Arg

Notations

Traditional name

Argument

Traditional notation

$\arg(z)$

Mathematica StandardForm notation

`Arg[z]`

Primary definition

12.02.02.0001.01

$$\arg(z) = -i \log\left(\frac{z}{|z|}\right)$$

$\operatorname{Arg}(z)$ is the argument of z, such that $z = |z|\, e^{i \operatorname{Arg}(z)}$. The argument of a complex number z is the phase angle (in radians) that the line from 0 to z makes with the positive real axis.

Specific values

Specialized values

12.02.03.0001.01

$$\arg(x) = 0 \;/;\; x \in \mathbb{R} \land x > 0$$

12.02.03.0020.01

$$\arg(x) = \pi \;/;\; x \in \mathbb{R} \land x < 0$$

12.02.03.0002.01

$$\arg(i\, x) = \frac{\pi}{2} \;/;\; x \in \mathbb{R} \land x > 0$$

12.02.03.0021.01

$$\arg(i\, x) = -\frac{\pi}{2} \;/;\; x \in \mathbb{R} \land x < 0$$

12.02.03.0003.01

$$\arg(x + i\, y) = \tan^{-1}(x, y) \;/;\; x \in \mathbb{R} \bigwedge y \in \mathbb{R}$$

Values at fixed points

12.02.03.0004.01

$\arg(0) \in (-\pi, \pi]$

Arg(0) is not a uniquely defined number. Depending on the argument of z, the limit $\lim_{|z|\to 0} \operatorname{Arg}(z)$ can take any value in the interval $(-\pi, \pi)$.

12.02.03.0005.01

$\arg(1) == 0$

12.02.03.0006.01

$\arg(-1) == \pi$

12.02.03.0007.01

$\arg(i) == \frac{\pi}{2}$

12.02.03.0008.01

$\arg(-i) == -\frac{\pi}{2}$

12.02.03.0022.01

$\arg(1+i) == \frac{\pi}{4}$

12.02.03.0023.01

$\arg(-1+i) == \frac{3\pi}{4}$

12.02.03.0024.01

$\arg(-1-i) == -\frac{3\pi}{4}$

12.02.03.0025.01

$\arg(1-i) == -\frac{\pi}{4}$

12.02.03.0026.01

$\arg\left(\sqrt{3}+i\right) == \frac{\pi}{6}$

12.02.03.0027.01

$\arg\left(1+i\sqrt{3}\right) == \frac{\pi}{3}$

12.02.03.0028.01

$\arg\left(-1+i\sqrt{3}\right) == \frac{2\pi}{3}$

12.02.03.0029.01

$\arg\left(-\sqrt{3}+i\right) == \frac{5\pi}{6}$

12.02.03.0030.01

$\arg\left(-\sqrt{3}-i\right) == -\frac{5\pi}{6}$

12.02.03.0031.01

$$\arg\left(-1 - i\sqrt{3}\right) = -\frac{2\pi}{3}$$

12.02.03.0032.01

$$\arg\left(1 - i\sqrt{3}\right) = -\frac{\pi}{3}$$

12.02.03.0033.01

$$\arg\left(\sqrt{3} - i\right) = -\frac{\pi}{6}$$

12.02.03.0009.01

$$\arg(2) = 0$$

12.02.03.0010.01

$$\arg(-2) = \pi$$

12.02.03.0011.01

$$\arg(\pi) = 0$$

12.02.03.0012.01

$$\arg(3\,i) = \frac{\pi}{2}$$

12.02.03.0013.01

$$\arg(-2\,i) = -\frac{\pi}{2}$$

12.02.03.0014.01

$$\arg(2 + i) = \tan^{-1}\left(\frac{1}{2}\right)$$

Values at infinities

12.02.03.0015.01

$$\arg(\infty) = 0$$

12.02.03.0016.01

$$\arg(-\infty) = \pi$$

12.02.03.0017.01

$$\arg(i\,\infty) = \frac{\pi}{2}$$

12.02.03.0018.01

$$\arg(-i\,\infty) = -\frac{\pi}{2}$$

12.02.03.0019.01

$$\arg(\tilde{\infty}) \in (-\pi, \pi]$$

General characteristics

Domain and analyticity

$\operatorname{Arg}(z)$ is a nonanalytical function; it is a real-analytic function of the complex variable z for $z \neq 0$.

12.02.04.0001.01

$z \longrightarrow \arg(z) :: \mathbb{C} \longrightarrow \mathbb{R}$

Symmetries and periodicities

Parity

Arg(z) is an odd function for almost all z.

12.02.04.0002.01

$\arg(-z) = -\arg(z) \;/;\; z \notin (-\infty, 0)$

12.02.04.0003.01

$\arg(-z) = \arg(z) - \frac{\sqrt{-z}}{\sqrt{z}}\, i\, \pi$

Mirror symmetry

12.02.04.0004.01

$\arg(\bar{z}) = -\arg(z) \;/;\; z \notin (-\infty, 0)$

Periodicity

No periodicity

Homogeneity

12.02.04.0005.01

$\arg(a\, z) = \arg(z) \;/;\; a \in \mathbb{R} \land a > 0$

Sets of discontinuity

The function Arg(z) is a single-valued, continuous function on the z-plane cut along the interval $(-\infty, 0)$, where it is continuous from above.

12.02.04.0006.01

$\mathcal{DS}_z(\arg(z)) = \{\{(-\infty, 0), -i\}\}$

12.02.04.0007.01

$\lim_{\epsilon \to +0} \arg(x + i\, \epsilon) = \arg(x) = \pi \;/;\; x \in \mathbb{R} \land x < 0$

12.02.04.0008.01

$\lim_{\epsilon \to +0} \arg(x - i\, \epsilon) = -\pi \;/;\; x \in \mathbb{R} \land x < 0$

Transformations

Transformations and argument simplifications

Argument involving complex characteristics

12.02.16.0032.01

$\arg(|z|) = 0$

12.02.16.0033.01

$$\arg\left(\frac{z}{|z|}\right) = \arg(z)$$

12.02.16.0034.01

$$\arg(\text{sgn}(z)) = \arg(z)$$

12.02.16.0006.01

$$\arg(\bar{z}) = -\arg(z) \;/;\; \arg(z) \neq \pi$$

12.02.16.0035.01

$$\arg(\bar{z}) = 2\pi \left\lfloor \frac{\arg(z) + \pi}{2\pi} \right\rfloor - \arg(z)$$

Argument involving basic arithmetic operations

12.02.16.0001.01

$$\arg(-z) = -\arg(z) \;/;\; z \notin (-\infty, 0)$$

12.02.16.0002.01

$$\arg(-z) = \arg(z) - \frac{\sqrt{-z}}{\sqrt{z}}\, i\, \pi$$

12.02.16.0036.01

$$\arg(-z) = \arg(z) + \pi\left(2\left\lfloor -\frac{\arg(z)}{2\pi} \right\rfloor + 1\right)$$

12.02.16.0037.01

$$\arg(i\, z) = \arg(z) + \frac{\pi}{2} \;/;\; \arg(z) \leq \frac{\pi}{2}$$

12.02.16.0038.01

$$\arg(i\, z) = \arg(z) - \frac{3\pi}{2} \;/;\; \arg(z) > \frac{\pi}{2}$$

12.02.16.0004.01

$$\arg(i\, z) = \arg(z) - \frac{\pi}{2} - \frac{(-1)^{3/4}\, \pi \sqrt{i\, z}}{\sqrt{z}}$$

12.02.16.0039.01

$$\arg(i\, z) = \arg(z) + 2\pi \left\lfloor \frac{1}{4} - \frac{\arg(z)}{2\pi} \right\rfloor + \frac{\pi}{2}$$

12.02.16.0040.01

$$\arg(-i\, z) = \arg(z) - \frac{\pi}{2} \;/;\; \arg(z) > -\frac{\pi}{2}$$

12.02.16.0041.01

$$\arg(-i\, z) = \arg(z) + \frac{3\pi}{2} \;/;\; \arg(z) \leq -\frac{\pi}{2}$$

12.02.16.0005.01

$$\arg(-i\, z) = \arg(z) + \frac{\pi}{2} - \frac{\sqrt[4]{-1}\; \pi \sqrt{-i\, z}}{\sqrt{z}}$$

12.02.16.0042.01

$$\arg(-i\,z) = \arg(z) + 2\pi \left\lfloor \frac{3}{4} - \frac{\arg(z)}{2\pi} \right\rfloor - \frac{\pi}{2}$$

12.02.16.0007.01

$$\arg\left(\frac{1}{z}\right) = -\arg(z) \;/;\; z \notin (-\infty, 0)$$

12.02.16.0008.01

$$\arg\left(\frac{1}{z}\right) = -\arg(z) \;/;\; \arg(z) \neq \pi$$

12.02.16.0043.01

$$\arg\left(\frac{1}{z}\right) = 2\pi - \arg(z) \;/;\; \arg(z) = \pi$$

12.02.16.0009.01

$$\arg\left(\frac{1}{z}\right) = -\sqrt{z}\,\sqrt{\frac{1}{z}}\,\arg(z)$$

12.02.16.0044.01

$$\arg\left(\frac{1}{z}\right) = 2\pi \left\lfloor \frac{\arg(z) + \pi}{2\pi} \right\rfloor - \arg(z)$$

12.02.16.0045.01

$$\arg\left(-\frac{1}{z}\right) = \pi - \arg(z) \;/;\; \mathrm{Im}(z) \geq 0$$

12.02.16.0046.01

$$\arg\left(-\frac{1}{z}\right) = -\arg(z) - \pi \;/;\; \mathrm{Im}(z) < 0$$

12.02.16.0047.01

$$\arg\left(-\frac{1}{z}\right) = -\arg(z) - \pi\,i\,\sqrt{-\frac{1}{z}}\,\sqrt{z}$$

12.02.16.0048.01

$$\arg\left(-\frac{1}{z}\right) = -\arg(z) + 2\pi \left\lfloor \frac{\arg(z)}{2\pi} \right\rfloor + \pi$$

12.02.16.0049.01

$$\arg\left(\frac{i}{z}\right) = \frac{\pi}{2} - \arg(z) \;/;\; \arg(z) \geq -\frac{\pi}{2}$$

12.02.16.0050.01

$$\arg\left(\frac{i}{z}\right) = -\arg(z) - \frac{3\pi}{2} \;/;\; \arg(z) < -\frac{\pi}{2}$$

12.02.16.0051.01

$$\arg\left(\frac{i}{z}\right) = -\arg(z) + 2\pi \left\lfloor \frac{\arg(z)}{2\pi} + \frac{1}{4} \right\rfloor + \frac{\pi}{2}$$

12.02.16.0052.01

$$\arg\left(-\frac{i}{z}\right) = -\frac{\pi\,i}{2} - \arg(z) \;/;\; \arg(z) < \frac{\pi}{2}$$

12.02.16.0053.01

$$\arg\left(-\frac{i}{z}\right) = \frac{3\pi}{2} - \arg(z) \;/;\; \arg(z) \geq \frac{\pi}{2}$$

12.02.16.0054.01

$$\arg\left(-\frac{i}{z}\right) = -\arg(z) + 2\pi \left\lfloor \frac{\arg(z)}{2\pi} + \frac{3}{4} \right\rfloor - \frac{\pi}{2}$$

Addition formulas

12.02.16.0010.01

$$\arg(x + i\, y) = \tan^{-1}(x, y) \;/;\; \operatorname{Im}(x) = 0 \bigwedge \operatorname{Im}(y) = 0$$

Multiple arguments

For products

12.02.16.0011.01

$$\arg(a\, z) = \arg(z) \;/;\; a \in \mathbb{R} \wedge a > 0$$

12.02.16.0014.01

$$\arg(z_1\, z_2) = \arg(z_1) + \arg(z_2) \;/;\; -\pi < \arg(z_1) + \arg(z_2) \leq \pi$$

12.02.16.0055.01

$$\arg\left(z - z^2\right) = \arg(1 - z) + \arg(z)$$

12.02.16.0056.01

$$\arg\left(-z - z^2\right) = \arg(1 + z) + \arg(-z)$$

12.02.16.0057.01

$$\arg(z_1\, z_2) = \arg(z_1) + \arg(z_2) \;/;\; \arg(z_1) \leq 0 \bigwedge -\arg(z_1) - \pi < \arg(z_2) \bigvee \arg(z_1) \geq 0 \bigwedge \arg(z_2) \leq \pi - \arg(z_1)$$

12.02.16.0058.01

$$\arg(z_1\, z_2) = \arg(z_1) + \arg(z_2) - 2\pi \;/;\; \arg(z_1) \geq 0 \bigwedge \arg(z_2) > \pi - \arg(z_1)$$

12.02.16.0059.01

$$\arg(z_1\, z_2) = \arg(z_1) + \arg(z_2) + 2\pi \;/;\; \arg(z_1) \leq 0 \bigwedge \arg(z_2) \leq -\arg(z_1) - \pi$$

12.02.16.0015.01

$$\arg(z_1\, z_2) = \arg(z_1) + \arg(z_2) + 2\pi \left\lfloor \frac{\pi - \arg(z_1) - \arg(z_2)}{2\pi} \right\rfloor$$

12.02.16.0060.01

$$\arg\left(\prod_{k=1}^{n} z_k\right) = \sum_{k=1}^{n} \arg(z_k) + 2\pi \left\lfloor \frac{\pi - \sum_{k=1}^{n} \arg(z_k)}{2\pi} \right\rfloor \;/;\; n \in \mathbb{N}^+$$

For quotients

12.02.16.0061.01

$$\arg\left(\frac{z}{z+1}\right) = \arg(z) - \arg(z + 1)$$

12.02.16.0062.01

$$\arg\left(\frac{z}{z-1}\right) = \arg(-z) - \arg(1 - z)$$

12.02.16.0016.01

$$\arg\left(\frac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2) \;/;\; -\pi < \arg(z_1) - \arg(z_2) \leq \pi$$

12.02.16.0063.01

$$\arg\left(\frac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2) \;/;\; \arg(z_1) \leq 0 \bigwedge \arg(z_2) < \arg(z_1) + \pi \bigvee \arg(z_1) > 0 \bigwedge \arg(z_2) \geq \arg(z_1) - \pi$$

12.02.16.0064.01

$$\arg\left(\frac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2) - 2\pi \;/;\; \arg(z_1) \geq 0 \bigwedge \arg(z_2) < \arg(z_1) - \pi$$

12.02.16.0065.01

$$\arg\left(\frac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2) + 2\pi \;/;\; \arg(z_1) \leq 0 \bigwedge \arg(z_2) \geq \arg(z_1) + \pi$$

12.02.16.0017.01

$$\arg\left(\frac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2) + 2\pi \left\lfloor \frac{\pi - \arg(z_1) + \arg(z_2)}{2\pi} \right\rfloor$$

Power of arguments

12.02.16.0066.01

$$\arg\left(\sqrt{z}\right) = \frac{\arg(z)}{2}$$

12.02.16.0067.01

$$\arg\left(\sqrt{z^2}\right) = \arg(z) \;/;\; \operatorname{Re}(z) > 0 \lor \operatorname{Re}(z) = 0 \land \operatorname{Im}(z) > 0$$

12.02.16.0068.01

$$\arg\left(\sqrt{z^2}\right) = \arg(z) - \frac{\pi i \left(\sqrt{z^2} - z\right)}{2\sqrt{-z^2}}$$

12.02.16.0069.01

$$\arg\left(z^{1/n}\right) = \frac{\arg(z)}{n} \;/;\; n \in \mathbb{Z} \land n \neq 0 \land n \neq -1$$

12.02.16.0070.01

$$\arg\left(z^2\right) = 2\arg(z) \;/;\; \operatorname{Re}(z) > 0 \lor \operatorname{Re}(z) = 0 \land \operatorname{Im}(z) > 0$$

12.02.16.0071.01

$$\arg\left(z^2\right) = 2\arg(z) + 2\pi \;/;\; -\pi < \arg(z) \leq -\frac{\pi}{2}$$

12.02.16.0072.01

$$\arg\left(z^2\right) = 2\arg(z) - 2\pi \;/;\; \frac{\pi}{2} < \arg(z) \leq \pi$$

12.02.16.0073.01

$$\arg\left(z^2\right) = 2\arg(z) + 2\pi \left\lfloor \frac{1}{2} - \frac{\arg(z)}{\pi} \right\rfloor$$

12.02.16.0074.01

$$\arg(z^2) = 2\arg(z) - \frac{\pi i\left(\sqrt{z^2} - z\right)}{\sqrt{-z^2}}$$

12.02.16.0018.01

$$\arg(x^a) = \tan^{-1}(\cos(\operatorname{Im}(a)\log(x)), \sin(\operatorname{Im}(a)\log(x))) \;/;\; x \in \mathbb{R} \land x > 0$$

12.02.16.0019.01

$$\arg(z^a) = a\arg(z) \;/;\; a \in \mathbb{R} \bigwedge -\pi < a\arg(z) \leq \pi$$

12.02.16.0075.01

$$\arg(z^a) = a\arg(z) + 2\pi k \;/;\; a \in \mathbb{R} \bigwedge -\pi - 2\pi k < a\arg(z) \leq \pi - 2\pi k \bigwedge k \in \mathbb{Z}$$

12.02.16.0076.01

$$\arg(z^a) = \operatorname{Im}(a\log(z)) \;/;\; -\pi < \operatorname{Im}(a\log(z)) \leq \pi$$

12.02.16.0077.01

$$\arg(z^a) = \operatorname{Im}(a\log(z)) + 2\pi k \;/;\; -2\pi k - \pi < \operatorname{Im}(a\log(z)) \leq \pi - 2\pi k \bigwedge k \in \mathbb{Z}$$

12.02.16.0020.01

$$\arg(z^a) = \arg\left(e^{i a \arg(z)}\right) \;/;\; a \in \mathbb{R}$$

12.02.16.0021.01

$$\arg(z^a) = \tan^{-1}\left(\cos\left(a\tan^{-1}(\operatorname{Re}(z), \operatorname{Im}(z))\right), \sin\left(a\tan^{-1}(\operatorname{Re}(z), \operatorname{Im}(z))\right)\right) \;/;\; a \in \mathbb{R}$$

12.02.16.0022.01

$$\arg(x^a) = \operatorname{Im}(a)\log(x) \;/;\; -\frac{\pi}{\log(x)} < \operatorname{Im}(a) \leq \frac{\pi}{\log(x)} \bigwedge x \in \mathbb{R} \bigwedge x > 0$$

12.02.16.0024.01

$$\arg(z^a) = a\arg(z) + 2\pi\left\lfloor\frac{\pi - a\arg(z)}{2\pi}\right\rfloor \;/;\; \operatorname{Im}(a) = 0$$

12.02.16.0078.01

$$\arg(z^a) = a\arg(z) + \operatorname{Im}(a)\,\overline{\log(z)} + 2\pi\left\lfloor\frac{\pi - \operatorname{Im}(a\log(z))}{2\pi}\right\rfloor$$

12.02.16.0079.01

$$\arg(z^a) = a\arg(z) + 2\pi\left\lfloor\frac{\pi - \operatorname{Im}(a\log(z))}{2\pi}\right\rfloor - i\,a\log(|z|) + i\operatorname{Re}(a\log(z))$$

12.02.16.0025.01

$$\arg(z^a) = 2\pi\left\lfloor\frac{\pi - \operatorname{Im}(a\log(z))}{2\pi}\right\rfloor + \operatorname{Im}(a\log(z))$$

12.02.16.0080.01

$$\arg(z^a) = \operatorname{Im}(a)\log(|z|) + \arg(z)\operatorname{Re}(a) + 2\pi\left\lfloor\frac{\pi - \operatorname{Im}(a\log(z))}{2\pi}\right\rfloor$$

12.02.16.0026.01

$$\arg(z^a) = \operatorname{Re}(a)\arg(z) + \operatorname{Im}(a)\log(|z|) + 2\pi\left\lfloor\frac{\pi - \operatorname{Im}(a)\log(|z|) - \arg(z)\operatorname{Re}(a)}{2\pi}\right\rfloor$$

12.02.16.0027.01

$$\arg(z^a) = \tan^{-1}\left(\cos\left(\operatorname{Im}(a)\log(|z|) + \tan^{-1}(\operatorname{Re}(z), \operatorname{Im}(z))\operatorname{Re}(a)\right), \sin\left(\operatorname{Im}(a)\log(|z|) + \tan^{-1}(\operatorname{Re}(z), \operatorname{Im}(z))\operatorname{Re}(a)\right)\right)$$

Exponent of arguments

12.02.16.0081.01

$$\arg\left(e^{x+i y}\right) = \tan^{-1}(\cos(y), \sin(y))$$

12.02.16.0082.01

$$\arg(e^z) = \operatorname{Im}(z) \;/;\; -\pi < \operatorname{Im}(z) \leq \pi$$

12.02.16.0083.01

$$\arg(e^z) = 2\pi k + \operatorname{Im}(z) \;/;\; -2\pi k - \pi < \operatorname{Im}(z) \leq \pi - 2\pi k \bigwedge k \in \mathbb{Z}$$

12.02.16.0084.01

$$\arg(e^z) = \operatorname{Im}(z) + 2\pi \left\lfloor \frac{\pi - \operatorname{Im}(z)}{2\pi} \right\rfloor$$

12.02.16.0085.01

$$\arg\left(e^{i z}\right) = \operatorname{Re}(z) + 2\pi \left\lfloor \frac{\pi - \operatorname{Re}(z)}{2\pi} \right\rfloor$$

12.02.16.0086.01

$$\arg(e^z) = \pi - (\pi - \operatorname{Im}(z)) \bmod (2\pi)$$

12.02.16.0087.01

$$\arg\left(e^{i z}\right) = \pi - (\pi - \operatorname{Re}(z)) \bmod (2\pi)$$

Some functions of arguments

12.02.16.0088.01

$$\arg(c\, z^a) = \arg(c) + \operatorname{Im}(a \log(z)) + 2\pi \left\lfloor \frac{\pi - \arg(c) - \operatorname{Im}(a \log(z))}{2\pi} \right\rfloor$$

12.02.16.0089.01

$$\arg(c\, e^z) = \arg(c) + \operatorname{Im}(z) + 2\pi \left\lfloor \frac{\pi - \arg(c) - \operatorname{Im}(z)}{2\pi} \right\rfloor$$

12.02.16.0090.01

$$\arg\left(x^a\, y^b\right) = a \arg(x) + b \arg(y) + 2\pi \left\lfloor \frac{-a \arg(x) - b \arg(y) + \pi}{2\pi} \right\rfloor \;/;\; a \in \mathbb{R} \bigwedge b \in \mathbb{R}$$

12.02.16.0091.01

$$\arg\left(x^a\, y^b\, z^c\right) = a \arg(x) + b \arg(y) + c \arg(z) + 2\pi \left\lfloor \frac{-a \arg(x) - b \arg(y) - c \arg(z) + \pi}{2\pi} \right\rfloor \;/;\; a \in \mathbb{R} \bigwedge b \in \mathbb{R} \bigwedge c \in \mathbb{R}$$

12.02.16.0092.01

$$\arg\left(\prod_{k=1}^{n} z_k^{a_k}\right) = \sum_{k=1}^{n} a_k \arg(z_k) + 2\pi \left\lfloor \frac{\pi - \sum_{k=1}^{n} a_k \arg(z_k)}{2\pi} \right\rfloor \;/;\; a_k \in \mathbb{R} \bigwedge 1 \leq k \leq n$$

12.02.16.0093.01

$$\arg\left(x^a\, y^b\right) = 2\pi \left\lfloor \frac{\pi - \operatorname{Im}(a \log(x)) - \operatorname{Im}(b \log(y))}{2\pi} \right\rfloor + \operatorname{Im}(a \log(x)) + \operatorname{Im}(b \log(y))$$

12.02.16.0094.01

$$\arg\left(x^a\, y^b\, z^c\right) = 2\pi \left\lfloor \frac{\pi - \operatorname{Im}(a \log(x)) - \operatorname{Im}(b \log(y)) - \operatorname{Im}(c \log(z))}{2\pi} \right\rfloor + \operatorname{Im}(a \log(x)) + \operatorname{Im}(b \log(y)) + \operatorname{Im}(c \log(z))$$

12.02.16.0095.01

$$\arg\left(\prod_{k=1}^{n} z_k^{a_k}\right) = 2\pi \left\lfloor \frac{\pi - \sum_{k=1}^{n} \operatorname{Im}(a_k \log(z_k))}{2\pi} \right\rfloor + \sum_{k=1}^{n} \operatorname{Im}(a_k \log(z_k))$$

Products, sums, and powers of the direct function

Sums of the direct function

12.02.16.0096.01

$$\arg(z_1) + \arg(z_2) = \arg(z_1 z_2) \;/;\; \arg(z_1) \leq 0 \land -\arg(z_1) - \pi < \arg(z_2) \lor \arg(z_1) \geq 0 \land \arg(z_2) \leq \pi - \arg(z_1)$$

12.02.16.0097.01

$$\arg(z_1) + \arg(z_2) = \arg(z_1 z_2) + 2\pi \;/;\; \arg(z_1) \geq 0 \land \arg(z_2) > \pi - \arg(z_1)$$

12.02.16.0098.01

$$\arg(z_1) + \arg(z_2) = \arg(z_1 z_2) - 2\pi \;/;\; \arg(z_1) \leq 0 \land \arg(z_2) \leq -\arg(z_1) - \pi$$

12.02.16.0028.01

$$\arg(z_1) + \arg(z_2) = \arg(z_1 z_2) - 2\pi \left\lfloor \frac{\pi - \arg(z_1) - \arg(z_2)}{2\pi} \right\rfloor$$

12.02.16.0099.01

$$\sum_{k=1}^{n} \arg(z_k) = \arg\left(\prod_{k=1}^{n} z_k\right) - 2\pi \left\lfloor \frac{\pi - \sum_{k=1}^{n} \arg(z_k)}{2\pi} \right\rfloor \;/;\; n \in \mathbb{N}^+$$

Differences of the direct function

12.02.16.0100.01

$$\arg(z_1) - \arg(z_2) = \arg\left(\frac{z_1}{z_2}\right) \;/;\; -\pi < \arg(z_1) - \arg(z_2) \leq \pi$$

12.02.16.0101.01

$$\arg(z_1) - \arg(z_2) = \arg\left(\frac{z_1}{z_2}\right) \;/;\; \arg(z_1) \leq 0 \land \arg(z_2) < \arg(z_1) + \pi \lor \arg(z_1) > 0 \land \arg(z_2) \geq \arg(z_1) - \pi$$

12.02.16.0102.01

$$\arg(z_1) - \arg(z_2) = \arg\left(\frac{z_1}{z_2}\right) + 2\pi \;/;\; \arg(z_1) \geq 0 \land \arg(z_2) < \arg(z_1) - \pi$$

12.02.16.0103.01

$$\arg(z_1) - \arg(z_2) = \arg\left(\frac{z_1}{z_2}\right) - 2\pi \;/;\; \arg(z_1) \leq 0 \land \arg(z_2) \geq \arg(z_1) + \pi$$

12.02.16.0104.01

$$\arg(z_1) - \arg(z_2) = \arg\left(\frac{z_1}{z_2}\right) - 2\pi \left\lfloor \frac{\arg(z_2) - \arg(z_1) + \pi}{2\pi} \right\rfloor$$

Linear combinations of the direct function

12.02.16.0105.01

$$a \arg(x) + b \arg(y) = \arg\left(x^a y^b\right) - 2\pi \left\lfloor \frac{\pi - a \arg(x) - b \arg(y)}{2\pi} \right\rfloor \;/;\; a \in \mathbb{R} \land b \in \mathbb{R}$$

12.02.16.0106.01

$$a \arg(x) + b \arg(y) + c \arg(z) = \arg\left(x^a y^b z^c\right) - 2\pi \left\lfloor \frac{\pi - a \arg(x) - b \arg(y) - c \arg(z)}{2\pi} \right\rfloor \;/;\; a \in \mathbb{R} \land b \in \mathbb{R} \land c \in \mathbb{R}$$

12.02.16.0107.01

$$\sum_{k=1}^{n} a_k \arg(z_k) = \arg\left(\prod_{k=1}^{n} z_k^{a_k}\right) - 2\pi \left\lfloor \frac{\pi - \sum_{k=1}^{n} a_k \arg(z_k)}{2\pi} \right\rfloor /; a_k \in \mathbb{R} \bigwedge 1 \leq k \leq n$$

Related transformations

12.02.16.0029.01

$$e^{i\arg(z)} = \frac{z}{|z|}$$

12.02.16.0030.01

$$e^{i\arg(z)} = \cos(\arg(z)) + i\sin(\arg(z))$$

12.02.16.0031.01

$$e^{i\arg(z)} = \cos\left(\tan^{-1}\left(\frac{\operatorname{Im}(z)}{\operatorname{Re}(z)}\right)\right) + i\sin\left(\tan^{-1}\left(\frac{\operatorname{Im}(z)}{\operatorname{Re}(z)}\right)\right) /; -\frac{\pi}{2} < \arg(\operatorname{Re}(z)) \leq \frac{\pi}{2}$$

Complex characteristics

Real part

12.02.19.0001.01

$$\operatorname{Re}(\arg(x + i y)) = \tan^{-1}(x, y)$$

12.02.19.0008.01

$$\operatorname{Re}(\arg(z)) = \arg(z)$$

Imaginary part

12.02.19.0002.01

$$\operatorname{Im}(\arg(x + i y)) = 0$$

12.02.19.0003.01

$$\operatorname{Im}(\arg(z)) = 0$$

Absolute value

12.02.19.0004.01

$$|\arg(x + i y)| = \sqrt{\tan^{-1}(x, y)^2}$$

12.02.19.0009.01

$$|\arg(z)| = \sqrt{\tan^{-1}(\operatorname{Re}(z), \operatorname{Im}(z))^2}$$

Argument

12.02.19.0005.01

$$\arg(\arg(x + i y)) = \tan^{-1}\left(\tan^{-1}(x, y), 0\right)$$

12.02.19.0010.01

$$\arg(\arg(z)) = \tan^{-1}\left(\tan^{-1}(\operatorname{Re}(z), \operatorname{Im}(z)), 0\right)$$

Conjugate value

12.02.19.0006.01

$$\overline{\arg(x + i\, y)} == \tan^{-1}(x,\, y)$$

12.02.19.0007.01

$$\overline{\arg(z)} == \arg(z)$$

Signum value

12.02.19.0011.01

$$\operatorname{sgn}(\arg(x + i\, y)) == \operatorname{sgn}\left(\tan^{-1}(x,\, y)\right)$$

12.02.19.0012.01

$$\operatorname{sgn}(\arg(x + i\, y)) == \frac{\tan^{-1}(x,\, y)}{\sqrt{\tan^{-1}(x,\, y)^2}}$$

12.02.19.0013.01

$$\operatorname{sgn}(\arg(z)) == \operatorname{sgn}\left(\tan^{-1}(\operatorname{Re}(z),\, \operatorname{Im}(z))\right)$$

12.02.19.0014.01

$$\operatorname{sgn}(\arg(z)) == \frac{\arg(z)}{\sqrt{\tan^{-1}(\operatorname{Re}(z),\, \operatorname{Im}(z))^2}}$$

Representations through equivalent functions

With related functions

With Re

12.02.27.0006.01

$$\arg(z) == \tan^{-1}(\operatorname{Re}(z),\, -i\,(z - \operatorname{Re}(z)))$$

With Im

12.02.27.0007.01

$$\arg(z) == \tan^{-1}(z - i\, \operatorname{Im}(z),\, \operatorname{Im}(z))$$

12.02.27.0004.01

$$\arg(z) == \tan^{-1}(\operatorname{Re}(z),\, \operatorname{Im}(z))$$

12.02.27.0005.01

$$\arg(z) == \tan^{-1}\left(\frac{\operatorname{Im}(z)}{\operatorname{Re}(z)}\right) /;\ \operatorname{Re}(z) > 0$$

With Abs

12.02.27.0001.01

$$\arg(z) == -i\, \log\left(\frac{z}{|z|}\right)$$

12.02.27.0008.01

$$\arg(z) == i\,(\log(|z|) - \log(z))$$

12.02.27.0002.01

$$\cos(\arg(z)) = \frac{\operatorname{Re}(z)}{|z|}$$

12.02.27.0003.01

$$\sin(\arg(z)) = \frac{\operatorname{Im}(z)}{|z|}$$

With Conjugate

12.02.27.0009.01

$$\arg(z) = \frac{1}{2}\, i\, (\log(z\, \bar{z}) - 2\log(z))$$

With Sign

12.02.27.0010.01

$$\arg(z) = -i \log(\operatorname{sgn}(z))$$

With inverse trigonometric functions

With ArcSin

12.02.27.0011.01

$$\arg(z) = \sin^{-1}\left(\frac{\operatorname{Im}(z)}{|z|}\right) + \frac{\pi}{4}\left(-\sqrt{\frac{1}{z}}\,\sqrt{z} + \frac{i\sqrt{-z^2}}{z}\left(1 - \frac{\sqrt{z^2}}{z}\right) - \sqrt{\frac{i}{z}}\,\sqrt{-i\,z} + 2\right)$$

12.02.27.0012.01

$$\arg(z) = \sin^{-1}\left(\frac{\operatorname{Re}(z)}{|z|}\right) + \frac{\pi}{4}\left(\frac{\sqrt{z^2}}{z}\left(1 + \frac{i\sqrt{-z^2}}{z}\right) - \sqrt{\frac{1}{z}}\,\sqrt{z} - \sqrt{-\frac{i}{z}}\,\sqrt{i\,z} + \frac{4\,(-1)^{3/4}\sqrt{i\,z}}{\sqrt{z}} + 4\right)$$

With ArcCos

12.02.27.0013.01

$$\arg(z) = -\cos^{-1}\left(\frac{\operatorname{Im}(z)}{|z|}\right) + \frac{\pi}{4}\left(-\sqrt{\frac{1}{z}}\,\sqrt{z} + \frac{i\sqrt{-z^2}}{z}\left(1 - \frac{\sqrt{z^2}}{z}\right) - \sqrt{\frac{i}{z}}\,\sqrt{-i\,z} + 4\right)$$

12.02.27.0014.01

$$\arg(z) = -\cos^{-1}\left(\frac{\operatorname{Re}(z)}{|z|}\right) + \frac{\pi}{4}\left(\frac{\sqrt{z^2}}{z}\left(1 + \frac{i\sqrt{-z^2}}{z}\right) - \sqrt{\frac{1}{z}}\,\sqrt{z} - \sqrt{-\frac{i}{z}}\,\sqrt{i\,z} + \frac{4\,(-1)^{3/4}\sqrt{i\,z}}{\sqrt{z}} + 6\right)$$

With ArcTan

12.02.27.0015.01

$$\arg(z) = \tan^{-1}(\operatorname{Re}(z), \operatorname{Im}(z))$$

With inverse hyperbolic functions

Inequalities

12.02.29.0001.01

$|\arg(z)| \leq \pi$

12.02.29.0002.01

$-\pi < \arg(z) \leq \pi$

12.02.29.0003.01

$-r < \arg(a + z) < R \;/;\; -r < \arg(z) < R \bigwedge -r < \arg(a) < R \bigwedge R - r \leq \pi$

Pavlyk O. (2006)

Zeros

12.02.30.0001.01

$\arg(z) == 0 \;/;\; z \in \mathbb{R} \land z > 0$

History

Arg is encountered often in mathematics and the natural sciences.

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