LCM

Notations

Traditional name

Least common multiple

Traditional notation

$\operatorname{lcm}(n_1, n_2, \ldots, n_m)$

Mathematica StandardForm notation

`LCM[`n_1`, `n_2`, …, `n_m`]`

Primary definition

04.10.02.0001.01

$$\operatorname{lcm}(n_1, n_2, \ldots, n_m) = p \;/;\; p \in \mathbb{N}^+ \bigwedge \frac{p}{n_k} \in \mathbb{Z} \bigwedge 1 \le k \le m \bigwedge \left(\neg \exists_q\, q < p \bigwedge p \in \mathbb{Z} \bigwedge \frac{q}{n_k} \in \mathbb{Z} \bigwedge 1 \le k \le m\right)$$

04.10.02.0002.01

$$\operatorname{lcm}(n_1, n_2, \ldots, n_m) = p \;/;\; \operatorname{Re}(p) \in \mathbb{Z} \bigwedge \operatorname{Im}(p) \in \mathbb{Z} \bigwedge \operatorname{Re}\left(\frac{p}{n_k}\right) \in \mathbb{Z} \bigwedge \operatorname{Im}\left(\frac{p}{n_k}\right) \in \mathbb{Z} \bigwedge 1 \le k \le m \bigwedge$$
$$\left(\neg \exists_q\, (\operatorname{RAbs}(q) < |p| \land \operatorname{Re}(q) \in \mathbb{Z} \land \operatorname{Im}(q) \in \mathbb{Z}) \bigwedge \operatorname{Re}\left(\frac{q}{n_k}\right) \in \mathbb{Z} \bigwedge \operatorname{Im}\left(\frac{q}{n_k}\right) \in \mathbb{Z} \bigwedge 1 \le k \le m\right)$$

$\operatorname{lcm}(n_1, n_2, \ldots, n_m)$ is the least common multiple of the integers (or rational) n_k. It is the minimal positive integer which divides to all n_k, $1 \le k \le m$.

For complex values $n_k$with rational $\operatorname{Re}(n_k)$and $\operatorname{Im}(n_k)$ the function $\operatorname{lcm}(n_1, n_2, \ldots, n_m)$ is also defined as shown above.

Examples: The least common multiple $\operatorname{lcm}(2, 4, 5)$ is 20; similar, other examples are $\operatorname{lcm}(27, 48, 36) = 432$, $\operatorname{lcm}(27 + 3 i, 48 - 6 i) = 222 + 216 i$, $\operatorname{lcm}\left(\frac{2}{3}, \frac{3}{4}\right) = 6$.

Specific values

Specialized values

04.10.03.0001.01

$$\operatorname{lcm}(n) = |n|$$

04.10.03.0028.01

$\operatorname{lcm}(0, n) == 0$

04.10.03.0002.01

$\operatorname{lcm}(n, n) == |n|$

04.10.03.0003.01

$\operatorname{lcm}(n, -n) == |n|$

04.10.03.0004.01

$\operatorname{lcm}(n_1, n_1, \ldots, n_1) == |n_1|$

04.10.03.0005.01

$\operatorname{lcm}(p_1, p_2) == p_1\, p_2 \;/;\; p_1 \neq p_2 \bigwedge p_1 \in \mathbb{P} \bigwedge p_2 \in \mathbb{P}$

04.10.03.0006.01

$\operatorname{lcm}(n, \gcd(m, n)) == n \;/;\; m \in \mathbb{N}^+ \bigwedge n \in \mathbb{N}^+$

04.10.03.0029.01

$\operatorname{lcm}(n, \gcd(p, q)) == \gcd(\operatorname{lcm}(n, p), \operatorname{lcm}(n, q)) \;/;\; n \in \mathbb{N}^+ \bigwedge p \in \mathbb{N}^+ \bigwedge q \in \mathbb{N}^+$

04.10.03.0007.01

$\operatorname{lcm}(\gcd(n, p), \gcd(n, q)) == \gcd(n, \operatorname{lcm}(p, q)) \;/;\; n \in \mathbb{N}^+ \bigwedge p \in \mathbb{N}^+ \bigwedge q \in \mathbb{N}^+$

04.10.03.0008.01

$\operatorname{lcm}(\gcd(k, m), \gcd(k, n), \gcd(m, n)) == \gcd(\operatorname{lcm}(k, m), \operatorname{lcm}(k, n), \operatorname{lcm}(m, n)) \;/;\; k \in \mathbb{N}^+ \bigwedge m \in \mathbb{N}^+ \bigwedge n \in \mathbb{N}^+$

Values at fixed points

04.10.03.0030.01

$\operatorname{lcm}(0, 0) == 0$

04.10.03.0009.01

$\operatorname{lcm}(1, 1) == 1$

04.10.03.0010.01

$\operatorname{lcm}(1, 2) == 2$

04.10.03.0011.01

$\operatorname{lcm}(2, 2) == 2$

04.10.03.0012.01

$\operatorname{lcm}(3, 2) == 6$

04.10.03.0013.01

$\operatorname{lcm}(4, 2) == 4$

04.10.03.0014.01

$\operatorname{lcm}(1, 3) == 3$

04.10.03.0015.01

$\operatorname{lcm}(2, 3) == 6$

04.10.03.0016.01

$\operatorname{lcm}(3, 3) == 3$

04.10.03.0017.01

$\operatorname{lcm}(4, 3) == 12$

04.10.03.0018.01

$\operatorname{lcm}(5, 3) == 15$

04.10.03.0019.01

$\operatorname{lcm}(6, 3) = 6$

04.10.03.0020.01

$\operatorname{lcm}(4, 6) = 12$

04.10.03.0021.01

$\operatorname{lcm}(36, 45) = 180$

04.10.03.0022.01

$\operatorname{lcm}(-36, 45) = 180$

04.10.03.0023.01

$\operatorname{lcm}(36, -45) = 180$

04.10.03.0024.01

$\operatorname{lcm}(-36, -45) = 180$

04.10.03.0025.01

$\operatorname{lcm}(-45, -36) = 9$

04.10.03.0026.01

$\operatorname{lcm}(30, 15, 5) = 30$

04.10.03.0027.01

$\operatorname{lcm}(2, 3, 4, 5) = 60$

General characteristics

Domain and analyticity

$\operatorname{lcm}(n_1, n_2, \ldots, n_m)$ is nonanalytical function defined on $\mathbb{Z}^m$with values from $\mathbb{Z}$.

04.10.04.0001.01

$(n_1 * n_2 * \ldots * n_m) \longrightarrow \operatorname{lcm}(n_1, n_2, \ldots, n_m) :: \mathbb{Z}^m \longrightarrow \mathbb{Z}$

Symmetries and periodicities

Parity

$\operatorname{lcm}(n_1, n_2, \ldots, n_m)$ is an even function.

04.10.04.0002.01

$\operatorname{lcm}(-n_1, -n_2, \ldots, -n_m) = \operatorname{lcm}(n_1, n_2, \ldots, n_m)$

04.10.04.0003.01

$\operatorname{lcm}(-n_1, n_2, \ldots, n_m) = \operatorname{lcm}(n_1, n_2, \ldots, n_m)$

Permutation symmetry

04.10.04.0004.01

$\operatorname{lcm}(m, n) = \operatorname{lcm}(n, m)$

04.10.04.0005.01

$\operatorname{lcm}(n_1, n_2, \ldots, n_k, \ldots, n_j, \ldots, n_m) = \operatorname{lcm}(n_1, n_2, \ldots, n_j, \ldots, n_k, \ldots, n_m) /; n_k \neq n_j \land k \neq j$

Periodicity

No periodicity

Product representations

04.10.08.0001.01

$$\operatorname{lcm}(n_1,\,n_2) = \prod_{j=1}^{j_k} p_{i,j}^{\max(\alpha_{1,j},\alpha_{2,j})} \;/;$$

$$n_1 \in \mathbb{N}^+ \bigwedge n_2 \in \mathbb{N}^+ \bigwedge \operatorname{factors}(n_k) = \left\{\{p_{k,1},\,\alpha_{k,1}\},\,\ldots,\,\left\{p_{k,j_k},\,\alpha_{k,j_k}\right\}\right\} \bigwedge p_{k,j} \in \mathbb{P} \bigwedge \alpha_{k,j} \in \mathbb{N}^+ \bigwedge 1 \le k \le 2$$

Transformations

Transformations and argument simplifications

04.10.16.0001.01

$$\operatorname{lcm}(-n_1,\,-n_2,\,\ldots,\,-n_m) = \operatorname{lcm}(n_1,\,n_2,\,\ldots,\,n_m)$$

04.10.16.0002.01

$$\operatorname{lcm}(-n_1,\,n_2,\,\ldots,\,n_m) = \operatorname{lcm}(n_1,\,n_2,\,\ldots,\,n_m)$$

Multiple arguments

04.10.16.0003.02

$$\operatorname{lcm}(p\,n_1,\,p\,n_2,\,\ldots,\,p\,n_m) = p\operatorname{lcm}(n_1,\,n_2,\,\ldots,\,n_m) \;/;\; p \in \mathbb{N}$$

Identities

Functional identities

04.10.17.0001.01

$$\operatorname{lcm}(\operatorname{lcm}(m,\,n),\,p) = \operatorname{lcm}(m,\,\operatorname{lcm}(n,\,p))$$

04.10.17.0002.01

$$\operatorname{lcm}(n_1,\,\operatorname{lcm}(n_2,\,n_3,\,\ldots,\,n_m)) = \operatorname{lcm}(n_1,\,n_2,\,n_3,\,\ldots,\,n_m)$$

04.10.17.0003.01

$$\operatorname{lcm}(m,\,n,\,p) = \operatorname{lcm}(m,\,\operatorname{lcm}(n,\,p))$$

04.10.17.0004.01

$$\operatorname{lcm}(n_1,\,n_2,\,n_3,\,\ldots,\,n_m) = \operatorname{lcm}(n_1,\,\operatorname{lcm}(n_2,\,n_3,\,\ldots,\,n_m))$$

Least common multiple of sequences

04.10.17.0005.01

$$\lim_{n\to\infty} \frac{\log(\operatorname{lcm}(h+k,\,h+2\,k,\,\ldots,\,h+n\,k))}{n} = \frac{k}{\phi(k)} \sum_{\substack{m=1 \\ \gcd(m,k)=1}}^{k} \frac{1}{m}$$

Representations through equivalent functions

With related functions

04.10.27.0001.01

$$\operatorname{lcm}(m, n) = \frac{m\,n}{\gcd(m, n)} \;/;\; m \in \mathbb{N}^+ \bigwedge n \in \mathbb{N}^+$$

04.10.27.0002.01

$$\operatorname{lcm}(n_1, n_2) = \prod_{j=1}^{j_k} p_{i,j}^{\max(\alpha_{1,j},\alpha_{2,j})} \;/;$$

$$n_1 \in \mathbb{N}^+ \bigwedge n_2 \in \mathbb{N}^+ \bigwedge \operatorname{factors}(n_k) = \{\{p_{k,1}, \alpha_{k,1}\}, \ldots, \{p_{k,j_k}, \alpha_{k,j_k}\}\} \bigwedge p_{k,j} \in \mathbb{P} \bigwedge \alpha_{k,j} \in \mathbb{N}^+ \bigwedge 1 \le k \le 2$$

04.10.27.0003.01

$$\gcd(n_1, n_2, \ldots, n_m) =$$

$$\left(\prod_{k_1=1}^{m} n_{k_1} \prod_{k_1=1}^{m} \prod_{k_2=k_1+1}^{m} \prod_{k_3=k_2+1}^{m} \operatorname{lcm}(n_{k_1}, n_{k_2}, n_{k_3}) \ldots\right) \Big/ \left(\prod_{k_1=1}^{m} \prod_{k_2=k_1+1}^{m} \operatorname{lcm}(n_{k_1}, n_{k_2}) \prod_{k_1=1}^{m} \prod_{k_2=k_1+1}^{m} \prod_{k_3=k_2+1}^{m} \prod_{k_4=k_3+1}^{m} \operatorname{lcm}(n_{k_1}, n_{k_2}, n_{k_3}, n_{k_4}) \ldots\right)$$

04.10.27.0004.01

$$\operatorname{lcm}(n, m, k) \gcd(n\,m, m\,k, k\,n) = n\,m\,k \;/;\; \{n, m, k\} \in \mathbb{Z}$$

04.10.27.0005.01

$$\gcd(\operatorname{lcm}(k, m), \operatorname{lcm}(k, n), \operatorname{lcm}(m, n)) = \operatorname{lcm}(\gcd(k, m), \gcd(k, n), \gcd(m, n)) \;/;\; \{n, m, k\} \in \mathbb{Z}$$

Inequalities

04.10.29.0001.01

$$\operatorname{lcm}(1, 2, \ldots, n) \ge 2^{n-2} \;/;\; n \in \mathbb{N}^+$$

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