DiracDelta2

Notations

Traditional name

Multidimensional Dirac delta function

Traditional notation

$\delta(x_1, x_2, \ldots)$

Mathematica StandardForm notation

`DiracDelta[`x_1`, `x_2`, …]`

Primary definition

14.04.02.0001.01

$$\delta(x_1, x_2, \ldots) = \frac{1}{\pi} \lim_{\varepsilon \to 0} \frac{\varepsilon}{\varepsilon^2 + x_1^2 + x_2^2 + \ldots} /; x_k \in \mathbb{R}$$

Specific values

Specialized values

14.04.03.0001.01

$$\delta(x_1, x_2, \ldots) = 0 /; x_1 \in \mathbb{R} \bigwedge x_2 \in \mathbb{R} \bigwedge \ldots \bigwedge x_1^2 + x_2^2 + \ldots \neq 0$$

Values at fixed points

14.04.03.0002.01

$$\delta(0, 0, \ldots, 0) = \infty$$

General characteristics

Domain and analyticity

$\delta(x_1, x_2, \ldots)$ is a nonanalytical function; it is a generalized function defined for $x_k \in \mathbb{R}$.

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$$(x_1 * x_2 * \ldots * x_n) \longrightarrow \delta(x_1, x_2, \ldots, x_n) :: \mathbb{R}^n \longrightarrow \{0, \infty\}$$

Symmetries and periodicities

Parity

$\delta(x_1, x_2, \ldots)$ is an even generalized function.

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$$\delta(-x_1, -x_2, \ldots, -x_n) = \delta(x_1, x_2, \ldots, x_n)$$

14.04.04.0003.01

$$\delta(x_1, x_2, \ldots, -x_k, x_{k+1}, \ldots, x_n) = \delta(x_1, x_2, \ldots, x_n)$$

Permutation symmetry

14.04.04.0004.01

$$\delta(x_2, x_1) = \delta(x_1, x_2)$$

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$$\delta(x_1, x_2, \ldots, x_k, \ldots, x_j, \ldots, x_n) = \delta(x_1, x_2, \ldots, x_j, \ldots, x_k, \ldots, x_n) /; x_k \neq x_j \land k \neq j$$

Periodicity

No periodicity

Integral representations

On the real axis

Of the direct function

14.04.07.0001.01

$$\delta(x_1, x_2, \ldots, x_n) = \frac{1}{(2\pi)^n} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\ldots\int_{-\infty}^{\infty} e^{i\sum_{k=1}^{n} t_k x_k} \, dt_1 \, dt_2 \ldots dt_n$$

Differentiation

Symbolic differentiation

With respect to x_n

In a distributional sense, for $x_k \in \mathbb{R}$:

14.04.20.0001.01

$$\delta^{(0,0,\ldots,0,n)}(x_1, x_2, \ldots, x_{n-1}, -x_n) = (-1)^n \, \delta(x_1, x_2, \ldots, x_n) /; n \in \mathbb{N}$$

14.04.20.0002.01

$$x_n^k \frac{\partial^m \delta(x_1, x_2, \ldots, x_n)}{\partial x_n^m} = 0 /; k \in \mathbb{N}^+ \land m \in \mathbb{N} \land m < k$$

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$$x_n^k \frac{\partial^k \delta(x_1, x_2, \ldots, x_n)}{\partial x_n^k} = (-1)^k \, k! \, \delta(x_1, x_2, \ldots, x_n) /; k \in \mathbb{N}$$

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$$x_n^k \frac{\partial^m \delta(x_1, x_2, \ldots, x_n)}{\partial x_n^m} = \frac{(-1)^k m!}{(m-k)!} \frac{\partial^{m-k} \delta(x_1, x_2, \ldots, x_n)}{\partial x_n^{m-k}} /; m \in \mathbb{N}^+ \bigwedge k \in \mathbb{N} \bigwedge k < m$$

Integration

Definite integration

Multiple integration

14.04.21.0001.01

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \ldots \int_{-\infty}^{\infty} \delta(t_1, t_2, \ldots, t_n) \, dt_1 \, dt_2 \ldots dt_n = 1$$

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$$\int_{-\infty}^{\infty} \int_{-\infty}^{d_2} \ldots \int_{-\infty}^{\infty} \delta(t_1 - a_1, t_2 - a_2, \ldots, t_n - a_n) \, f(t_1, t_2, \ldots, t_n) \, dt_1 \, dt_2 \ldots dt_n = f(a_1, a_2, \ldots, a_n) /;$$
$$-\infty \leq -d_k < a_k < d_k \leq \infty \bigwedge 1 \leq k \leq n$$

Integral transforms

Fourier exp transforms

14.04.22.0001.01

$$\mathcal{F}_t[\delta(x_1, x_2, \ldots, x_{n-1}, t)](z) = \frac{\delta(x_1, x_2, \ldots, x_{n-1})}{\sqrt{2\pi}}$$

Inverse Fourier exp transforms

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$$\mathcal{F}_t^{-1}[\delta(x_1, x_2, \ldots, x_{n-1}, t)](z) = \frac{\delta(x_1, x_2, \ldots, x_{n-1})}{\sqrt{2\pi}}$$

Fourier cos transforms

14.04.22.0003.01

$$\mathcal{F}c_t[\delta(x_1, x_2, \ldots, x_{n-1}, t)](z) = \sqrt{\frac{2}{\pi}} \, \delta(x_1, x_2, \ldots, x_{n-1})$$

Fourier sin transforms

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$$\mathcal{F}s_t[\delta(x_1, x_2, \ldots, x_{n-1}, t)](z) = 0$$

Laplace transforms

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$$\mathcal{L}_t[\delta(x_1, x_2, \ldots, x_{n-1}, t)](z) = \delta(x_1, x_2, \ldots, x_{n-1})$$

Representations through more general functions

Through Meijer *G*

Classical cases for the direct function itself

14.04.26.0001.01

$$\delta(x_1, x_2, \ldots, x_n) = \prod_{k=1}^{n} G_{0,0}^{0,0}\left(1 - x_k \middle| \right) /; x_k \neq x_j \bigwedge k \neq j \bigwedge x_k \in \mathbb{R} \bigwedge x_k < 2$$

14.04.26.0002.01

$$\delta(x_1, x_2, \ldots, x_n) = \prod_{k=1}^{n} G_{0,0}^{0,0}\left(x_k + 1 \middle| \right) /; x_k \neq x_j \bigwedge k \neq j \bigwedge x_k \in \mathbb{R} \bigwedge x_k > -2$$

Representations through equivalent functions

14.04.27.0001.01

$$\delta(x_1, x_2, \ldots, x_n) = \prod_{k=1}^{n} \delta(x_k) /; x_k \neq x_j \bigwedge k \neq j$$

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