MathieuSPrime

Notations

Traditional name

Derivative of the odd Mathieu function

Traditional notation

$\mathrm{Se}'(a, q, z)$

Mathematica StandardForm notation

`MathieuSPrime[a, q, z]`

Primary definition

11.04.02.0001.01

$$\mathrm{Se}'(a, q, z) = \frac{\partial \mathrm{Se}(a, q, z)}{\partial z}$$

Specific values

Specialized values

For fixed *a*, *z*

11.04.03.0001.01

$$\mathrm{Se}'(a, 0, z) = \sqrt{a}\,\cos\left(\sqrt{a}\,z\right)$$

General characteristics

Domain and analyticity

$\mathrm{Se}'(a, q, z)$ is an analytical function of a, q, z which is defined in $\mathbb{C}^3$.

11.04.04.0001.01

$$(a * q * z) \longrightarrow \mathrm{Se}'(a, q, z) :: (\mathbb{C} \otimes \mathbb{C} \otimes \mathbb{C}) \longrightarrow \mathbb{C}$$

Symmetries and periodicities

Parity

$\mathrm{Se}'(a, q, z)$ is an even function with respect to z.

11.04.04.0002.01

$$\mathrm{Se}'(a, q, -z) = \mathrm{Se}'(a, q, z)$$

Mirror symmetry

11.04.04.0003.01

$$\mathrm{Se}'(\bar{a}, \bar{q}, \bar{z}) = \overline{\mathrm{Se}'(a, q, z)}$$

Periodicity

No periodicity

Series representations

Generalized power series

Expansions at generic point $z = z_0$

For the function itself

11.04.06.0010.01

$$\begin{aligned}\mathrm{Se}'(a, q, z) \propto{}& \mathrm{Se}^{(0,0,1)}(a, q, z_0) + (2 q \cos(2 z_0) - a) \mathrm{Se}(a, q, z_0) (z - z_0) + \\ & \frac{1}{2} \left((2 q \cos(2 z_0) - a) \mathrm{Se}^{(0,0,1)}(a, q, z_0) - 4 q \sin(2 z_0) \mathrm{Se}(a, q, z_0)\right) (z - z_0)^2 + \\ & \frac{1}{6} \left(\left(a^2 + 4 q \cos(2 z_0) (-a + q \cos(2 z_0) - 2)\right) \mathrm{Se}(a, q, z_0) - 8 q \sin(2 z_0) \mathrm{Se}^{(0,0,1)}(a, q, z_0)\right) (z - z_0)^3 + \\ & \frac{1}{24} \left(\left(a^2 + 4 q \cos(2 z_0) (-a + q \cos(2 z_0) - 6)\right) \mathrm{Se}^{(0,0,1)}(a, q, z_0) + 16 q (a - 2 q \cos(2 z_0) + 1) \sin(2 z_0) \mathrm{Se}(a, q, z_0)\right) (z - z_0)^4 + \\ & \frac{1}{120} \left(\left(2 q \left(\left(3 q^2 + a (3 a + 28) + 16\right) \cos(2 z_0) + q (-3 (a + 4) - (3 a + 44) \cos(4 z_0) + q \cos(6 z_0))\right) - a^3\right) \mathrm{Se}(a, q, z_0) + \right. \\ & \left. 8 q (3 a - 6 q \cos(2 z_0) + 8) \sin(2 z_0) \mathrm{Se}^{(0,0,1)}(a, q, z_0)\right) (z - z_0)^5 + \ldots /; (z \to z_0)\end{aligned}$$

11.04.06.0011.01

$$\mathrm{Se}'(a, q, z) \propto \mathrm{Se}^{(0,0,1)}(a, q, z_0) (1 + O(z - z_0))$$

Expansions at $z = 0$

11.04.06.0001.01

$$\mathrm{Se}'(b_{2n+2}(q), q, z) = 2 \sum_{k=0}^{\infty} (k + 1) B_{2k+2}^{2n+2} \cos((2 k + 2) z) /;$$

$$(b_{2n}(q) - 4) B_2^{2n+2} - q B_4^{2n+2} = 0 \bigwedge \left(b_{2n}(q) - 4 k^2\right) B_{2k}^{2n} - q \left(B_{2k-2}^{2n+2} + B_{2k+2}^{2n+2}\right) = 0 \bigwedge \sum_{k=0}^{\infty} B_{2k+1}^{2n+1} = 1 \bigwedge n \in \mathbb{Z}$$

11.04.06.0002.01

$$\mathrm{Se}'(b_{2n+1}(q),\, q,\, z) \equiv \sum_{k=0}^{\infty} (2k+1)\, B_{2k+1}^{2n+1} \cos((2k+1)z) \,/;$$

$$(b_{2n+1}(q) - q - 1)\, B_1^{2n+1} - q\, B_3^{2n+1} \equiv 0 \bigwedge \left(b_{2n+1}(q) - (2k+1)^2\right) B_{2k+1}^{2n+1} - q\left(B_{2k-1}^{2n+1} + B_{2k+3}^{2n+1}\right) \equiv 0 \bigwedge \sum_{k=0}^{\infty} B_{2k+1}^{2n+1} \equiv 1 \bigwedge n \in \mathbb{Z}$$

Expansions at $q \equiv 0$

11.04.06.0003.01

$$\begin{aligned}
&\mathrm{Se}'(b_r(q),\, q,\, z) \propto -r\cos(rz) + \left(\frac{(2-r)\cos((r-2)z)}{4(r-1)} + \frac{(r+2)\cos((r+2)z)}{4(r+1)}\right) q + \\
&\quad \left(\frac{(4-r)\cos((r-4)z)}{32(r-2)(r-1)} + \frac{r\left(r^2+1\right)\cos(rz)}{16(r-1)^2(r+1)^2} - \frac{(r+4)\cos((r+4)z)}{32(r+1)(r+2)}\right) q^2 + \\
&\quad \left(\frac{(6-r)\cos((r-6)z)}{384(r-3)(r-2)(r-1)} + \left(\left(r^3-r^2-r-11\right)\cos((r-2)z)\right) \Big/ \left(128(r-1)^3(r+1)^2\right) - \right. \\
&\qquad \left. \left(\left(r^3+r^2-r+11\right)\cos((r+2)z)\right) \Big/ \left(128(r-1)^2(r+1)^3\right) + ((r+6)\cos((r+6)z))/(384(r+1)(r+2)(r+3))\right) q^3 + \\
&\quad \left(((8-r)\cos((r-8)z))/(6144(r-4)(r-3)(r-2)(r-1)) + \left((r-4)\left(r^3-r^2-7r-29\right)\cos((r-4)z)\right) \Big/ \right. \\
&\qquad \left(1536(r-3)(r-2)(r-1)^3(r+1)^2\right) - \left(r\left(r^8-15r^6-185r^4+675r^2+316\right)\cos(rz)\right) \Big/ \\
&\qquad \left(1024(r-2)^2(r-1)^4(r+1)^4(r+2)^2\right) + \left((r+4)\left(r^3+r^2-7r+29\right)\cos((r+4)z)\right) \Big/ \\
&\qquad \left. \left(1536(r-1)^2(r+1)^3(r+2)(r+3)\right) - ((r+8)\cos((r+8)z))/(6144(r+1)(r+2)(r+3)(r+4))\right) q^4 + \\
&\quad \left(((10-r)\cos((r-10)z))/(122\,880(r-5)(r-4)(r-3)(r-2)(r-1)) + \right. \\
&\qquad \left((r-6)\left(r^3-r^2-17r-55\right)\cos((r-6)z)\right) \Big/ \left(24\,576(r-4)(r-3)(r-2)(r-1)^3(r+1)^2\right) - \\
&\qquad \left(\left(r^9-r^8-31r^7-5r^6-273r^5+2457r^4+1931r^3-6335r^2-3572r-7564\right)\cos((r-2)z)\right) \Big/ \\
&\qquad \left(12\,288(r-3)(r-2)(r-1)^5(r+1)^4(r+2)^2\right) + \\
&\qquad \left(\left(r^9+r^8-31r^7+5r^6-273r^5-2457r^4+1931r^3+6335r^2-3572r+7564\right)\cos((r+2)z)\right) \Big/ \\
&\qquad \left(12\,288(r-2)^2(r-1)^4(r+1)^5(r+2)(r+3)\right) - \\
&\qquad \left((r+6)\left(r^3+r^2-17r+55\right)\cos((r+6)z)\right) \Big/ \left(24\,576(r-1)^2(r+1)^3(r+2)(r+3)(r+4)\right) + \\
&\qquad \left. ((r+10)\cos((r+10)z))/(122\,880(r+1)(r+2)(r+3)(r+4)(r+5))\right) q^5 + \\
&\quad \left(((12-r)\cos((r-12)z))/(2\,949\,120(r-6)(r-5)(r-4)(r-3)(r-2)(r-1)) + \right. \\
&\qquad \left((r-8)\left(r^3-r^2-31r-89\right)\cos((r-8)z)\right) \Big/ \left(491\,520(r-5)(r-4)(r-3)(r-2)(r-1)^3(r+1)^2\right) - \\
&\qquad \left(\left(r^{10}-3r^9-53r^8+69r^7+145r^6+8211r^5-16\,879r^4-32\,025r^3+32\,954r^2+16\,404r+71\,528\right)\cos((r-4)z)\right) \Big/ \\
&\qquad \left(196\,608(r-3)(r-2)^3(r-1)^5(r+1)^4(r+2)^2\right) + \\
&\qquad \left(r\left(r^{14}-72r^{12}+597r^{10}+75\,244r^8-718\,317r^6+153\,312r^4+4\,883\,287r^2+1\,329\,084\right)\cos(rz)\right) \Big/ \\
&\qquad \left(147\,456(r-3)^2(r-2)^2(r-1)^6(r+1)^6(r+2)^2(r+3)^2\right) - \\
&\qquad \left(\left(r^{10}+3r^9-53r^8-69r^7+145r^6-8211r^5-16\,879r^4+32\,025r^3+32\,954r^2-16\,404r+71\,528\right)\cos((r+4)z)\right) \Big/ \\
&\qquad \left(196\,608(r-2)^2(r-1)^4(r+1)^5(r+2)^3(r+3)\right) + \\
&\qquad \left((r+8)\left(r^3+r^2-31r+89\right)\cos((r+8)z)\right) \Big/ \left(491\,520(r-1)^2(r+1)^3(r+2)(r+3)(r+4)(r+5)\right) - \\
&\qquad \left. ((r+12)\cos((r+12)z))/(2\,949\,120(r+1)(r+2)(r+3)(r+4)(r+5)(r+6))\right) q^6 + O\left(q^7\right) \,/;\, \neg\,(r \in \mathbb{Z} \land -6 \le r \le 6)
\end{aligned}$$

11.04.06.0004.01

$$\mathrm{Se}'(b_1(q),\, q,\, z) \propto \cos(z) - \frac{3}{8}\cos(3\,z)\,q + \left(-\frac{\cos(z)}{128} + \frac{3}{64}\cos(3\,z) + \frac{5}{192}\cos(5\,z)\right)q^2 + \left(\frac{\cos(z)}{512} + \frac{\cos(3\,z)}{1024} - \frac{5\cos(5\,z)}{1152} - \frac{7\cos(7\,z)}{9216}\right)q^3 + \left(-\frac{37\cos(z)}{294\,912} + \frac{7\cos(7\,z)}{49\,152} + \frac{\cos(9\,z)}{81\,920} - \frac{49\cos(3\,z)}{24\,576}\right)q^4 + \left(-\frac{121\cos(z)}{1\,769\,472} + \frac{317\cos(3\,z)}{786\,432} + \frac{205\cos(5\,z)}{1\,179\,648} - \frac{\cos(9\,z)}{409\,600} - \frac{7\cos(7\,z)}{5\,898\,240} - \frac{11\cos(11\,z)}{88\,473\,600}\right)q^5 + \left(\frac{8105\cos(z)}{339\,738\,624} + \frac{103\cos(3\,z)}{18\,874\,368} + \frac{\cos(9\,z)}{31\,457\,280} + \frac{11\cos(11\,z)}{424\,673\,280} + \frac{13\cos(13\,z)}{14\,863\,564\,800} - \frac{731\cos(5\,z)}{18\,874\,368} - \frac{2653\cos(7\,z)}{471\,859\,200}\right)q^6 + O[q]^{11}$$

11.04.06.0005.01

$$\mathrm{Se}'(b_2(q),\, q,\, z) \propto 2\cos(2\,z) - \frac{1}{3}\cos(4\,z)\,q + \left(\frac{1}{64}\cos(6\,z) - \frac{1}{144}\cos(2\,z)\right)q^2 + \left(\frac{1}{384}\cos(4\,z) - \frac{\cos(8\,z)}{2880}\right)q^3 + \left(\frac{119\cos(2\,z)}{1\,327\,104} - \frac{19\cos(6\,z)}{122\,880} + \frac{\cos(10\,z)}{221\,184}\right)q^4 + \left(-\frac{1373\cos(4\,z)}{39\,813\,120} + \frac{\cos(8\,z)}{259\,200} - \frac{\cos(12\,z)}{25\,804\,800}\right)q^5 + \left(-\frac{1003\cos(2\,z)}{707\,788\,800} + \frac{8941\cos(6\,z)}{4\,246\,732\,800} - \frac{\cos(10\,z)}{18\,579\,456} + \frac{\cos(14\,z)}{4\,246\,732\,800}\right)q^6 + O\!\left(q^{11}\right)$$

11.04.06.0006.01

$$\mathrm{Se}'(b_3(q),\, q,\, z) \propto 3\cos(3\,z) + \left(\frac{\cos(z)}{8} - \frac{5}{16}\cos(5\,z)\right)q + \left(-\frac{\cos(z)}{64} - \frac{15}{512}\cos(3\,z) + \frac{7}{640}\cos(7\,z)\right)q^2 + \left(-\frac{\cos(z)}{4096} + \frac{3}{512}\cos(3\,z) + \frac{11\cos(5\,z)}{8192} - \frac{\cos(9\,z)}{5120}\right)q^3 + \left(\frac{21\cos(z)}{32\,768} - \frac{4863\cos(3\,z)}{13\,107\,200} - \frac{5\cos(5\,z)}{16\,384} - \frac{77\cos(7\,z)}{2\,949\,120} + \frac{11\cos(11\,z)}{5\,160\,960}\right)q^4 + \left(-\frac{14\,061\cos(z)}{104\,857\,600} - \frac{27\cos(3\,z)}{131\,072} + \frac{12\,329\cos(5\,z)}{377\,487\,360} + \frac{21\cos(7\,z)}{3\,276\,800} + \frac{\cos(9\,z)}{3\,670\,016} - \frac{13\cos(13\,z)}{825\,753\,600}\right)q^5 + \left(-\frac{699\cos(z)}{838\,860\,800} + \frac{13\,050\,583\cos(3\,z)}{181\,193\,932\,800} + \frac{533\cos(5\,z)}{41\,943\,040} - \frac{76\,679\cos(7\,z)}{75\,497\,472\,000} - \frac{17\cos(9\,z)}{235\,929\,600} - \frac{11\cos(11\,z)}{6\,606\,028\,800} + \frac{\cos(15\,z)}{11\,890\,851\,840}\right)q^6 + O\!\left(q^{11}\right)$$

11.04.06.0007.01

$$\mathrm{Se}'(b_4(q),\, q,\, z) \propto 4\cos(4\,z) + \left(\frac{1}{6}\cos(2\,z) - \frac{3}{10}\cos(6\,z)\right)q + \left(\frac{1}{120}\cos(8\,z) - \frac{17}{900}\cos(4\,z)\right)q^2 + \left(-\frac{1}{800}\cos(2\,z) + \frac{29\cos(6\,z)}{48\,000} - \frac{\cos(10\,z)}{8064}\right)q^3 + \left(\frac{18\,839\cos(4\,z)}{103\,680\,000} - \frac{\cos(8\,z)}{112\,000} + \frac{\cos(12\,z)}{860\,160}\right)q^4 + \left(\frac{40\,159\cos(2\,z)}{2\,488\,320\,000} - \frac{65\,633\cos(6\,z)}{9\,676\,800\,000} + \frac{67\cos(10\,z)}{928\,972\,800} - \frac{\cos(14\,z)}{132\,710\,400}\right)q^5 + \left(-\frac{1\,933\,949\cos(4\,z)}{677\,376\,000\,000} + \frac{23\,351\cos(8\,z)}{199\,065\,600\,000} - \frac{\cos(12\,z)}{3\,096\,576\,000} + \frac{\cos(16\,z)}{27\,869\,184\,000}\right)q^6 + O\!\left(q^{11}\right)$$

11.04.06.0008.01

$$\mathrm{Se}'(b_5(q),\,q,\,z) \propto 5\cos(5z) + \left(\frac{3}{16}\cos(3z) - \frac{7}{24}\cos(7z)\right)q + \left(\frac{\cos(z)}{384} - \frac{65\cos(5z)}{4608} + \frac{3}{448}\cos(9z)\right)q^2 + \left(-\frac{\cos(z)}{9216} - \frac{7\cos(3z)}{24576} + \frac{13\cos(7z)}{36864} - \frac{11\cos(11z)}{129024}\right)q^3 + \left(-\frac{\cos(z)}{589824} - \frac{\cos(3z)}{49152} + \frac{4015\cos(5z)}{2080899072} - \frac{\cos(9z)}{229376} + \frac{13\cos(13z)}{18579456}\right)q^4 + \left(-\frac{5\cos(z)}{42467328} - \frac{457\cos(3z)}{11098128384} + \frac{25\cos(5z)}{7077888} + \frac{1157\cos(7z)}{7134511104} + \frac{55\cos(11z)}{1783627776} - \frac{\cos(15z)}{247726080}\right)q^5 + \left(\frac{3407\cos(z)}{799065243648} + \frac{49\cos(3z)}{226492416} - \frac{9145\cos(5z)}{456608710656} - \frac{7\cos(7z)}{56623104} - \frac{1273\cos(9z)}{310747594752} - \frac{13\cos(13z)}{101921587200} + \frac{17\cos(17z)}{980995276800}\right)q^6 + O(q^{11})$$

11.04.06.0009.01

$$\mathrm{Se}'(b_6(q),\,q,\,z) \propto 6\cos(6z) + \left(\frac{1}{5}\cos(4z) - \frac{2}{7}\cos(8z)\right)q + \left(\frac{1}{320}\cos(2z) - \frac{111\cos(6z)}{9800} + \frac{5}{896}\cos(10z)\right)q^2 + \left(-\frac{163\cos(4z)}{784000} + \frac{257\cos(8z)}{1097600} - \frac{\cos(12z)}{16128}\right)q^3 + \left(-\frac{233\cos(2z)}{50176000} + \frac{286737\cos(6z)}{98344960000} - \frac{239\cos(10z)}{94832640} + \frac{\cos(14z)}{2211840}\right)q^4 + \left(-\frac{256941\cos(4z)}{983449600000} + \frac{170407\cos(8z)}{18587197440000} + \frac{41\cos(12z)}{2528870400} - \frac{\cos(16z)}{425779200}\right)q^5 + \left(-\frac{152641\cos(2z)}{62940774400000} + \frac{8712317791\cos(6z)}{234198687744000000} - \frac{553237\cos(10z)}{951664508928000} - \frac{31\cos(14z)}{476872704000} + \frac{\cos(18z)}{108999475200}\right)q^6 + O(q^{11})$$

Differential equations

Ordinary linear differential equations and wronskians

For the direct function itself

11.04.13.0001.01

$$(a - 2q\cos(2z))\,w''(z) - 4q\sin(2z)\,w'(z) + (a - 2q\cos(2z))^2\,w(z) = 0 \;/;\; w(z) = c_1\,\mathrm{Se}'(a,q,z) + c_2\,\mathrm{Ce}'(a,q,z)$$

11.04.13.0002.01

$$W_z\big(\mathrm{Se}'(a,q,z),\,\mathrm{Ce}'(a,q,z)\big) = (a - 2q\cos(2z))\big(\mathrm{Ce}'(a,q,0)\,\mathrm{Se}(a,q,0) - \mathrm{Ce}(a,q,0)\,\mathrm{Se}'(a,q,0)\big)$$

11.04.13.0003.01

$$w''(z) - \left(\frac{4q\sin(2g(z))\,g'(z)}{a - 2q\cos(2g(z))} + \frac{g''(z)}{g'(z)}\right)w'(z) + (a - 2q\cos(2g(z)))\,g'(z)^2\,w(z) = 0 \;/;\; w(z) = c_1\,\mathrm{Se}'(a,q,g(z)) + c_2\,\mathrm{Ce}'(a,q,g(z))$$

11.04.13.0004.01

$$W_z\big(\mathrm{Se}'(a,q,g(z)),\,\mathrm{Ce}'(a,q,g(z))\big) = g'(z)\,(a - 2q\cos(2g(z)))\,\big(\mathrm{Ce}'(a,q,0)\,\mathrm{Se}(a,q,0) - \mathrm{Ce}(a,q,0)\,\mathrm{Se}'(a,q,0)\big)$$

11.04.13.0005.01

$$w''(z) - \left(\frac{4\,q\sin(2\,g(z))\,g'(z)}{a-2\,q\cos(2\,g(z))} + \frac{2\,h'(z)}{h(z)} + \frac{g''(z)}{g'(z)}\right) w'(z) + \left((a-2\,q\cos(2\,g(z)))\,g'(z)^2 + \frac{2\,h'(z)^2}{h(z)^2} + \frac{1}{h(z)}\left(\frac{h'(z)}{g'(z)}\left(\frac{4\,q\sin(2\,g(z))\,g'(z)^2}{a-2\,q\cos(2\,g(z))} + g''(z)\right) - h''(z)\right)\right) w(z) = 0 \;/;\; w(z) = c_1\,h(z)\,\mathrm{Se}'(a,\,q,\,g(z)) + c_2\,h(z)\,\mathrm{Ce}'(a,\,q,\,g(z))$$

11.04.13.0006.01

$$W_z\left(h(z)\,\mathrm{Se}'(a,\,q,\,g(z)),\,h(z)\,\mathrm{Ce}'(a,\,q,\,g(z))\right) = h(z)^2\,g'(z)\,(a-2\,q\cos(2\,g(z)))\,\left(\mathrm{Ce}'(a,\,q,\,0)\,\mathrm{Se}(a,\,q,\,0) - \mathrm{Ce}(a,\,q,\,0)\,\mathrm{Se}'(a,\,q,\,0)\right)$$

11.04.13.0007.01

$$w''(z) - \left(\frac{4\,q\sin(2\,g(z))\,g'(z)}{a-2\,q\cos(2\,g(z))} + \frac{2\,h'(z)}{h(z)} + \frac{g''(z)}{g'(z)}\right) w'(z) + \left((a-2\,q\cos(2\,g(z)))\,g'(z)^2 + \frac{2\,h'(z)^2}{h(z)^2} + \frac{1}{h(z)}\left(\frac{h'(z)}{g'(z)}\left(\frac{4\,q\sin(2\,g(z))\,g'(z)^2}{a-2\,q\cos(2\,g(z))} + g''(z)\right) - h''(z)\right)\right) w(z) = 0 \;/;\; w(z) = c_1\,z^s\,\mathrm{Se}'(a,\,q,\,b\,z^r) + c_2\,z^s\,\mathrm{Ce}'(a,\,q,\,b\,z^r)$$

11.04.13.0008.01

$$W_z\left(z^s\,\mathrm{Se}'(a,\,q,\,b\,z^r),\,z^s\,\mathrm{Ce}'(a,\,q,\,b\,z^r)\right) = b\,r\,z^{r+2\,s-1}\,(a-2\,q\cos(2\,b\,z^r))\left(\mathrm{Ce}'(a,\,q,\,0)\,\mathrm{Se}(a,\,q,\,0) - \mathrm{Ce}(a,\,q,\,0)\,\mathrm{Se}'(a,\,q,\,0)\right)$$

11.04.13.0009.01

$$w''(z) - \left(\frac{4\,b\,q\log(r)\sin(2\,b\,r^z)\,r^z}{a-2\,q\cos(2\,b\,r^z)} + \log(r) + 2\log(s)\right) w'(z) + \left(\frac{4\,b\,q\log(r)\log(s)\sin(2\,b\,r^z)\,r^z}{a-2\,q\cos(2\,b\,r^z)} + b^2\,(a-2\,q\cos(2\,b\,r^z))\log^2(r)\,r^{2\,z} + \log(s)\,(\log(r)+\log(s))\right) w(z) = 0 \;/;\; w(z) = c_1\,s^z\,\mathrm{Se}'(a,\,q,\,b\,r^z) + c_2\,s^z\,\mathrm{Ce}'(a,\,q,\,b\,r^z)$$

11.04.13.0010.01

$$W_z\left(s^z\,\mathrm{Se}'(a,\,q,\,b\,r^z),\,s^z\,\mathrm{Ce}'(a,\,q,\,b\,r^z)\right) = b\,r^z\,s^{2\,z}\,(a-2\,q\cos(2\,b\,r^z))\log(r)\left(\mathrm{Ce}'(a,\,q,\,0)\,\mathrm{Se}(a,\,q,\,0) - \mathrm{Ce}(a,\,q,\,0)\,\mathrm{Se}'(a,\,q,\,0)\right)$$

Differentiation

Low-order differentiation

With respect to z

11.04.20.0001.01

$$\frac{\partial\,\mathrm{Se}'(a,\,q,\,z)}{\partial z} = (2\,q\cos(2\,z) - a)\,\mathrm{Se}(a,\,q,\,z)$$

11.04.20.0002.01

$$\frac{\partial^2\,\mathrm{Se}'(a,\,q,\,z)}{\partial z^2} = (2\,q\cos(2\,z) - a)\,\mathrm{Se}'(a,\,q,\,z) - 4\,q\sin(2\,z)\,\mathrm{Se}(a,\,q,\,z)$$

Integration

Indefinite integration

Involving only one direct function

11.04.21.0001.01

$$\int \mathrm{Se}'(a, q, z)\, dz = \mathrm{Se}(a, q, z)$$

Operations

Limit operation

11.04.25.0001.01

$$\lim_{a\to\infty} \frac{1}{\sqrt{a}} \mathrm{Se}'\left(a, q, \frac{z}{\sqrt{a}}\right) = \cos(z) \;/;\; q \in \mathbb{R}$$

Representations through equivalent functions

With related functions

11.04.27.0001.01

$$\mathrm{Se}'(a, q, z) = (-1)^n \mathrm{Se}'\left(a, -q, z + \frac{\pi}{2}\right) /; a = a_{2n+1}(q) \lor a = b_{2n+1}(q) \land n \in \mathbb{N}$$

History

– E. L. Mathieu (1868, 1873)
– H. Weber (1869)
– G.W. Hill (1877)
– E. Heine (1878)
– G. Floquet (1883)
– R. C. Maclaurin (1898)
– J. Dougall (1916, 1926)

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