

# CarmichaelLambda

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## Notations

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### Traditional name

Carmichael lambda function

### Traditional notation

 $\lambda(n)$ 

### Mathematica StandardForm notation

CarmichaelLambda[n]

## Primary definition

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13.09.02.0001.01

$$\lambda(n) = \lambda /; \forall m, \gcd(n,m)=1 \ m^\lambda \bmod n = 1 \wedge \forall \Lambda \left( \neg \left( \Lambda < \lambda \wedge m^\Lambda \bmod n = 1 \wedge \gcd(n, m) = 1 \right) \right)$$

For integer  $n$ , the Carmichael lambda function  $\lambda(n)$  is the smallest number such that for any  $m$  with  $\gcd(m, n) = 1$  the congruence  $m^{\lambda(n)} \bmod n = 1$  holds.

Examples:  $\lambda(3) = 2$ ,  $\lambda(5) = 4$ ,  $\lambda(11) = 10$ .

## Specific values

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### Specialized values

13.09.03.0001.01

$$\lambda(2^n) = \phi(2^n) /; 0 \leq n \leq 2$$

13.09.03.0002.01

$$\lambda(2^n) = 2^{n-2} /; n \geq 3$$

13.09.03.0003.01

$$\lambda(2^n) = \frac{1}{2} \phi(2^n) /; n \geq 3$$

13.09.03.0004.01

$$\lambda(p^n) = \phi(p^n) /; p \in \mathbb{P} \wedge p > 2 \wedge n \in \mathbb{N}^+$$

### Values at fixed points

13.09.03.0005.01  
 $\lambda(0) = 0$

13.09.03.0006.01  
 $\lambda(1) = 1$

13.09.03.0007.01  
 $\lambda(2) = 1$

13.09.03.0008.01  
 $\lambda(3) = 2$

13.09.03.0009.01  
 $\lambda(4) = 2$

13.09.03.0010.01  
 $\lambda(5) = 4$

13.09.03.0011.01  
 $\lambda(6) = 2$

13.09.03.0012.01  
 $\lambda(7) = 6$

13.09.03.0013.01  
 $\lambda(8) = 2$

13.09.03.0014.01  
 $\lambda(9) = 6$

13.09.03.0015.01  
 $\lambda(10) = 4$

13.09.03.0016.01  
 $\lambda(11) = 10$

13.09.03.0017.01  
 $\lambda(12) = 2$

13.09.03.0018.01  
 $\lambda(13) = 12$

13.09.03.0019.01  
 $\lambda(14) = 6$

13.09.03.0020.01  
 $\lambda(15) = 4$

13.09.03.0021.01  
 $\lambda(16) = 4$

13.09.03.0022.01  
 $\lambda(17) = 16$

13.09.03.0023.01  
 $\lambda(18) = 6$

13.09.03.0024.01  
 $\lambda(19) = 18$

13.09.03.0025.01  
 $\lambda(20) = 4$

13.09.03.0026.01  
 $\lambda(21) = 6$

13.09.03.0027.01  
 $\lambda(22) = 10$

13.09.03.0028.01  
 $\lambda(23) = 22$

13.09.03.0029.01  
 $\lambda(24) = 2$

13.09.03.0030.01  
 $\lambda(25) = 20$

13.09.03.0031.01  
 $\lambda(26) = 12$

13.09.03.0032.01  
 $\lambda(27) = 18$

13.09.03.0033.01  
 $\lambda(28) = 6$

13.09.03.0034.01  
 $\lambda(29) = 28$

13.09.03.0035.01  
 $\lambda(30) = 4$

13.09.03.0036.01  
 $\lambda(31) = 30$

13.09.03.0037.01  
 $\lambda(32) = 8$

13.09.03.0038.01  
 $\lambda(33) = 10$

13.09.03.0039.01  
 $\lambda(34) = 16$

13.09.03.0040.01  
 $\lambda(35) = 12$

13.09.03.0041.01  
 $\lambda(36) = 6$

13.09.03.0042.01  
 $\lambda(37) = 36$

13.09.03.0043.01  
 $\lambda(38) = 18$

13.09.03.0044.01  
 $\lambda(39) = 12$

13.09.03.0045.01  
 $\lambda(40) = 4$

13.09.03.0046.01  
 $\lambda(41) = 40$

13.09.03.0047.01  
 $\lambda(42) = 6$

13.09.03.0048.01  
 $\lambda(43) = 42$

13.09.03.0049.01  
 $\lambda(44) = 10$

13.09.03.0050.01  
 $\lambda(45) = 12$

13.09.03.0051.01  
 $\lambda(46) = 22$

13.09.03.0052.01  
 $\lambda(47) = 46$

13.09.03.0053.01  
 $\lambda(48) = 4$

13.09.03.0054.01  
 $\lambda(49) = 42$

13.09.03.0055.01  
 $\lambda(50) = 20$

## General characteristics

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### Domain and analyticity

$\lambda(n)$  is a non-analytical function which is defined for integer  $n$ .

13.09.04.0001.01  
 $n \rightarrow \lambda(n) : \mathbb{Z} \rightarrow \mathbb{Z}$

### Symmetries and periodicities

#### Parity

$\lambda(n)$  is an even function.

13.09.04.0002.01  
 $\lambda(-n) = \lambda(n)$

#### Periodicity

No periodicity

## Identities

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## Functional identities

13.09.17.0001.01

$$\lambda\left(2^\alpha \prod_{k=1}^n p_k^{\alpha_k}\right) = \text{lcm}(\lambda(2^\alpha), \lambda(p_1^{\alpha_1}), \dots, \lambda(p_n^{\alpha_n})); p_k \in \mathbb{P} \wedge \alpha \in \mathbb{N}^+ \wedge \alpha_k \in \mathbb{N}^+ \wedge p_k > 2$$

## Representations through equivalent functions

### With related functions

13.09.27.0001.01

$$\lambda(p^n) = \phi(p^n); p \in \mathbb{P} \wedge p > 2 \wedge n \in \mathbb{N}^+$$

## Zeros

13.09.30.0001.01

$$\lambda(0) = 0$$

## Other identities

### Congruence properties

13.09.32.0001.01

$$m^{\lambda(p^n)} \bmod p^n = 1; p \in \mathbb{P} \wedge n \in \mathbb{N}^+$$

13.09.32.0002.01

$$m^{\lambda(n)} \bmod n = 1; \text{gcd}(m, n) = 1 \wedge m \in \mathbb{N}^+ \wedge n \in \mathbb{N}^+$$

## Theorems

### Absolute pseudo-prime number

The number  $n$  is an absolute pseudoprime if and only if  $n \bmod \lambda(n) = 1$ .

## History

–R. D. Carmichael (1910)

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