

# Max

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## Notations

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### Traditional name

Maximum

### Traditional notation

$\max(x_1, x_2, \dots)$

### *Mathematica* StandardForm notation

`Max[x1, x2, ...]`

## Primary definition

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01.34.02.0001.01

$$\max(x_1, x_2) = \frac{1}{2} \left( x_1 + x_2 + \sqrt{(x_1 - x_2)^2} \right) /; x_1 \in \mathbb{R} \wedge x_2 \in \mathbb{R}$$

01.34.02.0002.01

$$\max(x_1, x_2, \dots, x_n) = \max(\max(x_1, x_2), x_3, \dots, x_n)$$

$\max(x_1, x_2, \dots)$  is the numerically largest of the real numbers  $x_k$ .

$\max(z_1, z_2, \dots)$  is not defined for complex numbers  $z_k$ .

## Specific values

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### Specialized values

01.34.03.0001.01

$$\max(x) = x$$

01.34.03.0002.01

$$\max(x_1, x_1, \dots, x_1) = x_1$$

01.34.03.0003.01

$$\max(x_1, x_2, x_3) = x_1 /; x_1 \in \mathbb{R} \wedge x_2 \in \mathbb{R} \wedge x_3 \in \mathbb{R} \wedge x_1 \geq x_2 \wedge x_1 \geq x_3$$

### Values at fixed points

01.34.03.0004.01

$$\max() = -\infty$$

01.34.03.0005.01

$$\max(1, 3) = 3$$

01.34.03.0006.01

$$\max(2, 6, 1, \pi, 8, 12, 8, 2, 6, 4, 6, 9) = 12$$

## Values at infinities

01.34.03.0007.01

$$\max(\infty, x_2, \dots, x_n) = \infty$$

01.34.03.0008.01

$$\max(-\infty, x_2, \dots, x_n) = \max(x_2, \dots, x_n)$$

01.34.03.0009.01

$$\max(-\infty, \infty) = \infty$$

## General characteristics

### Domain and analyticity

$\max$  is real valued function of an arbitrary number of real variables. In  $\mathbb{R}^n$  it is a piecewise linear function. The derivative of  $\max(x_1, x_2, \dots, x_k, \dots, x_j, \dots, x_n)$  is discontinuous at  $x_j = x_k$  for all  $j, k$ .

01.34.04.0001.01

$$(x_1 * x_2 * \dots * x_n) \rightarrow \max(x_1, x_2, \dots, x_n) :: \mathbb{R}^n \rightarrow \mathbb{R}$$

### Symmetries and periodicities

#### Permutation symmetry

01.34.04.0002.01

$$\max(x_1, x_2) = \max(x_2, x_1)$$

01.34.04.0003.01

$$\max(x_1, x_2, \dots, x_k, \dots, x_j, \dots, x_n) = \max(x_1, x_2, \dots, x_j, \dots, x_k, \dots, x_n)$$

#### Periodicity

No periodicity

### Sets of discontinuity

The function  $\max(x_1, x_2, \dots, x_n)$  is continuous function in  $\mathbb{R}^n$ .

01.34.04.0004.01

$$\mathcal{DS}_{x_k}(\max(x_1, x_2, \dots, x_n)) = \{ \} /; 1 \leq k \leq n$$

## Limit representations

01.34.09.0001.01

$$\max(x_1, x_2) = \lim_{\varepsilon \rightarrow 0} \varepsilon \log \left( e^{\frac{x_1}{\varepsilon}} + e^{\frac{x_2}{\varepsilon}} \right)$$

01.34.09.0002.01

$$\max(x_1, x_2, \dots, x_n) = \lim_{z \rightarrow \infty} \left( \frac{1}{n} \sum_{k=1}^n x_k^z \right)^{1/z}$$

## Transformations

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### Transformations and argument simplifications

01.34.16.0001.01

$$\max(-x_1, -x_2, \dots, -x_n) = -\min(x_1, x_2, \dots, x_n)$$

01.34.16.0002.01

$$\max(|x|, -|x|) = |x|$$

## Identities

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### Functional identities

01.34.17.0001.01

$$\max(x_1, x_2) = \max(x_2, x_1)$$

01.34.17.0002.01

$$\max(x_1, x_2, x_3, \dots) = \max(x_1, \max(x_2, x_3, \dots))$$

## Complex characteristics

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### Real part

01.34.19.0001.01

$$\operatorname{Re}(\max(x_1, x_2, \dots, x_n)) = \max(x_1, x_2, \dots, x_n)$$

### Imaginary part

01.34.19.0002.01

$$\operatorname{Im}(\max(x_1, x_2, \dots, x_n)) = 0$$

### Absolute value

01.34.19.0003.01

$$|\max(x_1, x_2, \dots, x_n)| = \sqrt{\max(x_1, x_2, \dots, x_n)^2}$$

### Argument

01.34.19.0004.01

$$\arg(\max(x_1, x_2, \dots, x_n)) = \tan^{-1}(\max(x_1, x_2, \dots, x_n), 0)$$

### Conjugate value

01.34.19.0005.01

$$\overline{\max(x_1, x_2, \dots, x_n)} = \max(x_1, x_2, \dots, x_n)$$

## Summation

01.34.23.0001.01

$$\sum_{k=0}^{m-1} \sum_{l=0}^{n-1} \max\left(\frac{k}{m}, \frac{l}{n}\right) = \frac{2mn}{3} - \frac{m+n+1}{4} - \frac{m^2+n^2-\text{gcd}(m,n)^2}{12mn} \quad ; m \in \mathbb{N}^+ \wedge n \in \mathbb{N}^+$$

01.34.23.0002.01

$$\sum_{j=0}^{m-1} \sum_{k=0}^{n-1} \sum_{l=0}^{o-1} \max\left(\frac{j}{m}, \frac{k}{n}, \frac{l}{o}\right) = \frac{3mno}{4} - \frac{1}{8}(m+n+o+1) + \frac{2om+m+o}{24n} - \frac{1}{6}(mn+on+mo) + \frac{2on+n+o}{24m} + \frac{2nm+m+n}{24o} - \frac{(n+1)\text{gcd}(m,o)^2}{24mo} - \frac{(m+1)\text{gcd}(n,o)^2}{24no} - \frac{(o+1)\text{gcd}(m,n)^2}{24mn} \quad ; m \in \mathbb{N}^+ \wedge n \in \mathbb{N}^+ \wedge o \in \mathbb{N}^+$$

## Integral transforms

### Fourier exp transforms

01.34.22.0001.01

$$\mathcal{F}_{(t_1, t_2)}[\max(t_1, t_2)](z_1, z_2) = -i\pi(\delta(z_2)\delta'(z_1) + \delta(z_1)\delta'(z_2)) - \frac{\delta(z_1 + z_2)}{z_1^2}$$

### Laplace transforms

01.34.22.0002.01

$$\mathcal{L}_{(t_1, t_2)}[\max(t_1, t_2)](z_1, z_2) = \frac{z_1^2 + z_2 z_1 + z_2^2}{z_1^2 z_2^2 (z_1 + z_2)}$$

## Representations through more general functions

### Through other functions

01.34.26.0001.01

$$\max(x_1, x_2) = \frac{1}{2} \left( x_1 + x_2 + \sqrt{(x_1 - x_2)^2} \right)$$

## Representations through equivalent functions

01.34.27.0001.01

$$\max(x_1, x_2) = x_2 + (x_1 - x_2)\theta(x_1 - x_2)$$

01.34.27.0002.01

$$\max(x_1, x_2, x_3) = x_2 + (x_1 - x_2)\theta(x_1 - x_2) + \theta(x_3 - x_2)(\theta(x_2 - x_1) + \theta(x_3 - x_1)\theta(x_1 - x_2)) + (x_3 - x_2 - (x_1 - x_2)\theta(x_1 - x_2))$$

01.34.27.0003.01

$$\begin{aligned} \max(x_1, x_2, \dots, x_n) = & \sum_{k_1=1}^n x_{k_1} - \sum_{k_1=1}^n \sum_{k_2=k_1+1}^n \min(x_{k_1}, x_{k_2}) + \sum_{k_1=1}^n \sum_{k_2=k_1+1}^n \sum_{k_3=k_2+1}^n \min(x_{k_1}, x_{k_2}, x_{k_3}) - \dots + \\ & (-1)^{j+1} \sum_{k_1=1}^n \sum_{k_2=k_1+1}^n \dots \sum_{k_j=k_{j-1}+1}^n \min(x_{k_1}, x_{k_2}, \dots, x_{k_j}) + \dots + (-1)^{n+1} \sum_{k_1=1}^n \sum_{k_2=k_1+1}^n \dots \sum_{k_n=k_{n-1}+1}^n \min(x_{k_1}, x_{k_2}, \dots, x_{k_n}) \end{aligned}$$

## History

The function max is encountered often in mathematics and the natural sciences.

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