

# DiracDelta2

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## Notations

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### Traditional name

Multidimensional Dirac delta function

### Traditional notation

$\delta(x_1, x_2, \dots)$

### Mathematica StandardForm notation

DiracDelta[ $x_1, x_2, \dots$ ]

## Primary definition

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14.04.02.0001.01

$$\delta(x_1, x_2, \dots) = \frac{1}{\pi} \lim_{\varepsilon \rightarrow 0} \frac{\varepsilon}{\varepsilon^2 + x_1^2 + x_2^2 + \dots} \quad ; x_k \in \mathbb{R}$$

## Specific values

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### Specialized values

14.04.03.0001.01

$$\delta(x_1, x_2, \dots) = 0 \quad ; x_1 \in \mathbb{R} \bigwedge x_2 \in \mathbb{R} \bigwedge \dots \bigwedge x_1^2 + x_2^2 + \dots \neq 0$$

### Values at fixed points

14.04.03.0002.01

$$\delta(0, 0, \dots, 0) = \infty$$

## General characteristics

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### Domain and analyticity

$\delta(x_1, x_2, \dots)$  is a nonanalytical function; it is a generalized function defined for  $x_k \in \mathbb{R}$ .

14.04.04.0001.01

$$(x_1 * x_2 * \dots * x_n) \rightarrow \delta(x_1, x_2, \dots, x_n) :: \mathbb{R}^n \rightarrow \{0, \infty\}$$

### Symmetries and periodicities

**Parity**

$\delta(x_1, x_2, \dots)$  is an even generalized function.

14.04.04.0002.01

$$\delta(-x_1, -x_2, \dots, -x_n) = \delta(x_1, x_2, \dots, x_n)$$

14.04.04.0003.01

$$\delta(x_1, x_2, \dots, -x_k, x_{k+1}, \dots, x_n) = \delta(x_1, x_2, \dots, x_n)$$

**Permutation symmetry**

14.04.04.0004.01

$$\delta(x_2, x_1) = \delta(x_1, x_2)$$

14.04.04.0005.01

$$\delta(x_1, x_2, \dots, x_k, \dots, x_j, \dots, x_n) = \delta(x_1, x_2, \dots, x_j, \dots, x_k, \dots, x_n) /; x_k \neq x_j \wedge k \neq j$$

**Periodicity**

No periodicity

**Integral representations****On the real axis****Of the direct function**

14.04.07.0001.01

$$\delta(x_1, x_2, \dots, x_n) = \frac{1}{(2\pi)^n} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} e^{i \sum_{k=1}^n t_k x_k} dt_1 dt_2 \dots dt_n$$

**Differentiation****Symbolic differentiation****With respect to  $x_n$** 

In a distributional sense, for  $x_k \in \mathbb{R}$ :

14.04.20.0001.01

$$\delta^{(0,0,\dots,0,n)}(x_1, x_2, \dots, x_{n-1}, -x_n) = (-1)^n \delta(x_1, x_2, \dots, x_n) /; n \in \mathbb{N}$$

14.04.20.0002.01

$$x_n^k \frac{\partial^m \delta(x_1, x_2, \dots, x_n)}{\partial x_n^m} = 0 /; k \in \mathbb{N}^+ \wedge m \in \mathbb{N} \wedge m < k$$

14.04.20.0003.01

$$x_n^k \frac{\partial^k \delta(x_1, x_2, \dots, x_n)}{\partial x_n^k} = (-1)^k k! \delta(x_1, x_2, \dots, x_n) /; k \in \mathbb{N}$$

14.04.20.0004.01

$$x_n^k \frac{\partial^m \delta(x_1, x_2, \dots, x_n)}{\partial x_n^m} = \frac{(-1)^k m!}{(m-k)!} \frac{\partial^{m-k} \delta(x_1, x_2, \dots, x_n)}{\partial x_n^{m-k}} ; m \in \mathbb{N}^+ \wedge k \in \mathbb{N} \wedge k < m$$

## Integration

### Definite integration

#### Multiple integration

14.04.21.0001.01

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \dots \int_{-\infty}^{\infty} \delta(t_1, t_2, \dots, t_n) dt_1 dt_2 \dots dt_n = 1$$

14.04.21.0002.01

$$\int_{-\infty}^{\infty} \int_{-\infty}^{d_2} \dots \int_{-\infty}^{\infty} \delta(t_1 - a_1, t_2 - a_2, \dots, t_n - a_n) f(t_1, t_2, \dots, t_n) dt_1 dt_2 \dots dt_n = f(a_1, a_2, \dots, a_n) ;$$

$$-\infty \leq -d_k < a_k < d_k \leq \infty \wedge 1 \leq k \leq n$$

## Integral transforms

### Fourier exp transforms

14.04.22.0001.01

$$\mathcal{F}_t[\delta(x_1, x_2, \dots, x_{n-1}, t)](z) = \frac{\delta(x_1, x_2, \dots, x_{n-1})}{\sqrt{2\pi}}$$

### Inverse Fourier exp transforms

14.04.22.0002.01

$$\mathcal{F}_t^{-1}[\delta(x_1, x_2, \dots, x_{n-1}, t)](z) = \frac{\delta(x_1, x_2, \dots, x_{n-1})}{\sqrt{2\pi}}$$

### Fourier cos transforms

14.04.22.0003.01

$$\mathcal{F}_c[\delta(x_1, x_2, \dots, x_{n-1}, t)](z) = \sqrt{\frac{2}{\pi}} \delta(x_1, x_2, \dots, x_{n-1})$$

### Fourier sin transforms

14.04.22.0004.01

$$\mathcal{F}_s[\delta(x_1, x_2, \dots, x_{n-1}, t)](z) = 0$$

### Laplace transforms

14.04.22.0005.01

$$\mathcal{L}_t[\delta(x_1, x_2, \dots, x_{n-1}, t)](z) = \delta(x_1, x_2, \dots, x_{n-1})$$

## Representations through more general functions

## Through Meijer G

### Classical cases for the direct function itself

14.04.26.0001.01

$$\delta(x_1, x_2, \dots, x_n) = \prod_{k=1}^n G_{0,0}^{0,0} \left( 1 - x_k \mid \right) /; x_k \neq x_j \wedge k \neq j \wedge x_k \in \mathbb{R} \wedge x_k < 2$$

14.04.26.0002.01

$$\delta(x_1, x_2, \dots, x_n) = \prod_{k=1}^n G_{0,0}^{0,0} \left( x_k + 1 \mid \right) /; x_k \neq x_j \wedge k \neq j \wedge x_k \in \mathbb{R} \wedge x_k > -2$$

## Representations through equivalent functions

14.04.27.0001.01

$$\delta(x_1, x_2, \dots, x_n) = \prod_{k=1}^n \delta(x_k) /; x_k \neq x_j \wedge k \neq j$$

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