GegenbauerC

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Notations

Traditional name

Renormalized Gegenbauer function

Traditional notation

$C_\nu^{(0)}(z)$

Mathematica StandardForm notation

`GegenbauerC[ν, z]`

Primary definition

07.13.02.0001.01

$$C_\nu^{(0)}(z) == \frac{2}{\nu} T_\nu(z) == \frac{2\cos\left(\nu \cos^{-1}(z)\right)}{\nu}$$

Specific values

Specialized values

For fixed ν

07.13.03.0001.01

$$C_\nu^{(0)}(0) == \frac{2}{\nu} \cos\left(\frac{\pi \nu}{2}\right)$$

07.13.03.0002.01

$$C_\nu^{(0)}(1) == \frac{2}{\nu}$$

07.13.03.0003.01

$$C_\nu^{(0)}(-1) == \frac{2}{\nu} \cos(\pi \nu)$$

For fixed z

07.13.03.0004.01

$$C_{-\frac{1}{2}}^{(0)}(z) == -4 \sqrt{\frac{z+1}{2}}$$

07.13.03.0005.01

$$C_{\frac{1}{2}}^{(0)}(z) = 4\sqrt{\frac{z+1}{2}}$$

07.13.03.0006.01

$$C_0^{(0)}(z) = \tilde{\infty}$$

07.13.03.0007.01

$$C_1^{(0)}(z) = 2z$$

07.13.03.0008.01

$$C_2^{(0)}(z) = 2z^2 - 1$$

07.13.03.0009.01

$$C_3^{(0)}(z) = \frac{2}{3}\left(4z^3 - 3z\right)$$

07.13.03.0010.01

$$C_4^{(0)}(z) = 8z^4 - 8z^2 + 1$$

07.13.03.0011.01

$$C_5^{(0)}(z) = \frac{2}{5}\left(16z^5 - 20z^3 + 5z\right)$$

07.13.03.0012.01

$$C_6^{(0)}(z) = \frac{1}{3}\left(32z^6 - 48z^4 + 18z^2 - 1\right)$$

07.13.03.0013.01

$$C_7^{(0)}(z) = \frac{2}{7}\left(64z^7 - 112z^5 + 56z^3 - 7z\right)$$

07.13.03.0014.01

$$C_8^{(0)}(z) = \frac{1}{4}\left(128z^8 - 256z^6 + 160z^4 - 32z^2 + 1\right)$$

07.13.03.0015.01

$$C_9^{(0)}(z) = \frac{2}{9}\left(256z^9 - 576z^7 + 432z^5 - 120z^3 + 9z\right)$$

07.13.03.0016.01

$$C_{10}^{(0)}(z) = \frac{1}{5}\left(512z^{10} - 1280z^8 + 1120z^6 - 400z^4 + 50z^2 - 1\right)$$

07.13.03.0017.01

$$C_n^{(0)}(z) = \frac{2^n}{n}z^n + \frac{\delta_{n,0}}{n} + \sum_{k=1}^{\left\lfloor \frac{n}{2} \right\rfloor} \frac{(-1)^k (n-k-1)!\,(2z)^{n-2k}}{k!\,(n-2k)!} \;/;\; n \in \mathbb{N}^+$$

07.13.03.0018.01

$$C_n^{(0)}(z) = \frac{2(-1)^n}{n}\cos\left(2n\sin^{-1}\left(\frac{\sqrt{z+1}}{\sqrt{2}}\right)\right) \;/;\; n \in \mathbb{N}^+$$

General characteristics

Domain and analyticity

$C_\nu^{(0)}(z)$ is an analytical function of ν and z which is defined over $\mathbb{C}^2$. For positive integer ν, $C_\nu^{(0)}(z)$ degenerates to a polynomial in z.

07.13.04.0001.01

$$(\nu * z) \longrightarrow C_\nu^{(0)}(z) :: (\mathbb{C} \otimes \mathbb{C}) \longrightarrow \mathbb{C}$$

Symmetries and periodicities

Parity

$C_\nu^{(0)}(z)$ is an odd function with respect to ν.

07.13.04.0002.01

$$C_{-\nu}^{(0)}(z) == -C_\nu^{(0)}(z)$$

07.13.04.0003.01

$$C_n^{(0)}(-z) == (-1)^n C_n^{(0)}(z) /; n \in \mathbb{N}^+$$

Mirror symmetry

07.13.04.0004.01

$$C_{\bar{\nu}}^{(0)}(\bar{z}) == \overline{C_\nu^{(0)}(z)} /; z \notin (-\infty, -1)$$

Periodicity

No periodicity

Poles and essential singularities

With respect to z

For fixed ν /; $\nu \notin \mathbb{Z}$, the function $C_\nu^{(0)}(z)$ does not have poles and essential singularities.

07.13.04.0005.01

$$\mathcal{S}ing_z\left(C_\nu^{(0)}(z)\right) == \{\} /; \nu \notin \mathbb{Z}$$

For integer $\nu \neq 0$, the function $C_\nu^{(0)}(z)$ is polynomial and has pole of order $|\nu|$ at $z = \tilde{\infty}$.

07.13.04.0006.01

$$\mathcal{S}ing_z\left(C_\nu^{(0)}(z)\right) == \{\{\tilde{\infty}, |\nu|\}\} /; \nu \in \mathbb{Z}$$

With respect to ν

For fixed z, the function $C_\nu^{(0)}(z)$ has two singular points:

a) $\nu == 0$ is the simple pole with residue 2;

b) $\nu == \tilde{\infty}$ is an essential singular point.

07.13.04.0007.01

$$\mathcal{S}ing_\nu\left(C_\nu^{(0)}(z)\right) == \{\{0, 1\}, \{\tilde{\infty}, \infty\}\}$$

07.13.04.0008.01

$$\operatorname{res}_{\nu}\left(C_{\nu}^{(0)}(z)\right)(0) = 2$$

Branch points

With respect to z

For fixed noninteger ν, the function $C_{\nu}^{(0)}(z)$ has two branch points: $z = -1$, $z = \tilde{\infty}$.

For fixed integer ν, the function $C_{\nu}^{(0)}(z)$ does not have branch points.

07.13.04.0009.01

$$\mathcal{BP}_{z}\left(C_{\nu}^{(0)}(z)\right) = \{-1, \tilde{\infty}\} /; \nu \notin \mathbb{Z}$$

07.13.04.0010.01

$$\mathcal{BP}_{z}\left(C_{\nu}^{(0)}(z)\right) = \{\} /; \nu \in \mathbb{Z} \land \nu \neq 0$$

07.13.04.0011.01

$$\mathcal{R}_{z}\left(C_{\nu}^{(0)}(z), -1\right) = 2 /; \nu \notin \mathbb{Z}$$

07.13.04.0012.01

$$\mathcal{R}_{z}\left(C_{\nu}^{(0)}(z), \tilde{\infty}\right) = \log /; \nu \notin \mathbb{Q}$$

07.13.04.0013.01

$$\mathcal{R}_{z}\left(C_{\nu}^{(0)}(z), \tilde{\infty}\right) = s /; \nu = \frac{r}{s} \bigwedge \{r, s\} \in \mathbb{Z} \bigwedge s > 1 \bigwedge \gcd(r, s) = 1$$

With respect to ν

For fixed z, the function $C_{\nu}^{(0)}(z)$ does not have branch points.

07.13.04.0014.01

$$\mathcal{BP}_{\nu}\left(C_{\nu}^{(0)}(z)\right) = \{\}$$

Branch cuts

With respect to z

For fixed noninteger ν, the function $C_{\nu}^{(0)}(z)$ is a single-valued function on the z-plane cut along the interval $(-\infty, -1)$, where it is continuous from above.

07.13.04.0015.01

$$\mathcal{BC}_{z}\left(C_{\nu}^{(0)}(z)\right) = \{\{(-\infty, -1), -i\}\} /; \nu \notin \mathbb{Z}$$

07.13.04.0016.01

$$\mathcal{BC}_{z}\left(C_{\nu}^{(0)}(z)\right) = \{\} /; \nu \in \mathbb{Z} \land \nu \neq 0$$

07.13.04.0017.01

$$\lim_{\epsilon \to +0} C_{\nu}^{(0)}(x + i\,\epsilon) = C_{\nu}^{(0)}(x) /; x < -1$$

07.13.04.0018.01

$$\lim_{\epsilon \to +0} C_{\nu}^{(0)}(x - i\,\epsilon) = 2\cos(\nu\,\pi)\, C_{\nu}^{(0)}(-x) - C_{\nu}^{(0)}(x) /; x < -1$$

With respect to ν

For fixed z, the function $C_\nu^{(0)}(z)$ does not have branch cuts.

07.13.04.0019.01

$$\mathcal{BC}_\nu\left(C_\nu^{(0)}(z)\right) = \{\}$$

Series representations

Generalized power series

Expansions at $\nu = 0$

For the function itself

07.13.06.0001.02

$$C_\nu^{(0)}(z) \propto \frac{2}{\nu} - \cos^{-1}(z)^2\,\nu + \frac{\cos^{-1}(z)^4}{12}\,\nu^3 - \ldots \;/;\; (\nu \to 0)$$

07.13.06.0031.01

$$C_\nu^{(0)}(z) \propto \frac{2}{\nu} - \cos^{-1}(z)^2\,\nu + \frac{\cos^{-1}(z)^4}{12}\,\nu^3 - O\left(\nu^6\right)$$

07.13.06.0002.01

$$C_\nu^{(0)}(z) = \frac{2}{\nu}\sum_{k=0}^{\infty}\frac{(-1)^k \cos^{-1}(z)^{2k}\,\nu^{2k}}{(2k)!}$$

07.13.06.0032.01

$$C_\nu^{(0)}(z) = \frac{2}{\nu}\,{}_0F_1\left(;\frac{1}{2}; -\frac{\cos^{-1}(z)^2}{4}\,\nu^2\right)$$

07.13.06.0003.01

$$C_\nu^{(0)}(z) \propto \frac{2}{\nu} + O(\nu) \;/;\; (\nu \to 0)$$

07.13.06.0033.01

$$C_\nu^{(0)}(z) = F_\infty(z, \nu) \;/;$$
$$\left(\left(F_n(z, \nu) = \frac{2}{\nu}\sum_{k=0}^{n}\frac{(-1)^k \cos^{-1}(z)^{2k}\,\nu^{2k}}{(2k)!} = C_\nu^{(0)}(z) + \frac{(-1)^n\sqrt{\pi}\,\cos^{-1}(z)^{2n+2}\,\nu^{2n+1}}{2^{2n+1}}\,{}_1\tilde{F}_2\left(1; n+\frac{3}{2}, n+2; -\frac{\cos^{-1}(z)^2\,\nu^2}{4}\right)\right) \bigwedge n \in \mathbb{N}\right)$$

Summed form of the truncated series expansion.

Expansions at generic point $z = z_0$

For the function itself

07.13.06.0034.01

$$C_\nu^{(0)}(z) \propto \frac{2}{\nu}\left(T_\nu(-z_0)\cos(\pi\nu)\left(-2\,i\,i^{\left\lfloor \frac{\arg(z-z_0)}{2\pi}\right\rfloor}\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor\left\lfloor\frac{\arg(z_0+1)+\pi}{2\pi}\right\rfloor+\left(\frac{1}{z_0+1}\right)^{\frac{1}{2}\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor}(z_0+1)^{\frac{1}{2}\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor}\right)+\right.$$
$$U_{\nu-1}(-z_0)\sin(\pi\nu)\left(\frac{1}{z_0+1}\right)^{\frac{1}{2}\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor}\sqrt{1-z_0^2}\,(z_0+1)^{\frac{1}{2}\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor}+$$
$$\left(\frac{\nu\sin(\pi\nu)}{\sqrt{1-z_0^2}}\left(\frac{1}{z_0+1}\right)^{\frac{1}{2}\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor}(z_0+1)^{\frac{1}{2}\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor}T_\nu(-z_0)-\nu\cos(\pi\nu)\,U_{\nu-1}(-z_0)\right.$$
$$\left.\left.\left(-2\,i\,i^{\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor}\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor\left\lfloor\frac{\arg(z_0+1)+\pi}{2\pi}\right\rfloor+\left(\frac{1}{z_0+1}\right)^{\frac{1}{2}\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor}(z_0+1)^{\frac{1}{2}\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor}\right)\right)(z-z_0)+\ldots\right)\,/;\,(z\to z_0)$$

07.13.06.0035.01

$$C_\nu^{(0)}(z) \propto \frac{2}{\nu}\left(T_\nu(-z_0)\cos(\pi\nu)\left(-2\,i\,i^{\left\lfloor \frac{\arg(z-z_0)}{2\pi}\right\rfloor}\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor\left\lfloor\frac{\arg(z_0+1)+\pi}{2\pi}\right\rfloor+\left(\frac{1}{z_0+1}\right)^{\frac{1}{2}\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor}(z_0+1)^{\frac{1}{2}\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor}\right)+\right.$$
$$U_{\nu-1}(-z_0)\sin(\pi\nu)\left(\frac{1}{z_0+1}\right)^{\frac{1}{2}\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor}\sqrt{1-z_0^2}\,(z_0+1)^{\frac{1}{2}\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor}+$$
$$\left(\frac{\nu\sin(\pi\nu)}{\sqrt{1-z_0^2}}\left(\frac{1}{z_0+1}\right)^{\frac{1}{2}\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor}(z_0+1)^{\frac{1}{2}\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor}T_\nu(-z_0)-\nu\cos(\pi\nu)\,U_{\nu-1}(-z_0)\right.$$
$$\left.\left.\left(-2\,i\,i^{\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor}\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor\left\lfloor\frac{\arg(z_0+1)+\pi}{2\pi}\right\rfloor+\left(\frac{1}{z_0+1}\right)^{\frac{1}{2}\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor}(z_0+1)^{\frac{1}{2}\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor}\right)\right)(z-z_0)\right)+O\left((z-z_0)^2\right)$$

07.13.06.0036.01

$$C_\nu^{(0)}(z) = \frac{\sin(2\pi\nu)}{\sqrt{\pi}}\sum_{k=0}^{\infty}\frac{1}{k!}\left(\frac{\pi\sec(\pi\nu)}{\sqrt{2}}(z_0+1)^{\frac{1}{2}-k}\left(\frac{1}{z_0+1}\right)^{\frac{1}{2}\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor}(z_0+1)^{\frac{1}{2}\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor}{}_2\tilde{F}_1\left(\nu+\frac{1}{2},\frac{1}{2}-\nu;\frac{3}{2}-k;\frac{z_0+1}{2}\right)-\right.$$
$$2^{-k}\,\Gamma(k-\nu)\,\Gamma(k+\nu)\left(-2\,i\,i^{\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor}\left\lfloor\frac{\arg(z_0+1)+\pi}{2\pi}\right\rfloor\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor+\left(\frac{1}{z_0+1}\right)^{\frac{1}{2}\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor}(z_0+1)^{\frac{1}{2}\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor}\right)$$
$$\left.{}_2\tilde{F}_1\left(k-\nu,k+\nu;k+\frac{1}{2};\frac{z_0+1}{2}\right)\right)(z-z_0)^k$$

07.13.06.0037.01

$$C_\nu^{(0)}(z) \propto \frac{2}{\nu}\left(\cos(\pi\nu)\left(-2\,i\,i^{\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor}\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor\left\lfloor\frac{\arg(z_0+1)+\pi}{2\pi}\right\rfloor+\left(\frac{1}{z_0+1}\right)^{\frac{1}{2}\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor}(z_0+1)^{\frac{1}{2}\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor}\right)T_\nu(-z_0)+\sin(\pi\nu)\sqrt{1-z_0^2}\left(\frac{1}{z_0+1}\right)^{\frac{1}{2}\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor}(z_0+1)^{\frac{1}{2}\left\lfloor\frac{\arg(z-z_0)}{2\pi}\right\rfloor}U_{\nu-1}(-z_0)\right)+O(z-z_0)$$

Expansions on branch cuts

For the function itself

07.13.06.0038.01

$$C_\nu^{(0)}(z) \propto \frac{2}{\nu}\left(\cos(\pi\nu)\left(-2\,i\,i^{\left\lfloor\frac{\arg(z-x)}{2\pi}\right\rfloor}\left\lfloor\frac{\arg(z-x)}{2\pi}\right\rfloor+e^{\pi i\left\lfloor\frac{\arg(z-x)}{2\pi}\right\rfloor}\right)T_\nu(-x)+\sqrt{1-x^2}\,e^{\pi i\left\lfloor\frac{\arg(z-x)}{2\pi}\right\rfloor}\sin(\pi\nu)\,U_{\nu-1}(-x)+\left(\frac{\nu\sin(\pi\nu)}{\sqrt{1-x^2}}e^{\pi i\left\lfloor\frac{\arg(z-x)}{2\pi}\right\rfloor}T_\nu(-x)-\nu\cos(\pi\nu)\left(-2\,i\,i^{\left\lfloor\frac{\arg(z-x)}{2\pi}\right\rfloor}\left\lfloor\frac{\arg(z-x)}{2\pi}\right\rfloor+e^{\pi i\left\lfloor\frac{\arg(z-x)}{2\pi}\right\rfloor}\right)U_{\nu-1}(-x)\right)(z-x)\right)+\ldots\;/;\;(z\to x)\land x\in\mathbb{R}\land x<-1$$

07.13.06.0039.01

$$C_\nu^{(0)}(z) \propto \frac{2}{\nu}\left(\cos(\pi\nu)\left(-2\,i\,i^{\left\lfloor\frac{\arg(z-x)}{2\pi}\right\rfloor}\left\lfloor\frac{\arg(z-x)}{2\pi}\right\rfloor+e^{\pi i\left\lfloor\frac{\arg(z-x)}{2\pi}\right\rfloor}\right)T_\nu(-x)+\sqrt{1-x^2}\,e^{\pi i\left\lfloor\frac{\arg(z-x)}{2\pi}\right\rfloor}\sin(\pi\nu)\,U_{\nu-1}(-x)+\left(\frac{\nu\sin(\pi\nu)}{\sqrt{1-x^2}}e^{\pi i\left\lfloor\frac{\arg(z-x)}{2\pi}\right\rfloor}T_\nu(-x)-\nu\cos(\pi\nu)\left(-2\,i\,i^{\left\lfloor\frac{\arg(z-x)}{2\pi}\right\rfloor}\left\lfloor\frac{\arg(z-x)}{2\pi}\right\rfloor+e^{\pi i\left\lfloor\frac{\arg(z-x)}{2\pi}\right\rfloor}\right)U_{\nu-1}(-x)\right)(z-x)\right)+O\left((z-x)^2\right)\;/;\;x\in\mathbb{R}\land x<-1$$

07.13.06.0040.01

$$C_\nu^{(0)}(z) = \frac{\sin(2\pi\nu)}{\sqrt{\pi}}\sum_{k=0}^{\infty}\frac{1}{k!}\left(\frac{\pi\sec(\pi\nu)}{\sqrt{2}}(x+1)^{\frac{1}{2}-k}e^{\pi i\left\lfloor\frac{\arg(z-x)}{2\pi}\right\rfloor}{}_2\tilde{F}_1\left(\nu+\frac{1}{2},\frac{1}{2}-\nu;\frac{3}{2}-k;\frac{x+1}{2}\right)-2^{-k}\,\Gamma(k-\nu)\,\Gamma(k+\nu)\left(-2\,i\,i^{\left\lfloor\frac{\arg(z-x)}{2\pi}\right\rfloor}\left\lfloor\frac{\arg(z-x)}{2\pi}\right\rfloor+e^{\pi i\left\lfloor\frac{\arg(z-x)}{2\pi}\right\rfloor}\right){}_2\tilde{F}_1\left(k-\nu,k+\nu;k+\frac{1}{2};\frac{x+1}{2}\right)\right)(z-x)^k\;/;\;x\in\mathbb{R}\land x<-1$$

07.13.06.0041.01

$$C_\nu^{(0)}(z) \propto \frac{2}{\nu}\left(\cos(\pi\nu)\left(-2\,i\,i^{\left\lfloor\frac{\arg(z-x)}{2\pi}\right\rfloor}\left\lfloor\frac{\arg(z-x)}{2\pi}\right\rfloor+e^{\pi i\left\lfloor\frac{\arg(z-x)}{2\pi}\right\rfloor}\right)T_\nu(-x)+\sqrt{1-x^2}\,e^{\pi i\left\lfloor\frac{\arg(z-x)}{2\pi}\right\rfloor}\sin(\pi\nu)\,U_{\nu-1}(-x)\right)+O(z-x)\;/;\;x\in\mathbb{R}\land x<-1$$

Expansions at $z = 0$

For the function itself

General case

07.13.06.0004.02

$$C_\nu^{(0)}(z) \propto \frac{2\cos\left(\frac{\pi\nu}{2}\right)}{\nu} + 2\sin\left(\frac{\pi\nu}{2}\right) z - \nu\cos\left(\frac{\pi\nu}{2}\right) z^2 + \ldots \;/;\; (z \to 0)$$

07.13.06.0042.01

$$C_\nu^{(0)}(z) \propto \frac{2\cos\left(\frac{\pi\nu}{2}\right)}{\nu} + 2\sin\left(\frac{\pi\nu}{2}\right) z - \nu\cos\left(\frac{\pi\nu}{2}\right) z^2 + O\left(z^3\right)$$

07.13.06.0005.01

$$C_\nu^{(0)}(z) = \frac{2}{\nu}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\frac{(-\nu)_{j+k}\,(\nu)_{j+k}\,(-z)^j}{\left(\frac{1}{2}\right)_{j+k}\, j!\, k!\, 2^{j+k}} \;/;\; |z| < 1$$

07.13.06.0006.02

$$C_\nu^{(0)}(z) = \frac{2}{\nu} F_{1\times 0\times 0}^{2\times 0\times 0}\left(\begin{matrix} -\nu, \nu;;; \\ \frac{1}{2};;; \end{matrix}\; \frac{1}{2}, -\frac{z}{2}\right)$$

07.13.06.0007.01

$$C_\nu^{(0)}(z) = \frac{2\sqrt{\pi}}{\nu}\sum_{k=0}^{\infty}\sum_{j=0}^{k}\frac{(-\nu)_k\,(\nu)_k\,(-z)^j}{\Gamma\left(k+\frac{1}{2}\right) j!\,(k-j)!\,2^k} \;/;\; |z| < 1$$

07.13.06.0043.01

$$C_\nu^{(0)}(z) = -\frac{\sin(\pi\nu)}{2\pi}\sum_{j=0}^{\infty}\frac{(-2)^j\,\Gamma\left(\frac{j}{2}-\frac{\nu}{2}\right)\Gamma\left(\frac{j}{2}+\frac{\nu}{2}\right) z^j}{j!} \;/;\; |z| < 1$$

07.13.06.0044.01

$$C_\nu^{(0)}(z) = \frac{2}{\nu}\cos\left(\frac{\pi\nu}{2}\right)\sum_{k=0}^{\infty}\frac{\left(-\frac{\nu}{2}\right)_k\left(\frac{\nu}{2}\right)_k}{\left(\frac{1}{2}\right)_k k!} z^{2k} + 2z\,\sin\left(\frac{\pi\nu}{2}\right)\sum_{k=0}^{\infty}\frac{\left(\frac{1-\nu}{2}\right)_k\left(\frac{\nu+1}{2}\right)_k}{\left(\frac{3}{2}\right)_k k!} z^{2k} \;/;\; |z| < 1$$

07.13.06.0045.01

$$C_\nu^{(0)}(z) = \frac{2}{\nu}\cos\left(\frac{\pi\nu}{2}\right) {}_2F_1\left(-\frac{\nu}{2}, \frac{\nu}{2}; \frac{1}{2}; z^2\right) + 2z\sin\left(\frac{\pi\nu}{2}\right) {}_2F_1\left(\frac{1-\nu}{2}, \frac{\nu+1}{2}; \frac{3}{2}; z^2\right)$$

07.13.06.0046.01

$$C_\nu^{(0)}(z) = \frac{2}{\nu}\cos\left(\frac{\pi\nu}{2}\right)\cos\left(\nu\sin^{-1}(z)\right) + \frac{2}{\nu}\sin\left(\frac{\pi\nu}{2}\right)\sin\left(\nu\sin^{-1}(z)\right)$$

07.13.06.0047.01

$$C_\nu^{(0)}(z) = \frac{2}{\nu}\cos\left(\frac{\nu}{2}\left(\pi - 2\sin^{-1}(z)\right)\right)$$

07.13.06.0008.02

$$C_\nu^{(0)}(z) \propto \frac{2}{\nu}\cos\left(\frac{\pi\nu}{2}\right)(1 + O(z))$$

07.13.06.0048.01

$$C_\nu^{(0)}(z) \equiv F_\infty(z,\nu) \;/;\; \left(\left(F_m(z,\nu) \equiv -\frac{\sin(\pi\nu)}{2\pi}\sum_{j=0}^{m}\frac{(-2)^j\,\Gamma\left(\frac{j}{2}-\frac{\nu}{2}\right)\Gamma\left(\frac{j}{2}+\frac{\nu}{2}\right)}{j!}z^j = C_\nu^{(0)}(z) - \frac{1}{\pi\,(m+2)!}\left(2^{m+1}(-1)^{m+1}z^{m+2}\sin(\pi\nu)\,\Gamma\left(\frac{1}{2}(m-\nu+2)\right)\Gamma\left(\frac{1}{2}(m+\nu+2)\right)\right){}_3F_2\left(1,\frac{m}{2}-\frac{\nu}{2}+1,\frac{m}{2}+\frac{\nu}{2}+1;\frac{m}{2}+\frac{3}{2},\frac{m}{2}+2;z^2\right) + \frac{1}{\pi\,(m+1)!}\left(2^m(-z)^{m+1}\sin(\pi\nu)\,\Gamma\left(\frac{1}{2}(m-\nu+1)\right)\Gamma\left(\frac{1}{2}(m+\nu+1)\right)\right){}_3F_2\left(1,\frac{m}{2}-\frac{\nu}{2}+\frac{1}{2},\frac{m}{2}+\frac{\nu}{2}+\frac{1}{2};\frac{m}{2}+1,\frac{m}{2}+\frac{3}{2};z^2\right)\right)\bigwedge m\in\mathbb{N}\right)$$

Summed form of the truncated series expansion.

Special cases

07.13.06.0009.01

$$C_n^{(0)}(z) \equiv \frac{2^n}{n}z^n + \frac{\delta_{n,0}}{n} + \sum_{k=1}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{(-1)^k\,(n-k-1)!\,(2z)^{n-2k}}{k!\,(n-2k)!} \;/;\; n\in\mathbb{N}^+$$

07.13.06.0049.01

$$C_n(z) = \sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{(-1)^{n+1}}{k!\,(n-2k)!}S_{n-k}^{(1)}\,(2z)^{n-2k} \;/;\; n\in\mathbb{N}^+$$

07.13.06.0010.01

$$C_n^{(0)}(z) \propto \frac{2}{n}(-1)^{\left\lfloor\frac{n}{2}\right\rfloor}(n\,z)^{n-2\left\lfloor\frac{n}{2}\right\rfloor}\left(1+O\left(z^2\right)\right) \;/;\; (z\to 0)\bigwedge n\in\mathbb{N}^+$$

Generic formulas for main term

07.13.06.0050.01

$$C_\nu^{(0)}(z) \propto \begin{cases} \tilde{\infty} & \nu = 0 \\ \frac{2(-1)^{\left\lfloor\frac{\nu}{2}\right\rfloor}(\nu z)^{\nu-2\left\lfloor\frac{\nu}{2}\right\rfloor}}{\nu} & \nu\in\mathbb{Z}\land\nu\neq 0 \\ \frac{2\cos\left(\frac{\pi\nu}{2}\right)}{\nu} & \text{True} \end{cases} \;/;\; (z\to 0)$$

Expansions at $z \equiv 1$

For the function itself

General case

07.13.06.0011.02

$$C_\nu^{(0)}(z) \propto \frac{2}{\nu} + 2\nu\,(z-1) - \frac{(1-\nu)\,\nu\,(1+\nu)}{8}(z-1)^2 + \ldots \;/;\; (z\to 1)$$

07.13.06.0051.01

$$C_\nu^{(0)}(z) \propto \frac{2}{\nu} + 2\,\nu\,(z-1) - \frac{(1-\nu)\,\nu\,(1+\nu)}{8}\,(z-1)^2 + O\big((z-1)^3\big)$$

07.13.06.0012.01

$$C_\nu^{(0)}(z) = \frac{2}{\nu}\sum_{k=0}^{\infty}\frac{(-\nu)_k\,(\nu)_k}{\left(\frac{1}{2}\right)_k\,k!}\left(\frac{1-z}{2}\right)^k \;/;\; \left|\frac{1-z}{2}\right| < 1$$

07.13.06.0013.01

$$C_\nu^{(0)}(z) = \frac{2}{\nu}\,{}_2F_1\!\left(-\nu,\,\nu;\,\frac{1}{2};\,\frac{1-z}{2}\right)$$

07.13.06.0052.01

$$C_\nu^{(0)}(z) = \frac{2}{\nu}\cos\!\left(2\,\nu\,\sin^{-1}\!\left(\frac{\sqrt{1-z}}{\sqrt{2}}\right)\right)$$

07.13.06.0015.02

$$C_\nu^{(0)}(z) \propto \frac{2}{\nu} + O(z-1)$$

07.13.06.0053.01

$$C_\nu^{(0)}(z) = F_\infty(z,\,\nu) \;/;\; \left(\left(F_m(z,\,\nu) = \frac{2}{\nu}\sum_{k=0}^{m}\frac{(-\nu)_k\,(\nu)_k}{\left(\frac{1}{2}\right)_k\,k!}\left(\frac{1-z}{2}\right)^k = C_\nu^{(0)}(z) - \frac{2^{-m}\,(-\nu)_{m+1}\,(\nu+1)_m\,(1-z)^{m+1}}{(m+1)!\left(\frac{1}{2}\right)_{m+1}}\,{}_3F_2\!\left(1,\,m-\nu+1,\,m+\nu+1;\,m+\frac{3}{2},\,m+2;\,\frac{1-z}{2}\right)\right)\bigwedge m \in \mathbb{N}\right)$$

Summed form of the truncated series expansion.

Special cases

07.13.06.0014.01

$$C_n^{(0)}(z) = \frac{2}{n}\sum_{k=0}^{n}\frac{(-n)_k\,(n)_k}{\left(\frac{1}{2}\right)_k\,k!}\left(\frac{1-z}{2}\right)^k \;/;\; n \in \mathbb{N}^+$$

Expansions at $z = -1$

For the function itself

General case

07.13.06.0016.02

$$C_\nu^{(0)}(z) \propto \frac{2}{\nu}\cos(\pi\,\nu)\left(1 - \nu^2(z+1) - \frac{(1-\nu)\,\nu^2\,(1+\nu)}{6}\,(z+1)^2 - \ldots\right) + 2\sqrt{2}\,\sqrt{z+1}\,\sin(\pi\,\nu)\left(1 + \frac{1}{3}\left(\frac{1}{2}-\nu\right)\left(\frac{1}{2}+\nu\right)(z+1) + \frac{1}{30}\left(\frac{1}{2}-\nu\right)\left(\frac{3}{2}-\nu\right)\left(\frac{1}{2}+\nu\right)\left(\frac{3}{2}+\nu\right)(z+1)^2 + \ldots\right) /;\; (z \to -1)$$

07.13.06.0054.01

$$C_{\nu}^{(0)}(z) \propto \frac{2}{\nu}\cos(\pi\nu)\left(1-\nu^2(z+1)-\frac{(1-\nu)\nu^2(1+\nu)}{6}(z+1)^2-O\left((z+1)^3\right)\right)+ 2\sqrt{2}\sqrt{z+1}\sin(\pi\nu)\left(1+\frac{1}{3}\left(\frac{1}{2}-\nu\right)\left(\frac{1}{2}+\nu\right)(z+1)+\frac{1}{30}\left(\frac{1}{2}-\nu\right)\left(\frac{3}{2}-\nu\right)\left(\frac{1}{2}+\nu\right)\left(\frac{3}{2}+\nu\right)(z+1)^2+O\left((z+1)^3\right)\right)$$

07.13.06.0017.01

$$C_{\nu}^{(0)}(z) = \frac{2}{\nu}\cos(\nu\pi)\sum_{k=0}^{\infty}\frac{(-\nu)_k(\nu)_k}{\left(\frac{1}{2}\right)_k k!}\left(\frac{z+1}{2}\right)^k+2\sqrt{2}\sqrt{z+1}\sin(\nu\pi)\sum_{k=0}^{\infty}\frac{\left(\frac{1}{2}-\nu\right)_k\left(\nu+\frac{1}{2}\right)_k}{\left(\frac{3}{2}\right)_k k!}\left(\frac{z+1}{2}\right)^k /; \left|\frac{z+1}{2}\right|<1$$

07.13.06.0018.01

$$C_{\nu}^{(0)}(z) = \frac{2}{\nu}\cos(\nu\pi)\,{}_2F_1\left(-\nu,\nu;\frac{1}{2};\frac{z+1}{2}\right)+2\sqrt{2}\sqrt{z+1}\sin(\nu\pi)\,{}_2F_1\left(\nu+\frac{1}{2},\frac{1}{2}-\nu;\frac{3}{2};\frac{z+1}{2}\right)$$

07.13.06.0055.01

$$C_{\nu}^{(0)}(z) = \frac{2}{\nu}\cos\left(\nu\left(\pi-2\sin^{-1}\left(\frac{\sqrt{z+1}}{\sqrt{2}}\right)\right)\right)$$

07.13.06.0019.02

$$C_{\nu}^{(0)}(z) \propto \frac{2}{\nu}\cos(\pi\nu)(1+O(z+1))+2\sqrt{2}\sin(\pi\nu)\sqrt{z+1}(1+O(z+1))$$

07.13.06.0056.01

$$C_{\nu}^{(0)}(z) = F_{\infty}(z,\nu) /; \left(\left(F_m(z,\nu) = \frac{2}{\nu}\cos(\nu\pi)\sum_{k=0}^{m}\frac{(-\nu)_k(\nu)_k}{\left(\frac{1}{2}\right)_k k!}\left(\frac{z+1}{2}\right)^k+2\sqrt{2}\sqrt{z+1}\sin(\nu\pi)\sum_{k=0}^{m}\frac{\left(\frac{1}{2}-\nu\right)_k\left(\nu+\frac{1}{2}\right)_k}{\left(\frac{3}{2}\right)_k k!}\left(\frac{z+1}{2}\right)^k = C_{\nu}^{(0)}(z)-\frac{2^{-m}\cos(\pi\nu)(-\nu)_{m+1}(\nu+1)_m}{(m+1)!\left(\frac{1}{2}\right)_{m+1}}(z+1)^{m+1}\,{}_3F_2\left(1,m-\nu+1,m+\nu+1;m+\frac{3}{2},m+2;\frac{z+1}{2}\right)- \frac{2^{-m+\frac{1}{2}}\sin(\pi\nu)\left(\frac{1}{2}-\nu\right)_{m+1}\left(\nu+\frac{1}{2}\right)_{m+1}}{(m+1)!\left(\frac{3}{2}\right)_{m+1}}(z+1)^{m+\frac{3}{2}}\,{}_3F_2\left(1,m-\nu+\frac{3}{2},m+\nu+\frac{3}{2};m+2,m+\frac{5}{2};\frac{z+1}{2}\right)\right)\bigwedge m\in\mathbb{N}\right)$$

Summed form of the truncated series expansion.

Special cases

07.13.06.0020.01

$$C_{n}^{(0)}(z) = \frac{2(-1)^n}{n}\sum_{k=0}^{n}\frac{(-n)_k(n)_k}{\left(\frac{1}{2}\right)_k k!}\left(\frac{z+1}{2}\right)^k /; n\in\mathbb{N}^+$$

07.13.06.0021.01

$$C_{n}^{(0)}(z) \propto \frac{2(-1)^n}{n}(1+O(z+1)) /; n\in\mathbb{N}^+$$

Expansions at $z = \infty$

For the function itself

Expansions in $1/z$

07.13.06.0057.01

$$C_\nu^{(0)}(z) \propto \frac{2^{-\nu}}{\nu} z^\nu \left(1 + \frac{\nu}{4 z^2} + \frac{(\nu+3)\nu}{32 z^4} + \ldots\right) + \frac{2^\nu}{\nu} z^{-\nu} \left(1 - \frac{\nu}{4 z^2} + \frac{(\nu-3)\nu}{32 z^4} + \ldots\right) /; (|z| \to \infty)$$

07.13.06.0058.01

$$C_\nu^{(0)}(z) \propto \frac{2^{-\nu}}{\nu} z^\nu \left(1 + \frac{\nu}{4 z^2} + \frac{(\nu+3)\nu}{32 z^4} + O\left(\frac{1}{z^6}\right)\right) + \frac{2^\nu}{\nu} z^{-\nu} \left(1 - \frac{\nu}{4 z^2} + \frac{(\nu-3)\nu}{32 z^4} + O\left(\frac{1}{z^6}\right)\right)$$

07.13.06.0059.01

$$C_\nu^{(0)}(z) = \frac{2^{-\nu}}{\nu} z^{-\nu} \sum_{k=0}^{\infty} \frac{\left(\frac{\nu}{2}\right)_k \left(\frac{\nu+1}{2}\right)_k}{(\nu+1)_k\, k!} z^{-2k} + \frac{2^\nu}{\nu} z^\nu \sum_{k=0}^{\infty} \frac{\left(-\frac{\nu}{2}\right)_k \left(\frac{1-\nu}{2}\right)_k}{(1-\nu)_k\, k!} z^{-2k} /; |z| > 1 \bigwedge 2\nu \notin \mathbb{Z}$$

07.13.06.0060.01

$$C_\nu^{(0)}(z) = \frac{2^{-\nu}}{\nu} z^{-\nu} {}_2F_1\left(\frac{\nu}{2}, \frac{\nu+1}{2}; \nu+1; \frac{1}{z^2}\right) + \frac{2^\nu}{\nu} z^\nu {}_2F_1\left(-\frac{\nu}{2}, \frac{1}{2} - \frac{\nu}{2}; 1-\nu; \frac{1}{z^2}\right) /; z \notin (-1, 0) \bigwedge \nu \notin \mathbb{Z}$$

07.13.06.0061.01

$$C_\nu^{(0)}(z) = \frac{1}{\nu} z^{-\nu} \left(\frac{\sqrt{z^2}\sqrt{z^2-1}}{z^2} + 1\right)^{-\nu} + \frac{1}{\nu} z^\nu \left(\frac{\sqrt{z^2}\sqrt{z^2-1}}{z^2} + 1\right)^{\nu} /; z \notin (-1, 0) \bigwedge \nu \notin \mathbb{Z}$$

07.13.06.0062.01

$$C_n^{(0)}(z) = \frac{2^n}{n} z^n \sum_{k=0}^{\left\lfloor \frac{n}{2} \right\rfloor} \frac{\left(\frac{1-n}{2}\right)_k \left(-\frac{n}{2}\right)_k z^{-2k}}{k!\,(1-n)_k} /; n \in \mathbb{N}^+$$

07.13.06.0063.01

$$C_n^{(0)}(z) = \frac{2^n}{n} z^n {}_2F_1\left(-\frac{n}{2}, \frac{1-n}{2}; 1-n; \frac{1}{z^2}\right) /; n-2 \in \mathbb{N}^+$$

07.13.06.0025.02

$$C_\nu^{(0)}(z) \propto \frac{2^{-\nu}}{\nu} z^{-\nu}\left(1 + O\left(\frac{1}{z}\right)\right) + \frac{2^\nu}{\nu} z^\nu \left(1 + O\left(\frac{1}{z}\right)\right) /; \nu \notin \mathbb{Z}$$

07.13.06.0027.02

$$C_n^{(0)}(z) \propto \frac{2^n}{n} z^n \left(1 + O\left(\frac{1}{z}\right)\right) /; n \in \mathbb{N}^+$$

07.13.06.0029.02

$$C_\nu^{(0)}(z) \propto \frac{2^{-|\nu|}}{\nu} z^{-|\nu|}\left(1 + O\left(\frac{1}{z}\right)\right) + \frac{2^{|\nu|}}{\nu} z^{|\nu|}\left(1 + O\left(\frac{1}{z}\right)\right) /; \nu - \frac{1}{2} \in \mathbb{Z}$$

07.13.06.0064.01

$$C_\nu^{(0)}(z) = F_\infty(z,\nu)\,/;\left(F_m(z,\nu) = \frac{2^{-\nu}}{\nu} z^{-\nu} \sum_{k=0}^{m} \frac{\left(\frac{\nu}{2}\right)_k \left(\frac{\nu+1}{2}\right)_k}{(\nu+1)_k\, k!} z^{-2k} + \frac{2^{\nu}}{\nu} z^{\nu} \sum_{k=0}^{m} \frac{\left(-\frac{\nu}{2}\right)_k \left(\frac{1-\nu}{2}\right)_k}{(1-\nu)_k\, k!} z^{-2k} = C_\nu^{(0)}(z) - \frac{2^{-2m-\nu-2} z^{-2(m+1)-\nu}\, \Gamma(2m+\nu+2)}{(m+1)!\, \Gamma(m+\nu+2)}\, {}_3F_2\!\left(1, m+\frac{\nu}{2}+1, m+\frac{\nu}{2}+\frac{3}{2}; m+2, m+\nu+2; \frac{1}{z^2}\right) + \frac{2^{-2m+\nu-2} z^{\nu-2(m+1)}\, \Gamma(2m-\nu+2)}{(m+1)!\, \Gamma(m-\nu+2)}\, {}_3F_2\!\left(1, m-\frac{\nu}{2}+1, m-\frac{\nu}{2}+\frac{3}{2}; m+2, m-\nu+2; \frac{1}{z^2}\right) \bigwedge m \in \mathbb{N}\right) \bigwedge \nu \notin \mathbb{Z}$$

Summed form of the truncated series expansion.

Expansions in $1/(1-z)$

07.13.06.0022.02

$$C_\nu^{(0)}(z) \propto \frac{2^{-\nu}}{\nu} (z-1)^{-\nu} \left(1 + \frac{\nu}{1-z} + \frac{\nu(3+2\nu)}{4(1-z)^2} + \ldots\right) + \frac{2^{\nu}}{\nu} (z-1)^{\nu} \left(1 - \frac{\nu}{1-z} - \frac{\nu(3-2\nu)}{4(1-z)^2} - \ldots\right) /; (|z| \to \infty) \bigwedge 2\nu \notin \mathbb{Z}$$

07.13.06.0065.01

$$C_\nu^{(0)}(z) \propto \frac{2^{-\nu}}{\nu} (z-1)^{-\nu} \left(1 + \frac{\nu}{1-z} + \frac{\nu(3+2\nu)}{4(1-z)^2} + O\!\left(\frac{1}{z^3}\right)\right) + \frac{2^{\nu}}{\nu} (z-1)^{\nu} \left(1 - \frac{\nu}{1-z} - \frac{\nu(3-2\nu)}{4(1-z)^2} - O\!\left(\frac{1}{z^3}\right)\right) /; 2\nu \notin \mathbb{Z}$$

07.13.06.0023.01

$$C_\nu^{(0)}(z) = \frac{2^{-\nu}}{\nu} (z-1)^{-\nu} \sum_{k=0}^{\infty} \frac{(\nu)_k \left(\nu+\frac{1}{2}\right)_k}{(2\nu+1)_k\, k!} \left(\frac{2}{1-z}\right)^k + \frac{2^{\nu}}{\nu} (z-1)^{\nu} \sum_{k=0}^{\infty} \frac{(-\nu)_k \left(\frac{1}{2}-\nu\right)_k}{(1-2\nu)_k\, k!} \left(\frac{2}{1-z}\right)^k /; \left|\frac{1-z}{2}\right| > 1 \bigwedge 2\nu \notin \mathbb{Z}$$

07.13.06.0024.01

$$C_\nu^{(0)}(z) = \frac{2^{-\nu}}{\nu} (z-1)^{-\nu}\, {}_2F_1\!\left(\nu, \nu+\frac{1}{2}; 2\nu+1; \frac{2}{1-z}\right) + \frac{2^{\nu}}{\nu} (z-1)^{\nu}\, {}_2F_1\!\left(-\nu, \frac{1}{2}-\nu; 1-2\nu; \frac{2}{1-z}\right) /; z \notin (0,1) \bigwedge 2\nu \notin \mathbb{Z}$$

07.13.06.0066.01

$$C_\nu^{(0)}(z) = \frac{2^{\nu}}{\nu} (z-1)^{-\nu} \left(1 + \sqrt{\frac{z+1}{z-1}}\right)^{-2\nu} + \frac{2^{-\nu}}{\nu} (z-1)^{\nu} \left(1 + \sqrt{\frac{z+1}{z-1}}\right)^{2\nu} /; z \notin (-1,1)$$

07.13.06.0067.01

$$C_n^{(0)}(z) = \frac{2^n}{n} (z-1)^n \sum_{k=0}^{n} \frac{(-n)_k \left(\frac{1}{2}-n\right)_k}{k!\,(1-2n)_k} \left(\frac{2}{1-z}\right)^k /;\ n \in \mathbb{N}^+$$

07.13.06.0068.01

$$C_n^{(0)}(z) = \frac{2^{1-n} \sqrt{\pi}\, (z-1)^n}{n!} \sum_{k=0}^{n} \frac{(2n-k-1)!\,(-n)_k}{k!\,\Gamma\!\left(n-k+\frac{1}{2}\right)} \left(\frac{2}{1-z}\right)^k /; n \in \mathbb{N}^+$$

07.13.06.0069.01

$$C_n^{(0)}(z) = \frac{2^n}{n} (z-1)^n\, {}_2F_1\!\left(-n, \frac{1}{2}-n; 1-2n; \frac{2}{1-z}\right) /; n-1 \in \mathbb{N}^+$$

07.13.06.0028.01

$$C_\nu^{(0)}(z) = \frac{2^{1-|\nu|}}{\nu}(z-1)^{-|\nu|}\,{}_2F_1\left(|\nu|, |\nu|+\frac{1}{2}; 2|\nu|+1; \frac{2}{1-z}\right) + \frac{2^{|\nu|}}{\nu}(z-1)^{|\nu|}\sum_{k=0}^{|\nu|-\frac{1}{2}} \frac{\left(\frac{1}{2}-|\nu|\right)_k (-|\nu|)_k}{k!\,(1-2|\nu|)_k}\left(\frac{2}{1-z}\right)^k /;\ \nu-\frac{1}{2}\in\mathbb{Z}$$

07.13.06.0070.01

$$C_\nu^{(0)}(z) = F_\infty(z,\nu) /; \left(\left(F_m(z,\nu) = \frac{2^{-\nu}}{\nu}(z-1)^{-\nu}\sum_{k=0}^{m}\frac{(\nu)_k\left(\nu+\frac{1}{2}\right)_k}{(2\nu+1)_k\,k!}\left(\frac{2}{1-z}\right)^k + \frac{2^\nu}{\nu}(z-1)^\nu\sum_{k=0}^{m}\frac{(-\nu)_k\left(\frac{1}{2}-\nu\right)_k}{(1-2\nu)_k\,k!}\left(\frac{2}{1-z}\right)^k = \right.\right.$$

$$C_\nu^{(0)}(z) + \frac{2^{1+m-\nu}(-1)^m(\nu+1)_m\left(\nu+\frac{1}{2}\right)_{m+1}(z-1)^{-m-\nu-1}}{(m+1)!\,(2\nu+1)_{m+1}}\,{}_3F_2\left(1, m+\nu+1, m+\nu+\frac{3}{2}; m+2, m+2\nu+2; \frac{2}{1-z}\right) -$$

$$\frac{2^{1+m+\nu}(-1)^m\left(\frac{1}{2}-\nu\right)_{m+1}(1-\nu)_m(z-1)^{-m+\nu-1}}{(m+1)!\,(1-2\nu)_{m+1}}$$

$$\left.\left.{}_3F_2\left(1, m-\nu+1, m-\nu+\frac{3}{2}; m+2, m-2\nu+2; \frac{2}{1-z}\right)\bigwedge m\in\mathbb{N}\right)\bigwedge \neg\, 2\nu\in\mathbb{Z}\right)$$

Summed form of the truncated series expansion.

Generic formulas for main term

07.13.06.0071.01

$$C_\nu^{(0)}(z) \propto \begin{cases} \frac{2^\nu z^\nu}{\nu} & \operatorname{Re}(\nu) > 0 \\ \frac{2^{-\nu} z^{-\nu}}{\nu} & \operatorname{Re}(\nu) < 0 \\ \frac{2^{-\nu} z^{-\nu}}{\nu} + \frac{2^\nu z^\nu}{\nu} & \text{True} \end{cases} /; (|z|\to\infty)$$

Asymptotic series expansions

Expansions at $\nu = \infty$

07.13.06.0072.01

$$C_\nu^{(0)}(z) \propto \begin{cases} \frac{1}{\nu} e^{i\nu\cos^{-1}(z)} & -\pi < \arg\left(\nu\cos^{-1}(z)\right) < 0 \\ \frac{1}{\nu} e^{-i\nu\cos^{-1}(z)} & 0 < \arg\left(\nu\cos^{-1}(z)\right) < \pi \\ \frac{2}{\nu}\cos\left(\nu\cos^{-1}(z)\right) & \text{True} \end{cases} /; (|\nu|\to\infty)$$

Other series representations

07.13.06.0030.01

$$C_n^{(0)}(z) = \frac{2}{n}\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}\binom{n}{2k} z^{n-2k}\left(z^2-1\right)^k /;\ n\in\mathbb{N}^+$$

Integral representations

On the real axis

Of the direct function

07.13.07.0001.01

$$C_n^{(0)}(x) = -\frac{2}{\pi n} \mathcal{P} \int_{-1}^{1} \frac{\sqrt{1-t^2}\, U_{n-1}(t)}{t-x} \, dt \,/;\, n \in \mathbb{N}^+ \bigwedge -1 < x < 1$$

Integral representations of negative integer order

07.13.07.0002.01

$$C_n^{(0)}(z) = \frac{(-1)^n \sqrt{\pi} \sqrt{1-z^2}}{n\, 2^{n-1}\, \Gamma\left(n+\frac{1}{2}\right)} \frac{\partial^n \left(1-z^2\right)^{n-\frac{1}{2}}}{\partial z^n} \,/;\, n \in \mathbb{N}^+$$

Limit representations

07.13.09.0001.01

$$C_\nu^{(0)}(z) = \lim_{\lambda \to 0} \frac{1}{\lambda} C_\nu^\lambda(z)$$

Generating functions

07.13.11.0001.01

$$C_n^{(0)}(z) = \frac{2}{n} \left([t^n] \frac{t(z-t)}{t^2 - 2zt + 1}\right) /;\, n \in \mathbb{N}^+ \bigwedge -1 < z < 1$$

07.13.11.0002.01

$$C_n^{(0)}(z) = -\left([t^n] \log\left(t^2 - 2zt + 1\right)\right) /;\, n \in \mathbb{N}^+ \bigwedge -1 < z < 1$$

Differential equations

Ordinary linear differential equations and wronskians

For the direct function itself

With respect to ν

07.13.13.0005.01

$$\nu\, w''(\nu) + 2\, w'(\nu) + \nu\, \cos^{-1}(z)^2\, w(\nu) = 0 \,/;\, w(\nu) = c_1\, C_\nu^{(0)}(z) + \frac{c_2}{\nu} U_\nu(z)$$

07.13.13.0006.01

$$W_\nu\left(C_\nu^{(0)}(z), \frac{1}{\nu} U_\nu(z)\right) = \frac{2\, z \cos^{-1}(z)}{\sqrt{1-z^2}\, \nu^2}$$

07.13.13.0007.01

$$w''(\nu) - \left(\frac{g''(\nu)}{g'(\nu)} - \frac{2\, g'(\nu)}{g(\nu)}\right) w'(\nu) + \cos^{-1}(z)^2\, g'(\nu)^2\, w(\nu) = 0 \,/;\, w(\nu) = c_1\, C_{g(\nu)}^{(0)}(z) + c_2 \frac{1}{g(\nu)} U_{g(\nu)}(z)$$

07.13.13.0008.01

$$W_\nu\left(C_{g(\nu)}^{(0)}(z),\ \frac{1}{g(\nu)}\,U_{g(\nu)}(z)\right) == \frac{2\,z\cos^{-1}(z)\,g'(\nu)}{\sqrt{1-z^2}\ g(\nu)^2}$$

07.13.13.0009.01

$$w''(\nu)-\left(-\frac{2\,g'(\nu)}{g(\nu)}+\frac{2\,h'(\nu)}{h(\nu)}+\frac{g''(\nu)}{g'(\nu)}\right)w'(\nu)+\left(\cos^{-1}(z)^2\,g'(\nu)^2-\frac{2\,h'(\nu)\,g'(\nu)}{g(\nu)\,h(\nu)}+\frac{2\,h'(\nu)^2}{h(\nu)^2}+\frac{h'(\nu)\,g''(\nu)}{h(\nu)\,g'(\nu)}-\frac{h''(\nu)}{h(\nu)}\right)w(\nu)=0\,/;$$
$$w(\nu)=c_1\,h(\nu)\,C_{g(\nu)}^{(0)}(z)+c_2\,\frac{h(\nu)}{g(\nu)}\,U_{g(\nu)}(z)$$

07.13.13.0010.01

$$W_\nu\left(h(\nu)\,C_{g(\nu)}^{(0)}(z),\ \frac{h(\nu)}{g(\nu)}\,U_{g(\nu)}(z)\right) == \frac{2\,z\cos^{-1}(z)\,h(\nu)^2\,g'(\nu)}{\sqrt{1-z^2}\ g(\nu)^2}$$

07.13.13.0011.01

$$\nu^2\,w''(\nu)\ +(r-2\,s+1)\,\nu\,w'(\nu)\ +\left(a^2\,r^2\cos^{-1}(z)^2\,\nu^{2\,r}+s\,(s-r)\right)w(\nu)=0\,/;\ w(\nu)=c_1\,\nu^s\,C_{a\,\nu^r}^{(0)}(z)+c_2\,\nu^{s-r}\,U_{a\,\nu^r}(z)$$

07.13.13.0012.01

$$W_\nu\left(\nu^s\,C_{a\,\nu^r}^{(0)}(z),\ \nu^{s-r}\,U_{a\,\nu^r}(z)\right) == \frac{2\,r\,z\,\nu^{-r+2\,s-1}\cos^{-1}(z)}{\sqrt{1-z^2}}$$

07.13.13.0013.01

$$w''(\nu)+(\log(r)-2\log(s))\,w'(\nu)+\left(a^2\cos^{-1}(z)^2\log^2(r)\,r^{2\,\nu}+\log(s)\,(\log(s)-\log(r))\right)w(\nu)=0\,/;$$
$$w(\nu)=c_1\,s^\nu\,C_{a\,r^\nu}^{(0)}(z)\ +c_2\,r^{-\nu}\,s^\nu\,U_{a\,r^\nu}(z)$$

07.13.13.0014.01

$$W_\nu\left(s^\nu\,C_{a\,r^\nu}^{(0)}(z)\,,\ r^{-\nu}\,s^\nu\,U_{a\,r^\nu}(z)\right) == \frac{2\,r^{-\nu}\,s^{2\,\nu}\,z\cos^{-1}(z)\log(r)}{\sqrt{1-z^2}}$$

With respect to *z*

07.13.13.0001.01

$$\left(1-z^2\right)w''(z)-z\,w'(z)+\nu^2\,w(z)==0\,/;\ w(z)==\ c_1\,C_\nu^{(0)}(z)+c_2\sin\left(\nu\cos^{-1}(z)\right)$$

07.13.13.0002.01

$$W_z\left(C_\nu^{(0)}(z),\ \sin\left(\nu\cos^{-1}(z)\right)\right)==-\frac{2}{\sqrt{1-z^2}}$$

07.13.13.0003.01

$$\left(1-z^2\right)w''(z)-z\,w'(z)+\nu^2\,w(z)==0\,/;\ w(z)==\ c_1\,C_\nu^{(0)}(z)+c_2\sinh\left(\nu\cosh^{-1}(z)\right)$$

07.13.13.0004.01

$$W_z\left(C_\nu^{(0)}(z),\ \sinh\left(\nu\cosh^{-1}(z)\right)\right)==\frac{2\cos\left(\nu\cos^{-1}(z)\right)\cosh\left(\nu\cosh^{-1}(z)\right)}{\sqrt{z-1}\ \sqrt{z+1}}-\frac{2\sin\left(\nu\cos^{-1}(z)\right)\sinh\left(\nu\cosh^{-1}(z)\right)}{\sqrt{1-z^2}}$$

07.13.13.0015.01

$$\left(1-z^2\right)w''(z)-z\,w'(z)+\nu^2\,w(z)\ =0\,/;\ w(z)=c_1\,C_\nu^{(0)}(z)+c_2\sqrt{1-z^2}\ U_{\nu-1}(z)$$

07.13.13.0016.01

$$W_z\left(C_\nu^{(0)}(z),\ \sqrt{1-z^2}\ U_{\nu-1}(z)\right) = -\frac{2}{\sqrt{1-z^2}}$$

07.13.13.0017.01

$$w''(z) - \left(\frac{g(z)\,g'(z)}{1-g(z)^2} + \frac{g''(z)}{g'(z)}\right) w'(z) + \frac{\nu^2\, g'(z)^2}{1-g(z)^2}\, w(z) = 0\ /;\ w(z) = c_1\, C_\nu^{(0)}(g(z)) + c_2 \sqrt{1-g(z)^2}\ U_{\nu-1}(g(z))$$

07.13.13.0018.01

$$W_z\left(C_\nu^{(0)}(g(z)),\ \sqrt{1-g(z)^2}\ U_{\nu-1}(g(z))\right) = -\frac{2\, g'(z)}{\sqrt{1-g(z)^2}}$$

07.13.13.0019.01

$$w''(z) - \left(\frac{g(z)\,g'(z)}{1-g(z)^2} + \frac{2\,h'(z)}{h(z)} + \frac{g''(z)}{g'(z)}\right) w'(z) +$$
$$\left(\frac{\nu^2\, g'(z)^2}{1-g(z)^2} + \frac{g(z)\,h'(z)\,g'(z)}{\left(1-g(z)^2\right) h(z)} + \frac{h(z)\,h'(z)\,g''(z) + g'(z)\left(2\,h'(z)^2 - h(z)\,h''(z)\right)}{h(z)^2\, g'(z)}\right) w(z) = 0\ /;$$
$$w(z) = c_1\, h(z)\, C_\nu^{(0)}(g(z)) + c_2\, h(z) \sqrt{1-g(z)^2}\ U_{\nu-1}(g(z))$$

07.13.13.0020.01

$$W_z\left(h(z)\, C_\nu^{(0)}(g(z)),\ h(z) \sqrt{1-g(z)^2}\ U_{\nu-1}(g(z))\right) = -\frac{2\, h(z)^2\, g'(z)}{\sqrt{1-g(z)^2}}$$

07.13.13.0021.01

$$z^2\left(a^2 z^{2r} - 1\right) w''(z) + \left(r - (2s-1)\left(a^2 z^{2r} - 1\right)\right) z\, w'(z) + \left(a^2 z^{2r}\left(s^2 - r^2 \nu^2\right) - s\,(r+s)\right) w(z) = 0\ /;$$
$$w(z) = c_1\, z^s\, C_\nu^{(0)}(a\, z^r) + c_2\, z^s \sqrt{1-a^2 z^{2r}}\ U_{\nu-1}(a\, z^r)$$

07.13.13.0022.01

$$W_z\left(z^s\, C_\nu^{(0)}(a\, z^r),\ z^s \sqrt{1-a^2 z^{2r}}\ U_{\nu-1}(a\, z^r)\right) = -\frac{2\,a\,r\, z^{r+2s-1}}{\sqrt{1-a^2 z^{2r}}}$$

07.13.13.0023.01

$$\left(a^2 r^{2z} - 1\right) w''(z) + \left(\log(r) - 2\left(a^2 r^{2z} - 1\right)\log(s)\right) w'(z) + \left(a^2 r^{2z}\left(\log^2(s) - \nu^2 \log^2(r)\right) - \log(s)\,(\log(r) + \log(s))\right) w(z) = 0\ /;$$
$$w(z) = c_1\, s^z\, C_\nu^{(0)}(a\, r^z) + c_2\, s^z \sqrt{1-a^2 r^{2z}}\ U_{\nu-1}(a\, r^z)$$

07.13.13.0024.01

$$W_z\left(s^z\, C_\nu^{(0)}(a\, r^z),\ s^z \sqrt{1-a^2 r^{2z}}\ U_{\nu-1}(a\, r^z)\right) = -\frac{2\,a\, r^z\, s^{2z} \log(r)}{\sqrt{1-a^2 r^{2z}}}$$

Transformations

Transformations and argument simplifications

Argument involving basic arithmetic operations

07.13.16.0001.01

$$C_{-\nu}^{(0)}(z) = C_{\nu}^{(0)}(z)$$

07.13.16.0002.01

$$C_{n}^{(0)}(-z) = (-1)^n C_{n}^{(0)}(z) /; n \in \mathbb{N}^+$$

Products, sums, and powers of the direct function

Products of the direct function

07.13.16.0003.01

$$C_{n}^{(0)}(z)\, C_{m}^{(0)}(z) = \frac{1}{m\, n} \left((n+m)\, C_{n+m}^{(0)}(z) + |n-m|\, C_{|n-m|}^{(0)}(z)\right) /; m \in \mathbb{N}^+ \bigwedge n \in \mathbb{N}^+$$

Related transformations

07.13.16.0004.01

$$C_{n}^{(0)}\left(\frac{m}{2}\, C_{m}^{(0)}(z)\right) = m\, C_{m\,n}^{(0)}(z) /; m \in \mathbb{N}^+ \bigwedge n \in \mathbb{N}^+$$

Identities

Recurrence identities

Consecutive neighbors

07.13.17.0001.01

$$C_{\nu}^{(0)}(z) = \frac{2\,(\nu+1)\, z}{\nu}\, C_{\nu+1}^{(0)}(z) - \frac{\nu+2}{\nu}\, C_{\nu+2}^{(0)}(z)$$

07.13.17.0002.01

$$C_{\nu}^{(0)}(z) = \frac{2\,(\nu-1)\, z}{\nu}\, C_{\nu-1}^{(0)}(z) - \frac{\nu-2}{\nu}\, C_{\nu-2}^{(0)}(z)$$

Distant neighbors

07.13.17.0005.01

$$C_{\nu}^{(0)}(z) = \mathcal{C}_m(\nu, z)\, C_{\nu+m}^{(0)}(z) - \frac{m+\nu+1}{m+\nu-1}\, \mathcal{C}_{m-1}(\nu, z)\, C_{\nu+m+1}^{(0)}(z) /;$$
$$\mathcal{C}_0(\nu, z) = 1 \bigwedge \mathcal{C}_1(\nu, z) = \frac{2\,(\nu+1)\, z}{\nu} \bigwedge \mathcal{C}_m(\nu, z) = \frac{2\, z\,(m+\nu)}{m+\nu-1}\, \mathcal{C}_{m-1}(\nu, z) - \frac{m+\nu}{m+\nu-2}\, \mathcal{C}_{m-2}(\nu, z) \bigwedge m \in \mathbb{N}^+$$

07.13.17.0006.01

$$C_{\nu}^{(0)}(z) = \mathcal{C}_m(\nu, z)\, C_{\nu-m}^{(0)}(z) - \frac{\nu-m-1}{\nu-m+1}\, \mathcal{C}_{m-1}(\nu, z)\, C_{\nu-m-1}^{(0)}(z) /;$$
$$\mathcal{C}_0(\nu, z) = 1 \bigwedge \mathcal{C}_1(\nu, z) = \frac{2\,(\nu-1)\, z}{\nu} \bigwedge \mathcal{C}_m(\nu, z) = \frac{2\, z\,(\nu-m)}{\nu-m+1}\, \mathcal{C}_{m-1}(\nu, z) - \frac{\nu-m}{\nu-m+2}\, \mathcal{C}_{m-2}(\nu, z) \bigwedge m \in \mathbb{N}^+$$

Functional identities

Relations between contiguous functions

07.13.17.0003.01

$$(\nu-1)\, C_{\nu-1}^{(0)}(z) + (\nu+1)\, C_{\nu+1}^{(0)}(z) = 2\, z\, \nu\, C_{\nu}^{(0)}(z)$$

07.13.17.0004.01

$$C_{\nu}^{(0)}(z) = \frac{1}{2\,\nu\,z}\left((\nu-1)\,C_{\nu-1}^{(0)}(z)+(\nu+1)\,C_{\nu+1}^{(0)}(z)\right)$$

07.13.17.0007.01

$$C_{\nu}^{(0)}(z) = \frac{2\,(\nu+1)\,z}{\nu}\,C_{\nu+1}^{(0)}(z)-\frac{\nu+2}{\nu}\,C_{\nu+2}^{(0)}(z)$$

Complex characteristics

Real part

07.13.19.0001.01

$$\mathrm{Re}\left(C_{n}^{(0)}(x+i\,y)\right) = C_{n}^{(0)}(x)+2\sum_{j=1}^{\left\lfloor\frac{n}{2}\right\rfloor}\frac{(-1)^{j}\,2^{2\,j-2}}{j}\,C_{n-2\,j}^{(2\,j)}(x)\,y^{2\,j}\;/;\;x\in\mathbb{R}\land y\in\mathbb{R}\land n\in\mathbb{N}^{+}$$

Imaginary part

07.13.19.0002.01

$$\mathrm{Im}\left(C_{n}^{(0)}(x+i\,y)\right) = 2\sum_{j=0}^{\left\lfloor\frac{n-1}{2}\right\rfloor}\frac{(-1)^{j}\,2^{2\,j}}{2\,j+1}\,C_{n-2\,j-1}^{(2\,j+1)}(x)\,y^{2\,j+1}\;/;\;x\in\mathbb{R}\land y\in\mathbb{R}\land n\in\mathbb{N}^{+}$$

Differentiation

Low-order differentiation

With respect to ν

07.13.20.0001.01

$$\frac{\partial C_{\nu}^{(0)}(z)}{\partial\nu} = -\frac{1}{\nu}\left(2\sqrt{1-z^{2}}\,\cos^{-1}(z)\,U_{\nu-1}(z)+C_{\nu}^{(0)}(z)\right)$$

07.13.20.0002.01

$$\frac{\partial^{2} C_{\nu}^{(0)}(z)}{\partial\nu^{2}} = \frac{1}{\nu^{2}}\left(4\sqrt{1-z^{2}}\,\cos^{-1}(z)\,U_{\nu-1}(z)+\left(2-\nu^{2}\cos^{-1}(z)^{2}\right)C_{\nu}^{(0)}(z)\right)$$

With respect to z

07.13.20.0003.01

$$\frac{\partial C_{\nu}^{(0)}(z)}{\partial z} = 2\,U_{\nu-1}(z)$$

07.13.20.0004.01

$$\frac{\partial^{2} C_{\nu}^{(0)}(z)}{\partial z^{2}} = \frac{\nu^{2}\,C_{\nu}^{(0)}(z)-2\,z\,U_{\nu-1}(z)}{z^{2}-1}$$

Symbolic differentiation

With respect to ν

07.13.20.0005.01

$$\frac{\partial^m C_\nu^{(0)}(z)}{\partial \nu^m} == 2\, m! \sum_{k=0}^{m} \frac{(-1)^{m-k}\, \nu^{k-m-1}}{k!} \cos^{-1}(z)^k \cos\left(\frac{\pi k}{2} + \nu \cos^{-1}(z)\right) /;\, m \in \mathbb{N}$$

07.13.20.0006.02

$$\frac{\partial^m C_\nu^{(0)}(z)}{\partial \nu^m} == \nu^{-m-1} \left(e^{i m \pi}\, \Gamma\left(m+1, -i\, \nu \cos^{-1}(z)\right) + e^{-i m \pi}\, \Gamma\left(m+1, i\, \nu \cos^{-1}(z)\right)\right) /;\, m \in \mathbb{N}$$

07.13.20.0007.02

$$\frac{\partial^m C_\nu^{(0)}(z)}{\partial \nu^m} == 2\,(-1)^m\, m! \sum_{k=0}^{m} \frac{(-i)^k\, \nu^{k-m-1} \cos^{-1}(z)^k}{k!} \left(T_\nu(z) - \left(T_\nu(z) - i\sqrt{1-z^2}\; U_{\nu-1}(z)\right)(k \bmod 2)\right) /;\, m \in \mathbb{N}$$

With respect to z

07.13.20.0008.01

$$\frac{\partial^m C_\nu^{(0)}(z)}{\partial z^m} == 2^m\, (m-1)!\, C_{\nu-m}^m(z) /;\, m \in \mathbb{N}^+$$

07.13.20.0009.02

$$\frac{\partial^m C_\nu^{(0)}(z)}{\partial z^m} == \frac{2\sqrt{\pi}}{\nu} (z-1)^{-m}\, {}_3\tilde{F}_2\left(1, -\nu, \nu; \frac{1}{2}, 1-m; \frac{1-z}{2}\right) /;\, m \in \mathbb{N}$$

Fractional integro-differentiation

With respect to ν

07.13.20.0010.01

$$\frac{\partial^\alpha C_\nu^{(0)}(z)}{\partial \nu^\alpha} == 2\, \mathcal{FC}_{\exp}^{(\alpha)}(\nu, -1)\, \nu^{-\alpha-1} - 2^{\alpha-1}\, \nu^{1-\alpha} \sqrt{\pi}\, \cos^{-1}(z)^2\, {}_2\tilde{F}_3\left(1, 1; 2, 1-\frac{\alpha}{2}, \frac{3-\alpha}{2}; -\frac{1}{4} \nu^2 \cos^{-1}(z)^2\right)$$

With respect to z

07.13.20.0011.01

$$\frac{\partial^\alpha C_\nu^{(0)}(z)}{\partial z^\alpha} == \frac{2\sqrt{\pi}}{\nu} z^{-\alpha}\, \tilde{F}_{1\times1\times0}^{2\times1\times0}\left(\begin{matrix} -\nu, \nu; 1; ; \\ \frac{1}{2}; 1-\alpha; ; \end{matrix} -\frac{z}{2}, \frac{1}{2}\right)$$

Integration

Indefinite integration

Involving only one direct function

07.13.21.0001.01

$$\int C_\nu^{(0)}(z)\, dz == \frac{1}{2\nu} \left(C_{\nu+1}^{(0)}(z) - C_{\nu-1}^{(0)}(z)\right)$$

Involving one direct function and elementary functions

Involving power function

07.13.21.0002.01

$$\int z^{\alpha-1} C_\nu^{(0)}(z)\,dz = \frac{2^{-\alpha}}{\nu} z^{\alpha-1} \left(T_{\alpha-1}(z) + i\sqrt{1-z^2}\; U_{\alpha-2}(z)\right)\left(z\left(z+i\sqrt{1-z^2}\right)\right)^{1-\alpha}$$
$$\left(\frac{1}{\alpha-\nu}\left(T_{\nu-\alpha}(z) + i\sqrt{1-z^2}\; U_{-\alpha+\nu-1}(z)\right) {}_2F_1\!\left(\frac{\nu-\alpha}{2}, 1-\alpha; \frac{\nu-\alpha}{2}+1; -2z^2 - 2i\sqrt{1-z^2}\; z+1\right) + \right.$$
$$\frac{1}{\alpha+\nu}\left(T_{\alpha+\nu}(z) - i\sqrt{1-z^2}\; U_{\alpha+\nu-1}(z)\right) {}_2F_1\!\left(-\frac{\alpha+\nu}{2}, 1-\alpha; 1-\frac{\alpha+\nu}{2}; -2z^2 - 2i\sqrt{1-z^2}\; z+1\right) +$$
$$\frac{1}{\nu-\alpha+2}\left(T_{2-\alpha+\nu}(z) + i\sqrt{1-z^2}\; U_{1-\alpha+\nu}(z)\right) {}_2F_1\!\left(\frac{\nu-\alpha}{2}+1, 1-\alpha; \frac{\nu-\alpha}{2}+2; -2z^2 - 2i\sqrt{1-z^2}\; z+1\right) -$$
$$\left.\frac{1}{\alpha+\nu-2}\left(T_{2-\alpha-\nu}(z) + i\sqrt{1-z^2}\; U_{1-\alpha-\nu}(z)\right) {}_2F_1\!\left(1-\frac{\alpha+\nu}{2}, 1-\alpha; 2-\frac{\alpha+\nu}{2}; -2z^2 - 2i\sqrt{1-z^2}\; z+1\right)\right)$$

Involving only one direct function with respect to ν

07.13.21.0003.01

$$\int C_\nu^{(0)}(z)\,d\nu = 2\,\mathrm{Ci}\!\left(\nu\cos^{-1}(z)\right)$$

Involving one direct function and elementary functions with respect to ν

Involving power function

07.13.21.0004.01

$$\int \nu^{\alpha-1} C_\nu^{(0)}(z)\,d\nu = \nu^{\alpha-1}\left(-\left(-i\nu\cos^{-1}(z)\right)^{1-\alpha}\Gamma\!\left(\alpha-1, -i\nu\cos^{-1}(z)\right) - \left(i\nu\cos^{-1}(z)\right)^{1-\alpha}\Gamma\!\left(\alpha-1, i\nu\cos^{-1}(z)\right)\right)$$

Definite integration

Involving the direct function

07.13.21.0005.01

$$\mathcal{P}\int_{-1}^{1}\frac{C_n^{(0)}(t)}{\sqrt{1-t^2}\,(t-x)}\,dt = \frac{2\pi}{n} U_{n-1}(x) \;/;\; n-1\in\mathbb{N}^+ \bigwedge -1<x<1$$

07.13.21.0006.01

$$\int_{-1}^{1}\frac{C_m^{(0)}(z)\,C_n^{(0)}(z)}{\sqrt{1-t^2}}\,dt = \frac{2\pi}{n^2}\delta_{m,n} \;/;\; m\in\mathbb{N}^+ \bigwedge n\in\mathbb{N}^+$$

Summation

Infinite summation

07.13.23.0001.01

$$\sum_{n=1}^{\infty} n\, C_n^{(0)}(z)\, w^n = \frac{2w(z-w)}{w^2-2zw+1} \;/;\; -1<z<1 \land |w|<1$$

07.13.23.0002.01

$$\sum_{n=1}^{\infty} C_n^{(0)}(z)\, w^n = -\log\!\left(w^2-2zw+1\right) \;/;\; -1<z<1 \land |w|<1$$

Representations through more general functions

Through hypergeometric functions

Involving $_2F_1$

07.13.26.0001.01

$$C_\nu^{(0)}(z) = \frac{2}{\nu}\, {}_2F_1\left(-\nu, \nu; \frac{1}{2}; \frac{1-z}{2}\right)$$

07.13.26.0002.01

$$C_\nu^{(0)}(z) = \frac{2}{\nu}\cos(\nu\pi)\, {}_2F_1\left(-\nu, \nu; \frac{1}{2}; \frac{z+1}{2}\right) + 2\sqrt{2}\,\sin(\nu\pi)\sqrt{z+1}\, {}_2F_1\left(\nu+\frac{1}{2}, \frac{1}{2}-\nu; \frac{3}{2}; \frac{z+1}{2}\right)$$

07.13.26.0003.01

$$C_\nu^{(0)}(z) = \frac{2^{-\nu}}{\nu}(z-1)^{-\nu}\, {}_2F_1\left(\nu, \nu+\frac{1}{2}; 2\nu+1; \frac{2}{1-z}\right) + \frac{2^\nu}{\nu}(z-1)^\nu\, {}_2F_1\left(-\nu, \frac{1}{2}-\nu; 1-2\nu; \frac{2}{1-z}\right) /; z \notin (0, 1) \bigwedge 2\nu \notin \mathbb{Z}$$

Through hypergeometric functions of two variables

07.13.26.0004.01

$$C_\nu^{(0)}(z) = F_{1\times 0\times 0}^{2\times 0\times 0}\left(\begin{matrix} -\nu, \nu;;; \\ \frac{1}{2};;; \end{matrix}\ \frac{1}{2}, -\frac{z}{2}\right)$$

Through Meijer G

Classical cases for the direct function itself

07.13.26.0005.01

$$C_\nu^{(0)}(z) = -\frac{2\sin(\pi\nu)}{\sqrt{\pi}}\, G_{2,2}^{1,2}\left(\frac{z-1}{2} \middle| \begin{matrix} \nu+1, 1-\nu \\ 0, \frac{1}{2} \end{matrix}\right) /; \nu \notin \mathbb{Z}$$

07.13.26.0006.01

$$C_n^{(0)}(z) = -\frac{2}{\sqrt{\pi}} \lim_{m\to n} \sin(\pi m)\, G_{2,2}^{1,2}\left(\frac{z-1}{2} \middle| \begin{matrix} m+1, 1-m \\ 0, \frac{1}{2} \end{matrix}\right) /; n \in \mathbb{Z}$$

07.13.26.0007.01

$$C_\nu^{(0)}(2z+1) = -\frac{2\sin(\pi\nu)}{\sqrt{\pi}}\, G_{2,2}^{1,2}\left(z \middle| \begin{matrix} \nu+1, 1-\nu \\ 0, \frac{1}{2} \end{matrix}\right) /; \nu \notin \mathbb{Z}$$

Classical cases involving algebraic functions

07.13.26.0008.01

$$(z+1)^{-\nu}\, C_\nu^{(0)}\left(\frac{1-z}{1+z}\right) = \frac{2^{2\nu+1}}{\Gamma(2\nu+1)}\, G_{2,2}^{1,2}\left(z \middle| \begin{matrix} \frac{1}{2}-\nu, 1-\nu \\ 0, \frac{1}{2} \end{matrix}\right) /; z \notin (-\infty, -1)$$

07.13.26.0009.01

$$(z+1)^{-\nu}\, C_\nu^{(0)}\left(\frac{z-1}{z+1}\right) = \frac{2^{2\nu+1}}{\Gamma(2\nu+1)}\, G_{2,2}^{2,1}\left(z \middle| \begin{matrix} 1-\nu, \frac{1}{2}-\nu \\ 0, \frac{1}{2} \end{matrix}\right) /; z \notin (-1, 0)$$

07.13.26.0010.01

$$(z+1)^{-\frac{\nu}{2}} C_{\nu}^{(0)}\left(\frac{1}{\sqrt{z+1}}\right) = \frac{2^{\nu}}{\Gamma(\nu+1)} G_{2,2}^{1,2}\left(z \middle| \begin{array}{c} 1-\frac{\nu}{2}, \frac{1-\nu}{2} \\ 0, \frac{1}{2} \end{array}\right)$$

07.13.26.0011.01

$$(z+1)^{-\frac{\nu}{2}} C_{\nu}^{(0)}\left(\sqrt{\frac{z}{z+1}}\right) = \frac{2^{\nu}}{\Gamma(\nu+1)} G_{2,2}^{2,1}\left(z \middle| \begin{array}{c} 1-\frac{\nu}{2}, \frac{1-\nu}{2} \\ 0, \frac{1}{2} \end{array}\right) /; z \notin (-1, 0)$$

Classical cases involving unit step θ

07.13.26.0012.01

$$\frac{\theta(1-|z|)}{\sqrt{1-z^2}} C_{\nu}^{(0)}(z) = \frac{2\sqrt{\pi}}{\nu} G_{2,2}^{2,0}\left(z, \frac{1}{2} \middle| \begin{array}{c} \frac{\nu+1}{2}, \frac{1-\nu}{2} \\ 0, \frac{1}{2} \end{array}\right) /; \nu \notin \mathbb{Z}$$

07.13.26.0013.01

$$\frac{\theta(1-|z|)}{\sqrt{1-z}} C_{\nu}^{(0)}(z) = \frac{2\sqrt{\pi}}{\nu} G_{2,2}^{2,0}\left(z \middle| \begin{array}{c} \frac{1}{2}-\nu, \nu+\frac{1}{2} \\ 0, \frac{1}{2} \end{array}\right) /; z \notin (-1, 0)$$

07.13.26.0014.01

$$\frac{\theta(|z|-1)}{\sqrt{z-1}} C_{\nu}^{(0)}(2z-1) = \frac{2\sqrt{\pi}}{\nu} G_{2,2}^{0,2}\left(z \middle| \begin{array}{c} \frac{1}{2}-\nu, \nu+\frac{1}{2} \\ 0, \frac{1}{2} \end{array}\right)$$

07.13.26.0015.01

$$\frac{\theta(1-|z|)}{\sqrt{1-z}} C_{\nu}^{(0)}\left(\frac{2}{z}-1\right) = \frac{2\sqrt{\pi}}{\nu} G_{2,2}^{2,0}\left(z \middle| \begin{array}{c} 0, \frac{1}{2} \\ -\nu, \nu \end{array}\right)$$

07.13.26.0016.01

$$\frac{\theta(|z|-1)}{\sqrt{z-1}} C_{\nu}^{(0)}\left(\frac{2}{z}-1\right) = \frac{2\sqrt{\pi}}{\nu} G_{2,2}^{0,2}\left(z \middle| \begin{array}{c} 0, \frac{1}{2} \\ -\nu, \nu \end{array}\right) /; z \notin (-\infty, -1)$$

07.13.26.0017.01

$$\frac{\theta(|z|-1)}{\sqrt{z-1}} C_{\nu}^{(0)}\left(8z^2-8z+1\right) = \frac{2\sqrt{\pi}}{\nu} G_{2,2}^{0,2}\left(z \middle| \begin{array}{c} \frac{1}{2}-2\nu, 2\nu+\frac{1}{2} \\ 0, \frac{1}{2} \end{array}\right) /; \operatorname{Re}(z) > 0$$

07.13.26.0018.01

$$\frac{\theta(1-|z|)}{\sqrt{1-z}} C_{\nu}^{(0)}\left(\frac{8}{z^2}-\frac{8}{z}+1\right) = \frac{2\sqrt{\pi}}{\nu} G_{2,2}^{2,0}\left(z \middle| \begin{array}{c} 0, \frac{1}{2} \\ -2\nu, 2\nu \end{array}\right) /; \operatorname{Re}(z) > 0$$

Generalized cases involving algebraic functions

07.13.26.0019.01

$$\left(z^2+1\right)^{-\frac{\nu}{2}} C_{\nu}^{(0)}\left(\frac{z}{\sqrt{z^2+1}}\right) = \frac{2^{\nu}}{\Gamma(\nu+1)} G_{2,2}^{2,1}\left(z, \frac{1}{2} \middle| \begin{array}{c} 1-\frac{\nu}{2}, \frac{1-\nu}{2} \\ 0, \frac{1}{2} \end{array}\right) /; \operatorname{Re}(z) > 0$$

Generalized cases involving unit step θ

07.13.26.0020.01

$$\frac{\theta(1-|z|)}{\sqrt{1-z^2}}\, C_\nu^{(0)}(z) = \frac{2\sqrt{\pi}}{\nu}\, G_{2,2}^{2,0}\left(z, \frac{1}{2} \,\middle|\, \begin{array}{c} \frac{\nu+1}{2}, \frac{1-\nu}{2} \\ 0, \frac{1}{2} \end{array}\right) /; \nu \notin \mathbb{Z}$$

07.13.26.0021.01

$$\frac{\theta(|z|-1)}{\sqrt{z^2-1}}\, C_\nu^{(0)}(z) = \frac{2\sqrt{\pi}}{\nu}\, \sqrt{z^2}\, G_{2,2}^{0,2}\left(z, \frac{1}{2} \,\middle|\, \begin{array}{c} -\frac{\nu}{2}, \frac{\nu}{2} \\ -\frac{1}{2}, 0 \end{array}\right)$$

07.13.26.0022.01

$$\frac{\theta(1-|z|)}{\sqrt{1-z^2}}\, C_\nu^{(0)}\left(\frac{1}{z}\right) = \frac{2\sqrt{\pi}}{\nu}\, G_{2,2}^{2,0}\left(z, \frac{1}{2} \,\middle|\, \begin{array}{c} 0, \frac{1}{2} \\ -\frac{\nu}{2}, \frac{\nu}{2} \end{array}\right)$$

07.13.26.0023.01

$$\frac{\theta(|z|-1)}{\sqrt{z^2-1}}\, C_\nu^{(0)}\left(\frac{1}{z}\right) = \frac{2\sqrt{\pi}}{\nu}\, \sqrt{z^2}\, G_{2,2}^{0,2}\left(z, \frac{1}{2} \,\middle|\, \begin{array}{c} -\frac{1}{2}, 0 \\ -\frac{\nu+1}{2}, \frac{\nu-1}{2} \end{array}\right)$$

Through other functions

Involving some hypergeometric-type functions

07.13.26.0024.01

$$C_\nu^{(0)}(z) = \sqrt{2\pi}\, \frac{\left(1-z^2\right)^{1/4}}{\nu}\, P_{\nu-\frac{1}{2}}^{\frac{1}{2}}(z)$$

07.13.26.0025.01

$$C_\nu^{(0)}(z) = \frac{\sqrt{2\pi}}{\nu}\, (z+1)^{1/4}\, (z-1)^{1/4}\, \mathcal{P}_{\nu-\frac{1}{2}}^{\frac{1}{2}}(z)$$

07.13.26.0026.01

$$C_\nu^{(0)}(z) = \frac{2\,(\nu-1)!}{\left(\frac{1}{2}\right)_\nu}\, P_\nu^{\left(-\frac{1}{2},-\frac{1}{2}\right)}(z)$$

07.13.26.0027.01

$$C_\nu(z) = \lim_{\lambda\to 0} \frac{1}{\lambda}\, C_\nu^\lambda(z)$$

Representations through equivalent functions

With related functions

07.13.27.0001.01

$$C_\nu^{(0)}(z) = \frac{2}{\nu}\, T_\nu(z)$$

07.13.27.0002.01

$$C_\nu^{(0)}(z) = \frac{2}{\nu}\, \left(U_\nu(z) - z\, U_{\nu-1}(z)\right)$$

07.13.27.0003.01

$$C_\nu^{(0)}(z) = \frac{1}{\nu}\left(U_\nu(z) - U_{\nu-2}(z)\right)$$

07.13.27.0004.01

$$C_\nu^{(0)}(z) = \frac{2}{\nu^2}\left(\frac{\partial\left(\left(z^2-1\right)U_{n-1}(z)\right)}{\partial z} - z\,U_{n-1}(z)\right)$$

With elementary functions

07.13.27.0005.01

$$C_\nu^{(0)}(z) = \frac{1}{\nu}\left(e^{\frac{i\pi\nu}{2}}\left(i\,z + \sqrt{1-z^2}\right)^{-\nu} + e^{-\frac{i\pi\nu}{2}}\left(i\,z + \sqrt{1-z^2}\right)^{\nu}\right)$$

07.13.27.0006.01

$$C_\nu^{(0)}(z) = \frac{2}{\nu}\cos\left(\nu\cos^{-1}(z)\right)$$

07.13.27.0007.01

$$C_n^{(0)}(z) = \frac{2^{-n}(-1)^n}{n}\left(\left(\sqrt{1-z} - \sqrt{-z-1}\right)^{2n} + \left(\sqrt{1-z} + \sqrt{-z-1}\right)^{2n}\right) /;\ n \in \mathbb{N}^+$$

07.13.27.0008.01

$$C_n^{(0)}(z) = \frac{1}{n}z^n\left(\left(1 - \sqrt{1-\frac{1}{z^2}}\right)^n + \left(1 + \sqrt{1-\frac{1}{z^2}}\right)^n\right) /;\ n \in \mathbb{N}^+$$

History

– F. G. Tricomi (1955)

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