DirectedInfinity

Notations

Traditional name

Directed infinity in the complex plane

Traditional notation

$z\,\infty$

Mathematica StandardForm notation

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DirectedInfinity[z]
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Primary definition

$z\,\infty$ represents an infinite numerical quantity that is a positive real multiple of the complex number z.

Specific values

Values at fixed points

02.13.03.0001.01

$0\,\infty = \tilde{\infty}$

02.13.03.0002.01

$1\,\infty = \infty$

02.13.03.0003.01

$-1\,\infty = -\infty$

02.13.03.0004.01

$i\,\infty = i\,\infty$

02.13.03.0005.01

$-i\,\infty = -i\,\infty$

02.13.03.0006.01

$$(1+i)\,\infty = \frac{1+i}{\sqrt{2}}\,\infty$$

Values at infinities

02.13.03.0007.01

$\infty\,\infty = \infty$

02.13.03.0008.01

$$-\infty\,\infty = -\infty$$

02.13.03.0009.01

$$i\,\infty\,\infty = i\,\infty$$

02.13.03.0010.01

$$-i\,\infty\,\infty = -i\,\infty$$

02.13.03.0011.01

$$\tilde{\infty}\,\infty = \tilde{\infty}$$

02.13.03.0012.01

$$¿\,\infty = \tilde{\infty}$$

General characteristics

$z\,\infty$ is a special symbol. On the Riemann sphere it is the north pole together with the direction z how to approach it. In the projective complex plane it is a point at the line at infinity.

Transformations

Transformations and argument simplifications

02.13.16.0001.01

$$-z\,\infty = -\operatorname{sgn}(z)\,\infty$$

02.13.16.0002.01

$$a\,z\,\infty = z\,\infty\; /;\; a > 0$$

02.13.16.0003.01

$$a\,z\,\infty = \operatorname{sgn}(z)\,\infty\; /;\; a > 0$$

02.13.16.0004.01

$$a\,z\,\infty = -\operatorname{sgn}(z)\,\infty\; /;\; a < 0$$

02.13.16.0005.01

$$i\,z\,\infty = i\operatorname{sgn}(z)\,\infty$$

02.13.16.0006.01

$$-i\,z\,\infty = -i\operatorname{sgn}(z)\,\infty$$

Argument involving complex components

02.13.16.0007.01

$$\frac{z}{|z|}\,\infty = z\,\infty$$

02.13.16.0008.01

$$\operatorname{sgn}(z)\,\infty = z\,\infty$$

Power of arguments

02.13.16.0009.01

$$e^{i x}\,\infty = e^{i x}\,\infty\; /;\; x \in \mathbb{R}$$

Products, sums, and powers of the direct function

Products involving the direct function

02.13.16.0010.01

$0\,(z\,\infty) = ¿$

02.13.16.0011.01

$a\,(z\,\infty) = z\,\infty \;/;\; a > 0$

02.13.16.0012.01

$a\,(z\,\infty) = -\text{sgn}(z)\,\infty \;/;\; a < 0$

02.13.16.0013.01

$$\frac{z\,\infty}{w\,\infty} = ¿$$

02.13.16.0014.01

$-(z\,\infty) = -\text{sgn}(z)\,\infty$

Related transformations

02.13.16.0015.01

$(z\,\infty)^0 = ¿$

02.13.16.0016.01

$1^{z\,\infty} = ¿$

Complex characteristics

Real part

02.13.19.0001.01

$\text{Re}(z\,\infty) = 0 \;/;\; \text{Re}(z) = 0$

02.13.19.0002.01

$\text{Re}(z\,\infty) = (\text{sgn}(\text{Re}(z))\,\infty) \;/;\; \text{Re}(z) \neq 0$

Imaginary part

02.13.19.0003.01

$\text{Im}(z\,\infty) = 0 \;/;\; \text{Im}(z) = 0$

02.13.19.0004.01

$\text{Im}(z\,\infty) = (\text{sgn}(\text{Im}(z))\,\infty) \;/;\; \text{Im}(z) \neq 0$

Absolute value

02.13.19.0005.01

$|z\,\infty| = \infty$

Argument

02.13.19.0006.01

$\arg(z\,\infty) = \arg(z)$

02.13.19.0007.01

$$\arg\left(e^{i x} \infty\right) == x + 2 \pi \left\lfloor \frac{\pi - x}{2 \pi} \right\rfloor /; x \in \mathbb{R}$$

Conjugate value

02.13.19.0008.01

$$\overline{z \infty} == \bar{z} \infty$$

Differentiation

Low-order differentiation

02.13.20.0001.01

$$\frac{\partial (a \infty)}{\partial z} == 0$$

Integration

Indefinite integration

02.13.21.0001.01

$$\int z \infty \, dz == z \infty$$

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