InverseErf2

Notations

Traditional name

Inverse of the generalized error function

Traditional notation

$\text{erf}^{-1}(z_1, z_2)$

Mathematica StandardForm notation

`InverseErf[`z_1`, `z_2`]`

Primary definition

06.30.02.0001.01

$z_2 == \text{erf}(z_1, w) \text{ /; } w == \text{erf}^{-1}(z_1, z_2)$

Specific values

Specialized values

06.30.03.0001.01

$\text{erf}^{-1}(0, z) == \text{erf}^{-1}(z)$

Values at fixed points

06.30.03.0002.01

$\text{erf}^{-1}(0, 0) == 0$

06.30.03.0003.01

$\text{erf}^{-1}(0, 1) == \infty$

06.30.03.0004.01

$\text{erf}^{-1}(1, 0) == 1$

Values at infinities

06.30.03.0005.01

$\text{erf}^{-1}(\infty, z) == \text{erfc}^{-1}(-z)$

General characteristics

Domain and analyticity

$\text{erf}^{-1}(z_1, z_2)$ is an analytical function of z_1 and z_2 which is defined in $\mathbb{C}^2$.

06.30.04.0001.01

$$(z_1 * z_2) \longrightarrow \text{erf}^{-1}(z_1, z_2) :: (\mathbb{C} \otimes \mathbb{C}) \longrightarrow \mathbb{C}$$

Symmetries and periodicities

Symmetry

No symmetry

Periodicity

No periodicity

Series representations

Generalized power series

Expansions at $z_1 = 0$

06.30.06.0001.01

$$\text{erf}^{-1}(z_1, z_2) \propto \text{erf}^{-1}(z_2) + e^{\text{erf}^{-1}(z_2)^2}\Big(z_1 + e^{\text{erf}^{-1}(z_2)^2}\,\text{erf}^{-1}(z_2)\, z_1^2 + \left(4\, e^{2\,\text{erf}^{-1}(z_2)^2}\,\text{erf}^{-1}(z_2)^2 + e^{2\,\text{erf}^{-1}(z_2)^2} - 1\right)\frac{z_1^3}{3} + e^{\text{erf}^{-1}(z_2)^2}\,\text{erf}^{-1}(z_2)\left(12\, e^{2\,\text{erf}^{-1}(z_2)^2}\,\text{erf}^{-1}(z_2)^2 + 7\, e^{2\,\text{erf}^{-1}(z_2)^2} - 4\right)\frac{z_1^4}{6} + \left(96\, e^{4\,\text{erf}^{-1}(z_2)^2}\,\text{erf}^{-1}(z_2)^4 - 40\, e^{2\,\text{erf}^{-1}(z_2)^2}\,\text{erf}^{-1}(z_2)^2 + 92\, e^{4\,\text{erf}^{-1}(z_2)^2}\,\text{erf}^{-1}(z_2)^2 - 10\, e^{2\,\text{erf}^{-1}(z_2)^2} + 7\, e^{4\,\text{erf}^{-1}(z_2)^2} + 3\right)\frac{z_1^5}{30} + \ldots\Big) /; (z_1 \to 0)$$

06.30.06.0002.01

$$\text{erf}^{-1}(z_1, z_2) \propto \text{erf}^{-1}(z_2) + e^{\text{erf}^{-1}(z_2)^2}\, z_1 (1 + O(z_1)) \;/; (z_1 \to 0)$$

Expansions at $z_2 = 0$

06.30.06.0003.01

$$\text{erf}^{-1}(z_1, z_2) \propto z_1 + \frac{1}{2} e^{z_1^2} \sqrt{\pi}\, z_2 + \frac{\pi z_1}{4} e^{2 z_1^2} z_2^2 + \frac{\pi^{3/2}(1 + 4 z_1^2)}{24} e^{3 z_1^2} z_2^3 + \frac{\pi^2 z_1 (7 + 12 z_1^2)}{96} e^{4 z_1^2} z_2^4 + \frac{\pi^{5/2}(7 + 8 z_1^2)(1 + 12 z_1^2)}{960} e^{5 z_1^2} z_2^5 + \frac{\pi^3 z_1 (127 + 652 z_1^2 + 480 z_1^4)}{5760} e^{6 z_1^2} z_2^6 + \frac{\pi^{7/2}(127 + 3480 z_1^2 + 10\,224 z_1^4 + 5760 z_1^6)}{80\,640} e^{7 z_1^2} z_2^7 + \frac{\pi^4 z_1 (4369 + 44\,808 z_1^2 + 88\,848 z_1^4 + 40\,320 z_1^6)}{645\,120} e^{8 z_1^2} z_2^8 + \ldots /; (z_2 \to 0)$$

06.30.06.0004.01

$$\text{erf}^{-1}(z_1, z_2) \propto z_1 (1 + O(z_2)) \;/; (z_2 \to 0)$$

Differential equations

Ordinary nonlinear differential equations

06.30.13.0001.01

$$w''(z_2) - 2\,w(z_2)\,w'(z_2)^2 == 0 \;/;\; w(z_2) == \operatorname{erf}^{-1}(z_1, z_2)$$

Differentiation

Low-order differentiation

With respect to z_1

06.30.20.0001.01

$$\frac{\partial \operatorname{erf}^{-1}(z_1, z_2)}{\partial z_1} == e^{\operatorname{erf}^{-1}(z_1,z_2)^2 - z_1^2}$$

06.30.20.0002.01

$$\frac{\partial^2 \operatorname{erf}^{-1}(z_1, z_2)}{\partial z_1^2} == e^{\operatorname{erf}^{-1}(z_1,z_2)^2 - z_1^2}\left(2\,e^{\operatorname{erf}^{-1}(z_1,z_2)^2 - z_1^2}\operatorname{erf}^{-1}(z_1, z_2) - 2\,z_1\right)$$

With respect to z_2

06.30.20.0003.01

$$\frac{\partial \operatorname{erf}^{-1}(z_1, z_2)}{\partial z_2} == \frac{\sqrt{\pi}}{2}\,e^{\operatorname{erf}^{-1}(z_1,z_2)^2}$$

06.30.20.0004.01

$$\frac{\partial^2 \operatorname{erf}^{-1}(z_1, z_2)}{\partial z_2^2} == \frac{\pi}{2}\,e^{2\operatorname{erf}^{-1}(z_1,z_2)^2}\operatorname{erf}^{-1}(z_1, z_2)$$

Symbolic differentiation

With respect to z_2

06.30.20.0005.01

$$\frac{\partial^n \operatorname{erf}^{-1}(z_1, z_2)}{\partial z_2^n} == \operatorname{erf}^{-1}(z_1, z_2)\,\delta_n + \frac{\pi^{n/2}}{2^n}\,e^{n\operatorname{erf}^{-1}(z_1,z_2)^2}\sum_{j_2=0}^{n}\cdots\sum_{j_n=0}^{n}\delta_{\sum_{i=2}^{n}(i-1)\,j_i,\,n-1}\,(-1)^{\sum_{i=2}^{n} j_i}\left(n+\sum_{i=2}^{n} j_i - 1\right)!\,\prod_{i=2}^{n}\frac{1}{j_i!}\left(\frac{2^{i-1}\,e^{\operatorname{erf}^{-1}(z_1,z_2)^2}\sqrt{\pi}\,\operatorname{erf}^{-1}(z_1, z_2)^{1-i}}{i!}\right)^{j_i}{}_2\tilde{F}_2\!\left(\frac{1}{2}, 1; 1-\frac{i}{2}, \frac{3-i}{2}; -\operatorname{erf}^{-1}(z_1, z_2)^2\right)^{j_i} \;/;\; n \in \mathbb{N}$$

Integration

Indefinite integration

Involving one direct function with respect to z_2

06.30.21.0001.01

$$\int \operatorname{erf}^{-1}(z_1, z_2)\,dz_2 == -\frac{1}{\sqrt{\pi}}\,e^{-\operatorname{erf}^{-1}(z_1,z_2)^2}$$

Representations through equivalent functions

With inverse function

06.30.27.0001.01

$$\operatorname{erf}\left(z_1, \operatorname{erf}^{-1}(z_1, z_2)\right) = z_2$$

Zeros

06.30.30.0001.01

$$\operatorname{erf}^{-1}(z_1, z_2) = 0 \;/;\; z_1 = 0 \land z_2 = 0$$

Copyright