GammaRegularized3

Notations

Traditional name

Generalized regularized incomplete gamma function

Traditional notation

$Q(a, z_1, z_2)$

Mathematica StandardForm notation

`GammaRegularized[a, z1, z2]`

Primary definition

06.09.02.0001.01

$$Q(a, z_1, z_2) = \frac{\Gamma(a, z_1, z_2)}{\Gamma(a)}$$

Specific values

Specialized values

06.09.03.0001.01

$$Q(-n, z_1, z_2) = 0 \;/;\; n \in \mathbb{N}$$

06.09.03.0002.01

$$Q(a, z_1, \infty) = Q(a, z_1)$$

06.09.03.0003.01

$$Q(a, 0, \infty) = 1 \;/;\; \mathrm{Re}(a) > 0$$

06.09.03.0004.01

$$Q(a, z_1, 0) = Q(a, z_1) - 1 \;/;\; \mathrm{Re}(a) > 0$$

06.09.03.0005.01

$$Q(a, 0, z_2) = 1 - Q(a, z_2) \;/;\; \mathrm{Re}(a) > 0$$

General characteristics

Domain and analyticity

$Q(a, z_1, z_2)$ is an analytical function of a, z_1, z_2 which is defined in $\mathbb{C}^3$. For fixed z_1, z_2, it is an entire function of a.

06.09.04.0001.01

$(a * z_1 * z_2) \longrightarrow Q(a, z_1, z_2) :: (\mathbb{C} \otimes \mathbb{C} \otimes \mathbb{C}) \longrightarrow \mathbb{C}$

Symmetries and periodicities

Mirror symmetry

06.09.04.0002.01

$Q(\overline{a}, \overline{z_1}, \overline{z_2}) = \overline{Q(a, z_1, z_2)} /; z_1 \notin (-\infty, 0) \bigwedge z_2 \notin (-\infty, 0)$

Permutation symmetry

06.09.04.0003.01

$Q(a, z_1, z_2) = -Q(a, z_2, z_1)$

Periodicity

No periodicity

Poles and essential singularities

With respect to z_k

For fixed a, the function $Q(a, z_1, z_2)$ has an essential singularity at $z_1 = \tilde{\infty}$ (for fixed z_2) and at $z_2 = \tilde{\infty}$ (for fixed z_1). At the same time, the points $z_k = \tilde{\infty} /; k = 1, 2$ are a branch points for generic a.

06.09.04.0004.01

$\mathcal{S}ing_{z_k}(Q(a, z_1, z_2)) = \{\{\tilde{\infty}, \infty\}\} /; k \in \{1, 2\}$

With respect to a

For fixed z_1, z_2, the function $Q(a, z_1, z_2)$ has only one singular point at $a = \tilde{\infty}$. It is an essential singular point.

06.09.04.0005.01

$\mathcal{S}ing_a(Q(a, z_1, z_2)) = \{\{\tilde{\infty}, \infty\}\}$

Branch points

With respect to z_k

The function $Q(a, z_1, z_2)$ has for fixed a, $z_1$or fixed a, z_2(with $a \notin \mathbb{N}^+$) two branch points with respect to $z_2$or z_1: $z_k = 0$, $z_k = \tilde{\infty}$, $k = 1, 2$. At the same time, the points $z_k = \tilde{\infty} /; k = 1, 2$ are an essential singularities.

06.09.04.0006.01

$\mathcal{BP}_{z_k}(Q(a, z_1, z_2)) = \{0, \tilde{\infty}\} /; k \in \{1, 2\} \bigwedge a \notin \mathbb{N}^+$

06.09.04.0007.01

$\mathcal{R}_{z_k}(Q(a, z_1, z_2), 0) = \log /; a \notin \mathbb{Q} \bigwedge k \in \{1, 2\}$

06.09.04.0008.01

$$\mathcal{R}_{z_k}\left(Q\left(\frac{p}{q}, z_1, z_2\right), 0\right) = q /; p \in \mathbb{Z} \bigwedge q - 1 \in \mathbb{N}^+ \bigwedge \gcd(p, q) = 1 \bigwedge k \in \{1, 2\}$$

06.09.04.0009.01

$\mathcal{R}_{z_k}(Q(a, z_1, z_2), \tilde{\infty}) = \log /; a \notin \mathbb{Q} \bigwedge k \in \{1, 2\}$

06.09.04.0010.01

$$\mathcal{R}_{z_k}\left(Q\left(\frac{p}{q}, z_1, z_2\right), \tilde{\infty}\right) == q\ /;\ p \in \mathbb{Z} \bigwedge q-1 \in \mathbb{N}^+ \bigwedge \gcd(p, q) == 1 \bigwedge k \in \{1, 2\}$$

With respect to a

For fixed z_1, z_2, the function $Q(a, z_1, z_2)$ does not have branch points.

06.09.04.0011.01

$$\mathcal{BP}_a(Q(a, z_1, z_2)) == \{\}$$

Branch cuts

With respect to z_2

For fixed z_1 and $a \notin \mathbb{N}^+$, the function $Q(a, z_1, z_2)$ is a single-valued function on the z_2-plane cut along the interval $(-\infty, 0)$, where it is continuous from above.

06.09.04.0012.01

$$\mathcal{BC}_{z_2}(Q(a, z_1, z_2)) == \{\{(-\infty, 0), -i\}\}$$

06.09.04.0013.01

$$\lim_{\epsilon \to +0} Q(a, z_1, x_2 + i\,\epsilon) == Q(a, z_1, x_2)\ /;\ x_2 < 0$$

06.09.04.0014.01

$$\lim_{\epsilon \to +0} Q(a, z_1, x_2 - i\,\epsilon) == Q(a, z_1, x_2) + \left(1 - e^{-2 i \pi a}\right) Q(a, x_2, 0)\ /;\ x_2 < 0$$

With respect to z_1

For fixed z_2 and $a \notin \mathbb{N}^+$, the function $Q(a, z_1, z_2)$ is a single-valued function on the z_1-plane cut along the interval $(-\infty, 0)$, where it is continuous from above.

06.09.04.0015.01

$$\mathcal{BC}_{z_1}(Q(a, z_1, z_2)) == \{\{(-\infty, 0), -i\}\}$$

06.09.04.0016.01

$$\lim_{\epsilon \to +0} Q(a, x_1 + i\,\epsilon, z_2) == Q(a, x_1, z_2)\ /;\ x_1 < 0$$

06.09.04.0017.01

$$\lim_{\epsilon \to +0} Q(a, x_1 - i\,\epsilon, z_2) == \left(1 - e^{-2 i \pi a}\right) Q(a, 0, x_1) + Q(a, x_1, z_2)\ /;\ x_1 < 0$$

With respect to a

For fixed z_1, z_2, the function $Q(a, z_1, z_2)$ does not have branch cuts.

06.09.04.0018.01

$$\mathcal{BC}_a(Q(a, z_1, z_2)) == \{\}$$

Series representations

Generalized power series

Expansions of $Q(\epsilon - n, z_1, z_2)$ at $\epsilon == 0$

For the function itself

06.09.06.0006.01

$$Q(\epsilon - n, z_1, z_2) \propto (-1)^n n!\,\Gamma(-n, z_1, z_2)\,\epsilon + (-1)^n n!\left(\Gamma(-n, z_1)\log(z_1) - \Gamma(-n, z_2)\log(z_2) + G_{2,3}^{3,0}\!\left(z_1 \,\middle|\, \begin{matrix}1, 1\\ 0, 0, -n\end{matrix}\right) - G_{2,3}^{3,0}\!\left(z_2 \,\middle|\, \begin{matrix}1, 1\\ 0, 0, -n\end{matrix}\right) - \Gamma(-n, z_1, z_2)\,\psi(n+1)\right)\epsilon^2 + \frac{(-1)^n n!}{6}\left(3\,\Gamma(-n, z_1)\log^2(z_1) - 3\,\Gamma(-n, z_2)\log^2(z_2) + 6\,(-H_n + \log(z_1) + \gamma)\,G_{2,3}^{3,0}\!\left(z_1 \,\middle|\, \begin{matrix}1, 1\\ 0, 0, -n\end{matrix}\right) - 6\,(-H_n + \log(z_2) + \gamma)\,G_{2,3}^{3,0}\!\left(z_2 \,\middle|\, \begin{matrix}1, 1\\ 0, 0, -n\end{matrix}\right) + 6\,G_{3,4}^{4,0}\!\left(z_1 \,\middle|\, \begin{matrix}1, 1, 1\\ 0, 0, 0, -n\end{matrix}\right) - 6\,G_{3,4}^{4,0}\!\left(z_2 \,\middle|\, \begin{matrix}1, 1, 1\\ 0, 0, 0, -n\end{matrix}\right) + (6\,\Gamma(-n, z_2)\log(z_2) - 6\,\Gamma(-n, z_1)\log(z_1))\,\psi(n+1) + \Gamma(-n, z_1, z_2)\left(3\,\psi(n+1)^2 - \pi^2 + 3\,\psi^{(1)}(n+1)\right)\right)\epsilon^3 + O\!\left(\epsilon^4\right) /;\, n \in \mathbb{N}$$

06.09.06.0007.01

$$Q(\epsilon - n, z_1, z_2) \propto \epsilon\,(-1)^n \sum_{k=0}^{\infty}\left(\sum_{j=0}^{k}\sum_{i=0}^{j} a_{k-j}\, b_{j-i,n}\, c_{i,n}\right)\epsilon^k /;$$

$$a_{2k} = \frac{(-1)^k \pi^{2k}}{(2k+1)!} \bigwedge a_{2k+1} = 0 \bigwedge b_{k,n} = \frac{(-1)^k}{k!}\,\Gamma^{(k)}(n+1) \bigwedge c_{k,n} = \frac{1}{k!}\,\Gamma^{(k,0,0)}(-n, z_1, z_2) \bigwedge k \in \mathbb{N} \bigwedge n \in \mathbb{N}$$

06.09.06.0008.01

$$Q(\epsilon - n, z_1, z_2) \propto (-1)^n n!\,\Gamma(-n, z_1, z_2)\,\epsilon\,(1 + O(\epsilon)) /;\, n \in \mathbb{N}$$

Expansions at $\{z_1, z_2\} == \{0, 0\}$

For the function itself

General case

06.09.06.0001.02

$$Q(a, z_1, z_2) \propto z_2^a\left(\frac{1}{\Gamma(a+1)} - \frac{a\,z_2}{\Gamma(a+2)} + \frac{a\,(a+1)\,z_2^2}{2\,\Gamma(a+3)} - \ldots\right) - z_1^a\left(\frac{1}{\Gamma(a+1)} - \frac{a\,z_1}{\Gamma(a+2)} + \frac{a\,(a+1)\,z_1^2}{2\,\Gamma(a+3)} - \ldots\right) /;\, (z_1 \to 0) \bigwedge (z_2 \to 0)$$

06.09.06.0009.01

$$Q(a, z_1, z_2) \propto z_2^a\left(\frac{1}{\Gamma(a+1)} - \frac{a\,z_2}{\Gamma(a+2)} + \frac{a\,(a+1)\,z_2^2}{2\,\Gamma(a+3)} + O\!\left(z_2^3\right)\right) - z_1^a\left(\frac{1}{\Gamma(a+1)} - \frac{a\,z_1}{\Gamma(a+2)} + \frac{a\,(a+1)\,z_1^2}{2\,\Gamma(a+3)} + O\!\left(z_1^3\right)\right)$$

06.09.06.0002.02

$$Q(a, z_1, z_2) == z_2^a \sum_{k=0}^{\infty} \frac{(a)_k\,(-z_2)^k}{\Gamma(a+k+1)\,k!} - z_1^a \sum_{k=0}^{\infty} \frac{(a)_k\,(-z_1)^k}{\Gamma(a+k+1)\,k!}$$

06.09.06.0003.02

$$Q(a, z_1, z_2) == z_2^a\, {}_1\tilde{F}_1(a; a+1; -z_2) - z_1^a\, {}_1\tilde{F}_1(a; a+1; -z_1)$$

06.09.06.0004.02

$$Q(a, z_1, z_2) \propto \frac{1}{\Gamma(a+1)}\, z_2^a\,(1 + O(z_2)) - \frac{1}{\Gamma(a+1)}\, z_1^a\,(1 + O(z_1)) /;\, (z_1 \to 0) \bigwedge (z_2 \to 0)$$

Special cases

06.09.06.0010.01

$$Q(1, z_1, z_2) \propto -z_1 + O(z_1^3) + z_2 + O(z_1^3)$$

06.09.06.0011.01

$$Q(n, z_1, z_2) \propto \frac{z_2^n - z_1^n}{n!} + O(z_1^{n+1}) + O(z_2^{n+1}) /; n \in \mathbb{N}^+$$

06.09.06.0005.01

$$Q(n, z_1, z_2) = e^{-z_1} \sum_{k=0}^{n-1} \frac{z_1^k}{k!} - e^{-z_2} \sum_{k=0}^{n-1} \frac{z_2^k}{k!} /; n \in \mathbb{N}^+$$

Integral representations

On the real axis

Of the direct function

06.09.07.0001.01

$$Q(a, z_1, z_2) = \frac{1}{\Gamma(a)} \int_{z_1}^{z_2} t^{a-1} e^{-t} \, dt$$

Differential equations

Ordinary linear differential equations and wronskians

For the direct function itself

With respect to z_1

06.09.13.0001.01

$$z_1 w''(z_1) + (1 - a + z_1) w'(z_1) = 0 /; w(z_1) = c_1 Q(a, z_1, z_2) + c_2$$

06.09.13.0003.01

$$W_{z_1}(1, Q(a, z_1, z_2)) = -\frac{e^{-z_1} z_1^{a-1}}{\Gamma(a)}$$

With respect to z_2

06.09.13.0002.01

$$z_2 w''(z_2) + (1 - a + z_2) w'(z_2) = 0 /; w(z_2) = c_1 Q(a, z_1, z_2) + c_2$$

06.09.13.0004.01

$$W_{z_2}(1, Q(a, z_1, z_2)) = \frac{e^{-z_2} z_2^{a-1}}{\Gamma(a)}$$

Transformations

Transformations and argument simplifications

Argument involving basic arithmetic operations

06.09.16.0001.01

$$Q(a+1, z_1, z_2) = Q(a, z_1, z_2) - \frac{e^{-z_2} z_2^a - e^{-z_1} z_1^a}{\Gamma(a+1)}$$

06.09.16.0002.01

$$Q(a-1, z_1, z_2) = Q(a, z_1, z_2) - \frac{z_1^{a-1} e^{-z_1} - z_2^{a-1} e^{-z_2}}{\Gamma(a)}$$

06.09.16.0003.01

$$Q(a+n, z_1, z_2) = Q(a, z_1, z_2) + \frac{1}{\Gamma(a)} \left(e^{-z_1} \sum_{k=1}^{n} \frac{z_1^{a+k-1}}{(a)_k} - e^{-z_2} \sum_{k=1}^{n} \frac{z_2^{a+k-1}}{(a)_k} \right) /; n \in \mathbb{N}$$

06.09.16.0004.01

$$Q(a-n, z_1, z_2) = Q(a, z_1, z_2) - \frac{1}{\Gamma(a-n)} \left(e^{-z_1} \sum_{k=1}^{n} \frac{z_1^{a+k-n-1}}{(a-n)_k} - e^{-z_2} \sum_{k=1}^{n} \frac{z_2^{a+k-n-1}}{(a-n)_k} \right) /; n \in \mathbb{N}$$

Identities

Recurrence identities

Consecutive neighbors

06.09.17.0001.01

$$Q(a, z_1, z_2) = Q(a+1, z_1, z_2) + \frac{e^{-z_2} z_2^a - e^{-z_1} z_1^a}{\Gamma(a+1)}$$

06.09.17.0002.01

$$Q(a, z_1, z_2) = Q(a-1, z_1, z_2) + \frac{z_1^{a-1} e^{-z_1} - z_2^{a-1} e^{-z_2}}{\Gamma(a)}$$

Distant neighbors

Functional identities

06.09.17.0003.02

$$Q(a, z_1, z_2) = Q(a+n, z_1, z_2) - e^{-z_1} \sum_{k=1}^{n} \frac{z_1^{a+k-1}}{\Gamma(a+k)} + e^{-z_2} \sum_{k=1}^{n} \frac{z_2^{a+k-1}}{\Gamma(a+k)} /; n \in \mathbb{N}$$

06.09.17.0004.02

$$Q(a, z_1, z_2) = Q(a-n, z_1, z_2) + e^{-z_1} \sum_{k=0}^{n-1} \frac{z_1^{a-k-1}}{\Gamma(a-k)} - e^{-z_2} \sum_{k=0}^{n-1} \frac{z_2^{a-k-1}}{\Gamma(a-k)} /; n \in \mathbb{N}$$

Differentiation

Low-order differentiation

With respect to a

06.09.20.0001.01

$$\frac{\partial Q(a, z_1, z_2)}{\partial a} = \Gamma(a)\, z_1^a\, {}_2\tilde{F}_2(a, a; a+1, a+1; -z_1) - \Gamma(a)\, z_2^a\, {}_2\tilde{F}_2(a, a; a+1, a+1; -z_2) + Q(a, z_1, 0)\log(z_1) - Q(a, z_2, 0)\log(z_2) - \psi(a)\, Q(a, z_1, z_2)$$

06.09.20.0002.01

$$\frac{\partial^2 Q(a, z_1, z_2)}{\partial a^2} = -2\,\Gamma(a)\, z_1^a \left(\Gamma(a)\, {}_3\tilde{F}_3(a, a, a; a+1, a+1, a+1; -z_1) + (\psi(a) - \log(z_1))\, {}_2\tilde{F}_2(a, a; a+1, a+1; -z_1)\right) + 2\,\Gamma(a)\, z_2^a \left(\Gamma(a)\, {}_3\tilde{F}_3(a, a, a; a+1, a+1, a+1; -z_2) + (\psi(a) - \log(z_2))\, {}_2\tilde{F}_2(a, a; a+1, a+1; -z_2)\right) + Q(a, z_1, 0)\left(\log^2(z_1) - 2\,\psi(a)\log(z_1) + \psi(a)^2 - \psi^{(1)}(a)\right) - Q(a, z_2, 0)\left(\log^2(z_2) - 2\,\psi(a)\log(z_2) + \psi(a)^2 - \psi^{(1)}(a)\right)$$

With respect to z_1

06.09.20.0003.01

$$\frac{\partial Q(a, z_1, z_2)}{\partial z_1} = -\frac{e^{-z_1}\, z_1^{a-1}}{\Gamma(a)}$$

06.09.20.0004.01

$$\frac{\partial^2 Q(a, z_1, z_2)}{\partial z_1^2} = \frac{z_1 - a + 1}{\Gamma(a)}\, e^{-z_1}\, z_1^{a-2}$$

With respect to z_2

06.09.20.0005.01

$$\frac{\partial Q(a, z_1, z_2)}{\partial z_2} = \frac{e^{-z_2}\, z_2^{a-1}}{\Gamma(a)}$$

06.09.20.0006.01

$$\frac{\partial^2 Q(a, z_1, z_2)}{\partial z_2^2} = \frac{a - z_2 - 1}{\Gamma(a)}\, e^{-z_2}\, z_2^{a-2}$$

Symbolic differentiation

With respect to a

06.09.20.0007.02

$$\frac{\partial^n Q(a, z_1, z_2)}{\partial a^n} = \frac{n!}{\Gamma(a)} \sum_{k=0}^{\infty} \frac{(-1)^{n-k-1}}{(a+k)^{n+1}\, k!}\, Q(n+1, -(a+k)\log(z_1), -(a+k)\log(z_2)) \;/;\; n \in \mathbb{N}$$

06.09.20.0008.02

$$\frac{\partial^n Q(a, z_1, z_2)}{\partial a^n} = n! \sum_{k=0}^{n} \left(z_2^a \sum_{i=0}^{n-k} (-1)^{n-i-k} \binom{n-k}{i} (n-i-k)!\, \Gamma(a)^{n-i-k+1} \log^i(z_2)\, {}_{n-k-i+1}\tilde{F}_{n-k-i+1}(a_1, a_2, \ldots, a_{n-k-i+1}; a_1+1, a_2+1, \ldots, a_{n-k-i+1}+1; -z_2) - z_1^a \sum_{i=0}^{n-k} (-1)^{n-i-k} \binom{n-k}{i} (n-i-k)!\, \Gamma(a)^{n-i-k+1} \log^i(z_1)\, {}_{n-k-i+1}\tilde{F}_{n-k-i+1}(a_1, a_2, \ldots, a_{n-k-i+1}; a_1+1, a_2+1, \ldots, a_{n-k-i+1}+1; -z_1) \right) \sum_{j=0}^{k} \frac{(-1)^j (k+1)\, \Gamma(a)^{-j-1}}{(j+1)!\,(n-k)!\,(k-j)!} \frac{\partial^k \Gamma(a)^j}{\partial a^k} \,/;\, a_1 == a_2 == \ldots == a_{n+1} == a \land n \in \mathbb{N}$$

06.09.20.0016.01

$$Q^{(n,0,0)}(-m, z_1, z_2) = (-1)^m\, n! \sum_{j=0}^{n-1} \sum_{i=0}^{j} a_{n-j-1}\, b_{j-i,m}\, c_{i,m} \,/;\, a_{2k} = \frac{(-1)^k \pi^{2k}}{(2k+1)!} \bigwedge a_{2k+1} = 0 \bigwedge b_{k,m} = \frac{(-1)^k}{k!} \Gamma^{(k)}(m+1) \bigwedge c_{k,m} = \frac{1}{k!} \Gamma^{(k,0,0)}(-m, z_1, z_2) \bigwedge k \in \mathbb{N} \bigwedge m \in \mathbb{N} \bigwedge n \in \mathbb{N}^+$$

With respect to z_1

06.09.20.0017.01

$$\frac{\partial^n Q(a, z_1, z_2)}{\partial z_1^n} == Q(a, z_1, z_2)\, \delta_n + e^{-z_1} \sum_{k=0}^{n-1} \frac{(-1)^{n-k} (n-1)!\, z_1^{a-k-1}}{\Gamma(a-k)\, k!\, (n-k-1)!} \,/;\, n \in \mathbb{N}$$

06.09.20.0009.02

$$\frac{\partial^n Q(a, z_1, z_2)}{\partial z_1^n} == -Q(a, z_2)\, \delta_n - a\, z_1^{-n} \sum_{k=0}^{n} (-1)^{n-k} \binom{n}{k} (-a-k+1)_{n-1}\, Q(a+k, z_1) \,/;\, n \in \mathbb{N}$$

With respect to z_2

06.09.20.0018.01

$$\frac{\partial^n Q(a, z_1, z_2)}{\partial z_2^n} == Q(a, z_1, z_2)\, \delta_n - e^{-z_2} \sum_{k=0}^{n-1} \frac{(-1)^{n-k} (n-1)!\, z_2^{a-k-1}}{\Gamma(a-k)\, k!\, (n-k-1)!} \,/;\, n \in \mathbb{N}$$

06.09.20.0010.02

$$\frac{\partial^n Q(a, z_1, z_2)}{\partial z_2^n} == Q(a, z_1)\, \delta_n + a\, z_2^{-n} \sum_{k=0}^{n} (-1)^{n-k} \binom{n}{k} (-a-k+1)_{n-1}\, Q(a+k, z_2) \,/;\, n \in \mathbb{N}$$

Fractional integro-differentiation

With respect to a

06.09.20.0011.01

$$\frac{\partial^\alpha Q(a, z_1, z_2)}{\partial a^\alpha} == \frac{a^{-\alpha}}{\Gamma(a)} \int_{z_1}^{z_2} t^{a-1} (a \log(t))^\alpha e^{-t}\, Q(-\alpha, 0, a \log(t))\, dt$$

With respect to z_1

06.09.20.0012.01

$$\frac{\partial^\alpha Q(a, z_1, z_2)}{\partial z_1^\alpha} = \frac{z_1^{-\alpha}}{\Gamma(1-\alpha)} (1 - Q(a, z_2)) - z_1^{a-\alpha} \, {}_1\tilde{F}_1(a; a-\alpha+1; -z_1) \,/;\, -a \notin \mathbb{N}^+$$

06.09.20.0013.01

$$\frac{\partial^\alpha Q(a, z_1, z_2)}{\partial z_1^\alpha} = \frac{z_1^{-\alpha}}{\Gamma(1-\alpha)} (1 - Q(a, z_2)) - \frac{1}{\Gamma(a)} \sum_{k=0}^{\infty} \frac{(-1)^k \mathcal{FC}_{\exp}^{(\alpha)}(z_1, a+k) \, z_1^{a+k-\alpha}}{(a+k) \, k!}$$

With respect to z_2

06.09.20.0014.01

$$\frac{\partial^\alpha Q(a, z_1, z_2)}{\partial z_2^\alpha} = z_2^{a-\alpha} \, {}_1\tilde{F}_1(a; a-\alpha+1; -z_2) + \frac{z_2^{-\alpha}}{\Gamma(1-\alpha)} (Q(a, z_1) - 1) \,/;\, -a \notin \mathbb{N}^+$$

06.09.20.0015.01

$$\frac{\partial^\alpha Q(a, z_1, z_2)}{\partial z_2^\alpha} = \frac{z_2^{-\alpha}}{\Gamma(1-\alpha)} (Q(a, z_1) - 1) + \frac{1}{\Gamma(a)} \sum_{k=0}^{\infty} \frac{(-1)^k \mathcal{FC}_{\exp}^{(\alpha)}(z_2, a+k) \, z_2^{a+k-\alpha}}{(a+k) \, k!}$$

Integration

Indefinite integration

Involving only one direct function with respect to a

06.09.21.0001.01

$$\int Q(a, z_1, z_2) \, da = \frac{1}{\Gamma(a)} \int_{z_1}^{z_2} \frac{t^{a-1} e^{-t}}{\log(t)} \, dt$$

Involving only one direct function with respect to z_1

06.09.21.0002.01

$$\int Q(a, z_1, z_2) \, dz_1 = z_1 \, Q(a, z_1, z_2) - a \, Q(a+1, z_1)$$

Involving one direct function and elementary functions with respect to z_1

Involving power function

06.09.21.0003.01

$$\int z_1^{\alpha-1} \, Q(a, z_1, z_2) \, dz_1 = \frac{1}{\alpha} \left(z_1^\alpha \, Q(a, z_1, z_2) - \frac{\Gamma(a+\alpha) \, Q(a+\alpha, z_1)}{\Gamma(a)} \right)$$

Involving only one direct function with respect to z_2

06.09.21.0004.01

$$\int \Gamma(a, z_1, z_2) \, dz_2 = \Gamma(a+1, z_2) + z_2 \, \Gamma(a, z_1, z_2)$$

Involving one direct function and elementary functions with respect to z_2

Involving power function

06.09.21.0005.01

$$\int z_2^{\alpha-1} Q(a, z_1, z_2)\, dz_2 = \frac{1}{\alpha}\left(Q(a, z_1, z_2)\, z_2^{\alpha} + \frac{\Gamma(a+\alpha)\, Q(a+\alpha, z_2)}{\Gamma(a)}\right)$$

Representations through more general functions

Through hypergeometric functions

Involving ${}_1\tilde{F}_1$

06.09.26.0001.01

$$Q(a, z_1, z_2) = z_2^a\, {}_1\tilde{F}_1(a; a+1; -z_2) - z_1^a\, {}_1\tilde{F}_1(a; a+1; -z_1)$$

Involving ${}_1F_1$

06.09.26.0002.01

$$Q(a, z_1, z_2) = \frac{z_2^a}{\Gamma(a+1)}\, {}_1F_1(a; a+1; -z_2) - \frac{z_1^a}{\Gamma(a+1)}\, {}_1F_1(a; a+1; -z_1) \;/;\; -a \notin \mathbb{N}$$

Involving hypergeometric U

06.09.26.0003.01

$$Q(a, z_1, z_2) = \frac{1}{\Gamma(a)}\left(e^{-z_1}\, U(1-a, 1-a, z_1) - e^{-z_2}\, U(1-a, 1-a, z_2)\right)$$

Through Meijer G

Classical cases for the direct function itself

06.09.26.0004.01

$$Q(a, z_1, z_2) = \frac{1}{\Gamma(a)}\left(G_{1,2}^{1,1}\left(z_2 \,\middle|\, \begin{matrix} 1 \\ a, 0 \end{matrix}\right) - G_{1,2}^{1,1}\left(z_1 \,\middle|\, \begin{matrix} 1 \\ a, 0 \end{matrix}\right)\right)$$

06.09.26.0005.01

$$Q(a, z_1, z_2) = \frac{1}{\Gamma(a)}\left(G_{1,2}^{2,0}\left(z_1 \,\middle|\, \begin{matrix} 1 \\ 0, a \end{matrix}\right) - G_{1,2}^{2,0}\left(z_2 \,\middle|\, \begin{matrix} 1 \\ 0, a \end{matrix}\right)\right)$$

06.09.26.0006.01

$$Q(a, 0, z) = \frac{1}{\Gamma(a)}\, G_{1,2}^{1,1}\left(z \,\middle|\, \begin{matrix} 1 \\ a, 0 \end{matrix}\right)$$

06.09.26.0007.01

$$Q\left(a, 0, \sqrt{z}\right) = \frac{2^{a-1}}{\sqrt{\pi}\, \Gamma(a)}\, G_{1,3}^{2,1}\left(\frac{z}{4} \,\middle|\, \begin{matrix} 1 \\ \frac{a}{2}, \frac{a+1}{2}, 0 \end{matrix}\right)$$

Classical cases involving exp

06.09.26.0008.01

$$e^z\, Q(a, 0, z) = -\pi \csc(\pi a)\, G_{2,3}^{1,1}\left(z \,\middle|\, \begin{matrix} a, 0 \\ a, 0, 0 \end{matrix}\right)$$

06.09.26.0009.01

$$e^z\, Q(a, 0, z) = z^a\, G_{1,2}^{1,1}\left(-z \,\middle|\, \begin{matrix} 0 \\ 0, -a \end{matrix}\right)$$

Generalized cases for the direct function itself

06.09.26.0010.01

$$Q(a, 0, z) = \frac{2^{a-1}}{\sqrt{\pi}\,\Gamma(a)} G_{1,3}^{2,1}\left(\frac{z}{2}, \frac{1}{2} \middle| \begin{matrix} 1 \\ \frac{a}{2}, \frac{a+1}{2}, 0 \end{matrix}\right)$$

Representations through equivalent functions

With inverse function

06.09.27.0001.01

$$Q\left(a, z_1, Q^{-1}(a, z_1, z_2)\right) = z_2$$

With related functions

06.09.27.0002.01

$$Q(a, z_1, z_2) = Q(a, z_1) - Q(a, z_2)$$

06.09.27.0003.01

$$Q(a, z_1, z_2) = \frac{\Gamma(a, z_1, z_2)}{\Gamma(a)}$$

06.09.27.0004.01

$$Q(a, z_1, z_2) = \frac{1}{\Gamma(a)} \left(z_1^a E_{1-a}(z_1) - z_2^a E_{1-a}(z_2)\right)$$

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