

# InverseGammaRegularized3

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## Notations

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### Traditional name

Inverse of the generalized regularized incomplete gamma function

### Traditional notation

$$Q^{-1}(a, z_1, z_2)$$

### Mathematica StandardForm notation

InverseGammaRegularized[ $a, z_1, z_2$ ]

## Primary definition

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06.13.02.0001.01

$$z_2 = Q(a, z_1, w) /; w = Q^{-1}(a, z_1, z_2)$$

## Specific values

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### Specialized values

06.13.03.0001.01

$$Q^{-1}(a, \infty, z) = Q^{-1}(a, -z)$$

## General characteristics

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### Domain and analyticity

$Q^{-1}(a, z_1, z_2)$  is an analytical function of  $a, z_1, z_2$  which is defined in  $\mathbb{C}^3$ . For fixed noninteger  $a$ , it has one infinitely long branch cut.

06.13.04.0001.01

$$(a * z_1 * z_2) \rightarrow Q^{-1}(a, z_1, z_2) :: (\mathbb{C} \otimes \mathbb{C} \otimes \mathbb{C}) \rightarrow \mathbb{C}$$

### Symmetries and periodicities

#### Symmetry

No symmetry

#### Periodicity

No periodicity

## Differential equations

### Ordinary nonlinear differential equations

06.13.13.0001.01

$$w(z_2) w''(z_2) - w'(z_2)^2 (-a + w(z_2) + 1) = 0 /; w(z_2) = Q^{-1}(a, z_1, z_2)$$

## Differentiation

### Low-order differentiation

With respect to  $a$

06.13.20.0001.01

$$\frac{\partial Q^{-1}(a, z_1, z_2)}{\partial a} = e^w w^{1-a} \left( \frac{1}{a^2} (w^a {}_2F_2(a, a; a+1, a+1; -w) - z_1^a {}_2F_2(a, a; a+1, a+1; -z_1)) + \Gamma(a, w, 0) \log(w) + \Gamma(a, 0, z_1) \log(z_1) + \Gamma(a, z_1, w) \psi(a) \right) /; w = Q^{-1}(a, z_1, z_2)$$

06.13.20.0002.01

$$\begin{aligned} \frac{\partial^2 Q^{-1}(a, z_1, z_2)}{\partial a^2} = & \frac{1}{a \Gamma(1-a)} \left( e^w w^{1-2a} \left( (a-w-1)(-a) e^w \Gamma(1-a) \Gamma(a)^2 (\log(w) - \log(z_1))^2 + \Gamma(a) (a e^w \pi \csc(a\pi) - a \Gamma(1-a) \right. \right. \\ & \left. \left. ((\log(z_1) - \log(w)) w^a + e^w \Gamma(a) + 2 e^w (1-a+w) \Gamma(a, w) (\log(w) - \psi(a)) - 2 e^w (1-a+w) \Gamma(a, \right. \right. \\ & \left. \left. z_1) (\log(z_1) - \psi(a))) (\log(w) - \log(z_1)) - a e^w (a-w-1) \Gamma(1-a) \Gamma(a, w)^2 (\log(w) - \psi(a))^2 - \right. \right. \\ & \left. \left. \Gamma(a, w) (a e^w \pi \csc(a\pi) (\log(w) - \psi(a)) + a \Gamma(1-a) ((\log^2(w) - 2 \psi(a) \log(w) + \psi(a)^2 + \psi^{(1)}(a)) w^a - \right. \right. \\ & \left. \left. e^w \Gamma(a) (\log(w) - \psi(a)) + 2 e^w (1-a+w) \Gamma(a, z_1) (\log(w) - \psi(a)) (\log(z_1) - \psi(a))) \right) + \right. \\ & \left. \Gamma(a, z_1) (a e^w \pi \csc(a\pi) (\log(z_1) - \psi(a)) + a \Gamma(1-a) ((\psi(a)^2 - 2 \log(w) \psi(a) + (2 \log(w) - \log(z_1)) \log(z_1) + \right. \right. \\ & \left. \left. \psi^{(1)}(a)) w^a + e^w (1-a+w) \Gamma(a, z_1) (\log(z_1) - \psi(a))^2 - e^w \Gamma(a) (\log(z_1) - \psi(a))) \right) \right) + \\ & \frac{1}{a^4} \left( e^w w \left( e^w (1-a+w) {}_2F_2(a, a; a+1, a+1; -z_1) z_1^a (2 {}_2F_2(a, a; a+1, a+1; -w) w^a + {}_2F_2(a, a; a+1, a+1; -z_1) z_1^a \right. \right. \\ & \left. \left. w^{-2a} + 2 a {}_3F_3(a, a, a; a+1, a+1, a+1; -z_1) z_1^a w^{-a} + e^w (1-a+w) {}_2F_2(a, a; a+1, a+1; -w)^2 - \right. \right. \\ & \left. \left. 2 a {}_3F_3(a, a, a; a+1, a+1, a+1; -w) + \frac{1}{\Gamma(1-a)} \left( w^{-2a} {}_2F_2(a, a; a+1, a+1; -z_1) \right. \right. \right. \\ & \left. \left. \left( a (a e^w \pi \csc(a\pi) - \Gamma(1-a) (a e^w \Gamma(a) + 2 a (\log(z_1) w^a - e^w \Gamma(a, z_1) \log(z_1) w + e^w \Gamma(a, z_1) \psi(a) w - \right. \right. \right. \\ & \left. \left. \left. w^a \log(w) - e^w (1-a+w) \Gamma(a) (\log(w) - \log(z_1)) + a e^w \Gamma(a, z_1) \log(z_1) - \right. \right. \right. \\ & \left. \left. \left. e^w \Gamma(a, z_1) \log(z_1) + e^w (1-a+w) \Gamma(a, w) (\log(w) - \psi(a)) - a e^w \Gamma(a, z_1) \psi(a) + \right. \right. \right. \\ & \left. \left. \left. e^w \Gamma(a, z_1) \psi(a)) \right) - 2 e^w w^a (1-a+w) \Gamma(1-a) {}_2F_2(a, a; a+1, a+1; -w) \right) z_1^a \right) + \\ & \left. \frac{1}{\Gamma(1-a)} \left( e^w w^{-a} {}_2F_2(a, a; a+1, a+1; -w) (2(a-w-1) \Gamma(1-a) {}_2F_2(a, a; a+1, a+1; -z_1) z_1^a + \right. \right. \\ & \left. \left. a (a \Gamma(1-a) (\Gamma(a) + 2(a-w-1) (\Gamma(a) (\log(w) - \log(z_1)) + \Gamma(a, z_1) (\log(z_1) - \psi(a)) + \right. \right. \\ & \left. \left. \left. \Gamma(a, w) (\psi(a) - \log(w)))) - a \pi \csc(a\pi)) \right) \right) \right) /; w = Q^{-1}(a, z_1, z_2) \end{aligned}$$

**With respect to  $z_1$**

06.13.20.0003.01

$$\frac{\partial Q^{-1}(a, z_1, z_2)}{\partial z_1} = e^{Q^{-1}(a, z_1, z_2) - z_1} \left( \frac{Q^{-1}(a, z_1, z_2)}{z_1} \right)^{1-a}$$

06.13.20.0004.01

$$\frac{\partial^2 Q^{-1}(a, z_1, z_2)}{\partial z_1^2} = \frac{1}{z_1} e^{w-2z_1} \left( \frac{w}{z_1} \right)^{1-2a} \left( e^w w - (a-1) \left( e^w - e^{z_1} \left( \frac{w}{z_1} \right)^a \right) - e^{z_1} z_1 \left( \frac{w}{z_1} \right)^a \right) /; w = Q^{-1}(a, z_1, z_2)$$

**With respect to  $z_2$**

06.13.20.0005.01

$$\frac{\partial Q^{-1}(a, z_1, z_2)}{\partial z_2} = e^{Q^{-1}(a, z_1, z_2)} \Gamma(a) Q^{-1}(a, z_1, z_2)^{1-a}$$

06.13.20.0006.01

$$\frac{\partial^2 Q^{-1}(a, z_1, z_2)}{\partial z_2^2} = e^{2w} \Gamma(a)^2 w^{1-2a} (1-a+w) /; w = Q^{-1}(a, z_1, z_2)$$

**Symbolic differentiation**

**With respect to  $z_2$**

06.13.20.0007.01

$$\frac{\partial^n Q^{-1}(a, z_1, z_2)}{\partial z_2^n} = w \delta_n + \left( \frac{\Gamma(a) e^w}{w^{a-1}} \right)^n \sum_{j_2=0}^n \dots \sum_{j_n=0}^n \delta_{\sum_{i=2}^n (i-1)j_i, n-1} (-1)^{\sum_{i=2}^n j_i} \left( n + \sum_{i=2}^n j_i - 1 \right)! \prod_{i=2}^n \frac{1}{j_i!} \left( \frac{\Gamma(a+1) e^w w^{1-a}}{i!} \right)^{j_i} \left( a w^{-i} \sum_{k=0}^i (-1)^{i-k} \binom{i}{k} (1-a-k)_{i-1} Q(a+k, w) + Q(a, z_1) \delta_i \right)^{j_i} /; w = Q^{-1}(a, z_1, z_2) \wedge n \in \mathbb{N}$$

**Integration**

**Indefinite integration**

**Involving one direct function with respect to  $z_2$**

06.13.21.0001.01

$$\int Q^{-1}(a, z_1, z_2) dz_2 = -a Q(a+1, Q^{-1}(a, z_1, z_2))$$

**Representations through equivalent functions**

**With inverse function**

06.13.27.0001.01

$$Q(a, z_1, Q^{-1}(a, z_1, z_2)) = z_2$$

06.13.27.0002.01

$$\Gamma(a, z_1, Q^{-1}(a, z_1, z_2)) = \Gamma(a) z_2$$

06.13.27.0004.01

$$Q(a, Q^{-1}(a, z_1, z_2)) = Q(a, z_1) - z_2$$

06.13.27.0005.01

$$\Gamma(a, Q^{-1}(a, z_1, z_2)) = \Gamma(a, z_1) - z_2 \Gamma(a)$$

### With related functions

06.13.27.0003.01

$$Q^{-1}(a, z_1, z_2) = Q^{-1}(a, Q(a, z_1) - z_2)$$

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