GoldenRatio

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Notations

Traditional name

Golden ratio

Traditional notation

ϕ

Mathematica StandardForm notation

`GoldenRatio`

Primary definition

02.02.02.0001.01

$$\phi == \frac{1}{2}\left(1+\sqrt{5}\right) == \exp\left(\operatorname{csch}^{-1}(2)\right)$$

Specific values

02.02.03.0001.01

$$\phi == 1.618033988749894848204586834365638117720309179805762862135448622705260462818902449707207204\ldots$$

Above approximate numerical value of ϕ shows 90 decimal digits.

General characteristics

The golden ratio ϕ is a constant. It is a positive quadratic irrational real number.

Limit representations

02.02.09.0001.01

$$\phi == \sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\ldots}}}}$$

02.02.09.0002.01

$$\phi == \lim_{n\to\infty} z_n \;/;\; z_{n+1} = \sqrt{1+z_n} \bigwedge z_0 == 1$$

02.02.09.0003.01

$$\phi == \lim_{\nu \to \infty} \frac{F_\nu}{F_{\nu-1}}$$

02.02.09.0004.01

$$\phi == \lim_{n \to \infty} \frac{\sum_{k=0}^{m-1} F_{\nu+k}}{F_{m+\nu} - F_\nu} = \phi \,/;\, m \in \mathbb{N}^+$$

02.02.09.0005.01

$$\phi = \lim_{\nu \to \infty} \frac{L_{\nu+1}}{L_\nu}$$

02.02.09.0006.01

$$\phi = \lim_{\nu \to \infty} \frac{\sum_{k=0}^{m-1} L_{k+\nu}}{L_{m+\nu} - L_\nu} \,/;\, m \in \mathbb{N}^+$$

Continued fraction representations

02.02.10.0001.01

$$\phi == 1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \ldots}}}}$$

02.02.10.0002.01

$$\phi == 1 + \mathrm{K}_k(1, 1)_1^\infty$$

Identities

Functional identities

02.02.17.0001.01

$$\phi^2 - \phi - 1 == 0$$

02.02.17.0002.01

$$\phi == 1 + \frac{1}{\phi}$$

02.02.17.0003.01

$$\phi == 1 + \frac{1}{1 + \frac{1}{\phi}}$$

02.02.17.0004.01

$$\phi == 1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{\phi}}}$$

02.02.17.0005.01

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02.02.17.0006.01

$$\phi == 1 + \frac{1}{\phi} == 1 + \frac{1}{1 + \frac{1}{\phi}} == 1 + \frac{1}{1 + \frac{1}{1+\frac{1}{\phi}}} == 1 + \frac{1}{1 + \frac{1}{1+\frac{1}{1+\frac{1}{\phi}}}} == \ldots$$

02.02.17.0007.01

$$\phi^{\phi^{2+\phi}} == \phi^{\phi^{\phi}\,(1+\phi)} == \phi^{\phi^{1+\phi^2}} == \phi^{\phi^{2+\phi}}$$

Udaya Chinthaka Jayatilake

02.02.17.0008.01

$$\phi^n == \phi^{n-1} + \phi^{n-2} \;/;\; n \in \mathbb{N}^+$$

Above Fibonacci recurrence allows any polynomial in ϕ to be reduced to a linear expression.

02.02.17.0009.01

$$\phi^n = F_{n-1} + \phi\, F_n \;/;\; n \in \mathbb{N}$$

Complex characteristics

Real part

02.02.19.0001.01

$$\mathrm{Re}(\phi) == \phi$$

Imaginary part

02.02.19.0002.01

$$\mathrm{Im}(\phi) == 0$$

Absolute value

02.02.19.0003.01

$$|\phi| == \phi$$

Argument

02.02.19.0004.01

$$\arg(\phi) == 0$$

Conjugate value

02.02.19.0005.01

$$\overline{\phi} == \phi$$

Signum value

02.02.19.0006.01

$$\operatorname{sgn}(\phi) == 1$$

Differentiation

Low-order differentiation

02.02.20.0001.01

$$\frac{\partial \phi}{\partial z} == 0$$

Fractional integro-differentiation

02.02.20.0002.01

$$\frac{\partial^{\alpha} \phi}{\partial z^{\alpha}} == \frac{z^{-\alpha}\, \phi}{\Gamma(1-\alpha)}$$

Integration

Indefinite integration

02.02.21.0001.01

$$\int \phi \, dz == \phi\, z$$

02.02.21.0002.01

$$\int z^{\alpha-1}\, \phi \, dz == \frac{z^{\alpha}\, \phi}{\alpha}$$

Integral transforms

Fourier exp transforms

02.02.22.0001.01

$$\mathcal{F}_t[\phi](z) == \phi \sqrt{2\pi}\ \delta(z)$$

Inverse Fourier exp transforms

02.02.22.0002.01

$$\mathcal{F}_t^{-1}[\phi](z) == \phi \sqrt{2\pi}\ \delta(z)$$

Fourier cos transforms

02.02.22.0003.01

$$\mathcal{F}c_t[\phi](z) == \phi \sqrt{\frac{\pi}{2}}\ \delta(z)$$

Fourier sin transforms

02.02.22.0004.01

$$\mathcal{F}s_t[\phi](z) == \sqrt{\frac{2}{\pi}}\,\frac{\phi}{z}$$

Laplace transforms

02.02.22.0005.01

$$\mathcal{L}_t[\phi](z) == \frac{\phi}{z}$$

Inverse Laplace transforms

02.02.22.0006.01

$$\mathcal{L}_t^{-1}[\phi](z) == \phi\,\delta(z)$$

Summation

Infinite summation

02.02.23.0001.01

$$\sum_{k=1}^{\infty} |F_k\,\phi - F_{k+1}| = \phi$$

Representations through more general functions

Through Meijer *G*

02.02.26.0002.01

$$\phi = \phi\,G_{0,1}^{1,0}(z \mid 0) + \phi\,G_{1,2}^{1,1}\left(z \,\middle|\, \begin{matrix} 1 \\ 1,\, 0 \end{matrix}\right)$$

Through other functions

02.02.26.0003.01

$$\phi == 2\cos\left(\frac{\pi}{5}\right)$$

02.02.26.0004.01

$$\phi == \frac{1}{2}\sec\left(\frac{2\pi}{5}\right)$$

02.02.26.0005.01

$$\phi == \frac{1}{2}\csc\left(\frac{\pi}{10}\right)$$

02.02.26.0006.01

$$\phi = 2\sin\left(\frac{\pi}{10}\right) + 1$$

02.02.26.0007.01

$$\phi = -2\sin(666\,°)$$

Above equation derived in 1994 connects the golden ratio to the Number of the Beast (666):

02.02.26.0008.01

$\phi = -2\cos(6\times 6\times 6\,°)$

Above equation derived in 1994 connects the golden ratio to the Number of the Beast (666):

02.02.26.0009.01

$\phi = -\cos(6\times 6\times 6\,°) - \sin(666\,°)$

Above equation derived in 1994 connects the golden ratio to the Number of the Beast (666):

02.02.26.0001.01

$$\phi = \frac{1}{2}\left(\sqrt{5}\;F_1 + \sqrt{5\,F_1^2 - 4}\right)$$

02.02.26.0010.01

$$\phi = 2^{-1/\nu}\left(\sqrt{5}\;F_\nu + \sqrt{5\,F_\nu^2 + 4\cos(\pi\,\nu)}\right)^{1/\nu} /;\ \nu\in\mathbb{R}\land\nu>0$$

02.02.26.0011.01

$$\phi = \exp\left(\operatorname{csch}^{-1}(2)\right)$$

02.02.26.0012.01

$$\phi = \left(z;\ z^2 - z - 1\right)_2^{-1}$$

Inequalities

02.02.29.0001.01

$$\frac{8}{5} < \phi < \frac{81}{50}$$

Theorems

Approximation of golden ratio theorem

If a number x agrees with ϕ to n decimal places,then $\frac{x^2+2\,x}{x^2+1}$ agrees with ϕ to $2\,n$ decimal places.

History

– known 2000–3000 years ago: Pythagoras (circa 580 BC – circa 500 BC),;Phidias (490–430 BC); Euclid (c. 325–c. 265 BC)

– Euclid (c. 325–c. 265 BC) names the ratio $1:\phi$ the "extreme and mean ratio" in Book VI of the *Elements*

– Leonardo of Pisa (1170s or 1180s-1250); Johannes Kepler (1571-1630)

– Luca Pacioli (1509) published book "Divina Proportione", which gave new impulse to the theory of the golden ratio, in particular
he illustrated the golden ratio as applied to the human faces of artists, architects, scientists, and mystics

– Gerolamo Cardano (1545) mentioned golden ratio in the famouse book "Ars Magna", where he solved quadratic and cubic
equations and was the first who explicitly made calculations with complex numbers

– M. Mästlin (1597) evaluated $1/\phi$ approximatly as 0.6180340 …

– J. Kepler (1608) independently shows that ratios of Fibonacci numbers approximate ϕ and describes the golden ratio as a "precious jewel"

– R. Simson (1753) gave simple limit representation of golden ratio based on its very simple continued fraction $\phi == 1 + \frac{1}{1+\frac{1}{1+\frac{1}{1+\ldots}}}$

– G.S. Ohm (1835) gives the first known use of the name "golden ratio," believed to have originated earlier in the century from an unknown source

– J. Sulley (1875) first used the term "golden ratio" in English

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