ from the poles of $\Gamma(b_i+s)$ at $s = -b_i - l$, $l \in \mathbb{N}$. Such a contour always exists in the cases $a_k - b_i - 1 \notin \mathbb{N}$.

There are three possibilities for the contour $\mathcal{L}$:

(i) $\mathcal{L}$ runs from γ-$i\infty$ to γ+$i\infty$ (where $\mathrm{Im}(\gamma) = 0$) so that all poles of $\Gamma(b_i + s)$, $i = 1, \ldots, m$ are to the left of $\mathcal{L}$, and all poles of $\Gamma(1 - a_i - s)$, $i = 1, \ldots, n$ are to the right of $\mathcal{L}$. This contour can be a straight line $(\gamma - i\infty, \gamma + i\infty)$ if $\mathrm{Re}(b_i - a_k) > -1$ (then $-\mathrm{Re}(b_i) < \gamma < 1 - \mathrm{Re}(a_k)$). (In this case, the integral converges if $p + q < 2(m + n)$, $|\mathrm{Arg}(z)| < \left(m + n - \frac{p+q}{2}\right)\pi$. If $m + n - \frac{p+q}{2} = 0$, then z must be real and positive and the additional condition $(q - p)\gamma + \mathrm{Re}(\mu) < 0$, $\mu == \sum_{l=1}^{q} b_l - \sum_{k=1}^{p} a_k + \frac{p-q}{2} + 1$ should be added.)

(ii) $\mathcal{L}$ is a left loop, starting and ending at -∞ and encircling all poles of $\Gamma(b_i + s)$, $i = 1, \ldots, m$, once in the positive direction, but none of the poles of $\Gamma(1 - a_i - s)$, $i = 1, \ldots, n$. In this case, the integral converges if $q \geq 1$ and one of the following conditions is satisfied:

- $q > p$ or $q = p$ and $|z| < 1$
- $q = p$ and $|z| == 1$ and $m + n - \frac{p+q}{2} \geq 0$ and $\mathrm{Re}(\mu) < 0$.

(iii) $\mathcal{L}$ is a right loop, starting and ending at +∞ and encircling all poles of $\Gamma(1 - a_i - s)$, $i = 1, \ldots, n$, once in the negative direction, but none of the poles of $\Gamma(b_i + s)$, $i = 1, \ldots, m$. In this case, the integral converges if $p \geq 1$ and one of the following conditions is satisfied:

- $p > q$ or $p = q$ and $|z| > 1$
- $q = p$ and $|z| == 1$ and $m + n - \frac{p+q}{2} \geq 0$ and $\mathrm{Re}(\mu) < 0$.

$$G_{p,q}^{m,n}\left(z, r \,\middle|\, \begin{matrix} a_1, \ldots, a_n, a_{n+1}, \ldots, a_p \\ b_1, \ldots, b_m, b_{m+1}, \ldots, b_q \end{matrix}\right)$$

The generalized Meijer G function: $G_{p,q}^{m,n}\left(z, r \,\middle|\, \begin{matrix} a_1, & a_2, & \ldots, & a_p \\ b_1, & b_2, & \ldots, & b_q \end{matrix}\right) == \frac{1}{2\pi i} \int_{\mathcal{L}} \frac{\left(\prod_{k=1}^{m} \Gamma(b_k + s)\right) \prod_{k=1}^{n} \Gamma(1 - a_k - s)}{\left(\prod_{k=n+1}^{p} \Gamma(a_k + s)\right) \prod_{k=m+1}^{q} \Gamma(1 - b_k - s)} z^{-\frac{s}{r}} \, ds \, /;$

$$r \in \mathbb{R} \bigwedge r \neq 0 \bigwedge m \in \mathbb{N} \bigwedge n \in \mathbb{N} \bigwedge p \in \mathbb{N} \bigwedge q \in \mathbb{N} \bigwedge m \leq q \bigwedge n \leq p.$$

For the description of the contour $\mathcal{L}$, see $G_{p,q}^{m,n}\left(z \,\middle|\, \begin{matrix} a_1, \ldots, a_n, a_{n+1}, \ldots, a_p \\ b_1, \ldots, b_m, b_{m+1}, \ldots, b_q \end{matrix}\right)$.

$$G_{p,q:p_1,q_1:p_2,q_2}^{m,n:m_1,n_1:m_2,n_2}\left(\begin{matrix} a_1, & \ldots, & a_p \\ b_1, & \ldots, & b_q \end{matrix} \,\middle|\, \begin{matrix} a_{1,1}, & \ldots, & a_{1,p_1} \\ b_{1,1}, & \ldots, & b_{1,q_1} \end{matrix} \,\middle|\, \begin{matrix} a_{2,1}, & \ldots, & a_{2,p_2} \\ b_{2,1}, & \ldots, & b_{2,q_2} \end{matrix} \,\middle|\, z, w\right)$$

The Meijer G function of two variables: $G_{p,q:p_1,q_1:p_2,q_2}^{m,n:m_1,n_1:m_2,n_2}\left(\begin{matrix} a_1, & \ldots, & a_p \\ b_1, & \ldots, & b_q \end{matrix} \,\middle|\, \begin{matrix} a_{1,1}, & \ldots, & a_{1,p_1} \\ b_{1,1}, & \ldots, & b_{1,q_1} \end{matrix} \,\middle|\, \begin{matrix} a_{2,1}, & \ldots, & a_{2,p_2} \\ b_{2,1}, & \ldots, & b_{2,q_2} \end{matrix} \,\middle|\, z, w\right) ==$

$$\frac{1}{(2\pi i)^2} \int_{\mathcal{L}} \int_{\mathcal{L}^*} \frac{\prod_{j=1}^{m} \Gamma(b_j + s + t) \prod_{j=1}^{n} \Gamma(1 - a_j - s - t)}{\prod_{j=n+1}^{p} \Gamma(a_j + s + t) \prod_{j=m+1}^{q} \Gamma(1 - b_j - s - t)} \frac{\prod_{j=1}^{m_1} \Gamma(b_{1,j} + s) \prod_{j=1}^{n_1} \Gamma(1 - a_{1,j} - s)}{\prod_{j=n_1+1}^{p_1} \Gamma(a_{1,j} + s) \prod_{j=m_1+1}^{q_1} \Gamma(1 - b_{1,j} - s)} \frac{\prod_{j=1}^{m_2} \Gamma(b_{2,j} + t) \prod_{j=1}^{n_2} \Gamma(1 - a_{2,j} - t)}{\prod_{j=n_2+1}^{p_2} \Gamma(a_{2,j} + t) \prod_{j=m_2+1}^{q_2} \Gamma(1 - b_{2,j} - t)} z^{-s} w^{-t} \, ds \, dt \, /;$$

$0 \leq m \leq q$, $0 \leq n \leq p$, $0 \leq m_1 \leq q_1$, $0 \leq n_1 \leq p_1$, $0 \leq m_2 \leq q_2$, $0 \leq n_2 \leq p_2$.

$\{g_2, g_3\} == \{g_2(\omega_1, \omega_3), g_3(\omega_1, \omega_3)\}$

The invariants $\{g_2, g_3\}$ for Weierstrass elliptic functions corresponding to the half-periods $\{\omega_1, \omega_3\}$:
$\{g_2(\omega_1, \omega_3), g_3(\omega_1, \omega_3)\} == \left\{60 \sum_{\substack{m, n=-\infty \\ \{m,n\}\neq\{0,0\}}}^{\infty} \frac{1}{(2 m \omega_1 + 2 n \omega_3)^4}, 140 \sum_{\substack{m, n=-\infty \\ \{m,n\}\neq\{0,0\}}}^{\infty} \frac{1}{(2 m \omega_1 + 2 n \omega_3)^6}\right\} /;\ \operatorname{Im}\left(\frac{\omega_3}{\omega_1}\right) > 0.$

$\gcd(n_1, n_2, \ldots, n_k)$

The greatest common divisor of the integers $n_1, \ldots, n_k$.

H

$h_\nu^{(1)}(z)$

The Hankel spherical function of the first kind H1: $h_\nu^{(1)}(z) = \sqrt{\frac{\pi}{2}} \frac{1}{\sqrt{z}} H_{\nu+\frac{1}{2}}^{(1)}(z)$.

$h_\nu^{(2)}(z)$

The Hankel spherical function of the second kind H2: $h_\nu^{(2)}(z) = \sqrt{\frac{\pi}{2}} \frac{1}{\sqrt{z}} H_{\nu+\frac{1}{2}}^{(2)}(z)$.

H_z

The z^{th} harmonic number: $H_z == \sum_{k=1}^{\infty} \left(\frac{1}{k} - \frac{1}{k+z}\right) == \psi(z+1) + \gamma$.

$H_z^{(r)}$

The generalized harmonic number of order r: $H_z^{(r)} == \zeta(r) - \zeta(r, z+1)$.

$H_\nu(z)$

The ν^{th} Hermite function in z: $H_\nu(z) == 2^\nu \sqrt{\pi} \left(\frac{1}{\Gamma\left(\frac{1-\nu}{2}\right)} {}_1F_1\left(-\frac{\nu}{2}; \frac{1}{2}; z^2\right) - \frac{2 z}{\Gamma\left(-\frac{\nu}{2}\right)} {}_1F_1\left(\frac{1-\nu}{2}; \frac{3}{2}; z^2\right)\right)$. For nonnegative integer ν it is a polynomial in z.

$\mathbf{H}_\nu(z)$

The Struve function H: $\mathbf{H}_\nu(z) == \left(\frac{z}{2}\right)^{\nu+1} \sum_{k=0}^{\infty} \frac{(-1)^k}{\Gamma\left(k+\frac{3}{2}\right) \Gamma\left(k+\nu+\frac{3}{2}\right)} \left(\frac{z}{2}\right)^{2 k}$.

$H_\nu^{(1)}(z)$

The Hankel function of the first kind H1: $H_\nu^{(1)}(z) == J_\nu(z) + i\, Y_\nu(z)$.

$H_\nu^{(2)}(z)$

The Hankel function of the second kind H2: $H_\nu^{(2)}(z) == J_\nu(z) - i\, Y_\nu(z)$.

$$H_{p,q}^{m,n}\left(z \,\middle|\, \begin{matrix} \{a_1, A_1\}, \ldots, \{a_n, A_n\}, \{a_{n+1}, A_{n+1}\}, \ldots, \{a_p, A_p\} \\ \{b_1, B_1\}, \ldots, \{b_m, B_m\}, \{b_{m+1}, B_{m+1}\}, \ldots, \{b_q, B_q\} \end{matrix}\right)$$

The Fox H function: $H_{p,q}^{m,n}\left(z \,\middle|\, \begin{matrix} \{a_1, A_1\}, \ldots, \{a_n, A_n\}, \{a_{n+1}, A_{n+1}\}, \ldots, \{a_p, A_p\} \\ \{b_1, B_1\}, \ldots, \{b_m, B_m\}, \{b_{m+1}, B_{m+1}\}, \ldots, \{b_q, B_q\} \end{matrix}\right) = \frac{1}{2\pi i} \int_{\mathcal{L}} \frac{\prod_{k=1}^{m} \Gamma(b_k + B_k s) \prod_{k=1}^{n} \Gamma(1 - a_k - A_k s)}{\prod_{k=n+1}^{p} \Gamma(a_k + A_k s) \prod_{k=m+1}^{q} \Gamma(1 - b_k - B_k s)} z^{-s} \, ds \,/;\ 0 \le m \le q,\ 0 \le n \le p.$

The infinite contour of integration $\mathcal{L}$ separates the poles of $\Gamma(1 - a_k - A_k s)$ at $s = (1 - a_k + j)/A_k,\ j \in \mathbb{N}$ from the poles of $\Gamma(b_i + B_i s)$ at points $s = -(b_i + l)/B_i,\ l \in \mathbb{N}$.

I

i

The imaginary unit i: $i = \sqrt{-1}$.

$I_\nu(z)$

The modified Bessel function of the first kind: $I_\nu(z) = \sum_{k=0}^{\infty} \frac{1}{\Gamma(k+\nu+1)\, k!} \left(\frac{z}{2}\right)^{2k+\nu} = \frac{1}{\Gamma(\nu+1)} \left(\frac{z}{2}\right)^{\nu} {}_0F_1\left(; \nu + 1; \frac{z^2}{4}\right)$.

$I_z(a, b)$

The regularized incomplete beta function: $I_z(a, b) = \frac{B_z(a,b)}{B(a,b)}$.

$I_z^{-1}(a, b)$

The inverse of the regularized incomplete beta function. The value of u such that $I_u(a, b) = z$.

$I_{(z_1,z_2)}(a, b)$

The generalized regularized incomplete beta function: $I_{(z_1,z_2)}(a, b) = \frac{B(z_1,z_2,a,b)}{B(a,b)}$.

$I_{(z_1,z_2)}^{-1}(a, b)$

The inverse of the generalized regularized incomplete beta function. The value of u such that $I_{(z_1,u)}(a, b) = z_2$.

$\mathrm{Im}(z)$

The imaginary part of the number z: $z = \mathrm{Re}(z) + i\, \mathrm{Im}(z),\ \mathrm{Im}(z) = \frac{z - \bar{z}}{2 i}$.

int (z)

The integer part of number z: $\mathrm{int}(x) = n \,/;\ x \in \mathbb{R} \land n \in \mathbb{Z} \land 0 \le \mathrm{sgn}(x)\,(x - n) < 1 \land x \ne 0$.

J

$j_\nu(z)$

The spherical Bessel function of the first kind: $j_\nu(z) = \sqrt{\frac{\pi}{2}} \frac{1}{\sqrt{z}} J_{\nu+\frac{1}{2}}(z) == \frac{\sqrt{\pi}}{2} \left(\frac{z}{2}\right)^\nu \sum_{k=0}^\infty \frac{(-1)^k z^{2k}}{4^k \Gamma\left(k+\nu+\frac{3}{2}\right) k!}$.

$j_{\nu,k}$

The k^{th} root of the equation $J_\nu(z) == 0$: $(J_\nu(z) /; z = j_{\nu,k}) == 0 /; \nu \in \mathbb{R} \bigwedge k \in \mathbb{N}^+$.

$J(z)$

The Klein invariant modular function: $J(z) == \frac{\left(\vartheta_2\left(0, e^{\pi i z}\right)^8 + \vartheta_3\left(0, e^{\pi i z}\right)^8 + \vartheta_4\left(0, e^{\pi i z}\right)^8\right)^3}{54\left(\vartheta_2\left(0, e^{\pi i z}\right) \vartheta_3\left(0, e^{\pi i z}\right) \vartheta_4\left(0, e^{\pi i z}\right)\right)^8} /; \operatorname{Im}(z) > 0$.

$J_\nu(z)$

The Bessel function of the first kind: $J_\nu(z) == \sum_{k=0}^\infty \frac{(-1)^k}{\Gamma(k+\nu+1)\, k!} \left(\frac{z}{2}\right)^{2k+\nu} == \frac{1}{\Gamma(\nu+1)} \left(\frac{z}{2}\right)^\nu {}_0F_1\left(; \nu+1; -\frac{z^2}{4}\right)$.

K

K

The Khinchin constant K: $K \approx 2.685452001 \ldots$

$K(z)$

The complete elliptic integral of the first kind: $K(z) == F\left(\frac{\pi}{2} \middle| z\right) == \int_0^{\frac{\pi}{2}} \frac{1}{\sqrt{1 - z \sin^2(t)}} \, dt /; |\operatorname{Arg}(1-z)| < \pi$.

$K_\nu(z)$

The modified Bessel function of the second kind: $K_\nu(z) == \frac{\pi \csc(\pi \nu)}{2} (I_{-\nu}(z) - I_\nu(z)) /; \nu \notin \mathbb{Z}$.

kei(z)

The Kelvin function of the second kind kei: $\operatorname{kei}(z) = -\frac{1}{4} i \left(2 K_0\left(\sqrt[4]{-1}\, z\right) + \pi Y_0\left(\sqrt[4]{-1}\, z\right) - 4 i \left(\log(z) - \log\left(\sqrt[4]{-1}\, z\right)\right) \operatorname{bei}(z) - i \pi \operatorname{ber}(z)\right); \operatorname{kei}(z) = \operatorname{kei}_0(z)$.

$\operatorname{kei}_\nu(z)$

The Kelvin function of the second kind kei: $\operatorname{kei}_\nu(z) = \frac{\pi}{2} (\csc(\pi \nu) \operatorname{bei}_{-\nu}(z) - \cot(\pi \nu) \operatorname{bei}_\nu(z) + \operatorname{ber}_\nu(z)) /; \nu \notin \mathbb{Z}$.

ker(z)

The Kelvin function of the second kind ker: $\operatorname{ker}(z) = \frac{1}{4} \left(2 K_0\left(\sqrt[4]{-1}\, z\right) - \pi Y_0\left(\sqrt[4]{-1}\, z\right) + \pi \operatorname{bei}(z) - 4 \left(\log(z) - \log\left(\sqrt[4]{-1}\, z\right)\right) \operatorname{ber}(z)\right); \operatorname{ker}(z) = \operatorname{ker}_0(z)$.

$\operatorname{ker}_\nu(z)$

The Kelvin function of the second kind ker: $\mathrm{ker}_\nu(z) = -\frac{1}{2}\pi\,(\mathrm{bei}_\nu(z) - \csc(\pi\,\nu)\,\mathrm{ber}_{-\nu}(z) + \cot(\pi\,\nu)\,\mathrm{ber}_\nu(z))\;/;\;\nu \notin \mathbb{Z}$.

L

L_ν

The ν^{th} Lucas number: $L_\nu = \phi^\nu + \phi^{-\nu}\cos(\pi\,\nu)$.

$L_\nu(z)$

The ν^{th} Laguerre function in z: $L_\nu(z) = {}_1F_1(-\nu; 1; z)$. For nonnegative integer ν it is a polynomial in z.

$L_\nu^\lambda(z)$

The ν^{th} generalized Laguerre polynomial in z for parameter λ: $L_\nu^\lambda(z) = \frac{\Gamma(\lambda+\nu+1)}{\nu!}\,{}_1\tilde{F}_1(-\nu; \lambda+1; z)$. For nonnegative integer ν it is a polynomial in z.

$\mathrm{L}_\nu(z)$

The modified Struve function: $\mathrm{L}_\nu(z) = \left(\frac{z}{2}\right)^{\nu+1}\sum_{k=0}^{\infty}\frac{1}{\Gamma\left(k+\frac{3}{2}\right)\Gamma\left(k+\nu+\frac{3}{2}\right)}\left(\frac{z}{2}\right)^{2k}$.

$\mathrm{lcm}(n_1, n_2, \ldots, n_m)$

The least common multiple of the integers (or rational) n_k.

$\mathrm{li}(z)$

$\mathrm{Li}_\nu(z)$

The polylogarithm function of order ν: $\mathrm{Li}_\nu(z) = \sum_{k=1}^{\infty}\frac{z^k}{k^\nu}\;/;\;|z| < 1$. For $\nu = 2$ it is a dilogarithm function in z.

$\log(z)$

The natural logarithm: $\log(z) = \sum_{k=1}^{\infty}\frac{(-1)^{k-1}(z-1)^k}{k}\;/;\;|z-1| < 1$.

$\log_a(z)$

The logarithm in base a: $\log_a(z) = \frac{\log(z)}{\log(a)}$.

$\log\Gamma(z)$

The logarithmic gamma function: $\log\Gamma(z) = \sum_{k=1}^{\infty}\left(\frac{z}{k} - \log\left(1+\frac{z}{k}\right)\right) - \gamma\,z - \log(z)$.

M

$M_{\nu,\mu}(z)$

The Whittaker hypergeometric function M: $M_{\nu,\mu}(z) = z^{\mu+\frac{1}{2}} e^{-\frac{z}{2}} {}_1F_1\left(\mu - \nu + \frac{1}{2}; 2\mu + 1; z\right)$.

$\max(x_1, x_2, \ldots, x_n)$

The maximum function (the numerically largest of the real numbers $x_1, x_2, \ldots, x_n$): $\max(x_1, x_2) == \frac{1}{2}\left(x_1 + x_2 + \sqrt{(x_1 - x_2)^2}\right) /; x_1 \in \mathbb{R} \bigwedge x_2 \in \mathbb{R};$

$\max(x_1, x_2, \ldots, x_n) == \max(\max(x_1, x_2), x_3, \ldots, x_n)$.

$\min(x_1, x_2, \ldots, x_n)$

The minimum function (the numerically smallest of the real numbers $x_1, x_2, \ldots, x_n$): $\min(x_1, x_2) == \frac{1}{2}\left(x_1 + x_2 - \sqrt{(x_1 - x_2)^2}\right) /; x_1 \in \mathbb{R} \bigwedge x_2 \in \mathbb{R}; \max(x_1, x_2, \ldots, x_n) == \max(\max(x_1, x_2), x_3, \ldots, x_n)$.

N

$\text{nc}(z \mid m)$

The Jacobi elliptic function nc: $\text{nc}(z \mid m) == \frac{1}{\text{cn}(z \mid m)}$.

$\text{nc}^{-1}(z \mid m)$

The inverse of the Jacobi elliptic function nc. The value of u such that $\text{nc}(u \mid m) == z: \ \text{nc}^{-1}(z \mid m) == \int_1^z \frac{1}{\sqrt{t^2-1}\sqrt{(1-m)t^2+m}} \, dt /; z \in \mathbb{R} \bigwedge z^2 > 1 \bigwedge (1-m) z^2 + m > 0$.

$\text{nd}(z \mid m)$

The Jacobi elliptic function nd: $\text{nd}(z \mid m) == \frac{1}{\text{dn}(z \mid m)}$.

$\text{nd}^{-1}(z \mid m)$

The inverse of the Jacobi elliptic function nd. The value of u such that $\text{nd}(u \mid m) == z: \ \text{nd}^{-1}(z \mid m) == \int_1^z \frac{1}{\sqrt{t^2-1}\sqrt{1-(1-m)t^2}} \, dt /; z \in \mathbb{R} \bigwedge z^2 > 1 \bigwedge (1-m) z^2 < 1 \bigwedge m > 0$.

$\text{ns}(z \mid m)$

The Jacobi elliptic function ns: $\text{ns}(z \mid m) == \frac{1}{\text{sn}(z \mid m)}$.

$\text{ns}^{-1}(z \mid m)$

The inverse of the Jacobi elliptic function ns. The value of u such that $\text{ns}(u \mid m) == z: \ \text{ns}^{-1}(z \mid m) == \int_z^\infty \frac{1}{\sqrt{t^2-1}\sqrt{t^2-m}} \, dt /; z \in \mathbb{R} \bigwedge z^2 > 1 \bigwedge z^2 > m$.

P

$p(n)$

The number of unrestricted partitions (independent of the order and with repetitions allowed) of the positive integer n into a sum of strictly positive integers that add up to n: $p(n) = \left([t^n] \prod_{k=1}^{\infty} \frac{1}{1-t^k}\right) /; n \in \mathbb{N}$.

$p_n(= \text{prime}(n))$

The n^{th} prime number (the smallest integer greater than p_{n-1} that cannot be divided by any integer greater than 1 and smaller than itself): $p_n = m /; n > 1 \bigwedge m \in \mathbb{Z} \bigwedge m > p_{n-1} \bigwedge \left(\neg \exists_{p,p\in\mathbb{P}}\, p_{n-1} < p < m\right) \bigwedge \left(\neg \exists_{k,k\in\mathbb{Z}\bigwedge 1<k<m}\, \frac{m}{k} \in \mathbb{Z}\right)$.

$P_\nu(z)$

The ν^{th} Legendre function in z: $P_\nu(z) = {}_2F_1\left(-\nu, \nu + 1; 1; \frac{1-z}{2}\right)$. For nonnegative integer ν it is a polynomial in z.

$P_\nu^\mu(z)$

The associated Legendre function of the first kind of type 2: $P_\nu^\mu(z) = \frac{(1+z)^{\mu/2}}{(1-z)^{\mu/2}}\, {}_2\tilde{F}_1\left(-\nu, \nu + 1; 1 - \mu; \frac{1-z}{2}\right)$.

$\mathcal{P}_\nu^\mu(z)$

The associated Legendre function of the second kind of type 3: $\mathcal{P}_\nu^\mu(z) = \frac{(z+1)^{\mu/2}}{(z-1)^{\mu/2}}\, {}_2\tilde{F}_1\left(-\nu, \nu + 1; 1 - \mu; \frac{1-z}{2}\right)$.

$P_\nu^{(a,b)}(z)$

The ν^{th} Jacobi function in z for parameters a and b: $P_\nu^{(a,b)}(z) = \frac{\Gamma(a+\nu+1)}{\Gamma(\nu+1)}\, {}_2\tilde{F}_1\left(-\nu, a + b + \nu + 1; a + 1; \frac{1-z}{2}\right)$. For nonnegative integer ν it is a polynomial in z.

$\mathcal{PhysicalQ}(j_1, m_1, j_2, m_2, j, m)$

A Boolean function that tests whether the angular momentum quantum numbers are physically realizable:

$$\mathcal{PhysicalQ}(j_1, m_1, j_2, m_2, j, m) = (2\, j_1 \in \mathbb{Z} \bigwedge 2\, j_2 \in \mathbb{Z} \bigwedge 2\, j \in \mathbb{Z} \bigwedge 2\, m_1 \in \mathbb{Z} \bigwedge 2\, m_2 \in \mathbb{Z} \bigwedge 2\, m \in \mathbb{Z} \bigwedge j_1 - m_1 \in \mathbb{Z} \bigwedge j_2 - m_2 \in \mathbb{Z} \bigwedge j - m \in \mathbb{Z} \bigwedge -j_1 \le m_1 \le j_1 \bigwedge -j_2 \le m_2 \le j_2 \bigwedge -j \le m \le j \bigwedge |j_1 - j_2| \le j \bigwedge |j| \le j_1 + j_2).$$

$PS_{\nu,\mu}(\gamma, z)$

The angular spheroidal function of the first kind with variable z and parameters ν, μ, γ.

$PS_{\nu,\mu}{}'(\gamma, z)$

The derivative with respect to z of the angular spheroidal function of the first kind with variable z and parameters ν, μ, γ: $PS_{\nu,\mu}{}'(\gamma, z) = \frac{\partial PS_{\nu,\mu}(\gamma,z)}{\partial z}$.

Q

$q(n)$

The number of ordered partitions (independent of the order and no repetitions allowed) of the positive integer n into a sum of strictly positive integers which add up to n: $q(n) = \left([t^n] \prod_{k=1}^{\infty} \left(1 + t^k\right)\right) /; n \in \mathbb{N}$.

$q(m)$

The elliptic nome q of the module m: $q(m) = \exp\left(-\pi \frac{K(1-m)}{K(m)}\right)$.

$q^{-1}(z)$

The module m of the nome z: $q^{-1}(z) = 16\, z \prod_{k=1}^{\infty} \left(\frac{1+z^{2k}}{1+z^{2k-1}}\right)^8 /; |z| < 1,\ q^{-1}(q(m)) = m$.

$Q_\nu(z)$

The ν^{th} Legendre function of the second kind: $Q_\nu(z) = Q_\nu^0(z)$.

$Q_\nu^\mu(z)$associated

The associated Legendre function of the second kind of type 2: $Q_\nu^\mu(z) = \frac{\pi \csc(\mu \pi)}{2} \left(\frac{(1+z)^{\mu/2}}{(1-z)^{\mu/2}} \cos(\mu \pi)\, {}_2\tilde{F}_1\left(-\nu, \nu+1; 1-\mu; \frac{1-z}{2}\right) - \frac{\Gamma(\mu+\nu+1)}{\Gamma(-\mu+\nu+1)} \frac{(1-z)^{\mu/2}}{(1+z)^{\mu/2}}\, {}_2\tilde{F}_1\left(-\nu, \nu+1; \mu+1; \frac{1-z}{2}\right)\right) /;$ $\mu \notin \mathbb{Z}$.

$\mathfrak{Q}_\nu^\mu(z)$

The associated Legendre function of the second kind of type 3: $\mathfrak{Q}_\nu^\mu(z) = \frac{\pi \csc(\mu \pi)}{2} e^{\pi i \mu} \left(\frac{(z+1)^{\mu/2}}{(z-1)^{\mu/2}}\, {}_2\tilde{F}_1\left(-\nu, \nu+1; 1-\mu; \frac{1-z}{2}\right) - \frac{\Gamma(\mu+\nu+1)}{\Gamma(-\mu+\nu+1)} \frac{(z-1)^{\mu/2}}{(z+1)^{\mu/2}}\, {}_2\tilde{F}_1\left(-\nu, \nu+1; \mu+1; \frac{1-z}{2}\right)\right) /; \mu \notin \mathbb{Z}$.

$Q(a, z)$

The regularized incomplete gamma function: $Q(a, z) = \frac{1}{\Gamma(a)} \int_z^{\infty} t^{a-1} e^{-t}\, dt = 1 - \frac{z^a}{\Gamma(a+1)} \sum_{k=0}^{\infty} \frac{a\,(-z)^k}{(a+k)\,k!}$.

$Q^{-1}(a, z)$

The inverse of the regularized incomplete gamma function. The value of u such that $Q(a, u) = z$.

$Q(a, z_1, z_2)$

The generalized regularized incomplete gamma function: $Q(a, z_1, z_2) = \frac{1}{\Gamma(a)} \int_{z_1}^{z_2} t^{a-1} e^{-t}\, dt$.

$Q^{-1}(a, z_1, z_2)$

The inverse of the generalized regularized incomplete gamma function. The value u such that $Q(a, z_1, u) = w$.

$QS_{\nu,\mu}(\gamma, z)$

The angular spheroidal function of the second kind with variable z and parameters ν, μ, γ.

$QS_{\nu,\mu}'(\gamma, z)$

The derivative with respect to z of the angular spheroidal function of the second kind with variable z and parameters ν, μ, γ: $QS_{\nu,\mu}'(\gamma, z) = \frac{\partial QS_{\nu,\mu}(\gamma,z)}{\partial z}$.

quotient(m, n)

The integer quotient of m and n: $\text{quotient}(m, n) = \lfloor \frac{m}{n} \rfloor$.

R

$r_m(n)$

The number of representations of n as a sum of m squares of different positive or negative integers.

$r(a, q)$

The characteristic exponent of the Mathieu functions. $\text{MathieuFunction}(a, q, z) = e^{i\, r(a,q)\, z} f(z)$ (where $f(z)$ has period 2π).

$R_n^m(z)$

$\text{Re}(z)$

The real part of the number z: $z = \text{Re}(z) + i\,\text{Im}(z)$, $\text{Re}(z) = \frac{z+\bar{z}}{2}$.

S

$S(z)$

The Fresnel integral S: $S(z) = \int_0^z \sin\left(\frac{\pi t^2}{2}\right) dt = z^3 \sum_{k=0}^{\infty} \frac{2^{-2k-1} \pi^{2k+1} \left(-z^4\right)^k}{(4k+3)(2k+1)!}$.

$S_{\nu,p}(z) = S_\nu^p(z)$

The Nielsen generalized polylogarithm: $S_{\nu,p}(z) = \sum_{k=1}^{\infty} \frac{(-1)^{k+p} S_k^{(p)} z^k}{k^\nu k!}$ /; $|z| < 1 \bigwedge p \in \mathbb{N}^+$.

$S_n^{(m)}$

The Stirling number of the first kind: $S_n^{(m)} = (-1)^n \left([t^m](-t)_n\right)$ /; $m, n \in \mathbb{N}$.

$\mathcal{S}_n^{(m)}$

The Stirling number of the second kind: $\mathcal{S}_n^{(m)} = \frac{1}{m!} \sum_{k=0}^{m} (-1)^{m-k} \binom{m}{k} k^n$ /; $m, n-1 \in \mathbb{N}$.

$S_{\nu,\mu}^{(1)}(\gamma, z)$

The radial spheroidal function of the first kind with variable z and parameters ν, μ, γ.

$S_{\nu,\mu}^{(1)\prime}(\gamma, z)$

The derivative with respect to z of the radial spheroidal function of the first kind with variable z and parameters ν, μ, γ: $S_{\nu,\mu}^{(1)\prime}(\gamma, z) = \frac{\partial S_{\nu,\mu}^{(1)}(\gamma,z)}{\partial z}$.

$S_{\nu,\mu}^{(2)}(\gamma, z)$

The radial spheroidal function of the second kind with variable z and parameters ν, μ, γ.

$S_{\nu,\mu}^{(2)\prime}(\gamma, z)$

The derivative with respect to z of the radial spheroidal function of the second kind with variable z and parameters ν, μ, γ: $S_{\nu,\mu}^{(2)\prime}(\gamma, z) = \frac{\partial S_{\nu,\mu}^{(2)}(\gamma,z)}{\partial z}$.

$\mathrm{sc}(z \mid m)$

The Jacobi elliptic function sc: $\mathrm{sc}(z \mid m) = \frac{\mathrm{sn}(z|m)}{\mathrm{cn}(z|m)} = \frac{1}{\mathrm{cs}(z|m)}$.

$\mathrm{sc}^{-1}(z \mid m)$

The inverse of the Jacobi elliptic function sc. The value of u such that $\mathrm{sc}(u \mid m) = z$: $\mathrm{sc}^{-1}(z \mid m) = \int_0^z \frac{1}{\sqrt{t^2+1}\,\sqrt{(1-m)\,t^2+1}}\,dt$ /; $z \in \mathbb{R} \bigwedge (1-m)\,z^2 > -1$.

$\mathrm{sd}(z \mid m)$

The Jacobi elliptic function sd: $\mathrm{sd}(z \mid m) = \frac{\mathrm{sn}(z|m)}{\mathrm{dn}(z|m)} = \frac{1}{\mathrm{ds}(z|m)}$.

$\mathrm{sd}^{-1}(z \mid m)$

The inverse of the Jacobi elliptic function sd. The value of u such that $\mathrm{sd}(u \mid m) = z$: $\mathrm{sd}^{-1}(z \mid m) = \int_0^z \frac{1}{\sqrt{m\,t^2+1}\,\sqrt{1-(1-m)\,t^2}}\,dt$ /; $z \in \mathbb{R} \bigwedge m\,z^2 > -1 \bigwedge (1-m)\,z^2 < 1$.

$\mathrm{Se}(a, q, z)$

The odd Mathieu function with characteristic value a and parameter q.

$\mathrm{Se}_z(a, q, z) = \mathrm{Se}'(a, q, z)$

The derivative with respect to z of the odd Mathieu function with characteristic value a and parameter q: $\text{Se}'(a, q, z) == \frac{\partial \text{Se}(a,q,z)}{\partial z}$.

$\sec(z)$

The secant function: $\sec(z) == \frac{1}{\cos(z)}$.

$\sec^{-1}(z)$

The inverse secant function: $\sec^{-1}(z) == \cos^{-1}\left(\frac{1}{z}\right)$.

$\text{sech}(z)$

The hyperbolic secant function: $\text{sech}(z) == \frac{1}{\cosh(z)} == \sec(i\, z)$.

$\text{sech}^{-1}(z)$

The inverse hyperbolic secant function: $\text{sech}^{-1}(z) == \cosh^{-1}\left(\frac{1}{z}\right) == \frac{\sqrt{1/z-1}}{\sqrt{1-1/z}} \sec^{-1}(z)$.

$\text{sgn}(z)$

The signum of the number z: $\text{sgn}(z) == \frac{z}{|z|}$ /; $z \neq 0$

$\text{Shi}(z)$

The hyperbolic sine integral function: $\text{Shi}(z) == \int_0^z \frac{\sinh(t)}{t}\, dt == z \sum_{k=0}^{\infty} \frac{z^{2k}}{(1+2k)^2 (2k)!}$.

$\text{Si}(z)$

The sine integral function: $\text{Si}(z) == \int_0^z \frac{\sin(t)}{t}\, dt == \sum_{k=0}^{\infty} \frac{(-1)^k z^{2k+1}}{(2k+1)(2k+1)!}$.

$\sin(z)$

The sine function: $\sin(z) == \frac{e^{iz} - e^{-iz}}{2i} == \sum_{k=0}^{\infty} \frac{(-1)^k z^{2k+1}}{(2k+1)!}$.

$\sin^{-1}(z)$

The inverse sine function: $\sin^{-1}(z) == -i \log\left(i\, z + \sqrt{1 - z^2}\right)$.

$\text{sinc}(z)$

The sinc (sampling) function: $\text{sinc}(z) = \frac{\sin(z)}{z}$ /; $z \neq 0$; $\text{sinc}(0) = 1$.

$\sinh(z)$

The hyperbolic sine function: $\sinh(z) == \frac{e^z - e^{-z}}{2} == -i\sin(i\,z) == \sum_{k=0}^{\infty} \frac{z^{2k+1}}{(2k+1)!}$.

$\sinh^{-1}(z)$

The inverse hyperbolic sine function: $\sinh^{-1}(z) == \log\left(z + \sqrt{1+z^2}\right) == -i\sin^{-1}(i\,z)$.

$\text{sn}(z \mid m)$

The Jacobi elliptic function sn: $\text{sn}(z \mid m) == \sin(\text{am}(z \mid m))$.

$\text{sn}^{-1}(z \mid m)$

The inverse of the Jacobi elliptic function sn. The value of u such that $\text{sn}(u \mid m) == z$: $\text{sn}^{-1}(z \mid m) == \int_0^z \frac{1}{\sqrt{1-t^2}\sqrt{1-m\,t^2}}\,dt$ /; $-1 < z < 1 \land m\,z^2 < 1$.

SpheroidalJoiningFactor(ν, μ, γ)

The spheroidal joining factor of degree ν and order μ appearing in the relations between radial and angular spheroidal functions.

SpheroidalRadialFactor(ν, μ, γ)

The spheroidal radial factor of degree ν and order μ appearing in expansions of radial spheroidal function of the first kind around $\gamma == 0$.

Subfactorial[z]

The subfactorial function (number of complete permutations) : Subfactorial[z] $== \frac{\Gamma(z+1,-1)}{e}$.

T

$T_\nu(z)$

The ν^{th} Chebyshev function of the first kind: $T_\nu(z) == \cos\left(\nu\cos^{-1}(z)\right) == {}_2F_1\left(-\nu, \nu; \frac{1}{2}; \frac{1-z}{2}\right)$. For nonnegative integer ν it is a polynomial in z.

$\tan(z)$

The tangent function: $\tan(z) == \frac{\sin(z)}{\cos(z)}$.

$\tan^{-1}(z)$

The inverse tangent function: $\tan^{-1}(z) == \frac{i}{2}(\log(1 - i\,z) - \log(1 + i\,z))$.

$\tan^{-1}(x, y)$

The inverse tangent function of two variables: $\tan^{-1}(x, y) == -i \log\left(\frac{x+i y}{\sqrt{x^2+y^2}}\right)$.

$\tanh(z)$

The hyperbolic tangent function: $\tanh(z) == \frac{\sinh(z)}{\cosh(z)} == -i \tan(i z)$.

$\tanh^{-1}(z)$

The inverse hyperbolic tangent function: $\tanh^{-1}(z) == \frac{1}{2} (\log(1+z) - \log(1-z)) == -i \tan^{-1}(i z)$.

U

$U_\nu(z)$

The ν^{th} Chebyshev function of the second kind: $U_\nu(z) == \frac{\sin((\nu+1)\cos^{-1}(z))}{\sqrt{1-z^2}} == (\nu+1)\, {}_2F_1\left(-\nu, \nu+2; \frac{3}{2}; \frac{1-z}{2}\right)$. For nonnegative integer ν it is a polynomial in z.

$U(a, b, z)$

The Tricomi hypergeometric function U: $U(a, b, z) == \frac{\Gamma(b-1)\, z^{1-b}}{\Gamma(a)}\, {}_1F_1(a-b+1; 2-b; z) + \frac{\Gamma(1-b)}{\Gamma(a-b+1)}\, {}_1F_1(a; b; z) \,/;\, b \notin \mathbb{Z}$.

$U(a, b, c, z)$

W

$W(z)$

The product log function on the principal sheet. The value of u such that $u e^u == z$.

$W_k(z)$

The product log function on the k^{th} sheet. The k^{th} value of u such that $u e^u == z$.

$W_{\nu,\mu}(z)$

The Whittaker hypergeometric function W: $W_{\nu,\mu}(z) = z^{\mu+\frac{1}{2}} e^{-\frac{z}{2}} U\left(\mu - \nu + \frac{1}{2}, 2\mu + 1, z\right)$.

Y

$y_\nu(z)$

The spherical Bessel function of the second kind: $y_\nu(z) = \sqrt{\frac{\pi}{2}} \frac{1}{\sqrt{z}} Y_{\nu+\frac{1}{2}}(z)$.

$y_{\nu,k}$

The k^{th} root of the equation $Y_\nu(z) == 0$: $(Y_\nu(z) \,/;\, z = y_{\nu,k}) == 0 \,/;\, \nu \in \mathbb{R} \bigwedge k \in \mathbb{N}^+$.

$Y_\nu(z)$

The Bessel function of the second kind: $Y_\nu(z) == \csc(\pi\nu)(\cos(\pi\nu) J_\nu(z) - J_{-\nu}(z)) \,/;\, \nu \notin \mathbb{Z}$.

$Y_\lambda^\mu(\vartheta, \varphi)$

The spherical harmonic function of θ and φ for parameters λ and μ: $Y_\lambda^\mu(\vartheta, \varphi) == \sqrt{\frac{2\lambda+1}{4\pi}} \frac{\sqrt{\Gamma(\lambda-\mu+1)}}{\sqrt{\Gamma(\lambda+\mu+1)}} e^{i\varphi\mu} \frac{\cos^2\left(\frac{\vartheta}{2}\right)^{\mu/2}}{\sin^2\left(\frac{\vartheta}{2}\right)^{\mu/2}} {}_2\tilde{F}_1\left(-\lambda, \lambda+1; 1-\mu; \sin^2\left(\frac{\vartheta}{2}\right)\right)$.

Z

$Z(z)$

The Riemann-Siegel Zeta function: $Z(z) == e^{i\vartheta(z)} \zeta\left(i z + \frac{1}{2}\right)$.

$\mathrm{Z}(z \mid m)$

The Jacobi Zeta function: $\mathrm{Z}(z \mid m) == E(z \mid m) - \frac{E(m)}{K(m)} F(z \mid m)$.

B

$\mathrm{B}(a, b)$

The Euler beta function: $\mathrm{B}(a, b) == \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)} == \int_0^1 t^{a-1}(1-t)^{b-1} \, dt \,/;\, \mathrm{Re}(a) > 0 \bigwedge \mathrm{Re}(b) > 0$.

$\mathrm{B}_z(a, b)$

The incomplete beta function: $\mathrm{B}_z(a, b) == \int_0^z t^{a-1}(1-t)^{b-1} \, dt \,/;\, \mathrm{Re}(a) > 0$; $\mathrm{B}_z(a, b) == z^a \Gamma(a) \, {}_2\tilde{F}_1(a, 1-b; a+1; z) \,/;\, -a \notin \mathbb{N}$.

$\mathrm{B}_{(z_1,z_2)}(a, b)$

The generalized incomplete beta function: $\mathrm{B}_{(z_1,z_2)}(a, b) == \int_{z_1}^{z_2} t^{a-1}(1-t)^{b-1} \, dt == \mathrm{B}_{z_2}(a, b) - \mathrm{B}_{z_1}(a, b)$.

Γ

γ

Euler gamma γ: $\gamma == \lim_{n\to\infty} \left(\sum_{k=1}^n \frac{1}{k} - \log(n)\right) \approx 0.5772156\ldots$

γ_n

The n^{th} Stieltjes constant: $\gamma_n == (-1)^n n! \left(\left[(s-1)^n\right] \left(\zeta(s) - \frac{1}{s-1}\right)\right) \,/;\, n \in \mathbb{N}$.

$\Gamma(z)$

The Euler gamma function: $\Gamma(z) == \int_0^\infty t^{z-1} e^{-t} \, dt \;/;\; \mathrm{Re}(z) > 0$.

$\Gamma(a, z)$

The incomplete gamma function: $\Gamma(a, z) == \int_z^\infty t^{a-1} e^{-t} \, dt == \Gamma(a) - z^a \sum_{k=0}^\infty \frac{(-z)^k}{(a+k)\, k!}$.

$\Gamma(a, z_1, z_2)$

The generalized incomplete gamma function: $\Gamma(a, z_1, z_2) == \int_{z_1}^{z_2} t^{a-1} e^{-t} \, dt == \Gamma(a, z_1) - \Gamma(a, z_2)$.

Δ

$\delta(x)$

The Dirac delta function: $\delta(x) == \frac{1}{\pi} \lim_{\varepsilon \to 0} \frac{\varepsilon}{x^2 + \varepsilon^2} \;/;\; x \in \mathbb{R}$.

$\delta(x_1, x_2, \ldots, x_m)$

$\delta(n)$

The discrete delta function: $\delta(n) = \begin{cases} 1 & n == 0 \\ 0 & \text{else} \end{cases}$

$\delta(n_1, n_2, \ldots, n_m)$

The multidimensional discrete delta function: $\delta(n_1, n_2, \ldots, n_m) = \prod_{k=1}^m \delta(n_j)$

$\delta_{n_1, n_2, \ldots, n_m}$

The Kronecker delta function: $\delta_{n_1, n_2, \ldots, n_m} = \begin{cases} 1 & n_1 == n_2 == \ldots == n_m \\ 0 & \text{else} \end{cases}$

E

$\varepsilon_{n_1, n_2, \ldots, n_d}$

Z

$\zeta(s)$

The Riemann zeta function: $\zeta(s) == \sum_{k=1}^\infty \frac{1}{k^s} \;/;\; \mathrm{Re}(s) > 1$.

$\zeta(s, a)$

The generalized Riemann zeta function: $\zeta(s, a) == \sum_{k=0}^\infty \frac{1}{\left((a+k)^2\right)^{s/2}} \;/;\; -a \notin \mathbb{N}$.

$\hat{\zeta}(s, a)$

The generalized classical Riemann zeta function: $\hat{\zeta}(s, a) == \sum_{k=0}^{\infty} \frac{1}{(a+k)^s}$ /; $\mathrm{Re}(s) > 1$.

$\tilde{\zeta}(s, a)$

The regularized generalized classical Riemann zeta function: $\tilde{\zeta}(s, a) == \sum_{k=0}^{\infty} \frac{1}{(a+k)^s}$ /; $-a \notin \mathbb{N}$;

$$\tilde{\zeta}(s, -n) == \sum_{k=0}^{n-1} \frac{z^k}{(k-n)^s} + \sum_{k=n+1}^{\infty} \frac{z^k}{(k-n)^s} \text{ /; } n \in \mathbb{N}.$$

$\zeta(z; g_2, g_3)$

The Weierstrass elliptic zeta function: $\zeta(z; g_2, g_3) == \frac{1}{z} + \sum_{\substack{m, n=-\infty \\ \{m,n\} \neq \{0,0\}}}^{\infty} \left(\frac{1}{z - 2 m \omega_1(g_2, g_3) - 2 n \omega_3(g_2, g_3)} + \frac{1}{2 m \omega_1(g_2, g_3) + 2 n \omega_3(g_2, g_3)} + \frac{z}{(2 m \omega_1(g_2, g_3) + 2 n \omega_3(g_2, g_3))^2} \right)$.

H

$\eta(z)$

The Dedekind eta modular function: $\eta(z) == e^{\pi i z/12} \prod_{k=1}^{\infty} \left(1 - e^{2 \pi i k z}\right)$ /; $\mathrm{Im}(z) > 0$.

$\{\eta_1, \eta_2, \eta_3\} == \{\eta_1(g_2, g_3), \eta_2(g_2, g_3), \eta_3(g_2, g_3)\}$

The values of the Weierstrass zeta function at the half-periods $\{\omega_1, \omega_2, \omega_3\}$: $\{\eta_1, \eta_2, \eta_3\} == \{\eta_1(g_2, g_3), \eta_2(g_2, g_3), \eta_3(g_2, g_3)) == \{\zeta(\omega_1; g_2, g_3), \zeta(\omega_2; g_2, g_3), \zeta(\omega_3; g_2, g_3)\}$.

Θ

$\theta(x)$

The unit step function: $\theta(x) == \begin{cases} 1 & x \geq 0 \\ 0 & x < 0 \end{cases}$ /; $x \in \mathbb{R}$.

$\theta(x_1, x_2, \ldots, x_n)$

The multidimensional unit step: $\theta(x_1, x_2, \ldots, x_n) = \prod_{k=1}^{n} \theta(x_k)$

$\vartheta(z)$

The Riemann-Siegel theta function: $\vartheta(z) == -\frac{z \log(\pi)}{2} - \frac{i}{2} \left(\log\Gamma\left(\frac{1}{4} + \frac{i z}{2}\right) - \log\Gamma\left(\frac{1}{4} - \frac{i z}{2}\right) \right)$.

$\vartheta_1(z, q)$

The first elliptic theta function: $\vartheta_1(z, q) == 2 \sqrt[4]{q} \sum_{k=0}^{\infty} (-1)^k q^{k(k+1)} \sin((2k+1) z)$ /; $|q| < 1$.

$\vartheta_1'(z, q)$

The first derivative with respect to z of the first elliptic theta function: $\vartheta_1'(z, q) = \frac{\partial \vartheta_1(z,q)}{\partial z} = 2\sqrt[4]{q}\, \sum_{k=0}^{\infty} (-1)^k q^{k(k+1)} (2k+1) \cos((2k+1) z) /; |q| < 1.$

$\vartheta_2(z, q)$

The second elliptic theta function: $\vartheta_2(z, q) = 2\sqrt[4]{q}\, \sum_{k=0}^{\infty} q^{k(k+1)} \cos((2k+1) z) /; |q| < 1.$

$\vartheta_2'(z, q)$

The first derivative with respect to z of the second elliptic theta function: $\vartheta_2'(z, q) = \frac{\partial \vartheta_2(z,q)}{\partial z} = -2\sqrt[4]{q}\, \sum_{k=0}^{\infty} q^{k(k+1)} (2k+1) \sin((2k+1) z) /; |q| < 1.$

$\vartheta_3(z, q)$

The third elliptic theta function: $\vartheta_3(z, q) = 1 + 2\sum_{k=1}^{\infty} q^{k^2} \cos(2kz) /; |q| < 1.$

$\vartheta_3'(z, q)$

The first derivative with respect to z of the third elliptic theta function: $\vartheta_3'(z, q) = \frac{\partial \vartheta_3(z,q)}{\partial z} = -4\sum_{k=1}^{\infty} q^{k^2} k \sin(2kz) /; |q| < 1.$

$\vartheta_4(z, q)$

The fourth elliptic theta function: $\vartheta_4(z, q) = 1 + 2\sum_{k=1}^{\infty} (-1)^k q^{k^2} \cos(2kz) /; |q| < 1.$

$\vartheta_4'(z, q)$

The first derivative with respect to z of the fourth elliptic theta function: $\vartheta_4'(z, q) = \frac{\partial 4(z,q)}{\partial z} = -4\sum_{k=1}^{\infty} (-1)^k k\, q^{k^2} \sin(2kz) /; |q| < 1.$

$\vartheta_c(z \mid m)$

The Neville elliptic theta function C: $\vartheta_c(z \mid m) = \sqrt{\frac{2\pi\sqrt{q(m)}}{\sqrt{m}\, K(m)}}\, \sum_{k=0}^{\infty} q(m)^{k(k+1)} \cos\left(\frac{(2k+1)\pi z}{2K(m)}\right).$

$\vartheta_d(z \mid m)$

The Neville elliptic theta function D: $\vartheta_d(z \mid m) = \sqrt{\frac{\pi}{2K(m)}}\, \left(1 + 2\sum_{k=1}^{\infty} q(m)^{k^2} \cos\left(\frac{k\pi z}{K(m)}\right)\right).$

$\vartheta_n(z \mid m)$

The Neville elliptic theta function N: $\vartheta_n(z \mid m) = \sqrt{\frac{\pi}{2\sqrt{1-m}\, K(m)}}\, \left(1 + 2\sum_{k=1}^{\infty} (-1)^k q(m)^{k^2} \cos\left(\frac{k\pi z}{K(m)}\right)\right).$

$\vartheta_s(z \mid m)$

The Neville elliptic theta function S: $\vartheta_s(z \mid m) = \sqrt{\frac{2\pi\sqrt{q(m)}}{\sqrt{m}\ \sqrt{1-m}\ K(m)}}\ \sum_{k=0}^{\infty}(-1)^k\, q(m)^{k(k+1)} \sin\left(\frac{(2k+1)\pi z}{2K(m)}\right)$.

$\Theta(\Omega, s)$

The Siegel theta function $\Theta(\Omega, s)$ with symmetric Riemann modular matrix $\Omega = \{\{m_{1,1}, \ldots, m_{1,r}\}, \ldots, \{m_{r,1}, \ldots, m_{r,r}\}\}$ with positive definite imaginary part and vector $s = \{s_1, \ldots, s_r\}$ is defined through $\sum_{n_1=-\infty}^{\infty} \ldots \sum_{n_r=-\infty}^{\infty} e^{i\pi\left(n\cdot\Omega^T\cdot n+2\,n\cdot s\right)}$, where Ω^T means transposed to Ω matrix (or vector) and n ranges over all possible vectors in the r-dimensional integer lattice:

$$\Theta(\Omega, s) = \Theta\left(\begin{pmatrix} m_{1,1} & \ldots & m_{1,r} \\ \ldots & \ldots & \ldots \\ m_{r,1} & \ldots & m_{r,r} \end{pmatrix}, \{s_1, \ldots, s_r\}\right) = \sum_{n_1=-\infty}^{\infty} \ldots \sum_{n_r=-\infty}^{\infty} e^{i\pi(n\cdot\Omega\cdot n+2\,n\cdot s)}\ /;$$

$$\Omega = \{\{m_{1,1}, \ldots, m_{1,r}\}, \ldots, \{m_{r,1}, \ldots, m_{r,r}\}\} \bigwedge s = \{s_1, \ldots, s_r\} \bigwedge n = \{n_1, \ldots, n_r\}.$$

$\Theta\left[\begin{matrix} u \\ v \end{matrix}\right](\Omega, s)$

The Siegel theta function $\Theta\left[\begin{matrix} u \\ v \end{matrix}\right](\Omega, s)$ with characteristic $\begin{pmatrix} u \\ v \end{pmatrix}$ /; $u = \{u_1, \ldots, u_r\} \bigwedge v = \{v_1, \ldots, v_r\}$, symmetric Riemann modular matrix $\Omega = \{\{m_{1,1}, \ldots, m_{1,r}\}, \ldots, \{m_{r,1}, \ldots, m_{r,r}\}\}$ with positive definite imaginary part and vector $s = \{s_1, \ldots, s_r\}$ is defined through $\sum_{n_1=-\infty}^{\infty} \ldots \sum_{n_r=-\infty}^{\infty} e^{i\pi\left((n+u)\cdot\Omega^T\cdot(n+u)+2\,(n+u)\cdot(s+v)\right)}$, where Ω^T means transposed to Ω matrix (or vector) and n ranges over all possible vectors in the r-dimensional integer lattice:

$$\Theta\left[\begin{matrix} u \\ v \end{matrix}\right](\Omega, s) = \Theta\left[\begin{matrix} \{u_1, \ldots, u_r\} \\ \{v_1, \ldots, v_r\} \end{matrix}\right]\left(\begin{pmatrix} m_{1,1} & \ldots & m_{1,r} \\ \ldots & \ldots & \ldots \\ m_{r,1} & \ldots & m_{r,r} \end{pmatrix}, \{s_1, \ldots, s_r\}\right) = \sum_{n_1=-\infty}^{\infty} \ldots \sum_{n_r=-\infty}^{\infty} e^{i\pi((n+u)\cdot\Omega\cdot(n+u)+2\,(n+u)\cdot(s+v))}\ /;$$

$$u = \{u_1, \ldots, u_r\} \bigwedge v = \{v_1, \ldots, v_r\} \bigwedge \Omega = \{\{m_{1,1}, \ldots, m_{1,r}\}, \ldots, \{m_{r,1}, \ldots, m_{r,r}\}\} \bigwedge s = \{s_1, \ldots, s_r\} \bigwedge n = \{n_1, \ldots, n_r\} \bigwedge n + u = \{n_1 + u_1, \ldots, n_r + u_r\} \bigwedge s + v = \{s_1 + v_1, \ldots, s_r + v_r\}.$$

Λ

$\lambda(n)$

The Carmichael lambda function: the smallest integer λ such that for any m with $\gcd(m, n) = 1$ the congruence $m^{\lambda(n)} \bmod n = 1$ holds.

$\lambda(z)$

The lambda modular function: $\lambda(z) = 16\, e^{i\pi z} \prod_{k=1}^{\infty}\left(\frac{1+e^{2k\pi i z}}{1+e^{(2k-1)\pi i z}}\right)^8$ /; $\operatorname{Im}(z) > 0$.

$\lambda_{\nu,\mu}(\gamma)$

The eigenvalue of the spheroidal wave functions (the spheroidal eigenvalue of degree ν and order μ of the corresponding Sturm-Liouville wave differential equation $\left(1-z^2\right) w''(z) - 2\,z\,w'(z) + \left(\lambda + \gamma^2\left(1-z^2\right) - \mu^2 / \left(1-z^2\right)\right) w(z) = 0$).

M

$\mu(n)$

The Möbius function μ: $\mu(n) = \begin{cases} 1 & \text{if} \quad n = 1 \\ 0 & \text{if} \quad \exists_m \frac{n}{m^2} \in \mathbb{Z} \\ (-1)^k & \text{if} \quad n = \prod_{j=1}^{k} p_j \;/;\; p_j \in \mathbb{P} \land p_{j-1} < p_j \end{cases}$.

Π

π

The constant pi: $\pi = 4 \sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1} \approx 3.141592\ldots$

$\pi(x)$

The number of primes less than or equal to x: $\pi(x) = \sum_{k=1}^{\lfloor x \rfloor} \theta(x - p_k) \;/;\; x \in \mathbb{R} \land x \geq 0 \land p_k \in \mathbb{P}$.

$\Pi(n \mid m)$

The complete elliptic integral of the third kind: $\Pi(n \mid m) = \Pi\left(n; \frac{\pi}{2} \,\middle|\, m\right) = \int_0^{\frac{\pi}{2}} \frac{1}{\left(1 - n \sin^2(t)\right) \sqrt{1 - m \sin^2(t)}} \, dt \;/;\; -\frac{\pi}{2} < z < \frac{\pi}{2}$.

$\Pi(n; z \mid m)$

The incomplete elliptic integral of the third kind: $\Pi(n; z \mid m) = \int_0^{z} \frac{1}{\left(1 - n \sin^2(t)\right) \sqrt{1 - m \sin^2(t)}} \, dt$.

P

ρ_k

The k^{th} nontrivial zero of the Riemann's zeta function $\zeta(s)$ on the critical half-line $s = \frac{1}{2} + i\,t \;/;\; t > 0$: $(\zeta(s) \;/;\; s = \rho_k) = 0 \;/;\; k \in \mathbb{N}^+$.

Σ

$\sigma_k(n)$

The sum of the k^{th} powers of the divisors of n: $\sigma_k(n) = \sum_{d \mid n} d^k \;/;\; n \in \mathbb{N}^+$.

$\sigma(z; g_2, g_3)$

The elliptic Weierstrass sigma function: $\sigma(z; g_2, g_3) = z \prod_{\substack{m, n = -\infty \\ \{m,n\} \neq \{0,0\}}}^{\infty} \left(1 - \frac{z}{2 m \omega_1(g_2, g_3) + 2 n \omega_3(g_2, g_3)}\right) \times \exp\left(\frac{z^2}{2\left(2 m \omega_1(g_2, g_3) + 2 n \omega_3(g_2, g_3)\right)^2} + \frac{z}{2 m \omega_1(g_2, g_3) + 2 n \omega_3(g_2, g_3)}\right)$.

$\sigma_n(z, g_2, g_3)$

The associated elliptic Weierstrass sigma function: $\sigma_n(z, g_2, g_3) = \frac{e^{-\eta_n z}\,\sigma(z+\omega_n; g_2, g_3)}{\sigma(\omega_n; g_2, g_3)}$ /; $n \in \{1, 2, 3\} \bigwedge$

$\{\omega_1, \omega_2, \omega_3\} = \{\omega_1(g_2, g_3), -\omega_1(g_2, g_3) - \omega_3(g_2, g_3), \omega_3(g_2, g_3)\} \bigwedge \eta_n = \zeta(\omega_n; g_2, g_3) \bigwedge n \in \{1, 2, 3\}$.

T

$\tau(n)$

The Ramanujan tau function of n: $\tau(n) = \frac{1}{2}\int_{i\gamma-1}^{i\gamma+1} e^{-2\pi i n z}\,\eta(z)^{24}\,dz$ /; $n \in \mathbb{Z} \bigwedge n \geq 0 \bigwedge \operatorname{Re}(\gamma) > 0$.

$\tau\mathrm{L}(z)$

The Ramanujan tau L function: $\tau L(z) = \sum_{n=1}^{\infty} \frac{\mathrm{RamanujanTau}(n)}{n^z}$ /; $\operatorname{Re}(z) > 1$.

$\tau Z(z)$

The Ramanujan tau Zeta function: $\tau Z(z) = 2^{-iz}\,\pi^{-iz-\frac{1}{2}}\,\Gamma(6 + i z)\,\tau L(6 + i z)\sqrt{\frac{\sinh(\pi z)}{z(z^2+1)(z^2+4)(z^2+9)(z^2+16)(z^2+25)}}$.

$\tau\theta(z)$

The Ramanujan tau theta function: $\tau\theta(z) = -\log(2\pi)\,z - \frac{i}{2}(\log\Gamma(6 + i z) - \log\Gamma(6 - i z))$.

Φ

$\Phi(z, s, a)$

The Lerch function: $\Phi(z, s, a) = \sum_{k=0}^{\infty} \frac{z^k}{((a+k)^2)^{s/2}}$ /; $(|z| < 1 \bigvee (|z| = 1 \bigwedge \operatorname{Re}(s) > 1)) \bigwedge -a \notin \mathbb{N}$;

$$\Phi(z, s, -n) = \sum_{k=0}^{n-1} \frac{z^k}{((k-n)^2)^{s/2}} + \sum_{k=n+1}^{\infty} \frac{z^k}{((k-n)^2)^{s/2}} \text{ /; } (|z| < 1 \bigvee (|z| = 1 \bigwedge \operatorname{Re}(s) > 1)) \bigwedge n \in \mathbb{N}.$$

$\hat{\Phi}(z, s, a)$

The Lerch classical transcendent phi function: $\hat{\Phi}(z, s, a) = \sum_{k=0}^{\infty} \frac{z^k}{(a+k)^s}$ /; $|z| < 1 \bigvee (|z| = 1 \bigwedge \operatorname{Re}(s) > 1)$.

$\tilde{\Phi}(z, s, a)$

The Lerch classical regularized transcendent phi function: $\tilde{\Phi}(z, s, a) = \sum_{k=0}^{\infty} \frac{z^k}{(a+k)^s}$ /; $(|z| < 1 \bigvee |z| = 1 \bigwedge \operatorname{Re}(s) > 1) \bigwedge -a \notin \mathbb{N}$;

$\tilde{\Phi}(z, s, -n) = \sum_{k=0}^{n-1} \frac{z^k}{(k-n)^s} + \sum_{k=n+1}^{\infty} \frac{z^k}{(k-n)^s}$ /; $(|z| < 1 \bigvee |z| = 1 \bigwedge \operatorname{Re}(s) > 1) \bigwedge n \in \mathbb{N}$.

ϕ

The golden ratio ϕ: $\phi = \frac{1}{2}\left(1 + \sqrt{5}\right) \approx 1.618033\ldots$

$\phi(n)$

The number of positive integers less than n ($n \geq 0$) and relatively prime to $\lfloor n \rfloor$ (the Euler totient function): $\phi(n) == \sum_{k=1}^{n} \delta_{\gcd(n,k),1} /; n \in \mathbb{N}$.

Ψ

$\psi(z)$

The digamma function $\psi(z)$: $\psi(z) == \sum_{k=1}^{\infty}\left(\frac{1}{k} - \frac{1}{k+z-1}\right) - \gamma$.

$\psi^{(\nu)}(z)$

The ν^{th} derivative of the digamma function:

$$\psi^{(\nu)}(z) = \begin{cases} \psi(z) & \nu = 0 \\ (-1)^{\nu+1}\,\nu!\,\sum_{k=0}^{\infty}\frac{1}{(k+z)^{\nu+1}} & \nu \in \mathbb{N}^+ \\ \frac{(-\gamma\,(z+\nu)+\nu\log(z)-\nu\,\psi(-\nu))\,z^{-\nu-1}}{\Gamma(1-\nu)} + \left(\sum_{k=1}^{\infty}\frac{1}{k^2}\,{}_2\tilde{F}_1\left(1, 2; 2-\nu; -\frac{z}{k}\right)\right) z^{1-\nu} & \text{True} \end{cases}.$$

Ω

$\{\omega_1, \omega_3\} == \{\omega_1(g_2, g_3), \omega_3(g_2, g_3)\}$

The half-periods $\{\omega_1, \omega_3\}$ for Weierstrass elliptic functions corresponding to the invariants $\{g_2, g_3\}$: $\{\omega_1(g_2, g_3), \omega_3(g_2, g_3)\} == \left\{i\left(\frac{60}{g_2}\sum_{\substack{m,\, n=-\infty \\ \{m,n\}\neq\{0,0\}}}^{\infty}\frac{1}{(2\,m+2\,n\,t)^4}\right)^{1/4}, i\,t\left(\frac{60}{g_2}\sum_{\substack{m,\, n=-\infty \\ \{m,n\}\neq\{0,0\}}}^{\infty}\frac{1}{(2\,m+2\,n\,t)^4}\right)^{1/4}\right\} /; J(t) == \frac{g_2^3}{g_2^3-27\,g_3^2}$.

$\{\omega_1, \omega_2, \omega_3\} == \{\omega_1(g_2, g_3), -\omega_1(g_2, g_3) - \omega_3(g_2, g_3), \omega_3(g_2, g_3)\}$

The half-periods for Weierstrass elliptic functions corresponding to the invariants $\{g_2, g_3\}$:
$\{\omega_1(g_2, g_3), \omega_2(g_2, g_3), \omega_3(g_2, g_3)\} == \{\omega_1, \omega_2, \omega_3\} /;$

$$\{\omega_1, \omega_3\} == \left\{i\left(\frac{60}{g_2}\sum_{\substack{m,\, n=-\infty \\ \{m,n\}\neq\{0,0\}}}^{\infty}\frac{1}{(2\,m+2\,n\,t)^4}\right)^{1/4}, i\,t\left(\frac{60}{g_2}\sum_{\substack{m,\, n=-\infty \\ \{m,n\}\neq\{0,0\}}}^{\infty}\frac{1}{(2\,m+2\,n\,t)^4}\right)^{1/4}\right\} /; J(t) == \frac{g_2^3}{g_2^3-27\,g_3^2} \bigwedge \omega_2 == -\omega_1 - \omega_3$$

℘

$\wp(z; g_2, g_3)$

The Weierstrass elliptic function $\wp$:

$$\wp(z; g_2, g_3) == \frac{1}{z^2} + \sum_{\substack{m,\, n=-\infty \\ \{m,n\}\neq\{0,0\}}}^{\infty}\left(\frac{1}{\left(z-2\,m\,\omega_1(g_2,g_3)-2\,n\,\omega_3(g_2,g_3)\right)^2} - \frac{1}{\left(2\,m\,\omega_1(g_2,g_3)+2\,n\,\omega_3(g_2,g_3)\right)^2}\right).$$

$\wp'(z; g_2, g_3)$

The derivative with respect to z of the Weierstrass elliptic function P:

$$\wp'(z; g_2, g_3) == \frac{\partial \wp(z; g_2, g_3)}{\partial z} == -2 \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \frac{1}{(z - 2\, m\, \omega_1(g_2, g_3) - 2\, n\, \omega_3(g_2, g_3))^3}.$$

$\wp^{-1}(z; g_2, g_3)$

The inverse of the Weierstrass elliptic function $\wp$. The value of u such that $\wp(u; g_2, g_3) == z$:

$$\wp^{-1}(z; g_2, g_3) == \int_{\infty}^{z} \frac{1}{\sqrt{4\, t^3 - g_2\, t - g_3}} \, dt \; /;\; z \in \mathbb{R} \bigwedge \operatorname{Re}\left(4\, z^3 - g_2\, z - g_3\right) > 0.$$

$\wp^{-1}(z_1, z_2; g_2, g_3)$

The inverse of the Weierstrass function $\wp$. The value of u such that $\wp(u; g_2, g_3) == z_1$ and $z_2 == \sqrt{4\, z_1^3 - g_2\, z_1 - g_3}$:

$$\wp^{-1}(z_1, z_2; g_2, g_3) == \int_{\infty}^{z_1} \frac{1}{\sqrt{4\, t^3 - g_2\, t - g_3}} \, dt \; /;\; z_2 == \sqrt{4\, z_1^3 - g_2\, z_1 - g_3}\,.$$

■

■ (x)

The generalized Dirac comb function ■(x) : ■ $(x) = \sum_{k=-\infty}^{\infty} \delta(x - k)$.

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