InverseErf

Notations

Traditional name

Inverse error function

Traditional notation

$\operatorname{erf}^{-1}(z)$

Mathematica StandardForm notation

InverseErf[z]

Primary definition

06.29.02.0001.01

$z = \operatorname{erf}(w) \;/;\; w = \operatorname{erf}^{-1}(z)$

Specific values

Values at fixed points

06.29.03.0001.01

$\operatorname{erf}^{-1}(0) = 0$

06.29.03.0002.01

$\operatorname{erf}^{-1}(1) = \infty$

General characteristics

Domain and analyticity

$\operatorname{erf}^{-1}(z)$ is an analytical function of z which is defined in the whole complex z-plane.

06.29.04.0001.01

$z \longrightarrow \operatorname{erf}^{-1}(z) :: \mathbb{C} \longrightarrow \mathbb{C}$

Symmetries and periodicities

Symmetry

No symmetry

Periodicity

No periodicity

Series representations

Generalized power series

Expansions at generic point $z == z_0$

For the function itself

06.29.06.0004.01

$$\operatorname{erf}^{-1}(z) \propto \operatorname{erf}^{-1}(z_0) + \frac{\sqrt{\pi}}{2} e^{\operatorname{erf}^{-1}(z_0)^2} (z - z_0) + \frac{\pi}{4} e^{2\operatorname{erf}^{-1}(z_0)^2} \operatorname{erf}^{-1}(z_0) (z - z_0)^2 + \frac{\pi^{3/2}}{24} e^{3\operatorname{erf}^{-1}(z_0)^2} \left(4 \operatorname{erf}^{-1}(z_0)^2 + 1\right) (z - z_0)^3 + \frac{\pi^2}{96} e^{4\operatorname{erf}^{-1}(z_0)^2} \operatorname{erf}^{-1}(z_0) \left(12 \operatorname{erf}^{-1}(z_0)^2 + 7\right) (z - z_0)^4 + \frac{\pi^{5/2}}{960} e^{5\operatorname{erf}^{-1}(z_0)^2} \left(8 \operatorname{erf}^{-1}(z_0)^2 + 7\right) \left(12 \operatorname{erf}^{-1}(z_0)^2 + 1\right) (z - z_0)^5 + \frac{\pi^3}{5760} e^{6\operatorname{erf}^{-1}(z_0)^2} \operatorname{erf}^{-1}(z_0) \left(480 \operatorname{erf}^{-1}(z_0)^4 + 652 \operatorname{erf}^{-1}(z_0)^2 + 127\right) (z - z_0)^6 + \frac{\pi^{7/2}}{80\,640} e^{7\operatorname{erf}^{-1}(z_0)^2} \left(5760 \operatorname{erf}^{-1}(z_0)^6 + 10\,224 \operatorname{erf}^{-1}(z_0)^4 + 3480 \operatorname{erf}^{-1}(z_0)^2 + 127\right) (z - z_0)^7 + \frac{\pi^4}{645\,120} e^{8\operatorname{erf}^{-1}(z_0)^2} \operatorname{erf}^{-1}(z_0) \left(40\,320 \operatorname{erf}^{-1}(z_0)^6 + 88\,848 \operatorname{erf}^{-1}(z_0)^4 + 44\,808 \operatorname{erf}^{-1}(z_0)^2 + 4369\right) (z - z_0)^8 + \frac{\pi^{9/2}}{11\,612\,160} e^{9\operatorname{erf}^{-1}(z_0)^2} \left(645\,120 \operatorname{erf}^{-1}(z_0)^8 + 1\,703\,808 \operatorname{erf}^{-1}(z_0)^6 + 1\,161\,168 \operatorname{erf}^{-1}(z_0)^4 + 204\,328 \operatorname{erf}^{-1}(z_0)^2 + 4369\right) (z - z_0)^9 + \frac{\pi^5}{116\,121\,600} e^{10\operatorname{erf}^{-1}(z_0)^2} \operatorname{erf}^{-1}(z_0) \left(5\,806\,080 \operatorname{erf}^{-1}(z_0)^8 + 17\,914\,752 \operatorname{erf}^{-1}(z_0)^6 + 15\,561\,936 \operatorname{erf}^{-1}(z_0)^4 + 4\,161\,288 \operatorname{erf}^{-1}(z_0)^2 + 243\,649\right) (z - z_0)^{10} + \ldots \;/;\; (z \to z_0)$$

06.29.06.0005.01

$$\operatorname{erf}^{-1}(z) \propto \operatorname{erf}^{-1}(z_0) \left(1 + O(z - z_0)\right)$$

Expansions at $z == 0$

06.29.06.0001.02

$$\operatorname{erf}^{-1}(z) \propto \frac{\sqrt{\pi}}{2} \left(z + \frac{\pi z^3}{12} + \frac{7 \pi^2 z^5}{480} + \ldots\right) /; (z \to 0)$$

06.29.06.0006.01

$$\operatorname{erf}^{-1}(z) \propto \frac{\sqrt{\pi}}{2} \left(z + \frac{\pi z^3}{12} + \frac{7 \pi^2 z^5}{480}\right) + O\left(z^7\right)$$

06.29.06.0007.01

$$\operatorname{erf}^{-1}(z) \propto \frac{\sqrt{\pi}}{2} z \left(1 + \frac{\pi}{12} z^2 + \frac{7 \pi^2}{480} z^4 + \frac{127 \pi^3}{40\,320} z^6 + \frac{4369 \pi^4}{5\,806\,080} z^8\right) + O\left(z^{11}\right)$$

06.29.06.0003.01

$$\text{erf}^{-1}(z) == \sum_{k=0}^{\infty} \frac{c_k}{2k+1} \left(\frac{\sqrt{\pi}}{2} z \right)^{2k+1} /; c_0 == 1 \bigwedge c_k == \sum_{m=0}^{k-1} \frac{c_m \, c_{k-1-m}}{(m+1)(2m+1)}$$

Gyorgy Steinbrecher

06.29.06.0008.01

$$\text{erf}^{-1}(z) == \frac{\sqrt{\pi} \, z}{2} + O(z^3)$$

Asymptotic series expansions

06.29.06.0002.01

$$\text{erf}^{-1}(z) \propto \frac{1}{\sqrt{2}} \sqrt{\log\left(\frac{2}{\pi (z-1)^2}\right) - \log\left(\log\left(\frac{2}{\pi (z-1)^2}\right)\right)} /; (z \to 1)$$

Differential equations

Ordinary nonlinear differential equations

06.29.13.0001.01

$$w''(z) - 2\, w(z)\, w'(z)^2 == 0 /; w(z) == \text{erf}^{-1}(z)$$

Differentiation

Low-order differentiation

06.29.20.0001.01

$$\frac{\partial \text{erf}^{-1}(z)}{\partial z} == \frac{\sqrt{\pi}}{2} e^{\text{erf}^{-1}(z)^2}$$

06.29.20.0002.01

$$\frac{\partial^2 \text{erf}^{-1}(z)}{\partial z^2} = \frac{1}{2} e^{2 w^2} \pi \, w /; w = \text{erf}^{-1}(z)$$

06.29.20.0003.01

$$\frac{\partial^3 \text{erf}^{-1}(z)}{\partial z^3} = \frac{1}{4} e^{3 w^2} \pi^{3/2} \left(4 w^2 + 1\right) /; w = \text{erf}^{-1}(z)$$

06.29.20.0004.01

$$\frac{\partial^4 \text{erf}^{-1}(z)}{\partial z^4} = \frac{1}{4} e^{4 w^2} \pi^2 \, w \left(12 w^2 + 7\right) /; w = \text{erf}^{-1}(z)$$

Symbolic differentiation

06.29.20.0005.01

$$\frac{\partial^n \operatorname{erf}^{-1}(z)}{\partial z^n} = \operatorname{erf}^{-1}(z)\,\delta_n + \frac{\pi^{n/2}}{2^n}\, e^{n\operatorname{erf}^{-1}(z)^2} \sum_{j_2=0}^{n} \ldots \sum_{j_n=0}^{n} \delta_{\sum_{i=2}^{n}(i-1)\,j_i,\,n-1}\, (-1)^{\sum_{i=2}^{n} j_i} \left(n + \sum_{i=2}^{n} j_i - 1\right)! \prod_{i=2}^{n} \frac{1}{j_i!} \left(\frac{2^{i-1}\, e^{\operatorname{erf}^{-1}(z)^2} \sqrt{\pi}\, \operatorname{erf}^{-1}(z)^{1-i}}{i!}\right)^{j_i} {}_2\tilde{F}_2\!\left(\frac{1}{2}, 1; 1-\frac{i}{2}, \frac{3-i}{2}; -\operatorname{erf}^{-1}(z)^2\right)^{j_i} /;\ n \in \mathbb{N}$$

Integration

Indefinite integration

Involving only one direct function

06.29.21.0001.01

$$\int \operatorname{erf}^{-1}(z)\, dz = -\frac{1}{\sqrt{\pi}}\, e^{-\operatorname{erf}^{-1}(z)^2}$$

Representations through more general functions

Through other functions

06.29.26.0001.01

$$\operatorname{erf}^{-1}(z) = \operatorname{erf}^{-1}(0, z)$$

Representations through equivalent functions

With inverse function

06.29.27.0001.01

$$\operatorname{erf}\left(\operatorname{erf}^{-1}(z)\right) = z$$

With related functions

06.29.27.0002.01

$$\operatorname{erf}^{-1}(z) = \operatorname{erfc}^{-1}(1-z)$$

Zeros

06.29.30.0001.01

$$\operatorname{erf}^{-1}(z) = 0 /;\ z = 0$$

History

– J. R. Philip (1960)

– A.J. Strecok (1968)

Copyright