

LCM

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Notations

Traditional name

Least common multiple

Traditional notation

$\text{lcm}(n_1, n_2, \dots, n_m)$

Mathematica StandardForm notation

$\text{LCM}[n_1, n_2, \dots, n_m]$

Primary definition

04.10.02.0001.01

$$\text{lcm}(n_1, n_2, \dots, n_m) = p \text{ ; } p \in \mathbb{N}^+ \wedge \frac{p}{n_k} \in \mathbb{Z} \wedge 1 \leq k \leq m \wedge \left(\neg \exists q < p \wedge p \in \mathbb{Z} \wedge \frac{q}{n_k} \in \mathbb{Z} \wedge 1 \leq k \leq m \right)$$

04.10.02.0002.01

$$\text{lcm}(n_1, n_2, \dots, n_m) = p \text{ ; } \text{Re}(p) \in \mathbb{Z} \wedge \text{Im}(p) \in \mathbb{Z} \wedge \text{Re}\left(\frac{p}{n_k}\right) \in \mathbb{Z} \wedge \text{Im}\left(\frac{p}{n_k}\right) \in \mathbb{Z} \wedge 1 \leq k \leq m \wedge$$

$$\left(\neg \exists q (\text{RAbs}(q) < |p| \wedge \text{Re}(q) \in \mathbb{Z} \wedge \text{Im}(q) \in \mathbb{Z}) \wedge \text{Re}\left(\frac{q}{n_k}\right) \in \mathbb{Z} \wedge \text{Im}\left(\frac{q}{n_k}\right) \in \mathbb{Z} \wedge 1 \leq k \leq m \right)$$

$\text{lcm}(n_1, n_2, \dots, n_m)$ is the least common multiple of the integers (or rational) n_k . It is the minimal positive integer which divides to all n_k , $1 \leq k \leq m$.

For complex values n_k with rational $\text{Re}(n_k)$ and $\text{Im}(n_k)$ the function $\text{lcm}(n_1, n_2, \dots, n_m)$ is also defined as shown above.

Examples: The least common multiple $\text{lcm}(2, 4, 5)$ is 20; similar, other examples are $\text{lcm}(27, 48, 36) = 432$, $\text{lcm}(27 + 3i, 48 - 6i) = 222 + 216i$, $\text{lcm}\left(\frac{2}{3}, \frac{3}{4}\right) = 6$.

Specific values

Specialized values

04.10.03.0001.01

$\text{lcm}(n) = |n|$

04.10.03.0028.01

$$\text{lcm}(0, n) = 0$$

04.10.03.0002.01

$$\text{lcm}(n, n) = |n|$$

04.10.03.0003.01

$$\text{lcm}(n, -n) = |n|$$

04.10.03.0004.01

$$\text{lcm}(n_1, n_1, \dots, n_1) = |n_1|$$

04.10.03.0005.01

$$\text{lcm}(p_1, p_2) = p_1 p_2 \text{ ; } p_1 \neq p_2 \wedge p_1 \in \mathbb{P} \wedge p_2 \in \mathbb{P}$$

04.10.03.0006.01

$$\text{lcm}(n, \text{gcd}(m, n)) = n \text{ ; } m \in \mathbb{N}^+ \wedge n \in \mathbb{N}^+$$

04.10.03.0029.01

$$\text{lcm}(n, \text{gcd}(p, q)) = \text{gcd}(\text{lcm}(n, p), \text{lcm}(n, q)) \text{ ; } n \in \mathbb{N}^+ \wedge p \in \mathbb{N}^+ \wedge q \in \mathbb{N}^+$$

04.10.03.0007.01

$$\text{lcm}(\text{gcd}(n, p), \text{gcd}(n, q)) = \text{gcd}(n, \text{lcm}(p, q)) \text{ ; } n \in \mathbb{N}^+ \wedge p \in \mathbb{N}^+ \wedge q \in \mathbb{N}^+$$

04.10.03.0008.01

$$\text{lcm}(\text{gcd}(k, m), \text{gcd}(k, n), \text{gcd}(m, n)) = \text{gcd}(\text{lcm}(k, m), \text{lcm}(k, n), \text{lcm}(m, n)) \text{ ; } k \in \mathbb{N}^+ \wedge m \in \mathbb{N}^+ \wedge n \in \mathbb{N}^+$$

Values at fixed points

04.10.03.0030.01

$$\text{lcm}(0, 0) = 0$$

04.10.03.0009.01

$$\text{lcm}(1, 1) = 1$$

04.10.03.0010.01

$$\text{lcm}(1, 2) = 2$$

04.10.03.0011.01

$$\text{lcm}(2, 2) = 2$$

04.10.03.0012.01

$$\text{lcm}(3, 2) = 6$$

04.10.03.0013.01

$$\text{lcm}(4, 2) = 4$$

04.10.03.0014.01

$$\text{lcm}(1, 3) = 3$$

04.10.03.0015.01

$$\text{lcm}(2, 3) = 6$$

04.10.03.0016.01

$$\text{lcm}(3, 3) = 3$$

04.10.03.0017.01

$$\text{lcm}(4, 3) = 12$$

04.10.03.0018.01

$$\text{lcm}(5, 3) = 15$$

04.10.03.0019.01

$$\text{lcm}(6, 3) = 6$$

04.10.03.0020.01

$$\text{lcm}(4, 6) = 12$$

04.10.03.0021.01

$$\text{lcm}(36, 45) = 180$$

04.10.03.0022.01

$$\text{lcm}(-36, 45) = 180$$

04.10.03.0023.01

$$\text{lcm}(36, -45) = 180$$

04.10.03.0024.01

$$\text{lcm}(-36, -45) = 180$$

04.10.03.0025.01

$$\text{lcm}(-45, -36) = 9$$

04.10.03.0026.01

$$\text{lcm}(30, 15, 5) = 30$$

04.10.03.0027.01

$$\text{lcm}(2, 3, 4, 5) = 60$$

General characteristics

Domain and analyticity

$\text{lcm}(n_1, n_2, \dots, n_m)$ is nonanalytical function defined on \mathbb{Z}^m with values from \mathbb{Z} .

04.10.04.0001.01

$$(n_1 * n_2 * \dots * n_m) \rightarrow \text{lcm}(n_1, n_2, \dots, n_m) :: \mathbb{Z}^m \rightarrow \mathbb{Z}$$

Symmetries and periodicities

Parity

$\text{lcm}(n_1, n_2, \dots, n_m)$ is an even function.

04.10.04.0002.01

$$\text{lcm}(-n_1, -n_2, \dots, -n_m) = \text{lcm}(n_1, n_2, \dots, n_m)$$

04.10.04.0003.01

$$\text{lcm}(-n_1, n_2, \dots, n_m) = \text{lcm}(n_1, n_2, \dots, n_m)$$

Permutation symmetry

04.10.04.0004.01

$$\text{lcm}(m, n) = \text{lcm}(n, m)$$

04.10.04.0005.01

$$\text{lcm}(n_1, n_2, \dots, n_k, \dots, n_j, \dots, n_m) = \text{lcm}(n_1, n_2, \dots, n_j, \dots, n_k, \dots, n_m) /; n_k \neq n_j \wedge k \neq j$$

Periodicity

No periodicity

Product representations

04.10.08.0001.01

$$\text{lcm}(n_1, n_2) = \prod_{j=1}^{j_k} p_{i,j}^{\max(\alpha_{1,j}, \alpha_{2,j})};$$

$$n_1 \in \mathbb{N}^+ \wedge n_2 \in \mathbb{N}^+ \wedge \text{factors}(n_k) = \{ \{p_{k,1}, \alpha_{k,1}\}, \dots, \{p_{k,j_k}, \alpha_{k,j_k}\} \} \wedge p_{k,j} \in \mathbb{P} \wedge \alpha_{k,j} \in \mathbb{N}^+ \wedge 1 \leq k \leq 2$$

Transformations

Transformations and argument simplifications

04.10.16.0001.01

$$\text{lcm}(-n_1, -n_2, \dots, -n_m) = \text{lcm}(n_1, n_2, \dots, n_m)$$

04.10.16.0002.01

$$\text{lcm}(-n_1, n_2, \dots, n_m) = \text{lcm}(n_1, n_2, \dots, n_m)$$

Multiple arguments

04.10.16.0003.02

$$\text{lcm}(p n_1, p n_2, \dots, p n_m) = p \text{lcm}(n_1, n_2, \dots, n_m); p \in \mathbb{N}$$

Identities

Functional identities

04.10.17.0001.01

$$\text{lcm}(\text{lcm}(m, n), p) = \text{lcm}(m, \text{lcm}(n, p))$$

04.10.17.0002.01

$$\text{lcm}(n_1, \text{lcm}(n_2, n_3, \dots, n_m)) = \text{lcm}(n_1, n_2, n_3, \dots, n_m)$$

04.10.17.0003.01

$$\text{lcm}(m, n, p) = \text{lcm}(m, \text{lcm}(n, p))$$

04.10.17.0004.01

$$\text{lcm}(n_1, n_2, n_3, \dots, n_m) = \text{lcm}(n_1, \text{lcm}(n_2, n_3, \dots, n_m))$$

Least common multiple of sequences

04.10.17.0005.01

$$\lim_{n \rightarrow \infty} \frac{\log(\text{lcm}(h+k, h+2k, \dots, h+nk))}{n} = \frac{k}{\phi(k)} \sum_{\substack{m=1 \\ \text{gcd}(m,k)=1}}^k \frac{1}{m}$$

Representations through equivalent functions

With related functions

04.10.27.0001.01

$$\text{lcm}(m, n) = \frac{m n}{\text{gcd}(m, n)} \quad ; m \in \mathbb{N}^+ \wedge n \in \mathbb{N}^+$$

04.10.27.0002.01

$$\text{lcm}(n_1, n_2) = \prod_{j=1}^{j_k} p_{i,j}^{\max(\alpha_{1,j}, \alpha_{2,j})} \quad ;$$

$$n_1 \in \mathbb{N}^+ \wedge n_2 \in \mathbb{N}^+ \wedge \text{factors}(n_k) = \{ \{p_{k,1}, \alpha_{k,1}\}, \dots, \{p_{k,j_k}, \alpha_{k,j_k}\} \} \wedge p_{k,j} \in \mathbb{P} \wedge \alpha_{k,j} \in \mathbb{N}^+ \wedge 1 \leq k \leq 2$$

04.10.27.0003.01

$$\text{gcd}(n_1, n_2, \dots, n_m) =$$

$$\left(\prod_{k_1=1}^m n_{k_1} \prod_{k_1=1}^m \prod_{k_2=k_1+1}^m \prod_{k_3=k_2+1}^m \text{lcm}(n_{k_1}, n_{k_2}, n_{k_3}) \dots \right) / \left(\prod_{k_1=1}^m \prod_{k_2=k_1+1}^m \text{lcm}(n_{k_1}, n_{k_2}) \prod_{k_1=1}^m \prod_{k_2=k_1+1}^m \prod_{k_3=k_2+1}^m \prod_{k_4=k_3+1}^m \text{lcm}(n_{k_1}, n_{k_2}, n_{k_3}, n_{k_4}) \dots \right)$$

04.10.27.0004.01

$$\text{lcm}(n, m, k) \text{gcd}(n m, m k, k n) = n m k \quad ; \{n, m, k\} \in \mathbb{Z}$$

04.10.27.0005.01

$$\text{gcd}(\text{lcm}(k, m), \text{lcm}(k, n), \text{lcm}(m, n)) = \text{lcm}(\text{gcd}(k, m), \text{gcd}(k, n), \text{gcd}(m, n)) \quad ; \{n, m, k\} \in \mathbb{Z}$$

Inequalities

04.10.29.0001.01

$$\text{lcm}(1, 2, \dots, n) \geq 2^{n-2} \quad ; n \in \mathbb{N}^+$$

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