MathieuCharacteristicB

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Notations

Traditional name

Characteristic value of an odd Mathieu function

Traditional notation

$b_r(q)$

Mathematica StandardForm notation

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MathieuCharacteristicB[r, q]
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Primary definition

11.06.02.0001.01

$b_r(q)$

$b_r(q)$ is the characteristic value b for odd Mathieu functions $w(z) == \text{Se}(a, q, z)$ with characteristic exponent r and parameter q, such that there exists a solution of the corresponding Mathieu differential equation $w''(z) + (a - 2\,q\cos(2\,z))\,w(z) == 0$ that is of the form $w(z) == e^{i\,r\,z}\,f(z)$, where $f(z)$ is an odd function of z with period $2\,\pi$.

Specific values

Specialized values

11.06.03.0001.01

$b_r(0) == r^2$

Values at fixed points

11.06.03.0002.01

$b_0(0) == 0$

General characteristics

Domain and analyticity

$b_r(q)$ is an analytical function of r, q which is defined in $\mathbb{C}^2$.

11.06.04.0001.01

$$(r * q) \longrightarrow b_r(q) :: (\mathbb{C} \otimes \mathbb{C}) \longrightarrow \mathbb{C}$$

Symmetries and periodicities

Parity

$b_r(q)$ is an even function.

11.06.04.0002.01

$$b_{-r}(q) == b_r(q)$$

11.06.04.0003.01

$$b_r(-q) == b_r(q)$$

11.06.04.0004.01

$$b_{-r}(-q) == b_r(q)$$

Mirror symmetry

11.06.04.0005.01

$$b_{\bar{r}}(\bar{q}) == \overline{b_r(q)}$$

Periodicity

No periodicity

Branch points

Branch points locations: complicated

Branch cuts

Branch cut locations: complicated

Series representations

Generalized power series

Expansions at $q == 0$

11.06.06.0001.01

$$b_r(q) \propto r^2 + \frac{q^2}{2(r-1)(r+1)} + \frac{(5r^2+7)q^4}{32(r-2)(r-1)^3(r+1)^3(r+2)} + \frac{(9r^4+58r^2+29)q^6}{64(r-3)(r-2)(r-1)^5(r+1)^5(r+2)(r+3)} + \frac{(1469r^{10}+9144r^8-140\,354r^6+64\,228r^4+827\,565r^2+274\,748)q^8}{8192(r-4)(r-3)(r-2)^3(r-1)^7(r+1)^7(r+2)^3(r+3)(r+4)} + \left((4471r^{12}+69\,361r^{10}-1\,039\,598r^8-2\,844\,430r^6+13\,541\,915r^4+20\,651\,309r^2+4\,453\,452)q^{10}\right)/\left(16\,384(r-5)(r-4)(r-3)(r-2)^3(r-1)^9(r+1)^9(r+2)^3(r+3)(r+4)(r+5)\right)+O(q^{11})\ /;\ \neg(r\in\mathbb{Z}\wedge -10\le r\le 10)$$

11.06.06.0012.01

$$b_0(q) \propto -\frac{q^2}{2} + \frac{7q^4}{128} - \frac{29q^6}{2304} + \frac{68\,687q^8}{18\,874\,368} - \frac{123\,707q^{10}}{104\,857\,600} + O(q^{11})$$

11.06.06.0002.01

$$b_1(q) \propto 1 - q - \frac{q^2}{8} + \frac{q^3}{64} - \frac{q^4}{1536} - \frac{11\,q^5}{36\,864} + \frac{49\,q^6}{589\,824} - \frac{55\,q^7}{9\,437\,184} - \frac{83\,q^8}{35\,389\,440} + \frac{12\,121\,q^9}{15\,099\,494\,400} - \frac{114\,299\,q^{10}}{1\,630\,745\,395\,200} - \frac{192\,151\,q^{11}}{7\,827\,577\,896\,960} + \frac{83\,513\,957\,q^{12}}{8\,766\,887\,244\,595\,200} - \frac{944\,750\,239\,q^{13}}{981\,891\,371\,394\,662\,400} - \frac{27\,587\,714\,461\,q^{14}}{94\,261\,571\,653\,887\,590\,400} + \frac{45\,487\,147\,753\,q^{15}}{361\,964\,435\,150\,928\,347\,136} - \frac{11\,583\,279\,236\,477\,q^{16}}{814\,419\,979\,089\,588\,781\,056\,000} - \frac{4\,401\,918\,060\,178\,787\,q^{17}}{1\,172\,764\,769\,889\,007\,844\,720\,640\,000} + \frac{20\,737\,942\,737\,397\,933\,q^{18}}{11\,727\,647\,698\,890\,078\,447\,206\,400\,000} - \frac{206\,649\,295\,526\,133\,419\,q^{19}}{938\,211\,815\,911\,206\,275\,776\,512\,000\,000} - \frac{230\,932\,630\,430\,735\,533\,q^{20}}{4\,586\,813\,322\,232\,564\,014\,907\,392\,000\,000} + O\left(q^{21}\right)$$

11.06.06.0003.01

$$b_2(q) \propto 4 - \frac{q^2}{12} + \frac{5\,q^4}{13\,824} - \frac{289\,q^6}{79\,626\,240} + \frac{21\,391\,q^8}{458\,647\,142\,400} - \frac{2\,499\,767\,q^{10}}{3\,698\,530\,556\,313\,600} + \frac{1\,046\,070\,973\,q^{12}}{99\,416\,501\,353\,709\,568\,000} - \frac{196\,784\,996\,207\,q^{14}}{1\,145\,278\,095\,594\,734\,223\,360\,000} + \frac{598\,543\,743\,703\,q^{16}}{206\,150\,057\,207\,052\,160\,204\,800\,000} - \frac{63\,122\,700\,730\,716\,751\,q^{18}}{1\,253\,920\,091\,965\,327\,187\,575\,308\,288\,000\,000} + \frac{49\,524\,045\,775\,449\,454\,931\,q^{20}}{55\,613\,863\,918\,846\,191\,423\,340\,073\,189\,376\,000\,000} + O\left(q^{21}\right)$$

11.06.06.0004.01

$$b_3(q) \propto 9 + \frac{q^2}{16} - \frac{q^3}{64} + \frac{13\,q^4}{20\,480} + \frac{5\,q^5}{16\,384} - \frac{1961\,q^6}{23\,592\,960} + \frac{609\,q^7}{104\,857\,600} + \frac{4\,957\,199\,q^8}{2\,113\,929\,216\,000} - \frac{872\,713\,q^9}{1\,087\,163\,596\,800} + \frac{421\,511\,q^{10}}{6\,012\,954\,214\,400} + \frac{16\,738\,435\,813\,q^{11}}{681\,869\,007\,912\,960\,000} - \frac{572\,669\,780\,189\,q^{12}}{60\,115\,798\,248\,652\,800\,000} + \frac{27\,992\,567\,161\,q^{13}}{29\,093\,077\,670\,952\,960\,000} + \frac{110\,350\,873\,865\,291\,q^{14}}{377\,046\,286\,615\,550\,361\,600\,000} - \frac{10\,234\,605\,999\,596\,669\,q^{15}}{81\,441\,997\,908\,958\,878\,105\,600\,000} + \frac{704\,720\,978\,382\,089\,561\,q^{16}}{49\,548\,909\,345\,104\,858\,185\,728\,000\,000} + \frac{195\,640\,795\,481\,512\,957\,q^{17}}{52\,122\,878\,661\,733\,681\,987\,584\,000\,000} - \frac{40\,878\,632\,822\,977\,874\,980\,039\,q^{18}}{23\,117\,539\,144\,052\,122\,635\,133\,255\,680\,000\,000} + \frac{38\,723\,118\,606\,468\,500\,148\,773\,q^{19}}{175\,807\,458\,181\,927\,253\,620\,272\,660\,480\,000\,000} + \frac{96\,836\,518\,225\,571\,706\,158\,506\,019\,q^{20}}{1\,923\,379\,256\,785\,136\,603\,243\,086\,872\,576\,000\,000\,000} + O\left(q^{21}\right)$$

11.06.06.0005.01

$$b_4(q) \propto 16 + \frac{q^2}{30} - \frac{317\,q^4}{864\,000} + \frac{10\,049\,q^6}{2\,721\,600\,000} - \frac{93\,824\,197\,q^8}{2\,006\,581\,248\,000\,000} + \frac{21\,359\,366\,443\,q^{10}}{31\,603\,654\,656\,000\,000\,000} - \frac{2\,860\,119\,307\,587\,541\,q^{12}}{271\,841\,995\,889\,049\,600\,000\,000\,000} + \frac{674\,066\,844\,771\,031\,q^{14}}{3\,923\,084\,197\,232\,640\,000\,000\,000\,000} - \frac{100\,817\,210\,359\,950\,705\,228\,637\,q^{16}}{34\,723\,400\,260\,812\,848\,234\,496\,000\,000\,000\,000\,000} + \frac{472\,428\,361\,549\,262\,608\,073\,838\,587\,q^{18}}{9\,384\,693\,388\,489\,888\,492\,337\,233\,920\,000\,000\,000\,000\,000} - \frac{154\,037\,203\,587\,442\,993\,906\,456\,195\,807\,519\,q^{20}}{172\,978\,668\,536\,645\,624\,690\,759\,895\,613\,440\,000\,000\,000\,000\,000\,000} + O\left(q^{21}\right)$$

11.06.06.0006.01

$$b_5(q) \propto 25 + \frac{q^2}{48} + \frac{11\,q^4}{774\,144} - \frac{q^5}{147\,456} + \frac{37\,q^6}{891\,813\,888} + \frac{7\,q^7}{339\,738\,624} + \frac{63\,439\,q^8}{201\,364\,441\,399\,296} + \frac{q^9}{2\,130\,840\,649\,728} - \frac{60\,609\,509\,q^{10}}{5\,799\,295\,912\,299\,724\,800} + \frac{6655\,q^{11}}{88\,370\,223\,425\,519\,616} + \frac{105\,674\,803\,279\,q^{12}}{2\,400\,630\,141\,488\,295\,680\,409\,600} + \frac{5\,574\,517\,q^{13}}{4\,988\,322\,371\,923\,731\,283\,968} - \frac{16\,772\,318\,839\,q^{14}}{395\,075\,131\,856\,359\,517\,690\,265\,600} - \frac{65\,998\,078\,949\,q^{15}}{2\,394\,394\,738\,523\,391\,016\,304\,640\,000} + \frac{670\,043\,657\,712\,623\,q^{16}}{3\,247\,054\,871\,864\,845\,067\,146\,864\,392\,929\,280} + \frac{6\,770\,772\,278\,721\,223\,q^{17}}{49\,062\,642\,294\,661\,120\,520\,076\,247\,695\,360\,000} + \frac{240\,333\,014\,990\,212\,253\,q^{18}}{56\,109\,108\,185\,824\,522\,760\,297\,816\,709\,817\,958\,400} - \frac{120\,574\,357\,589\,460\,233\,q^{19}}{452\,161\,311\,387\,596\,886\,713\,022\,698\,760\,437\,760\,000} - \frac{50\,669\,327\,402\,196\,350\,835\,443\,q^{20}}{565\,579\,810\,513\,111\,189\,423\,801\,992\,434\,965\,020\,672\,000\,000} + O\left(q^{21}\right)$$

11.06.06.0007.01

$$b_6(q) \propto 36 + \frac{q^2}{70} + \frac{187\,q^4}{43\,904\,000} - \frac{5\,861\,633\,q^6}{92\,935\,987\,200\,000} + \frac{2\,825\,925\,629\,q^8}{23\,315\,780\,468\,736\,000\,000} + \frac{45\,361\,065\,433\,q^{10}}{1\,595\,835\,640\,971\,264\,000\,000\,000} - \frac{5\,579\,610\,563\,247\,909\,577\,q^{12}}{6\,566\,958\,136\,066\,696\,858\,828\,800\,000\,000\,000} + \frac{210\,968\,021\,365\,243\,733\,001\,199\,q^{14}}{93\,702\,612\,251\,908\,484\,139\,256\,381\,440\,000\,000\,000\,000} - \frac{17\,871\,705\,224\,834\,685\,935\,771\,821\,q^{16}}{430\,982\,041\,632\,244\,675\,849\,038\,417\,887\,232\,000\,000\,000\,000\,000} - \frac{139\,633\,570\,904\,270\,043\,589\,485\,822\,895\,441\,q^{18}}{7\,389\,652\,564\,389\,797\,791\,681\,680\,636\,167\,910\,850\,560\,000\,000\,000\,000\,000} + \frac{399\,572\,262\,352\,900\,565\,424\,308\,968\,663\,357\,740\,251\,q^{20}}{6\,627\,749\,826\,390\,952\,080\,168\,132\,635\,856\,454\,570\,223\,861\,760\,000\,000\,000\,000\,000\,000} + O\left(q^{21}\right)$$

11.06.06.0008.01

$$b_7(q) \propto 49 + \frac{q^2}{96} + \frac{7\,q^4}{4\,423\,680} + \frac{17\,q^6}{20\,384\,317\,440} - \frac{q^7}{2\,123\,366\,400} + \frac{80\,617\,q^8}{103\,324\,028\,239\,872\,000} + \frac{q^9}{2\,174\,327\,193\,600} + \frac{22\,381\,q^{10}}{19\,044\,684\,885\,173\,207\,040} - \frac{121\,q^{11}}{1\,502\,894\,956\,216\,320\,000} + \frac{1\,585\,697\,167\,q^{12}}{475\,355\,334\,733\,923\,247\,718\,400\,000} + \frac{169\,q^{13}}{4\,155\,203\,974\,946\,881\,536\,000} - \frac{4\,087\,866\,435\,403\,q^{14}}{107\,331\,431\,740\,241\,997\,948\,832\,972\,800\,000} + \frac{619\,q^{15}}{11\,770\,607\,986\,095\,528\,345\,600\,000} + \frac{127\,886\,416\,305\,104\,603\,q^{16}}{2\,393\,782\,869\,301\,730\,012\,493\,396\,119\,126\,016\,000\,000} + \frac{710\,653\,159\,q^{17}}{5\,337\,927\,405\,856\,933\,273\,185\,288\,192\,000\,000} - \frac{4\,615\,810\,827\,596\,713\,259\,q^{18}}{165\,458\,271\,926\,135\,578\,463\,543\,539\,753\,990\,225\,920\,000\,000} + \frac{8\,903\,677\,513\,487\,q^{19}}{13\,856\,405\,477\,219\,661\,667\,865\,298\,500\,321\,280\,000\,000} + \frac{6\,648\,543\,660\,666\,518\,664\,511\,q^{20}}{648\,066\,959\,480\,287\,833\,726\,007\,336\,508\,428\,916\,883\,456\,000\,000\,000} + O\left(q^{21}\right)$$

11.06.06.0009.01

$$b_8(q) \propto 64 + \frac{q^2}{126} + \frac{109\,q^4}{160\,030\,080} + \frac{2707\,q^6}{13\,973\,506\,525\,440} - \frac{52\,492\,329\,667\,q^8}{22\,716\,763\,094\,823\,469\,056\,000} + \frac{9\,604\,120\,067\,q^{10}}{6\,088\,918\,573\,525\,228\,742\,246\,400} - \frac{52\,131\,336\,111\,051\,551\,q^{12}}{343\,105\,787\,905\,984\,995\,846\,250\,718\,822\,400\,000} + \frac{3\,037\,944\,758\,311\,201\,111\,q^{14}}{19\,065\,016\,210\,783\,962\,279\,192\,767\,442\,085\,478\,400\,000} - \frac{688\,644\,321\,431\,203\,907\,962\,783\,243\,q^{16}}{793\,447\,490\,813\,706\,069\,985\,104\,693\,586\,949\,714\,944\,720\,896\,000\,000} + \frac{2\,113\,914\,045\,440\,842\,540\,935\,295\,950\,474\,929\,q^{18}}{2\,526\,377\,173\,424\,697\,820\,060\,353\,486\,656\,610\,660\,515\,990\,570\,815\,979\,520\,000\,000} - \frac{222\,064\,082\,834\,050\,351\,792\,146\,883\,461\,654\,210\,383\,q^{20}}{834\,262\,291\,310\,042\,453\,898\,585\,976\,646\,535\,297\,604\,118\,819\,087\,309\,409\,878\,016\,000\,000\,000} + O\left(q^{21}\right)$$

11.06.06.0010.01

$$b_9(q) \propto 81 + \frac{q^2}{160} + \frac{103\,q^4}{315\,392\,000} + \frac{1993\,q^6}{36\,333\,158\,400\,000} + \frac{425\,125\,339\,q^8}{28\,676\,616\,703\,967\,232\,000\,000} - \frac{q^9}{106\,542\,032\,486\,400} + \frac{1\,130\,345\,443\,q^{10}}{203\,922\,607\,672\,655\,872\,000\,000\,000} + \frac{11\,q^{11}}{2\,727\,476\,031\,651\,840\,000} + \frac{62\,629\,265\,104\,555\,151\,q^{12}}{22\,563\,907\,986\,167\,495\,394\,538\,291\,200\,000\,000\,000} - \frac{31\,447\,q^{13}}{68\,997\,143\,004\,432\,039\,936\,000\,000} + \frac{80\,555\,146\,157\,400\,451\,q^{14}}{41\,259\,717\,460\,420\,563\,007\,155\,732\,480\,000\,000\,000\,000} + \frac{7\,339\,639\,q^{15}}{171\,686\,970\,880\,788\,333\,613\,547\,520\,000\,000} + \frac{405\,059\,575\,281\,583\,710\,161\,972\,141\,q^{16}}{167\,926\,821\,562\,095\,099\,968\,656\,489\,164\,199\,428\,096\,000\,000\,000\,000\,000} + \frac{15\,120\,710\,271\,341\,q^{17}}{2\,201\,990\,611\,321\,866\,125\,684\,585\,325\,920\,256\,000\,000\,000} - \frac{11\,523\,405\,880\,011\,634\,059\,270\,211\,650\,029\,q^{18}}{940\,175\,254\,416\,133\,078\,096\,516\,459\,013\,386\,622\,069\,637\,120\,000\,000\,000\,000\,000} + \frac{145\,650\,578\,734\,543\,q^{19}}{31\,317\,199\,805\,466\,540\,454\,180\,769\,079\,754\,752\,000\,000\,000\,000} + \frac{15\,089\,819\,507\,835\,055\,103\,567\,092\,482\,603\,681\,329\,q^{20}}{1\,936\,670\,767\,272\,810\,192\,103\,326\,639\,987\,511\,156\,347\,733\,782\,036\,480\,000\,000\,000\,000\,000\,000} + O\left(q^{21}\right)$$

11.06.06.0011.01

$$b_{10}(q) \propto 100 + \frac{q^2}{198} + \frac{169\,q^4}{993\,586\,176} + \frac{31\,943\,q^6}{1\,772\,341\,136\,197\,632} + \frac{1\,704\,670\,559\,q^8}{569\,203\,604\,713\,406\,176\,690\,176} - \frac{1\,768\,418\,729\,052\,839\,q^{10}}{62\,482\,162\,733\,716\,252\,102\,692\,647\,731\,200} + \frac{1\,119\,707\,449\,608\,181\,806\,019\,q^{12}}{123\,640\,255\,459\,938\,983\,842\,746\,782\,311\,963\,321\,958\,400} - \frac{24\,606\,893\,441\,988\,925\,522\,607\,q^{14}}{34\,245\,100\,789\,973\,865\,842\,579\,849\,356\,421\,640\,003\,728\,179\,200} + \frac{270\,635\,998\,049\,518\,041\,370\,843\,201\,q^{16}}{3\,368\,176\,723\,821\,395\,783\,072\,385\,039\,067\,573\,491\,742\,813\,047\,320\,739\,840} + \frac{11\,314\,282\,409\,772\,177\,042\,594\,165\,212\,832\,017\,q^{18}}{356\,159\,753\,797\,099\,085\,314\,066\,801\,692\,056\,277\,942\,540\,168\,039\,466\,400\,750\,134\,886\,400} - \left(47\,256\,590\,145\,830\,056\,043\,660\,165\,066\,696\,641\,992\,589\,q^{20}\right) \Big/ 446\,812\,383\,611\,567\,121\,300\,885\,596\,593\,131\,978\,254\,699\,031\,930\,215\,704\,800\,265\,218\,765\,619\,200\,000 + O\left(q^{21}\right)$$

Representations through equivalent functions

With related functions

11.06.27.0001.01

$$b_r(q) == a_r(q) \,/;\, r \notin \mathbb{Z} \bigwedge q \in \mathbb{R}$$

History

– E. L. Mathieu (1868, 1873)
– H. Weber (1869)
– G.W. Hill (1877)
– E. Heine (1878)
– G. Floquet (1883)
– R. C. Maclaurin (1898)
– J. Dougall (1916, 1926)

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