LegendreP3General

Notations

Traditional name

Assosiated Legendre function of the first kind of type 3

Traditional notation

$\mathrm{P}_{\nu}^{\mu}(z)$

Mathematica StandardForm notation

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LegendreP[ν, μ, 3, z]
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Primary definition

07.09.02.0001.01

$$\mathrm{P}_{\nu}^{\mu}(z) = \frac{(z+1)^{\mu/2}}{(z-1)^{\mu/2}}\, {}_2\tilde{F}_1\left(-\nu, \nu+1; 1-\mu; \frac{1-z}{2}\right)$$

Specific values

Specialized values

For fixed ν, μ

07.09.03.0001.01

$$\mathrm{P}_{\nu}^{\mu}(0) = \frac{2^{\mu}\, i^{-\mu} \sqrt{\pi}}{\Gamma\left(\frac{1-\mu-\nu}{2}\right)\Gamma\left(\frac{2-\mu+\nu}{2}\right)}$$

07.09.03.0002.01

$$\mathrm{P}_{\nu}^{\mu}(1) = 0 \;/;\; \mathrm{Re}(\mu) < 0 \bigvee \mu \in \mathbb{N}^{+}$$

07.09.03.0003.01

$$\mathrm{P}_{\nu}^{\mu}(1) = \tilde{\infty} \;/;\; \mathrm{Re}(\mu) > 0 \bigwedge \mu \notin \mathbb{N}^{+}$$

07.09.03.0004.01

$$\mathrm{P}_{\nu}^{\mu}(1) = ¿ \;/;\; \mathrm{Re}(\mu) = 0 \bigwedge \mu \neq 0$$

07.09.03.0005.01

$$\mathrm{P}_{\nu}^{\mu}(-1) = \tilde{\infty} \;/;\; \nu \notin \mathbb{Z}$$

For fixed ν, z

07.09.03.0006.01

$$\mathcal{P}_{\nu}^{0}(z) = P_{\nu}(z)$$

07.09.03.0007.01

$$\mathcal{P}_{\nu}^{-\nu-1}(z) = -\frac{2^{\nu+1}}{\Gamma(\nu+1)\,(1-z)^{\nu}}\,(z-1)^{\frac{\nu-1}{2}}\,(z+1)^{-\frac{\nu+1}{2}}\,B_{\frac{1-z}{2}}(\nu+1,\,\nu+1)$$

07.09.03.0008.01

$$\mathcal{P}_{\nu}^{-\nu}(z) = \frac{(z-1)^{\nu/2}\,(z+1)^{\nu/2}}{2^{\nu}\,\Gamma(\nu+1)}$$

07.09.03.0009.01

$$\mathcal{P}_{\nu}^{\nu}(z) = \frac{(1-z)^{\nu}}{2^{\nu}\,\Gamma(-\nu)}\,\frac{(z+1)^{\nu/2}}{(z-1)^{\nu/2}}\,B_{\frac{1-z}{2}}(-\nu,\,-\nu)$$

07.09.03.0010.01

$$\mathcal{P}_{\nu}^{\nu+1}(z) = \frac{2^{\nu+1}}{\Gamma(-\nu)}\,(z-1)^{-\frac{\nu+1}{2}}\,(z+1)^{-\frac{\nu+1}{2}}$$

07.09.03.0093.01

$$P_{\nu-\frac{1}{2}}^{\frac{1}{2}}(z) = \sqrt{\frac{2}{\pi}}\,(z-1)^{-1/4}\,(z+1)^{-1/4}\,T_{\nu}(z)$$

07.09.03.0094.01

$$\mathcal{P}_{\nu}^{\frac{1}{2}}(z) = \sqrt{\frac{2}{\pi}}\,\frac{\cos\left(\left(\nu+\frac{1}{2}\right)\cos^{-1}(z)\right)}{\sqrt[4]{z-1}\,\sqrt[4]{z+1}}$$

For fixed μ, z

07.09.03.0011.01

$$\mathcal{P}_{0}^{\mu}(z) = \frac{1}{\Gamma(1-\mu)}\,\frac{(z+1)^{\mu/2}}{(z-1)^{\mu/2}}$$

07.09.03.0012.01

$$\mathcal{P}_{1}^{\mu}(z) = \frac{z-\mu}{\Gamma(2-\mu)}\,\frac{(z+1)^{\mu/2}}{(z-1)^{\mu/2}}$$

07.09.03.0013.01

$$\mathcal{P}_{2}^{\mu}(z) = \frac{3\,z^2-3\,\mu\,z+\mu^2-1}{\Gamma(1-\mu)}\,\frac{(z+1)^{\mu/2}}{(z-1)^{\mu/2}}$$

07.09.03.0014.01

$$\mathcal{P}_{3}^{\mu}(z) = \frac{15\,z^3-15\,\mu\,z^2+3\left(2\,\mu^2-3\right)z-\mu^3+4\,\mu}{\Gamma(4-\mu)}\,\frac{(z+1)^{\mu/2}}{(z-1)^{\mu/2}}$$

07.09.03.0015.01

$$\mathcal{P}_{4}^{\mu}(z) = \frac{9+105\,z^4-105\,z^3\,\mu-10\,\mu^2+\mu^4+45\,z^2\left(\mu^2-2\right)+z\left(55\,\mu-10\,\mu^3\right)}{\Gamma(5-\mu)}\,\frac{(z+1)^{\mu/2}}{(z-1)^{\mu/2}}$$

07.09.03.0016.01

$$\mathrm{P}_5^{\mu}(z) == \frac{1}{\Gamma(6-\mu)} \left(945 z^5 - 64\mu - 945 z^4 \mu + 20\mu^3 - \mu^5 - 105 z^2 \mu \left(-7+\mu^2\right) + 210 z^3 \left(-5+2\mu^2\right) + 15 z \left(15 - 13\mu^2 + \mu^4\right)\right) \frac{(z+1)^{\mu/2}}{(z-1)^{\mu/2}}$$

07.09.03.0017.01

$$\mathrm{P}_6^{\mu}(z) == \frac{1}{\Gamma(7-\mu)} \left(10\,395 z^6 - 10\,395 z^5 \mu + (-5+\mu)(-3+\mu)(-1+\mu)(1+\mu)(3+\mu)(5+\mu) + 4725 z^4 \left(-3+\mu^2\right) - 630 z^3 \mu \left(-17+2\mu^2\right) - 21 z \mu \left(99 - 25\mu^2 + \mu^4\right) + 105 z^2 \left(45 - 32\mu^2 + 2\mu^4\right)\right) \frac{(z+1)^{\mu/2}}{(z-1)^{\mu/2}}$$

07.09.03.0018.01

$$\mathrm{P}_7^{\mu}(z) == \frac{1}{\Gamma(8-\mu)} \left(\left(135\,135 z^7 + 2304\mu - 135\,135 z^6 \mu - \mu^3 \left(-28+\mu^2\right)^2 - 17\,325 z^4 \mu \left(-10+\mu^2\right) + 31\,185 z^5 \left(-7+2\mu^2\right) - 189 z^2 \mu \left(283 - 60\mu^2 + 2\mu^4\right) + 1575 z^3 \left(63 - 38\mu^2 + 2\mu^4\right) + 7 z \left(-1575 + 1516\mu^2 - 170\mu^4 + 4\mu^6\right)\right)\right) \frac{(z+1)^{\mu/2}}{(z-1)^{\mu/2}}$$

07.09.03.0019.01

$$P_8^{\mu}(z) == \frac{1}{\Gamma(9-\mu)} \left(11\,025 + 2\,027\,025 z^8 - 2\,027\,025 z^7 \mu - 12\,916\mu^2 + 1974\mu^4 - 84\mu^6 + \mu^8 + 945\,945 z^6 \left(-4+\mu^2\right) - 135\,135 z^5 \mu \left(-23+2\mu^2\right) + 51\,975 z^4 \left(42 - 22\mu^2 + \mu^4\right) - 3465 z^3 \mu \left(383 - 70\mu^2 + 2\mu^4\right) + 315 z^2 \left(-1260 + 1043\mu^2 - 100\mu^4 + 2\mu^6\right) - 9 z \mu \left(-15\,159 + 4396\mu^2 - 266\mu^4 + 4\mu^6\right)\right) \frac{(z+1)^{\mu/2}}{(z-1)^{\mu/2}}$$

07.09.03.0020.01

$$\mathrm{P}_9^{\mu}(z) == \frac{1}{\Gamma(10-\mu)} \left(34\,459\,425 z^9 - 34\,459\,425 z^8 \mu - 4\,729\,725 z^6 \mu \left(-13+\mu^2\right) + 8\,108\,100 z^7 \left(-9+2\mu^2\right) - 135\,135 z^4 \mu \left(249 - 40\mu^2 + \mu^4\right) + 945\,945 z^5 \left(54 - 25\mu^2 + \mu^4\right) - 495 z^2 \mu \left(-11\,601 + 2933\mu^2 - 154\mu^4 + 2\mu^6\right) + 6930 z^3 \left(-1890 + 1373\mu^2 - 115\mu^4 + 2\mu^6\right) - \mu \left(147\,456 - 52\,480\mu^2 + 4368\mu^4 - 120\mu^6 + \mu^8\right) + 45 z \left(19\,845 - 20\,217\mu^2 + 2674\mu^4 - 98\mu^6 + \mu^8\right)\right) \frac{(z+1)^{\mu/2}}{(z-1)^{\mu/2}}$$

07.09.03.0021.01

$$\mathrm{P}_{10}^{\mu}(z) == \frac{1}{\Gamma(11-\mu)} \left(-893\,025 + 654\,729\,075 z^{10} - 654\,729\,075 z^9 \mu + 1\,057\,221\mu^2 - 172\,810\mu^4 + 8778\mu^6 - 165\mu^8 + \mu^{10} + 310\,134\,825 z^8 \left(-5+\mu^2\right) - 45\,945\,900 z^7 \mu \left(-29+2\mu^2\right) - 2\,837\,835 z^5 \mu \left(314 - 45\mu^2 + \mu^4\right) + 9\,459\,450 z^6 \left(135 - 56\mu^2 + 2\mu^4\right) + 315\,315 z^4 \left(-1350 + 874\mu^2 - 65\mu^4 + \mu^6\right) - 12\,870 z^3 \mu \left(-16\,830 + 3773\mu^2 - 175\mu^4 + 2\mu^6\right) - 55 z \mu \left(251\,865 - 78\,877\mu^2 + 5754\mu^4 - 138\mu^6 + \mu^8\right) + 1485 z^2 \left(33\,075 - 29\,828\mu^2 + 3479\mu^4 - 112\mu^6 + \mu^8\right)\right) \frac{(z+1)^{\mu/2}}{(z-1)^{\mu/2}}$$

07.09.03.0022.01

$$\mathrm{P}_n^{\mu}(z) == \frac{(z+1)^{\mu/2}}{(z-1)^{\mu/2}} \sum_{k=0}^{n} \frac{(-n)_k \, (n+1)_k}{\Gamma(k-\mu+1)\, k!} \left(\frac{1-z}{2}\right)^k /;\, n \in \mathbb{N}$$

07.09.03.0023.01

$$P_{-n}^{\mu}(z) = \frac{(z+1)^{\mu/2}}{(z-1)^{\mu/2}} \sum_{k=0}^{n-1} \frac{(1-n)_k\,(n)_k}{\Gamma(k-\mu+1)\,k!} \left(\frac{1-z}{2}\right)^k \;/;\; n \in \mathbb{N}^+$$

07.09.03.0024.01

$$P_n^{\mu}(z) = \frac{\Gamma(-\mu)}{\Gamma(-n-\mu)\,\Gamma(n-\mu+1)} \frac{(z+1)^{\mu/2}}{(z-1)^{\mu/2}} \sum_{k=0}^{|n|+\theta(n)-1} \frac{(-n)_k\,(n+1)_k}{(\mu+1)_k\,k!} \left(\frac{z+1}{2}\right)^k \;/;\; n \in \mathbb{Z}$$

07.09.03.0025.01

$$P_n^{\mu}(z) = \frac{1}{2^n\,n!} (z-1)^{n-\mu/2}\,(1+z)^{\mu/2} \sum_{k=0}^{n} \frac{(2n-k)!\,(-n)_k}{k!\,\Gamma(n-k-\mu+1)} \left(\frac{2}{1-z}\right)^k \;/;\; n \in \mathbb{N}$$

07.09.03.0026.01

$$P_n^{\mu}(z) = 0 \;/;\; n \in \mathbb{N} \bigwedge n \in \mathbb{N}^+ \bigwedge n < m$$

For fixed z

07.09.03.0027.01

$$P_0^0(z) = 1$$

07.09.03.0028.01

$$P_1^0(z) = z$$

07.09.03.0029.01

$$P_1^1(z) = \sqrt{z-1}\,\sqrt{z+1}$$

07.09.03.0030.01

$$P_2^0(z) = \frac{1}{2}\left(3z^2-1\right)$$

07.09.03.0031.01

$$P_2^1(z) = 3z\,\sqrt{z-1}\,\sqrt{z+1}$$

07.09.03.0032.01

$$P_2^2(z) = 3\left(z^2-1\right)$$

07.09.03.0033.01

$$P_3^0(z) = \frac{1}{2} z\left(5z^2-3\right)$$

07.09.03.0034.01

$$P_3^1(z) = \frac{3}{2}\sqrt{z-1}\,\sqrt{z+1}\,\left(5z^2-1\right)$$

07.09.03.0035.01

$$P_3^2(z) = 15z\left(z^2-1\right)$$

07.09.03.0036.01

$$P_3^3(z) = 15\,(z-1)^{3/2}\,(z+1)^{3/2}$$

07.09.03.0037.01

$$P_4^0(z) = \frac{1}{8}\left(3-30z^2+35z^4\right)$$

07.09.03.0038.01

$$P_4^1(z) = \frac{5}{2} z \sqrt{z-1} \sqrt{z+1} \left(7 z^2 - 3\right)$$

07.09.03.0039.01

$$P_4^2(z) = \frac{15}{2} \left(z^2 - 1\right) \left(7 z^2 - 1\right)$$

07.09.03.0040.01

$$P_4^3(z) = 105 z (z-1)^{3/2} (z+1)^{3/2}$$

07.09.03.0041.01

$$P_4^4(z) = 105 \left(z^2 - 1\right)^2$$

07.09.03.0042.01

$$P_5^0(z) = \frac{1}{8} z \left(15 - 70 z^2 + 63 z^4\right)$$

07.09.03.0043.01

$$P_5^1(z) = \frac{15}{8} \sqrt{z-1} \sqrt{z+1} \left(1 - 14 z^2 + 21 z^4\right)$$

07.09.03.0044.01

$$P_5^2(z) = \frac{105}{2} z \left(z^2 - 1\right) \left(3 z^2 - 1\right)$$

07.09.03.0045.01

$$P_5^3(z) = \frac{105}{2} (z-1)^{3/2} (z+1)^{3/2} \left(9 z^2 - 1\right)$$

07.09.03.0046.01

$$P_5^4(z) = 945 z \left(z^2 - 1\right)^2$$

07.09.03.0047.01

$$P_5^5(z) = 945 (z-1)^{5/2} (z+1)^{5/2}$$

07.09.03.0048.01

$$P_6^0(z) = \frac{1}{16} \left(231 z^6 - 315 z^4 + 105 z^2 - 5\right)$$

07.09.03.0049.01

$$P_6^1(z) = \frac{21}{8} z \sqrt{z-1} \sqrt{z+1} \left(5 - 30 z^2 + 33 z^4\right)$$

07.09.03.0050.01

$$P_6^2(z) = \frac{105}{8} \left(z^2 - 1\right) \left(33 z^4 - 18 z^2 + 1\right)$$

07.09.03.0051.01

$$P_6^3(z) = \frac{315}{2} z (z-1)^{3/2} (z+1)^{3/2} \left(11 z^2 - 3\right)$$

07.09.03.0052.01

$$P_6^4(z) = \frac{945}{2} \left(z^2 - 1\right)^2 \left(11 z^2 - 1\right)$$

07.09.03.0053.01

$$\mathrm{P}_6^5(z) = 10\,395\,z\,(z-1)^{5/2}\,(z+1)^{5/2}$$

07.09.03.0054.01

$$\mathrm{P}_6^6(z) = 10\,395\,(z^2-1)^3$$

07.09.03.0055.01

$$\mathrm{P}_7^0(z) = \frac{1}{16}\,z\,(429\,z^6 - 693\,z^4 + 315\,z^2 - 35)$$

07.09.03.0056.01

$$\mathrm{P}_7^1(z) = \frac{7}{16}\,\sqrt{z-1}\,\sqrt{z+1}\,(429\,z^6 - 495\,z^4 + 135\,z^2 - 5)$$

07.09.03.0057.01

$$\mathrm{P}_7^2(z) = \frac{63}{8}\,z\,(z^2-1)\,(143\,z^4 - 110\,z^2 + 15)$$

07.09.03.0058.01

$$\mathrm{P}_7^3(z) = \frac{315}{8}\,(z-1)^{3/2}\,(z+1)^{3/2}\,(3 - 66\,z^2 + 143\,z^4)$$

07.09.03.0059.01

$$\mathrm{P}_7^4(z) = \frac{3465}{2}\,z\,(z^2-1)^2\,(13\,z^2 - 3)$$

07.09.03.0060.01

$$\mathrm{P}_7^5(z) = \frac{10\,395}{2}\,(z-1)^{5/2}\,(z+1)^{5/2}\,(-1 + 13\,z^2)$$

07.09.03.0061.01

$$\mathrm{P}_7^6(z) = 135\,135\,z\,(z^2-1)^3$$

07.09.03.0062.01

$$\mathrm{P}_7^7(z) = 135\,135\,(z-1)^{7/2}\,(z+1)^{7/2}$$

07.09.03.0063.01

$$\mathrm{P}_8^0(z) = \frac{1}{128}\,(6435\,z^8 - 12\,012\,z^6 + 6930\,z^4 - 1260\,z^2 + 35)$$

07.09.03.0064.01

$$\mathrm{P}_8^1(z) = \frac{9}{16}\,z\,\sqrt{z-1}\,\sqrt{z+1}\,(715\,z^6 - 1001\,z^4 + 385\,z^2 - 35)$$

07.09.03.0065.01

$$\mathrm{P}_8^2(z) = \frac{315}{16}\,(z^2-1)\,(143\,z^6 - 143\,z^4 + 33\,z^2 - 1)$$

07.09.03.0066.01

$$\mathrm{P}_8^3(z) = \frac{3465}{8}\,z\,(z-1)^{3/2}\,(z+1)^{3/2}\,(3 - 26\,z^2 + 39\,z^4)$$

07.09.03.0067.01

$$\mathrm{P}_8^4(z) = \frac{10\,395}{8}\,(z^2-1)^2\,(65\,z^4 - 26\,z^2 + 1)$$

07.09.03.0068.01

$$\mathrm{P}_8^5(z) = \frac{135135}{2} z (z-1)^{5/2} (z+1)^{5/2} \left(5 z^2 - 1\right)$$

07.09.03.0069.01

$$\mathrm{P}_8^6(z) = \frac{135135}{2} \left(z^2 - 1\right)^3 \left(15 z^2 - 1\right)$$

07.09.03.0070.01

$$\mathrm{P}_8^7(z) = 2027025 z (z-1)^{7/2} (z+1)^{7/2}$$

07.09.03.0071.01

$$\mathrm{P}_8^8(z) = 2027025 \left(z^2 - 1\right)^4$$

07.09.03.0072.01

$$\mathrm{P}_9^0(z) = \frac{1}{128} z \left(12155 z^8 - 25740 z^6 + 18018 z^4 - 4620 z^2 + 315\right)$$

07.09.03.0073.01

$$\mathrm{P}_9^1(z) = \frac{45}{128} \sqrt{z-1} \sqrt{z+1} \left(2431 z^8 - 4004 z^6 + 2002 z^4 - 308 z^2 + 7\right)$$

07.09.03.0074.01

$$\mathrm{P}_9^2(z) = \frac{495}{16} z \left(z^2 - 1\right) \left(221 z^6 - 273 z^4 + 91 z^2 - 7\right)$$

07.09.03.0075.01

$$\mathrm{P}_9^3(z) = \frac{3465}{16} (z-1)^{3/2} (z+1)^{3/2} \left(221 z^6 - 195 z^4 + 39 z^2 - 1\right)$$

07.09.03.0076.01

$$\mathrm{P}_9^4(z) = \frac{135135}{8} \left(z^2 - 1\right)^2 \left(17 z^5 - 10 z^3 + z\right)$$

07.09.03.0077.01

$$\mathrm{P}_9^5(z) = \frac{135135}{8} (z-1)^{5/2} (z+1)^{5/2} \left(85 z^4 - 30 z^2 + 1\right)$$

07.09.03.0078.01

$$\mathrm{P}_9^6(z) = \frac{675675}{2} z \left(z^2 - 1\right)^3 \left(17 z^2 - 3\right)$$

07.09.03.0079.01

$$\mathrm{P}_9^7(z) = \frac{2027025}{2} (z-1)^{7/2} (z+1)^{7/2} \left(17 z^2 - 1\right)$$

07.09.03.0080.01

$$\mathrm{P}_9^8(z) = 34459425 z \left(z^2 - 1\right)^4$$

07.09.03.0081.01

$$\mathrm{P}_9^9(z) = 34459425 (z-1)^{9/2} (z+1)^{9/2}$$

07.09.03.0082.01

$$\mathrm{P}_{10}^0(z) = \frac{1}{256} \left(46189 z^{10} - 109395 z^8 + 90090 z^6 - 30030 z^4 + 3465 z^2 - 63\right)$$

07.09.03.0083.01

$$\mathrm{P}_{10}^{1}(z) = \frac{55}{128} z \sqrt{z-1} \sqrt{z+1} \left(4199 z^8 - 7956 z^6 + 4914 z^4 - 1092 z^2 + 63\right)$$

07.09.03.0084.01

$$\mathrm{P}_{10}^{2}(z) = \frac{495}{128} \left(z^2 - 1\right) \left(4199 z^8 - 6188 z^6 + 2730 z^4 - 364 z^2 + 7\right)$$

07.09.03.0085.01

$$\mathrm{P}_{10}^{3}(z) = \frac{6435}{16} z (z-1)^{3/2} (z+1)^{3/2} \left(323 z^6 - 357 z^4 + 105 z^2 - 7\right)$$

07.09.03.0086.01

$$\mathrm{P}_{10}^{4}(z) = \frac{45\,045}{16} \left(z^2 - 1\right)^2 \left(323 z^6 - 255 z^4 + 45 z^2 - 1\right)$$

07.09.03.0087.01

$$\mathrm{P}_{10}^{5}(z) = \frac{135\,135}{8} z (z-1)^{5/2} (z+1)^{5/2} \left(323 z^4 - 170 z^2 + 15\right)$$

07.09.03.0088.01

$$\mathrm{P}_{10}^{6}(z) = \frac{675\,675}{8} \left(z^2 - 1\right)^3 \left(323 z^4 - 102 z^2 + 3\right)$$

07.09.03.0089.01

$$\mathrm{P}_{10}^{7}(z) = \frac{11\,486\,475}{2} z (z-1)^{7/2} (z+1)^{7/2} \left(19 z^2 - 3\right)$$

07.09.03.0090.01

$$\mathrm{P}_{10}^{8}(z) = \frac{34\,459\,425}{2} \left(z^2 - 1\right)^4 \left(19 z^2 - 1\right)$$

07.09.03.0091.01

$$\mathrm{P}_{10}^{9}(z) = 654\,729\,075\, z (z-1)^{9/2} (z+1)^{9/2}$$

07.09.03.0092.01

$$\mathrm{P}_{10}^{10}(z) = 654\,729\,075 \left(z^2 - 1\right)^5$$

General characteristics

Domain and analyticity

$\mathrm{P}_{\nu}^{\mu}(z)$ is an analytical function of ν, μ and z which is defined over $\mathbb{C}^3$. For integer ν, $\mathrm{P}_{\nu}^{\mu}(z)$ degenerates to a polynomial in z multiplied on function $\frac{(z+1)^{\mu/2}}{(z-1)^{\mu/2}}$.

07.09.04.0001.01

$$(\nu * \mu * 3 * z) \longrightarrow \mathrm{P}_{\nu}^{\mu}(z) :: (\mathbb{C} \otimes \mathbb{C} \otimes \{3\} \otimes \mathbb{C}) \longrightarrow \mathbb{C}$$

Symmetries and periodicities

Parity

07.09.04.0002.01

$$\mathcal{P}_{-\nu}^{\mu}(z) = \mathcal{P}_{\nu-1}^{\mu}(z)$$

Mirror symmetry

07.09.04.0003.01

$$\mathcal{P}_{\bar{\nu}}^{\bar{\mu}}(\bar{z}) = \overline{\mathcal{P}_{\nu}^{\mu}(z)} \,/;\, z \notin (-\infty, 1)$$

Periodicity

No periodicity

Poles and essential singularities

With respect to z

For fixed ν, μ /; $\frac{\mu}{2} \notin \mathbb{Z}$, the function $\mathcal{P}_{\nu}^{\mu}(z)$ does not have poles and essential singularities.

07.09.04.0004.01

$$\mathcal{S}ing_z\left(\mathcal{P}_{\nu}^{\mu}(z)\right) = \{\} \,/;\, \frac{\mu}{2} \notin \mathbb{Z}$$

For integer ν and integer $\frac{\mu}{2}$, the function $\mathcal{P}_{\nu}^{\mu}(z)$ is polynomial and has pole of order ν at $z = \tilde{\infty}$ (for $\nu \in \mathbb{N}^+$) or order $-\nu - 1$ at $z = \tilde{\infty}$ (for $-\nu \in \mathbb{N}^+$).

07.09.04.0005.01

$$\mathcal{S}ing_z\left(\mathcal{P}_{\nu}^{\mu}(z)\right) = \{\{\tilde{\infty}, \nu\}\} \,/;\, \frac{\mu}{2} \in \mathbb{Z} \bigwedge \nu \in \mathbb{N}^+$$

07.09.04.0006.01

$$\mathcal{S}ing_z\left(\mathcal{P}_{\nu}^{\mu}(z)\right) = \{\{\tilde{\infty}, -\nu - 1\}\} \,/;\, \frac{\mu}{2} \in \mathbb{Z} \bigwedge -\nu \in \mathbb{N}^+$$

With respect to μ

For fixed ν, z, the function $\mathcal{P}_{\nu}^{\mu}(z)$ has only one singular point at $\mu = \tilde{\infty}$. It is an essential singular point. .

07.09.04.0007.01

$$\mathcal{S}ing_{\mu}\left(\mathcal{P}_{\nu}^{\mu}(z)\right) = \{\{\tilde{\infty}, \infty\}\}$$

With respect to ν

For fixed μ, z, the function $\mathcal{P}_{\nu}^{\mu}(z)$ has only one singular point at $\nu = \tilde{\infty}$. It is an essential singular point. .

07.09.04.0008.01

$$\mathcal{S}ing_{\nu}\left(\mathcal{P}_{\nu}^{\mu}(z)\right) = \{\{\tilde{\infty}, \infty\}\}$$

Branch points

With respect to z

For fixed generic ν, μ /; $\nu \notin \mathbb{Z} \bigwedge \frac{\mu}{2} \notin \mathbb{Z}$, the function $\mathrm{P}_\nu^\mu(z)$ has three branch points: $z == \pm 1$and $z == \tilde{\infty}$.

For fixed noninteger ν and integer $\frac{\mu}{2}$, the function $\mathrm{P}_\nu^\mu(z)$ has two branch points: $z == -1$and $z == \tilde{\infty}$.

For fixed integer ν and noninteger $\frac{\mu}{2}$, the function $\mathrm{P}_\nu^\mu(z)$ has two branch points: $z == -1$and $z == 1$.

For fixed integers ν and integers $\frac{\mu}{2}$, the function $\mathrm{P}_\nu^\mu(z)$does not have branch points.

07.09.04.0009.01

$$\mathcal{BP}_z\left(\mathrm{P}_\nu^\mu(z)\right) == \{-1, 1, \tilde{\infty}\} \text{ /; } \nu \notin \mathbb{Z} \bigwedge \frac{\mu}{2} \notin \mathbb{Z}$$

07.09.04.0010.01

$$\mathcal{BP}_z\left(\mathrm{P}_\nu^\mu(z)\right) == \{-1, \tilde{\infty}\} \text{ /; } \nu \notin \mathbb{Z} \bigwedge \frac{\mu}{2} \in \mathbb{Z}$$

07.09.04.0011.01

$$\mathcal{BP}_z\left(\mathrm{P}_\nu^\mu(z)\right) == \{-1, 1\} \text{ /; } \nu \in \mathbb{Z} \bigwedge \frac{\mu}{2} \notin \mathbb{Z}$$

07.09.04.0012.01

$$\mathcal{BP}_z\left(\mathrm{P}_\nu^\mu(z)\right) == \{\} \text{ /; } \nu \in \mathbb{Z} \bigwedge \frac{\mu}{2} \in \mathbb{Z}$$

07.09.04.0013.01

$$\mathcal{R}_z\left(\mathrm{P}_\nu^\mu(z), -1\right) == \log \text{ /; } \mu \in \mathbb{Z} \bigvee \mu \notin \mathbb{Q}$$

07.09.04.0014.01

$$\mathcal{R}_z\left(\mathrm{P}_\nu^\mu(z), -1\right) == s \text{ /; } \mu == \frac{r}{s} \bigwedge r \in \mathbb{Z} \bigwedge s - 1 \in \mathbb{N}^+ \bigwedge \gcd(r, s) == 1$$

07.09.04.0015.01

$$\mathcal{R}_z\left(\mathrm{P}_\nu^\mu(z), 1\right) == \log \text{ /; } \frac{\mu}{2} \notin \mathbb{Z} \bigwedge \frac{\mu}{2} \notin \mathbb{Q}$$

07.09.04.0016.01

$$\mathcal{R}_z\left(\mathrm{P}_\nu^\mu(z), \tilde{\infty}\right) == \log \text{ /; } \nu + \frac{1}{2} \in \mathbb{Z} \bigvee \nu \notin \mathbb{Q}$$

07.09.04.0017.01

$$\mathcal{R}_z\left(\mathrm{P}_\nu^\mu(z), \tilde{\infty}\right) == s \text{ /; } \nu == \frac{r}{s} \bigwedge \{r, s\} \in \mathbb{Z} \bigwedge s > 2 \bigwedge \gcd(r, s) == 1$$

With respect to μ

For fixed ν, z, the function $\mathrm{P}_\nu^\mu(z)$ does not have branch points.

07.09.04.0018.01

$$\mathcal{BP}_\mu\left(\mathrm{P}_\nu^\mu(z)\right) == \{\}$$

With respect to ν

For fixed μ, z, the function $\mathrm{P}_\nu^\mu(z)$ does not have branch points.

07.09.04.0019.01

$$\mathcal{BP}_\nu\left(\mathrm{P}_\nu^\mu(z)\right) == \{\}$$

Branch cuts

With respect to z

For fixed $\nu, \mu /; \nu \notin \mathbb{Z} \bigwedge \frac{\mu}{2} \notin \mathbb{Z}$, the function $\mathrm{P}_\nu^\mu(z)$ is a single-valued function on the z-plane cut along the intervals $(-\infty, -1)$ and $(-1, 1)$.
The function $\mathrm{P}_\nu^\mu(z)$is continuous from above on the interval $(-\infty, -1)$ and on the interval $(-1, 1)$.

For fixed noninteger ν and integer $\frac{\mu}{2}$, the function $\mathrm{P}_\nu^\mu(z)$ is a single-valued function on the z-plane cut along the interval $(-\infty, -1)$, where it is continuous from above.

For fixed integer ν and fixed $\mu /; \frac{\mu}{2} \notin \mathbb{Z}$, the function $\mathrm{P}_\nu^\mu(z)$ is a single-valued function on the z-plane cut along the interval $(-1, 1)$, where it is continuous from above.

For fixed integers ν and $\frac{\mu}{2}$, the function $\mathrm{P}_\nu^\mu(z)$ is a polynomial and does not have branch cuts.

07.09.04.0020.01

$$\mathcal{BC}_z\left(\mathrm{P}_\nu^\mu(z)\right) = \{\{(-\infty, -i), -i\}, \{(-1, 1), -i\}\} /; \nu \notin \mathbb{Z} \bigwedge \frac{\mu}{2} \notin \mathbb{Z}$$

07.09.04.0021.01

$$\mathcal{BC}_z\left(\mathrm{P}_\nu^\mu(z)\right) = \{\{(-\infty, -i), -i\}\} /; \nu \notin \mathbb{Z} \bigwedge \frac{\mu}{2} \in \mathbb{Z}$$

07.09.04.0022.01

$$\mathcal{BC}_z\left(\mathrm{P}_\nu^\mu(z)\right) = \{\{(-1, 1), -i\}\} /; \nu \in \mathbb{Z} \bigwedge \frac{\mu}{2} \notin \mathbb{Z}$$

07.09.04.0023.01

$$\mathcal{BC}_z\left(\mathrm{P}_\nu^\mu(z)\right) = \{\} /; \nu \in \mathbb{Z} \bigwedge \frac{\mu}{2} \in \mathbb{Z}$$

07.09.04.0024.01

$$\lim_{\epsilon \to +0} \mathrm{P}_\nu^\mu(x + i\,\epsilon) = \mathrm{P}_\nu^\mu(x) /; x < -1$$

07.09.04.0025.01

$$\lim_{\epsilon \to +0} \mathrm{P}_\nu^\mu(x - i\,\epsilon) = e^{2\,i\,\mu\,\pi}\, \mathrm{P}_\nu^\mu(x) + \frac{2\,i\,\pi\,e^{i\,\mu\,\pi}}{\Gamma(1-\mu+\nu)\,\Gamma(-\mu-\nu)}\, \mathrm{P}_\nu^{-\mu}(-x) /; x < -1$$

07.09.04.0026.01

$$\lim_{\epsilon \to +0} \mathrm{P}_\nu^\mu(x + i\,\epsilon) = \mathrm{P}_\nu^\mu(x) /; -1 < x < 1$$

07.09.04.0027.01

$$\lim_{\epsilon \to +0} \mathrm{P}_\nu^\mu(x - i\,\epsilon) = e^{i\,\pi\,\mu}\, \mathrm{P}_\nu^\mu(x) /; -1 < x < 1$$

With respect to μ

For fixed ν, z, the function $\mathrm{P}_\nu^\mu(z)$ does not have branch cuts.

07.09.04.0028.01

$$\mathcal{BC}_\mu\left(\mathrm{P}_\nu^\mu(z)\right) = \{\}$$

With respect to ν

For fixed μ, z, the function $\mathfrak{P}_\nu^\mu(z)$ does not have branch cuts.

07.09.04.0029.01

$$\mathcal{BC}_\nu\left(\mathfrak{P}_\nu^\mu(z)\right) = \{\}$$

Series representations

Generalized power series

Expansions at $z = 0$

07.09.06.0001.01

$$\mathfrak{P}_\nu^\mu(z) = \frac{(1-z)^{\mu/2}}{(z-1)^{\mu/2}} \sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{j=0}^{\infty} \frac{(-\nu)_k\,(\nu+1)_k\left(\frac{\mu}{2}-k\right)_m\left(-\frac{\mu}{2}\right)_j(-1)^j\,z^{j+m}}{\Gamma(k-\mu+1)\,k!\,m!\,j!\,2^k} \;/;\; |z|<1 \land \mu \notin \mathbb{N}^+$$

07.09.06.0002.01

$$\mathfrak{P}_\nu^\mu(z) = \frac{(1-z)^{\mu/2}}{(z-1)^{\mu/2}} \left(\frac{\mu}{2}\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{j=0}^{\infty} \frac{(-\nu)_k\,(\nu+1)_k\left(\frac{\mu}{2}+1\right)_m\left(1-\frac{\mu}{2}\right)_k\left(-\frac{\mu}{2}\right)_j(-1)^{j+k}\,z^{j+k+m+1}}{(2)_{k+m}\,\Gamma(k-\mu+1)\,k!\,j!\,2^k} + \right.$$

$$\left.\sum_{k=0}^{\infty}\sum_{m=0}^{\infty}\sum_{j=0}^{\infty} \frac{(-\nu)_{k+m}\,(\nu+1)_{k+m}\left(m-\frac{\mu}{2}+1\right)_k\left(-\frac{\mu}{2}\right)_j(-z)^{j+k}}{\Gamma(k+m-\mu+1)\,(k+m)!\;k!\,j!\,2^{k+m}}\right) /;\; |z|<1$$

07.09.06.0003.01

$$\mathfrak{P}_\nu^\mu(z) = \frac{(1-z)^{\mu/2}}{(z-1)^{\mu/2}} \left(\frac{\mu z}{2}\sum_{k=0}^{\infty}(k+1)!\left(\frac{\mu}{2}+1\right)_k z^k\;{}_1F_0\!\left(-\frac{\mu}{2};;-z\right){}_3\tilde{F}_2\!\left(-\nu,\nu+1,1-\frac{\mu}{2};k+2,1-\mu;-\frac{z}{2}\right)+\right.$$

$$\left.\Gamma\!\left(1-\frac{\mu}{2}\right)\sum_{k=0}^{\infty}\left(-\frac{\mu}{2}\right)_k(-z)^k\,\tilde{F}_{2\times1\times0}^{3\times1\times0}\!\left(\begin{matrix}-\nu,\nu+1,1-\frac{\mu}{2};1;;\\ 1,1-\mu;1-\frac{\mu}{2};;\end{matrix}\;\frac{1}{2},-\frac{z}{2}\right)\right)$$

07.09.06.0004.01

$$\mathfrak{P}_\nu^\mu(z) \propto \frac{2^\mu\sqrt{\pi}}{\Gamma\!\left(\frac{1-\mu-\nu}{2}\right)\Gamma\!\left(1-\frac{\mu-\nu}{2}\right)}\,\frac{(z+1)^{\mu/2}}{(z-1)^{\mu/2}}\,(1+O(z)) \;/;\; (z\to 0)$$

07.09.06.0005.01

$$\mathfrak{P}_n^m(z) = 2^{-n}\,(z-1)^{m/2}\,(z+1)^{m/2}\sum_{k=0}^{\left\lfloor\frac{n}{2}\right\rfloor}(-1)^k\binom{n}{k}\binom{2n-2k}{n}(n-m-2k+1)_m\,z^{n-m-2k} \;/;\; m\in\mathbb{N}\land n\in\mathbb{N}$$

07.09.06.0006.01

$$\mathfrak{P}_n^m(z) \propto (-1)^{m+\left\lfloor\frac{n-m}{2}\right\rfloor}\,\frac{(z+1)^{m/2}}{(z-1)^{m/2}}\,2^{-n}\binom{n}{\left\lfloor\frac{n-m}{2}\right\rfloor}\binom{2n-2\left\lfloor\frac{n-m}{2}\right\rfloor}{n}\left(n-m-2\left\lfloor\frac{n-m}{2}\right\rfloor+1\right)_m z^{n-m-2\left\lfloor\frac{n-m}{2}\right\rfloor}(1+O(z)) \;/;$$

$$(z\to 0)\land m\in\mathbb{N}\land n\in\mathbb{N}\land n\geq m$$

Expansions at $z = 1$

07.09.06.0007.01

$$\mathfrak{P}_\nu^\mu(z) = \frac{(z+1)^{\mu/2}}{(z-1)^{\mu/2}}\left(\frac{1}{\Gamma(1-\mu)} - \frac{(-\nu)\,(\nu+1)\,(z-1)}{2\,\Gamma(2-\mu)} + \frac{(-\nu)\,(1-\nu)\,(\nu+1)\,(\nu+2)\,(z-1)^2}{8\,\Gamma(3-\mu)} - \ldots\right) /;\; \left|\frac{1-z}{2}\right|<1$$

07.09.06.0008.01

$$\mathcal{P}_\nu^\mu(z) = \frac{(z+1)^{\mu/2}}{(z-1)^{\mu/2}} \sum_{k=0}^{\infty} \frac{(-\nu)_k (\nu+1)_k}{\Gamma(k-\mu+1)\, k!} \left(\frac{1-z}{2}\right)^k /; \left|\frac{1-z}{2}\right| < 1$$

07.09.06.0009.01

$$\mathcal{P}_\nu^\mu(z) = \frac{(z+1)^{\mu/2}}{(z-1)^{\mu/2}} \, {}_2\tilde{F}_1\left(-\nu, \nu+1; 1-\mu; \frac{1-z}{2}\right)$$

07.09.06.0010.01

$$\mathcal{P}_\nu^\mu(z) \propto \frac{2^{\mu/2}}{\Gamma(1-\mu)} (z-1)^{-\frac{\mu}{2}} (1 + O(z-1)) /; (z \to 1) \bigwedge \mu \notin \mathbb{N}^+$$

07.09.06.0011.01

$$\mathcal{P}_\nu^m(z) = (-1)^m\, 2^{-m} (-\nu)_m (\nu+1)_m (z+1)^{m/2} (z-1)^{m/2} \, {}_2\tilde{F}_1\left(m-\nu, m+\nu+1; m+1; \frac{1-z}{2}\right) /; m \in \mathbb{N}$$

07.09.06.0012.01

$$\mathcal{P}_\nu^m(z) \propto -\frac{2^{-\frac{m}{2}}\, \Gamma(m-\nu)\, \Gamma(m+\nu+1) \sin(\pi\nu)}{\pi\, m!} (-1)^m (z+1)^{m/2} (z-1)^{m/2} (1 + O(z-1)) /; m \in \mathbb{N}$$

07.09.06.0013.01

$$\mathcal{P}_n^\mu(z) = \frac{(z+1)^{\mu/2}}{(z-1)^{\mu/2}} \sum_{k=0}^{n} \frac{(-n)_k (n+1)_k}{\Gamma(k-\mu+1)\, k!} \left(\frac{1-z}{2}\right)^k /; n \in \mathbb{N}$$

07.09.06.0014.01

$$\mathcal{P}_{-n}^\mu(z) = \frac{(z+1)^{\mu/2}}{(z-1)^{\mu/2}} \sum_{k=0}^{n-1} \frac{(1-n)_k (n)_k}{\Gamma(k-\mu+1)\, k!} \left(\frac{1-z}{2}\right)^k /; n \in \mathbb{N}^+$$

Expansions at $z = -1$

07.09.06.0015.01

$$\mathcal{P}_\nu^\mu(z) = \frac{\Gamma(-\mu)}{\Gamma(-\mu-\nu)\,\Gamma(\nu-\mu+1)} \frac{(z+1)^{\mu/2}}{(z-1)^{\mu/2}} \left(1 - \frac{\nu(1+\nu)}{2(1+\mu)}(1+z) - \frac{(1-\nu)\,\nu\,(1+\nu)(2+\nu)}{8(1+\mu)(2+\mu)}(1+z)^2 + \ldots\right) - \frac{2^\mu}{\pi} \sin(\nu\pi)\,\Gamma(\mu)\,(z+1)^{-\mu/2} (z-1)^{-\mu/2} \left(1 + \frac{(-\mu-\nu)(1-\mu+\nu)}{2(1-\mu)}(1+z) + \frac{(-\mu-\nu)(1-\mu-\nu)(1-\mu+\nu)(2-\mu+\nu)}{8(1-\mu)(2-\mu)}(1+z)^2 + \ldots\right) /; \left|\frac{z+1}{2}\right| < 1 \bigwedge \mu \notin \mathbb{Z}$$

07.09.06.0016.01

$$\mathcal{P}_\nu^\mu(z) = \frac{\Gamma(-\mu)}{\Gamma(-\mu-\nu)\,\Gamma(\nu-\mu+1)} \frac{(z+1)^{\mu/2}}{(z-1)^{\mu/2}} \sum_{k=0}^{\infty} \frac{(-\nu)_k (\nu+1)_k}{(\mu+1)_k\, k!} \left(\frac{z+1}{2}\right)^k - \frac{2^\mu}{\pi} \sin(\nu\pi)\,\Gamma(\mu)\,(z+1)^{-\mu/2} (z-1)^{-\mu/2} \sum_{k=0}^{\infty} \frac{(\nu-\mu+1)_k (-\mu-\nu)_k}{(1-\mu)_k\, k!} \left(\frac{z+1}{2}\right)^k /; \left|\frac{z+1}{2}\right| < 1 \bigwedge \mu \notin \mathbb{Z}$$

07.09.06.0017.01

$$\mathcal{P}_\nu^\mu(z) = \frac{\Gamma(-\mu)}{\Gamma(-\mu-\nu)\,\Gamma(\nu-\mu+1)} \frac{(z+1)^{\mu/2}}{(z-1)^{\mu/2}} \, {}_2F_1\left(-\nu, \nu+1; \mu+1; \frac{z+1}{2}\right) - \frac{2^\mu}{\pi} \sin(\nu\pi)\,\Gamma(\mu)\,(z+1)^{-\mu/2} (z-1)^{-\mu/2} \, {}_2F_1\left(\nu-\mu+1, -\mu-\nu; 1-\mu; \frac{z+1}{2}\right) /; \mu \notin \mathbb{Z}$$

07.09.06.0018.01

$$\mathrm{P}_\nu^\mu(z) = \frac{2^{-\frac{\mu}{2}}\,\Gamma(-\mu)}{\Gamma(-\mu-\nu)\,\Gamma(\nu-\mu+1)}\,\frac{(1-z)^{\mu/2}\,(1+z)^{\mu/2}}{(z-1)^{\mu/2}}\sum_{k=0}^{\infty}\sum_{j=0}^{k}\frac{\left(\frac{\mu}{2}\right)_{k-j}(-\nu)_j\,(\nu+1)_j\,2^{-k}\,(z+1)^k}{(k-j)!\,j!\,(\mu+1)_j} - \frac{2^{\mu/2}\sin(\pi\,\nu)\,\Gamma(\mu)}{\pi}\,\frac{(1-z)^{\mu/2}}{(z-1)^{\mu/2}\,(1+z)^{\mu/2}}\sum_{k=0}^{\infty}\sum_{j=0}^{k}\frac{\left(\frac{\mu}{2}\right)_{k-j}(-\mu-\nu)_j\,(\nu-\mu+1)_j\,2^{-k}\,(z+1)^k}{(k-j)!\,j!\,(1-\mu)_j}\;/;\;\mu\notin\mathbb{Z}$$

07.09.06.0019.01

$$\mathrm{P}_\nu^\mu(z) \propto \frac{2^{-\mu/2}\,\Gamma(-\mu)}{\Gamma(-\mu-\nu)\,\Gamma(\nu-\mu+1)}\,\frac{(1-z)^{\mu/2}\,(1+z)^{\mu/2}}{(z-1)^{\mu/2}}\,(1+O(z+1)) - \frac{2^{\mu/2}\sin(\pi\,\nu)\,\Gamma(\mu)}{\pi}\,\frac{(1-z)^{\mu/2}}{(z-1)^{\mu/2}\,(1+z)^{\mu/2}}\,(1+O(z+1))\;/;\;(z\to-1)\bigwedge\mu\notin\mathbb{Z}$$

07.09.06.0020.01

$$\mathrm{P}_n^\mu(z) = \frac{\Gamma(-\mu)}{\Gamma(-n-\mu)\,\Gamma(n-\mu+1)}\,\frac{(z+1)^{\mu/2}}{(z-1)^{\mu/2}}\sum_{k=0}^{|n|+\theta(n)-1}\frac{(-n)_k\,(n+1)_k}{(\mu+1)_k\,k!}\left(\frac{z+1}{2}\right)^k\;/;\;n\in\mathbb{Z}$$

07.09.06.0021.01

$$\mathrm{P}_\nu^m(z) = \frac{(-1)^{m-1}}{m!\,\Gamma(-m-\nu)\,\Gamma(\nu-m+1)}\log\left(\frac{z+1}{2}\right)\frac{(z+1)^{m/2}}{(z-1)^{m/2}}\,{}_2F_1\left(-\nu,\,\nu+1;\,m+1;\,\frac{z+1}{2}\right) - \frac{2^m\sin(\pi\,\nu)\,(m-1)!}{\pi\,(z-1)^{m/2}\,(1+z)^{m/2}}\sum_{k=0}^{m-1}\frac{(-m-\nu)_k\,(\nu-m+1)_k}{k!\,(1-m)_k}\left(\frac{z+1}{2}\right)^k + \frac{(-1)^m}{\Gamma(-m-\nu)\,\Gamma(-m+\nu+1)}\,\frac{(z+1)^{m/2}}{(z-1)^{m/2}}\sum_{k=0}^{\infty}\frac{(-\nu)_k\,(\nu+1)_k}{k!\,(k+m)!}\,(\psi(k+1)+\psi(k+m+1)-\psi(k+\nu+1)-\psi(k-\nu))\left(\frac{z+1}{2}\right)^k\;/;\;\left|\frac{z+1}{2}\right|<1\bigwedge m\in\mathbb{N}^+\bigwedge\nu\notin\mathbb{Z}$$

07.09.06.0022.01

$$\mathrm{P}_\nu^m(z) \propto \frac{(-1)^{m-1}\,2^{-\frac{m}{2}}}{m!\,\Gamma(-m-\nu)\,\Gamma(\nu-m+1)}\,(1-z)^{m/2}\,(z-1)^{-m/2}\,(z+1)^{m/2}\left(\log\left(\frac{z+1}{2}\right)-\psi(m+1)+\psi(-\nu)+\psi(\nu+1)+\gamma\right)(1+O(z+1)) - \frac{2^{m/2}}{\pi}\sin(\nu\,\pi)\,(m-1)!\,(1-z)^{m/2}\,(z-1)^{-m/2}\,(z+1)^{-m/2}\,(1+O(z+1))\;/;\;(z\to-1)\bigwedge m\in\mathbb{N}^+\bigwedge\nu\notin\mathbb{Z}$$

07.09.06.0023.01

$$\mathrm{P}_\nu^0(z) = \frac{\sin(\pi\,\nu)}{\pi}\log\left(\frac{z+1}{2}\right){}_2F_1\left(-\nu,\,\nu+1;\,1;\,\frac{z+1}{2}\right) - \frac{\sin(\pi\,\nu)}{\pi}\sum_{k=0}^{\infty}\frac{(-\nu)_k\,(\nu+1)_k\,(2\,\psi(k+1)-\psi(k+\nu+1)-\psi(k-\nu))}{k!^2}\left(\frac{z+1}{2}\right)^k\;/;\;\left|\frac{z+1}{2}\right|<1\bigwedge\nu\notin\mathbb{Z}$$

07.09.06.0024.01

$$\mathrm{P}_\nu^0(z) \propto \frac{\sin(\pi\,\nu)}{\pi}\log\left(\frac{z+1}{2}\right)(1+O(z+1)) + \frac{\sin(\pi\,\nu)}{\pi}\,(-\pi\cot(\pi\,\nu)+2\,\psi(-\nu)+2\,\gamma)\,(1+O(z+1))\;/;\;(z\to-1)\bigwedge\nu\notin\mathbb{Z}$$

07.09.06.0025.01

$$\mathfrak{P}_\nu^{-m}(z) = \frac{(-1)^m \sin(\nu\pi)}{2^m \pi m!} (z+1)^{m/2} (z-1)^{m/2} \log\left(\frac{z+1}{2}\right) {}_2F_1\left(m+\nu+1, m-\nu; m+1; \frac{z+1}{2}\right) + \frac{(m-1)!}{\Gamma(m-\nu)\Gamma(m+\nu+1)} \frac{(z-1)^{m/2}}{(z+1)^{m/2}} \sum_{k=0}^{m-1} \frac{(-\nu)_k (\nu+1)_k}{k! (1-m)_k} \left(\frac{z+1}{2}\right)^k - \frac{(-1)^m \sin(\nu\pi)}{2^m \pi} (z+1)^{m/2} (z-1)^{m/2} \sum_{k=0}^{\infty} \frac{(m-\nu)_k (m+\nu+1)_k}{k! (k+m)!} (\psi(k+1) + \psi(k+m+1) - \psi(k+m+\nu+1) - \psi(k+m-\nu)) \left(\frac{z+1}{2}\right)^k /; m \in \mathbb{N}^+ \bigwedge \nu \notin \mathbb{Z}$$

07.09.06.0026.01

$$\mathfrak{P}_\nu^{-m}(z) \propto \frac{(-1)^m i^m \sin(\nu\pi)}{2^{m/2} \pi m!} (z+1)^{m/2} \left(\log\left(\frac{z+1}{2}\right) + (\gamma - \psi(m+1) + \psi(m-\nu) + \psi(m+\nu+1))\right)(1 + O(z+1)) + \frac{i^m 2^{m/2} (m-1)!}{\Gamma(m-\nu)\Gamma(m+\nu+1)} (z+1)^{-m/2} (1 + O(z+1)) /; (z \to -1) \bigwedge m \in \mathbb{N}^+ \bigwedge \nu \notin \mathbb{Z}$$

Expansions at $z == \infty$

07.09.06.0027.01

$$\mathfrak{P}_\nu^{\mu}(z) = \frac{1}{\sqrt{\pi}} \frac{(z+1)^{\mu/2}}{(z-1)^{\mu/2}} \left(\frac{2^\nu (z-1)^\nu}{\Gamma(\nu-\mu+1)} \Gamma\left(\nu+\frac{1}{2}\right)\left(1 + \frac{\mu-\nu}{1-z} + \frac{(1-\nu)(\mu-\nu)(1+\mu-\nu)}{(1-2\nu)(1-z)^2} + \ldots\right) + \frac{2^{-\nu-1} (z-1)^{-\nu-1}}{\Gamma(-\mu-\nu)} \Gamma\left(-\nu-\frac{1}{2}\right)\left(1 + \frac{1+\mu+\nu}{1-z} + \frac{(2+\nu)(1+\mu+\nu)(2+\mu+\nu)}{(3+2\nu)(z-1)^2} + \ldots\right)\right) /; \left|\frac{1-z}{2}\right| > 1 \bigwedge 2\nu \notin \mathbb{Z}$$

07.09.06.0028.01

$$\mathfrak{P}_\nu^{\mu}(z) = \frac{1}{\sqrt{\pi}} \frac{(z+1)^{\mu/2}}{(z-1)^{\mu/2}} \left(\frac{2^\nu (z-1)^\nu}{\Gamma(\nu-\mu+1)} \Gamma\left(\nu+\frac{1}{2}\right) \sum_{k=0}^{\infty} \frac{(-\nu)_k (\mu-\nu)_k}{(-2\nu)_k k!} \left(\frac{2}{1-z}\right)^k + \frac{2^{-\nu-1} (z-1)^{-\nu-1}}{\Gamma(-\mu-\nu)} \Gamma\left(-\nu-\frac{1}{2}\right) \sum_{k=0}^{\infty} \frac{(\nu+1)_k (\mu+\nu+1)_k}{(2\nu+2)_k k!} \left(\frac{2}{1-z}\right)^k\right) /; \left|\frac{1-z}{2}\right| > 1 \bigwedge 2\nu \notin \mathbb{Z}$$

07.09.06.0029.01

$$\mathfrak{P}_\nu^{\mu}(z) = \frac{1}{\sqrt{\pi}} \frac{(z+1)^{\mu/2}}{(z-1)^{\mu/2}} \left(\frac{2^\nu (z-1)^\nu}{\Gamma(\nu-\mu+1)} \Gamma\left(\nu+\frac{1}{2}\right) {}_2F_1\left(\mu-\nu, -\nu; -2\nu; \frac{2}{1-z}\right) + \frac{2^{-\nu-1} (z-1)^{-\nu-1}}{\Gamma(-\mu-\nu)} \Gamma\left(-\nu-\frac{1}{2}\right) {}_2F_1\left(\nu+1, \mu+\nu+1; 2\nu+2; \frac{2}{1-z}\right)\right) /; z \notin (-1, 1) \bigwedge 2\nu \notin \mathbb{Z}$$

07.09.06.0030.01

$$\mathfrak{P}_\nu^{\mu}(z) \propto \frac{1}{\sqrt{\pi}} \left(\frac{2^\nu z^\nu}{\Gamma(\nu-\mu+1)} \Gamma\left(\nu+\frac{1}{2}\right)\left(1 + O\left(\frac{1}{z}\right)\right) + \frac{2^{-\nu-1} z^{-\nu-1}}{\Gamma(-\mu-\nu)} \Gamma\left(-\nu-\frac{1}{2}\right)\left(1 + O\left(\frac{1}{z}\right)\right)\right) /; (|z| \to \infty) \bigwedge 2\nu \notin \mathbb{Z}$$

07.09.06.0031.01

$$\mathfrak{P}_n^{\mu}(z) = \frac{1}{2^n n!} (z-1)^{n-\mu/2} (z+1)^{\mu/2} \sum_{k=0}^{n} \frac{(2n-k)! (-n)_k}{k! \Gamma(n-k-\mu+1)} \left(\frac{2}{1-z}\right)^k /; n \in \mathbb{N}$$

07.09.06.0032.01

$$\mathfrak{P}_\nu^\mu(z) = \frac{2^{\nu+1}\sin(\pi(\mu-\nu))\,\Gamma(\mu+\nu+1)}{\pi\,\Gamma(-\nu)\,\Gamma(2\nu+2)}(z-1)^{-\nu-1}\frac{(z+1)^{\mu/2}}{(z-1)^{\mu/2}}\log\left(\frac{z-1}{2}\right){}_2F_1\left(\nu+1,\,\mu+\nu+1;\,2\nu+2;\,\frac{2}{1-z}\right)+ \frac{2^{-\nu}(z-1)^{\nu}}{\Gamma(\nu+1)}\frac{(z+1)^{\mu/2}}{(z-1)^{\mu/2}}\sum_{k=0}^{2\nu}\frac{(2\nu-k)!\,(-\nu)_k}{k!\,\Gamma(\nu-\mu-k+1)}\left(\frac{2}{1-z}\right)^k+\frac{2^{-\nu}\sin(\pi(\nu-\mu))\sin(\pi\nu)\,\Gamma(\mu+\nu+1)}{\pi^{3/2}\,\Gamma\left(\nu+\frac{3}{2}\right)}(z-1)^{-\nu-1}\frac{(z+1)^{\mu/2}}{(z-1)^{\mu/2}} \sum_{k=0}^{\infty}\frac{(\nu+1)_k\,(\mu+\nu+1)_k}{k!\,(2\nu+2)_k}(\psi(k+1)-\psi(k+\nu+1)+\psi(k+2\nu+2)-\psi(-k-\mu-\nu))\left(\frac{2}{1-z}\right)^k /;\ 2\nu+1\in\mathbb{N}\bigwedge\nu-\mu\notin\mathbb{Z}$$

07.09.06.0033.01

$$\mathfrak{P}_\nu^\mu(z)\propto\frac{2^\nu\,\Gamma\left(\nu+\frac{1}{2}\right)}{\sqrt{\pi}\,\Gamma(\nu-\mu+1)}z^\nu\left(1+O\left(\frac{1}{z}\right)\right)+ \frac{2^{\nu+1}\,\Gamma(\mu+\nu+1)\sin(\pi(\mu-\nu))}{\pi\,\Gamma(-\nu)\,\Gamma(2\nu+2)}z^{-\nu-1}\left(\log\left(\frac{z}{2}\right)-\psi(-\mu-\nu)-\psi(\nu+1)+\psi(2\nu+2)-\gamma\right)\left(1+O\left(\frac{1}{z}\right)\right) /;\ 2\nu\in\mathbb{N}\bigwedge\nu-\mu\notin\mathbb{Z}$$

07.09.06.0034.01

$$\mathfrak{P}_{-\frac{1}{2}}^\mu(z)\propto\frac{\sqrt{2}\,\cos(\pi\mu)\,\Gamma\left(\mu+\frac{1}{2}\right)}{\pi^{3/2}\sqrt{z}}\left(\log(2z)-\psi\left(\frac{1}{2}-\mu\right)-\gamma\right)\left(1+O\left(\frac{1}{z}\right)\right) /;\ \mu+\frac{1}{2}\notin\mathbb{Z}$$

07.09.06.0035.01

$$\mathfrak{P}_\nu^\mu(z) = \frac{(-1)^{\mu-\nu-1}\,2^{\nu+1}\,(\mu+\nu)!}{(2\nu+1)!\,\Gamma(-\nu)}\frac{(z+1)^{\mu/2}}{(z-1)^{\mu/2+\nu+1}}{}_2F_1\left(\nu+1,\,\mu+\nu+1;\,2\nu+2;\,\frac{2}{1-z}\right) /;\ 2\nu+1\in\mathbb{N}\bigwedge\mu-\nu\in\mathbb{N}^+$$

07.09.06.0036.01

$$\mathfrak{P}_\nu^\mu(z)\propto\frac{(-1)^{\mu-\nu-1}\,2^{\nu+1}\,(\mu+\nu)!}{(2\nu+1)!\,\Gamma(-\nu)}z^{-\nu-1}\left(1+O\left(\frac{1}{z}\right)\right) /;\ (|z|\to\infty)\bigwedge 2\nu+1\in\mathbb{N}\bigwedge\mu-\nu\in\mathbb{N}^+$$

07.09.06.0037.01

$$\mathfrak{P}_\nu^\mu(z) = \frac{2^\nu\,\Gamma\left(\nu+\frac{1}{2}\right)}{\sqrt{\pi}\,\Gamma(\nu-\mu+1)}(z-1)^{\nu-\mu/2}(z+1)^{\mu/2}\sum_{k=0}^{\nu-\mu}\frac{(\mu-\nu)_k\,(-\nu)_k}{k!\,(-2\nu)_k}\left(\frac{2}{1-z}\right)^k-\frac{(-1)^{\nu-\mu}\,2^{\nu+1}\,\Gamma(\mu+\nu+1)}{\Gamma(-\nu)\,\Gamma(2\nu+2)} (z-1)^{-\mu/2-\nu-1}(z+1)^{\mu/2}{}_2F_1\left(\nu+1,\,\mu+\nu+1;\,2\nu+2;\,\frac{2}{1-z}\right) /;\ 2\nu+1\in\mathbb{N}\bigwedge\nu-\mu\in\mathbb{Z}\bigwedge-\nu\leq\mu\leq\nu+1$$

07.09.06.0038.01

$$\mathfrak{P}_\nu^\mu(z)\propto\frac{2^\nu\,\Gamma\left(\nu+\frac{1}{2}\right)z^\nu}{\sqrt{\pi}\,\Gamma(\nu-\mu+1)}\left(1+O\left(\frac{1}{z}\right)\right)-\frac{(-1)^{\nu-\mu}\,2^{\nu+1}\,\Gamma(\mu+\nu+1)\,z^{-\nu-1}}{\Gamma(-\nu)\,\Gamma(2\nu+2)}\left(1+O\left(\frac{1}{z}\right)\right) /; (|z|\to\infty)\bigwedge 2\nu+1\in\mathbb{N}\bigwedge\nu-\mu\in\mathbb{Z}\bigwedge-\nu\leq\mu\leq\nu+1$$

07.09.06.0039.01

$$\mathfrak{P}_\nu^\mu(z) = \frac{(-1)^{2\nu+1}\,2^{\nu+1}}{(-\mu-\nu-1)!\,\Gamma(-\nu)}\,(z-1)^{\mu/2-\nu-1}\,(z+1)^{\mu/2} \sum_{k=0}^{-\mu-\nu-1} \frac{(\nu+1)_k\,(\mu+\nu+1)_k}{k!\,(2\nu+k+1)!} \left(\text{Log}\left[\frac{z-1}{2}\right]+\psi(k+1)-\psi(-k-\mu-\nu)-\psi(k+\nu+1)+\psi(k+2\nu+2)\right)\left(\frac{2}{1-z}\right)^k + \frac{2^\nu\,\Gamma\left(\nu+\frac{1}{2}\right)}{\sqrt{\pi}\,\Gamma(\nu-\mu+1)}\,(z-1)^{\nu-\mu/2}\,(z+1)^{\mu/2}\sum_{k=0}^{2\nu}\frac{(\mu-\nu)_k\,(-\nu)_k}{k!\,(-2\nu)_k}\left(\frac{2}{1-z}\right)^k + \frac{(-1)^{2\nu}\,2^{1-\mu}\,\sin(\nu\pi)\,\Gamma(1-\mu)}{\pi\,\Gamma(1-\mu-\nu)\,\Gamma(\nu-\mu+2)}\,(z-1)^{\mu/2-1}\,(z+1)^{\mu/2}\,{}_3F_2\left(1,1,1-\mu;-\mu-\nu+1,-\mu+\nu+2;\frac{2}{1-z}\right) /;\ 2\nu+1\in\mathbb{N}\bigwedge\nu-\mu\in\mathbb{Z}\bigwedge\mu+\nu\leq 0$$

07.09.06.0040.01

$$\mathfrak{P}_\nu^\mu(z) \propto \frac{(-1)^{2\nu+1}\,2^{\nu+1}\,z^{-\nu-1}}{(-\mu-\nu-1)!\,\Gamma(-\nu)\,\Gamma(2\nu+2)}\left(\log\left(\frac{z}{2}\right)-\psi(-\mu-\nu)-\psi(\nu+1)+\psi(2\nu+2)-\gamma\right)\left(1+O\left(\frac{1}{z}\right)\right) + \frac{2^\nu\,\Gamma\left(\nu+\frac{1}{2}\right)z^\nu}{\sqrt{\pi}\,\Gamma(\nu-\mu+1)}\left(1+O\left(\frac{1}{z}\right)\right) + \frac{(-1)^{2\nu}\,2^{1-\mu}\,\sin(\pi\nu)\,\Gamma(1-\mu)}{\pi\,\Gamma(1-\mu-\nu)\,\Gamma(2-\mu+\nu)}\,z^{\mu-1}\left(1+O\left(\frac{1}{z}\right)\right) /;\ (|z|\to\infty)\bigwedge 2\nu+1\in\mathbb{N}\bigwedge\nu-\mu\in\mathbb{Z}\bigwedge\mu+\nu\leq 0$$

07.09.06.0041.01

$$\mathfrak{P}_{-\frac{1}{2}}^\mu(z) \propto \sqrt{\frac{2}{\pi}}\,\frac{\log(2z)-\psi\left(\frac{1}{2}-\mu\right)-\gamma}{\Gamma\left(\frac{1}{2}-\mu\right)\sqrt{z}}\left(1+O\left(\frac{1}{z}\right)\right) /;\ (|z|\to\infty)\bigwedge -\mu-\frac{1}{2}\in\mathbb{N}$$

Integral representations

On the real axis

Of the direct function

07.09.07.0001.01

$$\mathfrak{P}_\nu^\mu(z) = \frac{(-1)^{-\nu}\,2^{\mu-2\nu}}{\Gamma(-\mu-\nu)\,\Gamma(\nu+1)}\left(\frac{1}{1-z}\right)^{-\nu}\frac{(z+1)^{\mu/2}}{(z-1)^{\mu/2}}\int_{-1}^{1}\frac{(t-1)^\nu}{(t+1)^{\mu+\nu+1}}\left(t-\frac{z+3}{z-1}\right)^\nu dt /;\ -1<\text{Re}(\nu)<-\text{Re}(\mu)$$

07.09.07.0002.01

$$\mathfrak{P}_\nu^m(z) = \frac{(-\nu)_m}{\pi}\int_0^\pi \frac{\cos(m\,t)}{\left(z+\sqrt{z^2-1}\,\cos(t)\right)^{\nu+1}}\,dt /;\ m\in\mathbb{N}\land\text{Re}(z)>0$$

07.09.07.0003.01

$$\mathfrak{P}_\nu^\mu(z) = \frac{1}{\Gamma(-\mu)}\,(z+1)^{\mu/2}\,(z-1)^{\mu/2}\int_1^z P_\nu(t)\,(t-z)^{-\mu-1}\,dt /;\ \text{Re}(\mu)<0$$

Integral representations of negative integer order

07.09.07.0004.01

$$\mathfrak{P}_\nu^m(z) = (z+1)^{m/2}\,(z-1)^{m/2}\,\frac{\partial^m P_\nu(z)}{\partial z^m} /;\ m\in\mathbb{N}$$

Differential equations

Ordinary linear differential equations and wronskians

For the direct function itself

07.09.13.0001.01

$$\left(1-z^2\right) w''(z) - 2\, z\, w'(z) + \left(\nu\,(\nu+1) - \frac{\mu^2}{1-z^2}\right) w(z) = 0 \;/;\; w(z) = c_1\, \mathfrak{P}_\nu^\mu(z) + c_2\, \mathfrak{Q}_\nu^\mu(z)$$

07.09.13.0002.02

$$W_z\left(\mathfrak{P}_\nu^\mu(z), \mathfrak{Q}_\nu^\mu(z)\right) = \frac{e^{i\pi\mu}\, \Gamma(\mu+\nu+1)}{\left(1-z^2\right) \Gamma(-\mu+\nu+1)}$$

07.09.13.0003.01

$$g'(z)\, w''(z) - \left(\frac{2\, g(z)\, g'(z)^2}{1-g(z)^2} + g''(z)\right) w'(z) - \frac{\left(\mu^2 - \nu\,(\nu+1)\left(1-g(z)^2\right)\right) g'(z)^3}{\left(1-g(z)^2\right)^2}\, w(z) = 0 \;/;\; w(z) = c_1\, P_\nu^\mu(g(z)) + c_2\, Q_\nu^\mu(g(z))$$

07.09.13.0004.01

$$W_z\left(P_\nu^\mu(g(z)), Q_\nu^\mu(g(z))\right) = \frac{e^{i\pi\mu}\, \Gamma(\mu+\nu+1)}{\Gamma(1-\mu+\nu)}\, g'(z)\, (-g(z)-1)^{-\frac{\mu}{2}}\, (g(z)-1)^{-\frac{\mu}{2}} \left(1-g(z)^2\right)^{\frac{\mu}{2}-1}$$

07.09.13.0005.01

$$g'(z)\, h(z)^2\, w''(z) - \left(\left(\frac{2\, g(z)\, g'(z)^2}{1-g(z)^2} + g''(z)\right) h(z)^2 + 2\, g'(z)\, h'(z)\, h(z)\right) w'(z) + \left(-\frac{\mu^2 - \nu\,(\nu+1)\left(1-g(z)^2\right)}{\left(1-g(z)^2\right)^2}\, h(z)^2\, g'(z)^3 + 2\, h'(z)^2\, g'(z) + h(z)\left(h'(z)\left(\frac{2\, g(z)\, g'(z)^2}{1-g(z)^2} + g''(z)\right) - g'(z)\, h''(z)\right)\right) w(z) = 0 \;/;\; w(z) = c_1\, h(z)\, P_\nu^\mu(g(z)) + c_2\, h(z)\, Q_\nu^\mu(g(z))$$

07.09.13.0006.01

$$W_z\left(h(z)\, P_\nu^\mu(g(z)), h(z)\, Q_\nu^\mu(g(z))\right) = \frac{e^{i\pi\mu}\, \Gamma(\mu+\nu+1)}{\Gamma(1-\mu+\nu)}\, h(z)^2\, g'(z)\, (-g(z)-1)^{-\frac{\mu}{2}}\, (g(z)-1)^{-\frac{\mu}{2}} \left(1-g(z)^2\right)^{\frac{\mu}{2}-1}$$

07.09.13.0007.01

$$z^2\, w''(z) - z\left(2\, s + \frac{r\left(a^2\, z^{2r}+1\right)}{1-a^2\, z^{2r}} - 1\right) w'(z) + \left(-\frac{a^2\, r^2\left(\mu^2 + \left(a^2\, z^{2r}-1\right) \nu\,(\nu+1)\right) z^{2r}}{\left(1-a^2\, z^{2r}\right)^2} + s^2 + \frac{r\, s\left(a^2\, z^{2r}+1\right)}{1-a^2\, z^{2r}}\right) w(z) = 0 \;/;\; w(z) = c_1\, z^s\, P_\nu^\mu(a\, z^r) + c_2\, z^s\, Q_\nu^\mu(a\, z^r)$$

07.09.13.0008.01

$$W_z\left(z^s\, P_\nu^\mu(a\, z^r), z^s\, Q_\nu^\mu(a\, z^r)\right) = \frac{a\, e^{i\pi\mu}\, r\, z^{r+2s-1}\, \Gamma(\mu+\nu+1)}{\Gamma(-\mu+\nu+1)}\, (-a\, z^r-1)^{-\frac{\mu}{2}}\, (a\, z^r-1)^{-\frac{\mu}{2}} \left(1-a^2\, z^{2r}\right)^{\frac{\mu}{2}-1}$$

07.09.13.0009.01

$$w''(z) - \frac{a^2 (\log(r) - 2\log(s))\, r^{2z} + \log(r) + 2\log(s)}{1 - a^2 r^{2z}} w'(z) + \left(-\frac{a^2\left(\mu^2 - \left(1 - a^2 r^{2z}\right)\nu(\nu+1)\right)\log^2(r)\, r^{2z}}{\left(1 - a^2 r^{2z}\right)^2} + \log^2(s) + \frac{\left(a^2 r^{2z} + 1\right)\log(r)\log(s)}{1 - a^2 r^{2z}}\right) w(z) = 0 \;/;\; w(z) = c_1 s^z P_\nu^\mu(a r^z) + c_2 s^z Q_\nu^\mu(a r^z)$$

07.09.13.0010.01

$$W_z\left(s^z P_\nu^\mu(a r^z), s^z Q_\nu^\mu(a r^z)\right) = \frac{a e^{i\pi\mu} r^z (-a r^z - 1)^{-\frac{\mu}{2}} (a r^z - 1)^{-\frac{\mu}{2}} \left(1 - a^2 r^{2z}\right)^{\frac{\mu}{2}-1} s^{2z}\, \Gamma(\mu + \nu + 1)\log(r)}{\Gamma(-\mu + \nu + 1)}$$

Transformations

Transformations and argument simplifications

Argument involving basic arithmetic operations

07.09.16.0001.01

$$\mathrm{P}_{-\nu-1}^\mu(z) = \mathrm{P}_\nu^\mu(z)$$

07.09.16.0002.01

$$\mathrm{P}_\nu^{-m}(z) = \frac{\Gamma(\nu - m + 1)}{\Gamma(\nu + m + 1)} \mathrm{P}_\nu^m(z) \;/;\; m \in \mathbb{Z}$$

07.09.16.0004.01

$$\mathrm{P}_\nu^{-\mu}(z) = -\frac{1}{\pi}\Gamma(-\mu - \nu)\,\Gamma(\nu - \mu + 1)\frac{(z-1)^{\mu/2}}{(1-z)^{\mu/2}}\left(\sin(\nu\pi)\frac{(z-1)^{\mu/2}}{(1-z)^{\mu/2}}\mathrm{P}_\nu^\mu(z) + \sin(\mu\pi)\frac{(-z-1)^{\mu/2}}{(z+1)^{\mu/2}}\mathrm{P}_\nu^\mu(-z)\right)$$

07.09.16.0003.01

$$\mathrm{P}_\nu^\mu(-z) = -\csc(\mu\pi)\frac{\left(1 - z^2\right)^{\mu/2}}{(-z-1)^{\mu/2}(z-1)^{\mu/2}}\left(\sin(\pi\nu)\frac{(z-1)^\mu}{(1-z)^\mu}\mathrm{P}_\nu^\mu(z) + \frac{\pi}{\Gamma(-\mu-\nu)\,\Gamma(\nu-\mu+1)}\mathrm{P}_\nu^{-\mu}(z)\right) /;\; \mu \notin \mathbb{Z}$$

Identities

Recurrence identities

Consecutive neighbors

07.09.17.0001.01

$$\mathrm{P}_\nu^\mu(z) = \frac{(2\nu+3)z}{\mu+\nu+1}\mathrm{P}_{\nu+1}^\mu(z) + \frac{\mu-\nu-2}{\mu+\nu+1}\mathrm{P}_{\nu+2}^\mu(z)$$

07.09.17.0002.01

$$\mathrm{P}_\nu^\mu(z) = \frac{(2\nu-1)z}{\nu-\mu}\mathrm{P}_{\nu-1}^\mu(z) - \frac{\mu+\nu-1}{\nu-\mu}\mathrm{P}_{\nu-2}^\mu(z)$$

07.09.17.0003.01

$$\mathrm{P}_\nu^\mu(z) = \frac{2(\mu+1)z}{(\mu(\mu+1) - \nu(\nu+1))\sqrt{-z-1}\sqrt{1-z}}\mathrm{P}_\nu^{\mu+1}(z) - \frac{1}{\mu(\mu+1) - \nu(\nu+1)}\mathrm{P}_\nu^{\mu+2}(z)$$

07.09.17.0004.01

$$\mathrm{P}_\nu^\mu(z) = \frac{2(\mu-1)z}{\sqrt{-1-z}\sqrt{1-z}}\mathrm{P}_\nu^{\mu-1}(z) - ((\mu-2)(\mu-1)-\nu(\nu+1))\mathrm{P}_\nu^{\mu-2}(z)$$

Distant neighbors

07.09.17.0009.01

$$\mathrm{P}_\nu^\mu(z) = C_n(\nu,\mu,z)\mathrm{P}_{\nu+n}^\mu(z) + \frac{\mu-\nu-n-1}{n+\mu+\nu}C_{n-1}(\nu,\mu,z)\mathrm{P}_{\nu+n+1}^\mu(z)\;/;\;C_0(\nu,\mu,z) = 1 \bigwedge C_1(\nu,\mu,z) = \frac{(2\nu+3)z}{\mu+\nu+1} \bigwedge C_n(\nu,\mu,z) = \frac{z(2n+2\nu+1)}{n+\mu+\nu}C_{n-1}(\nu,\mu,z) + \frac{\mu-\nu-n}{n+\mu+\nu-1}C_{n-2}(\nu,\mu,z) \bigwedge n \in \mathbb{N}^+$$

07.09.17.0010.01

$$\mathrm{P}_\nu^\mu(z) = C_n(\nu,\mu,z)\mathrm{P}_{\nu-n}^\mu(z) - \frac{\mu+\nu-n}{\nu-\mu-n+1}C_{n-1}(\nu,\mu,z)\mathrm{P}_{\nu-n-1}^\mu(z)\;/;\;C_0(\nu,\mu,z) = 1 \bigwedge C_1(\nu,\mu,z) = \frac{(2\nu-1)z}{\nu-\mu} \bigwedge C_n(\nu,\mu,z) = \frac{z(2n-2\nu-1)}{n+\mu-\nu-1}C_{n-1}(\nu,\mu,z) - \frac{\mu+\nu-n+1}{\nu-\mu-n+2}C_{n-2}(\nu,\mu,z) \bigwedge n \in \mathbb{N}^+$$

07.09.17.0011.01

$$\mathrm{P}_\nu^\mu(z) = C_n(\nu,\mu,z)\mathrm{P}_\nu^{\mu+n}(z) - \frac{1}{(n+\mu-1)(n+\mu)-\nu(\nu+1)}C_{n-1}(\nu,\mu,z)\mathrm{P}_\nu^{\mu+n+1}(z)\;/;\;C_0(\nu,\mu,z) = 1 \bigwedge C_1(\nu,\mu,z) = \frac{2(\mu+1)z}{(\mu(\mu+1)-\nu(\nu+1))\sqrt{-z-1}\sqrt{1-z}} \bigwedge C_n(\nu,\mu,z) = \frac{2z(n+\mu)}{\sqrt{-z-1}\sqrt{1-z}((n+\mu-1)(n+\mu)-\nu(\nu+1))}C_{n-1}(\nu,\mu,z) - \frac{1}{(n+\mu-2)(n+\mu-1)-\nu(\nu+1)}C_{n-2}(\nu,\mu,z) \bigwedge n \in \mathbb{N}^+$$

07.09.17.0012.01

$$\mathrm{P}_\nu^\mu(z) = C_n(\nu,\mu,z)\mathrm{P}_\nu^{\mu-n}(z) - ((\mu-n-1)(\mu-n)-\nu(\nu+1))C_{n-1}(\nu,\mu,z)\mathrm{P}_\nu^{\mu-n-1}(z)\;/;\;C_0(\nu,\mu,z) = 1 \bigwedge C_1(\nu,\mu,z) = \frac{2(\mu-1)z}{\sqrt{-z-1}\sqrt{1-z}} \bigwedge C_n(\nu,\mu,z) = \frac{2z(\mu-n)}{\sqrt{-z-1}\sqrt{1-z}}C_{n-1}(\nu,\mu,z) - ((\mu-n)(\mu-n+1)-\nu(\nu+1))C_{n-2}(\nu,\mu,z) \bigwedge n \in \mathbb{N}^+$$

Functional identities

Relations between contiguous functions

07.09.17.0005.01

$$(\mu+\nu)\mathrm{P}_{\nu-1}^\mu(z) + (\nu-\mu+1)\mathrm{P}_{\nu+1}^\mu(z) = (2\nu+1)z\,\mathrm{P}_\nu^\mu(z)$$

07.09.17.0006.01

$$\mathrm{P}_\nu^\mu(z) = \frac{(\mu+\nu)\mathrm{P}_{\nu-1}^\mu(z) + (\nu-\mu+1)\mathrm{P}_{\nu+1}^\mu(z)}{(2\nu+1)z}$$

07.09.17.0007.01

$$\mathrm{P}_\nu^{\mu+1}(z) + (\mu(\mu-1)-\nu(\nu+1))\mathrm{P}_\nu^{\mu-1}(z) + \frac{2\mu z}{\sqrt{z-1}\sqrt{z+1}}\mathrm{P}_\nu^\mu(z) = 0$$

07.09.17.0013.01

$$z(\mu+\nu+1)P_\nu^\mu(z) + \sqrt{z-1}\sqrt{z+1}\,P_\nu^{\mu+1}(z) - (-\mu+\nu+1)P_{\nu+1}^\mu(z) = 0$$

Pavlyk O. (2006)

07.09.17.0014.01

$$P_\nu^{\mu+1}(z) - z\, P_{\nu+1}^{\mu+1}(z) + \sqrt{z-1}\; \sqrt{z+1}\; (-\mu+\nu+1)\, P_{\nu+1}^{\mu}(z) = 0$$

Pavlyk O. (2006)

Additional relations between contiguous functions

07.09.17.0008.01

$$\mathcal{P}_\nu^{\mu+1}(z)\, \mathcal{P}_{\nu_1}^{\mu_1+1}(z) - \mathcal{P}_{\nu-1}^{\mu+1}(z)\, \mathcal{P}_{\nu_1-1}^{\mu_1+1}(z) - (\nu-\mu)\,(\nu_1-\mu_1)\, \mathcal{P}_\nu^\mu(z)\, \mathcal{P}_{\nu_1}^{\mu_1}(z) + (\mu+\nu)\,(\mu_1+\nu_1)\, \mathcal{P}_{\nu-1}^{\mu}(z)\, \mathcal{P}_{\nu_1-1}^{\mu_1}(z) = 0$$

Differentiation

Low-order differentiation

With respect to ν

07.09.20.0001.01

$$\frac{\partial \mathcal{P}_\nu^\mu(z)}{\partial \nu} = \pi \cot(\pi\nu)\, \mathcal{P}_\nu^\mu(z) - \frac{(z+1)^{\mu/2}}{(z-1)^{\mu/2}} \sum_{k=0}^{\infty} \frac{(-\nu)_k\, (\nu+1)_k}{\Gamma(k-\mu+1)\, k!} \left(\psi(k-\nu) - \psi(k+\nu+1)\right) \left(\frac{1-z}{2}\right)^k /;\ \left|\frac{1-z}{2}\right| < 1 \bigwedge \nu \notin \mathbb{Z}$$

07.09.20.0002.01

$$\frac{\partial \mathcal{P}_\nu^\mu(z)}{\partial \nu} = \frac{(z+1)^{\mu/2}}{(z-1)^{\mu/2}} \sum_{k=0}^{\infty} \frac{1}{\Gamma(k-\mu+1)\, k!} \left(\frac{1-z}{2}\right)^k \sum_{j=1}^{k} S_k^{(j)}\, \nu^j \sum_{r=1}^{k} (-1)^r\, S_k^{(r)} \left(\frac{j}{\nu} + \frac{r}{\nu+1}\right) (\nu+1)^r /;\ \left|\frac{1-z}{2}\right| < 1$$

07.09.20.0003.01

$$\frac{\partial \mathcal{P}_\nu^\mu(z)}{\partial \nu} = \frac{2\nu+1}{2\,\Gamma(2-\mu)}\, (z-1)^{1-\mu/2}\, (z+1)^{\mu/2}\, F_{2\times0\times2}^{2\times1\times3}\left(\begin{matrix} 1-\nu,\ \nu+2;\ 1;\ 1,\ -\nu,\ \nu+1; \\ 2,\ 2-\mu;;\ \nu+2,\ 1-\nu; \end{matrix}\ \frac{1-z}{2},\ \frac{1-z}{2}\right)$$

07.09.20.0004.01

$$\frac{\partial^2 \mathcal{P}_\nu^\mu(z)}{\partial \nu^2} = \frac{(z+1)^{\mu/2}}{(z-1)^{\mu/2}} \sum_{k=0}^{\infty} \frac{(-\nu)_k\, (\nu+1)_k}{k!\, \Gamma(k-\mu+1)} \Big(\psi(k-\nu)^2 - 2\,(\pi\cot(\pi\nu) + \psi(k+\nu+1))\, \psi(k-\nu) + \psi(k+\nu+1)^2 + 2\pi\cot(\pi\nu)\, \psi(k+\nu+1) + \psi^{(1)}(k-\nu) + \psi^{(1)}(k+\nu+1)\Big) \left(\frac{1-z}{2}\right)^k - \pi^2\, \mathcal{P}_\nu^\mu(z) /;\ \left|\frac{1-z}{2}\right| < 1$$

07.09.20.0005.01

$$\frac{\partial^2 \mathcal{P}_\nu^\mu(z)}{\partial \nu^2} = \frac{(z+1)^{\mu/2}}{(z-1)^{\mu/2}} \sum_{k=0}^{\infty} \frac{1}{\Gamma(k-\mu+1)\, k!} \left(\frac{1-z}{2}\right)^k \sum_{i=1}^{k} \nu^{i-2}\, S_k^{(i)} \sum_{r=1}^{k} (-1)^r\, (\nu+1)^{r-2} \left((r-1)\, r\, \nu^2 + i^2\, (\nu+1)^2 + ((2r-1)\,\nu - 1)\, i\, (\nu+1)\right) S_k^{(r)} /;\ \left|\frac{1-z}{2}\right| < 1$$

With respect to μ

07.09.20.0006.01

$$\frac{\partial \mathrm{P}_\nu^\mu(z)}{\partial \mu} = \frac{1}{2}\left(\log(z+1)-\log(z-1)\right)\mathrm{P}_\nu^\mu(z) + \frac{(z+1)^{\mu/2}}{(z-1)^{\mu/2}}\sum_{k=0}^{\infty}\frac{(-\nu)_k\,(\nu+1)_k}{\Gamma(k-\mu+1)\,k!}\,\psi(k-\mu+1)\left(\frac{1-z}{2}\right)^k /;\left|\frac{1-z}{2}\right|<1\bigwedge \mu\notin\mathbb{N}^+$$

07.09.20.0007.01

$$\frac{\partial \mathrm{P}_\nu^\mu(z)}{\partial \mu} = \frac{\nu(\nu+1)(z-1)}{2(1-\mu)\,\Gamma(2-\mu)}\,\frac{(z+1)^{\mu/2}}{(z-1)^{\mu/2}}\,F_{2\times0\times1}^{2\times1\times2}\left(\begin{matrix}1-\nu,\ \nu+2;\ 1;\ 1,\ 1-\mu; \\ 2,\ 2-\mu;;\ 2-\mu;\end{matrix}\ \frac{1-z}{2},\ \frac{1-z}{2}\right) + \left(\psi(1-\mu)-\frac{1}{2}\left(\log(z+1)-\log(z-1)\right)\right)\mathrm{P}_\nu^\mu(z)$$

07.09.20.0008.01

$$\frac{\partial^2 \mathrm{P}_\nu^\mu(z)}{\partial \mu^2} = \frac{1}{4}\left(\log(z+1)-\log(z-1)\right)^2\,\mathrm{P}_\nu^\mu(z) + \frac{(z+1)^{\mu/2}}{(z-1)^{\mu/2}}\sum_{k=0}^{\infty}\frac{(-\nu)_k\,(\nu+1)_k}{k!\,\Gamma(k-\mu+1)}\left(\frac{1-z}{2}\right)^k\left(\psi(k-\mu+1)^2+\left(\log(z+1)-\log(z-1)\right)\psi(k-\mu+1)-\psi^{(1)}(k-\mu+1)\right) /;\left|\frac{1-z}{2}\right|<1\bigwedge \mu\notin\mathbb{N}^+$$

With respect to z

07.09.20.0009.01

$$\frac{\partial \mathrm{P}_\nu^\mu(z)}{\partial z} = \frac{1}{z^2-1}\left(z\,\nu\,\mathrm{P}_\nu^\mu(z)-(\mu+\nu)\,\mathrm{P}_{\nu-1}^\mu(z)\right)$$

07.09.20.0010.01

$$\frac{\partial^2 \mathrm{P}_\nu^\mu(z)}{\partial z^2} = \frac{2\,z\,(\mu+\nu)\,\mathrm{P}_{\nu-1}^\mu(z)+\left(\mu^2+\left((\nu-1)\,z^2-\nu-1\right)\nu\right)\mathrm{P}_\nu^\mu(z)}{\left(z^2-1\right)^2}$$

Symbolic differentiation

With respect to ν

07.09.20.0011.02

$$\frac{\partial^m \mathrm{P}_\nu^\mu(z)}{\partial \nu^m} = \frac{(z+1)^{\mu/2}}{(z-1)^{\mu/2}}\sum_{k=0}^{\infty}\frac{1}{\Gamma(k-\mu+1)\,k!}\left(\frac{1-z}{2}\right)^k\sum_{j=0}^{m}\binom{m}{j}\sum_{i=1}^{k}S_k^{(i)}\,(i-j+1)_j\,\nu^{i-j}\sum_{r=1}^{k}(-1)^r\,S_k^{(r)}\,(j-m+r+1)_{m-j}\,(\nu+1)^{j-m+r} /;\left|\frac{1-z}{2}\right|<1\bigwedge m\in\mathbb{N}$$

With respect to z

07.09.20.0012.02

$$\frac{\partial^m \mathrm{P}_\nu^\mu(z)}{\partial z^m} = \frac{(z+1)^{\mu/2}}{(z-1)^{\mu/2+m}}\,\Gamma\left(\frac{\mu}{2}+1\right)\sum_{j=0}^{m}\binom{m}{j}\,{}_2\tilde{F}_1\left(-j,\ \frac{\mu}{2};\ \frac{\mu}{2}-j+1;\ \frac{z+1}{z-1}\right){}_3\tilde{F}_2\left(1,\ -\nu,\ \nu+1;\ j-m+1,\ 1-\mu;\ \frac{1-z}{2}\right)\left(\frac{z-1}{z+1}\right)^j /; m\in\mathbb{N}$$

07.09.20.0014.01

$$\frac{\partial^m \mathfrak{P}_\nu^\mu(z)}{\partial z^m} = \frac{\Gamma\left(1-\frac{\mu}{2}\right)\Gamma(\mu+\nu+1)}{\Gamma(1-\mu+\nu)} \sum_{k=0}^{m}\sum_{j=0}^{k} \frac{2^{2j-k}\binom{m}{k} k!\,\Gamma(1-k+m-\mu+\nu)}{(k-j)!\,(2j-k)!\,\Gamma\left(1-j-\frac{\mu}{2}\right)\Gamma(k-m+\mu+\nu+1)} z^{2j-k}(z-1)^{\frac{1}{2}(-2j+k-m)}(z+1)^{\frac{1}{2}(-2j+k-m)}\mathfrak{P}_\nu^{k-m+\mu}(z) \;/;\; m\in\mathbb{N}$$

07.09.20.0015.01

$$\frac{\partial^m \mathfrak{P}_\nu^\mu(z)}{\partial z^m} = \sqrt{\pi}\sum_{k=0}^{m}(-1)^{m-k}(z-1)^{\frac{k-m}{2}} z^{-k}(z+1)^{\frac{k-m}{2}}\binom{m}{k} (-\mu-\nu)_{m-k}\, {}_3\tilde{F}_2\left(1,-k,\frac{\mu}{2};\frac{1-k}{2},1-\frac{k}{2};\frac{z^2}{z^2-1}\right)(1-\mu+\nu)_{m-k}\,\mathfrak{P}_\nu^{k-m+\mu}(z) \;/;\; m\in\mathbb{N}$$

Fractional integro-differentiation

With respect to z

07.09.20.0013.01

$$\frac{\partial^\alpha \mathfrak{P}_\nu^\mu(z)}{\partial z^\alpha} = \frac{(1-z)^{\mu/2}}{(z-1)^{\mu/2}}\left(\frac{\mu}{2} z^{1-\alpha}\sum_{k=0}^{\infty}(k+1)!\left(\frac{\mu}{2}+1\right)_k z^k\,\tilde{F}_{1\times 0\times 2}^{1\times 1\times 3}\left(\begin{matrix}k+2;-\frac{\mu}{2};-\nu,\nu+1,1-\frac{\mu}{2};\\ k-\alpha+2;;k+2,1-\mu;\end{matrix}\;-z,-\frac{z}{2}\right)+\Gamma\left(1-\frac{\mu}{2}\right)z^{-\alpha}\sum_{k=0}^{\infty}\left(-\frac{\mu}{2}\right)_k(-z)^k\,\tilde{F}_{2\times 1\times 1}^{3\times 1\times 1}\left(\begin{matrix}-\nu,\nu+1,1-\frac{\mu}{2};1;k+1;\\ 1,1-\mu;1-\frac{\mu}{2};k-\alpha+1;\end{matrix}\;\frac{1}{2},-\frac{z}{2}\right)\right)$$

Integration

Indefinite integration

Involving only one direct function

07.09.21.0001.01

$$\int \mathfrak{P}_\nu^\mu(z)\,dz = \frac{(1-z)^{\mu/2}}{(z-1)^{\mu/2}}\left(\frac{\mu}{2} z^2\sum_{k=0}^{\infty}(k+1)!\left(\frac{\mu}{2}+1\right)_k z^k\,\tilde{F}_{1\times 0\times 2}^{1\times 1\times 3}\left(\begin{matrix}k+2;-\frac{\mu}{2};-\nu,\nu+1,1-\frac{\mu}{2};\\ k+3;;k+2,1-\mu;\end{matrix}\;-z,-\frac{z}{2}\right)+\Gamma\left(1-\frac{\mu}{2}\right)z\sum_{k=0}^{\infty}\left(-\frac{\mu}{2}\right)_k(-z)^k\,\tilde{F}_{2\times 1\times 1}^{3\times 1\times 1}\left(\begin{matrix}-\nu,\nu+1,1-\frac{\mu}{2};1;k+1;\\ 1,1-\mu;1-\frac{\mu}{2};k+2;\end{matrix}\;\frac{1}{2},-\frac{z}{2}\right)\right)$$

Definite integration

Involving the direct function

07.09.21.0002.01

$$\int_{-1}^{1}\mathfrak{P}_n^m(t)^2\,dt = \frac{(-1)^m\,2\,(m+n)!}{(2n+1)\,(n-m)!} \;/;\; m\in\mathbb{N}\land n\in\mathbb{N}\land m\le n$$

07.09.21.0003.01

$$\int_{-1}^{1}\mathfrak{P}_n^m(t)\,\mathfrak{P}_l^m(t)\,dt = \frac{(-1)^m\,2\,(m+n)!\,\delta_{n,l}}{(2n+1)\,(n-m)!} \;/;\; l\in\mathbb{N}\bigwedge m\in\mathbb{N}\bigwedge n\in\mathbb{N}$$

Summation

Finite summation

07.09.23.0001.01

$$\sum_{m=-n}^{n} (-1)^m \, \mathrm{P}_n^m(z) \, \mathrm{P}_n^{-m}(z) == 1 \,/;\, n \in \mathbb{N}$$

Operations

Limit operation

07.09.25.0001.01

$$\lim_{\nu\to\infty} \nu^{-\mu} \, \mathrm{P}_\nu^\mu\left(\cosh\left(\frac{z}{\nu}\right)\right) == I_{-\mu}(z)$$

Representations through more general functions

Through hypergeometric functions

Involving ${}_2\tilde{F}_1$

07.09.26.0001.01

$$\mathrm{P}_\nu^\mu(z) == \frac{(z+1)^{\mu/2}}{(z-1)^{\mu/2}} \, {}_2\tilde{F}_1\left(-\nu, \nu+1; 1-\mu; \frac{1-z}{2}\right)$$

07.09.26.0081.01

$$\mathrm{P}_\nu^\mu(z) == -\pi \csc(\pi \mu) \left(\frac{z-1}{z+1}\right)^{-\frac{\mu}{2}} \left(\frac{2^\mu \sin(\pi \nu)}{\pi} (z+1)^{-\mu} \, {}_2\tilde{F}_1\left(-\mu+\nu+1, -\mu-\nu; 1-\mu; \frac{z+1}{2}\right) + \frac{1}{\Gamma(-\mu-\nu)\,\Gamma(-\mu+\nu+1)} \, {}_2\tilde{F}_1\left(-\nu, \nu+1; \mu+1; \frac{z+1}{2}\right)\right) /;\, \mu \notin \mathbb{Z}$$

07.09.26.0003.01

$$\mathrm{P}_\nu^\mu(z) == 2^{-\nu} \frac{(z+1)^{\mu/2+\nu}}{(z-1)^{\mu/2}} \, {}_2\tilde{F}_1\left(-\nu, -\mu-\nu; 1-\mu; \frac{z-1}{z+1}\right)$$

07.09.26.0082.01

$$\mathrm{P}_\nu^\mu(z) = \pi (z-1)^{-\frac{\mu}{2}} (z+1)^{\mu/2} \csc(2\nu\pi) \left(\frac{2^{\nu+1} (z-1)^{-\nu-1}}{\Gamma(-\mu-\nu)\,\Gamma(-\nu)} \, {}_2\tilde{F}_1\left(\nu+1, \mu+\nu+1; 2\nu+2; \frac{2}{1-z}\right) - \frac{2^{-\nu} (z-1)^\nu}{\Gamma(\nu+1)\,\Gamma(-\mu+\nu+1)} \, {}_2\tilde{F}_1\left(-\nu, \mu-\nu; -2\nu; \frac{2}{1-z}\right)\right) /;\, z \notin (-1, 1) \bigwedge 2\nu \notin \mathbb{Z}$$

07.09.26.0083.01

$$\mathcal{P}_\nu^\mu(z) = 2^\mu \pi (z-1)^{-\frac{\mu}{2}} (z+1)^{-\frac{\mu}{2}} \left(\frac{1}{\Gamma\left(\frac{1}{2}(-\mu-\nu+1)\right)\Gamma\left(\frac{1}{2}(-\mu+\nu+2)\right)} \, {}_2\tilde{F}_1\left(\frac{1}{2}(-\mu+\nu+1), -\frac{1}{2}(\mu+\nu); \frac{1}{2}; z^2\right) - \frac{z}{\Gamma\left(\frac{1}{2}(-\mu-\nu)\right)\Gamma\left(\frac{1}{2}(-\mu+\nu+1)\right)} \, {}_2\tilde{F}_1\left(\frac{1}{2}(-\mu-\nu+1), \frac{1}{2}(-\mu+\nu+2); \frac{3}{2}; z^2\right)\right)$$

07.09.26.0084.01

$$\mathcal{P}_\nu^\mu(z) == \frac{2^{-\nu-3}(\cos(2\pi\mu)-\cos(2\pi\nu))\sec(\pi\nu)}{\sqrt{\pi}} (z-1)^{-\frac{\mu}{2}} (z+1)^{-\frac{\mu}{2}} \left(\frac{1}{z}\Gamma(\mu+\nu+1)\left(\frac{\sqrt{-z^2}}{z}\sec\left(\frac{1}{2}\pi(\mu-\nu)\right) - \csc\left(\frac{1}{2}\pi(\mu-\nu)\right)\right)\left(-z^2\right)^{\frac{\mu-\nu}{2}} {}_2\tilde{F}_1\left(\frac{1}{2}(-\mu+\nu+1), \frac{1}{2}(-\mu+\nu+2); \nu+\frac{3}{2}; \frac{1}{z^2}\right) + 2^{2\nu+1}\Gamma(\mu-\nu)\left(\frac{\sqrt{-z^2}}{z}\sec\left(\frac{1}{2}\pi(\mu+\nu)\right) - \csc\left(\frac{1}{2}\pi(\mu+\nu)\right)\right) \left(-z^2\right)^{\frac{\mu+\nu}{2}} {}_2\tilde{F}_1\left(\frac{1}{2}(-\mu-\nu), \frac{1}{2}(-\mu-\nu+1); \frac{1}{2}-\nu; \frac{1}{z^2}\right)\right) /; z \notin (-1, 0)$$

Involving ${}_2F_1$

07.09.26.0004.01

$$\mathcal{P}_\nu^\mu(z) == \frac{1}{\Gamma(1-\mu)} \frac{(z+1)^{\mu/2}}{(z-1)^{\mu/2}} \, {}_2F_1\left(-\nu, \nu+1; 1-\mu; \frac{1-z}{2}\right) /; \mu \notin \mathbb{N}^+$$

07.09.26.0005.01

$$\mathcal{P}_\nu^\mu(z) == \frac{\Gamma(-\mu)}{\Gamma(-\mu-\nu)\Gamma(\nu-\mu+1)} \frac{(z+1)^{\mu/2}}{(z-1)^{\mu/2}} \, {}_2F_1\left(-\nu, \nu+1; \mu+1; \frac{z+1}{2}\right) - \frac{2^\mu \sin(\pi\nu)\Gamma(\mu)}{\pi} (z+1)^{-\mu/2}(z-1)^{-\mu/2} \, {}_2F_1\left(\nu-\mu+1, -\mu-\nu; 1-\mu; \frac{z+1}{2}\right) /; \mu \notin \mathbb{Z} \bigwedge \nu \notin \mathbb{Z}$$

07.09.26.0085.01

$$\mathcal{P}_\nu^\mu(z) == \frac{2^{-\nu}}{\Gamma(1-\mu)} \frac{(z+1)^{\mu/2+\nu}}{(z-1)^{\mu/2}} \, {}_2F_1\left(-\nu, -\mu-\nu; 1-\mu; \frac{z-1}{z+1}\right) /; \mu \notin \mathbb{N}^+$$

07.09.26.0006.01

$$\mathcal{P}_\nu^\mu(z) == \frac{2^\nu \Gamma(\nu+1/2)}{\sqrt{\pi}\,\Gamma(\nu-\mu+1)} \frac{(z+1)^{\mu/2}}{(z-1)^{\mu/2-\nu}} \, {}_2F_1\left(-\nu, \mu-\nu; -2\nu; \frac{2}{1-z}\right) + \frac{2^{-\nu-1}\Gamma(-\nu-1/2)}{\sqrt{\pi}\,\Gamma(-\mu-\nu)} \frac{(z+1)^{\mu/2}}{(z-1)^{\mu/2+\nu+1}} \, {}_2F_1\left(\nu+1, \mu+\nu+1; 2\nu+2; \frac{2}{1-z}\right) \Big/ /; z \notin (-1, 1) \bigwedge 2\nu \notin \mathbb{Z}$$

07.09.26.0086.01

$$\mathbf{P}_\nu^\mu(z) = 2^\mu \sqrt{\pi}\, (z-1)^{-\frac{\mu}{2}} (z+1)^{-\frac{\mu}{2}} \left(\frac{1}{\Gamma\left(\frac{1}{2}(-\mu-\nu+1)\right) \Gamma\left(\frac{1}{2}(-\mu+\nu+2)\right)} \,_2F_1\left(\frac{1}{2}(-\mu+\nu+1), -\frac{1}{2}(\mu+\nu); \frac{1}{2}; z^2\right) - \frac{2\,z}{\Gamma\left(\frac{1}{2}(-\mu-\nu)\right) \Gamma\left(\frac{1}{2}(-\mu+\nu+1)\right)} \,_2F_1\left(\frac{1}{2}(-\mu-\nu+1), \frac{1}{2}(-\mu+\nu+2); \frac{3}{2}; z^2\right) \right)$$

07.09.26.0087.01

$$\mathbf{P}_\nu^\mu(z) \equiv \frac{2^{-\nu-2} (\cos(2\pi\mu) - \cos(2\pi\nu))}{\pi\sqrt{\pi}} (z-1)^{-\frac{\mu}{2}} (z+1)^{-\frac{\mu}{2}} \left(\frac{\Gamma(\mu+\nu+1)}{z(2\nu+1)} \Gamma\left(\frac{1}{2}-\nu\right) \left(\frac{\sqrt{-z^2}}{z} \sec\left(\frac{1}{2}\pi(\mu-\nu)\right) - \csc\left(\frac{1}{2}\pi(\mu-\nu)\right) \right) \left(-z^2\right)^{\frac{\mu-\nu}{2}} \,_2F_1\left(\frac{1}{2}(-\mu+\nu+1), \frac{1}{2}(-\mu+\nu+2); \nu+\frac{3}{2}; \frac{1}{z^2}\right) + 2^{2\nu} \Gamma(\mu-\nu) \Gamma\left(\nu+\frac{1}{2}\right) \left(\frac{\sqrt{-z^2}}{z} \sec\left(\frac{1}{2}\pi(\mu+\nu)\right) - \csc\left(\frac{1}{2}\pi(\mu+\nu)\right) \right) \left(-z^2\right)^{\frac{\mu+\nu}{2}} \,_2F_1\left(\frac{1}{2}(-\mu-\nu), \frac{1}{2}(-\mu-\nu+1); \frac{1}{2}-\nu; \frac{1}{z^2}\right) \right) /; z \notin (-1, 0)$$

Through Meijer *G*

Classical cases for the direct function itself

07.09.26.0007.01

$$\mathbf{P}_\nu^\mu(z) \equiv -\frac{\sin(\pi\nu)}{\pi} \frac{(z+1)^{\mu/2}}{(z-1)^{\mu/2}} G_{2,2}^{1,2}\left(\frac{z-1}{2} \middle| \begin{matrix} \nu+1, -\nu \\ 0, \mu \end{matrix}\right) /; \nu \notin \mathbb{Z}$$

07.09.26.0008.01

$$\mathbf{P}_n^\mu(z) \equiv -\frac{1}{\pi} \frac{(z+1)^{\mu/2}}{(z-1)^{\mu/2}} \lim_{\nu\to n} \sin(\pi\nu)\, G_{2,2}^{1,2}\left(\frac{z-1}{2} \middle| \begin{matrix} \nu+1, -\nu \\ 0, \mu \end{matrix}\right) /; n \in \mathbb{Z}$$

07.09.26.0009.01

$$\mathbf{P}_\nu^\mu(2z+1) \equiv -\frac{\sin(\pi\nu)}{\pi} \frac{(z+1)^{\mu/2}}{z^{\mu/2}} G_{2,2}^{1,2}\left(z \middle| \begin{matrix} \nu+1, -\nu \\ 0, \mu \end{matrix}\right) /; \nu \notin \mathbb{Z}$$

Classical cases involving algebraic functions

07.09.26.0010.01

$$(z+1)^{\mu/2}\, \mathbf{P}_\nu^\mu(2z+1) \equiv \frac{1}{\Gamma(1-\mu+\nu)\,\Gamma(-\mu-\nu)} G_{2,2}^{1,2}\left(z \middle| \begin{matrix} \frac{\mu}{2}-\nu, \frac{\mu}{2}+\nu+1 \\ -\frac{\mu}{2}, \frac{\mu}{2} \end{matrix}\right) /; z \notin (-1, 0)$$

07.09.26.0011.01

$$(z+1)^{-\frac{\mu}{2}}\, \mathbf{P}_\nu^\mu(2z+1) \equiv -\frac{\sin(\pi\nu)}{\pi} G_{2,2}^{1,2}\left(z \middle| \begin{matrix} 1-\frac{\mu}{2}+\nu, -\frac{\mu}{2}-\nu \\ -\frac{\mu}{2}, \frac{\mu}{2} \end{matrix}\right) /; z \notin (-1, 0)$$

07.09.26.0012.01

$$(z+1)^{\mu/2}\, \mathbf{P}_\nu^\mu\left(1+\frac{2}{z}\right) \equiv \frac{1}{\Gamma(1-\mu+\nu)\,\Gamma(-\mu-\nu)} G_{2,2}^{2,1}\left(z \middle| \begin{matrix} \mu+1, 1 \\ \nu+1, -\nu \end{matrix}\right) /; z \notin (-1, 0)$$

07.09.26.0013.01

$$(z+1)^{-\frac{\mu}{2}}\,\mathfrak{P}_{\nu}^{\mu}\left(1+\frac{2}{z}\right) == -\frac{\sin(\pi\,\nu)}{\pi}\,G_{2,2}^{2,1}\left(z\,\middle|\,\begin{array}{c}1,\,1-\mu\\-\nu,\,\nu+1\end{array}\right)$$

07.09.26.0014.01

$$\mathfrak{P}_{\nu}^{\mu}\left(\sqrt{z+1}\right) == \frac{2^{\mu}}{\Gamma\left(\frac{1-\mu+\nu}{2}\right)\Gamma\left(-\frac{\mu+\nu}{2}\right)}\,G_{2,2}^{1,2}\left(z\,\middle|\,\begin{array}{c}\frac{1-\nu}{2},\,\frac{\nu}{2}+1\\-\frac{\mu}{2},\,\frac{\mu}{2}\end{array}\right)/;\, z\notin(-1,\,0)$$

07.09.26.0015.01

$$\mathfrak{P}_{\nu}^{\mu}\left(\sqrt{\frac{z+1}{z}}\right) == \frac{2^{\mu}}{\Gamma\left(\frac{1-\mu+\nu}{2}\right)\Gamma\left(-\frac{\mu+\nu}{2}\right)}\,G_{2,2}^{2,1}\left(z\,\middle|\,\begin{array}{c}\frac{\mu}{2}+1,\,1-\frac{\mu}{2}\\\frac{\nu+1}{2},\,-\frac{\nu}{2}\end{array}\right)/;\, z\notin(-1,\,0)$$

07.09.26.0016.01

$$\frac{1}{\sqrt{z+1}}\,\mathfrak{P}_{\nu}^{\mu}\left(\sqrt{z+1}\right) == \frac{2^{\mu}}{\Gamma\left(\frac{\nu-\mu}{2}+1\right)\Gamma\left(\frac{1-\mu-\nu}{2}\right)}\,G_{2,2}^{1,2}\left(z\,\middle|\,\begin{array}{c}-\frac{\nu}{2},\,\frac{\nu+1}{2}\\-\frac{\mu}{2},\,\frac{\mu}{2}\end{array}\right)/;\, z\notin(-1,\,0)$$

07.09.26.0017.01

$$\frac{1}{\sqrt{z+1}}\,\mathfrak{P}_{\nu}^{\mu}\left(\sqrt{\frac{z+1}{z}}\right) == \frac{2^{\mu}}{\Gamma\left(\frac{\nu-\mu}{2}+1\right)\Gamma\left(\frac{1-\mu-\nu}{2}\right)}\,G_{2,2}^{2,1}\left(z\,\middle|\,\begin{array}{c}\frac{\mu+1}{2},\,\frac{1-\mu}{2}\\\frac{\nu+1}{2},\,-\frac{\nu}{2}\end{array}\right)/;\, z\notin(-1,\,0)$$

07.09.26.0018.01

$$(z+1)^{\nu/2}\,\mathfrak{P}_{\nu}^{\mu}\left(\frac{z+2}{2\sqrt{z+1}}\right) == \frac{1}{\sqrt{\pi}\,\Gamma(-\mu-\nu)}\,G_{2,2}^{1,2}\left(z\,\middle|\,\begin{array}{c}\nu+1,\,\frac{1}{2}\\-\mu,\,\mu\end{array}\right)/;\, \mathrm{Re}(z)>0$$

07.09.26.0019.01

$$(z+1)^{\nu/2}\,\mathfrak{P}_{\nu}^{\mu}\left(\frac{2\,z+1}{2\sqrt{z^2+z}}\right) == \frac{1}{\sqrt{\pi}\,\Gamma(-\mu-\nu)}\,G_{2,2}^{2,1}\left(z\,\middle|\,\begin{array}{c}\mu+\frac{\nu}{2}+1,\,1-\mu+\frac{\nu}{2}\\\frac{\nu+1}{2},\,-\frac{\nu}{2}\end{array}\right)/;\, \mathrm{Re}(z)>0$$

Classical cases involving unit step θ

07.09.26.0020.01

$$\theta(|z|-1)\,(z-1)^{-\frac{\mu}{2}}\,\mathfrak{P}_{\nu}^{\mu}(2\,z-1) == G_{2,2}^{0,2}\left(z\,\middle|\,\begin{array}{c}1-\frac{\mu}{2}+\nu,\,-\frac{\mu}{2}-\nu\\-\frac{\mu}{2},\,\frac{\mu}{2}\end{array}\right)$$

07.09.26.0021.01

$$\theta(1-|z|)\,(1-z)^{-\frac{\mu}{2}}\,\mathfrak{P}_{\nu}^{\mu}\left(\frac{2}{z}-1\right) == G_{2,2}^{2,0}\left(z\,\middle|\,\begin{array}{c}1,\,1-\mu\\-\nu,\,\nu+1\end{array}\right)$$

07.09.26.0022.01

$$\theta(|z|-1)\,(z-1)^{-\frac{\mu}{2}}\,\mathfrak{P}_{\nu}^{\mu}\left(\sqrt{z}\right) == 2^{\mu}\,G_{2,2}^{0,2}\left(z\,\middle|\,\begin{array}{c}\frac{1-\mu-\nu}{2},\,\frac{\nu-\mu}{2}+1\\0,\,\frac{1}{2}\end{array}\right)$$

07.09.26.0023.01

$$\theta(1-|z|)\,(1-z)^{-\frac{\mu}{2}}\,\mathfrak{P}_{\nu}^{\mu}\left(\frac{1}{\sqrt{z}}\right) == 2^{\mu}\,G_{2,2}^{2,0}\left(z\,\middle|\,\begin{array}{c}1-\frac{\mu}{2},\,\frac{1-\mu}{2}\\\frac{\nu+1}{2},\,-\frac{\nu}{2}\end{array}\right)/;\, z\notin(-1,\,0)$$

07.09.26.0024.01

$$\theta(1-|z|)\,(1-z)^{-\mu}\,\mathfrak{P}_\nu^\mu\!\left(\frac{z+1}{2\sqrt{z}}\right)=\frac{\Gamma\left(\frac{1}{2}-\mu\right)}{\sqrt{\pi}}\,G_{2,2}^{2,0}\!\left(z\,\middle|\,\begin{array}{c}1-\mu+\frac{\nu}{2},\ \frac{1-\nu}{2}-\mu\\ -\frac{\nu}{2},\ \frac{\nu+1}{2}\end{array}\right)/;\ z\notin(-1,0)$$

07.09.26.0025.01

$$\theta(|z|-1)\,(z-1)^{-\mu}\,\mathfrak{P}_\nu^\mu\!\left(\frac{z+1}{2\sqrt{z}}\right)=\frac{\Gamma\left(\frac{1}{2}-\mu\right)}{\sqrt{\pi}}\,G_{2,2}^{0,2}\!\left(z\,\middle|\,\begin{array}{c}1-\mu+\frac{\nu}{2},\ \frac{1-\nu}{2}-\mu\\ -\frac{\nu}{2},\ \frac{\nu+1}{2}\end{array}\right)$$

Classical cases involving sgn

07.09.26.0026.01

$$(\operatorname{sgn}(1-|z|)\,(1-z))^{\nu}\,\mathfrak{P}_\nu^\mu\!\left(\frac{z+1}{\operatorname{sgn}(1-|z|)\,(1-z)}\right)=\frac{\Gamma(\nu+1)}{\Gamma(-\mu-\nu)}\,G_{2,2}^{1,1}\!\left(z\,\middle|\,\begin{array}{c}\frac{\mu}{2}+\nu+1,\ 1-\frac{\mu}{2}+\nu\\ -\frac{\mu}{2},\ \frac{\mu}{2}\end{array}\right)$$

07.09.26.0027.01

$$(\operatorname{sgn}(|z|-1)\,(z-1))^{\mu}\,\mathfrak{P}_\nu^\mu\!\left(\frac{z+1}{2\sqrt{z}}\right)=\frac{\cos(\pi\mu)\,\Gamma\left(\mu+\frac{1}{2}\right)}{\pi^{3/2}\,\Gamma(-\mu-\nu)\,\Gamma(1-\mu+\nu)}\,G_{2,2}^{2,2}\!\left(z\,\middle|\,\begin{array}{c}\mu+\frac{\nu}{2}+1,\ \mu+\frac{1-\nu}{2}\\ -\frac{\nu}{2},\ \frac{\nu+1}{2}\end{array}\right)$$

07.09.26.0028.01

$$(\operatorname{sgn}(|z|-1)\,(z-1))^{-\mu}\,\mathfrak{P}_\nu^\mu\!\left(\frac{z+1}{2\sqrt{z}}\right)=$$

$$\frac{\Gamma\left(\frac{1}{2}-\mu\right)}{\sqrt{\pi}\,\cos(\pi\nu)}\left(\sin(\pi(\mu-\nu))\,G_{2,2}^{1,1}\!\left(z\,\middle|\,\begin{array}{c}1-\mu+\frac{\nu}{2},\ \frac{1-\nu}{2}-\mu\\ -\frac{\nu}{2},\ \frac{\nu+1}{2}\end{array}\right)+\sin(\pi(\mu+\nu))\,G_{2,2}^{1,1}\!\left(z\,\middle|\,\begin{array}{c}\frac{1-\nu}{2}-\mu,\ 1-\mu+\frac{\nu}{2}\\ \frac{\nu+1}{2},\ -\frac{\nu}{2}\end{array}\right)\right)/;\ z\notin(-1,0)$$

Classical cases for powers of Legendre *P*

07.09.26.0029.01

$$\mathfrak{P}_\nu^\mu\!\left(\sqrt{z+1}\right)^2=\frac{1}{\sqrt{\pi}\,\Gamma(-\mu+\nu+1)\,\Gamma(-\mu-\nu)}\,G_{3,3}^{1,3}\!\left(z\,\middle|\,\begin{array}{c}\nu+1,\ -\nu,\ \frac{1}{2}\\ -\mu,\ 0,\ \mu\end{array}\right)/;\ z\notin(-1,0)$$

07.09.26.0030.01

$$\mathfrak{P}_\nu^\mu\!\left(\sqrt{\frac{z+1}{z}}\right)^2=\frac{1}{\sqrt{\pi}\,\Gamma(1-\mu+\nu)\,\Gamma(-\mu-\nu)}\,G_{3,3}^{3,1}\!\left(z\,\middle|\,\begin{array}{c}\mu+1,\ 1,\ 1-\mu\\ -\nu,\ \nu+1,\ \frac{1}{2}\end{array}\right)/;\ z\notin(-1,0)$$

Classical cases involving products of Legendre *P*

07.09.26.0031.01

$$\mathfrak{P}_\nu^\mu\!\left(\sqrt{z+1}\right)\mathfrak{P}_\nu^{-\mu}\!\left(\sqrt{z+1}\right)=-\frac{\sin(\pi\nu)}{\pi^{3/2}}\,G_{3,3}^{1,3}\!\left(z\,\middle|\,\begin{array}{c}\nu+1,\ -\nu,\ \frac{1}{2}\\ 0,\ -\mu,\ \mu\end{array}\right)$$

07.09.26.0032.01

$$\mathfrak{P}_\nu^\mu\!\left(\sqrt{\frac{z+1}{z}}\right)\mathfrak{P}_\nu^{-\mu}\!\left(\sqrt{\frac{z+1}{z}}\right)=-\frac{\sin(\pi\nu)}{\pi^{3/2}}\,G_{3,3}^{3,1}\!\left(z\,\middle|\,\begin{array}{c}1,\ \mu+1,\ 1-\mu\\ \nu+1,\ -\nu,\ \frac{1}{2}\end{array}\right)$$

07.09.26.0033.01

$$\frac{1}{\sqrt{z+1}}\,\mathfrak{P}_\nu^\mu\!\left(\sqrt{z+1}\right)\mathfrak{P}_{-\nu}^\mu\!\left(\sqrt{z+1}\right)=\frac{1}{\sqrt{\pi}\,\Gamma(1-\mu+\nu)\,\Gamma(1-\mu-\nu)}\,G_{3,3}^{1,3}\!\left(z\,\middle|\,\begin{array}{c}\nu,\ -\nu,\ \frac{1}{2}\\ -\mu,\ 0,\ \mu\end{array}\right)/;\ z\notin(-1,0)$$

07.09.26.0034.01

$$\frac{1}{\sqrt{z+1}}\,\mathfrak{P}_{\nu}^{\mu}\left(\sqrt{\frac{z+1}{z}}\right)\mathfrak{P}_{-\nu}^{\mu}\left(\sqrt{\frac{z+1}{z}}\right)=\frac{1}{\sqrt{\pi}\;\Gamma(1-\mu+\nu)\,\Gamma(1-\mu-\nu)}\,G_{3,3}^{3,1}\left(z\,\middle|\,\begin{array}{c}\mu+\frac{1}{2},\frac{1}{2},\frac{1}{2}-\mu\\ \frac{1}{2}-\nu,\nu+\frac{1}{2},0\end{array}\right)/;\,z\notin(-1,0)$$

07.09.26.0035.01

$$\frac{1}{\sqrt{z+1}}\,\mathfrak{P}_{\nu}^{\mu}\left(\sqrt{z+1}\right)\mathfrak{P}_{\nu}^{\mu+1}\left(\sqrt{z+1}\right)=\frac{1}{\sqrt{\pi}\;\Gamma(1-\mu+\nu)\,\Gamma(-\mu-\nu)}\,G_{3,3}^{1,3}\left(z\,\middle|\,\begin{array}{c}0,\nu+\frac{1}{2},-\nu-\frac{1}{2}\\ -\mu-\frac{1}{2},-\frac{1}{2},\mu+\frac{1}{2}\end{array}\right)/;\,z\notin(-1,0)$$

07.09.26.0036.01

$$\frac{1}{\sqrt{z+1}}\,\mathfrak{P}_{\nu}^{\mu}\left(\sqrt{\frac{z+1}{z}}\right)\mathfrak{P}_{\nu}^{\mu+1}\left(\sqrt{\frac{z+1}{z}}\right)=\frac{1}{\sqrt{\pi}\;\Gamma(1-\mu+\nu)\,\Gamma(-\mu-\nu)}\,G_{3,3}^{3,1}\left(z\,\middle|\,\begin{array}{c}\mu+1,1,-\mu\\ \frac{1}{2},-\nu,\nu+1\end{array}\right)/;\,z\notin(-1,0)$$

07.09.26.0037.01

$$\mathfrak{P}_{\nu}^{\mu}\left(\sqrt{z+1}\right)\mathfrak{P}_{-\mu-\frac{1}{2}}^{-\nu-\frac{1}{2}}\left(\sqrt{1+\frac{1}{z}}\right)=\frac{1}{\sqrt{2}\;\pi\,\Gamma(1-\mu+\nu)}\,G_{3,3}^{2,2}\left(z\,\middle|\,\begin{array}{c}\frac{3}{4},\frac{1}{4}-\nu,\nu+\frac{5}{4}\\ \frac{1}{4},\frac{1}{4}-\mu,\mu+\frac{1}{4}\end{array}\right)/;\,z\notin(-\infty,-1)$$

07.09.26.0038.01

$$\mathfrak{P}_{\nu}^{\mu}\left(\sqrt{z+1}\right)\mathfrak{P}_{\mu-\frac{1}{2}}^{\nu+\frac{1}{2}}\left(\sqrt{1+\frac{1}{z}}\right)=\frac{1}{\sqrt{2}\;\pi\,\Gamma(-\mu-\nu)}\,G_{3,3}^{2,2}\left(z\,\middle|\,\begin{array}{c}\frac{3}{4},\nu+\frac{5}{4},\frac{1}{4}-\nu\\ \frac{1}{4},\frac{1}{4}-\mu,\mu+\frac{1}{4}\end{array}\right)/;\,z\notin(-\infty,-1)$$

Classical cases involving Legendre *Q*

07.09.26.0039.01

$$\mathfrak{P}_{\nu}^{\mu}\left(\sqrt{z+1}\right)\mathfrak{Q}_{\nu}^{\mu}\left(\sqrt{z+1}\right)=\frac{e^{\pi i\mu}\,\Gamma(\mu+\nu+1)}{2\sqrt{\pi}\;\Gamma(1-\mu+\nu)}\,G_{3,3}^{2,2}\left(z\,\middle|\,\begin{array}{c}\frac{1}{2},-\nu,\nu+1\\ 0,-\mu,\mu\end{array}\right)/;\,z\notin(-\infty,-1)$$

07.09.26.0040.01

$$\mathfrak{P}_{\nu}^{\mu}\left(\sqrt{\frac{z+1}{z}}\right)\mathfrak{Q}_{\nu}^{\mu}\left(\sqrt{\frac{z+1}{z}}\right)=\frac{e^{\pi i\mu}\,\Gamma(\mu+\nu+1)}{2\sqrt{\pi}\;\Gamma(1-\mu+\nu)}\,G_{3,3}^{2,2}\left(z\,\middle|\,\begin{array}{c}1,\mu+1,1-\mu\\ \frac{1}{2},\nu+1,-\nu\end{array}\right)/;\,z\notin(-\infty,-1)$$

07.09.26.0041.01

$$\mathfrak{P}_{\nu}^{-\mu}\left(\sqrt{z+1}\right)\mathfrak{Q}_{\nu}^{\mu}\left(\sqrt{z+1}\right)=\frac{e^{\pi i\mu}}{2\sqrt{\pi}}\,G_{3,3}^{2,2}\left(z\,\middle|\,\begin{array}{c}\frac{1}{2},-\nu,\nu+1\\ 0,\mu,-\mu\end{array}\right)/;\,z\notin(-\infty,-1)$$

07.09.26.0042.01

$$\mathfrak{P}_{\nu}^{-\mu}\left(\sqrt{\frac{z+1}{z}}\right)\mathfrak{Q}_{\nu}^{\mu}\left(\sqrt{\frac{z+1}{z}}\right)=\frac{e^{\pi i\mu}}{2\sqrt{\pi}}\,G_{3,3}^{2,2}\left(z\,\middle|\,\begin{array}{c}1,1-\mu,\mu+1\\ \frac{1}{2},\nu+1,-\nu\end{array}\right)/;\,z\notin(-\infty,-1)$$

07.09.26.0043.01

$$\mathfrak{P}_{-\mu-\frac{1}{2}}^{-\nu-\frac{1}{2}}\left(\sqrt{1+\frac{1}{z}}\right)\mathfrak{Q}_{\nu}^{\mu}\left(\sqrt{z+1}\right)=\frac{e^{\pi i\mu}}{\sqrt{2}\;\Gamma(1-\mu+\nu)}\,G_{3,3}^{3,1}\left(z\,\middle|\,\begin{array}{c}\frac{1}{4}-\nu,\nu+\frac{5}{4},\frac{3}{4}\\ \frac{1}{4},\mu+\frac{1}{4},\frac{1}{4}-\mu\end{array}\right)/;\,z\notin(-1,0)$$

07.09.26.0044.01

$$\mathfrak{P}_{-\mu-\frac{1}{2}}^{-\nu-\frac{1}{2}}\left(\sqrt{z+1}\right)\mathfrak{Q}_{\nu}^{\mu}\left(\sqrt{\frac{z+1}{z}}\right)=\frac{e^{\pi i\mu}}{\sqrt{2}\;\Gamma(1-\mu+\nu)}\,G_{3,3}^{1,3}\left(z\,\middle|\,\begin{array}{c}\frac{3}{4},\frac{3}{4}-\mu,\mu+\frac{3}{4}\\ \nu+\frac{3}{4},-\nu-\frac{1}{4},\frac{1}{4}\end{array}\right)/;\,z\notin(-1,0)$$

07.09.26.0045.01

$$\mathfrak{P}_{\mu-\frac{1}{2}}^{\nu+\frac{1}{2}}\left(\sqrt{1+\frac{1}{z}}\right)\mathfrak{Q}_{\nu}^{\mu}\left(\sqrt{z+1}\right)=\frac{e^{\pi i\mu}\cos(\pi\mu)\,\Gamma(\mu+\nu+1)}{\pi\sqrt{2}}\,G_{3,3}^{3,1}\left(z\,\middle|\,\begin{matrix}\frac{3}{4},\nu+\frac{5}{4},\frac{1}{4}-\nu\\ \frac{1}{4},\mu+\frac{1}{4},\frac{1}{4}-\mu\end{matrix}\right)/;\,z\notin(-1,0)$$

07.09.26.0046.01

$$\mathfrak{P}_{\mu-\frac{1}{2}}^{\nu+\frac{1}{2}}\left(\sqrt{z+1}\right)\mathfrak{Q}_{\nu}^{\mu}\left(\sqrt{\frac{z+1}{z}}\right)=\frac{e^{\pi i\mu}\cos(\pi\mu)\,\Gamma(\mu+\nu+1)}{\pi\sqrt{2}}\,G_{3,3}^{1,3}\left(z\,\middle|\,\begin{matrix}\frac{3}{4},\frac{3}{4}-\mu,\mu+\frac{3}{4}\\ \frac{1}{4},-\nu-\frac{1}{4},\nu+\frac{3}{4}\end{matrix}\right)/;\,z\notin(-1,0)$$

07.09.26.0047.01

$$\frac{1}{\sqrt{z+1}}\mathfrak{P}_{\mu-\frac{1}{2}}^{-\nu-\frac{3}{2}}\left(\sqrt{1+\frac{1}{z}}\right)\mathfrak{Q}_{\nu}^{\mu}\left(\sqrt{z+1}\right)=\frac{e^{\pi i\mu}}{\sqrt{2}\,(\mu+\nu+1)\,\Gamma(2-\mu+\nu)}\,G_{3,3}^{3,1}\left(z\,\middle|\,\begin{matrix}-\nu-\frac{3}{4},\frac{3}{4},\nu+\frac{5}{4}\\ \frac{1}{4},\mu+\frac{1}{4},\frac{1}{4}-\mu\end{matrix}\right)/;\,z\notin(-1,0)$$

07.09.26.0048.01

$$\frac{1}{\sqrt{z+1}}\mathfrak{P}_{\mu-\frac{1}{2}}^{-\nu-\frac{3}{2}}\left(\sqrt{z+1}\right)\mathfrak{Q}_{\nu}^{\mu}\left(\sqrt{\frac{z+1}{z}}\right)=\frac{e^{\pi i\mu}}{\sqrt{2}\,(\mu+\nu+1)\,\Gamma(2-\mu+\nu)}\,G_{3,3}^{1,3}\left(z\,\middle|\,\begin{matrix}\frac{1}{4},\frac{1}{4}-\mu,\mu+\frac{1}{4}\\ \nu+\frac{5}{4},-\frac{1}{4},-\nu-\frac{3}{4}\end{matrix}\right)/;\,z\notin(-1,0)$$

07.09.26.0049.01

$$\frac{1}{\sqrt{z+1}}\mathfrak{P}_{\mu-\frac{1}{2}}^{\frac{1}{2}-\nu}\left(\sqrt{1+\frac{1}{z}}\right)\mathfrak{Q}_{\nu}^{\mu}\left(\sqrt{z+1}\right)=\frac{e^{\pi i\mu}}{\sqrt{2}\,\Gamma(1-\mu+\nu)}\,G_{3,3}^{3,1}\left(z\,\middle|\,\begin{matrix}\frac{1}{4}-\nu,\frac{3}{4},\nu+\frac{1}{4}\\ \frac{1}{4},\mu+\frac{1}{4},\frac{1}{4}-\mu\end{matrix}\right)/;\,z\notin(-1,0)$$

07.09.26.0050.01

$$\frac{1}{\sqrt{z+1}}\mathfrak{P}_{\mu-\frac{1}{2}}^{\frac{1}{2}-\nu}\left(\sqrt{z+1}\right)\mathfrak{Q}_{\nu}^{\mu}\left(\sqrt{\frac{z+1}{z}}\right)=\frac{e^{\pi i\mu}}{\sqrt{2}\,\Gamma(1-\mu+\nu)}\,G_{3,3}^{1,3}\left(z\,\middle|\,\begin{matrix}\frac{1}{4},\frac{1}{4}-\mu,\mu+\frac{1}{4}\\ \nu+\frac{1}{4},-\frac{1}{4},\frac{1}{4}-\nu\end{matrix}\right)/;\,z\notin(-1,0)$$

07.09.26.0051.01

$$\frac{1}{\sqrt{z+1}}\mathfrak{P}_{\mu+\frac{1}{2}}^{-\nu-\frac{1}{2}}\left(\sqrt{1+\frac{1}{z}}\right)\mathfrak{Q}_{\nu}^{\mu}\left(\sqrt{z+1}\right)=\frac{e^{\pi i\mu}}{\sqrt{2}\,(\mu+\nu+1)\,\Gamma(1-\mu+\nu)}\,G_{3,3}^{3,1}\left(z\,\middle|\,\begin{matrix}-\nu-\frac{1}{4},\frac{1}{4},\nu+\frac{3}{4}\\ -\frac{1}{4},\mu+\frac{3}{4},-\mu-\frac{1}{4}\end{matrix}\right)/;\,z\notin(-1,0)$$

07.09.26.0052.01

$$\frac{1}{\sqrt{z+1}}\mathfrak{P}_{\mu+\frac{1}{2}}^{-\nu-\frac{1}{2}}\left(\sqrt{z+1}\right)\mathfrak{Q}_{\nu}^{\mu}\left(\sqrt{\frac{z+1}{z}}\right)=\frac{e^{\pi i\mu}}{\sqrt{2}\,(\mu+\nu+1)\,\Gamma(1-\mu+\nu)}\,G_{3,3}^{1,3}\left(z\,\middle|\,\begin{matrix}\frac{3}{4},-\mu-\frac{1}{4},\mu+\frac{3}{4}\\ \nu+\frac{3}{4},\frac{1}{4},-\nu-\frac{1}{4}\end{matrix}\right)/;\,z\notin(-1,0)$$

Generalized cases involving algebraic functions

07.09.26.0053.01

$$\mathfrak{P}_{\nu}^{\mu}\left(\frac{\sqrt{z^2+1}}{z}\right)=\frac{2^{\mu}}{\Gamma\left(\frac{1-\mu+\nu}{2}\right)\Gamma\left(-\frac{\mu+\nu}{2}\right)}\,G_{2,2}^{2,1}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{\mu}{2}+1,1-\frac{\mu}{2}\\ \frac{\nu+1}{2},-\frac{\nu}{2}\end{matrix}\right)/;\,\mathrm{Re}(z)>0$$

07.09.26.0054.01

$$\frac{1}{\sqrt{z^2+1}}\mathfrak{P}_{\nu}^{\mu}\left(\frac{\sqrt{z^2+1}}{z}\right)=\frac{2^{\mu}}{\Gamma\left(\frac{\nu-\mu}{2}+1\right)\Gamma\left(\frac{1-\mu-\nu}{2}\right)}\,G_{2,2}^{2,1}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{\mu+1}{2},\frac{1-\mu}{2}\\ \frac{\nu+1}{2},-\frac{\nu}{2}\end{matrix}\right)/;\,\mathrm{Re}(z)>0$$

07.09.26.0055.01

$$\left(z^2+1\right)^{\nu/2}\mathfrak{P}_{\nu}^{\mu}\left(\frac{2z^2+1}{2z\sqrt{z^2+1}}\right)=\frac{1}{\sqrt{\pi}\,\Gamma(-\mu-\nu)}\,G_{2,2}^{2,1}\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\mu+\frac{\nu}{2}+1,1-\mu+\frac{\nu}{2}\\ \frac{\nu+1}{2},-\frac{\nu}{2}\end{matrix}\right)/;\,\mathrm{Re}(z)>0$$

Generalized cases involving unit step θ

07.09.26.0056.01

$$\theta(|z|-1)\left(z^2-1\right)^{-\frac{\mu}{2}}\,\mathfrak{P}_\nu^\mu(z)=2^\mu\,G_{2,2}^{0,2}\!\left(z,\,\frac{1}{2}\,\middle|\,\begin{array}{c}\frac{1-\mu-\nu}{2},\,\frac{\nu-\mu}{2}+1\\0,\,\frac{1}{2}\end{array}\right)/;\,\mathrm{Re}(z)>0$$

07.09.26.0057.01

$$\theta(1-|z|)\left(1-z^2\right)^{-\frac{\mu}{2}}\,\mathfrak{P}_\nu^\mu\!\left(\frac{1}{z}\right)=2^\mu\,G_{2,2}^{2,0}\!\left(z,\,\frac{1}{2}\,\middle|\,\begin{array}{c}1-\frac{\mu}{2},\,\frac{1-\mu}{2}\\\frac{\nu+1}{2},\,-\frac{\nu}{2}\end{array}\right)$$

07.09.26.0058.01

$$\theta(1-|z|)\left(1-z^2\right)^{-\mu}\,\mathfrak{P}_\nu^\mu\!\left(\frac{z^2+1}{2\,z}\right)=\frac{\Gamma\left(\frac{1}{2}-\mu\right)}{\sqrt{\pi}}\,G_{2,2}^{2,0}\!\left(z,\,\frac{1}{2}\,\middle|\,\begin{array}{c}1-\mu+\frac{\nu}{2},\,\frac{1-\nu}{2}-\mu\\-\frac{\nu}{2},\,\frac{\nu+1}{2}\end{array}\right)$$

07.09.26.0059.01

$$\theta(|z|-1)\left(z^2-1\right)^{-\mu}\,\mathfrak{P}_\nu^\mu\!\left(\frac{z^2+1}{2\,z}\right)=\frac{\Gamma\left(\frac{1}{2}-\mu\right)}{\sqrt{\pi}}\,G_{2,2}^{0,2}\!\left(z,\,\frac{1}{2}\,\middle|\,\begin{array}{c}1-\mu+\frac{\nu}{2},\,\frac{1-\nu}{2}-\mu\\-\frac{\nu}{2},\,\frac{\nu+1}{2}\end{array}\right)/;\,\mathrm{Re}(z)>0$$

Generalized cases involving sgn

07.09.26.0060.01

$$\left(\mathrm{sgn}(|z|-1)\left(z^2-1\right)\right)^\mu\,\mathfrak{P}_\nu^\mu\!\left(\frac{z^2+1}{2\,z}\right)=\frac{\cos(\pi\,\mu)\,\Gamma\left(\mu+\frac{1}{2}\right)}{\pi^{3/2}\,\Gamma(-\mu-\nu)\,\Gamma(1-\mu+\nu)}\,G_{2,2}^{2,2}\!\left(z,\,\frac{1}{2}\,\middle|\,\begin{array}{c}\mu+\frac{\nu}{2}+1,\,\mu+\frac{1-\nu}{2}\\-\frac{\nu}{2},\,\frac{\nu+1}{2}\end{array}\right)/;\,\mathrm{Re}(z)>0$$

07.09.26.0061.01

$$\left(\mathrm{sgn}(|z|-1)\left(z^2-1\right)\right)^{-\mu}\,\mathfrak{P}_\nu^\mu\!\left(\frac{z^2+1}{2\,z}\right)=$$

$$\frac{\Gamma\left(\frac{1}{2}-\mu\right)}{\sqrt{\pi}\,\cos(\pi\,\nu)}\left(\sin(\pi(\mu-\nu))\,G_{2,2}^{1,1}\!\left(z,\,\frac{1}{2}\,\middle|\,\begin{array}{c}1-\mu+\frac{\nu}{2},\,\frac{1-\nu}{2}-\mu\\-\frac{\nu}{2},\,\frac{\nu+1}{2}\end{array}\right)+\sin(\pi(\mu+\nu))\,G_{2,2}^{1,1}\!\left(z,\,\frac{1}{2}\,\middle|\,\begin{array}{c}\frac{1-\nu}{2}-\mu,\,1-\mu+\frac{\nu}{2}\\\frac{\nu+1}{2},\,-\frac{\nu}{2}\end{array}\right)\right)/;\,\mathrm{Re}(z)>0$$

Generalized cases for powers of Legendre P

07.09.26.0062.01

$$\mathfrak{P}_\nu^\mu\!\left(\frac{\sqrt{z^2+1}}{z}\right)^2=\frac{1}{\sqrt{\pi}\,\Gamma(1-\mu+\nu)\,\Gamma(-\mu-\nu)}\,G_{3,3}^{3,1}\!\left(z,\,\frac{1}{2}\,\middle|\,\begin{array}{c}\mu+1,\,1,\,1-\mu\\-\nu,\,\nu+1,\,\frac{1}{2}\end{array}\right)/;\,\mathrm{Re}(z)>0$$

Classical cases involving products of Legendre P

07.09.26.0063.01

$$\mathfrak{P}_\nu^\mu\!\left(\frac{\sqrt{z^2+1}}{z}\right)\mathfrak{P}_\nu^{-\mu}\!\left(\frac{\sqrt{z^2+1}}{z}\right)=-\frac{\sin(\pi\,\nu)}{\pi^{3/2}}\,G_{3,3}^{3,1}\!\left(z,\,\frac{1}{2}\,\middle|\,\begin{array}{c}1,\,\mu+1,\,1-\mu\\\nu+1,\,-\nu,\,\frac{1}{2}\end{array}\right)/;\,\mathrm{Re}(z)>0$$

07.09.26.0064.01

$$\frac{1}{\sqrt{z^2+1}}\,\mathfrak{P}_\nu^\mu\!\left(\frac{\sqrt{z^2+1}}{z}\right)\mathfrak{P}_{-\nu}^\mu\!\left(\frac{\sqrt{z^2+1}}{z}\right)=\frac{1}{\sqrt{\pi}\,\Gamma(1-\mu+\nu)\,\Gamma(1-\mu-\nu)}\,G_{3,3}^{3,1}\!\left(z,\,\frac{1}{2}\,\middle|\,\begin{array}{c}\mu+\frac{1}{2},\,\frac{1}{2},\,\frac{1}{2}-\mu\\\frac{1}{2}-\nu,\,\nu+\frac{1}{2},\,0\end{array}\right)/;\,\mathrm{Re}(z)>0$$

07.09.26.0065.01

$$\frac{1}{\sqrt{z^2+1}}\,\mathfrak{P}_\nu^\mu\left(\frac{\sqrt{z^2+1}}{z}\right)\mathfrak{P}_\nu^{\mu+1}\left(\frac{\sqrt{z^2+1}}{z}\right)=\frac{1}{\sqrt{\pi}\,\Gamma(1-\mu+\nu)\,\Gamma(-\mu-\nu)}\,G_{3,3}^{3,1}\!\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\mu+1,1,-\mu\\ \frac{1}{2},-\nu,\nu+1\end{matrix}\right)\text{/; Re}(z)>0$$

07.09.26.0066.01

$$\mathfrak{P}_\nu^\mu\left(\sqrt{z^2+1}\right)\mathfrak{P}_{-\mu-\frac{1}{2}}^{-\nu-\frac{1}{2}}\left(\frac{\sqrt{z^2+1}}{z}\right)=\frac{1}{\sqrt{2}\,\pi\,\Gamma(1-\mu+\nu)}\,G_{3,3}^{2,2}\!\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{3}{4},\frac{1}{4}-\nu,\nu+\frac{5}{4}\\ \frac{1}{4},\frac{1}{4}-\mu,\mu+\frac{1}{4}\end{matrix}\right)\text{/; Re}(z)>0$$

07.09.26.0067.01

$$\mathfrak{P}_\nu^\mu\left(\sqrt{z^2+1}\right)\mathfrak{P}_{\mu-\frac{1}{2}}^{\nu+\frac{1}{2}}\left(\frac{\sqrt{z^2+1}}{z}\right)=\frac{1}{\sqrt{2}\,\pi\,\Gamma(-\mu-\nu)}\,G_{3,3}^{2,2}\!\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{3}{4},\nu+\frac{5}{4},\frac{1}{4}-\nu\\ \frac{1}{4},\frac{1}{4}-\mu,\mu+\frac{1}{4}\end{matrix}\right)\text{/; Re}(z)>0$$

Generalized cases involving Legendre *Q*

07.09.26.0068.01

$$\mathfrak{P}_\nu^\mu\left(\frac{\sqrt{z^2+1}}{z}\right)\mathfrak{Q}_\nu^\mu\left(\frac{\sqrt{z^2+1}}{z}\right)=\frac{e^{\pi i\mu}\,\Gamma(\mu+\nu+1)}{2\sqrt{\pi}\,\Gamma(1-\mu+\nu)}\,G_{3,3}^{2,2}\!\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}1,\mu+1,1-\mu\\ \frac{1}{2},\nu+1,-\nu\end{matrix}\right)$$

07.09.26.0069.01

$$\mathfrak{P}_\nu^{-\mu}\left(\frac{\sqrt{z^2+1}}{z}\right)\mathfrak{Q}_\nu^\mu\left(\frac{\sqrt{z^2+1}}{z}\right)=\frac{e^{\pi i\mu}}{2\sqrt{\pi}}\,G_{3,3}^{2,2}\!\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}1,1-\mu,\mu+1\\ \frac{1}{2},\nu+1,-\nu\end{matrix}\right)$$

07.09.26.0070.01

$$\mathfrak{P}_{-\mu-\frac{1}{2}}^{-\nu-\frac{1}{2}}\left(\frac{\sqrt{z^2+1}}{z}\right)\mathfrak{Q}_\nu^\mu\left(\sqrt{z^2+1}\right)=\frac{e^{\pi i\mu}}{\sqrt{2}\,\Gamma(1-\mu+\nu)}\,G_{3,3}^{3,1}\!\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{1}{4}-\nu,\nu+\frac{5}{4},\frac{3}{4}\\ \frac{1}{4},\mu+\frac{1}{4},\frac{1}{4}-\mu\end{matrix}\right)\text{/; Re}(z)>0$$

07.09.26.0071.01

$$\mathfrak{P}_{-\mu-\frac{1}{2}}^{-\nu-\frac{1}{2}}\left(\sqrt{z^2+1}\right)\mathfrak{Q}_\nu^\mu\left(\frac{\sqrt{z^2+1}}{z}\right)=\frac{e^{\pi i\mu}}{\sqrt{2}\,\Gamma(1-\mu+\nu)}\,G_{3,3}^{1,3}\!\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{3}{4},\frac{3}{4}-\mu,\mu+\frac{3}{4}\\ \nu+\frac{3}{4},-\nu-\frac{1}{4},\frac{1}{4}\end{matrix}\right)\text{/; Re}(z)>0$$

07.09.26.0072.01

$$\mathfrak{P}_{\mu-\frac{1}{2}}^{\nu+\frac{1}{2}}\left(\frac{\sqrt{z^2+1}}{z}\right)\mathfrak{Q}_\nu^\mu\left(\sqrt{z^2+1}\right)=\frac{e^{\pi i\mu}\cos(\pi\mu)\,\Gamma(\mu+\nu+1)}{\pi\sqrt{2}}\,G_{3,3}^{3,1}\!\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{3}{4},\nu+\frac{5}{4},\frac{1}{4}-\nu\\ \frac{1}{4},\mu+\frac{1}{4},\frac{1}{4}-\mu\end{matrix}\right)\text{/; Re}(z)>0$$

07.09.26.0073.01

$$\mathfrak{P}_{\mu-\frac{1}{2}}^{\nu+\frac{1}{2}}\left(\sqrt{z^2+1}\right)\mathfrak{Q}_\nu^\mu\left(\frac{\sqrt{z^2+1}}{z}\right)=\frac{e^{\pi i\mu}\cos(\pi\mu)\,\Gamma(\mu+\nu+1)}{\pi\sqrt{2}}\,G_{3,3}^{1,3}\!\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}\frac{3}{4},\frac{3}{4}-\mu,\mu+\frac{3}{4}\\ \frac{1}{4},-\nu-\frac{1}{4},\nu+\frac{3}{4}\end{matrix}\right)\text{/; Re}(z)>0$$

07.09.26.0074.01

$$\frac{1}{\sqrt{z^2+1}}\,\mathfrak{P}_{\mu-\frac{1}{2}}^{-\nu-\frac{3}{2}}\left(\frac{\sqrt{z^2+1}}{z}\right)\mathfrak{Q}_\nu^\mu\left(\sqrt{z^2+1}\right)=\frac{e^{\pi i\mu}}{\sqrt{2}\,(\mu+\nu+1)\,\Gamma(2-\mu+\nu)}\,G_{3,3}^{3,1}\!\left(z,\frac{1}{2}\,\middle|\,\begin{matrix}-\nu-\frac{3}{4},\frac{3}{4},\nu+\frac{5}{4}\\ \frac{1}{4},\mu+\frac{1}{4},\frac{1}{4}-\mu\end{matrix}\right)\text{/; Re}(z)>0$$

07.09.26.0075.01

$$\frac{1}{\sqrt{z^2+1}}\,\mathfrak{P}_{\mu-\frac{1}{2}}^{-\nu-\frac{3}{2}}\left(\sqrt{z^2+1}\right)\mathfrak{Q}_\nu^\mu\left(\frac{\sqrt{z^2+1}}{z}\right) = \frac{e^{\pi i \mu}}{\sqrt{2}\,(\mu+\nu+1)\,\Gamma(2-\mu+\nu)}\,G_{3,3}^{1,3}\left(z,\ \frac{1}{2}\,\middle|\,\begin{array}{c}\frac{1}{4},\ \frac{1}{4}-\mu,\ \mu+\frac{1}{4}\\ \nu+\frac{5}{4},\ -\frac{1}{4},\ -\nu-\frac{3}{4}\end{array}\right)\ /;\ \mathrm{Re}(z)>0$$

07.09.26.0076.01

$$\frac{1}{\sqrt{z^2+1}}\,\mathfrak{P}_{\mu-\frac{1}{2}}^{\frac{1}{2}-\nu}\left(\frac{\sqrt{z^2+1}}{z}\right)\mathfrak{Q}_\nu^\mu\left(\sqrt{z^2+1}\right) = \frac{e^{\pi i \mu}}{\sqrt{2}\,\Gamma(1-\mu+\nu)}\,G_{3,3}^{3,1}\left(z,\ \frac{1}{2}\,\middle|\,\begin{array}{c}\frac{1}{4}-\nu,\ \frac{3}{4},\ \nu+\frac{1}{4}\\ \frac{1}{4},\ \mu+\frac{1}{4},\ \frac{1}{4}-\mu\end{array}\right)\ /;\ \mathrm{Re}(z)>0$$

07.09.26.0077.01

$$\frac{1}{\sqrt{z^2+1}}\,\mathfrak{P}_{\mu-\frac{1}{2}}^{\frac{1}{2}-\nu}\left(\sqrt{z^2+1}\right)\mathfrak{Q}_\nu^\mu\left(\frac{\sqrt{z^2+1}}{z}\right) = \frac{e^{\pi i \mu}}{\sqrt{2}\,\Gamma(1-\mu+\nu)}\,G_{3,3}^{1,3}\left(z,\ \frac{1}{2}\,\middle|\,\begin{array}{c}\frac{1}{4},\ \frac{1}{4}-\mu,\ \mu+\frac{1}{4}\\ \nu+\frac{1}{4},\ -\frac{1}{4},\ \frac{1}{4}-\nu\end{array}\right)\ /;\ \mathrm{Re}(z)>0$$

07.09.26.0078.01

$$\frac{1}{\sqrt{z^2+1}}\,\mathfrak{P}_{\mu+\frac{1}{2}}^{-\nu-\frac{1}{2}}\left(\frac{\sqrt{z^2+1}}{z}\right)\mathfrak{Q}_\nu^\mu\left(\sqrt{z^2+1}\right) = \frac{e^{\pi i \mu}}{\sqrt{2}\,(\mu+\nu+1)\,\Gamma(1-\mu+\nu)}\,G_{3,3}^{3,1}\left(z,\ \frac{1}{2}\,\middle|\,\begin{array}{c}-\nu-\frac{1}{4},\ \frac{1}{4},\ \nu+\frac{3}{4}\\ -\frac{1}{4},\ \mu+\frac{3}{4},\ -\mu-\frac{1}{4}\end{array}\right)\ /;\ \mathrm{Re}(z)>0$$

07.09.26.0079.01

$$\frac{1}{\sqrt{z^2+1}}\,\mathfrak{P}_{\mu+\frac{1}{2}}^{-\nu-\frac{1}{2}}\left(\sqrt{z^2+1}\right)\mathfrak{Q}_\nu^\mu\left(\frac{\sqrt{z^2+1}}{z}\right) = \frac{e^{\pi i \mu}}{\sqrt{2}\,(\mu+\nu+1)\,\Gamma(1-\mu+\nu)}\,G_{3,3}^{1,3}\left(z,\ \frac{1}{2}\,\middle|\,\begin{array}{c}\frac{3}{4},\ -\mu-\frac{1}{4},\ \mu+\frac{3}{4}\\ \nu+\frac{3}{4},\ \frac{1}{4},\ -\nu-\frac{1}{4}\end{array}\right)\ /;\ \mathrm{Re}(z)>0$$

Through other functions

Involving some hypergeometric-type functions

07.09.26.0080.01

$$\mathfrak{P}_\nu^\mu(z) = \frac{\Gamma(\nu+1)}{\Gamma(\nu-\mu+1)}\,\frac{(z+1)^{\mu/2}}{(z-1)^{\mu/2}}\,P_\nu^{(-\mu,\mu)}(z)$$

Involving spheroidal functions

07.09.26.0088.01

$$P_\nu^\mu(z) = \frac{(1-z)^{\mu/2}}{(z-1)^{\mu/2}}\,PS_{\nu,\mu}(0,\ z)$$

Representations through equivalent functions

With related functions

Involving Gegenbauer functions

07.09.27.0001.01

$$\mathfrak{P}_\nu^\mu(z) = \frac{2^{-\mu}\,\Gamma\left(\frac{1}{2}-\mu\right)\Gamma(\mu+\nu+1)}{\sqrt{\pi}\,\Gamma(1-\mu+\nu)}\,(z+1)^{-\mu/2}\,(z-1)^{-\mu/2}\,C_{\mu+\nu}^{\frac{1}{2}-\mu}(z)$$

Involving Legendre functions

07.09.27.0002.01

$$\mathfrak{P}_\nu^\mu(z) = \frac{(1-z)^{\mu/2}}{(z-1)^{\mu/2}} P_\nu^\mu(z)$$

07.09.27.0003.01

$$\mathfrak{P}_\nu^\mu(z) = -\csc(\pi\,\mu)\,\frac{(1-z)^{\mu/2}}{(z-1)^{\mu/2}}\left(\sin(\pi\,\nu)\,P_\nu^\mu(-z) + \frac{\pi}{\Gamma(-\mu-\nu)\,\Gamma(\nu-\mu+1)}\,P_\nu^{-\mu}(-z)\right) /;\ \mu \notin \mathbb{Z}$$

07.09.27.0004.01

$$\mathfrak{P}_\nu^\mu(x) = e^{\frac{\pi i \mu}{2}} \lim_{z\to x - i\,0} P_\nu^\mu(z) /;\ x > 1$$

07.09.27.0010.01

$$\mathfrak{P}_\nu^\mu(x) = e^{-\frac{\pi i \mu}{2}} \lim_{z\to x + i\,0} P_\nu^\mu(z) /;\ x < -1$$

07.09.27.0005.01

$$\mathfrak{P}_\nu^\mu(x) = e^{-\frac{\pi i \mu}{2}} \lim_{z\to x + i\,0} P_\nu^\mu(z) /;\ -1 < x < 1$$

07.09.27.0006.01

$$\mathfrak{P}_\nu^\mu(x) = e^{-\frac{\pi i \mu}{2}} P_\nu^\mu(x) /;\ x < -1$$

07.09.27.0007.01

$$\mathfrak{P}_\nu^\mu(x) = e^{\frac{\pi i \mu}{2}} P_\nu^\mu(x) /;\ x > 1$$

07.09.27.0011.01

$$\mathfrak{P}_\nu^\mu(x) = e^{-\frac{\pi i \mu}{2}} P_\nu^\mu(x) /;\ -1 < x < 1$$

07.09.27.0008.01

$$\mathfrak{P}_\nu^\mu(\coth(z)) = \frac{i\,e^{i\pi\nu}}{\Gamma(-\mu-\nu)}\sqrt{\frac{2}{\pi}}\,\sinh^{\frac{1}{2}}(z)\,\mathfrak{Q}_{-\mu-\frac{1}{2}}^{-\nu-\frac{1}{2}}(\cosh(z))$$

Involving spherical harmonic functions

07.09.27.0009.01

$$\mathfrak{P}_\nu^\mu(z) = \frac{2\sqrt{\pi}\,\sqrt{\Gamma(\mu+\nu+1)}}{\sqrt{2\,\nu+1}\,\sqrt{\Gamma(\nu-\mu+1)}}\,\frac{(1-z)^{\mu/2}}{(z-1)^{\mu/2}}\,Y_\nu^\mu\left(\cos^{-1}(z),\,0\right)$$

History

– D. Bernoulli (1748)
– A. M. Legendre (1782)
– E. Heine (1842)

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