CarmichaelLambda

Notations

Traditional name

Carmichael lambda function

Traditional notation

$\lambda(n)$

Mathematica StandardForm notation

CarmichaelLambda[n]

Primary definition

13.09.02.0001.01

$$\lambda(n) = \lambda \;/;\; \forall_{m,\, \gcd(n,m)=1}\, m^{\lambda} \bmod n = 1 \bigwedge \forall_{\Lambda} \left(\neg \left(\Lambda < \lambda \bigwedge m^{\lambda} \bmod n = 1 \bigwedge \gcd(n, m) = 1\right)\right)$$

For integer n, the Carmichael lambda function $\lambda(n)$ is the smallest number such that for any m with $\gcd(m, n) = 1$ the congruence $m^{\lambda(n)} \bmod n = 1$ holds.

Examples: $\lambda(3) = 2$, $\lambda(5) = 4$, $\lambda(11) = 10$.

Specific values

Specialized values

13.09.03.0001.01

$$\lambda(2^n) = \phi(2^n) \;/;\; 0 \le n \le 2$$

13.09.03.0002.01

$$\lambda(2^n) = 2^{n-2} \;/;\; n \ge 3$$

13.09.03.0003.01

$$\lambda(2^n) = \frac{1}{2}\, \phi(2^n) \;/;\; n \ge 3$$

13.09.03.0004.01

$$\lambda(p^n) = \phi(p^n) \;/;\; p \in \mathbb{P} \land p > 2 \land n \in \mathbb{N}^+$$

Values at fixed points

13.09.03.0005.01

$\lambda(0) == 0$

13.09.03.0006.01

$\lambda(1) == 1$

13.09.03.0007.01

$\lambda(2) == 1$

13.09.03.0008.01

$\lambda(3) == 2$

13.09.03.0009.01

$\lambda(4) == 2$

13.09.03.0010.01

$\lambda(5) == 4$

13.09.03.0011.01

$\lambda(6) == 2$

13.09.03.0012.01

$\lambda(7) == 6$

13.09.03.0013.01

$\lambda(8) == 2$

13.09.03.0014.01

$\lambda(9) == 6$

13.09.03.0015.01

$\lambda(10) == 4$

13.09.03.0016.01

$\lambda(11) == 10$

13.09.03.0017.01

$\lambda(12) == 2$

13.09.03.0018.01

$\lambda(13) == 12$

13.09.03.0019.01

$\lambda(14) == 6$

13.09.03.0020.01

$\lambda(15) == 4$

13.09.03.0021.01

$\lambda(16) == 4$

13.09.03.0022.01

$\lambda(17) == 16$

13.09.03.0023.01

$\lambda(18) == 6$

13.09.03.0024.01

$\lambda(19) == 18$

13.09.03.0025.01

$\lambda(20) = 4$

13.09.03.0026.01

$\lambda(21) = 6$

13.09.03.0027.01

$\lambda(22) = 10$

13.09.03.0028.01

$\lambda(23) = 22$

13.09.03.0029.01

$\lambda(24) = 2$

13.09.03.0030.01

$\lambda(25) = 20$

13.09.03.0031.01

$\lambda(26) = 12$

13.09.03.0032.01

$\lambda(27) = 18$

13.09.03.0033.01

$\lambda(28) = 6$

13.09.03.0034.01

$\lambda(29) = 28$

13.09.03.0035.01

$\lambda(30) = 4$

13.09.03.0036.01

$\lambda(31) = 30$

13.09.03.0037.01

$\lambda(32) = 8$

13.09.03.0038.01

$\lambda(33) = 10$

13.09.03.0039.01

$\lambda(34) = 16$

13.09.03.0040.01

$\lambda(35) = 12$

13.09.03.0041.01

$\lambda(36) = 6$

13.09.03.0042.01

$\lambda(37) = 36$

13.09.03.0043.01

$\lambda(38) = 18$

13.09.03.0044.01

$\lambda(39) = 12$

13.09.03.0045.01
$\lambda(40) == 4$

13.09.03.0046.01
$\lambda(41) == 40$

13.09.03.0047.01
$\lambda(42) == 6$

13.09.03.0048.01
$\lambda(43) == 42$

13.09.03.0049.01
$\lambda(44) == 10$

13.09.03.0050.01
$\lambda(45) == 12$

13.09.03.0051.01
$\lambda(46) == 22$

13.09.03.0052.01
$\lambda(47) == 46$

13.09.03.0053.01
$\lambda(48) == 4$

13.09.03.0054.01
$\lambda(49) == 42$

13.09.03.0055.01
$\lambda(50) == 20$

General characteristics

Domain and analyticity

$\lambda(n)$ is a non-analytical function which is defined for integer n.

13.09.04.0001.01
$n \longrightarrow \lambda(n) :: \mathbb{Z} \longrightarrow \mathbb{Z}$

Symmetries and periodicities

Parity

$\lambda(n)$ is an even function.

13.09.04.0002.01
$\lambda(-n) == \lambda(n)$

Periodicity

No periodicity

Identities

Functional identities

13.09.17.0001.01

$$\lambda\left(2^{\alpha}\prod_{k=1}^{n} p_k^{\alpha_k}\right) = \operatorname{lcm}\left(\lambda(2^{\alpha}),\ \lambda\left(p_1^{\alpha_1}\right),\ \ldots,\ \lambda\left(p_n^{\alpha_n}\right)\right) /;\ p_k \in \mathbb{P} \bigwedge \alpha \in \mathbb{N}^+ \bigwedge \alpha_k \in \mathbb{N}^+ \bigwedge p_k > 2$$

Representations through equivalent functions

With related functions

13.09.27.0001.01

$$\lambda(p^n) = \phi(p^n) /;\ p \in \mathbb{P} \bigwedge p > 2 \bigwedge n \in \mathbb{N}^+$$

Zeros

13.09.30.0001.01

$$\lambda(0) = 0$$

Other identities

Congruence properties

13.09.32.0001.01

$$m^{\lambda(p^n)} \bmod p^n = 1 /;\ p \in \mathbb{P} \bigwedge n \in \mathbb{N}^+$$

13.09.32.0002.01

$$m^{\lambda(n)} \bmod n = 1 /;\ \gcd(m, n) = 1 \bigwedge m \in \mathbb{N}^+ \bigwedge n \in \mathbb{N}^+$$

Theorems

Absolute pseudo-prime number

The number n is an absolute pseudoprime if and only if $n \bmod \lambda(n) = 1$.

History

– R. D. Carmichael (1910)

Copyright