

Glaisher

View the online version at
● functions.wolfram.com

Download the
● PDF File

Notations

Traditional name

Glaisher constant

Traditional notation

A

Mathematica StandardForm notation

Glaisher

Primary definition

02.08.02.0001.01

$$A = \exp\left(\frac{1}{12} - \zeta'(-1)\right)$$

Specific values

02.08.03.0001.01

$A = 1.28242712910062263687534256886979172776768892732500119206374002174040630885882646112973649 \dots$

Above approximate numerical value of A shows 90 decimal digits.

General characteristics

The Glaisher number A is a constant. It is a positive real number.

Series representations

02.08.06.0001.01

$$\log(A) = \frac{1}{2} \sum_{k=0}^{\infty} \frac{1}{k+1} \sum_{j=0}^k (-1)^{j+1} \binom{k}{j} (j+1)^2 \log(j+1) + \frac{1}{8}$$

Integral representations

On the real axis

Of the direct function

02.08.07.0001.01

$$A = \frac{2^{7/36}}{\sqrt[6]{\pi}} \exp\left(\frac{2}{3} \int_0^{\frac{1}{2}} \log(\Gamma(t+1)) dt + \frac{1}{3}\right)$$

Product representations

02.08.08.0001.01

$$A = e^{\gamma/12} \sqrt[12]{2\pi} \left(\prod_{k=1}^{\infty} k^{k^2} \right)^{\frac{1}{2\pi^2}}$$

02.08.08.0002.01

$$A = \sqrt[9]{2} e^{\gamma/12} \sqrt[12]{\pi} \left(\prod_{k=1}^{\infty} (2k+1)^{\frac{1}{(2k+1)^2}} \right)^{\frac{2}{3\pi^2}}$$

02.08.08.0003.01

$$A = 2^{9/32} e^{\frac{3\zeta(3)}{64\pi^2} + \zeta'(-1) - 2\zeta^{(1,0)}(-2, \frac{1}{4}) + \frac{29}{192} + \frac{\gamma}{96}} \sqrt[32]{\pi} \left(\frac{\prod_{k=1}^{\infty} (4k+1)^{\frac{1}{(4k+1)^3}}}{\prod_{j=1}^{\infty} (4j+3)^{\frac{1}{(4j+3)^3}} \right)^{\frac{1}{\pi^3}}$$

Limit representations

02.08.09.0001.01

$$A = \lim_{n \rightarrow \infty} n^{-\frac{n^2}{2} - \frac{n}{2} - \frac{1}{12}} e^{\frac{n^2}{4}} \prod_{k=1}^n k^k$$

02.08.09.0002.01

$$A = \lim_{n \rightarrow \infty} e^{\frac{n^2}{4}} n^{-\frac{n^2}{2} - \frac{n}{2} - \frac{1}{12}} \Gamma(n+1)^n \prod_{k=1}^n \frac{1}{\Gamma(k)}$$

02.08.09.0003.01

$$A = \lim_{n \rightarrow \infty} \exp\left(\frac{1}{12} \log(2\pi) - \frac{1}{2\pi^2} \left(2^{1 - \lceil n \log_8(10) + 1 \rceil} \sum_{j=0}^{2 \lceil n \log_8(10) + 1 \rceil - 1} \frac{1}{(j+1)^2} \log(2(j+1)) (-1)^j \left(\sum_{k=0}^{j - \lceil n \log_8(10) + 1 \rceil} \binom{\lceil n \log_8(10) + 1 \rceil}{k} - 2^{\lceil n \log_8(10) + 1 \rceil} \right) \right) + \frac{\gamma}{12} \right)$$

The above formula is used for the numerical computation of Glaisher's constant in *Mathematica*.

Complex characteristics

Real part

02.08.19.0001.01

Re(A) = A

Imaginary part

02.08.19.0002.01

$$\operatorname{Im}(A) = 0$$

Absolute value

02.08.19.0003.01

$$|A| = A$$

Argument

02.08.19.0004.01

$$\operatorname{arg}(A) = 0$$

Conjugate value

02.08.19.0005.01

$$\bar{A} = A$$

Signum value

02.08.19.0006.01

$$\operatorname{sgn}(A) = 1$$

Differentiation

Low-order differentiation

02.08.20.0001.01

$$\frac{\partial A}{\partial z} = 0$$

Fractional integro-differentiation

02.08.20.0002.01

$$\frac{\partial^\alpha A}{\partial z^\alpha} = \frac{z^{-\alpha} A}{\Gamma(1 - \alpha)}$$

Integration

Indefinite integration

02.08.21.0001.01

$$\int A dz = A z$$

02.08.21.0002.01

$$\int z^{\alpha-1} A dz = \frac{z^\alpha A}{\alpha}$$

Integral transforms

Fourier exp transforms

02.08.22.0001.01

$$\mathcal{F}_i[A](z) = \sqrt{2\pi} A \delta(z)$$

Inverse Fourier exp transforms

02.08.22.0002.01

$$\mathcal{F}_i^{-1}[A](z) = \sqrt{2\pi} A \delta(z)$$

Fourier cos transforms

02.08.22.0003.01

$$\mathcal{F}_c[A](z) = \sqrt{\frac{\pi}{2}} A \delta(z)$$

Fourier sin transforms

02.08.22.0004.01

$$\mathcal{F}_s[A](z) = \sqrt{\frac{2}{\pi}} \frac{A}{z}$$

Laplace transforms

02.08.22.0005.01

$$\mathcal{L}_i[A](z) = \frac{A}{z}$$

Inverse Laplace transforms

02.08.22.0006.01

$$\mathcal{L}_i^{-1}[A](z) = A \delta(z)$$

Representations through more general functions

Through Meijer G

02.08.26.0002.01

$$A = A G_{0,1}^{1,0}(z | 0) + A G_{1,2}^{1,1}\left(z \left| \begin{matrix} 1 \\ 1, 0 \end{matrix} \right. \right)$$

Through other functions

02.08.26.0001.01

$$A = \exp\left(\frac{1}{12} - \zeta'(-1)\right)$$

$$A = \exp\left(\frac{96\pi\zeta^{(1,0)}\left(-1, \frac{1}{4}\right) + \pi - 24C}{12\pi}\right)$$

$$A = e^{\frac{\gamma}{12} - \frac{\zeta'(2)}{2\pi^2} \sqrt[12]{2\pi}}$$

Inequalities

$$\frac{5}{4} < A < \frac{13}{10}$$

History

- H. Kinkelin (1860)
- J. W. L. Glaisher (1877–1878)

Copyright

This document was downloaded from functions.wolfram.com, a comprehensive online compendium of formulas involving the special functions of mathematics. For a key to the notations used here, see <http://functions.wolfram.com/Notations/>.

Please cite this document by referring to the functions.wolfram.com page from which it was downloaded, for example:

<http://functions.wolfram.com/Constants/E/>

To refer to a particular formula, cite functions.wolfram.com followed by the citation number.

e.g.: <http://functions.wolfram.com/01.03.03.0001.01>

This document is currently in a preliminary form. If you have comments or suggestions, please email comments@functions.wolfram.com.

© 2001-2008, Wolfram Research, Inc.