Glaisher

Notations

Traditional name

Glaisher constant

Traditional notation

A

Mathematica StandardForm notation

`Glaisher`

Primary definition

02.08.02.0001.01

$$A = \exp\left(\frac{1}{12} - \zeta'(-1)\right)$$

Specific values

02.08.03.0001.01

$$A = 1.28242712910062263687534256886979172776768892732500119206374002174040630885882646112973649\ldots$$

Above approximate numerical value of A shows 90 decimal digits.

General characteristics

The Glaisher number A is a constant. It is a positive real number.

Series representations

02.08.06.0001.01

$$\log(A) = \frac{1}{2} \sum_{k=0}^{\infty} \frac{1}{k+1} \sum_{j=0}^{k} (-1)^{j+1} \binom{k}{j} (j+1)^2 \log(j+1) + \frac{1}{8}$$

Integral representations

On the real axis

Of the direct function

02.08.07.0001.01

$$A = \frac{2^{7/36}}{\sqrt[6]{\pi}} \exp\left(\frac{2}{3}\int_0^{\frac{1}{2}} \log(\Gamma(t+1))\, dt + \frac{1}{3}\right)$$

Product representations

02.08.08.0001.01

$$A = e^{\gamma/12}\, \sqrt[12]{2\pi} \left(\prod_{k=1}^{\infty} k^{\frac{1}{k^2}}\right)^{\frac{1}{2\pi^2}}$$

02.08.08.0002.01

$$A = \sqrt[9]{2}\; e^{\gamma/12}\, \sqrt[12]{\pi} \left(\prod_{k=1}^{\infty} (2k+1)^{\frac{1}{(2k+1)^2}}\right)^{\frac{2}{3\pi^2}}$$

02.08.08.0003.01

$$A = 2^{9/32}\, e^{\frac{3\zeta(3)}{64\pi^2} + \zeta'(-1) - 2\zeta^{(1,0)}\left(-2, \frac{1}{4}\right) + \frac{29}{192} + \frac{\gamma}{96}}\, \sqrt[32]{\pi} \left(\frac{\prod_{k=1}^{\infty} (4k+1)^{\frac{1}{(4k+1)^3}}}{\prod_{j=1}^{\infty} (4j+3)^{\frac{1}{(4j+3)^3}}}\right)^{\frac{1}{\pi^3}}$$

Limit representations

02.08.09.0001.01

$$A = \lim_{n\to\infty} n^{-\frac{n^2}{2} - \frac{n}{2} - \frac{1}{12}}\, e^{\frac{n^2}{4}} \prod_{k=1}^{n} k^k$$

02.08.09.0002.01

$$A = \lim_{n\to\infty} e^{\frac{n^2}{4}}\, n^{-\frac{n^2}{2} - \frac{n}{2} - \frac{1}{12}}\, \Gamma(n+1)^n \prod_{k=1}^{n} \frac{1}{\Gamma(k)}$$

02.08.09.0003.01

$$A = \lim_{n\to\infty} \exp\left(\frac{1}{12}\log(2\pi) - \frac{1}{2\pi^2}\left(2^{1-\lceil n\log_8(10)+1\rceil} \sum_{j=0}^{2\lceil n\log_8(10)+1\rceil - 1} \frac{1}{(j+1)^2} \log(2(j+1))\, (-1)^j \left(\sum_{k=0}^{j-\lceil n\log_8(10)+1\rceil} \binom{\lceil n\log_8(10)+1\rceil}{k} - 2^{\lceil n\log_8(10)+1\rceil}\right)\right) + \frac{\gamma}{12}\right)$$

The above formula is used for the numerical computation of Glaisher's constant in *Mathematica*.

Complex characteristics

Real part

02.08.19.0001.01

$$\operatorname{Re}(A) = A$$

Imaginary part

02.08.19.0002.01

$\operatorname{Im}(A) = 0$

Absolute value

02.08.19.0003.01

$|A| = A$

Argument

02.08.19.0004.01

$\arg(A) = 0$

Conjugate value

02.08.19.0005.01

$\overline{A} = A$

Signum value

02.08.19.0006.01

$\operatorname{sgn}(A) = 1$

Differentiation

Low-order differentiation

02.08.20.0001.01

$$\frac{\partial A}{\partial z} = 0$$

Fractional integro-differentiation

02.08.20.0002.01

$$\frac{\partial^\alpha A}{\partial z^\alpha} = \frac{z^{-\alpha} A}{\Gamma(1-\alpha)}$$

Integration

Indefinite integration

02.08.21.0001.01

$$\int A \, dz = A z$$

02.08.21.0002.01

$$\int z^{\alpha-1} A \, dz = \frac{z^\alpha A}{\alpha}$$

Integral transforms

Fourier exp transforms

02.08.22.0001.01

$$\mathcal{F}_t[A](z) == \sqrt{2\pi}\; A\,\delta(z)$$

Inverse Fourier exp transforms

02.08.22.0002.01

$$\mathcal{F}_t^{-1}[A](z) == \sqrt{2\pi}\; A\,\delta(z)$$

Fourier cos transforms

02.08.22.0003.01

$$\mathcal{F}c_t[A](z) == \sqrt{\frac{\pi}{2}}\; A\,\delta(z)$$

Fourier sin transforms

02.08.22.0004.01

$$\mathcal{F}s_t[A](z) == \sqrt{\frac{2}{\pi}}\; \frac{A}{z}$$

Laplace transforms

02.08.22.0005.01

$$\mathcal{L}_t[A](z) == \frac{A}{z}$$

Inverse Laplace transforms

02.08.22.0006.01

$$\mathcal{L}_t^{-1}[A](z) == A\,\delta(z)$$

Representations through more general functions

Through Meijer *G*

02.08.26.0002.01

$$A = A\,G_{0,1}^{1,0}(z \mid 0) + A\,G_{1,2}^{1,1}\left(z \,\middle|\, \begin{matrix} 1 \\ 1,\, 0 \end{matrix}\right)$$

Through other functions

02.08.26.0001.01

$$A == \exp\left(\frac{1}{12} - \zeta'(-1)\right)$$

02.08.26.0003.01

$$A == \exp\left(\frac{96\,\pi\,\zeta^{(1,0)}\left(-1,\,\frac{1}{4}\right)+\pi-24\,C}{12\,\pi}\right)$$

02.08.26.0004.01

$$A == e^{\frac{\gamma}{12}-\frac{\zeta'(2)}{2\pi^2}}\,\sqrt[12]{2\,\pi}$$

Inequalities

02.08.29.0001.01

$$\frac{5}{4} < A < \frac{13}{10}$$

History

–H. Kinkelin (1860)

–J. W. L. Glaisher (1877–1878)

Copyright