JacobiNS

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Notations

Traditional name

Jacobi elliptic function ns

Traditional notation

$\mathrm{ns}(z \mid m)$

Mathematica StandardForm notation

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JacobiNS[z, m]
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Primary definition

09.33.02.0001.01

$$\mathrm{ns}(z \mid m) = \frac{1}{\mathrm{sn}(z \mid m)}$$

Specific values

Specialized values

For fixed *z*

Case $m = 0$

09.33.03.0001.01

$$\mathrm{ns}(z \mid 0) = \csc(z)$$

09.33.03.0002.01

$$\mathrm{ns}\left(z + \frac{\pi}{2} \middle| 0\right) = \sec(z)$$

09.33.03.0026.01

$$\mathrm{ns}\left(z + \frac{\pi k}{2} \middle| 0\right) = \csc\left(z + \frac{\pi k}{2}\right) /; k \in \mathbb{Z}$$

Case $m = 1$

09.33.03.0003.01

$$\mathrm{ns}(z \mid 1) = \coth(z)$$

09.33.03.0004.01

$$\operatorname{ns}\left(z + \frac{\pi i}{2} \,\middle|\, 1\right) = \tanh(z)$$

09.33.03.0027.01

$$\operatorname{ns}\left(z + \frac{i \pi k}{2} \,\middle|\, 1\right) = \coth\left(z + \frac{i \pi k}{2}\right) /; k \in \mathbb{Z}$$

For fixed *m*

Values at quarter-period points in the fundamental period parallelogram

09.33.03.0005.01

$$\operatorname{ns}(0 \mid m) = \tilde{\infty}$$

09.33.03.0006.01

$$\operatorname{ns}(K(m) \mid m) = 1$$

09.33.03.0007.01

$$\operatorname{ns}(2 K(m) \mid m) = \tilde{\infty}$$

09.33.03.0008.01

$$\operatorname{ns}(3 K(m) \mid m) = -1$$

09.33.03.0009.01

$$\operatorname{ns}(4 K(m) \mid m) = \tilde{\infty}$$

09.33.03.0010.01

$$\operatorname{ns}(i K(1-m) \mid m) = 0$$

09.33.03.0011.01

$$\operatorname{ns}(2 i K(1-m) \mid m) = \tilde{\infty}$$

09.33.03.0012.01

$$\operatorname{ns}(3 i K(1-m) \mid m) = 0$$

09.33.03.0013.01

$$\operatorname{ns}(4 i K(1-m) \mid m) = \tilde{\infty}$$

09.33.03.0014.01

$$\operatorname{ns}(K(m) + i K(1-m) \mid m) = \sqrt{m}$$

09.33.03.0015.01

$$\operatorname{ns}(2 K(m) + i K(1-m) \mid m) = 0$$

09.33.03.0016.01

$$\operatorname{ns}(3 K(m) + i K(1-m) \mid m) = -\sqrt{m}$$

09.33.03.0017.01

$$\operatorname{ns}(4 K(m) + i K(1-m) \mid m) = 0$$

09.33.03.0018.01

$$\operatorname{ns}(K(m) + 2 i K(1-m) \mid m) = 1$$

09.33.03.0019.01

$$\operatorname{ns}(2 K(m) + 2 i K(1-m) \mid m) = \tilde{\infty}$$

09.33.03.0020.01

$$\operatorname{ns}(3 K(m) + 2 i K(1-m) \mid m) = -1$$

09.33.03.0021.01

$$\operatorname{ns}(4\,K(m) + 2\,i\,K(1-m) \mid m) = \tilde{\infty}$$

09.33.03.0022.01

$$\operatorname{ns}(2\,r\,K(m) + 2\,i\,s\,K(1-m) \mid m) = \tilde{\infty} \;/;\; \{r,\, s\} \in \mathbb{Z}$$

Values at half-quarter-period points

09.33.03.0023.01

$$\operatorname{ns}\left(\frac{K(m)}{2} \,\middle|\, m\right) = \sqrt{1 + \sqrt{1-m}}$$

09.33.03.0024.01

$$\operatorname{ns}\left(\frac{i\,K(1-m)}{2} \,\middle|\, m\right) = -i\,\sqrt[4]{m}$$

09.33.03.0025.01

$$\operatorname{ns}\left(\frac{K(m)}{2} + \frac{i\,K(1-m)}{2} \,\middle|\, m\right) = \frac{\sqrt{2}\,\sqrt[4]{m}}{\sqrt{1+\sqrt{m}} + i\sqrt{1-\sqrt{m}}}$$

General characteristics

Domain and analyticity

$\operatorname{ns}(z \mid m)$ is a meromorphic function of z and m which is defined over $\mathbb{C}^2$.

09.33.04.0001.01

$$(z * m) \longrightarrow \operatorname{ns}(z \mid m) :: (\mathbb{C} \otimes \mathbb{C}) \longrightarrow \mathbb{C}$$

Symmetries and periodicities

Parity

$\operatorname{ns}(z \mid m)$ is an odd function with respect to z.

09.33.04.0002.01

$$\operatorname{ns}(-z \mid m) = -\operatorname{ns}(z \mid m)$$

Mirror symmetry

09.33.04.0003.01

$$\operatorname{ns}(\bar{z} \mid \overline{m}) = \overline{\operatorname{ns}(z \mid m)}$$

Periodicity

$\operatorname{ns}(z \mid m)$ is a doubly periodic function with respect to z with periods $2\,i\,K(1-m)$ and $4\,K(m)$.

09.33.04.0004.01

$$\operatorname{ns}(z + 2\,K(m) \mid m) = -\operatorname{ns}(z \mid m)$$

09.33.04.0005.01

$$\operatorname{ns}(z + 4\,K(m) \mid m) = \operatorname{ns}(z \mid m)$$

09.33.04.0006.01

$$\mathrm{ns}(z + 2\,i\,K(1-m) \mid m) = \mathrm{ns}(z \mid m)$$

09.33.04.0007.01

$$\mathrm{ns}(z + 2\,K(m) + 2\,i\,K(1-m) \mid m) = -\mathrm{ns}(z \mid m)$$

09.33.04.0008.01

$$\mathrm{ns}(z + 2\,i\,s\,K(1-m) + 2\,r\,K(m) \mid m) = (-1)^r\,\mathrm{ns}(z \mid m) \;/;\; \{r, s\} \in \mathbb{Z}$$

Poles and essential singularities

With respect to z

For fixed m, the function $\mathrm{ns}(z \mid m)$ has an infinite set of singular points:

a) $z = 2\,r\,K(m) + 2\,s\,i\,K(1-m)$, $\{r, s\} \in \mathbb{Z}$, are the simple poles with residues $(-1)^r$;

b) $z = \tilde{\infty}$ is an essential singular point.

09.33.04.0009.01

$$\mathcal{Sing}_z(\mathrm{ns}(z \mid m)) = \{\{\{2\,s\,i\,K(1-m) + 2\,r\,K(m), 1\} \;/;\; \{r, s\} \in \mathbb{Z}\}, \{\tilde{\infty}, \infty\}\}$$

09.33.04.0010.01

$$\mathrm{res}_z(\mathrm{ns}(z \mid m))\,(2\,s\,i\,K(1-m) + 2\,r\,K(m)) = (-1)^r \;/;\; \{r, s\} \in \mathbb{Z}$$

Branch points

With respect to m

For fixed z, the function $\mathrm{ns}(z \mid m)$ is a meromorphic function in m that has no branch points.

09.33.04.0013.01

$$\mathcal{BP}_m(\mathrm{ns}(z \mid m)) = \{\}$$

P. Walker

With respect to z

For fixed m, the function $\mathrm{ns}(z \mid m)$ does not have branch points.

09.33.04.0011.01

$$\mathcal{BP}_z(\mathrm{ns}(z \mid m)) = \{\}$$

Branch cuts

With respect to m

For fixed z, the function $\mathrm{ns}(z \mid m)$ is a meromorphic function in m that has no branch cuts.

09.33.04.0014.01

$$\mathcal{BC}_m(\mathrm{ns}(z \mid m)) = \{\}$$

P. Walker

With respect to z

For fixed m, the function $\mathrm{ns}(z \mid m)$ does not have branch cuts.

09.33.04.0012.01

$\mathcal{BC}_z(\text{ns}(z \mid m)) == \{\}$

Series representations

Generalized power series

Expansions at $z == 0$

09.33.06.0005.01

$$\text{ns}(z \mid m) \propto \frac{1}{z} + \frac{1}{6}(1+m)\,z + \frac{1}{360}\left(7 - 22\,m + 7\,m^2\right)z^3 + \ldots \;/;\; (z \to 0)$$

09.33.06.0001.02

$$\text{ns}(z \mid m) \propto \frac{1}{z} + \frac{1}{6}(1+m)\,z + \frac{1}{360}\left(7 - 22\,m + 7\,m^2\right)z^3 + \frac{\left(31 - 15\,m - 15\,m^2 + 31\,m^3\right)z^5}{15\,120} + \frac{\left(127 - 284\,m + 186\,m^2 - 284\,m^3 + 127\,m^4\right)z^7}{604\,800} + \frac{1}{23\,950\,080}\left(\left(511 - 1261\,m + 1006\,m^2 + 1006\,m^3 - 1261\,m^4 + 511\,m^5\right)z^9\right) + O\left(z^{11}\right)$$

09.33.06.0006.01

$$\text{ns}(z \mid m) = \sum_{k=0}^{\infty}(k+1)\sum_{r=0}^{k}\frac{(-1)^r}{r+1}\binom{k}{r}p_{r,k}\,z^{2k-1} \;/;\; p_{j,0} = 1 \bigwedge p_{j,k} = \frac{1}{k}\sum_{i=1}^{k}\frac{(j\,i + i - k)\,(-1)^i\,\text{sn}_i(m)\,p_{j,k-i}}{(2\,i+1)!} \bigwedge k \in \mathbb{N}^+ \bigwedge \text{sn}_0(m) = 1 \bigwedge \text{sn}_n(m) = \sum_{j=0}^{n}\sum_{k=0}^{n}\binom{2\,n}{2\,j}\text{cn}_j(m)\,\text{dn}_k(m)\,\delta_{j+k-n} \bigwedge \text{cn}_0(m) = 1 \bigwedge \text{cn}_n(m) = \sum_{j=0}^{n-1}\sum_{k=0}^{n-1}\binom{2\,n-1}{2\,j+1}\text{sn}_j(m)\,\text{dn}_k(m)\,\delta_{j+k-n+1} \bigwedge \text{dn}_0(m) = 1 \bigwedge \text{dn}_n(m) = m\sum_{j=0}^{n-1}\sum_{k=0}^{n-1}\binom{2\,n-1}{2\,j+1}\text{sn}_j(m)\,\text{cn}_k(m)\,\delta_{j+k-n+1}$$

09.33.06.0007.01

$$\text{ns}(z \mid m) \propto \frac{1}{z}\left(1 + O\left(z^2\right)\right)$$

Expansions at $z == 2\,r\,K(m) + 2\,i\,s\,K(1-m)$

09.33.06.0008.01

$$\text{ns}(z \mid m) \propto (-1)^r\left(\frac{1}{z - z_0} + \frac{1}{6}(m+1)(z - z_0) + \frac{1}{360}\left(7\,m^2 - 22\,m + 7\right)(z - z_0)^3 + \ldots\right) /; (z \to z_0) \bigwedge z_0 = 2\,r\,K(m) + 2\,i\,s\,K(1-m) \bigwedge r \in \mathbb{Z} \bigwedge s \in \mathbb{Z}$$

09.33.06.0009.01

$$\operatorname{ns}(z \mid m) = (-1)^r \sum_{k=0}^{\infty} (k+1) \sum_{r=0}^{k} \frac{(-1)^r}{r+1} \binom{k}{r} p_{r,k} (z-z_0)^{2k-1} \;/;$$

$$z_0 = 2 r K(m) + 2 i s K(1-m) \bigwedge r \in \mathbb{Z} \bigwedge s \in \mathbb{Z} \bigwedge p_{j,0} = 1 \bigwedge p_{j,k} = \frac{1}{k} \sum_{i=1}^{k} \frac{(j i + i - k)(-1)^i \operatorname{sn}_i(m) p_{j,k-i}}{(2i+1)!} \bigwedge$$

$$k \in \mathbb{N}^+ \bigwedge \operatorname{sn}_0(m) = 1 \bigwedge \operatorname{sn}_n(m) = \sum_{j=0}^{n} \sum_{k=0}^{n} \binom{2n}{2j} \operatorname{cn}_j(m) \operatorname{dn}_k(m) \delta_{j+k-n} \bigwedge \operatorname{cn}_0(m) = 1 \bigwedge$$

$$\operatorname{cn}_n(m) = \sum_{j=0}^{n-1} \sum_{k=0}^{n-1} \binom{2n-1}{2j+1} \operatorname{sn}_j(m) \operatorname{dn}_k(m) \delta_{j+k-n+1} \bigwedge \operatorname{dn}_0(m) = 1 \bigwedge \operatorname{dn}_n(m) = m \sum_{j=0}^{n-1} \sum_{k=0}^{n-1} \binom{2n-1}{2j+1} \operatorname{sn}_j(m) \operatorname{cn}_k(m) \delta_{j+k-n+1}$$

09.33.06.0010.01

$$\operatorname{ns}(z \mid m) \propto \frac{(-1)^r}{z-z_0} \left(1 + O\left((z-z_0)^2\right)\right) /; z_0 = 2 r K(m) + 2 i s K(1-m) \wedge r \in \mathbb{Z} \wedge s \in \mathbb{Z}$$

Expansions at $m == 0$

09.33.06.0011.01

$$\operatorname{ns}(z \mid m) \propto \csc(z) + \frac{1}{4} \cot(z)(z - \cos(z)\sin(z))\csc(z)\, m +$$
$$\frac{1}{512}\left(8\cos(2z) z^2 + 24 z^2 - 4\sin(2z) z + 4\sin(4z) z + 5\cos(4z) - 5\right)\csc^3(z)\, m^2 + \ldots /; (m \to 0)$$

09.33.06.0012.01

$$\operatorname{ns}(z \mid m) \propto \csc(z) + \frac{1}{4} \cot(z)(z - \cos(z)\sin(z))\csc(z)\, m +$$
$$\frac{1}{512}\left(8\cos(2z) z^2 + 24 z^2 - 4\sin(2z) z + 4\sin(4z) z + 5\cos(4z) - 5\right)\csc^3(z)\, m^2 +$$
$$\frac{1}{49152}\left(32 z\left(23 z^2 - 6\right)\cos(z) + 8 z\left(4 z^2 + 39\right)\cos(3z) - 120 z\cos(5z) + \right.$$
$$\left. 6\left(80\cos(2z) z^2 - 8\cos(4z) z^2 + 120 z^2 - \cos(2z) + 42\cos(4z) + \cos(6z) - 42\right)\sin(z)\right)\csc^4(z)\, m^3 + \frac{1}{1572864}$$
$$\left(3680 z^4 + 2760 z^2 + 4\left(1976 z^2 - 1527\right)\sin(2z) z + 4\left(56 z^2 + 1227\right)\sin(4z) z + 4\left(8 z^2 - 297\right)\sin(6z) z - 36\sin(8z) z + \right.$$
$$\left.\left(2432 z^4 + 312 z^2 + 1167\right)\cos(2z) + 2\left(16 z^4 - 1716 z^2 + 1287\right)\cos(4z) + 3\left(120 z^2 - 389\right)\cos(6z) - 60\cos(8z) - 2514\right)$$
$$\csc^5(z)\, m^4 + \frac{1}{251658240}\left(16 z\left(13456 z^4 + 28940 z^2 - 23745\right)\cos(z) + 24 z\left(1264 z^4 - 18980 z^2 + 27975\right)\cos(3z) + \right.$$
$$8 z\left(16 z^4 - 540 z^2 - 44145\right)\cos(5z) - 20 z\left(160 z^2 - 2859\right)\cos(7z) + 4500 z\cos(9z) +$$
$$840\left(476 z^4 + 87 z^2 - 411\right)\sin(z) + 30\left(6288 z^4 + 10116 z^2 - 2755\right)\sin(3z) + 5\left(736 z^4 - 44904 z^2 + 41811\right)\sin(5z) -$$
$$\left. 10\left(16 z^4 - 1848 z^2 + 5835\right)\sin(7z) + 30\left(36 z^2 - 161\right)\sin(9z) - 15\sin(11z)\right)\csc^6(z)\, m^5 +$$
$$\frac{1}{24159191040}\left(3014144 z^6 + 17125920 z^4 - 2772000 z^2 + 24\left(485296 z^4 + 1407720 z^2 - 1672185\right)\sin(2z) z + \right.$$
$$60\left(31808 z^4 - 276960 z^2 + 638391\right)\sin(4z) z + 180\left(32 z^4 - 408 z^2 - 80953\right)\sin(6z) z +$$
$$24\left(16 z^4 - 2760 z^2 + 62415\right)\sin(8z) z - 180\left(72 z^2 - 1381\right)\sin(10z) z +$$
$$900\sin(12z) z + 2\left(1349504 z^6 - 4620960 z^4 + 7884360 z^2 + 4624785\right)\cos(2z) +$$
$$4\left(46208 z^6 - 1922880 z^4 - 5262840 z^2 + 2644965\right)\cos(4z) +$$
$$\left(256 z^6 - 204480 z^4 + 8508960 z^2 - 9467235\right)\cos(6z) + 30\left(400 z^4 - 11904 z^2 + 58617\right)\cos(8z) -$$

$$45\left(2160\,z^2-4837\right)\cos(10\,z)+1620\cos(12\,z)-12\,339\,990\big)\csc^7(z)\,m^6\;+$$

$$\frac{1}{5\,411\,658\,792\,960}\Big(8\,z\left(33\,244\,544\,z^6+169\,790\,880\,z^4+373\,682\,400\,z^2-460\,109\,475\right)\cos(z)+$$

$$12\,z\left(5\,176\,064\,z^6-94\,597\,440\,z^4-370\,032\,320\,z^2+594\,245\,715\right)\cos(3\,z)+$$

$$16\,z\left(139\,456\,z^6-13\,918\,800\,z^4+88\,750\,830\,z^2-293\,798\,925\right)\cos(5\,z)+$$

$$4\,z\left(256\,z^6-94\,080\,z^4+9\,042\,600\,z^2+336\,907\,305\right)\cos(7\,z)-$$

$$420\,z\left(192\,z^4+20\,072\,z^2+162\,363\right)\cos(9\,z)+2520\,z\left(1260\,z^2-11\,297\right)\cos(11\,z)-$$

$$258\,300\,z\cos(13\,z)+63\left(9\,614\,336\,z^6+67\,529\,280\,z^4-48\,535\,200\,z^2-46\,854\,555\right)\sin(z)+$$

$$140\left(3\,269\,504\,z^6-1\,174\,224\,z^4+27\,202\,662\,z^2-992\,079\right)\sin(3\,z)+$$

$$70\left(478\,976\,z^6-10\,396\,272\,z^4-37\,828\,116\,z^2+25\,073\,235\right)\sin(5\,z)+$$

$$35\left(1792\,z^6-490\,272\,z^4+20\,050\,128\,z^2-27\,450\,513\right)\sin(7\,z)-7\left(256\,z^6+84\,960\,z^4-1\,830\,960\,z^2-16\,994\,475\right)$$

$$\sin(9\,z)+630\left(432\,z^4-22\,276\,z^2+35\,213\right)\sin(11\,z)-2520\left(25\,z^2-109\right)\sin(13\,z)+315\sin(15\,z)\Big)\csc^8(z)\,m^7+$$

$$\frac{1}{173\,173\,081\,374\,720}\Big(1\,196\,803\,584\,z^8+11\,621\,272\,320\,z^6+59\,265\,712\,800\,z^4-51\,670\,006\,920\,z^2+$$

$$28\left(225\,685\,504\,z^6+1\,385\,289\,216\,z^4+3\,419\,854\,200\,z^2-5\,152\,437\,585\right)\sin(2\,z)\,z+$$

$$4\left(464\,069\,120\,z^6-3\,558\,439\,584\,z^4-18\,625\,005\,840\,z^2+39\,050\,472\,195\right)\sin(4\,z)\,z+$$

$$144\left(480\,320\,z^6-23\,703\,624\,z^4+113\,082\,060\,z^2-540\,952\,755\right)\sin(6\,z)\,z+$$

$$4\left(5888\,z^6-6\,093\,024\,z^4+375\,391\,800\,z^2+4\,243\,004\,955\right)\sin(8\,z)\,z+$$

$$32\left(32\,z^6+129\,948\,z^4-12\,802\,230\,z^2+2\,131\,605\right)\sin(10\,z)\,z-$$

$$756\left(864\,z^4-95\,200\,z^2+578\,675\right)\sin(12\,z)\,z+420\left(1000\,z^2-16\,371\right)\sin(14\,z)\,z-8820\sin(16\,z)\,z+$$

$$2\left(636\,233\,728\,z^8-1\,416\,993\,536\,z^6-30\,708\,972\,000\,z^4+53\,216\,291\,520\,z^2+17\,945\,797\,275\right)\cos(2\,z)+$$

$$4\left(42\,446\,336\,z^8-2\,036\,204\,800\,z^6-2\,051\,632\,800\,z^4-22\,241\,422\,350\,z^2+6\,187\,089\,195\right)\cos(4\,z)+$$

$$63\left(53\,248\,z^8-10\,174\,720\,z^6+163\,077\,600\,z^4+668\,248\,200\,z^2-553\,255\,345\right)\cos(6\,z)+$$

$$2\left(256\,z^8-760\,704\,z^6+12\,390\,000\,z^4-3\,620\,465\,100\,z^2+6\,758\,870\,265\right)\cos(8\,z)+$$

$$35\left(1792\,z^6+2\,028\,000\,z^4-25\,849\,368\,z^2-29\,453\,985\right)\cos(10\,z)-1260\left(8640\,z^4-195\,426\,z^2+251\,863\right)\cos(12\,z)+$$

$$1260\left(2300\,z^2-4459\right)\cos(14\,z)-16\,380\cos(16\,z)-37\,948\,733\,550\Big)\csc^9(z)\,m^8\;+\;\frac{1}{49\,873\,847\,435\,919\,360}$$

$$\Big(8\,z\left(17\,769\,803\,264\,z^8+154\,337\,008\,896\,z^6+662\,044\,192\,992\,z^4+1\,413\,285\,787\,620\,z^2-2\,207\,834\,702\,955\right)\cos(z)+4\,z$$

$$\left(11\,114\,481\,664\,z^8-218\,002\,922\,496\,z^6-1\,813\,216\,327\,776\,z^4-4\,986\,817\,337\,880\,z^2+9\,150\,568\,206\,255\right)\cos(3\,z)+$$

$$20\,z\left(179\,849\,216\,z^8-17\,458\,283\,520\,z^6+75\,235\,589\,856\,z^4+516\,684\,231\,000\,z^2-1\,408\,419\,633\,915\right)\cos(5\,z)+$$

$$64\,z\left(629\,536\,z^8-211\,179\,024\,z^6+6\,915\,622\,266\,z^4-21\,606\,985\,575\,z^2+173\,396\,382\,075\right)\cos(7\,z)+$$

$$128\,z\left(16\,z^8-22\,536\,z^6+88\,216\,317\,z^4-3\,070\,844\,595\,z^2-14\,032\,657\,485\right)\cos(9\,z)-$$

$$36\,z\left(10\,240\,z^6+65\,446\,752\,z^4-2\,648\,877\,000\,z^2+3\,662\,620\,605\right)\cos(11\,z)+$$

$$11\,340\,z\left(23\,328\,z^4-1\,054\,072\,z^2+5\,123\,503\right)\cos(13\,z)-56\,700\,z\left(3400\,z^2-23\,959\right)\cos(15\,z)+4\,524\,660\,z\cos(17\,z)+$$

$$63\left(5\,765\,895\,168\,z^8+64\,101\,931\,520\,z^6+358\,754\,198\,880\,z^4-461\,814\,328\,800\,z^2-210\,208\,477\,185\right)\sin(z)+$$

$$216\left(1\,654\,016\,128\,z^8+3\,795\,293\,152\,z^6-30\,335\,368\,280\,z^4+117\,788\,351\,835\,z^2+5\,787\,713\,820\right)\sin(3\,z)+$$

$$720\left(74\,894\,880\,z^8-1\,610\,434\,336\,z^6-3\,479\,640\,570\,z^4-22\,032\,817\,380\,z^2+9\,879\,190\,965\right)\sin(5\,z)+$$

$$63\left(17\,245\,696\,z^8-1\,590\,476\,032\,z^6+22\,333\,139\,520\,z^4+86\,714\,291\,880\,z^2-90\,423\,595\,035\right)\sin(7\,z)+$$

$$9\left(24\,064\,z^8+26\,967\,808\,z^6-5\,753\,905\,920\,z^4-53\,348\,533\,560\,z^2+184\,980\,199\,635\right)\sin(9\,z)-$$

$$36\left(128\,z^8+2\,892\,288\,z^6-583\,461\,480\,z^4+5\,821\,526\,340\,z^2+1\,616\,170\,815\right)\sin(11\,z)+$$

$$189\left(62\,208\,z^6-12\,659\,840\,z^4+187\,133\,400\,z^2-213\,575\,595\right)\sin(13\,z)-$$

$$945\left(20\,000\,z^4-800\,040\,z^2+997\,737\right)\sin(15\,z)+5670\left(196\,z^2-839\right)\sin(17\,z)-2835\sin(19\,z)\Big)\csc^{10}(z)\,m^9+$$

$$\frac{1}{7\,979\,815\,589\,747\,097\,600}\Big(3\,127\,485\,374\,464\,z^{10}+43\,994\,001\,300\,480\,z^8+330\,560\,432\,294\,400\,z^6+1\,585\,067\,158\,382\,400\,z^4-2\,197\,145\,541\,729\,600\,z^2+1080\left(18\,545\,000\,960\,z^8+183\,880\,478\,592\,z^6+874\,548\,867\,248\,z^4+1\,915\,061\,306\,760\,z^2-3\,460\,899\,395\,415\right)\sin(2z)\,z+120\left(69\,804\,123\,904\,z^8-410\,456\,968\,704\,z^6-5\,520\,309\,884\,208\,z^4-16\,449\,365\,164\,680\,z^2+36\,940\,206\,271\,815\right)\sin(4z)\,z+160\left(4\,557\,007\,168\,z^8-197\,971\,975\,584\,z^6+530\,033\,745\,528\,z^4+4\,517\,746\,166\,430\,z^2-16\,561\,877\,387\,265\right)\sin(6z)\,z+40\left(206\,488\,576\,z^8-31\,450\,871\,808\,z^6+739\,283\,503\,392\,z^4-653\,416\,076\,880\,z^2+21\,042\,320\,833\,485\right)\sin(8z)\,z+320\left(992\,z^8-17\,632\,080\,z^6+6\,037\,860\,528\,z^4-148\,398\,278\,175\,z^2-293\,396\,228\,055\right)\sin(10z)\,z+20\left(512\,z^8+53\,001\,216\,z^6-18\,679\,375\,008\,z^4+500\,341\,983\,120\,z^2-880\,724\,436\,795\right)\sin(12z)\,z-540\left(186\,624\,z^6-62\,952\,736\,z^4+1\,786\,255\,800\,z^2-7\,697\,400\,375\right)\sin(14z)\,z+18\,900\left(20\,000\,z^4-1\,596\,816\,z^2+7\,072\,143\right)\sin(16z)\,z-18\,900\left(2744\,z^2-41\,535\right)\sin(18z)\,z+510\,300\sin(20z)\,z+\left(3\,714\,757\,763\,072\,z^{10}-725\,097\,369\,600\,z^8-264\,064\,801\,674\,240\,z^6-2\,031\,769\,019\,901\,600\,z^4+4\,031\,463\,722\,718\,600\,z^2+969\,296\,803\,112\,925\right)\cos(2z)+2\left(365\,160\,251\,392\,z^{10}-18\,368\,995\,703\,040\,z^8-80\,431\,555\,034\,880\,z^6+126\,842\,760\,399\,600\,z^4-1\,454\,744\,457\,558\,000\,z^2+200\,711\,093\,812\,425\right)\cos(4z)+\left(37\,339\,713\,536\,z^{10}-6\,399\,354\,193\,920\,z^8+85\,954\,169\,468\,160\,z^6+302\,198\,583\,304\,800\,z^4+1\,412\,940\,934\,326\,600\,z^2-861\,620\,257\,661\,625\right)\cos(6z)+32\left(7\,556\,864\,z^{10}-4\,110\,213\,600\,z^8+266\,226\,090\,480\,z^6-3\,641\,863\,424\,850\,z^4-10\,988\,271\,798\,525\,z^2+15\,273\,874\,335\,825\right)\cos(8z)+\left(4096\,z^{10}-32\,532\,480\,z^8-133\,951\,507\,200\,z^6+9\,643\,178\,109\,600\,z^4-4\,911\,089\,891\,400\,z^2-110\,501\,743\,056\,075\right)\cos(10z)+90\left(11\,520\,z^8+344\,065\,792\,z^6-27\,964\,693\,680\,z^4+238\,785\,870\,960\,z^2-6\,743\,301\,075\right)\cos(12z)-4725\left(622\,080\,z^6-47\,775\,520\,z^4+557\,948\,568\,z^2-597\,795\,471\right)\cos(14z)+4725\left(1\,120\,000\,z^4-18\,727\,968\,z^2+17\,561\,343\right)\cos(16z)-2\,778\,300\left(124\,z^2-221\right)\cos(18z)+963\,900\cos(20z)-889\,662\,247\,584\,075\Big)\csc^{11}(z)\,m^{10}+O\left(m^{11}\right)$$

09.33.06.0013.01

$$\operatorname{ns}(z\mid m)\propto\csc(z)\,(1+O(m))$$

Expansions at $m == 1$

09.33.06.0014.01

$$\operatorname{ns}(z\mid m)\propto\coth(z)+\frac{1}{4}\left(\coth(z)-z\operatorname{csch}^2(z)\right)(m-1)+\frac{1}{512}\left(4\left(8z^2+3\right)\cosh(z)-13\cosh(3z)+\cosh(5z)+8z\sinh(z)\right)\operatorname{csch}^3(z)\,(m-1)^2+\ldots\;/;\;(m\to 1)$$

09.33.06.0015.01

$$\operatorname{ns}(z\mid m)\propto\coth(z)+\frac{1}{4}\left(\coth(z)-z\operatorname{csch}^2(z)\right)(m-1)+\frac{1}{512}\left(4\left(8z^2+3\right)\cosh(z)-13\cosh(3z)+\cosh(5z)+8z\sinh(z)\right)\operatorname{csch}^3(z)\,(m-1)^2-\frac{1}{12\,288}\left(128z^3+2\left(32z^2-9\right)\cosh(2z)\,z+24\cosh(4z)\,z-6\cosh(6z)\,z+3\left(48z^2+59\right)\sinh(2z)-102\sinh(4z)+9\sinh(6z)\right)\operatorname{csch}^4(z)\,(m-1)^3+\frac{1}{1\,572\,864}\left(\left(5632z^4-5568z^2-7821\right)\cosh(z)+\left(512z^4+5952z^2+12\,381\right)\cosh(3z)-3\left(160z^2+1663\right)\cosh(5z)+6\left(16z^2+71\right)\cosh(7z)+3\cosh(9z)+96z\left(88z^2+51\right)\sinh(z)+16z\left(176z^2-255\right)\sinh(3z)+2040z\sinh(5z)-408z\sinh(7z)\right)\operatorname{csch}^5(z)\,(m-1)^4-\frac{1}{62\,914\,560}\left(33\,792z^5+102\,400\sinh(2z)\,z^4+10\,240\sinh(4z)\,z^4+1920\cosh(6z)\,z^3-320\cosh(8z)\,z^3-\right.$$

$$126\,720\,z^3 - 168\,960 \sinh(2\,z)\,z^2 + 102\,960 \sinh(4\,z)\,z^2 - 15\,840 \sinh(6\,z)\,z^2 + 2640 \sinh(8\,z)\,z^2 + 4\left(6656\,z^4 + 22\,400\,z^2 + 48\,585\right)\cosh(2\,z)\,z + 8\left(128\,z^4 + 4440\,z^2 - 14\,895\right)\cosh(4\,z)\,z + 44\,280 \cosh(6\,z)\,z - 7170 \cosh(8\,z)\,z - 60 \cosh(10\,z)\,z - 112\,230\,z - 303\,870 \sinh(2\,z) + 261\,870 \sinh(4\,z) - 82\,005 \sinh(6\,z) + 6405 \sinh(8\,z) + 105 \sinh(10\,z)\Big)\operatorname{csch}^6(z)\,(m-1)^5 +$$

$$\frac{1}{24\,159\,191\,040}\Big(4\left(1\,236\,992\,z^6 - 4\,312\,320\,z^4 + 6\,395\,040\,z^2 + 11\,557\,035\right)\cosh(z) + 3\left(311\,296\,z^6 + 5\,135\,360\,z^4 - 14\,041\,920\,z^2 - 28\,700\,655\right)\cosh(3\,z) + \left(16\,384\,z^6 + 1\,889\,280\,z^4 + 19\,742\,400\,z^2 + 52\,368\,975\right)\cosh(5\,z) - 30\left(1792\,z^4 + 123\,648\,z^2 + 448\,491\right)\cosh(7\,z) + 30\left(256\,z^4 + 16\,896\,z^2 + 31\,137\right)\cosh(9\,z) + 45\left(128\,z^2 + 565\right)\cosh(11\,z) + 45 \cosh(13\,z) + 92\,160\,z\left(112\,z^4 - 579\,z^2 - 850\right)\sinh(z) + 1440\,z\left(4480\,z^4 + 7152\,z^2 + 39\,465\right)\sinh(3\,z) + 144\,z\left(1792\,z^4 + 25\,440\,z^2 - 197\,015\right)\sinh(5\,z) + 5040\,z\left(144\,z^2 + 1705\right)\sinh(7\,z) - 720\,z\left(144\,z^2 + 1573\right)\sinh(9\,z) - 23\,760\,z \sinh(11\,z)\Big)\operatorname{csch}^7(z)\,(m-1)^6 -$$

$$\frac{1}{1\,352\,914\,698\,240}\Big(32\,z\left(1\,236\,992\,z^6 - 9\,370\,368\,z^4 + 39\,264\,960\,z^2 + 64\,244\,565\right) + 6\,z\left(6\,504\,448\,z^6 + 18\,464\,768\,z^4 - 256\,231\,360\,z^2 - 586\,516\,665\right)\cosh(2\,z) + 96\,z\left(40\,960\,z^6 + 1\,895\,936\,z^4 + 2\,352\,560\,z^2 + 22\,632\,855\right)\cosh(4\,z) + 2\,z\left(16\,384\,z^6 + 3\,451\,392\,z^4 + 15\,536\,640\,z^2 - 458\,842\,545\right)\cosh(6\,z) + 672\,z\left(256\,z^4 + 40\,480\,z^2 + 348\,465\right)\cosh(8\,z) - 42\,z\left(512\,z^4 + 75\,200\,z^2 + 595\,815\right)\cosh(10\,z) - 3360\,z\left(16\,z^2 + 279\right)\cosh(12\,z) - 1890\,z \cosh(14\,z) + 35\left(5\,218\,304\,z^6 - 22\,364\,160\,z^4 + 44\,972\,928\,z^2 + 86\,951\,799\right)\sinh(2\,z) + 28\left(1\,490\,944\,z^6 + 11\,304\,960\,z^4 - 52\,434\,000\,z^2 - 113\,726\,745\right)\sinh(4\,z) + 7\left(106\,496\,z^6 + 7\,680\,000\,z^4 + 83\,145\,600\,z^2 + 216\,095\,715\right)\sinh(6\,z) - 840\left(4096\,z^4 + 133\,152\,z^2 + 391\,941\right)\sinh(8\,z) + 105\left(4096\,z^4 + 116\,736\,z^2 + 189\,585\right)\sinh(10\,z) + 23\,940\left(16\,z^2 + 33\right)\sinh(12\,z) + 3465 \sinh(14\,z)\Big)\operatorname{csch}^8(z)\,(m-1)^7 +$$

$$\frac{1}{173\,173\,081\,374\,720}\Big(2\left(1\,023\,606\,784\,z^8 - 7\,012\,999\,168\,z^6 + 21\,516\,284\,160\,z^4 - 42\,858\,547\,200\,z^2 - 82\,105\,338\,105\right)\cosh(z) + 9\left(62\,521\,344\,z^8 + 1\,202\,962\,432\,z^6 - 6\,670\,164\,480\,z^4 + 18\,532\,442\,880\,z^2 + 37\,330\,928\,005\right)\cosh(3\,z) + 2\left(16\,187\,392\,z^8 + 1\,569\,505\,280\,z^6 + 6\,832\,358\,400\,z^4 - 57\,050\,022\,960\,z^2 - 126\,632\,390\,535\right)\cosh(5\,z) + \left(131\,072\,z^8 + 61\,243\,392\,z^6 + 3\,616\,757\,760\,z^4 + 39\,545\,624\,160\,z^2 + 99\,139\,214\,475\right)\cosh(7\,z) - 63\left(16\,384\,z^6 + 4\,984\,320\,z^4 + 113\,688\,000\,z^2 + 296\,165\,235\right)\cosh(9\,z) + 224\left(512\,z^6 + 136\,560\,z^4 + 2\,695\,050\,z^2 + 4\,300\,785\right)\cosh(11\,z) + 105\left(8192\,z^4 + 362\,784\,z^2 + 500\,313\right)\cosh(13\,z) + 315\left(288\,z^2 + 1243\right)\cosh(15\,z) + 315 \cosh(17\,z) + 1120\,z\left(4\,444\,160\,z^6 - 39\,086\,208\,z^4 + 201\,035\,160\,z^2 + 402\,688\,125\right)\sinh(z) + 2016\,z\left(2\,158\,592\,z^6 - 2\,344\,320\,z^4 - 50\,634\,360\,z^2 - 165\,406\,515\right)\sinh(3\,z) + 56\,z\left(8\,634\,368\,z^6 + 197\,434\,368\,z^4 + 275\,753\,280\,z^2 + 3\,182\,522\,625\right)\sinh(5\,z) + 8\,z\left(507\,904\,z^6 + 43\,760\,640\,z^4 - 208\,071\,360\,z^2 - 8\,131\,828\,635\right)\sinh(7\,z) + 6048\,z\left(4736\,z^4 + 321\,640\,z^2 + 2\,350\,155\right)\sinh(9\,z) - 336\,z\left(9472\,z^4 + 505\,680\,z^2 + 3\,608\,835\right)\sinh(11\,z) - 840\,z\left(11\,008\,z^2 + 85\,701\right)\sinh(13\,z) - 370\,440\,z \sinh(15\,z)\Big)\operatorname{csch}^9(z)\,(m-1)^8 -$$

$$\frac{1}{12\,468\,461\,858\,979\,840}\Big(131\,596\,288 \cosh(6\,z)\,z^9 + 262\,144 \cosh(8\,z)\,z^9 + 20\,472\,135\,680\,z^9 + 120\,246\,239\,232 \sinh(2\,z)\,z^8 + 42\,955\,702\,272 \sinh(4\,z)\,z^8 + 2\,611\,740\,672 \sinh(6\,z)\,z^8 + 10\,616\,832 \sinh(8\,z)\,z^8 + 23\,458\,480\,128 \cosh(6\,z)\,z^7 + 184\,909\,824 \cosh(8\,z)\,z^7 + 2\,949\,120 \cosh(10\,z)\,z^7 - 294\,912 \cosh(12\,z)\,z^7 - 243\,194\,757\,120\,z^7 - 962\,553\,102\,336 \sinh(2\,z)\,z^6 + 298\,540\,376\,064 \sinh(4\,z)\,z^6 + 118\,394\,486\,784 \sinh(6\,z)\,z^6 + 2\,691\,440\,640 \sinh(8\,z)\,z^6 - 108\,380\,160 \sinh(10\,z)\,z^6 + 10\,838\,016 \sinh(12\,z)\,z^6 + 372\,219\,273\,216 \cosh(6\,z)\,z^5 + 8\,509\,197\,312 \cosh(8\,z)\,z^5 + 1\,496\,033\,280 \cosh(10\,z)\,z^5 - 115\,540\,992 \cosh(12\,z)\,z^5 - 6\,193\,152 \cosh(14\,z)\,z^5 + 1\,476\,800\,640\,000\,z^5 + 3\,207\,601\,797\,120 \sinh(2\,z)\,z^4 - 2\,269\,879\,234\,560 \sinh(4\,z)\,z^4 + 281\,550\,366\,720 \sinh(6\,z)\,z^4 + 136\,169\,026\,560 \sinh(8\,z)\,z^4 -$$

$$
\begin{aligned}
&12\,444\,364\,800\sinh(10z)z^4+733\,501\,440\sinh(12z)z^4+92\,897\,280\sinh(14z)z^4+696\,311\,925\,120\cosh(6z)z^3-\\
&159\,170\,719\,680\cosh(8z)z^3+69\,201\,216\,000\cosh(10z)z^3-3\,783\,326\,400\cosh(12z)z^3-\\
&563\,794\,560\cosh(14z)z^3-1\,632\,960\cosh(16z)z^3-7\,154\,466\,480\,000z^3-7\,897\,343\,287\,680\sinh(2z)z^2+\\
&8\,833\,984\,513\,200\sinh(4z)z^2-4\,876\,025\,182\,560\sinh(6z)z^2+1\,494\,827\,233\,200\sinh(8z)z^2-\\
&244\,240\,920\,000\sinh(10z)z^2+14\,486\,305\,680\sinh(12z)z^2+1\,762\,780\,320\sinh(14z)z^2+11\,022\,480\sinh(16z)z^2+\\
&4\left(5\,782\,503\,424z^8+7\,668\,744\,192z^6-326\,379\,594\,240z^4+2\,624\,672\,272\,320z^2+6\,486\,583\,627\,755\right)\cosh(2z)z+\\
&2\left(1\,914\,699\,776z^8+94\,436\,868\,096z^6-276\,692\,516\,352z^4-1\,973\,108\,138\,400z^2-8\,390\,490\,202\,395\right)\cosh(4z)z+\\
&7\,873\,011\,299\,160\cosh(6z)z-2\,518\,261\,223\,940\cosh(8z)z+475\,221\,751\,200\cosh(10z)z-\\
&31\,297\,804\,650\cosh(12z)z-2\,847\,349\,260\cosh(14z)z-25\,560\,360\cosh(16z)z-22\,680\cosh(18z)z-\\
&14\,961\,155\,195\,700z-15\,485\,073\,557\,625\sinh(2z)+18\,235\,162\,121\,085\sinh(4z)-10\,895\,064\,726\,735\sinh(6z)+\\
&3\,632\,088\,305\,160\sinh(8z)-599\,330\,138\,715\sinh(10z)+25\,809\,304\,185\sinh(12z)+\\
&1\,888\,813\,080\sinh(14z)+20\,383\,650\sinh(16z)+42\,525\sinh(18z)\Big)\operatorname{csch}^{10}(z)(m-1)^9+\\
&\frac{1}{7\,979\,815\,589\,747\,097\,600}\Big(2\left(2\,748\,011\,511\,808z^{10}-29\,125\,921\,996\,800z^8+157\,454\,330\,019\,840z^6-\right.\\
&\left.448\,907\,881\,766\,400z^4+1\,111\,733\,772\,559\,200z^2+2\,164\,413\,964\,533\,975\right)\cosh(z)+\\
&2\left(954\,606\,813\,184z^{10}+18\,720\,609\,730\,560z^8-206\,405\,127\,290\,880z^6+764\,016\,547\,795\,200z^4-\right.\\
&\left.2\,359\,871\,063\,071\,200z^2-4\,712\,117\,404\,911\,525\right)\cosh(3z)+10\left(20\,065\,550\,336z^{10}+1\,954\,603\,008\,000z^8+\right.\\
&\left.5\,939\,132\,645\,376z^6-70\,263\,594\,858\,240z^4+388\,125\,336\,886\,560z^2+813\,651\,319\,331\,235\right)\cosh(5z)+\\
&4\left(1\,062\,207\,488z^{10}+314\,797\,916\,160z^8+9\,391\,436\,328\,960z^6+7\,984\,617\,984\,000z^4-\right.\\
&\left.452\,547\,712\,184\,400z^2-1\,013\,014\,530\,506\,475\right)\cosh(7z)+4\left(1\,048\,576z^{10}+1\,365\,442\,560z^8+\right.\\
&\left.247\,200\,952\,320z^6+10\,978\,948\,915\,200z^4+123\,177\,730\,158\,000z^2+294\,044\,394\,487\,725\right)\cosh(9z)-\\
&45\left(1\,441\,792z^8+1\,042\,055\,168z^6+77\,228\,659\,200z^4+1\,566\,052\,488\,000z^2+3\,817\,653\,876\,225\right)\cosh(11z)+\\
&45\left(131\,072z^8+50\,692\,096z^6-329\,710\,080z^4+51\,470\,354\,880z^2+132\,716\,507\,175\right)\cosh(13z)+\\
&315\left(1\,048\,576z^6+172\,316\,160z^4+2\,063\,619\,360z^2+1\,866\,547\,935\right)\cosh(15z)+\\
&14\,175\left(13\,824z^4+514\,208z^2+593\,979\right)\cosh(17z)+14\,175\left(512z^2+2177\right)\cosh(19z)+\\
&14\,175\cosh(21z)+240z\left(60\,865\,380\,352z^8-813\,022\,838\,784z^6+\right.\\
&\left.5\,255\,225\,644\,032z^4-29\,643\,273\,777\,600z^2-68\,653\,370\,969\,505\right)\sinh(z)+480z\\
&\left(32\,971\,882\,496z^8-124\,073\,791\,488z^6-446\,811\,757\,056z^4+8\,746\,254\,697\,320z^2+26\,144\,882\,285\,445\right)\sinh(3z)+\\
&960z\left(3\,158\,540\,288z^8+64\,913\,602\,560z^6-294\,922\,236\,672z^4-1\,463\,761\,178\,460z^2-7\,482\,523\,932\,135\right)\sinh(5z)+\\
&480z\left(224\,362\,496z^8+18\,346\,008\,576z^6+231\,438\,340\,224z^4+591\,295\,420\,800z^2+6\,232\,106\,981\,835\right)\sinh(7z)+\\
&160z\left(1\,343\,488z^8+370\,778\,112z^6+10\,596\,942\,720z^4-450\,710\,870\,400z^2-5\,291\,217\,495\,585\right)\sinh(9z)+\\
&63\,360z\left(48\,128z^6+7\,194\,432z^4+308\,621\,250z^2+2\,186\,282\,385\right)\sinh(11z)-\\
&2880z\left(96\,256z^6+712\,320z^4+87\,346\,980z^2+2\,285\,491\,635\right)\sinh(13z)-\\
&7560z\left(868\,352z^4+32\,539\,680z^2+125\,224\,455\right)\sinh(15z)-\\
&6\,690\,600z\left(288z^2+1885\right)\sinh(17z)-29\,484\,000z\sinh(19z)\Big)\operatorname{csch}^{11}(z)(m-1)^{10}+O\!\left((m-1)^{11}\right)
\end{aligned}
$$

09.33.06.0016.01

$$\operatorname{ns}(z\mid m)\propto\coth(z)\,(1+O(m-1))$$

q-series

09.33.06.0002.01

$$\operatorname{ns}(z\mid m)=\frac{\pi}{2K(m)}\csc\left(\frac{\pi z}{2K(m)}\right)+\frac{2\pi}{K(m)}\sum_{k=0}^{\infty}\frac{q(m)^{2k+1}}{1-q(m)^{2k+1}}\sin\left(\frac{(2k+1)\pi z}{2K(m)}\right)$$

Other series representations

09.33.06.0003.01

$$\text{ns}(z \mid m) = \frac{\pi}{2\,K(1-m)} \sum_{k=-\infty}^{\infty} (-1)^k \coth\left(\pi \frac{K(m)}{K(1-m)} \left(k + \frac{z}{2\,K(m)}\right)\right)$$

09.33.06.0004.01

$$\text{ns}(z \mid m) \propto \frac{(-1)^r}{z - 2\,s\,i\,K(1-m) - 2\,r\,K(m)} + O(1) \;/;\; (z \to 2\,s\,i\,K(1-m) + 2\,r\,K(m)) \bigwedge \{r, s\} \in \mathbb{Z}$$

Product representations

09.33.08.0001.01

$$\text{ns}(z \mid m) = \frac{1}{2} \frac{\sqrt[4]{m}}{\sqrt[4]{q(m)}} \csc\left(\frac{\pi z}{2\,K(m)}\right) \prod_{k=1}^{\infty} \frac{1 - 2\,q(m)^{2k-1} \cos\left(\frac{\pi z}{K(m)}\right) + q(m)^{4k-2}}{1 - 2\,q(m)^{2k} \cos\left(\frac{\pi z}{K(m)}\right) + q(m)^{4k}}$$

Differential equations

Ordinary nonlinear differential equations

09.33.13.0001.01

$$w''(z) - w(z)\left(2\,w(z)^2 - m - 1\right) = 0 \;/;\; w(z) = \text{ns}(z \mid m)$$

Transformations

Transformations and argument simplifications

Argument involving basic arithmetic operations

09.33.16.0001.01

$$\text{ns}(i\,z \mid m) = -i\,\text{cs}(z \mid 1-m)$$

09.33.16.0002.01

$$\text{ns}(z \mid 1-m) = i\,\text{cs}(i\,z \mid m)$$

09.33.16.0003.01

$$\text{ns}(i\,z \mid 1-m) = -i\,\text{cs}(z \mid m)$$

09.33.16.0011.01

$$\text{ns}(x + i\,y \mid m) = \left(\text{cn}(y \mid 1-m)^2 + m\,\text{sn}(x \mid m)^2\,\text{sn}(y \mid 1-m)^2\right) \Big/ \\ (\text{dn}(y \mid 1-m)\,\text{sn}(x \mid m) + i\,\text{cn}(x \mid m)\,\text{cn}(y \mid 1-m)\,\text{dn}(x \mid m)\,\text{sn}(y \mid 1-m)) \;/;\; \{x, y\} \in \mathbb{R}$$

09.33.16.0012.01

$$\text{ns}\left(\sqrt{1-m}\; z \;\middle|\; \frac{m}{m-1}\right) = \frac{1}{\sqrt{1-m}}\,\text{ds}(z \mid m)$$

09.33.16.0013.01

$$\text{ns}\left(\sqrt{m}\; z \;\middle|\; \frac{1}{m}\right) = \frac{1}{\sqrt{m}}\,\text{ns}(z \mid m)$$

09.33.16.0014.01

$$\operatorname{ns}\left(i \sqrt{1-m}\, z \,\middle|\, \frac{1}{1-m}\right) = -\frac{i}{\sqrt{1-m}} \operatorname{cs}(z \mid m)$$

09.33.16.0015.01

$$\operatorname{ns}\left(i \sqrt{m}\, z \,\middle|\, \frac{m-1}{m}\right) = -\frac{i}{\sqrt{m}} \operatorname{ds}(z \mid m)$$

Landen's transformation:

09.33.16.0016.01

$$\operatorname{sn}\left(\left(1+\sqrt{1-m}\right) z \,\middle|\, \left(\frac{1-\sqrt{1-m}}{1+\sqrt{1-m}}\right)^2\right) = \frac{1}{1+\sqrt{1-m}} \frac{\operatorname{dn}(z \mid m)}{\operatorname{sn}(z \mid m) \operatorname{cn}(z \mid m)}$$

09.33.16.0017.01

$$\operatorname{sn}\left(\left(1+\sqrt{1-m}\right) z \,\middle|\, \left(\frac{1-\sqrt{1-m}}{1+\sqrt{1-m}}\right)^2\right) = \frac{1}{1+\sqrt{1-m}} \frac{\operatorname{ns}(z \mid m) \operatorname{nc}(z \mid m)}{\operatorname{nd}(z \mid m)}$$

Gauss' transformation:

09.33.16.0018.01

$$\operatorname{ns}\left(\left(1+\sqrt{m}\right) z \,\middle|\, \frac{4\sqrt{m}}{\left(1+\sqrt{m}\right)^2}\right) = \frac{1}{1+\sqrt{m}} \frac{1+\sqrt{m} \operatorname{sn}(z \mid m)^2}{\operatorname{sn}(z \mid m)}$$

09.33.16.0019.01

$$\operatorname{ns}\left(\left(1+\sqrt{m}\right) z \,\middle|\, \frac{4\sqrt{m}}{\left(1+\sqrt{m}\right)^2}\right) = \frac{1}{1+\sqrt{m}} \frac{\sqrt{m} + \operatorname{ns}(z \mid m)^2}{\operatorname{ns}(z \mid m)}$$

n th degree transformations:

09.33.16.0020.01

$$\operatorname{ns}\left(\frac{z}{M} \,\middle|\, l\right) = M \operatorname{ns}(z \mid m) \prod_{r=1}^{\frac{n-1}{2}} \frac{\operatorname{ns}(z \mid m)^2 \operatorname{ns}\left(\frac{2 r K(m)}{n} \,\middle|\, m\right)^2 - m}{\operatorname{ns}(z \mid m)^2 \operatorname{ns}\left(\frac{2 r K(m)}{n} \,\middle|\, m\right)^2 - \operatorname{ns}\left(\frac{2 r K(m)}{n} \,\middle|\, m\right)^4} /;$$

$$\frac{n+1}{2} \in \mathbb{Z}^+ \bigwedge l = m^n \prod_{r=1}^{\frac{n-1}{2}} \frac{1}{\operatorname{ns}\left(\frac{(2r-1) K(m)}{n} \,\middle|\, m\right)^8} \bigwedge M = \prod_{r=1}^{\frac{n-1}{2}} \frac{\operatorname{ns}\left(\frac{2 r K(m)}{n} \,\middle|\, m\right)^2}{\operatorname{ns}\left(\frac{(2r-1) K(m)}{n} \,\middle|\, m\right)^2}$$

09.33.16.0021.01

$$\operatorname{ns}\left(\frac{z}{M} + \frac{K(m)}{n M} \,\middle|\, l\right) = \prod_{r=1}^{\frac{n}{2}} \frac{\operatorname{ns}(z \mid m)^2 \operatorname{ns}\left(\frac{(2r-1) K(m)}{n} \,\middle|\, m\right)^2 - m}{\operatorname{ns}(z \mid m)^2 \operatorname{ns}\left(\frac{(2r-1) K(m)}{n} \,\middle|\, m\right)^2 - \operatorname{ns}\left(\frac{(2r-1) K(m)}{n} \,\middle|\, m\right)^4} /;$$

$$\frac{n}{2} \in \mathbb{Z}^+ \bigwedge l = m^n \prod_{r=1}^{\frac{n}{2}} \frac{1}{\operatorname{ns}\left(\frac{(2r-1) K(m)}{n} \,\middle|\, m\right)^8} \bigwedge M = \prod_{r=1}^{\frac{n}{2}} \frac{\operatorname{ns}\left(\frac{2 r K(m)}{n} \,\middle|\, m\right)^2}{\operatorname{ns}\left(\frac{(2r-1) K(m)}{n} \,\middle|\, m\right)^2}$$

Argument involving half-periods

09.33.16.0008.01

$$\operatorname{ns}(z+K(m)\mid m)=\operatorname{dc}(z\mid m)$$

09.33.16.0030.01

$$\operatorname{ns}(z-K(m)\mid m)=-\operatorname{dc}(z\mid m)$$

09.33.16.0009.01

$$\operatorname{ns}(z+3\,K(m)\mid m)=-\operatorname{dc}(z\mid m)$$

09.33.16.0031.01

$$\operatorname{ns}(z+(2\,r+1)\,K(m)\mid m)=(-1)^r\operatorname{dc}(z\mid m)\;/;\;r\in\mathbb{Z}$$

09.33.16.0010.01

$$\operatorname{ns}(z+i\,K(1-m)\mid m)=\sqrt{m}\,\operatorname{sn}(z\mid m)$$

09.33.16.0032.01

$$\operatorname{ns}(z-i\,K(1-m)\mid m)=\sqrt{m}\,\operatorname{sn}(z\mid m)$$

09.33.16.0033.01

$$\operatorname{ns}(z+3\,i\,K(1-m)\mid m)=\sqrt{m}\,\operatorname{sn}(z\mid m)\;/;\;s\in\mathbb{Z}$$

09.33.16.0034.01

$$\operatorname{ns}(z+(2\,s+1)\,i\,K(1-m)\mid m)=\sqrt{m}\,\operatorname{sn}(z\mid m)\;/;\;s\in\mathbb{Z}$$

09.33.16.0004.01

$$\operatorname{ns}(z+K(m)+i\,K(1-m)\mid m)=\sqrt{m}\,\operatorname{cd}(z\mid m)$$

09.33.16.0035.01

$$\operatorname{ns}(z-i\,K(1-m)+K(m)\mid m)=\sqrt{m}\,\operatorname{cd}(z\mid m)$$

09.33.16.0036.01

$$\operatorname{ns}(z+i\,K(1-m)-K(m)\mid m)=-\sqrt{m}\,\operatorname{cd}(z\mid m)$$

09.33.16.0037.01

$$\operatorname{ns}(z-i\,K(1-m)-K(m)\mid m)=-\sqrt{m}\,\operatorname{cd}(z\mid m)$$

09.33.16.0005.01

$$\operatorname{ns}(z+3\,K(m)+i\,K(1-m)\mid m)=-\sqrt{m}\,\operatorname{cd}(z\mid m)$$

09.33.16.0006.01

$$\operatorname{ns}(z+(4\,r+1)\,K(m)+i\,(2\,s+1)\,K(1-m)\mid m)=\sqrt{m}\,\operatorname{cd}(z\mid m)\;/;\;\{r,s\}\in\mathbb{Z}$$

09.33.16.0007.01

$$\operatorname{ns}(z+(4\,r-1)\,K(m)+i\,(2\,s+1)\,K(1-m)\mid m)=-\sqrt{m}\,\operatorname{cd}(z\mid m)\;/;\;\{r,s\}\in\mathbb{Z}$$

09.33.16.0038.01

$$\operatorname{ns}(z+(2\,s+1)\,i\,K(1-m)+(2\,r+1)\,K(m)\mid m)=(-1)^r\sqrt{m}\,\operatorname{cd}(z\mid m)\;/;\;\{r,s\}\in\mathbb{Z}$$

Argument involving inverse Jacobi functions

09.33.16.0039.01

$$\operatorname{ns}\left(\operatorname{cd}^{-1}(z\mid m)\mid m\right)^2=\frac{m\,z^2-1}{z^2-1}$$

09.33.16.0040.01

$$\operatorname{ns}\left(\operatorname{cn}^{-1}(z\mid m)\mid m\right)^2=\frac{1}{1-z^2}$$

09.33.16.0041.01

$$\operatorname{ns}\left(\operatorname{cs}^{-1}(z \mid m) \,\middle|\, m\right)^2 = z^2 + 1$$

09.33.16.0042.01

$$\operatorname{ns}\left(\operatorname{dc}^{-1}(z \mid m) \,\middle|\, m\right)^2 = \frac{m - z^2}{1 - z^2}$$

09.33.16.0043.01

$$\operatorname{ns}\left(\operatorname{dn}^{-1}(z \mid m) \,\middle|\, m\right)^2 = \frac{m}{1 - z^2}$$

09.33.16.0044.01

$$\operatorname{ns}\left(\operatorname{ds}^{-1}(z \mid m) \,\middle|\, m\right)^2 = z^2 + m$$

09.33.16.0045.01

$$\operatorname{ns}\left(\operatorname{nc}^{-1}(z \mid m) \,\middle|\, m\right)^2 = \frac{z^2}{z^2 - 1}$$

09.33.16.0046.01

$$\operatorname{ns}\left(\operatorname{nd}^{-1}(z \mid m) \,\middle|\, m\right)^2 = \frac{m\, z^2}{z^2 - 1}$$

09.33.16.0047.01

$$\operatorname{ns}\left(\operatorname{sc}^{-1}(z \mid m) \,\middle|\, m\right)^2 = \frac{z^2 + 1}{z^2}$$

09.33.16.0048.01

$$\operatorname{ns}\left(\operatorname{sd}^{-1}(z \mid m) \,\middle|\, m\right)^2 = \frac{m\, z^2 + 1}{z^2}$$

09.33.16.0049.01

$$\operatorname{ns}\left(\operatorname{sn}^{-1}(z \mid m) \,\middle|\, m\right) = \frac{1}{z}$$

Addition formulas

09.33.16.0022.01

$$\operatorname{ns}(u + v \mid m) == \frac{1 - m \operatorname{sn}(u \mid m)^2 \operatorname{sn}(v \mid m)^2}{\operatorname{cn}(v \mid m) \operatorname{dn}(v \mid m) \operatorname{sn}(u \mid m) + \operatorname{cn}(u \mid m) \operatorname{dn}(u \mid m) \operatorname{sn}(v \mid m)}$$

09.33.16.0023.01

$$\operatorname{ns}(u + v \mid m) \operatorname{ns}(u - v \mid m) == \frac{1 - m \operatorname{sn}(u \mid m)^2 \operatorname{sn}(v \mid m)^2}{\operatorname{sn}(u \mid m)^2 - \operatorname{sn}(v \mid m)^2}$$

09.33.16.0024.01

$$\operatorname{ns}(u + v \mid m) \operatorname{ns}(u - v \mid m) == \frac{m - \operatorname{ns}(u \mid m)^2 \operatorname{ns}(v \mid m)^2}{\operatorname{ns}(u \mid m)^2 - \operatorname{ns}(v \mid m)^2}$$

Half-angle formulas

09.33.16.0025.01

$$\operatorname{ns}\left(\frac{z}{2} \,\middle|\, m\right)^2 == \frac{1 + \operatorname{dn}(z \mid m)}{1 - \operatorname{cn}(z \mid m)}$$

Multiple arguments

Double angle formulas

09.33.16.0026.01

$$\operatorname{ns}(2z \mid m) = \frac{1 - m\operatorname{sn}(z \mid m)^4}{2\operatorname{sn}(z \mid m)\operatorname{cn}(z \mid m)\operatorname{dn}(z \mid m)}$$

09.33.16.0027.01

$$\operatorname{ns}(2z \mid m) = \frac{\operatorname{cn}(z \mid m)^2 + \operatorname{dn}(z \mid m)^2 \operatorname{sn}(z \mid m)^2}{2\operatorname{sn}(z \mid m)\operatorname{cn}(z \mid m)\operatorname{dn}(z \mid m)}$$

Multiple angle formulas

09.33.16.0028.01

$$\operatorname{ns}(n z, m) = (-1)^{\frac{n-1}{2}} m^{\frac{1-n^2}{4}} \prod_{\mu=-\frac{n-1}{2}}^{\frac{n-1}{2}} \prod_{\nu=-\frac{n-1}{2}}^{\frac{n-1}{2}} \operatorname{ns}\left(z + 2\frac{\mu K(m) + i\nu K(1-m)}{n} \,\middle|\, m\right) /; \frac{n+1}{2} \in \mathbb{Z}^+$$

09.33.16.0029.01

$$\operatorname{ns}\left(\frac{2n}{\pi} K\left(\lambda\left(\frac{n}{\pi i}\log(q(m))\right)\right) z \,\middle|\, \lambda\left(\frac{n}{\pi i}\log(q(m))\right)\right) = \frac{\sqrt[4]{q(m)^n}}{q(m)^{n/4}} \frac{\sqrt[4]{\lambda\left(\frac{n}{\pi i}\log(q(m))\right)}}{\left(\sqrt[4]{m}\right)^n} \prod_{r=0}^{n-1} \operatorname{ns}\left(\frac{2K(m)}{\pi}\left(z + \frac{r\pi}{n}\right) \,\middle|\, m\right) /; n \in \mathbb{Z}^+$$

Identities

Functional identities

09.33.17.0001.01

$$4 w(z)^2 \left(w(z)^2 - 1\right)\left(w(z)^2 - m\right) w(2z)^2 - \left(m - w(z)^4\right)^2 = 0 /; w(z) = \operatorname{ns}(z \mid m)$$

Complex characteristics

Real part

09.33.19.0001.01

$$\operatorname{Re}(\operatorname{ns}(x + i y \mid m)) = \frac{\operatorname{dn}(y \mid 1-m)\operatorname{sn}(x \mid m)\left(\operatorname{cn}(y \mid 1-m)^2 + m\operatorname{sn}(x \mid m)^2 \operatorname{sn}(y \mid 1-m)^2\right)}{\operatorname{dn}(y \mid 1-m)^2 \operatorname{sn}(x \mid m)^2 + \operatorname{cn}(x \mid m)^2 \operatorname{cn}(y \mid 1-m)^2 \operatorname{dn}(x \mid m)^2 \operatorname{sn}(y \mid 1-m)^2} /; \{x, y, m\} \in \mathbb{R}$$

Imaginary part

09.33.19.0002.01

$$\operatorname{Im}(\operatorname{ns}(x + i y \mid m)) = -\frac{\operatorname{cn}(x \mid m)\operatorname{cn}(y \mid 1-m)\operatorname{dn}(x \mid m)\operatorname{sn}(y \mid 1-m)\left(\operatorname{cn}(y \mid 1-m)^2 + m\operatorname{sn}(x \mid m)^2 \operatorname{sn}(y \mid 1-m)^2\right)}{\operatorname{dn}(y \mid 1-m)^2 \operatorname{sn}(x \mid m)^2 + \operatorname{cn}(x \mid m)^2 \operatorname{cn}(y \mid 1-m)^2 \operatorname{dn}(x \mid m)^2 \operatorname{sn}(y \mid 1-m)^2} /; \{x, y, m\} \in \mathbb{R}$$

Absolute value

09.33.19.0003.01

$$|\operatorname{ns}(x+i\,y\mid m)| = \left(\sqrt{\left(\left(\operatorname{dn}(y\mid 1-m)^2\operatorname{sn}(x\mid m)^2+\operatorname{cn}(x\mid m)^2\operatorname{cn}(y\mid 1-m)^2\operatorname{dn}(x\mid m)^2\operatorname{sn}(y\mid 1-m)^2\right)\left(\operatorname{cn}(y\mid 1-m)^2+m\operatorname{sn}(x\mid m)^2\operatorname{sn}(y\mid 1-m)^2\right)^2\right)}\right)\Big/ \left(\operatorname{dn}(y\mid 1-m)^2\operatorname{sn}(x\mid m)^2+\operatorname{cn}(x\mid m)^2\operatorname{cn}(y\mid 1-m)^2\operatorname{dn}(x\mid m)^2\operatorname{sn}(y\mid 1-m)^2\right)\ /;\ \{x,y,m\}\in\mathbb{R}$$

Argument

09.33.19.0004.01

$$\arg(\operatorname{ns}(x+i\,y\mid m)) = \tan^{-1}\Big(\operatorname{dn}(y\mid 1-m)\operatorname{sn}(x\mid m)\left(\operatorname{cn}(y\mid 1-m)^2+m\operatorname{sn}(x\mid m)^2\operatorname{sn}(y\mid 1-m)^2\right), -\left(\operatorname{cn}(x\mid m)\operatorname{cn}(y\mid 1-m)\operatorname{dn}(x\mid m)\operatorname{sn}(y\mid 1-m)\left(\operatorname{cn}(y\mid 1-m)^2+m\operatorname{sn}(x\mid m)^2\operatorname{sn}(y\mid 1-m)^2\right)\right)\Big)\ /;\ \{x,y,m\}\in\mathbb{R}$$

Conjugate value

09.33.19.0005.01

$$\overline{\operatorname{ns}(x+i\,y\mid m)} = \frac{\operatorname{cn}(y\mid 1-m)^2+m\operatorname{sn}(x\mid m)^2\operatorname{sn}(y\mid 1-m)^2}{\operatorname{dn}(y\mid 1-m)\operatorname{sn}(x\mid m)-i\operatorname{cn}(x\mid m)\operatorname{cn}(y\mid 1-m)\operatorname{dn}(x\mid m)\operatorname{sn}(y\mid 1-m)}\ /;\ \{x,y,m\}\in\mathbb{R}$$

Differentiation

Low-order differentiation

With respect to *z*

09.33.20.0001.01

$$\frac{\partial \operatorname{ns}(z,m)}{\partial z} = -\operatorname{cs}(z\mid m)\operatorname{ds}(z\mid m)$$

09.33.20.0002.01

$$\frac{\partial^2 \operatorname{ns}(z\mid m)}{\partial z^2} = \left(\operatorname{cs}(z\mid m)^2+\operatorname{ds}(z\mid m)^2\right)\operatorname{ns}(z\mid m)$$

With respect to *m*

09.33.20.0003.01

$$\frac{\partial \operatorname{ns}(z\mid m)}{\partial m} = \frac{1}{2\,m\,(1-m)}\left(\operatorname{ds}(z\mid m)\operatorname{cs}(z\mid m)\left(-(1-m)\,z+E(\operatorname{am}(z\mid m)\mid m)-m\operatorname{sn}(z\mid m)\operatorname{cd}(z\mid m)\right)\right)$$

09.33.20.0004.01

$$\frac{\partial^2 \mathrm{ns}(z \mid m)}{\partial m^2} = \frac{1}{4(m-1)^2 m^2} \Big(\mathrm{ns}(z \mid m)((m-1)z + E(\mathrm{am}(z \mid m) \mid m) - m\,\mathrm{dn}(z \mid m)\,\mathrm{sc}(z \mid m))((m-1)z + E(\mathrm{am}(z \mid m) \mid m) - m\,\mathrm{cd}(z \mid m)\,\mathrm{sn}(z \mid m))\,\mathrm{cs}(z \mid m)^2 + 2(m-1)\,\mathrm{ds}(z \mid m)((m-1)z + E(\mathrm{am}(z \mid m) \mid m) - m\,\mathrm{cd}(z \mid m)\,\mathrm{sn}(z \mid m))\,\mathrm{cs}(z \mid m) + 2m\,\mathrm{ds}(z \mid m)((m-1)z + E(\mathrm{am}(z \mid m) \mid m) - m\,\mathrm{cd}(z \mid m)\,\mathrm{sn}(z \mid m))\,\mathrm{cs}(z \mid m) + (1-m)\,m\,\mathrm{ds}(z \mid m) \left(2z + \frac{E(\mathrm{am}(z \mid m) \mid m) - F(\mathrm{am}(z \mid m) \mid m)}{m} - 2\,\mathrm{cd}(z \mid m)\,\mathrm{sn}(z \mid m) - ((m-1)z + E(\mathrm{am}(z \mid m) \mid m))\,\mathrm{nd}(z \mid m)\,\mathrm{sd}(z \mid m)\,\mathrm{sn}(z \mid m) + \frac{1}{m-1}(\mathrm{cd}(z \mid m)\,\mathrm{cn}(z \mid m)\,\mathrm{dn}(z \mid m)(-m z + z - E(\mathrm{am}(z \mid m) \mid m) + m\,\mathrm{cd}(z \mid m)\,\mathrm{sn}(z \mid m))) + \frac{1}{(m-1)\,m}\left(((m-1)z + E(\mathrm{am}(z \mid m) \mid m))\,\mathrm{dn}(z \mid m) - m\,\mathrm{cn}(z \mid m)\,\mathrm{sn}(z \mid m))\sqrt{1 - m\,\mathrm{sn}(z \mid m)^2}\right)\right)\mathrm{cs}(z \mid m) + \mathrm{ds}(z \mid m)^2\,\mathrm{ns}(z \mid m)((m-1)z + E(\mathrm{am}(z \mid m) \mid m) - m\,\mathrm{cd}(z \mid m)\,\mathrm{sn}(z \mid m))^2\Big)$$

Symbolic differentiation

With respect to *z*

09.33.20.0007.01

$$\frac{\partial^n \mathrm{ns}(z \mid m)}{\partial z^n} = \mathrm{ns}(z \mid m)\,\delta_n - \sum_{j=0}^{n-1} \binom{n-1}{j} \frac{\partial^j \mathrm{ds}(z \mid m)}{\partial z^j} \frac{\partial^{-j+n-1} \mathrm{cs}(z \mid m)}{\partial z^{-j+n-1}} \;/;\; n \in \mathbb{N}$$

09.33.20.0005.01

$$\frac{\partial^n \mathrm{ns}(z \mid m)}{\partial z^n} = (-1)^n\, n!\, z^{-n-1} + z^{-n-1} \sum_{k=1}^{\infty} \frac{(-1)^{k-1}\left(2^{2k-1}-1\right) B_{2k}}{k\,(2k-n-1)!} \left(\frac{\pi z}{2K(m)}\right)^{2k} + \frac{2^{1-n}\,\pi^{n+1}}{K(m)^{n+1}} \sum_{k=0}^{\infty} \frac{(2k+1)^n\, q(m)^{2k+1}}{1-q(m)^{2k+1}} \sin\left(\frac{\pi n}{2} + \frac{(2k+1)\pi z}{2K(m)}\right) /;\; n \in \mathbb{N}^+$$

Fractional integro-differentiation

With respect to *z*

09.33.20.0006.01

$$\frac{\partial^\alpha \mathrm{ns}(z \mid m)}{\partial z^\alpha} = \mathcal{FC}_{\exp}^{(\alpha)}(z, -1)\, z^{-\alpha-1} + \sum_{k=1}^{\infty} \frac{(-1)^{k-1}\, 2^{1-2k}\left(2^{2k-1}-1\right)\pi^{2k}\, z^{2k-\alpha-1}\, B_{2k}\, K(m)^{-2k}}{(2k+1)\,\Gamma(2k-\alpha)} + \frac{2^{\alpha-1}\,\pi^{5/2}\, z^{1-\alpha}}{K(m)^2} \sum_{k=0}^{\infty} \frac{(2k+1)\, q(m)^{2k+1}}{1-q(m)^{2k+1}}\; {}_1\tilde{F}_2\left(1;\, 1-\frac{\alpha}{2}, \frac{3-\alpha}{2};\, -\frac{(2k+1)^2\,\pi^2\, z^2}{16\,K(m)^2}\right)$$

Integration

Indefinite integration

Involving only one direct function

09.33.21.0001.01

$$\int \operatorname{ns}(z \mid m)\, dz = \log(\operatorname{ds}(z \mid m) - \operatorname{cs}(z \mid m))$$

Involving functions of the direct function

Involving elementary functions of the direct function

Involving powers of the direct function

09.33.21.0002.01

$$\int \operatorname{ns}(z \mid m)^2\, dz = z - \frac{\operatorname{cn}(z \mid m)\operatorname{dn}(z \mid m)}{\operatorname{sn}(z \mid m)} + \frac{\left(m \operatorname{sn}(z \mid m)^2 - 1\right) E(\operatorname{am}(z \mid m) \mid m)}{\operatorname{dn}(z \mid m)\sqrt{1 - m \operatorname{sn}(z \mid m)^2}}$$

Representations through equivalent functions

With inverse function

09.33.27.0001.01

$$\operatorname{ns}\left(\operatorname{ns}^{-1}(z \mid m) \,\middle|\, m\right) = z$$

With related functions

Involving am

09.33.27.0028.01

$$\operatorname{ns}(z \mid m) = \csc(\operatorname{am}(z \mid m))$$

Involving one other Jacobi elliptic function

Involving cd

09.33.27.0002.01

$$\operatorname{ns}(z \mid m)^2 = \frac{m \operatorname{cd}(z \mid m)^2 - 1}{\operatorname{cd}(z \mid m)^2 - 1}$$

Involving cn

09.33.27.0005.01

$$\operatorname{ns}(z \mid m)^2 = \frac{1}{1 - \operatorname{cn}(z \mid m)^2}$$

Involving cs

09.33.27.0007.01

$$\operatorname{ns}(z \mid m) = i \operatorname{cs}(i z \mid 1 - m)$$

09.33.27.0029.01

$$\operatorname{ns}(z \mid m)^2 = \operatorname{cs}(z \mid m)^2 + 1$$

Involving dc

09.33.27.0008.01

$$\operatorname{ns}(z \mid m)^2 = \frac{m - \operatorname{dc}(z \mid m)^2}{1 - \operatorname{dc}(z \mid m)^2}$$

Involving dn

09.33.27.0011.01

$$\operatorname{ns}(z \mid m)^2 = \frac{m}{1 - \operatorname{dn}(z \mid m)^2}$$

Involving ds

09.33.27.0013.01

$$\operatorname{ns}(z \mid m)^2 = \operatorname{ds}(z \mid m)^2 + m$$

Involving nc

09.33.27.0015.01

$$\operatorname{ns}(z \mid m)^2 = \frac{\operatorname{nc}(z \mid m)^2}{\operatorname{nc}(z \mid m)^2 - 1}$$

Involving nd

09.33.27.0017.01

$$\operatorname{ns}(z \mid m)^2 = \frac{m \operatorname{nd}(z \mid m)^2}{\operatorname{nd}(z \mid m)^2 - 1}$$

Involving sc

09.33.27.0018.01

$$\operatorname{ns}(z \mid m) = \frac{i}{\operatorname{sc}(i z \mid 1 - m)}$$

09.33.27.0019.01

$$\operatorname{ns}(z \mid m)^2 = \frac{\operatorname{sc}(z \mid m)^2 + 1}{\operatorname{sc}(z \mid m)^2}$$

Involving sd

09.33.27.0020.01

$$\operatorname{ns}(z \mid m)^2 = \frac{m \operatorname{sd}(z \mid m)^2 + 1}{\operatorname{sd}(z \mid m)^2}$$

Involving sn

09.33.27.0021.01

$$\operatorname{ns}(z \mid m) = \frac{1}{\operatorname{sn}(z \mid m)}$$

Involving two other Jacobi elliptic functions

Involving cn and cs

09.33.27.0003.01

$$\operatorname{ns}(z \mid m) = \frac{\operatorname{cs}(z \mid m)}{\operatorname{cn}(z \mid m)}$$

09.33.27.0030.01

$$\operatorname{ns}(z \mid m) = \frac{\operatorname{cn}(z \mid m)\left(\operatorname{cs}(z \mid m)^2 + 1\right)}{\operatorname{cs}(z \mid m)}$$

Involving cn and sc

09.33.27.0004.01

$$\operatorname{ns}(z \mid m) = \frac{1}{\operatorname{cn}(z \mid m)\operatorname{sc}(z \mid m)}$$

09.33.27.0031.01

$$\operatorname{ns}(z \mid m) = \frac{\operatorname{cn}(z \mid m)\left(\operatorname{sc}(z \mid m)^2 + 1\right)}{\operatorname{sc}(z \mid m)}$$

Involving cs and sn

09.33.27.0032.01

$$\operatorname{ns}(z \mid m) = \left(\operatorname{cs}(z \mid m)^2 + 1\right)\operatorname{sn}(z \mid m)$$

Involving dn and ds

09.33.27.0009.01

$$\operatorname{ns}(z \mid m) = \frac{\operatorname{ds}(z \mid m)}{\operatorname{dn}(z \mid m)}$$

Involving dn and sd

09.33.27.0010.01

$$\operatorname{ns}(z \mid m) = \frac{1}{\operatorname{dn}(z \mid m)\operatorname{sd}(z \mid m)}$$

Involving dn and sn

09.33.27.0033.01

$$\operatorname{ns}(z \mid m) = -\frac{m\operatorname{sn}(z \mid m)}{(\operatorname{dn}(z \mid m) - 1)(\operatorname{dn}(z \mid m) + 1)}$$

Involving ds and nd

09.33.27.0012.01

$$\text{ns}(z \mid m) = \text{nd}(z \mid m)\,\text{ds}(z \mid m)$$

Involving nc and sc

09.33.27.0014.01

$$\text{ns}(z \mid m) = \frac{\text{nc}(z \mid m)}{\text{sc}(z \mid m)}$$

09.33.27.0006.01

$$\text{ns}(z \mid m) = \text{nc}(z \mid m)\,\text{cs}(z \mid m)$$

Involving nc and sn

09.33.27.0034.01

$$\text{ns}(z \mid m) = \frac{\text{nc}(z \mid m)^2\,\text{sn}(z \mid m)}{(\text{nc}(z \mid m) - 1)\,(\text{nc}(z \mid m) + 1)}$$

Involving nd and sd

09.33.27.0016.01

$$\text{ns}(z \mid m) = \frac{\text{nd}(z \mid m)}{\text{sd}(z \mid m)}$$

09.33.27.0035.01

$$\text{ns}(z \mid m) = \frac{m\,\text{nd}(z \mid m)\,\text{sd}(z \mid m)}{(\text{nd}(z \mid m) - 1)\,(\text{nd}(z \mid m) + 1)}$$

Involving sc and sn

09.33.27.0036.01

$$\text{ns}(z \mid m) = \frac{\left(\text{sc}(z \mid m)^2 + 1\right)\text{sn}(z \mid m)}{\text{sc}(z \mid m)^2}$$

Involving three other Jacobi elliptic functions

09.33.27.0037.01

$$\text{ns}(z \mid m) = \frac{\text{cd}(z \mid m)\left(\text{cs}(z \mid m)^2 - m + 1\right)}{\text{cs}(z \mid m)\,\text{dn}(z \mid m)}$$

09.33.27.0038.01

$$\text{ns}(z \mid m) = \frac{\text{cd}(z \mid m)\left(\text{cs}(z \mid m)^2 + 1\right)\text{dn}(z \mid m)}{\text{cs}(z \mid m)}$$

09.33.27.0039.01

$$\text{ns}(z \mid m) = \frac{\text{cs}(z \mid m)\left(\text{cs}(z \mid m)^2 + 1\right)\text{dn}(z \mid m)}{\text{cd}(z \mid m)\left(\text{cs}(z \mid m)^2 - m + 1\right)}$$

09.33.27.0040.01

$$\operatorname{ns}(z \mid m) = \frac{\left(\operatorname{cs}(z \mid m)^2 + 1\right) \operatorname{dn}(z \mid m)}{\operatorname{cs}(z \mid m) \operatorname{dc}(z \mid m)}$$

09.33.27.0041.01

$$\operatorname{ns}(z \mid m) = -\frac{m \operatorname{cn}(z \mid m)}{\operatorname{cs}(z \mid m) (\operatorname{dn}(z \mid m) - 1) (\operatorname{dn}(z \mid m) + 1)}$$

09.33.27.0042.01

$$\operatorname{ns}(z \mid m) = \frac{m \operatorname{cd}(z \mid m) \operatorname{cs}(z \mid m) \operatorname{dn}(z \mid m)}{\operatorname{dn}(z \mid m)^2 + m - 1}$$

09.33.27.0043.01

$$\operatorname{ns}(z \mid m) = -\frac{\operatorname{dn}(z \mid m)}{(\operatorname{cn}(z \mid m) - 1) (\operatorname{cn}(z \mid m) + 1) \operatorname{ds}(z \mid m)}$$

09.33.27.0044.01

$$\operatorname{ns}(z \mid m) = \frac{\left(\operatorname{cs}(z \mid m)^2 + 1\right) \operatorname{dn}(z \mid m)}{\operatorname{ds}(z \mid m)}$$

09.33.27.0045.01

$$\operatorname{ns}(z \mid m) = \frac{\operatorname{cn}(z \mid m) \left(\operatorname{dc}(z \mid m)^2 + \operatorname{ds}(z \mid m)^2\right)}{\operatorname{dc}(z \mid m) \operatorname{ds}(z \mid m)}$$

09.33.27.0046.01

$$\operatorname{ns}(z \mid m) = \frac{\operatorname{dn}(z \mid m) \left(\operatorname{dc}(z \mid m)^2 + \operatorname{ds}(z \mid m)^2\right)}{\operatorname{dc}(z \mid m)^2 \operatorname{ds}(z \mid m)}$$

09.33.27.0047.01

$$\operatorname{ns}(z \mid m) = \frac{\operatorname{dc}(z \mid m) \operatorname{nc}(z \mid m)}{\operatorname{ds}(z \mid m) (\operatorname{nc}(z \mid m) - 1) (\operatorname{nc}(z \mid m) + 1)}$$

09.33.27.0048.01

$$\operatorname{ns}(z \mid m) = \frac{\operatorname{dn}(z \mid m) \operatorname{nc}(z \mid m)^2}{\operatorname{ds}(z \mid m) (\operatorname{nc}(z \mid m) - 1) (\operatorname{nc}(z \mid m) + 1)}$$

09.33.27.0049.01

$$\operatorname{ns}(z \mid m) = \frac{\operatorname{cd}(z \mid m) \left(\operatorname{cs}(z \mid m)^2 + 1\right)}{\operatorname{cs}(z \mid m) \operatorname{nd}(z \mid m)}$$

09.33.27.0050.01

$$\operatorname{ns}(z \mid m) = \frac{m \operatorname{cd}(z \mid m) \operatorname{nd}(z \mid m)}{\operatorname{cs}(z \mid m) (\operatorname{nd}(z \mid m) - 1) (\operatorname{nd}(z \mid m) + 1)}$$

09.33.27.0051.01

$$\operatorname{ns}(z \mid m) = \frac{m \operatorname{cn}(z \mid m)}{\left(\operatorname{dn}(z \mid m)^2 + m - 1\right) \operatorname{sc}(z \mid m)}$$

09.33.27.0052.01

$$\operatorname{ns}(z \mid m) = -\frac{\operatorname{sc}(z \mid m)}{\operatorname{cn}(z \mid m) - \operatorname{nc}(z \mid m)}$$

09.33.27.0053.01

$$\operatorname{ns}(z \mid m) = -\frac{\left(\operatorname{dn}(z \mid m)^2 + m - 1\right) \operatorname{nc}(z \mid m) \operatorname{sc}(z \mid m)}{(\operatorname{dn}(z \mid m) - 1)(\operatorname{dn}(z \mid m) + 1)}$$

09.33.27.0054.01

$$\operatorname{ns}(z \mid m) = \operatorname{cn}(z \mid m)(\operatorname{cs}(z \mid m) + \operatorname{sc}(z \mid m))$$

09.33.27.0055.01

$$\operatorname{ns}(z \mid m) = \frac{\operatorname{dn}(z \mid m)\left(\operatorname{sc}(z \mid m)^2 + 1\right)}{\operatorname{ds}(z \mid m) \operatorname{sc}(z \mid m)^2}$$

09.33.27.0056.01

$$\operatorname{ns}(z \mid m) = \frac{\operatorname{dn}(z \mid m)\left(\operatorname{sc}(z \mid m)^2 + 1\right)}{\operatorname{dc}(z \mid m) \operatorname{sc}(z \mid m)}$$

09.33.27.0057.01

$$\operatorname{ns}(z \mid m) = \frac{\operatorname{cd}(z \mid m)\left(\operatorname{sc}(z \mid m)^2 + 1\right)}{\operatorname{nd}(z \mid m) \operatorname{sc}(z \mid m)}$$

09.33.27.0058.01

$$\operatorname{ns}(z \mid m) = -\frac{\operatorname{dn}(z \mid m)\left(\operatorname{sc}(z \mid m)^2 + 1\right)}{\operatorname{cd}(z \mid m) \operatorname{sc}(z \mid m)\left(m \operatorname{sc}(z \mid m)^2 - \operatorname{sc}(z \mid m)^2 - 1\right)}$$

09.33.27.0059.01

$$\operatorname{ns}(z \mid m) = -\frac{\left(\operatorname{cs}(z \mid m)^2 + 1\right) \operatorname{dn}(z \mid m)}{\left(-\operatorname{cs}(z \mid m)^2 + m - 1\right) \operatorname{sd}(z \mid m)}$$

09.33.27.0060.01

$$\operatorname{ns}(z \mid m) = -\frac{\operatorname{dn}(z \mid m) \operatorname{nc}(z \mid m)^2}{\left(m \operatorname{nc}(z \mid m)^2 - \operatorname{nc}(z \mid m)^2 - m\right) \operatorname{sd}(z \mid m)}$$

09.33.27.0061.01

$$\operatorname{ns}(z \mid m) = -\frac{\operatorname{dn}(z \mid m)\left(\operatorname{sc}(z \mid m)^2 + 1\right)}{\left(m \operatorname{sc}(z \mid m)^2 - \operatorname{sc}(z \mid m)^2 - 1\right) \operatorname{sd}(z \mid m)}$$

09.33.27.0062.01

$$\operatorname{ns}(z \mid m) = -\frac{\left(-\operatorname{cs}(z \mid m)^2 + m - 1\right) \operatorname{sd}(z \mid m)}{\operatorname{dn}(z \mid m)}$$

09.33.27.0063.01

$$\operatorname{ns}(z \mid m) = \frac{\operatorname{dc}(z \mid m) \operatorname{nc}(z \mid m) \operatorname{sd}(z \mid m)}{(\operatorname{nc}(z \mid m) - 1)(\operatorname{nc}(z \mid m) + 1)}$$

09.33.27.0064.01

$$\operatorname{ns}(z \mid m) = -\frac{m \operatorname{sd}(z \mid m)}{\operatorname{dn}(z \mid m) - \operatorname{nd}(z \mid m)}$$

09.33.27.0065.01

$$\operatorname{ns}(z \mid m) = -\frac{\operatorname{sd}(z \mid m)}{(\operatorname{cn}(z \mid m) - 1)(\operatorname{cn}(z \mid m) + 1) \operatorname{nd}(z \mid m)}$$

09.33.27.0066.01

$$\operatorname{ns}(z \mid m) = \frac{\left(\operatorname{cs}(z \mid m)^2 + 1\right) \operatorname{sd}(z \mid m)}{\operatorname{nd}(z \mid m)}$$

09.33.27.0067.01

$$\operatorname{ns}(z \mid m) = \frac{\operatorname{nc}(z \mid m)^2 \operatorname{sd}(z \mid m)}{(\operatorname{nc}(z \mid m) - 1)(\operatorname{nc}(z \mid m) + 1) \operatorname{nd}(z \mid m)}$$

09.33.27.0068.01

$$\operatorname{ns}(z \mid m) = -\left(-\operatorname{cs}(z \mid m)^2 + m - 1\right) \operatorname{nd}(z \mid m) \operatorname{sd}(z \mid m)$$

09.33.27.0069.01

$$\operatorname{ns}(z \mid m) = \frac{\operatorname{dc}(z \mid m)^2 \operatorname{nd}(z \mid m) \operatorname{sd}(z \mid m)}{(\operatorname{dc}(z \mid m) \operatorname{nd}(z \mid m) - 1)(\operatorname{dc}(z \mid m) \operatorname{nd}(z \mid m) + 1)}$$

09.33.27.0070.01

$$\operatorname{ns}(z \mid m) = \frac{\left(\operatorname{sc}(z \mid m)^2 + 1\right) \operatorname{sd}(z \mid m)}{\operatorname{nd}(z \mid m) \operatorname{sc}(z \mid m)^2}$$

09.33.27.0071.01

$$\operatorname{ns}(z \mid m) = \frac{\operatorname{cn}(z \mid m) \left(\operatorname{dc}(z \mid m)^2 \operatorname{sd}(z \mid m)^2 + 1\right)}{\operatorname{dc}(z \mid m) \operatorname{sd}(z \mid m)}$$

09.33.27.0072.01

$$\operatorname{ns}(z \mid m) = \frac{\operatorname{dn}(z \mid m) \left(\operatorname{dc}(z \mid m)^2 \operatorname{sd}(z \mid m)^2 + 1\right)}{\operatorname{dc}(z \mid m)^2 \operatorname{sd}(z \mid m)}$$

09.33.27.0073.01

$$\operatorname{ns}(z \mid m) = \frac{\operatorname{dc}(z \mid m)^2 \operatorname{sn}(z \mid m)}{(\operatorname{dc}(z \mid m) - \operatorname{dn}(z \mid m))(\operatorname{dc}(z \mid m) + \operatorname{dn}(z \mid m))}$$

09.33.27.0074.01

$$\operatorname{ns}(z \mid m) = -\frac{\operatorname{sn}(z \mid m)}{(\operatorname{cd}(z \mid m) \operatorname{dn}(z \mid m) - 1)(\operatorname{cd}(z \mid m) \operatorname{dn}(z \mid m) + 1)}$$

09.33.27.0075.01

$$\operatorname{ns}(z \mid m) = \frac{\left(\operatorname{dc}(z \mid m)^2 + \operatorname{ds}(z \mid m)^2\right) \operatorname{sn}(z \mid m)}{\operatorname{dc}(z \mid m)^2}$$

09.33.27.0076.01

$$\operatorname{ns}(z \mid m) = -\frac{\operatorname{nd}(z \mid m)^2 \operatorname{sn}(z \mid m)}{(\operatorname{cd}(z \mid m) - \operatorname{nd}(z \mid m))(\operatorname{cd}(z \mid m) + \operatorname{nd}(z \mid m))}$$

09.33.27.0077.01

$$\operatorname{ns}(z \mid m) = \frac{\operatorname{dc}(z \mid m)^2 \operatorname{nd}(z \mid m)^2 \operatorname{sn}(z \mid m)}{(\operatorname{dc}(z \mid m) \operatorname{nd}(z \mid m) - 1)(\operatorname{dc}(z \mid m) \operatorname{nd}(z \mid m) + 1)}$$

09.33.27.0078.01

$$\operatorname{ns}(z \mid m) = \frac{(\operatorname{cs}(z \mid m) + \operatorname{sc}(z \mid m)) \operatorname{sn}(z \mid m)}{\operatorname{sc}(z \mid m)}$$

09.33.27.0079.01

$$\operatorname{ns}(z \mid m) = \frac{\left(\operatorname{dc}(z \mid m)^2 \operatorname{sd}(z \mid m)^2 + 1\right) \operatorname{sn}(z \mid m)}{\operatorname{dc}(z \mid m)^2 \operatorname{sd}(z \mid m)^2}$$

09.33.27.0080.01

$$\operatorname{ns}(z \mid m) = \frac{\operatorname{cn}(z \mid m) + \operatorname{sc}(z \mid m) \operatorname{sn}(z \mid m)}{\operatorname{sc}(z \mid m)}$$

Involving four other Jacobi elliptic functions

09.33.27.0081.01

$$\operatorname{ns}(z \mid m) = \frac{\operatorname{cn}(z \mid m) (\operatorname{dc}(z \mid m) + \operatorname{cs}(z \mid m) \operatorname{ds}(z \mid m))}{\operatorname{ds}(z \mid m)}$$

09.33.27.0082.01

$$\operatorname{ns}(z \mid m) = \frac{\operatorname{dn}(z \mid m) (\operatorname{dc}(z \mid m) + \operatorname{cs}(z \mid m) \operatorname{ds}(z \mid m))}{\operatorname{dc}(z \mid m) \operatorname{ds}(z \mid m)}$$

09.33.27.0083.01

$$\operatorname{ns}(z \mid m) = \frac{\operatorname{cn}(z \mid m) \operatorname{ds}(z \mid m)^2 + \operatorname{dc}(z \mid m) \operatorname{dn}(z \mid m)}{\operatorname{dc}(z \mid m) \operatorname{ds}(z \mid m)}$$

09.33.27.0084.01

$$\operatorname{ns}(z \mid m) = -\frac{\operatorname{dc}(z \mid m)}{\operatorname{ds}(z \mid m) (\operatorname{cn}(z \mid m) - \operatorname{nc}(z \mid m))}$$

09.33.27.0085.01

$$\operatorname{ns}(z \mid m) = \frac{\operatorname{cd}(z \mid m) \operatorname{ds}(z \mid m)^2 + \operatorname{dc}(z \mid m)}{\operatorname{ds}(z \mid m) \operatorname{nc}(z \mid m)}$$

09.33.27.0086.01

$$\operatorname{ns}(z \mid m) = -\frac{\operatorname{dn}(z \mid m) \operatorname{nc}(z \mid m)}{\operatorname{ds}(z \mid m) (\operatorname{cn}(z \mid m) - \operatorname{nc}(z \mid m))}$$

09.33.27.0087.01

$$\operatorname{ns}(z \mid m) = \frac{\operatorname{cd}(z \mid m) \operatorname{ds}(z \mid m)^2 + \operatorname{dn}(z \mid m) \operatorname{nc}(z \mid m)}{\operatorname{ds}(z \mid m) \operatorname{nc}(z \mid m)}$$

09.33.27.0088.01

$$\operatorname{ns}(z \mid m) = \frac{m \operatorname{cd}(z \mid m) \operatorname{cs}(z \mid m)}{\operatorname{dn}(z \mid m) + m \operatorname{nd}(z \mid m) - \operatorname{nd}(z \mid m)}$$

09.33.27.0089.01

$$\operatorname{ns}(z \mid m) = \frac{m \operatorname{cd}(z \mid m) \operatorname{sc}(z \mid m)}{\operatorname{cd}(z \mid m) \operatorname{nc}(z \mid m) - \operatorname{dn}(z \mid m)}$$

09.33.27.0090.01

$$\operatorname{ns}(z \mid m) = \frac{\operatorname{dc}(z \mid m) \operatorname{sc}(z \mid m)}{\operatorname{dc}(z \mid m) \operatorname{nc}(z \mid m) - \operatorname{dn}(z \mid m)}$$

09.33.27.0091.01

$$\operatorname{ns}(z \mid m) = -\frac{m \operatorname{cd}(z \mid m) \operatorname{sc}(z \mid m)}{\operatorname{dn}(z \mid m) - \operatorname{nd}(z \mid m)}$$

09.33.27.0092.01

$$\operatorname{ns}(z \mid m) = -\frac{\operatorname{nc}(z \mid m)\,(\operatorname{dn}(z \mid m) + m\operatorname{nd}(z \mid m) - \operatorname{nd}(z \mid m))\operatorname{sc}(z \mid m)}{\operatorname{dn}(z \mid m) - \operatorname{nd}(z \mid m)}$$

09.33.27.0093.01

$$\operatorname{ns}(z \mid m) = \frac{\operatorname{nd}(z \mid m)\operatorname{sc}(z \mid m)}{\operatorname{nc}(z \mid m)\operatorname{nd}(z \mid m) - \operatorname{cd}(z \mid m)}$$

09.33.27.0094.01

$$\operatorname{ns}(z \mid m) = \frac{\operatorname{dn}(z \mid m)\,(\operatorname{cs}(z \mid m) + \operatorname{sc}(z \mid m))}{\operatorname{dc}(z \mid m)}$$

09.33.27.0095.01

$$\operatorname{ns}(z \mid m) = \frac{\operatorname{cd}(z \mid m)\,(\operatorname{cs}(z \mid m) + \operatorname{sc}(z \mid m))}{\operatorname{nd}(z \mid m)}$$

09.33.27.0096.01

$$\operatorname{ns}(z \mid m) = \frac{\operatorname{dn}(z \mid m)\,(\operatorname{cs}(z \mid m) + \operatorname{sc}(z \mid m))}{\operatorname{ds}(z \mid m)\operatorname{sc}(z \mid m)}$$

09.33.27.0097.01

$$\operatorname{ns}(z \mid m) = \frac{\operatorname{cd}(z \mid m)\operatorname{ds}(z \mid m) + \operatorname{sc}(z \mid m)}{\operatorname{nc}(z \mid m)}$$

09.33.27.0098.01

$$\operatorname{ns}(z \mid m) = \frac{\left(\operatorname{cs}(z \mid m)^2 + 1\right)\operatorname{dn}(z \mid m)}{\operatorname{cd}(z \mid m)\,(\operatorname{cs}(z \mid m) - m\operatorname{sc}(z \mid m) + \operatorname{sc}(z \mid m))}$$

09.33.27.0099.01

$$\operatorname{ns}(z \mid m) = \frac{\operatorname{cd}(z \mid m)\,(\operatorname{cs}(z \mid m) - m\operatorname{sc}(z \mid m) + \operatorname{sc}(z \mid m))}{\operatorname{dn}(z \mid m)}$$

09.33.27.0100.01

$$\operatorname{ns}(z \mid m) = \frac{\operatorname{cn}(z \mid m)\operatorname{ds}(z \mid m) + \operatorname{dn}(z \mid m)\operatorname{sc}(z \mid m)}{\operatorname{ds}(z \mid m)\operatorname{sc}(z \mid m)}$$

09.33.27.0101.01

$$\operatorname{ns}(z \mid m) = -\frac{\operatorname{dn}(z \mid m)\,(\operatorname{cs}(z \mid m) + \operatorname{sc}(z \mid m))}{\operatorname{cd}(z \mid m)\left(m\operatorname{sc}(z \mid m)^2 - \operatorname{sc}(z \mid m)^2 - 1\right)}$$

09.33.27.0102.01

$$\operatorname{ns}(z \mid m) = \frac{m\operatorname{cd}(z \mid m) - \operatorname{dn}(z \mid m)\operatorname{nc}(z \mid m)}{(m-1)\operatorname{nc}(z \mid m)\operatorname{sd}(z \mid m)}$$

09.33.27.0103.01

$$\operatorname{ns}(z \mid m) = \frac{\operatorname{dn}(z \mid m)\,(\operatorname{cs}(z \mid m) + \operatorname{sc}(z \mid m))}{(\operatorname{cs}(z \mid m) - m\operatorname{sc}(z \mid m) + \operatorname{sc}(z \mid m))\operatorname{sd}(z \mid m)}$$

09.33.27.0104.01

$$\operatorname{ns}(z \mid m) = -\frac{\operatorname{dc}(z \mid m)\operatorname{sd}(z \mid m)}{\operatorname{cn}(z \mid m) - \operatorname{nc}(z \mid m)}$$

09.33.27.0105.01

$$\mathrm{ns}(z\mid m)=\frac{\mathrm{dc}(z\mid m)^2\,\mathrm{sd}(z\mid m)}{\mathrm{dc}(z\mid m)\,\mathrm{nc}(z\mid m)-\mathrm{dn}(z\mid m)}$$

09.33.27.0106.01

$$\mathrm{ns}(z\mid m)=-\frac{\mathrm{nc}(z\mid m)\,\mathrm{sd}(z\mid m)}{(\mathrm{cn}(z\mid m)-\mathrm{nc}(z\mid m))\,\mathrm{nd}(z\mid m)}$$

09.33.27.0107.01

$$\mathrm{ns}(z\mid m)=\frac{m\,\mathrm{cd}(z\mid m)\,\mathrm{sd}(z\mid m)}{\mathrm{cd}(z\mid m)\,\mathrm{nd}(z\mid m)-\mathrm{cn}(z\mid m)}$$

09.33.27.0108.01

$$\mathrm{ns}(z\mid m)=\frac{\mathrm{dc}(z\mid m)\,\mathrm{sd}(z\mid m)}{\mathrm{dc}(z\mid m)\,\mathrm{nd}(z\mid m)-\mathrm{cn}(z\mid m)}$$

09.33.27.0109.01

$$\mathrm{ns}(z\mid m)=\frac{\mathrm{dc}(z\mid m)^2\,\mathrm{sd}(z\mid m)}{\mathrm{dc}(z\mid m)^2\,\mathrm{nd}(z\mid m)-\mathrm{dn}(z\mid m)}$$

09.33.27.0110.01

$$\mathrm{ns}(z\mid m)=\frac{\mathrm{nc}(z\mid m)\,\mathrm{sd}(z\mid m)}{\mathrm{nc}(z\mid m)\,\mathrm{nd}(z\mid m)-\mathrm{cd}(z\mid m)}$$

09.33.27.0111.01

$$\mathrm{ns}(z\mid m)=\mathrm{cn}(z\mid m)\,(\mathrm{cs}(z\mid m)+\mathrm{dc}(z\mid m)\,\mathrm{sd}(z\mid m))$$

09.33.27.0112.01

$$\mathrm{ns}(z\mid m)=\frac{\mathrm{dn}(z\mid m)\,(\mathrm{cs}(z\mid m)+\mathrm{dc}(z\mid m)\,\mathrm{sd}(z\mid m))}{\mathrm{dc}(z\mid m)}$$

09.33.27.0113.01

$$\mathrm{ns}(z\mid m)=\frac{\mathrm{cd}(z\mid m)+\mathrm{sc}(z\mid m)\,\mathrm{sd}(z\mid m)}{\mathrm{nd}(z\mid m)\,\mathrm{sc}(z\mid m)}$$

09.33.27.0114.01

$$\mathrm{ns}(z\mid m)=\frac{\mathrm{cd}(z\mid m)+\mathrm{sc}(z\mid m)\,\mathrm{sd}(z\mid m)}{\mathrm{nc}(z\mid m)\,\mathrm{sd}(z\mid m)}$$

09.33.27.0115.01

$$\mathrm{ns}(z\mid m)=\frac{\mathrm{cn}(z\mid m)\,\mathrm{nd}(z\mid m)+\mathrm{sc}(z\mid m)\,\mathrm{sd}(z\mid m)}{\mathrm{nd}(z\mid m)\,\mathrm{sc}(z\mid m)}$$

09.33.27.0116.01

$$\mathrm{ns}(z\mid m)=\frac{\mathrm{dn}(z\mid m)\,\mathrm{nc}(z\mid m)+m\,\mathrm{sc}(z\mid m)\,\mathrm{sd}(z\mid m)}{\mathrm{nc}(z\mid m)\,\mathrm{sd}(z\mid m)}$$

09.33.27.0117.01

$$\mathrm{ns}(z\mid m)=\frac{\mathrm{cn}(z\mid m)+\mathrm{dn}(z\mid m)\,\mathrm{sc}(z\mid m)\,\mathrm{sd}(z\mid m)}{\mathrm{sc}(z\mid m)}$$

09.33.27.0118.01

$$\mathrm{ns}(z\mid m)=\frac{1}{\mathrm{sd}(z\mid m)}\left(\mathrm{dn}(z\mid m)\,\mathrm{sc}(z\mid m)^2+m\,\mathrm{nc}(z\mid m)\,\mathrm{sd}(z\mid m)\,\mathrm{sc}(z\mid m)-\mathrm{nc}(z\mid m)\,\mathrm{sd}(z\mid m)\,\mathrm{sc}(z\mid m)+\mathrm{dn}(z\mid m)\right)$$

09.33.27.0119.01

$$\operatorname{ns}(z \mid m) = \frac{\operatorname{dc}(z \mid m)\, \operatorname{sd}(z \mid m)^2 + \operatorname{cd}(z \mid m)}{\operatorname{nc}(z \mid m)\, \operatorname{sd}(z \mid m)}$$

09.33.27.0120.01

$$\operatorname{ns}(z \mid m) = \frac{\operatorname{dc}(z \mid m)\, \operatorname{dn}(z \mid m)\, \operatorname{sd}(z \mid m)^2 + \operatorname{cn}(z \mid m)}{\operatorname{dc}(z \mid m)\, \operatorname{sd}(z \mid m)}$$

09.33.27.0121.01

$$\operatorname{ns}(z \mid m) = \frac{(\operatorname{dc}(z \mid m) + \operatorname{cs}(z \mid m)\, \operatorname{ds}(z \mid m))\, \operatorname{sn}(z \mid m)}{\operatorname{dc}(z \mid m)}$$

09.33.27.0122.01

$$\operatorname{ns}(z \mid m) = \frac{\operatorname{dc}(z \mid m)\, \operatorname{nc}(z \mid m)\, \operatorname{sn}(z \mid m)}{\operatorname{dc}(z \mid m)\, \operatorname{nc}(z \mid m) - \operatorname{dn}(z \mid m)}$$

09.33.27.0123.01

$$\operatorname{ns}(z \mid m) = \frac{\operatorname{nc}(z \mid m)\, \operatorname{nd}(z \mid m)\, \operatorname{sn}(z \mid m)}{\operatorname{nc}(z \mid m)\, \operatorname{nd}(z \mid m) - \operatorname{cd}(z \mid m)}$$

09.33.27.0124.01

$$\operatorname{ns}(z \mid m) = \frac{(\operatorname{cs}(z \mid m) + \operatorname{dc}(z \mid m)\, \operatorname{sd}(z \mid m))\, \operatorname{sn}(z \mid m)}{\operatorname{dc}(z \mid m)\, \operatorname{sd}(z \mid m)}$$

09.33.27.0125.01

$$\operatorname{ns}(z \mid m) = \operatorname{cd}(z \mid m)\, \operatorname{cs}(z \mid m)\, \operatorname{dn}(z \mid m) + \operatorname{sn}(z \mid m)$$

09.33.27.0126.01

$$\operatorname{ns}(z \mid m) = \operatorname{dn}(z \mid m)\, \operatorname{ds}(z \mid m)\, \operatorname{cd}(z \mid m)^2 + \operatorname{sn}(z \mid m)$$

09.33.27.0127.01

$$\operatorname{ns}(z \mid m) = \frac{\operatorname{cs}(z \mid m)\, \operatorname{dn}(z \mid m) + \operatorname{dc}(z \mid m)\, \operatorname{sn}(z \mid m)}{\operatorname{dc}(z \mid m)}$$

09.33.27.0128.01

$$\operatorname{ns}(z \mid m) = \frac{\operatorname{cn}(z \mid m)\, \operatorname{ds}(z \mid m) + \operatorname{dc}(z \mid m)\, \operatorname{sn}(z \mid m)}{\operatorname{dc}(z \mid m)}$$

09.33.27.0129.01

$$\operatorname{ns}(z \mid m) = \frac{\operatorname{sn}(z \mid m)\, \operatorname{dc}(z \mid m)^2 + \operatorname{dn}(z \mid m)\, \operatorname{ds}(z \mid m)}{\operatorname{dc}(z \mid m)^2}$$

09.33.27.0130.01

$$\operatorname{ns}(z \mid m) = \frac{\operatorname{cd}(z \mid m)\, \operatorname{ds}(z \mid m) + \operatorname{nc}(z \mid m)\, \operatorname{sn}(z \mid m)}{\operatorname{nc}(z \mid m)}$$

09.33.27.0131.01

$$\operatorname{ns}(z \mid m) = \frac{\operatorname{cd}(z \mid m)\, \operatorname{cs}(z \mid m) + \operatorname{nd}(z \mid m)\, \operatorname{sn}(z \mid m)}{\operatorname{nd}(z \mid m)}$$

09.33.27.0132.01

$$\operatorname{ns}(z \mid m) = \frac{\operatorname{cs}(z \mid m) + \operatorname{dc}(z \mid m)\, \operatorname{nd}(z \mid m)\, \operatorname{sn}(z \mid m)}{\operatorname{dc}(z \mid m)\, \operatorname{nd}(z \mid m)}$$

09.33.27.0133.01

$$\operatorname{ns}(z \mid m) = \frac{\operatorname{cn}(z \mid m) + m \operatorname{cd}(z \mid m) \operatorname{sd}(z \mid m) \operatorname{sn}(z \mid m)}{\operatorname{cd}(z \mid m) \operatorname{sd}(z \mid m)}$$

09.33.27.0134.01

$$\operatorname{ns}(z \mid m) = \frac{\operatorname{cn}(z \mid m) + \operatorname{dc}(z \mid m) \operatorname{sd}(z \mid m) \operatorname{sn}(z \mid m)}{\operatorname{dc}(z \mid m) \operatorname{sd}(z \mid m)}$$

Involving five other Jacobi elliptic functions

09.33.27.0135.01

$$\operatorname{ns}(z \mid m) = \frac{(m \operatorname{cd}(z \mid m) + \operatorname{nc}(z \mid m) \operatorname{nd}(z \mid m)) \operatorname{sd}(z \mid m)}{\operatorname{nc}(z \mid m) \operatorname{nd}(z \mid m)^2 - \operatorname{cn}(z \mid m)}$$

09.33.27.0136.01

$$\operatorname{ns}(z \mid m) = \frac{m \operatorname{cd}(z \mid m) \operatorname{sd}(z \mid m) + \operatorname{nc}(z \mid m) \operatorname{nd}(z \mid m) \operatorname{sd}(z \mid m) - \operatorname{nc}(z \mid m) \operatorname{sn}(z \mid m)}{\operatorname{nc}(z \mid m) (\operatorname{nd}(z \mid m) - 1) (\operatorname{nd}(z \mid m) + 1)}$$

Involving Weierstrass functions

09.33.27.0022.01

$$\operatorname{ns}(z \mid m) = \frac{1}{\sqrt{e_1 - e_3}} \frac{\sigma_3\left(\frac{z}{\sqrt{e_1-e_3}}; g_2, g_3\right)}{\sigma\left(\frac{z}{\sqrt{e_1-e_3}}; g_2, g_3\right)} /;$$

$$\{\omega_1, \omega_2, \omega_3\} = \{\omega_1(g_2, g_3), -\omega_1(g_2, g_3) - \omega_3(g_2, g_3), \omega_3(g_2, g_3)\} \bigwedge m = \lambda\left(\frac{\omega_3}{\omega_1}\right) \bigwedge e_n = \wp(\omega_n; g_2, g_3) \bigwedge n \in \{1, 2, 3\}$$

09.33.27.0023.01

$$\operatorname{ns}(z \mid m)^2 = \frac{\wp\left(\frac{z}{\sqrt{e_1-e_3}}; g_2, g_3\right) - e_3}{e_1 - e_3} /;$$

$$\{\omega_1, \omega_2, \omega_3\} = \{\omega_1(g_2, g_3), -\omega_1(g_2, g_3) - \omega_3(g_2, g_3), \omega_3(g_2, g_3)\} \bigwedge m = \lambda\left(\frac{\omega_3}{\omega_1}\right) \bigwedge e_n = \wp(\omega_n; g_2, g_3) \bigwedge n \in \{1, 2, 3\}$$

09.33.27.0024.01

$$\operatorname{ns}\left(z \,\middle|\, \frac{e_2 - e_3}{e_1 - e_3}\right) = \sqrt{\frac{\wp\left(\frac{z}{\sqrt{e_1-e_3}}; g_2, g_3\right) - e_3}{e_1 - e_3}} /;$$

$$\{\omega_1, \omega_2, \omega_3\} = \{\omega_1(g_2, g_3), -\omega_1(g_2, g_3) - \omega_3(g_2, g_3), \omega_3(g_2, g_3)\} \land e_n = \wp(\omega_n; g_2, g_3) \land n \in \{1, 2, 3\}$$

Involving theta functions

09.33.27.0025.01

$$\operatorname{ns}(z \mid m) = \sqrt[4]{m} \frac{\vartheta_4\left(\frac{\pi z}{2 K(m)}, q(m)\right)}{\vartheta_1\left(\frac{\pi z}{2 K(m)}, q(m)\right)}$$

09.33.27.0026.01

$$\operatorname{ns}(z \mid m) = \frac{\vartheta_2(0, q(m))}{\vartheta_3(0, q(m))} \frac{\vartheta_4\left(\frac{z}{\vartheta_3(0,q(m))^2}, q(m)\right)}{\vartheta_1\left(\frac{z}{\vartheta_3(0,q(m))^2}, q(m)\right)}$$

09.33.27.0027.01

$$\operatorname{ns}(z \mid m) = \frac{\vartheta_n(z \mid m)}{\vartheta_s(z \mid m)}$$

Zeros

09.33.30.0001.01

$$\operatorname{ns}((2 s + 1) i K(1 - m) + 2 r K(m) \mid m) = 0 \;/;\; \{r, s\} \in \mathbb{Z}$$

History

– C. G. J. Jacobi (1827)

– N. H. Abel (1827)

– J. Glaisher (1882) introduced the notation ns

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