EulerE2

Notations

Traditional name

Euler polynomial

Traditional notation

$E_n(z)$

Mathematica StandardForm notation

`EulerE[n, z]`

Primary definition

05.13.02.0001.01

$$E_n(z) == 2\, n!\left([t^n]\, \frac{e^{z\,t}}{e^t+1}\right) /;\, n \in \mathbb{N}$$

Specific values

Specialized values

For fixed *n*

05.13.03.0001.01

$$E_n(0) == -\frac{2\left(2^{n+1}-1\right)}{n+1}\, B_{n+1}$$

05.13.03.0002.01

$$E_n(0) == -2^{1-n}\left(2^{n+1}-1\right)\pi^{-n-1}\, n!\, \sin\left(\frac{n\,\pi}{2}\right)\zeta(n+1) /;\, n \in \mathbb{N}^+$$

05.13.03.0003.01

$$E_{2\,n-1}\left(\frac{1}{3}\right) == -\frac{\left(1-3^{1-2\,n}\right)\left(2^{2\,n}-1\right)}{2\,n}\, B_{2\,n} /;\, n \in \mathbb{N}^+$$

05.13.03.0004.01

$$E_n\left(\frac{1}{2}\right) == 2^{-n}\, E_n$$

05.13.03.0005.01

$$E_{2n-1}\left(\frac{2}{3}\right) = \frac{\left(1-3^{1-2n}\right)\left(2^{2n}-1\right)}{2n} B_{2n} /; n \in \mathbb{N}^+$$

05.13.03.0006.01

$$E_n(1) = \frac{2\left(2^{n+1}-1\right)}{n+1} B_{n+1} /; n \in \mathbb{N}^+$$

05.13.03.0007.01

$$E_n\left(\frac{3}{2}\right) = 2^{1-n} - 2^{-n} E_n$$

05.13.03.0008.01

$$E_n\left(m+\frac{1}{2}\right) = 2^{-n}\left((-1)^m E_n + 2\sum_{k=0}^{m-1} (-1)^{-k+m-1} (2k+1)^n\right) /; m \in \mathbb{N}^+$$

05.13.03.0009.01

$$E_n\left(\frac{1}{2}-m\right) = 2^{-n}\left((-1)^m E_n - 2\sum_{k=0}^{m-1} (-1)^{k-1} (2k-2m+1)^n\right) /; m \in \mathbb{N}^+$$

05.13.03.0010.01

$$E_n\left(\frac{p}{q}\right) = \frac{4\,n!}{(2\pi q)^{n+1}} \sum_{k=1}^{q} \zeta\left(n+1, \frac{2k-1}{2q}\right) \sin\left(\frac{(2k-1)p\pi}{q} - \frac{\pi n}{2}\right) /; n-1 \in \mathbb{N}^+ \bigwedge p \in \mathbb{N} \bigwedge q \in \mathbb{N}^+ \bigwedge p \le q$$

For fixed z

05.13.03.0011.01

$$E_0(z) = 1$$

05.13.03.0012.01

$$E_1(z) = z - \frac{1}{2}$$

05.13.03.0013.01

$$E_2(z) = z^2 - z$$

05.13.03.0014.01

$$E_3(z) = z^3 - \frac{3z^2}{2} + \frac{1}{4}$$

05.13.03.0015.01

$$E_4(z) = z^4 - 2z^3 + z$$

05.13.03.0016.01

$$E_5(z) = z^5 - \frac{5z^4}{2} + \frac{5z^2}{2} - \frac{1}{2}$$

05.13.03.0017.01

$$E_6(z) = z^6 - 3z^5 + 5z^3 - 3z$$

05.13.03.0018.01

$$E_7(z) = z^7 - \frac{7z^6}{2} + \frac{35z^4}{4} - \frac{21z^2}{2} + \frac{17}{8}$$

05.13.03.0019.01

$$E_8(z) == z^8 - 4z^7 + 14z^5 - 28z^3 + 17z$$

05.13.03.0020.01

$$E_9(z) == z^9 - \frac{9z^8}{2} + 21z^6 - 63z^4 + \frac{153z^2}{2} - \frac{31}{2}$$

05.13.03.0021.01

$$E_{10}(z) == z^{10} - 5z^9 + 30z^7 - 126z^5 + 255z^3 - 155z$$

General characteristics

Domain and analyticity

$E_n(z)$ is a polynomial of z and as such an analytical function of z. $E_n(z)$ is defined in the whole complex z-plane and for $n \in \mathbb{N}$.

05.13.04.0001.01

$$(n * z) \longrightarrow E_n(z) :: (\mathbb{N} \otimes \mathbb{C}) \longrightarrow \mathbb{C}$$

Symmetries and periodicities

Mirror symmetry

05.13.04.0002.01

$$E_n(\bar{z}) == \overline{E_n(z)}$$

Periodicity

No periodicity

Poles and essential singularities

With respect to z

The function $E_n(z)$ has a pole of order n at $z == \tilde{\infty}$.

05.13.04.0003.01

$$\mathcal{S}ing_z(E_n(z)) == \{\{\tilde{\infty}, n\}\}$$

Branch points

With respect to z

The function $E_n(z)$ does not have branch points.

05.13.04.0004.01

$$\mathcal{BP}_z(E_n(z)) == \{\}$$

Branch cuts

With respect to z

The function $E_n(z)$ does not have branch cuts.

05.13.04.0005.01

$$\mathcal{BC}_z(E_n(z)) == \{\}$$

Series representations

Generalized power series

Expansions at generic point $z == z_0$

For the function itself

05.13.06.0010.01

$$E_n(z) \propto E_n(z_0) + n\,E_{n-1}(z_0)\,(z-z_0) + \frac{1}{2}\,(n-1)\,n\,E_{n-2}(z_0)\,(z-z_0)^2 + \ldots \;/;\; (z \to z_0)$$

05.13.06.0011.01

$$E_n(z) \propto E_n(z_0) + n\,E_{n-1}(z_0)\,(z-z_0) + \frac{1}{2}\,(n-1)\,n\,E_{n-2}(z_0)\,(z-z_0)^2 + O\big((z-z_0)^3\big)$$

05.13.06.0012.01

$$E_n(z) == \sum_{k=0}^{n} \frac{(n-k+1)_k\,E_{n-k}(z_0)}{k!}\,(z-z_0)^k$$

05.13.06.0013.01

$$E_n(z) \propto E_n(z_0)\,(1 + O\,(z-z_0))$$

Expansions at $z == 0$

For the function itself

05.13.06.0014.01

$$E_n(z) \propto \frac{\left(2-2^{n+2}\right)B_{n+1}}{n+1} + \left(2-2^{n+1}\right)B_n\,z + \frac{(2-2^n)\,n}{2}\,B_{n-1}\,z^2 + \ldots \;/;\; (z \to 0)$$

05.13.06.0004.01

$$E_n(z) \propto n!\sum_{k=0}^{n} \frac{\left(2-2^{n-k+2}\right)B_{n-k+1}\,z^k}{k!\,(n-k+1)!} \;/;\; (z \to 0)$$

05.13.06.0005.01

$$E_n(z) \propto -\frac{\left(2\left(2^{n+1}-1\right)B_{n+1}\right)}{n+1}\,(1 + O(z)) \;/;\; (z \to 0)$$

Exponential Fourier series

05.13.06.0001.01

$$E_n(x) == \frac{4\,n!}{\pi^{n+1}} \sum_{k=0}^{\infty} \frac{1}{(2\,k+1)^{n+1}} \sin\!\left((2\,k+1)\,\pi\,x - \frac{\pi\,n}{2}\right) \;/;\; -1 < x < 1 \bigwedge n \in \mathbb{N}^+$$

05.13.06.0002.01

$$E_{2n-1}(x) == \frac{(-1)^n\, 4\,(2n-1)!}{\pi^{2n}} \sum_{k=0}^{\infty} \frac{\cos((2k+1)\,\pi\,x)}{(2k+1)^{2n}} \;/;\; -1 \leq x \leq 1 \bigwedge n \in \mathbb{N}^+$$

05.13.06.0003.01

$$E_{2n}(x) == \frac{(-1)^n\, 4\,(2n)!}{\pi^{2n+1}} \sum_{k=0}^{\infty} \frac{\sin((2k+1)\,\pi\,x)}{(2k+1)^{2n+1}} \;/;\; -1 \leq x \leq 1 \bigwedge n \in \mathbb{N}^+$$

Asymptotic series expansions

05.13.06.0006.02

$$E_n(z) \propto n!\, z^n \sum_{k=0}^{n} \frac{\left(2-2^{k+2}\right) B_{k+1}\, z^{-k}}{(n-k)!\,(k+1)!} \;/;\; (|z| \to \infty)$$

05.13.06.0007.01

$$E_n(z) \propto z^n \left(1 + O\left(\frac{1}{z}\right)\right) /;\; (|z| \to \infty)$$

05.13.06.0008.01

$$E_n(z) \propto z^n \left(1 - \frac{n}{2z} + O\left(\frac{1}{z^3}\right)\right) /;\; (|z| \to \infty)$$

05.13.06.0015.01

$$E_n(z) \propto z^n \;/;\; (|z| \to \infty)$$

Other series representations

05.13.06.0009.01

$$E_n(z) == n! \sum_{k=0}^{n} \frac{2-2^{n-k+2}}{k!\,(n-k+1)!} B_{n-k+1}\, z^k$$

Integral representations

On the real axis

Of the direct function

05.13.07.0001.01

$$E_n(z) == (-1)^{\frac{3n}{2}}\, 2 \int_0^{\infty} \frac{\left(\sin(\pi\,(z-i\,t)) - (-1)^{n-1} \sin(\pi\,(i\,t+z))\right) t^n}{\cosh(2\,\pi\,t) - \cos(2\,\pi\,z)}\, dt \;/;\; 0 < \mathrm{Re}(z) < 1 \land n \in \mathbb{N}$$

05.13.07.0002.01

$$E_n(x) == \frac{4}{\pi^{n+1}} \int_0^1 \frac{\log^n\left(\frac{1}{t}\right)\left(t^2 \sin\left(\pi\left(\frac{n}{2}+x\right)\right) - \sin\left(\pi\left(\frac{n}{2}-x\right)\right)\right)}{t^4 - 2\cos(2\,\pi\,x)\, t^2 + 1}\, dt \;/;\; 0 < x < 1 \land n \in \mathbb{N}$$

05.13.07.0003.01

$$E_{2n}(x) == \frac{4\,(-1)^n \sin(\pi\,x)}{\pi^{2n+1}} \int_0^1 \frac{\log^{2n}(t)\left(t^2+1\right)}{t^4 - 2\cos(2\,\pi\,x)\, t^2 + 1}\, dt \;/;\; 0 \leq x \leq 1 \land n \in \mathbb{N}$$

05.13.07.0004.01

$$E_{2n+1}(x) = \frac{(-1)^{n+1}(2n+1)}{\pi^{2n+2}} \int_0^1 \frac{\log^{2n}(t)}{t} \log\left(\frac{t^2 + 2\cos(\pi x)\,t + 1}{t^2 - 2\cos(\pi x)\,t + 1}\right) dt \;/;\; 0 \le x \le 1 \land n \in \mathbb{N}$$

Generating functions

05.13.11.0001.01

$$E_n(z) = 2\,n!\left([t^n]\,\frac{e^{z t}}{e^t + 1}\right) /;\; n \in \mathbb{N}$$

Transformations

Transformations and argument simplifications

Argument involving basic arithmetic operations

05.13.16.0001.01

$$E_n(1+z) = 2z^n - E_n(z)$$

05.13.16.0002.01

$$E_n(1+z) = \sum_{k=0}^{n} \binom{n}{k} E_k(z)$$

05.13.16.0003.01

$$E_n(1-z) = (-1)^n E_n(z)$$

05.13.16.0004.01

$$E_n(-z) = (-1)^{n+1}\left(E_n(z) - 2z^n\right)$$

05.13.16.0005.01

$$E_n(z-1) = 2(z-1)^n - E_n(z)$$

05.13.16.0006.01

$$E_n(z+m) = (-1)^m E_n(z) + 2\sum_{k=0}^{m-1} (-1)^{m-k-1}(z+k)^n \;/;\; m \in \mathbb{N}^+$$

05.13.16.0007.01

$$E_n(z-m) = (-1)^m E_n(z) - 2\sum_{k=0}^{m-1} (-1)^{k-1}(z+k-m)^n \;/;\; m \in \mathbb{N}^+$$

Addition formulas

05.13.16.0008.01

$$E_n(z+w) = \sum_{k=0}^{n} \binom{n}{k} E_k(z)\, w^{n-k}$$

Multiple arguments

05.13.16.0009.01

$$E_n(m z) = m^n \sum_{k=0}^{m-1} (-1)^k E_n\left(z + \frac{k}{m}\right) /;\; \frac{m-1}{2} \in \mathbb{N}^+$$

05.13.16.0010.01

$$E_n(2\,z) = -\frac{2^{n+1}}{n+1}\left(B_{n+1}(z) - B_{n+1}\left(z + \frac{1}{2}\right)\right)$$

05.13.16.0011.01

$$E_n(m\,z) = -\frac{2\,m^n}{n+1}\sum_{k=0}^{m-1}(-1)^k\,B_{n+1}\left(\frac{k}{m} + z\right) /;\ \frac{m}{2} \in \mathbb{N}^+$$

Identities

Recurrence identities

Consecutive neighbors

05.13.17.0001.01

$$E_n(z) = 2\,z^n - E_n(z+1)$$

05.13.17.0002.01

$$E_n(z) = 2\,(z-1)^n - E_n(z-1)$$

Distant neighbors

05.13.17.0003.01

$$E_n(z) = (-1)^m\,E_n(z+m) - (-1)^m\,2\sum_{k=0}^{m-1}(-1)^{m-k-1}\,(k+z)^n /;\ m \in \mathbb{N}^+$$

05.13.17.0004.01

$$E_n(z) = (-1)^m\,E_n(z-m) + 2\,(-1)^m\sum_{k=0}^{m-1}(-1)^{k-1}\,(k-m+z)^n /;\ m \in \mathbb{N}^+$$

Functional identities

Relations of special kind

05.13.17.0005.01

$$E_n(z) + E_n(z+1) = 2\,z^n$$

05.13.17.0006.01

$$E_n(m) = (-1)^m\,E_n(0) - 2\sum_{k=1}^{m-1}(-1)^{m-k}\,k^n /;\ m \in \mathbb{N}$$

05.13.17.0007.01

$$E_n(z+1) = \sum_{k=0}^{n}\binom{n}{k}E_k(z)$$

Complex characteristics

Real part

05.13.19.0001.01

$$\operatorname{Re}(E_n(x+i\,y)) = \frac{1}{2}\left(E_n\left(x - x\sqrt{-\frac{y^2}{x^2}}\right) + E_n\left(x + x\sqrt{-\frac{y^2}{x^2}}\right)\right)$$

Imaginary part

05.13.19.0002.01

$$\operatorname{Im}(E_n(x+i\,y)) = \frac{x}{2\,y}\sqrt{-\frac{y^2}{x^2}}\left(E_n\left(x - x\sqrt{-\frac{y^2}{x^2}}\right) - E_n\left(x + x\sqrt{-\frac{y^2}{x^2}}\right)\right)$$

Absolute value

05.13.19.0003.01

$$|E_n(x+i\,y)| = \sqrt{E_n\left(x - x\sqrt{-\frac{y^2}{x^2}}\right) E_n\left(\sqrt{-\frac{y^2}{x^2}}\;x + x\right)}$$

Argument

05.13.19.0004.01

$$\arg(E_n(x+i\,y)) = \tan^{-1}\left(\frac{1}{2}\left(E_n\left(x - x\sqrt{-\frac{y^2}{x^2}}\right) + E_n\left(x + x\sqrt{-\frac{y^2}{x^2}}\right)\right), \frac{x}{2\,y}\sqrt{-\frac{y^2}{x^2}}\left(E_n\left(x - x\sqrt{-\frac{y^2}{x^2}}\right) - E_n\left(x + x\sqrt{-\frac{y^2}{x^2}}\right)\right)\right)$$

Conjugate value

05.13.19.0005.01

$$\overline{E_n(x+i\,y)} = \frac{1}{2}\left(E_n\left(x - x\sqrt{-\frac{y^2}{x^2}}\right) + E_n\left(x + x\sqrt{-\frac{y^2}{x^2}}\right)\right) - \frac{i\,x}{2\,y}\sqrt{-\frac{y^2}{x^2}}\left(E_n\left(x - x\sqrt{-\frac{y^2}{x^2}}\right) - E_n\left(x + x\sqrt{-\frac{y^2}{x^2}}\right)\right)$$

Signum value

05.13.19.0006.01

$$\operatorname{sgn}(E_n(x+i\,y)) = \left(\frac{i\,x\sqrt{-\frac{y^2}{x^2}}\left(E_n\left(x - x\sqrt{-\frac{y^2}{x^2}}\right) - E_n\left(\sqrt{-\frac{y^2}{x^2}}\;x + x\right)\right)}{y} + E_n\left(\sqrt{-\frac{y^2}{x^2}}\;x + x\right) + E_n\left(x - x\sqrt{-\frac{y^2}{x^2}}\right)\right) \Big/ \left(2\sqrt{E_n\left(x - x\sqrt{-\frac{y^2}{x^2}}\right) E_n\left(\sqrt{-\frac{y^2}{x^2}}\;x + x\right)}\right)$$

Differentiation

Low-order differentiation

05.13.20.0001.01

$$\frac{\partial E_n(z)}{\partial z} == n\, E_{n-1}(z)$$

05.13.20.0002.01

$$\frac{\partial^2 E_n(z)}{\partial z^2} == n\,(n-1)\, E_{n-2}(z)$$

Symbolic differentiation

05.13.20.0003.02

$$\frac{\partial^m E_n(z)}{\partial z^m} == (n-m+1)_m\, E_{n-m}(z) \,/;\, m \in \mathbb{N}$$

Fractional integro-differentiation

05.13.20.0004.01

$$\frac{\partial^\alpha E_n(z)}{\partial z^\alpha} == n!\sum_{k=0}^{n} \frac{\left(2-2^{n-k+2}\right) z^{k-\alpha}}{(n-k+1)!\,\Gamma(k-\alpha+1)}\, B_{n-k+1}$$

Integration

Indefinite integration

Involving only one direct function

05.13.21.0001.01

$$\int E_n(z)\, dz == \frac{E_{n+1}(z)}{n+1}$$

Definite integration

For the direct function itself

05.13.21.0002.01

$$\int_0^1 E_m(t)\, E_n(t)\, dt == \frac{(-1)^n\, 4\left(2^{m+n+2}-1\right) m!\, n!}{(m+n+2)!}\, B_{m+n+2}$$

Involving the direct function

05.13.21.0003.01

$$\int_0^1 E_n(t) \sec(\pi\, t)\, dt == (-1)^{\frac{n+1}{2}} \frac{\pi^{-n-1}\, n!}{4^n} \left(\zeta\left(n+1, \frac{1}{4}\right) - \zeta\left(n+1, \frac{3}{4}\right)\right) /;\, \frac{n+1}{2} \in \mathbb{N}^+$$

Integral transforms

Fourier exp transforms

05.13.22.0001.01

$$\mathcal{F}_t[E_n(t)](x) = \sqrt{2\pi}\; n! \sum_{k=0}^{n} \frac{\left(2-2^{n-k+2}\right)(-i)^k}{k!\,(n-k+1)!} B_{n-k+1}\, \delta^{(k)}(x)$$

Inverse Fourier exp transforms

05.13.22.0002.01

$$\mathcal{F}_t^{-1}[E_n(t)](x) = \sqrt{2\pi}\; n! \sum_{k=0}^{n} \frac{\left(2-2^{n-k+2}\right) i^k}{k!\,(n-k+1)!} B_{n-k+1}\, \delta^{(k)}(x)$$

Fourier cos transforms

05.13.22.0003.01

$$\mathcal{F}c_t[E_n(t)](x) = \sqrt{2\pi}\; n! \sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} \frac{(-1)^k \left(2-2^{n-2k+2}\right)}{(2k)!\,(n-2k+1)!} B_{n-2k+1}\, \delta^{(2k)}(x) - \sqrt{\frac{2}{\pi}}\; n! \sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} \frac{\left(2-2^{-2k+n+1}\right)(-1)^k}{(n-2k)!\, x^{2k+2}} B_{n-2k}$$

Fourier sin transforms

05.13.22.0004.01

$$\mathcal{F}s_t[E_n(t)](x) = \sqrt{\frac{2}{\pi}}\; n! \sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} \frac{(-1)^k \left(2-2^{n-2k+2}\right)}{(n-2k+1)!\, x^{2k+1}} B_{n-2k+1} - \sqrt{2\pi}\; n! \sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} \frac{(-1)^k \left(2-2^{n-2k+1}\right)}{(2k+1)!\,(n-2k)!} B_{n-2k}\, \delta^{(2k+1)}(x)$$

Laplace transforms

05.13.22.0005.01

$$\mathcal{L}_t[E_n(t)](z) = n! \sum_{k=0}^{n} \frac{\left(2-2^{n-k+2}\right) B_{n-k+1}\, z^{-k-1}}{(n-k+1)!} \;/;\; \mathrm{Re}(z) > 0$$

Summation

Finite summation

05.13.23.0001.01

$$\sum_{k=0}^{n} \binom{n}{k} E_k(z) = E_n(z+1)$$

05.13.23.0002.01

$$\sum_{k=0}^{n} \binom{n}{k} E_k(z)\, w^k = w^n E_n\!\left(z+\frac{1}{w}\right)$$

05.13.23.0003.01

$$\sum_{k=0}^{m} (-1)^k E_n\!\left(z+\frac{k}{m+1}\right) = (m+1)^{-n} E_n(m z + z) \;/;\; \frac{m}{2} \in \mathbb{N}^+$$

05.13.23.0004.01

$$\sum_{k=0}^{n} \binom{n}{k} E_k(z)\, E_{n-k}(w) = 2\, E_{n+1}(z+w) + 2\,(1-z-w)\, E_n(z+w) \;/;\; n \in \mathbb{N}$$

05.13.23.0005.01

$$\sum_{k=0}^{n} \binom{n}{k} B_{n-k} E_k(z) = 2^n B_n\left(\frac{z}{2}\right)$$

Infinite summation

05.13.23.0006.01

$$\sum_{n=0}^{\infty} \frac{w^n}{n!} E_n(z) = \frac{2 e^{z w}}{e^w + 1} /; |w| < \pi$$

Representations through more general functions

Through other functions

Involving Stirling numbers

05.13.26.0001.01

$$E_n(z) = 2 \sum_{m=0}^{n} (-1)^m (-z)_m \, {}_2F_1(1, z+1; -m+z+1; -1) \, \mathcal{S}_n^{(m)}$$

Involving zeta functions

05.13.26.0002.01

$$E_n(z) = 2 \left(2^{n+1} \zeta\left(-n, \frac{z}{2}\right) - \zeta(-n, z)\right) /; n \in \mathbb{N}$$

05.13.26.0003.01

$$E_n(z) = 2^{n+1} \left(\zeta\left(-n, \frac{z}{2}\right) - \zeta\left(-n, \frac{z+1}{2}\right)\right) /; n \in \mathbb{N}$$

Representations through equivalent functions

With related functions

05.13.27.0001.01

$$E_n(z) = \sum_{k=0}^{n} \binom{n}{k} 2^{-k} E_k \left(z - \frac{1}{2}\right)^{n-k}$$

05.13.27.0002.01

$$E_n(z) = \frac{2}{n+1} \left(B_{n+1}(z) - 2^{n+1} B_{n+1}\left(\frac{z}{2}\right)\right)$$

05.13.27.0003.01

$$E_n(z) = \frac{2^{n+1}}{n+1} \left(B_{n+1}\left(\frac{z+1}{2}\right) - B_{n+1}\left(\frac{z}{2}\right)\right)$$

05.13.27.0004.01

$$E_n(z) = n! \sum_{k=0}^{n} \frac{\left(2 - 2^{n-k+2}\right) z^k}{k! \, (n-k+1)!} B_{n-k+1}$$

05.13.27.0005.01

$$E_n(z) == \frac{4}{(n+1)(n+2)} \sum_{k=0}^{n} \binom{n+2}{k} \left(2^{n-k+2}-1\right) B_{n-k+2} B_k(z)$$

05.13.27.0006.01

$$E_n(z) == -\frac{2 j^n}{n+1} \sum_{k=0}^{j-1} (-1)^k B_{n+1}\left(\frac{k+z}{j}\right) /; 2 j \in \mathbb{N}^+$$

Inequalities

05.13.29.0001.01

$$0 < E_{2n}(x) < 4^{-n} \left|E_{2n}\right| /; n \in \mathbb{N}^+ \bigwedge 0 < x < \frac{1}{2}$$

05.13.29.0002.01

$$0 < (-1)^n E_{2n-1}(x) < \frac{4(2n-1)!}{\pi^{2n}} \left(1 + \frac{1}{2^{2n}-2}\right) /; n \in \mathbb{N}^+ \bigwedge 0 < x < \frac{1}{2}$$

Other identities

Congruence properties

05.13.32.0001.01

$$q^n \left(E_n\left(\frac{p}{q}\right) - (-1)^{p q} E_n(0)\right) \in \mathbb{Z} /; n \in \mathbb{N}^+ \land p \in \mathbb{Z} \land q \in \mathbb{Z} \land q \neq 0$$

Theorems

The Boole summation formula

$$f(1) \approx \frac{1}{2} \sum_{k=0}^{n} \frac{E_k(1)}{k!} \left(f^{(k)}(1) + f^{(k)}(0)\right).$$

History

– L. Euler
– H. F. Scherk (1825) suggested the name and calculated the first 30 numbers
– G. Chrystal (1889) introduced modern notations
– L. Saalschütz (1893) found the relation between Euler and Bernoulli numbers

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