DiscreteDelta2

Notations

Traditional name

Multivariate discrete delta function

Traditional notation

$\delta(n_1, n_2, \ldots)$

Mathematica StandardForm notation

`DiscreteDelta[`n_1`, `n_2`, …]`

Primary definition

04.19.02.0001.01

$\delta(n_1, n_2, \ldots) = 1 /; n_1 = n_2 = \ldots = 0$

04.19.02.0002.01

$\delta(n_1, n_2, \ldots) = 0 /; \neg\, n_1 = n_2 = \ldots = 0$

Discrete delta function equal to 1 if all its arguments are zero, and 0 otherwise.

Specific values

Values at fixed points

04.19.03.0001.01

$\delta(0) = 1$

04.19.03.0002.01

$\delta(1) = 0$

04.19.03.0003.01

$\delta(0, 0) = 1$

04.19.03.0004.01

$\delta(0, 1) = 0$

04.19.03.0005.01

$\delta(0, 0, 0) = 1$

04.19.03.0006.01

$\delta(0, 0, 1) = 0$

04.19.03.0007.01

$\delta(0, 1, 0) == 0$

04.19.03.0008.01

$\delta(1, 0, 0) == 0$

04.19.03.0009.01

$\delta(0, 0, 0, 0) == 1$

04.19.03.0010.01

$\delta(0, 0, 0, 1) == 0$

04.19.03.0011.01

$\delta(0, 0, 1, 0) == 0$

Values at infinities

04.19.03.0012.01

$\delta(\infty) == 0$

04.19.03.0013.01

$\delta(-\infty) == 0$

04.19.03.0014.01

$\delta(\infty, -\infty) == 0$

04.19.03.0015.01

$\delta(-\infty, \infty) == 0$

General characteristics

Domain and analyticity

$\delta(n_1, n_2, \ldots, n_m)$ is a nonanalytical function defined on $\mathbb{Q}^m$. Its possible values are 0 and 1. The value 1 is taken in case all arguments are 0.

04.19.04.0001.01

$(n_1 * n_2 * \ldots * n_m) \longrightarrow \delta(n_1, n_2, \ldots, n_m) :: \mathbb{Q}^m \longrightarrow \{0, 1\}$

Symmetries and periodicities

Parity

$\delta(n_1, n_2, \ldots, n_m)$ is an even function.

04.19.04.0002.01

$\delta(-n_1, -n_2, \ldots, -n_m) == \delta(n_1, n_2, \ldots, n_m)$

Permutation symmetry

04.19.04.0003.01

$\delta(m, n) == \delta(n, m)$

04.19.04.0004.01

$\delta(n_1, n_2, \ldots, n_k, \ldots, n_j, \ldots, n_m) == \delta(n_1, n_2, \ldots, n_j, \ldots, n_k, \ldots, n_m) /; n_k \neq n_j \bigwedge k \neq j$

Periodicity

No periodicity

Transformations

Transformations and argument simplifications

04.19.16.0001.01

$$\delta(-n_1, -n_2, \ldots, -n_m) = \delta(n_1, n_2, \ldots, n_m)$$

Products, sums, and powers of the direct function

04.19.16.0002.01

$$\delta(n_1, n_2, \ldots, n_m)\,\delta(n_{m+1}, n_{m+2}, \ldots, n_{m+r}) = \delta(n_1, n_2, \ldots, n_m, n_{m+1}, n_{m+2}, \ldots, n_{m+r})$$

Identities

Functional identities

04.19.17.0001.01

$$\delta(n_1, n_2, \ldots, n_m)\,\delta(n_{m+1}, n_{m+2}, \ldots, n_{m+r}) = \delta(n_1, n_2, \ldots, n_m, n_{m+1}, n_{m+2}, \ldots, n_{m+r})$$

Complex characteristics

Real part

04.19.19.0001.01

$$\operatorname{Re}(\delta(n_1, n_2, \ldots, n_m)) = \delta(n_1, n_2, \ldots, n_m)$$

Imaginary part

04.19.19.0002.01

$$\operatorname{Im}(\delta(n_1, n_2, \ldots, n_m)) = 0$$

Absolute value

04.19.19.0003.01

$$|\delta(n_1, n_2, \ldots, n_m)| = \delta(n_1, n_2, \ldots, n_m)$$

Argument

04.19.19.0004.01

$$\arg(\delta(n_1, n_2, \ldots, n_m)) = \tan^{-1}(\delta(n_1, n_2, \ldots, n_m), 0)$$

Conjugate value

04.19.19.0005.01

$$\overline{\delta(n_1, n_2, \ldots, n_m)} = \delta(n_1, n_2, \ldots, n_m)$$

Summation

Infinite summation

04.19.23.0001.01

$$\sum_{k=-\infty}^{\infty} \delta(k, n)\, a_k = a_0$$

Above relation represents the sifting property of discrete delta function.

Representations through equivalent functions

04.19.27.0001.01

$$\delta(n_1, n_2, \ldots, n_m) == \delta_{n_1, n_2, \ldots, n_m, 0}$$

History

– L. Kronecker (1866, 1903)

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