

Khinchin

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Notations

Traditional name

Khinchin constant

Traditional notation

K

Mathematica StandardForm notation

Khinchin

Primary definition

02.09.02.0001.01

$$K = \prod_{k=1}^{\infty} \left(1 + \frac{1}{k(k+2)} \right)^{\log_2(k)}$$

Specific values

02.09.03.0001.01

$K = 2.68545200106530644530971483548179569382038229399446295305115234555721885953715200280114117 \dots$

Above approximate numerical value of K shows 90 decimal digits.

General characteristics

The Khinchine's number K is a constant. It is a positive real number.

Series representations

Generalized power series

02.09.06.0001.01

$$K = \exp \left(\frac{1}{\log(2)} \sum_{k=2}^{\infty} \log \left(\frac{k}{k-1} \right) \log \left(\frac{k+1}{k} \right) \right)$$

02.09.06.0002.01

$$K = \exp\left(\log(2) + \frac{1}{2\log(2)} \sum_{k=1}^{\infty} \frac{1}{k} \left(\psi\left(k + \frac{1}{2}\right) - \psi(k)\right) (\zeta(2k) - 1)\right)$$

02.09.06.0003.01

$$K = \exp\left(\frac{1}{\log(2)} \sum_{k=2}^{\infty} \frac{(-1)^k (2 - 2^k)}{k} \zeta'(k)\right)$$

02.09.06.0004.01

$$K = \exp\left(\frac{1}{\log(2)} \left(\log^2(2) + \text{Li}_2\left(-\frac{1}{2}\right) + \frac{1}{2} \sum_{k=2}^{\infty} (-1)^k \text{Li}_2\left(\frac{4}{k^2}\right)\right)\right)$$

02.09.06.0005.01

$$K = \exp\left(\frac{1}{\log(2)} \left(\frac{\pi^2}{6} - \frac{1}{2} \log^2(2) + \sum_{k=2}^{\infty} \text{Li}_2\left(-\frac{1}{k^2 - 1}\right)\right)\right)$$

02.09.06.0006.01

$$K = \exp\left(\frac{1}{\log(2)} \left(\sum_{k=1}^{\infty} \frac{\zeta(2k, n+1)}{k} \left(\log(2) + \frac{1}{2} \left(\psi\left(k + \frac{1}{2}\right) - \psi(k)\right)\right) - \sum_{k=2}^n \log\left(1 - \frac{1}{k}\right) \log\left(1 + \frac{1}{k}\right)\right)\right); n \in \mathbb{N}^+$$

02.09.06.0007.01

$$K = \exp\left(\frac{2}{\log(2)} \sum_{k=0}^{\infty} (-1)^k \left(\frac{(2^{k+1} - 1) \log(k+1) (k+1)^{-k-2}}{k+2} - \frac{(k+1)^{-k-3} \log(k+1)}{k+3}\right)\right. \\ \left. \left(2^{k+2} {}_2F_1\left(1, k+3; k+4; -\frac{2}{k+1}\right) - {}_2F_1\left(1, k+3; k+4; -\frac{1}{k+1}\right)\right) - \frac{2^{k+1} - 1}{k+2} \zeta^{(1,0)}(k+2, k+2)\right)$$

02.09.06.0008.01

$$K = \exp\left(\frac{1}{\log(2)} \sum_{k=1}^{\infty} \frac{\zeta(2k) - 1}{k} \sum_{j=1}^{2k-1} \frac{(-1)^{j-1}}{j}\right)$$

Integral representations

On the real axis

Of the direct function

02.09.07.0001.01

$$K = \exp\left(\frac{\log^2(2)}{2} + \frac{1}{\log(2)} \int_0^{\pi} \frac{\log(t |\cot(t)|)}{t} dt + \frac{\pi^2}{12 \log(2)}\right)$$

02.09.07.0002.01

$$K = \exp\left(\frac{1}{\log(2)} \int_1^{\infty} \frac{\log(\lfloor t \rfloor)}{t(t+1)} dt\right)$$

02.09.07.0003.01

$$K = 2 \exp\left(\frac{1}{\log(2)} \int_0^1 \frac{1}{t(t+1)} \log\left(\frac{\pi t(1-t^2)}{\sin(\pi t)}\right) dt\right)$$

$$02.09.07.0005.01$$

$$K = 2 \exp\left(\frac{1}{\log(2)} \int_0^1 \frac{1}{t(t+1)} \log(\Gamma(2+t)\Gamma(2-t)) dt\right)$$

$$02.09.07.0004.01$$

$$K = \exp\left(\frac{1}{\log(2)} \int_0^1 \frac{1}{t+1} \log\left(\left[\frac{1}{t}\right]\right) dt\right)$$

Limit representations

$$02.09.09.0001.01$$

$$K = \lim_{n \rightarrow \infty} e^{\frac{1}{\log(2)} \sum_{m=1}^n \log_d(10)^{m+1} \frac{1}{m} (\zeta(2m-1) \sum_{k=1}^{2m-1} \frac{(-1)^{k+1}}{k}}$$

The above formula is used for the numerical computation of Khinchin's constant in *Mathematica*.

Complex characteristics

Real part

$$02.09.19.0001.01$$

$$\operatorname{Re}(K) = K$$

Imaginary part

$$02.09.19.0002.01$$

$$\operatorname{Im}(K) = 0$$

Absolute value

$$02.09.19.0003.01$$

$$|K| = K$$

Argument

$$02.09.19.0004.01$$

$$\arg(K) = 0$$

Conjugate value

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$$\bar{K} = K$$

Signum value

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$$\operatorname{sgn}(K) = 1$$

Differentiation

Low-order differentiation

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$$\frac{\partial K}{\partial z} = 0$$

Fractional integro-differentiation

02.09.20.0002.01

$$\frac{\partial^\alpha K}{\partial z^\alpha} = \frac{z^{-\alpha} K}{\Gamma(1-\alpha)}$$

Integration

Indefinite integration

02.09.21.0001.01

$$\int K dz = K z$$

02.09.21.0002.01

$$\int z^{\alpha-1} K dz = \frac{z^\alpha K}{\alpha}$$

Integral transforms

Fourier exp transforms

02.09.22.0001.01

$$\mathcal{F}_i[K](z) = \sqrt{2\pi} K \delta(z)$$

Inverse Fourier exp transforms

02.09.22.0002.01

$$\mathcal{F}_i^{-1}[K](z) = \sqrt{2\pi} K \delta(z)$$

Fourier cos transforms

02.09.22.0003.01

$$\mathcal{F}_c[K](z) = \sqrt{\frac{\pi}{2}} K \delta(z)$$

Fourier sin transforms

02.09.22.0004.01

$$\mathcal{F}_s[K](z) = \sqrt{\frac{2}{\pi}} \frac{K}{z}$$

Laplace transforms

02.09.22.0005.01

$$\mathcal{L}_i[K](z) = \frac{K}{z}$$

Inverse Laplace transforms

02.09.22.0006.01

$$\mathcal{L}_t^{-1}[K](z) = K \delta(z)$$

Inequalities

02.09.29.0001.01

$$\frac{53}{20} < K < \frac{27}{10}$$

Theorems

The continued fraction mean theorem

For almost all $x \in \mathbb{R}$:

$$\lim_{n \rightarrow \infty} \left(\prod_{k=0}^n q_k \right)^{1/n} = K \quad ; \quad x = q_0 + \frac{1}{q_1 + \frac{1}{q_2 + \frac{1}{q_3 + \dots}}} \wedge q_k \in \mathbb{N}^+.$$

This relation fails for $x = e$, rational numbers, solutions of quadratic equations with rational coefficients and quadratic irrationals, such as ϕ , $\sqrt{2}$, $\sqrt{3}$.

Numerical verifications show that this relation can be valid for $x = \pi$, $x = \gamma$, and $x = K$. But it was not accurately proved.

So, K is the limit of the geometric mean of the first n partial quotients of the simple continued fraction for almost any real number x , when n tends to infinity. This limit exists and is a Khinchin constant independent from x for almost all numbers.

History

– A. Khinchin (1934)

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