InverseWeierstrassP4

Notations

Traditional name

Generalized inverse Weierstrass elliptic function

Traditional notation

$\wp^{-1}(z_1, z_2; g_2, g_3)$

Mathematica StandardForm notation

`InverseWeierstrassP[{z1, z2}, {g2, g3}]`

Primary definition

09.23.02.0001.01

$$\wp(w; g_2, g_3) = z_1 \;/;\; w = \wp^{-1}(z_1, z_2; g_2, g_3) \bigwedge z_2 = \sqrt{4 z_1^3 - g_2 z_1 - g_3}$$

09.23.02.0002.01

$$\wp^{-1}(z_1, z_2; g_2, g_3) = \int_{\infty}^{z_1} \frac{1}{\sqrt{4 t^3 - g_2 t - g_3}} \, dt \;/;\; z_2 = \sqrt{4 z_1^3 - g_2 z_1 - g_3}$$

$\wp^{-1}(z_1, z_2; g_2, g_3)$ is the unique value of u for which $z_1 = \wp(u; g_2, g_3)$ and $z_2 = \wp'(u; g_2, g_3)$. For $\wp^{-1}(z_1, z_2; g_2, g_3)$ to exist, z_1 and z_2 must be related by $z_2^2 = 4 z_1^3 - g_2 z_1 - g_3$.

General characteristics

Domain and analyticity

$\wp^{-1}(z_1, z_2; g_2, g_3)$ is an analytical function of z_1, z_2, g_2, g_3 which is defined in $\mathbb{C}^4$.

09.23.04.0001.01

$$(\{z_1 * z_2\} * \{g_2 * g_3\}) \longrightarrow \wp^{-1}(z_1, z_2; g_2, g_3) :: (\{\mathbb{C} \otimes \{-1, 1\}\} \otimes \{\mathbb{C} \otimes \mathbb{C}\}) \longrightarrow \mathbb{C}$$

Symmetries and periodicities

Symmetry

No symmetry

Periodicity

No periodicity

Poles and essential singularities

With respect to g_3

The function $\wp^{-1}(z_1, z_2; g_2, g_3)$ does not have poles and essential singularities with respect to g_3.

09.23.04.0002.01

$$\mathcal{Sing}_{g_3}\left(\wp^{-1}(z_1, z_2; g_2, g_3)\right) == \{\}$$

With respect to g_2

The function $\wp^{-1}(z_1, z_2; g_2, g_3)$ does not have poles and essential singularities with respect to g_2.

09.23.04.0003.01

$$\mathcal{Sing}_{g_2}\left(\wp^{-1}(z_1, z_2; g_2, g_3)\right) == \{\}$$

With respect to z_1

The function $\wp^{-1}(z_1, z_2; g_2, g_3)$ does not have poles and essential singularities with respect to z_1.

09.23.04.0004.01

$$\mathcal{Sing}_{z_1}\left(\wp^{-1}(z_1, z_2; g_2, g_3)\right) == \{\}$$

Branch cuts

Branch cut locations: complicated

Integral representations

On the real axis

Of the direct function

09.23.07.0001.01

$$\wp^{-1}(z_1, z_2; g_2, g_3) == \int_{\infty}^{z_1} \frac{1}{\sqrt{4\,t^3 - g_2\,t - g_3}}\,dt \;/;\; z_2 == \sqrt{4\,z_1^3 - g_2\,z_1 - g_3}$$

Differential equations

Ordinary nonlinear differential equations

09.23.13.0001.01

$$\left(4\,z_1^3 - g_2\,z_1 - g_3\right) w'(z_1)^2 - 1 == 0 \;/;\; w(z_1) == \wp^{-1}(z_1, z_2; g_2, g_3)$$

Differentiation

Low-order differentiation

09.23.20.0001.01

$$\frac{\partial \wp^{-1}(z_1, z_2; g_2, g_3)}{\partial z_1} = \frac{1}{\wp'\left(\wp^{-1}(z_1; g_2, g_3); g_2, g_3\right)} \;/;\; z_2 = \sqrt{4 z_1^3 - g_2 z_1 - g_3}$$

09.23.20.0002.01

$$\frac{\partial \wp^{-1}(z_1, z_2; g_2, g_3)}{\partial z_1} = \frac{1}{\sqrt{4 z_1^3 - g_2 z_1 - g_3}} \;/;\; z_2 = \sqrt{4 z_1^3 - g_2 z_1 - g_3}$$

09.23.20.0003.01

$$\frac{\partial^2 \wp^{-1}(z_1, z_2; g_2, g_3)}{\partial z_1^2} = \frac{g_2 - 12 \wp\left(\wp^{-1}(z_1; g_2, g_3); g_2, g_3\right)^2}{2 \wp'\left(\wp^{-1}(z_1; g_2, g_3); g_2, g_3\right)^3} \;/;\; z_2 = \sqrt{4 z_1^3 - g_2 z_1 - g_3}$$

Symbolic differentiation

09.23.20.0004.01

$$\frac{\partial^n \wp^{-1}(z_1, z_2; g_2, g_3)}{\partial z_1^n} = \frac{\delta_{n-1}}{\sqrt{4 z_1^3 - g_2 z_1 - g_3}} + \wp^{-1}(z_1, z_2; g_2, g_3)\, \delta_n + \sum_{m=1}^{n-1} \frac{1}{m!} \left(\frac{1}{2} - m\right)_m \sum_{j=0}^{m-1} (-1)^j \binom{m}{j} \left(4 z_1^3 - g_2 z_1 - g_3\right)^{j-m-\frac{1}{2}} \sum_{k_1=0}^{m-j} \sum_{k_2=0}^{m-j} \sum_{k_3=0}^{m-j} (-1)^{n+k_2+k_3-1} \delta_{m-j, k_1+k_2+k_3} (k_1 + k_2 + k_3; k_1, k_2, k_3)\, 4^{k_1} g_2^{k_2} g_3^{k_3} (-3 k_1 - k_2)_{n-1} z_1^{-n+3 k_1+k_2+1} \;/;\; n \in \mathbb{N} \bigwedge z_2 = \sqrt{4 z_1^3 - g_2 z_1 - g_3}$$

Representations through equivalent functions

With inverse function

09.23.27.0001.01

$$\wp\left(\wp^{-1}(z_1, z_2; g_2, g_3); g_2, g_3\right) = z_1 \;/;\; z_2 = \sqrt{4 z_1^3 - g_2 z_1 - g_3}$$

09.23.27.0002.01

$$\wp'\left(\wp^{-1}(z_1, z_2; g_2, g_3); g_2, g_3\right) = z_2 \;/;\; z_2 = \sqrt{4 z_1^3 - g_2 z_1 - g_3}$$

History

– L. Euler (1761)

– J.-L. Lagrange (1769)

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