Hypergeometric5F4

Notations

Traditional name

Generalized hypergeometric function ${}_5F_4$

Traditional notation

${}_5F_4(a_1, a_2, a_3, a_4, a_5; b_1, b_2, b_3, b_4; z)$

Mathematica StandardForm notation

HypergeometricPFQ[{a_1, a_2, a_3, a_4, a_5}, {b_1, b_2, b_3, b_4}, z]

Primary definition

07.29.02.0001.01

$${}_5F_4(a_1, a_2, a_3, a_4, a_5; b_1, b_2, b_3, b_4; z) = \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k (a_3)_k (a_4)_k (a_5)_k z^k}{(b_1)_k (b_2)_k (b_3)_k (b_4)_k k!} \;/;\; |z| < 1 \bigvee |z| = 1 \bigwedge \operatorname{Re}\left(\sum_{j=1}^{4} b_j - \sum_{j=1}^{5} a_j\right) > 0$$

For $a_i = -n, b_j = -m \;/;\; m \geq n$ being nonpositive integers and $\nexists_{a_k} (a_k > -n \bigwedge a_k \in \mathbb{N}) \bigwedge \nexists_{b_k} (b_k > -m \bigwedge b_k \in \mathbb{N})$ the function ${}_5F_4(a_1, a_2, a_3, a_4, a_5; b_1, b_2, b_3, b_4; z)$ cannot be uniquely defined by a limiting procedure based on the above definition because the two variables a_i, b_j can approach nonpositive integers $-n, -m; m \geq n$ at different speeds. For the above conditions we define:

07.29.02.0002.01

$${}_5F_4\left(a_1, \ldots, a_i, \ldots, a_5; b_1, \ldots, b_j, \ldots, b_4; z\right) = \sum_{k=0}^{n} \frac{(a_1)_k (a_2)_k (a_3)_k (a_4)_k (a_5)_k z^k}{(b_1)_k (b_2)_k (b_3)_k (b_4)_k k!} \;/;\; a_i = -n \bigwedge b_j = -m \bigwedge m \in \mathbb{N} \bigwedge n \in \mathbb{N} \bigwedge m \geq n$$

Specific values

Values at $z = 0$

07.29.03.0001.01

$${}_5F_4(a_1, a_2, a_3, a_4, a_5; b_1, b_2, b_3, b_4; 0) = 1$$

Values at $z = 1$

For fixed a_1, a_2, a_3, a_4

07.29.03.0002.01

$${}_5F_4\left(a, b, c, d, a+\frac{1}{2}; \frac{a}{2}, a-b+1, a-c+1, a-d+1; 1\right) = (\Gamma(a-b+1)\,\Gamma(a-c+1)\,\Gamma(a-d+1)\,\Gamma(a-b-c-d+1)) / (\Gamma(a+1)\,\Gamma(a-b-c+1)\,\Gamma(a-b-d+1)\,\Gamma(a-c-d+1)) \;/;\; \mathrm{Re}(a-b-c-d) > -1$$

For fixed a_2, a_3, a_4, a_5

07.29.03.0003.01

$${}_5F_4(1, b, c, d, e; b+1, c+1, d+1, e+1; 1) = -b\,c\,d\,e\left(\frac{\psi(b)}{(c-b)\,(d-b)\,(e-b)} + \frac{\psi(c)}{(b-c)\,(d-c)\,(e-c)} + \frac{\psi(d)}{(b-d)\,(c-d)\,(e-d)} + \frac{\psi(e)}{(b-e)\,(c-e)\,(d-e)}\right) /; b \neq c \bigwedge b \neq d \bigwedge b \neq e \bigwedge c \neq d \bigwedge c \neq e \bigwedge d \neq e$$

For fixed a_2, a_3, a_4, b_1

07.29.03.0004.01

$${}_5F_4\left(-n, b, c, d, b+\frac{1}{2}; f, \frac{c-n}{2}, \frac{c-n+1}{2}, 2\,b+d-f+1; 1\right) = \frac{(2\,b-c+1)_n}{(1-c)_n}\,{}_4F_3(-n, 2\,b, f-d, 2\,b-f+1; f, 2\,b-c+1, 2\,b+d-f+1; 1)$$

For fixed a_2, a_3, a_4

07.29.03.0005.01

$${}_5F_4\left(-n, b, c, d, b+\frac{1}{2}; 2\,b, \frac{c+d}{2}, \frac{c+d+1}{2}, 2\,b-c-d-n+1; 1\right) = \frac{(d)_n\,(-2\,b+2\,c+d)_n}{(c+d)_n\,(-2\,b+c+d)_n}\,{}_5F_4\left(-n, \frac{c}{2}, \frac{c+1}{2}, c+d-2\,b, 2\,c+d-2\,b+n; c+d+n, 1-d-n, \frac{d}{2}+c-b, \frac{d+1}{2}-b+c; 1\right)$$

07.29.03.0006.01

$${}_5F_4(1, b, c, d, b; b+1, c+1, d+1, b+1; 1) = b^2\,c\,d\left(\frac{(c+d-2\,b)\,\psi(b)}{(b-c)^2\,(b-d)^2} + \frac{\psi^{(1)}(b)}{(b-c)\,(b-d)} - \frac{\psi(c)}{(b-c)^2\,(d-c)} - \frac{\psi(d)}{(b-d)^2\,(c-d)}\right) /; b \neq c \bigwedge b \neq d \bigwedge c \neq d$$

For fixed a_2, a_3

07.29.03.0007.01

$${}_5F_4\left(1, b, c, b+\frac{1}{2}, c+\frac{1}{2}; \frac{1}{2}-b+n, -b+n+1, \frac{1}{2}-c+n, 1-c+n; 1\right) = 1 - \frac{(1+(-1)^n)\,(2\,b-2\,n)_n\,(2\,c-2\,n)_n}{4\,(1-2\,b)_n\,(1-2\,c)_n} + \frac{\Gamma(1-2\,b)\,\Gamma(1-2\,c)}{4\,\Gamma(1-2\,b-2\,c+2\,n)} \left(\left(\sqrt{\pi}\;\Gamma\left(\frac{1}{2}-2\,b-2\,c+2\,n\right)(1-4\,b+2\,n)_{2\,n}\,(1-4\,c+2\,n)_{2\,n}\right) \Big/ \left(2^{4\,n}\,\Gamma\left(\frac{1}{2}-2\,b+2\,n\right)\Gamma\left(\frac{1}{2}-2\,c+2\,n\right)\right) + (-1)^n\,(1-2\,b+n)_n\,(1-2\,c+n)_n\right) - \sum_{k=0}^{\left\lfloor \frac{n-1}{2} \right\rfloor} \frac{(2\,b-2\,n)_{2\,k}\,(2\,c-2\,n)_{2\,k}}{(1-2\,b)_{2\,k}\,(1-2\,c)_{2\,k}} /; \mathrm{Re}(b+c) < n+\frac{1}{4} \bigwedge n \in \mathbb{N}^+$$

07.29.03.0008.01

$${}_5F_4(1, b, c, b, c; b+1, c+1, b+1, c+1; 1) = \frac{b^2\,c^2}{(b-c)^3}\left(2\,(\psi(c)-\psi(b)) + (b-c)\left(\psi^{(1)}(b)+\psi^{(1)}(c)\right)\right) /; b \neq c$$

07.29.03.0009.01

$${}_5F_4(1,\,b,\,c,\,b,\,b;\,b+1,\,c+1,\,b+1,\,b+1;\,1) = \frac{b^3\,c}{2\,(b-c)^3}\left(2\,(\psi(b)-\psi(c))+(b-c)\left((b-c)\,\psi^{(2)}(b)-2\,\psi^{(1)}(b)\right)\right)\,/;\,b\neq c$$

07.29.03.0010.01

$${}_5F_4(1,\,b,\,c,\,-b,\,-c;\,b+1,\,1-b,\,c+1,\,1-c;\,1) = \frac{\pi\,b\,c}{2\left(b^2-c^2\right)}\,(b\cot(\pi\,c)-c\cot(\pi\,b))+\frac{1}{2}\,/;\,b\neq c$$

For fixed a_2

07.29.03.0011.01

$${}_5F_4(1,\,b,\,b,\,b,\,b;\,b+1,\,b+1,\,b+1,\,b+1;\,1) = \frac{1}{6}\,b^4\,\psi^{(3)}(b)$$

07.29.03.0012.01

$${}_5F_4(1,\,b,\,b,\,-b,\,-b;\,b+1,\,b+1,\,1-b,\,1-b;\,1) = \frac{1}{4}\left(b^2\,\pi^2\,\csc^2(b\,\pi)+b\,\pi\cot(b\,\pi)+2\right)$$

07.29.03.0013.01

$${}_5F_4(1,\,b,\,b,\,1-b,\,1-b;\,b+1,\,b+1,\,2-b,\,2-b;\,1) = \frac{(b-1)^2\,b^2\,\pi\csc^2(b\,\pi)\,(\pi\,(2\,b-1)+\sin(2\,b\,\pi))}{(2\,b-1)^3}\,/;\,b\neq\frac{1}{2}$$

07.29.03.0014.01

$${}_5F_4(2,\,b,\,b,\,2-b,\,2-b;\,b+1,\,b+1,\,3-b,\,3-b;\,1) = \frac{(2-b)^2\,b^2}{4\,(1-b)}\left(\psi^{(1)}(b-1)-\psi^{(1)}(1-b)\right)\,/;\,b\neq 1$$

For fixed a_4

07.29.03.0015.01

$${}_5F_4\!\left(\frac{1}{2},\,\frac{1}{2},\,1,\,d,\,1-d;\,\frac{3}{2},\,\frac{3}{2},\,d+1,\,2-d;\,1\right) = \frac{\pi\,d\,(1-d)}{(1-2\,d)^3}\left(\cot(\pi\,d)-\pi\left(\frac{1}{2}-d\right)\right)\,/;\,d\neq\frac{1}{2}$$

07.29.03.0016.01

$${}_5F_4(1,\,1,\,1,\,d,\,2-d;\,2,\,2,\,d+1,\,3-d;\,1) = \frac{(d-2)\,d}{6\,(d-1)^4}\left(\pi^2\,(d-1)^2+3\,\pi\cot(d\,\pi)\,(d-1)-3\right)\,/;\,d\neq 1$$

Values at $z = -1$

For fixed a_2, a_3, a_4, a_5

07.29.03.0017.01

$${}_5F_4(1,\,b,\,c,\,d,\,e;\,b+1,\,c+1,\,d+1,\,e+1;\,-1) = \frac{b\,c\,d\,e}{2}\left(\frac{\psi\left(\frac{b+1}{2}\right)-\psi\left(\frac{b}{2}\right)}{(c-b)\,(d-b)\,(e-b)}+\frac{\psi\left(\frac{c+1}{2}\right)-\psi\left(\frac{c}{2}\right)}{(b-c)\,(d-c)\,(e-c)}+\frac{\psi\left(\frac{d+1}{2}\right)-\psi\left(\frac{d}{2}\right)}{(b-d)\,(c-d)\,(e-d)}+\frac{\psi\left(\frac{e+1}{2}\right)-\psi\left(\frac{e}{2}\right)}{(b-e)\,(c-e)\,(d-e)}\right)/;$$
$$b\neq c\land b\neq d\land b\neq e\land c\neq d\land c\neq e\land d\neq e$$

For fixed a_2, a_3, a_4

07.29.03.0018.01

$$ {}_5F_4(1, b, c, d, b; b+1, c+1, d+1, b+1; -1) = -\frac{b^2 c\, d}{2} \left(\frac{(c+d-2b)\left(\psi\left(\frac{b+1}{2}\right) - \psi\left(\frac{b}{2}\right)\right)}{(b-c)^2 (b-d)^2} + \frac{\psi^{(1)}\left(\frac{b+1}{2}\right) - \psi^{(1)}\left(\frac{b}{2}\right)}{2(b-c)(b-d)} - \frac{\psi\left(\frac{c+1}{2}\right) - \psi\left(\frac{c}{2}\right)}{(b-c)^2 (d-c)} - \frac{\psi\left(\frac{d+1}{2}\right) - \psi\left(\frac{d}{2}\right)}{(b-d)^2 (c-d)} \right) /; b \neq c \land b \neq d \land c \neq d $$

For fixed a_2, a_3

07.29.03.0019.01

$$ {}_5F_4(1, b, c, b, c; b+1, c+1, b+1, c+1; -1) = -\frac{b^2 c^2}{(b-c)^3} \left(-\psi\left(\frac{b+1}{2}\right) + \psi\left(\frac{c+1}{2}\right) + \psi\left(\frac{b}{2}\right) - \psi\left(\frac{c}{2}\right) + \frac{b-c}{4} \left(\psi^{(1)}\left(\frac{b+1}{2}\right) + \psi^{(1)}\left(\frac{c+1}{2}\right) - \psi^{(1)}\left(\frac{b}{2}\right) - \psi^{(1)}\left(\frac{c}{2}\right) \right) \right) /; b \neq c $$

07.29.03.0020.01

$$ {}_5F_4(1, b, c, b, b; b+1, c+1, b+1, b+1; -1) = -\frac{b^3 c}{2(b-c)^3} \left(\psi\left(\frac{b+1}{2}\right) - \psi\left(\frac{c+1}{2}\right) - \psi\left(\frac{b}{2}\right) + \psi\left(\frac{c}{2}\right) + \frac{b-c}{2} \left(-\psi^{(1)}\left(\frac{b+1}{2}\right) + \psi^{(1)}\left(\frac{b}{2}\right) + \frac{b-c}{4} \left(\psi^{(2)}\left(\frac{b+1}{2}\right) - \psi^{(2)}\left(\frac{b}{2}\right) \right) \right) \right) /; b \neq c $$

For fixed a_2

07.29.03.0021.01

$$ {}_5F_4(1, b, b, b, b; b+1, b+1, b+1, b+1; -1) = -\frac{1}{96} b^4 \left(\psi^{(3)}\left(\frac{b+1}{2}\right) - \psi^{(3)}\left(\frac{b}{2}\right) \right) $$

07.29.03.0022.01

$$ {}_5F_4(1, b, b, -b, -b; b+1, b+1, 1-b, 1-b; -1) = \frac{\pi^2 \cos(\pi b)\, b^2}{4 \sin^2(\pi b)} + \frac{\pi b}{4 \sin(\pi b)} + \frac{1}{2} $$

For fixed a_4

07.29.03.0023.01

$$ {}_5F_4\left(\frac{1}{2}, \frac{1}{2}, 1, d, 1-d; \frac{3}{2}, \frac{3}{2}, d+1, 2-d; -1\right) = \frac{(d-1)\, d}{2(2d-1)^3} \left(8\, C\, (2d-1) - 2\pi \csc(d\pi) - 2\psi\left(\frac{d}{2}\right) + 2\psi\left(\frac{d+1}{2}\right) \right) $$

07.29.03.0024.01

$$ {}_5F_4(1, 1, 1, d, 2-d; 2, 2, d+1, 3-d; -1) = \frac{1}{24(d-1)^4} \left(2(d-2)\, d \csc(d\pi) \left(6\pi (d-1) + \left(\pi^2 (d-1)^2 + 6\right) \sin(d\pi) \right) \right) $$

Values at other z

Values at $z = \frac{1}{4}$

07.29.03.0025.01

$$ {}_5F_4\left(-n, b, c, \frac{b}{3}+1, 1-c; \frac{b}{3}, b+2n+1, \frac{b-c}{2}+1, \frac{b+c+1}{2}; \frac{1}{4}\right) = \frac{\left(\frac{b+1}{2}\right)_n \left(\frac{b}{2}+1\right)_n}{\left(\frac{b-c}{2}+1\right)_n \left(\frac{b+c+1}{2}\right)_n} /; n \in \mathbb{N} $$

Values at $z = 4$

07.29.03.0026.01

$$ {}_5F_4\left(-n, b, c, \frac{2b}{3}+1, b-c+\frac{1}{2}; \frac{2b}{3}, b+\frac{n}{2}+1, 2c, 2b-2c+1; 4\right) = \frac{(1+(-1)^n)\, n!\, (b+1)_{\frac{n}{2}}}{2^{n+1}\, \frac{n}{2}!\, \left(c+\frac{1}{2}\right)_{\frac{n}{2}} (b-c+1)_{\frac{n}{2}}} /; n \in \mathbb{N} $$

For fixed *z*

07.29.03.0027.01

$$ {}_5F_4\left(\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 1; \frac{3}{2}, \frac{7}{4}, 2, \frac{9}{4}; z\right) = \frac{5}{4 z^{5/4}} \left(4 \sqrt[4]{z} (\log(1-z)-1) - 8 z^{3/4} \tanh^{-1}\left(\sqrt{z}\right) + 2\left(z - 6\sqrt{z}+1\right) \tan^{-1}\left(\sqrt[4]{z}\right) + \left(z+6\sqrt{z}+1\right)\left(\log\left(\sqrt[4]{z}+1\right) - \log\left(1-\sqrt[4]{z}\right)\right)\right) $$

07.29.03.0028.01

$$ {}_5F_4(1, 1, 1, 3, 3; 2, 2, 4, 4; z) = \frac{9}{16 z^3} \left(-3z^2 - 8z + 4\left(z^2-1\right)\log(1-z) + 4\left(z^2+1\right)\mathrm{Li}_2(z)\right) $$

Values at fixed points

Values at *z* == 1

07.29.03.0029.01

$$ {}_5F_4\left(-n, \frac{1}{2}-n, -\frac{1}{4}, \frac{1}{4}, \frac{9}{8}; \frac{1}{4}-n, \frac{3}{4}-n, \frac{1}{8}, \frac{3}{2}; 1\right) = 0 /; n \in \mathbb{N}^+ $$

07.29.03.0030.01

$$ {}_5F_4\left(\frac{1}{8}, \frac{3}{8}, \frac{5}{8}, \frac{7}{8}, 1; \frac{9}{8}, \frac{11}{8}, \frac{13}{8}, \frac{15}{8}; 1\right) = \frac{35}{64}\left(2-\sqrt{2}\right)\pi $$

07.29.03.0031.01

$$ {}_5F_4\left(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{2}; \frac{5}{4}, \frac{5}{4}, \frac{5}{4}, \frac{5}{4}; 1\right) = \frac{\left(48 C \pi + 5\pi^3\right)}{1536\sqrt{2\pi}} \Gamma\left(\frac{1}{4}\right)^2 $$

07.29.03.0032.01

$$ {}_5F_4\left(\frac{1}{4}, \frac{1}{4}, \frac{3}{4}, \frac{3}{4}, 1; \frac{5}{4}, \frac{5}{4}, \frac{7}{4}, \frac{7}{4}; 1\right) = \frac{9}{32}\left(\pi^2 - 2\pi\right) $$

07.29.03.0033.01

$$ {}_5F_4\left(\frac{1}{4}, \frac{1}{2}, \frac{1}{2}, \frac{3}{4}, 1; \frac{5}{4}, \frac{3}{2}, \frac{3}{2}, \frac{7}{4}; 1\right) = \frac{3}{8}\left(4\pi - \pi^2\right) $$

07.29.03.0034.01

$$ {}_5F_4\left(\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \frac{3}{4}, 1; \frac{5}{4}, \frac{3}{2}, \frac{7}{4}, \frac{7}{4}; 1\right) = \frac{9}{16}\left(8\log(2) + \pi^2 - 2\pi - 8C\right) $$

07.29.03.0035.01

$$ {}_5F_4\left(\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 1; \frac{5}{4}, \frac{3}{2}, \frac{7}{4}, 2; 1\right) = 6\log(2) - \pi $$

07.29.03.0036.01

$$ {}_5F_4\left(\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 1; \frac{5}{4}, \frac{3}{2}, 2, \frac{11}{4}; 1\right) = \frac{7}{90}(144\log(2) - 27\pi - 2) $$

07.29.03.0037.01

$${}_5F_4\left(\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 1; \frac{5}{4}, \frac{7}{4}, 2, \frac{5}{2}; 1\right) = \frac{3}{5} (22 \log(2) - 4 \pi - 1)$$

07.29.03.0038.01

$${}_5F_4\left(\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 1; \frac{5}{4}, 2, \frac{5}{2}, \frac{11}{4}; 1\right) = \frac{7}{15} (54 \log(2) - 9 \pi - 7)$$

07.29.03.0039.01

$${}_5F_4\left(\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 1; \frac{3}{2}, \frac{7}{4}, 2, \frac{9}{4}; 1\right) = \frac{5}{2} (8 \log(2) - \pi - 2)$$

07.29.03.0040.01

$${}_5F_4\left(\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 1; \frac{3}{2}, 2, \frac{9}{4}, \frac{11}{4}; 1\right) = \frac{7}{18} (108 \log(2) - 9 \pi - 44)$$

07.29.03.0041.01

$${}_5F_4\left(\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 1; \frac{7}{4}, 2, \frac{9}{4}, \frac{5}{2}; 1\right) = 3 (18 \log(2) - \pi - 9)$$

07.29.03.0042.01

$${}_5F_4\left(\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 1; 2, \frac{9}{4}, \frac{5}{2}, \frac{11}{4}; 1\right) = \frac{7}{3} (54 \log(2) - 37)$$

07.29.03.0043.01

$${}_5F_4\left(\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, \frac{7}{4}; \frac{5}{4}, \frac{7}{4}, \frac{5}{2}, \frac{11}{4}; 1\right) = \frac{7}{20} (12 \log(2) - 3 \pi + 4)$$

07.29.03.0044.01

$${}_5F_4\left(\frac{1}{4}, \frac{1}{2}, 1, 1, \frac{7}{4}; \frac{5}{4}, 2, \frac{5}{2}, \frac{11}{4}; 1\right) = \frac{7}{45} (19 - 18 \log(2))$$

07.29.03.0045.01

$${}_5F_4\left(\frac{1}{4}, \frac{1}{2}, 1, 1, \frac{7}{4}; \frac{3}{2}, 2, \frac{9}{4}, \frac{11}{4}; 1\right) = \frac{7}{54} (86 - 72 \log(2) - 9 \pi)$$

07.29.03.0046.01

$${}_5F_4\left(\frac{1}{4}, \frac{1}{2}, 1, 1, \frac{7}{4}; 2, \frac{9}{4}, \frac{5}{2}, \frac{11}{4}; 1\right) = \frac{7}{9} (67 - 54 \log(2) - 9 \pi)$$

07.29.03.0047.01

$${}_5F_4\left(\frac{1}{4}, \frac{3}{4}, 1, 1, \frac{3}{2}; \frac{5}{4}, \frac{7}{4}, 2, \frac{5}{2}; 1\right) = \frac{3}{5} (-14 \log(2) + 3 \pi + 2)$$

07.29.03.0048.01

$${}_5F_4\left(\frac{1}{4}, \frac{3}{4}, 1, 1, \frac{3}{2}; \frac{5}{4}, 2, \frac{5}{2}, \frac{11}{4}; 1\right) = \frac{7}{30} (26 - 72 \log(2) + 9 \pi)$$

07.29.03.0049.01

$${}_5F_4\left(\frac{1}{4}, \frac{3}{4}, 1, 1, \frac{3}{2}; \frac{7}{4}, 2, \frac{9}{4}, \frac{5}{2}; 1\right) = \frac{3}{2} (26 - 32 \log(2) - \pi)$$

07.29.03.0050.01

$${}_5F_4\left(\frac{1}{4}, \frac{3}{4}, 1, 1, \frac{3}{2}; 2, \frac{9}{4}, \frac{5}{2}, \frac{11}{4}; 1\right) = \frac{7}{6} (104 - 108 \log(2) - 9 \pi)$$

07.29.03.0051.01

$${}_5F_4\left(\frac{1}{4}, 1, 1, \frac{3}{2}, \frac{7}{4}; 2, \frac{9}{4}, \frac{5}{2}, \frac{11}{4}; 1\right) = \frac{7}{18} (144 \log(2) + 27 \pi - 182)$$

07.29.03.0052.01

$${}_5F_4\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}; \frac{3}{2}, \frac{3}{2}, \frac{3}{2}, \frac{3}{2}; 1\right) = \frac{1}{96} \pi \left(\log^3(4) + \pi^2 \log(4) + 12 \zeta(3)\right)$$

07.29.03.0053.01

$${}_5F_4\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, 1; \frac{3}{2}, \frac{3}{2}, \frac{3}{2}, \frac{3}{2}; 1\right) = \frac{\pi^4}{96}$$

07.29.03.0054.01

$${}_5F_4\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, 1; \frac{5}{2}, \frac{5}{2}, \frac{5}{2}, \frac{5}{2}; 1\right) = \frac{27}{256} \left(30 \pi^2 + \pi^4 - 384\right)$$

07.29.03.0055.01

$${}_5F_4\left(\frac{1}{2}, \frac{1}{2}, 1, \frac{5}{2}, \frac{5}{2}; \frac{3}{2}, \frac{3}{2}, \frac{7}{2}, \frac{7}{2}; 1\right) = \frac{25}{576} \left(-64 + 9 \pi^2\right)$$

07.29.03.0056.01

$${}_5F_4\left(\frac{1}{2}, \frac{3}{4}, 1, 1, \frac{5}{4}; \frac{3}{2}, \frac{7}{4}, 2, \frac{9}{4}; 1\right) = 5 (4 - 10 \log(2) + \pi)$$

07.29.03.0057.01

$${}_5F_4\left(\frac{1}{2}, \frac{3}{4}, 1, 1, \frac{5}{4}; \frac{3}{2}, 2, \frac{9}{4}, \frac{11}{4}; 1\right) = \frac{7}{6} (58 - 96 \log(2) + 3 \pi)$$

07.29.03.0058.01

$${}_5F_4\left(\frac{1}{2}, \frac{3}{4}, 1, 1, \frac{5}{4}; \frac{7}{4}, 2, \frac{9}{4}, \frac{5}{2}; 1\right) = 15 (7 - 10 \log(2))$$

07.29.03.0059.01

$${}_5F_4\left(\frac{1}{2}, \frac{3}{4}, 1, 1, \frac{5}{4}; 2, \frac{9}{4}, \frac{5}{2}, \frac{11}{4}; 1\right) = 7 (47 - 54 \log(2) - 3 \pi)$$

07.29.03.0060.01

$${}_5F_4\left(\frac{1}{2}, 1, 1, 1, \frac{3}{2}; 2, 2, \frac{5}{2}, \frac{5}{2}; 1\right) = \frac{3}{4} \left(84 - 5 \pi^2 - 24 \,\text{Log}[4]\right)$$

07.29.03.0061.01

$${}_5F_4\left(\frac{1}{2}, 1, 1, \frac{5}{4}, \frac{7}{4}; 2, \frac{9}{4}, \frac{5}{2}, \frac{11}{4}; 1\right) = \frac{7}{3} (66 \log(2) + 12 \pi - 83)$$

07.29.03.0062.01

$${}_5F_4\left(\frac{2}{3}, 1, 1, 1, \frac{4}{3}; \frac{5}{3}, 2, 2, \frac{7}{3}; 1\right) = \frac{4}{3} \left(27 - 3 \sqrt{3} - \pi^2\right)$$

07.29.03.0063.01

$${}_5F_4\left(\frac{3}{4}, 1, 1, \frac{5}{4}, \frac{3}{2}; \frac{7}{4}, 2, \frac{9}{4}, \frac{5}{2}; 1\right) = 15 (10 \log(2) + \pi - 10)$$

07.29.03.0064.01

$${}_5F_4\left(\frac{3}{4}, 1, 1, \frac{5}{4}, \frac{3}{2}; 2, \frac{9}{4}, \frac{5}{2}, \frac{11}{4}; 1\right) = \frac{35}{2} (24 \log(2) + 3 \pi - 26)$$

07.29.03.0065.01

$${}_5F_4(1, 1, 1, 1, 1; 3, 3, 3, 3; 1) == \frac{16}{45} \left(150 \pi^2 + \pi^4 - 1575\right)$$

07.29.03.0066.01

$${}_5F_4\left(1, 1, 1, \frac{3}{2}, \frac{3}{2}; \frac{5}{2}, \frac{5}{2}, 3, 3; 1\right) == 12 \left(-69 + 7 \pi^2\right)$$

07.29.03.0067.01

$${}_5F_4(1, 1, 1, 3, 3; 2, 2, 4, 4; 1) == \frac{3}{16} \left(4 \pi^2 - 33\right)$$

Values at $z == -1$

07.29.03.0068.01

$${}_5F_4\left(\frac{1}{4}, \frac{1}{2}, \frac{1}{2}, \frac{3}{4}, 1; \frac{5}{4}, \frac{3}{2}, \frac{3}{2}, \frac{7}{4}; -1\right) == -\frac{3}{\sqrt{2}} \log\left(3 - 2 \sqrt{2}\right) - 3 C$$

07.29.03.0069.01

$${}_5F_4\left(\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 1; \frac{5}{4}, \frac{3}{2}, \frac{7}{4}, 2; -1\right) == \pi \left(-\frac{3}{2} + 2 \sqrt{2}\right) - \log(2) + \sqrt{2} \log\left(3 - 2 \sqrt{2}\right)$$

07.29.03.0070.01

$${}_5F_4\left(\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1, 1; \frac{3}{2}, \frac{7}{4}, 2, \frac{9}{4}; -1\right) == \frac{5}{4} \left(\pi \left(-2 + 3 \sqrt{2}\right) - 4 \log(2) - 6 \sqrt{2} \log\left(1 + \sqrt{2}\right) + 4\right)$$

07.29.03.0071.01

$${}_5F_4\left(\frac{1}{2}, \frac{1}{2}, 1, \frac{5}{2}, \frac{5}{2}; \frac{3}{2}, \frac{3}{2}, \frac{7}{2}, \frac{7}{2}; -1\right) == \frac{25}{144} (18 C - 11)$$

07.29.03.0072.01

$${}_5F_4\left(\frac{1}{2}, 1, \frac{3}{2}, \frac{3}{2}, \frac{9}{4}; \frac{5}{4}, 2, 2, 3; -1\right) == \frac{16}{5} \left(\frac{4}{\pi} - 1\right)$$

07.29.03.0073.01

$${}_5F_4\left(\frac{2}{3}, 1, 1, 1, \frac{4}{3}; \frac{5}{3}, 2, 2, \frac{7}{3}; -1\right) == 8 \sqrt{3} \pi - \frac{2 \pi^2}{3} - 36$$

07.29.03.0074.01

$${}_5F_4\left(1, 1, 1, \frac{3}{2}, \frac{3}{2}; \frac{5}{2}, \frac{5}{2}, 3, 3; -1\right) == 828 - 432 \log(2) - 576 C$$

07.29.03.0075.01

$${}_5F_4(1, 1, 1, 3, 3; 2, 2, 4, 4; -1) == \frac{3}{16} \left(2 \pi^2 - 15\right)$$

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