

MathieuCPrime

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Notations

Traditional name

Derivative of the even Mathieu function

Traditional notation

$Ce'(a, q, z)$

Mathematica StandardForm notation

`MathieuCPrime[a, q, z]`

Primary definition

11.03.02.0001.01

$$Ce'(a, q, z) = \frac{\partial Ce(a, q, z)}{\partial z}$$

Specific values

Specialized values

For fixed a, z

11.03.03.0001.01

$$Ce'(a, 0, z) = -\sqrt{a} \sin(\sqrt{a} z)$$

General characteristics

Domain and analyticity

$Ce'(a, q, z)$ is an analytical function of a, q, z which is defined in \mathbb{C}^3 .

11.03.04.0001.01

$$(a * q * z) \rightarrow Ce'(a, q, z) :: (\mathbb{C} \otimes \mathbb{C} \otimes \mathbb{C}) \rightarrow \mathbb{C}$$

Symmetries and periodicities

Parity

$Ce'(a, q, z)$ is an odd function with respect to z .

11.03.04.0002.01

$$Ce'(a, q, -z) = -Ce'(a, q, z)$$

Mirror symmetry

11.03.04.0003.01

$$Ce'(\bar{a}, \bar{q}, \bar{z}) = \overline{Ce'(a, q, z)}$$

Periodicity

No periodicity

Series representations

Generalized power series

Expansions at generic point $z = z_0$

For the function itself

11.03.06.0011.01

$$\begin{aligned}
 Ce'(a, q, z) \propto & Ce^{(0,0,1)}(a, q, z_0) + (2q \cos(2z_0) - a) Ce(a, q, z_0) (z - z_0) + \\
 & \frac{1}{2} ((2q \cos(2z_0) - a) Ce^{(0,0,1)}(a, q, z_0) - 4q \sin(2z_0) Ce(a, q, z_0)) (z - z_0)^2 + \\
 & \frac{1}{6} ((a^2 + 4q \cos(2z_0)(-a + q \cos(2z_0) - 2)) Ce(a, q, z_0) - 8q Ce^{(0,0,1)}(a, q, z_0) \sin(2z_0)) (z - z_0)^3 + \\
 & \frac{1}{24} ((a^2 + 4q \cos(2z_0)(-a + q \cos(2z_0) - 6)) Ce^{(0,0,1)}(a, q, z_0) + 16q(a - 2q \cos(2z_0) + 1) Ce(a, q, z_0) \sin(2z_0)) (z - z_0)^4 + \\
 & \frac{1}{120} ((2q((3q^2 + a(3a + 28) + 16) \cos(2z_0) + q(-3(a + 4) - (3a + 44) \cos(4z_0) + q \cos(6z_0))) - a^3) Ce(a, q, z_0) + \\
 & 8q(3a - 6q \cos(2z_0) + 8) Ce^{(0,0,1)}(a, q, z_0) \sin(2z_0)) (z - z_0)^5 + \dots /; (z \rightarrow z_0)
 \end{aligned}$$

11.03.06.0012.01

$$Ce'(a, q, z) \propto Ce^{(0,0,1)}(a, q, z_0) (1 + O(z - z_0))$$

Expansions at $z = 0$

11.03.06.0001.01

$$\begin{aligned}
 Ce'(a_{2n}(q), q, z) = & -2 \sum_{k=0}^{\infty} k A_{2k}^{2n} \sin(2kz) /; a_{2n}(q) A_0^{2n} - q A_2^{2n} = 0 \wedge (a_{2n}(q) - 4) A_2^{2n} - q (A_4^{2n} - 2 A_0^{2n}) = 0 \wedge \\
 & (a_{2n}(q) - 4k^2) A_{2k}^{2n} - q (A_{2k-2}^{2n} + A_{2k+2}^{2n}) = 0 \wedge 2 (A_0^{2n})^2 + \sum_{k=1}^{\infty} A_{2k}^{2n} = 1 \wedge n \in \mathbb{Z}
 \end{aligned}$$

11.03.06.0002.01

$$\text{Ce}'(a_{2n+1}(q), q, z) = - \sum_{k=0}^{\infty} (2k+1) A_{2k+1}^{2n+1} \sin((2k+1)z) /;$$

$$(a_{2n+1}(q) - q - 1) A_1^{2n+1} - q A_3^{2n+1} = 0 \wedge (a_{2n+1}(q) - (2k+1)^2) A_{2k+1}^{2n+1} - q (A_{2k-1}^{2n+1} + A_{2k+3}^{2n+1}) = 0 \wedge \sum_{k=0}^{\infty} A_{2k+1}^{2n+1} = 1 \wedge n \in \mathbb{Z}$$

Expansions at $q = 0$

11.03.06.0003.01

$$\begin{aligned}
 & \text{Ce}'(a_r(q), q, z) - r \sin(rz) + \frac{1}{4} \left(\frac{(2-r) \sin((r-2)z)}{r-1} + \frac{(r+2) \sin((r+2)z)}{r+1} \right) q + \\
 & \frac{1}{32} \left(\frac{(4-r) \sin((r-4)z)}{(r-2)(r-1)} + \frac{2r(r^2+1) \sin(rz)}{(r-1)^2(r+1)^2} - \frac{(r+4) \sin((r+4)z)}{(r+1)(r+2)} \right) q^2 + \\
 & \frac{1}{384} \left(-\frac{(r-6) \sin((r-6)z)}{(r-3)(r-2)(r-1)} + \frac{3(r^3-r^2-r-11) \sin((r-2)z)}{(r-1)^3(r+1)^2} - \frac{3(r^3+r^2-r+11) \sin((r+2)z)}{(r-1)^2(r+1)^3} + \frac{(r+6) \sin((r+6)z)}{(r+1)(r+2)(r+3)} \right) \\
 & q^3 + \frac{1}{6144} \left(-\frac{(r-8) \sin((r-8)z)}{(r-4)(r-3)(r-2)(r-1)} + \frac{4(r-4)(r^3-r^2-7r-29) \sin((r-4)z)}{(r-3)(r-2)(r-1)^3(r+1)^2} - \right. \\
 & \left. \frac{6r(r^8-15r^6-185r^4+675r^2+316) \sin(rz)}{(r-2)^2(r-1)^4(r+1)^4(r+2)^2} + \right. \\
 & \left. \frac{4(r+4)(r^3+r^2-7r+29) \sin((r+4)z)}{(r-1)^2(r+1)^3(r+2)(r+3)} - \frac{(r+8) \sin((r+8)z)}{(r+1)(r+2)(r+3)(r+4)} \right) q^4 + \\
 & \frac{1}{122880} \left(-\frac{(r-10) \sin((r-10)z)}{(r-5)(r-4)(r-3)(r-2)(r-1)} + \frac{5(r-6)(r^3-r^2-17r-55) \sin((r-6)z)}{(r-4)(r-3)(r-2)(r-1)^3(r+1)^2} - \right. \\
 & \left. \frac{10(r^9-r^8-31r^7-5r^6-273r^5+2457r^4+1931r^3-6335r^2-3572r-7564) \sin((r-2)z)}{(r-3)(r-2)(r-1)^5(r+1)^4(r+2)^2} + \right. \\
 & \left. \frac{10(r^9+r^8-31r^7+5r^6-273r^5-2457r^4+1931r^3+6335r^2-3572r+7564) \sin((r+2)z)}{(r-2)^2(r-1)^4(r+1)^5(r+2)(r+3)} - \right. \\
 & \left. \frac{5(r+6)(r^3+r^2-17r+55) \sin((r+6)z)}{(r-1)^2(r+1)^3(r+2)(r+3)(r+4)} + \frac{(r+10) \sin((r+10)z)}{(r+1)(r+2)(r+3)(r+4)(r+5)} \right) q^5 + \\
 & \frac{1}{2949120} \left(-\frac{(r-12) \sin((r-12)z)}{(r-6)(r-5)(r-4)(r-3)(r-2)(r-1)} + \frac{6(r-8)(r^3-r^2-31r-89) \sin((r-8)z)}{(r-5)(r-4)(r-3)(r-2)(r-1)^3(r+1)^2} - \right. \\
 & \left. \frac{15(r^{10}-3r^9-53r^8+69r^7+145r^6+8211r^5-16879r^4-32025r^3+32954r^2+16404r+71528) \sin((r-4)z)}{(r-3)(r-2)^3(r-1)^5(r+1)^4(r+2)^2} + \right. \\
 & \left. \frac{20r(r^{14}-72r^{12}+597r^{10}+75244r^8-718317r^6+153312r^4+4883287r^2+1329084) \sin(rz)}{(r-3)^2(r-2)^2(r-1)^6(r+1)^6(r+2)^2(r+3)^2} - \right. \\
 & \left. \frac{15(r^{10}+3r^9-53r^8-69r^7+145r^6-8211r^5-16879r^4+32025r^3+32954r^2-16404r+71528) \sin((r+4)z)}{(r-2)^2(r-1)^4(r+1)^5(r+2)^3(r+3)} + \frac{6(r+8)(r^3+r^2-31r+89) \sin((r+8)z)}{(r-1)^2(r+1)^3(r+2)(r+3)(r+4)(r+5)} - \right. \\
 & \left. \frac{(r+12) \sin((r+12)z)}{(r+1)(r+2)(r+3)(r+4)(r+5)(r+6)} \right) q^6 + O(q^7) ; \neg (r \in \mathbb{Z} \wedge -6 \leq r \leq 6)
 \end{aligned}$$

11.03.06.0004.01

$Ce'(a_0(q), q, z) \propto$

$$\begin{aligned} & \frac{1}{\sqrt{2}} \left(1 - \frac{1}{2} \cos(2z) q + \left(\frac{1}{32} \cos(4z) - \frac{1}{16} \right) q^2 + \left(\frac{11}{128} \cos(2z) - \frac{\cos(6z)}{1152} \right) q^3 + \left(-\frac{29 \cos(4z)}{4608} + \frac{\cos(8z)}{73728} + \frac{79}{4096} \right) q^4 + \right. \\ & \left(-\frac{1891 \cos(2z)}{73728} + \frac{55 \cos(6z)}{294912} - \frac{\cos(10z)}{7372800} \right) q^5 + \left(\frac{8941 \cos(4z)}{4718592} - \frac{89 \cos(8z)}{29491200} + \frac{\cos(12z)}{1061683200} - \frac{36919}{5308416} \right) q^6 + \\ & \left(\frac{1521691 \cos(2z)}{169869312} - \frac{26641 \cos(6z)}{471859200} + \frac{131 \cos(10z)}{4246732800} - \frac{\cos(14z)}{208089907200} \right) q^7 + \\ & \left(-\frac{11239489 \cos(4z)}{16986931200} + \frac{62299 \cos(8z)}{67947724800} - \frac{181 \cos(12z)}{832359628800} + \frac{\cos(16z)}{53271016243200} + \frac{58304143}{21743271936} \right) q^8 + \\ & \left(-\frac{3664886951 \cos(2z)}{1087163596800} + \frac{48249209 \cos(6z)}{2446118092800} - \frac{125119 \cos(10z)}{13317754060800} + \frac{239 \cos(14z)}{213084064972800} - \frac{\cos(18z)}{17259809262796800} \right) \\ & q^9 + \left(\frac{7785932591 \cos(4z)}{31310311587840} - \frac{153642299 \cos(8z)}{479439146188800} + \frac{226201 \cos(12z)}{3409345039564800} - \right. \\ & \left. \frac{61 \cos(16z)}{13807847410237440} + \frac{\cos(20z)}{6903923705118720000} - \frac{233552934751}{217432719360000} \right) q^{10} + O(q^{11}) \end{aligned}$$

11.03.06.0005.01

$$\begin{aligned}
 \text{Ce}'(a_1(q), q, z) \propto & -\sin(z) + \frac{3}{8} \sin(3z)q + \left(\frac{\sin(z)}{128} + \frac{3}{64} \sin(3z) - \frac{5}{192} \sin(5z) \right) q^2 + \\
 & \left(\frac{\sin(z)}{512} - \frac{\sin(3z)}{1024} - \frac{5 \sin(5z)}{1152} + \frac{7 \sin(7z)}{9216} \right) q^3 + \left(\frac{37 \sin(z)}{294912} - \frac{49 \sin(3z)}{24576} + \frac{7 \sin(7z)}{49152} - \frac{\sin(9z)}{81920} \right) q^4 + \\
 & \left(-\frac{121 \sin(z)}{1769472} - \frac{317 \sin(3z)}{786432} + \frac{205 \sin(5z)}{1179648} + \frac{7 \sin(7z)}{5898240} - \frac{\sin(9z)}{409600} + \frac{11 \sin(11z)}{88473600} \right) q^5 + \\
 & \left(-\frac{8105 \sin(z)}{339738624} + \frac{103 \sin(3z)}{18874368} + \frac{731 \sin(5z)}{18874368} - \frac{2653 \sin(7z)}{471859200} - \frac{\sin(9z)}{31457280} + \frac{11 \sin(11z)}{424673280} - \frac{13 \sin(13z)}{14863564800} \right) q^6 + \\
 & \left(-\frac{481 \sin(z)}{226492416} + \frac{102547 \sin(3z)}{4529848320} + \frac{659 \sin(5z)}{2264924160} - \frac{88739 \sin(7z)}{67947724800} + \right. \\
 & \quad \left. \frac{181 \sin(9z)}{1887436800} + \frac{11 \sin(11z)}{26424115200} - \frac{13 \sin(13z)}{69363302400} + \frac{\sin(15z)}{221962567680} \right) q^7 + \\
 & \left(\frac{1237783 \sin(z)}{1449551462400} + \frac{940781 \sin(3z)}{181193932800} - \frac{322897 \sin(5z)}{163074539520} - \frac{65891 \sin(7z)}{3261490790400} + \frac{2143 \sin(9z)}{93952409600} - \right. \\
 & \quad \left. \frac{13519 \sin(11z)}{13317754060800} - \frac{13 \sin(13z)}{3805072588800} + \frac{\sin(15z)}{1014686023680} - \frac{17 \sin(17z)}{958878292377600} \right) q^8 + \\
 & \left(\frac{11221967 \sin(z)}{32614907904000} - \frac{506831 \sin(3z)}{34789235097600} - \frac{1550665 \sin(5z)}{3131031158784} + \frac{5039101 \sin(7z)}{78275778969600} + \right. \\
 & \quad \frac{16013 \sin(9z)}{35514010828800} - \frac{34837 \sin(11z)}{142056043315200} + \frac{6253 \sin(13z)}{852336259891200} + \\
 & \quad \left. \frac{\sin(15z)}{51140175593472} - \frac{17 \sin(17z)}{4314952315699200} + \frac{19 \sin(19z)}{345196185255936000} \right) q^9 + \\
 & \left(\frac{4539285691 \sin(z)}{125241246351360000} - \frac{1282939901 \sin(3z)}{4174708211712000} - \frac{10304813 \sin(5z)}{1168918299279360} + \frac{48740801 \sin(7z)}{2922295748198400} - \right. \\
 & \quad \frac{7516703 \sin(9z)}{681869079129600} - \frac{338503 \sin(11z)}{61368210712166400} + \frac{329927 \sin(13z)}{184104632136499200} - \frac{2839 \sin(15z)}{73641852854599680} - \\
 & \quad \left. \frac{17 \sin(17z)}{204560702373888000} + \frac{19 \sin(19z)}{1534205267804160000} - \frac{\sin(21z)}{7232681976791040000} \right) q^{10} + O(q^{11})
 \end{aligned}$$

11.03.06.0006.01

$$\begin{aligned}
 \text{Ce}'(a_2(q), q, z) \propto & -2 \sin(2z) + \frac{1}{3} \sin(4z) q + \left(\frac{19}{144} \sin(2z) - \frac{1}{64} \sin(6z) \right) q^2 + \left(\frac{\sin(8z)}{2880} - \frac{11 \sin(4z)}{1152} \right) q^3 + \\
 & \left(-\frac{51191 \sin(2z)}{1327104} + \frac{29 \sin(6z)}{122880} - \frac{\sin(10z)}{221184} \right) q^4 + \left(\frac{181193 \sin(4z)}{39813120} - \frac{23 \sin(8z)}{8294400} + \frac{\sin(12z)}{25804800} \right) q^5 + \\
 & \left(\frac{88995077 \sin(2z)}{637099200} - \frac{778189 \sin(6z)}{4246732800} + \frac{\sin(10z)}{61931520} - \frac{\sin(14z)}{4246732800} \right) q^6 + \\
 & \left(-\frac{81026723 \sin(4z)}{45864714240} + \frac{332933 \sin(8z)}{89181388800} - \frac{\sin(12z)}{46242201600} + \frac{\sin(16z)}{936404582400} \right) q^7 + \\
 & \left(-\frac{4746016900871 \sin(2z)}{880602513408000} + \frac{2526784843 \sin(6z)}{34245653299200} - \frac{25811 \sin(10z)}{561842749440} - \frac{79 \sin(14z)}{205473919795200} - \frac{\sin(18z)}{266355081216000} \right) \\
 & q^8 + \left(\frac{25918360893277 \sin(4z)}{36985305563136000} - \frac{16587823349 \sin(8z)}{10787380789248000} + \right. \\
 & \left. \frac{362869 \sin(12z)}{958878292377600} + \frac{19 \sin(16z)}{5393690394624000} + \frac{\sin(20z)}{94928950945382400} \right) q^9 + \\
 & \left(\frac{67083347965282273 \sin(2z)}{31067656673034240000} - \frac{163850664620431 \sin(6z)}{5523138964094976000} + \frac{31842087803 \sin(10z)}{1656941689228492800} - \right. \\
 & \left. \frac{13981 \sin(14z)}{6262062317568000} - \frac{73 \sin(18z)}{4219064486461440000} - \frac{\sin(22z)}{41423542230712320000} \right) q^{10} + O(q^{11})
 \end{aligned}$$

11.03.06.0007.01

$$\begin{aligned}
 \text{Ce}'(a_3(q), q, z) \propto & -3 \sin(3z) + \left(\frac{5}{16} \sin(5z) - \frac{\sin(z)}{8} \right) q + \left(-\frac{\sin(z)}{64} + \frac{15}{512} \sin(3z) - \frac{7}{640} \sin(7z) \right) q^2 + \\
 & \left(\frac{\sin(z)}{4096} + \frac{3}{512} \sin(3z) - \frac{11 \sin(5z)}{8192} + \frac{\sin(9z)}{5120} \right) q^3 + \left(\frac{21 \sin(z)}{32768} + \frac{4863 \sin(3z)}{13107200} - \frac{5 \sin(5z)}{16384} + \frac{77 \sin(7z)}{2949120} - \frac{11 \sin(11z)}{5160960} \right) q^4 + \\
 & \left(\frac{14061 \sin(z)}{104857600} - \frac{27 \sin(3z)}{131072} - \frac{12329 \sin(5z)}{377487360} + \frac{21 \sin(7z)}{3276800} - \frac{\sin(9z)}{3670016} + \frac{13 \sin(13z)}{825753600} \right) q^5 + \\
 & \left(-\frac{699 \sin(z)}{838860800} - \frac{13050583 \sin(3z)}{181193932800} + \frac{533 \sin(5z)}{41943040} + \frac{76679 \sin(7z)}{75497472000} - \frac{17 \sin(9z)}{235929600} + \frac{11 \sin(11z)}{6606028800} - \frac{\sin(15z)}{11890851840} \right) \\
 & q^6 + \left(-\frac{31826419 \sin(z)}{4348654387200} - \frac{210369 \sin(3z)}{33554432000} + \frac{326021051 \sin(5z)}{60881161420800} - \frac{9233 \sin(7z)}{27179089920} - \right. \\
 & \left. \frac{7831 \sin(9z)}{469762048000} + \frac{407 \sin(11z)}{832359628800} - \frac{13 \sin(13z)}{2283043553280} + \frac{17 \sin(17z)}{49941577728000} \right) q^7 + \\
 & \left(-\frac{300245939 \sin(z)}{173946175488000} + \frac{1193766593741 \sin(3z)}{454579338608640000} + \frac{10194121 \sin(5z)}{17394617548800} - \frac{55617547 \sin(7z)}{347892350976000} + \frac{30253 \sin(9z)}{5919001804800} + \right. \\
 & \left. \frac{311971 \sin(11z)}{1826434842624000} - \frac{221 \sin(13z)}{106542032486400} + \frac{\sin(15z)}{213084064972800} - \frac{19 \sin(19z)}{17579435360256000} \right) q^8 + \\
 & \left(-\frac{64306539779 \sin(z)}{10909904126607360000} + \frac{1087026917 \sin(3z)}{1043677052928000} - \frac{767116375621 \sin(5z)}{4363961650642944000} - \frac{468897223 \sin(7z)}{24352464568320000} + \right. \\
 & \left. \frac{21665887 \sin(9z)}{8349416423424000} - \frac{84293 \sin(11z)}{1704672519782400} - \frac{2457689 \sin(13z)}{2045607023738880000} + \right. \\
 & \left. \frac{23 \sin(15z)}{4602615803412480} + \frac{17 \sin(17z)}{281270965764096000} + \frac{\sin(21z)}{361634098839552000} \right) q^9 + \\
 & \left(\frac{78221421983189 \sin(z)}{785513097115729920000} + \frac{3555989290829 \sin(3z)}{33249231623946240000} - \frac{1595308088447 \sin(5z)}{19637827427893248000} + \right. \\
 & \left. \frac{16802983705367 \sin(7z)}{3366484701924556800000} + \frac{50000417 \sin(9z)}{151526446202880000} - \frac{488862979 \sin(11z)}{18410463213649920000} + \right. \\
 & \left. \frac{1474993 \sin(13z)}{4418511171275980800} + \frac{74069 \sin(15z)}{12000894539268096000} - \frac{17 \sin(17z)}{13807847410237440000} - \right. \\
 & \left. \frac{19 \sin(19z)}{48603622884035788800} - \frac{23 \sin(23z)}{3949044359327907840000} \right) q^{10} + O(q^{11})
 \end{aligned}$$

11.03.06.0008.01

$Ce'(a_4(q), q, z) \propto$

$$\begin{aligned}
 & -4 \sin(4z) + \left(\frac{3}{10} \sin(6z) - \frac{1}{6} \sin(2z) \right) q + \left(\frac{17}{900} \sin(4z) - \frac{1}{120} \sin(8z) \right) q^2 + \left(-\frac{7 \sin(2z)}{14400} - \frac{29 \sin(6z)}{48000} + \frac{\sin(10z)}{8064} \right) q^3 + \\
 & \left(\frac{22411 \sin(4z)}{103680000} + \frac{\sin(8z)}{112000} - \frac{\sin(12z)}{860160} \right) q^4 + \left(\frac{39491 \sin(2z)}{2488320000} - \frac{97117 \sin(6z)}{9676800000} - \frac{67 \sin(10z)}{928972800} + \frac{\sin(14z)}{132710400} \right) q^5 + \\
 & \left(-\frac{10022209 \sin(4z)}{6096384000000} + \frac{39649 \sin(8z)}{199065600000} + \frac{\sin(12z)}{3096576000} - \frac{\sin(16z)}{27869184000} \right) q^6 + \\
 & \left(\frac{88961041 \sin(2z)}{877879296000000} + \frac{68332987 \sin(6z)}{1300561920000000} - \frac{211243 \sin(10z)}{93640458240000} - \frac{\sin(14z)}{1911029760000} + \frac{\sin(18z)}{7629963264000} \right) q^7 + \\
 & \left(-\frac{3503676485287 \sin(4z)}{67421129932800000000} - \frac{29028749 \sin(8z)}{35115171840000000} + \right. \\
 & \quad \left. \frac{18517 \sin(12z)}{1109812838400000} - \frac{23 \sin(16z)}{7725337804800000} - \frac{\sin(20z)}{2636915304038400} \right) q^8 + \\
 & \left(-\frac{5763181587047 \sin(2z)}{1618107118387200000000} + \frac{2686845093031 \sin(6z)}{898948399104000000000} + \frac{194272651 \sin(10z)}{25170555174912000000} - \right. \\
 & \quad \left. \frac{164327 \sin(14z)}{1883510931456000000} + \frac{23 \sin(18z)}{878971768012800000} + \frac{\sin(22z)}{1121887602081792000} \right) q^9 + \\
 & \left(\frac{618233476217857 \sin(4z)}{1699012474306560000000000} - \frac{5461479238529 \sin(8z)}{75511665524736000000000} - \frac{43505923 \sin(12z)}{922920356413440000000} + \right. \\
 & \quad \left. \frac{4117 \sin(16z)}{12108284559360000000} - \frac{41 \sin(20z)}{370222908686991360000} - \frac{\sin(24z)}{575902302401986560000} \right) q^{10} + O(q^{11})
 \end{aligned}$$

11.03.06.0009.01

$$\begin{aligned}
 \text{Ce}'(a_5(q), q, z) \propto & -5 \sin(5z) + \left(\frac{7}{24} \sin(7z) - \frac{3}{16} \sin(3z) \right) q + \left(-\frac{\sin(z)}{384} + \frac{65 \sin(5z)}{4608} - \frac{3}{448} \sin(9z) \right) q^2 + \\
 & \left(-\frac{\sin(z)}{9216} + \frac{7 \sin(3z)}{24576} - \frac{13 \sin(7z)}{36864} + \frac{11 \sin(11z)}{129024} \right) q^3 + \left(\frac{\sin(z)}{589824} - \frac{\sin(3z)}{49152} - \frac{4015 \sin(5z)}{2080899072} + \frac{\sin(9z)}{229376} - \frac{13 \sin(13z)}{18579456} \right) q^4 + \\
 & \left(-\frac{5 \sin(z)}{42467328} + \frac{457 \sin(3z)}{11098128384} + \frac{25 \sin(5z)}{7077888} - \frac{1157 \sin(7z)}{7134511104} - \frac{55 \sin(11z)}{1783627776} + \frac{\sin(15z)}{247726080} \right) q^5 + \\
 & \left(-\frac{3407 \sin(z)}{799065243648} + \frac{49 \sin(3z)}{226492416} + \frac{9145 \sin(5z)}{456608710656} - \frac{7 \sin(7z)}{56623104} + \frac{1273 \sin(9z)}{310747594752} + \frac{13 \sin(13z)}{101921587200} - \right. \\
 & \left. \frac{17 \sin(17z)}{980995276800} \right) q^6 + \left(\frac{71521 \sin(z)}{19177565847552} + \frac{2237 \sin(3z)}{17046725197824} - \frac{205 \sin(5z)}{10871635968} - \frac{263827 \sin(7z)}{230130790170624} + \right. \\
 & \left. \frac{5 \sin(9z)}{2466250752} - \frac{953513 \sin(11z)}{20136444139929600} - \frac{\sin(15z)}{4185579847680} + \frac{19 \sin(19z)}{329614413004800} \right) q^7 + \\
 & \left(\frac{718283 \sin(z)}{3682092642729984} - \frac{64811 \sin(3z)}{102280351186944} + \frac{755718235 \sin(5z)}{2886760631900307456} + \frac{3161 \sin(7z)}{5479304527872} + \right. \\
 & \left. \frac{294521 \sin(9z)}{11932707638476800} - \frac{253 \sin(11z)}{12785043898368} + \frac{3609307 \sin(13z)}{10632042505882828800} - \right. \\
 & \left. \frac{17 \sin(17z)}{31642983648460800} - \frac{\sin(21z)}{6529504562380800} \right) q^8 + \\
 & \left(-\frac{187567 \sin(z)}{29456741141839872} + \frac{889319687 \sin(3z)}{15396056703468306432} + \frac{7955 \sin(5z)}{424274049368064} - \frac{10349972261 \sin(7z)}{742309876774364774400} - \right. \\
 & \left. \frac{157 \sin(9z)}{17046725197824} - \frac{27393431 \sin(11z)}{92788734596795596800} + \frac{715 \sin(13z)}{5523138964094976} - \right. \\
 & \left. \frac{9607 \sin(15z)}{5670422669804175360} + \frac{19 \sin(19z)}{3290870299439923200} + \frac{23 \sin(23z)}{69108276288238387200} \right) q^9 + \\
 & \left(\frac{1135692325 \sin(z)}{3325548247949154189312} + \frac{592465 \sin(3z)}{1099718335962021888} - \frac{18970709026319 \sin(5z)}{1662774123974577094656000} - \right. \\
 & \left. \frac{13721 \sin(7z)}{117826964567359488} + \frac{21353891011 \sin(9z)}{71129781970023575715840} + \frac{6919 \sin(11z)}{75745905793302528} + \right. \\
 & \left. \frac{112350719 \sin(13z)}{48992451867108075110400} - \frac{1123 \sin(15z)}{1841046321364992000} + \frac{187 \sin(17z)}{29784038265638092800} - \right. \\
 & \left. \frac{\sin(21z)}{42123139832831016960} - \frac{\sin(25z)}{1658598630917721292800} \right) q^{10} + O(q^{11})
 \end{aligned}$$

11.03.06.0010.01

$$\begin{aligned}
 \text{Ce}'(a_6(q), q, z) \propto & -5 \sin(5z) + \left(\frac{7}{24} \sin(7z) - \frac{3}{16} \sin(3z) \right) q + \left(-\frac{\sin(z)}{384} + \frac{65 \sin(5z)}{4608} - \frac{3}{448} \sin(9z) \right) q^2 + \\
 & \left(-\frac{\sin(z)}{9216} + \frac{7 \sin(3z)}{24576} - \frac{13 \sin(7z)}{36864} + \frac{11 \sin(11z)}{129024} \right) q^3 + \left(\frac{\sin(z)}{589824} - \frac{\sin(3z)}{49152} - \frac{4015 \sin(5z)}{2080899072} + \frac{\sin(9z)}{229376} - \frac{13 \sin(13z)}{18579456} \right) q^4 + \\
 & \left(-\frac{5 \sin(z)}{42467328} + \frac{457 \sin(3z)}{11098128384} + \frac{25 \sin(5z)}{7077888} - \frac{1157 \sin(7z)}{7134511104} - \frac{55 \sin(11z)}{1783627776} + \frac{\sin(15z)}{247726080} \right) q^5 + \\
 & \left(-\frac{3407 \sin(z)}{799065243648} + \frac{49 \sin(3z)}{226492416} + \frac{9145 \sin(5z)}{456608710656} - \frac{7 \sin(7z)}{56623104} + \frac{1273 \sin(9z)}{310747594752} + \frac{13 \sin(13z)}{101921587200} - \right. \\
 & \left. \frac{17 \sin(17z)}{980995276800} \right) q^6 + \left(\frac{71521 \sin(z)}{19177565847552} + \frac{2237 \sin(3z)}{17046725197824} - \frac{205 \sin(5z)}{10871635968} - \frac{263827 \sin(7z)}{230130790170624} + \right. \\
 & \left. \frac{5 \sin(9z)}{2466250752} - \frac{953513 \sin(11z)}{20136444139929600} - \frac{\sin(15z)}{4185579847680} + \frac{19 \sin(19z)}{329614413004800} \right) q^7 + \\
 & \left(\frac{718283 \sin(z)}{3682092642729984} - \frac{64811 \sin(3z)}{102280351186944} + \frac{755718235 \sin(5z)}{2886760631900307456} + \frac{3161 \sin(7z)}{5479304527872} + \right. \\
 & \left. \frac{294521 \sin(9z)}{11932707638476800} - \frac{253 \sin(11z)}{12785043898368} + \frac{3609307 \sin(13z)}{10632042505882828800} - \right. \\
 & \left. \frac{17 \sin(17z)}{31642983648460800} - \frac{\sin(21z)}{6529504562380800} \right) q^8 + \\
 & \left(-\frac{187567 \sin(z)}{29456741141839872} + \frac{889319687 \sin(3z)}{15396056703468306432} + \frac{7955 \sin(5z)}{424274049368064} - \frac{10349972261 \sin(7z)}{742309876774364774400} - \right. \\
 & \left. \frac{157 \sin(9z)}{17046725197824} - \frac{27393431 \sin(11z)}{92788734596795596800} + \frac{715 \sin(13z)}{5523138964094976} - \right. \\
 & \left. \frac{9607 \sin(15z)}{5670422669804175360} + \frac{19 \sin(19z)}{3290870299439923200} + \frac{23 \sin(23z)}{69108276288238387200} \right) q^9 + \\
 & \left(\frac{1135692325 \sin(z)}{3325548247949154189312} + \frac{592465 \sin(3z)}{1099718335962021888} - \frac{18970709026319 \sin(5z)}{1662774123974577094656000} - \right. \\
 & \left. \frac{13721 \sin(7z)}{117826964567359488} + \frac{21353891011 \sin(9z)}{71129781970023575715840} + \frac{6919 \sin(11z)}{75745905793302528} + \right. \\
 & \left. \frac{112350719 \sin(13z)}{48992451867108075110400} - \frac{1123 \sin(15z)}{1841046321364992000} + \frac{187 \sin(17z)}{29784038265638092800} - \right. \\
 & \left. \frac{\sin(21z)}{42123139832831016960} - \frac{\sin(25z)}{1658598630917721292800} \right) q^{10} + O(q^{11})
 \end{aligned}$$

Differential equations

Ordinary linear differential equations and wronskians

For the direct function itself

11.03.13.0001.01

$$(a - 2q \cos(2z)) w''(z) - 4q \sin(2z) w'(z) + (a - 2q \cos(2z))^2 w(z) = 0 /; w(z) = c_1 \text{Ce}'(a, q, z) + c_2 \text{Se}'(a, q, z)$$

11.03.13.0002.01

$$W_z(\text{Ce}'(a, q, z), \text{Se}'(a, q, z)) = -(a - 2q \cos(2z)) (\text{Ce}'(a, q, 0) \text{Se}(a, q, 0) - \text{Ce}(a, q, 0) \text{Se}'(a, q, 0))$$

11.03.13.0003.01

$$w''(z) - \left(\frac{4q \sin(2g(z)) g'(z)}{a - 2q \cos(2g(z))} + \frac{g''(z)}{g'(z)} \right) w'(z) + (a - 2q \cos(2g(z))) g'(z)^2 w(z) = 0 /; w(z) = c_1 \text{Ce}'(a, q, g(z)) + c_2 \text{Se}'(a, q, g(z))$$

11.03.13.0004.01

$$W_z(\text{Ce}'(a, q, g(z)), \text{Se}'(a, q, g(z))) = -g'(z) (a - 2q \cos(2g(z))) (\text{Ce}'(a, q, 0) \text{Se}(a, q, 0) - \text{Ce}(a, q, 0) \text{Se}'(a, q, 0))$$

11.03.13.0005.01

$$w''(z) - \left(\frac{4q \sin(2g(z)) g'(z)}{a - 2q \cos(2g(z))} + \frac{2h'(z)}{h(z)} + \frac{g''(z)}{g'(z)} \right) w'(z) + \left((a - 2q \cos(2g(z))) g'(z)^2 + \frac{2h'(z)^2}{h(z)^2} + \frac{1}{h(z)} \left(\frac{h'(z)}{g'(z)} \left(\frac{4q \sin(2g(z)) g'(z)^2}{a - 2q \cos(2g(z))} + g''(z) \right) - h''(z) \right) \right) w(z) = 0 /;$$

$$w(z) = c_1 h(z) \text{Ce}'(a, q, g(z)) + c_2 h(z) \text{Se}'(a, q, g(z))$$

11.03.13.0006.01

$$W_z(h(z) \text{Ce}'(a, q, g(z)), h(z) \text{Se}'(a, q, g(z))) = -h(z)^2 g'(z) (a - 2q \cos(2g(z))) (\text{Ce}'(a, q, 0) \text{Se}(a, q, 0) - \text{Ce}(a, q, 0) \text{Se}'(a, q, 0))$$

11.03.13.0007.01

$$w''(z) - \left(\frac{4q \sin(2g(z)) g'(z)}{a - 2q \cos(2g(z))} + \frac{2h'(z)}{h(z)} + \frac{g''(z)}{g'(z)} \right) w'(z) + \left((a - 2q \cos(2g(z))) g'(z)^2 + \frac{2h'(z)^2}{h(z)^2} + \frac{1}{h(z)} \left(\frac{h'(z)}{g'(z)} \left(\frac{4q \sin(2g(z)) g'(z)^2}{a - 2q \cos(2g(z))} + g''(z) \right) - h''(z) \right) \right) w(z) = 0 /;$$

$$w(z) = c_1 z^s \text{Ce}'(a, q, b z^r) + c_2 z^s \text{Se}'(a, q, b z^r)$$

11.03.13.0008.01

$$W_z(z^s \text{Ce}'(a, q, b z^r), z^s \text{Se}'(a, q, b z^r)) = -b r z^{r+2s-1} (a - 2q \cos(2b z^r)) (\text{Ce}'(a, q, 0) \text{Se}(a, q, 0) - \text{Ce}(a, q, 0) \text{Se}'(a, q, 0))$$

11.03.13.0009.01

$$w''(z) - \left(\frac{4bq \log(r) \sin(2b r^z) r^z}{a - 2q \cos(2b r^z)} + \log(r) + 2 \log(s) \right) w'(z) + \left(\frac{4bq \log(r) \log(s) \sin(2b r^z) r^z}{a - 2q \cos(2b r^z)} + b^2 (a - 2q \cos(2b r^z)) \log^2(r) r^{2z} + \log(s) (\log(r) + \log(s)) \right) w(z) = 0 /;$$

$$w(z) = c_1 s^z \text{Ce}'(a, q, b r^z) + c_2 s^z \text{Se}'(a, q, b r^z)$$

11.03.13.0010.01

$$W_z(s^z \text{Ce}'(a, q, b r^z), s^z \text{Se}'(a, q, b r^z)) = -b r^z s^{2z} (a - 2q \cos(2b r^z)) \log(r) (\text{Ce}'(a, q, 0) \text{Se}(a, q, 0) - \text{Ce}(a, q, 0) \text{Se}'(a, q, 0))$$

Differentiation

Low-order differentiation

With respect to z

11.03.20.0001.01

$$\frac{\partial \text{Ce}'(a, q, z)}{\partial z} = (2q \cos(2z) - a) \text{Ce}(a, q, z)$$

$$\frac{\partial^2 \text{Ce}'(a, q, z)}{\partial z^2} = (2q \cos(2z) - a) \text{Ce}'(a, q, z) - 4q \sin(2z) \text{Ce}(a, q, z)$$

Integration

Indefinite integration

Involving only one direct function

$$\int \text{Ce}'(a, q, z) dz = \text{Ce}(a, q, z)$$

Operations

Limit operation

$$\lim_{a \rightarrow \infty} \frac{1}{\sqrt{a}} \text{Ce}'\left(a, q, \frac{z}{\sqrt{a}}\right) = -\sin(z) /; q \in \mathbb{R}$$

Representations through equivalent functions

With related functions

$$\text{Ce}'(a, q, z) = (-1)^n \text{Se}'\left(a, -q, z + \frac{\pi}{2}\right) /; a = a_{2n+1}(q) \vee a = b_{2n+1}(q) \wedge n \in \mathbb{N}$$

History

- E. L. Mathieu (1868, 1873)
- H. Weber (1869)
- G. W. Hill (1877)
- E. Heine (1878)
- G. Floquet (1883)
- R. C. Maclaurin (1898)
- J. Dougall (1916, 1926)

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