

HeavisideTheta2

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Notations

Traditional name

Multivariate Heaviside step function

Traditional notation

$\theta(x_1, x_2, \dots)$

Mathematica StandardForm notation

HeavisideTheta[x₁, x₂, ...]

Primary definition

14.06.02.0001.01

$$\theta(x_1, x_2, \dots, x_n) = \begin{cases} 1 & x_1 > 0 \wedge x_2 > 0 \wedge \dots \wedge x_n > 0 \text{ ; } x_k \in \mathbb{R} \wedge x_k \neq 0 \wedge 1 \leq k \leq n \end{cases}$$

14.06.02.0002.01

$$\frac{\partial \theta(x_1, x_2, \dots, x_k, \dots, x_n)}{\partial x_k} = \delta(x_k) \theta(x_1, x_2, \dots, x_{k-1}, x_{k+1}, \dots, x_n) \text{ ; } x_k \in \mathbb{R} \wedge x_k \neq 0 \wedge 1 \leq k \leq n$$

Specific values

Specialized values

14.06.03.0001.01

$$\theta(x_1, x_1, \dots, x_1) = \theta(x_1)$$

Values at fixed points

14.06.03.0002.01

$$\theta(1, 1, \dots, 1) = 1$$

14.06.03.0003.01

$$\theta(-1, 1, \dots, 1) = 0$$

Values at infinities

14.06.03.0004.01

$$\theta(\infty, \infty, \dots, \infty) = 1$$

14.06.03.0005.01

$$\theta(-\infty, -\infty, \dots, -\infty) = 0$$

General characteristics

Domain and analyticity

$\theta(x_1, x_2, \dots)$ is a nonanalytical function; it is a piecewise constant function which is defined for all nonzero real x_k .

14.06.04.0001.01

$$(x_1 * x_2 * \dots * x_n) \rightarrow \theta(x_1, x_2, \dots) :: \mathbb{R}^n \rightarrow \mathbb{Z}$$

Symmetries and periodicities

Permutation symmetry

14.06.04.0002.01

$$\theta(x_2, x_1) = \theta(x_1, x_2)$$

14.06.04.0003.01

$$\theta(x_1, x_2, \dots, x_k, \dots, x_j, \dots, x_n) = \theta(x_1, x_2, \dots, x_j, \dots, x_k, \dots, x_n) /; x_k \neq x_j \wedge k \neq j$$

Periodicity

No periodicity

Sets of discontinuity

The function $\theta(x_1, x_2, \dots)$ has discontinuity at the point $x_1 = x_2 = \dots = 0$.

Transformations

Transformations and argument simplifications

Argument involving basic arithmetic operations

14.06.16.0001.01

$$\theta(-x_1, x_2, \dots) = \theta(x_2, \dots) - \theta(x_1, x_2, \dots) /; x_1 \neq 0$$

Products, sums, and powers of the direct function

Powers of the direct function

14.06.16.0002.01

$$\theta(x_1, x_2, \dots)^a = \theta(x_1, x_2, \dots) /; a > 0$$

Identities

Functional identities

14.06.17.0001.01

$$\theta(x_1, x_2, \dots, x_n) = \theta(x_1, x_2, \dots, x_{n-1}) \theta(x_n)$$

Complex characteristics

Real part

14.06.19.0001.01

$$\operatorname{Re}(\theta(x_1, x_2, \dots, x_n)) = \theta(x_1, x_2, \dots, x_n)$$

Imaginary part

14.06.19.0002.01

$$\operatorname{Im}(\theta(x_1, x_2, \dots, x_n)) = 0$$

Absolute value

14.06.19.0003.01

$$|\theta(x_1, x_2, \dots, x_n)| = \theta(x_1, x_2, \dots, x_n)$$

Argument

14.06.19.0004.01

$$\arg(\theta(x_1, x_2, \dots, x_n)) = \tan^{-1}(\theta(x_1, x_2, \dots, x_n), 0)$$

Conjugate value

14.06.19.0005.01

$$\overline{\theta(x_1, x_2, \dots, x_n)} = \theta(x_1, x_2, \dots, x_n)$$

Differentiation

Low-order differentiation

In a distributional sense.

14.06.20.0001.01

$$\frac{\partial \theta(x_1, x_2, \dots)}{\partial x_1} = \delta(x_1) \theta(x_2, \dots)$$

14.06.20.0002.01

$$\frac{\partial \theta(x_1, x_2, \dots, x_k, \dots, x_n)}{\partial x_k} = \delta(x_k) \theta(x_1, x_2, \dots, x_{k-1}, x_{k+1}, \dots, x_n) /; x_k \in \mathbb{R} \wedge x_k \neq 0 \wedge 1 \leq k \leq n$$

Integration

Indefinite integration

Involving only one direct function

14.06.21.0001.01

$$\int \theta(x_1, x_2, \dots) dx_1 = x_1 \theta(x_1, x_2, \dots)$$

Involving one direct function and elementary functions

Involving power function

14.06.21.0002.01

$$\int x_1^{\alpha-1} \theta(x_1, x_2, \dots) dx_1 = \frac{x_1^\alpha}{\alpha} \theta(x_1, x_2, \dots)$$

14.06.21.0003.01

$$\int \theta(x_1, x_2, \dots) dx_1 = x_1 \theta(x_1, x_2, \dots)$$

Integral transforms

Fourier exp transforms

14.06.22.0001.01

$$\mathcal{F}_t[\theta(x_1, x_2, \dots, x_{n-1}, t)](x) = \theta(x_1, x_2, \dots, x_{n-1}) \left(\frac{i}{\sqrt{2\pi} x} + \sqrt{\frac{\pi}{2}} \delta(x) \right)$$

Inverse Fourier exp transforms

14.06.22.0002.01

$$\mathcal{F}_t^{-1}[\theta(x_1, x_2, \dots, x_{n-1}, t)](x) = \theta(x_1, x_2, \dots, x_{n-1}) \left(-\frac{i}{\sqrt{2\pi} x} + \sqrt{\frac{\pi}{2}} \delta(x) \right)$$

Fourier cos transforms

14.06.22.0003.01

$$\mathcal{F}_t[\theta(x_1, x_2, \dots, x_{n-1}, t)](x) = \sqrt{\frac{\pi}{2}} \theta(x_1, x_2, \dots, x_{n-1}) \delta(x)$$

Fourier sin transforms

14.06.22.0004.01

$$\mathcal{F}_{St}[\theta(x_1, x_2, \dots, x_{n-1}, t)](z) = \sqrt{\frac{2}{\pi}} \frac{\theta(x_1, x_2, \dots, x_{n-1})}{z}$$

Laplace transforms

14.06.22.0005.01

$$\mathcal{L}_t[\theta(x_1, x_2, \dots, x_{n-1}, t)](z) = \frac{\theta(x_1, x_2, \dots, x_{n-1})}{z}$$

Representations through more general functions

Through Meijer G

Classical cases for the direct function itself

14.06.26.0001.01

$$\theta(x_1, x_2, \dots, x_n) = \prod_{k=1}^n G_{1,1}^{1,0} \left(1 - x_k \left| \begin{matrix} 1 \\ 0 \end{matrix} \right. \right); x_k < 2$$

14.06.26.0002.01

$$\theta(x_1, x_2, \dots, x_n) = \prod_{k=1}^n G_{1,1}^{0,1} \left(x_k + 1 \left| \begin{matrix} 1 \\ 0 \end{matrix} \right. \right); x_k > -2$$

14.06.26.0003.01

$$\theta(1 - |z_1|, 1 - |z_2|, \dots, 1 - |z_n|) = \prod_{k=1}^n G_{1,1}^{1,0} \left(z_k \left| \begin{matrix} 1 \\ 0 \end{matrix} \right. \right)$$

14.06.26.0004.01

$$\theta(|z_1| - 1, |z_2| - 1, \dots, |z_n| - 1) = \prod_{k=1}^n G_{1,1}^{0,1} \left(z_k \left| \begin{matrix} 1 \\ 0 \end{matrix} \right. \right)$$

Representations through equivalent functions

14.06.27.0001.01

$$\theta(x_1, x_2, \dots, x_n) = \prod_{k=1}^n \theta(x_k)$$

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