

InverseWeierstrassP4

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Notations

Traditional name

Generalized inverse Weierstrass elliptic function

Traditional notation

$$\wp^{-1}(z_1, z_2; g_2, g_3)$$

Mathematica StandardForm notation

$$\text{InverseWeierstrassP}[\{z_1, z_2\}, \{g_2, g_3\}]$$

Primary definition

09.23.02.0001.01

$$\wp(w; g_2, g_3) = z_1 /; w = \wp^{-1}(z_1, z_2; g_2, g_3) \wedge z_2 = \sqrt{4z_1^3 - g_2z_1 - g_3}$$

09.23.02.0002.01

$$\wp^{-1}(z_1, z_2; g_2, g_3) = \int_{\infty}^{z_1} \frac{1}{\sqrt{4t^3 - g_2t - g_3}} dt /; z_2 = \sqrt{4z_1^3 - g_2z_1 - g_3}$$

$\wp^{-1}(z_1, z_2; g_2, g_3)$ is the unique value of u for which $z_1 = \wp(u; g_2, g_3)$ and $z_2 = \wp'(u; g_2, g_3)$. For $\wp^{-1}(z_1, z_2; g_2, g_3)$ to exist, z_1 and z_2 must be related by $z_2^2 = 4z_1^3 - g_2z_1 - g_3$.

General characteristics

Domain and analyticity

$\wp^{-1}(z_1, z_2; g_2, g_3)$ is an analytical function of z_1, z_2, g_2, g_3 which is defined in \mathbb{C}^4 .

09.23.04.0001.01

$$(\{z_1 * z_2\} * \{g_2 * g_3\}) \rightarrow \wp^{-1}(z_1, z_2; g_2, g_3) :: (\{\mathbb{C} \otimes \{-1, 1\}\} \otimes \{\mathbb{C} \otimes \mathbb{C}\}) \rightarrow \mathbb{C}$$

Symmetries and periodicities

Symmetry

No symmetry

Periodicity

No periodicity

Poles and essential singularities

With respect to g_3

The function $\wp^{-1}(z_1, z_2; g_2, g_3)$ does not have poles and essential singularities with respect to g_3 .

09.23.04.0002.01

$$\text{Sing}_{g_3}(\wp^{-1}(z_1, z_2; g_2, g_3)) = \{\}$$

With respect to g_2

The function $\wp^{-1}(z_1, z_2; g_2, g_3)$ does not have poles and essential singularities with respect to g_2 .

09.23.04.0003.01

$$\text{Sing}_{g_2}(\wp^{-1}(z_1, z_2; g_2, g_3)) = \{\}$$

With respect to z_1

The function $\wp^{-1}(z_1, z_2; g_2, g_3)$ does not have poles and essential singularities with respect to z_1 .

09.23.04.0004.01

$$\text{Sing}_{z_1}(\wp^{-1}(z_1, z_2; g_2, g_3)) = \{\}$$

Branch cuts

Branch cut locations: complicated

Integral representations

On the real axis

Of the direct function

09.23.07.0001.01

$$\wp^{-1}(z_1, z_2; g_2, g_3) = \int_{\infty}^{z_1} \frac{1}{\sqrt{4t^3 - g_2 t - g_3}} dt /; z_2 = \sqrt{4z_1^3 - g_2 z_1 - g_3}$$

Differential equations

Ordinary nonlinear differential equations

09.23.13.0001.01

$$(4z_1^3 - g_2 z_1 - g_3) w'(z_1)^2 - 1 = 0 /; w(z_1) = \wp^{-1}(z_1, z_2; g_2, g_3)$$

Differentiation

Low-order differentiation

09.23.20.0001.01

$$\frac{\partial \varphi^{-1}(z_1, z_2; g_2, g_3)}{\partial z_1} = \frac{1}{\varphi'(\varphi^{-1}(z_1; g_2, g_3); g_2, g_3)} /; z_2 = \sqrt{4 z_1^3 - g_2 z_1 - g_3}$$

09.23.20.0002.01

$$\frac{\partial \varphi^{-1}(z_1, z_2; g_2, g_3)}{\partial z_1} = \frac{1}{\sqrt{4 z_1^3 - g_2 z_1 - g_3}} /; z_2 = \sqrt{4 z_1^3 - g_2 z_1 - g_3}$$

09.23.20.0003.01

$$\frac{\partial^2 \varphi^{-1}(z_1, z_2; g_2, g_3)}{\partial z_1^2} = \frac{g_2 - 12 \varphi(\varphi^{-1}(z_1; g_2, g_3); g_2, g_3)^2}{2 \varphi'(\varphi^{-1}(z_1; g_2, g_3); g_2, g_3)^3} /; z_2 = \sqrt{4 z_1^3 - g_2 z_1 - g_3}$$

Symbolic differentiation

09.23.20.0004.01

$$\frac{\partial^n \varphi^{-1}(z_1, z_2; g_2, g_3)}{\partial z_1^n} = \frac{\delta_{n-1}}{\sqrt{4 z_1^3 - g_2 z_1 - g_3}} + \varphi^{-1}(z_1, z_2; g_2, g_3) \delta_n +$$

$$\sum_{m=1}^{n-1} \frac{1}{m!} \left(\frac{1}{2} - m\right)_m \sum_{j=0}^{m-1} (-1)^j \binom{m}{j} (4 z_1^3 - g_2 z_1 - g_3)^{j-m-\frac{1}{2}} \sum_{k_1=0}^{m-j} \sum_{k_2=0}^{m-j-k_1} \sum_{k_3=0}^{m-j-k_1-k_2} (-1)^{n+k_2+k_3-1} \delta_{m-j, k_1+k_2+k_3}$$

$$(k_1 + k_2 + k_3; k_1, k_2, k_3) 4^{k_1} g_2^{k_2} g_3^{k_3} (-3 k_1 - k_2)_{n-1} z_1^{-n+3 k_1+k_2+1} /; n \in \mathbb{N} \wedge z_2 = \sqrt{4 z_1^3 - g_2 z_1 - g_3}$$

Representations through equivalent functions

With inverse function

09.23.27.0001.01

$$\varphi(\varphi^{-1}(z_1, z_2; g_2, g_3); g_2, g_3) = z_1 /; z_2 = \sqrt{4 z_1^3 - g_2 z_1 - g_3}$$

09.23.27.0002.01

$$\varphi'(\varphi^{-1}(z_1, z_2; g_2, g_3); g_2, g_3) = z_2 /; z_2 = \sqrt{4 z_1^3 - g_2 z_1 - g_3}$$

History

-L. Euler (1761)

-J.-L. Lagrange (1769)

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