InverseBetaRegularized4

Notations

Traditional name

Inverse of the generalized regularized incomplete beta function

Traditional notation

$I^{-1}_{(z_1,z_2)}(a, b)$

Mathematica StandardForm notation

`InverseBetaRegularized[`z_1`, `z_2`, `a`, `b`]`

Primary definition

06.24.02.0001.01

$z_2 = I_{(z_1,w)}(a, b) \;/;\; w = I^{-1}_{(z_1,z_2)}(a, b)$

Specific values

Specialized values

06.24.03.0001.01

$I^{-1}_{(0,z_2)}(a, b) = I^{-1}_{z_2}(a, b)$

06.24.03.0002.01

$I^{-1}_{(z_1,0)}(a, b) = z_1$

General characteristics

Domain and analyticity

$I^{-1}_{(z_1,z_2)}(a, b)$ is an analytical function of z_1, z_2, a, b which is defined in $\mathbb{C}^4$.

06.24.04.0001.01

$(z_1 * z_2 * a * b) \longrightarrow I^{-1}_{(z_1,z_2)}(a, b) :: (\mathbb{C}^4) \longrightarrow \mathbb{C}$

Symmetries and periodicities

Symmetry

No symmetry

Periodicity

No periodicity

Differential equations

Ordinary nonlinear differential equations

06.24.13.0001.01

$$w(z_2)\,(1-w(z_2))\,w''(z_2)-(1-a+(a+b-2)\,w(z_2))\,w'(z_2)^2=0\;/;\;w(z_2)=I^{-1}_{(z_1,z_2)}(a,\,b)$$

Differentiation

Low-order differentiation

With respect to z_1

06.24.20.0001.01

$$\frac{\partial I^{-1}_{(z_1,z_2)}(a,\,b)}{\partial z_1}=(1-w)^{1-b}\,w^{1-a}\,(1-z_1)^{b-1}\,z_1^{a-1}\;/;\;w=I^{-1}_{(z_1,z_2)}(a,\,b)$$

06.24.20.0002.01

$$\frac{\partial^2 I^{-1}_{(z_1,z_2)}(a,\,b)}{\partial z_1^2}=(1-w)^{1-2b}\,w^{1-2a}\,(1-z_1)^{b-2}\,z_1^{a-2}\left((a-1)\,(1-w)^b\,w^a-(a+b-2)\,(1-w)^b\,z_1\,w^a+(a\,(w-1)+(b-2)\,w+1)\,(1-z_1)^b\,z_1^a\right)\;/;\;w=I^{-1}_{(z_1,z_2)}(a,\,b)$$

With respect to z_2

06.24.20.0003.01

$$\frac{\partial I^{-1}_{(z_1,z_2)}(a,\,b)}{\partial z_2}=(1-w)^{1-b}\,w^{1-a}\,\mathrm{B}(a,\,b)\;/;\;w=I^{-1}_{(z_1,z_2)}(a,\,b)$$

06.24.20.0004.01

$$\frac{\partial^2 I^{-1}_{(z_1,z_2)}(a,\,b)}{\partial z_2^2}=(1-w)^{1-2b}\,w^{1-2a}\,((w-1)\,a+(b-2)\,w+1)\,\mathrm{B}(a,\,b)^2\;/;\;w=I^{-1}_{(z_1,z_2)}(a,\,b)$$

With respect to a

06.24.20.0005.01

$$\frac{\partial I^{-1}_{(z_1,z_2)}(a,\,b)}{\partial a}=(1-w)^{1-b}\,w^{1-a}\left(\Gamma(a)^2\,w^a\,{}_3\tilde{F}_2(a,\,a,\,1-b;\,a+1,\,a+1;\,w)-\Gamma(a)^2\,z_1^a\,{}_3\tilde{F}_2(a,\,a,\,1-b;\,a+1,\,a+1;\,z_1)+\mathrm{B}(a,\,b)\left(\log(z_1)\,I_{z_1}(a,\,b)-\log(w)\,I_w(a,\,b)+I_{(z_1,w)}(a,\,b)\,(\psi(a)-\psi(a+b))\right)\right)\;/;\;w=I^{-1}_{(z_1,z_2)}(a,\,b)$$

With respect to b

06.24.20.0006.01

$$\frac{\partial I_{(z_1,z_2)}^{-1}(a,b)}{\partial b} == \mathrm{B}(a,b)\,(1-w)^{1-b}\,w^{1-a}\left(\frac{1}{b^2\,\mathrm{B}(a,b)}\left({}_3F_2(1-a,b,b;b+1,b+1;1-z_1)\,(1-z_1)^b - {}_3F_2(1-a,b,b;b+1,b+1;1-w)\,(1-w)^b\right) + (\psi(b)-\psi(a+b))\,z_2 - \log(1-w)\left(I_{z_1}(a,b)+z_2-1\right) - I_{1-z_1}(b,a)\log(1-z_1)\right) /;\; w == I_{(z_1,z_2)}^{-1}(a,b)$$

Integration

Indefinite integration

Involving one direct function with respect to z_2

06.24.21.0001.01

$$\int I_{(z_1,z_2)}^{-1}(a,b)\,dz_2 == \frac{1}{(a+1)\,\mathrm{B}(a,b)}\,{}_2F_1\left(a+1,1-b;a+2;I_{(z_1,z_2)}^{-1}(a,b)\right)I_{(z_1,z_2)}^{-1}(a,b)^{a+1}$$

Representations through equivalent functions

With inverse function

06.24.27.0001.01

$$I_{\left(z_1,I_{(z_1,z_2)}^{-1}(a,b)\right)}(a,b) == z_2$$

06.24.27.0002.01

$$\mathrm{B}_{\left(z_1,I_{(z_1,z_2)}^{-1}(a,b)\right)}(a,b) == \mathrm{B}(a,b)\,z_2$$

With related functions

06.24.27.0003.01

$$I_{(z_1,z_2)}^{-1}(a,b) == I_{I_{z_1}(a,b)+z_2}^{-1}(a,b)$$

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