Conjugate

Notations

Traditional name

Complex conjugate

Traditional notation

$\bar{z}$

Mathematica StandardForm notation

`Conjugate[z]`

Primary definition

12.05.02.0001.01

$\bar{z} == \mathrm{Re}(z) - i\, \mathrm{Im}(z)$

$\bar{z}$ is the complex conjugate of the complex number z.

Specific values

Specialized values

12.05.03.0001.01

$\overline{x} == x \,/;\, x \in \mathbb{R}$

12.05.03.0002.01

$\overline{i\, x} == -i\, x \,/;\, x \in \mathbb{R}$

12.05.03.0003.01

$\overline{x + i\, y} == x - i\, y \,/;\, x \in \mathbb{R} \bigwedge y \in \mathbb{R}$

Values at fixed points

12.05.03.0004.01

$\overline{0} == 0$

12.05.03.0005.01

$\overline{1} == 1$

12.05.03.0006.01

$\overline{-1} == -1$

12.05.03.0007.01

$\overline{i} == -i$

12.05.03.0008.01

$\overline{-i} == i$

12.05.03.0020.01

$\overline{1+i} == 1-i$

12.05.03.0021.01

$\overline{-1+i} == -1-i$

12.05.03.0022.01

$-\overline{1-i} == -1+i$

12.05.03.0023.01

$\overline{1-i} == 1+i$

12.05.03.0024.01

$\overline{\sqrt{3}+i} == \sqrt{3}-i$

12.05.03.0025.01

$\overline{1+i\sqrt{3}} == 1-i\sqrt{3}$

12.05.03.0026.01

$\overline{-1+i\sqrt{3}} == -1-i\sqrt{3}$

12.05.03.0027.01

$\overline{-\sqrt{3}+i} == -\sqrt{3}-i$

12.05.03.0028.01

$\overline{-\sqrt{3}-i} == -\sqrt{3}+i$

12.05.03.0029.01

$\overline{-1-i\sqrt{3}} == -1+i\sqrt{3}$

12.05.03.0030.01

$\overline{1-i\sqrt{3}} == 1+i\sqrt{3}$

12.05.03.0031.01

$\overline{\sqrt{3}-i} == \sqrt{3}+i$

12.05.03.0009.01

$\overline{2} == 2$

12.05.03.0010.01

$\overline{-2} == -2$

12.05.03.0011.01

$\overline{\pi} == \pi$

12.05.03.0012.01

$\overline{3\,i} == -3\,i$

12.05.03.0013.01

$\overline{-2\,i} == 2\,i$

12.05.03.0014.01

$\overline{2+i} == 2-i$

Values at infinities

12.05.03.0015.01

$\text{Conjugate}(\infty) = \infty$

12.05.03.0016.01

$\text{Conjugate}(-\infty) = -\infty$

12.05.03.0017.01

$\text{Conjugate}(i\,\infty) = -i\,\infty$

12.05.03.0018.01

$\text{Conjugate}(-i\,\infty) = i\,\infty$

12.05.03.0019.01

$\text{Conjugate}(\tilde{\infty}) = \tilde{\infty}$

General characteristics

Domain and analyticity

$\bar{z}$ is a nonanalytical function. The real and the imaginary parts of $\bar{z}$ are real-analytic functions of the variable z.

12.05.04.0001.01

$z \longrightarrow \bar{z} :: \mathbb{C} \longrightarrow \mathbb{C}$

Symmetries and periodicities

Parity

$\bar{z}$ is an odd function.

12.05.04.0002.01

$\overline{-z} = -\bar{z}$

Mirror symmetry

12.05.04.0003.01

$\bar{\bar{z}} = z$

Periodicity

No periodicity

Homogeneity

12.05.04.0005.01

$\overline{a\,z} = \bar{a}\,\bar{z}$

Sets of discontinuity

The function $\bar{z}$ is continuous function in $\mathbb{C}$.

12.05.04.0004.01

$\mathcal{DS}_z(\bar{z}) = \{\}$

Transformations

Transformations and argument simplifications

Argument involving basic arithmetic operations

12.05.16.0001.01

$\overline{-z} == -\bar{z}$

12.05.16.0002.01

$\overline{a\,z} == a\,\bar{z} \;/;\; a \in \mathbb{R}$

12.05.16.0003.01

$\overline{i\,x} == -i\,x \;/;\; x \in \mathbb{R}$

12.05.16.0004.01

$\overline{i\,z} == -i\,\bar{z}$

12.05.16.0005.01

$\overline{-i\,z} == i\,\bar{z}$

12.05.16.0006.01

$$\overline{\frac{1}{z}} == \frac{1}{\bar{z}}$$

Addition formulas

12.05.16.0007.01

$\overline{x + i\,y} == x - i\,y \;/;\; x \in \mathbb{R} \bigwedge y \in \mathbb{R}$

12.05.16.0008.01

$$\overline{\sum_{k=1}^{n} z_k} == \sum_{k=1}^{n} \overline{z_k}$$

12.05.16.0009.01

$\overline{z_1 + z_2} == \overline{z_1} + \overline{z_2}$

Multiple arguments

12.05.16.0010.01

$\overline{a\,z} == a\,\bar{z} \;/;\; a \in \mathbb{R}$

12.05.16.0011.01

$\overline{i\,z} == -i\,\bar{z}$

12.05.16.0012.01

$\overline{-i\,z} == i\,\bar{z}$

12.05.16.0013.01

$$\overline{\prod_{k=1}^{n} z_k} == \prod_{k=1}^{n} \overline{z_k}$$

12.05.16.0014.01

$\overline{z_1\,z_2} == \overline{z_1}\,\overline{z_2}$

Ratio of arguments

12.05.16.0025.01

$$\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\overline{z_1}}{\overline{z_2}}$$

Power of arguments

12.05.16.0015.01

$$\overline{x^a} = x^{\bar{a}} \;/;\; x \in \mathbb{R} \land x > 0$$

12.05.16.0016.01

$$\overline{x^a} = x^{\mathrm{Re}(a)} \left(\cos(\mathrm{Im}(a) \log(x)) - i \sin(\mathrm{Im}(a) \log(x))\right) \;/;\; x \in \mathbb{R} \land x > 0$$

12.05.16.0017.01

$$\overline{z^a} = \bar{z}^a \;/;\; a \in \mathbb{R}$$

12.05.16.0018.01

$$\overline{z^a} = |z|^a \left(\cos\left(a \tan^{-1}(\mathrm{Re}(z), \mathrm{Im}(z))\right) - i \sin\left(a \tan^{-1}(\mathrm{Re}(z), \mathrm{Im}(z))\right)\right) \;/;\; a \in \mathbb{R}$$

12.05.16.0019.01

$$\overline{z^a} = \bar{z}^{\bar{a}} \;/;\; \arg(z) \neq \pi$$

12.05.16.0026.01

$$\overline{z^a} = \left(\frac{1}{\bar{z}}\right)^{-\bar{a}}$$

12.05.16.0020.01

$$\overline{z^a} = |z|^{\mathrm{Re}(a)} \exp(-\arg(z) \,\mathrm{Im}(a) - i\,(\mathrm{Im}(a) \log(|z|) + \arg(z) \,\mathrm{Re}(a)))$$

12.05.16.0021.01

$$\overline{z^a} = \exp\left(-\mathrm{Im}(a) \tan^{-1}(\mathrm{Re}(z), \mathrm{Im}(z))\right) |z|^{\mathrm{Re}(a)} \left(\cos\left(\mathrm{Im}(a) \log(|z|) + \tan^{-1}(\mathrm{Re}(z), \mathrm{Im}(z)) \,\mathrm{Re}(a)\right) - i \sin\left(\mathrm{Im}(a) \log(|z|) + \tan^{-1}(\mathrm{Re}(z), \mathrm{Im}(z)) \,\mathrm{Re}(a)\right)\right)$$

Exponent of arguments

12.05.16.0027.01

$$\overline{e^{x+i y}} = e^{x-i y}$$

12.05.16.0028.01

$$\overline{e^z} = e^{\bar{z}}$$

12.05.16.0029.01

$$\overline{e^{i z}} = e^{-i \bar{z}}$$

Products, sums, and powers of the direct function

Products of the direct function

12.05.16.0022.01

$$\overline{z_1}\,\overline{z_2} = \overline{z_1 z_2}$$

Sums of the direct function

12.05.16.0023.01

$$\overline{z_1} + \overline{z_2} = \overline{z_1 + z_2}$$

Powers of the direct function

12.05.16.0024.01

$\bar{z}^a = \overline{z^a} \;/;\; a \in \mathbb{R}$

Complex characteristics

Real part

12.05.19.0001.01

$\operatorname{Re}(\overline{x + i\, y}) = x$

12.05.19.0002.01

$\operatorname{Re}(\bar{z}) = \operatorname{Re}(z)$

Imaginary part

12.05.19.0003.01

$\operatorname{Im}(\overline{x + i\, y}) = -y$

12.05.19.0004.01

$\operatorname{Im}(\bar{z}) = -\operatorname{Im}(z)$

Absolute value

12.05.19.0005.01

$|\overline{x + i\, y}| = \sqrt{x^2 + y^2}$

12.05.19.0006.01

$|\bar{z}| = |z|$

Argument

12.05.19.0007.01

$\arg(\overline{x + i\, y}) = \tan^{-1}(x, -y)$

12.05.19.0008.01

$\arg(\bar{z}) = -\arg(z) \;/;\; \arg(z) \neq \pi$

Conjugate value

12.05.19.0009.01

$\overline{\overline{x + i\, y}} = x + i\, y$

12.05.19.0010.01

$\bar{\bar{z}} = z$

Signum value

12.05.19.0012.01

$\operatorname{sgn}(\overline{x + i\, y}) = \frac{x - i\, y}{\sqrt{x^2 + y^2}}$

12.05.19.0011.01

$$\operatorname{sgn}(\bar{z}) = \frac{\bar{z}}{|z|}$$

Differentiation

Low-order differentiation

In a distributional sense, for $x \in \mathbb{R}$.

12.05.20.0001.01

$$\frac{\partial \overline{x}}{\partial x} = 1$$

Fractional integro-differentiation

12.05.20.0002.01

$$\frac{\partial^{\alpha} \overline{x}}{\partial z^{\alpha}} = \frac{x^{1-\alpha}}{\Gamma(2-\alpha)}$$

Representations through equivalent functions

With related functions

With Re

12.05.27.0003.01

$$\bar{z} = 2 \operatorname{Re}(z) - z$$

With Im

12.05.27.0004.01

$$\bar{z} = z - 2 i \operatorname{Im}(z)$$

12.05.27.0002.01

$$\bar{z} = \operatorname{Re}(z) - i \operatorname{Im}(z)$$

With Abs

12.05.27.0005.01

$$\bar{z} = \frac{|z|^2}{z}$$

With Arg

12.05.27.0006.01

$$\bar{z} = e^{-2 i \arg(z)} z$$

12.05.27.0007.01

$$\bar{z} = z\, e^{-2 i \left(\arg(z) - \pi \left\lfloor \frac{\arg(z)+\pi}{2 \pi} \right\rfloor\right)}$$

12.05.27.0001.01

$$\bar{z} = |z|\, e^{-i \arg(z)}$$

12.05.27.0008.01

$$\bar{z} == |z| \cos(\arg(z)) - i\, |z| \sin(\arg(z))$$

With Sign

12.05.27.0009.01

$$\bar{z} == \frac{z}{\operatorname{sgn}(z)^2}$$

Zeros

12.05.30.0001.01

$$\bar{z} == 0 \;/;\; z == 0$$

History

– A. L. Cauchy (1821) (used the word "conjugate" the first time in the current sense)

The function $\bar{z}$ is encountered often in mathematics and the natural sciences.

Copyright