

Hypergeometric6F5

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Notations

Traditional name

Generalized hypergeometric function ${}_6F_5$

Traditional notation

${}_6F_5(a_1, a_2, a_3, a_4, a_5, a_6; b_1, b_2, b_3, b_4, b_5; z)$

Mathematica StandardForm notation

HypergeometricPFQ[$\{a_1, a_2, a_3, a_4, a_5, a_6\}, \{b_1, b_2, b_3, b_4, b_5\}, z]$

Primary definition

07.30.02.0001.01

$${}_6F_5(a_1, a_2, a_3, a_4, a_5, a_6; b_1, b_2, b_3, b_4, b_5; z) = \sum_{k=0}^{\infty} \frac{(a_1)_k (a_2)_k (a_3)_k (a_4)_k (a_5)_k (a_6)_k z^k}{(b_1)_k (b_2)_k (b_3)_k (b_4)_k (b_5)_k k!} /;$$

$$|z| < 1 \vee |z| = 1 \wedge \operatorname{Re}\left(\sum_{j=1}^5 b_j - \sum_{j=1}^6 a_j\right) > 0$$

For $a_i = -n, b_j = -m /; m \geq n$ being nonpositive integers and $\nexists_{a_k} (a_k > -n \wedge a_k \in \mathbb{N}) \wedge \nexists_{b_k} (b_k > -m \wedge b_k \in \mathbb{N})$ the function ${}_6F_5(a_1, a_2, a_3, a_4, a_5, a_6; b_1, b_2, b_3, b_4, b_5; z)$ cannot be uniquely defined by a limiting procedure based on the above definition because the two variables a_i, b_j can approach nonpositive integers $-n, -m; m \geq n$ at different speeds. For the above conditions we define:

07.30.02.0002.01

$${}_6F_5(a_1, \dots, a_i, \dots, a_6; b_1, \dots, b_j, \dots, b_5; z) = \sum_{k=0}^n \frac{(a_1)_k (a_2)_k (a_3)_k (a_4)_k (a_5)_k (a_6)_k z^k}{(b_1)_k (b_2)_k (b_3)_k (b_4)_k (b_5)_k k!} /;$$

$$a_i = -n \wedge b_j = -m \wedge m \in \mathbb{N} \wedge n \in \mathbb{N} \wedge m \geq n$$

Specific values

Values at $z = 0$

07.30.03.0001.01

$${}_6F_5(a_1, a_2, a_3, a_4, a_5, a_6; b_1, b_2, b_3, b_4, b_5; 0) = 1$$

Values at z = 1

For fixed a₃, a₄, b₁

07.30.03.0002.01

$${}_6F_5\left(-n, 1 - \frac{n}{3}, c, d, 1 - 2c - 2d - n, \frac{1}{2} - c - d - n; -\frac{n}{3}, 1 - 2c - n, 1 - 2d - n, 1 - c - d - n, c + d + \frac{1}{2}; 1\right) = 0 /;$$

$$n \in \mathbb{N}^+ \wedge \left\lfloor \frac{n+1}{3} \right\rfloor = \left\lfloor \frac{n+2}{3} \right\rfloor$$

For fixed a₃, a₄

07.30.03.0003.01

$${}_6F_5(-n, 1, c, d, 2 - c, 2 - d; n + 2, c + 1, 3 - c, d + 1, 3 - d; 1) = \frac{(n + 2)(2 - c)c(2 - d)d}{2(n + 1)(c - 1)^2(d - 1)^2} +$$

$$\frac{(n + 2)!n!}{2(d - c)(c + d - 2)} \left(\frac{c(2 - c)}{(d - 1)^2(d + 1)_n(3 - d)_n} - \frac{d(2 - d)}{(c - 1)^2(c + 1)_n(3 - c)_n} \right) /; c \neq d \wedge c \neq 1 \wedge d \neq 1$$

07.30.03.0004.01

$${}_6F_5\left(-n, 1 - \frac{n}{3}, c, d, 1 - 2c - 2d - n, \frac{1}{2} - c - d - n; -\frac{n}{3}, 1 - 2c - n, 1 - 2d - n, 1 - c - d - n, c + d + \frac{1}{2}; 1\right) = 0 /;$$

$$n \in \mathbb{N}^+ \wedge \left\lfloor \frac{n+1}{3} \right\rfloor = \left\lfloor \frac{n+2}{3} \right\rfloor$$

For fixed a₃

07.30.03.0005.01

$${}_6F_5(2, 2, c, c, 2 - c, 2 - c; 1, c + 1, c + 1, 3 - c, 3 - c; 1) = \frac{\pi c^2(2 - c)^2}{4(1 - c)} \left(\frac{\pi(1 - c)}{\sin^2(\pi c)} + \cot(\pi c) \right)$$

Values at z = -1

For fixed a₁

07.30.03.0006.01

$${}_6F_5\left(a, a + \frac{1}{3}, a + \frac{2}{3}, 3a - \frac{1}{3}, 3a, 3a + \frac{1}{3}; \frac{1}{3}, \frac{2}{3}, 2a, 2a + \frac{1}{3}, 2a + \frac{2}{3}; -1\right) =$$

$$\frac{\sqrt{\pi} \Gamma(6a)}{3} \left(\frac{4 \cdot 3^{-\frac{9a}{2}} \sqrt{\pi} \cos\left(\frac{9a-1}{6} \pi\right)}{\Gamma\left(\frac{2}{3}\right)\Gamma(3a)\Gamma\left(3a + \frac{1}{3}\right)} + \frac{2^{1-9a}}{\Gamma\left(\frac{3a+1}{2}\right)\Gamma\left(\frac{9a}{2}\right)} \right)$$

For fixed a₃

07.30.03.0007.01

$${}_6F_5\left(\frac{1}{2}, 1, c, c, 1 - c, 1 - c; \frac{3}{2}, c + 1, c + 1, 2 - c, 2 - c; -1\right) =$$

$$\frac{\pi(c - 1)^2 c^2 \csc^2(c \pi)}{(1 - 2c)^4} (2 - (2c - 1)\pi \cos(c \pi) - 2 \cos(2c \pi) - 4 \sin(c \pi)) /; c \neq \frac{1}{2}$$

07.30.03.0008.01

$${}_6F_5\left(1, \frac{3}{2}, c, c, 1-c, 1-c; \frac{1}{2}, c+1, c+1, 2-c, 2-c; -1\right) = \frac{(c-1)^2 c^2 \pi^2 \cot(c\pi) \csc(c\pi)}{1-2c} /; c \neq \frac{1}{2}$$

07.30.03.0009.01

$${}_6F_5(2, 2, c, c, 2-c, 2-c; 1, c+1, c+1, 3-c, 3-c; -1) = \frac{(c-2)^2 c^2 \pi \csc^2(c\pi) ((c-1)\pi \cos(c\pi) - \sin(c\pi))}{4(c-1)} /; c \neq 1$$

Specialized values

For fixed a_1, a_2, b_3, z

07.30.03.0010.01

$${}_6F_5\left(a, b, a + \frac{1}{3}, a + \frac{2}{3}, b + \frac{1}{3}, b + \frac{2}{3}; \frac{1}{3}, \frac{2}{3}, g, g + \frac{1}{3}, g + \frac{2}{3}; z\right) = \frac{1}{3} \sum_{k=0}^2 {}_2F_1\left(3a, 3b; 3g; e^{\frac{2\pi i k}{3}} \sqrt[3]{z}\right)$$

Values at fixed points

Values at $z = 1$

07.30.03.0011.01

$${}_6F_5\left(\frac{1}{2}, 1, \frac{3}{2}, \frac{3}{2}, \frac{3}{2}, \frac{9}{4}; \frac{5}{4}, 2, 2, 2, 3; 1\right) = \frac{32}{5} \left(1 - \frac{8}{\pi^2}\right)$$

Values at $z = -1$

07.30.03.0012.01

$${}_6F_5\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, 1; \frac{3}{2}, \frac{3}{2}, \frac{3}{2}, \frac{3}{2}, \frac{3}{2}; -1\right) = \frac{5\pi^5}{1536}$$

07.30.03.0013.01

$${}_6F_5\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{5}{4}; \frac{1}{4}, 1, 1, 1, 1; -1\right) = \frac{2}{\Gamma\left(\frac{3}{4}\right)^4}$$

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