

# DirectedInfinity

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## Notations

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### Traditional name

Directed infinity in the complex plane

### Traditional notation

$z \infty$

### Mathematica StandardForm notation

`DirectedInfinity[z]`

## Primary definition

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$z \infty$  represents an infinite numerical quantity that is a positive real multiple of the complex number  $z$ .

## Specific values

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### Values at fixed points

02.13.03.0001.01

$$0 \infty = \infty$$

02.13.03.0002.01

$$1 \infty = \infty$$

02.13.03.0003.01

$$-1 \infty = -\infty$$

02.13.03.0004.01

$$i \infty = i \infty$$

02.13.03.0005.01

$$-i \infty = -i \infty$$

02.13.03.0006.01

$$(1 + i) \infty = \frac{1 + i}{\sqrt{2}} \infty$$

### Values at infinities

02.13.03.0007.01

$$\infty \infty = \infty$$

02.13.03.0008.01

$$-\infty \infty = -\infty$$

02.13.03.0009.01

$$i \infty \infty = i \infty$$

02.13.03.0010.01

$$-i \infty \infty = -i \infty$$

02.13.03.0011.01

$$\tilde{\infty} \infty = \tilde{\infty}$$

02.13.03.0012.01

$$i \infty = \tilde{\infty}$$

## General characteristics

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$z \infty$  is a special symbol. On the Riemann sphere it is the north pole together with the direction  $z$  how to approach it. In the projective complex plane it is a point at the line at infinity.

## Transformations

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### Transformations and argument simplifications

02.13.16.0001.01

$$-z \infty = -\operatorname{sgn}(z) \infty$$

02.13.16.0002.01

$$a z \infty = z \infty ; a > 0$$

02.13.16.0003.01

$$a z \infty = \operatorname{sgn}(z) \infty ; a > 0$$

02.13.16.0004.01

$$a z \infty = -\operatorname{sgn}(z) \infty ; a < 0$$

02.13.16.0005.01

$$i z \infty = i \operatorname{sgn}(z) \infty$$

02.13.16.0006.01

$$-i z \infty = -i \operatorname{sgn}(z) \infty$$

### Argument involving complex components

02.13.16.0007.01

$$\frac{z}{|z|} \infty = z \infty$$

02.13.16.0008.01

$$\operatorname{sgn}(z) \infty = z \infty$$

### Power of arguments

02.13.16.0009.01

$$e^{ix} \infty = e^{ix} \infty ; x \in \mathbb{R}$$

## Products, sums, and powers of the direct function

### Products involving the direct function

02.13.16.0010.01

$$0(z^\infty) = i$$

02.13.16.0011.01

$$a(z^\infty) = z^\infty /; a > 0$$

02.13.16.0012.01

$$a(z^\infty) = -\operatorname{sgn}(z^\infty) /; a < 0$$

02.13.16.0013.01

$$\frac{z^\infty}{w^\infty} = i$$

02.13.16.0014.01

$$-(z^\infty) = -\operatorname{sgn}(z^\infty)$$

### Related transformations

02.13.16.0015.01

$$(z^\infty)^0 = i$$

02.13.16.0016.01

$$1^{z^\infty} = i$$

## Complex characteristics

### Real part

02.13.19.0001.01

$$\operatorname{Re}(z^\infty) = 0 /; \operatorname{Re}(z) = 0$$

02.13.19.0002.01

$$\operatorname{Re}(z^\infty) = (\operatorname{sgn}(\operatorname{Re}(z)^\infty)) /; \operatorname{Re}(z) \neq 0$$

### Imaginary part

02.13.19.0003.01

$$\operatorname{Im}(z^\infty) = 0 /; \operatorname{Im}(z) = 0$$

02.13.19.0004.01

$$\operatorname{Im}(z^\infty) = (\operatorname{sgn}(\operatorname{Im}(z)^\infty)) /; \operatorname{Im}(z) \neq 0$$

### Absolute value

02.13.19.0005.01

$$|z^\infty| = \infty$$

### Argument

02.13.19.0006.01

$$\arg(z^\infty) = \arg(z)$$

02.13.19.0007.01

$$\arg(e^{ix}) = x + 2\pi \left\lfloor \frac{\pi - x}{2\pi} \right\rfloor; x \in \mathbb{R}$$

## Conjugate value

02.13.19.0008.01

$$\overline{z^\infty} = \bar{z}^\infty$$

## Differentiation

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### Low-order differentiation

02.13.20.0001.01

$$\frac{\partial (a^\infty)}{\partial z} = 0$$

## Integration

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### Indefinite integration

02.13.21.0001.01

$$\int z^\infty dz = z^\infty$$

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